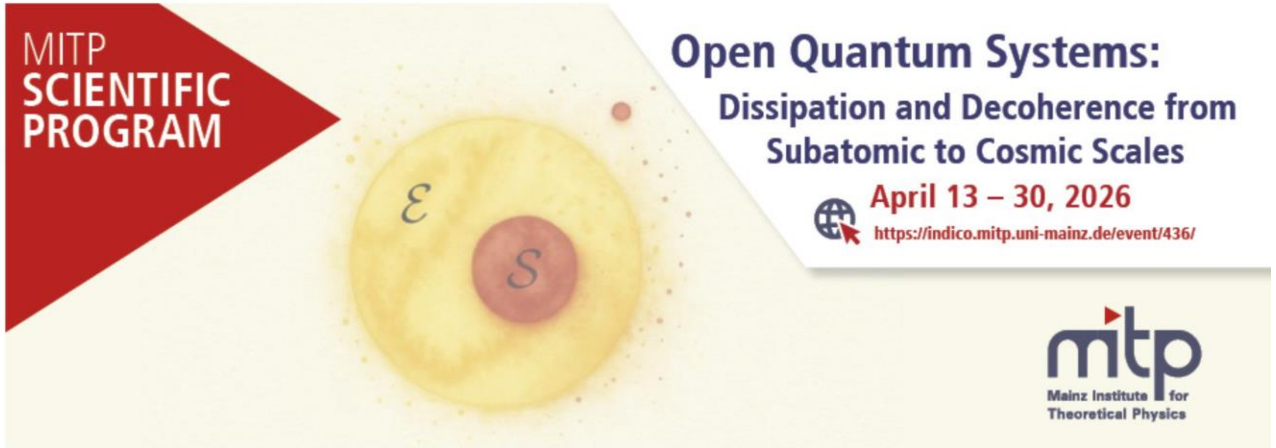
Heavy quarkonium spin transport in a hot medium

Shuo Fang (TUM &USTC)

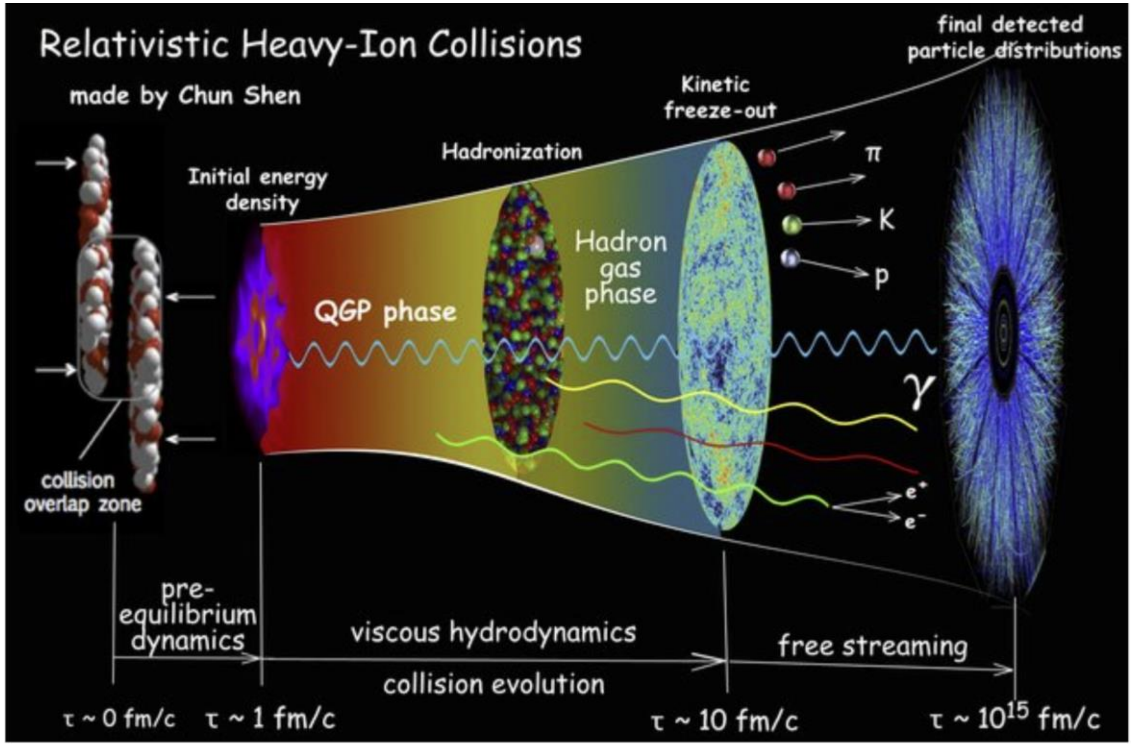
In collaboration with **Nora Brambilla, Tom Magorsch, Panayiotis Panayiotiou, Antonio Vairo**

Open Quantum Systems: Dissipation and Decoherence from Subatomic to Cosmic Scales

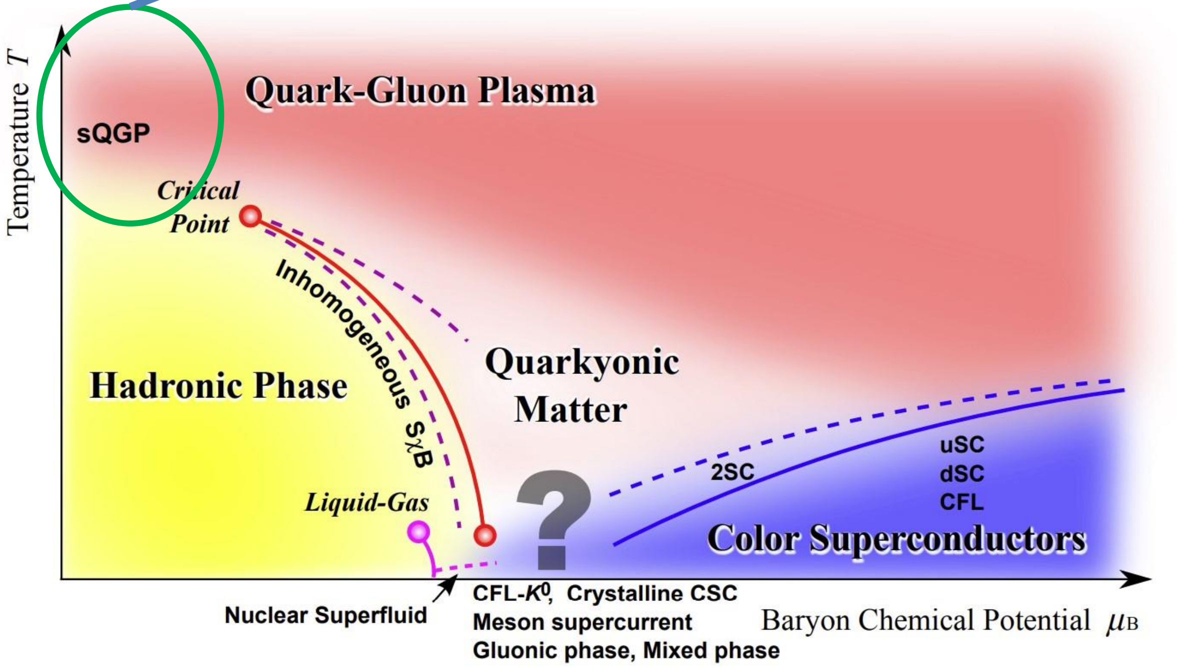
April 13 – 30, 2026 @Mainz, Germany



Deconfined quark matter at high temperature



Low-μ high-T matter produced in heavy ion collisions



Soft probes: Strangeness, EM

Hard probes: Jet, heavy quark(onia)

Collective flow, HBT, polarization …

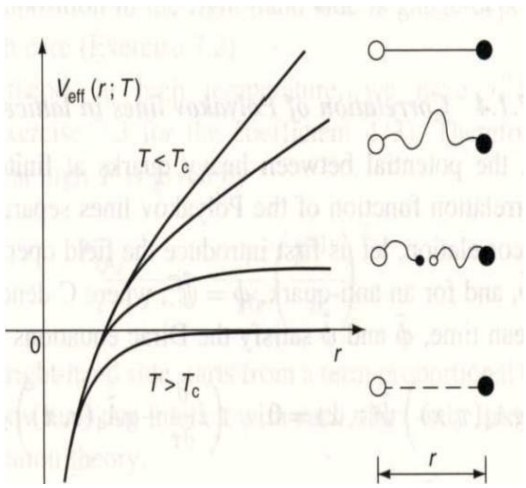
Recent review on QGP:
Harris, Muller, EPJC 84 (2024) 3, 247

Gyulassy, nucl-th/0403032;Shuryak, NPA 750, 64 (2005)

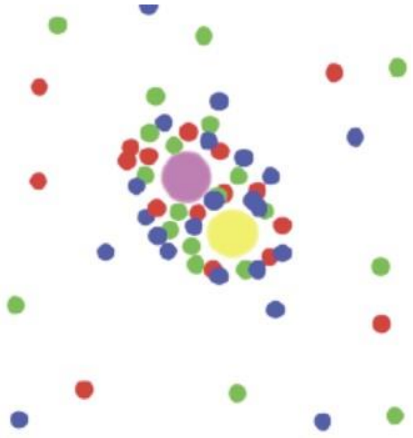
Fukushima, Hatsuda, Rept.Prog.Phys. 74 (2011) 014001

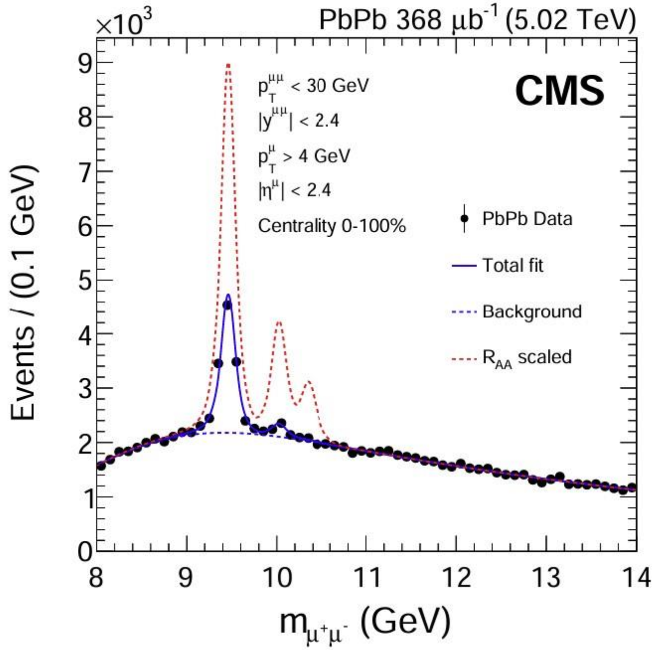
Heavy quarkonium in medium (I)

- One of smoking guns of deconfined phase of QCD matter: quarkonium yield suppression.



Matsui, Satz, PLB 178(1986), 416





[CMS collaboration] PRL 120 (2018)

$$V_{\rm eff}(r)=\sigma r-\frac{\alpha_{\rm eff}}{r}$$

Softening string tension →

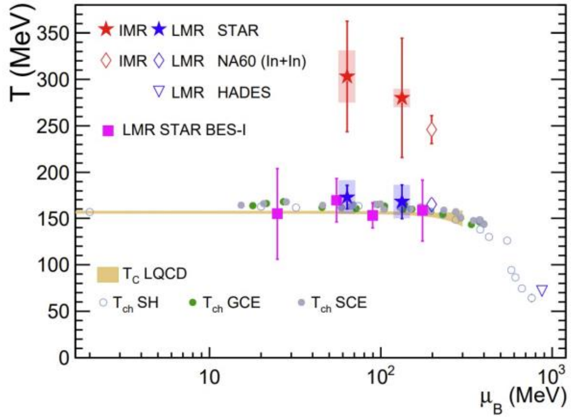
$$V_{\rm eff}(r) = -\frac{\alpha_{\rm eff}}{r}e^{-r\lambda_{\rm D}^{-1}}$$

In vacuum

Debye screening

Heavy quarkonium in medium (II)

- Debye screening not valid enough:
  - Temperature is not that high;
  - R_{AA} almost not changed with higher temperature.
- Modern view: Dynamical thermometer of strongly-coupled medium.
  - Dissociation/regeneration before screening due to imaginary part of effective potential: Landau damping and singlet ↔ octet transition (Dominant for $E \gtrsim T$); open heavy flavor recombination;
  - Non-eq. dynamics coupled with fluid evolution;
  - Spectral function and diffusion coefficients.

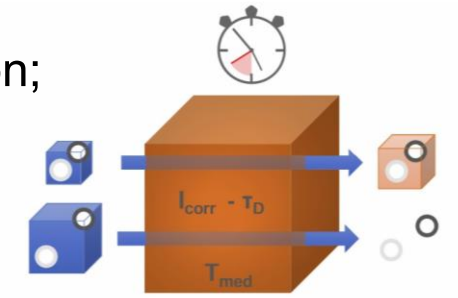


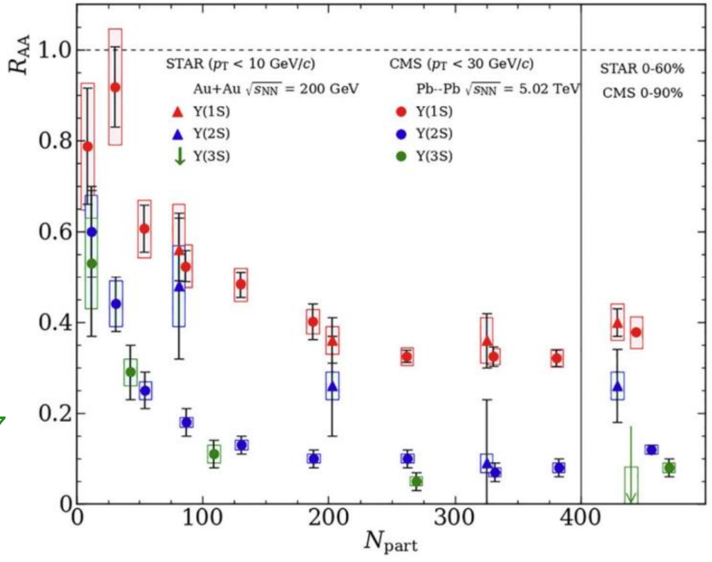
[STAR] Nature Commun. 16 (2025) 1, 9098

Equilibrium medium

$$V_{\rm eff}(r) = -\frac{\alpha_{\rm eff}}{r} e^{-r\lambda_D^{-1}} + i{\rm Im}V_{\rm eff}(r)$$

Debye screening + thermal dissociation/regeneration



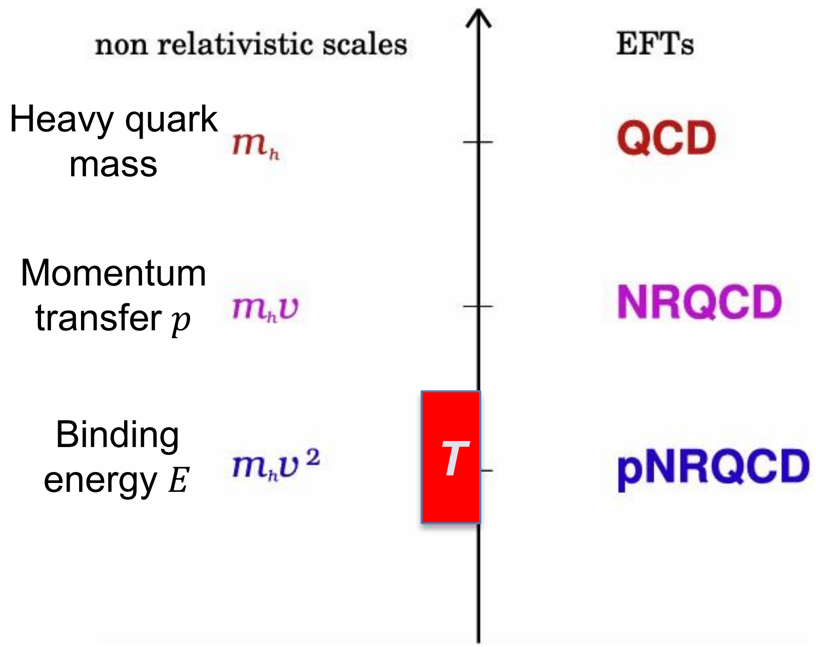


Rothkopf, Phys. Rept. 858 (2020) 1-117
He, van Hess, Rapp, PPNP 130 (2023) 104020
Andronic et al. EPJA 60 (2024) 4, 88

Laine,et al. JHEP 03 (2007) 054; Beraudo et al. NPA 806(2008), 312
Brambilla,et al. PRD 78,014017 (2008)

[CMS] PLB 790,270(2019);
[STAR] PRL, 130, 112301 (2023)

Description of heavy quarkonium with QCD EFT



Brambilla, Pineda, Soto, Vairo, Nucl.Phys.B 566 (2000) 275
Brambilla, Pineda, Soto, Vairo, Rev.Mod.Phys. 77 (2005)

Inverse mass expansion: M^{-1}

Multipole expansion: r

(Weakly-coupled pNRQCD)

Picture taken from *EPJC 83 (2023), 1125*

$$M \gg r^{-1} \gg E$$

E.g. b quark mass 4.85 GeV, $v \sim \alpha_s(a_0^{-1}) \sim 0.28$

Medium temperature: $T \sim 0.5$ GeV

- Degrees of freedom:
  quark-antiquark pair $3 \otimes \bar{3} = 8 \oplus 1$;

  S : color-singlet field O : color-octet field

  ultrasoft gluons $\boldsymbol{E}, \boldsymbol{B}$; light quarks

- Lagrangian up to NLO multipole expansion:

$$\mathcal{L} = \int \mathrm{d}^3\boldsymbol{r} \mathrm{S}^\dagger(i\partial_0 - h_s)\mathrm{S} + \mathrm{O}^\dagger(iD_0 - h_o)\mathrm{O}$$

$$O(r^1) \quad + V_\mathrm{A}\left(\mathrm{S}^\dagger \boldsymbol{r}\cdot g\tilde{\boldsymbol{E}}\mathrm{O} + \mathrm{H.c.}\right) + V_\mathrm{B}\mathrm{O}^\dagger\{\boldsymbol{r}\cdot g\tilde{\boldsymbol{E}}, \mathrm{O}\}$$

$h_{s/o}$: singlet/octet Hamiltonian

Introduction to open quantum system formalism

$$H_{\rm tot} = H_{\rm sys} + H_{\rm env} + H_{\rm int} \qquad H_{\rm int} = \sum_\alpha O^\alpha_{\rm sys} \otimes O^\alpha_{\rm env},$$

$$\frac{\mathrm{d}\rho_{\rm tot}}{\mathrm{d}t} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

von Neumann equation (unitary)

Trace over env.

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{\rm sys}(t) = -\int_{t_0}^{t} \mathrm{d}s \mathrm{Tr}[H_{\rm int}(t), [H_{\rm int}(s), \rho_{\rm tot}(s)]]$$

Markovian limit

$$\rho_{\rm tot}(s) \approx \rho_{\rm sys}(t) \otimes \rho_{\rm env}(t_0)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -i[H, \rho] + \sum_n \left(C_n \rho C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho\} \right)$$

Lindblad equation (non-unitary)

C_i: collapse operators, $H = H_{\rm sys} + \mathrm{Lamb\ shift}$

Breuer and Petruccuibe, The theory of open quantum systems;
Rivas and Huelga, Open Quantum Systems;
Akamatsu, PPNP 123(2022) 103932;

- Born-Markovian approximation: system is weakly coupled with environment.
- Lindblad eq. is trace preserving, time irreversibility, and complete positive.
- If perturbatively treating system-environment coupling, positivity may be jeopardized.

Microscopic derivation of quantum master equations (I)

- Various time scales from multiple energy scales: system intrinsic time τ_{S}, environment relaxation time τ_{E}, open system in-environment relaxation time τ_{R}.
- Relative relations among these time scales allow for the derivation of quantum master equations from microscopic theories: the first-principle QCD EFT.

Akamatsu, Rothkopf, PRD (2012); Akamatsu, PRD (2015); Brambilla et al. PRD(2017), PRD(2018);Blaizot, Escobedo, JHEP(2018); Delorme, et al. APPBPS. (2023); Yao, Mehen PRD(2019), JHEP(2021);

System	Heavy quark-antiquark pair ($\rho_{s/o}$)	$H_{\text{sys}} = h_{s/o}$	$\tau_{\mathrm{S}} \sim E^{-1}$
Environment	In-medium light DoFs (gluons, light quarks)	$H_{\text{env}} = H_{\text{light}}$	$\tau_{\mathrm{E}} \sim T^{-1}$
system-environment interaction	Dipole transition arranged by $\{r, M^{-1}\}$ expansion (C_n)	$H_{\text{int}} \sim \mathrm{S}^{\dagger}\boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{O}$	$\tau_{\mathrm{R}} \sim \Gamma_{Q\bar{Q}}^{-1}$

$\tau_{\mathrm{R}} \gg \tau_{\mathrm{E}}$, Markovian limit

$\tau_{\mathrm{S}} \gg \tau_{\mathrm{E}}$, Quantum Browian regime → High-temperature limit, resolving single quarks in the bound state

$\bar{Q}$ Q Probing medium long-wavelength excitations

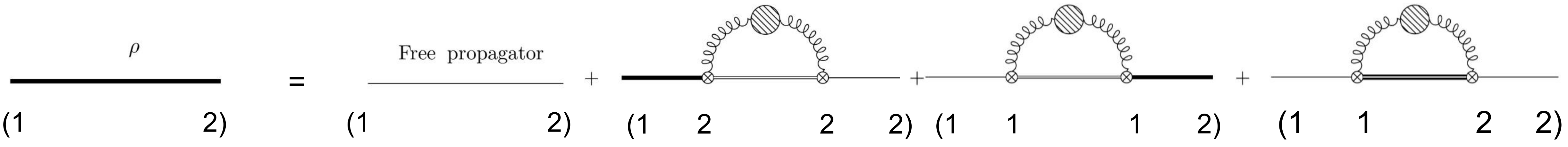
$\tau_{\mathrm{R}} \gg \tau_{\mathrm{S}}$, Quantum optical regime → Low-temperature limit, quarkonium bound-state as basic D.o.F.

$\bar{Q}Q$ Well-defined energy eigenstates

Akamatsu, PPNP 123(2022) 103932;
Yao, IJMPA 36(2021) 2130010

Microscopic derivation of quantum master equations(II)

- A non-equilibrium field-theory derivation:



➢ The density operator is identified as "12" component of real-time propagator:

$$\langle T_{12} S(\boldsymbol{r}, \boldsymbol{R}, t) S^{\dagger}(\boldsymbol{r}', \boldsymbol{R}', t') \rangle = \langle \boldsymbol{r}', \boldsymbol{R}' | \rho_s(t'; t) | \boldsymbol{r}, \boldsymbol{R} \rangle$$

➢ In the dilute limit: keep linear order in density of heavy quarkonium.

➢ Markovian limit: in the r.h.s. the "11" and "22" propagator are approximated as tree-level propagator, $e^{-ih_s(r)(t-t_0)} \rho_s(t_0; t_0) e^{ih_s(r)(t-t_0)} \rightarrow \rho_s(t; t)$

$$\langle T_{11} S(\boldsymbol{r}, \boldsymbol{R}, t) S^{\dagger}(\boldsymbol{r}', \boldsymbol{R}', t') \rangle = \theta(t - t') \delta^{(3)}(\boldsymbol{r} - \boldsymbol{r}') \delta^{(3)}(\boldsymbol{R} - \boldsymbol{R}') e^{-ih_s(\hat{r}, \hat{R})(t-t')} + \mathcal{O}(\rho)$$

Quarkonium real-time dynamics as an OQS (I)

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -i[H,\rho] + \sum_{nm} h_{nm}\left(L_i^n \rho L_i^{m\dagger} - \frac{1}{2}\{L_i^{m\dagger}L_i^n,\rho\}\right),\qquad h = \begin{pmatrix} 0 & 1 & & \\ 1 & 0 & & \\ & & 0 & 1 \\ & & 1 & 0 \end{pmatrix}$$

$$L_i^n \sim A_i^{uv} = \frac{g^2}{2N_c}\int_{t_0}^{t} \mathrm{d}t_2 e^{ih_u(t_2-t)} r^j e^{ih_v(t-t_2)} \langle \boldsymbol{E}^{a,j}(t_2)\boldsymbol{E}^{a,j}(t)\rangle$$

$$\Gamma_{Q\bar{Q}} \sim \alpha_s^2 r^2 T^3 \quad \ll \quad T,\ (E \ll T)$$

Thermal pQCD calculation: Brambilla, et al. PRD(2008);

$$\tau_S \gg \tau_E \quad , \quad \text{Quantum Brownian limit}$$

Lindblad eq. *Brambilla, et al. PRD(2017), PRD(2018)*

$$\Gamma_{Q\bar{Q}} \sim \alpha_s r^2 E^3 \quad \ll \quad E,\ (T \lesssim E)$$

Brambilla, et al. JHEP 12 (2011) 116

$$\tau_R \gg \tau_S \quad , \quad \text{Quantum optical limit}$$

Boltzmann eq. *Yao, Mehen, PRD(2019), JHEP(2021)*

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -i[H,\rho] + \sum_n \left(C_n \rho C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n,\rho\}\right) \qquad \rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{\boldsymbol{r}^2}{2}\gamma(t)\begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

$$\kappa + i\gamma = \frac{g^2}{6N_c}\int_{-\infty}^{+\infty} \mathrm{d}s \langle T_{11}\boldsymbol{E}^a(s,\boldsymbol{0})\cdot \boldsymbol{E}^a(0,\boldsymbol{0})\rangle$$

Up to LO E/T expansion

$$\boldsymbol{C}^0 = \sqrt{\frac{\kappa(t)}{N_c^2-1}}\boldsymbol{r}\begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix},\quad \boldsymbol{C}^1 = \sqrt{\frac{(N_c^2-4)\kappa(t)}{2(N_c^2-1)}}\boldsymbol{r}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

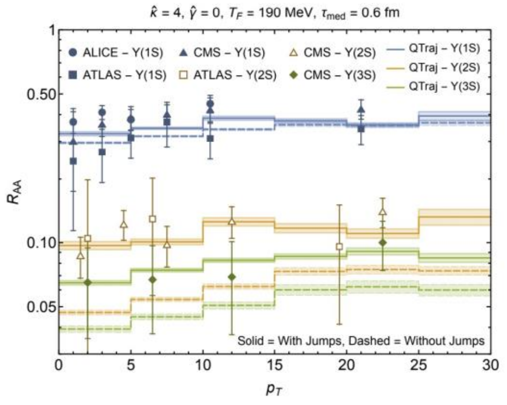
$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}}_Q\cdot\nabla_{\boldsymbol{x}_Q} + \dot{\boldsymbol{x}}_{\bar{Q}}\cdot\nabla_{\boldsymbol{x}_{\bar{Q}}}\right) f_{Q\bar{Q}}(\boldsymbol{x}_Q,\boldsymbol{p}_Q,\boldsymbol{x}_{\bar{Q}},\boldsymbol{p}_{\bar{Q}},t) = \mathcal{C}_{Q\bar{Q}} - \mathcal{C}_{Q\bar{Q}}^{+} + \mathcal{C}_{Q\bar{Q}}^{-}$$

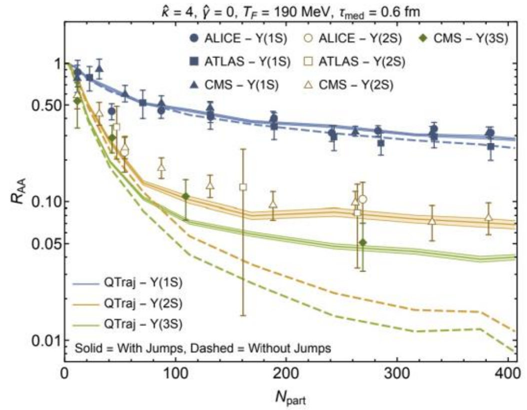
$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}}\cdot\nabla_{\boldsymbol{x}}\right) f_{nls}(\boldsymbol{x},\boldsymbol{p},t) = \mathcal{C}_{nls}^{+} - \mathcal{C}_{nls}^{-},$$

Distribution functions for states specified by definite quantum number

Quarkonium real-time dynamics as an OQS (II)

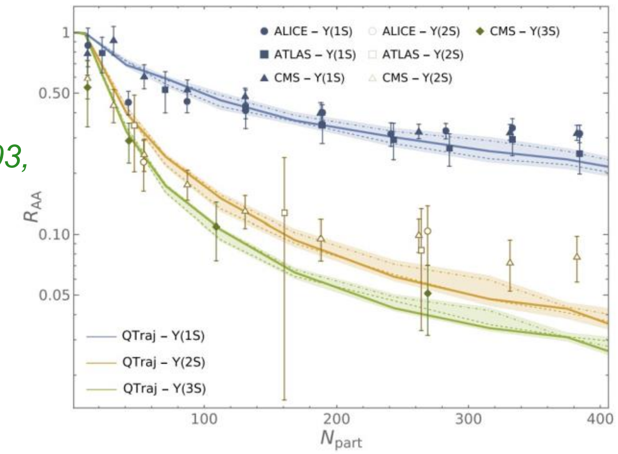
Lindblad eq. Solved with Qtraj.





Lindblad eq. with C_n in NLO E/T

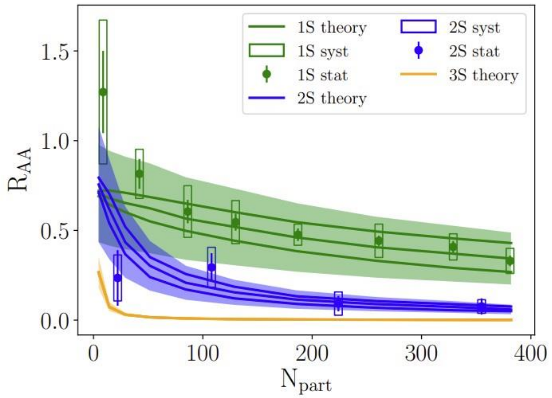
Brambilla, et al. JHEP 08 (2022) 303, PRD 108 (2023) 1, L011502

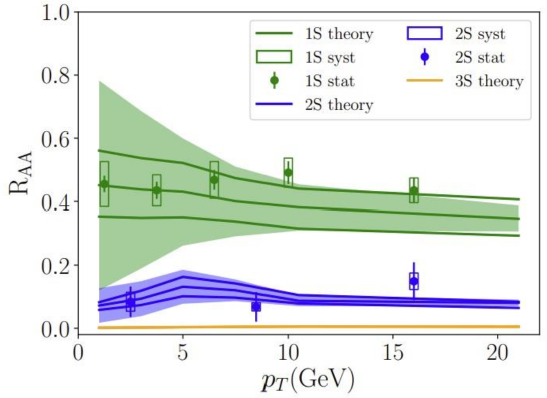


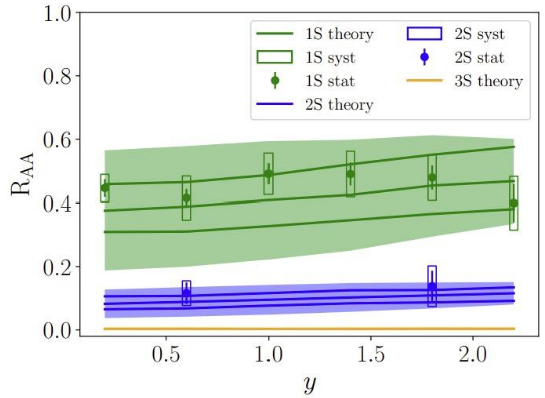
Three-loop QCD potential

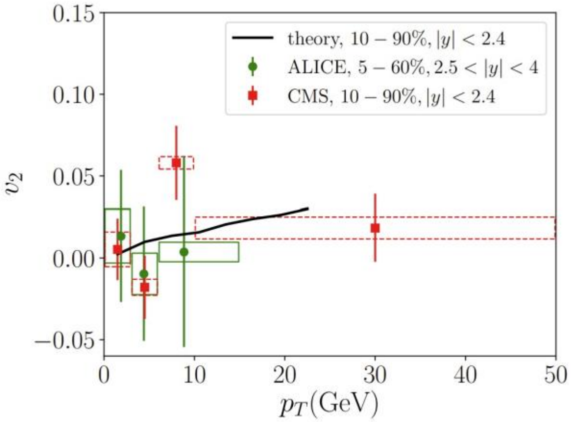
Brambilla, Magorsch, Vairo, PRD 109 (2024) 11, 114016

Boltzmann eq. *Yao, et al. JHEP 01 (2021) 046*

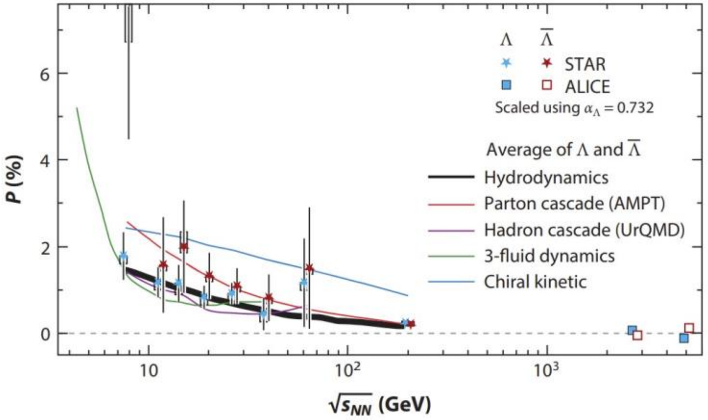




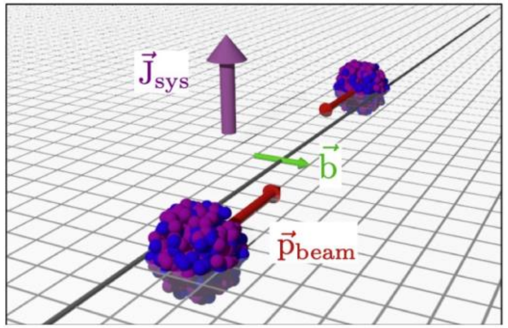




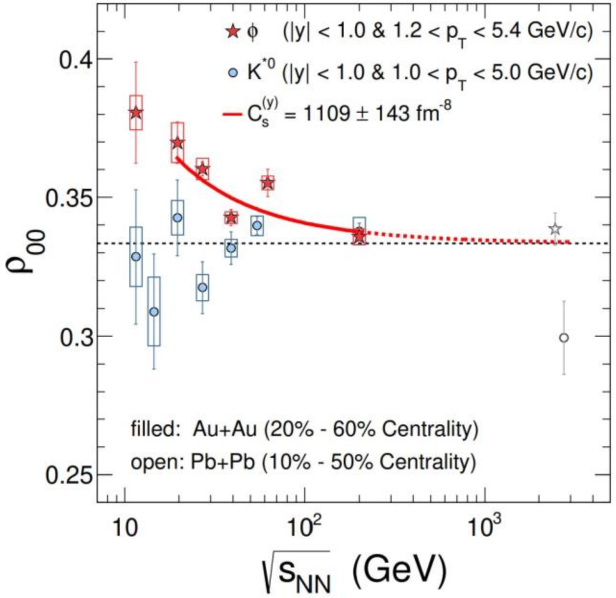
Polarization as a probe of hot quark matter



[STAR Collaboration] Nature 548(2017) 62;
Becattini, Lisa, ARNPS 70 (2020) 395;



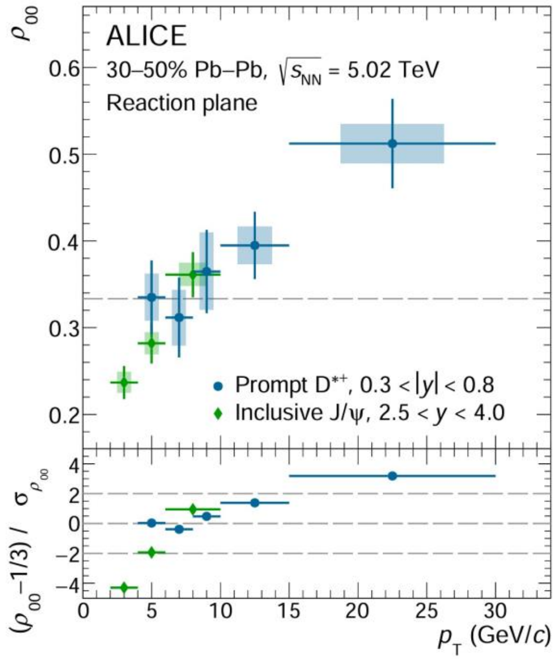
Hyperon polarization:
strong vorticity



[STAR Collaboration] Nature 614(2023) 244;
Sheng et al, PRL 131 (2023) 4, 042304

$$\delta\rho_{00} \sim C_1(\boldsymbol{p})\langle \boldsymbol{B}^{\phi}\cdot \boldsymbol{B}^{\phi}\rangle + C_2(\boldsymbol{p})\langle \boldsymbol{E}^{\phi}\cdot \boldsymbol{E}^{\phi}\rangle$$

Vector meson polarization:
meson field correlation (?)



[ALICE Collaboration]
PRL 131 (2023) 042303;2504.00714

Heavy VM polarization:
???
How to relate with QCD?

Heavy quarkonium polarization from dilepton decay (I)

- Dilepton production: a clean probe of QGP medium,

$$-Qe\int \mathrm{d}^4x J_\mu(x)A^\mu(x),$$

$$\frac{\mathrm{d}\mathcal{R}_{l\bar{l}}}{\mathrm{d}^4X\mathrm{d}^4P} = -\frac{32\pi^3\alpha_e^2}{3P^2}\sum_{i=1}^{N_\mathrm{f}} Q_i^2\left(\frac{2m_l^2}{P^2}+1\right)\left(1-\frac{4m_l^2}{P^2}\right)^{-\frac{1}{2}} \boxed{\rho_\mathrm{V}(X,P)} n_\mathrm{B}(P_0)$$

All thermal bath information are encoded in spectral functions: $\rho_\mathrm{V}(\frac{x+x'}{2},P) = \int \mathrm{d}^4(x-x')e^{-iP\cdot(x-x')}\langle[J^\mu(x),J_\mu(x')]\rangle$

Kapusta, Gale, Finite-Temperature Field Theory; Le Bellac, Thermal Field Theory

- Near threshold energy $P_0 \sim 2M$: heavy quarkonium decay channel. J^μ becomes heavy-quark current

$$\boldsymbol{J}^\mathrm{HQ}(x) = e^{2iMt}\psi^\dagger\boldsymbol{\sigma}\chi + e^{-2iMt}\chi^\dagger\boldsymbol{\sigma}\psi$$

in terms of pNRQCD current: $\langle J_\mu^\mathrm{HQ}(x)J_\nu^\mathrm{HQ}(x')\rangle \to \langle \mathrm{tr}_s\left(\sigma_\mu S^\dagger(\boldsymbol{x},\boldsymbol{0},t)\right)\mathrm{tr}_s\left(S(\boldsymbol{x}',\boldsymbol{0},t')\sigma_\nu\right)\rangle$

With spin degrees of freedom, $\frac{1}{2}\otimes\frac{1}{2} = 1\oplus 0$:

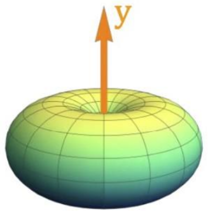
$$S(\boldsymbol{R},\boldsymbol{r},t) = \frac{1}{\sqrt{2}}\sigma_\mu S^\mu = \frac{1}{\sqrt{2}}S_\mathrm{sin} + \frac{\boldsymbol{\sigma}}{\sqrt{2}}\cdot\sum_\lambda(\boldsymbol{\epsilon}_\lambda S_\lambda)$$

Heavy quarkonium polarization from dilepton decay (II)

- The angular distribution uncovering off-diagonal elements of vector spectral function.

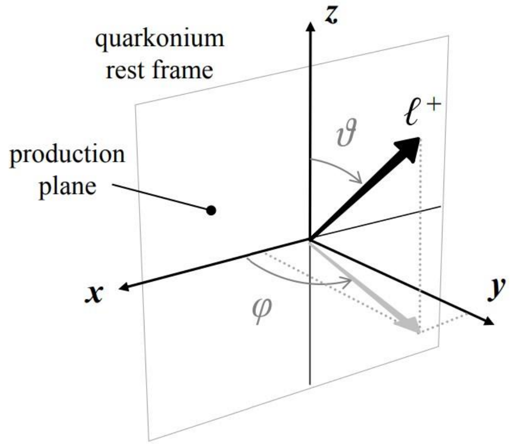
$$\frac{\mathrm{d}\mathcal{R}_{l\bar{l}}}{\mathrm{d}^4X\mathrm{d}^4P\mathrm{d}\cos\theta} = \frac{\alpha_e^2 \mathrm{Tr}\rho_{\lambda\lambda'}}{2(2\pi)^3P^2}\sum_{i=1}^{N_\mathrm{f}} Q_i^2\left[1+\rho_{00}^{\mathrm{N}}(P)\right]\left[1+\lambda_\theta\cos^2\theta\right]$$

$$\lambda_\theta = \frac{1-3\rho_{00}^{\mathrm{N}}(q)}{1+\rho_{00}^{\mathrm{N}}(q)}$$



$\rho_{00} < 1/3, \quad \lambda_\theta > 0,$
transverse polarization

$\rho_{00} > 1/3, \quad \lambda_\theta < 0,$
longitudinal polarization



Polarization kinematics for e.g. $\Upsilon(\bar{b}b) \to l^+l^-$

where spin density matrix elements are projections of pNRQCD correlator

$$\rho_{\lambda\lambda'}(q,X) = \varepsilon_\lambda^\mu(q)\varepsilon_{\lambda'}^{*\rho}(q)\rho_{\mu\rho}(q,X) \qquad \varepsilon_\lambda^\mu(q) \; : \; \text{Polarization vectors}$$

$$\rho_{\mu\nu}(q,\frac{R+R'}{2}) = -2\int \mathrm{d}^4\delta R e^{i(2Mv-q)\cdot\delta R}\langle S_\mu^\dagger(R,\mathbf{0})S_\nu(R',\mathbf{0})\rangle$$

✓ Total Decay rate only depends on $\mathrm{Tr}\rho_{\lambda\lambda'} = -\rho_\mu^\mu$

✓ Polarization probes the off-diagonal correlator.

Spin-dependent interactions of heavy quarkonium

- In NRQCD, spin-magnetic and spin-orbital vertices are of same order; moving particle carries OAM w.r.t. medium: induces polarization;
  - ✓ Center-of-mass motion may be important.

- Only spin-dependent interactions contribute to off-diagonal color-singlet **spin-triplet** correlators:

$$\mathcal{L}^{(\mathrm{LO})}_{\mathrm{pNRQCD}} = \int \mathrm{d}^3\boldsymbol{r}\,\left[S^{\dagger,i}(i\partial_t\delta^{ij} - \underline{h_s^{\mathrm{tri},ij}})S^j + O^{\dagger,a,i}(i\partial_t\delta^{ij} - \underline{h_o^{\mathrm{tri},ij}})O^{a,j}\right]$$

Brambilla, Pineda, Soto, Vairo, Rev.Mod.Phys. 77 (2005)

w. spin dependent potential

$$\mathcal{L}^{\mathrm{spin}}_{\mathrm{pNRQCD}} = 2\sqrt{\frac{T_{\mathrm{F}}}{N_c}}\int \mathrm{d}^3\boldsymbol{r}\left[S^\dagger_{\sin}\left(\frac{c_{\mathrm{F}}V^{(1,0)}_{\mathrm{SO}b}}{2M}g\boldsymbol{B}^a + c_{\mathrm{S}}V^{(2,0)}_{\mathrm{SO}a}\frac{[\boldsymbol{P}\times, g\boldsymbol{E}^a]}{16M^2}\right)^i O^{a,i} + S^{\dagger,i}\left(\frac{c_{\mathrm{F}}V^{(1,0)}_{\mathrm{SO}b}}{2M}g\boldsymbol{B}^a + c_{\mathrm{S}}V^{(2,0)}_{\mathrm{SO}a}\frac{[\boldsymbol{P}\times, g\boldsymbol{E}^a]}{16M^2}\right)^i O^a_{\sin} + \mathrm{H.c.}\right],$$

(leading order)

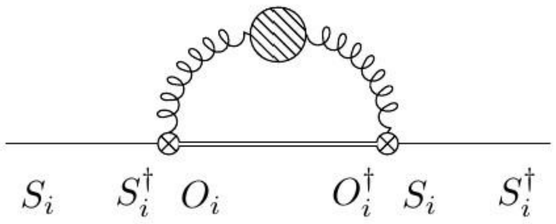
Brambilla, Gromes, Vairo, PLB 576, 314(2003)

Spin-chromomagnetic coupling

Spin-orbital coupling

✓ Chromoelectric dipole transition only contributes diagonal correlators

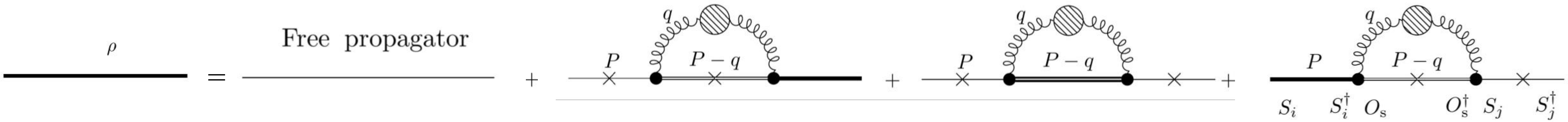
$$\mathrm{Tr}_s(\mathrm{S}^\dagger\boldsymbol{r}\cdot g\tilde{\boldsymbol{E}}\mathrm{O})$$
$$\propto \quad S_i^\dagger\boldsymbol{r}\cdot g\tilde{\boldsymbol{E}}O_i$$

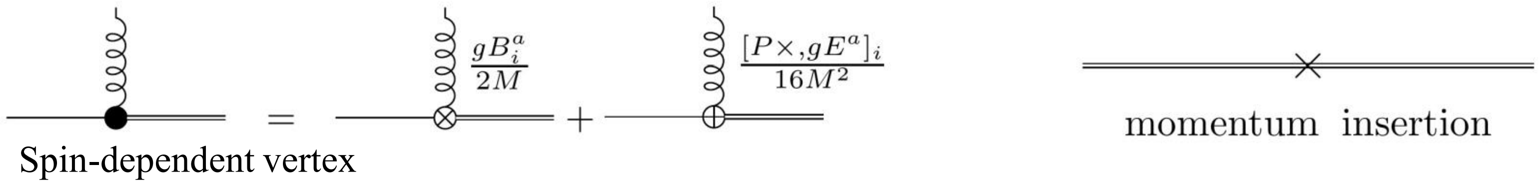


5/4/2026

Real-time spin evolution of heavy quarkonium (I)

Up to leading inverse-mass expansion and recoiling effects, from the real-time Dyson-Schwinger equation, we derive a spin Boltzmann equation:





Also see: *Yang, Yao PRD110 (2024) 7, 074037* for another derivation

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\delta\rho^s_{ji}(t,\boldsymbol{R},\boldsymbol{P}) + i[h_s, \delta\rho^s_{ji}(t,\boldsymbol{R},\boldsymbol{P})] \\
&= -\frac{g^2 T_{\mathrm{F}}}{M^2 N_c (d-1)} \int_{t_0}^{t} \mathrm{d}t_2 \int_q e^{-i(q_0+\Delta E^{\mathrm{cm}}_{\boldsymbol{P},\boldsymbol{P}-\boldsymbol{q}})(t-t_2)} e^{-ih_o(t-t_2)} e^{ih_s(t-t_2)} \star \mathcal{G}^{ji}_{11}(q,P,\boldsymbol{\nabla}_{\boldsymbol{R}})\rho^s_{kk}(t,\boldsymbol{R},\boldsymbol{P}) + \mathrm{H.c.} \\
&+ \frac{g^2 T_{\mathrm{F}}}{M^2 N_c (N_c^2-1)} \int_{t_0}^{t} \mathrm{d}t_2 \int_q e^{-i(q_0+\Delta E^{\mathrm{cm}}_{\boldsymbol{P},\boldsymbol{P}-\boldsymbol{q}})(t-t_2)} e^{-ih_s(t-t_2)} e^{ih_o(t-t_2)} \star \mathcal{G}^{ji}_{11}(q,P,\boldsymbol{\nabla}_{\boldsymbol{R}})\rho^o_{\mathrm{sin}}(t,\boldsymbol{R},\boldsymbol{P}-\boldsymbol{q}) + \mathrm{H.c.}
\end{aligned}
$$

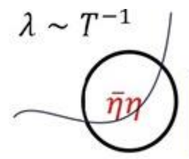
Spin damping

Spin regeneration

Real-time spin evolution of heavy quarkonium (II)

- In the quantum optical regime: $M \gg \quad r^{-1} \gg \quad E \gtrsim T$ we have well-defined basis $\{n, \lambda\}$ for bound state.

$\bar{Q}Q$ Well-defined energy eigenstates



Rotating-wave approximation (RWA)

The projection of density operator $\langle n|\rho(t)|n'\rangle \sim e^{-i\Delta E^{n}_{n'}t}\langle n|\rho(0)|n'\rangle$ vanishes unless $n = n'$ as $\Delta E^{n}_{n'}t \sim \tau_{\rm S}^{-1}\tau_{\rm R} \gg 1 \to$ fast oscillating exponential.[a]

[a] It only holds for finite-gaped energy levels and is not applicable to scattering state.

Sandwiching with $|n, \lambda\rangle$,
Rotating-wave approximation

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle n,\lambda|\delta\rho_{ji}(t,\boldsymbol{R})|n,\lambda\rangle = \mathcal{C}^{+}_{ji,n\lambda}[\rho^{o}_{\rm sin}] - \Gamma_{ji}(t,\boldsymbol{R},\boldsymbol{\nabla}_{\boldsymbol{R}})\langle n,\lambda|\rho^{s,l=1}_{kk}(t,\boldsymbol{R})|n,\lambda\rangle$$

Observable:

$$\rho^{n}_{\lambda\lambda'}(P) - \frac{1}{3}\delta_{\lambda\lambda'}\rho^{n}_{\lambda_1\lambda_1}(P) = \varepsilon^{i}_{\lambda}(P)\varepsilon^{*j}_{\lambda'}(P)\frac{\int \mathrm{d}^3\boldsymbol{R}_1\sum_{\lambda_1}|\phi_{n,\lambda_1}(\boldsymbol{0})|^2\langle n,\lambda|\delta\rho_{ji}(t_{\rm F},\boldsymbol{R},\boldsymbol{P})|n,\lambda\rangle}{\int \mathrm{d}^3\boldsymbol{R}_2\sum_{\lambda_2}|\tilde{\phi}_{n,\lambda_2}(\boldsymbol{0})|^2\langle n,\lambda|\rho_{kk}(t_{\rm F},\boldsymbol{R},\boldsymbol{P})|n,\lambda\rangle},$$

Real-time spin evolution of heavy quarkonium (III)

$$\Gamma_{ji}(t,\boldsymbol{R},\boldsymbol{\nabla}_{\boldsymbol{R}}) = -\frac{g^2 T_{\mathrm{F}}}{(d-1)M^2 N_c} 2\mathrm{Re}\int_{\boldsymbol{p}} \left|{}_s\langle n,\lambda|\boldsymbol{p}\rangle_o\right|^2 \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{-i}{q_0 + \frac{\boldsymbol{q}^2}{4M} - \Delta E^{so}_{n\lambda,\boldsymbol{p}} - i\epsilon} \star \mathcal{G}^{ji}_{11}(q,0,\boldsymbol{\nabla}_{\boldsymbol{R}})$$

Wavefunction overlap:

${}_s\langle n,\lambda|\boldsymbol{p}\rangle_o$

$$\mathcal{C}^{+}_{ji,n\lambda}[\rho^{o}_{\mathrm{sin}}] = \frac{g^2 T_{\mathrm{F}}}{M^2 N_c (N_c^2-1)} 2\mathrm{Re}\int_{\boldsymbol{p}_1,\boldsymbol{p}_2} \left[{}_s\langle n,\lambda|\boldsymbol{p}_1\rangle_o\right]\left[{}_o\langle \boldsymbol{p}_2|n,\lambda\rangle_s\right]$$
$$\times \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{i}{q_0 + \frac{\boldsymbol{q}^2}{4M} - \Delta E^{so}_{n\lambda,\boldsymbol{p}} + i\epsilon} \star {}^{*}\mathcal{G}^{ji}_{11}(q,0,\boldsymbol{\nabla}_{\boldsymbol{R}})\langle \boldsymbol{p}_1|\rho^{o,l=0}_{\mathrm{sin}}(t,\boldsymbol{R})|\boldsymbol{p}_2\rangle$$

Similar derivation of quarkonia density transport:
Yao, Mehen, JHEP(2021);

- Factorization of collision kernel up to $O(\frac{T}{M})$:

  strongly-coupled environment + perturbative dipole transition + quarkonium distribution.

- Quantum corrections: Moyal product for semiclassical ($\hbar$) expansion (for complete leading T/M expansion).

- Spin-chromomagnetic and spin-orbital interactions contribute at same order.

Overlooked by *Yang, Yao PRD110 (2024) 7, 074037; Chen, Lin, PRD 111 (2025) 7, 074002*

Real-time spin evolution of heavy quarkonium (IV)

- What is new from in-medium quarkonium polarization?
  - New spectral functions and chromo-field correlators: other aspects of finite-temperature QCD transport properties

Brambilla, et al, PRD 96 (2017) 3, 034021; PRD 97 (2018) 7, 074009; Yao, Mehen, JHEP 02 (2021) 062

Quarkonia transport: $$\kappa + i\gamma = \frac{g^2}{6N_c}\int_{-\infty}^{+\infty} \mathrm{d}s \langle T_{11}\Omega^\dagger(s)\boldsymbol{E}^a(s,\mathbf{0})\Omega(s)\cdot\Omega^\dagger(0)\boldsymbol{E}^a(0,\mathbf{0})\Omega(0)\rangle$$

Spin transport: $$\mathcal{G}^{ji}_{\alpha\beta}(q) = G^{ji}_{BB,\alpha\beta}(q) + \epsilon^{irs}\frac{q^s}{8M}G^{jr}_{BE,\alpha\beta}(q) + \epsilon^{jpq}\frac{q^q}{8M}G^{pi}_{EB,\alpha\beta}(q) + \frac{\epsilon^{irs}\epsilon^{jpq}q^q q^s}{64M^2}G^{pr}_{EE,\alpha\beta}(q),$$

  - Interplay of dynamics between heavy quarkonium and medium: perturbative collision term is driven by relative motions and chromo fields are defined in the quarkonium-rest frame.

$$\boldsymbol{E}^a = \gamma_v(\boldsymbol{E}^a_{\rm rf} - \boldsymbol{v}\times\boldsymbol{B}^a_{\rm rf}) - \frac{\gamma_v^2}{\gamma_v+1}\boldsymbol{v}(\boldsymbol{v}\cdot\boldsymbol{E}^a_{\rm rf}),$$
$$\boldsymbol{B}^a = \gamma_v(\boldsymbol{B}^a_{\rm rf} + \boldsymbol{v}\times\boldsymbol{E}^a_{\rm rf}) - \frac{\gamma_v^2}{\gamma_v+1}\boldsymbol{v}(\boldsymbol{v}\cdot\boldsymbol{B}^a_{\rm rf}),$$

Quarkonium-medium relative velocity

$$\boldsymbol{v} = -\frac{\boldsymbol{P}_{Q\bar{Q}} - \boldsymbol{u}(\boldsymbol{u}\cdot\boldsymbol{P}_{Q\bar{Q}})}{M_{Q\bar{Q}}} - \gamma_u\frac{E_{Q\bar{Q}} + (\boldsymbol{u}\cdot\boldsymbol{P}_{Q\bar{Q}})}{M_{Q\bar{Q}}}\boldsymbol{u}$$

$$\text{collision term} = \mathcal{C}^1_{ij}\boldsymbol{v}^i\boldsymbol{v}^j + \mathcal{C}^2_{ij}\boldsymbol{v}^{(i}\nabla^{j)}_{\boldsymbol{P}} + \mathcal{C}^3_{ij}\nabla^i_{\boldsymbol{P}}\nabla^j_{\boldsymbol{P}}$$

Simplification and numerical estimation (I)

- Plugging in perturbative correlators and neglecting momentum evolution, the leading-order equation is $O(\frac{T}{M})$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\delta n^{s}_{ji,1\mathrm{S}}(t,\boldsymbol{P},\boldsymbol{u}) = \frac{\boldsymbol{v}^i(\boldsymbol{P},\boldsymbol{u})\boldsymbol{v}^j(\boldsymbol{P},\boldsymbol{u})}{d-1}\int_{\boldsymbol{p}} \left|{}_s\langle 1\mathrm{S}|0\boldsymbol{p}\rangle_o\right|^2 w^3_{1\mathrm{S},\boldsymbol{p}}\left(1+n_{\mathrm{B}}^{<}(\Delta E^{so}_{1\mathrm{S},\boldsymbol{p}})\right) n^s_{1\mathrm{S}}(t) - \frac{\boldsymbol{v}^i(\boldsymbol{P},\boldsymbol{u})\boldsymbol{v}^j(\boldsymbol{P},\boldsymbol{u})}{N_c^2-1}\int_{\boldsymbol{p}_1}\int_{\boldsymbol{p}_2}\left[{}_s\langle 1\mathrm{S}|0\boldsymbol{p}_1\rangle_o\right]\left[{}_o\langle 0\boldsymbol{p}_2|1\mathrm{S}\rangle_s\right] w^3_{1\mathrm{S},\boldsymbol{p}} n_{\mathrm{B}}^{<}(\Delta E^{so}_{1\mathrm{S},\boldsymbol{p}_1})\,{}_o\langle \boldsymbol{p}_1|\hat{\rho}^{\mathrm{sin}}_{o,l=0}(t)|\boldsymbol{p}_2\rangle_o$$

$$n^s_{n\lambda}(t) = {}_s\langle n,\lambda|\hat{\rho}^{\mathrm{tri}}_s(t)|n,\lambda\rangle_s$$

  - ✓ Polarization originates from relative motion between quarkonium and medium. See also discussions in *Chen, Lin, PRD 111 (2025) 7, 074002*.

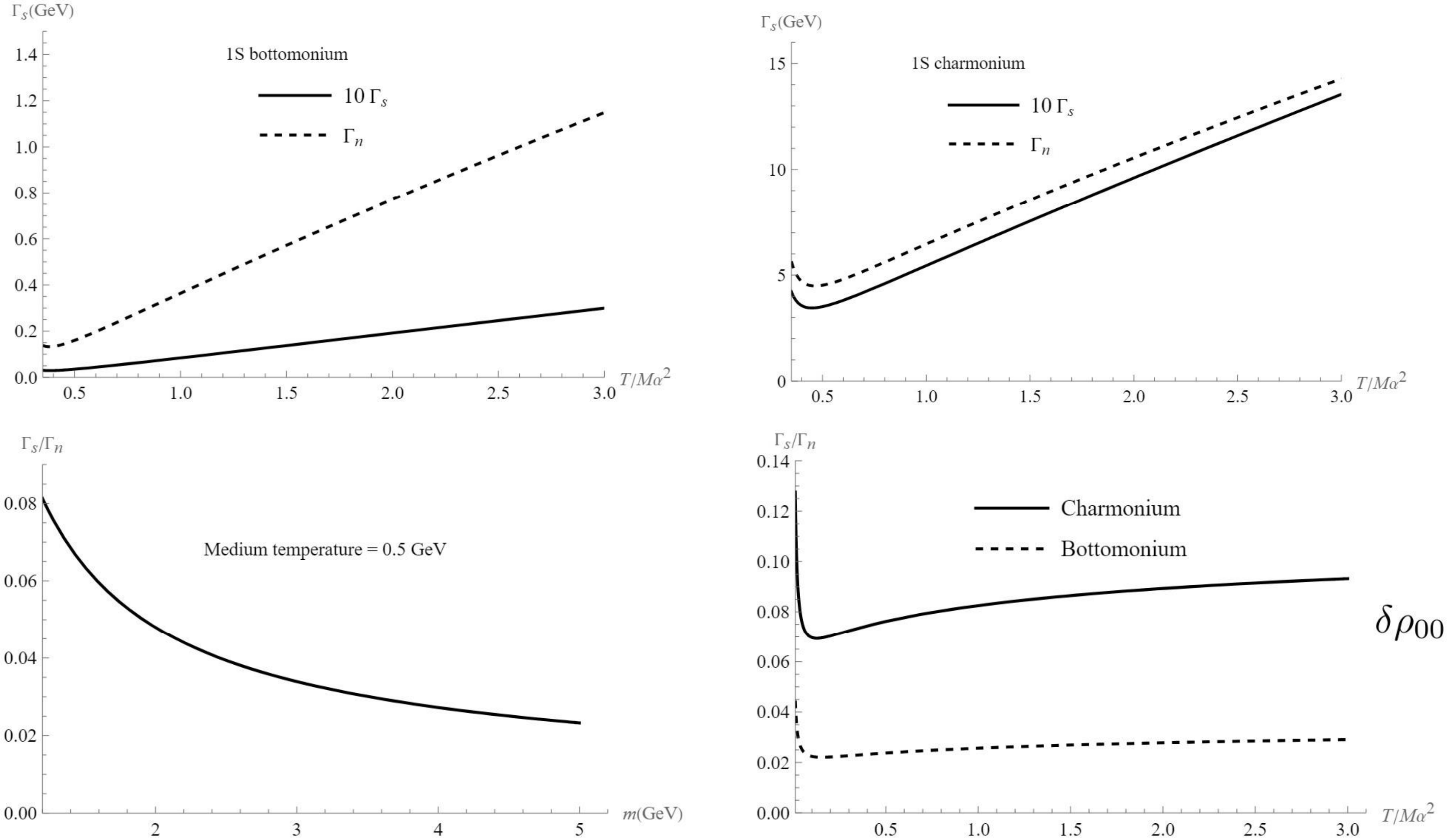
- Different from quarkonia damping rate governed by EE correlator only, spin damping rate governs spin dynamics, receiving contribution from EE, EB, BE and BB correlators

$$\Gamma^s_{n\lambda} = \frac{8\alpha_s C_F}{9M^2}\int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\left|{}_s\langle n,\lambda|\boldsymbol{p}\rangle_o\right|^2(\Delta E^{so}_{n\lambda,\boldsymbol{p}})^3\left(1+n_{\mathrm{B}}^{<}(\Delta E^{so}_{n\lambda,\boldsymbol{p}})\right)$$

$$\Gamma_n = \frac{2\alpha_s(N_c^2-1)}{3N_c}\int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\left|{}_s\langle n|\boldsymbol{r}|\boldsymbol{p}\rangle_o\right|^2(\Delta E^{so}_{n\lambda,\boldsymbol{p}})^3\left(1+n_{\mathrm{B}}^{<}(\Delta E^{so}_{n\lambda,\boldsymbol{p}})\right)$$

Brambilla, et al. JHEP 12 (2011) 116
Biondini, et al. JHEP 07 (2023) 006

Simplification and numerical estimation (II)



Γ_s: spin damping rate
Γ_n: number damping rate

$M_c = 1.27$ GeV, $M_b = 4.85$ GeV
α_s determined from Coulombic system: $\alpha_s(a_0^{-1}) = \frac{3}{2Ma_0}$

Estimation of polarization:

$$\delta\rho_{00} \sim (\boldsymbol{v}\cdot\boldsymbol{\varepsilon})^2 \delta n(t) n^{-1}(t) \sim \boldsymbol{v}^2 \Gamma_s \Gamma_n^{-1}$$
$$\sim \begin{cases} \lesssim 0.01 & , \text{ bottomonia } \boldsymbol{v}^2 \sim 0.1 \\ 0.04 & , \text{ charmonia } \boldsymbol{v}^2 \sim 0.3 \end{cases}$$

Simplification and numerical estimation (III)

$$\frac{\mathrm{d}}{\mathrm{d}t}\delta n^s_{ji,1S}(t,\boldsymbol{P},\boldsymbol{u}) = \frac{\boldsymbol{v}^i(\boldsymbol{P},\boldsymbol{u})\boldsymbol{v}^j(\boldsymbol{P},\boldsymbol{u})}{d-1}\int_{\boldsymbol{p}} |{}_s\langle 1S|0\boldsymbol{p}\rangle_o|^2\, w^3_{1S,\boldsymbol{p}}\left(1+n_B^{\lessgtr}(\Delta E^{so}_{1S,\boldsymbol{p}})\right) n^s_{1S}(t) - \frac{\boldsymbol{v}^i(\boldsymbol{P},\boldsymbol{u})\boldsymbol{v}^j(\boldsymbol{P},\boldsymbol{u})}{N_c^2-1}\int_{\boldsymbol{p}_1}\int_{\boldsymbol{p}_2} [{}_s\langle 1S|0\boldsymbol{p}_1\rangle_o]\,[{}_o\langle 0\boldsymbol{p}_2|1S\rangle_s]\, w^3_{1S,\boldsymbol{p}} n_B^{\lessgtr}(\Delta E^{so}_{1S,\boldsymbol{p}_1})\,{}_o\langle \boldsymbol{p}_1|\hat{\rho}^{\sin}_{o,l=0}(t)|\boldsymbol{p}_2\rangle_o$$

- Initial conditions: We assume no significant polarization at $t = 0$ for $p_T < 10$ GeV region similar to pp collisions. (No significant modifications from CNM effects, k_T broadening and nPDF, using ICEM, see *Cheung, Vogt, PRC 105 (2022) 5, 055202*).

- Bound-state distribution function is determined from 1S overlap of $\hat{\rho}_s^{\text{tri}}(t)$ and scattering distribution from Fourier transform of $\hat{\rho}_o^{\sin}(t, r, r')$: using Qtraj simulation for Lindblad equations.

- Relative velocity: $\boldsymbol{v}(\boldsymbol{q},\boldsymbol{u})$ gives $\{q_T, Y\}$ dependence; lab-frame fluid velocity $\boldsymbol{u}$ from hydrodynamical simulation (*in progress*…): for bottomonium, the polarization from dissociation with solution of Lindblad equation is

  ✓ Υ(1S) nearly no polarized.
  $$\delta\rho^{\Upsilon(1S)}_{\lambda\lambda'} = \varepsilon^i_\lambda \varepsilon^{*j}_{\lambda'}\boldsymbol{v}^i\boldsymbol{v}^j \times 0.0377 \approx 0.0029,$$

Summary

1. Heavy quarkonium is an important hard probe of QGP. Its complex in-medium behavior reveals aspects of strongly-coupled quark matter: from hydrodynamical property to non-perturbative spectral functions.

2. Heavy quarkonium spin polarization provides a unique window to study the remaining chromo-correlators (BB, BE). Revealing an interplay between fluid dynamics and quarkonia dynamics.

3. We derived a complete $O(\frac{T}{M})$ spin transport equation from QCD EFT within OQS approach including spin-orbital interactions and recoiling effects, which is factorized.

4. Heavy quarkonium spin relaxes much slower than the number transport. Heavier quarkonium has less polarization than lighter ones. We also predict that Upsilon may be not polarized at LHC energy.

Thanks for your attention！

Backup slides: Perturbative collision kernel

The momentum-dependent Boltzmann equation in quarkonium rest frame is

$$\frac{\mathrm{d}}{\mathrm{d}t}\delta f^s_{ji,n\lambda}(t,\boldsymbol{R},\boldsymbol{q},\boldsymbol{u}) \quad = \quad \mathcal{C}^+_{ji,n\lambda}[f^o_{\mathrm{sin}}] - \boldsymbol{v}^i\boldsymbol{v}^j\Gamma^s_{n\lambda}f^s_{n\lambda}(t,\boldsymbol{R},\boldsymbol{q},\boldsymbol{u})$$

where

$$\begin{aligned}\mathcal{C}^+_{ji,n\lambda}[f^o_{\mathrm{sin}}] = &-\frac{1}{N_c^2-1}\boldsymbol{v}^i\boldsymbol{v}^j\int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}W^3_{n\lambda,\boldsymbol{p}}f^o_{\mathrm{sin}}(t,\boldsymbol{R},\boldsymbol{P}_{\mathrm{cm}}=\boldsymbol{0};\boldsymbol{p})n^<_{\mathrm{B}}(\Delta E^{so}_{n\lambda,\boldsymbol{p}})\\ &+\frac{1}{4(N_c^2-1)}\left(\boldsymbol{v}^j\boldsymbol{\nabla}^i_{\boldsymbol{P}_{\mathrm{cm}}}+\boldsymbol{v}^i\boldsymbol{\nabla}^j_{\boldsymbol{P}_{\mathrm{cm}}}\right)\int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}W^4_{n\lambda,\boldsymbol{p}}f^o_{\mathrm{sin}}(t,\boldsymbol{R},\boldsymbol{P}_{\mathrm{cm}}=\boldsymbol{0};\boldsymbol{p})n^<_{\mathrm{B}}(\Delta E^{so}_{n\lambda,\boldsymbol{p}})\\ &-\frac{\boldsymbol{\nabla}^i_{\boldsymbol{P}_{\mathrm{cm}}}\boldsymbol{\nabla}^j_{\boldsymbol{P}_{\mathrm{cm}}}}{20(N_c^2-1)}\int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}W^5_{n\lambda,\boldsymbol{p}}f^o_{\mathrm{sin}}(t,\boldsymbol{R},\boldsymbol{P}_{\mathrm{cm}}=\boldsymbol{0};\boldsymbol{p})n^<_{\mathrm{B}}(\Delta E^{so}_{n\lambda,\boldsymbol{p}}),\end{aligned}$$

$$\Gamma^s_{n\lambda} \quad = \quad -\frac{1}{3}\int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}W^3_{n\lambda,\boldsymbol{p}}\left(1+n^<_{\mathrm{B}}(\Delta E^{so}_{n\lambda,\boldsymbol{p}})\right)$$

The scattering matrix elements is $W^k_{n\lambda,\boldsymbol{p}} \quad = \quad \frac{8\alpha_{\mathrm{s}}C_{\mathrm{F}}}{3M^2}\left|{}_s\langle n,\lambda|\boldsymbol{p}\rangle_o\right|^2\left(\Delta E^{so}_{n\lambda,\boldsymbol{p}}\right)^k$