An Open System Approach to Cosmology

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Acknowledgments

- Based on JHEP 10 (2024) 248 (2404.15416), JHEP 03 (2025) 138 (2412.12299) and 2507.03103. In collaboration with and

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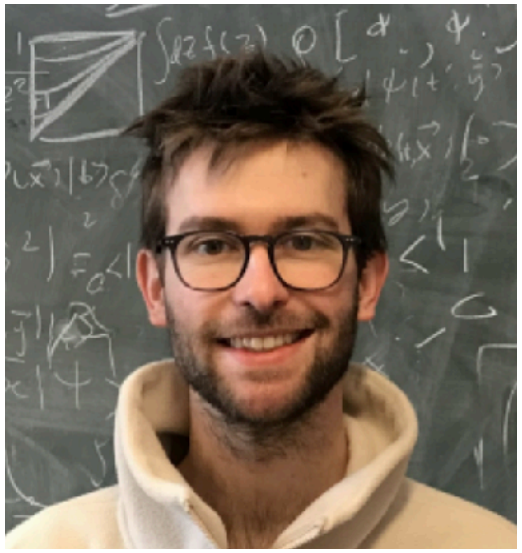

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- Two more papers on Open dark energy and counting degrees of freedom to appear

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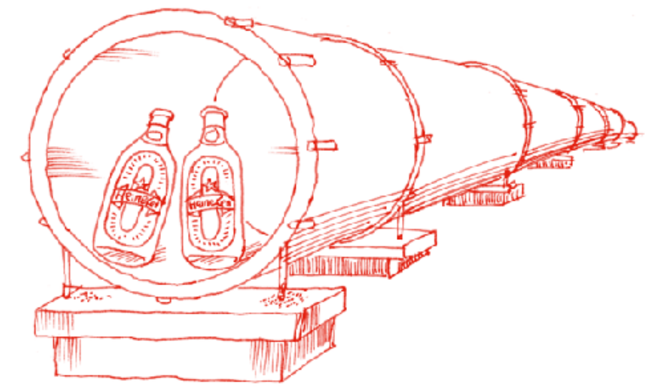
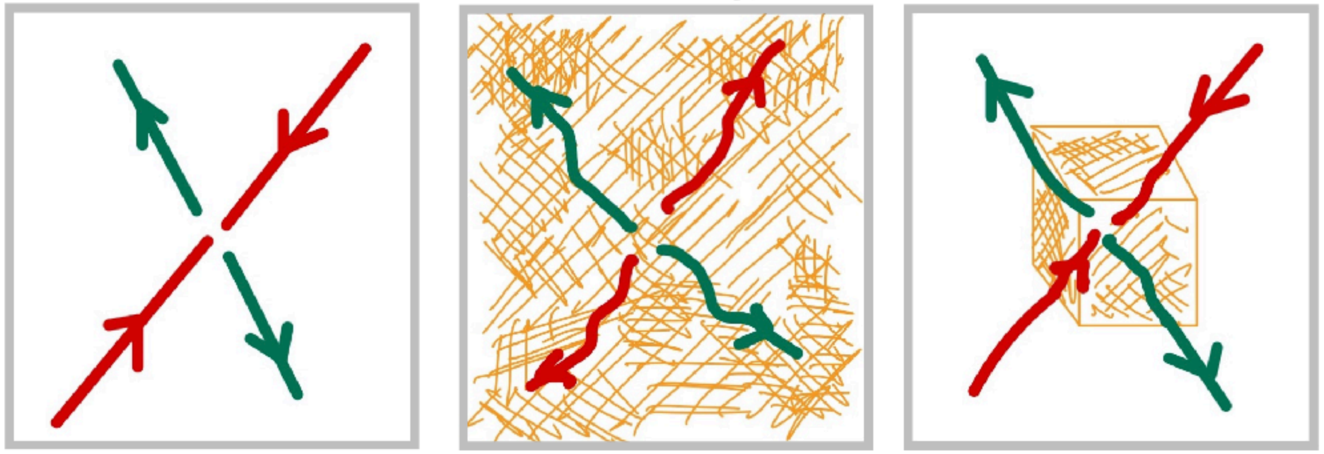
Open problems



- Major open problems in gravity/cosmology: inflation, dark matter and dark energy
- What do these problems have in common?

1. They all involve a *space-time filling medium*. Contrast this with:
   - particle physics where engineers spend years to make sure centre of detector is clean an empty. Cosmo cares about different states
   - condensed matter, where we control the “sample”. We can move it, cut it, heat it, radiate it, etc. Cosmo cares about different (curved) asymptotic
2. The microscopic description of the medium is *unknown*:
   - This is in contrast with condensed matter and QCD where microphysics is known but intractable
3. We can only probe the medium *gravitationally*. All of our measurements and constraints exist only at finite G_N, in the presence of dynamical gravity

An open system approach



- We want to describe gravity ($g_{\mu\nu}$) interacting with an unknown medium (inflation/dark matter/dark energy). Toolkit from particle physics is inadequate, this is an Open System.

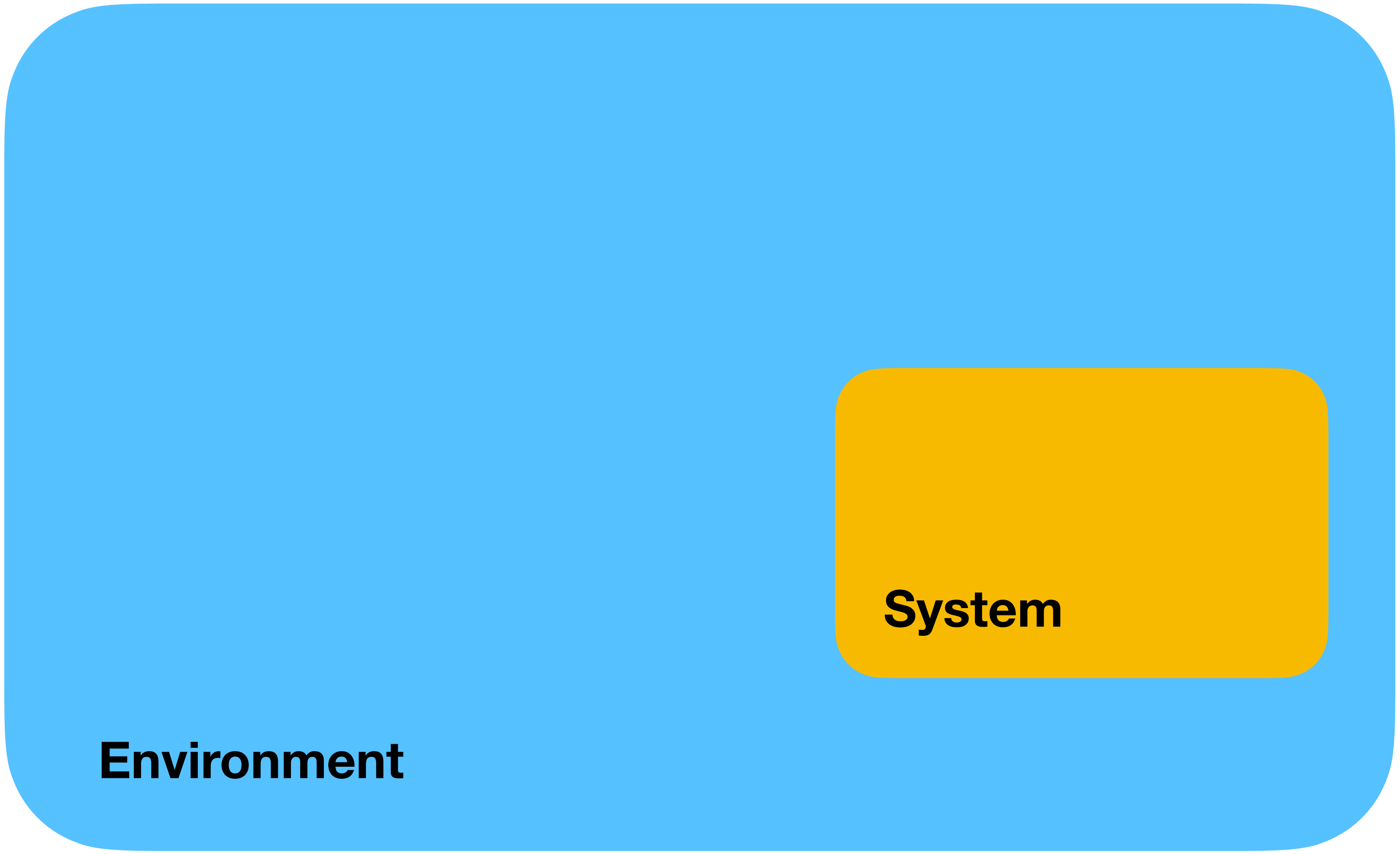
- We want to specify *coarse-grained properties* of the medium: symmetries, local density/pressure, background time dependence.

- Since we don't know the microscopic, we want an Effective Field Theory (EFT)

- Goal: develop Open System EFTs for gravity and cosmology.

- Observables: $g_{\mu\nu}(\mathbf{x}, t)$ and its correlators

A brief intro to Open Systems



A toy model

- Let's start with a *heuristic* introduction to open systems at the classical level
- Open system: Let ϕ be the deg. of freedom of a system that is in contact with a homogenous, isotropic and stationary environment
- Local EFT: assume a separation of scale, i.e. the environment has d.o.f. only at high frequency/wavenumber so the effective dynamics of ϕ is *local*
- Prototypical dynamics is captured by the *Langevin equation*

  $$\ddot{\phi}+\Gamma\dot{\phi}+k^2\phi + V'(\phi) = \xi$$

  where $\Gamma\dot{\phi}$ represents dissipation (to leading order in derivatives).

- ξ is a stochastic variable that models noise fluctuations of the environment. It's specified through its correlation functions, $\langle \xi(x)^n \rangle$. For example

$$\langle \xi(x)\xi(y) \rangle = \beta_1 \delta^{(4)}(x-y) + \beta_2 \partial_x^2 \delta^{(4)}(x-y) + \dots$$

Open Effective Action

- It is useful to derive the Eq.o.M. from an action principle (conserved currents/charges from symmetries, generalises to Quantum Mechanics). Usual actions cannot do it.

- The trick is to double the fields, $\phi(x) \to \phi_+(x)\,,\phi_-(x)$, and define an *Open functional*
$$S[\phi_+, \phi_-] = \int L(\phi_+) - L(\phi_-) + F(\phi_+, \phi_-)$$
where $L(\phi)$ is the usual action and $F(\phi_+, \phi_-)$ is the *Feynman-Vernon influence functional.* This encodes all dissipative and noise phenomena.

- Perform the Keldysh rotation to "*retarded*" and "*advanced*" fields
$\phi_r = \frac{1}{2}(\phi_+ + \phi_-)$ and $\phi_a = \phi_+ - \phi_-$

- Minimise S w.r.t. advanced fields to find the Eq.o.M (noise from HS transform)

$$S = \int (\partial_\mu \phi_a \partial^\mu \phi_r) + \Gamma \phi_r \dot{\phi}_a + \beta \phi_a^2 \Rightarrow \frac{\delta S}{\delta \phi_a}\Big|_{\phi_a=0} = \ddot{\phi} + k^2\phi + \Gamma \dot{\phi} = \xi$$

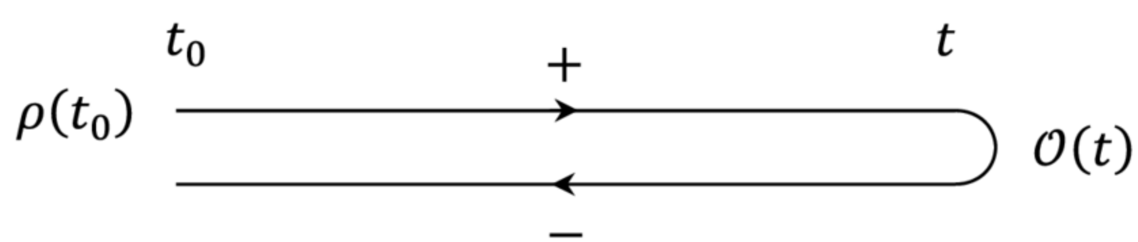
Open Quantum Systems

- For a more precise derivation consider Quant. Mech. with Hilbert space $\mathscr{H}$. Any linear *density operator* ρ satisfying
$$\rho^\dagger = \rho\,, \quad \rho \geq 0\,, \quad \text{Tr}\rho = 1$$
defines a state of the system, which is pure iff $\text{Tr}\rho^2 = 1$ otherwise its *a mixed state*.

- Expectation values of operators are
$$\langle \phi^n(t) \rangle = \text{Tr}[\rho(t)\phi^n(t)] = \text{Tr}[U(t,t_i)\rho_i U^\dagger(t,t_i)\phi^n(t)]$$
$$= \int_{\phi_i \phi_i' \phi} \rho_i(\phi_i, \phi_i')\langle \phi_i' \,|\, U \,|\, \phi \rangle \phi^n \langle \phi \,|\, U^\dagger \,|\, \phi_i \rangle$$
where the two time evolutions $U(t,t_i)$ and $U^\dagger(t,t_i)$ can be written as a path integral with a forward and a backward branch

The closed-time contour

- This leads to the following path integral
$$\langle \phi^n \rangle = \int d\phi \int_{I.C.}^{\phi} D\Phi_+ \int_{I.C.}^{\phi} D\Phi\_ e^{iS[\Phi_+]} e^{-iS[\Phi_-]} \Phi^n$$
which goes by the many names *Schwinger-Keldysh formalism*, Closed-time path, in-in formalism



- The initial conditions "I.C." can be any initial density matrix ρ_i. For us this will always be a pure state (e.g. Bunch-Davies).

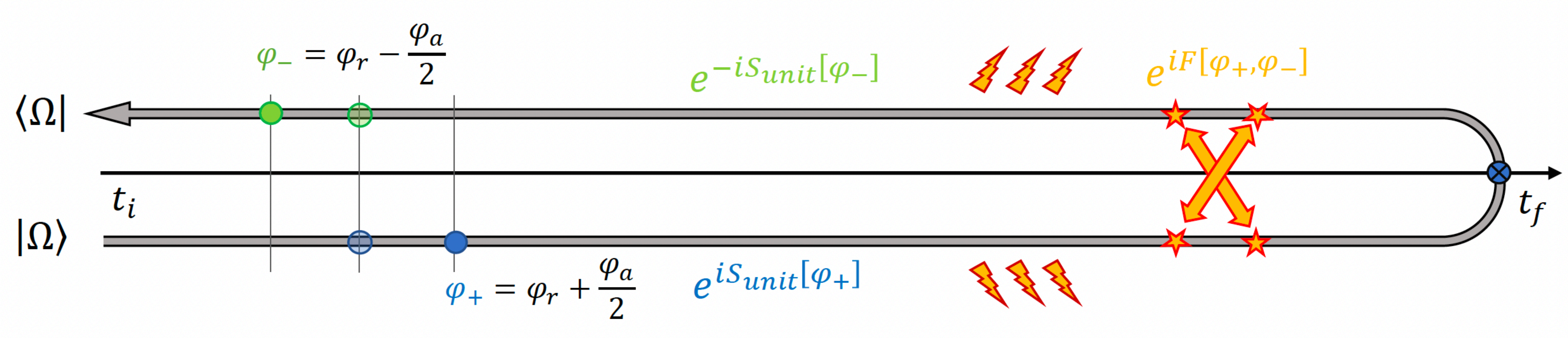
- The SK path integral prepares and evolves the density matrix. It's the functional analog of the master equation in the operator formalism

Integrating out the environment

- To get an *open* system we have to integrate out an environment. Split $\Phi(x)$ into a system $\pi(x)$ and an environment $\sigma(x)$.

- Integrating out (a.k.a. "tracing out") $\sigma_\pm$ changes the action $S[\pi, \sigma]$ into an open effective functional and crucially induces a Feynman-Vernon influence functional
$$\int D\sigma_\pm D\pi_\pm e^{iS[\pi_+,\sigma_+]-iS[\pi_-,\sigma_-]} = \int D\pi_\pm e^{iS_{\text{eff}}[\pi_+,\pi_-]}$$

- The new *open effective functional (a.k.a. "Open action")* $S_{\text{eff}}[\pi_+, \pi_-]$ now describes non-Hamiltonian, dissipative, non-conservative evolution of the system



Consistency

- We want our open EFT to come from a unitary "closed" UV theory. Does a given S_{eff} obey this constraint?? No

- A necessary set of conditions is that $\text{Tr}(\rho) = 1$, $\rho = \rho^\dagger$ and $\rho \geq 0$. Since the SK path integral prepares ρ, this leads to *unitarity constraints*

$$S_{\text{eff}}[\pi_+, \pi_+] = 0 \qquad S_{\text{eff}}[\pi_r, \pi_a = 0] = 0\,,$$

$$S_{\text{eff}}[\pi_+, \pi_-] = -S^*_{\text{eff}}[\pi_-, \pi_+] \qquad S_{\text{eff}}[\pi_r, \pi_a] = -S^*_{\text{eff}}[\pi_r, -\pi_a]$$

$$\Im\text{m}\, S_{\text{eff}}[\pi_+, \pi_-] \geq 0 \qquad \Im\text{m}\, S_{\text{eff}}[\pi_r, \pi_a] \geq 0.$$

- Hence S_{eff} starts linear in π_a

- Note that S_{eff} now has imaginary coupling constants, namely every interactions that is even in π_a, such as $S_{\text{eff}} \supset \int i\beta\pi_a^2$

Open EFT of inflation

[Agui Salcedo, Colas, E.P. '24]

- Idea: use the principles of Effective Field Theory (EFT) to build large classes of bottom-up open quantum systems for inflation (see [Bereira 90s; Boyanovsky; Holman; Burgess; Lopez-Nacir, Porto, Senatore, Zaldarriaga 11; Hong et al 19] for seminal work on this)

- EFT rules:

  1. Identify the low-energy *deg. of freedom*

  2. Choose *symmetries* & principles (e.g. locality). Write down the most generic symmetric action (∞ possibilities)

  3. Choose a radiatively-stable *power counting* and truncate EFT to the desired precision with a *finite number* of operators

Symmetries and fundamental principles

- Let's build an Open EFT for a single deg. of freedom $\pi(x)$, interpreted as the scalar Nambu-Goldstone boson of broke time translation in cosmology, as in the decoupling limit of the EFT of Inflation [Cheung et al '07]

- In practice π is the adiabatic mode that source all structures in the Universe

- Fields: Double the fields
$$\pi_r = \frac{\pi_+ + \pi_-}{2} \quad \text{and} \quad \pi_a = \pi_+ - \pi_- \Leftrightarrow \pi_\pm = \pi_r \pm \frac{1}{2}\pi_a \,.$$

- Principles: *unitarity* constraints and *local* interactions

- *Global symmetries* are doubled but off-diagonal combination is broken (see [Hongo, Kim , Noumi & Ota 19; Akyuz, Goon & Penco 23; Firat et al 25]). π_r is a goldstone, not π_a

$$\pi_r(x) \to \pi_r(\Lambda x + \epsilon) + \epsilon^0 + (\Lambda x)^0 - t \,, \qquad \pi_a(x) \to \pi_a(\Lambda x + \epsilon)$$

Free theory

- The free theory of open inflationary perturbations up to two derivatives is (see also [Lopez-Nacir et al '11])

$$S_{\rm eff}^{(2)} = \int a^2 \pi_r' \pi_a' - c_s^2 a^2 \partial_i \pi_r \partial^i \pi_a - a^3 \gamma \pi_r' \pi_a + i \left[\beta_1 a^4 \pi_a^2 + \beta_2 a^2 \left(\partial_i \pi_a \right)^2 \right]$$

- Black: Conservative dynamics, same as in the EFT of I
- Dissipation γ and fluctuations β_i
- Three 2-pt functions: the Keldysh propagator G_K capturing the effect of noise and giving the *power spectrum* $\langle \pi_r(x) \pi_r(t) \rangle = - i G_K(x, y)$
- the retarded/advanced propagators $G_{R,A}$ $\theta(t-t')\langle [\pi(x), \pi(x')] \rangle = G_R(x, x') = G_A(x', x)$ These are independent of the state of the system
- π_a does not propagate

Dissipative & Warm inflation

- By scale invariance $P(k) \propto k^{-3}$. Direct calculation gives

$$P(k) = \frac{2\pi^2 \Delta_\zeta^2(k)}{k^3} = \frac{1}{4k^3} \frac{\beta_1}{H^2} \frac{H^4}{f_\pi^4} 2^{2\nu_\gamma} \frac{\Gamma\left(\nu_\gamma - 1\right) \Gamma\left(\nu_\gamma\right)^2}{\Gamma\left(\nu_\gamma - \frac{1}{2}\right) \Gamma\left(2\nu_\gamma - \frac{1}{2}\right)}$$

- The origin of perturbations is classical, not quantum!

- For strong or weak dissipation this simplifies to

$$\Delta_\zeta^2(k) \propto \begin{cases} \frac{\beta_1}{H^2} \frac{H^4}{f_\pi^4} + \mathcal{O}\left(\frac{\gamma}{H}\right), & \gamma \ll H, \\ \frac{\beta_1}{H^2} \frac{H^4}{f_\pi^4} \sqrt{\frac{H}{\gamma}} \left[1 + \mathcal{O}\left(\frac{H}{\gamma}\right)\right], & \gamma \gg H. \end{cases}$$

- If we assume thermal equilibrium $\beta = 2\pi\gamma T$, as in warm inflation [Berera Fang '95; Berera]

Interactions

- To cubic order in fluctuations the interactions are

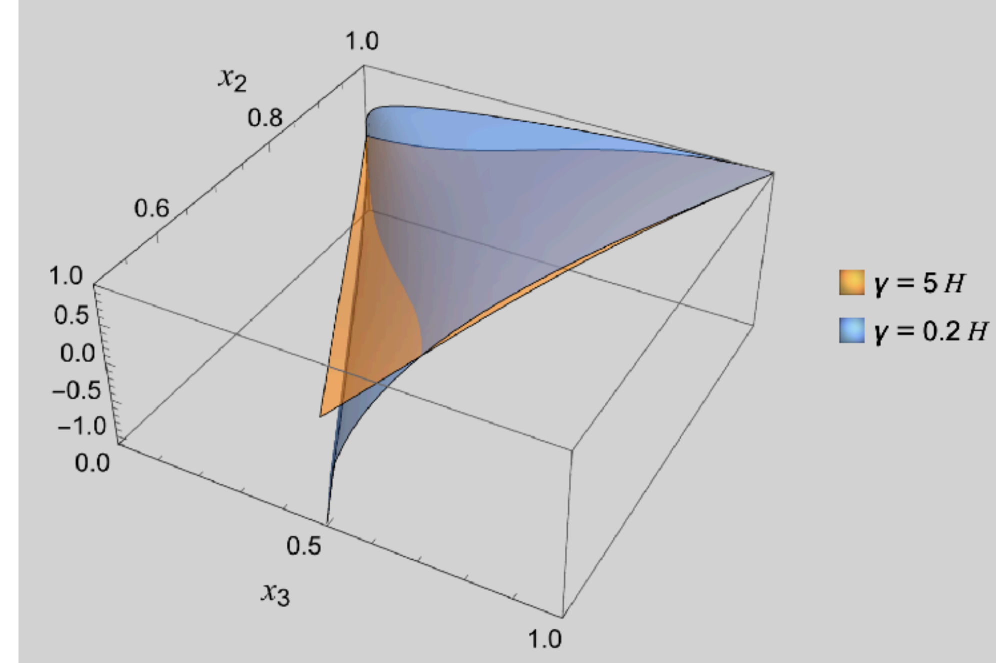
$$S \supset \int \partial\pi_r^2\pi_a + \partial\pi_r^2\partial\pi_a + \partial\pi_r(\pi_a^2 + \pi_a\partial\pi_a + \partial\pi_a^2) + \pi_a^3 + \pi_a^2\partial\pi_a + \pi_a\partial\pi_a^2 + \partial\pi_a^3$$

- Each term gives a different tree-level bispectrum (PNG). Only certain tuned combination correspond to the EFTofI.

- This contains the explicit UV-completion in [Creminelli, Kumar, Salehian & Santoni ’23].

- The EFT relates operators at different orders because of the non-linearly realised boost [Lopez-Nacir et al ’11]. Hence dissipation γ and speed of sound c_s fix the size of some bispectra such as $\gamma\partial_\mu\pi_r^2\pi_a$. So interactions grow with γ and $1/c_s^2$ (as in the closed EFToI)

Primordial non-Gaussianity



- Primordial perturbations appear Gaussian to better than 0.01% precision. But we know gravity is non-linear so there is a gravitational floor of non-Gaussianity 2 orders of magnitude away.
  Non-Gaussianities are probed by the primordial 3-point function, a.k.a. bispectrum $B(k)$

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle \sim \delta(\sum_a \mathbf{k}_a) f_{NL}\, B(k_1, k_2, k_3)$$

- A detection of B for (i) $k_1 \ll k_2 \sim k_3$ implies extra fields; (ii) $k_1 \sim k_2 \sim k_3$ implies self-interactions; (iii) $k_1 + k_2 \sim k_3$ implies Open systems/non-Bunch-Davies

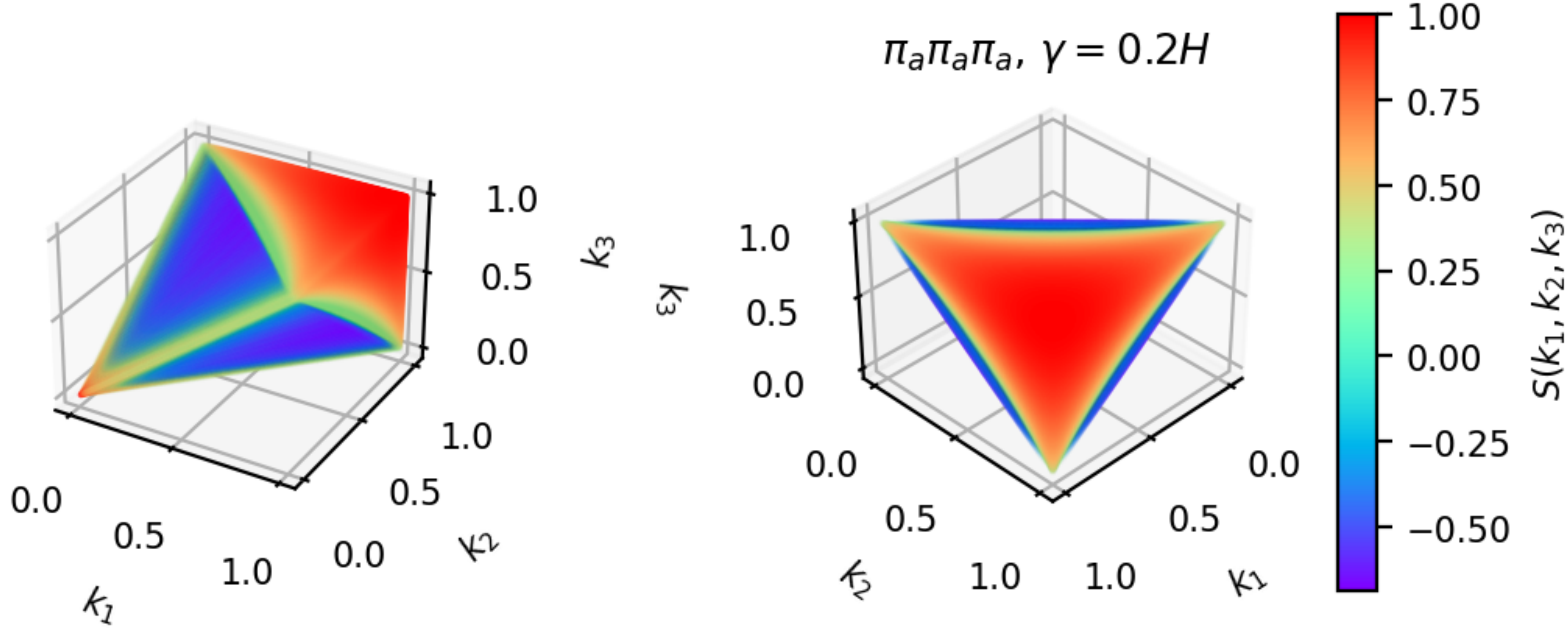
- For strong dissipation $\gamma \gg H$ the bispectrum peaks on *equilateral* configurations $k_1 \sim k_2 \sim k_3$

- For weak dissipation, $\gamma \ll H$, the bispectrum peaks on *folded* configuration $k_i + k_j - k_l \sim 0$.

- dissipation $\gamma \neq 0$ regulates the growth in folded configurations and there are no folded divergences (contrast with non-Bunch-Davies initial states)

multifield | Open system / non Bunch-Davies | self-interactions

CMB data



- The open system bispectrum is being searched for in Planck CMB data [Zhang, Suman, Colas, Salcedo WIP] using the model pipeline [Fergusson, Shellard, …]

- For interaction π_a^3 CMB gives ->

γ/H	$f_{\rm NL}$ (68% CL)
0.10	-19.7 ± 37.3
0.20	-20.8 ± 39.3
0.30	-21.6 ± 41.1
0.40	-22.2 ± 42.6
0.50	-22.5 ± 43.9
0.60	-22.6 ± 45.2
0.70	-22.4 ± 46.2
0.80	-22.1 ± 47.1
0.90	-21.6 ± 47.8

Open Electromagnetism

[Agui Salcedo, Colas, E.P. '24]

Open E&M

- How do we describe Open gauge theories with constraints such as GR? We warm up with E&M
- Deg. of freedom: $a^\mu = A^\mu_+ - A^\mu_-$ (adv) and $A^\mu = \frac{1}{2}(A^\mu_+ + A^\mu_-)$ (ret)
- Gauge transformations: $A^\mu \to A^\mu + \partial^\mu \epsilon$, with a^μ invariant
- Most general free theory to all orders in derivatives (omitting noise for now)

$$S_1 = \int_{\omega,\boldsymbol{k}} \left[a^0 i k_i F^{0i} + a_i \left(\gamma_2 F^{0i} + \gamma_3 i k_j F^{ij} + \gamma_4 \epsilon^i{}_{jl} F^{jl} \right) \right] \equiv \int_{\omega,\boldsymbol{k}} a^\mu M_{\mu\nu} A^\nu ,$$

where $\gamma_a = \gamma_a(\omega, k)$ describe generic higher derivatives terms

- Note that $\det M = 0$ because k^μ is e-vector by gauge invariance
- There is a "*deformed*" *advanced gauge invariance* $a^\mu \to a^\mu + v^\mu$ with A^μ invariant. Explicit calculation shows $v^\mu = (i\gamma_2, \vec{k})$.
- [Kaplanek, Mylova & Tolley '25] found (see Maria's talk) $\mathrm{U}_r(1) \times \mathrm{U}_a(1)$ gauge. In our theory the $\mathrm{U}_a(1)$ appears deformed.

Dynamics

- Coupling a current $S \supset \int a_\mu J^\mu$, Maxwell's equations are (with $\gamma_2 = \Gamma - i\omega$ and $\gamma_3 = -v^2$)

$$\frac{\delta S}{\delta a_\mu} \quad \Rightarrow \quad \partial^\mu F_{\mu\nu} + \delta^i_\nu(\Gamma F_{0i} + \gamma_4 \epsilon_{ijl} F^{jl}) = J_\nu$$

- No ghosts. 2 deg. Of freedom: transverse part of A^μ is dynamical with dispersion relation

$$i\gamma_2\omega + \gamma_3 k^2 + \pm 2\gamma_4 k = 0$$

- To lowest order in derivatives:
  - $\gamma_3 = -v^2$ is the *speed of light* in the medium
  - γ_4 is *birefringence*
  - $\gamma_2 = \Gamma - i\omega$ is *dissipative* propagation giving $A_\perp \propto e^{-\Gamma t}$
- The choice $\gamma_2 = -i\omega$, $\gamma_3 = -c^2$, $\gamma_4 = 0$ is Maxwell in vacuum
- Quantization is easiest in covariant gauges and we derived all dissipative propagators (Keldysh, advanced/retarded) to all orders in derivatives

Noise constraint

- The current $J^\mu = j^\mu + \xi^\mu$ contains an external current j^μ and stochastic noise ξ^μ. *Current conservation* is unfamiliar and gives a *noise constraint*

$$\partial^\mu J_\mu = \Gamma J_0$$

- This looks strange but it is consistent with *total* charge conservation:

$$\partial^\mu F_{\mu i} + \Gamma F_{0i} = J_i\,, \quad \partial^\mu F_{\mu 0} = J_0$$

$$\partial^\mu (\text{E.o.M})_\mu = 0 \quad \Rightarrow \partial^\mu J_\mu = \Gamma J_0$$

- We interpret the ΓF_{0i} term as the expectation value of the charge current of the medium, which in the EFT is a function of $F_{\mu\nu}$. This is key for gravity

Open Gravity

[Agui Salcedo, Colas, (1+Dufner), E.P. ‘25]

Open General Relativity

- We now aim at building the most general dynamics for general relativity in the presence of a medium (see also [Lau, Nishii, Noumi '24] for a nice related construction). More precisely we want: two massless spin-2 deg. of freedom + a minimally-coupled matter sector
- Let $g_{\mu\nu} = \frac{1}{2}(g^{+}_{\mu\nu} + g^{-}_{\mu\nu})$ and $a^{\mu\nu} = g^{\mu\nu}_{+} - g^{\mu\nu}_{-}$
- For today we focus on the classical/deterministic regime, namely an open EFT to *linear* order in the advanced fields $a_{\mu\nu}$
- The open functional is $S = \int \mathrm{EE}_{\mu\nu} a^{\mu\nu}$ where $\mathrm{EE}(g)$ are the Einstein Equations for the retarded metric $g_{\mu\nu}$. What should they be? How can we ensure we have a graviton and no ghosts?
- We organise $\mathrm{EE}_{\mu\nu}$ in a derivative expansion.

Open General Relativity

- (Very) long story (very) short the most general classical/deterministic theory of GR with a medium is (up to ∂^2. Here $K_{\mu\nu}$ = extrinsic curvature, R = Riemann/Ricci tensor)

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \sum_{\ell=0} (g^{00}+1)^\ell \left[M_{\mu\nu,\ell} a^{\mu\nu} + i N_{\mu\nu\rho\sigma,\ell} a^{\mu\nu} a^{\rho\sigma} + \ldots \right]$$

$$M_{00,\ell} = \gamma_1^{tt} + \gamma_2^{tt} K + \gamma_3^{tt} K^2 + \gamma_4^{tt} K_{\alpha\beta} K^{\alpha\beta} + \gamma_5^{tt} \nabla^0 K + \gamma_6^{tt} R + \gamma_7^{tt} R^{00}$$

$$M_{0\mu,\ell} = \gamma_1^{ts} R^0{}_\mu + \gamma_2^{ts} \nabla_\mu K + \gamma_3^{ts} \nabla_\beta K^\beta{}_\mu$$

$$\begin{aligned} M_{\mu\nu,\ell} = {} & g_{\mu\nu} \left(\gamma_1^{ss} + \gamma_2^{ss} K + \gamma_3^{ss} K^2 + \gamma_4^{ss} K_{\alpha\beta} K^{\alpha\beta} + \gamma_5^{ss} \nabla^0 K + \gamma_6^{ss} R + \gamma_7^{ss} R^{00} \right) \\ & + \gamma_8^{ss} K_{\mu\nu} + \gamma_9^{ss} \nabla^0 K_{\mu\nu} + \gamma_{10}^{ss} K_{\mu\alpha} K^\alpha{}_\nu + \gamma_{11}^{ss} K K_{\mu\nu} + \gamma_{12}^{ss} R_{\mu\nu} + \gamma_{13}^{ss} R_\mu{}^0{}_\nu{}^0 \\ & + \gamma_{1,\ell}^{PO} \epsilon_\mu{}^{\alpha\beta 0} \nabla_\alpha K_{\beta\nu} + \gamma_{2,\ell}^{PO} \epsilon_\mu{}^{\alpha\beta 0} R_{\alpha\beta}{}^0{}_\nu \end{aligned}$$

- and similar expressions for the noise matrix $N_{\mu\nu\rho\sigma}$
- Deg. of freedom: two massless spin 2 + 1 scalar. The scalar can be made explicit introducing the Goldsone boson of time translations. Very different from E&M, U(1) vs diffs.
- Very rich pheno. Without additional matter this is the Open EFT of Inflation away from the decoupling limit

Friedmann equations

- Our modified Einstein's equation give new Friedmann equations

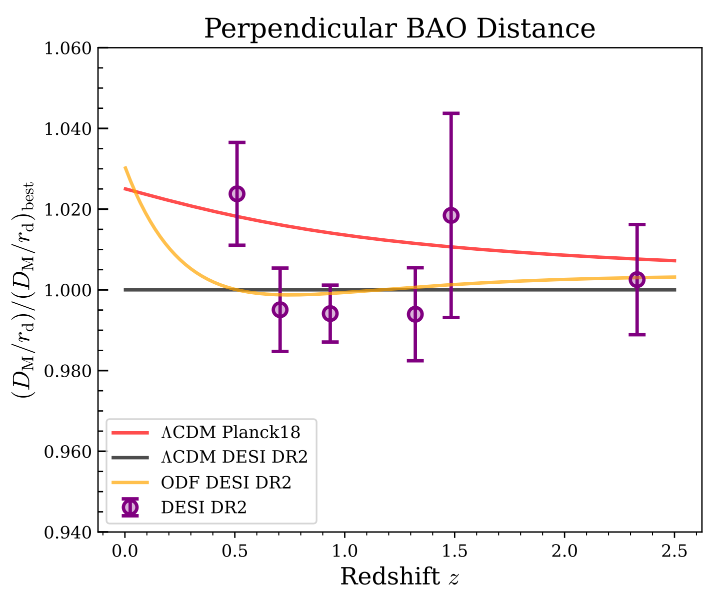
$$3M_P^2 H^2 = \alpha_1 + \alpha_2 H\,, \quad 2M_P^2 \dot{H} = \alpha_3 + \alpha_4 H$$

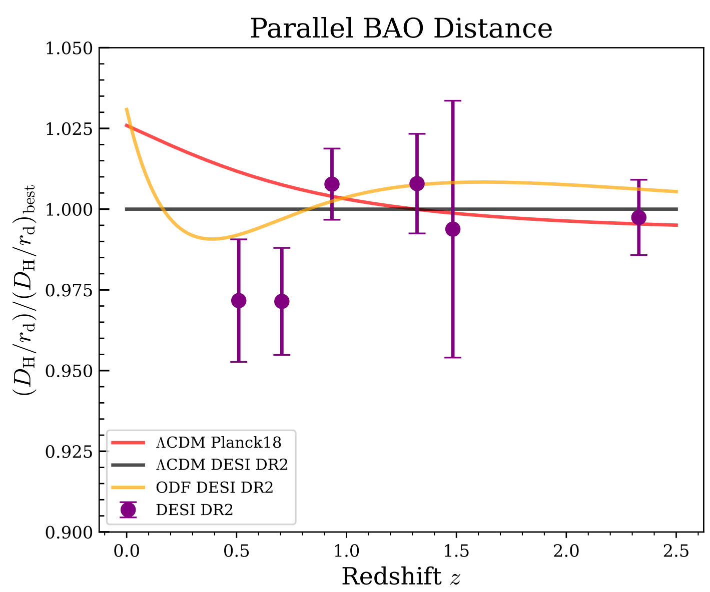
- Here α_a are combination of the γs. (may be constrained by energy conditions)

- Some concrete UV models: α_4 models shear viscosity, so open GR captures *dissipative* hydro; $\alpha_2 =$ const: appears in Dvali-Gabadaze-Porrati model of brane-work gravity

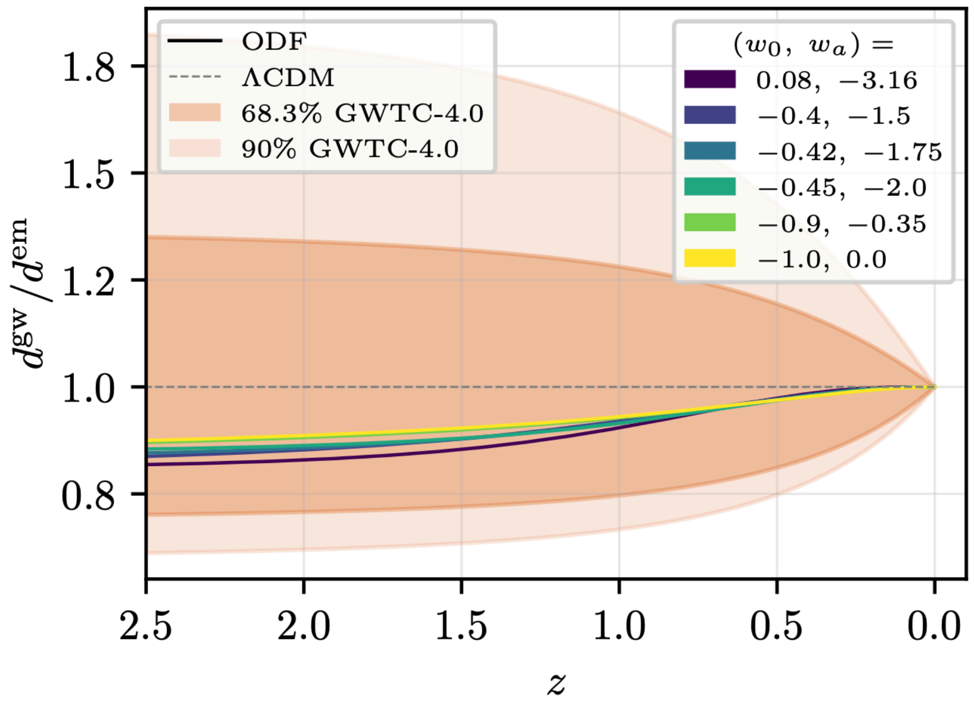
- Modified Friedmann eq can give *late-time accelerated expansion and fit Baryon Acoustic Oscillation data (DESI DR2)*

$$\frac{\ddot{a}}{a} = \frac{1}{2M_P^2}\left(\alpha_3 + \frac{2}{3}\alpha_1\right) + \frac{1}{2M_P^2}\left(\alpha_4 + \frac{2}{3}\alpha_2\right)\frac{\alpha_2}{6M_P^2}\left(1 + \sqrt{1 + \frac{12M_p^2}{\alpha_2^2}\alpha_1}\right)$$





Open GR



- Linearising the Einstein Equations we find gravity waves (gravitons)

$$\ddot{\gamma}_{ij} + (3H + \Gamma)\dot{\gamma}_{ij} + c_T^2 k^2 \gamma_{ij} + \beta \epsilon_{ijl} \dot{\gamma}_{ij} k_l = 0$$

- Propagation at speed c_T (highly constrained). Dissipation Γ much less

- *Dissipative birefringence β is a new phenomenon*: in E&M the leading birefringence is conservative (1derivative). For GR that violates the equivalence principle. Disspative birefringence is leading (2 derivatives)

- Applied to inflation this gives *chiral primordial gravitational waves*

Problems

- Christodoulidis & Gong ’25 found that for generic operators our Open GR does not have scalar degrees of freedom at linear order. Only for specific operators one scalar appears:

$$G_{\mu\nu} + \Gamma\left(K_{\mu\nu} - KP_{\mu\nu}\right)\sqrt{-g^{00}} = \frac{1}{M^2}T_{\mu\nu}\,,$$

- This is possibly related to not imposing the double (deformed?) gauge symmetry of Kaplanek, Mylova and Tolley ’25

- To make progress we want to learn to count degrees of freedom and detect gauge symmetries in open theories

Counting degrees of freedom in open theories

[Lausdei & Pajer to appear]

Let's play a game

- How many degrees of freedom?

$$\ddot{\phi} + \phi = 0$$

- Two initial conditions, $\phi(0)$ and $\dot{\phi}(0)$ means $2/2 = 1$ degree of freedom

Let's play a game

- How many degrees of freedom?

$$\ddot{\phi} + \phi = 0,$$

$$\ddot{\chi} + \lambda\chi = 0$$

- Four initial conditions, $\phi(0)$, $\dot{\phi}(0)$, $\chi(0)$ and $\dot{\chi}(0)$ means $4/2 = 2$ degrees of freedom

Let’s play a game

- How many degrees of freedom?

$$\ddot{\phi} + \phi = 0, \qquad \ddot{\chi} + \lambda\chi = 0, \qquad \dot{\phi} + \chi = 0$$

- The constraint makes it tricky. Naively three initial conditions, $\phi(0)$, $\dot{\phi}(0)$, and $\dot{\chi}(0)$ with $\chi(0)$ fixed by the constraint.

- But the time derivative of the constraint gives another constraint $-\phi + \dot{\chi} = 0$. New independent constraints

$$\dot{\phi} + \lambda^n\chi = 0 \text{ and } -\phi + \lambda^n\dot{\chi} = 0$$

- keep appearing unless $\lambda = 1$ in which case only first two constraints are independent

- $\lambda \neq 1$ zero deg. of freedom; $\lambda = 1$ one deg of freedom

Let’s play a game

- How many degrees of freedom?

$$3H\dot{A} + \frac{k^2 A}{a^2} + 2Hk^2(a\dot{B} - F) + \alpha\left(\frac{3}{2}\dot{A} + k^2(a\dot{B} - F)\right) = 0\,,$$

$$\left(H + \frac{\alpha}{2}\right)E - \dot{A} = 0\,,$$

$$\left(H + \frac{\alpha}{2}\right)\dot{E} - (3H + \alpha)\,\dot{A} - \ddot{A} - \frac{k^2(E + A)}{2a^2} - k^2\dot{(a\dot{B} - F)} - (2H + \alpha)k^2(a\dot{B} - F) = 0\,,$$

$$-\frac{1}{2}(E + A) - a^2\Big(\dot{(a\dot{B} - F)} + (2H + \alpha)(a\dot{B} - F)\Big) = 0\,.$$

- Very hard to tell by eye. We have many fields, equations and constraints (as well as gauge redundancies). We need a systematic procedure…

l, g and e

- Consider a system of linear, 2nd order ode's for ϕ_J with $J = 1,2,\ldots,N_r$ and $i = 1,2,\ldots N_a$

$$E_i = W_{iJ}\ddot{\phi}_J + K_i(\phi, \dot{\phi}) = 0\,,$$

- When this comes from a Lagrangian $N_a = N_r$ and

$$\#_{\text{d.o.f.}} = N_r - \frac{1}{2}(l + g + e)\,.$$

- where l is the number of constraints, g the number of gauge identities and e the number of gauge transformations

The algorithm

- **Step n**: find left null e-vectors of W_{iJ} to get *constraints* (valid on-shell)

$$\mathscr{C} \equiv v_i E_i = v_i (W_{iJ} \phi_J + K_i) = v_i K_i(\phi, \dot{\phi}) = 0\,.$$

- If constraints are not independent you have a *gauge identity* (valid off-shell)

$$\mathscr{G} \equiv \sum_{\alpha} a_\alpha \mathscr{C}_\alpha = 0$$

- For each gauge identity

$$\mathscr{G} = \sum_{i=1}^{N_r} \sum_{m=0}^{M} \rho_{i,m} \left(\frac{d}{dt}\right)^m E_i = 0 \quad \Rightarrow \quad g \to g+1\,, \quad e \to e+1+M\,.$$

- gauge identities imply invariance under gauge transformations

- **Step n+1**: iterate using $(E_i, \dot{\mathscr{C}})$ as new equations. Stop when no new constraints

Open theories

- The eq. of motion of open theories do not come from a Lagrangian and the previous formula gives the wrong results. Moreover, evolution is not Hamiltonian, so the well-known Dirac-Bergman algorithm doesn't work either
- We developed a new method. First, find the "dual" N_r equations of motion for N_a new "advanced" fields
- $$E_i^r = M_{iI}\phi_I^r = 0 \quad \leftrightarrow \quad E_I^a = (M^\dagger)_{Ii}\phi_i^a = 0\,.$$
- This is simply the classical/deterministic Schwinger-Keldysh
- $$S_{\text{SK}} = \int \phi_a E_i^r = \int \phi_i^a\, M_{iI}\, \phi_I^r\,,$$

The new algorithm

- Now apply the old algorithm to $E_i^r = 0$ and $E_J^a = 0$. This gives (l^r, g^r, e^r) and (l^a, g^a, e^a), respectively.

- adv. (ret.) gauge identities $\leftrightarrow$ ret. (adv.) gauge transform.

$$\mathcal{G}^a = \sum_{I=1}^{N_a} \sum_{m=0}^{M} \rho^a_{I,m} \left(\frac{d}{dt}\right)^m E_I^a = 0\,, \qquad \Delta\phi_r = \sum_{m=0} (-1)^m \left(\frac{d}{dt}\right)^m \left(\rho^a_{Im}\epsilon\right)$$

$$S_S K \simeq \int \phi_a M_{iI} \Delta\phi_r \simeq \int \Delta\phi_r E_I^a \simeq \int \epsilon\, \mathcal{G}^a = 0$$

- Claim: the number of deg. of freedom is

Open system: $\quad \#_{\text{dof}} = N_r - \frac{1}{2}(l_r + g_a + e_a)$

Open gravity

- Using the *deformed* advanced+retarded gauge symmetry, we derived a class of all-order modifications to Einstein equations
- $G_{\mu\nu} + \frac{X}{f}\left(K_{\mu\nu} - Kg_{\mu\nu} - n_\mu a_\nu - n_\nu a_\mu\right) - \frac{\dot{X}}{f}P_{\mu\nu} = \frac{1}{M^2}T_{\mu\nu}$
- where $X = \dot{f}/N$ and $a_\mu = n^\rho \nabla_\rho n_\mu$ is the so-called acceleration
- Using our algorithm we confirmed they all have exactly one scalar deg. of freedom (plus the graviton)!

Summary and Outlook

- The major questions in cosmology, related to inflation, dark matter and dark energy, have in common the need for an open system approach
- We have developed and adapted the necessary formalism and moved the first steps into modelling open gravity for inflation and the dark sector
- Many questions are still open:
  - What is the meaning of the two metrics $g^{\pm}_{\mu\nu}$ in differential geometry?
  - Can I always deform the gauge symmetry?
  - are there constraints from consistent completions to an open system?