Neutrino quantum kinetics:
Theory and (some) results

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Classical transport

Distribution function

$$f(\mathbf{x}, \mathbf{p}, t)$$

Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \dot{\mathbf{p}} \cdot \frac{\partial f}{\partial \mathbf{p}} = \mathcal{C}[f]$$

Collision term:

$$\mathcal{C}[f_1] = \frac{1}{2p_1} \int \frac{\mathrm{d}^3\mathbf{p}_2}{(2\pi)^3\, 2E_2} \frac{\mathrm{d}^3\mathbf{p}_3}{(2\pi)^3\, 2E_3} \frac{\mathrm{d}^3\mathbf{p}_4}{(2\pi)^3\, 2E_4} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \left\langle |\mathcal{M}|^2 \right\rangle F(1,2,3,4)$$

Classical transport

Distribution function

$$f(\mathbf{x}, \mathbf{p}, t)$$

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$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \dot{\mathbf{p}} \cdot \frac{\partial f}{\partial \mathbf{p}} = \mathcal{C}[f]$$

Collision term:

Scattering amplitude

$$\mathcal{C}[f_1] = \frac{1}{2p_1} \int \frac{\mathrm{d}^3\mathbf{p}_2}{(2\pi)^3\, 2E_2} \frac{\mathrm{d}^3\mathbf{p}_3}{(2\pi)^3\, 2E_3} \frac{\mathrm{d}^3\mathbf{p}_4}{(2\pi)^3\, 2E_4} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \langle |\mathcal{M}|^2 \rangle\, F(1,2,3,4)$$

Energy-momentum conservation *Statistical factor*

$$F(1,2,3,4) = (1 - f_1)(1 - f_2)\, f_3 f_4 - (1 - f_3)(1 - f_4)\, f_1 f_2$$

$$f_i = f_{\phi_i}(\mathbf{x}, \mathbf{p}_i, t)$$

Electrons, nucleons, neutrinos…

Neutrino flavor oscillations

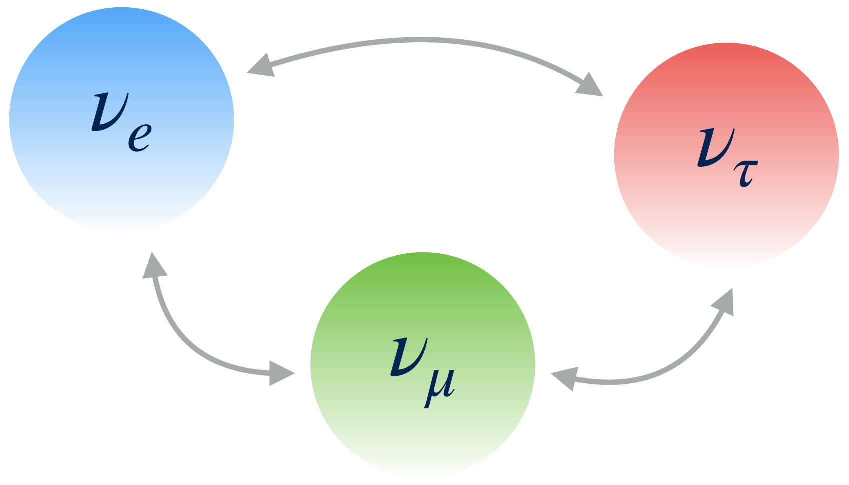
- Standard Model: 3 species of massless neutrinos ν_L

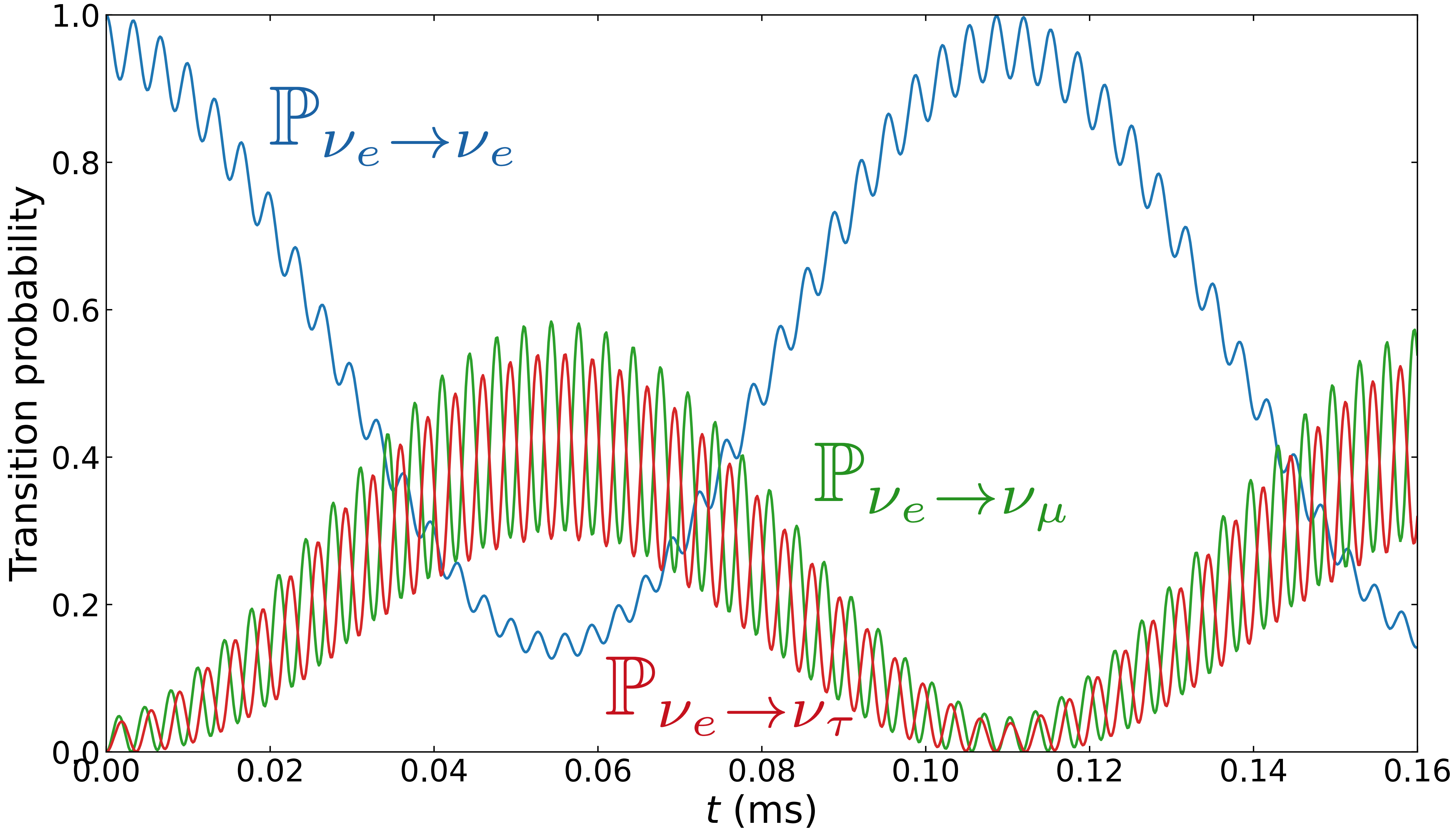
- Experimental evidence in the second half of the 20th century
  $\rightarrow$ massive neutrinos

$$\text{Flavor states} \quad \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad \text{Mass states}$$

PMNS mixing matrix

$\Longrightarrow$ neutrino oscillations





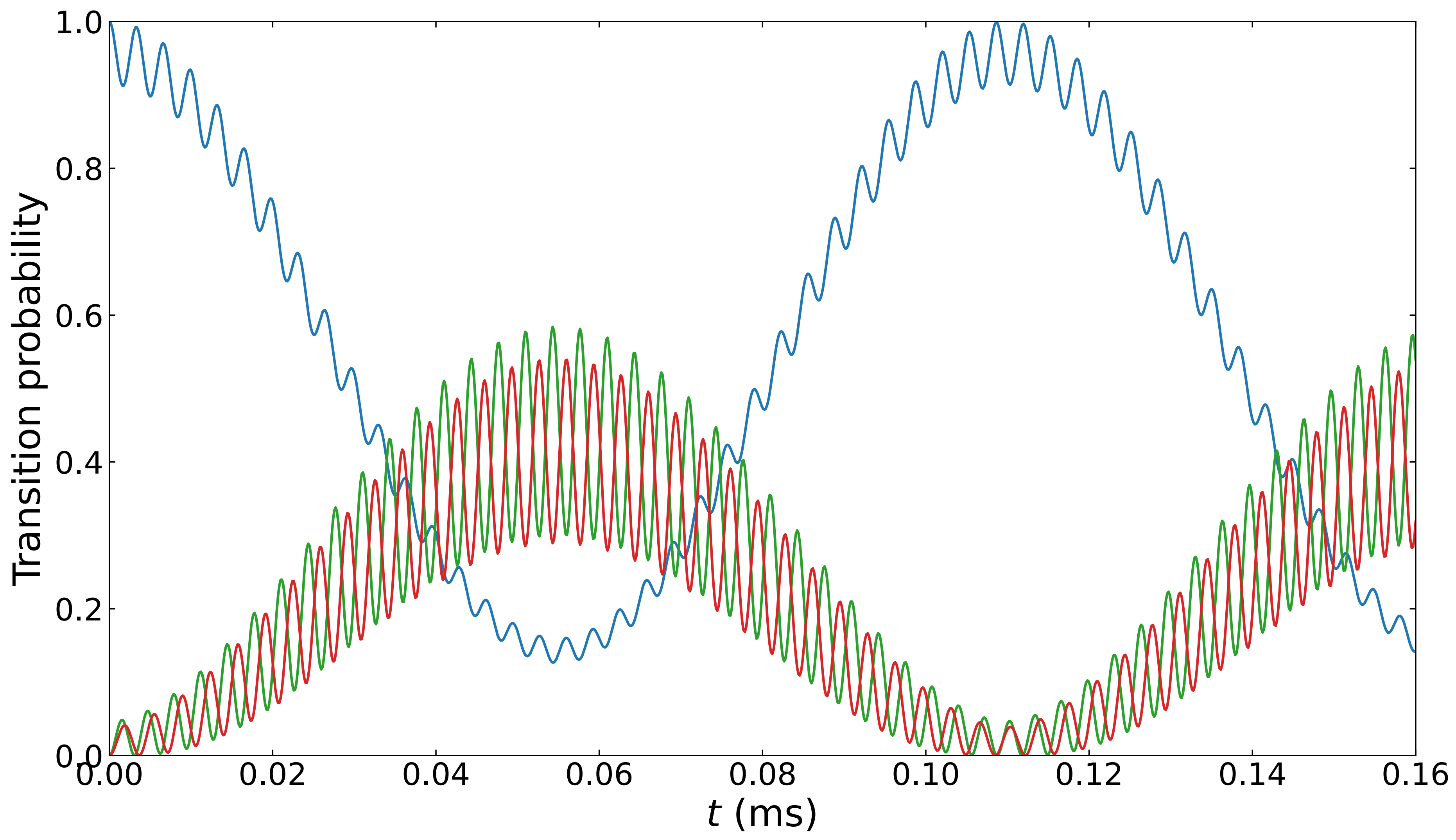
Neutrino quantum kinetic theory?

Boltzmann equation

$$\frac{\partial f_{\nu_e}}{\partial t} + \mathbf{v} \cdot \frac{\partial f_{\nu_e}}{\partial \mathbf{x}} + \dot{\mathbf{p}} \cdot \frac{\partial f_{\nu_e}}{\partial \mathbf{p}} = \mathcal{C}_{\nu_e}$$

$$\frac{\partial f_{\nu_\mu}}{\partial t} + \mathbf{v} \cdot \frac{\partial f_{\nu_\mu}}{\partial \mathbf{x}} + \dot{\mathbf{p}} \cdot \frac{\partial f_{\nu_\mu}}{\partial \mathbf{p}} = \mathcal{C}_{\nu_\mu}$$

$$\frac{\partial f_{\nu_\tau}}{\partial t} + \mathbf{v} \cdot \frac{\partial f_{\nu_\tau}}{\partial \mathbf{x}} + \dot{\mathbf{p}} \cdot \frac{\partial f_{\nu_\tau}}{\partial \mathbf{p}} = \mathcal{C}_{\nu_\tau}$$



Superposition of flavor states

Quantum Kinetic Equation

G. Sigl and G. Raffelt, *Nucl. Phys. B* **406** (1993)
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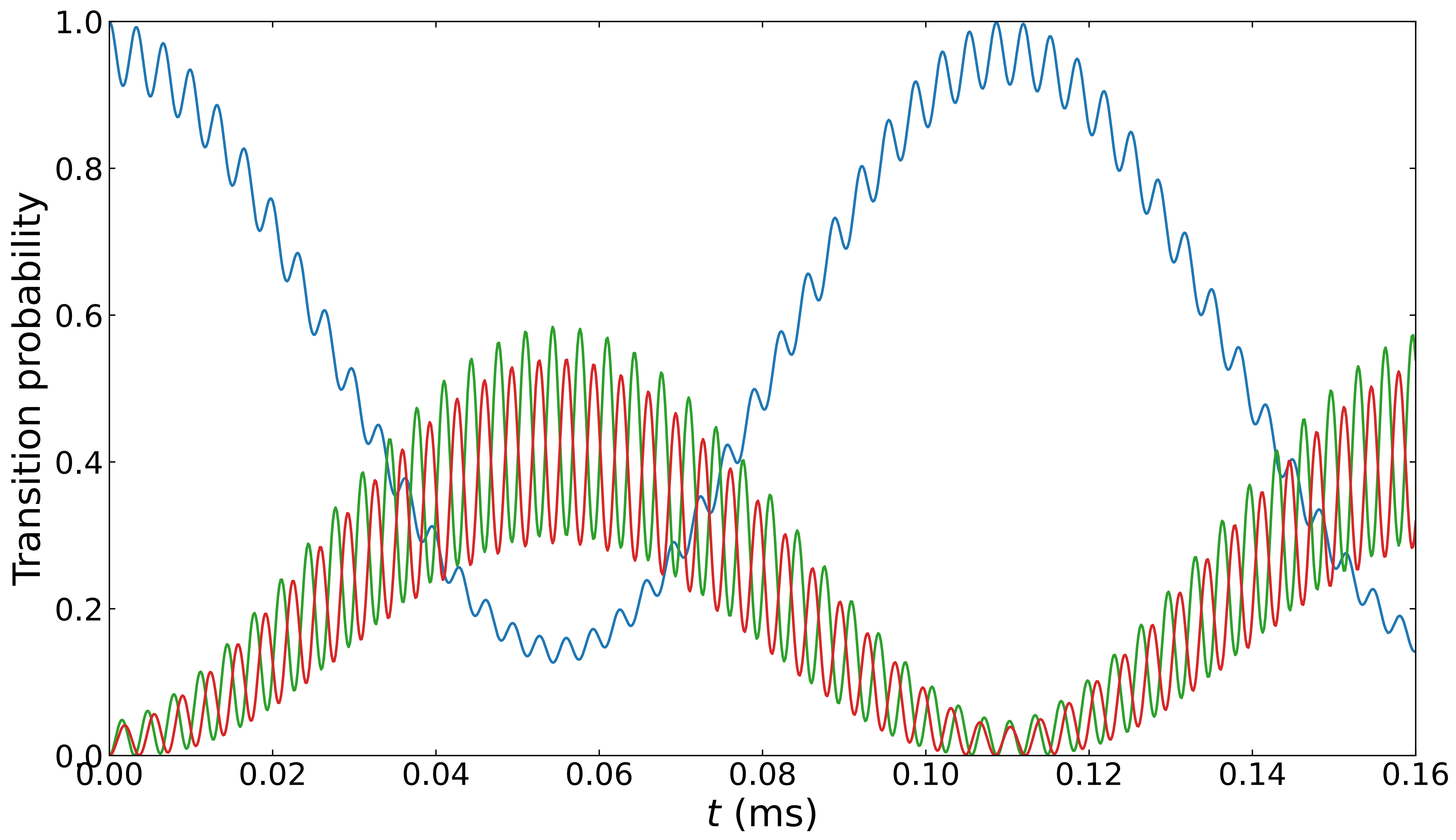
$$\frac{\partial f_{\nu_\mu}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_{\nu_\mu}}{\partial \mathbf{x}} + \dot{\mathbf{p}}\cdot\frac{\partial f_{\nu_\mu}}{\partial \mathbf{p}} = \mathcal{C}_{\nu_\mu}$$

$$\frac{\partial f_{\nu_\tau}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_{\nu_\tau}}{\partial \mathbf{x}} + \dot{\mathbf{p}}\cdot\frac{\partial f_{\nu_\tau}}{\partial \mathbf{p}} = \mathcal{C}_{\nu_\tau}$$



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BBGKY approach

Blackboard:

* Hamiltonian (second quantization)
* One-body, two-body… density matrices
* Ehrenfest equation and BBGKY hierarchy
* Mean-field approximation and beyond: separation of scales and collision term

Example: $\nu - \nu$ interactions

Neutrino-neutrino interaction Hamiltonian

$$H_{\nu\nu} = \frac{G_F}{4\sqrt{2}} \sum_{\alpha,\beta} \int \mathrm{d}^3\mathbf{x} \left[\bar{\psi}_{\nu_\alpha} \gamma_\mu (1-\gamma_5) \psi_{\nu_\alpha}\right] \left[\bar{\psi}_{\nu_\beta} \gamma^\mu (1-\gamma_5) \psi_{\nu_\beta}\right]$$

$$\downarrow$$

$$\psi_{\nu_\alpha}(\mathbf{x}) = \sum_h \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3 2E_p} \left[a_{\nu_\alpha}(\mathbf{p},h) u_{\mathbf{p}}^{(h)} e^{i\mathbf{p}\cdot\mathbf{x}} + b^\dagger_{\nu_\alpha}(\mathbf{p},h) v_{\mathbf{p}}^{(h)} e^{-i\mathbf{p}\cdot\mathbf{x}}\right]$$

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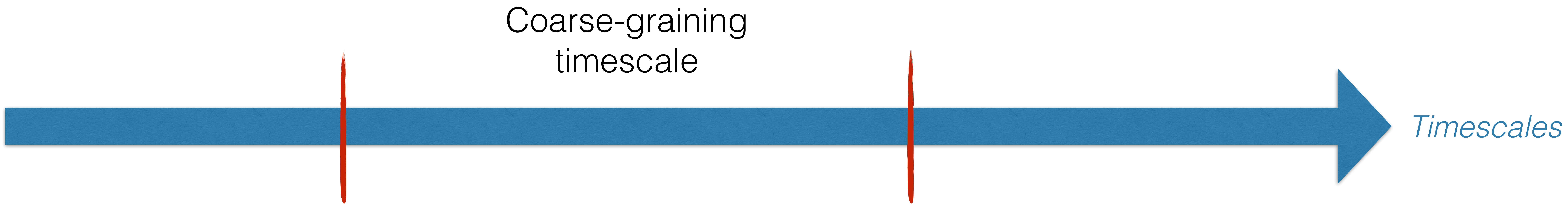
One obtains the interaction coefficient:

$$v^{\nu_\alpha(1)\nu_\beta(2)}_{\nu_\alpha(3)\nu_\beta(4)} = \frac{G_F}{2\sqrt{2}} (2\pi)^3 \delta^{(3)}(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \left[\bar{u}_{\mathbf{p}_1}^{(h_1)} \gamma_\mu (1-\gamma_5) u_{\mathbf{p}_3}^{(h_3)}\right] \left[\bar{u}_{\mathbf{p}_2}^{(h_2)} \gamma^\mu (1-\gamma_5) u_{\mathbf{p}_4}^{(h_4)}\right]$$

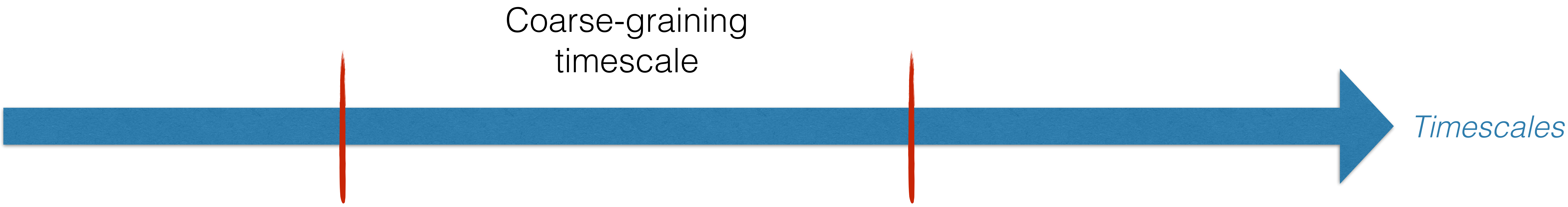
A problem of many scales



$$\tau_{\text{coll}} \ll t \ll \tau_{\text{evol. macro}}$$

A problem of many scales

Typical energy ~ temperature ~ $10\,\mathrm{MeV}$



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Interaction length ~ de Broglie wavelength

$$\lambda_{\mathrm{dB}} \sim \frac{1}{10\,\mathrm{MeV}} \sim 10^{-13}\,\mathrm{m} \longleftrightarrow \tau_{\mathrm{coll}} \sim 10^{-21}\,\mathrm{s}$$

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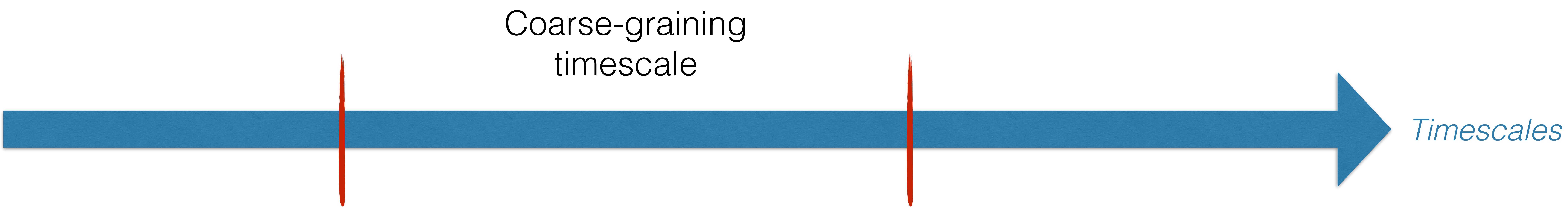

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Duration between collisions = Mean-free path

$$\tau_{\mathrm{mfp}} \sim \left(G_F^2 T^5\right)^{-1} \sim 10^{-4}\,\mathrm{s}$$

A problem of many scales

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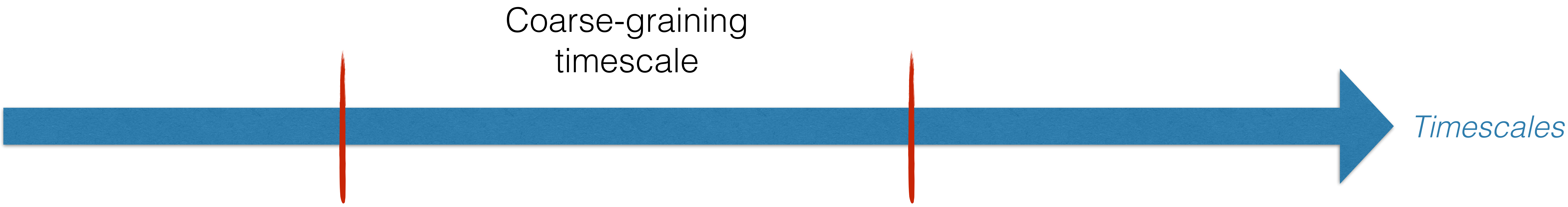

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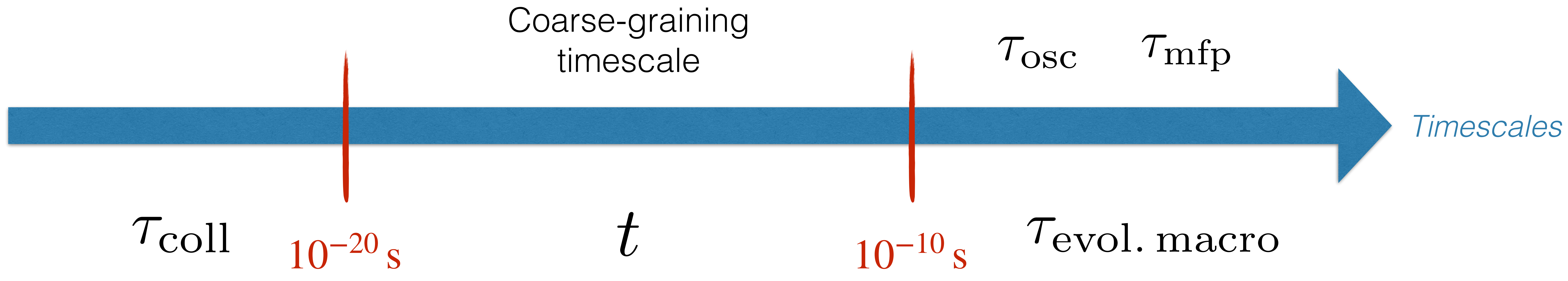
Oscillation timescales

$$\tau_{\mathrm{osc,\,vac}} \sim \left(\frac{\Delta m^2}{2E}\right)^{-1} \sim \left(\frac{2\times 10^{-3}\,\mathrm{eV}^2}{2\times 10\,\mathrm{MeV}}\right)^{-1} \sim 10^{-5}\,\mathrm{s}$$

$$\tau_{\mathrm{osc,\,self}} \sim \left(\sqrt{2}G_F n_\nu\right)^{-1} \sim 10^{-10}\,\mathrm{s}$$

A problem of many scales

Typical energy ~ temperature ~ $10\,\text{MeV}$



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Homogeneous Quantum Kinetic Equation

$$\mathrm{i}\frac{\mathrm{d}\varrho^i_j}{\mathrm{d}t} = [t+\Gamma, \varrho]^i_j + \mathrm{i}\,\mathcal{C}^i_j$$

$$\begin{aligned}\mathcal{C}^{i_1}_{i'_1} \propto \frac{1}{4}\Big(&\tilde{v}^{i_1 i_2}_{i_3 i_4}\,\varrho^{i_3}_{j_3}\,\varrho^{i_4}_{j_4}\,\tilde{v}^{j_3 j_4}_{j_1 j_2}(\hat{1}-\varrho)^{j_1}_{i'_1}(\hat{1}-\varrho)^{j_2}_{i_2} - \tilde{v}^{i_1 i_2}_{i_3 i_4}(\hat{1}-\varrho)^{i_3}_{j_3}(\hat{1}-\varrho)^{i_4}_{j_4}\,\tilde{v}^{j_3 j_4}_{j_1 j_2}\,\varrho^{j_1}_{i'_1}\,\varrho^{j_2}_{i_2} \\ &+(\hat{1}-\varrho)^{i_1}_{j_1}(\hat{1}-\varrho)^{i_2}_{j_2}\,\tilde{v}^{j_1 j_2}_{j_3 j_4}\,\varrho^{j_3}_{i_3}\,\varrho^{j_4}_{i_4}\,\tilde{v}^{i_3 i_4}_{i'_1 i_2} - \varrho^{i_1}_{j_1}\,\varrho^{i_2}_{j_2}\,\tilde{v}^{j_1 j_2}_{j_3 j_4}(\hat{1}-\varrho)^{j_3}_{i_3}(\hat{1}-\varrho)^{j_4}_{i_4}\,\tilde{v}^{i_3 i_4}_{i'_1 i_2}\Big)\end{aligned}$$

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$$+(\hat{1}-\varrho)^{i_1}_{j_1}(\hat{1}-\varrho)^{i_2}_{j_2}\,\tilde{v}^{j_1 j_2}_{j_3 j_4}\,\varrho^{j_3}_{i_3}\,\varrho^{j_4}_{i_4}\,\tilde{v}^{i_3 i_4}_{i_1' i_2} - \varrho^{i_1}_{j_1}\,\varrho^{i_2}_{j_2}\,\tilde{v}^{j_1 j_2}_{j_3 j_4}(\hat{1}-\varrho)^{j_3}_{i_3}(\hat{1}-\varrho)^{j_4}_{i_4}\,\tilde{v}^{i_3 i_4}_{i_1' i_2}\Big)$$

Gain **Loss**

Homogeneous Quantum Kinetic Equation

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$$+(\hat{1}-\varrho)^{i_1}_{j_1}(\hat{1}-\varrho)^{i_2}_{j_2}\,\tilde{v}^{j_1 j_2}_{j_3 j_4}\,\varrho^{j_3}_{i_3}\,\varrho^{j_4}_{i_4}\,\tilde{v}^{i_3 i_4}_{i'_1 i_2} - \varrho^{i_1}_{j_1}\,\varrho^{i_2}_{j_2}\,\tilde{v}^{j_1 j_2}_{j_3 j_4}(\hat{1}-\varrho)^{j_3}_{i_3}(\hat{1}-\varrho)^{j_4}_{i_4}\,\tilde{v}^{i_3 i_4}_{i'_1 i_2}\Big)$$

$\langle|\mathscr{M}|^2\rangle$

Homogeneous Quantum Kinetic Equation

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$$\langle|\mathscr{M}|^2\rangle$$

<u>Homogeneous ensemble of ultrarelativistic neutrinos:</u>

$$\varrho^{\nu_\alpha(\mathbf{p},h)}_{\nu_\beta(\mathbf{p}',h')} = (2\pi)^3\, 2E_{\mathbf{p}}\,\delta^{(3)}(\mathbf{p}-\mathbf{p}')\,\delta_{h-}\delta_{h'-}\,\boxed{\rho_{\alpha\beta}(\mathbf{p})}$$

Homogeneous Quantum Kinetic Equation

$$\mathrm{i}\frac{\mathrm{d}\rho}{\mathrm{d}t} = [\mathcal{H}_{\mathrm{vac}} + \mathcal{H}_{\mathrm{mat}} + \mathcal{H}_{\mathrm{self}}, \rho] + \mathrm{i}\,\mathcal{C}$$

$$\mathcal{H}_{\mathrm{vac}} = U\frac{\mathbb{M}^2}{2|\mathbf{p}|}U^\dagger$$

Vacuum term

$$\mathcal{H}_{\mathrm{mat}} = \sqrt{2}G_F\,(n_{e^-} - n_{e^+})\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

Matter mean-field potential

$$\mathcal{H}_{\mathrm{self}} = \sqrt{2}G_F\int\frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}\,(1-\hat{p}\cdot\hat{q})\,[\rho(\mathbf{q}) - \bar{\rho}(\mathbf{q})]$$

Self-interaction mean-field potential

Homogeneous Quantum Kinetic Equation

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Self-interaction mean-field potential

Rich phenomenology of flavor conversion mechanisms, including:

Vacuum oscillations

MSW resonance

Matter-neutrino resonance

Slow instabilities

Fast instabilities

Weak inhomogeneities and advection term

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Homogeneous case $\rho_{ij}(\mathbf{p}) = \left\langle a_j^\dagger(\mathbf{p}) a_i(\mathbf{p}) \right\rangle$

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Homogeneous case $\rho_{ij}(\mathbf{p}) = \left\langle a_j^\dagger(\mathbf{p}) a_i(\mathbf{p}) \right\rangle$

Wigner transform

$$\rho_{ij}(\mathbf{x},\mathbf{p}) = \langle \mathcal{D}_{ij}(\mathbf{x},\mathbf{p}) \rangle = \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} e^{i\boldsymbol{\Delta}\cdot\mathbf{x}} \left\langle a_j^\dagger\left(\mathbf{p} - \frac{\boldsymbol{\Delta}}{2}\right) a_i\left(\mathbf{p} + \frac{\boldsymbol{\Delta}}{2}\right)\right\rangle$$

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Weak inhomogeneities: $\ell \gg \lambda_{\mathrm{dB}} \sim |\mathbf{p}|^{-1} \sim 10^{-13}\,\mathrm{m}$

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lengthscale of variation of ρ

$$|\Delta| \sim \ell^{-1} \ll |\mathbf{p}|$$

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Weak inhomogeneities: $\ell \gg \lambda_{\mathrm{dB}} \sim |\mathbf{p}|^{-1} \sim 10^{-13}\,\mathrm{m}$

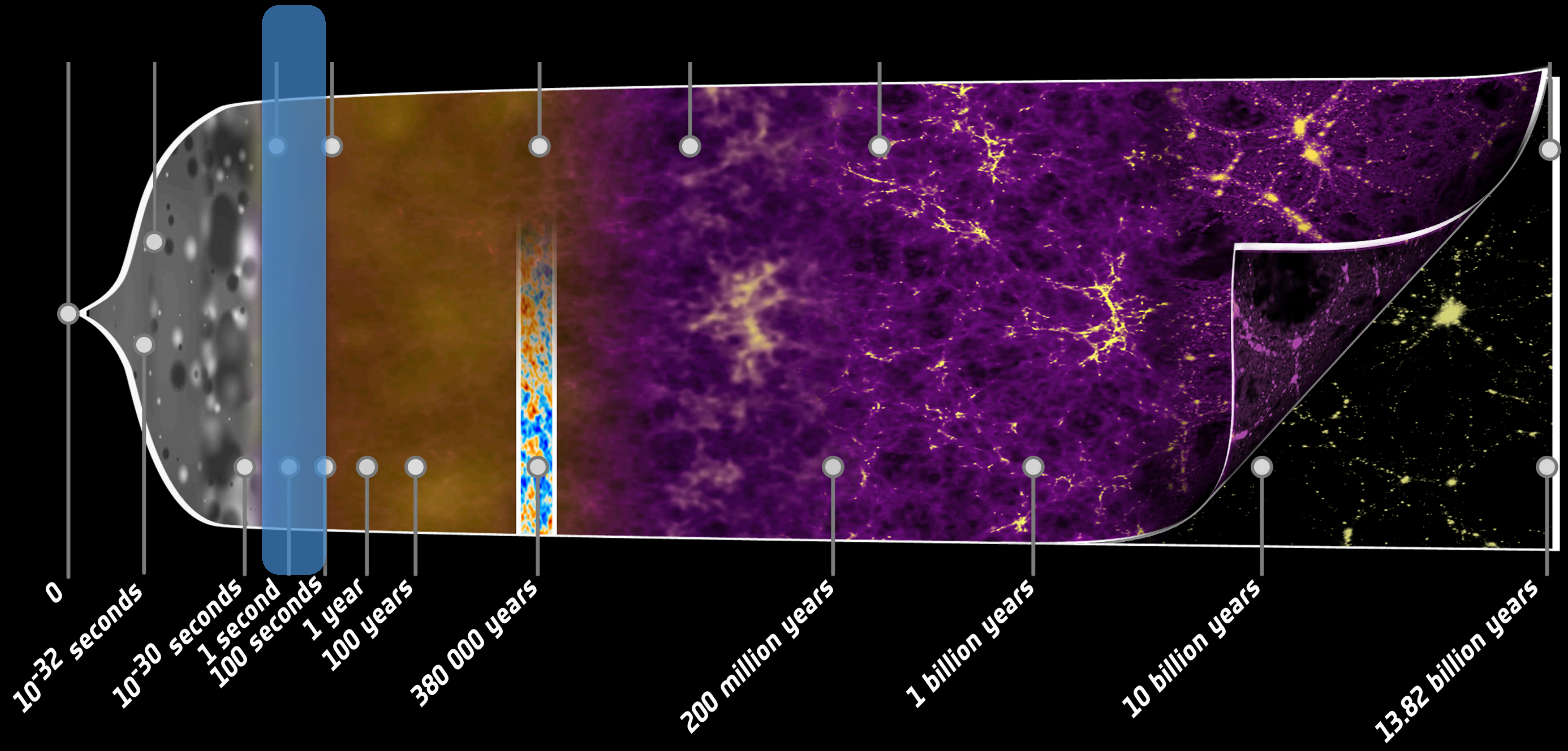
lengthscale of variation of ρ

$|\Delta| \sim \ell^{-1} \ll |\mathbf{p}|$

→ $\langle [\mathcal{D}_{ij}(\mathbf{x},\mathbf{p}), H_0] \rangle \simeq -\mathrm{i}\,\mathbf{v}\cdot\partial_{\mathbf{x}}\rho_{ij}(\mathbf{x},\mathbf{p})$

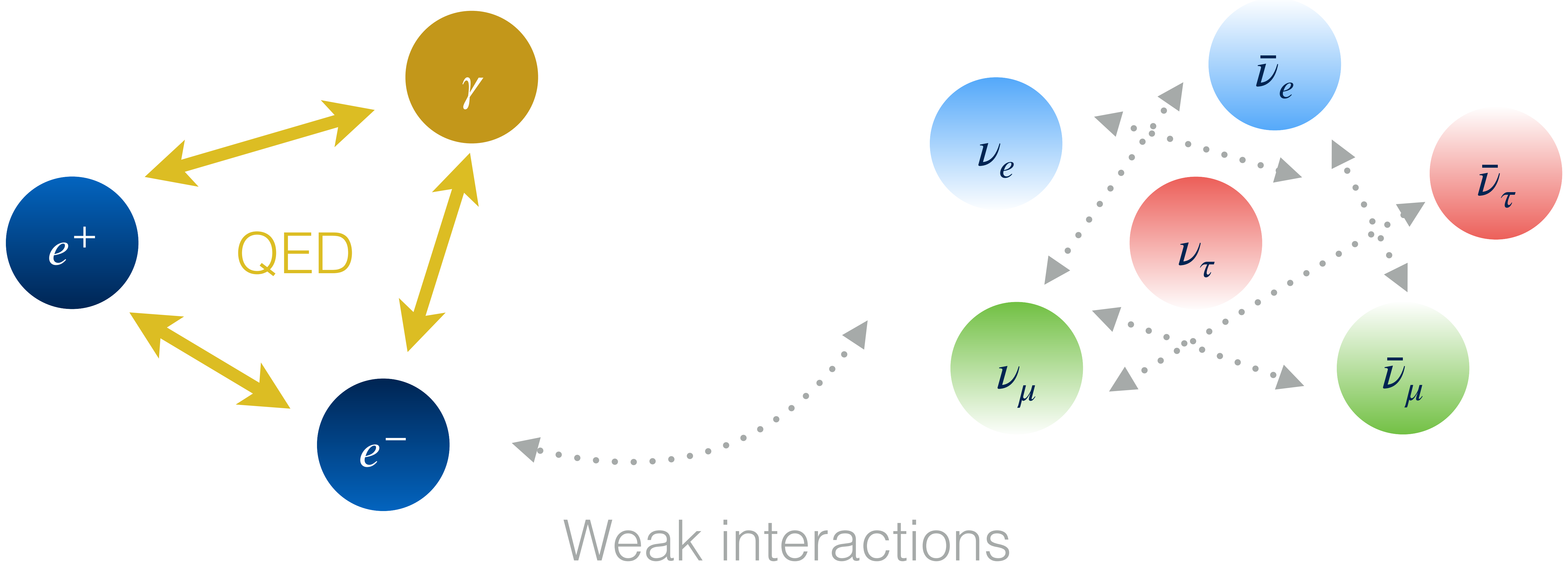
$k_B T \sim 1\,\mathrm{MeV} \sim 10^{10}\,\mathrm{K}$

‘MeV age’



Credits: ESA

Neutrinos in the MeV age

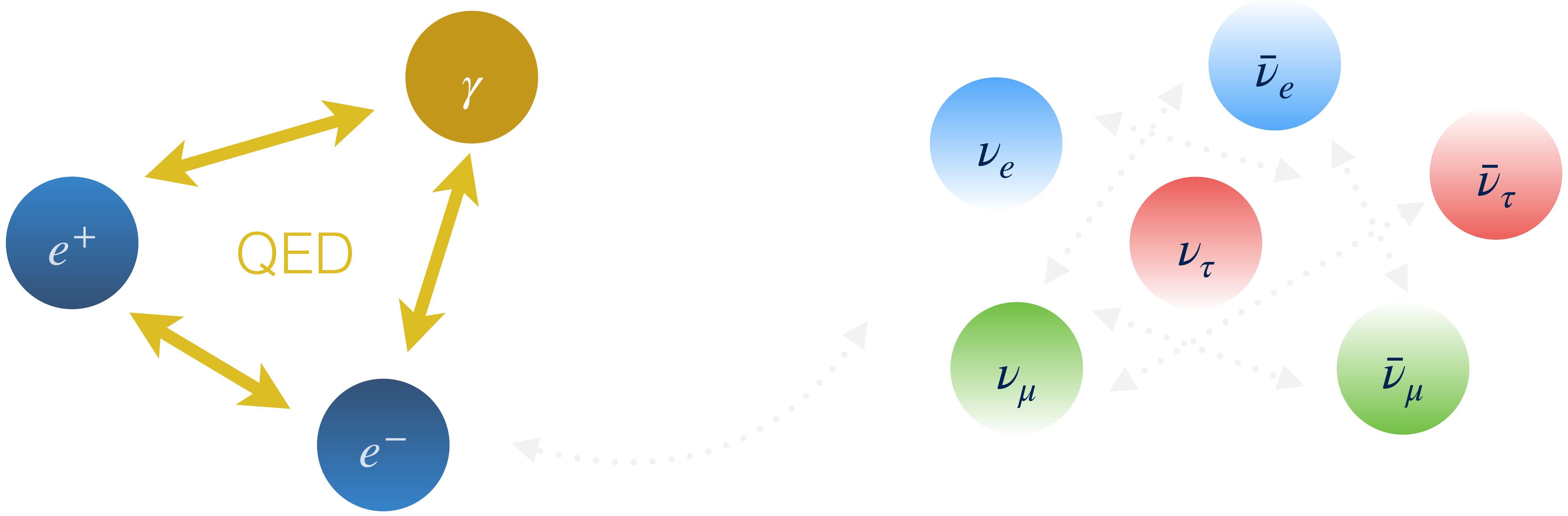


Neutrinos in the MeV age

Neutrino decoupling

$$\frac{\text{collision rate}}{\text{expansion rate}} \qquad \frac{\Gamma}{H} \simeq \frac{G_F^2 T^5}{\sqrt{g_*} T^2 / M_{\rm Pl}}$$

$$\Gamma/H = 1 \longleftrightarrow T_{\rm dec} \simeq 1\,{\rm MeV}$$



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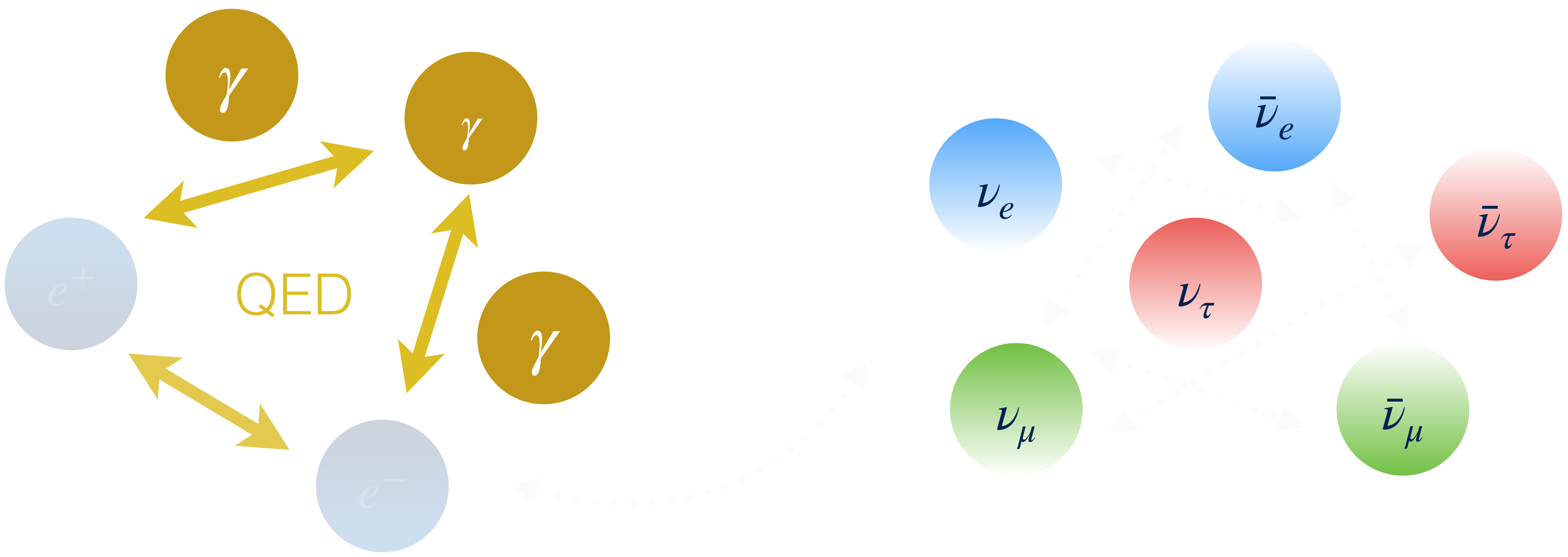

ELECTRON-POSITRON ANNIHILATIONS

→ entropy release

$$T_{e^\pm} = 0.511\,{\rm MeV}$$

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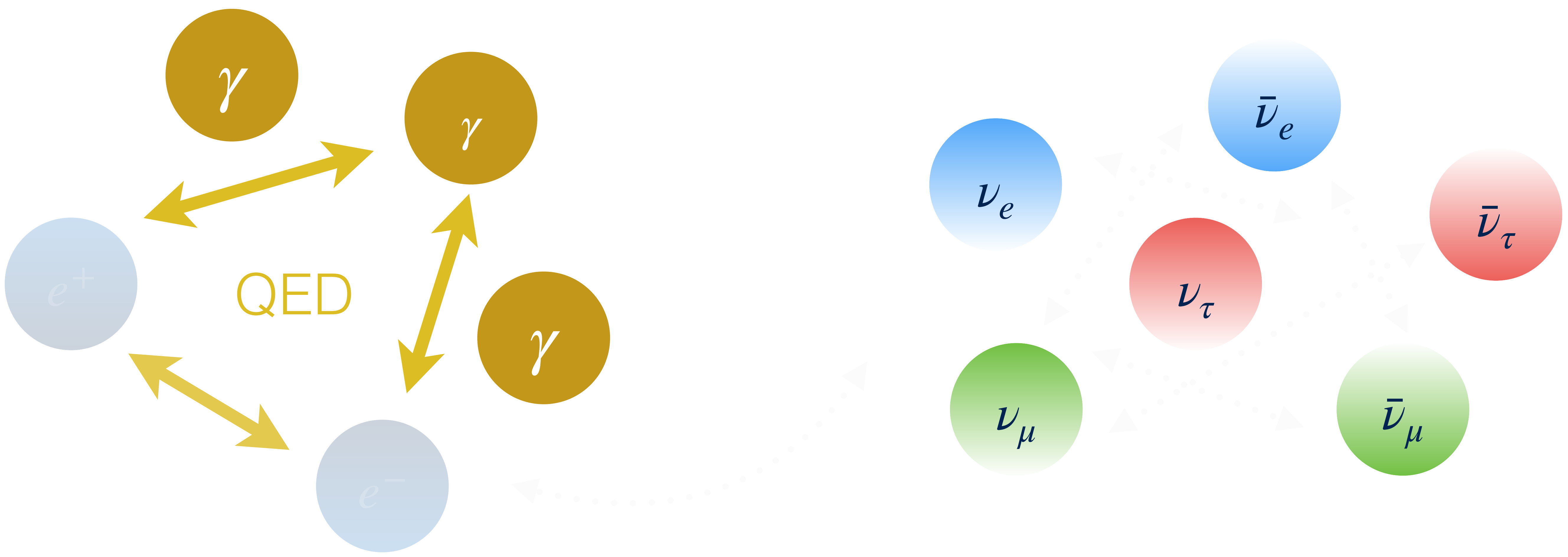

Neutrino oscillations

$$\Omega_{\rm vac} = \frac{\Delta m^2}{2E}$$

$$\Omega_{\rm matt} = \frac{\sqrt{2}G_F}{m_W^2}E\left[\rho_{e^\pm} + P_{e^\pm}\right] = \frac{7\pi^2}{45}\,\frac{\sqrt{2}G_F}{m_W^2}\,E\,T^4$$

MSW transition for $\Omega_{\rm vac} = \Omega_{\rm matt}$

$$T_{\rm MSW} \in [12\,\mathrm{MeV},\,2.8\,\mathrm{MeV}]$$

Electron-positron annihilations

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Neutrino transport in the early Universe

$$\frac{\partial \varrho}{\partial t} - Hp\frac{\partial \varrho}{\partial p} = -\mathrm{i}\left[\mathcal{H}_{\mathrm{vac}} + \mathcal{H}_{\mathrm{mat}} + \mathcal{H}_{\nu\nu}, \varrho\right] + \mathcal{C}$$

JF, C. Pitrou, M.C. Volpe [2008.01074]
J. Bennett *et al.* [2012.02726]
M. Drewes *et al.* [2402.18481]

- Calculation of **standard neutrino decoupling**: $N_{\mathrm{eff}} = 3.044$

$$\rho_{\mathrm{tot}} = \rho_\gamma + \rho_{\nu,\bar{\nu}} = \left[1 + N_{\mathrm{eff}} \times \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right]\rho_\gamma$$

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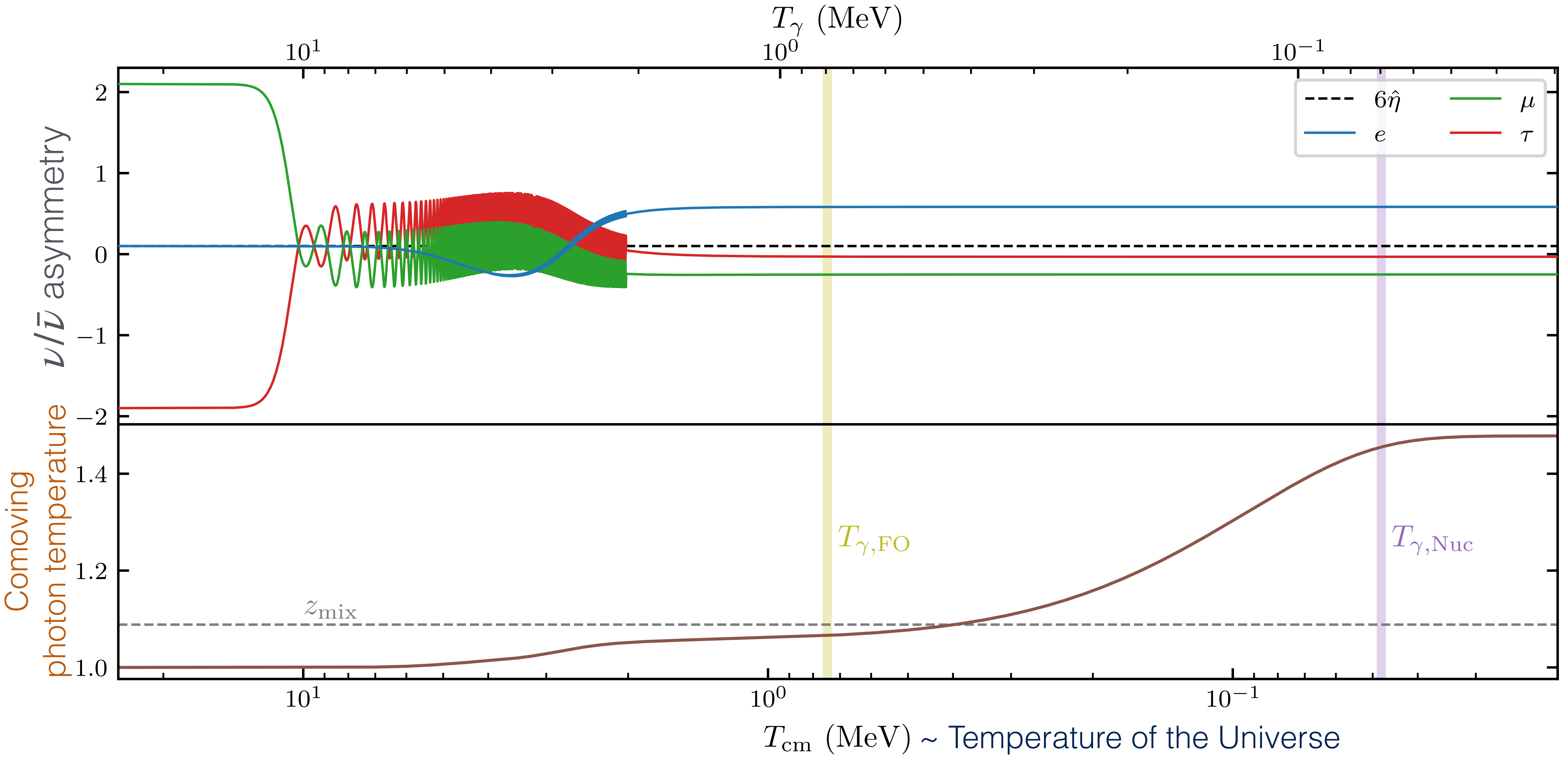
- Extension to the case of **nonzero neutrino/antineutrino asymmetries**



$$\eta_\alpha \equiv \frac{n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{T^3}$$

Initial values

$$\eta_{\mathrm{av}} = \eta_e = 0.1$$
$$\eta_\mu - \eta_\tau = 4$$

JF, C. Pitrou [2110.11889]
JF, C. Pitrou [2405.06509]

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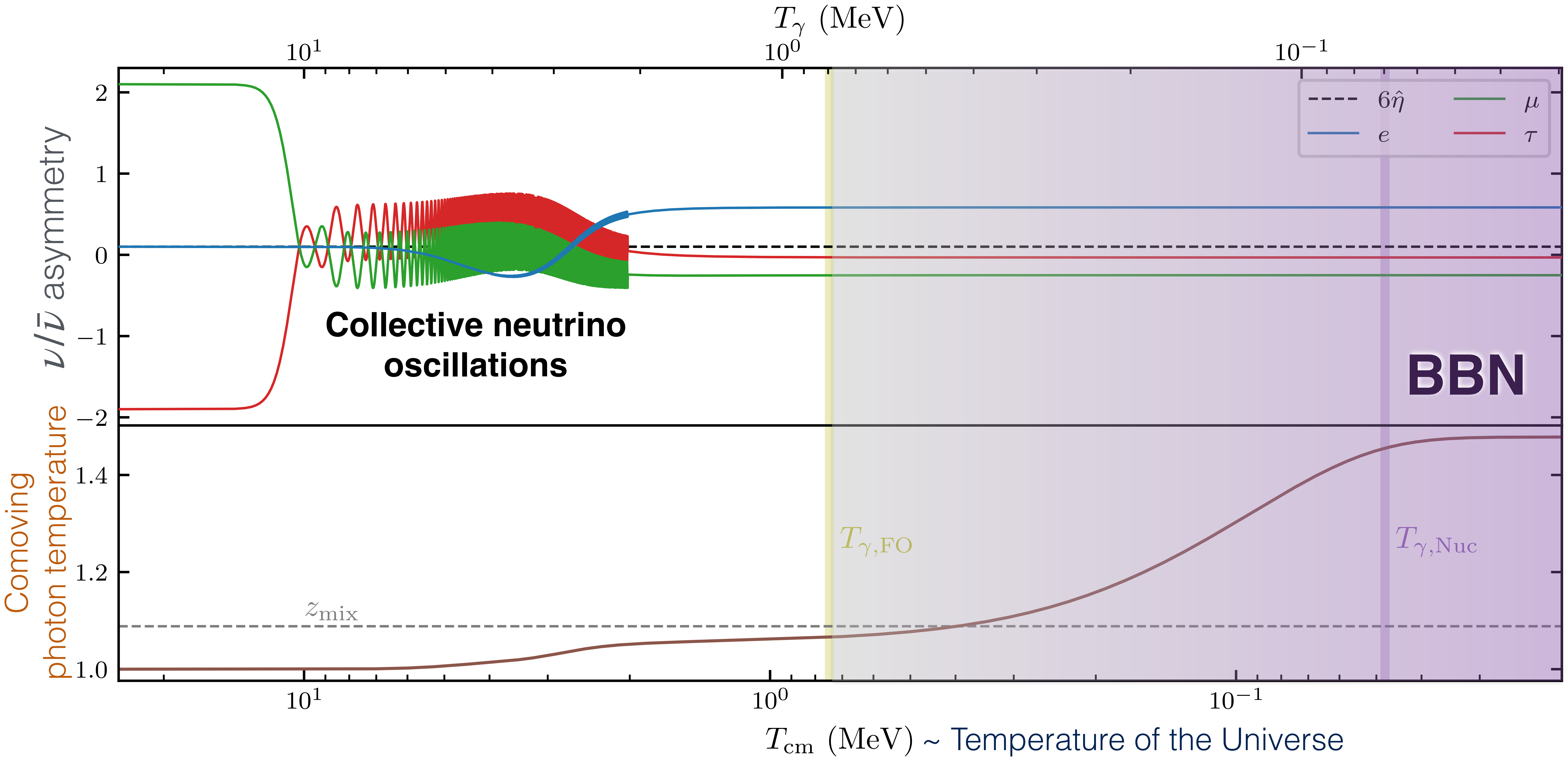

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Summary and prospects

- The quantum kinetic approach consists in describing the evolution **at the one-body level**.

  Formally similar to OQS equations with system = 'one neutrino' and environment = 'the rest'

- It relies on a **separation of scales**:
  - quasi-instantaneous scatterings compared to a 'long' macroscopic evolution $\tau_{\mathrm{coll}} \ll \tau_{\mathrm{macro\,evol.}}$
  - Weak inhomogeneities, on large scales compared to the de Broglie wavelength $|\mathbf{p}|^{-1} \ll \ell$

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- Ongoing debate: **entanglement is neglected** in the QKE approach. *Is that always justified?*