Universal Description of Decoherence in Scale-Invariant Environments

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Abstract

When a quantum system couples to a scale-invariant environment, what form must its decoherence take? We prove that the answer is unique: under locality, Lorentz invariance, unitarity, and continuous scale invariance, the effect of any such environment is mathematically equivalent to that of an *unparticle bath* [1] — a scale-invariant continuum of states — characterized entirely by the scaling dimension $d_{\mathcal{U}}$ of the coupled operator. This is not a modelling choice but a consequence of conformal symmetry. All decoherence and dissipation exponents are fixed by $d_{\mathcal{U}}$ through exact consistency relations, providing falsifiable predictions independent of microscopic details. We validate the framework using multi-channel transport data from the unitary Fermi gas, where two genuinely independent observables yield a consistent $d_{\mathcal{U}} = 7/4$. We further show that quantum Ising criticality, inflationary cosmology, and high-energy astrophysical neutrinos—spanning more than 25 orders of magnitude in energy—are unified as specific realizations of the same structure. A decoherence phase transition at $d_{\mathcal{U}} = 5/2$, where quantum coherence is *protected* rather than destroyed at long times, is a qualitative prediction inaccessible to any memoryless dynamical description.

Introduction

Decoherence—the irreversible loss of quantum coherence through environmental entanglement—underpins the quantum-to-classical transition and sets fundamental limits on quantum technologies. A central open question is: *what constraints does symmetry place on the form of decoherence?*

For generic environments the answer is: very few. The Lindblad formalism [2] —the standard framework for open quantum dynamics— treats dissipation rates as free parameters, while the Caldeira-Leggett model [3] —which models the environment as a bath of harmonic oscillators— engineers spectral densities to match desired phenomenology. These approaches capture enormous variety but obscure universal structure.

Here we identify one case in which symmetry is maximally constraining: when the environment is *scale-invariant*. Scale-invariant environments arise ubiquitously—at quantum critical points in condensed matter, in the approximately de Sitter geometry of inflationary cosmology, and potentially in quantum gravity. Despite this prevalence, the consequences for open-system dynamics have not been systematically established.

We prove that under physically natural assumptions, a scale-invariant environment coupled to any quantum system *must* be described by an unparticle bath [1]. Originally proposed as speculative beyond-Standard-Model phenomenology, unparticles are here elevated to a *universal characterization* of quantum systems coupled to scale-invariant environments. This universality has a sharp experimental consequence: the power-law exponents of multiple independently measured observables are not free parameters but are fixed by a single number $d_{\mathcal{U}}$, obeying exact consistency relations that can be tested and falsified.

The Uniqueness Theorem

What can we actually know about an environment we cannot directly observe, armed only with the knowledge that it has no intrinsic scale? Surprisingly, the answer is: almost everything. The argument below is a uniqueness proof—we show not merely that unparticle baths *can* describe scale-invariant environments, but that they *must*. The chain of reasoning is tight: scale invariance forces conformal symmetry, conformal symmetry fixes the correlators of environmental fluctuations, and fixed correlators determine frequency dependence of environmental noise completely. No free functions, no adjustable spectral shapes remain. A single number, the scaling dimension $d_{\mathcal{U}}$, determines all decoherence and dissipation exponents exactly. The proof is short but technical; the experimental consequences are developed in the following sections independently of its details.

Theorem 1 (Unparticle Universality)**.** *Let a quantum system S couple locally to an environment E in d spatial dimensions. Assume: (i) E exhibits exact continuous scale invariance; (ii) the theory is Lorentz-invariant; (iii) the coupling is local, $H_{\text{int}} = g\, A_S(x)\, \mathcal{O}_E(x)$; (iv) the full system evolves unitarily. Then:*

1. *E is described by a conformal field theory (CFT).*
2. *The noise spectrum of environmental fluctuations, $J(\omega)$, takes the unique form*
$$J(\omega) = A\,\omega^{2\Delta-d-1}, \tag{1}$$
where Δ is the scaling dimension of $\mathcal{O}_E$[1].
3. *This is equivalent to an unparticle bath with*
$$d_{\mathcal{U}} = \Delta - \tfrac{d-2}{2}. \tag{3}$$
4. *All dynamical exponents are uniquely determined by $d_{\mathcal{U}}$ (Table 1).*

The proof follows in four steps. *Step 1.* In a relativistic QFT, continuous scale invariance combined with Lorentz invariance and energy-momentum conservation implies full conformal invariance [4][2]: tracelessness of $T^{\mu\nu}$ and Poincaré symmetry together generate the full conformal group $SO(d+1,1)$. *Step 2.* Conformal Ward identities fix the two-point function of any primary operator $\mathcal{O}_E$ with dimension Δ up to normalization: $\langle \mathcal{O}_E(x)\mathcal{O}_E(0)\rangle = C_{\mathcal{O}}/(x_E^2)^{\Delta}$. There is no freedom to choose an alternative functional form. *Step 3.* Analytic continuation to Lorentzian signature and Fourier transformation at zero spatial momentum give the retarded Green's function $G_R(\omega) \propto (-i\omega)^{2\Delta-d}$, yielding $J(\omega) = -2\,\mathrm{Im}[G_R(\omega)] \propto \omega^{2\Delta-d-1}$. *Step 4.* Matching to the unparticle spectral form $\rho_{\mathcal{U}}(\omega) \propto \omega^{2d_{\mathcal{U}}-3}$ gives Eq. (3), and the memory kernels follow by Fourier transformation (Table 1).

The theorem is not merely that unparticles provide a *convenient* parametrization: they are the *only* possible description.

[1] Throughout this work we adopt the convention
$$J(\omega) = -2\,\mathrm{Im}\,G_R(\omega, \mathbf{k}=0), \quad \omega > 0, \tag{2}$$
where $G_R(\omega)$ is the retarded Green's function of the environmental operator $\mathcal{O}_E$, evaluated at zero spatial momentum appropriate for a local coupling. The overall normalization constant A in Eq. (1) absorbs the coupling strength g^2 and the CFT coefficient $C_{\mathcal{O}}$; its precise value depends on the microscopic theory and is not fixed by scale invariance alone. Since all physical predictions of the framework depend only on the *scaling exponent* $2d_{\mathcal{U}}-3$ and not on A, we do not specify a normalization convention beyond Eq. (2), keeping the analysis general and independent of microscopic details.

[2] In $d=2$, this follows rigorously [4]; in $d \geq 3$ it holds under the additional assumption of unitarity and absence of a virial current, supported by strong evidence in $d=4$ [5, 6].

Table 1: **Complete set of scaling exponents for any scale-invariant environment.** All quantities are determined by the single parameter $d_{\mathcal{U}}$, providing consistency relations that can be independently measured and tested.

Observable	**Scaling**	**Exponent**
Spectral density	$J(\omega) \propto \omega^s$	$s = 2d_{\mathcal{U}} - 3$
Dissipation kernel	$\eta(t) \propto t^{-\alpha_\eta}$	$\alpha_\eta = 2d_{\mathcal{U}} - 2$
Noise kernel (high-T)	$\nu(t) \propto T\, t^{-\alpha_\nu}$	$\alpha_\nu = 2d_{\mathcal{U}} - 3$
Damping function	$\Gamma_{\text{damp}} \propto t^{\beta}$	$\beta = 3 - 2d_{\mathcal{U}}$
Decoherence functional	$\Gamma_{\text{decoh}} \propto t^{\gamma}$	$\gamma = 5 - 2d_{\mathcal{U}}$

Falsifiability and consistency relations. The exponents in Table 1 are not independent. They satisfy exact algebraic relations:

$$s + \gamma_{\text{decoh}} = 2, \tag{4}$$
$$\alpha_\eta + \delta_{\text{decoh}} = 2, \tag{5}$$
$$\alpha_\nu + \beta_{\text{damp}} = 0, \tag{6}$$

where $\delta_{\text{decoh}} = 4 - 2d_{\mathcal{U}}$ is the instantaneous decoherence rate exponent. These are predictions, not fits: each relation connects an independently measurable bath property to an independently measurable system response, so neither side is derived from the other. Measuring any two exponents independently extracts $d_{\mathcal{U}}$ from each; consistency tests the scale-invariance assumption. *Inconsistency falsifies scale invariance* and reveals intrinsic scales, non-locality, or multiple competing sectors in the environment.

A genuine CFT bath is further distinguished from a phenomenological power-law mimic by the absence of a UV cutoff function modifying the spectral density. A generic oscillator bath with $J(\omega) \propto \omega^s f(\omega/\Lambda)$ reproduces the same exponent but introduces a rolloff at scale Λ; the unparticle bath has $f \equiv 1$ over the entire scaling window. This can be tested by checking that the consistency relations (4)–(6) hold simultaneously across the full frequency range of the experiment.

Loopholes. The theorem requires *continuous* scale invariance. Physical deviations arise from: (i) infrared cutoffs (finite temperature T, system size L, mass gap m) breaking scale invariance below ω_{IR}; (ii) ultraviolet cutoffs (e.g., lattice spacing, Planck length) above ω_{UV}; (iii) discrete scale invariance (e.g., Efimov states in ultra cold atoms, hierarchical models), which produces log-periodic modulations; (iv) multiple unparticle sectors summing with different $d_{\mathcal{U}}^{(i)}$, causing crossover behavior. The unparticle description is valid within the scaling window $\omega_{\text{IR}} < \omega < \omega_{\text{UV}}$. Crucially, thermal corrections modify amplitudes but not exponents: both the vacuum ($t \ll \beta$) and thermal ($t \gg \beta$) regimes exhibit power-law behavior with the *same* $d_{\mathcal{U}}$, only the prefactor changes.

Experimental Validation: Unitary Fermi Gas

The unitary Fermi gas —a strongly interacting gas of cold fermionic atoms tuned to a scattering resonance— at $T \gtrsim T_F$ provides a rare platform in which exact scale invariance holds over a measurable window and multiple independent transport channels are experimentally accessible. At unitarity, the interparticle scattering length diverges and T is the only energy scale, enforcing scale-invariant fluid dynamics. This allows a non-trivial test of the theorem's central prediction: that a single $d_{\mathcal{U}}$ governs two physically independent observables—shear viscosity and thermal conductivity—of the same many-body state.

In a scale-invariant fluid, conformal symmetry fixes the frequency dependence of stress fluctuations —and through the theorem, this frequency dependence is controlled by $d_{\mathcal{U}}$— so shear viscosity, which measures resistance to flow, obeys $\eta(T) \propto T^{2d_{\mathcal{U}}-2}$. Cao *et al.* [7] measure $\eta \propto T^{3/2}$ via anisotropic expansion over $E = 2.3$–$4.6\,E_F$, firmly within the conformal window, giving

$$2d_{\mathcal{U}} - 2 = \tfrac{3}{2} \quad \Rightarrow \quad d_{\mathcal{U}} = \tfrac{7}{4}. \tag{7}$$

A direct power-law fit yields $d_{\mathcal{U}} = 1.72 \pm 0.03$, within 1σ of 7/4. Wang *et al.* [8], using a uniform box potential that eliminates trap-inhomogeneity systematics, independently recover $d_{\mathcal{U}} = 1.71 \pm 0.07$ from shear viscosity in the high-temperature subset ($T \gtrsim 0.45\,T_F$). Both measurements probe the same physical quantity—shear viscosity— and their agreement constitutes an internal reproducibility check.

The thermal conductivity $\kappa(T)$, which measures heat transport and is experimentally independent of the viscosity measurements, provides a genuinely independent channel. Wang *et al.* [8] measure κ in the same dataset, finding $d_{\mathcal{U}} = 1.60 \pm 0.09$ from the same high-temperature cut—consistent with 7/4 at the $\lesssim 1.5\sigma$ level and consistent with the shear channel. The slight downward shift is expected when the fit window extends below the strict scale-invariant regime. Sound diffusivity D_s from Patel *et al.* [9], a composite quantity sensitive to both viscosity and heat transport, provides an additional cross-check. Restricting to $T \gtrsim 0.75\,T_F$ gives $d_{\mathcal{U}} = 1.63 \pm 0.13$.

Taken together, shear viscosity and thermal conductivity yield mutually consistent, independently obtained values of $d_{\mathcal{U}} \approx 7/4$ (Fig. 1). We emphasize that neither measurement accesses the spectral density directly: $d_{\mathcal{U}}$ is inferred from the temperature scaling of transport coefficients, under the assumption that this scaling reflects the underlying power-law structure of environmental fluctuations. Furthermore, the unitary Fermi gas is conformal only approximately, in the regime $T \gtrsim T_F$ where T is the sole energy scale and quantum degeneracy effects are suppressed; all fits are therefore restricted to this window. The consistency between independent channels is strong evidence that a single scaling dimension governs the system within this regime, though we stress that this constitutes a test of the framework under approximate rather than exact scale invariance

The open-system side of the theorem is tested independently by Sun *et al.* [10], who engineer baths with power-law spectral densities $J(\omega) \propto \omega^s$ in a trapped-ion platform and directly measure the decoherence of a coupled spin (Fig. 2, right). This is a direct experimental probe of the consistency relation $s + \gamma = 2$ (Eq. (4)), independent of any assumption about the microscopic origin of the bath. For $s = 0.5$ and $s = 1.0$ we extract $s+\gamma = 1.80 \pm 0.19$ and 1.97 ± 0.03, in good agreement with the prediction. The $s = 2$ case ($d_{\mathcal{U}} = 5/2$) is the marginal dimension where asymptotic power-law scaling gives way to logarithmic growth; finite bandwidth of the engineered bath accelerates this crossover, and the extracted effective exponent should be interpreted as pre-asymptotic rather than a violation of the scaling relation. We note that these baths are not themselves scale-invariant — $J(\omega) \propto \omega^s$ holds only over a narrow bandwidth — so this test probes the mathematical structure of the framework under approximate conditions. Together with the Fermi gas results—which test the theorem through he properties of the bath itself—, the spin-boson results test it through the response of the coupled system, constituting the first two-sided experimental test of the universality theorem.

Beyond exponent matching, a genuine CFT bath is subject to an exact constraint from comformal symmetry: the stress tensor must be tracelessness ($T^\mu_\mu = 0$), which implies vanishing bulk viscosity $\zeta = 0$ [11]—a prediction that no phenomenological power-law bath carries. A bath engineered to reproduce $J(\omega) \propto \omega^{1/2}$ matches the power-law behaviour but is free to have any ζ. Elliott *et al.* [12] measure $\zeta = 0.005(16)\,\hbar n$, consistent with *exactly* zero. This measurement directly tests whether the unitary Fermi gas is a genuine CFT bath rather than a power-law mimic—and confirms that it is.

Three Physical Realizations

The unparticle dimension $d_{\mathcal{U}}$ can be derived from first principles for any conformal field theory (CFT). The procedure is systematic: identify the CFT describing the environment, determine the scaling dimension Δ of the coupled operator, apply Eq. (3), and read off all exponents from Table 1. Three realizations spanning 25 orders of magnitude in energy illustrate the universality.

Quantum Ising criticality (millikelvin scales). The two-dimensional quantum Ising CFT has two relevant primary operators that couple to external probes: the spin σ ($\Delta = 1/8$) and the energy density ε ($\Delta = 1$). For a probe spin coupled to the energy operator of the $(2+1)$-dimensional Ising model ($d = 2$):

$$d_{\mathcal{U}} = 1 - \tfrac{2-2}{2} = 1, \quad J(\omega) \propto \omega^{-1}. \tag{8}$$

This is the $1/f$ noise spectrum, providing a field-theoretic explanation for its prevalence near two-dimensional quan-

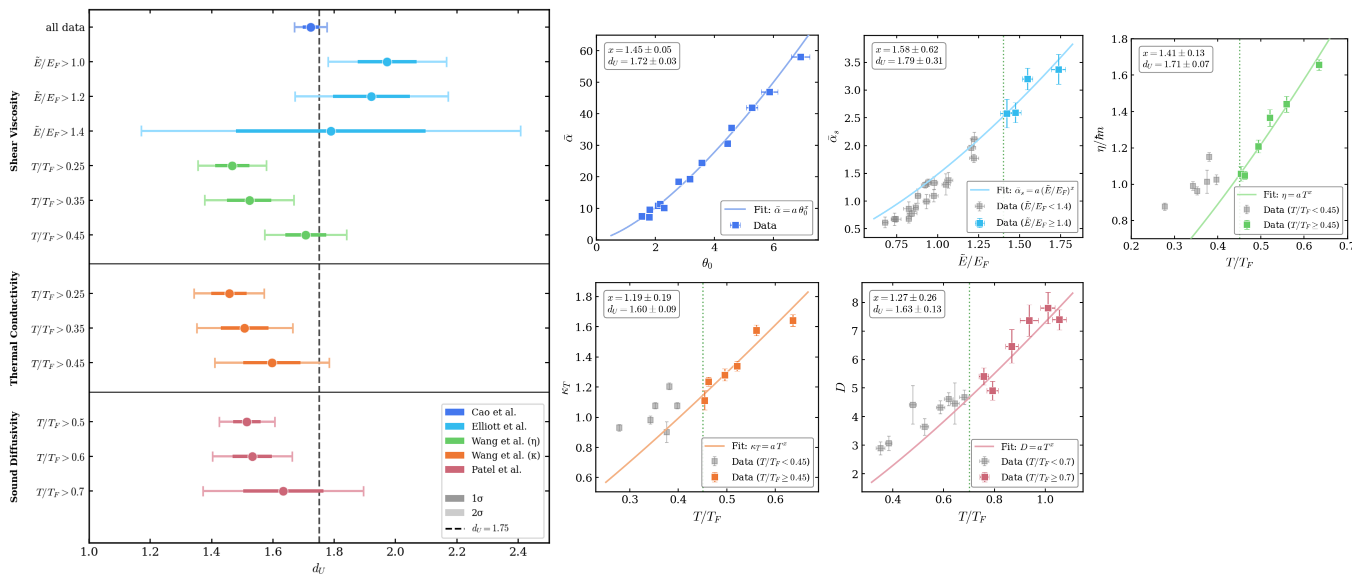

Figure 1: **Bath-side experimental validation.** Consistency check in the unitary Fermi gas. Extracted values of $d_{\mathcal{U}}$ from shear viscosity (Cao *et al.* [7], Elliott *et al.* [12], Wang *et al.* [8]), thermal conductivity (Wang *et al.* [8]), and sound diffusivity (Patel *et al.* [9]), shown under progressively conservative high-T cuts. Shear viscosity and thermal conductivity probe independent Green's functions ($G^R_{T_{xy}}$ and $G^R_{J_E J_E}$). The dashed line marks $d_{\mathcal{U}} = 7/4$.

tum critical points [13]. For the $(1+1)$-dimensional chain coupled to ε: $d_{\mathcal{U}} = 3/2$, predicting quadratic decoherence growth $\Gamma_{\text{decoh}} \propto t^2$, testable in trapped-ion quantum simulators via Ramsey interferometry.

Inflationary cosmology (10^{13} GeV scales). In the exponentially expanding spacetime of inflation—well approximated by a $3+1$ de Sitter geometry—a massless scalar field has scaling dimension $\Delta = 3/2$ with respect to de Sitter isometries, which differ from the flat-space conformal group, so the relation (3) must be derived anew in this geometry; the result is

$$d_{\mathcal{U}} = 2, \quad J(\omega) \propto \omega. \tag{9}$$

This is exactly Ohmic—meaning the noise spectrum is linear in frequency— predicting linear decoherence growth $\Gamma_{\text{decoh}} \propto Ht$, in agreement with established results on the quantum-to-classical transition during inflation [14, 15]. De Sitter isometries enforce scale invariance even in the thermal Bunch-Davies vacuum, making inflation the most robust application of the framework. Deviations from $d_{\mathcal{U}} = 2$ signal massive fields or exotic couplings and are constrained by CMB measurements of the primordial spectral index.

High-energy astrophysical neutrinos (TeV–PeV scales). If high-energy astrophysical neutrinos detected by IceCube couple to a scale-invariant sector—quantum gravity, a conformal fixed point, or other beyond-Standard-Model physics—the decoherence rate per unit distance is

$$\frac{\Gamma_{\text{decoh}}}{L} \sim \frac{g^2}{M_*^2}\, E^{2d_{\mathcal{U}}-3}, \tag{10}$$

where M_* is the energy scale suppressing the interactions—for instance the Planck scale for quantum gravity scenarios— and g is a dimensionless coupling. The energy dependence is a direct imprint of $d_{\mathcal{U}}$, making astrophysical neutrino data a powerful probe of scale-invariant new physics.

IceCube neutrinos with $E \sim 10^{12}$–10^{15} eV have energies far exceeding the temperature of any scale-invariant sector thermalized with the diffuse cosmological or intergalactic environment ($T_{\mathcal{U}} \lesssim$ keV). The relevant comparison is E versus $T_{\mathcal{U}}$, the effective temperature of the scale-invariant bath itself, not that of Standard Model backgrounds such as the CMB or the cosmic neutrino background. In this regime, thermal occupation numbers of bath modes at $\omega \sim E$ are exponentially suppressed, and the decoherence rate is dominated by its vacuum contribution, providing a clean imprint of $d_{\mathcal{U}}$ free of thermal contamination. For scale-invariant sectors localized near energetic astrophysical sources such as AGN or GRBs, $T_{\mathcal{U}}$ is a priori unspecified and the vacuum approximation must be assessed case by case.

Decoherence Phase Transition

A striking consequence of the framework is a *decoherence phase transition* at $d_{\mathcal{U}} = 5/2$. For $d_{\mathcal{U}} < 5/2$, the decoherence exponent $\gamma = 5 - 2d_{\mathcal{U}} > 0$ and coherence is irreversibly lost. At $d_{\mathcal{U}} = 5/2$, $\gamma = 0$ and decoherence

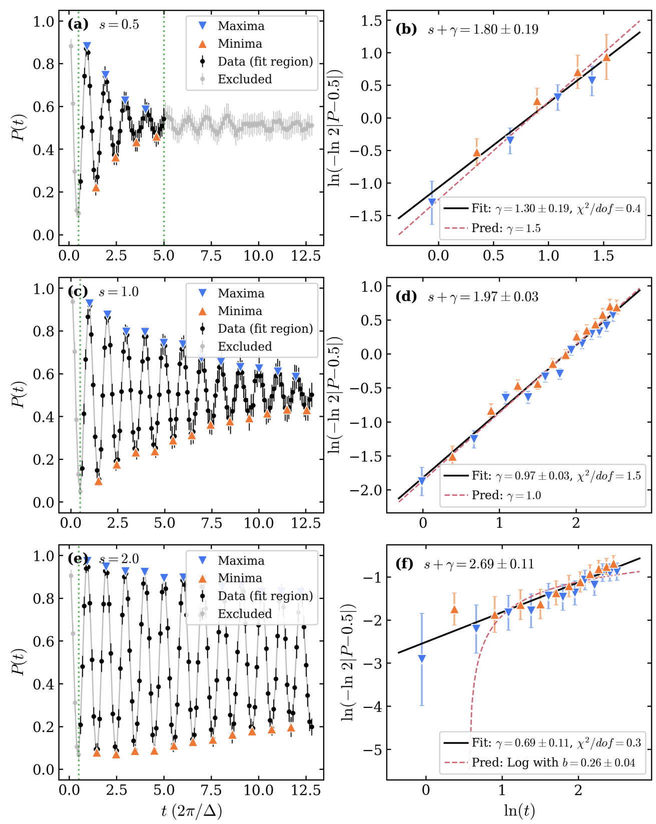

Figure 2: **Open-system-side validation.** Engineered spin-boson baths (Sun *et al.* [10]). Coherence decay envelopes for spectral exponents $s = 0.5, 1.0, 2.0$ and corresponding linear fits extracting γ, testing $s + \gamma = 2$. Agreement is good for $s = 0.5$ (1.80 ± 0.19) and $s = 1.0$ (1.97 ± 0.03). The $s = 2$ case corresponds to the marginal dimension $d_{\mathcal{U}} = 5/2$, where finite bandwidth induces a crossover before the asymptotic regime.

grows only logarithmically. For $d_{\mathcal{U}} > 5/2$, $\gamma < 0$: the decoherence functional *decreases* with time, and quantum coherence is protected at long times.

This phenomenon is impossible in any memoryless (Markovian) description: no standard master equation can produce a decoherence functional that saturates or reverses. Its physical origin is the extremely short correlation time of super-Ohmic baths—high-frequency modes dominate but their rapid oscillations average to zero on the system's timescale, causing the bath to effectively decouple at late times. The transition is equivalent to the result of Ref. [16] that coherence survives for $s > 2$ in the spin-boson model, here derived from first principles as a consequence of conformal symmetry rather than a model-specific calculation.

Discussion

The unparticle framework does not merely provide a convenient parametrization of scale-invariant decoherence: it is its unique characterization. This elevates the role of $d_{\mathcal{U}}$ to that of an order parameter for a universality class of non-equilibrium open quantum systems—analogous to how the central charge c classifies conformal field theories at equilibrium.

The consistency relations (4)–(6) are the primary experimental handles. An experiment that measures, for instance, the noise spectrum and the decoherence rate independently extracts two values of $d_{\mathcal{U}}$ and tests their consistency. Inconsistency is not merely a failure of the model but a diagnostic: it signals specific physics beyond scale invariance—a UV cutoff, a mass gap, non-locality, or multiple competing sectors—and points toward what new physics is present.

The full fluctuation-dissipation structure that distinguishes a genuine CFT bath from a power-law mimic requires the simultaneous absence of a UV cutoff function across the scaling window. This is testable in the unitary Fermi gas by verifying that the thermal equilibrium relation $\tilde{\nu}(\omega)/\tilde{\eta}(\omega) = \coth(\hbar\omega/2k_BT)$—known as the KMS relation— holds with no measurable rolloff over the frequency range $\omega \sim k_BT/\hbar$, where the quantum structure of the coth factor is operative. We encourage experimentalists to pursue this measurement as the definitive test of CFT bath identity.

Three interconnected directions call for further development. First, the connection to holography: AdS/CFT maps strongly coupled CFTs to gravitational theories, and the unparticle bath description of near-horizon fluctuations may provide a novel link between quantum information and black hole physics [17]. Second, strange metals and spin liquids near quantum critical points [18] are natural condensed matter realizations where the framework's predictions have not yet been systematically confronted with data. Third, the extension to gravitational open quantum systems, where the environment itself may exhibit approximate scale invariance through asymptotic safety, remains largely unexplored.

Across all these settings, the unparticle language is not exotic speculation but the natural effective description dictated by symmetry—as inevitable, given its assumptions, as thermodynamics is for equilibrium systems.

Methods

Proof of Theorem 1. We prove the four conclusions in sequence.

Step 1: Scale invariance implies conformal invariance. In a relativistic quantum field theory, the energy-momentum tensor $T^{\mu\nu}$ satisfies $\partial_\mu T^{\mu\nu} = 0$ from translation invariance. Scale invariance under $x^\mu \to \lambda x^\mu$ requires tracelessness: $T^\mu_{\ \mu} = 0$. These two conditions, together with Poincaré invariance, imply invariance under the full conformal group $SO(d+1,1)$ [4], which includes translations, rotations, dilatations, and special conformal transformations. The argument fails for two classes of exception, which we exclude by assumption: theories with exactly marginal operators (beta function identically zero, not just approximately), and theories with only discrete scale invariance. For theories with continuous scale invariance, full conformal invariance follows.

Step 2: CFT two-point functions are fixed by Ward identities. Let $\mathcal{O}_E$ be a primary operator of the CFT with scaling dimension Δ. Under $x \to \lambda x$ it transforms as $\mathcal{O}_E(\lambda x) = \lambda^{-\Delta}\mathcal{O}_E(x)$. Conformal Ward identities fix the Euclidean two-point function up to a single normalization constant [19]:

$$\langle \mathcal{O}_E(x)\, \mathcal{O}_E(0)\rangle_{\mathrm{E}} = \frac{C_{\mathcal{O}}}{(x_E^2)^{\Delta}}, \tag{11}$$

where $x_E^2 = \sum_\mu (x^\mu)^2$. This is the unique functional form consistent with conformal symmetry; no alternative is possible for a primary operator.

Step 3: Spectral density from analytic continuation. Continuing to Lorentzian signature via $\tau = it - \epsilon$ (Feynman prescription):

$$\langle \mathcal{O}_E(x,t)\, \mathcal{O}_E(0,0)\rangle_{\mathrm{L}} = \frac{C_{\mathcal{O}}}{(-t^2 + \mathbf{x}^2 + i\epsilon)^{\Delta}}. \tag{12}$$

The retarded Green's function at zero spatial momentum ($\mathbf{k} = 0$), appropriate for a local coupling, is

$$G_R(\omega, \mathbf{k} = 0) \propto (-i\omega)^{2\Delta - d}. \tag{13}$$

Dimensional analysis fixes this form; the only freedom is an overall constant. The spectral density $J(\omega) = -2\,\mathrm{Im}[G_R(\omega)]$ then gives

$$J(\omega) \propto \omega^{2\Delta - d - 1}, \qquad \omega > 0, \tag{14}$$

which is Eq. (1).

Step 4: Identification with the unparticle form. The unparticle spectral density [1] is $\rho_{\mathcal{U}}(\omega) \propto \omega^{2d_{\mathcal{U}}-3}$. Matching exponents with Eq. (14):

$$2d_{\mathcal{U}} - 3 = 2\Delta - d - 1 \quad \Rightarrow \quad d_{\mathcal{U}} = \Delta - \tfrac{d-2}{2}, \tag{15}$$

which is Eq. (3). The dynamical exponents in Table 1 follow by Fourier transformation of the memory kernels

Memory kernels and scaling exponents. Tracing out the environmental degrees of freedom in the path integral yields a non-Markovian master equation whose dissipation and noise kernels are the imaginary and real parts of the retarded Green's function, respectively. Substituting $J(\omega) \propto \omega^{2\Delta-d-1}$ and applying the Fourier identity

$$\int_0^\infty d\omega\, \omega^\mu \sin(\omega t) = \Gamma(\mu+1) \sin\!\Big[\tfrac{\pi(\mu+1)}{2}\Big]\, t^{-(\mu+1)}, \tag{16}$$

gives the dissipation kernel $\eta(t) \propto t^{-(2d_{\mathcal{U}}-2)}$ and, in the high-temperature limit ($t \gg \beta \equiv 1/T$), the noise kernel $\nu(t) \propto T\, t^{-(2d_{\mathcal{U}}-3)}$. Successive time integration yields the damping and decoherence functionals:

$$\Gamma_{\mathrm{damp}}(t) \propto t^{3-2d_{\mathcal{U}}}, \tag{17}$$

$$\Gamma_{\mathrm{decoh}}(t) \propto t^{5-2d_{\mathcal{U}}}, \tag{18}$$

establishing all entries of Table 1. The consistency relations (4)–(6) are algebraic identities among these exponents and require no further proof. The exact noise kernel valid for all t and T via Matsubara summation.

Loopholes. Five classes of physical deviation from the theorem's assumptions are identified. *(i) Approximate scale invariance*: infrared cutoffs (finite temperature T, system size L, mass gap m) and ultraviolet cutoffs (lattice spacing, Planck length) restrict the unparticle description to the scaling window $\omega_{\mathrm{IR}} < \omega < \omega_{\mathrm{UV}}$; outside this window the spectral density acquires cutoff functions $f_{\mathrm{IR}}(\omega/\omega_{\mathrm{IR}})$ and $f_{\mathrm{UV}}(\omega/\omega_{\mathrm{UV}})$ approaching unity in the scaling regime. *(ii) Non-local coupling*: a coupling kernel $K(x-y)$ introduces a momentum-dependent form factor modifying the pure power law. *(iii) Discrete scale invariance*: invariance only under $x \to \lambda_0^n x$ (e.g., Efimov states) produces log-periodic modulations $J(\omega) \propto \omega^s[1 + A\cos(b\ln\omega + \varphi)]$. *(iv) Multiple competing sectors*: if the environment contains several unparticle sectors with dimensions $d_{\mathcal{U}}^{(i)}$, the total spectral density is $J_{\mathrm{tot}}(\omega) = \sum_i A_i\, \omega^{2d_{\mathcal{U}}^{(i)}-3}$, producing observable crossovers between sectors. *(v) Quantum anomalies*: classical scale invariance may be broken by quantum effects (e.g., the trace anomaly), and the unparticle description applies only above the anomaly-generated scale. In all cases, thermal corrections to the noise kernel modify the amplitude but not the scaling exponents: both the vacuum ($t \ll \beta$) and thermal ($t \gg \beta$) regimes exhibit power-law behavior with the same $d_{\mathcal{U}}$, differing only in prefactor.

Contrapositive (no-go theorem). The logical contrapositive of the theorem provides an experimental diagnostic: if the independently measured exponents s and γ_{decoh} are inconsistent with any single real value of $d_{\mathcal{U}}$ (i.e., $s + \gamma_{\mathrm{decoh}} \neq 2$), then at least one of the following holds: the environment is not scale-invariant; the coupling is non-local; Lorentz invariance or unitarity is violated; or multiple competing sectors are present. Inconsistency is therefore not a failure of the model but a diagnostic pointing toward specific new physics.

Stress-tensor channels and extraction of $d_{\mathcal{U}}$. In a conformal fluid, the retarded Green's function of the stress tensor $G^R_{T_{xy}}(\omega) \propto (-i\omega)^{2d_{\mathcal{U}}-1}$ determines the shear viscosity via the Kubo relation

$$\eta = \lim_{\omega\to 0} \frac{1}{\omega}\, \mathrm{Im}\, G^R_{T_{xy}}(\omega), \tag{19}$$

giving $\eta(T) \propto T^{2d_{\mathcal{U}}-2}$ by dimensional analysis. Similarly, thermal conductivity is related to the energy-current correlator $G^R_{J_E J_E}$ via an analogous Kubo relation, giving $\kappa(T) \propto T^{2d_{\mathcal{U}}-2}$. The two channels are genuinely independent: they couple to distinct components of the stress tensor and are measured in separate experiments. The unparticle dimension is extracted from power-law fits $X(T) = a\,(T/T_F)^x$ via

$$d_{\mathcal{U}} = \frac{x+2}{2}, \tag{20}$$

which follows directly from matching the temperature scaling to the CFT prediction $\eta(T) \propto T^{2d_{\mathcal{U}}-2}$.

Unitary Fermi gas fits. Power-law fits to transport data follow the form $X(T) = a\,(T/T_F)^x$, with $d_{\mathcal{U}} = (x+2)/2$ for shear viscosity and thermal conductivity. For Cao *et al.* [7] we fit the dimensionless viscosity coefficient $\bar{\alpha}$ as a function of the reduced temperature parameter θ_0, using all data in the conformal window $E \geq 2.3\,E_F$. For Wang *et al.* [8] and Patel *et al.* [9] we apply conservative high-T cuts ($T/T_F \geq 0.45$ and 0.75 respectively) to restrict to the approximately scale-invariant regime. Uncertainties are propagated from the original experimental error bars via orthogonal distance regression.

Decoherence exponent extraction (Sun et al.). Coherence decay envelopes are extracted from donor-population traces by identifying oscillation extrema $A(t) = |P_{\text{peak}}(t) - 1/2|$ and fitting to a stretched exponential $A(t) = \frac{1}{2}\exp(-Ct^{\gamma})$ in the region $t > 0.5\,(2\pi/\Delta)$, removing the initial transient. The exponent γ is extracted from the slope of $\ln(-\ln 2A)$ versus $\ln t$.[3]

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[3] This method relies on the approximation that the exponential envelope varies much more slowly than the oscillations. To check the validity of the approximation, we also perform a full non-linear fit of the data to $P(t) = \frac{1}{2} + \frac{1}{2}e^{-Ct^{\gamma}}\cos(\omega t + \phi)$. With this method, we obtain results that are compatible with those of the linear fit. In the $s = 0.5$ case, where the approximation starts being imperfect, the non-linear fit method becomes more precise and yields $\gamma = 1.41 \pm 0.14$, which has a smaller error and is closer to the predicted value of $\gamma = 1.5$.

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