Intro to Cosmology and OQS

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PennState
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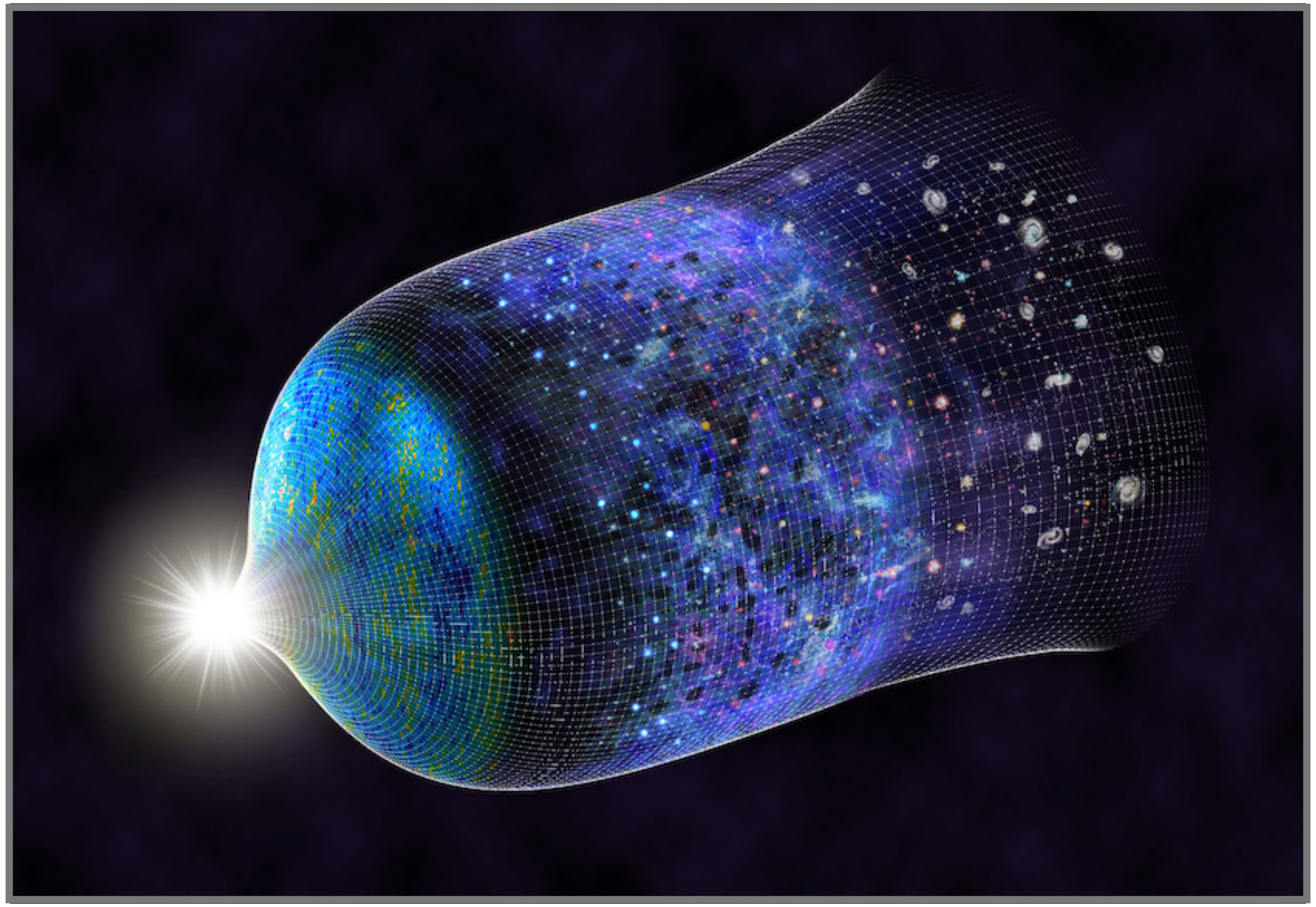
Primordial cosmology

Would like a 'natural' explanation for correlations on large scales

Early-time inhomogeneities were small (definitely perturbative)



Goals and tools of elementary-particle cosmology

- What matter was responsible for the evolution of the primordial universe?
- What is the origin of the density inhomogeneities in the universe?
- How can (early universe) cosmology constrain beyond-standard model physics?
- What does (early universe) cosmology say about quantum gravity?

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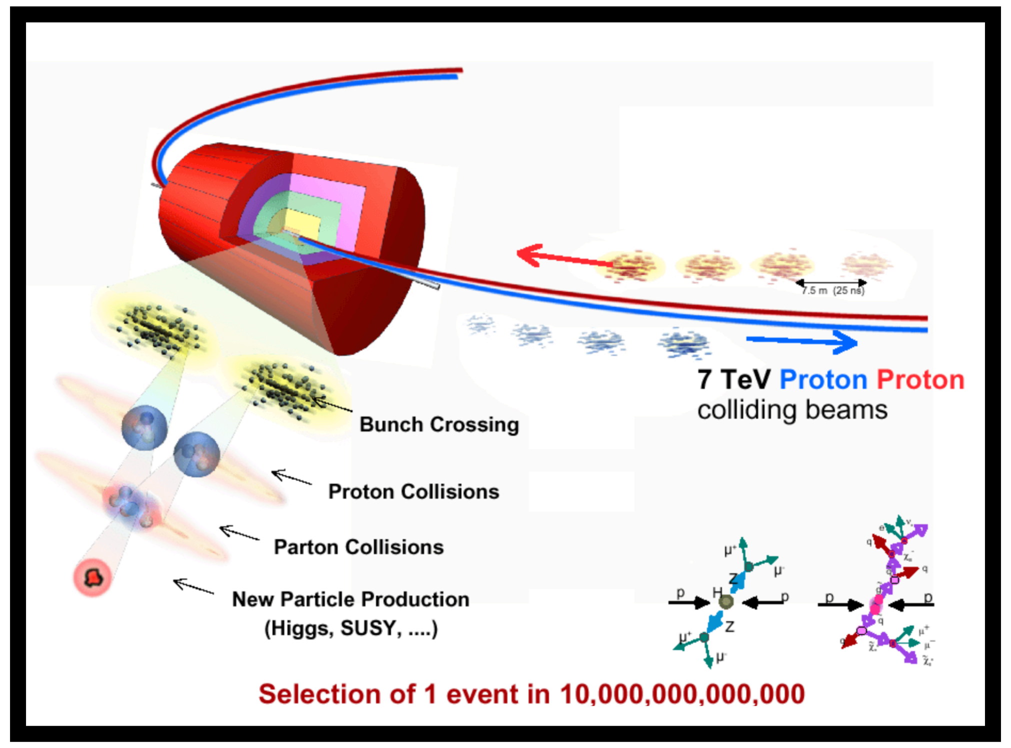
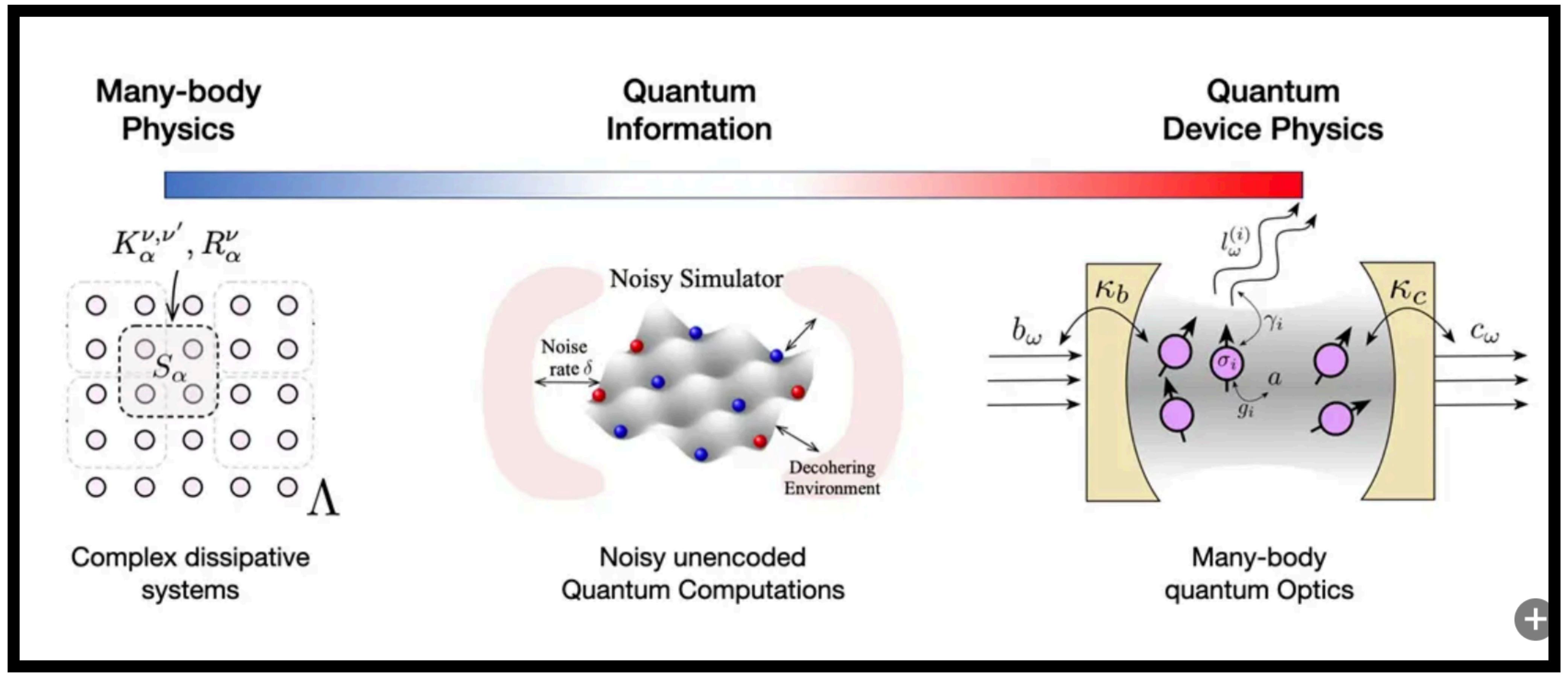
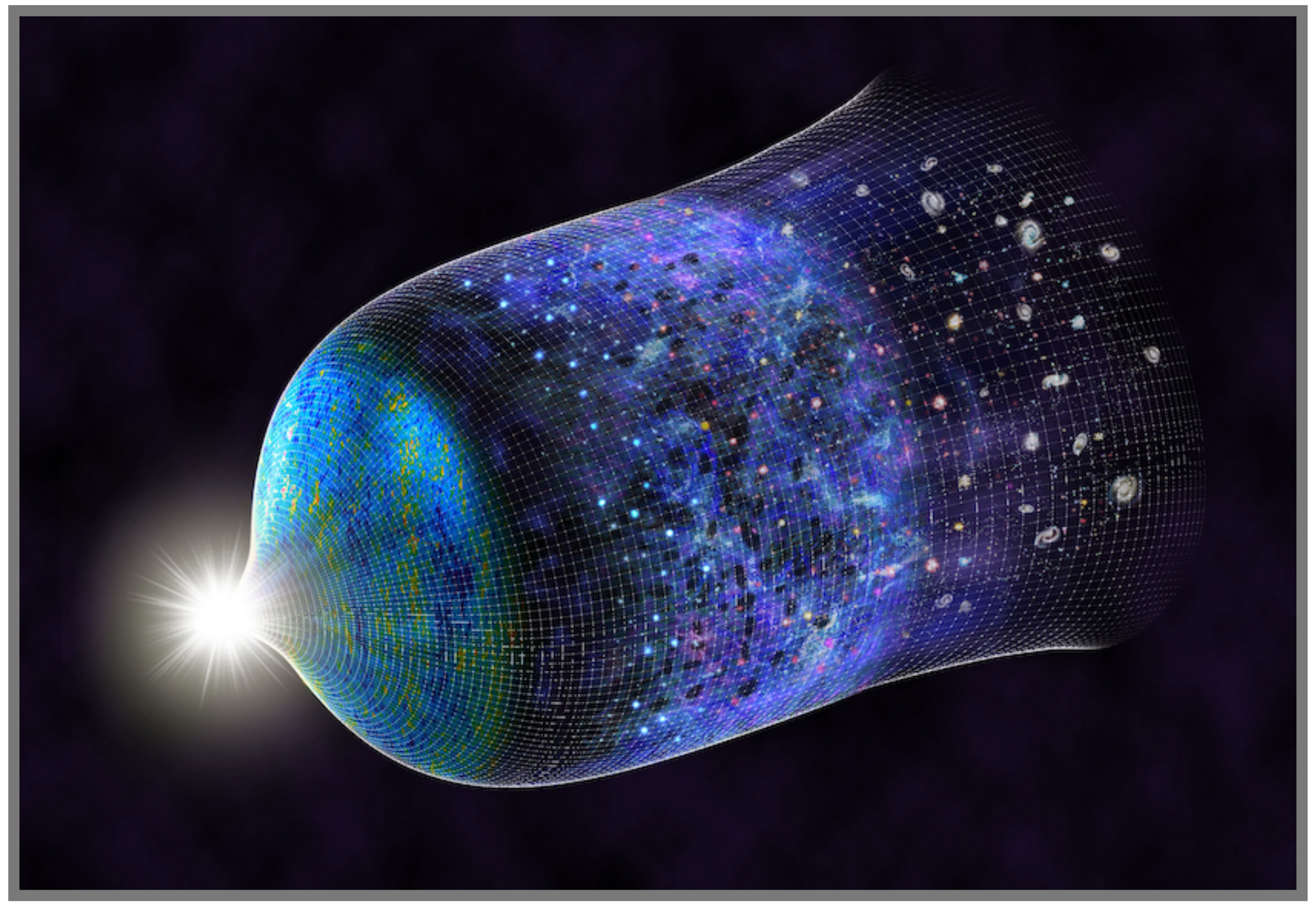
For decades, we've based our Beyond-Standard-Model ideas on successes of coldider physics (low-energy EFTs, SUSY, strings, …)

Updating our theory framework







R. Trivedi

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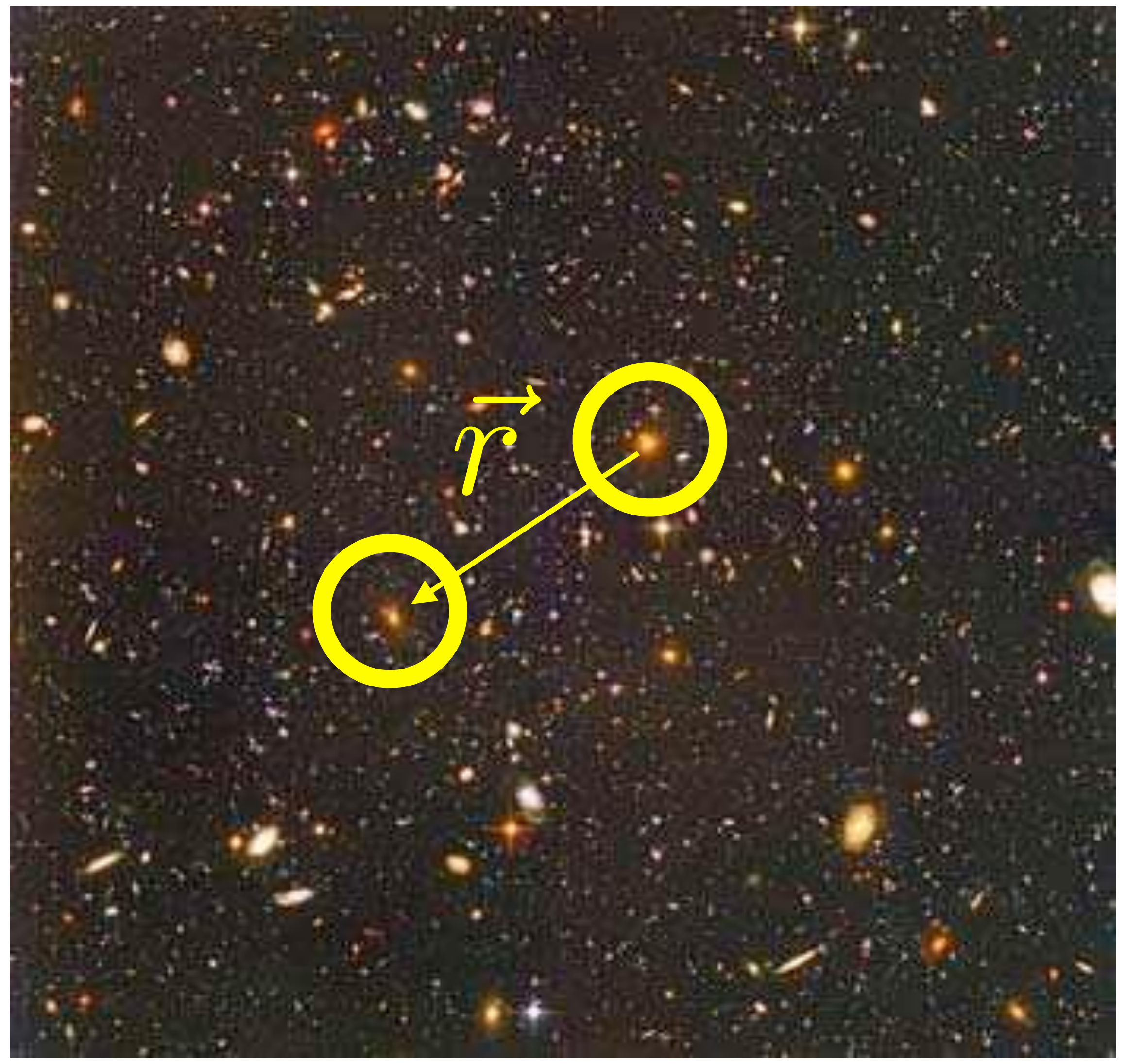
Primary tool: semi-classical gravity, where metric perturbations are treated as usual quantum fields (mostly scalar curvature, tensor gravitational waves)

Classical gravity for the non-linear evolution

How do we test an early universe model?



Hubble Telescope image

Statistically

Correlation function:

Consider galaxy at position $\vec{x}$.

Is their also a galaxy at position $\vec{x} + \vec{r}$?

Planck satellite CMB temperature fluctuation map

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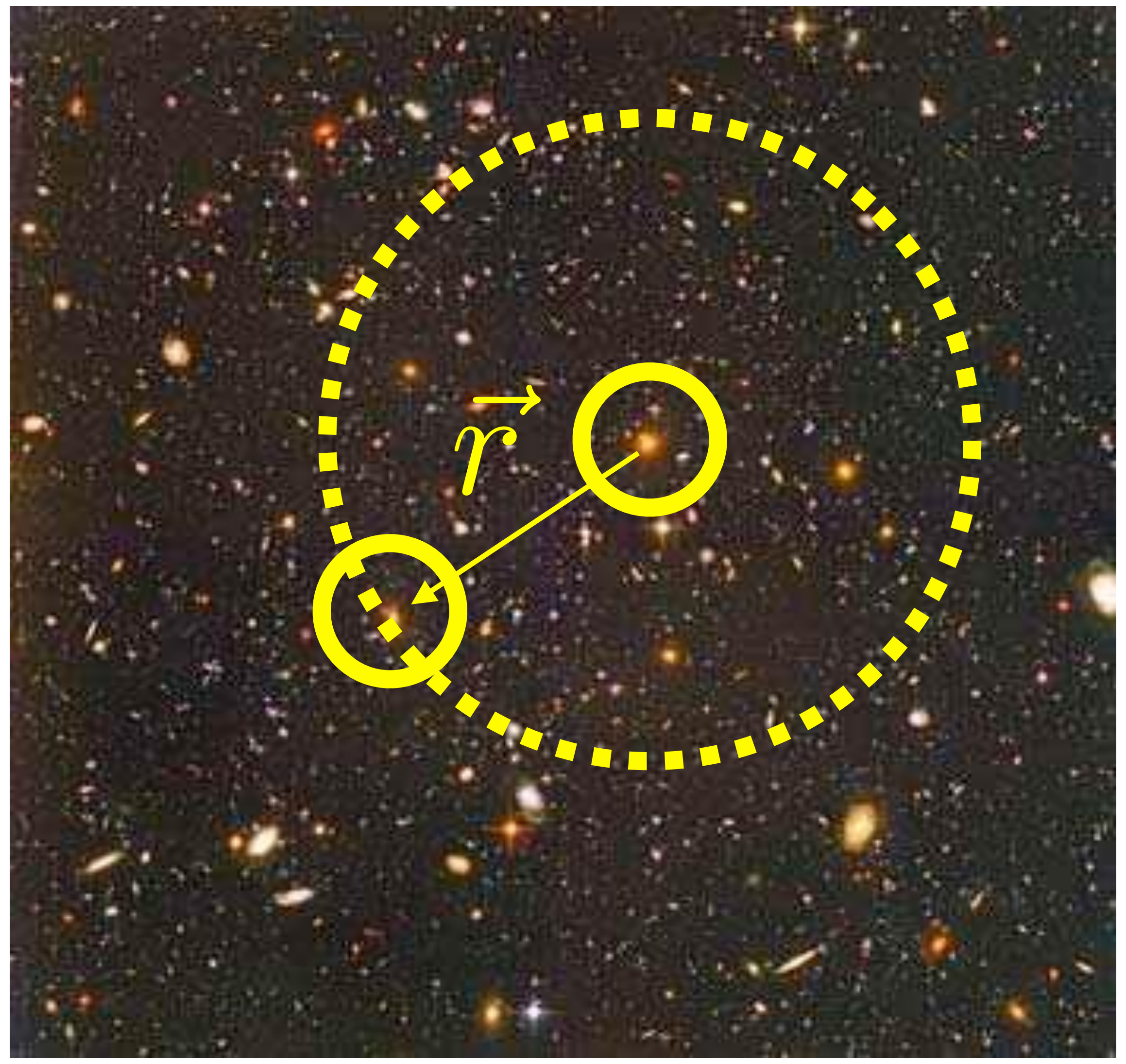

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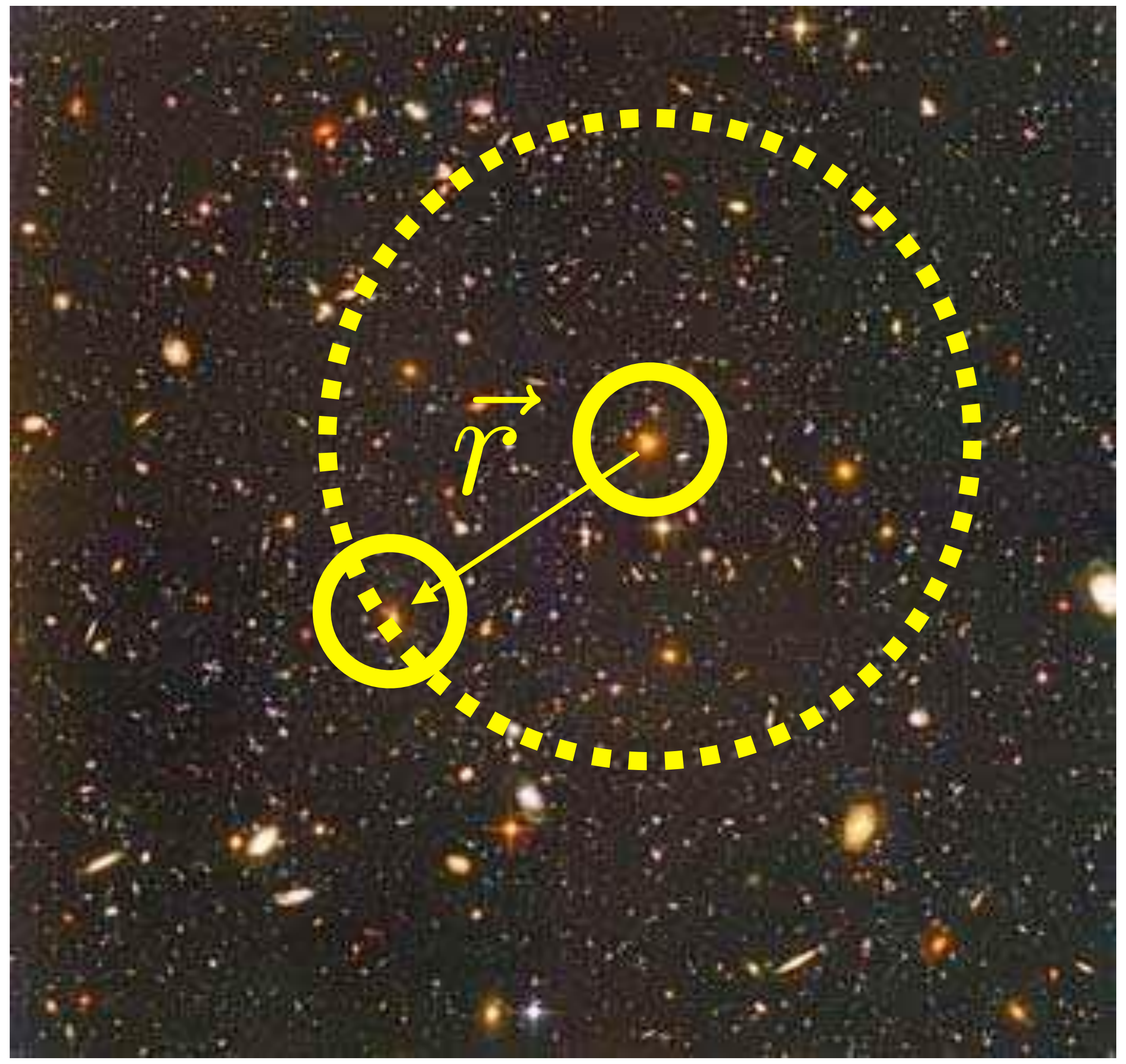

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$$\langle\delta(\vec{x})\delta(\vec{x}+\vec{r})\rangle$$

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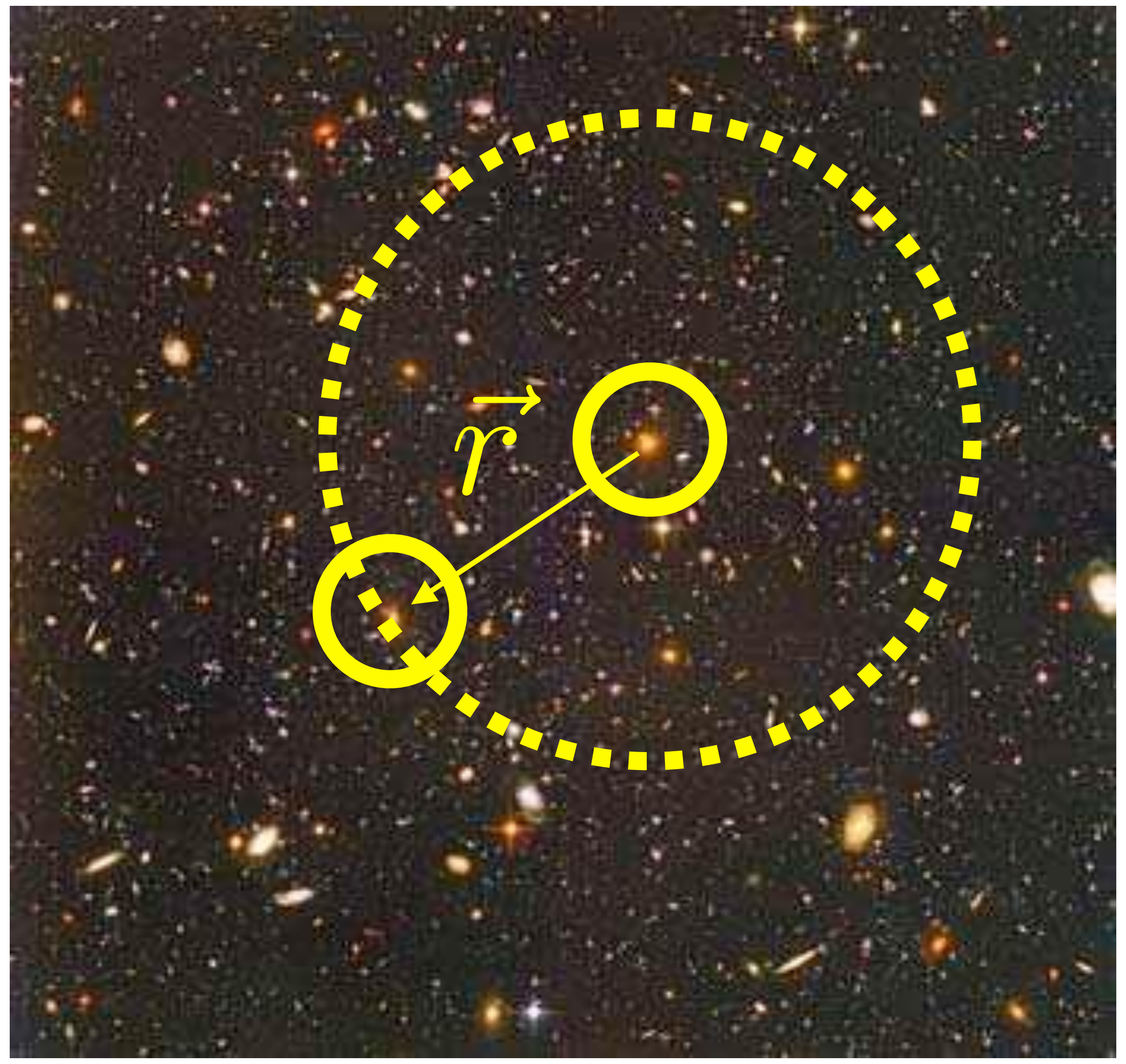

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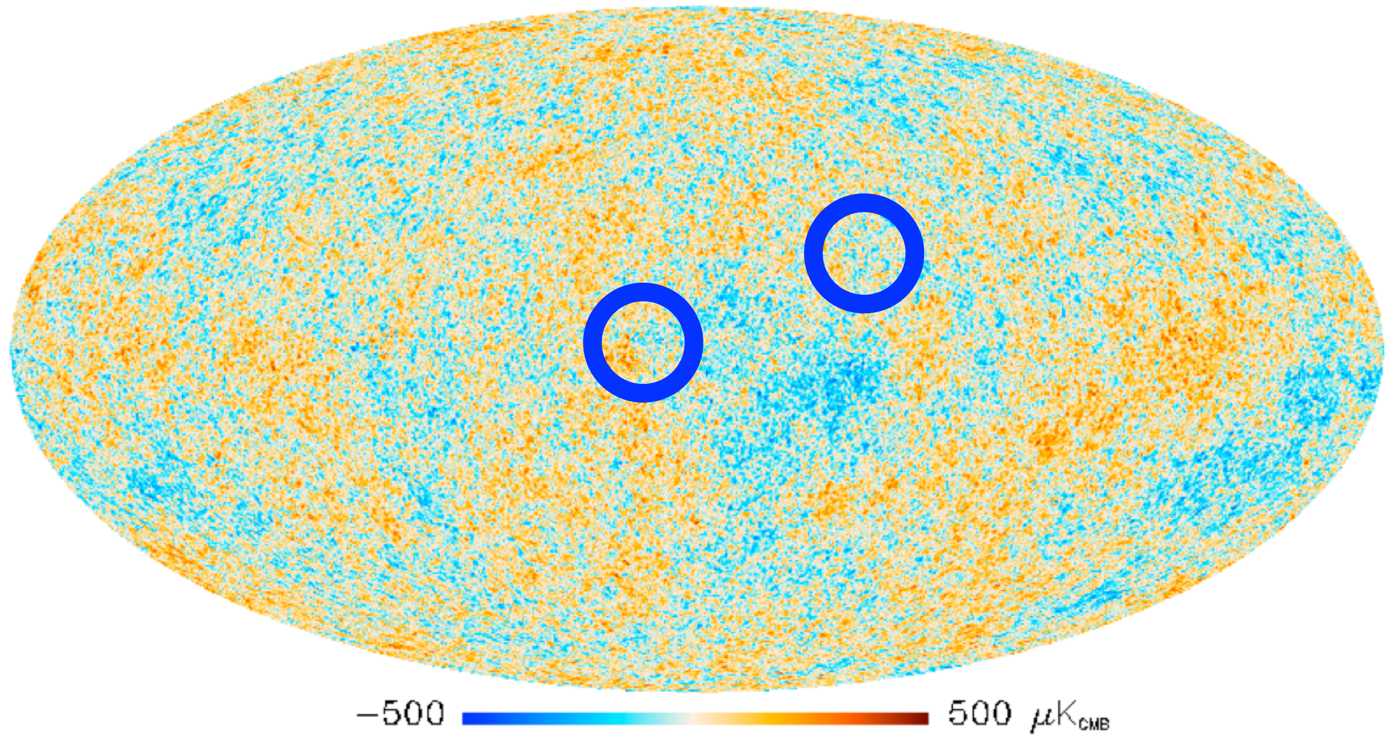

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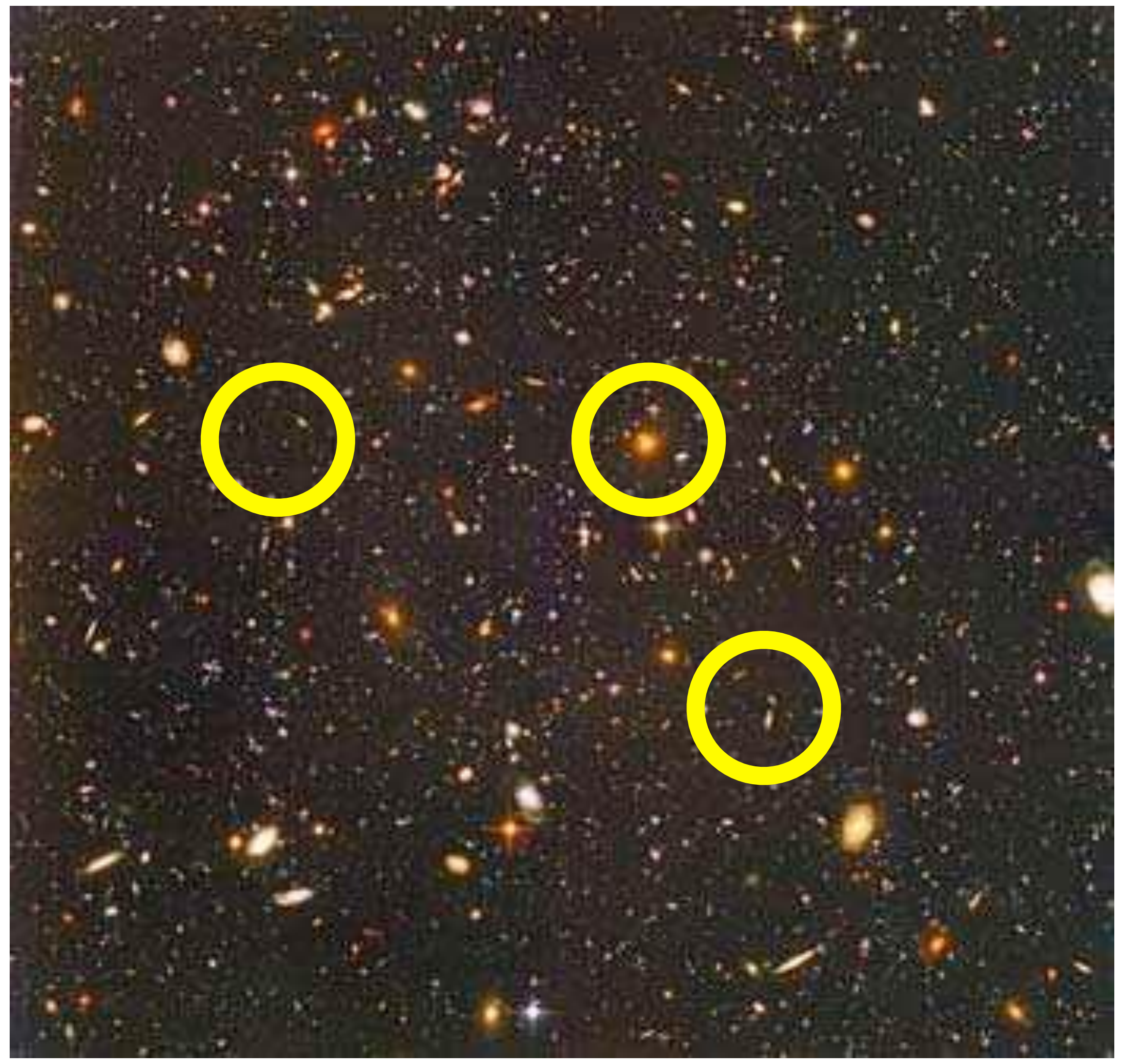
$$\langle \delta(\vec{x}) \delta(\vec{x} + \vec{r}) \rangle$$

Statistics, cont'd

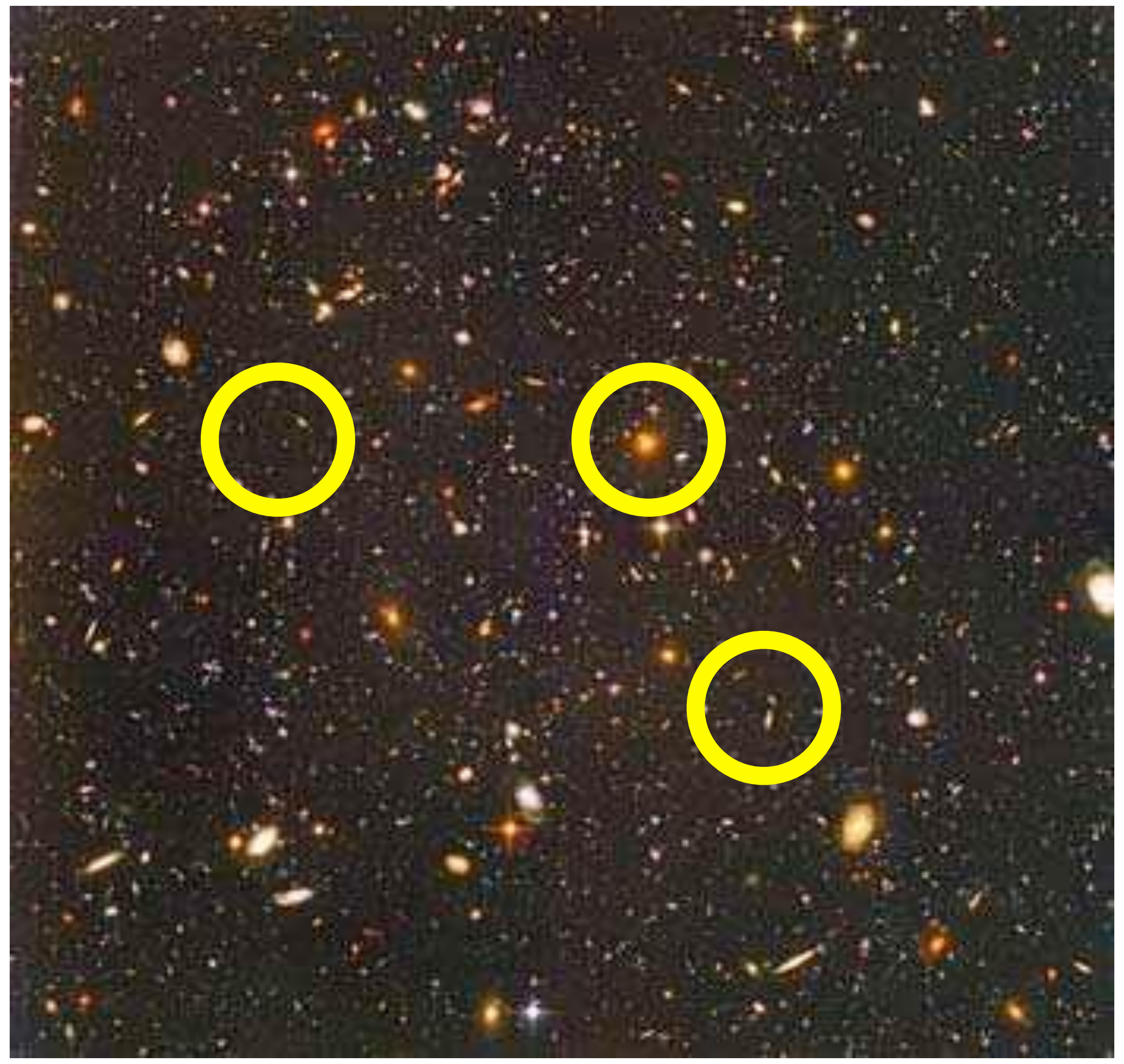


Higher order correlation functions:

3-point: $\langle \delta(\vec{x}_1)\delta(\vec{x}_2)\delta(\vec{x}_3) \rangle$

n-point: $\langle \delta(\vec{x}_1)\delta(\vec{x}_2)\ldots\delta(\vec{x}_n) \rangle$

Statistics, cont'd



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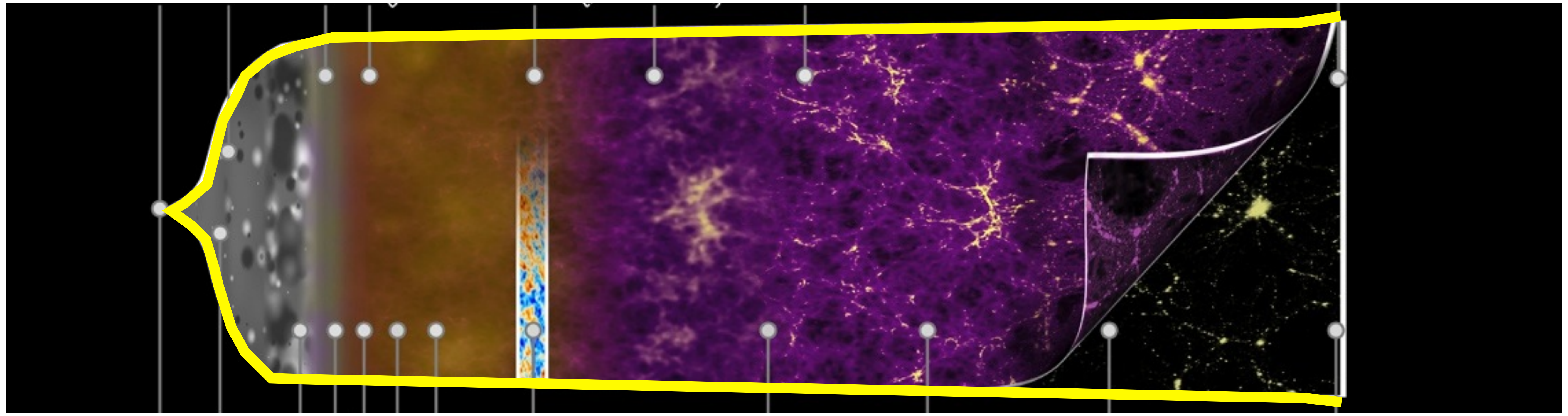
Fourier space statistics: $\langle \tilde{\delta}(\vec{k}_1)\tilde{\delta}(\vec{k}_2)\ldots\tilde{\delta}(\vec{k}_n) \rangle$

How do we model the physics?

General relativity: (Geometry) = (Matter)



(Rule for defining how far apart things are)

Background metric $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$

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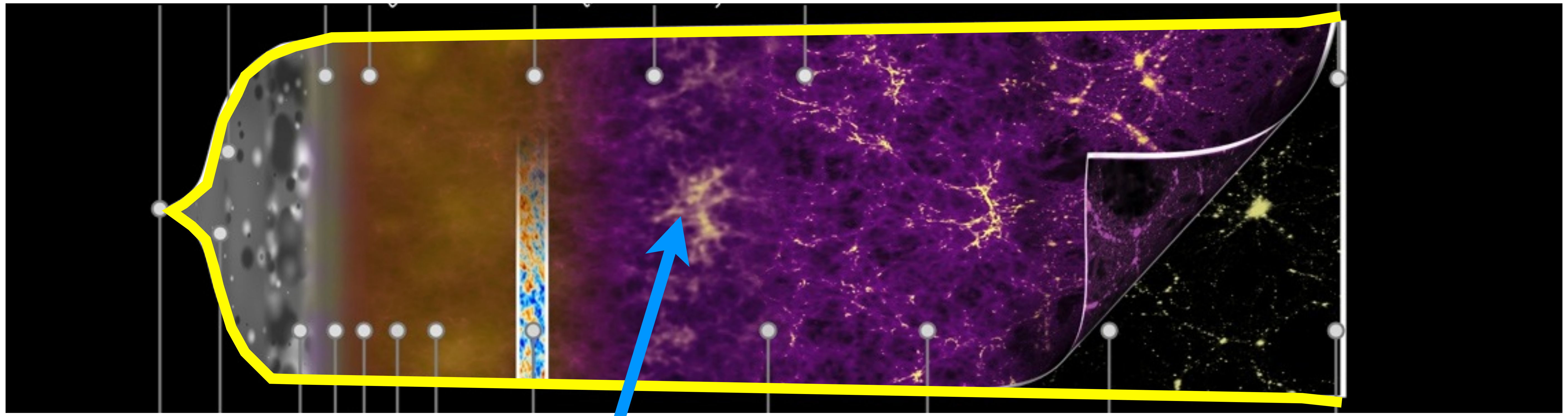

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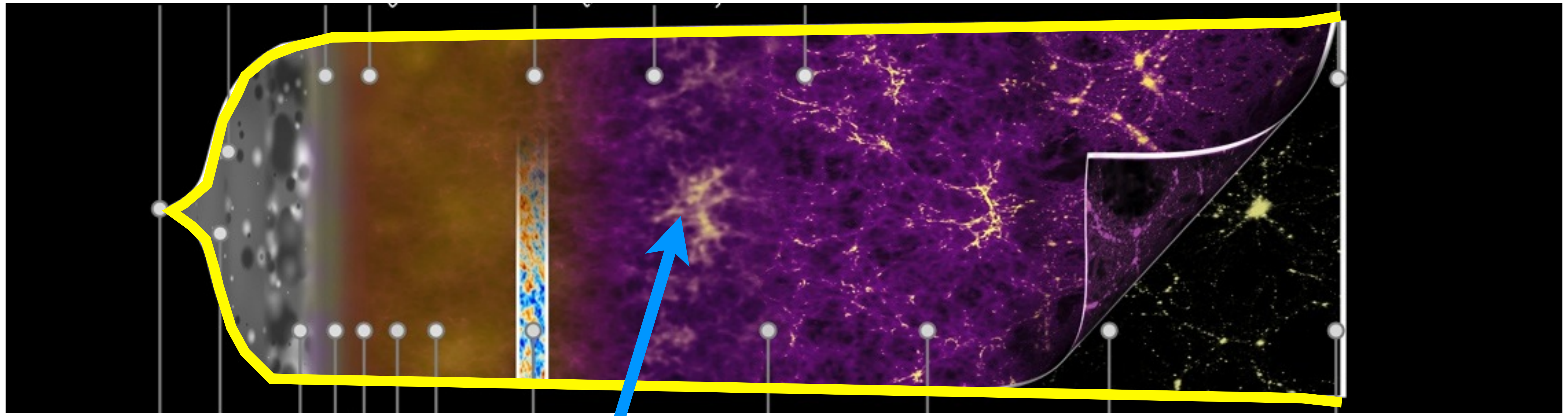

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$a(t) \rightarrow a(t)[1 + \zeta(\vec{x}, t)]$

$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2) \ldots \zeta(\vec{k}_n) \rangle$

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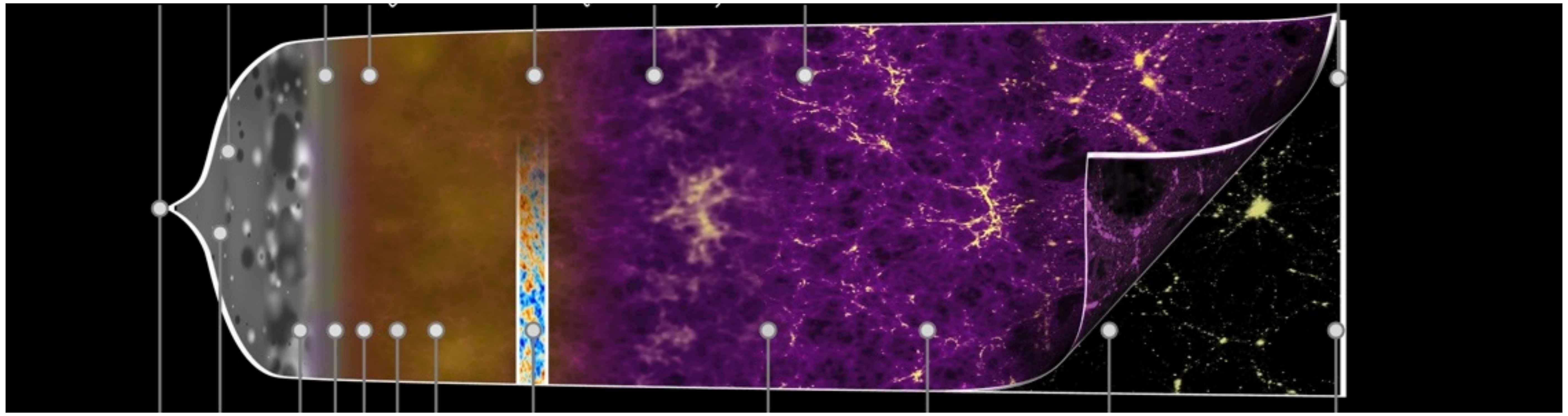

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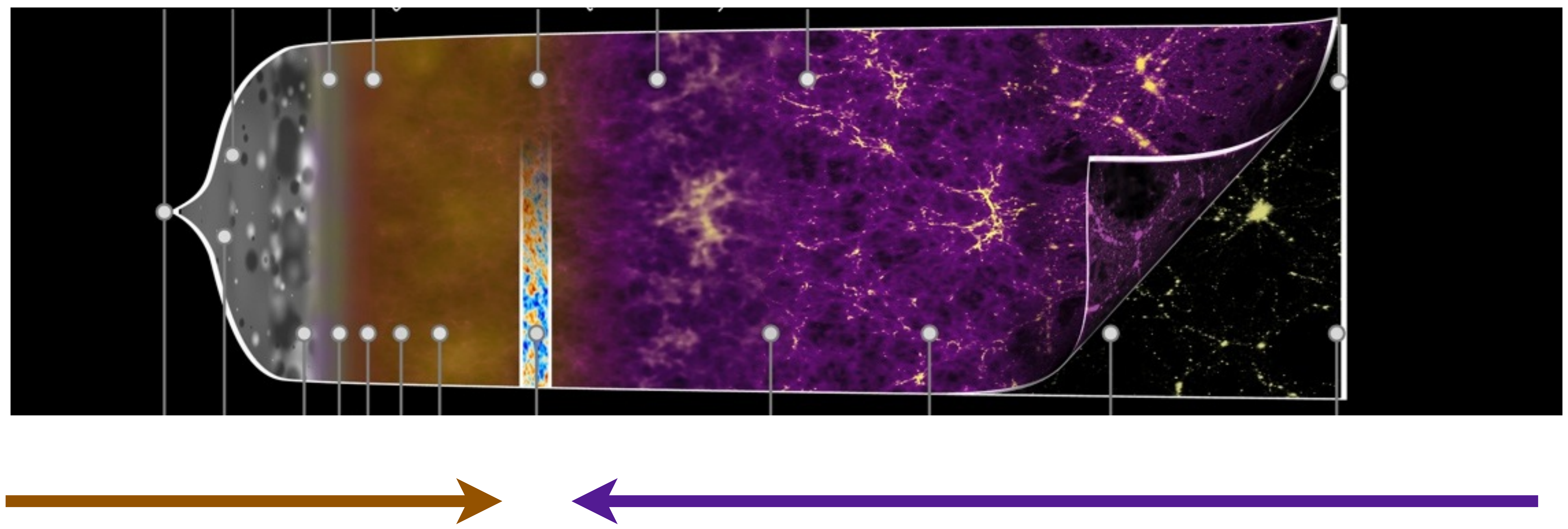

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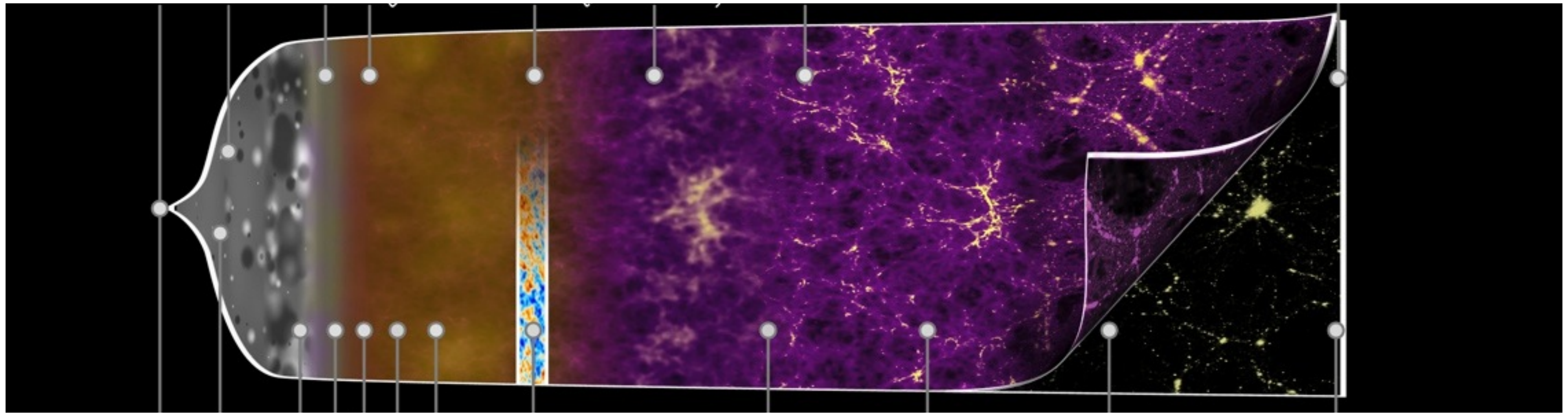

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$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\ldots\zeta(\vec{k}_n)\rangle \stackrel{?}{=} \langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\ldots\zeta(\vec{k}_n)\rangle$$

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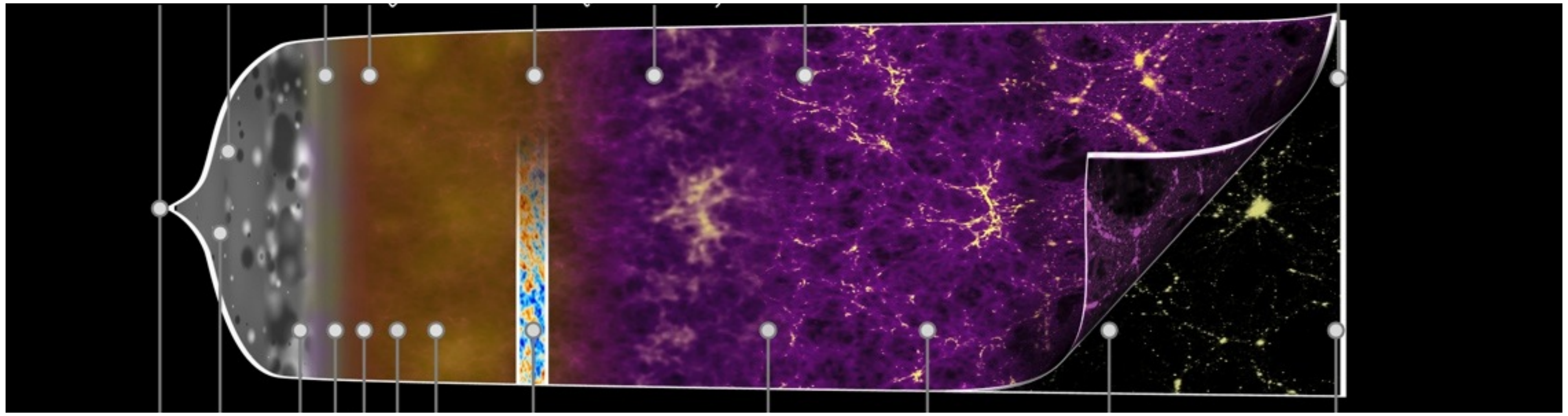

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actually, can only ask for statistical consistency

Parameterize the observed statistics

The observed power spectrum:

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\rangle = \delta(\vec{k}_1 + \vec{k}_2)k_1^{-3}P(k_1)$$

$$P(k_1) = A_0 \left(\frac{k}{k_0}\right)^{n_s - 1}$$

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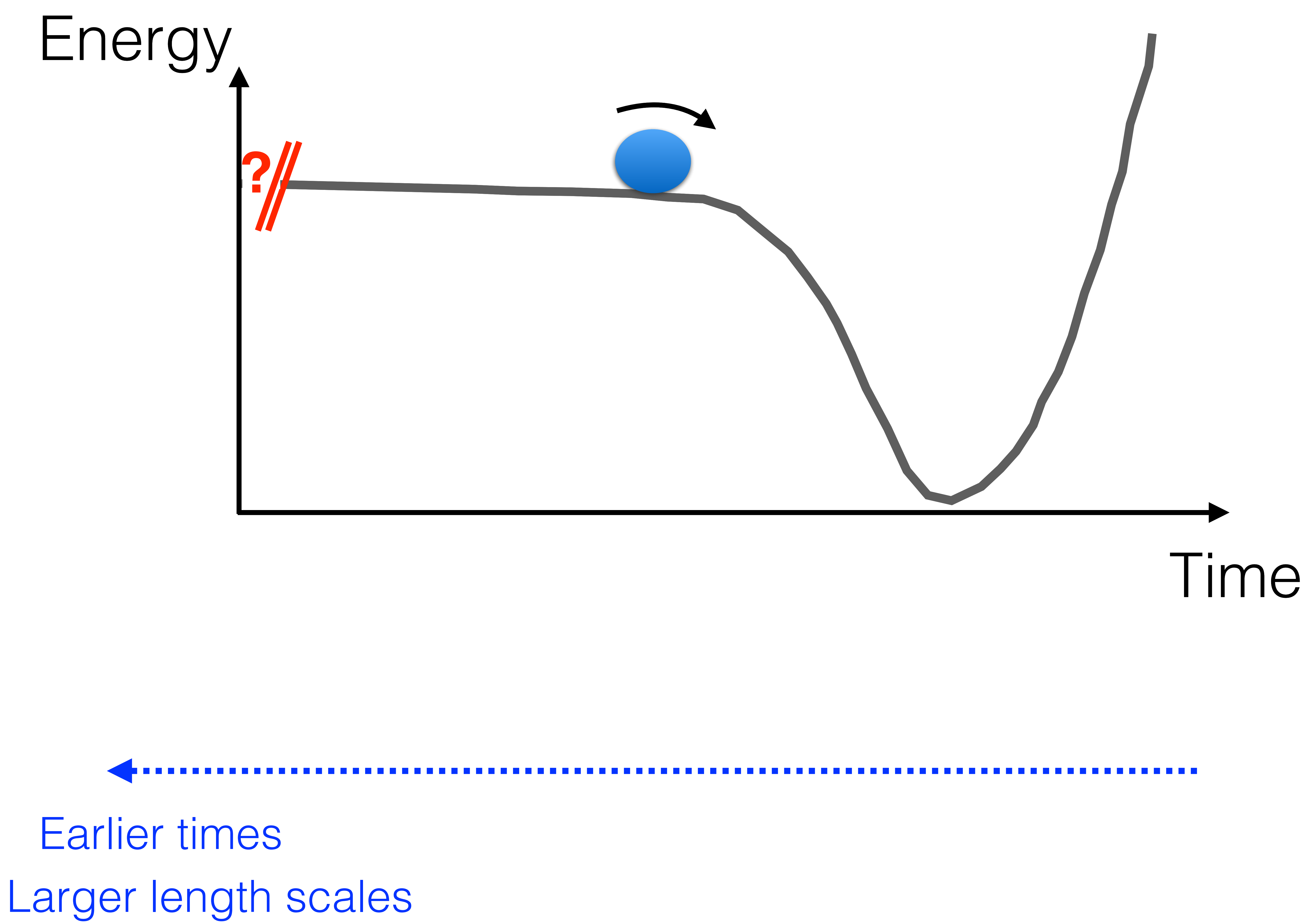
What theory of matter + gravity can give us the same statistics, “naturally”?

Inflation (?)

Inflation:

The canonical cartoon



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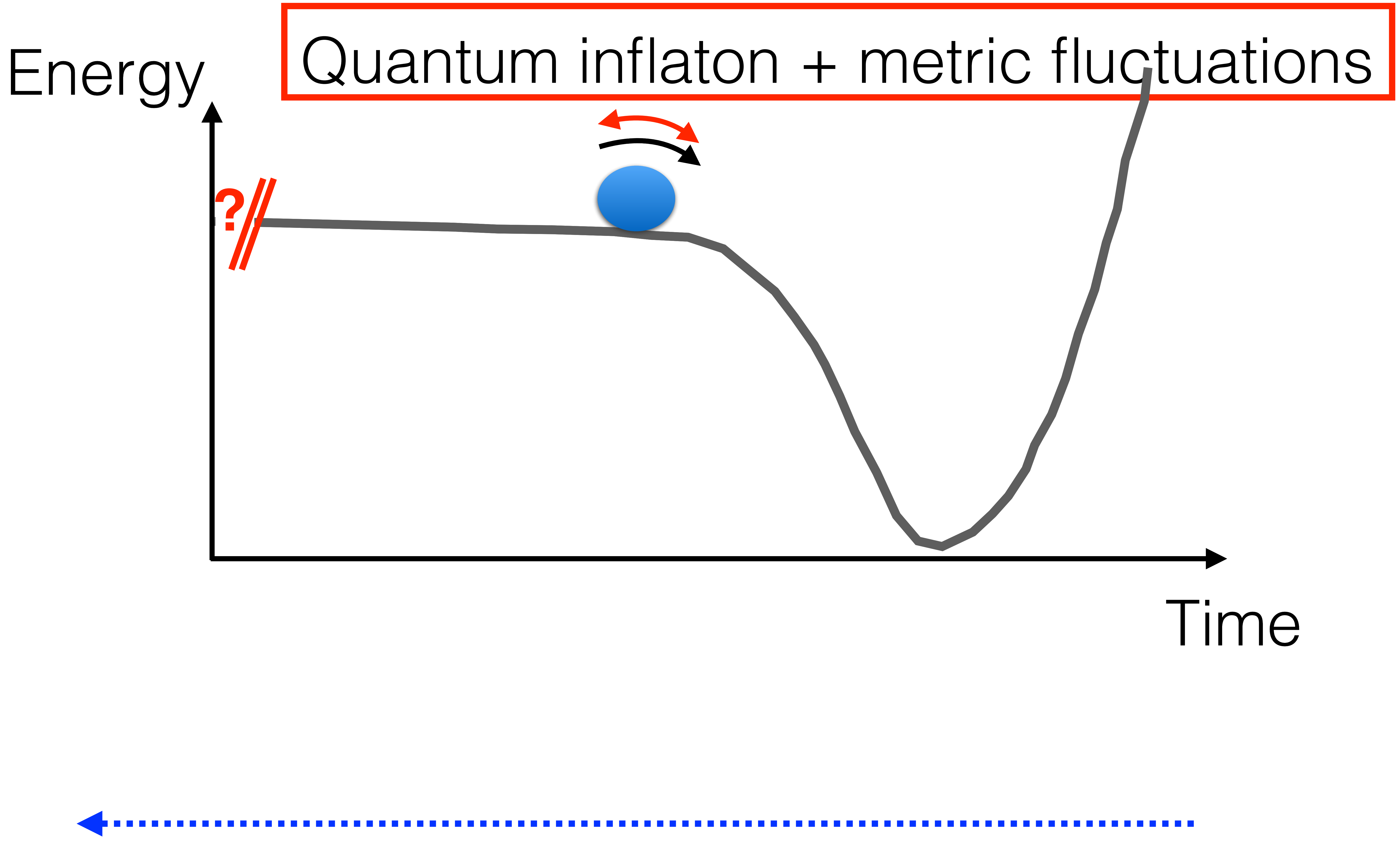

Earlier times
Larger length scales

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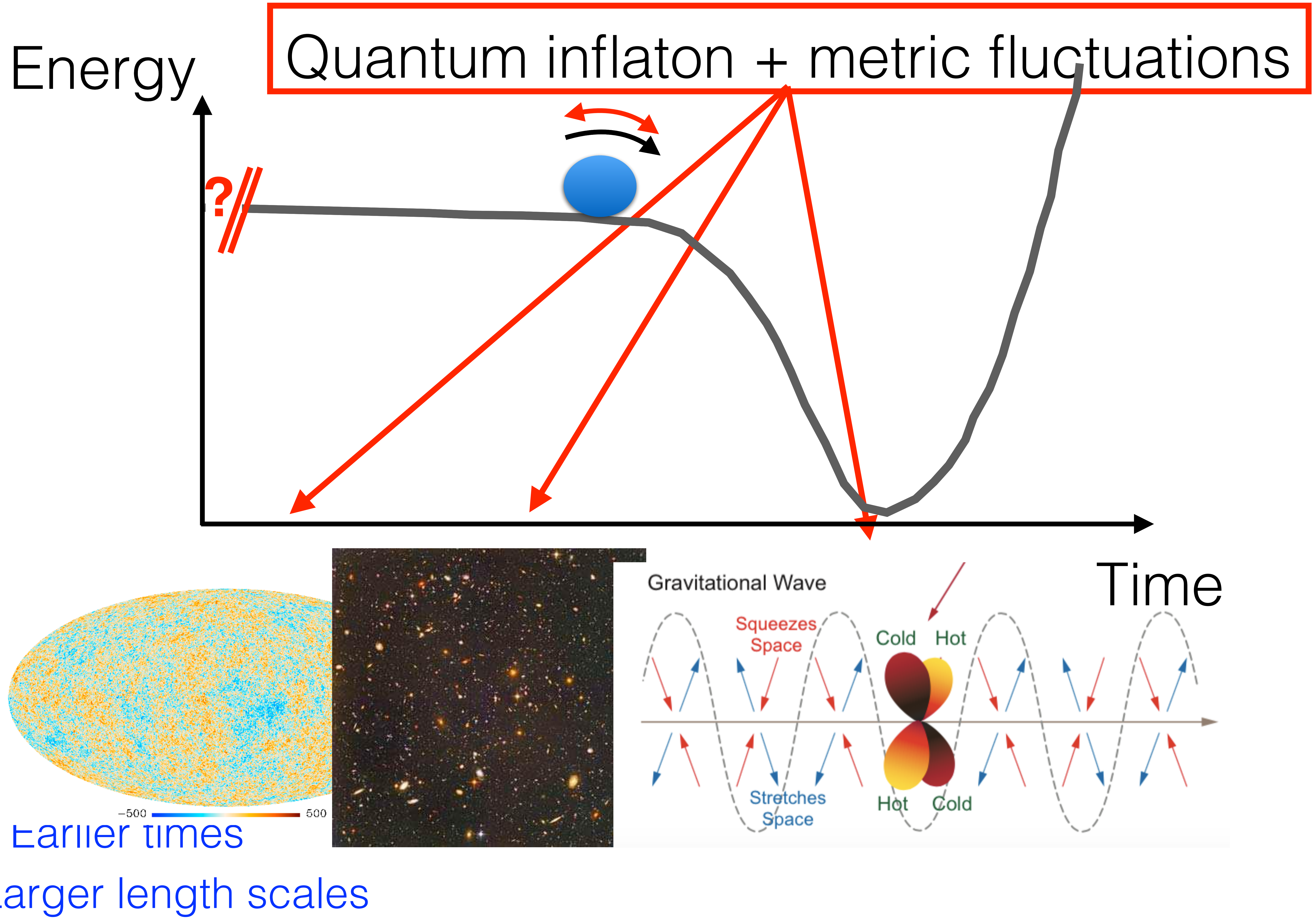

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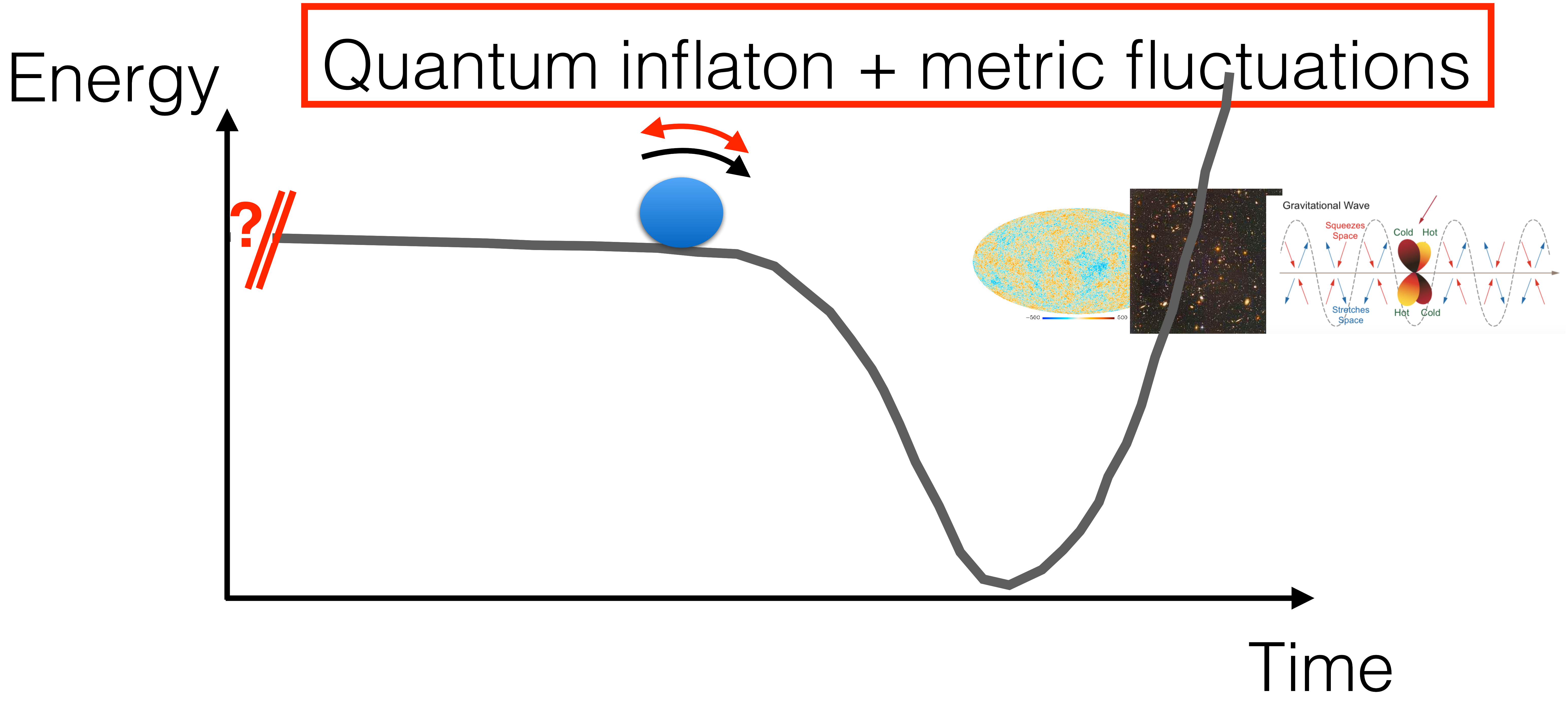

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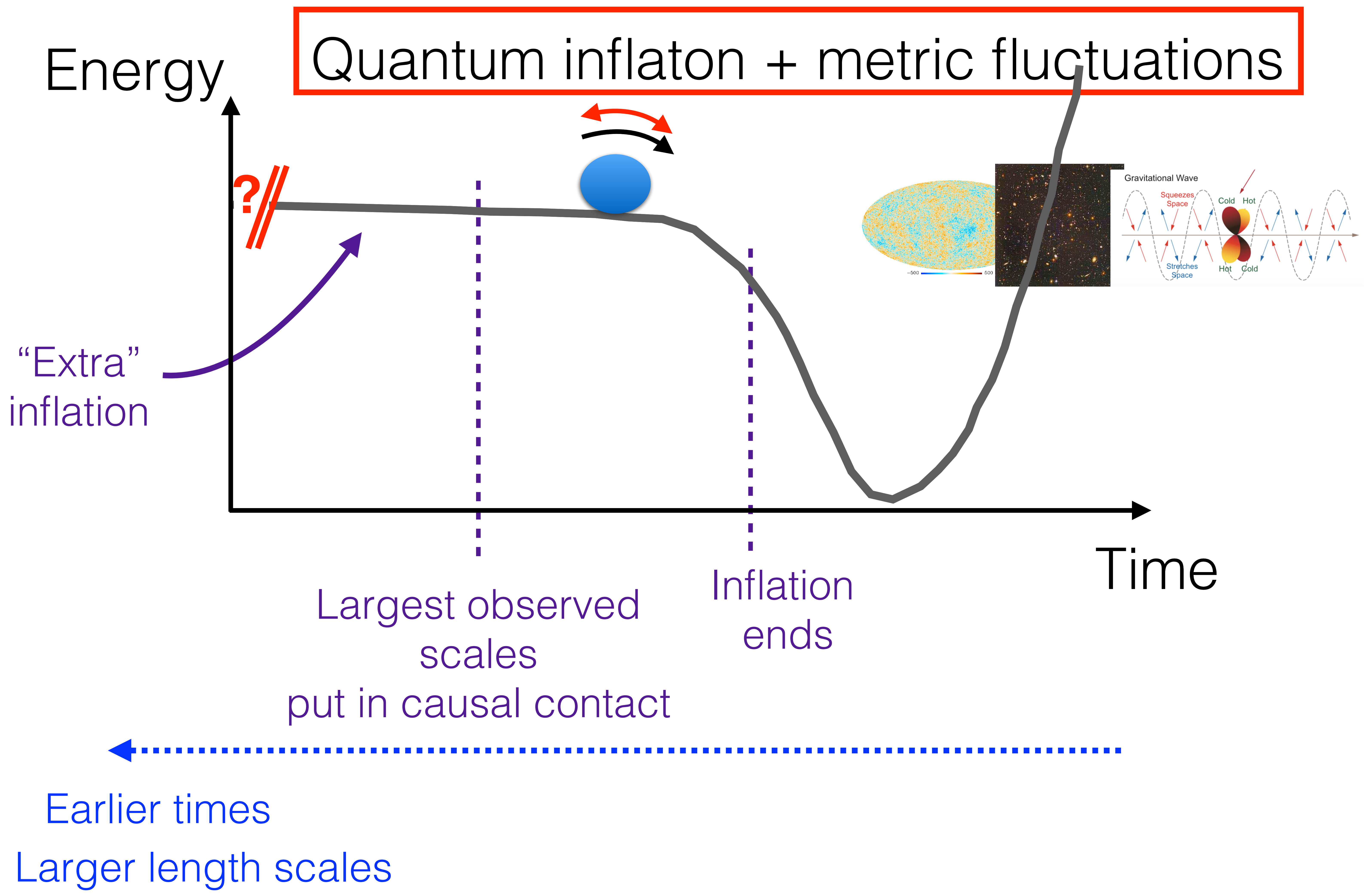
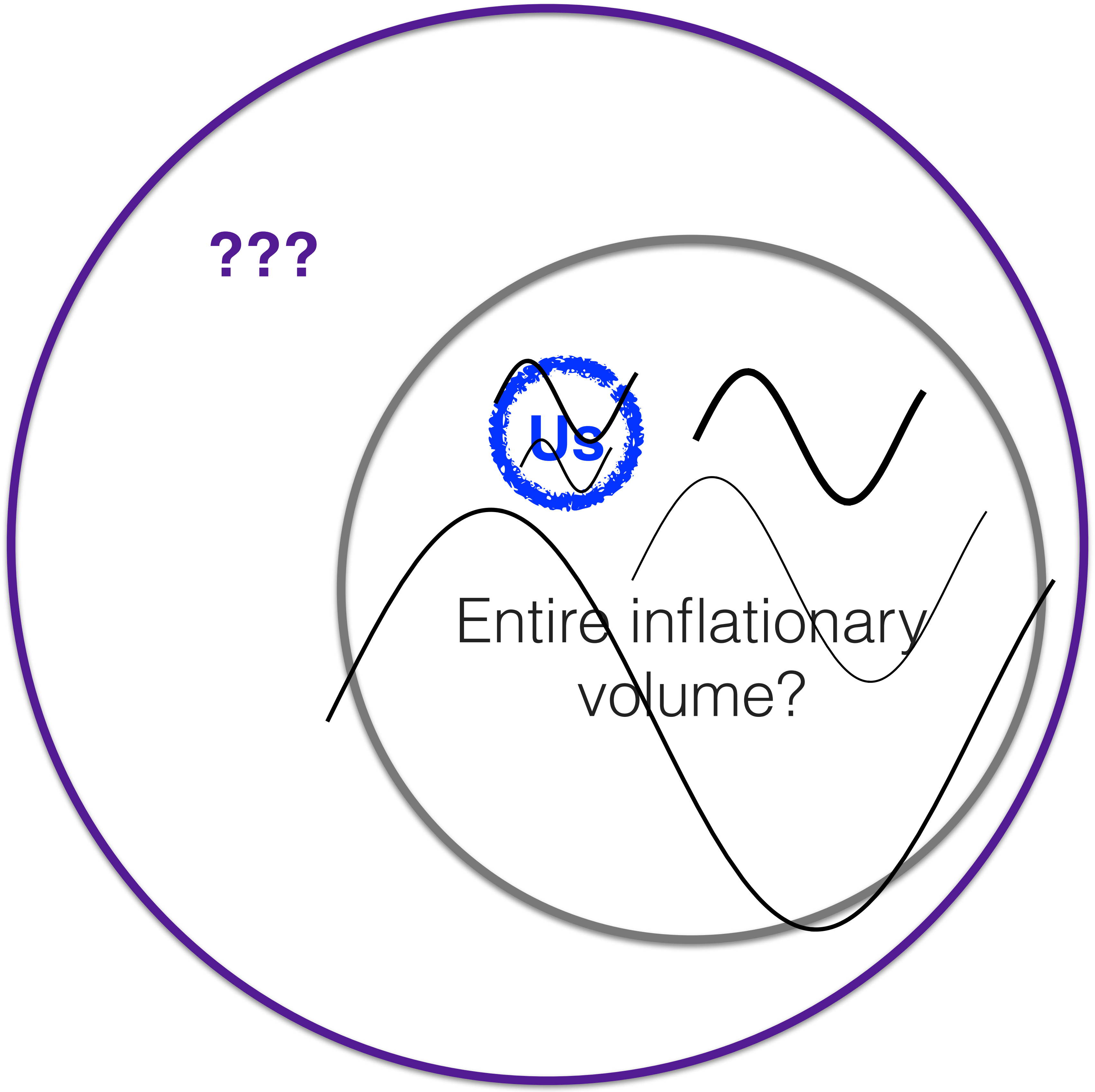


Then, the universe….



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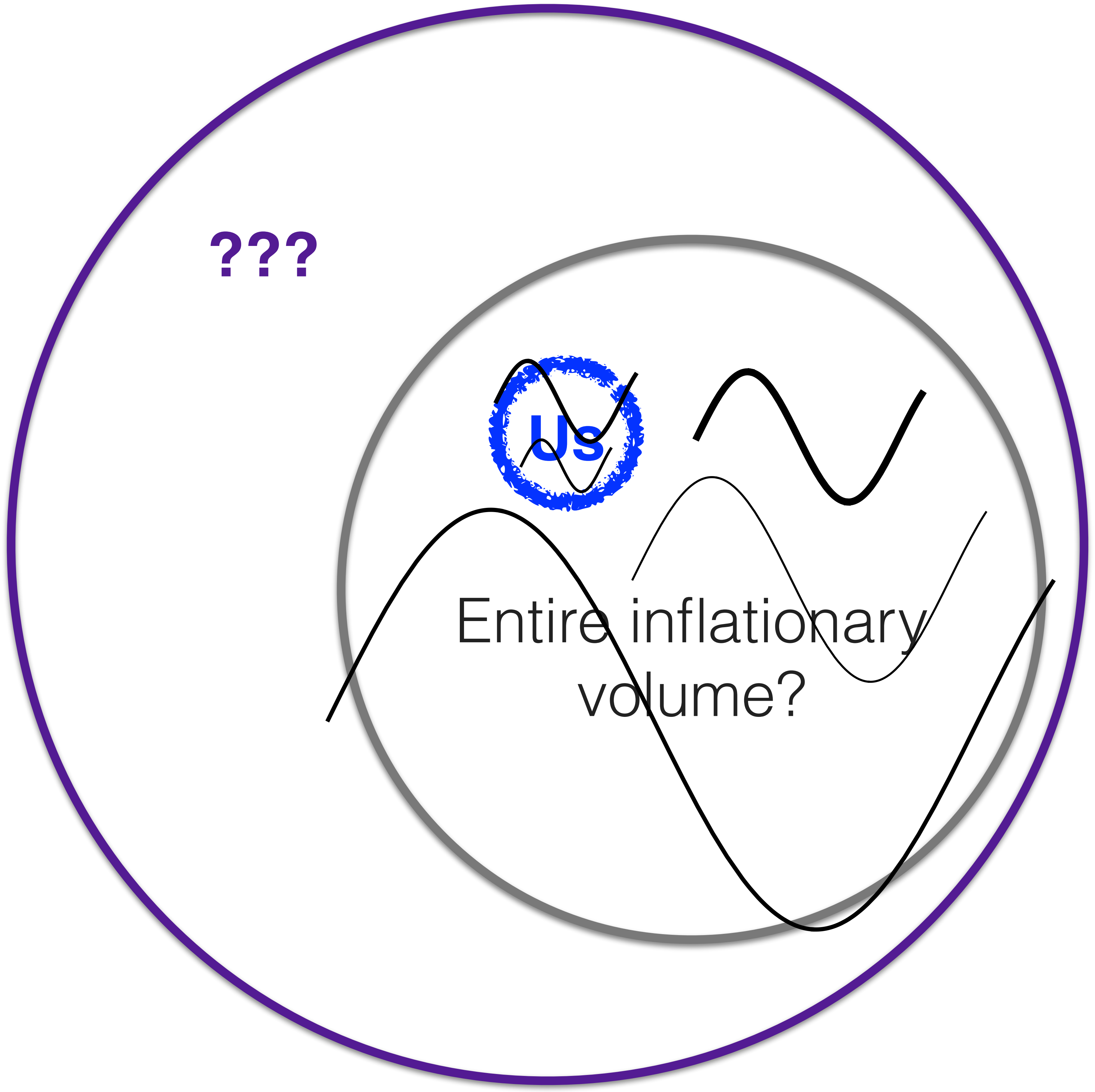

We can observe fluctuations on scales spanning about 10 e-folds (~4 decades in k-modes)

Total inflation is ~60 e-folds

Few prospects for accessing much smaller scales

What we can learn

- Scalar amplitude (two-point): quadratic action for gravity (perturbed metric) + scalar field (background + perturbations)

- At quadratic order, each Fourier mode is independent

$$A_0 \propto \frac{H^2}{M_p^2 \epsilon} \qquad \epsilon = -\frac{\dot{H}}{H^2}$$

One representation of the physics at quadratic order (standard single-field slow-roll)

$$\chi = z\zeta = \frac{\sqrt{2}\epsilon a}{c_s}\zeta$$

$$\hat{H} = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left[c_s k\left(\hat{c}_{\vec{k}}\hat{c}^\dagger_{\vec{k}} + \hat{c}_{-\vec{k}}\hat{c}^\dagger_{-\vec{k}}\right) - i\frac{z'}{z}\left(\hat{c}_{\vec{k}}\hat{c}_{-\vec{k}} - \hat{c}^\dagger_{\vec{k}}\hat{c}^\dagger_{-\vec{k}}\right)\right]$$

Two-mode squeezing
Gravity is a zero-momentum pump

Grishchuk et al 1992;
Albrecht et al 1994;

Beyond quadratic

- Gravity is non-linear, and matter can have interaction terms

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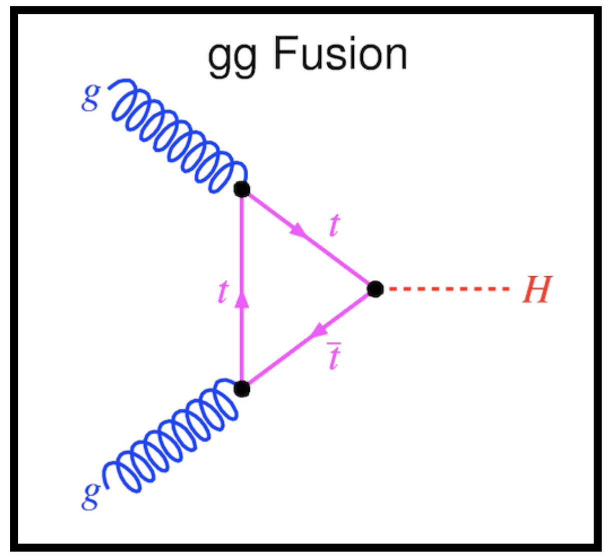
- Higgs



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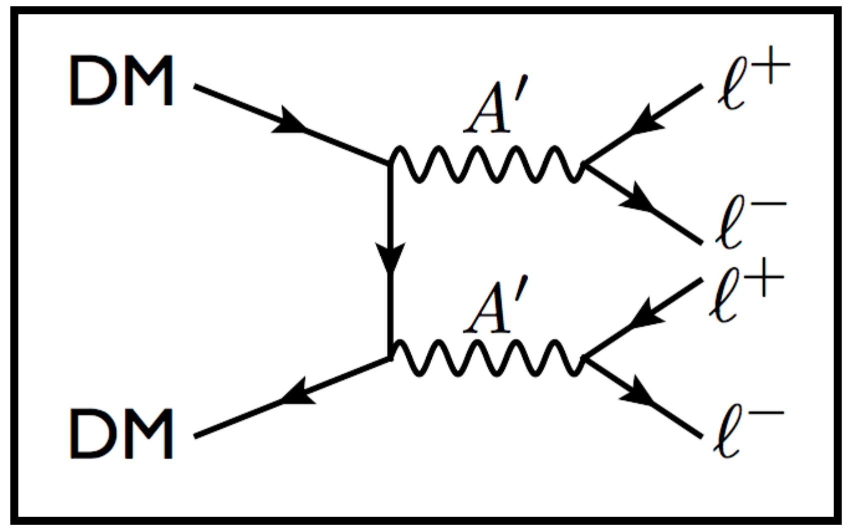
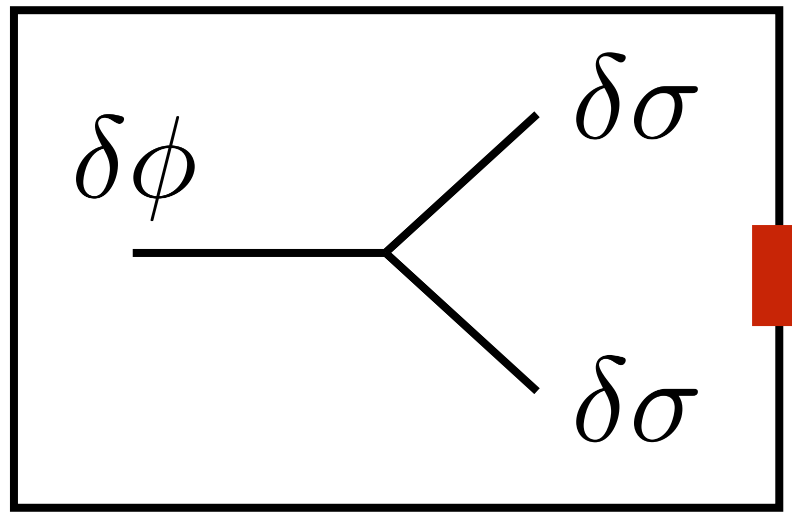


Dynamical
Repeatable

- Cosmology



Observational
One sky

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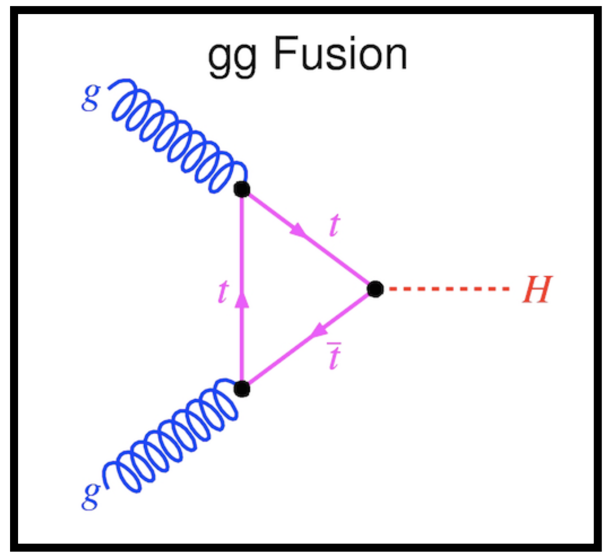

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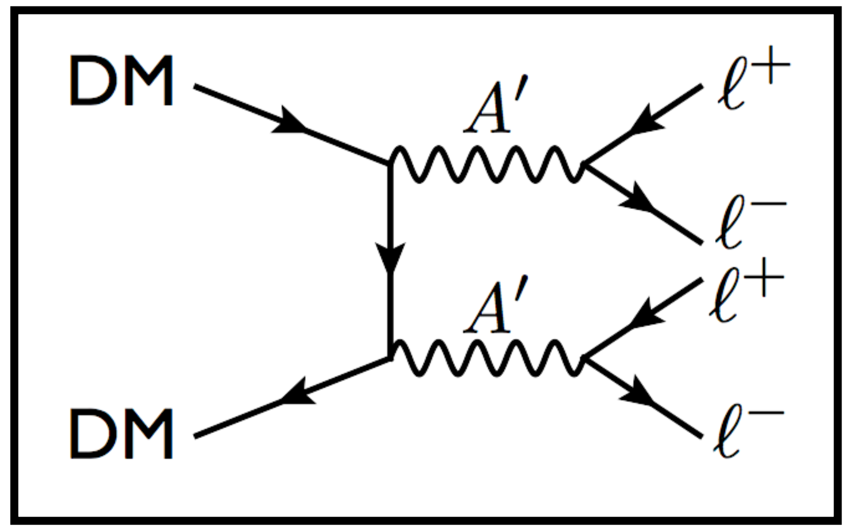

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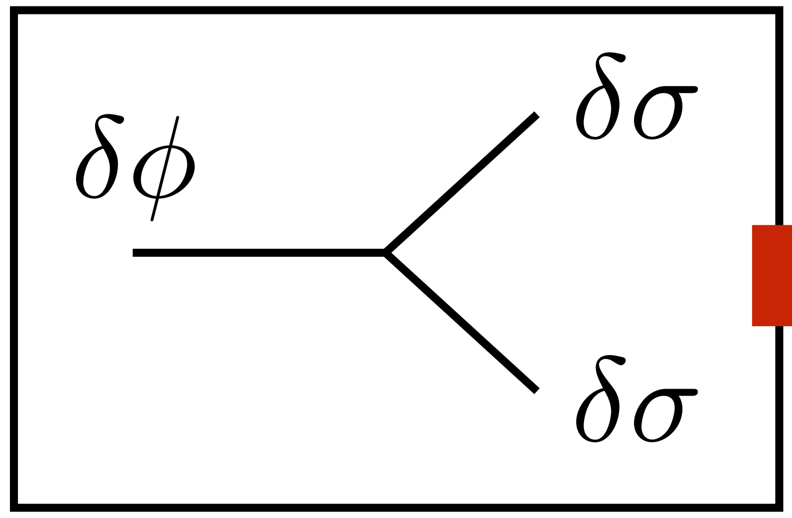

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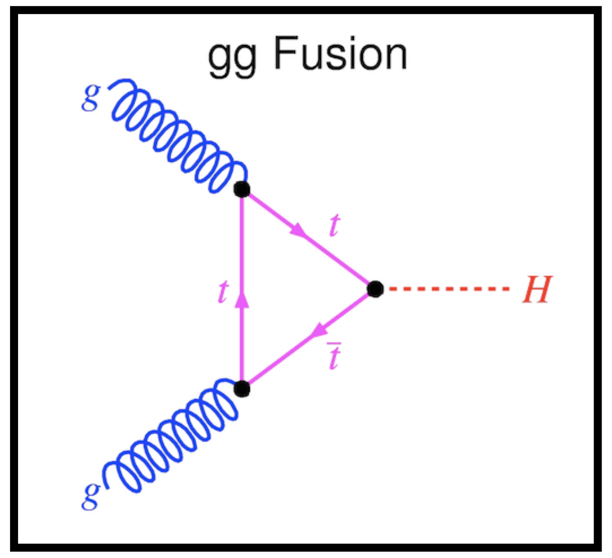

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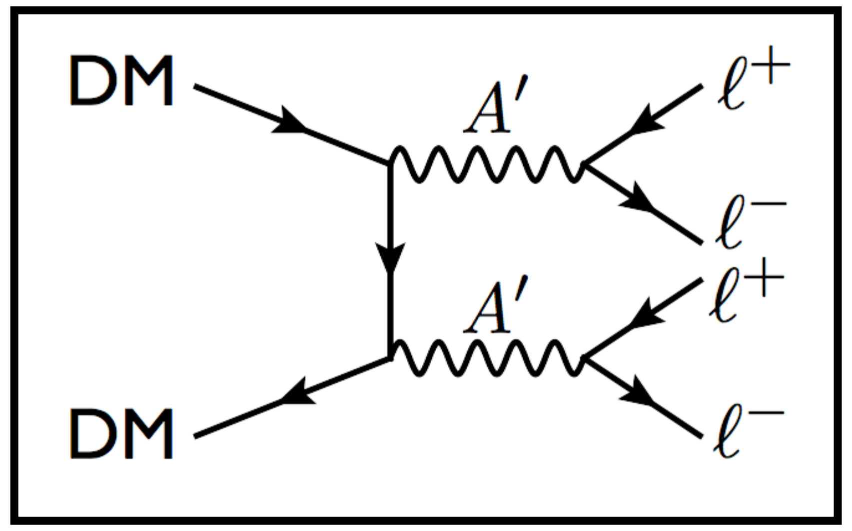

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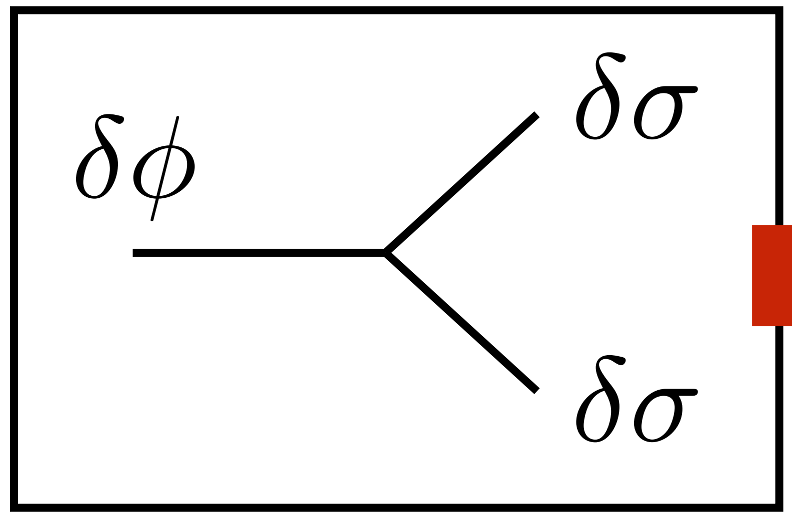

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Non-Gaussian statistics

We use in-in formalism to compute correlations from inflation

Open systems in inflation

- One approach for **vacuum** fluctuations: choose some Fourier modes to be system, some to be environment
- Choose some interaction between system and environment
- We can solve for all the behavior of all the modes. So, a choice of system + environment should correspond to some relevant scale for processes after inflation
- Generally, there is no natural separation of scales corresponding to the system/environment split: non-Markovianity is generic

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Suppose modes of very different wavelengths are correlated

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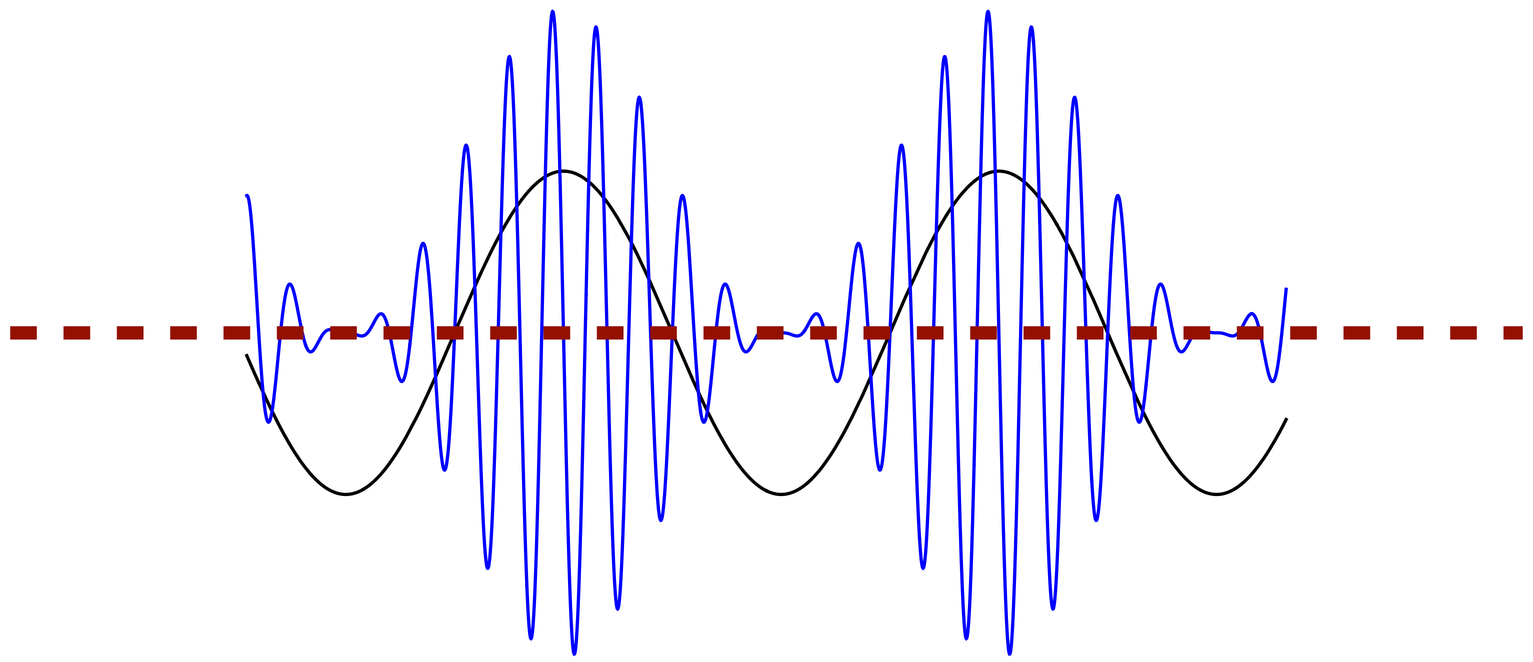
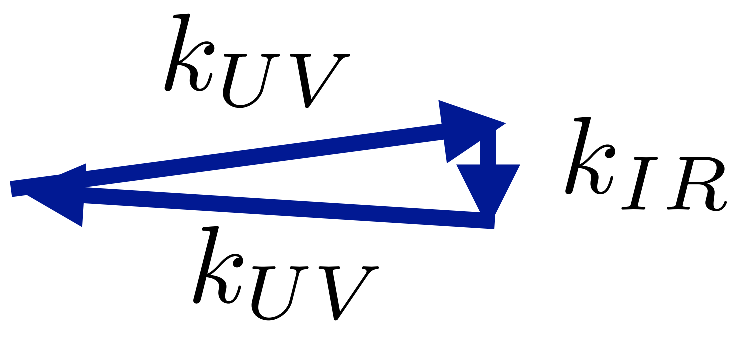
Bispectrum: squeezed triangles





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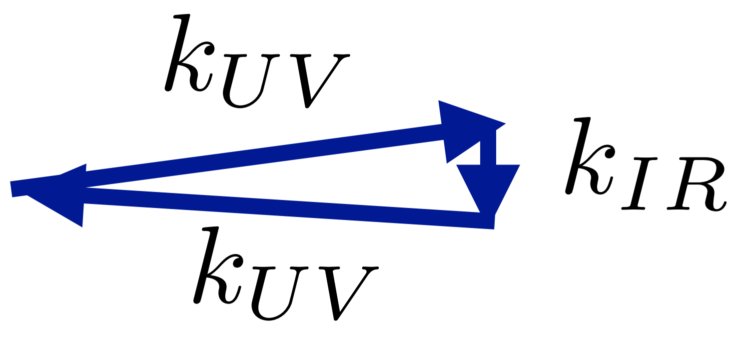

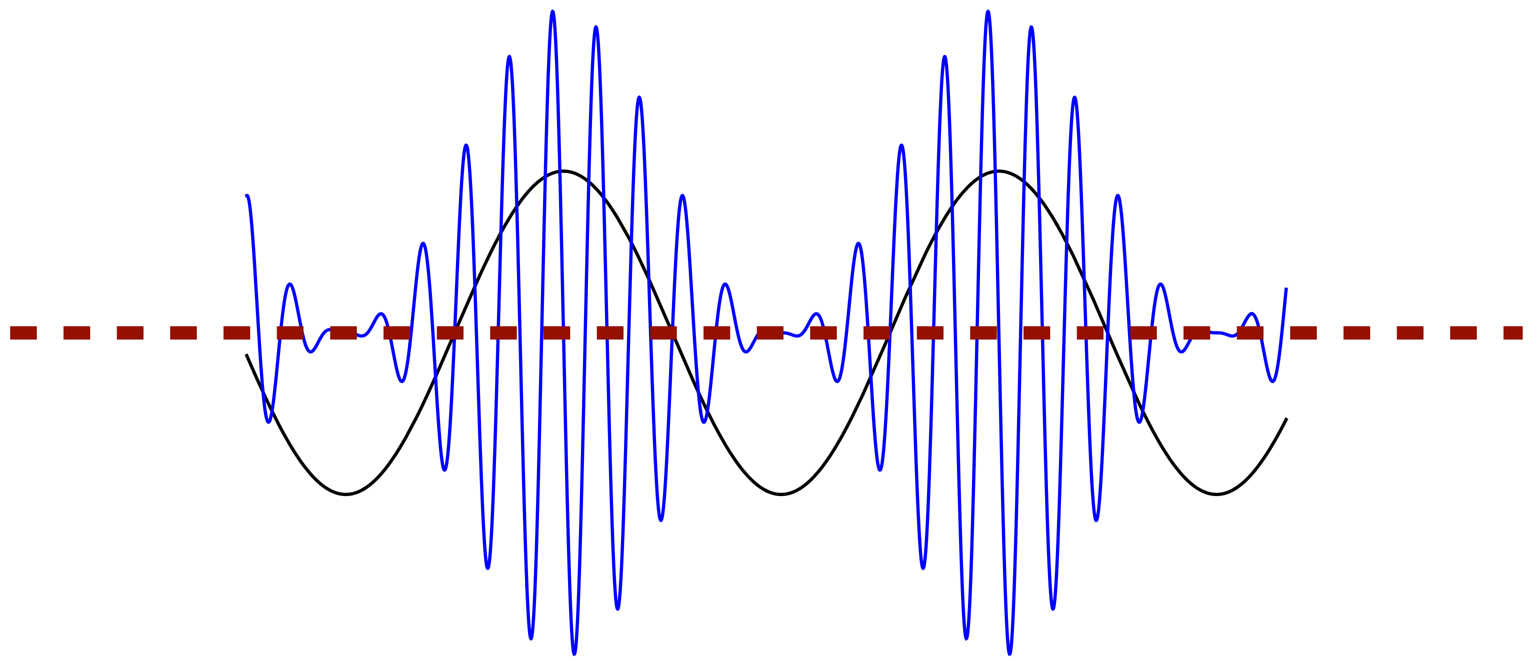

Then local statistics tend to be biased by unobservable long-wavelength modes

Shifting the amplitude of fluctuations

$$\Delta_\zeta^2 = A_0 \left(\frac{k}{k_0}\right)^{n_s - 1}$$

Phenomenology (an ansatz)

LoVerde, Nelson, Shandera, 1303.3549;
Bonga, Brahma, Deutsch, Shandera, 1512.05365

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$$\sigma(A_0) \sim \mathcal{O}(10^{-10})$$

Data

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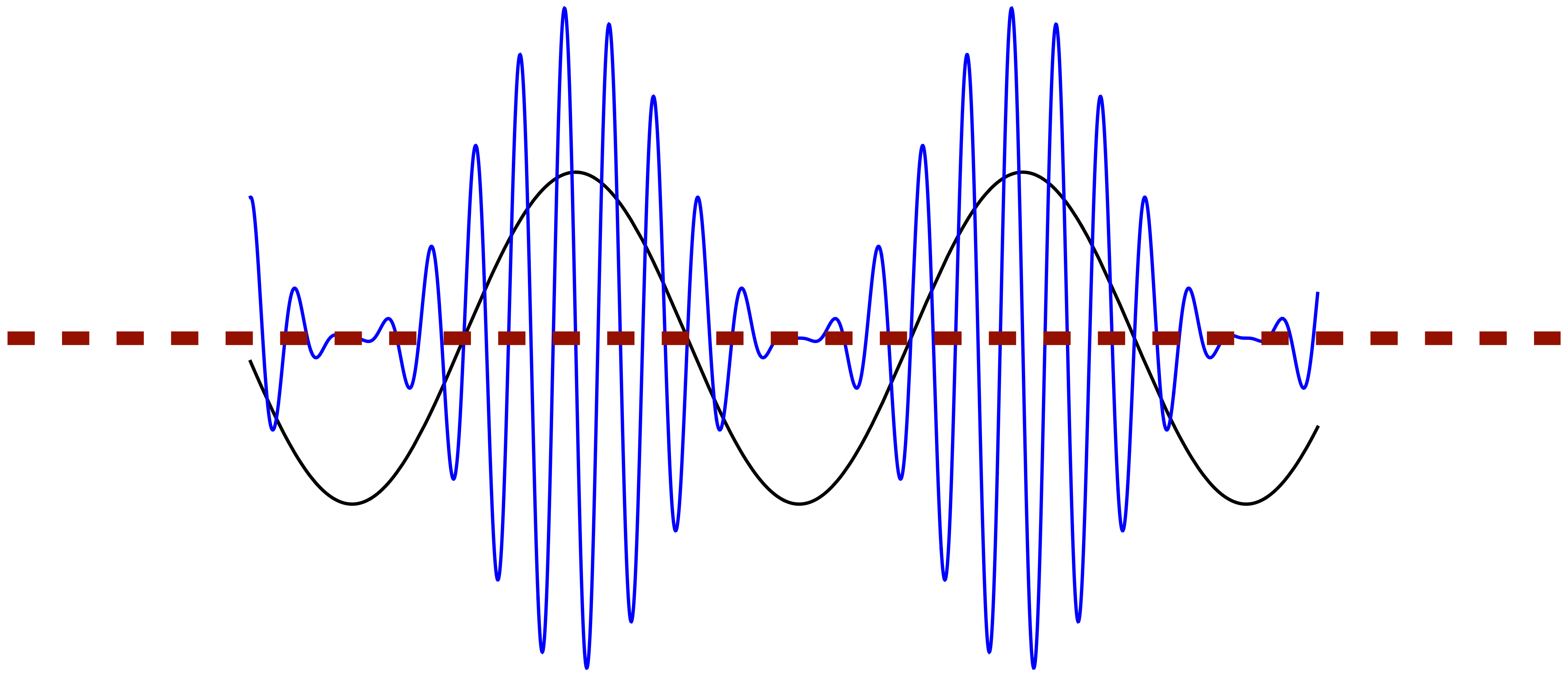

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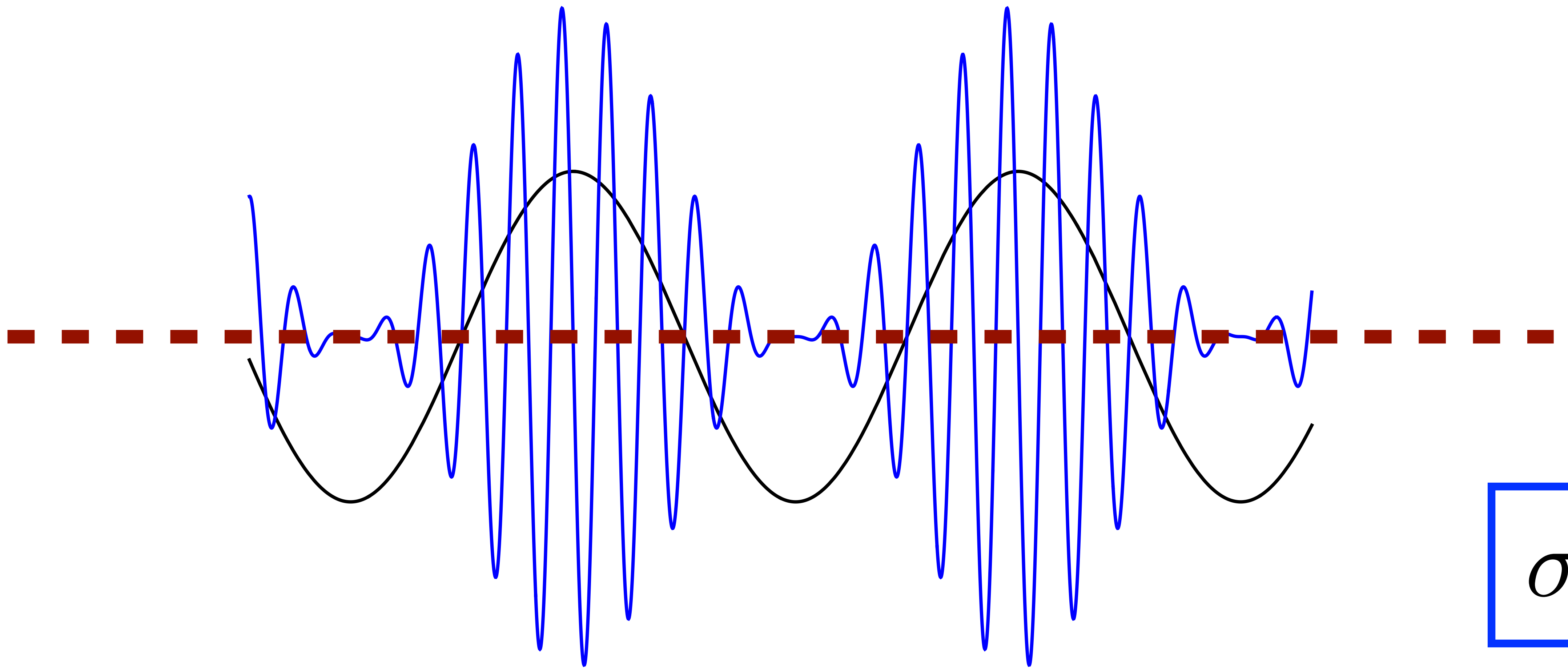

$$\sigma_{C.V.}(A_0) \sim \mathcal{O}(10^{-9})$$

Model/theory dependent (C.V.=cosmic variance)

LoVerde, Nelson, Shandera, 1303.3549;
Bonga, Brahma, Deutsch, Shandera, 1512.05365

And everything else…

Scalar spectral index:	$\sigma(n_s) \sim 0.005$ $\sigma_{C.V.}(n_s) \sim 0.04$
Amplitude for a particular shape can depend on long-wavelength modes:	$\sigma(f_{\rm NL}^{\rm local}) \sim 6$ $\sigma_{C.V.}(f_{\rm NL}^{\rm local}) \lesssim 10^4$
Shape of bispectrum depends on long-wavelength modes:	$\sigma(f_{\rm NL}^{\rm equil}) \sim 40$ $\sigma_{C.V.}(f_{\rm NL}^{\rm equil}) \sim 10^3$

Nelson, Shandera, 1212.4550
LoVerde, Nelson, Shandera, 1303.3549;
Bramante, Kumar, Nelson, Shandera (1307.5083)

Baytas, Kesavan, Nelson, Park, Shandera, 1502.01009; Adhikari, Jeong, Shandera, 1608.05139; Isotropy breaking: Adhikari, Shandera, Erickcek 1508.06489; Adhikari, Deutsch, Shandera 1805.00037;

And everything else…

Give me an n-point correlation…..
…….I'll give you a family of (n+1)-point correlations that induces it in biased sub-volumes

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Trace this back to inflation

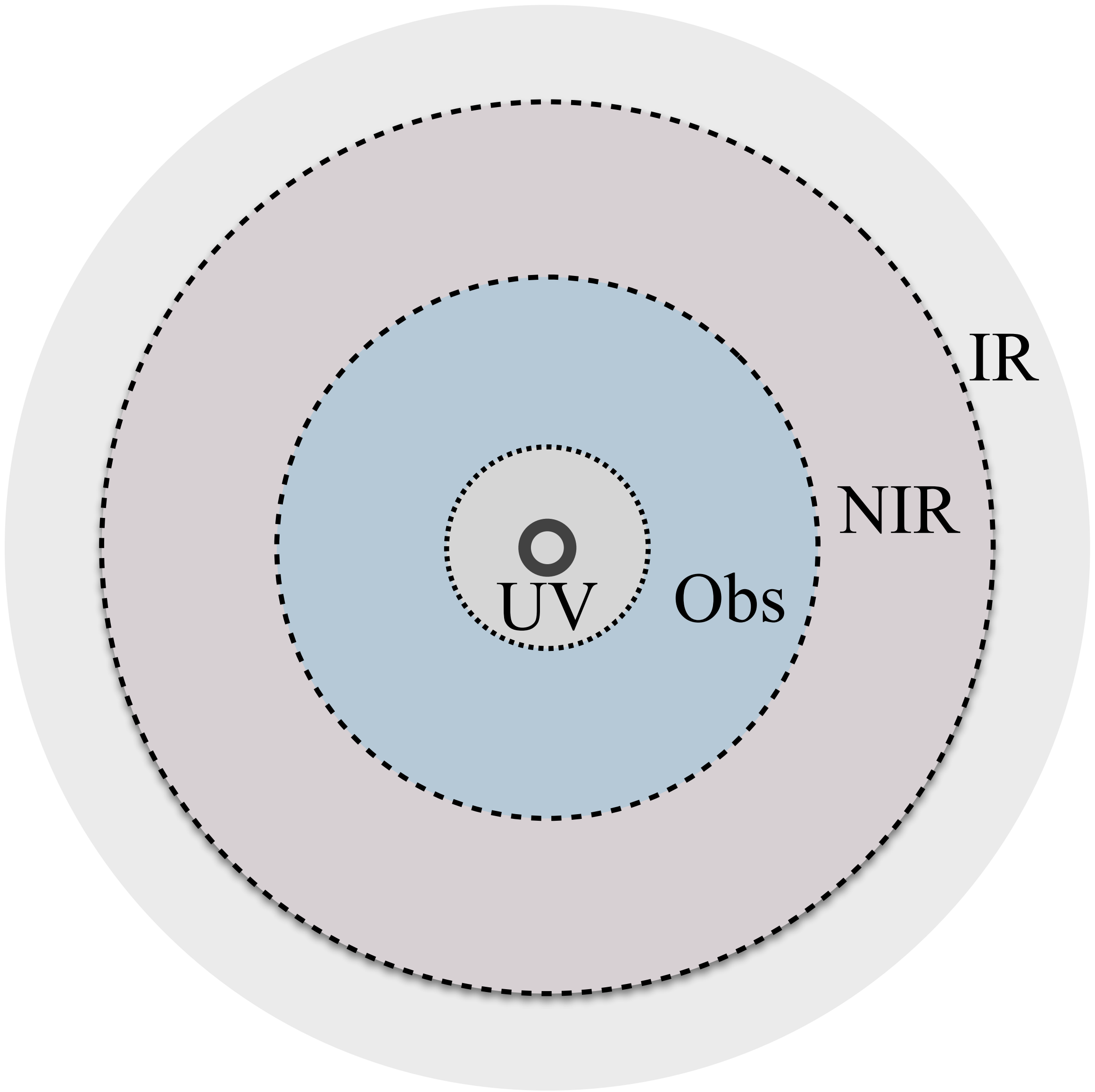
An open system with time-dependent dynamics at quadratic order, time-dependent interaction

Shandera, Agarwal, Kamal, 1708.00493

Trace this back to inflation

An open system with time-dependent dynamics at quadratic order, time-dependent interaction

System: modes inside the Hubble scale today;
Environment: near IR modes



$\eta \gg \eta_0$

$(aH)^{-1} \ll k_{\rm UV}^{-1}$

Shandera, Agarwal, Kamal, 1708.00493

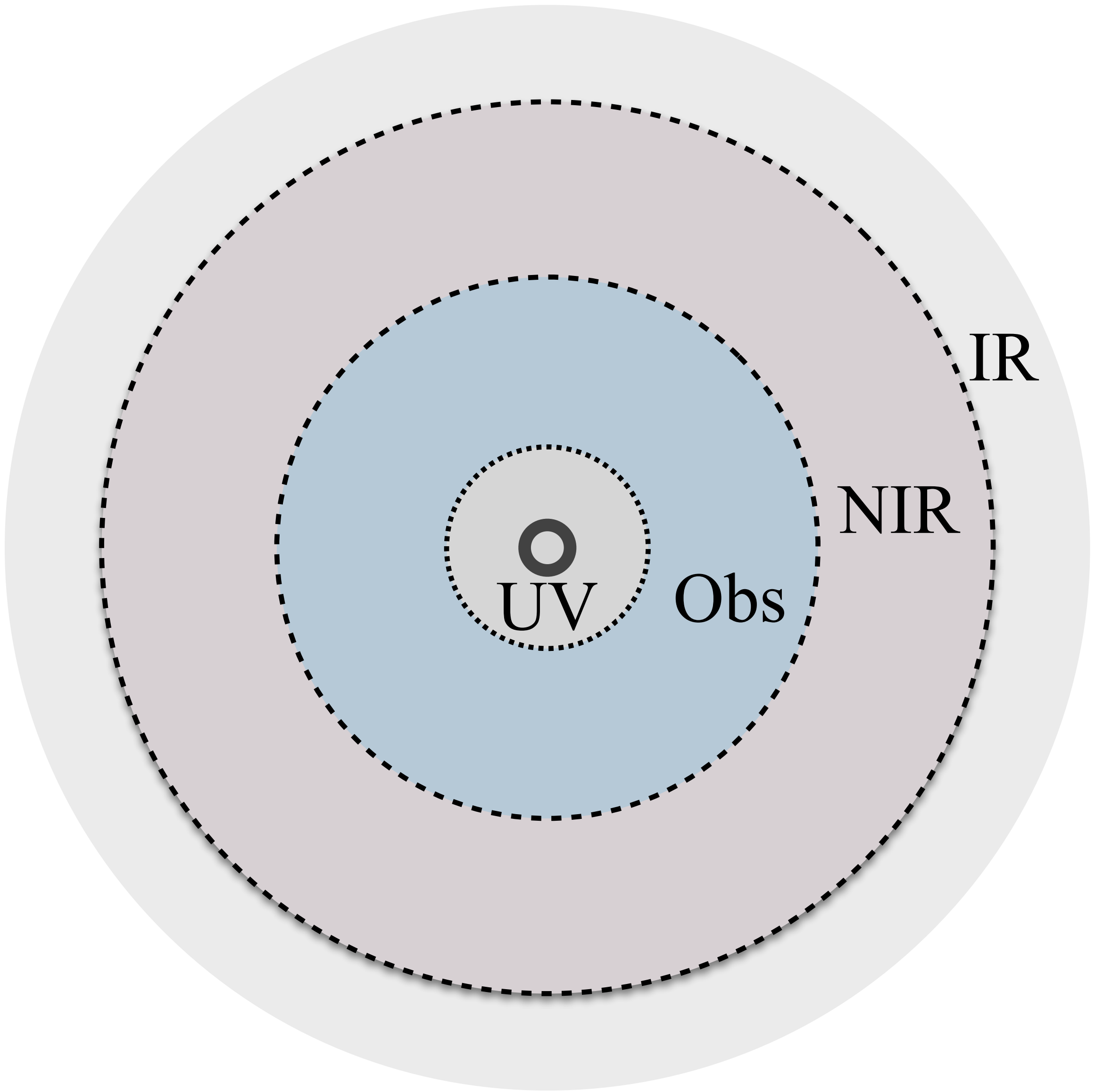
Trace this back to inflation

An open system with time-dependent dynamics at quadratic order, time-dependent interaction

System: modes inside the Hubble scale today;
Environment: near IR modes

Look at a single interaction term:

$$S_3 = \int d^3x\, d\eta\, a^4 \frac{3\epsilon(c_s^2 - 1)}{c_s^2} \zeta \dot{\zeta}^2$$



$\eta \gg \eta_0$

$(aH)^{-1} \ll k_{\mathrm{UV}}^{-1}$

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Trace this back to inflation

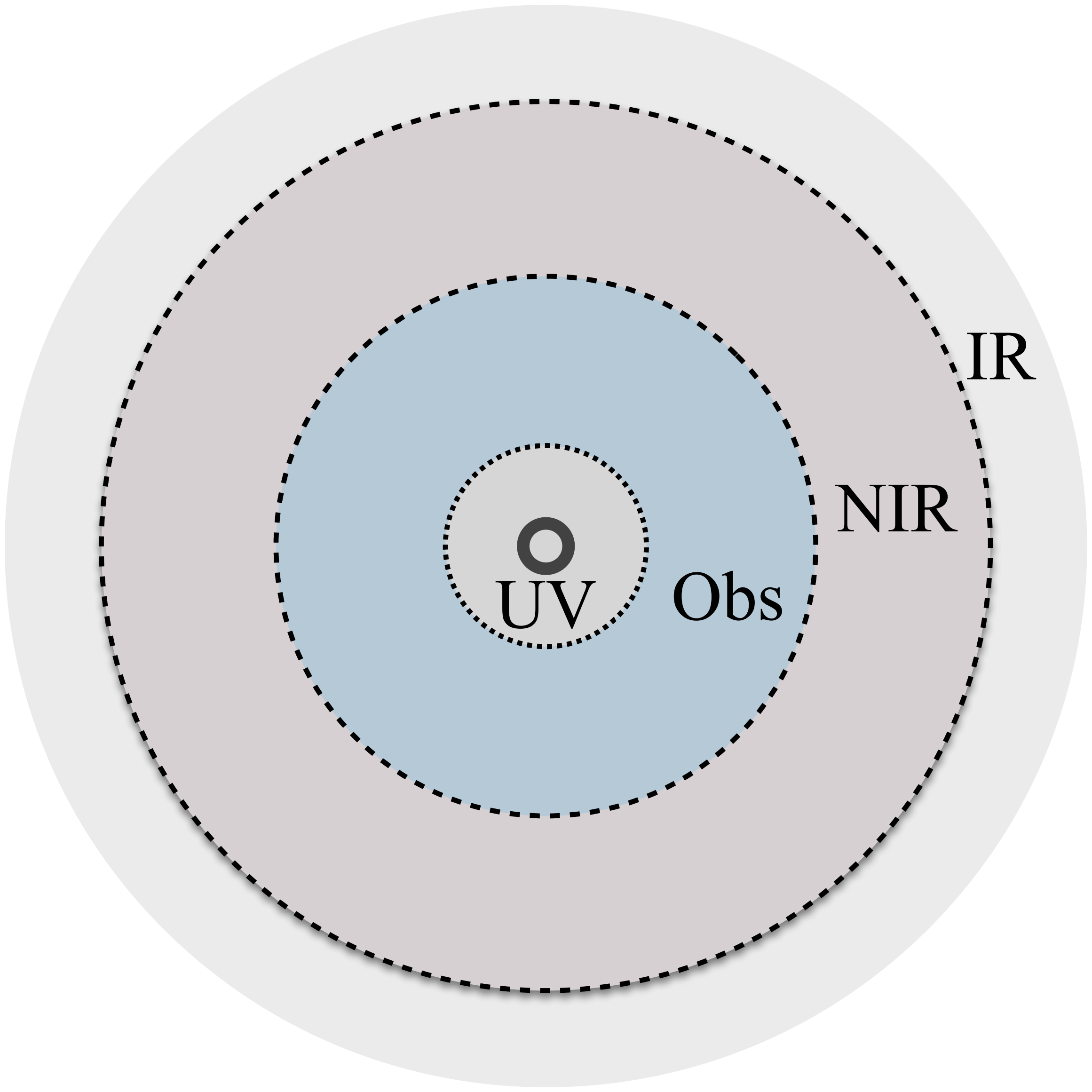
An open system with time-dependent dynamics at quadratic order, time-dependent interaction

System: modes inside the Hubble scale today;
Environment: near IR modes

Look at a single interaction term:

$$S_3 = \int d^3x\, d\eta\, a^4 \frac{3\epsilon(c_s^2 - 1)}{c_s^2} \zeta \dot{\zeta}^2$$

But two different inflationary dynamics that change the relevance of the interaction



$\eta \gg \eta_0$

$(aH)^{-1} \ll k_{\rm UV}^{-1}$

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Simplest context to try it:

slow-roll:

$$\epsilon \approx \text{constant},$$

$$\frac{z'}{z} \approx \frac{a'}{a}$$

non-attractor:

W. Kinney (ultra-slow-roll)

$$\dot{\phi} + 3H\phi = \text{constant}$$

$$\epsilon \propto a^{-6}$$

$$\frac{z'}{z} \approx -2\frac{a'}{a}$$

$$\hat{H}^{\text{sq.}} = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left[-i\left(\frac{z'}{z}\right)\left(\hat{c}_{\vec{k}}\hat{c}_{-\vec{k}} - \hat{c}^{\dagger}_{\vec{k}}\hat{c}^{\dagger}_{-\vec{k}}\right)\right]$$

- Squeezing parameters have a different time dependence in two cases

A single interaction term:

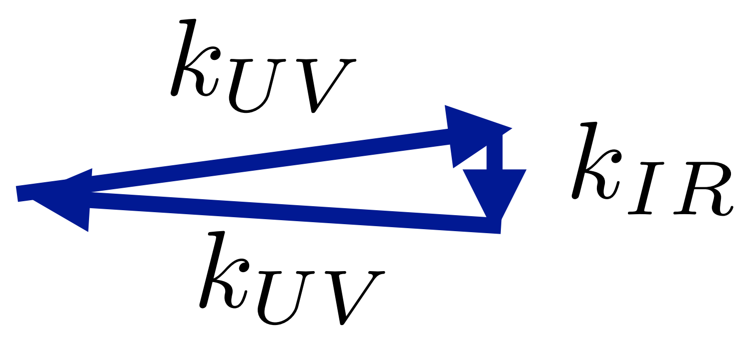
$$\lambda(\eta)\hat{H}_I = \frac{3(c_s^2-1)}{8M_p c_s^2 a\sqrt{\epsilon}}\int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)$$
$$\times\left[\sqrt{\frac{k_2k_3}{k_1}}\left(\hat{c}^\dagger_{-\vec{k}_1}\hat{c}^\dagger_{-\vec{k}_2}\hat{c}^\dagger_{-\vec{k}_3}+\hat{c}_{\vec{k}_1}\hat{c}^\dagger_{-\vec{k}_2}\hat{c}^\dagger_{-\vec{k}_3}+\ldots\right)+\text{symm}\right]$$

Time-dependence differs in slow-roll vs non-attractor

Long-short mode-coupling differs…

Namjoo et al 2012
Chen et al 2013;

$$\frac{3}{5}f_{\rm NL}^{\rm local} = \frac{3}{4c_s^2}(1+c_s^2)$$



Deriving the "Lindblad" operators

Calculate $\boxed{\frac{d\rho}{dt}}$, given

$$\boxed{\begin{aligned} \rho(t) &= \sum_n \langle B_n|\sigma(t)|B_n\rangle \\ \sigma(t) &= \hat{U}(t,t_0)\sigma(t_0)\hat{U}^\dagger(t,t_0) \end{aligned}}$$

✳ Treat in perturbation theory….first interesting stuff happens at second order

The “Lindblad”-like operators

$$\hat{L}_{N1}(\eta) = \lambda(\eta)\langle N|\hat{H}_I(\eta_0)|SQ(\eta)\rangle$$

$$\hat{L}_{N2}(\eta) = \int_{\eta_0}^{\eta} d\eta_1 \lambda(\eta_1)\langle N|\hat{H}_{I,int}(\eta_1 - \eta)|SQ(\eta)\rangle$$

These will take a different form depending on how many of the interaction modes are in the system (observable) vs the bath (near-infrared)

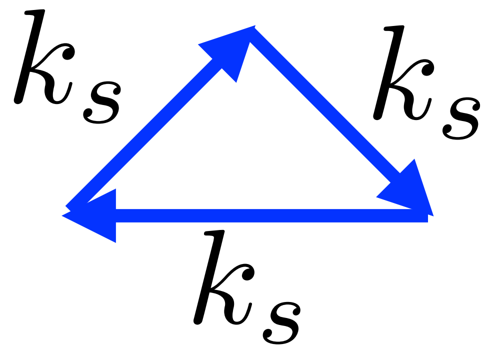
Evolution depends on terms like: $\sum_N \hat{L}_N \rho^{(0)}(\eta)\hat{L}_N^\dagger$

(compare: Agon et al 1412.3148)

The upshot

1. Correlation function shapes tell us how the system and bath are coupled:

(Large momentum, short wavelength system)



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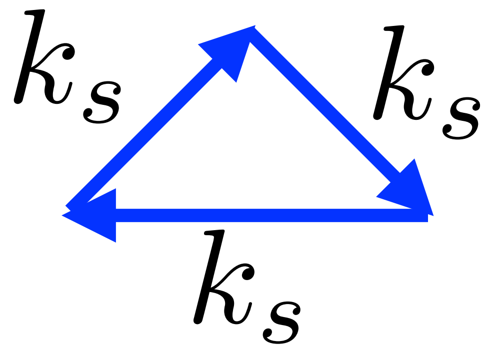

Coupling within system

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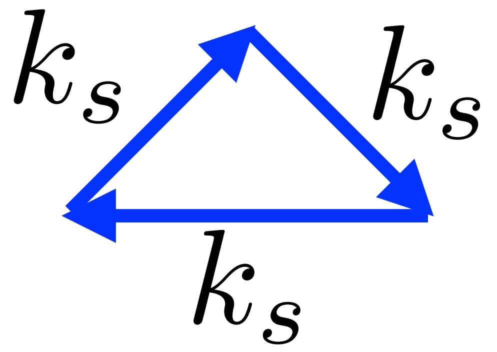

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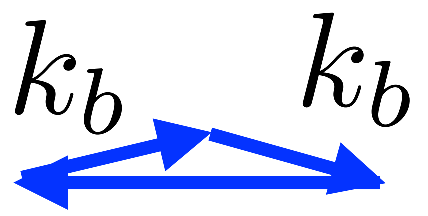

Coupling of one system mode (near boundary) to two bath modes (linear dissipation)

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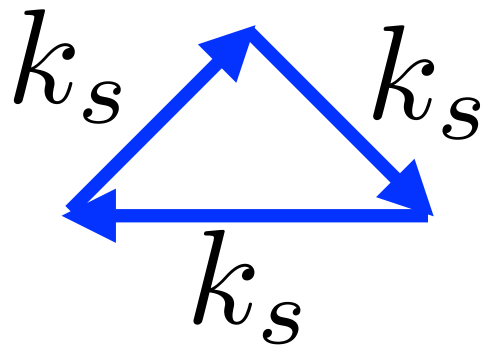

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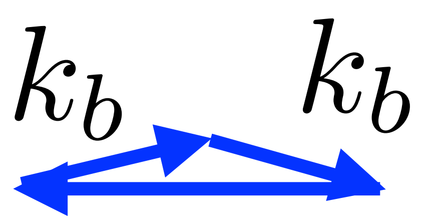

Coupling of one system mode (near boundary) to two bath modes (linear dissipation)

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Shandera, Agarwal, Kamal, 1708.00493

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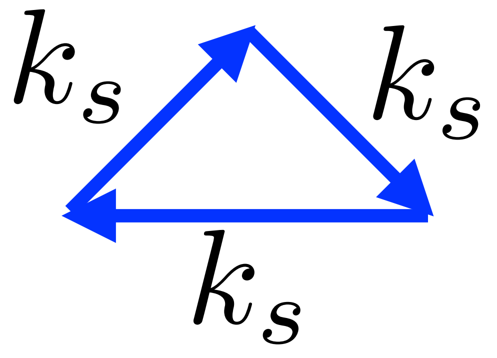

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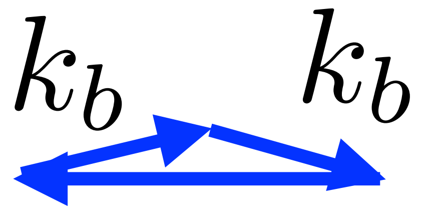

Coupling of one system mode (near boundary) to two bath modes (linear dissipation)

Coupling of two system modes to one bath modes (non-linear dissipation)

2. System-bath correlations do not rapidly dissipate; dissipators oscillate in sign

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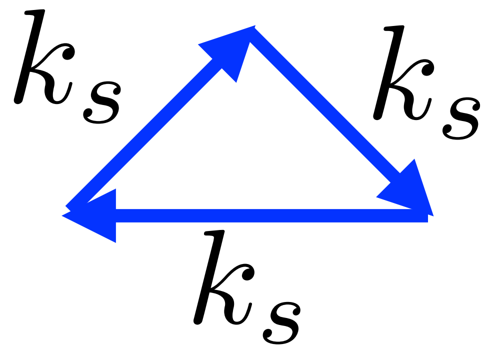
The upshot

1. Correlation function shapes tell us how the system and bath are coupled:

(Large momentum, short wavelength system)



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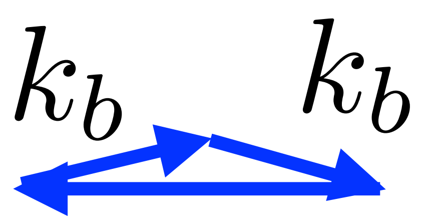

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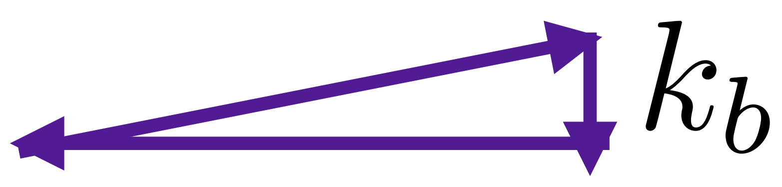

Coupling of two system modes to one bath modes (non-linear dissipation)

2. System-bath correlations do not rapidly dissipate; dissipators oscillate in sign

3. Different scalar field dynamics change the length of time for which the “Lindbladian” terms are relevant

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Cosmology: inference to discover elementary matter, laws of physics

- OQS is the right framework for this inference: even more, need how to put partial observations together for inference in quantum systems

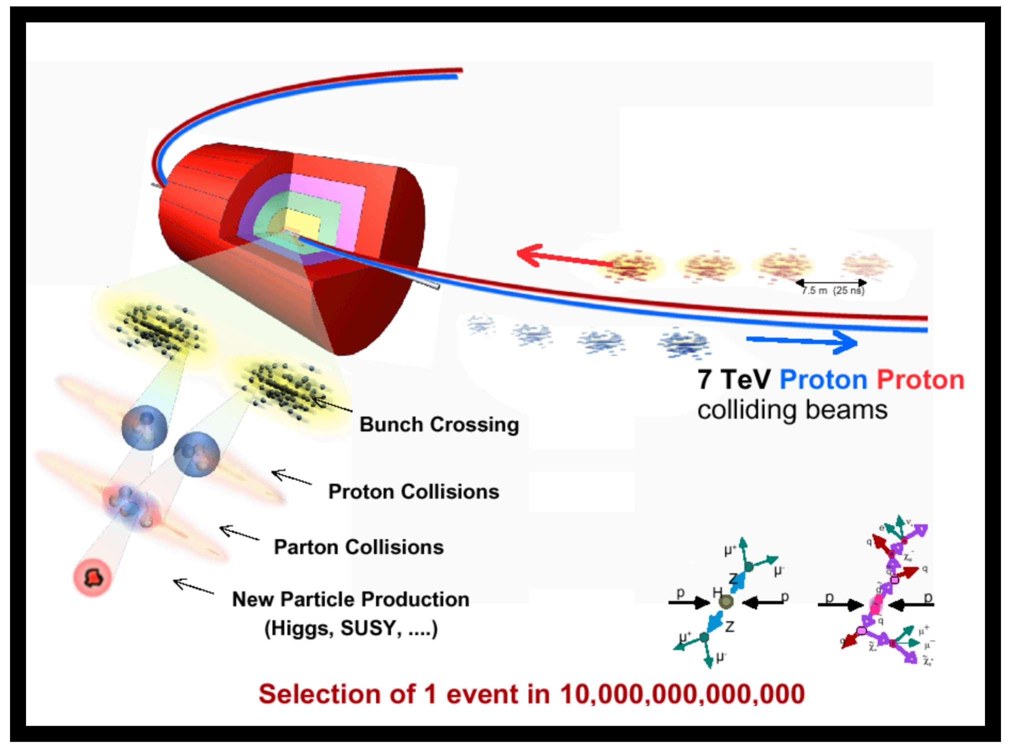
- What can be gained for inflation? (Fluctuations, vacuum + other; background?)

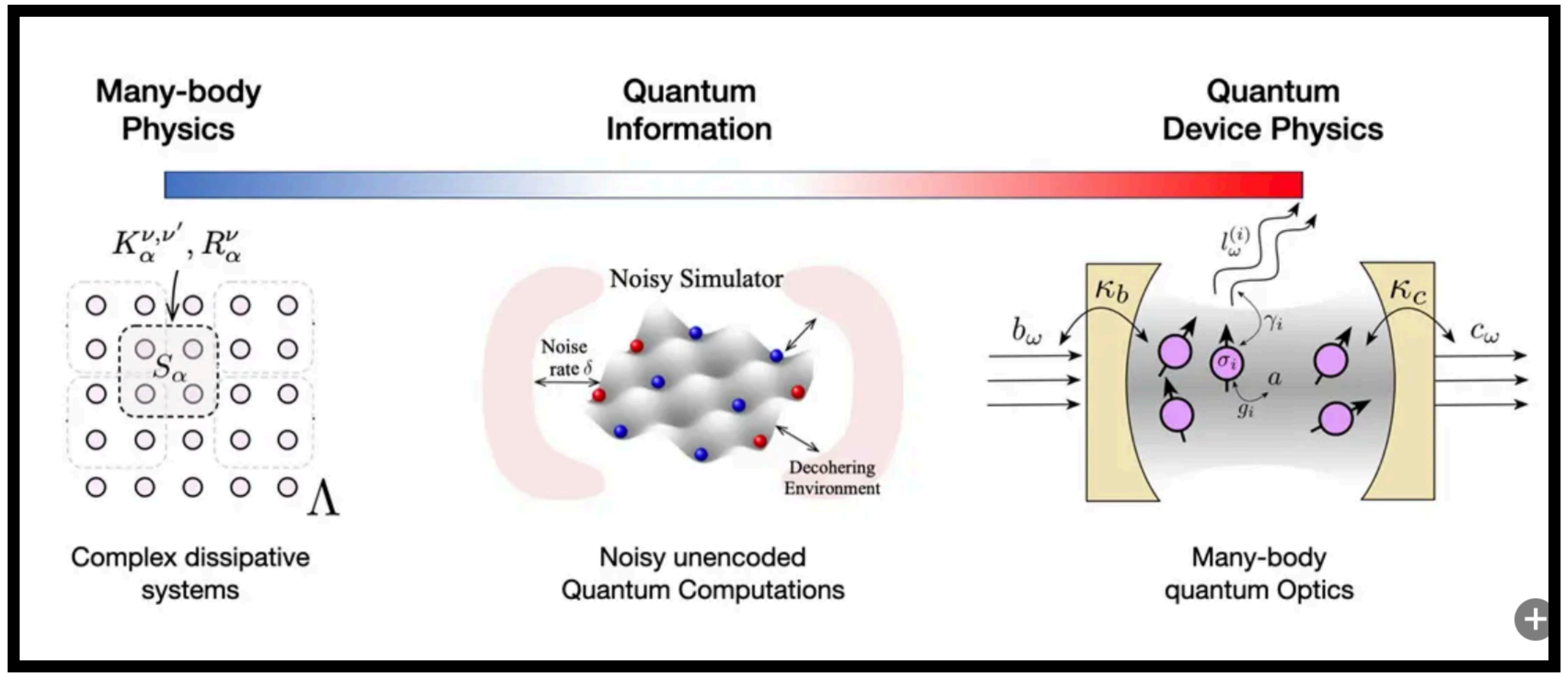
- What can be gained beyond inflation?

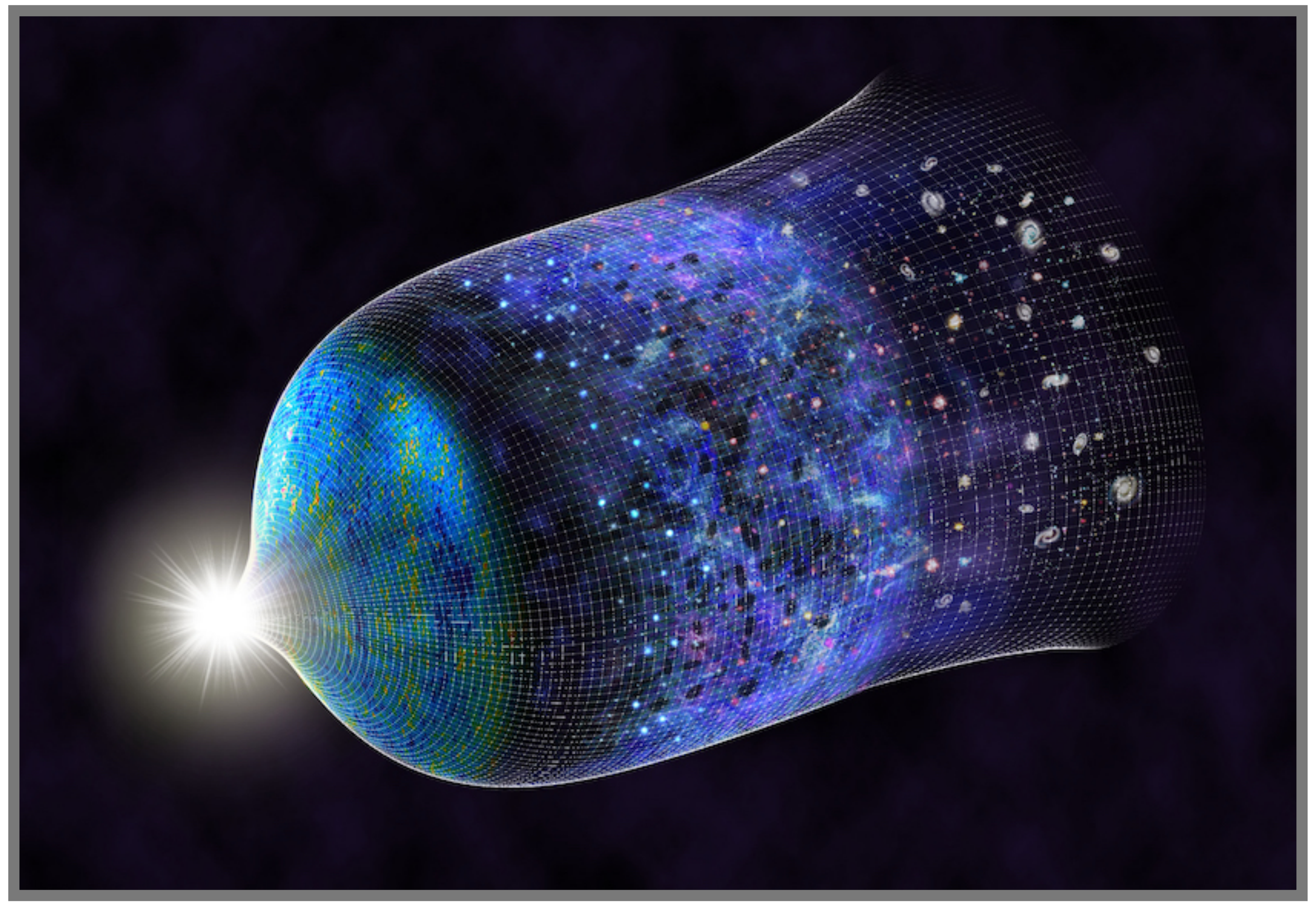
- How much do we need to understand how to go beyond 'simple' open systems? (Role of correlations, non-Markovianity, inference)

- What techniques/principles do we need to develop to do so?

Updating our theory framework



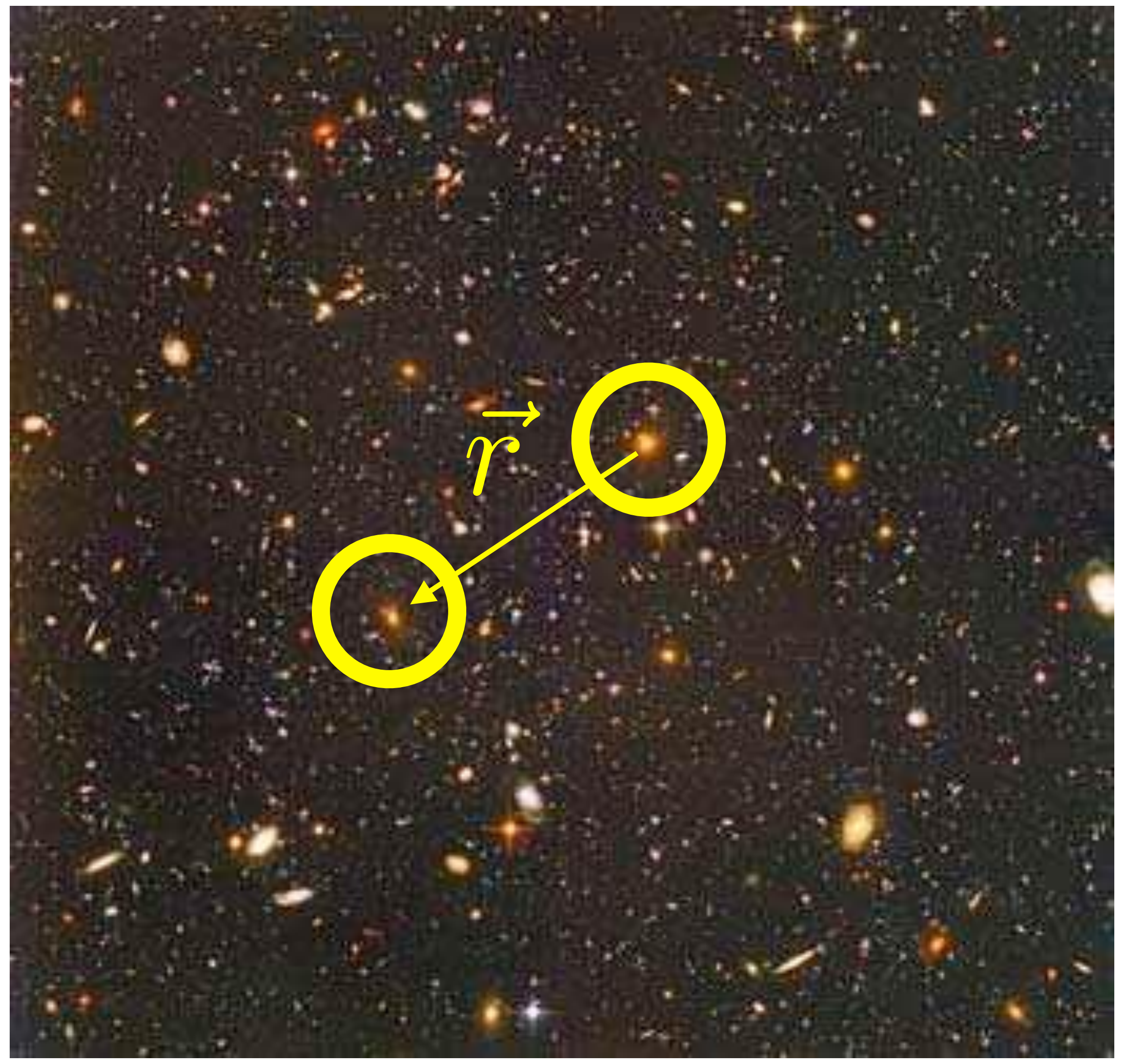




R. Trivedi

Correlation functions assuming homogeneity and isotropy

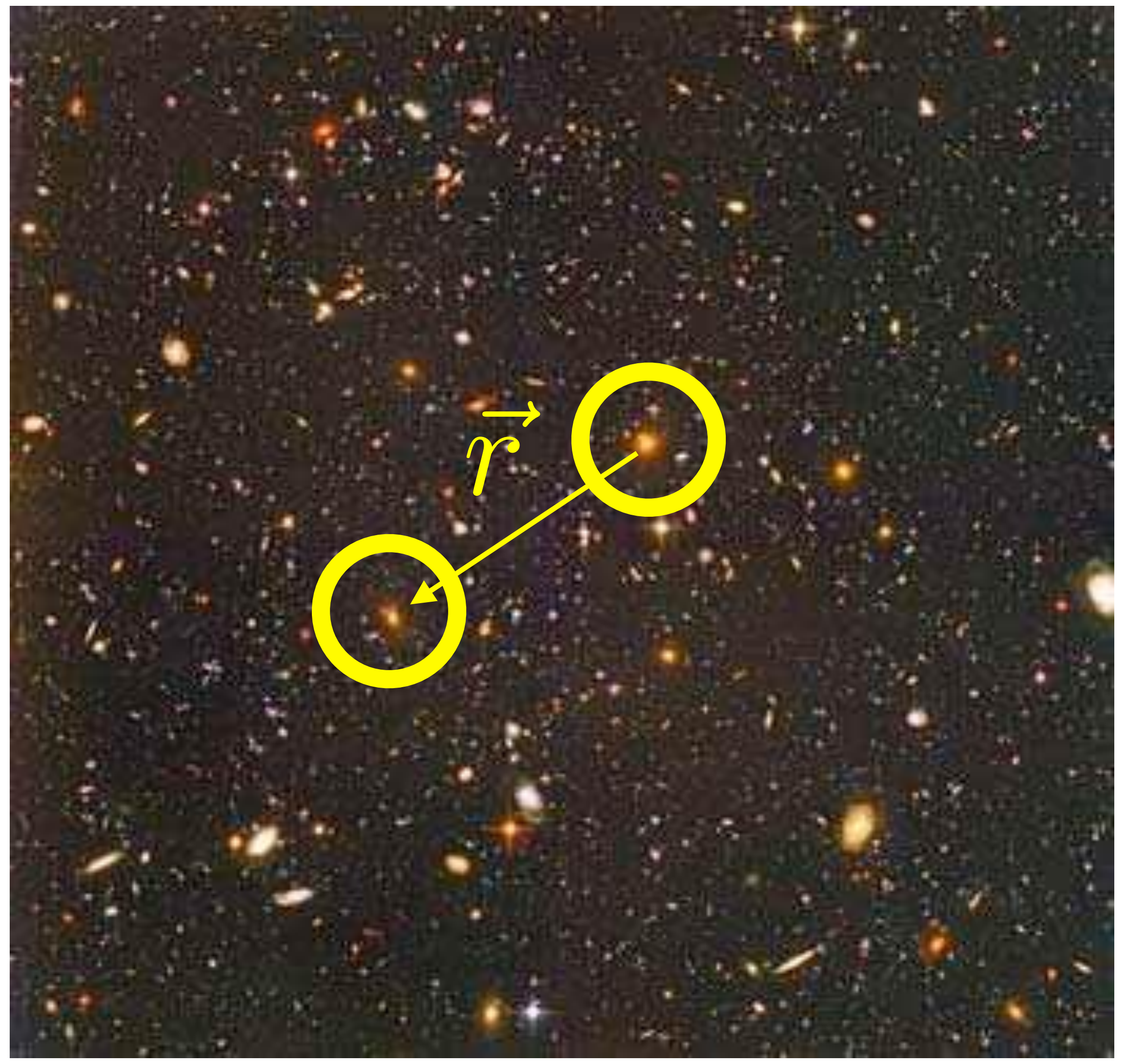
$$\langle \delta(\vec{x})\delta(\vec{x}+\vec{r})\rangle$$



Hubble Telescope image

Correlation functions assuming homogeneity and isotropy

$$\langle \delta(\vec{x})\delta(\vec{x}+\vec{r})\rangle \propto f(|\vec{r}|)$$



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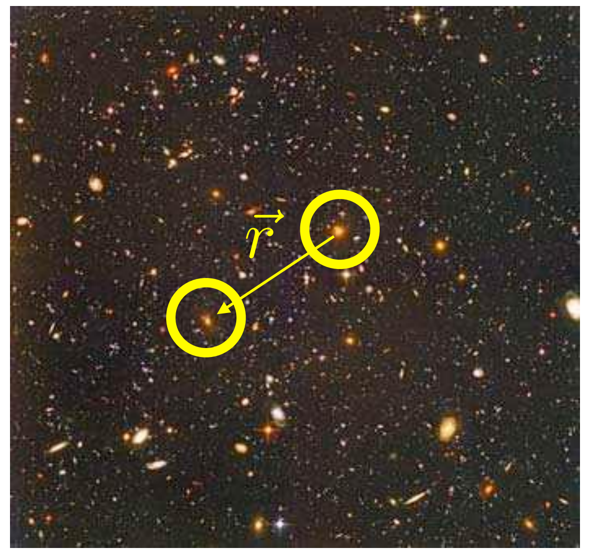
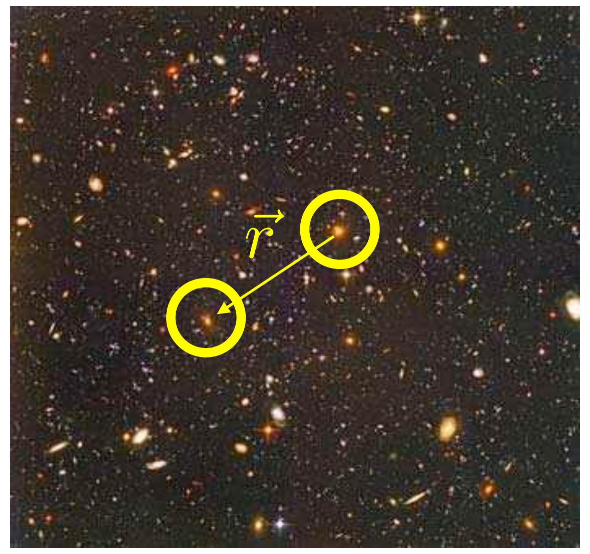


Hubble Telescope image

Power spectrum

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2) P(k_1)$$

Correlation functions assuming homogeneity and isotropy



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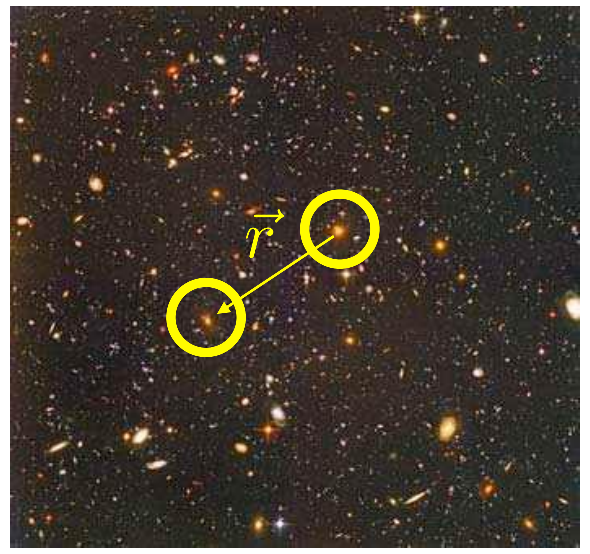

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$$\Delta_\zeta^2 = A_0 \left(\frac{k}{k_0} \right)^{n_s - 1}$$

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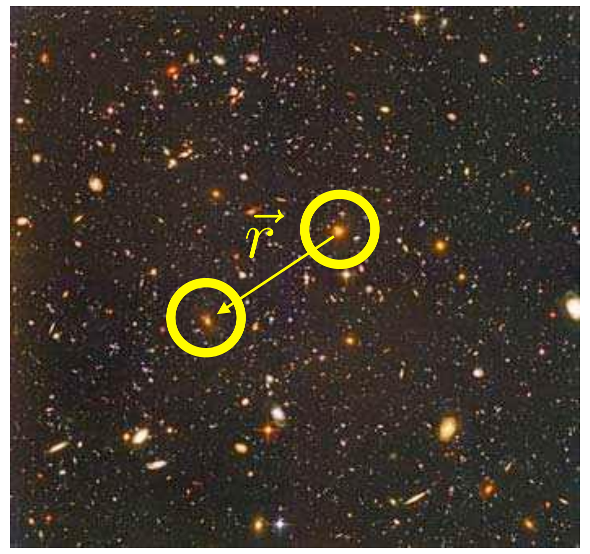

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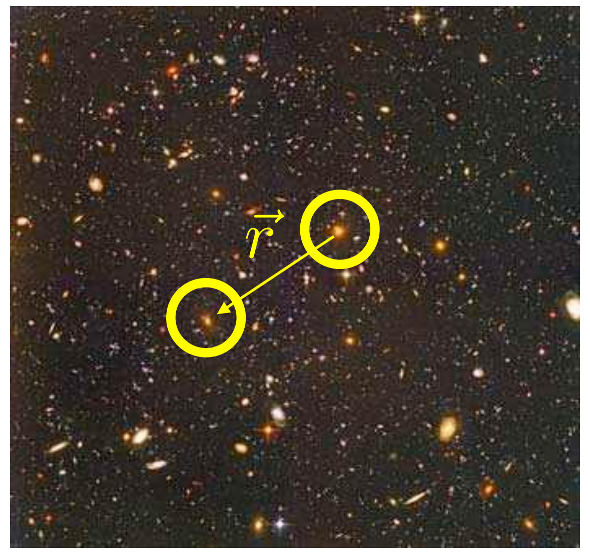
"Bispectrum"

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

$$B(k, k, k) \propto f_{\rm NL} \frac{[\Delta^2(k)]^2}{k^6}$$

Correlation functions assuming homogeneity and isotropy



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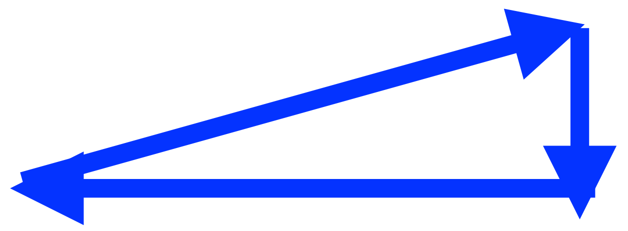

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Inflationary quadratic evolution

$$\hat{U}_0(\eta,\eta_0)|0_{\vec{k}},0_{-\vec{k}}\rangle = e^{-i\int_{\eta_0}^{\eta}\hat{H}_0(k,\eta_1)d\eta_1}|0_{\vec{k}},0_{-\vec{k}}\rangle$$

Inflationary quadratic evolution

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$$= \hat{S}_k(\eta)\hat{R}_k(\eta)|0_{\vec{k}},0_{-\vec{k}}\rangle$$

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$$= \hat{S}_k(\eta)\hat{R}_k(\eta)|0_{\vec{k}},0_{-\vec{k}}\rangle$$

$$= \frac{1}{\cosh r_k}\sum_{n=0}^{\infty} e^{-2in\phi_k}\tanh^n r_k|n_{\vec{k}},n_{-\vec{k}}\rangle$$

$$\equiv |SQ(k,\eta)\rangle = \sum_n c_n^{(sq)}(\eta)|n_{\vec{k}},n_{-\vec{k}}\rangle$$

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$r_k(\eta)$, $\phi_k(\eta)$ differ in slow-roll vs non-attractor

The evolution:

Evolution of the full system:

$$\sigma(\eta) = \hat{U}(\eta,\eta_0)\sigma(\eta_0)\hat{U}^\dagger(\eta,\eta_0)$$

$$\hat{U}(\eta,\eta_0) = e^{-i\int_{\eta_0}^{\eta}\hat{H}_0(\eta_1)d\eta_1} T e^{-i\int_{\eta_0}^{\eta}\hat{H}_{\mathrm{I,i}}(\eta_1)d\eta_1}$$

Describing only the observable modes

The full density matrix:

$$\sigma(\eta_0) = |\overline{NIR}\rangle|\psi_{\rm obs.}(\eta_0)\rangle\langle\psi_{\rm obs.}(\eta_0)|\langle\overline{NIR}|$$

The reduced density matrix:

$$\rho(\eta) = Tr_{NIR}\,\sigma(\eta)$$

$$= \sum_N \langle N|\hat{U}(\eta,\eta_0)|\overline{NIR}\rangle|\psi_{\rm obs.}(\eta_0)\rangle\langle\psi_{\rm obs.}(\eta_0)|\langle\overline{NIR}|\hat{U}^\dagger(\eta,\eta_0)|N\rangle$$

The evolution:

For just the observable modes, interesting part is second order in interaction strength:

$$\partial_\eta \rho^{(2)}(\eta) = -i\left[\hat{H}_0^{\rm obs}, \rho^{(2)}(\eta)\right] - i\left[\hat{H}^{(2)}_{eff}, \rho^{(0)}(\eta)\right] + \{\hat{A}(\eta), \rho^{(0)}(\eta)\} + \sum_N \left[\hat{L}_{N1}\rho^{(0)}(\eta)\hat{L}^\dagger_{N2} + \hat{L}_{N2}\rho^{(0)}(\eta)\hat{L}^\dagger_{N1}\right]$$

$$\hat{H}^{(2)}_{eff} = -\frac{i}{2}\sum_N (\hat{L}^\dagger_{N1}\hat{L}_{N2} - \hat{L}^\dagger_{N2}\hat{L}_{N1})$$

$$\hat{A}(\eta) = -\frac{1}{2}\sum_N \left(\hat{L}^\dagger_{N1}\hat{L}_{N2} + \hat{L}^\dagger_{N2}\hat{L}_{N1}\right)$$

(H.P. Breuer)