



E. PARRA



S. KUHN



G. MAGNIFICO



JAD HALIMEH

DYNAMICS ON AN OPEN SU(2) LATTICE GAUGE THEORY

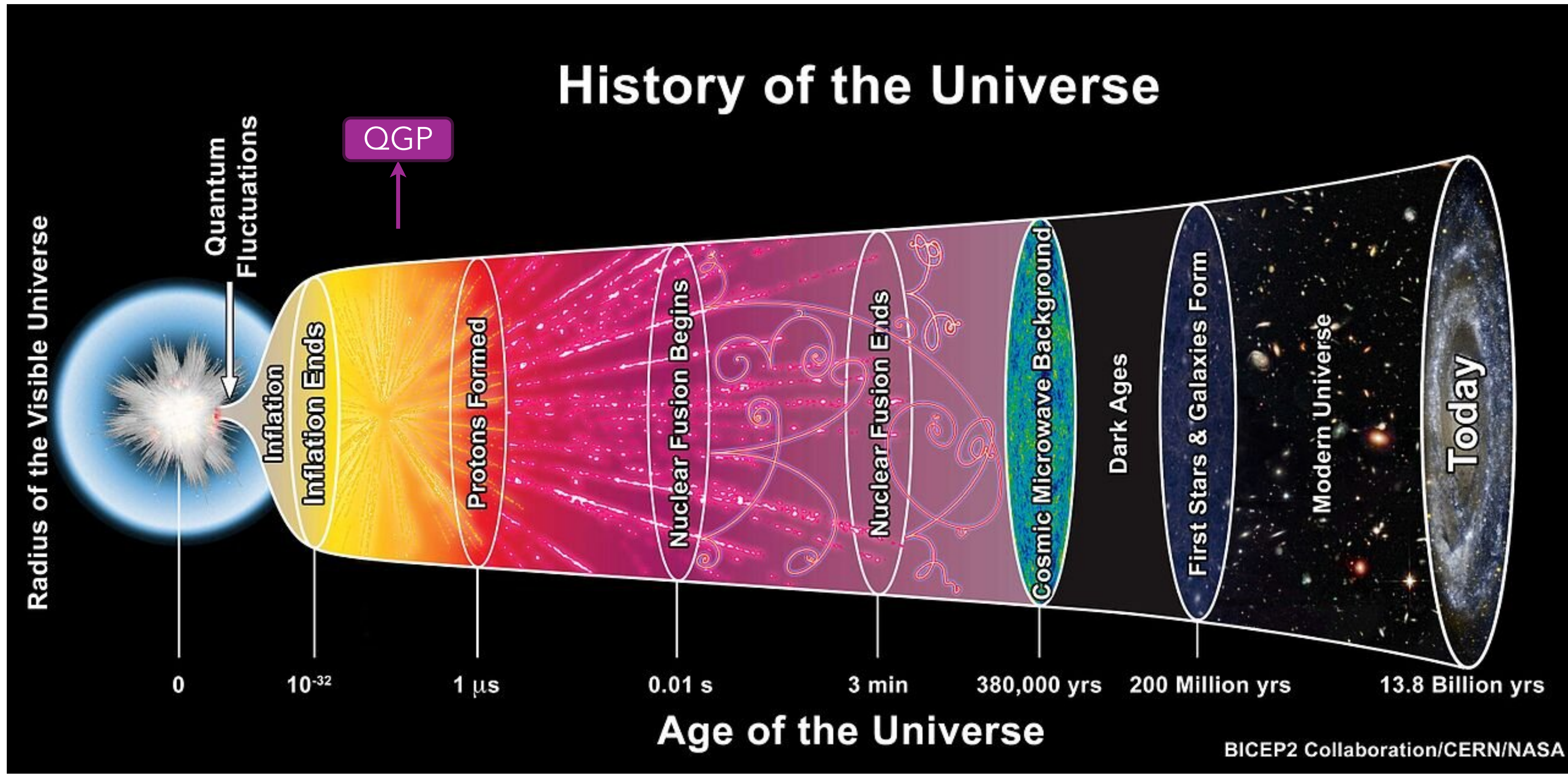
OPEN QUANTUM SYSTEMS: DISSIPATION AND DECOHERENCE FROM SUBATOMIC TO COSMIC SCALES

GIOVANNI CATALDI

MAINZ. 27/05/26

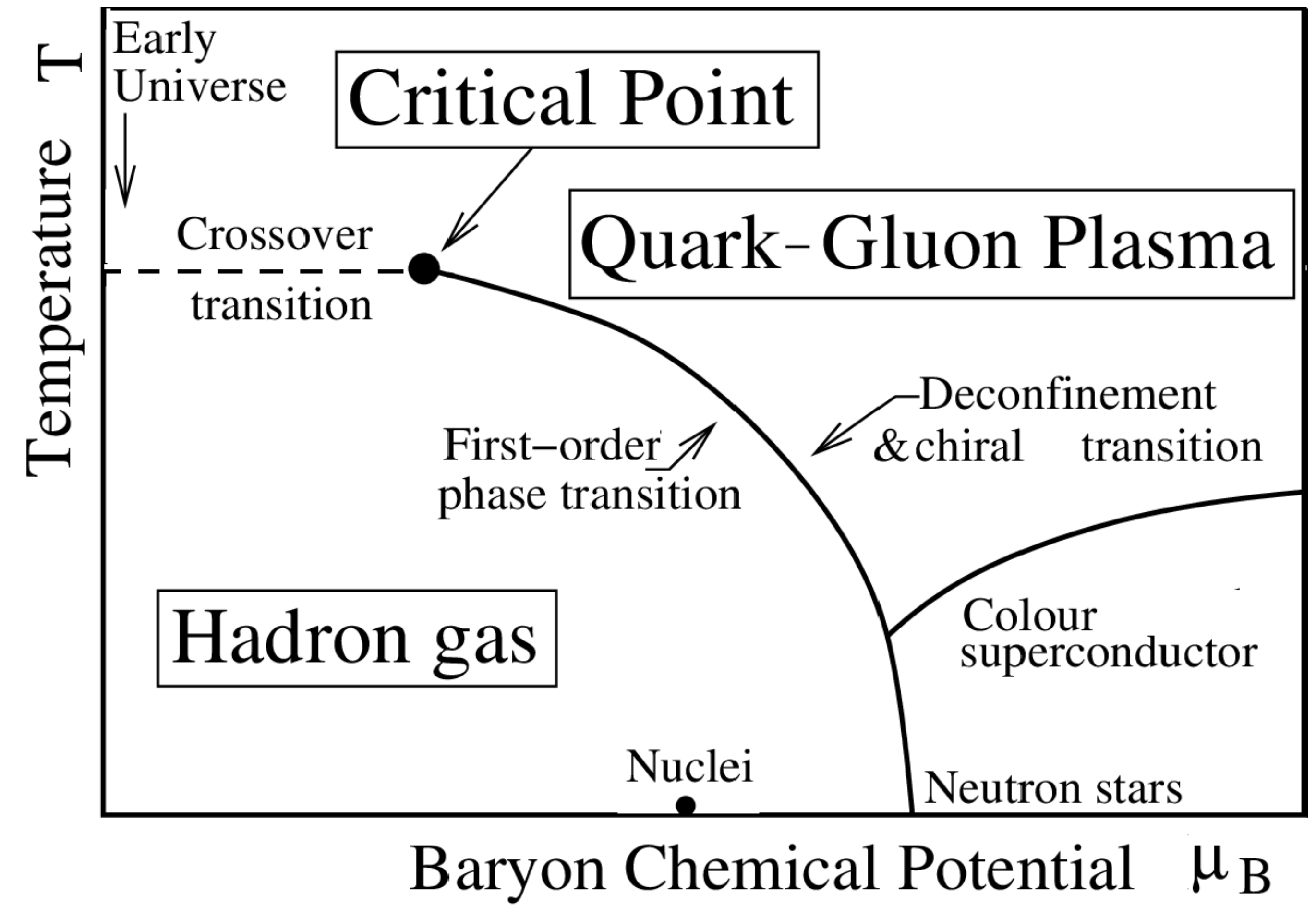
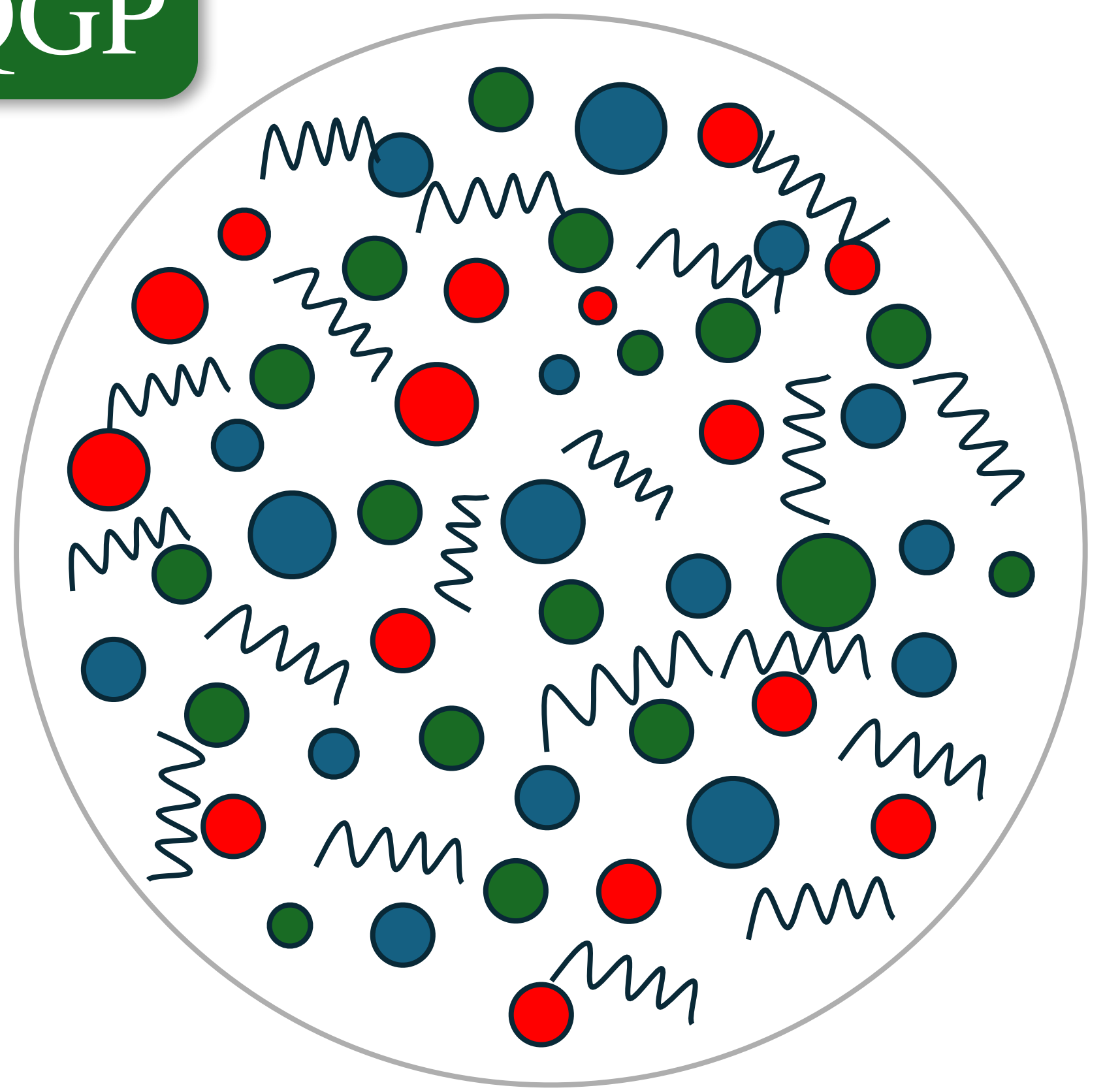


THE PROBLEM



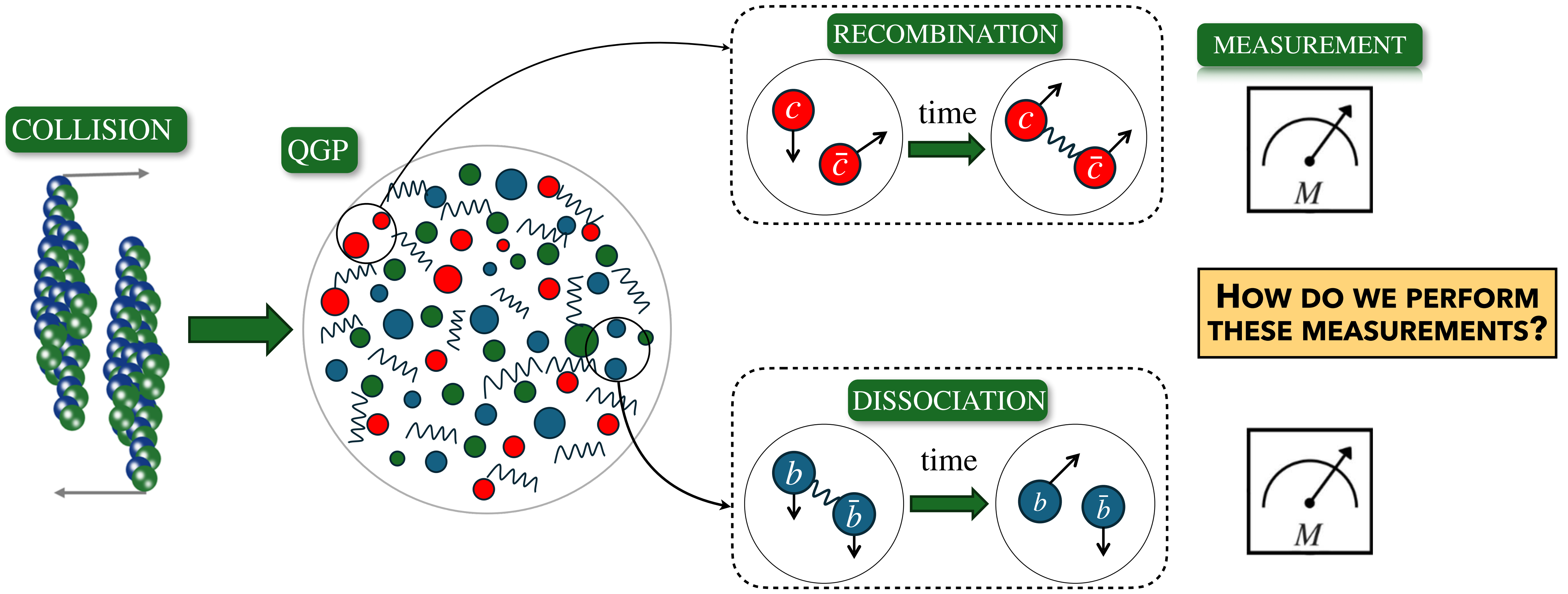
QUARK-GLUON PLASMA

QGP



PROBING QUARK-GLUON PLASMA

LOOKING AT HEAVY-ION COLLISIONS

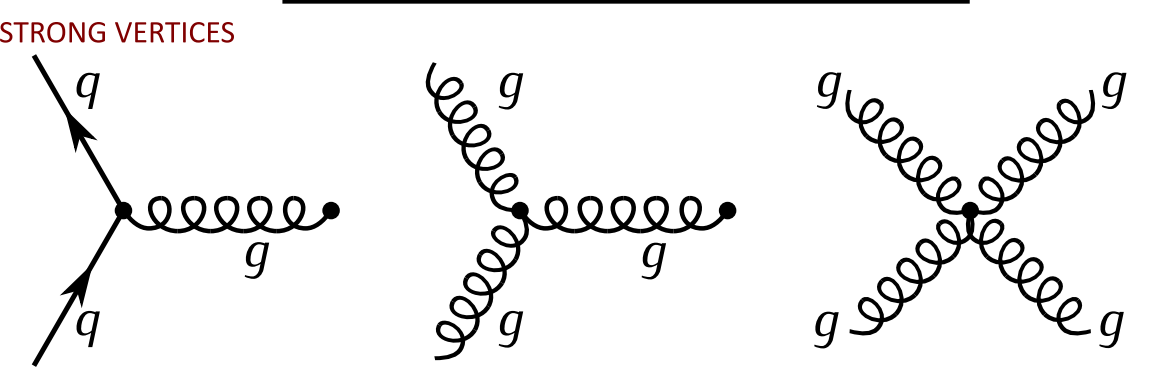


HOW TO STUDY GAUGE THEORIES

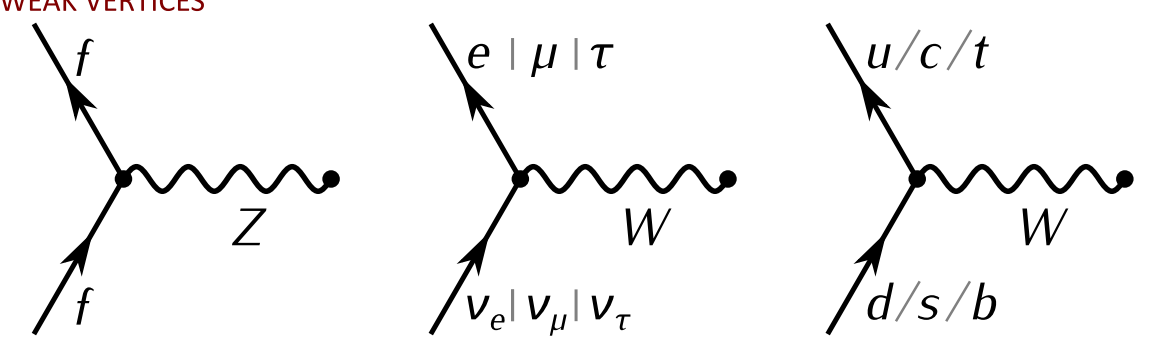
THEORETICAL PARTICLE PHYSICS

PERTURBATION THEORY VIA FEYNMAN DIAGRAMS

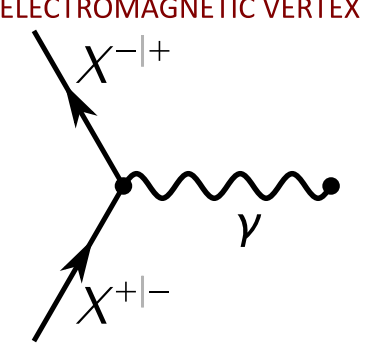
STRONG VERTICES



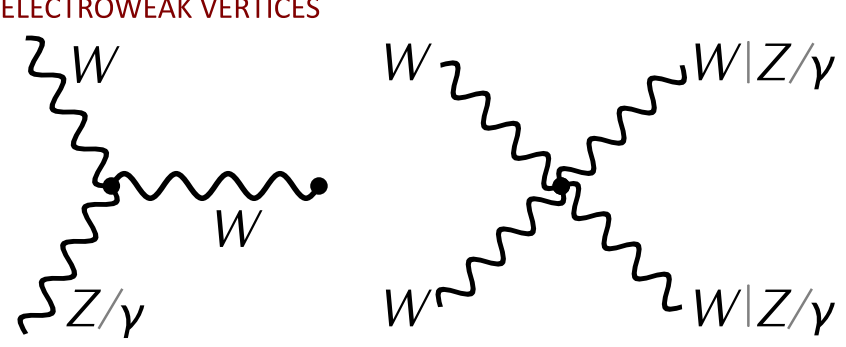
WEAK VERTICES



ELECTROMAGNETIC VERTEX



ELECTROWEAK VERTICES



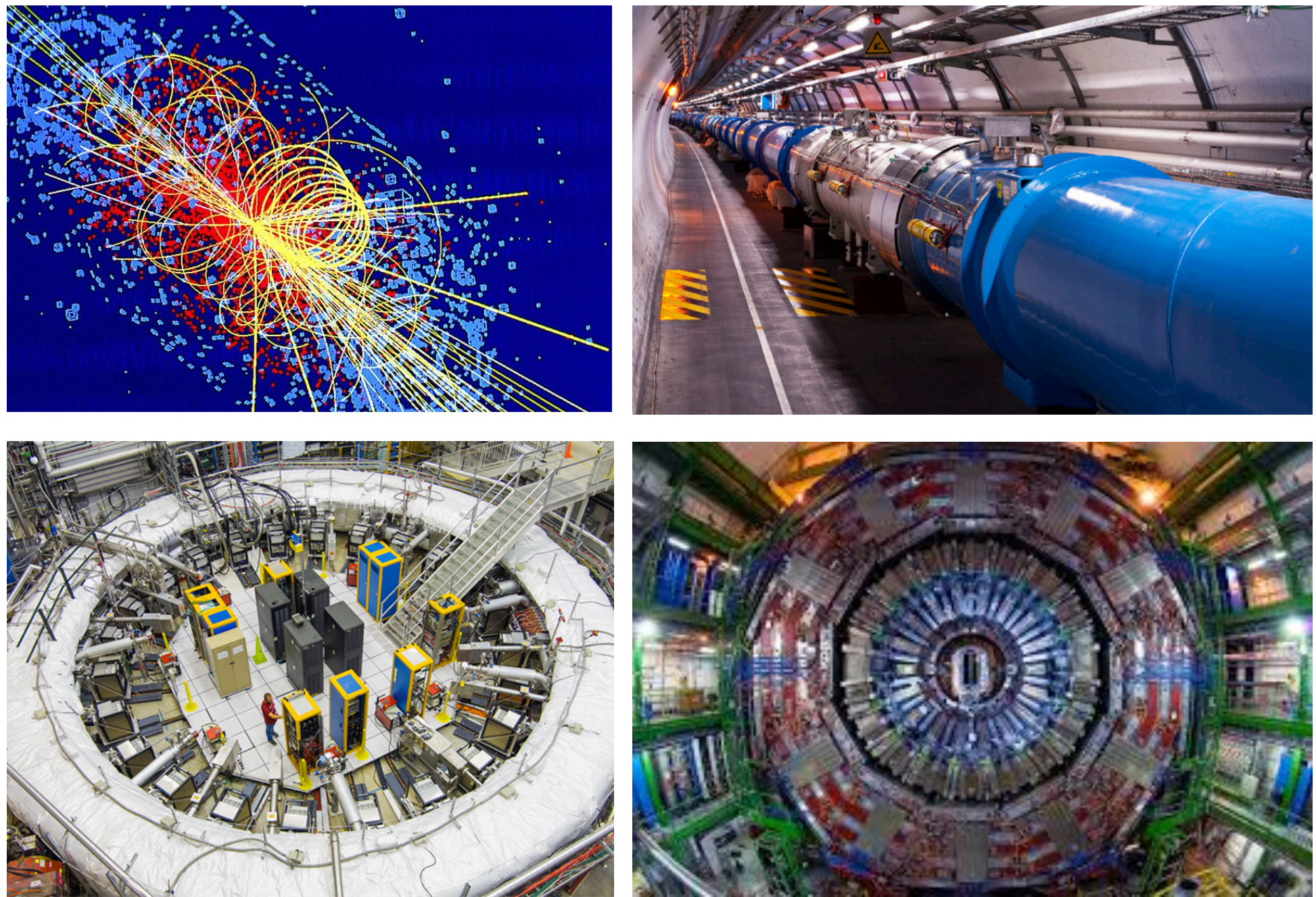
NUMERICAL SIMULATIONS
BEYOND EXPERIMENTAL AND PERTURBATIVE APPROACHES:
LATTICE GAUGE THEORIES



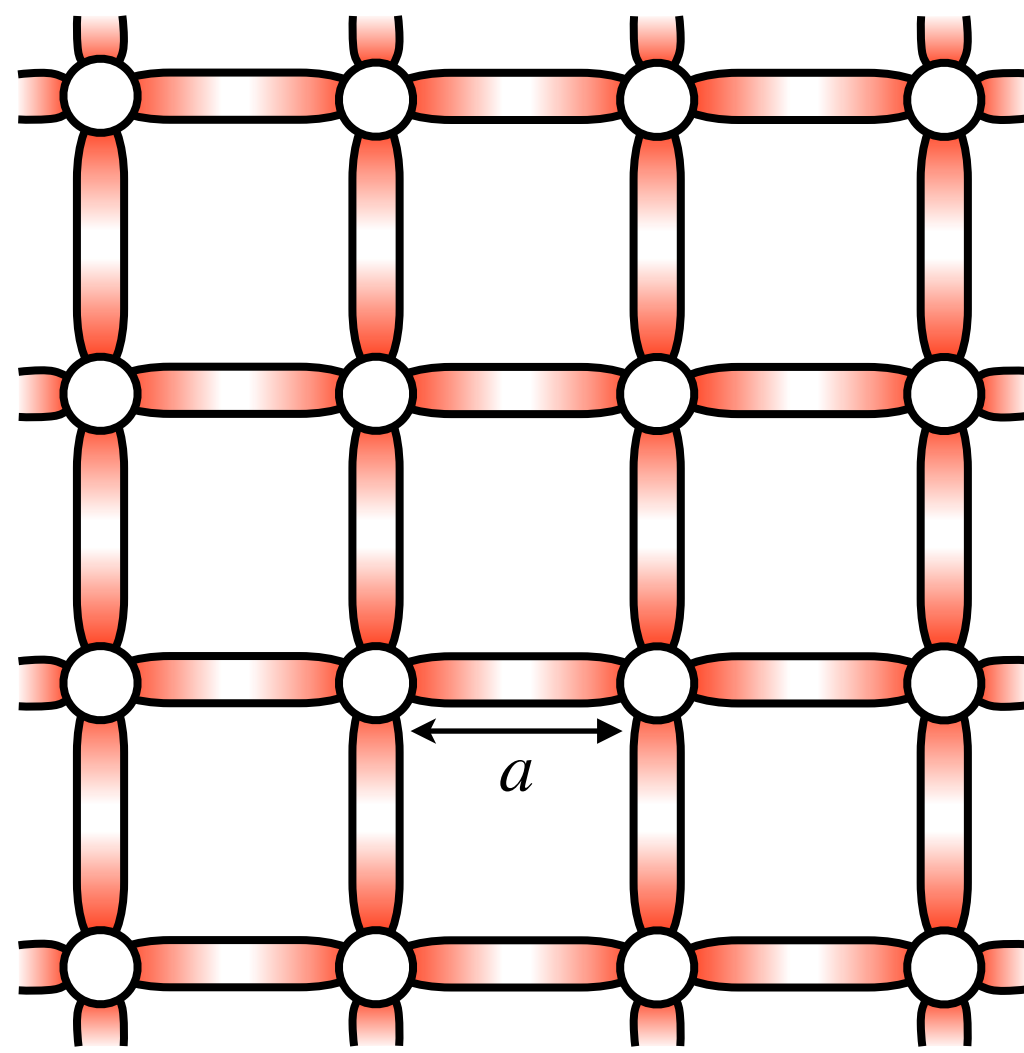
[Rothe, Lattice Gauge Theories (2012)]



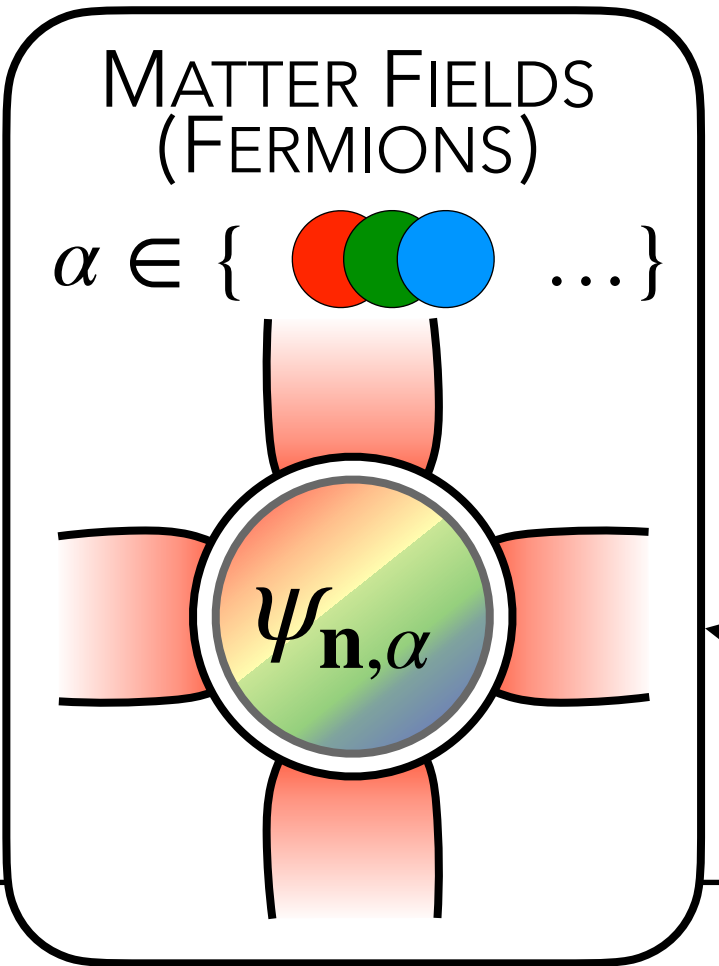
EXPERIMENTAL PARTICLE PHYSICS



LATTICE GAUGE THEORY

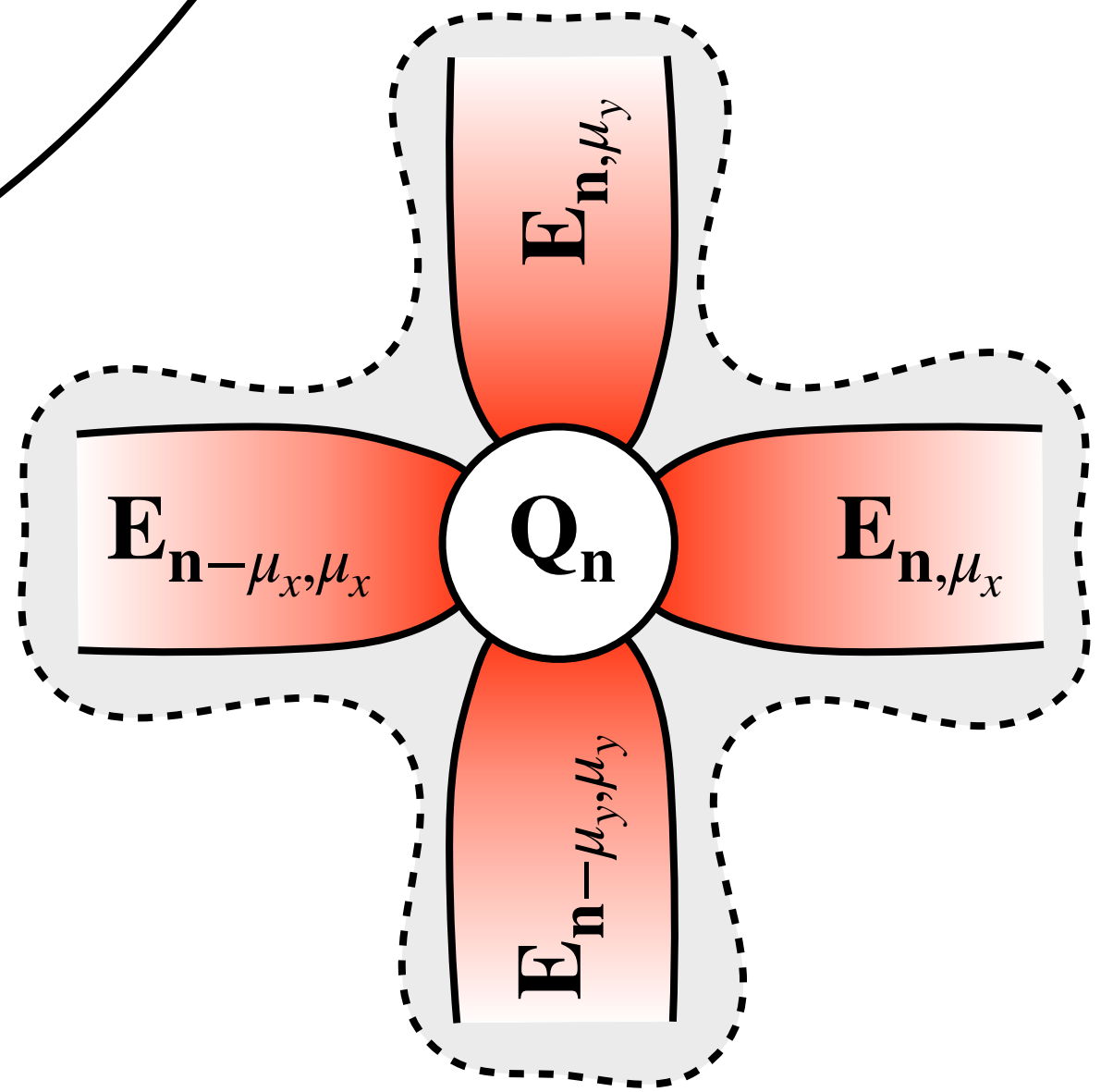
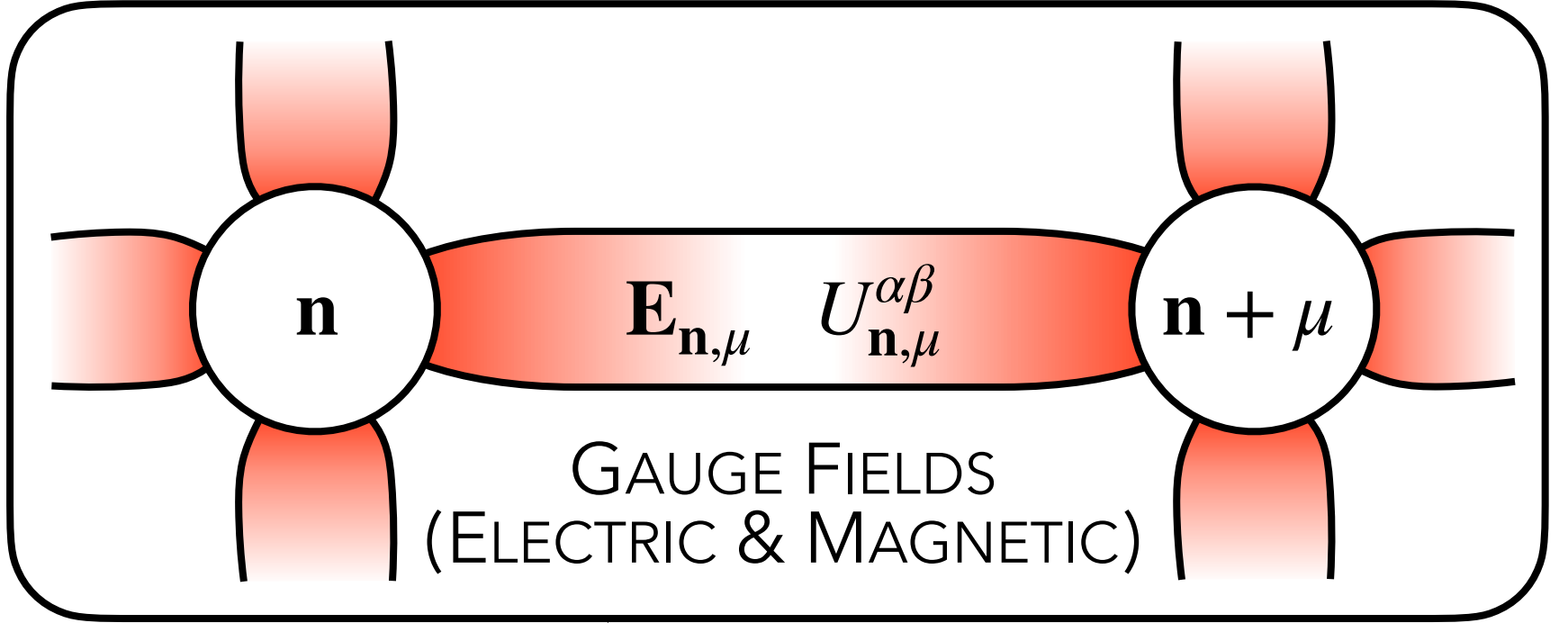


Hyper-cubic spatial Lattice Λ



		three generations of matter (fermions)		
		I	II	III
QUARKS	mass	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$
	charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
	spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
		u up	c charm	t top
		d down	s strange	b bottom
		e electron	μ muon	τ tau
LEPTONS	mass	$< 1.0 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 18.2 \text{ MeV}/c^2$
	charge	0	0	0
	spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	

		interactions / force carriers (bosons)	
GAUGE BOSONS VECTOR BOSONS	mass	0	$\approx 125.11 \text{ GeV}/c^2$
	charge	0	0
	spin	1	0
		g gluon	H higgs
SCALAR BOSONS	mass	0	$\approx 91.19 \text{ GeV}/c^2$
	charge	0	0
	spin	1	1
	γ photon	Z Z boson	W W boson



GAUSS LAW

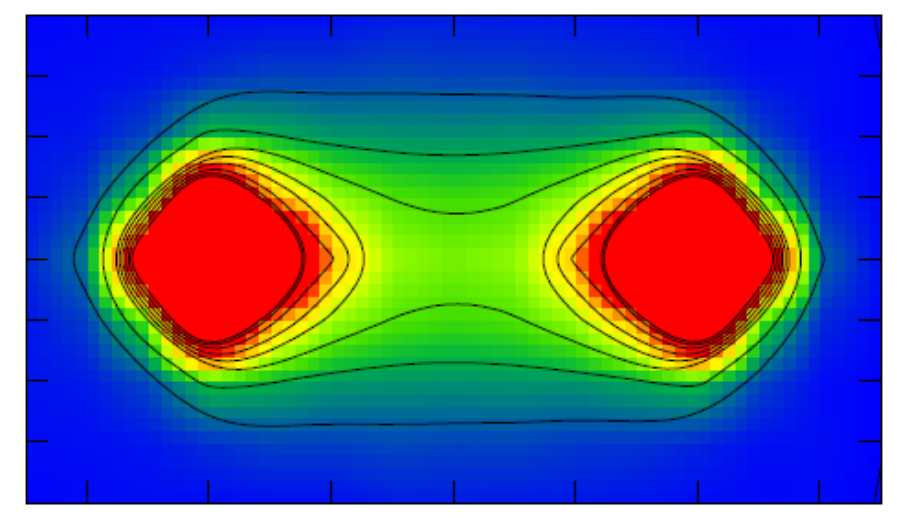
$$\nabla \cdot \mathbf{E}(\mathbf{r}, t) - \rho(\mathbf{r}, t) = 0$$

GAUSS LAW: $\mathbf{G}_n |\psi\rangle = 0$

$$\mathbf{G}_n = \sum_{k=x,y} [\mathbf{E}_{n+\mu_k} - \mathbf{E}_n] - Q_n$$

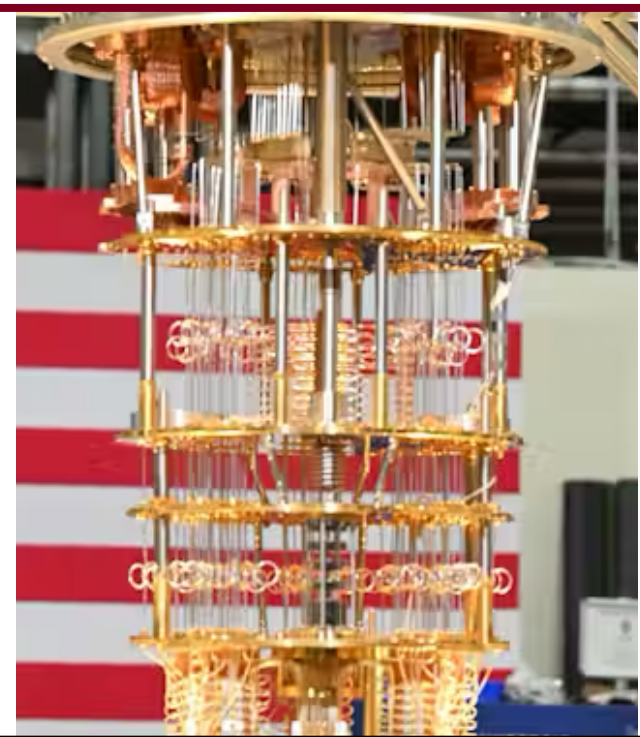
HOW TO SIMULATE LGTS?

MONTE CARLO SIMULATIONS



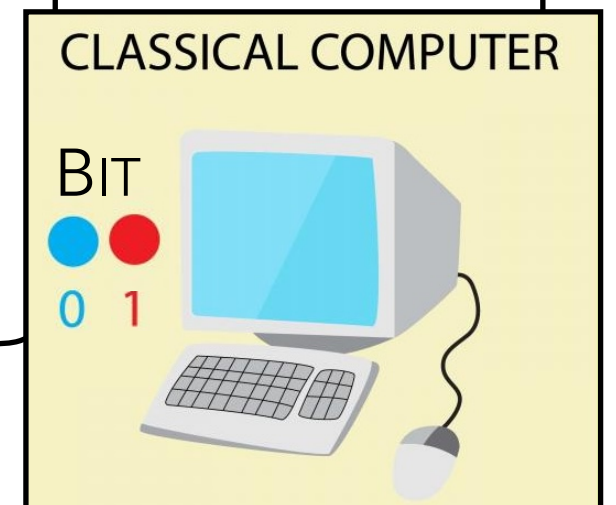
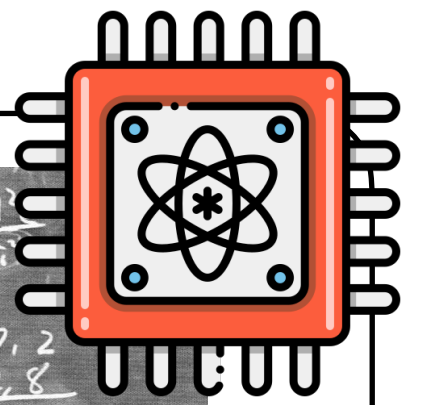
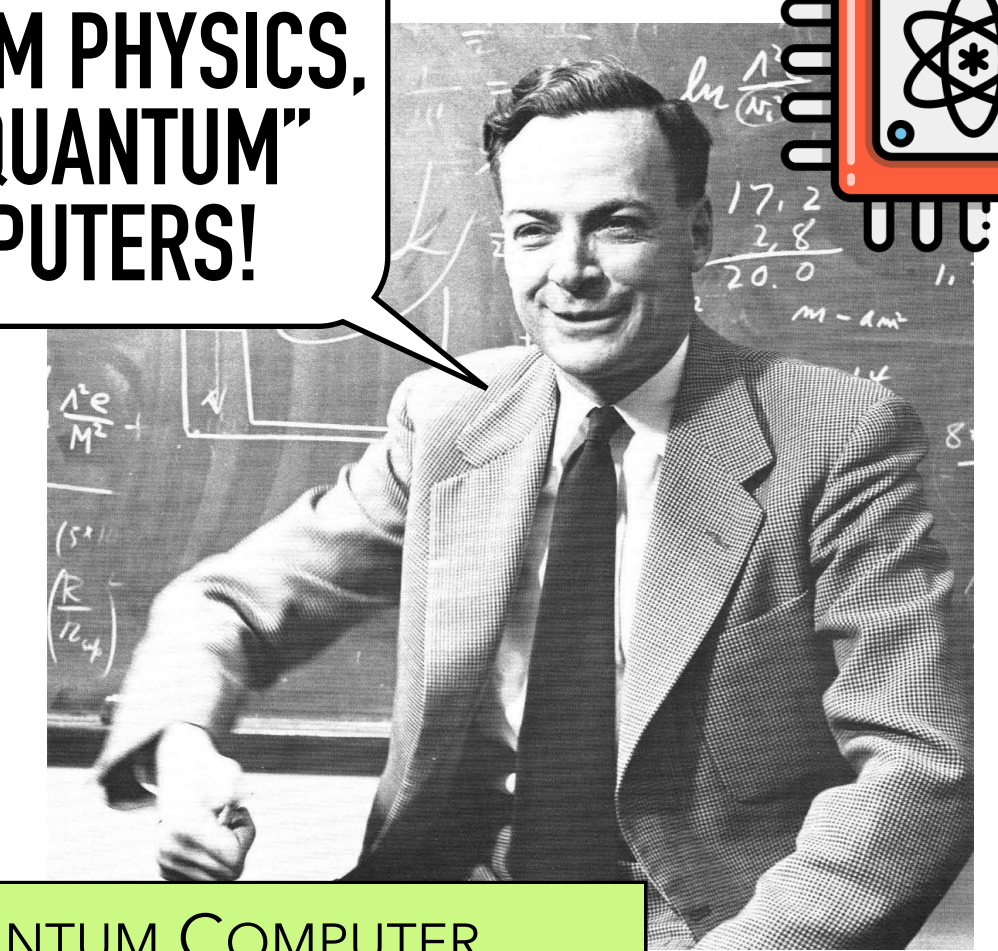
- LAGRANGIAN FORMALISM
- PARTITION FUNCTION
- GOOD RESULTS:
 - CONFINEMENT, ✓
 - HADRONIC SPECTRUM, ✓
 - CHYRAL-SYMMETRY BREAKING ✓
- SIGN PROBLEM:
 - REAL-TIME DYNAMICS, ✗
 - FINITE DENSITY OR CHEMICAL POTENTIAL ✗

NOISY INTERMEDIATE SCALE QUANTUM ERA
Preskill, Quantum 2, 79 (2018)

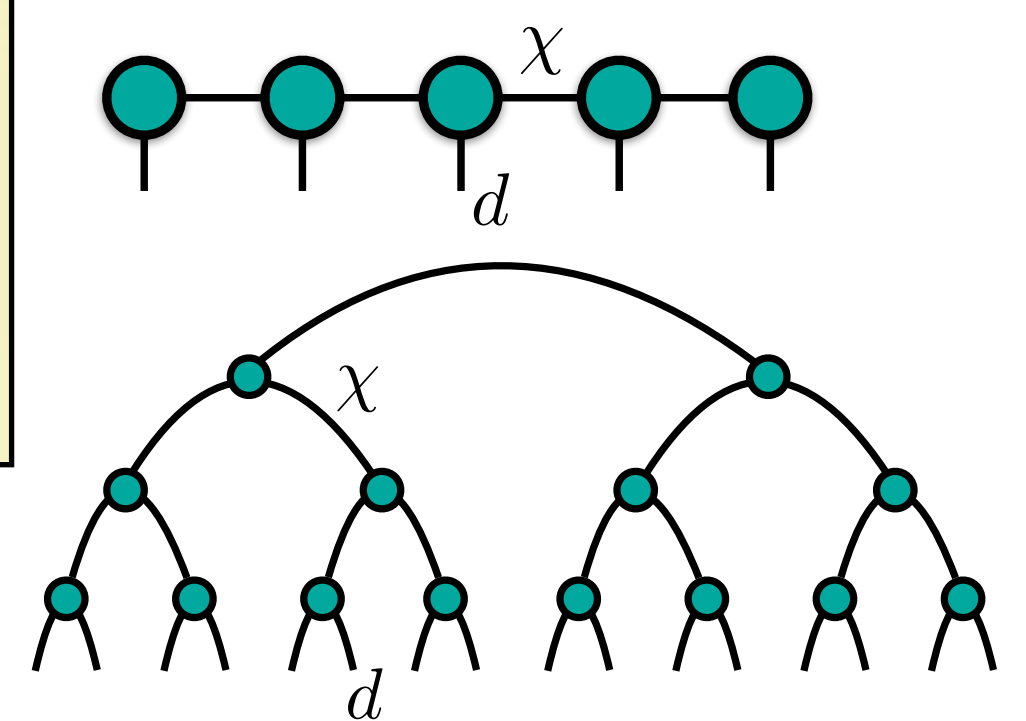


NOISE & SCALABILITY PROBLEMS:
NOT READY, BUT SOON ...

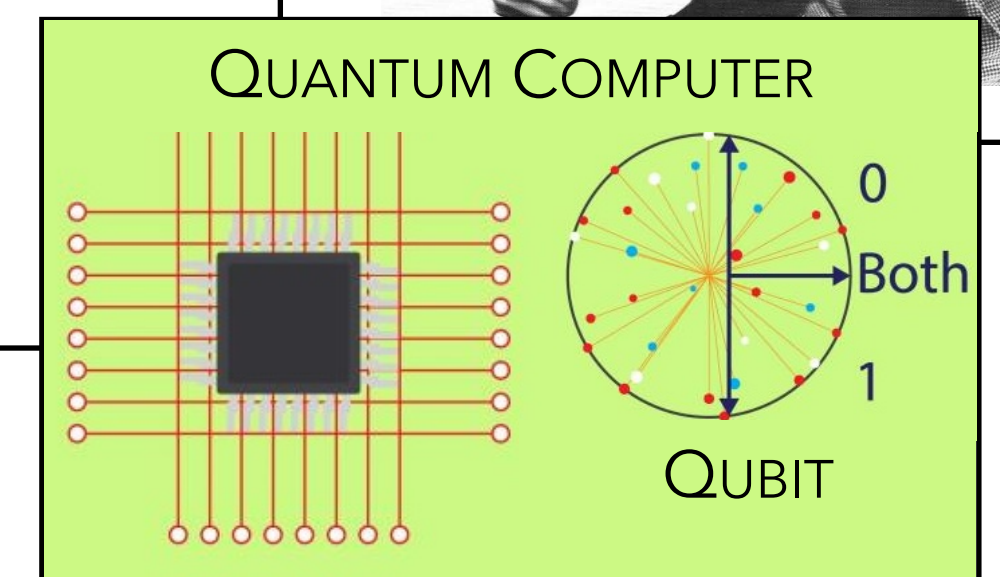
TO SIMULATE QUANTUM PHYSICS,
USE "QUANTUM" COMPUTERS!



TENSOR NETWORK METHODS

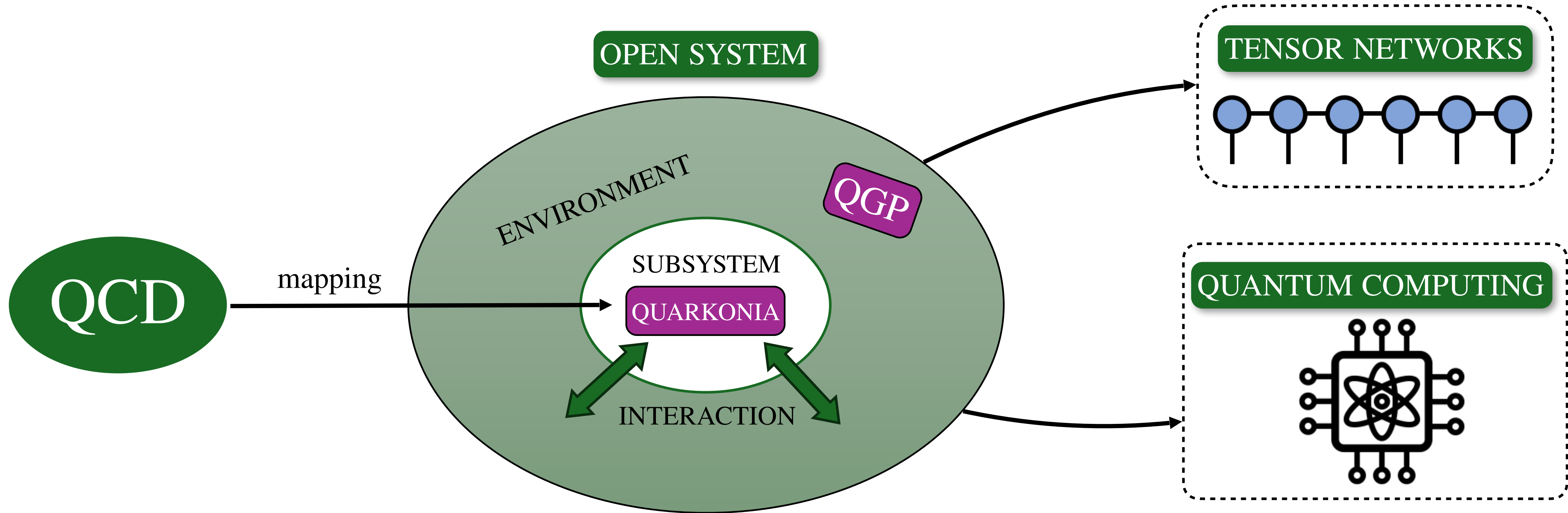


- HAMILTONIAN FORMALISM
- VARIATIONAL APPROACH
- SIGN PROBLEM FREE!
- REAL-TIME DYNAMICS ✓
- FINITE DENSITY ✓



QGP IN LATTICE GAUGE THEORIES

SIMULATE THE LATTICE GAUGE THEORY AS AN OPEN QUANTUM SYSTEM



SUBSYSTEM APPROXIMATION $(3 + 1)\text{D } \text{SU}(3) \longrightarrow (1 + 1)\text{D } \text{SU}(2)$

OVERVIEW



HAMILTONIAN LATTICE GAUGE THEORIES

- ◆ Kogut-Susskind formulation for SU(2) Yang-Mills
- ◆ Matter & Gauge SU(2) Hilbert space



OPEN QUANTUM SYSTEMS THEORY

- ◆ Lindblad Master Equation
- ◆ Timescales and Brownian Limit
- ◆ Physics-informed approximations



TENSOR NETWORK SIMULATIONS

- ◆ Time evolution for open quantum systems



THERMALIZATION TIME

- ◆ Empirical vs Liouvillian
- ◆ Dependence on Environment, Temperature etc
- ◆ Comparison with other approaches (pNRQCD)
- ◆ Anomalies



ENTROPY



LOCAL OBSERVABLES

- ◆ Casimir evolution
- ◆ Matter production



OUTLOOK AND CONCLUSIONS

HAMILTONIAN LGTS: INGREDIENTS

$$H = H_{\text{matter}} + H_{\text{gauge}} + H_{\text{interaction}}$$

$$= \int \psi^\dagger \left[-i\vec{\gamma} \cdot \vec{\nabla} + \gamma^0 m \right] \psi - \int \left[E^2 + B^2 \right] + \int g \psi^\dagger i(\vec{\gamma} \cdot \vec{A}) \psi$$

DIRAC HAMILTONIAN OF FREE FERMIONS

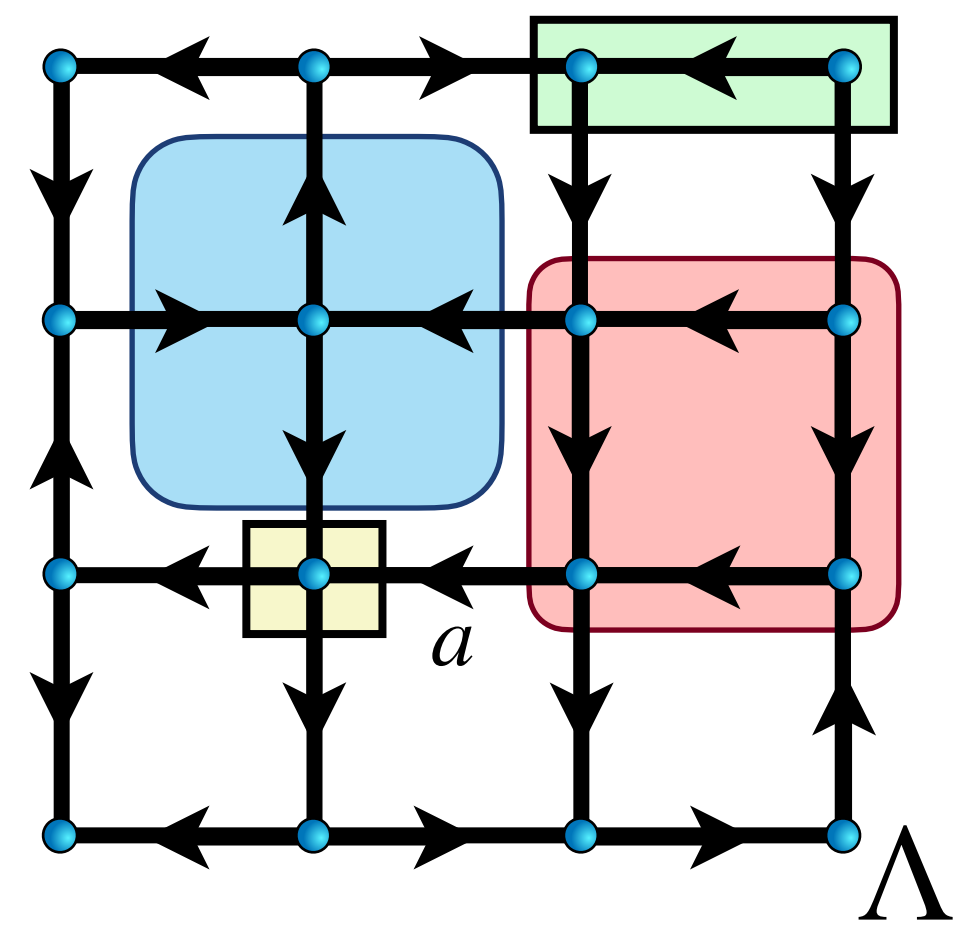
ELECTRIC CONTRIBUTION

MAGNETIC CONTRIBUTION

COVARIANT DERIVATIVE:
 $\partial_k \rightarrow \partial_k - gA_k$

SPATIAL DISCRETIZATION. TIME IS CONTINUOUS

PARALLEL TRANSPORTER
 $-i\psi^\dagger \gamma^k [\partial_k - gA_k] \psi \rightarrow -i\psi^\dagger \gamma^k U_k \psi$



$$H = -\frac{1}{2a} \sum_{\mathbf{n}, \mu} \left[\psi_{\mathbf{n}}^\dagger U_{\mathbf{n}, \mathbf{n}+\mu} \psi_{\mathbf{n}+\mu} + \text{H.c.} \right] + m \sum_{\mathbf{n}} \psi_{\mathbf{n}}^\dagger \psi_{\mathbf{n}} + H_{\text{Pure}}$$

$$H_{\text{Pure}} = \frac{g^2}{2a} \sum_{\mathbf{n}, k} E_{\mathbf{n}, \mu_k}^2 - \frac{1}{2ag^2} \sum_{\square} \text{Tr} \left[U_{\square} + U_{\square}^\dagger \right]$$

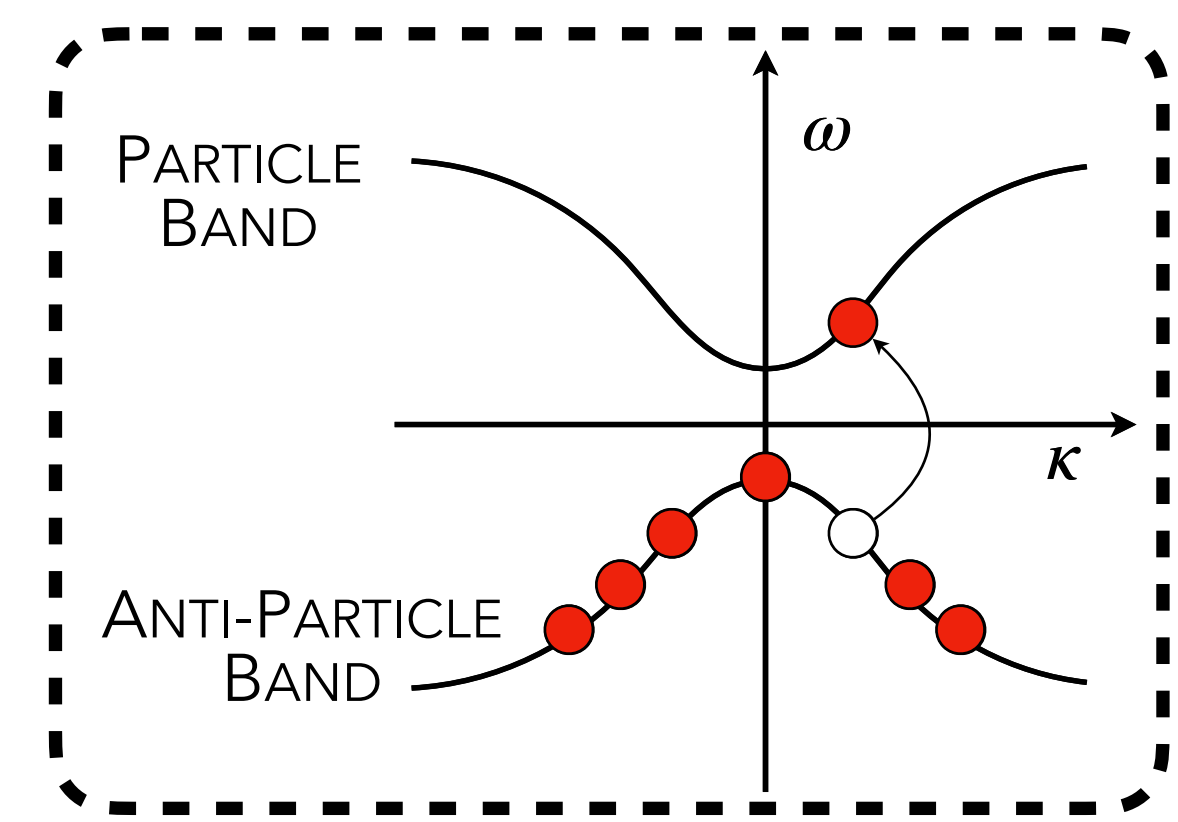
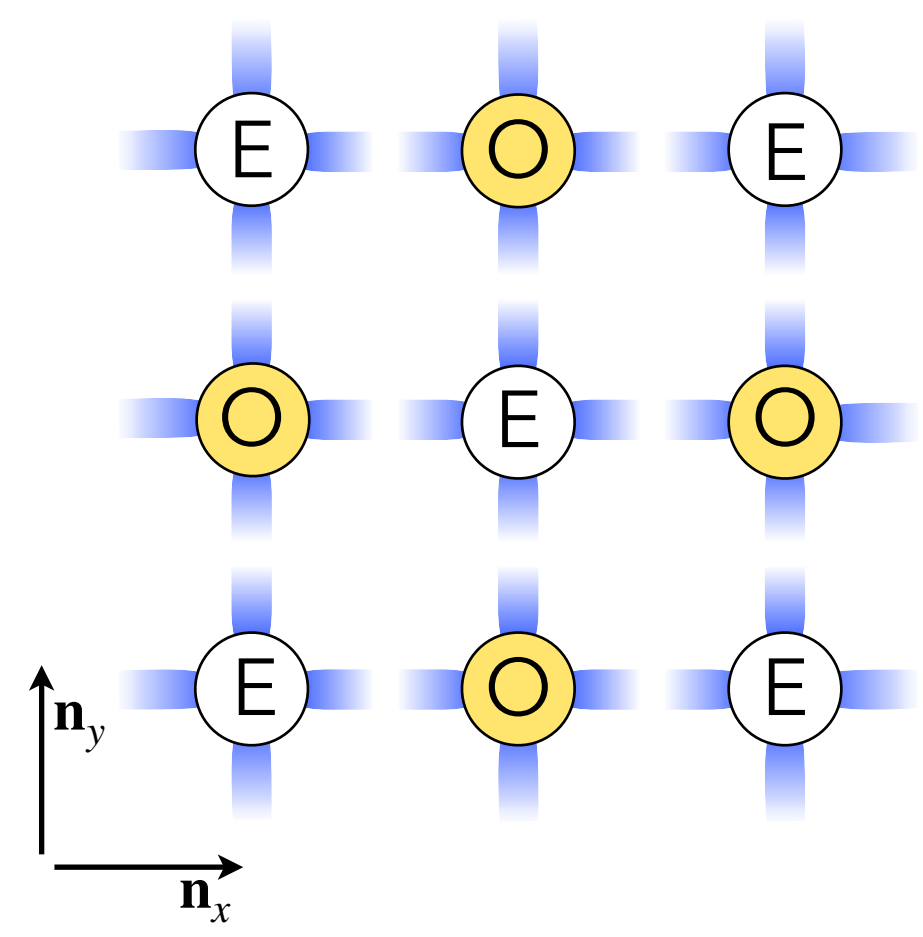
$$U_{\square} = \begin{pmatrix} & U^\dagger & \\ U^\dagger & & \\ & U & \\ U & & \end{pmatrix}$$

MATTER HILBERT SPACE

$$\mathcal{H} = \left[\mathcal{H}_{\text{matter}} \otimes \mathcal{H}_{\text{gauge}} \right]_{\text{Gauss Law}}$$

STUDYING FERMIONS ON THE LATTICE IS CHALLENGING [**FERMION DOUBLING PROBLEM**]

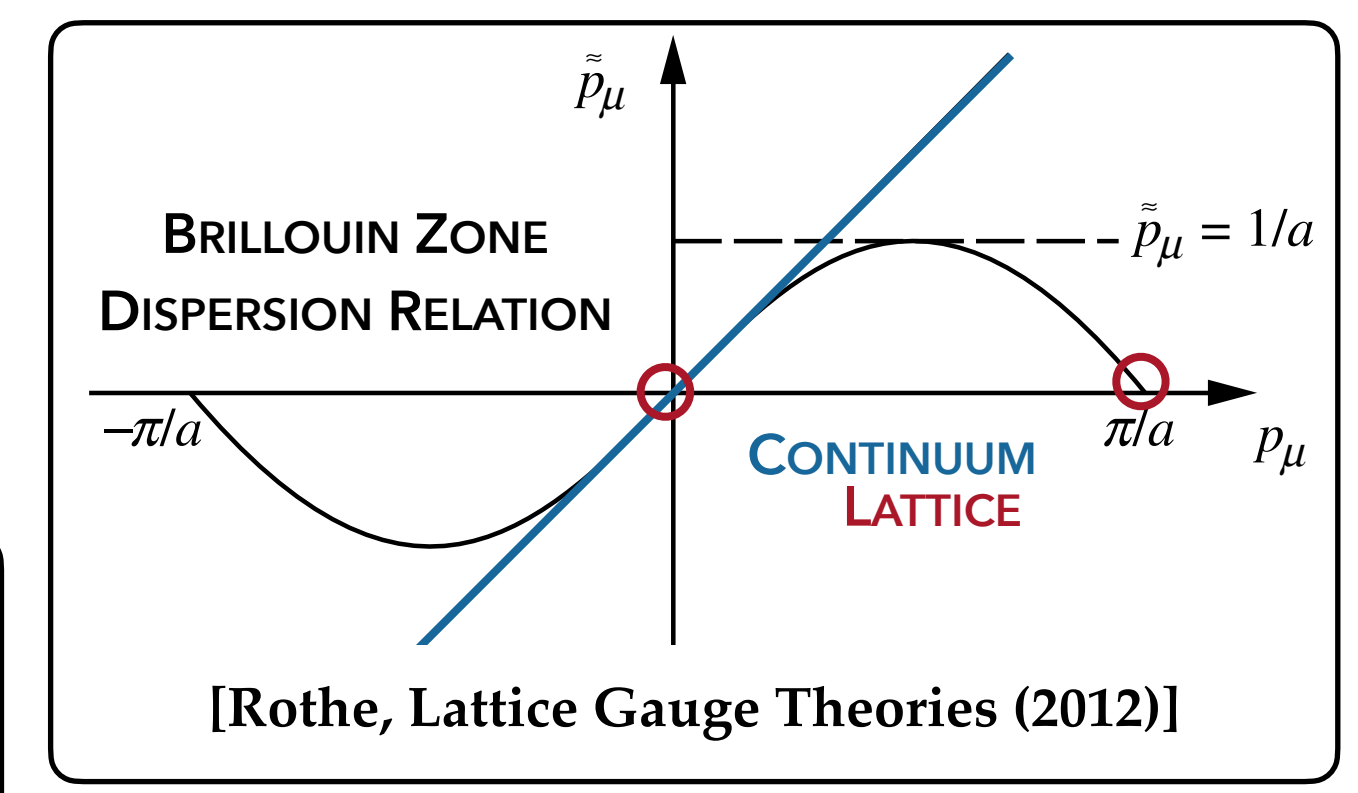
WE USE **STAGGERED FERMIONS** AND **SPLIT SPINOR COMPONENTS** ON SUBLATTICES



ABELIAN CASE: U(1)

ELECTRONS q , POSITRONS \bar{q}

$(-1)^{\mathbf{n}_x + \mathbf{n}_y} = +1$	<div style="display: flex; flex-direction: column; align-items: center; gap: 5px;"> <div style="border: 1px solid black; border-radius: 50%; width: 20px; height: 20px; margin: 2px;"></div> <div style="border: 1px solid black; border-radius: 50%; width: 20px; height: 20px; background-color: black; margin: 2px;"></div> </div>	\emptyset q
$(-1)^{\mathbf{n}_x + \mathbf{n}_y} = -1$	<div style="display: flex; flex-direction: column; align-items: center; gap: 5px;"> <div style="border: 1px solid black; border-radius: 50%; width: 20px; height: 20px; background-color: yellow; margin: 2px;"></div> <div style="border: 1px solid black; border-radius: 50%; width: 20px; height: 20px; margin: 2px;"></div> </div>	\emptyset \bar{q}



DESPITE THESE STRATEGIES, FERMIONS ARE STILL PROBLEMATIC [ANTI-COMMUTE]

GAUGE LINK HILBERT SPACE

FOR A **CONTINUOUS GAUGE GROUP** [U(1), SU(2), SU(3)]
THE CORRESPONDING **HILBERT SPACE IS INFINITE**: IT MUST BE TRUNCATED!

📌 QUANTUM LINK MODELS

[\[Chandrasekharan et al, Nucl.Phys B 492, 1-2 \(1997\)\]](#)

📌 USE FINITE SUBGROUPS (\mathbb{Z}_N, D_6)

[\[Ercolessi et al, PRD 98, 074503 \(2018\)\]](#)

📌 ALGEBRA DEFORMATION

[\[Zache et al, PRL 131, 171902 \(2023\)\]](#)

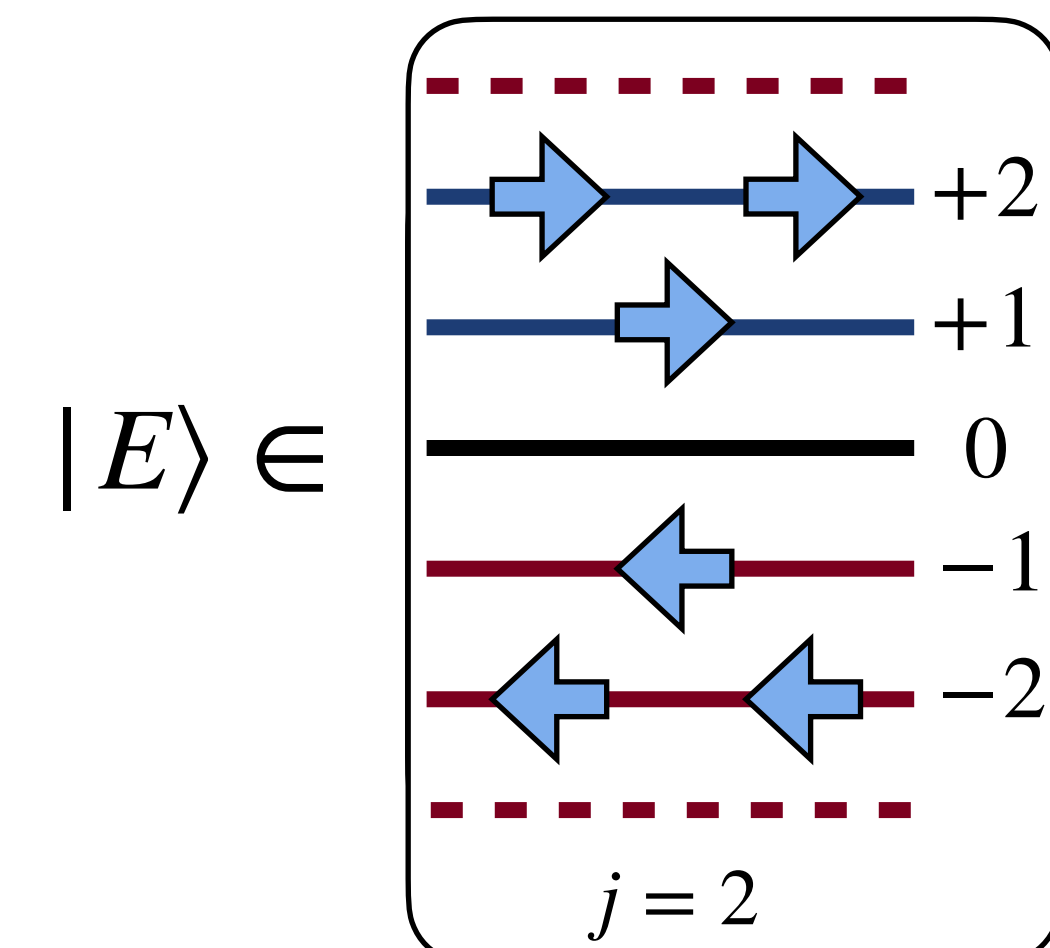
📌 INTRODUCE AN ENERGY CUTOFF

CORE IDEA

USE SPIN-LIKE OPERATORS IN
THE j -IRREP. EXAMPLE: QED
(E, U, U^\dagger) \rightarrow (S^z, S^+, S^-)

ABELIAN CASE

U(1) LINK HILBERT SPACE \mathcal{H}_G



$$\dim \mathcal{H}_G = (2j_{\max} + 1)$$

ABELIAN CASE: (1+1)D U(1) LGT, SCHWINGER MODEL

$$\hat{H}^{\text{U}(1)} = -\frac{1}{2} \sum_{\mathbf{n}} \left[i\hat{\psi}_{\mathbf{n}}^\dagger \hat{S}_{\mathbf{n},+\mu}^+ \hat{\psi}_{\mathbf{n}+\mu} + \text{H.c.} \right] + m \sum_{\mathbf{n}} (-1)^{\mathbf{n}} \hat{\psi}_{\mathbf{n}}^\dagger \hat{\psi}_{\mathbf{n}} + \frac{g^2}{2} \sum_{\mathbf{n}} \left(\hat{S}_{\mathbf{n},+\mu}^z \right)^2$$

(1+1)D SU(2) YANG-MILLS LGT

$$\hat{H}^{\text{SU}(2)} = -\frac{1}{2} \sum_{\mathbf{n}} \sum_{\alpha, \beta} \left[i \hat{\psi}_{\alpha, \mathbf{n}}^\dagger \hat{U}_{\mathbf{n}, \mathbf{n}+\mu}^{\alpha, \beta} \hat{\psi}_{\beta, \mathbf{n}+\mu} + \text{H.c.} \right] + m \sum_{\mathbf{n}} (-1)^{\mathbf{n}} \sum_{\alpha} \hat{\psi}_{\alpha, \mathbf{n}}^\dagger \hat{\psi}_{\alpha, \mathbf{n}} + \frac{g^2}{2} \sum_{\mathbf{n}} \hat{E}_{\mathbf{n}, +\mu}^2$$

MATTER HILBERT SPACE

STAGGERED FLAVORLESS SU(2) COLOR FERMION FIELDS

4 matter states

quarks q , baryons b			
\emptyset	q	q	b
			even sites
\emptyset	\bar{q}	\bar{q}	\bar{b}
			odd sites

$$\mathcal{H} = \mathcal{H}_{\text{matter}} \otimes \mathcal{H}_{\text{gauge}} \Big|_{\text{Gauss Law}}$$

GAUGE HILBERT SPACE

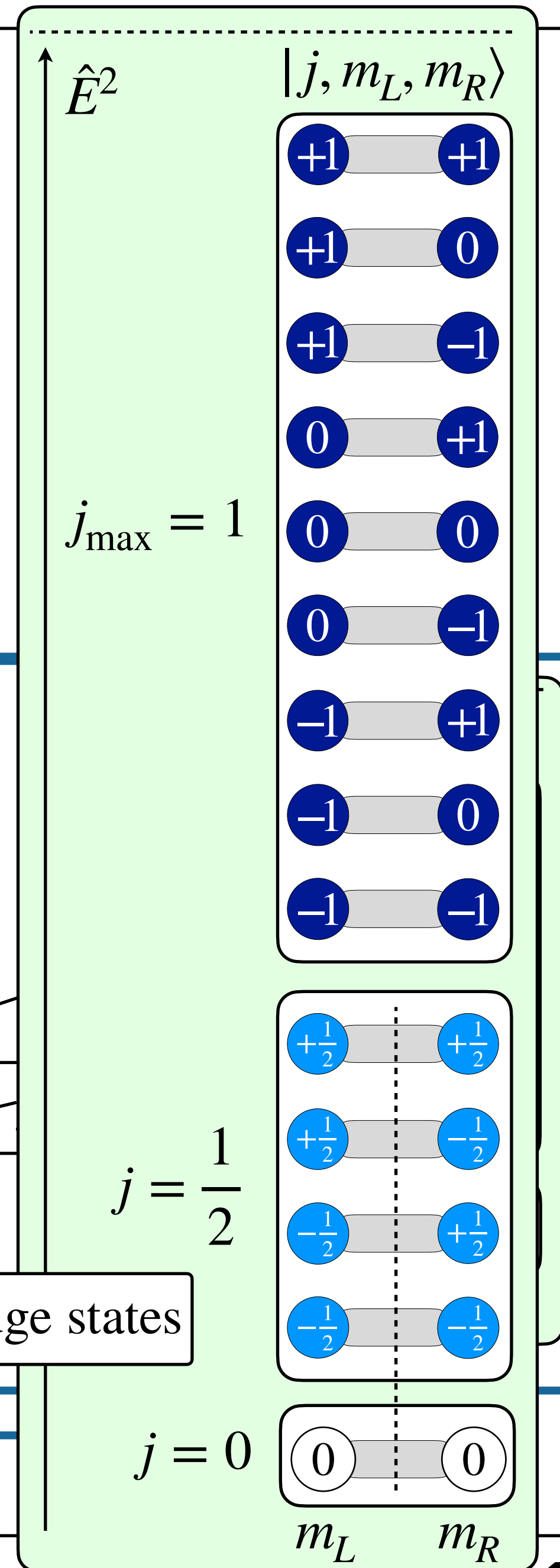
infinite in the Electric basis

$$\hat{E}^2 |j, m_L, m_R\rangle = j(j+1) |j, m_L, m_R\rangle$$

$j \in \mathbb{N}/2$

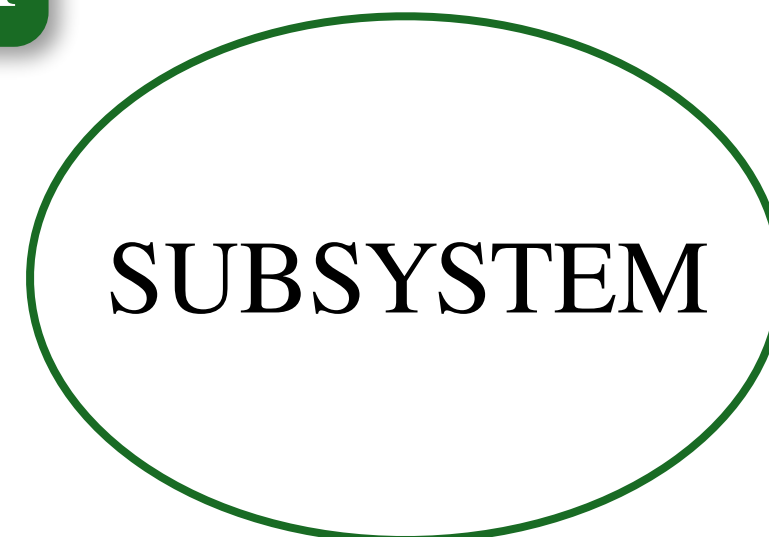
energy cutoff

14 gauge states



OPEN SYSTEM VS CLOSED SYSTEM

CLOSED SYSTEM



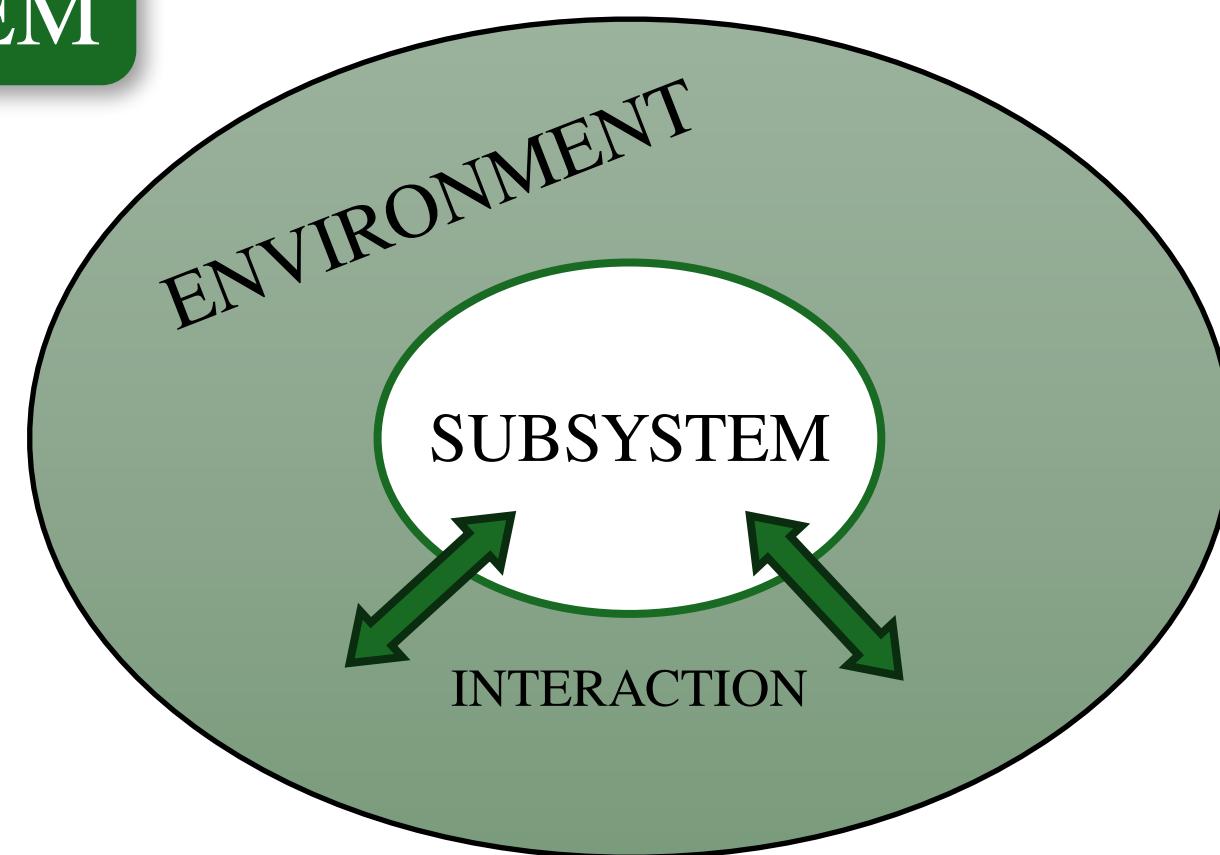
$$\hat{H}_{\text{tot}} = \hat{H}_S$$



VON NEUMANN EQUATION

$$\frac{d\hat{\rho}(t)}{dt} = -i \left[\hat{H}, \hat{\rho}(t) \right]$$

OPEN SYSTEM



$$\hat{H}_{\text{tot}} = \hat{H}_S + \hat{H}_E + \hat{H}_I$$



LINBLAD MASTER EQUATION

$$\frac{d\hat{\rho}_S(t)}{dt} = -i \left[\hat{H}_S + \Delta\hat{H}_S, \hat{\rho}_S(t) \right] + \sum_{\mathbf{n}_1, \mathbf{n}_2} D_{\mathbf{n}_1, \mathbf{n}_2}(\omega = 0) \left(\tilde{O}_{\mathbf{n}_2} \hat{\rho}_S(t) \tilde{O}_{\mathbf{n}_1}^\dagger - \frac{1}{2} \left\{ \tilde{O}_{\mathbf{n}_1}^\dagger \tilde{O}_{\mathbf{n}_2}, \hat{\rho}_S(t) \right\} \right)$$

ASSUMPTIONS

RELEVANT TIMESCALES

RELAXATION TIME

$$\tau_R \sim \frac{T}{\left(H_I^{(\text{int})}\right)^2}$$

SUBSYSTEM TIMESCALE

$$\tau_S \sim \frac{1}{H_S}$$

ENVIRONMENT CORRELATION TIME

$$\tau_E \sim \frac{1}{T}$$

BROWNIAN MOTION LIMIT

$$\tau_R \gg \tau_E$$

$$\tau_S \gg \tau_E$$

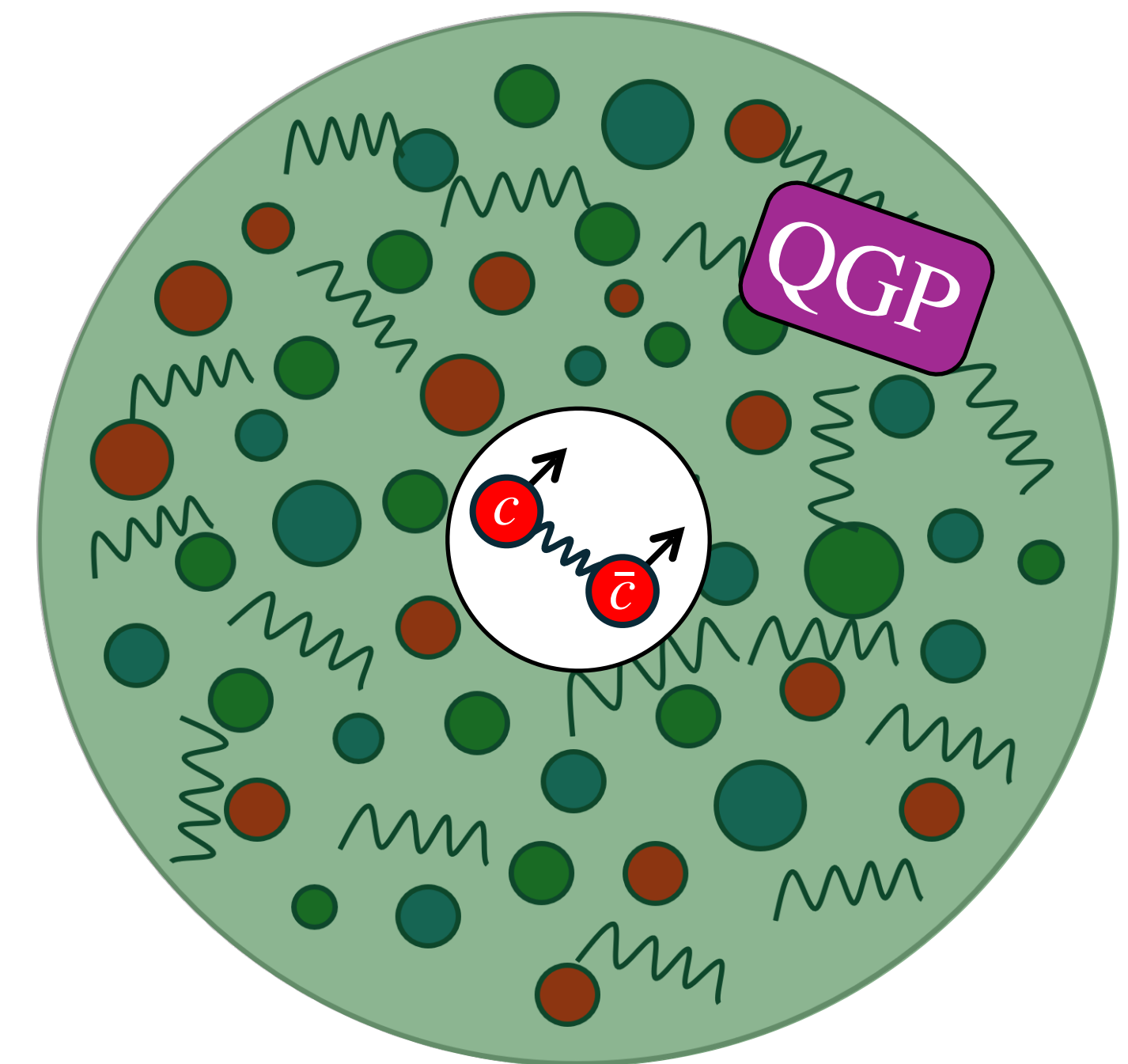
WEAK COUPLING LIMIT

$$\hat{\rho}(t) = \hat{\rho}_S(t) \otimes \hat{\rho}_E$$

BROWNIAN MOTION LIMIT

$$T \gg H_I^{(\text{int})}$$

$$T \gg H_S$$

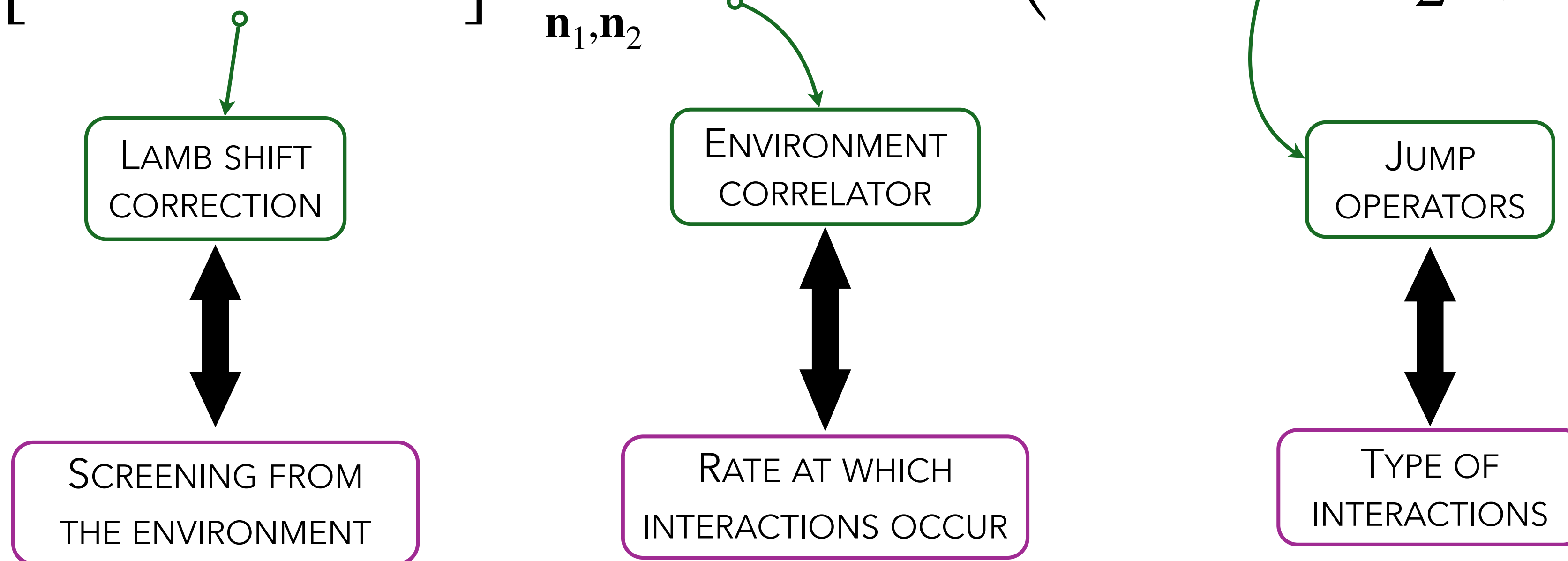


$$T \sim 10^{12} \text{ [Kelvin]}$$

TIME EVOLUTION EQUATION

LINBLAD MASTER EQUATION IN THE BROWNIAN MOTION LIMIT

$$\frac{d\hat{\rho}_S(t)}{dt} = -i \left[\hat{H}_S + \Delta\hat{H}_S, \hat{\rho}_S(t) \right] + \sum_{\mathbf{n}_1, \mathbf{n}_2} D_{\mathbf{n}_1, \mathbf{n}_2}(\omega = 0) \left(\tilde{O}_{\mathbf{n}_2} \hat{\rho}_S(t) \tilde{O}_{\mathbf{n}_1}^\dagger - \frac{1}{2} \left\{ \tilde{O}_{\mathbf{n}_1}^\dagger \tilde{O}_{\mathbf{n}_2}, \hat{\rho}_S(t) \right\} \right)$$



INTERACTION TYPE & LAMB SHIFT

YUKAWA POTENTIAL

$$\hat{H}_I = \lambda \sum_{n,\alpha} (-1)^n \hat{\phi}_{n,\alpha} \hat{\psi}_{n,\alpha}^\dagger \hat{\psi}_{n,\alpha}$$

JUMP OPERATOR

$$\hat{O}_n = (-1)^n \hat{\psi}_{n,\alpha}^\dagger \hat{\psi}_{n,\alpha}$$

$$\Delta \hat{H}_S = \sum_{\alpha,\beta} S_{\alpha\beta} \hat{O}_\alpha^{(S)} \hat{O}_\beta^{(S)} + \frac{i}{8T} \sum_{\alpha,\beta} D_{\alpha,\beta} \left\{ \hat{O}_\alpha^{(S)}, \left[H_S, \hat{O}_\beta^{(S)} \right] \right\}$$

LAMB SHIFT

PHENOMENOLOGICALLY EXPECTED TO BE IRRELEVANT:

X. Yao, Int. J. Mod. Phys. A 36, 2130010 (2021)

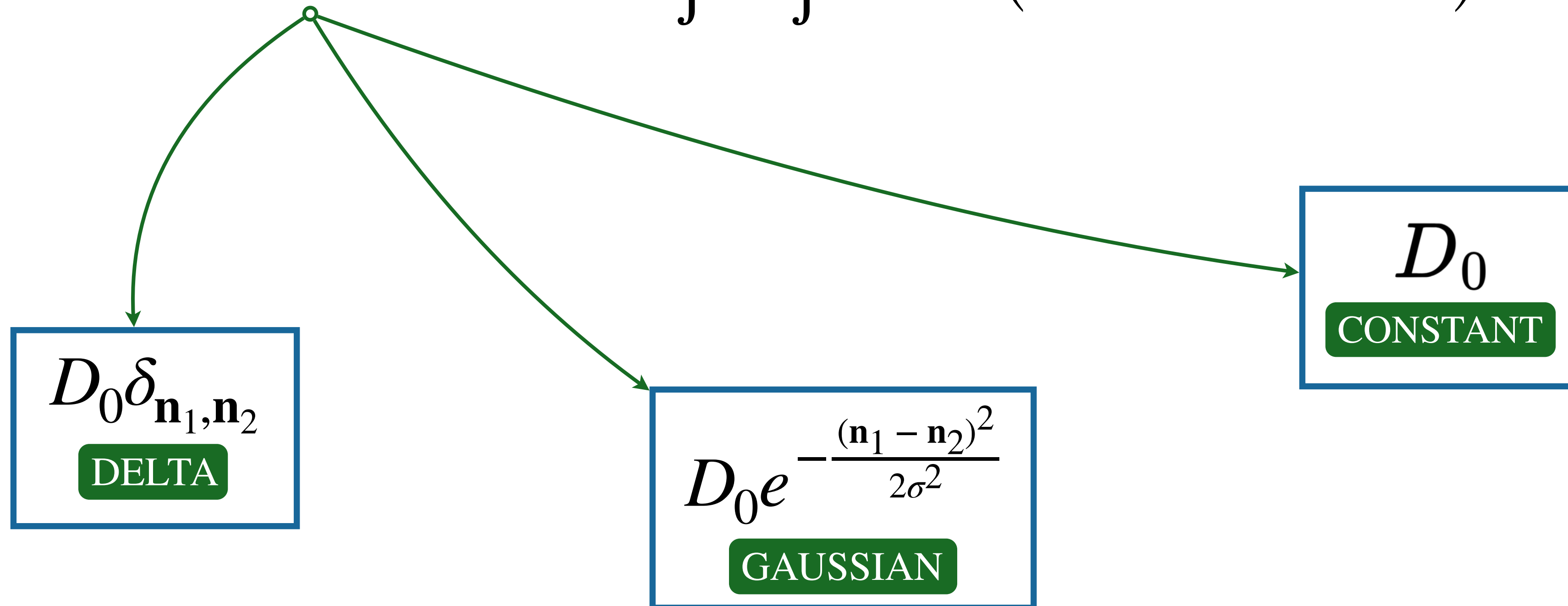
Lee et al., PRD 108, 094518 (2023)

Angelides et al., JHEP 04, 195 (2025)

ENVIRONMENT CORRELATORS

ENVIRONMENT CORRELATOR

$$D_{\mathbf{n}_1, \mathbf{n}_2}(\omega = 0) = \lambda^2 \int dt_1 \int dt_2 \text{Tr}_E \left(\tilde{O}_{\mathbf{n}_1}^{(E)}(t_1) \tilde{O}_{\mathbf{n}_2}^{(E)}(t_2) \hat{\rho}_E \right)$$



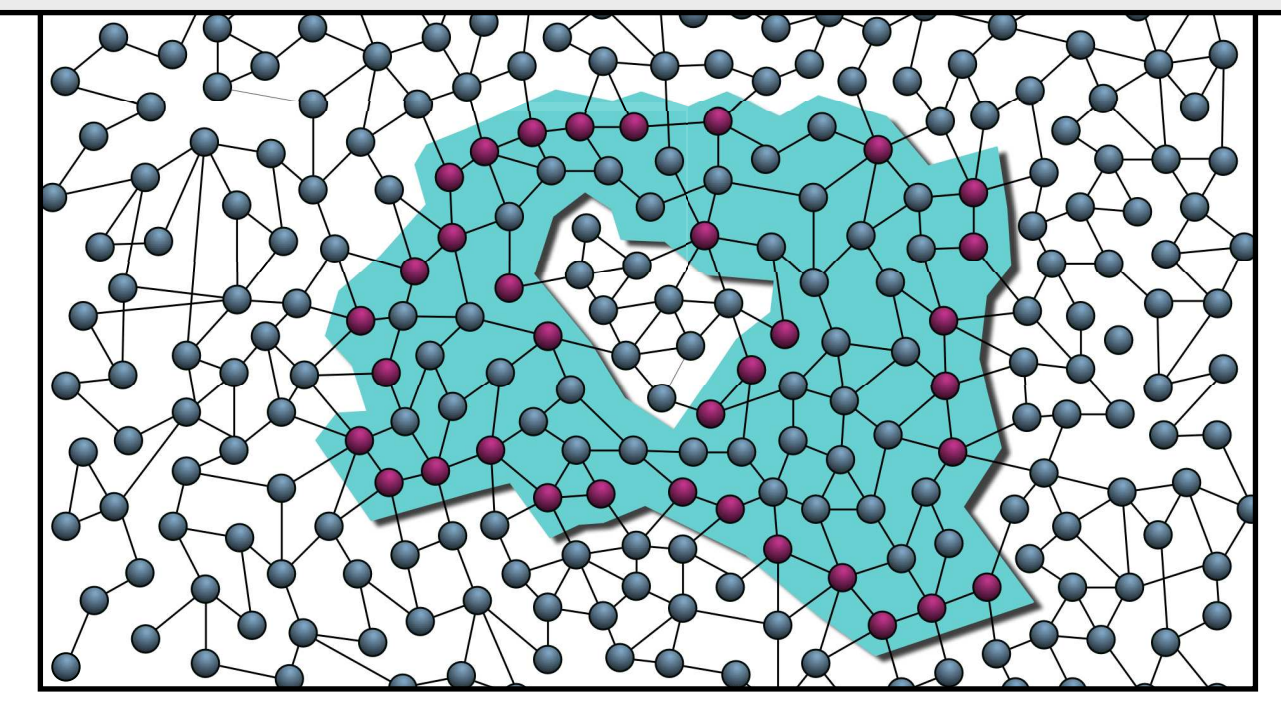
TENSOR NETWORK METHODS

The Hilbert space of N-Body Systems grows exponentially. Exact Diagonalization (ED) is not sustainable for large N

$$T_{\alpha_1 \dots \alpha_N} |\Psi_{\text{QMB}}\rangle = \sum_{\alpha_1 \dots \alpha_N} T_{\alpha_1 \dots \alpha_N} |\alpha_1 \dots \alpha_N\rangle$$

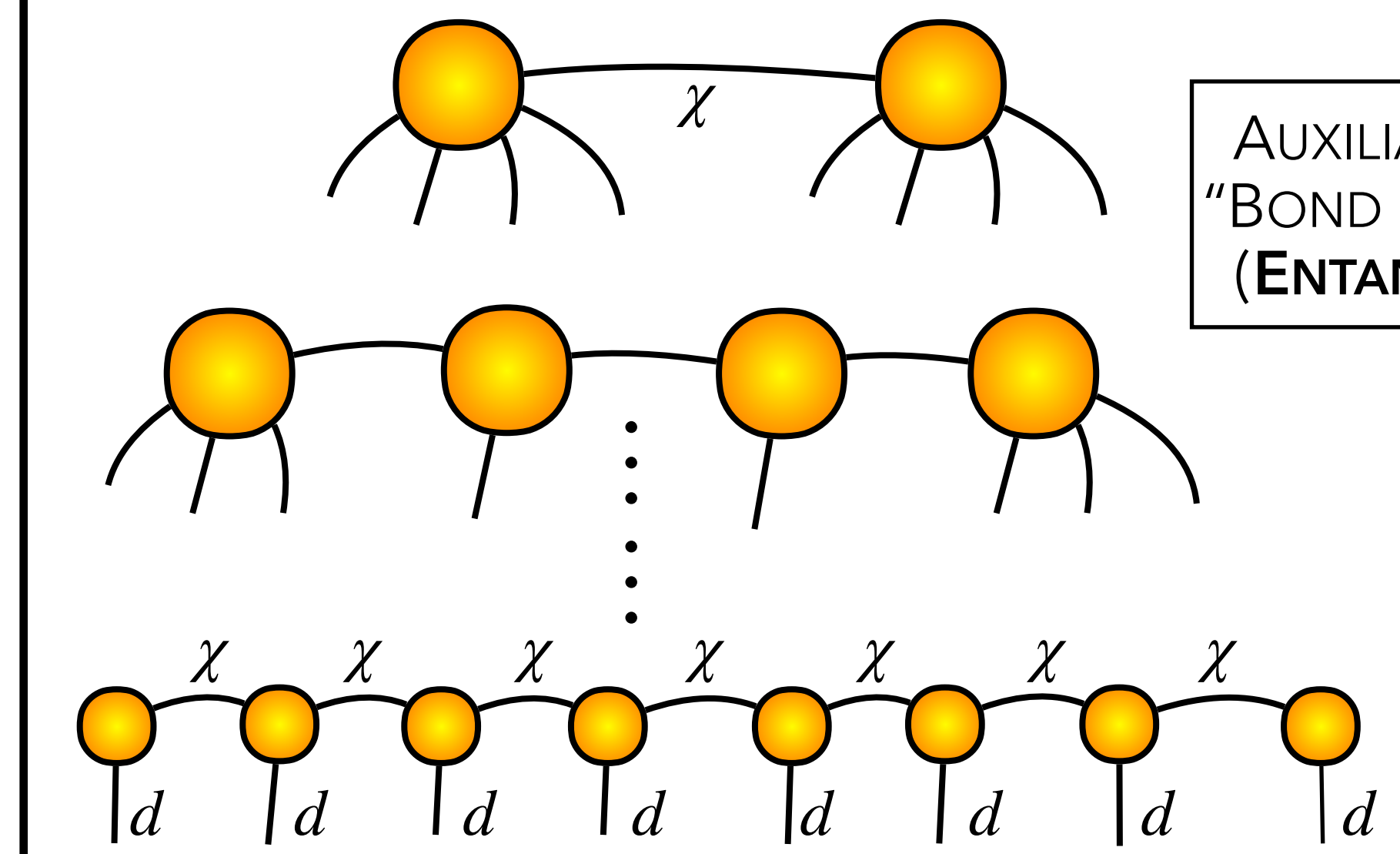
$$\dim \mathcal{H} = d^N$$

FOR GROUND STATES, THERE IS AREA LAW!



[Eisert et al, Rev. Mod. Phys. 82, 277 (2010)]

Trunc SVD



AUXILIARY LINKS χ
"BOND DIMENSION"
(ENTANGLEMENT)

$$\dim \mathcal{H}_{\text{trunc}} \sim N\chi^2 d$$

FOR TIME EVOLUTION

TIME-EVOLVING
BLOCK DECOMPOSITION
(TEBD)

[G. Vidal, PRL 93,040502 (2004)]

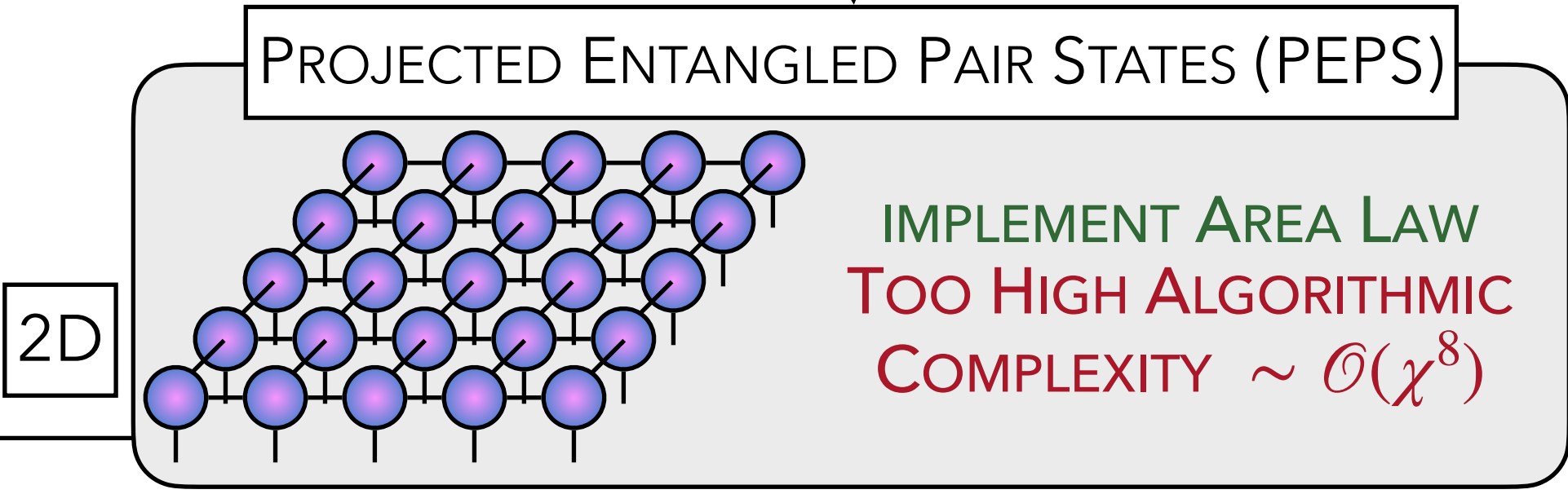
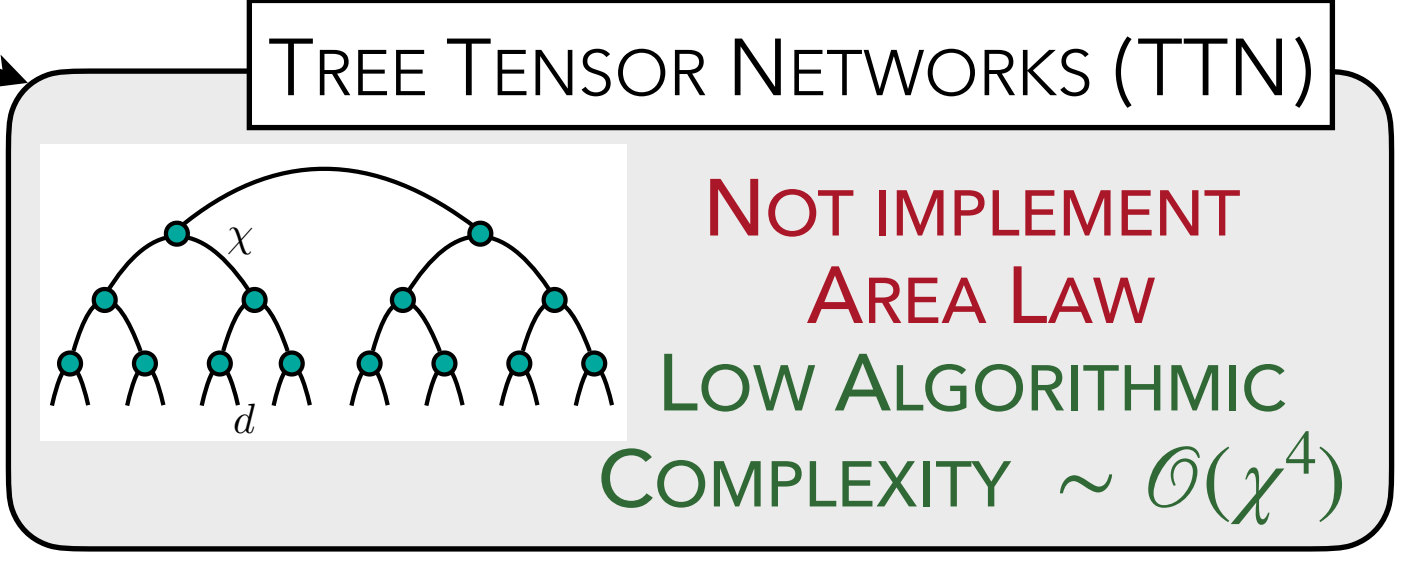
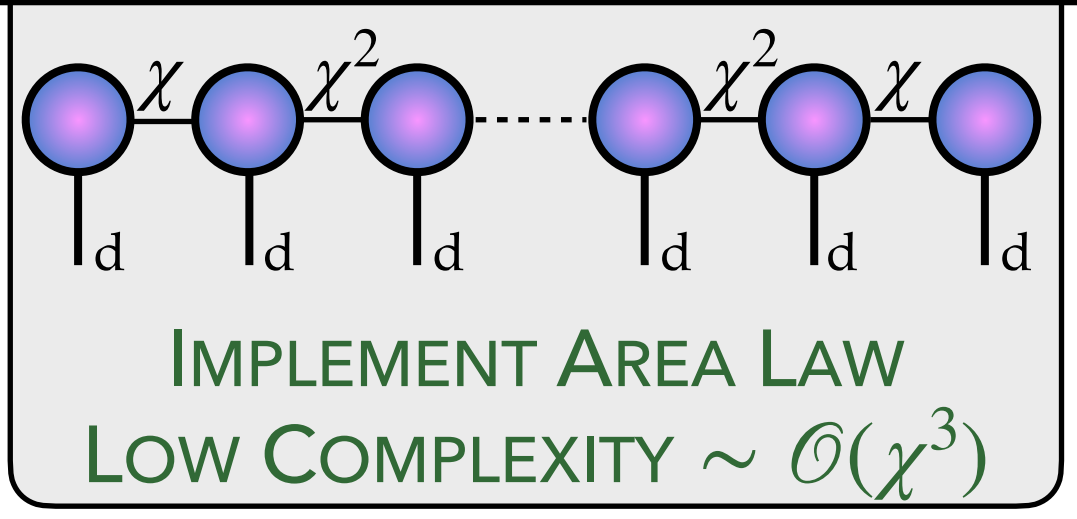
TIME-DEPENDENT
VARIATIONAL PRINCIPLE
(TVDP)

[J. Haegeman et al, PRL 107, 070601 (2011)]

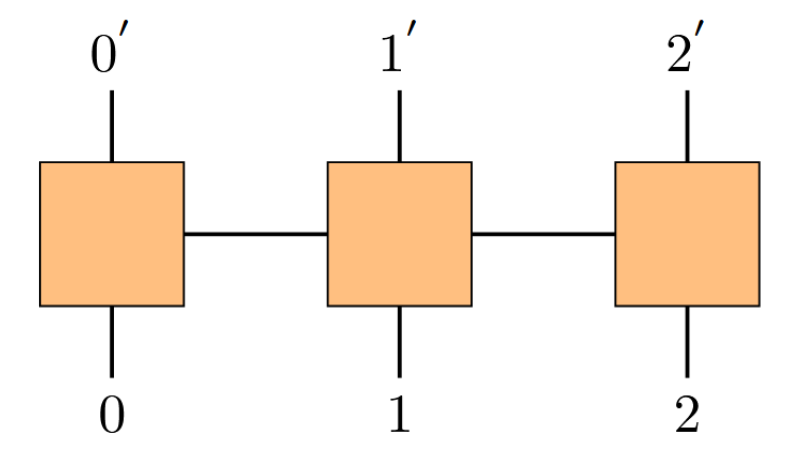
TENSOR NETWORK ANSÄTZE

POLYNOMIAL COMPLEXITY: MANIPULATIONS $\sim \mathcal{O}(\chi^k)$
[Silvi et al, SciPostPhysLectNotes.8 (2019)]

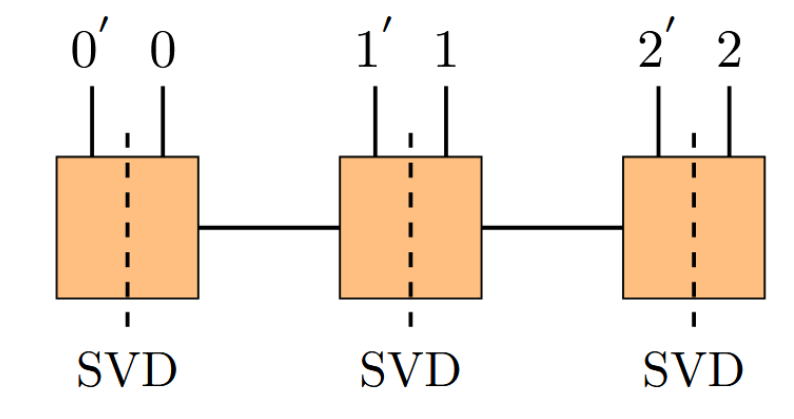
1D MATRIX PRODUCT STATES (MPS)



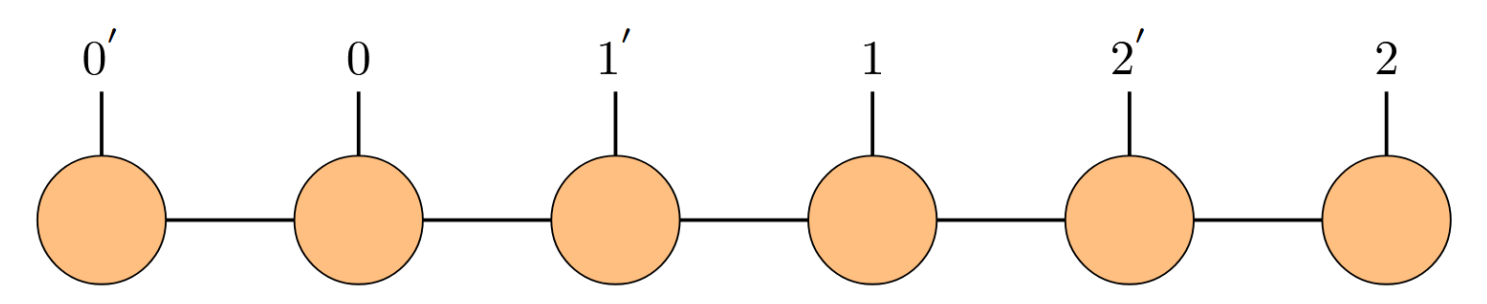
MPS FOR OPEN QUANTUM MANY-BODY SYSTEMS



(a) Initial MPO representing ρ_S .



(b) Separating the legs on each site with SVD.



(c) MPS representing ρ_S equivalent to the initial MPO in (a).

Angelides et al., JHEP 04, 195 (2025)

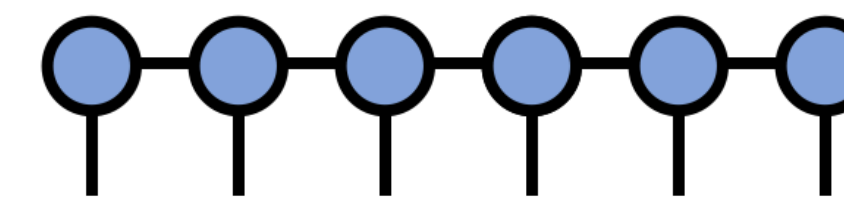
TENSOR NETWORKS FOR OPEN SYSTEMS

(1 + 1)D U(1) LGT

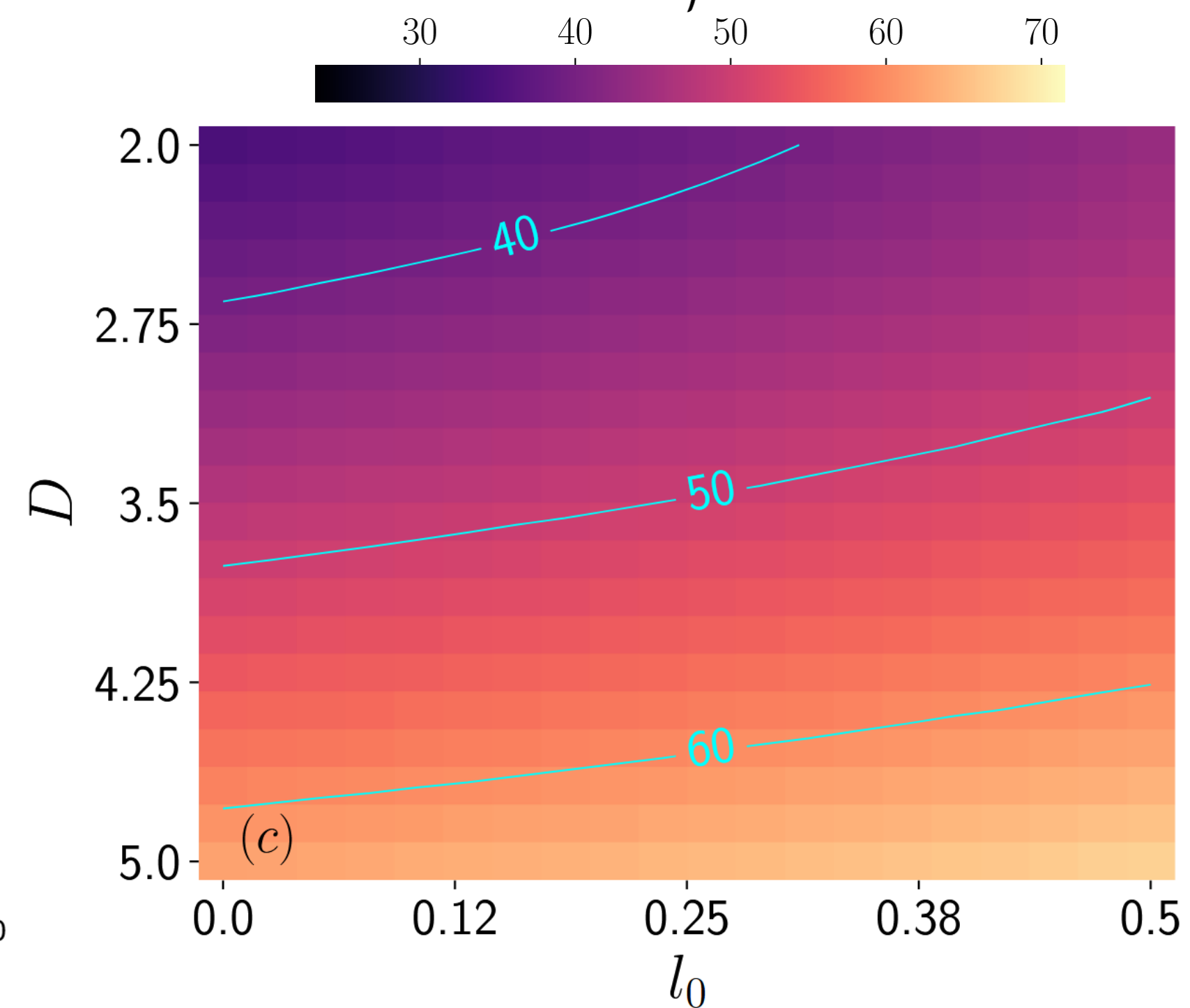
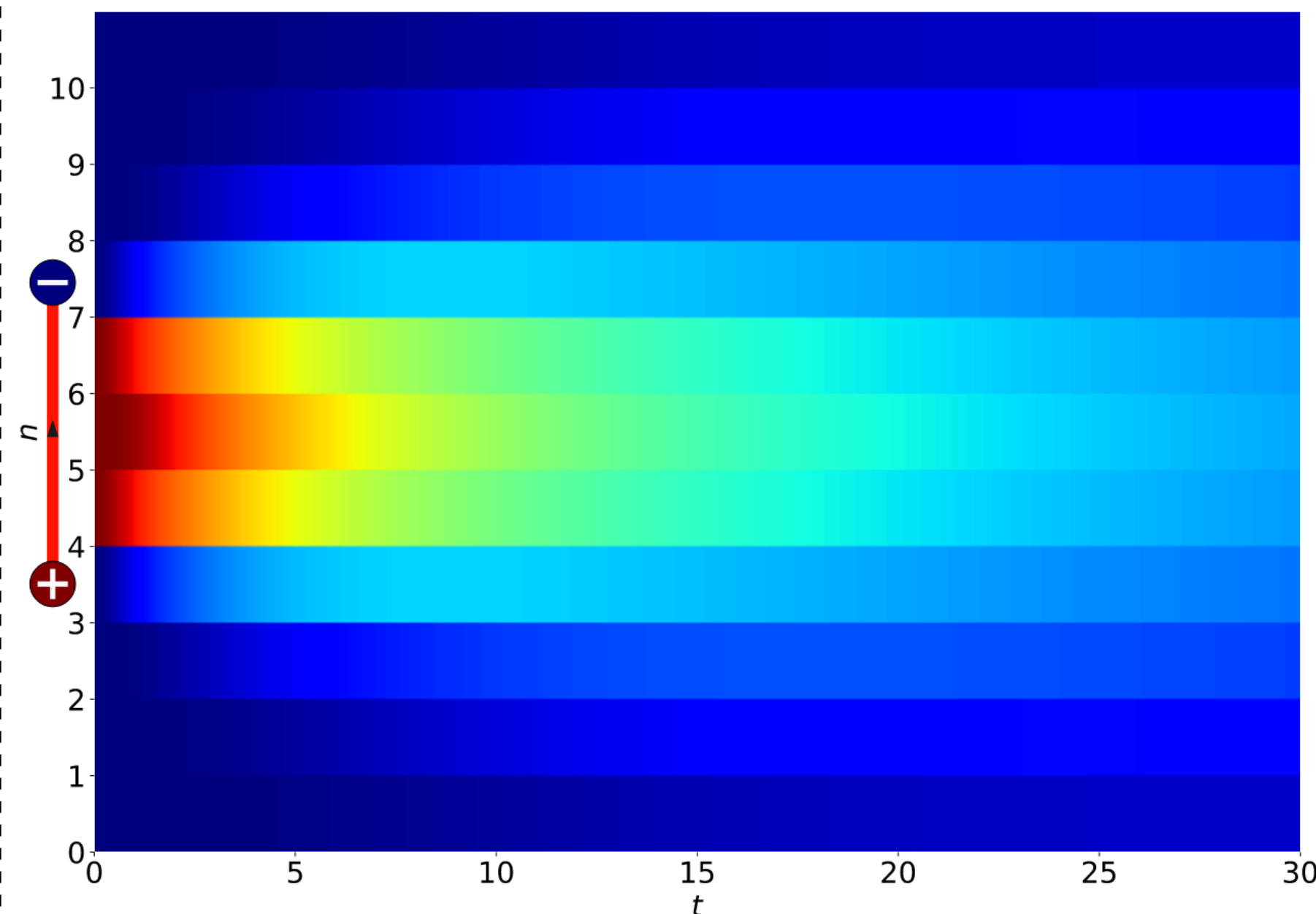
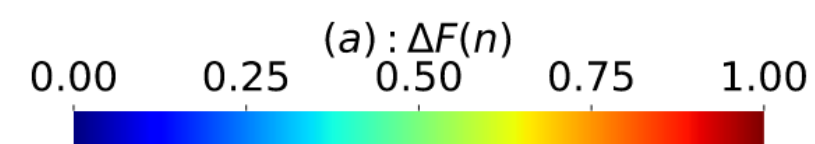
DELTA CORRELATOR

$$D_{\alpha,\beta}(\omega = 0) = D\delta_{\alpha,\beta}$$

MATRIX PRODUCT STATES



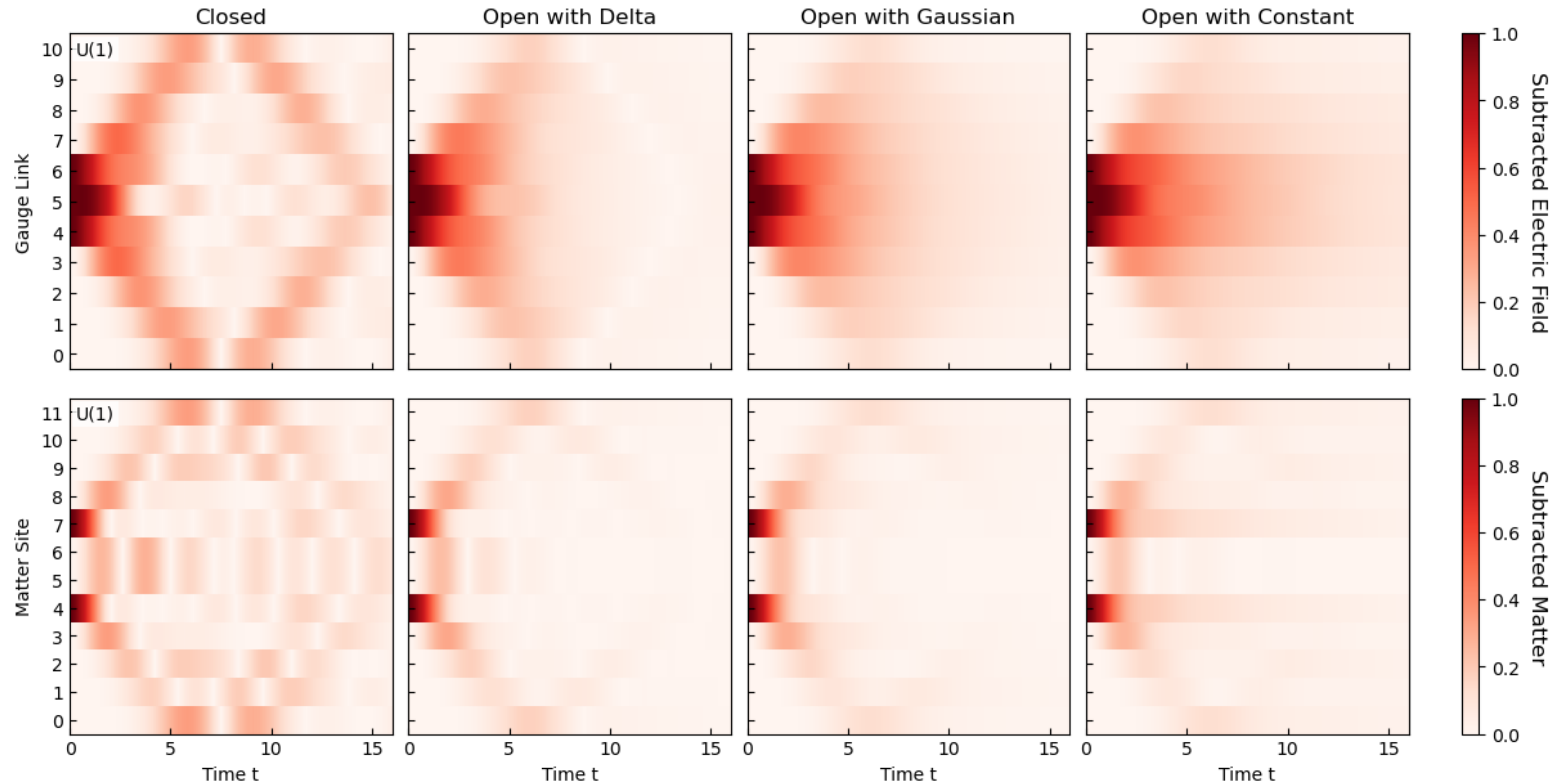
OPEN



Angelides et al., JHEP 04, 195 (2025)

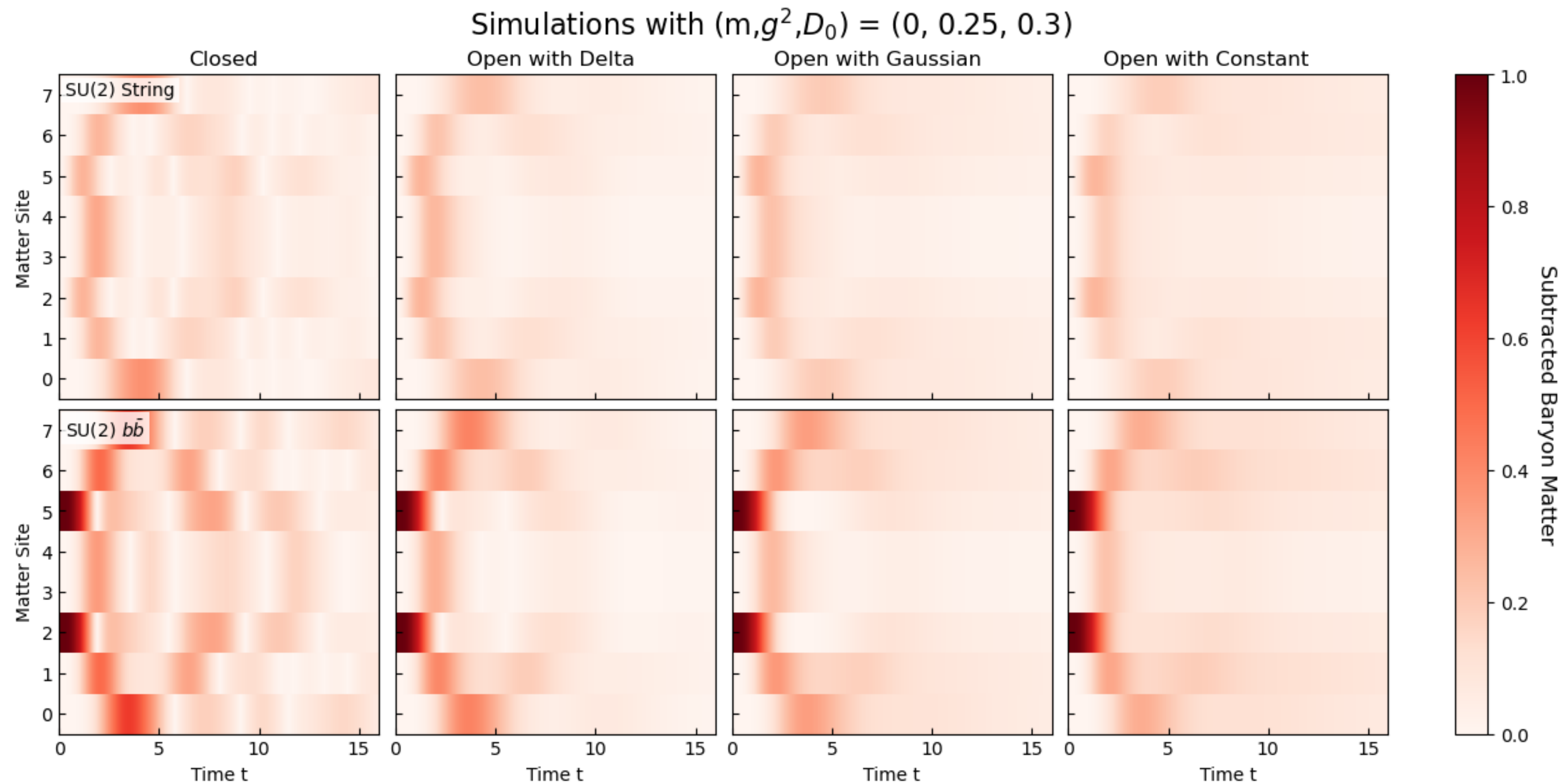
SITE-RESOLVED $U(1)$ DYNAMICS

Simulations with $(m, g^2, D_0) = (0, 0.25, 0.3)$



A CONSTANT
ENVIRONMENT
TRIES TO KEEP
THE MATTER
WHERE IT
BEGAN MOST

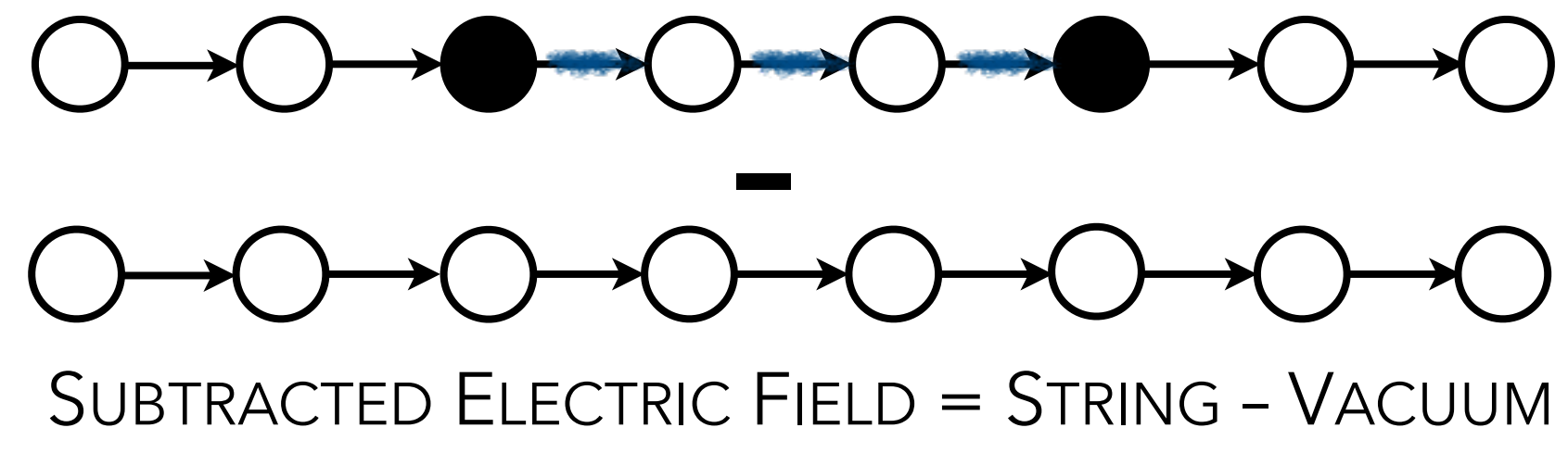
SITE-RESOLVED SU(2) DYNAMICS



EMPIRICAL THERMALIZATION TIME

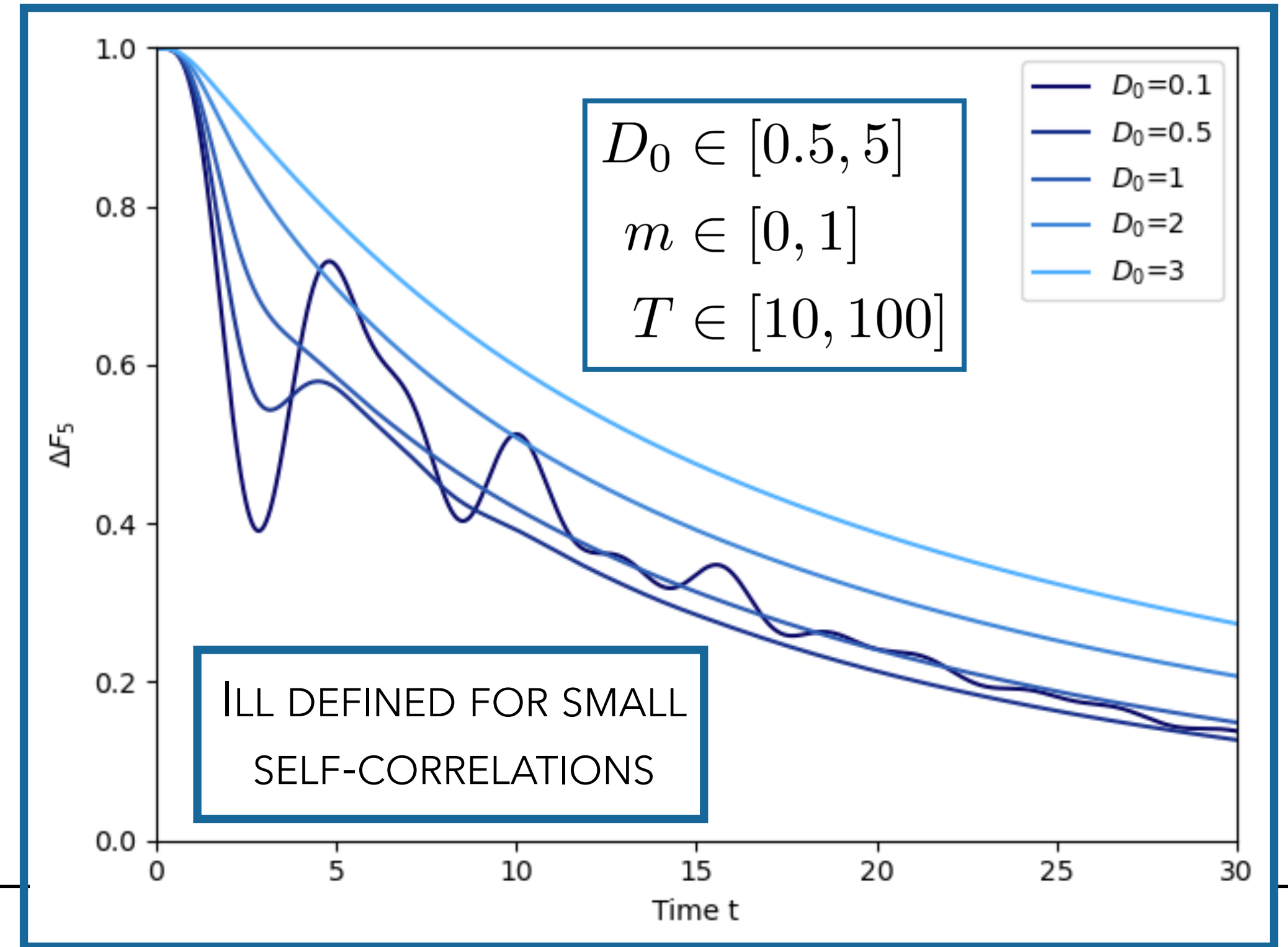
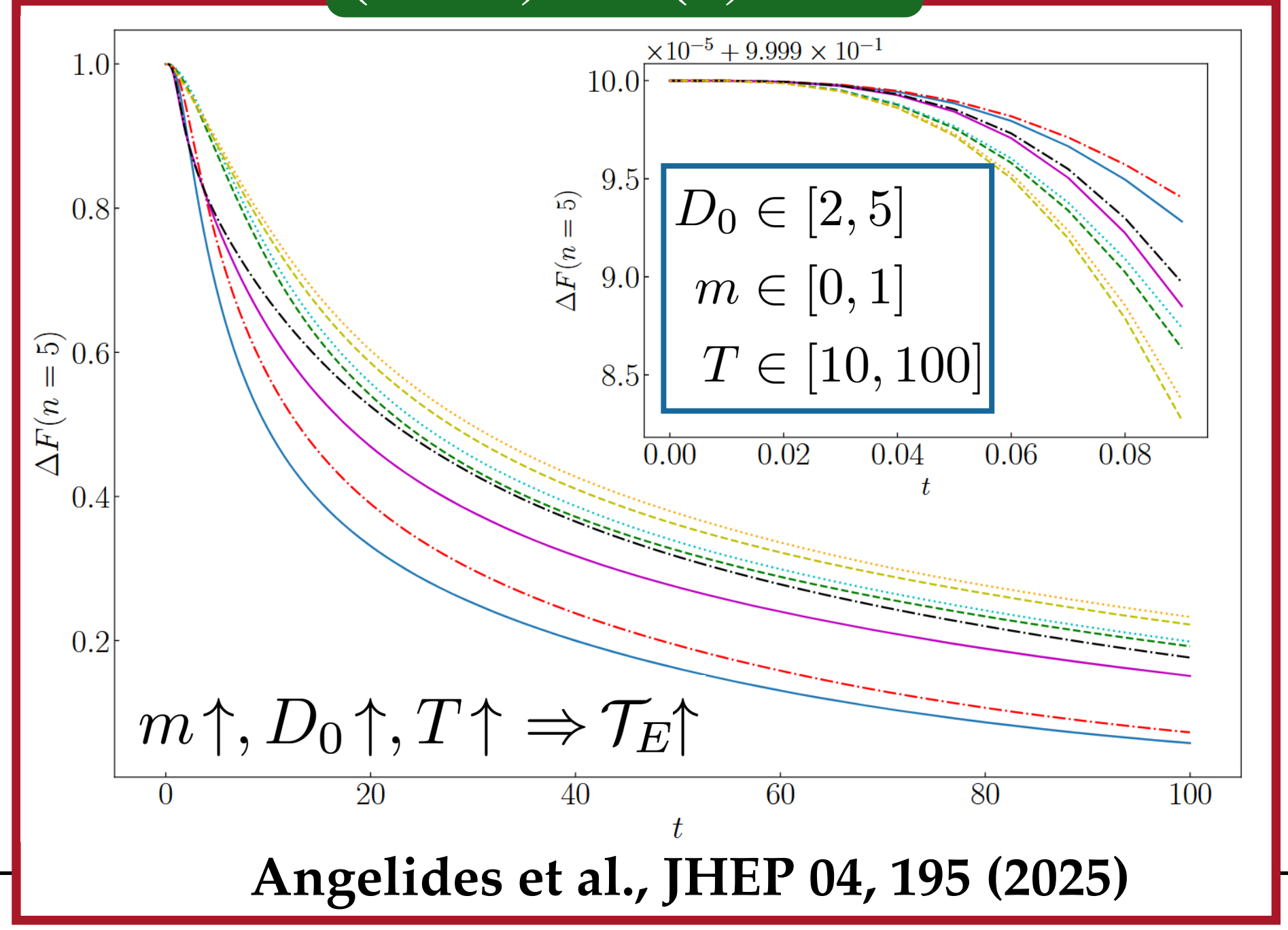
EMPIRICAL THERMALIZATION TIME

$\mathcal{T}_E =$ TIME FOR SEF IN THE CENTRAL GAUGE LINK TO DECREASE BY A GIVEN %



SEF

(1 + 1)D U(1) LGT



LIOUVILLIAN THERMALIZATION TIME

$$\frac{d\hat{\rho}}{dt} = \mathcal{L}\hat{\rho} \Rightarrow \hat{\rho}(t) = e^{\mathcal{L}t} \hat{\rho}(0)$$

THE LIOUVILLIAN IS
NOT-HERMITIAN!

LIOUVILLIAN DIAGONALIZATION

\hat{r}_k = Right Eigenvectors

$\hat{\ell}_k$ = Left Eigenvectors

λ_k = Eigenvalues

$\lambda_0 = 0$
steady state

$\text{Re}(\lambda_k) < 0 \quad \forall k > 0$
decay mode

ASSUMPTION
iff $\text{Re}(\lambda_1) \gg \text{Re}(\lambda_{k>1}) \quad \forall k$

$$\begin{aligned} \hat{\rho}(t) &= \hat{\rho}_{\text{ss}} + \sum_k e^{\lambda_k t} C(\hat{\ell}_k^\dagger) \hat{r}_k \\ &= \hat{\rho}_{\text{ss}} + e^{\lambda_1 t} C(\hat{\ell}_1) \hat{r}_1 + \mathcal{O}(e^{\lambda_2 t}) \end{aligned}$$

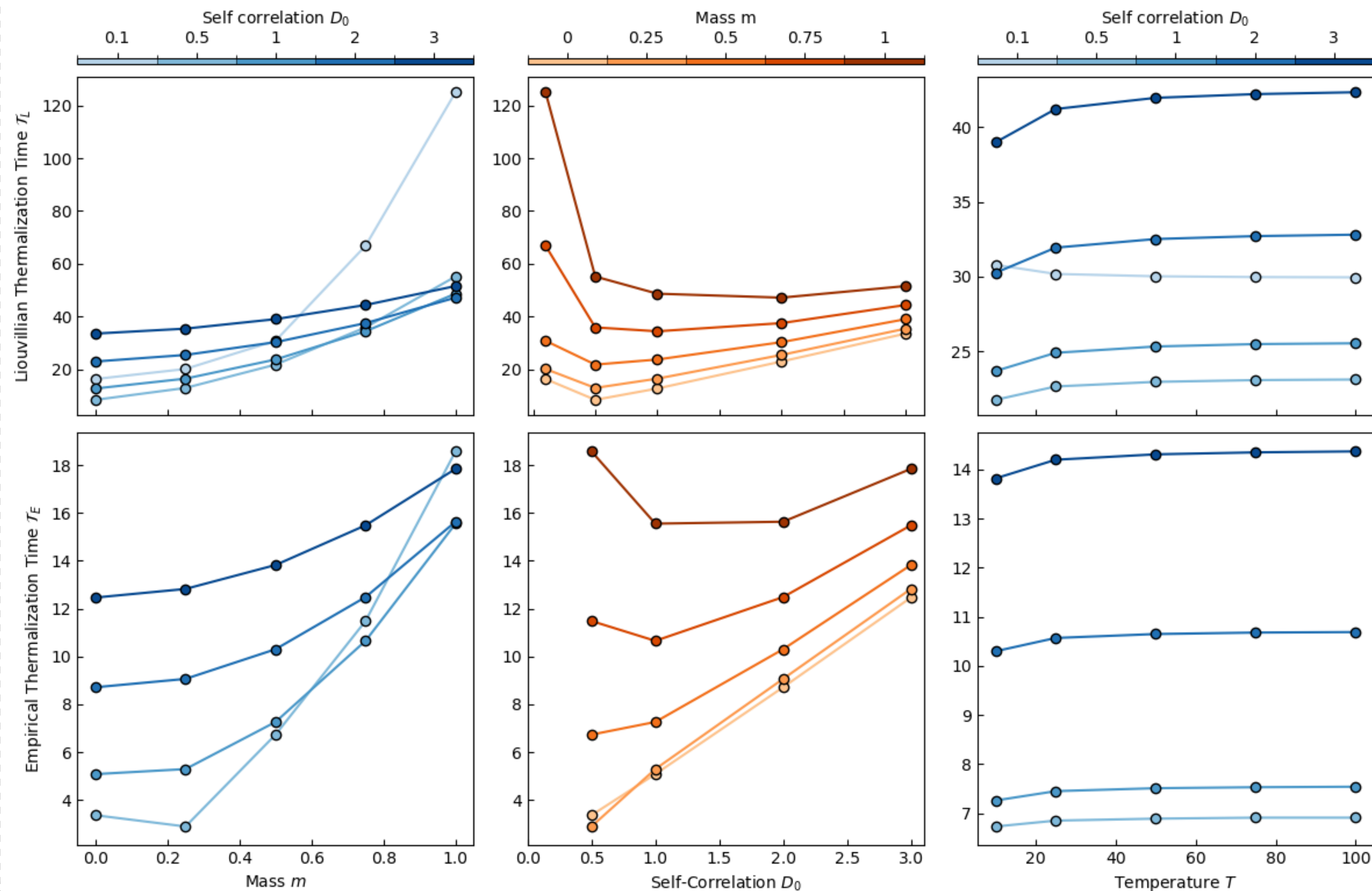
LIOUVILLIAN THERMALIZATION TIME

$$\mathcal{T}_L = \frac{1}{|\text{Re}(\lambda_1)|}$$

INVERSE OF THE REAL PART OF
THE FIRST NON-ZERO
EIGENVALUE OF THE LIOUVILLIAN

U(1) THERMALIZATION

U(1) Simulations with $g^2 = 1$



U(1)

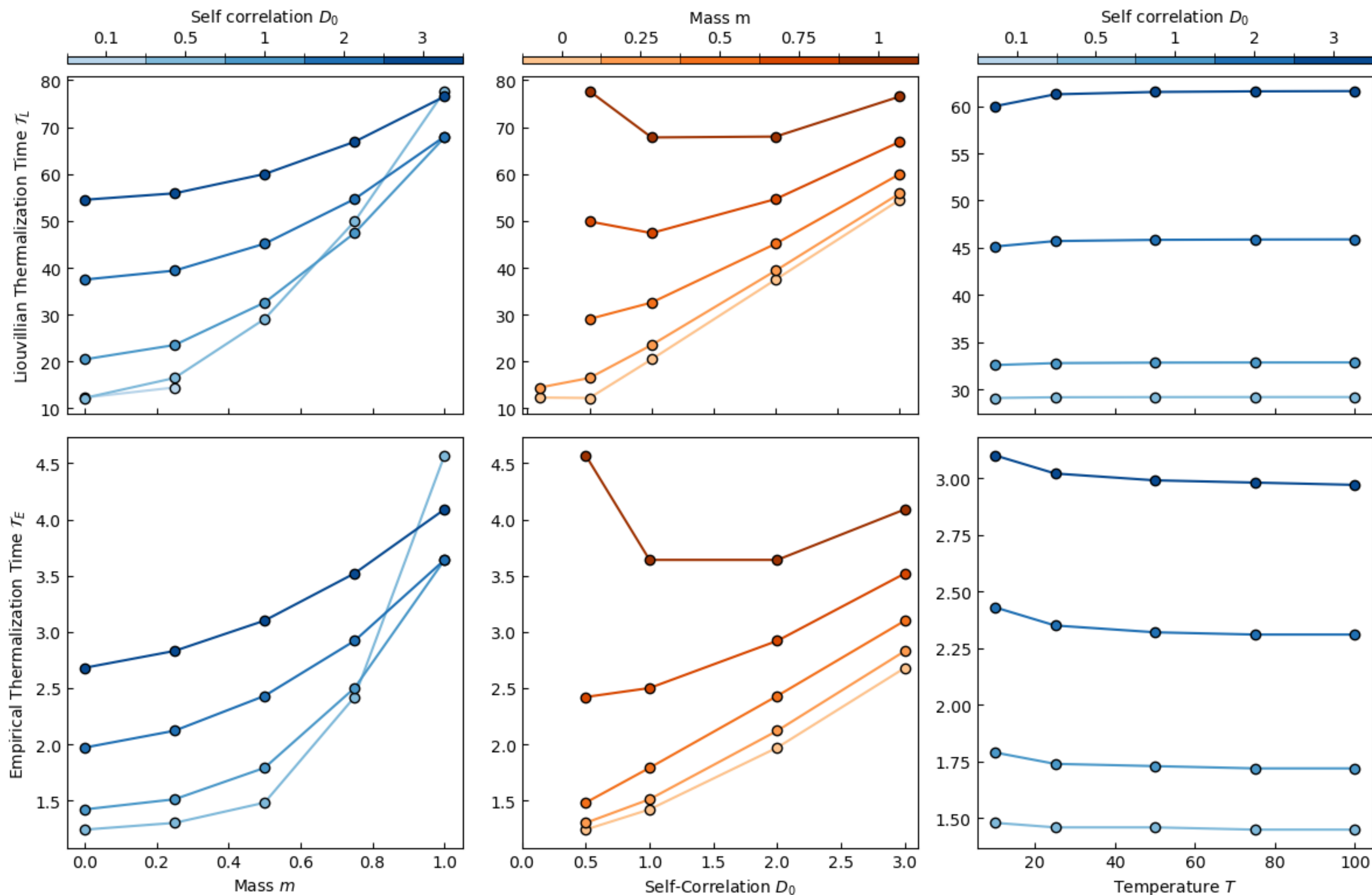
$(D_0 \sim 1)$

$$[T \uparrow, D_0 \uparrow] \rightarrow \mathcal{T}_E \uparrow$$

Angelides et al., JHEP 04, 195 (2025)

SU2 THERMALIZATION

SU(2) Simulations with $g^2 = 1$



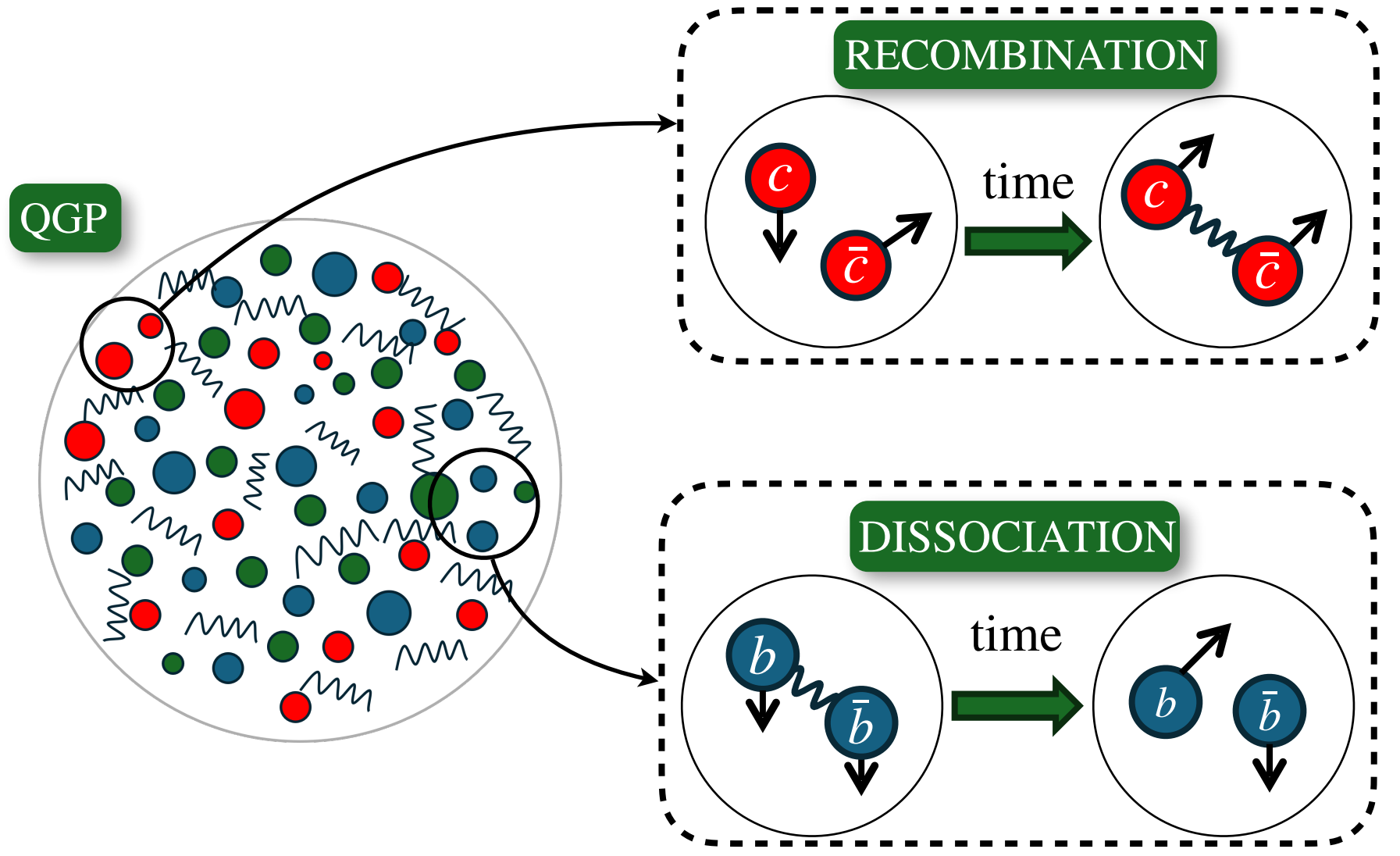
SU(2)

$(D_0 \ll 1)$

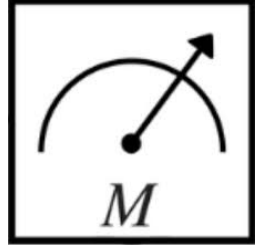
$$[T \uparrow, D_0 \uparrow] \rightarrow \tau_E \downarrow$$

POTENTIAL NON-RELATIVISTIC QCD

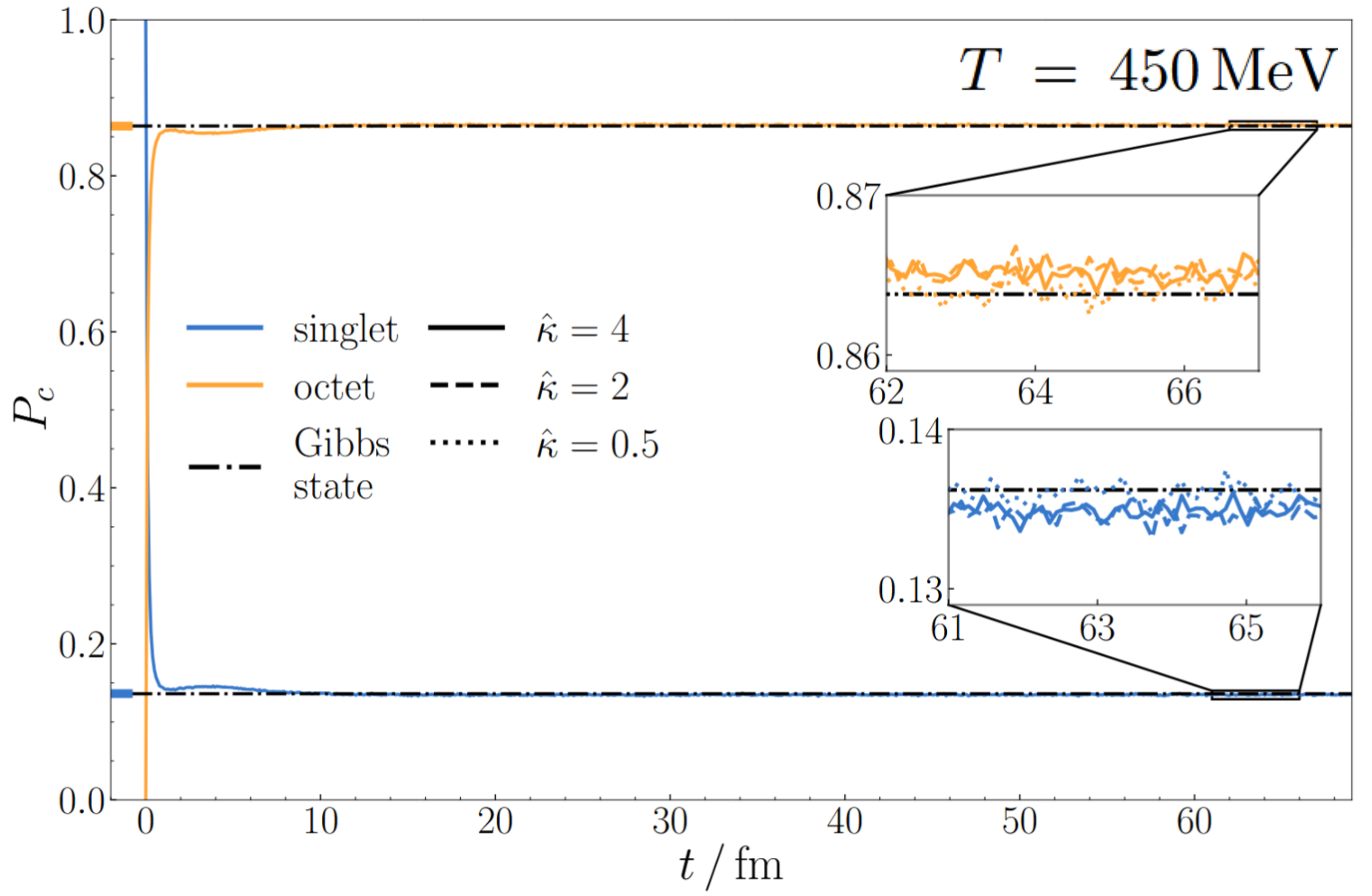
Brambilla et al., arXiv:2508.11743 (2025)



— octet $P_{octet} = \text{Free quark}$
 — singlet $P_{singlet} = \text{Quarkonia}$



How? $\hat{\rho}_{ss}$ vs. $\hat{\rho}_{Gibbs} = \frac{1}{Z} e^{-\beta \hat{H}_s}$

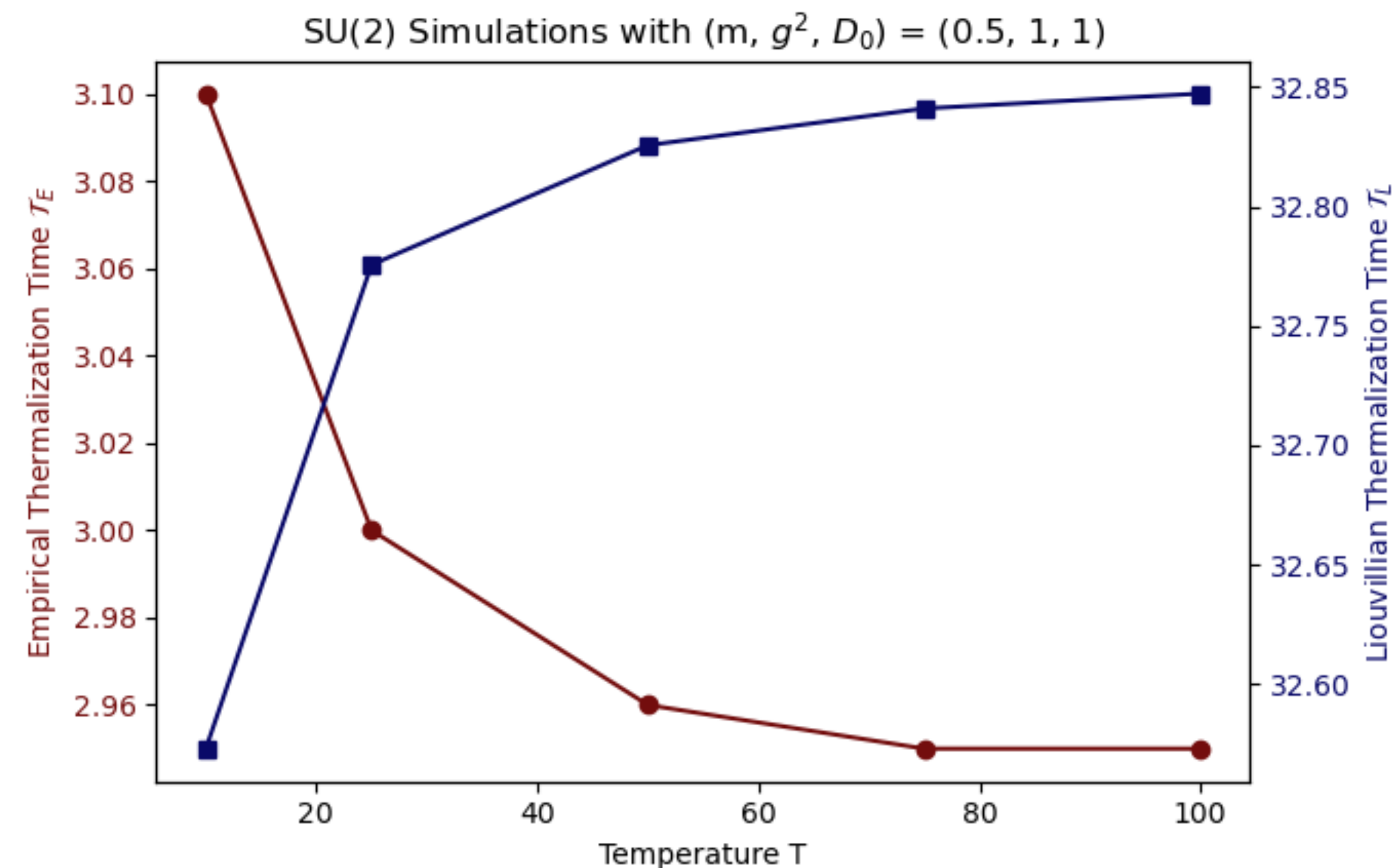


TEMPERATURE DEPENDENT CONSTANT CORRELATOR $D_{n,m} = \hat{\kappa} T^3$

DISCREPANCIES

THERMALIZATION TIME DEPENDS:

1. ON DEFINITION (EMPIRICAL VS LIOUVILLIAN)
2. ON PARAMETER REGIMES (m, g, T, D_0)
3. ON THE LATTICE GAUGE THEORY



U(1)

$(D_0 \sim 1)$

$$[T \uparrow, D_0 \uparrow] \rightarrow \mathcal{T}_E \uparrow$$

Angelides et al., JHEP 04, 195 (2025)

SU(2)

$(D_0 \ll 1)$

$$[T \uparrow, D_0 \uparrow] \rightarrow \mathcal{T}_E \downarrow$$

PNRQCD

$(D_0 \propto T)$

$$[T \uparrow, D_0 \uparrow] \rightarrow \mathcal{T}_E \downarrow$$

Brambilla et al., arXiv:2508.11743 (2025)

QUESTION: WHICH BEHAVIOUR IS EXPECTED IN QGP PHYSICS?

SUMMARY

EMPIRICAL THERMALIZATION TIME

$\mathcal{T}_E =$ TIME IT TAKES FOR SEF IN THE CENTRAL GAUGE LINK TO DECREASE BY A GIVEN%

PRO:

- EXPERIMENTALLY FRIENDLY
- TENSOR NETWORK FRIENDLY

CONS:

- MANY POSSIBLE ARBITRARY DEFINITIONS
- OBSERVABLE DEPENDENT
- ILL DEFINED IN CERTAIN PARAMETER REGIONS

LIOUVILLIAN THERMALIZATION TIME

$\mathcal{T}_L = \frac{1}{|\text{Re}(\lambda_1)|}$ INVERSE OF THE REAL PART OF THE FIRST NON-ZERO EIGENVALUE OF THE LIOUVILLIAN

PRO:

- OBJECTIVE/UNIVERSAL PROBE
- WELL BEHAVED IN ALL PARAMETER SPACE
- INDEPENDENT TO THE INITIAL STATE

CONS:

- DIFFICULT WITH TENSOR NETWORK
- DIFFICULT WITH EXPERIMENTS
- SUBJECT TO ANOMALIES

QUESTION: WHICH ONE IS MORE RELEVANT TO QUARKONIA IN QGP?

ANOMALOUS THERMALIZATION TIME

LIUVILLIAN DIAGONALIZATION

\hat{r}_k = Right Eigenvectors

$\hat{\ell}_k$ = Left Eigenvectors

λ_k = Eigenvalues

$$\begin{aligned}\hat{\rho}(t) &= \hat{\rho}_{\text{ss}} + \sum_k e^{\lambda_k t} C(\hat{\ell}_k^\dagger) \hat{r}_k \\ &= \hat{\rho}_{\text{ss}} + e^{\lambda_1 t} C(\hat{\ell}_1) \hat{r}_1 + \mathcal{O}(e^{\lambda_2 t})\end{aligned}$$

LIUVILLIAN
THERMALIZATION
TIME IS RELIABLE

$$\mathcal{T}_L = \frac{1}{|\text{Re}(\lambda_1)|}$$

iff $\lambda_1 \gg \lambda_{k>1} \forall k$

ANOMALIES

QUANTUM MPEMBA EFFECT

THE FIRST C_k ARE SMALL

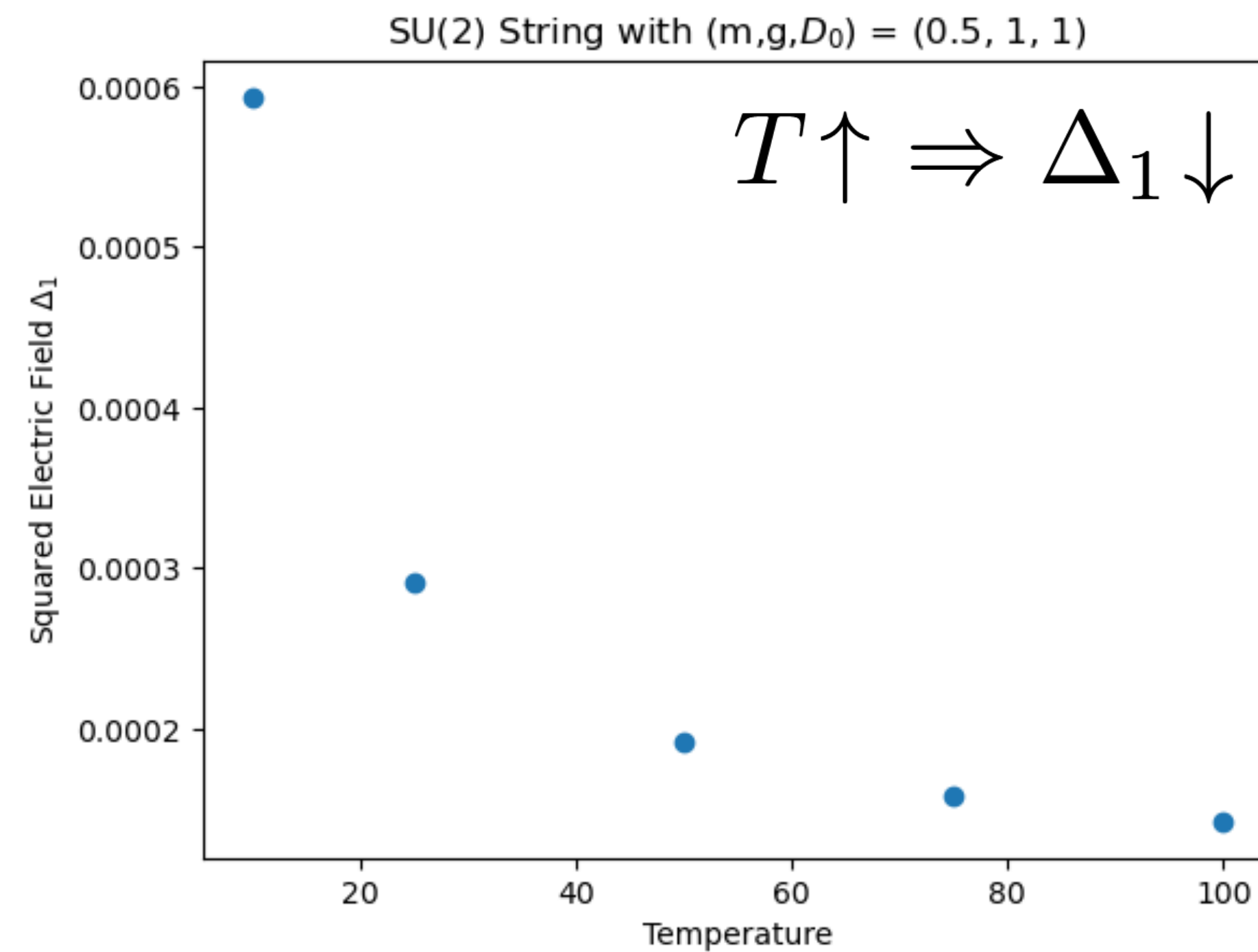
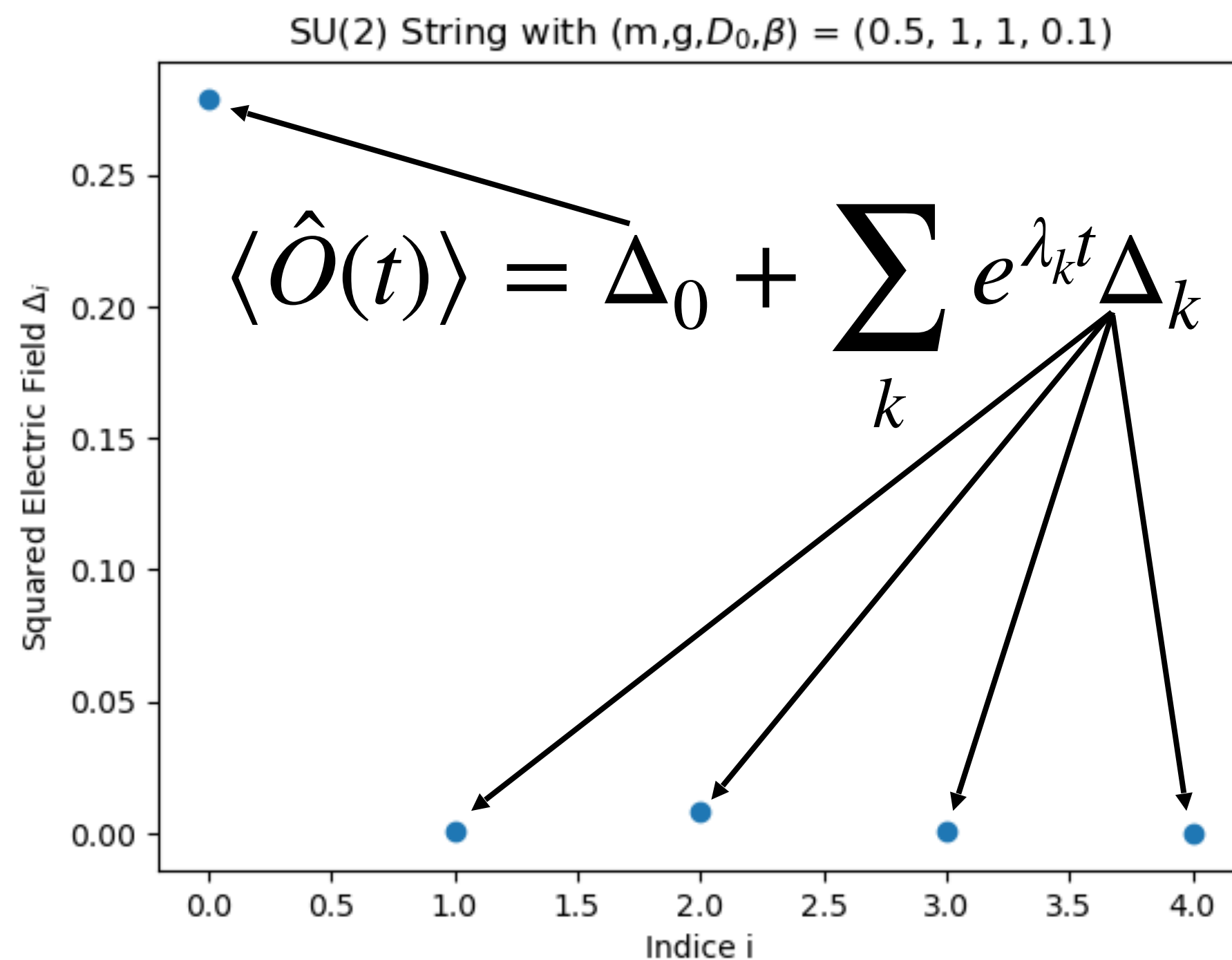
SUPER-EXPONENTIAL DECAY MODE

MANY LARGE C_k ARRIVES LATER

EFFECTS ON LOCAL OBSERVABLES

THESE ANOMALIES CAN BE
OBSERVABLE DEPENDENT

$$\langle \hat{O}(t) \rangle = \Delta_0 + \sum_k e^{\lambda_k t} \text{Tr}(\hat{O}^\dagger \hat{r}_k) C_k = \Delta_0 + \sum_k e^{\lambda_k t} \Delta_k$$



REAL-TIME DYNAMICS OF ENTROPY

VON NEUMANN ENTROPY

$$S_{vN} = -\text{tr}(\rho \log(\rho))$$

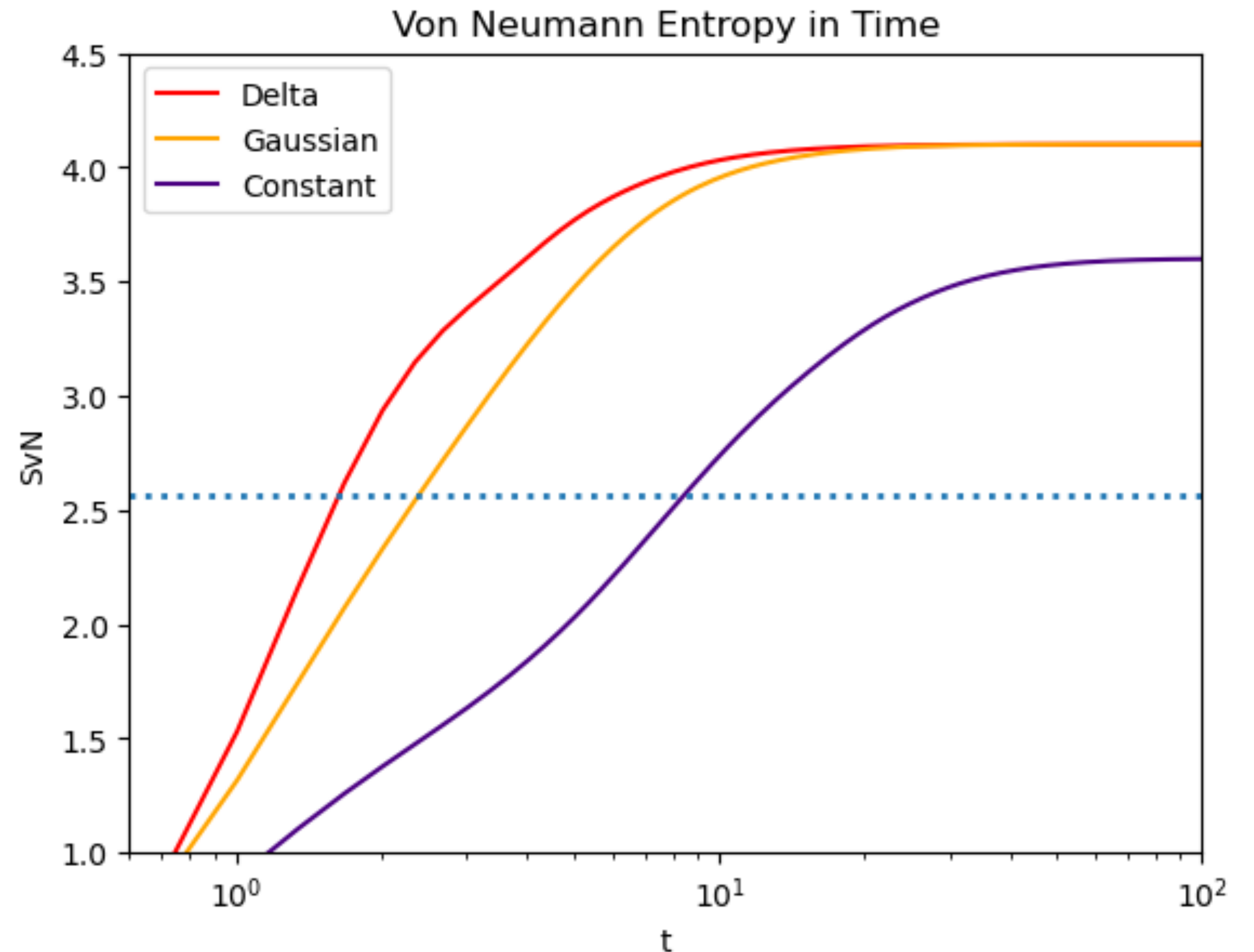
IS BOUNDED BY:

$$0 \leq S_{vN} \leq \log d$$

IN THE LIMIT OF INFINITE TEMPERATURE, THE ENTROPY GOES TO

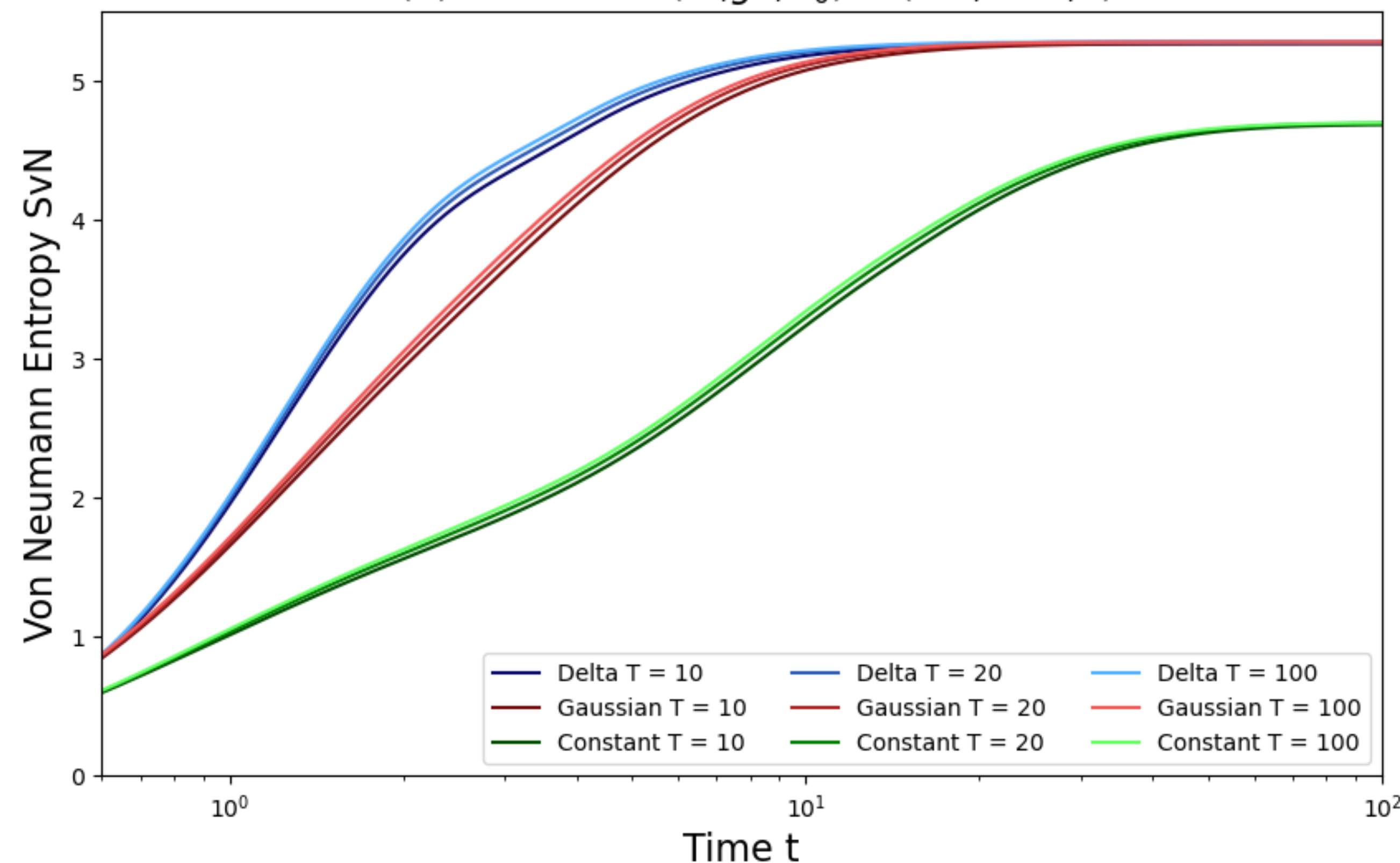
$$S_{vN} \rightarrow \log d$$

IN THE STEADY STATE.

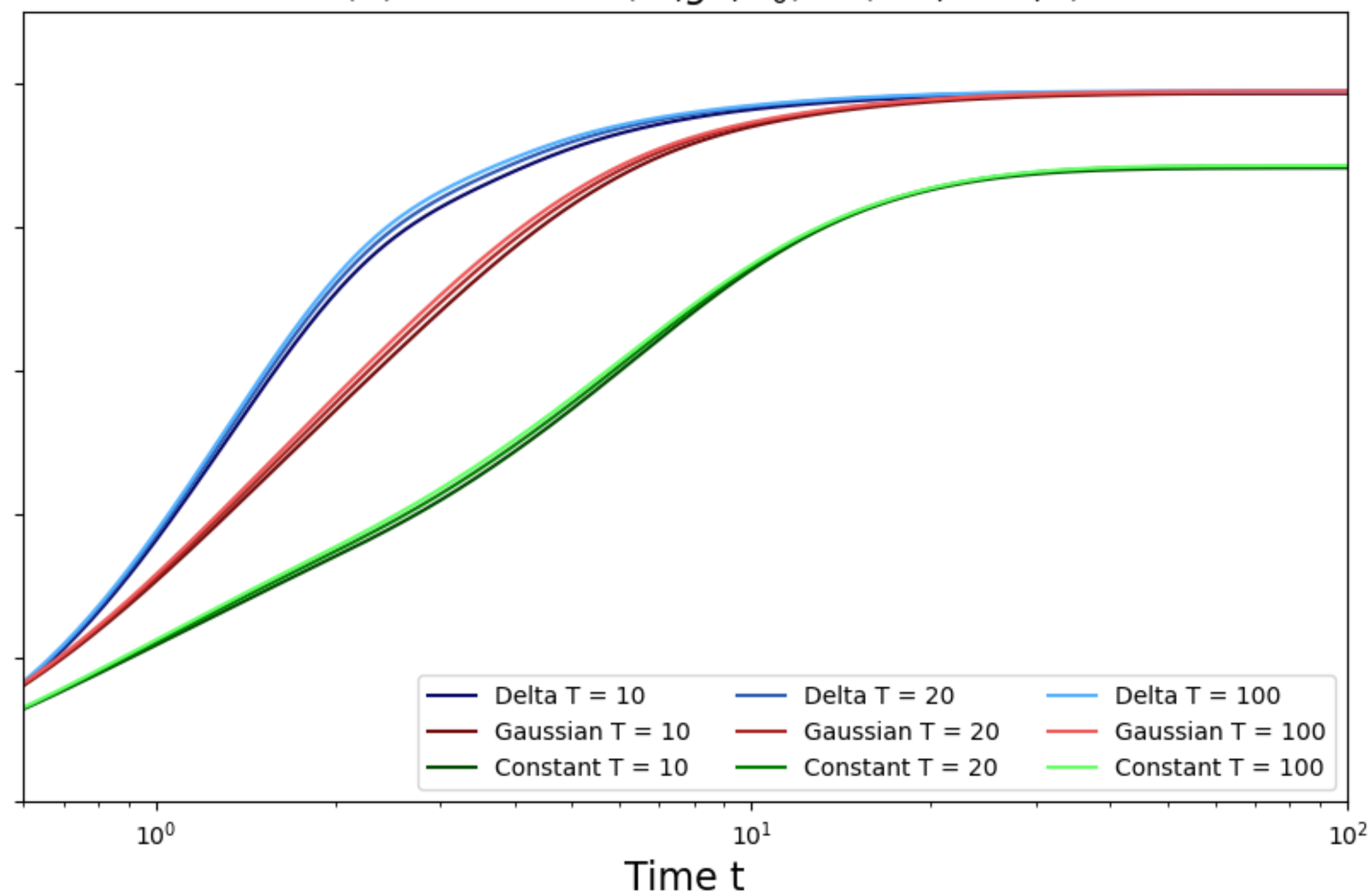


REAL-TIME DYNAMICS OF ENTROPY

U(1) Simulation $(m, g^2, D_0) = (0.5, 0.64, 1)$

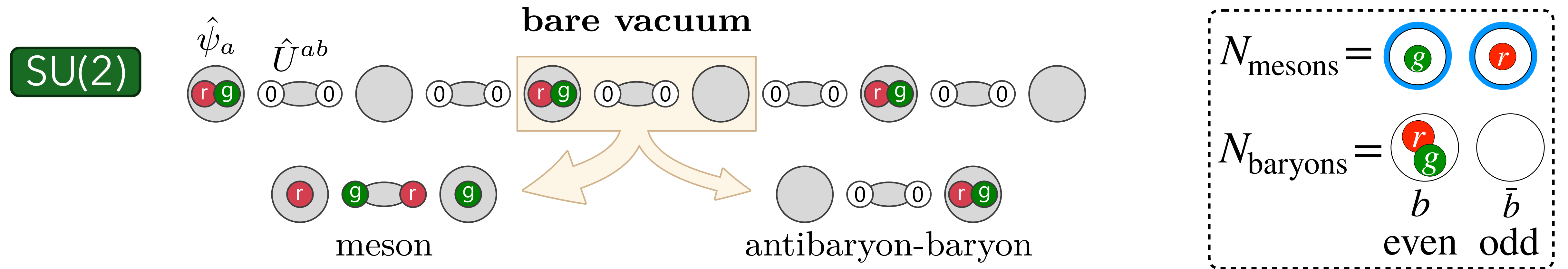


SU(2) Simulation $(m, g^2, D_0) = (0.5, 0.64, 1)$



ENTROPY GROWS FAST WITH TEMPERATURE!

MATTER DENSITY IMBALANCE



How do different environments change the final composition of matter density?

MATTER PROPORTION $\mathcal{M}\mathcal{P} = \frac{N_{\text{mesons}}}{N_{\text{baryons}}}$

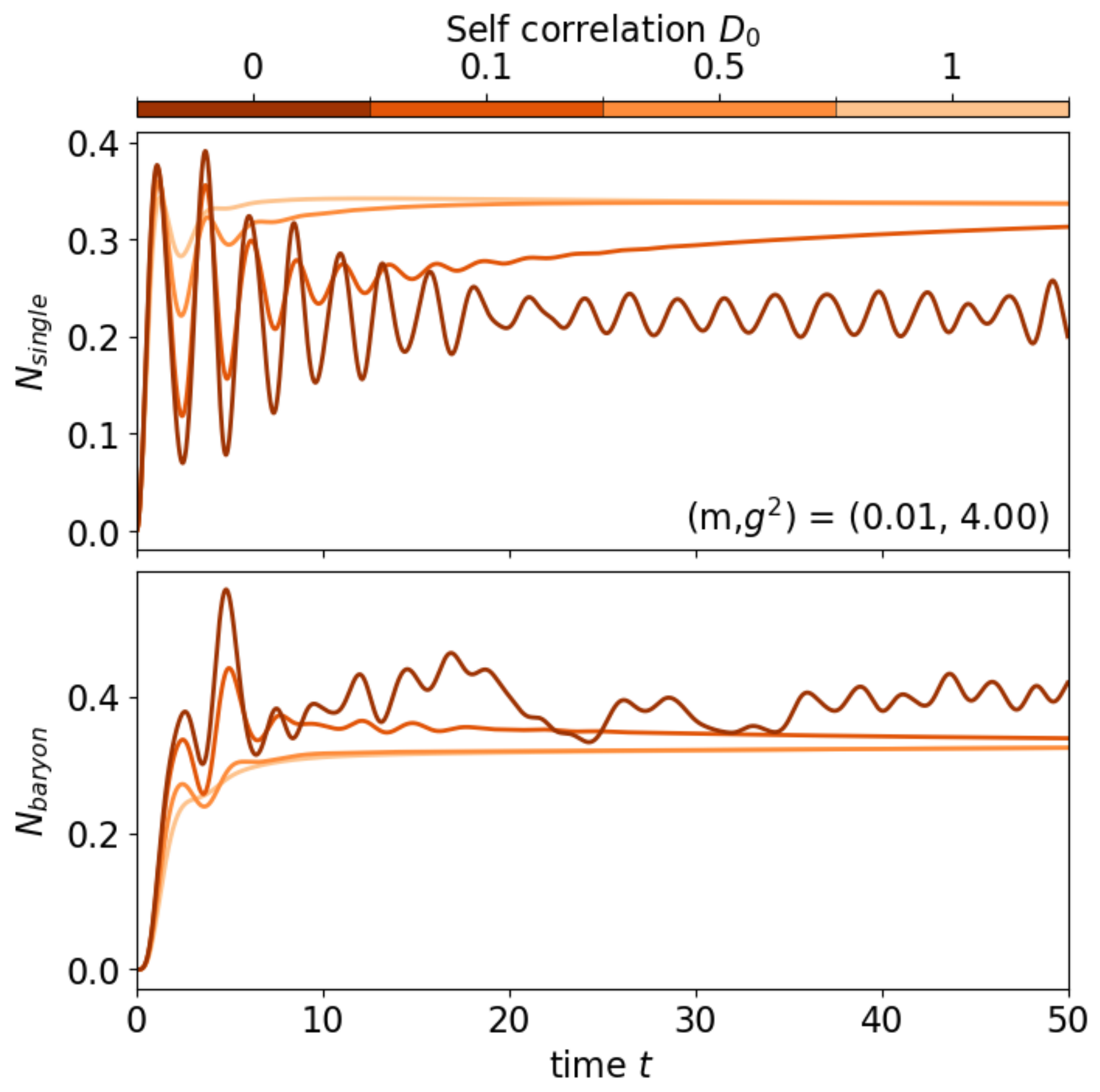
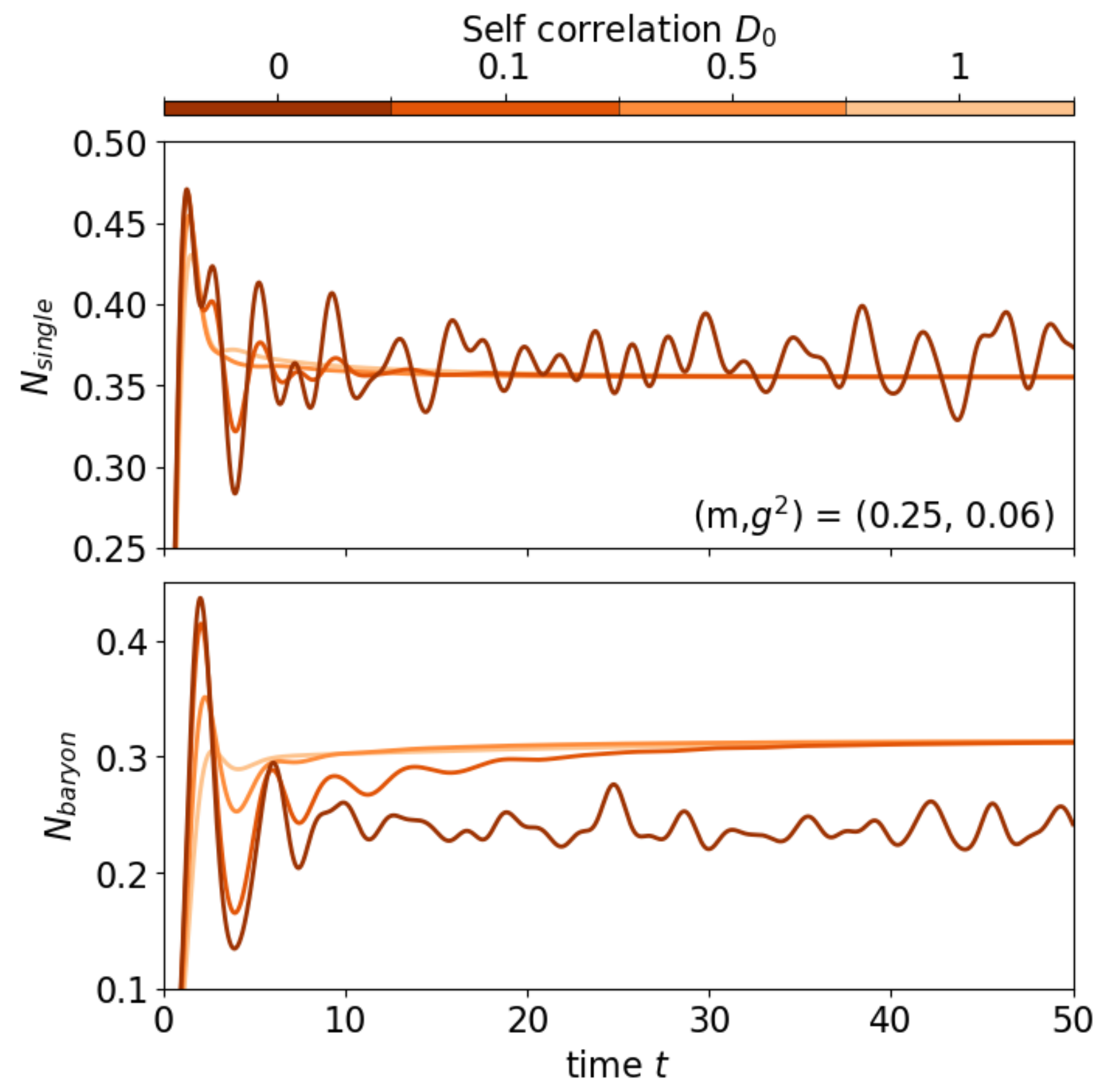
$\mathcal{M}\mathcal{P}\mathcal{D} = \mathcal{M}\mathcal{P}_{\text{open}} - \mathcal{M}\mathcal{P}_{\text{closed}}$

$\langle \mathcal{O} \rangle = \text{Tr}(\mathcal{O} \rho_{ss}) \quad \text{Tr}(\mathcal{O} \rho_{\text{Gibbs}})$

- $= 0 \Rightarrow$ No change
- $\mathcal{M}\mathcal{P}\mathcal{D} > 0 \Rightarrow$ Environment induces mesons
- $< 0 \Rightarrow$ Environment induces baryons

MATTER DENSITY IMBALANCE

DELTA CORRELATOR

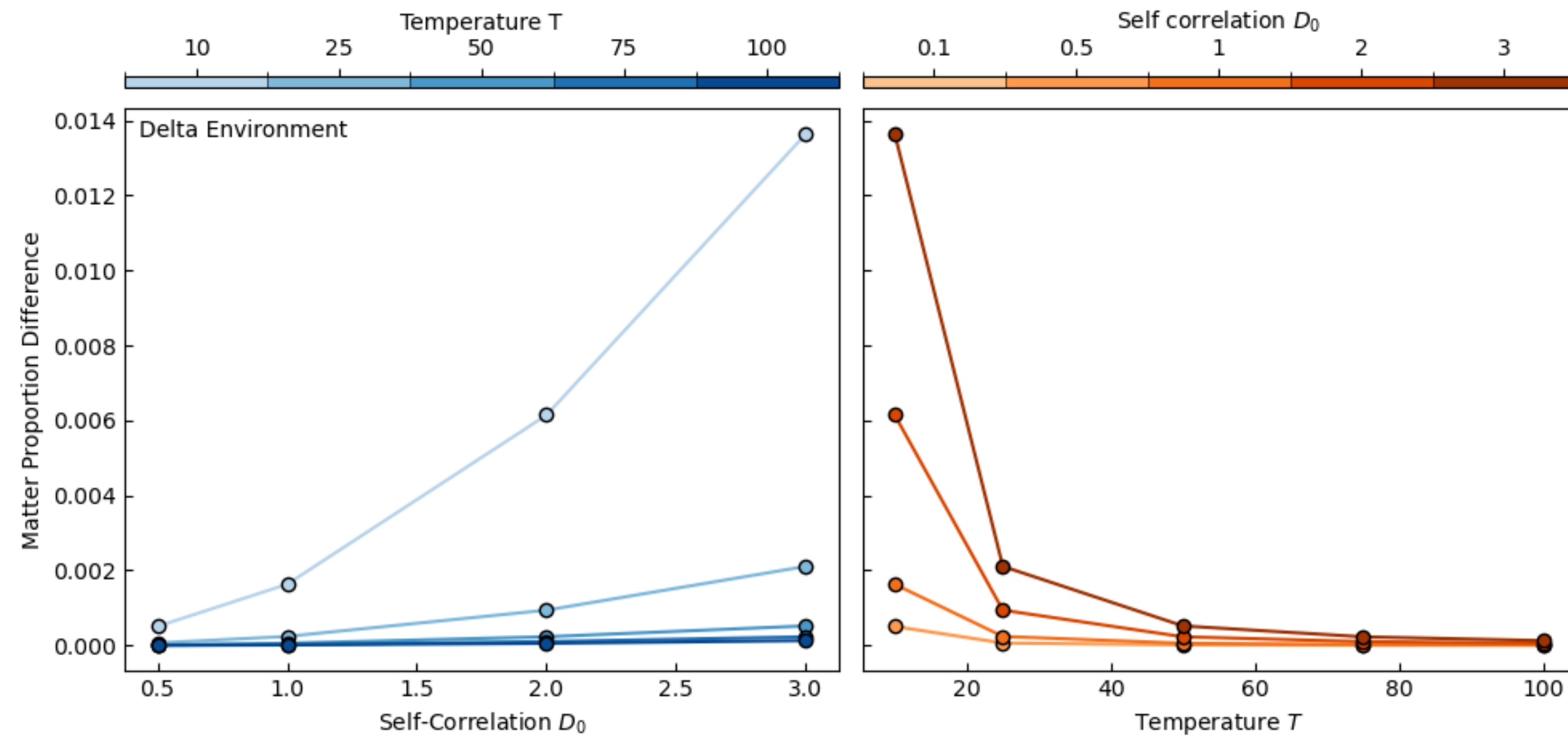


RESULTS CHANGES WITH HAMILTONIAN PARAMETERS

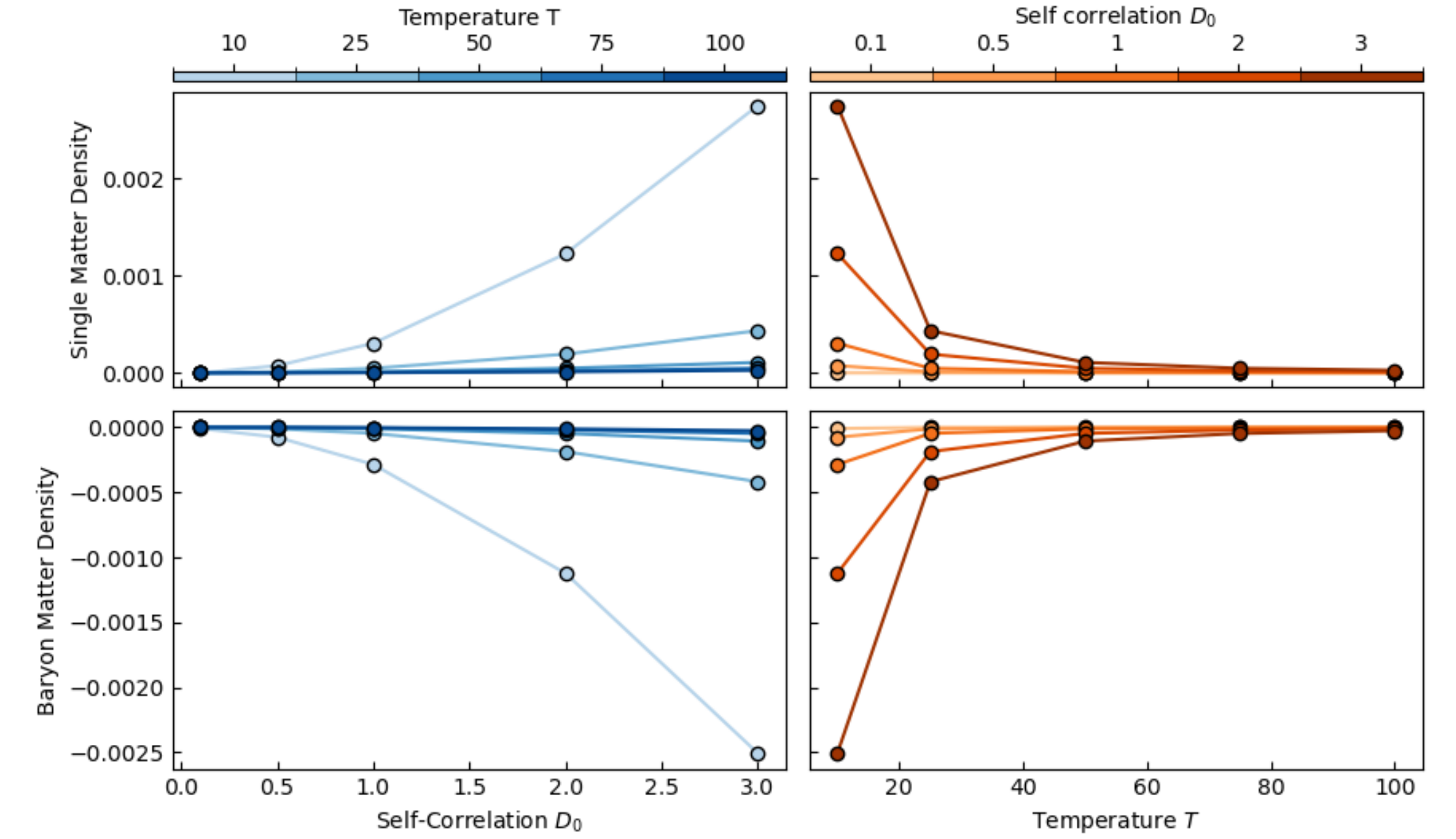
m, g^2

MATTER COMPOSITION ANALYSIS

SU(2) Simulations with $(m, g^2) = (1, 1)$



SU(2) Simulations (Steady State - Gibbs) with $g^2 = 1$

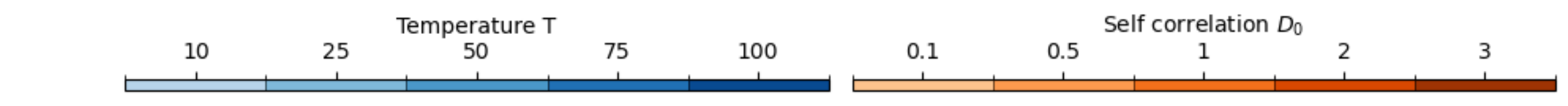


CAN WE FIND A REGION WITH MPD < 0?

MATTER COMPOSITION ANALYSIS

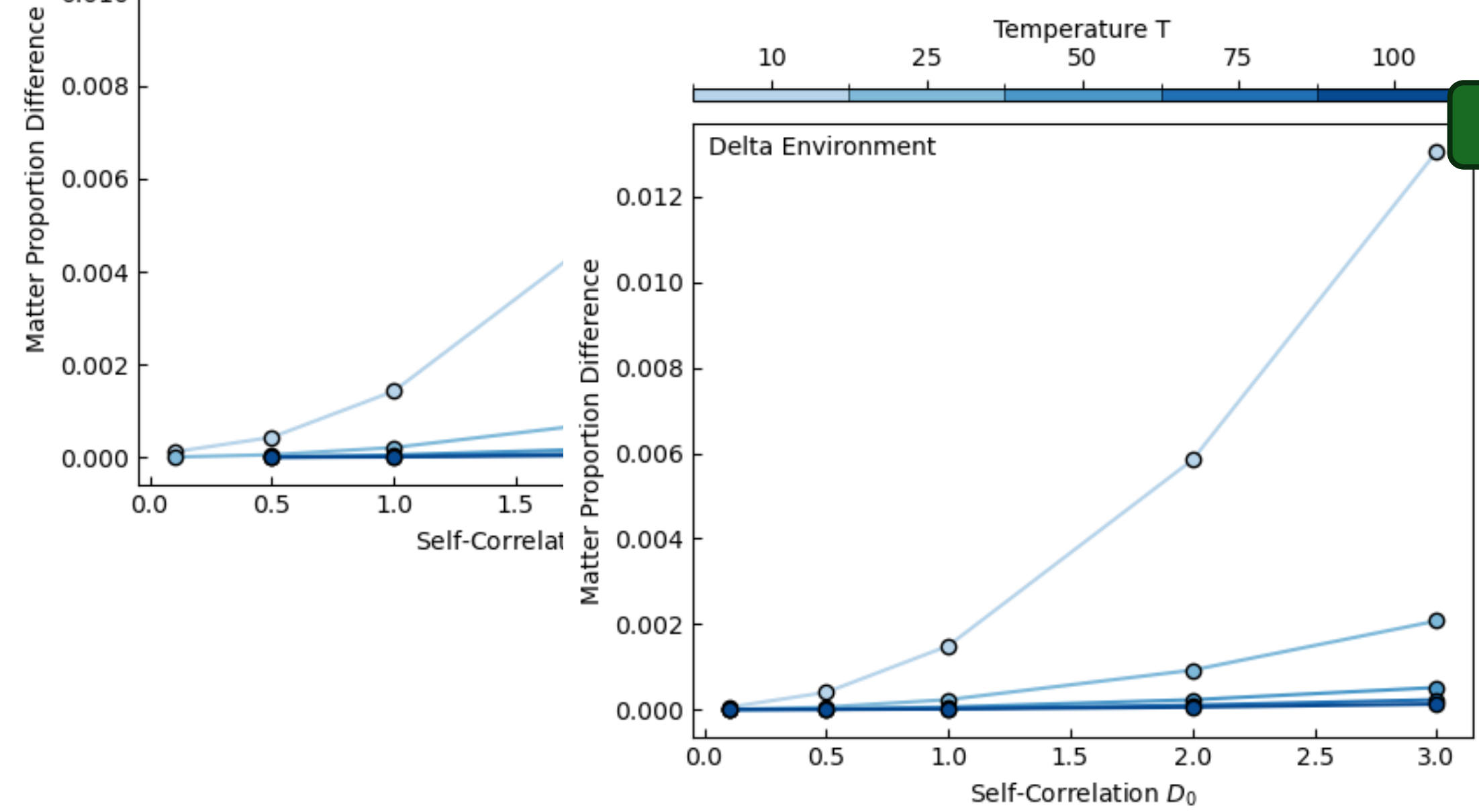
FINITE DENSITY

SU(2) Simulations with $(m, g^2) = (1, 1)$



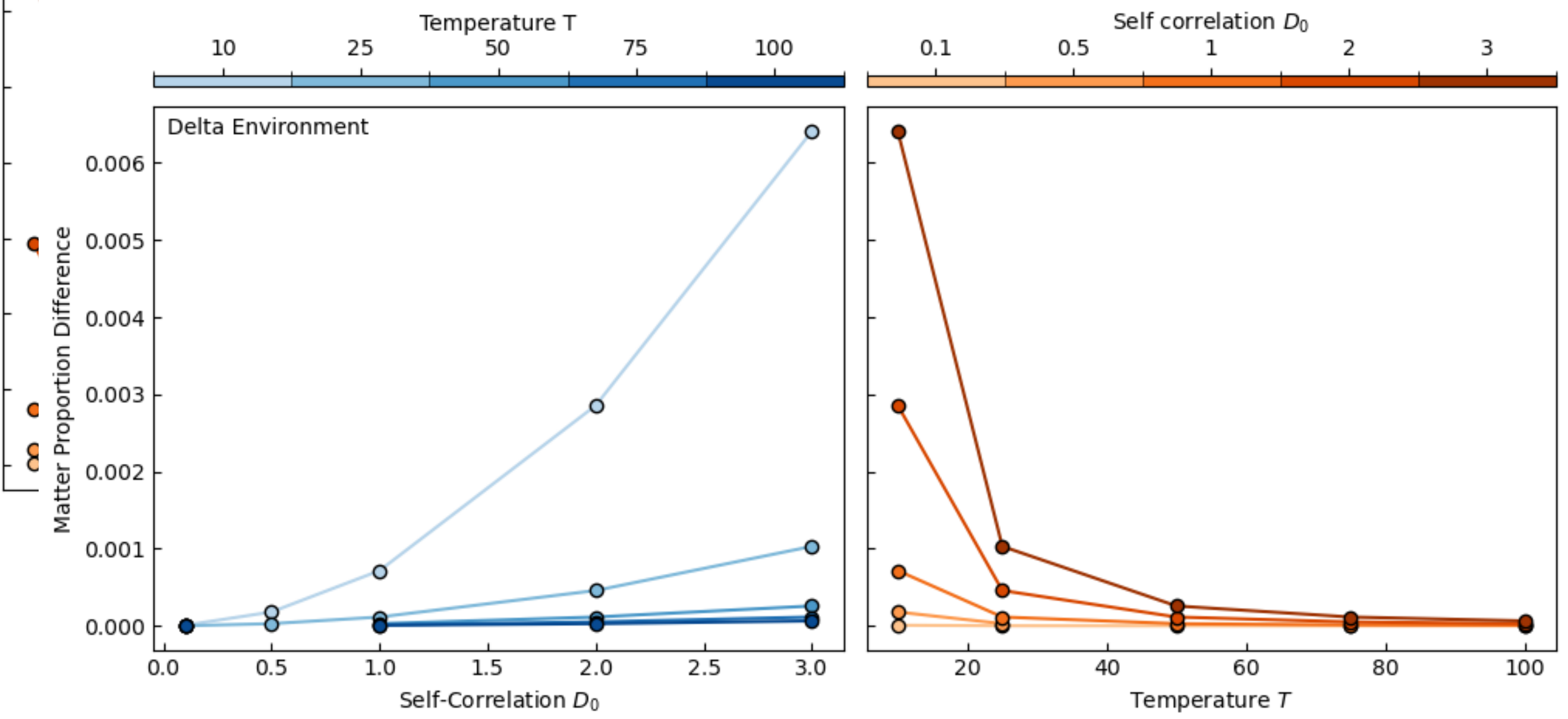
DIFFERENT g

SU(2) Simulations with $(m, g^2) = (0.5, 0.1)$



WITH LAMB-SHIFT

SU(2) Simulations with $(m, g^2) = (0, 0.1)$



SUMMARY:



HAMILTONIAN LATTICE GAUGE THEORIES

OPEN QUANTUM SYSTEMS THEORY

TENSOR NETWORK SIMULATIONS

THERMALIZATION TIME

ENTROPY & LOCAL OBSERVABLES

THANKS!



E. PARRA



S. KUHN



JAD HALIMEH



G. MAGNIFICO

