Quantum Cosmology and Gravity group

Quantum master equations: Interacting QFTs and critical environments

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- B. Bowen, N. A., and A. Kamal, PRR 7, 043311 (2025) [arXiv:2403.18907]
- A. Keefe, N. A., and A. Kamal, Quantum 9, 1863 (2025) [arXiv:2405.01722]
- *A. Keefe, *B. Bowen, A. Lawrence, N. A., and A. Kamal, In preparation

MITP program on open quantum systems

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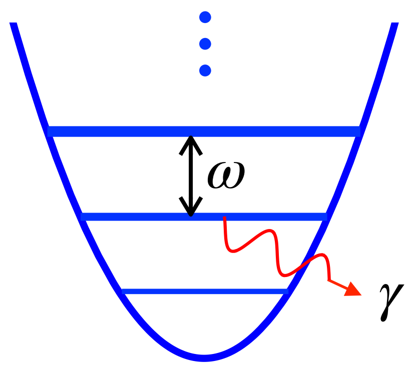
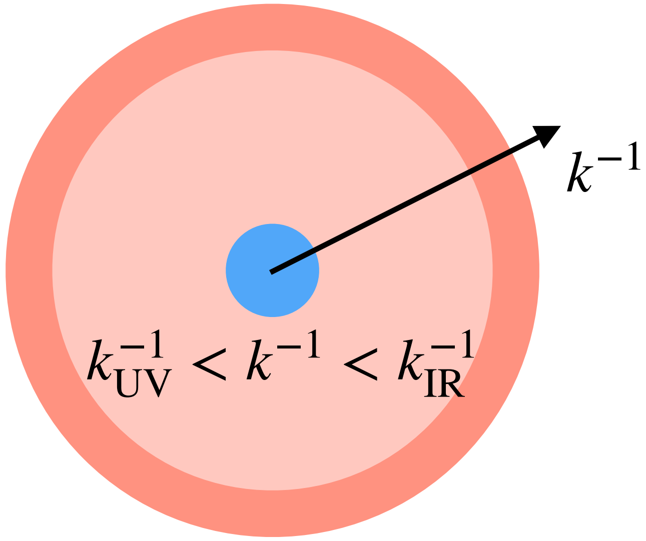
Motivation

- Most physical systems that we encounter are open, since they interact with some uncontrollable/unknown environment
- This is especially relevant for quantum systems, that typically decohere due to this interaction



- The open quantum system framework is also particularly relevant for EFT and cosmology —
  - There is a cosmological horizon beyond which we cannot make observations
  - We probe inflation only through light fields as any heavy fields decay away



S. Shandera, *N. A., and *A. Kamal, PRD 98, 083535 (2018) [arXiv:1708.00493]

F. Lopez and N. Bartolo (2025) [arXiv:2503.23150]

S. A. Salcedo, T. Colas, and E. Pajer, JHEP 10, 248 (2024) [arXiv:2404.15416]

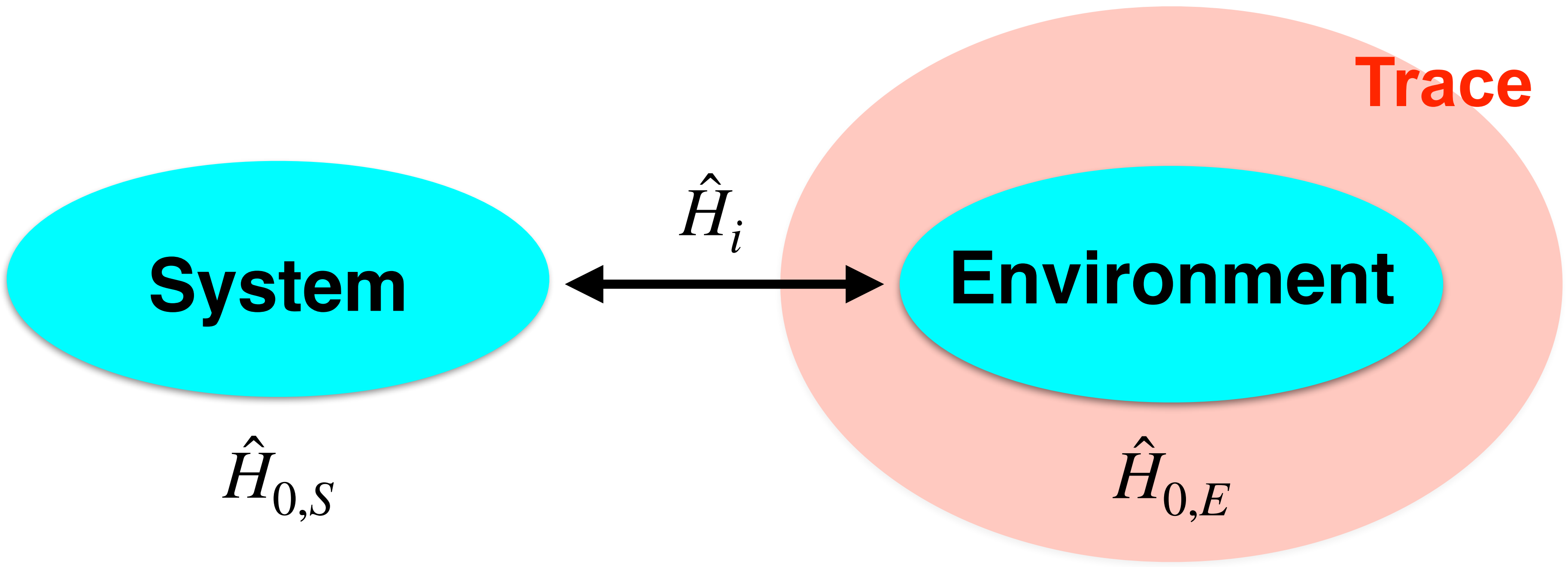
Motivation

- Decoherence can further be memory-less (Markovian) or have memory (non-Markovian) depending on exact properties of the environment and the system-environment interaction
- Understanding the distinction between Markovian and non-Markovian dissipation allows us to use it as a *resource*
  - Part 1: To resum correlations in interacting QFTs
  - Part 2: To probe criticality in a many-body environment

Outline

- Introduction
- Redfield master equation construction
- Part 1: Resummations in interacting QFTs
- Interacting QFTs (System: ϕ, Environment: χ)
- System observables: ϕ two-point correlations
- Part 2: Probing critical environments
- Qubit coupled to TFIM (System: Qubit, Environment: TFIM)
- System observables: Qubit dephasing rate, spectrum
- Discussion

Introduction



- The system+environment evolves unitarily with the full Hamiltonian $\hat{\sigma}(t) = \hat{U}(t, t_0)\hat{\sigma}(t_0)\hat{U}^\dagger(t, t_0)$, where

$$\hat{U}(t, t_0) = e^{-i\hat{H}_0(t-t_0)} T e^{-i\int_{t'=t_0}^{t} \hat{H}_{i,I}(t')}$$

Interaction picture

- Effective system dynamics are described by the reduced density operator $\hat{\rho}(t) = \mathrm{Tr}_E \hat{\sigma}(t)$

Introduction

- For the special case of a quantum Markov process, the evolution of $\hat{\rho}(t)$ is described by the Gorini-Kossakowski-Lindblad-Sudarshan (GKLS) master equation,

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = -i\left[\hat{H}, \hat{\rho}(t)\right] + \sum_{\alpha} \gamma_{\alpha} \left[\hat{L}_{\alpha}\hat{\rho}(t)L_{\alpha}^{\dagger} - \frac{1}{2}\left\{L_{\alpha}^{\dagger}L_{\alpha}, \hat{\rho}(t)\right\}\right]$$

Lindblad or jump operators

Decay rates for different decay channels

- More generally, one can carry out a microscopic derivation by explicitly tracing out the environment under a series of approximations, yielding an intermediate non-Markovian master equation called the Redfield equation

Redfield master equation construction

- Assume that the system and environment are initialized in a product state: $\hat{\sigma}(t_0) = \hat{\rho}(t_0) \otimes \hat{\rho}_E(t_0)$

- Using the von Neumann equation $\partial_t \hat{\sigma}_I(t) = -i[\hat{H}_{i,I}(t), \hat{\sigma}_I(t)]$, we can write an equation for the reduced density operator,

$$\frac{\mathrm{d}\hat{\rho}_I}{\mathrm{d}t} = -i\mathrm{Tr}_E\left[\hat{H}_{i,I}(t), \hat{\sigma}(t_0)\right] - \int_{t_1=t_0}^{t} \mathrm{Tr}_E\left[\hat{H}_{i,I}(t), \left[\hat{H}_{i,I}(t_1), \hat{\sigma}_I(t_1)\right]\right]$$

Contains an environment correlation function!

- This is still an *exact* expression

- We now make two approximations, on the interaction strength and nature of the environment correlation

Redfield master equation construction

- First, make the Born approximation (that the interaction is weak): $\hat{\sigma}_I(t_1) \approx \hat{\rho}_I(t_1) \otimes \hat{\rho}_E$
- Second, make the Markov approximation (that the environment correlation decays or is oscillatory with $\tau_E \ll \tau_S$): $\hat{\rho}_I(t_1) \to \hat{\rho}_I(t)$
- This yields the (non-Markovian, time-local) Redfield equation

$$\frac{\mathrm{d}\hat{\rho}_I}{\mathrm{d}t} = -i\left[\hat{H}_{\mathrm{eff}}(t), \hat{\rho}(t_0)\right] - \int_{t_1=t_0}^{t} \mathrm{Tr}_E\left[\hat{H}_{i,I}(t), \left[\hat{H}_{i,I}(t_1), \hat{\rho}_I(t) \otimes \rho_E\right]\right]$$

$\approx \hat{\rho}(t_0)$ in Dyson series perturbation theory

where $\hat{H}_{\mathrm{eff}}(t) = \mathrm{Tr}_E\left\{\hat{H}_{i,I}(t)\hat{\rho}_E\right\}$

- Caution: The Redfield equation violates positivity
- The Markovian approximation further sets $t - t_0 \to \infty$ in the integral limit

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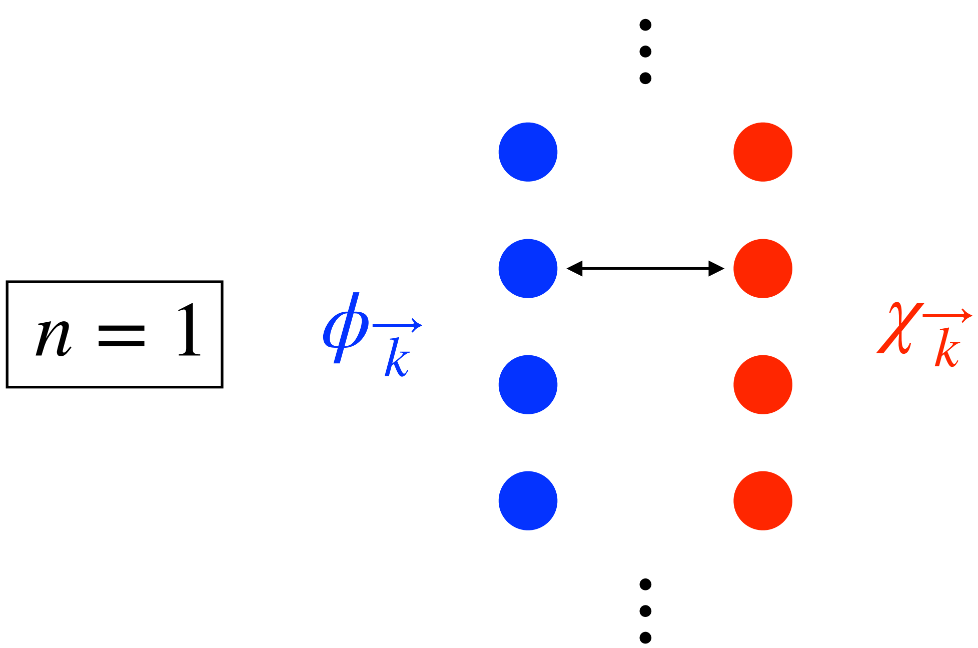
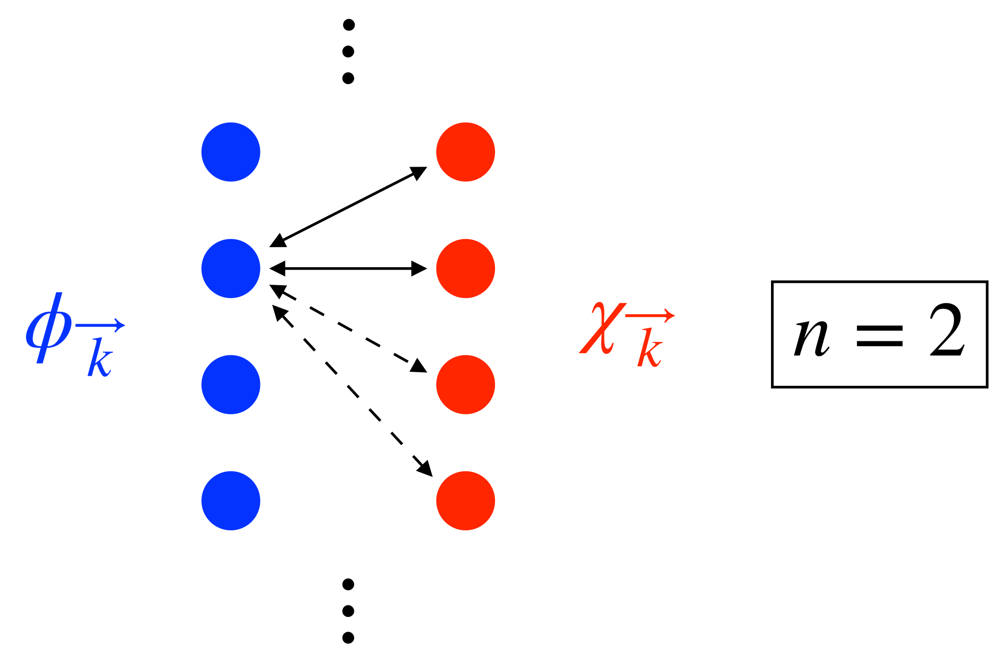
Interacting QFTs

- We will consider two interacting theories in $3+1D$ Minkowski. In the mostly-plus metric signature,

$$\mathscr{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}M^2\chi^2 - \frac{\lambda}{n!}\phi\chi^n \quad (n = 1, 2)$$





- Are the reduced dynamics of $\hat{\phi}$ Markovian or non-Markovian? What is the $\hat{\phi}$ equal-time two-point correlation?

B. Bowen, N. A., and A. Kamal, PRR 7, 043311 (2025) [arXiv:2403.18907]

Interacting QFTs: Redfield master equation

- It is convenient to write the interaction Hamiltonian as

$$\hat{H}_i = \frac{\lambda}{n!}\int_{\vec{x}} \hat{\phi}\hat{\chi}^n = \lambda \sum_{\alpha=1}^{2} \int_{\vec{k}} \hat{L}_{\vec{k},\alpha}\hat{\mathcal{O}}_{-\vec{k}}$$

where $\int_{\vec{k}} \equiv \int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{\omega_k^3}$ with dimensionless system operators

$$\hat{L}_{\vec{k},1} = \omega_k^{3/2}\hat{a}_{\vec{k}}, \qquad \hat{L}_{\vec{k},2} = \omega_k^{3/2}\hat{a}^{\dagger}_{-\vec{k}}$$

and environment operator

$$\hat{\mathcal{O}}_{-\vec{k}} = \frac{\omega_k}{\sqrt{2n!}}\hat{\chi}^n_{-\vec{k}}$$

- Now trace out $\hat{\chi}$ as outlined earlier to find the Redfield equation

Interacting QFTs: Redfield master equation

- The Redfield equation (in Schrödinger picture) for $\hat{\phi}$ is

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = -i\left[\hat{H}_{\text{eff}}(t), \hat{\rho}_0(t)\right] - i\left[\hat{H}_S(t), \hat{\rho}(t)\right]$$

$$+ \sum_{\alpha,\beta} \int_{\vec{k}} \gamma_{k,\alpha\beta}(t) \left(\hat{L}_{\vec{k},\beta} \hat{\rho}(t) \hat{L}^{\dagger}_{\vec{k},\alpha} - \frac{1}{2} \left\{ \hat{L}^{\dagger}_{\vec{k},\alpha} \hat{L}_{\vec{k},\beta}, \hat{\rho}(t) \right\} \right)$$

$\hat{H}_S(t)$: $= \hat{H}_{0,S} + \sum_{\alpha,\beta} \int_{\vec{k}} S_{k,\alpha\beta}(t) \hat{L}^{\dagger}_{\vec{k},\alpha} \hat{L}_{\vec{k},\beta}$

Lamb shift-like correction

$\gamma_{k,\alpha\beta}(t)$: Time-dependent decay rate

- The rotating wave approximation (RWA) can be made by dropping off-diagonal ($\alpha \neq \beta$) terms

Interacting QFTs: Redfield master equation

- The Lamb shift and decay rate can further be written in terms of the environment correlation,

$$S_{k,\alpha\beta}(t) = \frac{1}{2i}\left[\Gamma_{k,\beta}(t) - \Gamma^*_{k,\alpha}(t)\right]$$

$$\gamma_{k,\alpha\beta}(t) = \Gamma_{k,\beta}(t) + \Gamma^*_{k,\alpha}(t)$$

where

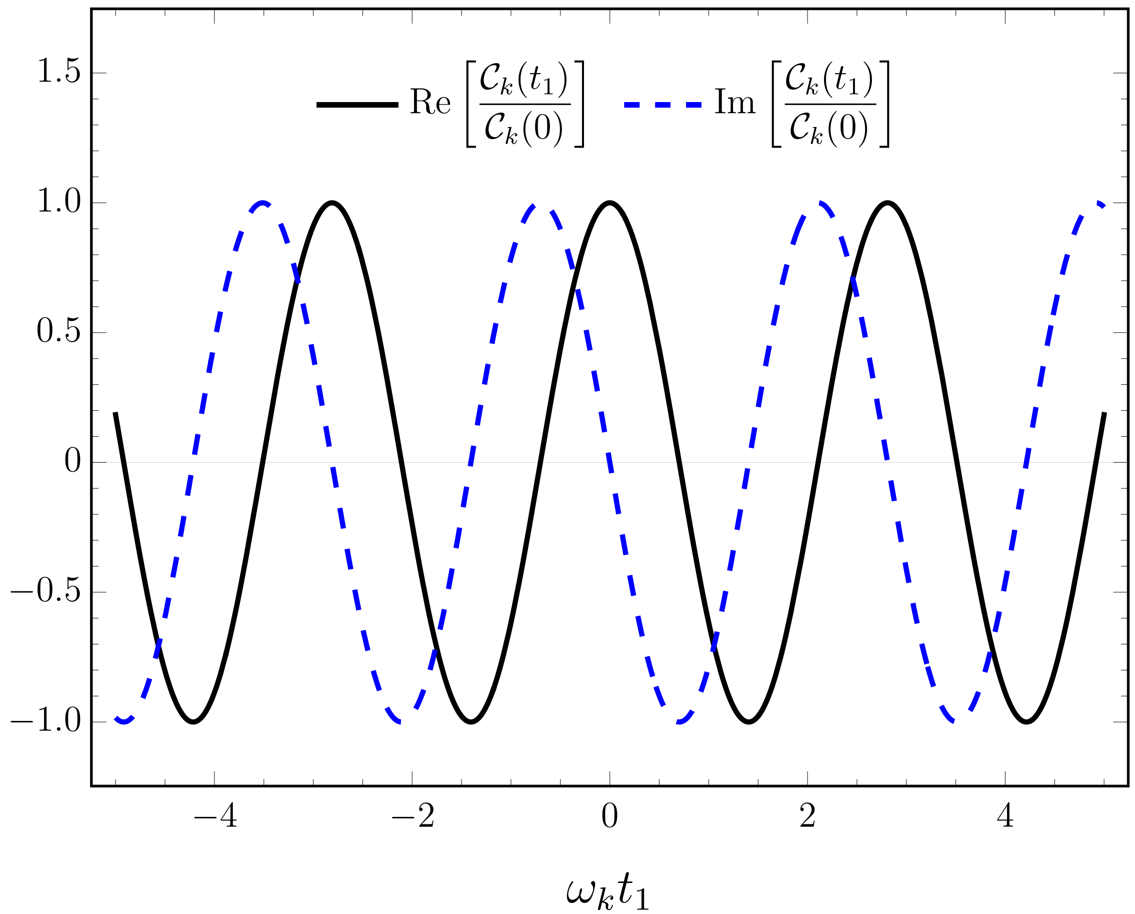
$$\Gamma_{k,\beta}(t) = \lambda^2 \int_{t_1=0}^{t-t_0} \frac{e^{-i(-1)^\beta \omega_k t_1}}{2\omega_k} \frac{8\pi^3}{n!} \int \delta^3\left(\vec{k} + \sum_i \vec{p}_i\right) \prod_{j=1}^{n} \frac{\mathrm{d}^3 p_j}{(2\pi)^3} \frac{e^{-i\Omega_{p_j} t_1}}{2\Omega_{p_j}}$$

Environment correlation, $C_k(t_1) \sim \left\langle \chi^n_{-\vec{k}}(t)\chi^n_{-\vec{k}'}(t-t_1)\right\rangle$

- If $C_k(t_1) \sim \delta(t_1)$, then the Lamb shift and decay rate are time-independent, and the Markovian approximation is justified!

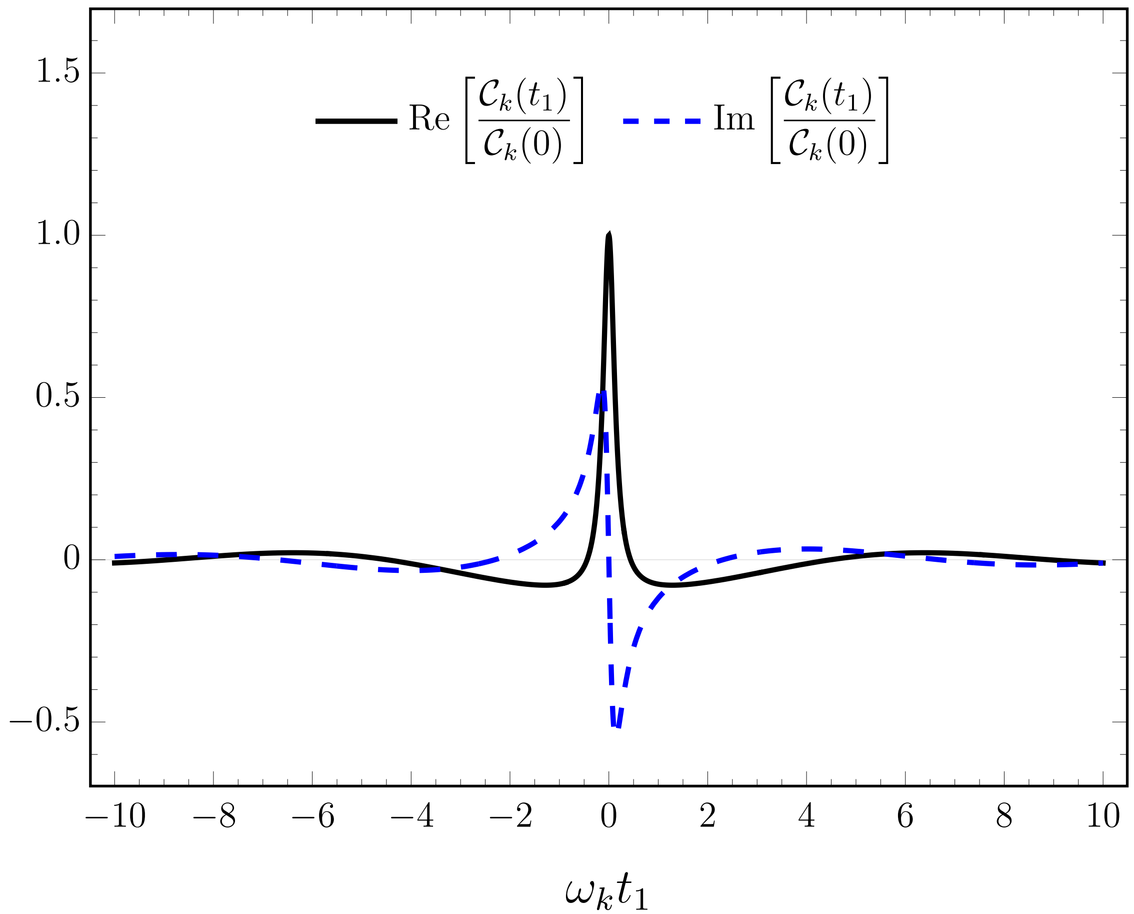
Environment correlation function

$n = 1$



$$C_k(t_1) = \frac{e^{-i\Omega_k t_1}}{2\Omega_k}$$

$n = 2$



$$C_k(t_1) = -\frac{i}{32\pi^2}\frac{e^{-ikt_1}}{t_1 - i\epsilon}$$

$$= -\frac{1}{32\pi}\delta(t_1) - \frac{ie^{-ikt_1}}{32\pi^2}P\left(\frac{1}{t_1}\right)$$

Two-point correlation: Calculation and approximations

- The equal-time two-point correlation function $\mathcal{G}_k(t)$ is defined by

$$\left\langle \hat{\phi}_{\overrightarrow{k}} \hat{\phi}_{\overrightarrow{k}'} \hat{\rho}(t) \right\rangle = \mathcal{G}_k(t)(2\pi)^3 \delta^3\left(\overrightarrow{k} + \overrightarrow{k}'\right)$$

$$\mathcal{G}_k(t) = \frac{1}{2\omega_k}[2\xi_{k,1}(t) + \xi_{k,3}(t)]$$

where $\xi_k(t) = \xi_{k,1}(t) + i\xi_{k,2}(t)$ is the $\hat{a}\hat{a}$ correlator and $\xi_{k,3}(t)$ is the symmetrized $\hat{a}\hat{a}^\dagger$ correlator

- We can now use the Redfield equation (in Schrödinger picture) to write coupled differential equations for $\xi_k(t)$ and $\xi_{k,3}(t)$

$$\dot{\xi}(t) = -[\gamma_-(t) + 2i\omega_S(t)]\xi(t) - 2iS_o(t)\xi_3(t) - \gamma_o(t)$$

$$\dot{\xi}_3(t) = -\gamma_-(t)\xi_3(t) + 8\mathrm{Im}[S_o(t)\xi^*(t)] + \gamma_+(t)$$

Two-point correlation: Calculation and approximations

- Different approximations (perturbation theory, Markovian, RWA) simplify the coefficients in the differential equations, e.g.,

Vanish under RWA

$$\dot{\xi}(t) = -\left[\gamma_-(t) + 2i\omega_S(t)\right]\xi(t) - 2iS_o(t)\xi_3(t) - \gamma_o(t)$$

Constant in Markovian approximation

Initial-time values in perturbation theory (except for free part)

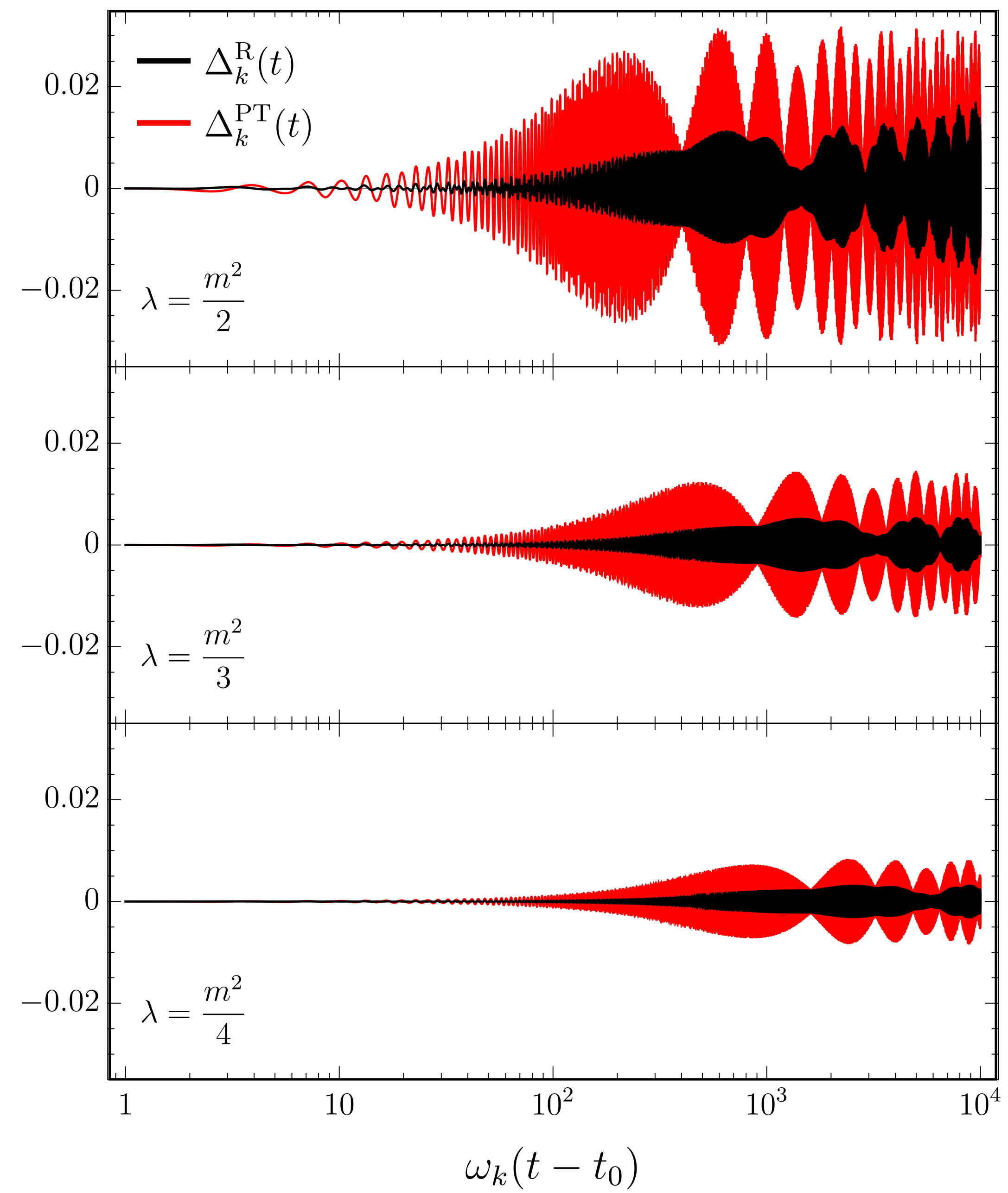
- The equations decouple under perturbation theory, RWA
- Solving the coupled equations provides a perturbative resummation to standard loop corrections

Two-point correlation: Results for a $\lambda\hat{\phi}\hat{\chi}$ interaction

- The $\lambda\hat{\phi}\hat{\chi}$ interaction is *exactly solvable*
- We compare the Redfield (R) and perturbation theory (PT) solutions against the exact one,

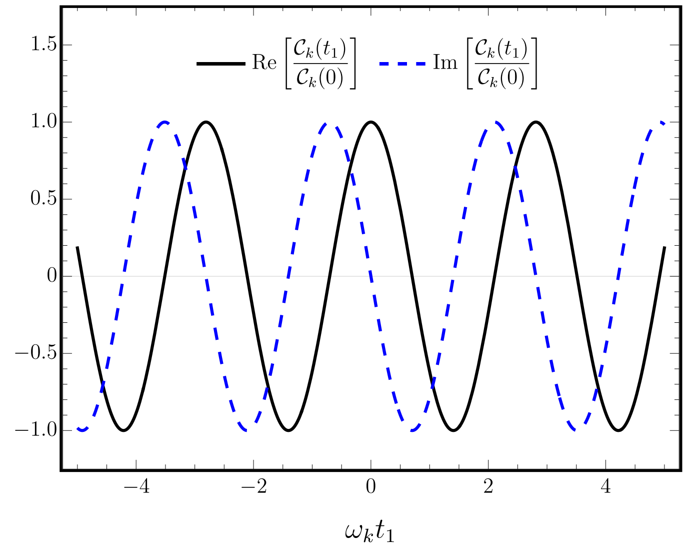
$$\Delta_k^R(t) = \frac{\mathcal{G}_k^R(t) - \mathcal{G}_k^{\text{exact}}}{\mathcal{G}_k^{\text{exact}}}$$

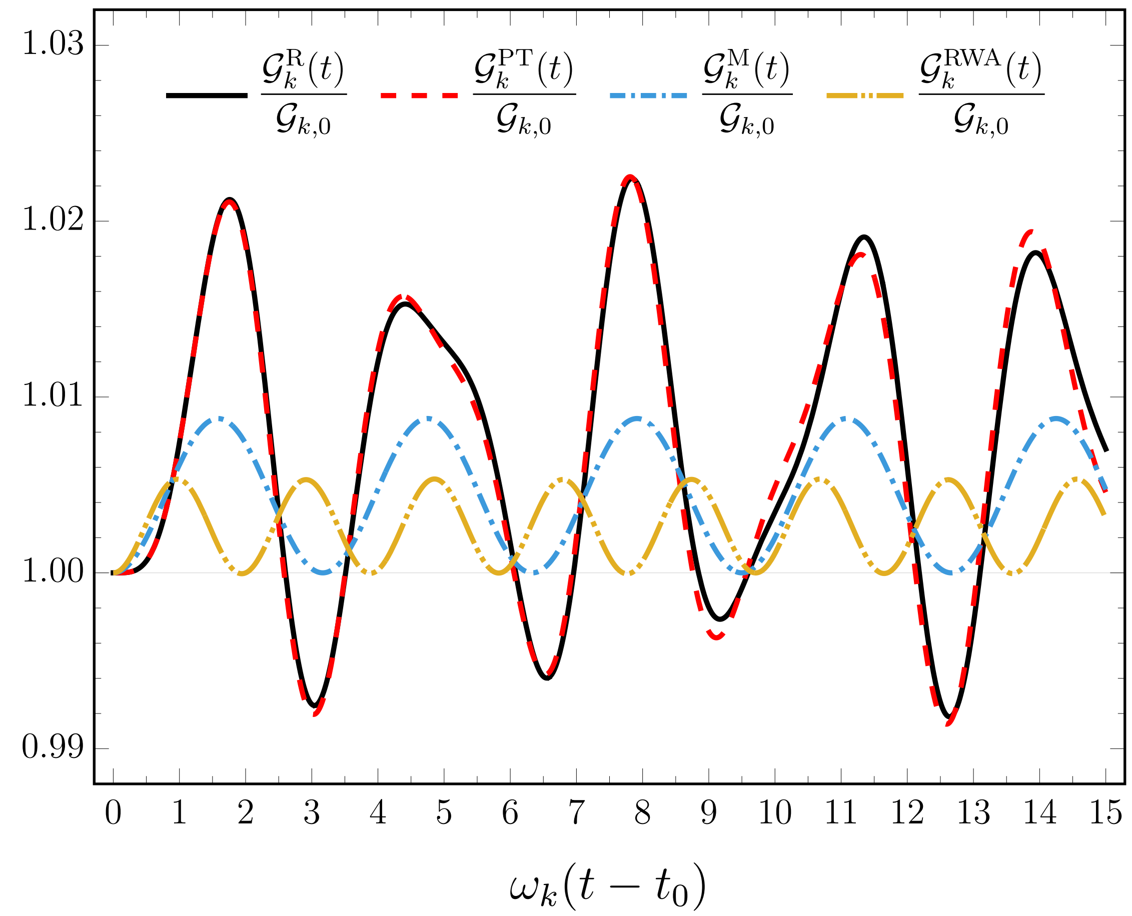
- The Redfield solution breaks down slower than the perturbation theory one



Two-point correlation: Results for a $\lambda\hat{\phi}\hat{\chi}$ interaction

- We next compare the Redfield solution against the Markovian approximation (M) and RWA
- The Markovian approximation is not a good approximation, as expected from the environment correlation

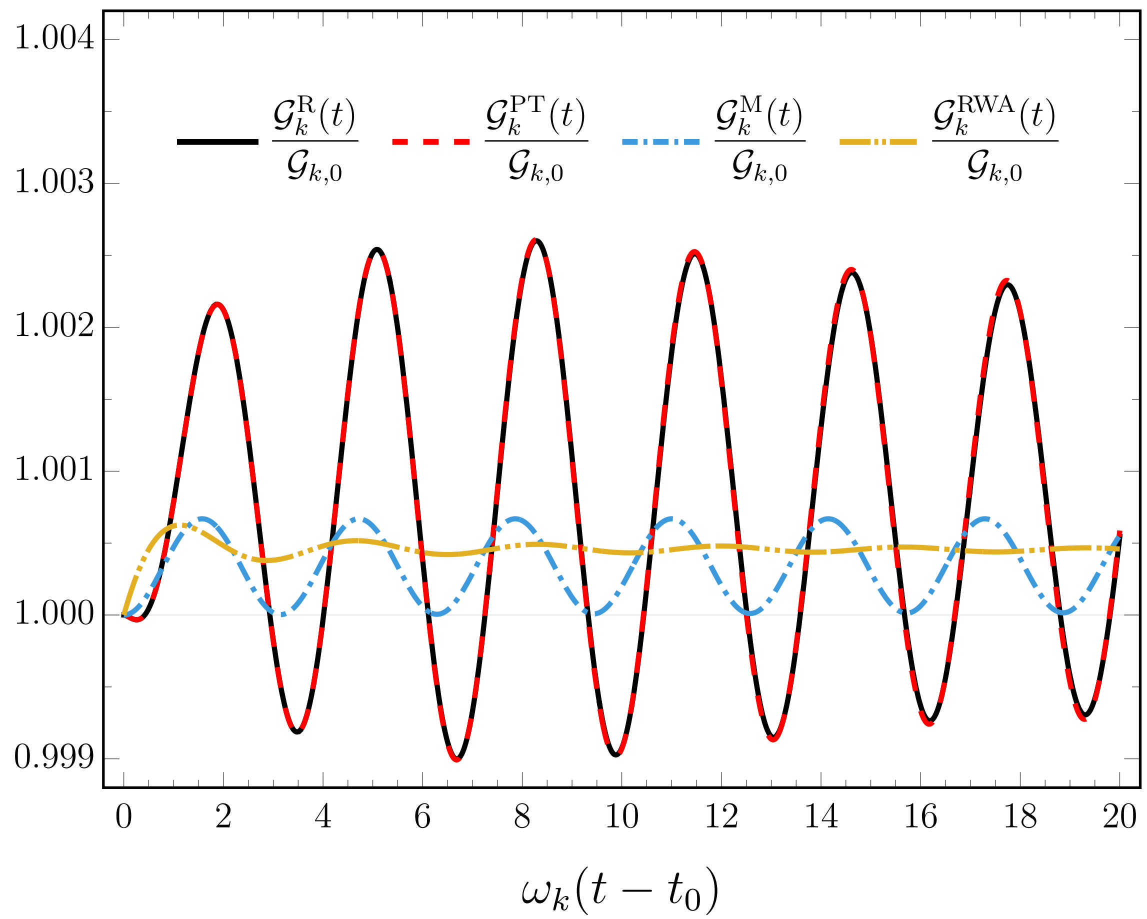


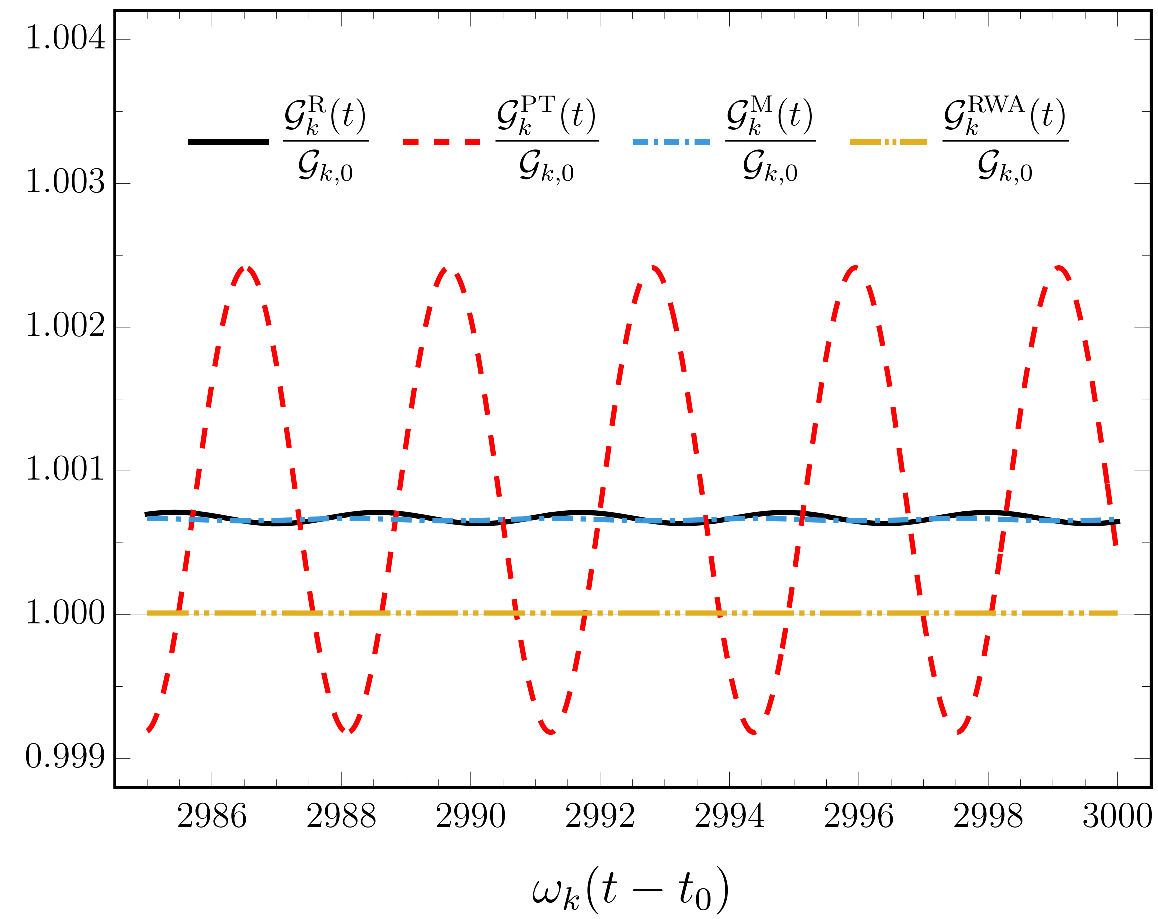


Two-point correlation: Results for a $\lambda\hat{\phi}\hat{\chi}^2$ interaction

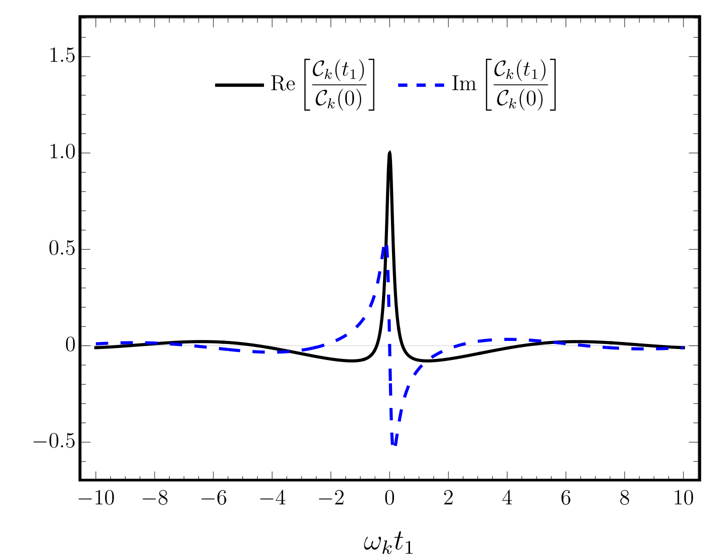
- The $\lambda\hat{\phi}\hat{\chi}^2$ interaction is *not* exactly solvable, but we again expect the Redfield equation to provide a perturbative resummation
- The environment correlation is now UV-divergent
- The system correlation is renormalized by standard renormalization counterterms in the Hamiltonian
- The solution depends on the renormalization scale, however, and breaks uncertainty at late times

Two-point correlation: Results for a $\lambda\hat{\phi}\hat{\chi}^2$ interaction





- The Markovian approximation is a good approximation in the late-time limit, as expected from the environment correlation



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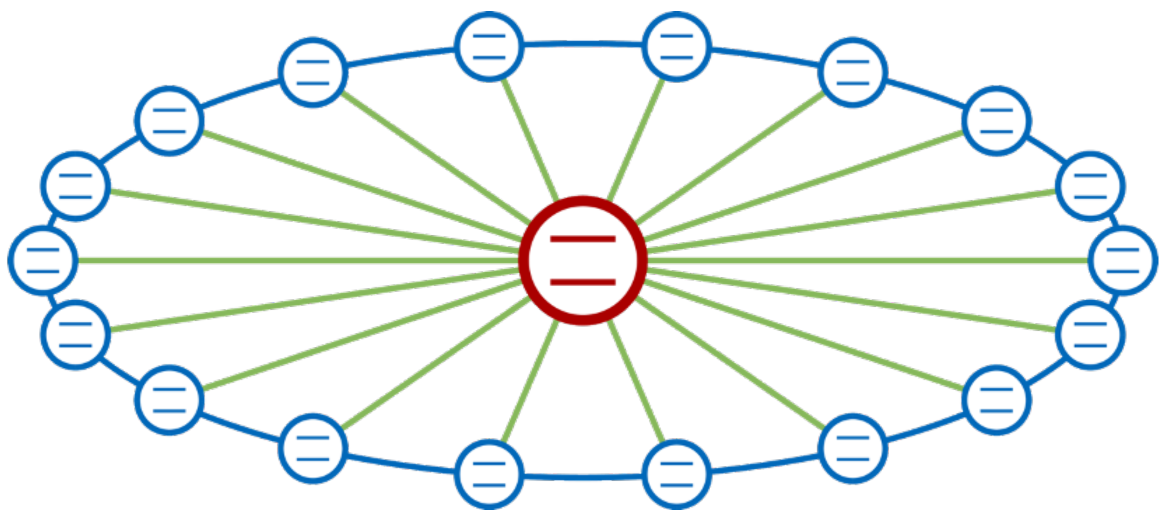
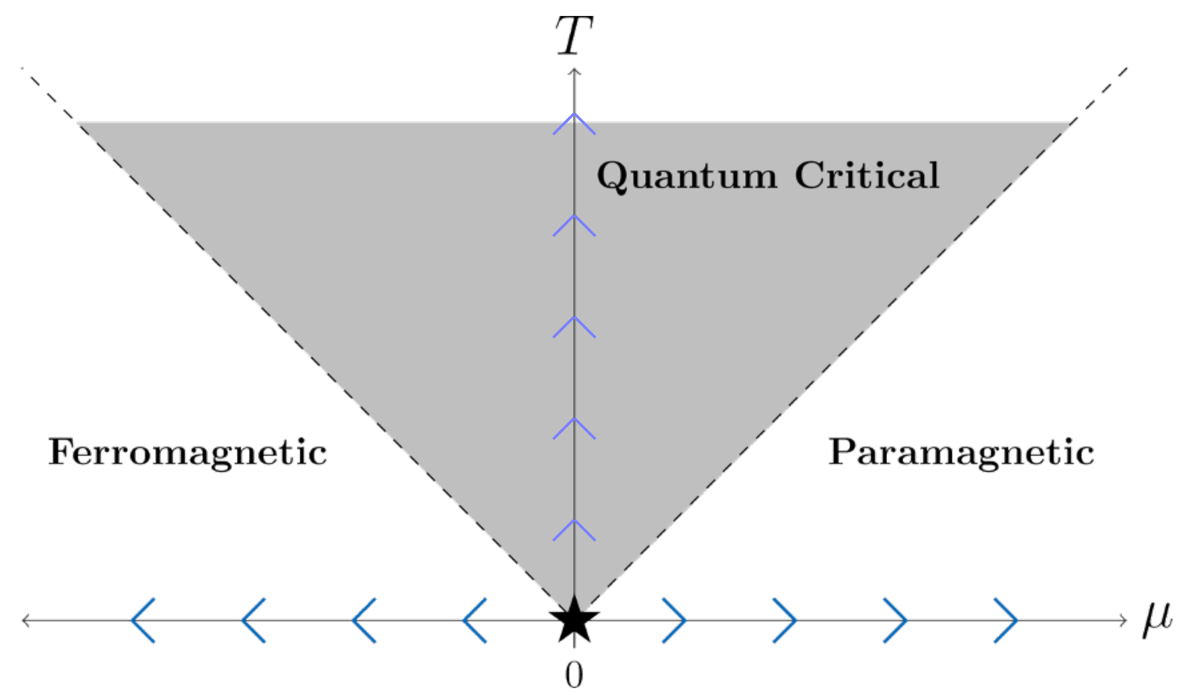
Qubit coupled to TFIM

- Consider a probe qubit coupled to a 1D transverse-field Ising model (TFIM), that has an *unstable* quantum critical point at $\{\mu, T\} = \{0{,}0\}$ and *stable* fixed points at $\{\infty{,}0\}$ and $\{0{,}\infty\}$



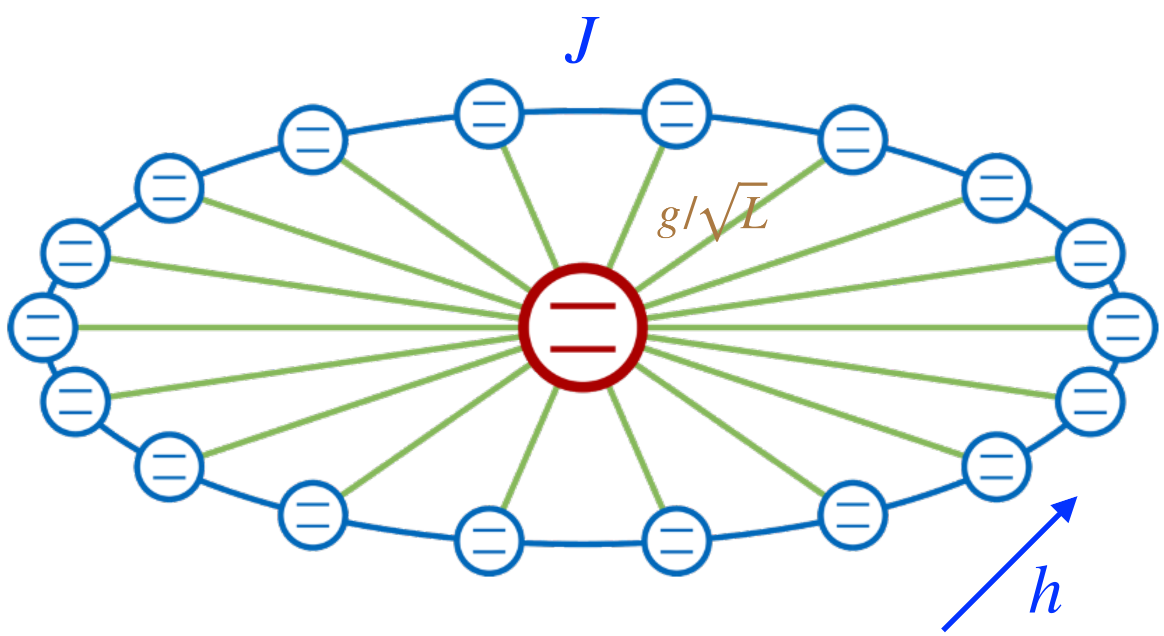


- Are the reduced dynamics of the qubit Markovian or non-Markovian? Can the qubit dephasing rate and spectrum probe criticality and RG flow in the TFIM?

*A. Keefe, *B. Bowen, A. Lawrence, N. A., and A. Kamal, In preparation

P. Haikka, J. Goold, S. McEndoo, F. Plastina, and S. Maniscalco, PRA 85, 060101 (R) (2012) [arXiv:1202.2997]

Qubit coupled to TFIM



System

$$\hat{H}_S = \frac{\omega_S}{2}\hat{\sigma}_S^z$$

Environment

$$\hat{H}_E = -J\sum_{j=1}^{N}\hat{\sigma}_j^z\hat{\sigma}_{j+1}^z - h\sum_{j=1}^{N}\hat{\sigma}_j^x$$

Interaction

$$\hat{H}_i = \frac{g}{\sqrt{L}}\hat{\sigma}_S^z\sum_{j=1}^{N}\hat{\sigma}_j^x$$

- TFIM lattice spacing $a = L/N$ and quantum critical point at $h = J$

Qubit coupled to TFIM

- In the thermodynamic ($N \to \infty$) and continuum ($a \to 0$) limits, the TFIM maps to a free fermionic QFT through the Jordan-Wigner transformation. In the diagonal basis,

S. Sachdev, *Quantum Phase Transitions*, Cambridge University Press (2011)

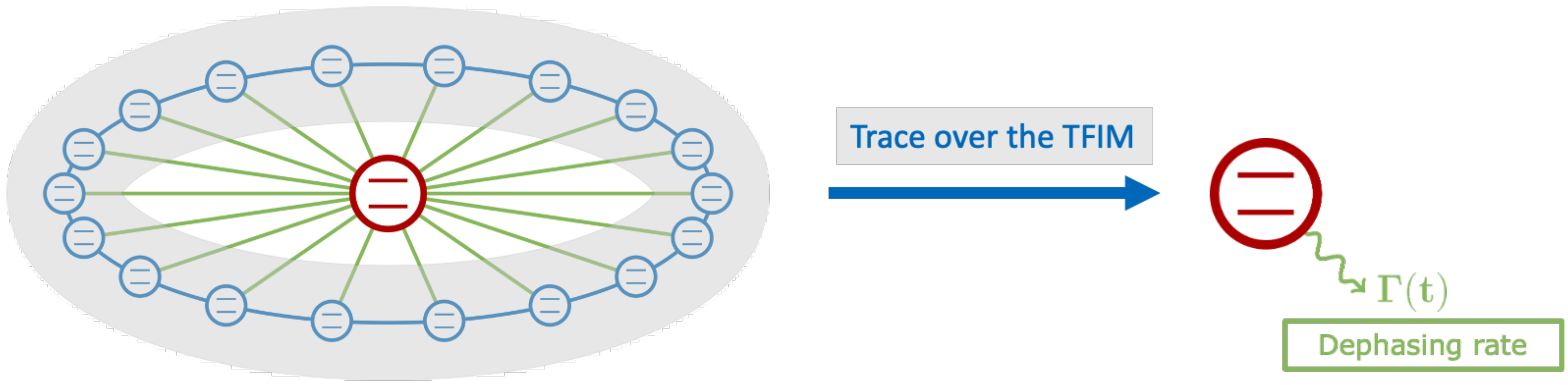
$$\hat{H}_E = \int \frac{\mathrm{d}k}{2\pi} \epsilon_k : \hat{\gamma}_k^\dagger \hat{\gamma}_k :$$

where $\epsilon_k = \sqrt{k^2 + \mu^2}$, $\{\gamma_k, \gamma_{k'}^\dagger\} = 2\pi\delta(k - k')$, $\mu = (1 - h/J)/a$, and $v = 2Ja \equiv 1$

- Under the same transformation, the interaction Hamiltonian becomes $\hat{H}_i = \hat{\sigma}_S^z \hat{\mathcal{O}}_E$ with

$$\hat{\mathcal{O}}_E = \frac{ig}{\sqrt{L}} \int \frac{\mathrm{d}k}{2\pi} \frac{k}{\epsilon_k} \left(\hat{\gamma}_{-k}^\dagger \hat{\gamma}_k^\dagger - \hat{\gamma}_k \hat{\gamma}_{-k} \right)$$

Redfield master equation



- The Redfield equation (in Schrödinger picture) for the qubit is

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = -i\frac{\omega_S}{2}[\hat{\sigma}_S^z, \hat{\rho}(t)] + \Gamma_{\mu,T}(t)\left\{\hat{\sigma}_S^z\hat{\rho}(t)\hat{\sigma}_S^z - \hat{\rho}(t)\right\}$$

where $\Gamma_{\mu,T}(t) = \int_{t_0}^{t} \mathrm{d}t_1 \mathrm{Re}\left[C_{\mu,T}(t,t_1)\right]$ is the dephasing rate and $C_{\mu,T}(t,t_1) = 2\mathrm{Tr}_E\left\{\hat{\mathcal{O}}_E^\dagger(t)\hat{\mathcal{O}}_E(t_1)\hat{\rho}_E(T)\right\}$ is the environment correlation

Environment correlation function

- We can calculate the environment correlation function $C_{\mu,T}(t,t_1)$ explicitly for our given $\hat{\mathcal{O}}_E$
- At the unstable quantum critical point $\{\mu, T\} = \{0{,}0\}$,

$$C_{0,0}(t,t_1) =$$

- At the stable fixed point $\{\mu, T\} = \{0,\infty\}$,

$$C_{0,\infty}(t,t_1) =$$

- At the stable fixed point $\{\mu, T\} = \{\infty,0\}$,

$$C_{\infty,0}(t,t_1) =$$

Environment correlation function: Connection to CFT

- At the quantum critical point $\{\mu, T\} = \{0,0\}$, the TFIM maps to a free fermion conformal field theory with power law correlations,

$$\Delta_{0,0}(x,t,x_1,t_1) = \frac{1}{\pi^2}\frac{1}{\left(x-x_1\right)^2-\left(t-t_1\right)^2}$$

- The environment correlation in the Redfield equation is

$$C_{0,0}(t,t_1) = \lim_{L\to\infty}\frac{2g^2}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}}\mathrm{d}x\int_{-\frac{L}{2}}^{\frac{L}{2}}\mathrm{d}x_1\Delta_{0,0}(x,t,x_1,t_1)$$

Regulating by setting $t-t_1 \to t-t_1-i\epsilon$ gives the same result as before

Qubit dephasing rate

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Qubit spectrum

- The qubit spectrum — the Fourier transform of the unequal-time correlation $\left\langle \hat{\sigma}_{+}^{\dagger}(t)\hat{\sigma}\_(0) \right\rangle_{\text{ss}}$ in the steady state — is a more sensitive probe of non-Markovianity
- In fact, the spectrum can be calculated without making the time-local (Markov) approximation using the frequency-domain master equation construction

A. Keefe, N. A., and A. Kamal, Quantum 9, 1863 (2025) [arXiv:2405.01722]

- For Markovian dynamics, the spectrum is Lorentzian,

$$S_M^{\xi}[\Delta] = \frac{1}{\pi}\frac{\xi}{\Delta^2 + \xi^2}, \text{ with } \Delta = \omega - \omega_S$$

where $\xi = \Gamma_{0,0} \equiv \Gamma$ at the quantum critical point $\{\mu, T\} = \{0,0\}$, $\xi = \Gamma/2$ at the fixed point $\{0,\infty\}$, and $\xi = 0$ at the fixed point $\{\infty,0\}$

Qubit spectrum

- Further, the spectrum can be used to define a spectral measure of non-Markovianity,

$$\mathcal{N}_s = \min_\xi \left\{ \mathrm{JSD}(S \,||\, S_M^\xi) \right\}$$

where the Jensen-Shannon divergence (JSD) is the average of the Kullback-Leibler (KL) divergences,

$$D_{\mathrm{KL}}(S_1 \,||\, S_2) = \int_{\omega=-\infty}^{\infty} S_1[\omega] \, \log_2 \frac{S_1[\omega]}{S_2[\omega]}$$

A. Keefe, N. A., and A. Kamal, Quantum 9, 1863 (2025) [arXiv:2405.01722]

Qubit spectrum

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Reconstructing the phase diagram and RG flow

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Summary and discussion — 1

- The Redfield master equation is a tractable non-Markovian equation, though it is known to violate positivity*
- It provides a perturbative resummation to standard loop corrections in interacting QFTs, by setting up coupled differential equations
- The reduced dynamics of $\hat{\phi}$ are non-Markovian for a $\lambda\hat{\phi}\hat{\chi}$ theory but Markovian (in the late-time limit) for a $\lambda\hat{\phi}\hat{\chi}^2$ theory, as inferred from environment and system correlations

What diagrams does the Redfield master equation resum? Is fluctuation-dissipation satisfied? How can we restore uncertainty?

C. Käding et al., In progress

*The frequency-domain master equation preserves positivity, at least for a qubit coupled to a lossy cavity

Summary and discussion — 2

- A qubit coupled to a 1D TFIM can be used to probe criticality and even the RG flow of the TFIM
- The reduced dynamics of the qubit are Markovian only at the critical/fixed points, as inferred from the environment correlation, qubit dephasing rate, and qubit spectrum
- In *fixed point space*, we see a flow towards the stable fixed points

Can these results be extended to other critical theories? What is the general connection to CFTs?

L. S. Rathore et al., In progress