Università degli Studi di Milano

High-mass Drell-Yan and its role in precision EW tests

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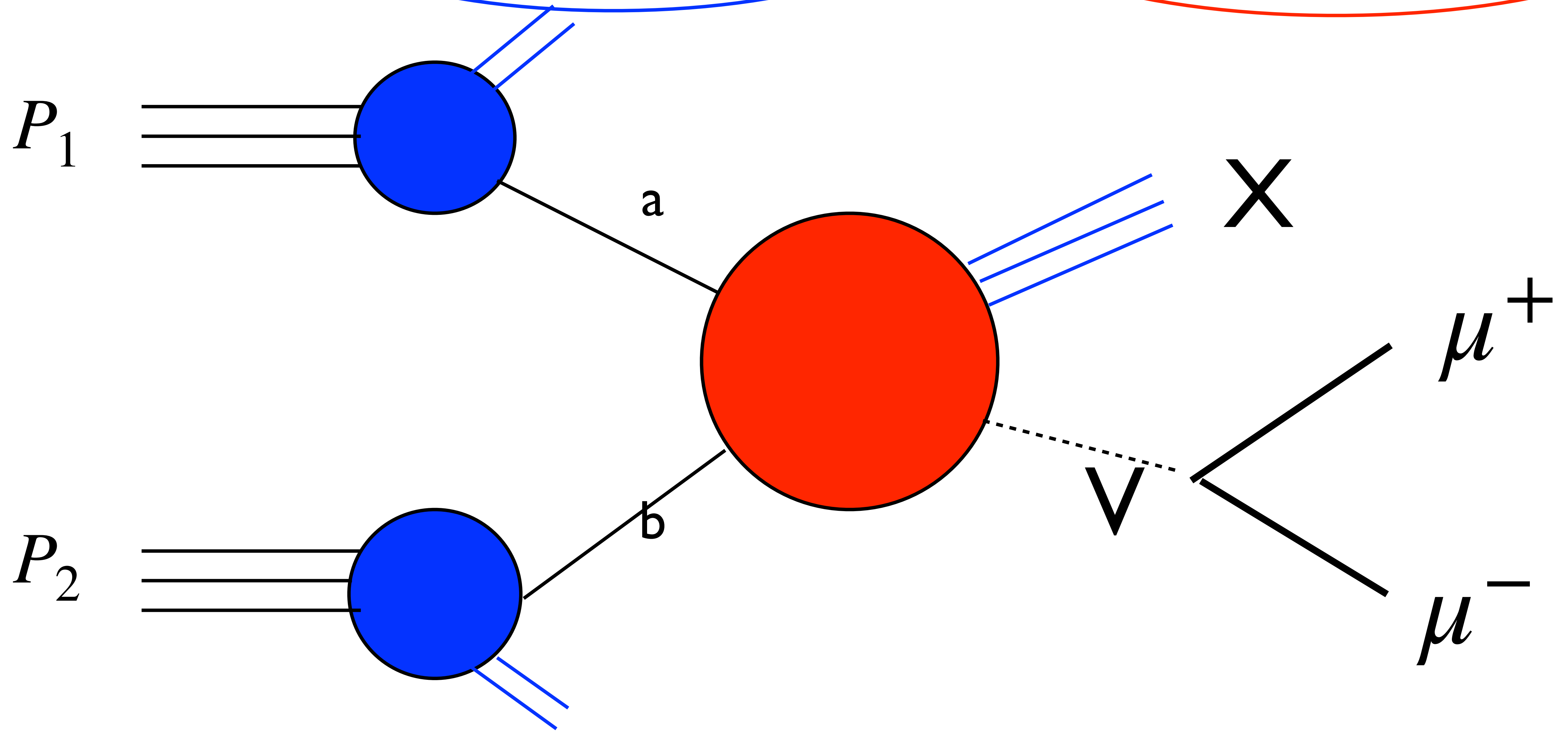
The Neutral-Current Drell-Yan process

$$\sigma(P_1, P_2; m_V) = \sum_{a,b} \int_0^1 dx_1 dx_2 f_{h_1,a}(x_1, M_F) f_{h_2,b}(x_2, M_F) \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2, \alpha_s(\mu), M_F)$$



Particles $P_{1,2}$ can be protons (→ Drell-Yan @ LHC) or leptons (→ FCC-ee, muon collider)

The partonic content of the scattering particles can be expressed in terms of PDFs
proton PDFs: ABM, CT18, MSHT, NNPDF,… lepton PDFs: Frixione et al. arXiv:1911.12040, 2207.03265

The partonic scattering can be computed in perturbation theory, in the full QCD+EW theory

Factorisation theorems guarantee the validity of the above picture up to power correction effects

The lepton-pair invariant mass distribution in the Drell-Yan process

Measured with high precision across 2 orders of magnitude: ~ [50-5000] GeV

Sensitivity to a variety of fundamental aspects of the EW interaction:

the Z resonance

→ mass, width, couplings of the Z boson to fermions (the weak mixing angle)

The cross section receives important radiative corrections:

large QED effects,

large EW Sudakov logarithms,

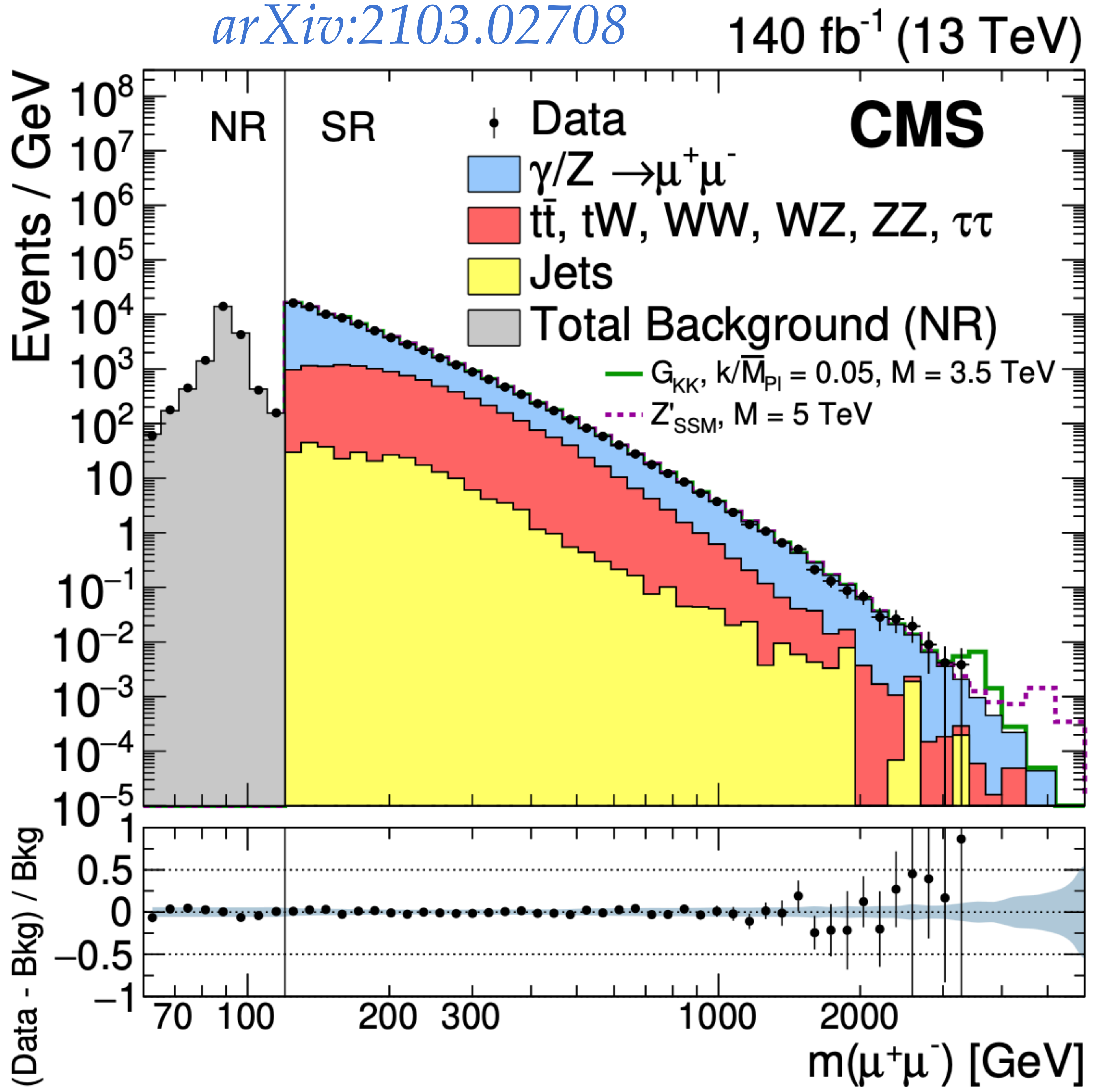
QCD corrections and the proton structure

Very large invariant mass range

→ test the SM

search for deviations in the energy dependence

arXiv:2103.02708



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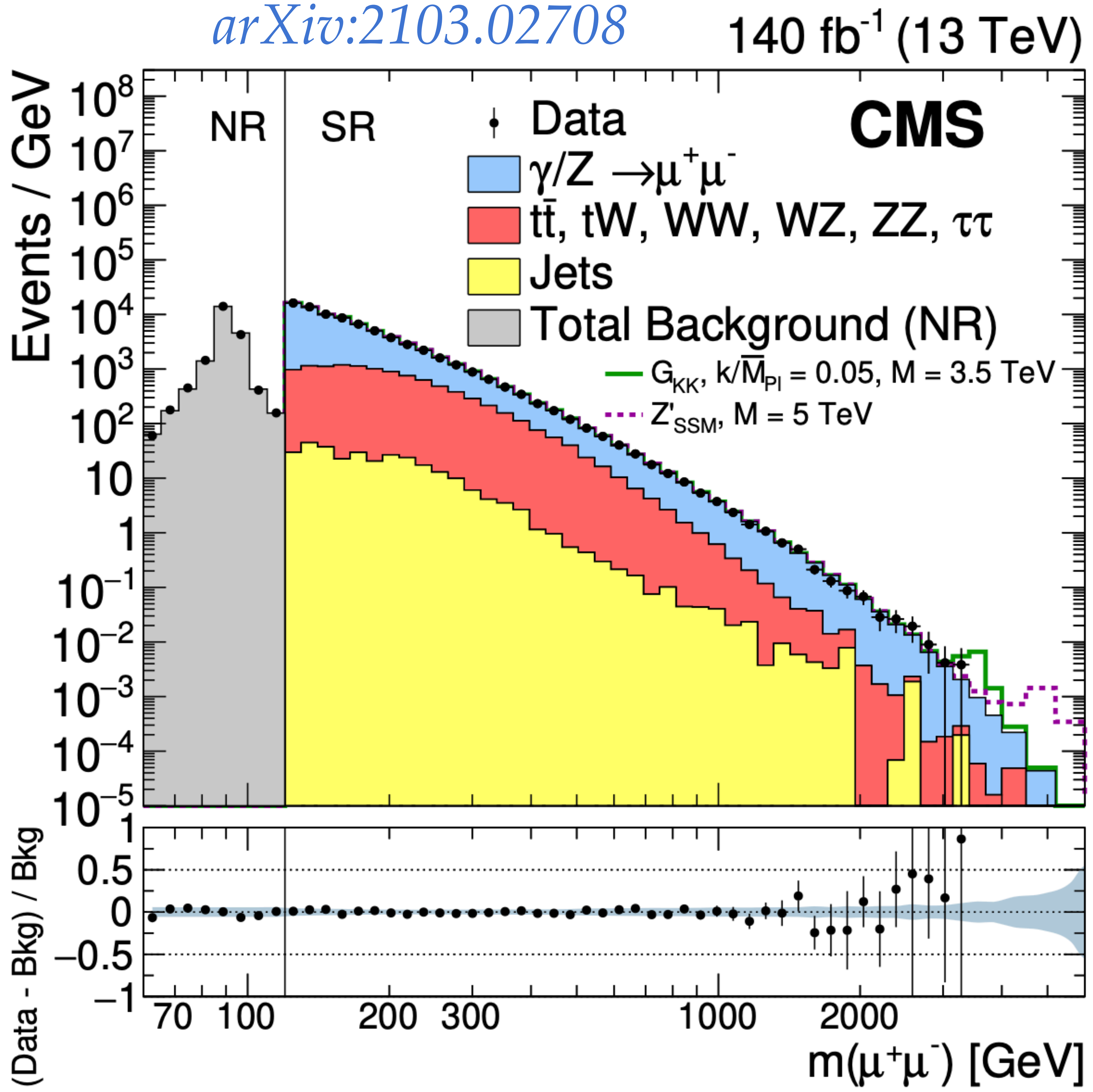

Tests of the Standard Model
→ comparison of th vs exp cross sections / asymmetries

→ determination of SM parameters with a fit to the data
comparison with the corresponding th predictions

Statistical precision from small to large fermion-pair invariant masses

Statistical errors

LHC and HL-LHC $\sigma(pp \to \mu^+\mu^- + X)$
arXiv:2106.11953

bin range (GeV)	% error 140 fb^{-1}	% error 3 ab^{-1}
91-92	0.03	6 10^{-3}
120-400	0.1	0.02
400-600	0.6	0.13
600-900	1.4	0.30
900-1300	3.2	0.69

FCC-ee $\sigma(e^+e^- \to \mu^+\mu^- + X)$
arXiv:2206.08326

sqrt(S) (GeV)	luminosity (ab^{-1})	σ (fb)	% error
91	150	2.17595 10^6	0.0002
240	5	1870.84 ± 0.612	0.03
365	1,5	787.74 ± 0.725	0.09

Theoretical systematics

proton PDFs
increasingly large QCD, QCD-EW and EW corrections

EW input parameters
large QED corrections
increasingly large EW corrections

Are we able to reach (at least) the 0.1% precision throughout the whole invariant mass range?
Claims of a significant discrepancy theory vs data require a reduction of the theoretical uncertainties

Structure of the lepton-pair invariant mass distribution in the Drell-Yan process

The triple-differential cross section at LO

$$\frac{\mathrm{d}^3\sigma}{\mathrm{dm}_{\ell\ell}\mathrm{d}y_{\ell\ell}\mathrm{d}\cos\theta_{CS}} = \frac{\pi\alpha^2}{3\mathrm{m}_{\ell\ell}s}\Bigg((1+\cos^2\theta_{CS})\sum_q S_q[f_q(x_1,Q^2)f_{\overline{q}}(x_2,Q^2)+f_q(x_2,Q^2)f_{\overline{q}}(x_1,Q^2)] + \cos\theta_{CS}\sum_q A_q\mathrm{sign}(y_{\ell\ell})\cdot[f_q(x_1,Q^2)f_{\overline{q}}(x_2,Q^2)-f_q(x_2,Q^2)f_{\overline{q}}(x_1,Q^2)]\Bigg)$$

$$S_q = e_\ell^2 e_q^2 + P_{\gamma\mathrm{Z}}\cdot e_\ell v_\ell e_q v_q + P_{\mathrm{ZZ}}\cdot(v_\ell^2+a_\ell^2)(v_q^2+a_q^2)$$

$$A_q = P_{\gamma\mathrm{Z}}\cdot 2e_\ell a_\ell e_q a_q + P_{\mathrm{ZZ}}\cdot 8v_\ell a_\ell v_q a_q,$$

$$P_{\gamma\mathrm{Z}}(\mathrm{m}_{\ell\ell}) = \frac{2\mathrm{m}_{\ell\ell}^2(\mathrm{m}_{\ell\ell}^2-\mathrm{m}_\mathrm{Z}^2)}{\sin^2\theta_W\cos^2\theta_W[(\mathrm{m}_{\ell\ell}^2-\mathrm{m}_\mathrm{Z}^2)^2+\Gamma_\mathrm{Z}^2\mathrm{m}_\mathrm{Z}^2]}$$

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Decomposing the invariant mass distribution into a forward (F) and a backward (B) components,

$$F(M_{\ell\ell}) \equiv \int_0^1 d\cos\theta_{CS}\,\frac{d\sigma}{d\cos\theta_{CS}}(M_{\ell\ell}) \qquad B(M_{\ell\ell}) \equiv \int_{-1}^0 d\cos\theta_{CS}\,\frac{d\sigma}{d\cos\theta_{CS}}(M_{\ell\ell})$$

we have two combinations $F+B$ and $F-B$, with complementary information

→ we consider $\frac{d\sigma}{dM_{\ell\ell}}$ and $A_{FB}(M_{\ell\ell}) \equiv \frac{F(M_{\ell\ell})-B(M_{\ell\ell})}{F(M_{\ell\ell})+B(M_{\ell\ell})}$

Sensitivity to the coupling constants at different invariant masses

The $A_{FB}(M_{\ell\ell})$ asymmetry $\leftrightarrow$ parity violation due to axial-vector coupling

but $A_{FB}(M_{\ell\ell}) \neq 0$ does not imply sensitivity to $\sin^2\theta_W$

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sensitivity to $\sin^2\theta_W$

	Z resonance	large invariant mass
$\frac{d\sigma}{dM_{\ell\ell}}$	no relevant for the overall normalisation	yes combination of different couplings
$A_{FB}(M_{\ell\ell})$	yes $3\,\frac{g_V^e/g_A^e}{1+(g_V^e/g_A^e)^2}\,\frac{g_V^f/g_A^f}{1+(g_V^f/g_A^f)^2}$ from $\lvert\mathcal{M}_Z\rvert^2$	no $\sim e^2 Q_e Q_f g_A^e g_A^f$ (photon-Z interference)

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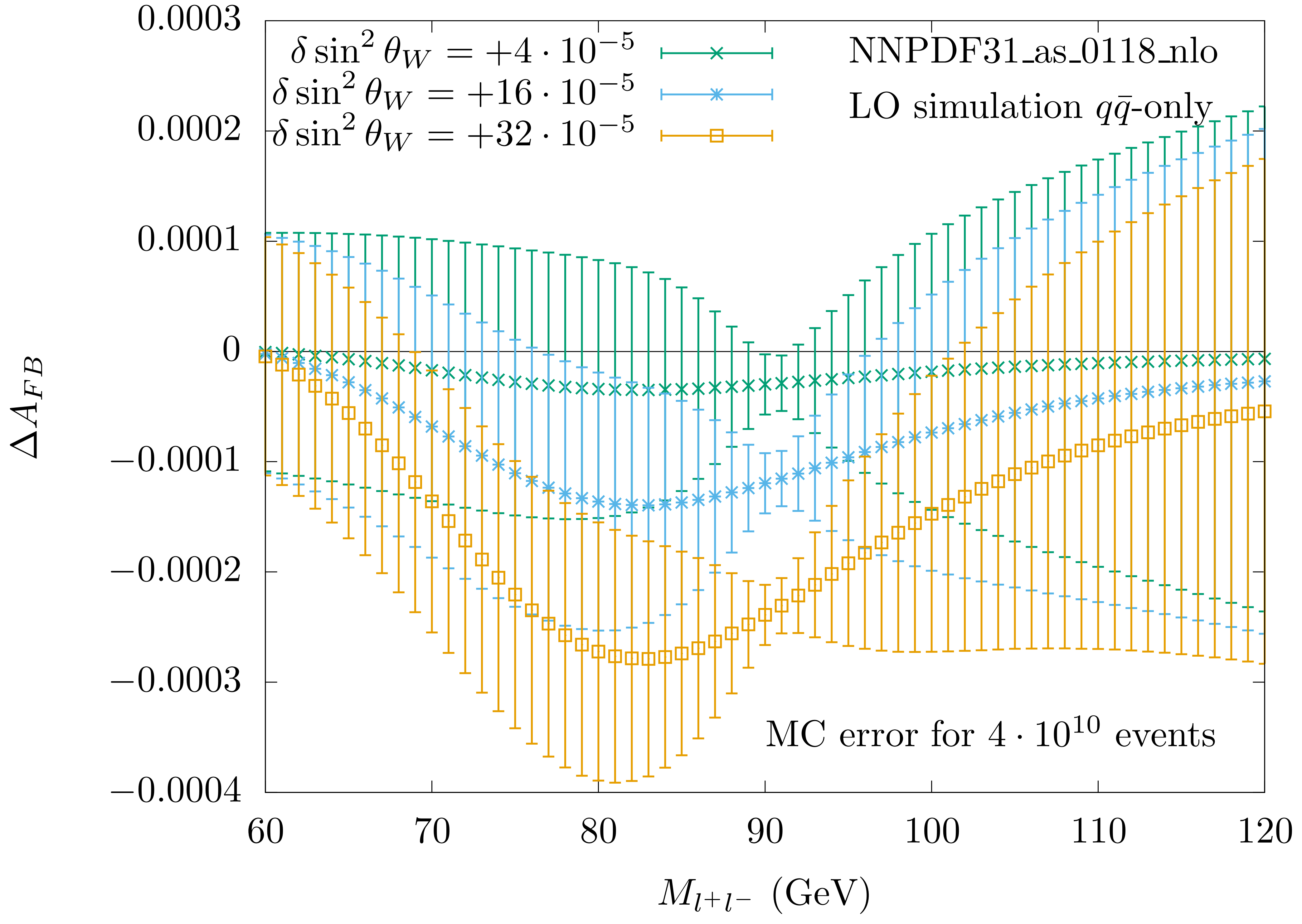
sensitivity to $\sin^2\theta_W$

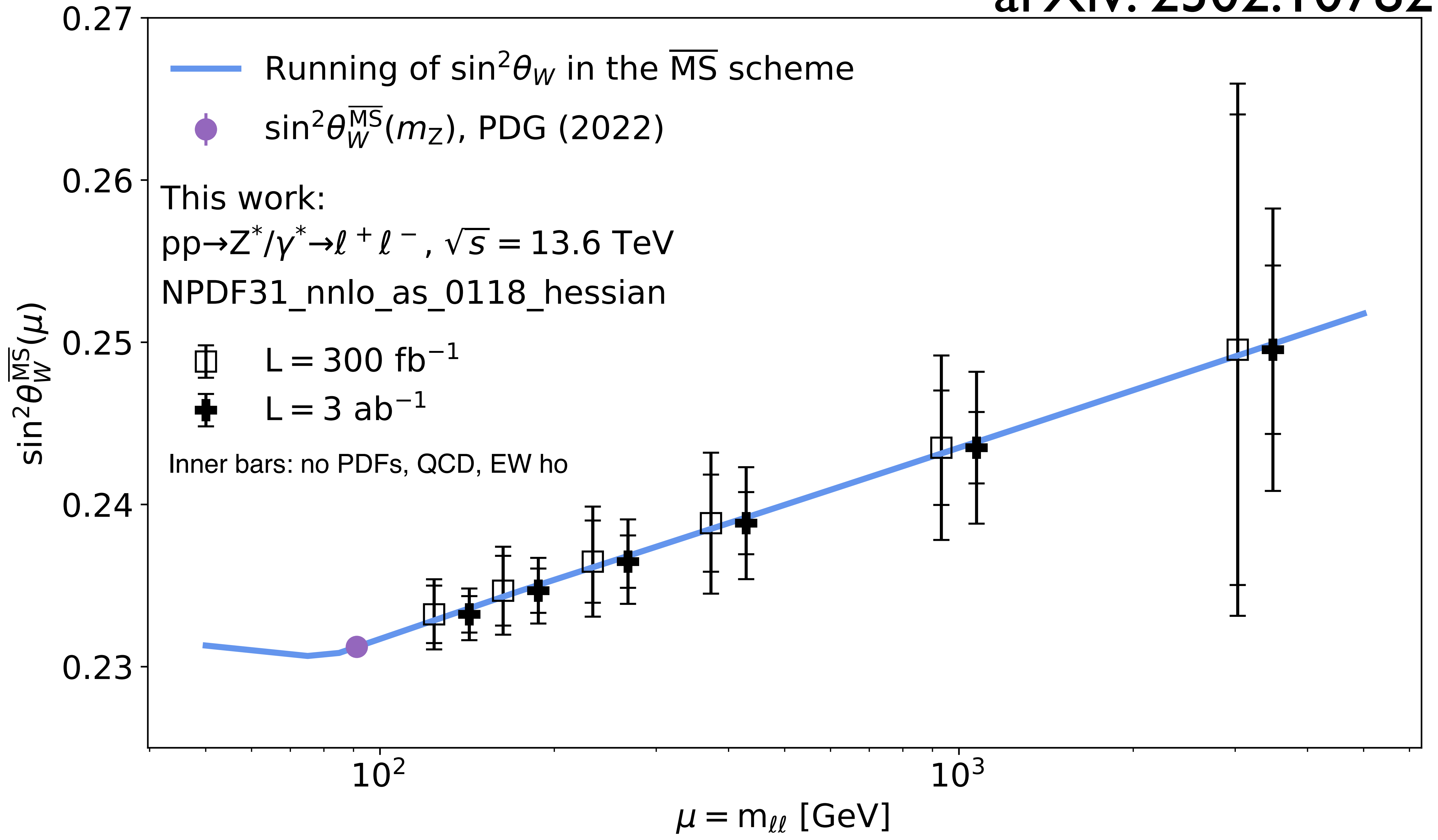
from $A_{FB}(M_{\ell\ell})$



maximal below the Z resonance

from $\frac{d\sigma}{dM_{\ell\ell}}$

arXiv: 2302.10782



not negligible at all $M_{\ell\ell}$

The Drell-Yan cross section in a fixed-order expansion

Drell-Yan (1970)

$$\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} +$$

Baur, Brein, Hollik, Schappacher, Wackeroth (2001)

Altarelli, Ellis, Martinelli (1979)

$$\alpha_s\,\sigma^{(1,0)} + \alpha\,\sigma^{(0,1)} +$$

$$\alpha_s^2\,\sigma^{(2,0)} + \alpha\,\alpha_s\,\sigma^{(1,1)} + \alpha^2\,\sigma^{(0,2)} +$$

$$\alpha_s^3\,\sigma^{(3,0)} + \ldots$$

Hamberg, Matsuura, van Nerveen, (1991)
Anastasiou, Dixon, Melnikov, Petriello, (2003)
Catani, Cieri, Ferrera, de Florian, Grazzini (2009)

C.Duhr, B.Mistlberger, arXiv:2111.10379
X.Chen, T.Gehrmann, N.Glover, A.Huss, T.Yang, H.X.Zhu , arXiv:2107.09085
X.Chen, T.Gehrmann, N.Glover, A.Huss, P.F.Monni, E.Re, L.Rottoli, P.Torrielli, arXiv:2203.01565

still missing
Sudakov high-energy approximations

Neutral Current

R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, (2021)
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, (2022)
F.Buccioni, F.Caola, H.Chawdhry, F.Devoto, M.Heller, A.von Manteuffel, K.Melnikov, R.Röntsch, C.Signorile-Signorile, (2022)
T.Armadillo, R.Bonciani, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, AV, (2024)

Charged-current

L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, (2021)
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, (2024)

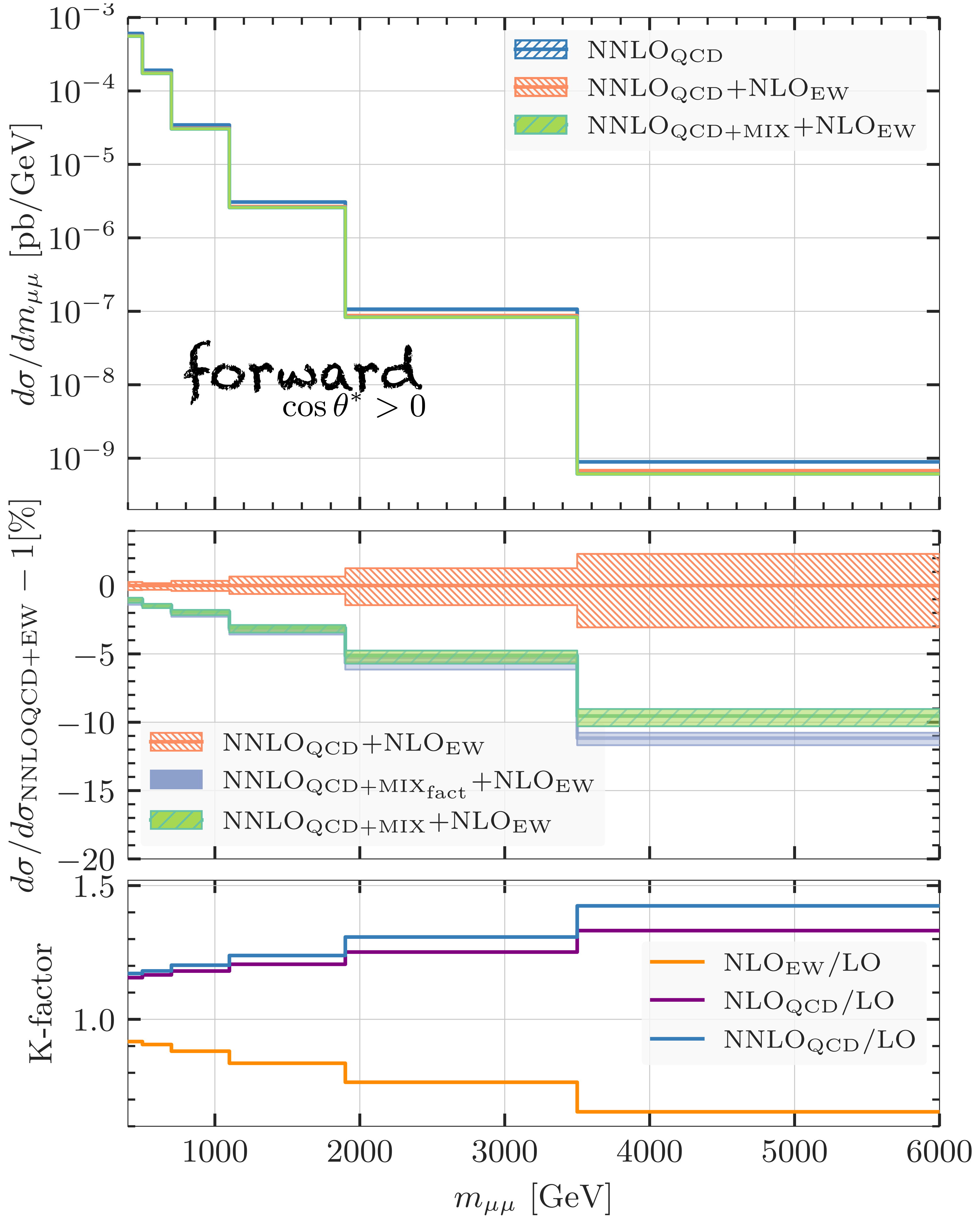
Contributions to NC-DY at large invariant masses

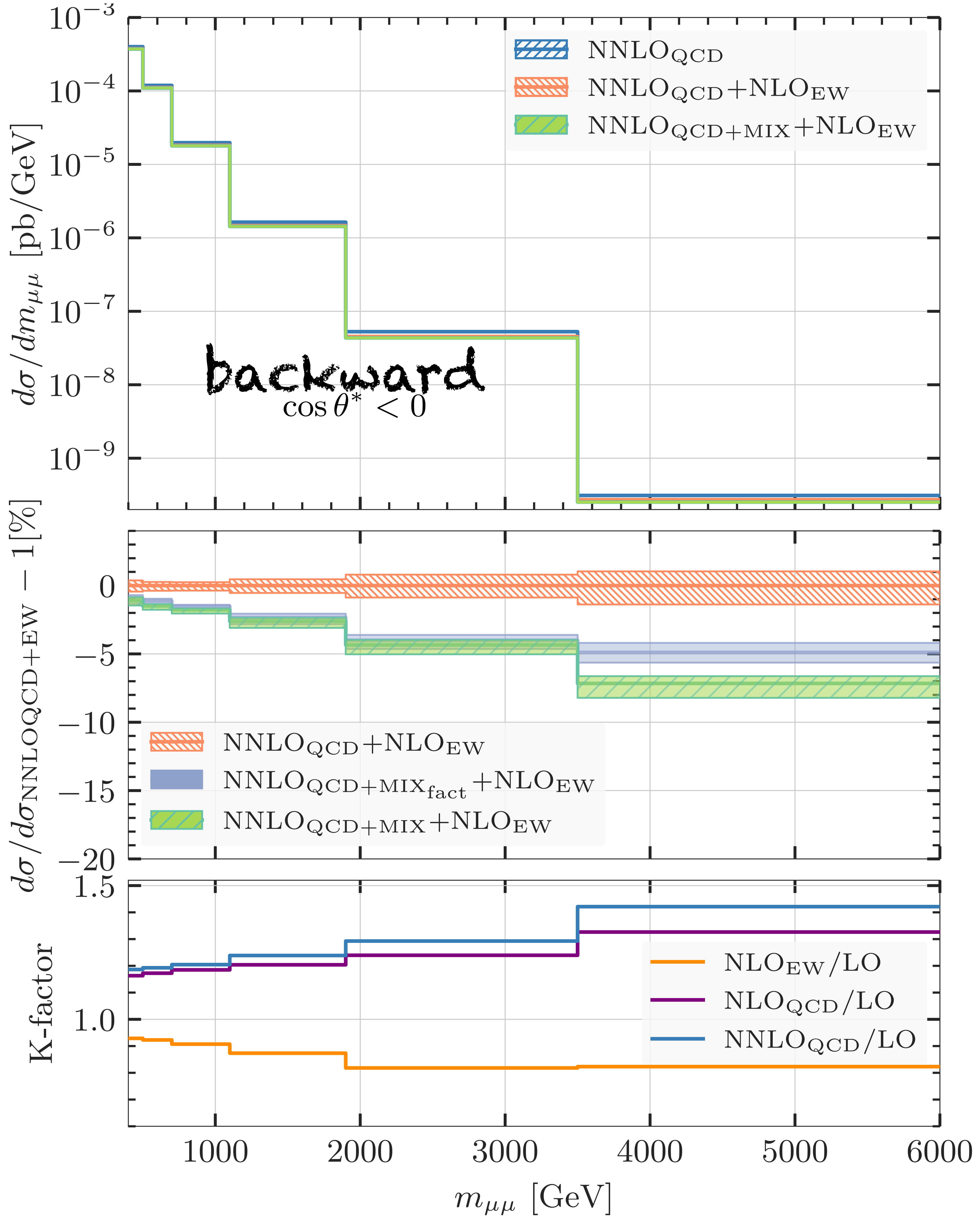
$\hat{\alpha}(\mu_R)$	overall positive growing contribution
$\sin^2\hat{\theta}(\mu_R)$	growing in size, the impact depends on the different coupling combination
EW Sudakov logs	negative growing contribution, non-trivial subleading effects
QCD	overall positive growing contribution

non-trivial combination of several competing effects

Phenomenology of Neutral Current Drell-Yan including exact NNLO QCD-EW corrections

T.Armadillo, R.Bonciani, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, arXiv:2106.11953 , Phys.Rev.Lett. 128 (2022) 1, 012002 and arXiv:2412.16095



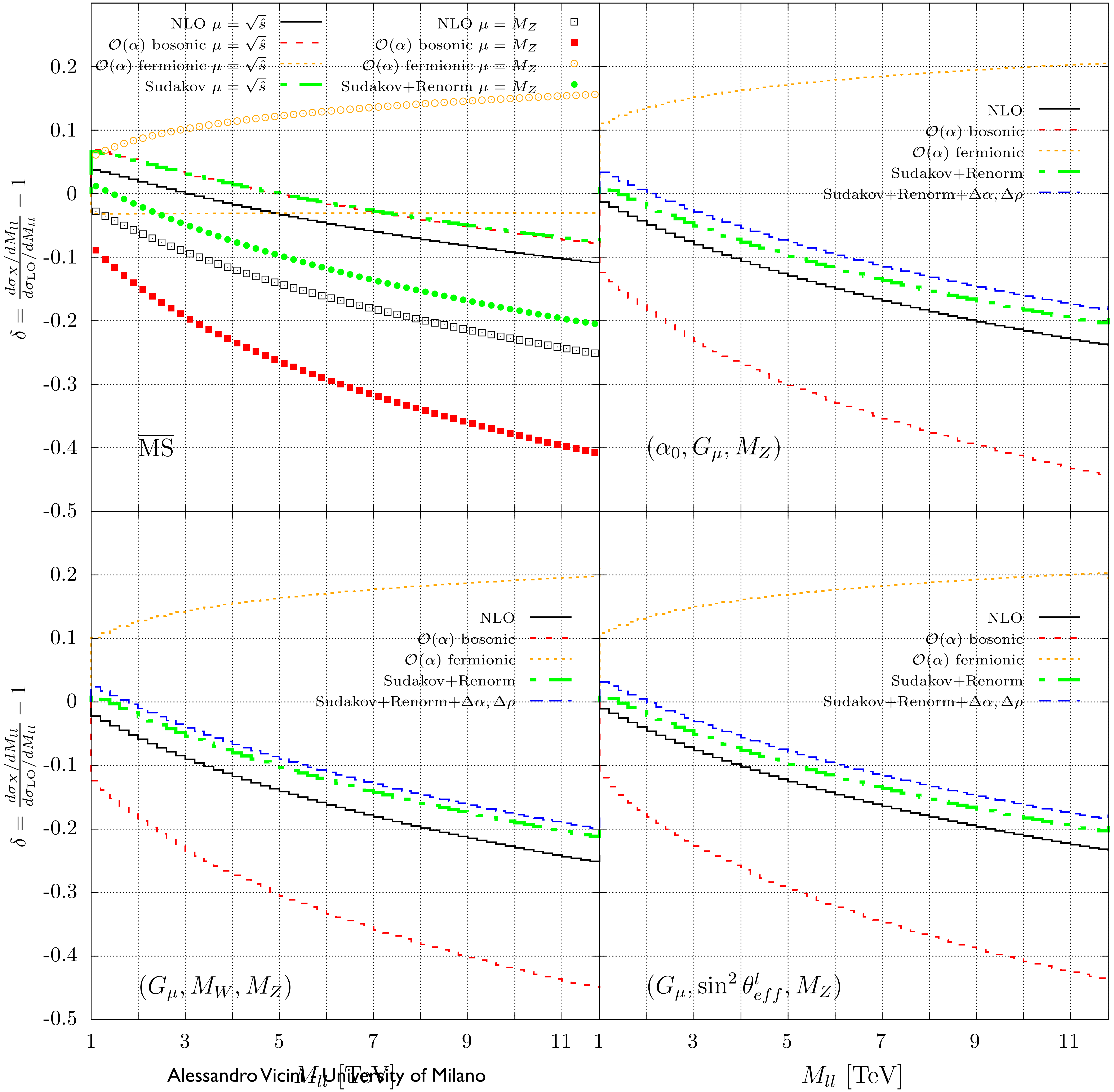


Negative mixed NNLO QCD-EW effects (-3% or more) at large invariant masses,

absent in any additive combination → impact on the searches for new physics

Need for a full NNLO-EW calculation to reduce the uncertainties to sub-percent level

M.Chiesa, C.Del Pio, F.Piccinini, arXiv:2402.14659



NLO-EW analysis + universal higher orders

corrections of different origin:

- electroweak Sudakov logarithms
- corrections to the definition of renormalised couplings
- choice of the renormalisation scale

different input schemes

→ spread of the predictions ranging from
1% (at 1 TeV) to 10% (at 12 TeV)

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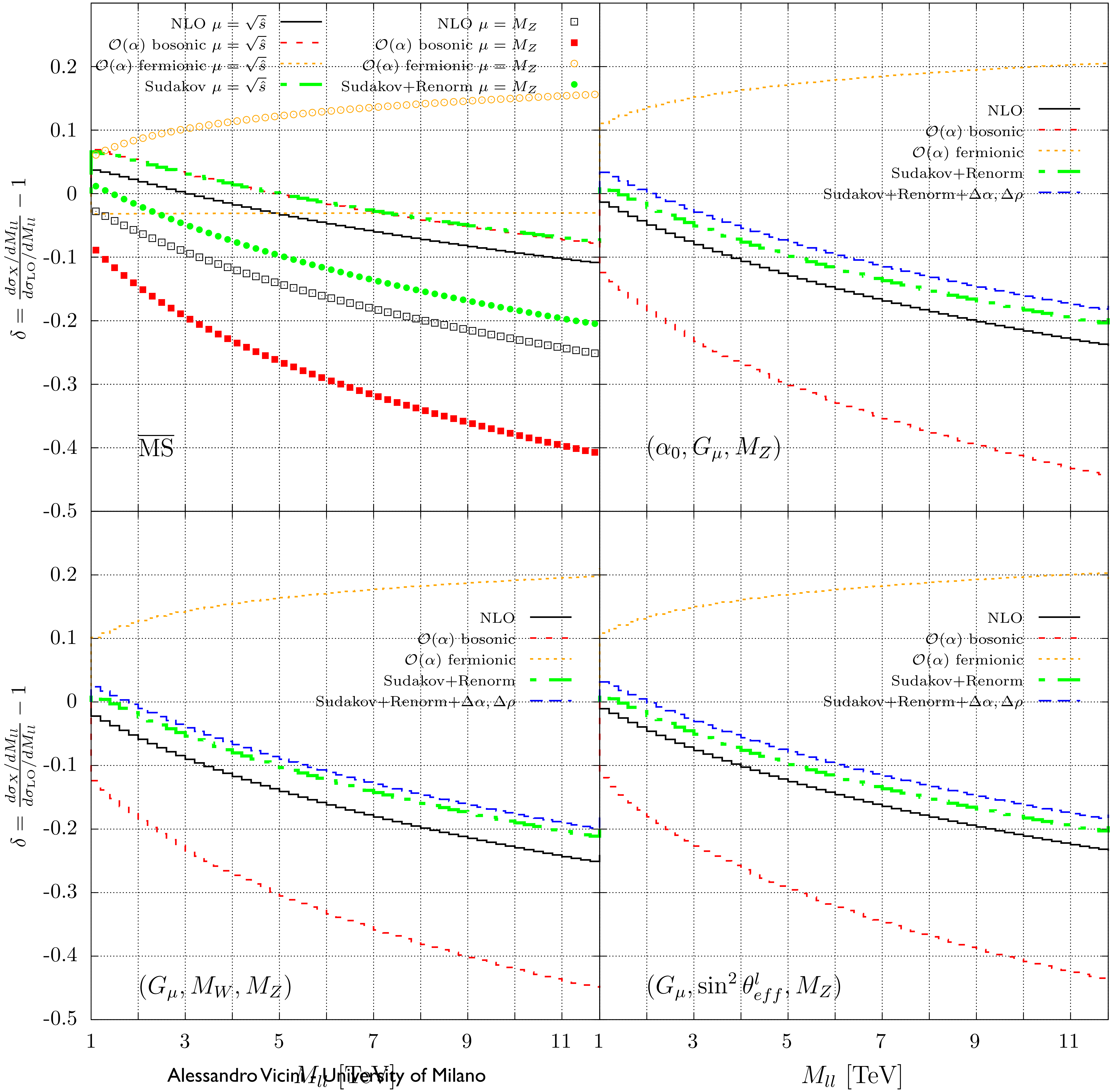

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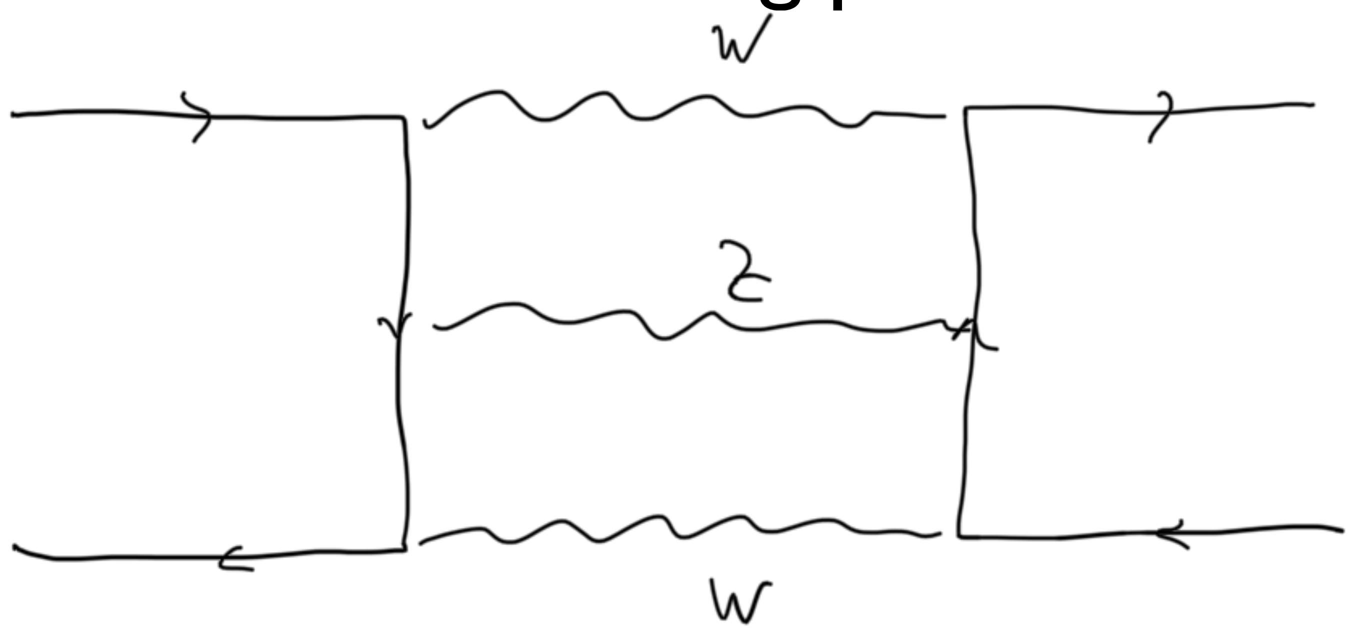
These ambiguities can be reduced
with a complete NNLO-EW calculation

Need for a full NNLO-EW calculation to reduce the uncertainties to sub-percent level

The NNLO-EW corrections to scattering processes are still today one of the frontiers in QFT

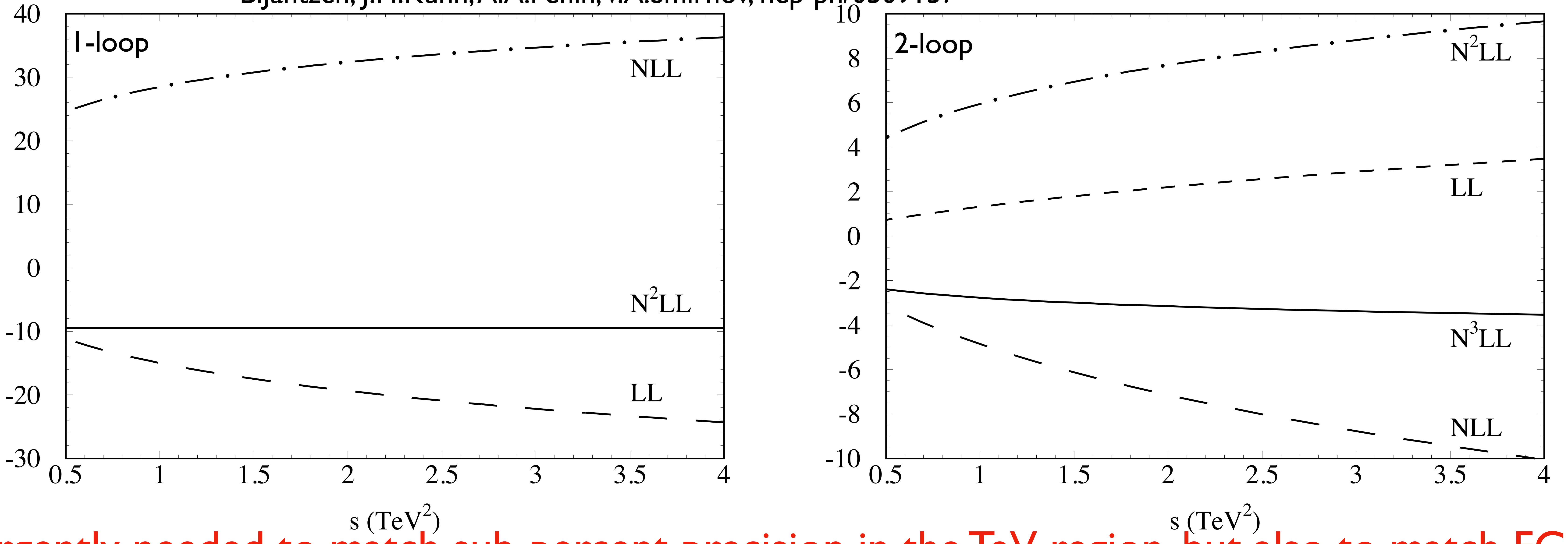


Large logarithmic corrections (EW Sudakov logs) appear in the virtual corrections at large invariant masses

At two-loop level, we have up to the fourth power of $\log(s/m_V^2)$

Sensitivity to the weak charges

B.Jantzen, J.H.Kühn, A.A.Penin, V.A.Smirnov, hep-ph/0509157

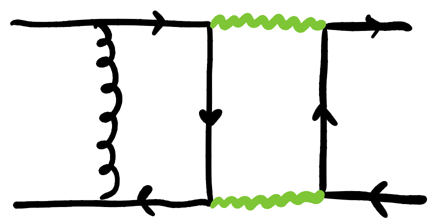
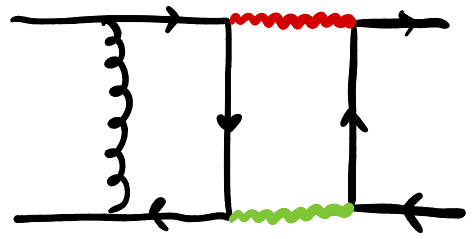
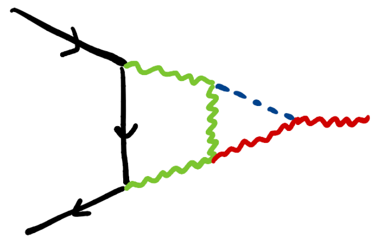
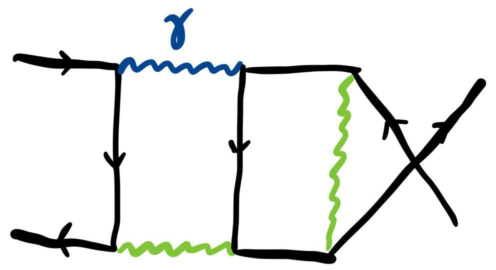
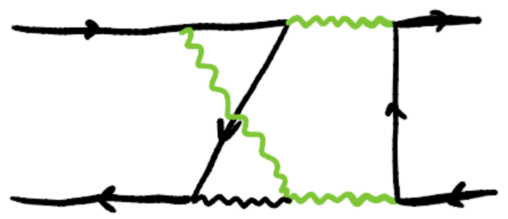
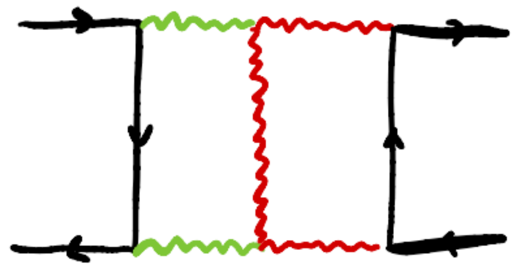
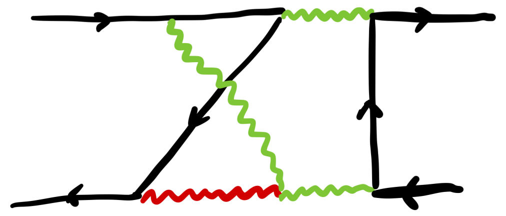


corrections to $e^+e^- \to q\bar{q}$ due to EW Sudakov logs

urgently needed to match sub-percent precision in the TeV region, but also to match FCC-ee precision

Challenges in the Master Integrals evaluation

courtesy of T.Armadillo

	NCDY - 2L Mixed	CCDY - 2L Mixed	Z on shell - 2L EW	NCDY - 2L EW	NCDY - 2L EW	NCDY - 2L EW	NCDY - 2L EW
Example Topology							
Number of masters	36	56	51	47	104	126	140
Reduction Kira + Firefly	12 hours (32 core)	16 hours (32 core)	1 day (32 core)	1.5 m (96 core + Ratracer)	30 m (120 core + Ratracer)	8 h (120 core + Ratracer)	54 h (120 core + Ratracer)
AMFlow 1 point	50 min (32 core)	75 min (32 core)	6 h 45 m (32 core)	15 min (96 core)	1 hour (120 core)	4 hours (120 core)	X
Dimension equations	700 Kb	2.1 Mb	/	4 Mb	45 Mb	350 Mb	??
SeaSyde 3250 points	5 days	10 days	/	??	??	??	??

key to overcome these problems: the improvement in the choice of the Feynman Integrals
systematic usage of polynomial reconstruction

Interpretation of cross sections: strategies

The cross section is a convolution of proton PDFs with the partonic cross section

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2) Assume that the SM partonic xsecs are "optimal" (with their intrinsic uncertainties)
   → proton PDF determination (including high-mass data) within the SM hypothesis
   → alternative studies including SMEFT operators and consistency checks of the global PDF determination
   arXiv:2104.02723

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3) Having a good data description based on the SM cross sections,
   we can prepare templates, letting one of the SM input parameters vary
   → perform a template fit to the data
   the template with best agreement selects the "experimental" value of the parameter

Strategy 3: Determination of the weak mixing angle from the high-mass data arXiv:2302.10782

$$\mathscr{L}_{SM} = \mathscr{L}_{SM}(\hat{\alpha}(\mu_R), \sin^2\hat{\theta}(\mu_R), m_Z)$$

Only the input parameters are free and can be fit to the data

A determination of the weak mixing angle requires that $\sin^2\theta_W$ is one of the SM input parameters

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POWHEG Z_ew-BMNNPV describes NC DY at NLO-(QCD+EW) matched with QCD and QED showers

the $(\hat{\alpha}(\mu_R), \sin^2\hat{\theta}(\mu_R), m_Z)$ input scheme is available arXiv:1302.4606, arXiv:2402.14659

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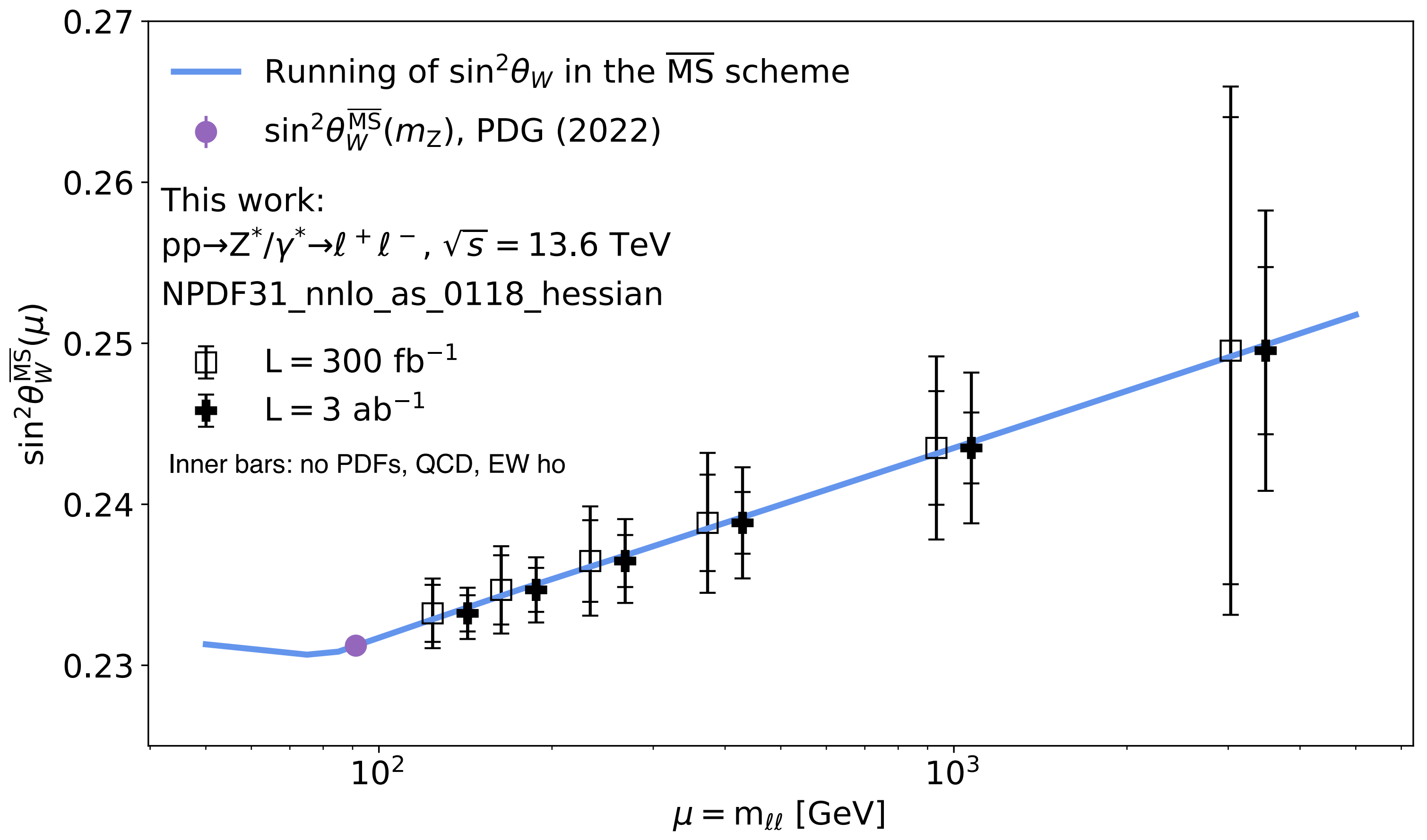

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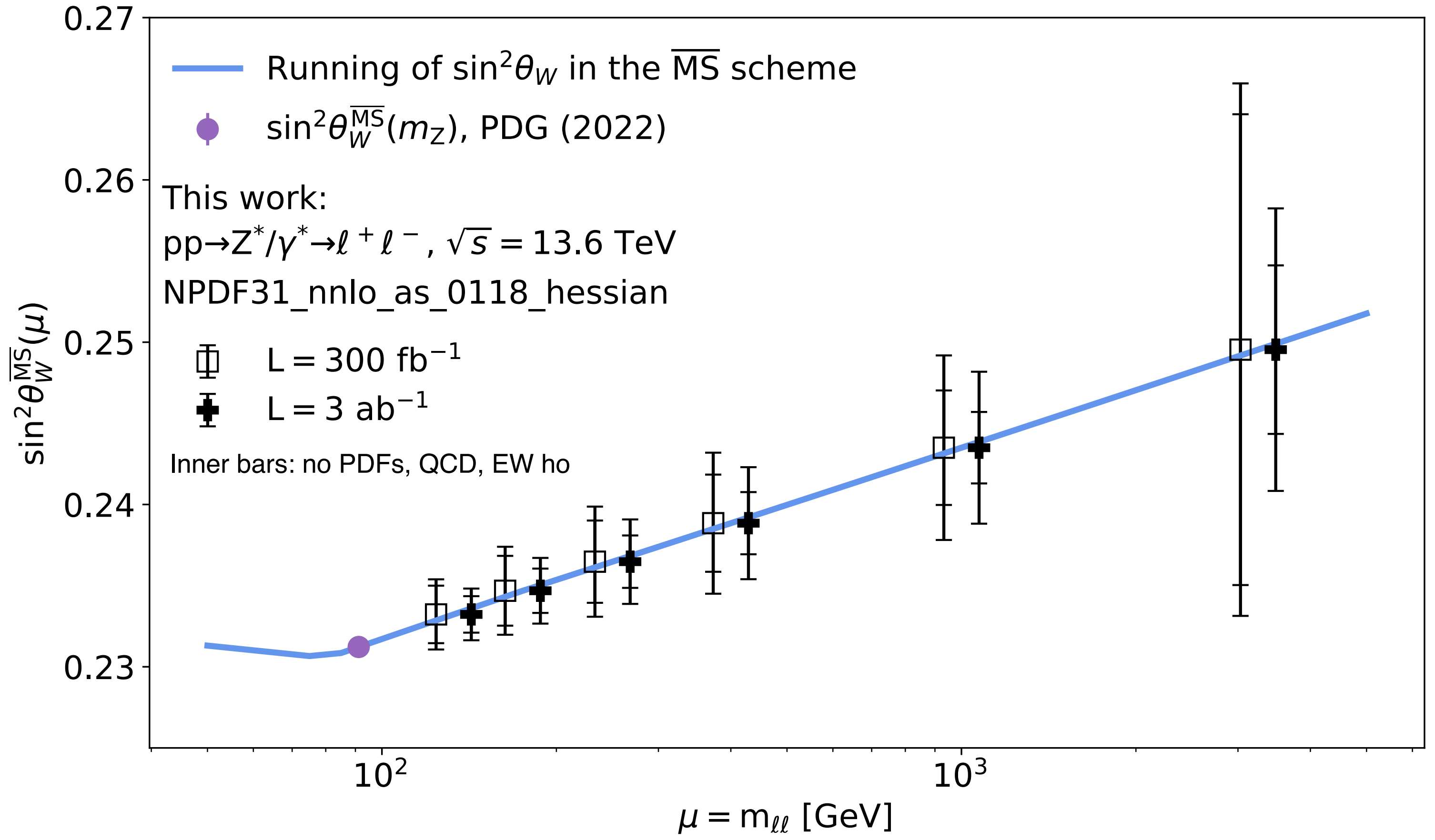


CAVEATS:

ok for a sensitivity study

but

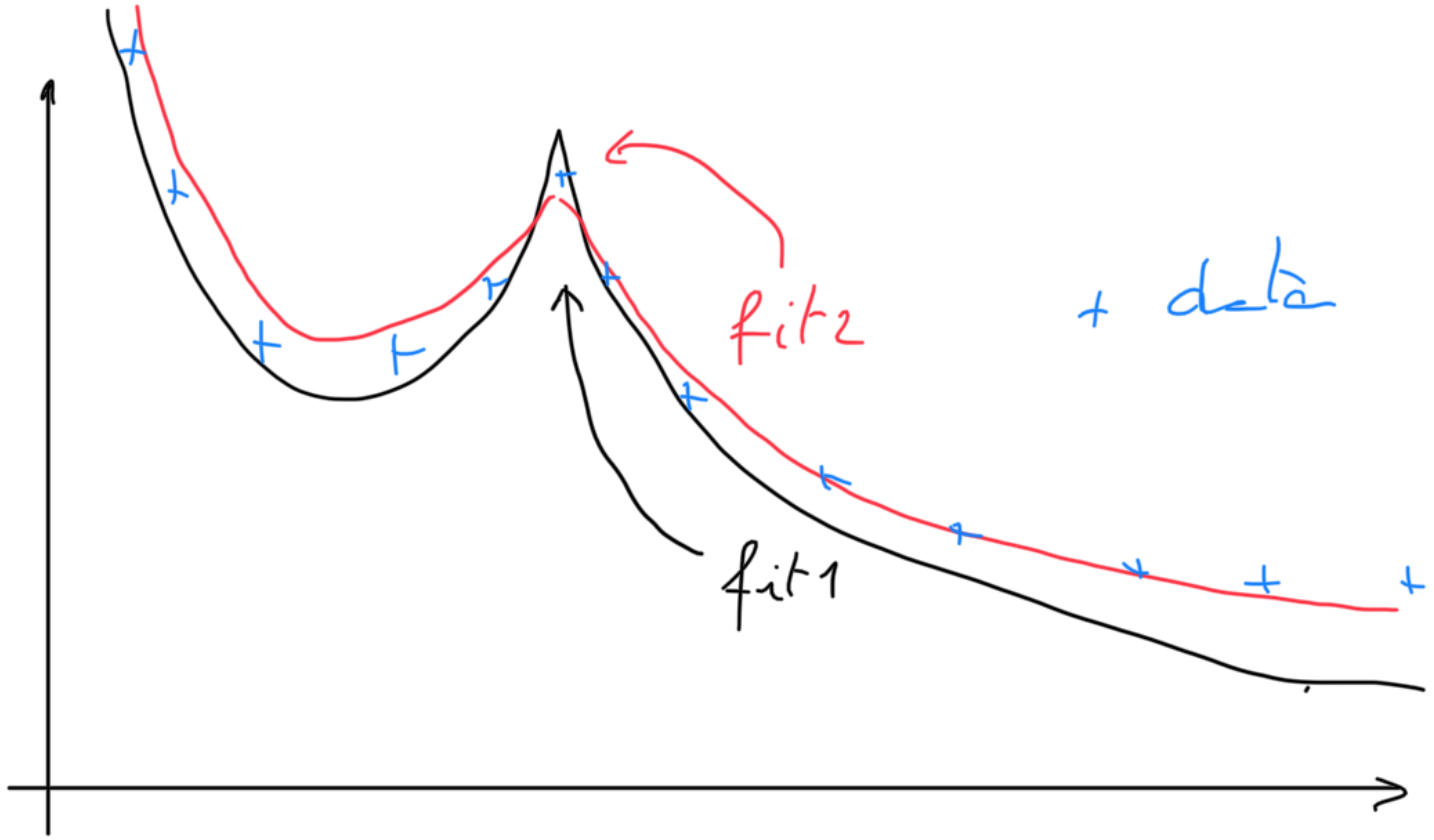
the accurate estimate of the central value might suffer of missing higher-order effects

Templates to be fitted to the data (1)

The templates are a set of predictions of our observable, computed with different numerical values of the input parameter.

Example 1: fitting $\sin^2\hat{\theta}_W(\mu_R = m_Z)$

we compute the NC DY invariant mass distribution in the range $M_{\ell\ell} \in [m_Z, 1\ \mathrm{TeV}]$
we stick to plain NLO-EW and renormalize at $\mu_R = m_Z$
we assign to $\sin^2\hat{\theta}_W(\mu_R = m_Z)$ several values in a range, e.g. $[0.22600, 0.23600]$
by fitting the data, we identify which value globally best describes the data



since the input is defined at $\mu_R = m_Z$,
the good description at high invariant masses depends on the diagrammatic content of the fitting formula
the fitted parameter is always defined at $\mu_R = m_Z$, but its value is affected also by high-mass data

with this approach we experimentally determine $\sin^2\hat{\theta}_W(\mu_R = m_Z)$, i.e. only the boundary condition of the RGE

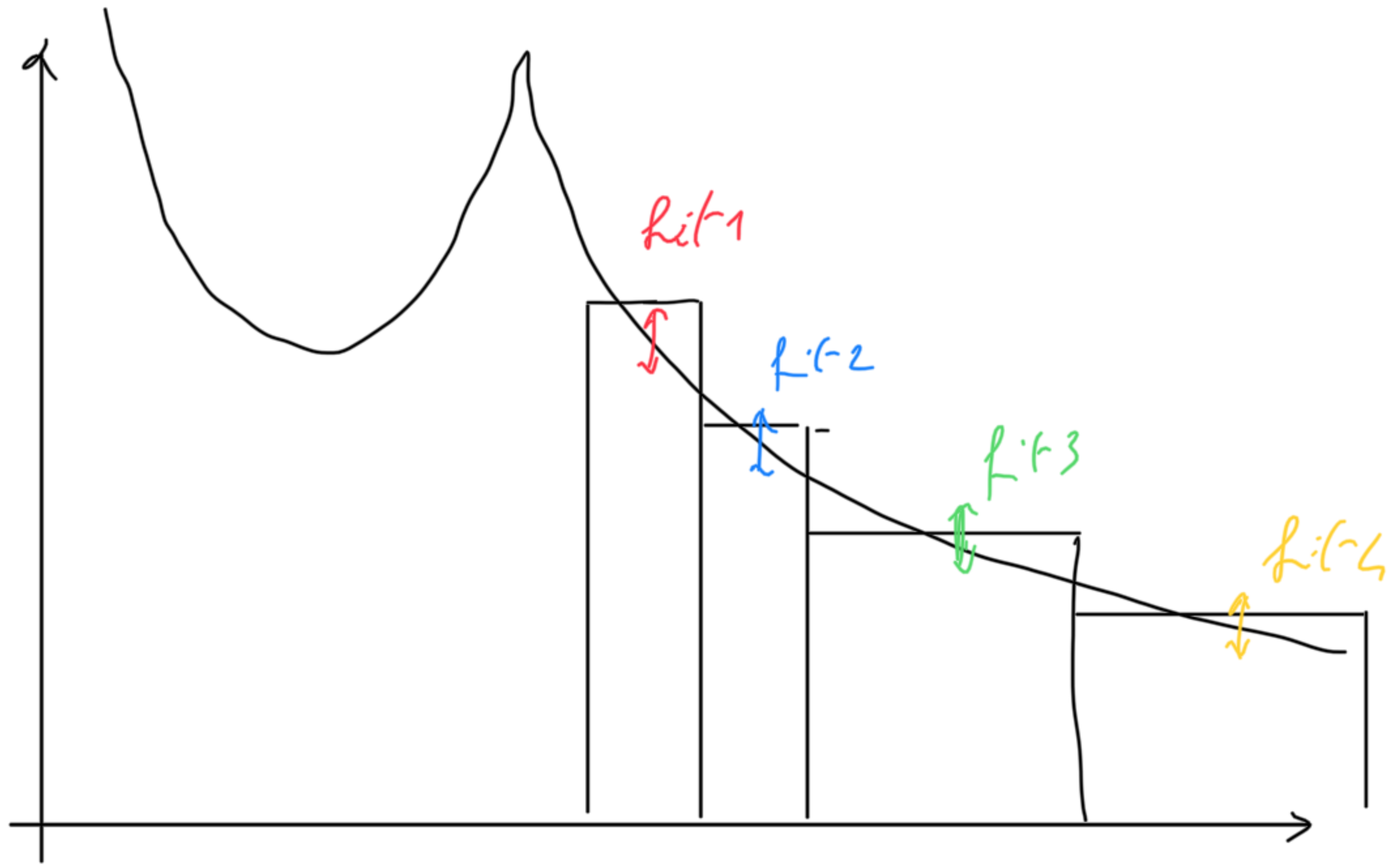
Templates to be fitted to the data (2)

The templates are a set of predictions of our observable, computed with different numerical values of the input parameter.

Example 2: fitting $\sin^2\hat\theta_W(\mu_R = M_{\ell\ell})$ in several invariant mass bins

we compute the NC DY invariant mass distribution in one single bin of invariant mass $[M_{\ell\ell}, M_{\ell\ell}+\delta]$
we stick to plain NLO-EW and we renormalize $\sin^2\theta_W$ at $\mu_R = M_{\ell\ell}$
we assign to $\sin^2\hat\theta_W(\mu_R = M_{\ell\ell})$ several values in a range, e.g. $[0.22600, 0.25600]$
by fitting only in that $M_{\ell\ell}$ bin, we identify which value best describes that data point

we repeat the above procedure in different mass bins
in each bin we might expect to find a different best fit value



the sequence of best fit values, as a function of $M_{\ell\ell}$, can be compared with the solution of the RGE

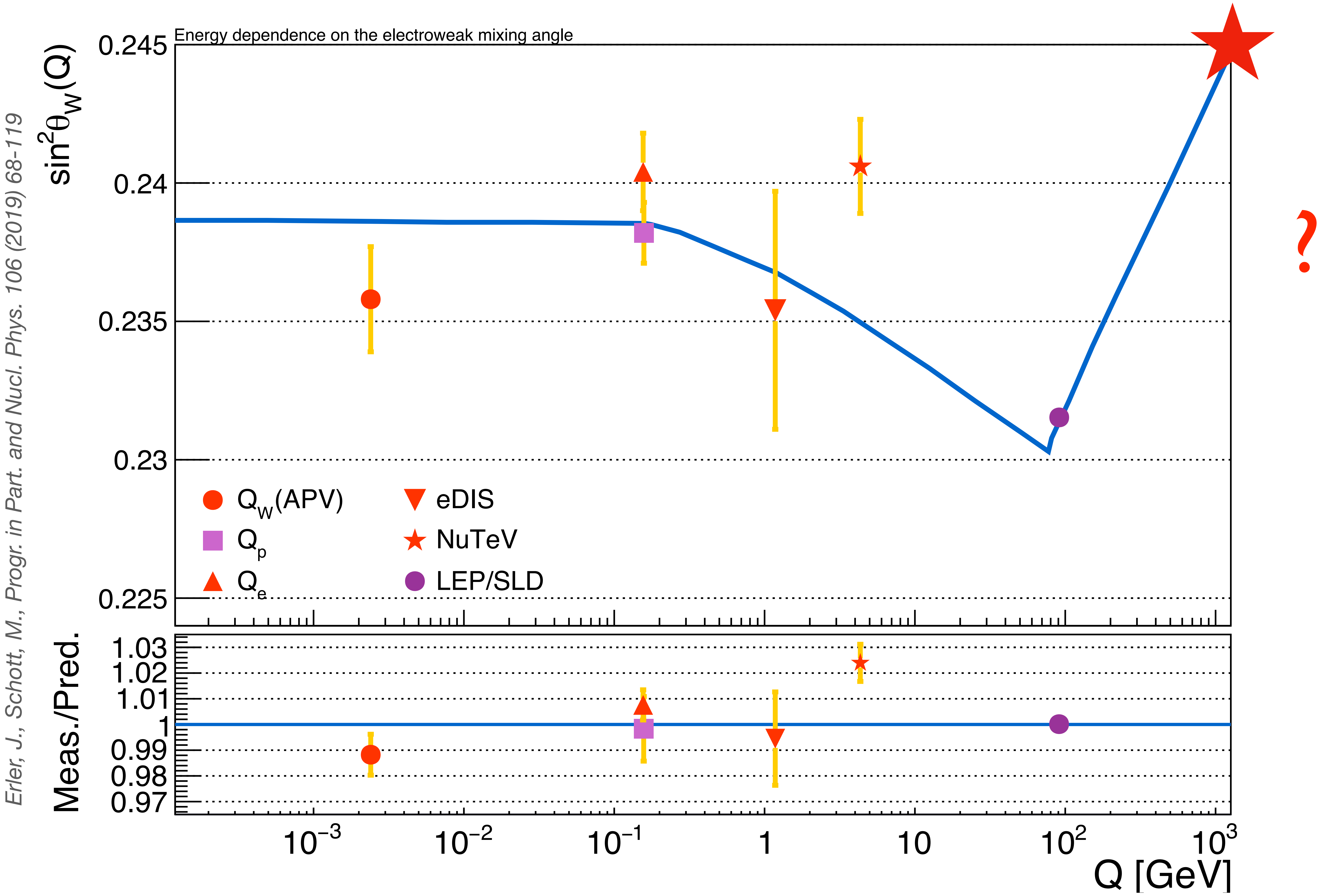
with this approach, we extract information about: 1) the fact that $\sin^2\hat\theta_W(\mu)$ indeed runs
2) the slope of the running (cfr. with SM β function)

Interplay of precision measurements at Z resonance, low-, and high-energy

The determination of EW parameters at the Z resonance is a cornerstone of the whole precision program but there is more…

The SM predicts the running of $\sin^2 \hat{\theta}(\mu_R = Q)$

J.Erler, M.J.Ramsey-Musolf, arXiv:0409169



low-energy (sub-GeV) determinations (P2 in Mainz, Møller at JLab) + high-energy (TeV) determinations (CMS, ATLAS, LHCb)

→ stringent test of the SM, complementary to the results at the Z resonance

→ sensitivity to different BSM scenarios

Comments and conclusions on the high-mass tail of $d\sigma/dM_{\ell\ell}$ in NC DY

Sensitivity to $\sin^2\hat{\theta}(\mu_R = M_{\ell\ell})$ test a SM prediction
probe of heavy BSM physics
complementary to the studies at low invariant masses

Determination with a template-fit approach → $\sin^2\hat{\theta}(\mu_R = M_{\ell\ell})$ has to be a SM input parameter
the comment holds both at large masses but also at the Z resonance

Theoretical systematic errors: missing higher order can bias the fit → need for full NNLO-EW + best QCD

The evaluation of the full two-loop EW amplitude challenging:
large size of the amplitude
large size of the differential equations satisfied by multi-scale Master Integrals

Thank you