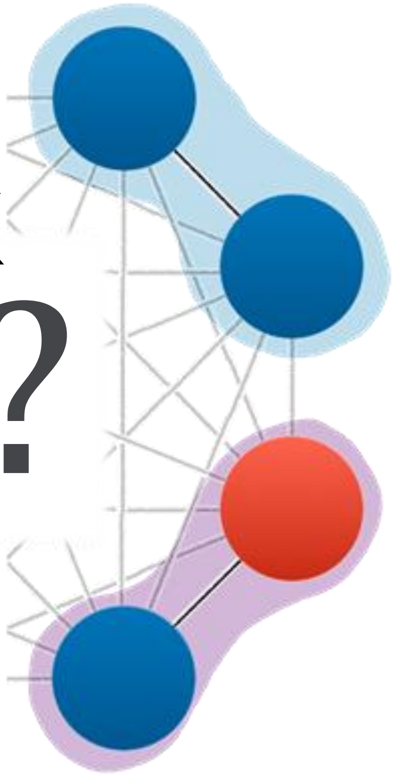
Tommy Chin Sarah Shandera

¿ Nicht-Markowsche Dynamik und Fisher Information von Qubits ?



PennState
Institute for Gravitation and the Cosmos
IGC

Partial Observations

Large Autonomous
System

Partial Observations

$\hat{\rho}_S(t_1 \to t_2)$

Large Autonomous System

Partial Observations

$\hat{\rho}_{S_1}(t_1^{S_1} \to t_2^{S_1})$

$\hat{\rho}_{S_2}(t_1^{S_2} \to t_2^{S_2})$

Large Autonomous System

Correlated Partial Observations

$\hat{\rho}_{S_1}\left(t_1^{S_1} \to t_2^{S_1}\right)$

$\hat{\rho}_{S_2}\left(t_1^{S_2} \to t_2^{S_2}\right)$

Large Autonomous System

Correlated Partial Observations

$\hat{\rho}_{S_1}\left(t_1^{S_1} \to t_2^{S_1}\right)$

$\hat{\rho}_{S_2}\left(t_1^{S_2} \to t_2^{S_2}\right)$ $\hat{\rho}_{tot}(t_0) \neq \hat{\rho}_{S_i}(t_0) \otimes \hat{\rho}_{E_i}$

Large Autonomous System

Correlated Partial Observations

$\hat{\rho}_{S_1}(t_1^{S_1} \to t_2^{S_1})$

Global Properties

$\hat{\rho}_{S_2}(t_1^{S_2} \to t_2^{S_2})$

$$\hat{\rho}_{tot}(t_0) \neq \hat{\rho}_{S_i}(t_0) \otimes \hat{\rho}_{E_i}$$

Large Autonomous System

Correlated Partial Observations

$\hat{\rho}_{S_1}(t_1^{S_1} \to t_2^{S_1})$

$$\frac{d\hat{\rho}_S}{dt} = [\hat{H}, \hat{\rho}_S] + \sum_i \gamma_i \left(\hat{L}_i \, \hat{\rho}_S \, \hat{L}_i^\dagger + \frac{1}{2}\{\hat{L}_i^\dagger \hat{L}_i, \hat{\rho}_S\} \right)$$

Lindblad Equation

? Global Properties

$\hat{\rho}_{S_2}(t_1^{S_2} \to t_2^{S_2})$

$$\hat{\rho}_{tot}(t_0) \neq \hat{\rho}_{S_i}(t_0) \otimes \hat{\rho}_{E_i}$$

Large Autonomous System

Correlated Partial Observations

$\hat{\rho}_{S_1}\left(t_1^{S_1} \to t_2^{S_1}\right)$

?

Global Properties

$\hat{\rho}_{S_2}\left(t_1^{S_2} \to t_2^{S_2}\right)$

Large Autonomous System

$$\frac{d\hat{\rho}_S}{dt} = \left[\hat{H}, \hat{\rho}_S\right] + \sum_i \gamma_i \left(\hat{L}_i\, \hat{\rho}_S\, \hat{L}_i^\dagger + \frac{1}{2}\left\{\hat{L}_i^\dagger \hat{L}_i, \hat{\rho}_S\right\} \right)$$

Lindblad Equation assumes

$$\hat{\rho}_{tot}(0) = \hat{\rho}_S(0) \otimes \hat{\rho}_E$$

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Correlated Partial Observations

$\hat{\rho}_{S_1}\left(t_1^{S_1} \to t_2^{S_1}\right)$

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Correlated Partial Observations

$\hat{\rho}_{S_1}\left(t_1^{S_1} \rightarrow t_2^{S_1}\right)$

? Global Properties

$$\cancel{\frac{d\hat{\rho}_S}{dt} = \left[\hat{H}, \hat{\rho}_S\right] + \sum_i \gamma_i \left(\hat{L}_i \hat{\rho}_S \hat{L}_i^{\dagger} + \frac{1}{2}\left\{\hat{L}_i^{\dagger} \hat{L}_i, \hat{\rho}_S\right\}\right)}$$

Lindblad Equation assumes

$$\hat{\rho}_{tot}(0) = \hat{\rho}_S(0) \otimes \hat{\rho}_E$$

$\hat{\rho}_{S_2}\left(t_1^{S_2} \rightarrow t_2^{S_2}\right)$

$$\hat{\rho}_{tot}(t_0) \neq \hat{\rho}_{S_i}(t_0) \otimes \hat{\rho}_{E_i}$$

Large Autonomous System

Correlated Partial Observations

$$\xcancel{\frac{d\hat{\rho}_S}{dt} = [\hat{H}, \hat{\rho}_S] + \sum_i \gamma_i \left(\hat{L}_i \hat{\rho}_S \hat{L}_i^\dagger + \frac{1}{2}\{\hat{L}_i^\dagger \hat{L}_i, \hat{\rho}_S\} \right)}$$

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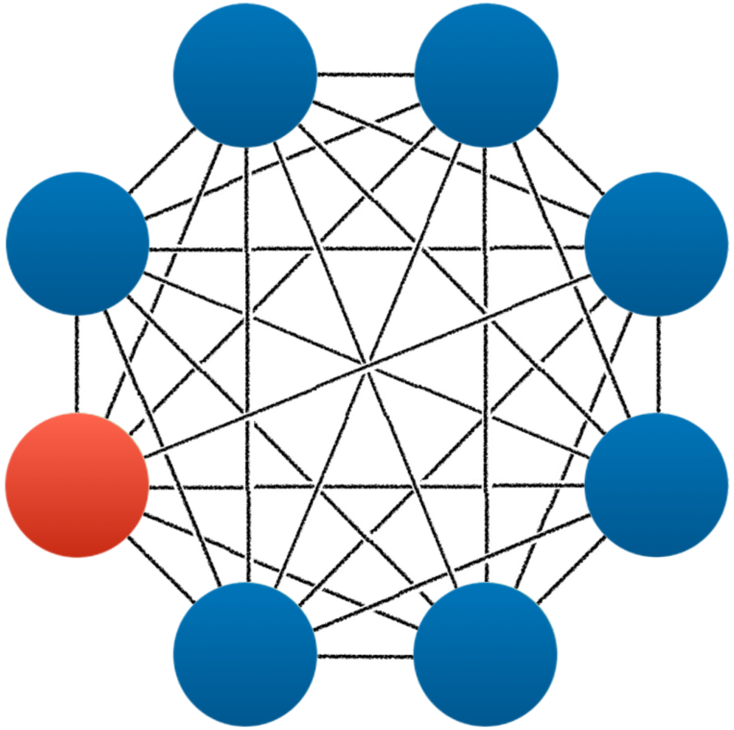
$$\hat{\rho}_{tot}(0) = \hat{\rho}_S(0) \otimes \hat{\rho}_E$$



N Qubits $\quad \hat{\rho}_{tot}(t_0) \neq \hat{\rho}_{S_i}(t_0) \otimes \hat{\rho}_{E_i}$

Large Autonomous System

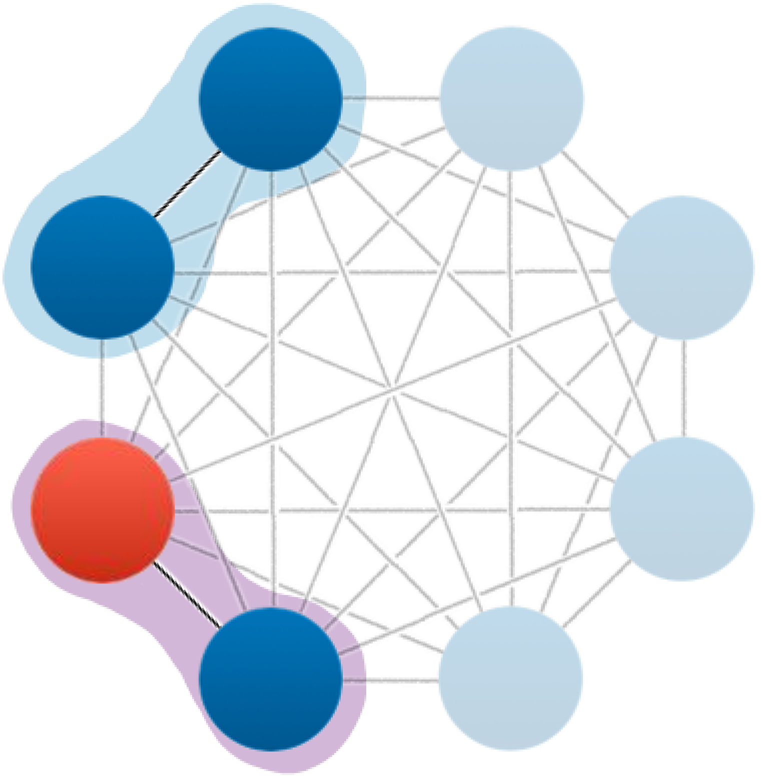
Correlated Partial Observations

$$\xcancel{\frac{d\hat{\rho}_S}{dt} = \left[\hat{H}, \hat{\rho}_S\right] + \sum_i \gamma_i \left(\hat{L}_i \hat{\rho}_S \hat{L}_i^\dagger + \frac{1}{2}\{\hat{L}_i^\dagger \hat{L}_i, \hat{\rho}_S\} \right)}$$

Lindblad Equation assumes

$$\hat{\rho}_{tot}(0) = \hat{\rho}_S(0) \otimes \hat{\rho}_E$$

$$\hat{\rho}_S(t_2) = \Phi_S(t_1, t_2)[\hat{\rho}_S(t_1)]$$



K Qubits

Partial Systems

$$\hat{\rho}_{tot}(t_0) \neq \hat{\rho}_{S_i}(t_0) \otimes \hat{\rho}_{E_i}$$

Correlated Partial Observations

$$\xcancel{\frac{d\hat{\rho}_S}{dt} = \left[\hat{H}, \hat{\rho}_S\right] + \sum_i \gamma_i \left(\hat{L}_i \hat{\rho}_S \hat{L}_i^\dagger + \frac{1}{2}\{\hat{L}_i^\dagger \hat{L}_i, \hat{\rho}_S\}\right)}$$

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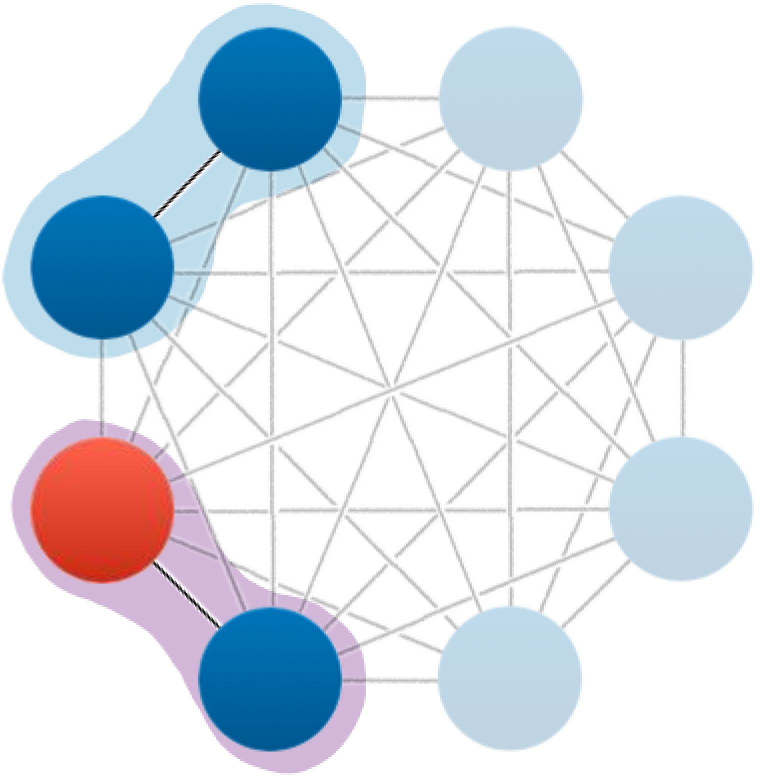


$$\hat{\rho}_S(t_2) = \Phi_S(t_1, t_2)[\hat{\rho}_S(t_1)]$$

$$\left\{\Phi_{S_i}\left(t_1^{S_i}, t_2^{S_i}\right)\right\}_i$$

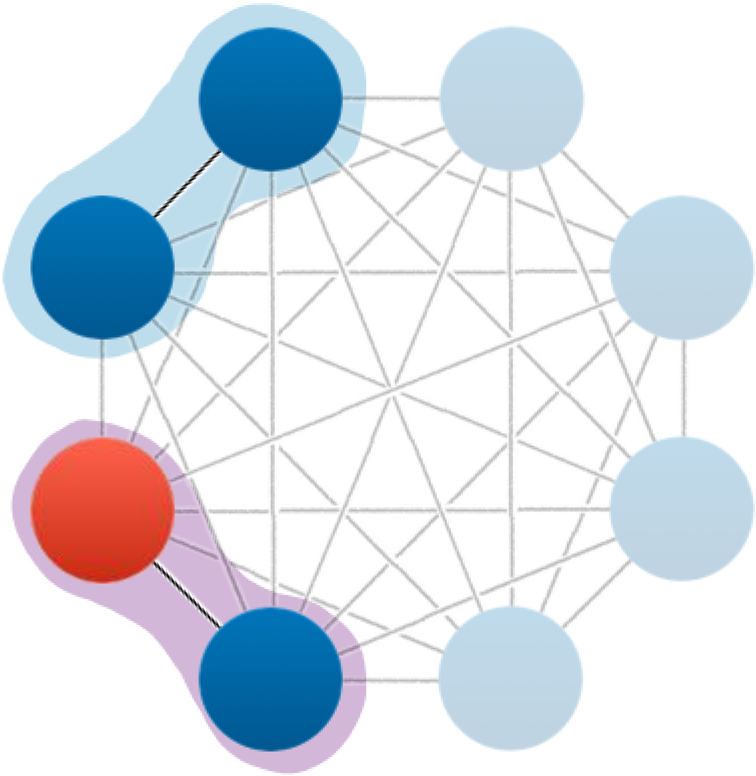
K Qubits

Partial Systems

$$\hat{\rho}_{tot}(t_0) \neq \hat{\rho}_{S_i}(t_0) \otimes \hat{\rho}_{E_i}$$

Ensemble of Correlated Open Dynamics

Correlated Partial Observations



K Qubits

Partial Systems

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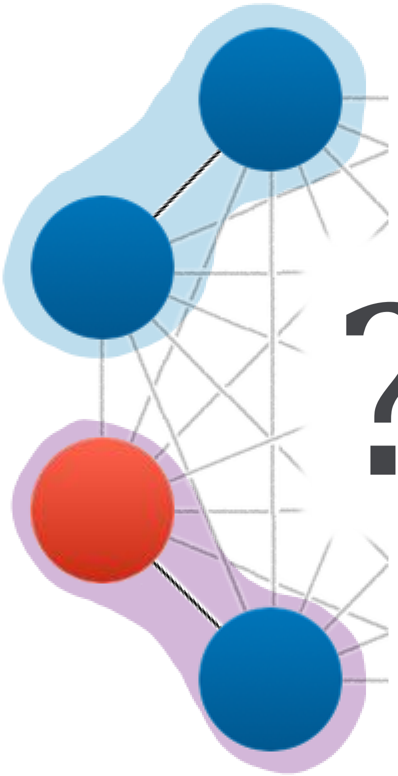
Ensemble of Correlated Open Dynamics

Correlated Partial Observations



? Global Properties

N Qubits

$$\boxed{\hat{\rho}_{tot}(t_0) \neq \hat{\rho}_{S_i}(t_0) \otimes \hat{\rho}_{E_i}}$$

Large Autonomous System

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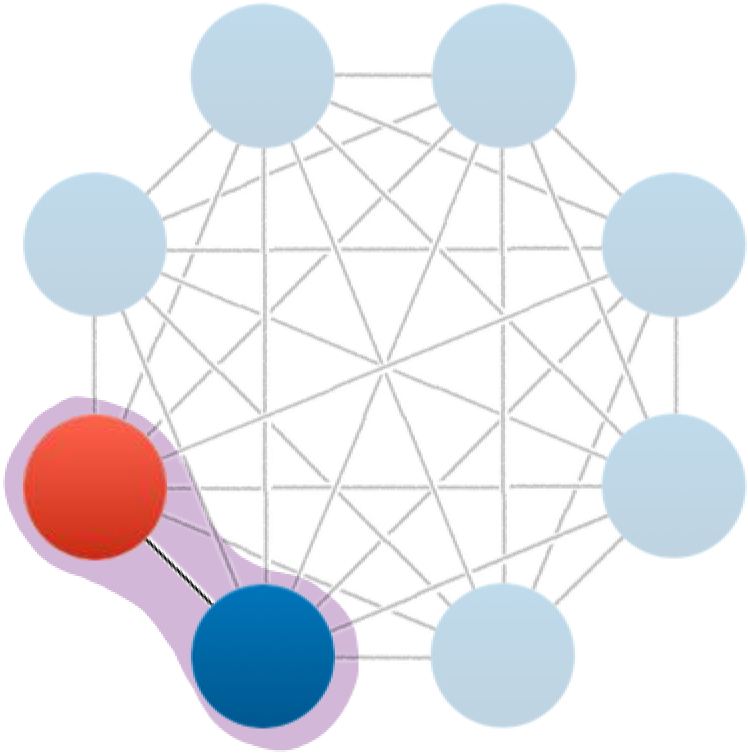
Ensemble of Correlated Open Dynamics

Non-Markovian Dynamics



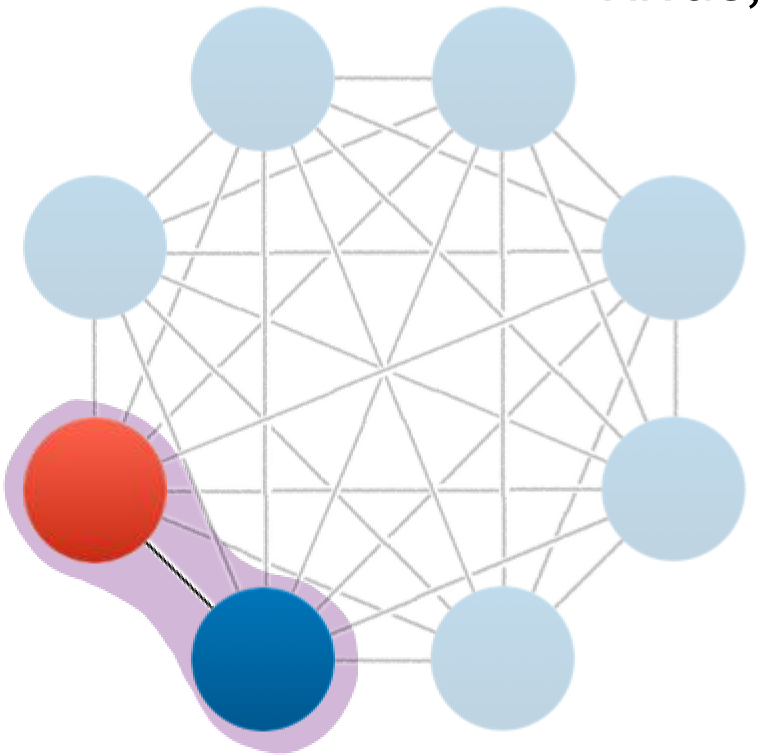
K Qubits

Partial Systems

Non-Markovian Dynamics

Non-CP-Divisibility

Rivas, Huelga, Plenio, PRL **105**, 050403 (2010)



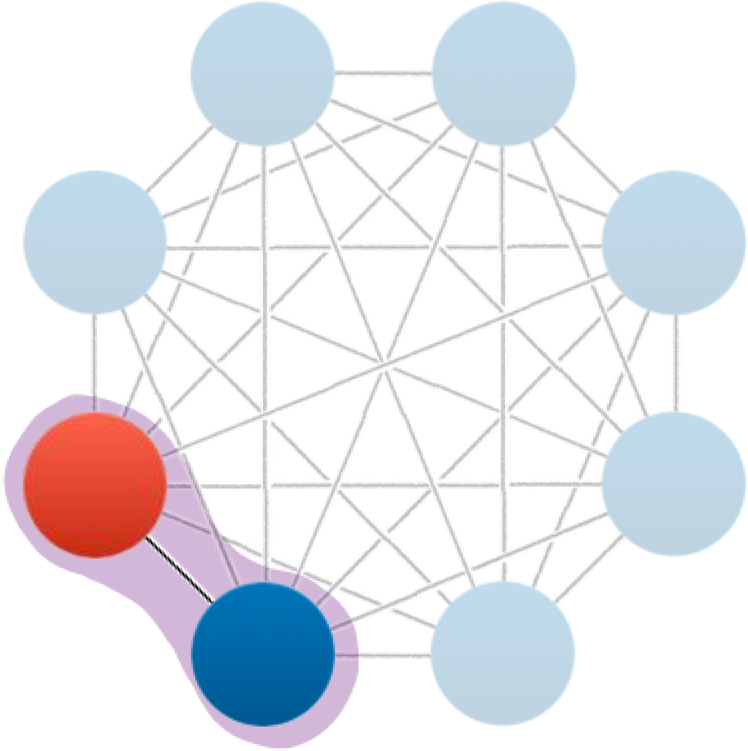
K Qubits

Partial Systems

Non-Markovian Dynamics

Non-CP-Divisibility $\Longrightarrow$ Non-Markovianity

Rivas, Huelga, Plenio, PRL **105**, 050403 (2010)



K Qubits

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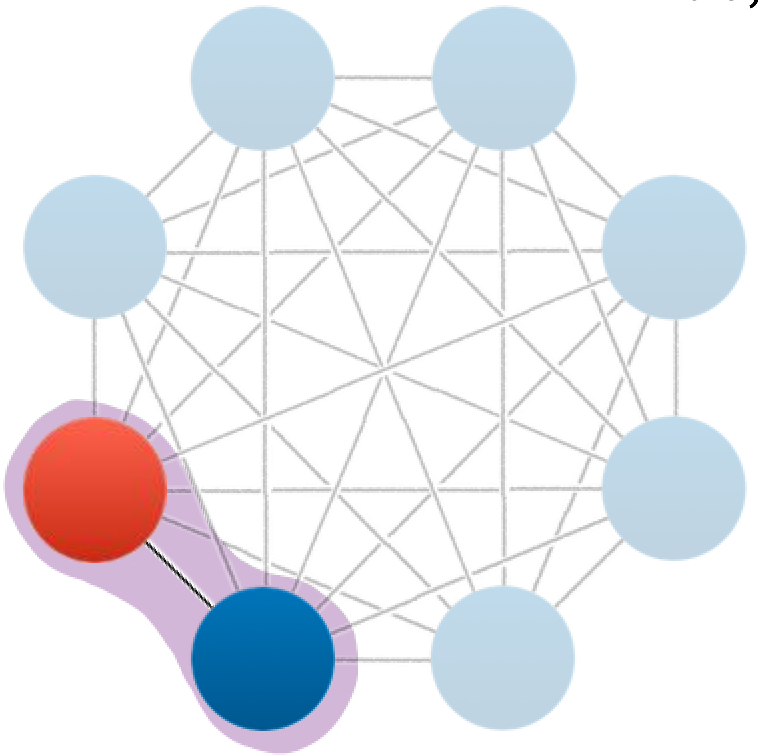


P

K Qubits

Partial Systems

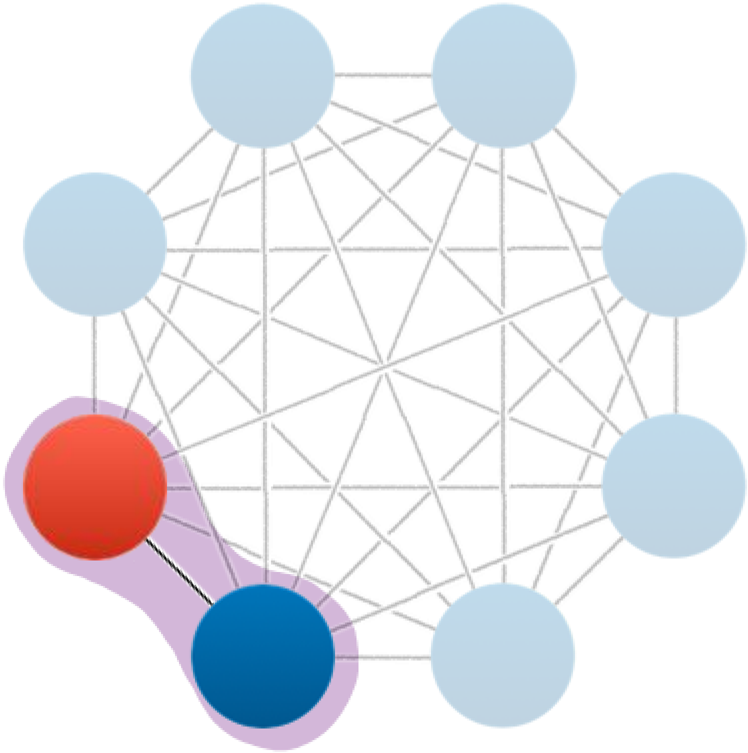
Non-Markovian Dynamics

Non-CP-Divisibility $\Longrightarrow$ Non-Markovianity

Rivas, Huelga, Plenio, PRL **105**, 050403 (2010)

$$\hat{\rho} = \sum_{\mu} p_{\mu} |\mu\rangle\langle\mu|$$

P



K Qubits

Partial Systems

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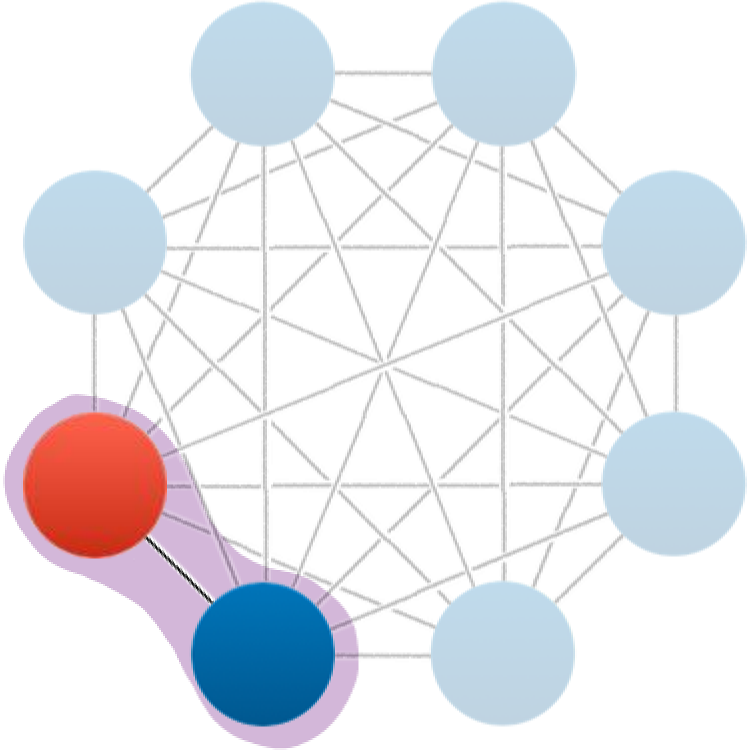

P

$|\mu\rangle\langle\mu|$

Pure States

K Qubits

Partial Systems

Non-Markovian Dynamics

Non-CP-Divisibility $\Longrightarrow$ Non-Markovianity

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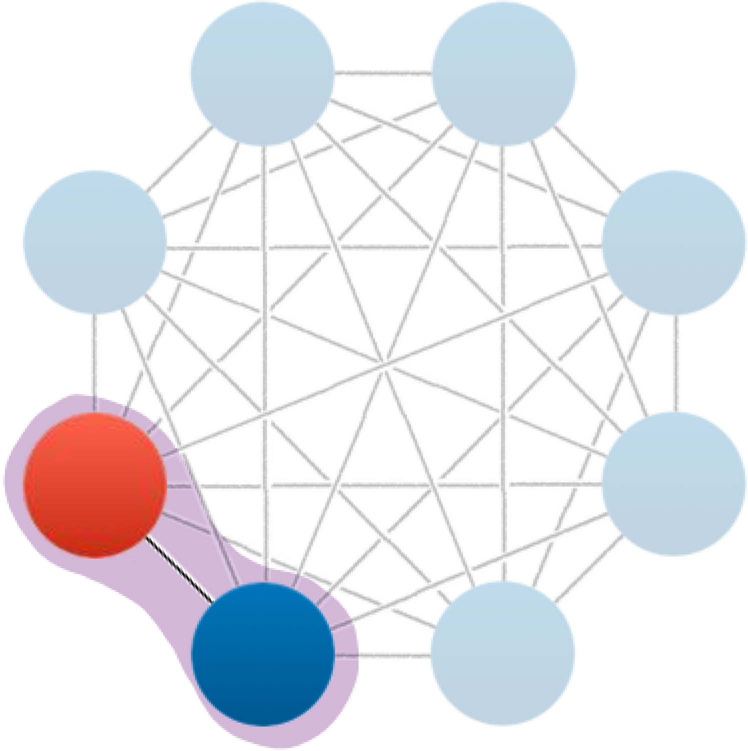

K Qubits

Partial Systems

P

$\Phi[|\mu\rangle\langle\mu|]$

Pure States

Non-Markovian Dynamics

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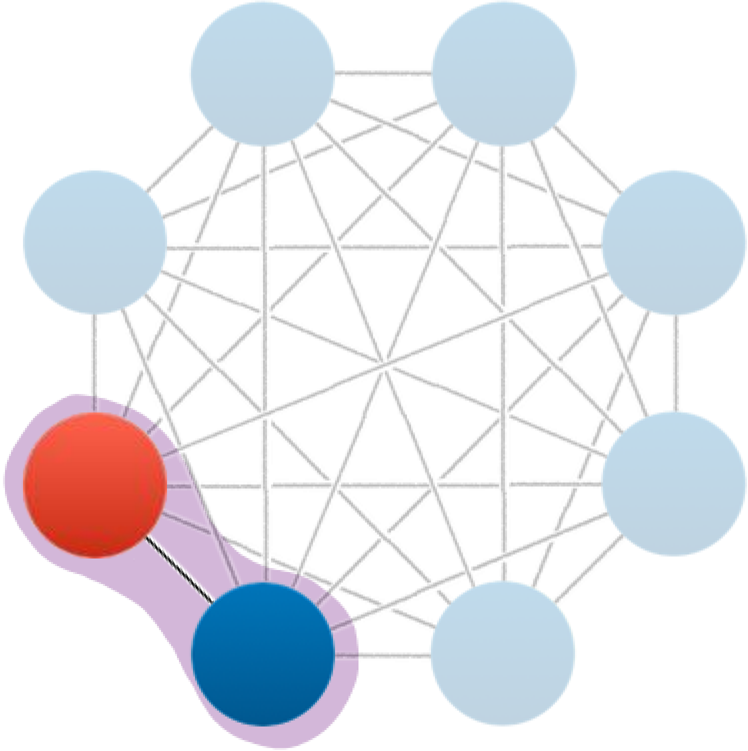

P

$$\Phi[|\mu\rangle\langle\mu|] \not\geqslant 0$$

Pure States

K Qubits

Partial Systems

Non-Markovian Dynamics

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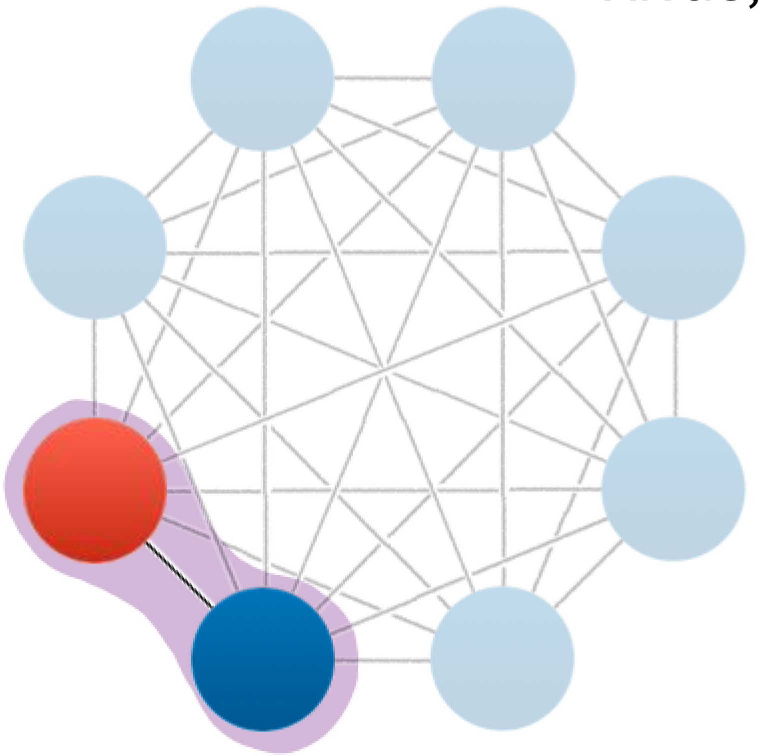

K Qubits

Partial Systems

$\Phi(t_1, t_2)$ is P $\Longleftrightarrow$ $\Phi[|\mu\rangle\langle\mu|] \succcurlyeq 0$

Pure States

Non-Markovian Dynamics

Non-CP-Divisibility $\Longrightarrow$ Non-Markovianity

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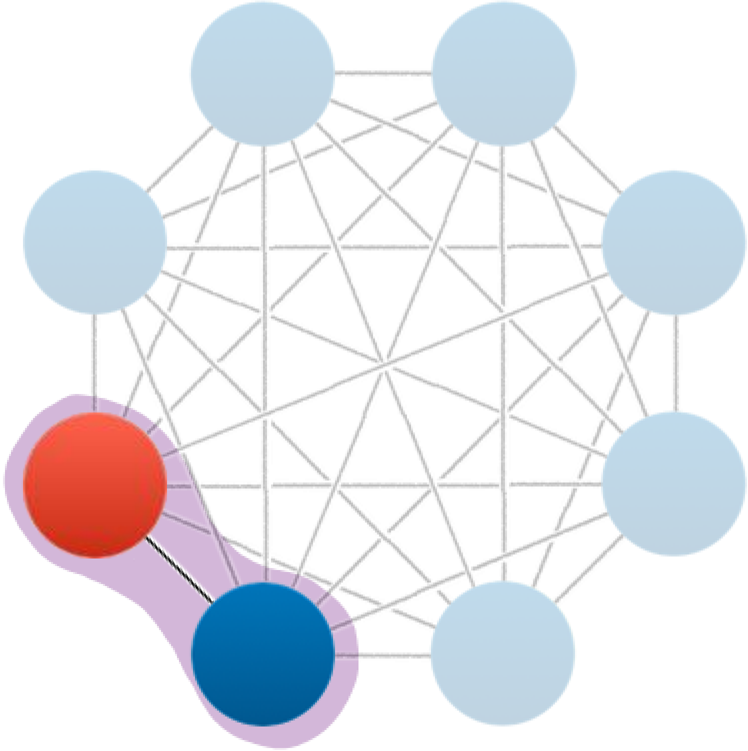

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Pure States

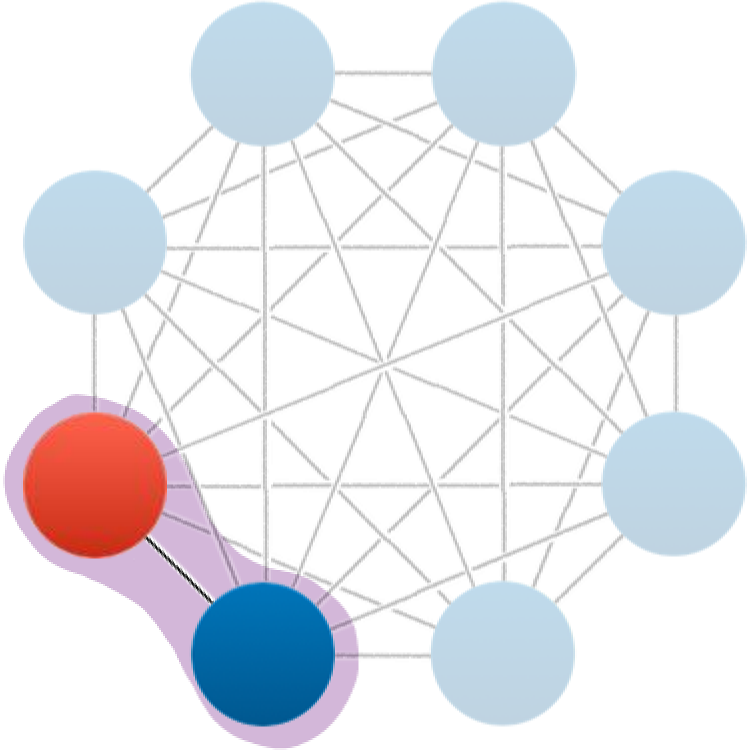
CP $\quad (\Phi \otimes \hat{\mathbb{1}})[\hat{\rho}_{SE}]$

Choi, Linear Algebra and its Applications **10**, 285 (1975)

K Qubits

Partial Systems

Non-Markovian Dynamics



K Qubits

Partial Systems

Non-CP-Divisibility $\Longrightarrow$ Non-Markovianity

Rivas, Huelga, Plenio, PRL **105**, 050403 (2010)

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$\Phi(t_1, t_2)$ is P $\iff$ $\Phi[|\mu\rangle\langle\mu|] \succcurlyeq 0$

Pure States

CP $\quad$ $\Phi[\hat{\rho}_S] \otimes \hat{\rho}_E$

Product

Choi, Linear Algebra and its Applications **10**, 285 (1975)

Non-Markovian Dynamics

Non-CP-Divisibility $\Longrightarrow$ Non-Markovianity

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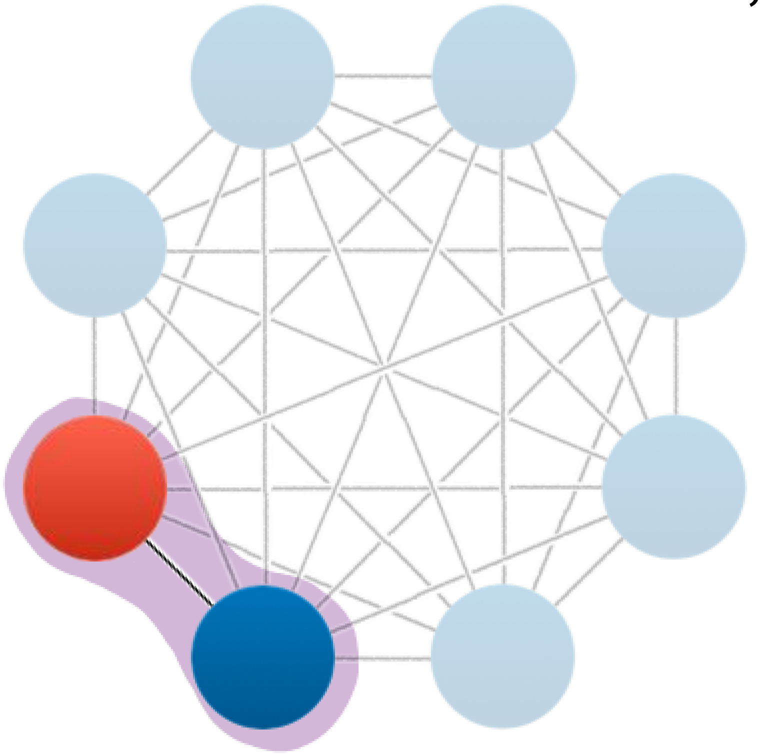

$\Phi(t_1, t_2)$ is P $\quad\Longleftrightarrow\quad$ $\Phi[|\mu\rangle\langle\mu|] \succcurlyeq 0$

Pure States

CP $\quad \sum_{\mu,\nu} |\mu\rangle\langle\nu| \otimes |\mu\rangle\langle\nu|$

Max Entangled

Choi, Linear Algebra and its Applications **10**, 285 (1975)

K Qubits

Partial Systems

Non-Markovian Dynamics

Non-CP-Divisibility $\Longrightarrow$ Non-Markovianity

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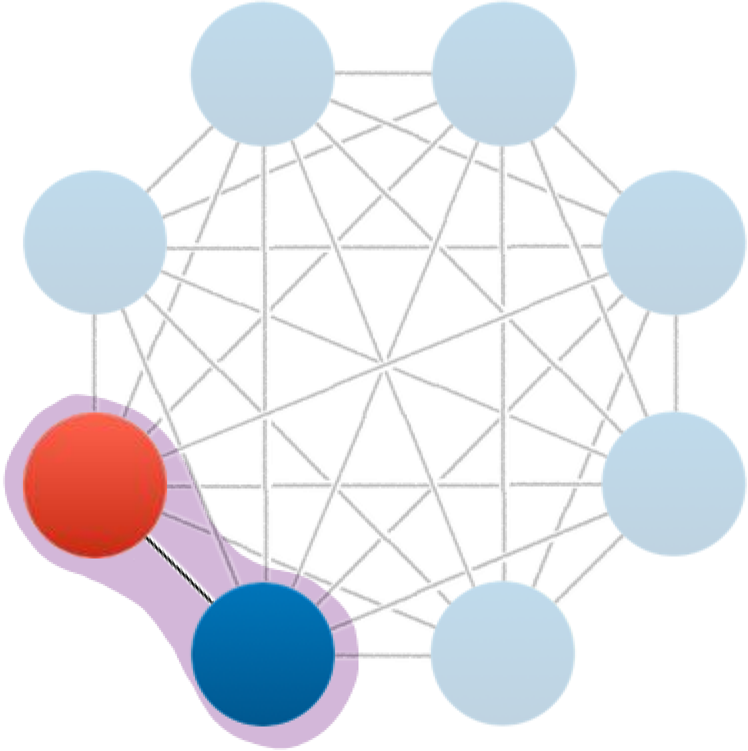

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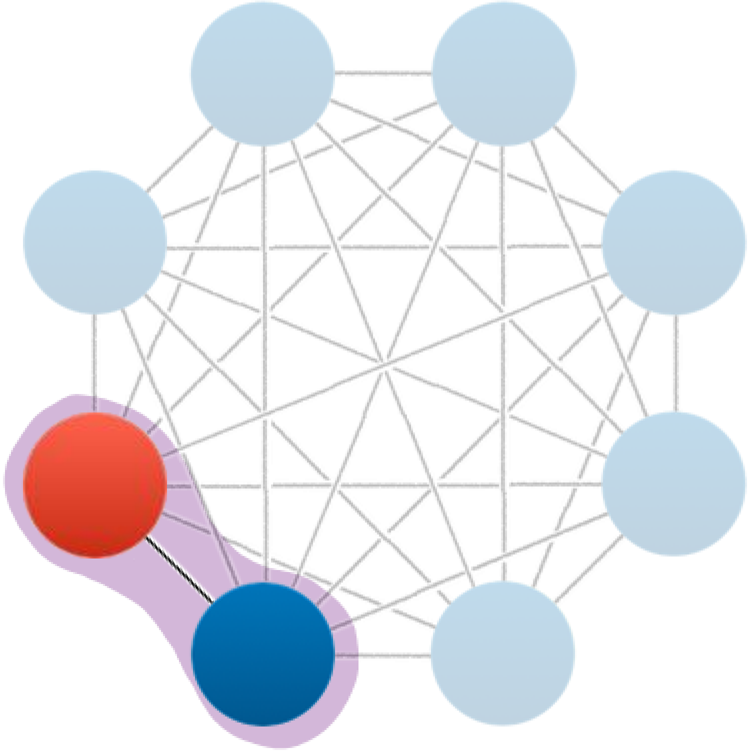


K Qubits

Partial Systems

$\Phi(t_1, t_2)$ is P $\Longleftrightarrow$ $\Phi[|\mu\rangle\langle\mu|] \geqslant 0$

Pure States

CP $\sum_{\mu,\nu} \Phi[|\mu\rangle\langle\nu|] \otimes |\mu\rangle\langle\nu| \geqslant 0$

Max Entangled

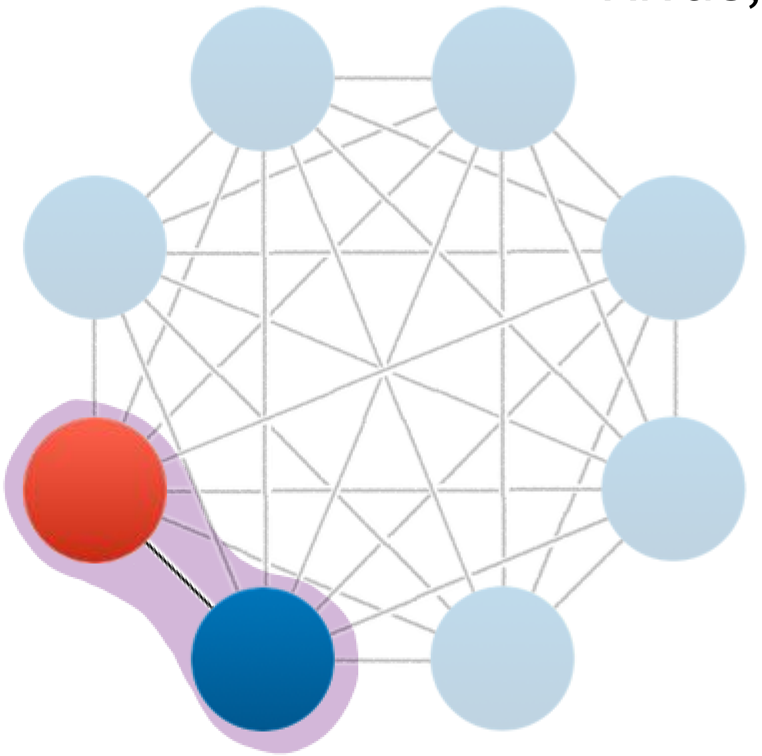
Choi, Linear Algebra and its Applications **10**, 285 (1975)

Non-Markovian Dynamics

Non-CP-Divisibility $\implies$ Non-Markovianity

Rivas, Huelga, Plenio, PRL **105**, 050403 (2010)

$$\hat{\rho} = \sum_{\mu} p_\mu |\mu\rangle\langle\mu|$$



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Pure States

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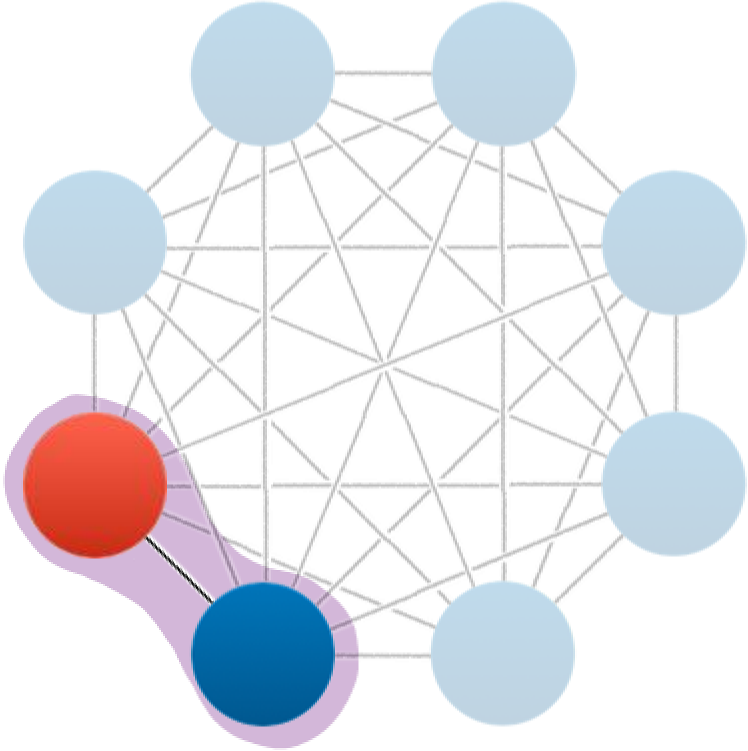

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Max Entangled

Choi, Linear Algebra and its Applications **10**, 285 (1975)

K Qubits

Partial Systems

$\Phi(t_1, t_2)$ is NCP $\forall\, t_1, t_2$

Non-Markovian Dynamics

Non-CP-Divisibility $\Longrightarrow$ Non-Markovianity

Rivas, Huelga, Plenio, PRL **105**, 050403 (2010)

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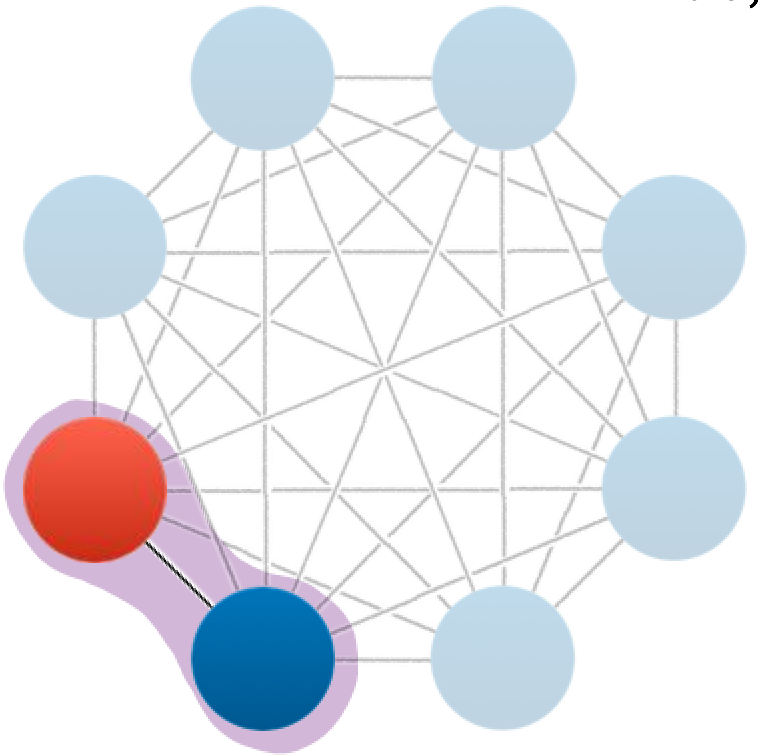

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Max Entangled

Choi, Linear Algebra and its Applications **10**, 285 (1975)

K Qubits

Partial Systems

$\Phi(t_1, t_2)$ is NCP $\forall\, t_1, t_2$ $\Longleftrightarrow$ Non-CP-Divisibility

Non-Markovian Dynamics

Non-CP-Divisibility $\Longrightarrow$ Non-Markovianity

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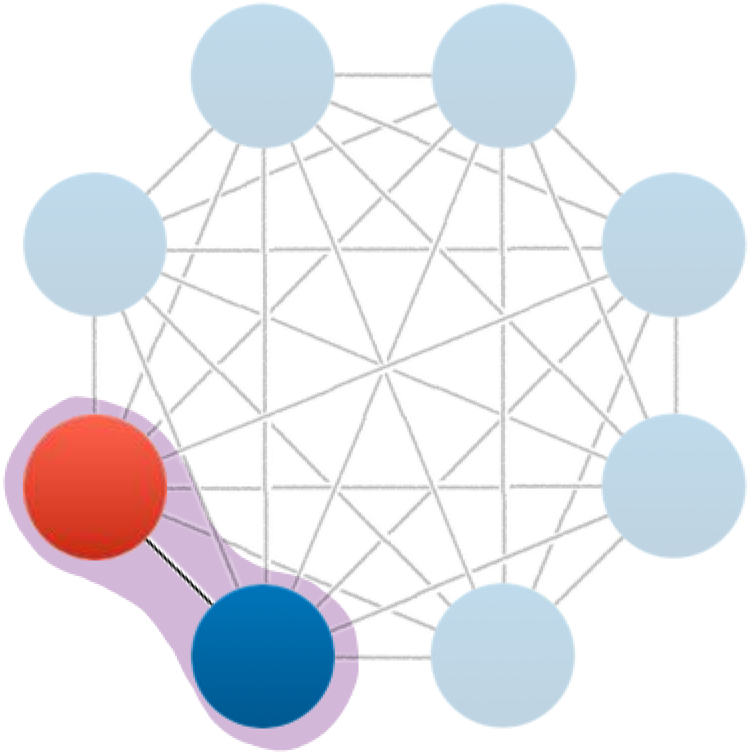

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Max Entangled

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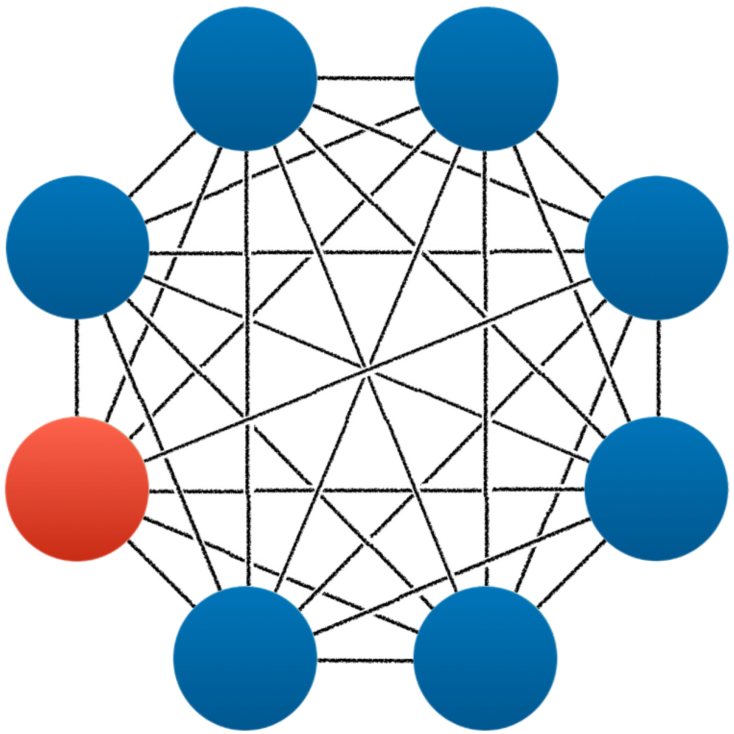
K Qubits

Partial Systems

$\Phi(t_1, t_2)$ is NCP $\forall\, t_1, t_2$ $\Longleftrightarrow$ Non-CP-Divisibility $\Longrightarrow$ Non-Markovianity

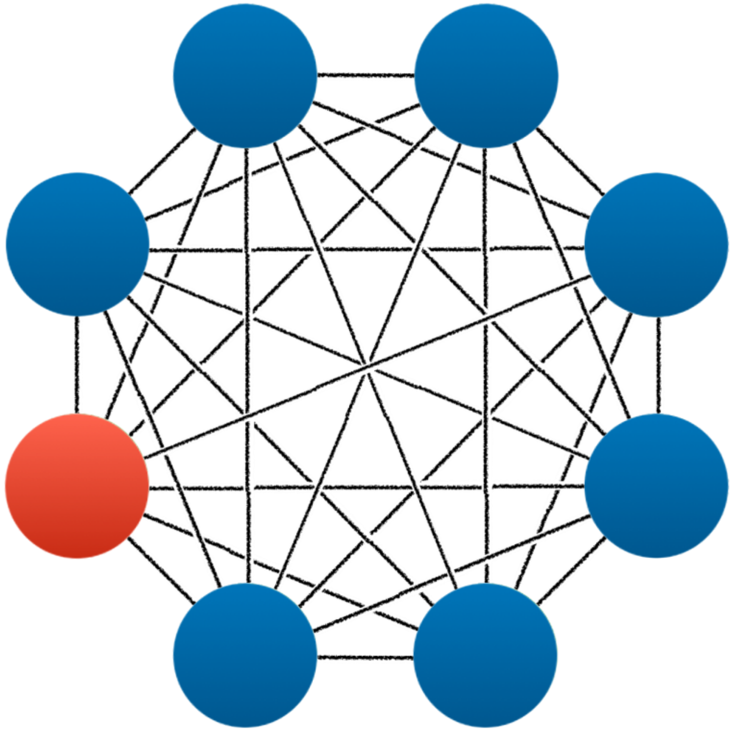
Non-Markovian Dynamics



N Qubits

Large Autonomous System

Non-Markovian Dynamics



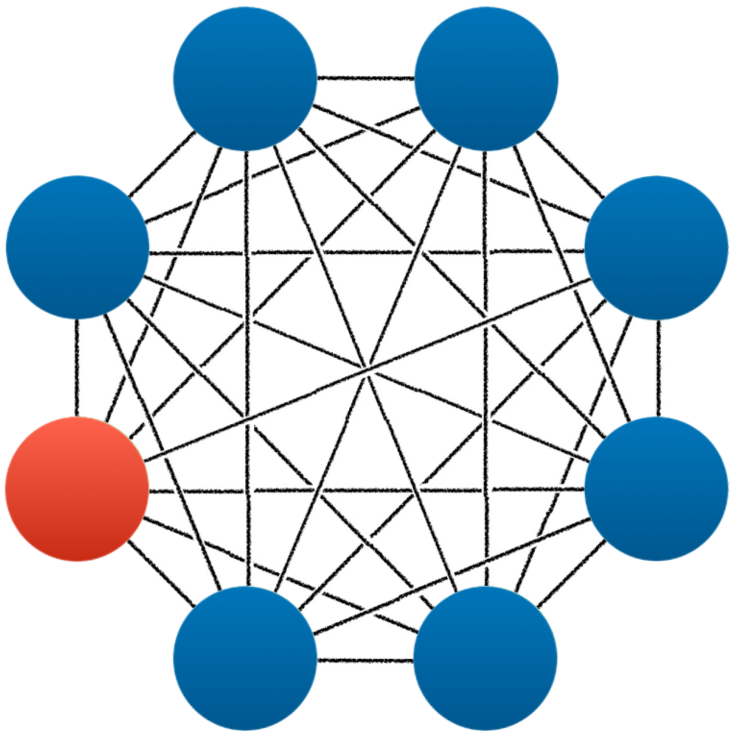
N Qubits

Large Autonomous System

$$\mathcal{U}(\hat{H})[\hat{\rho}] = e^{-i\hat{H}t}\,\hat{\rho}\,e^{i\hat{H}t}$$

Unitary

Non-Markovian Dynamics



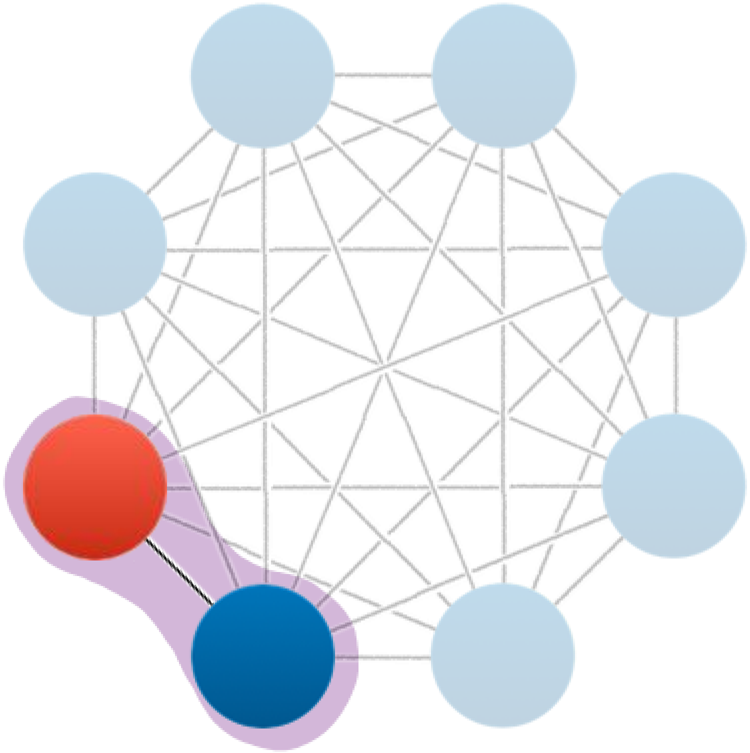
N Qubits

Large Autonomous System

$$\mathcal{U}(\hat{H})[\hat{\rho}] = e^{-i\hat{H}t}\,\hat{\rho}\,e^{i\hat{H}t}$$

Unitary

Non-Markovian Dynamics



K Qubits

Partial System

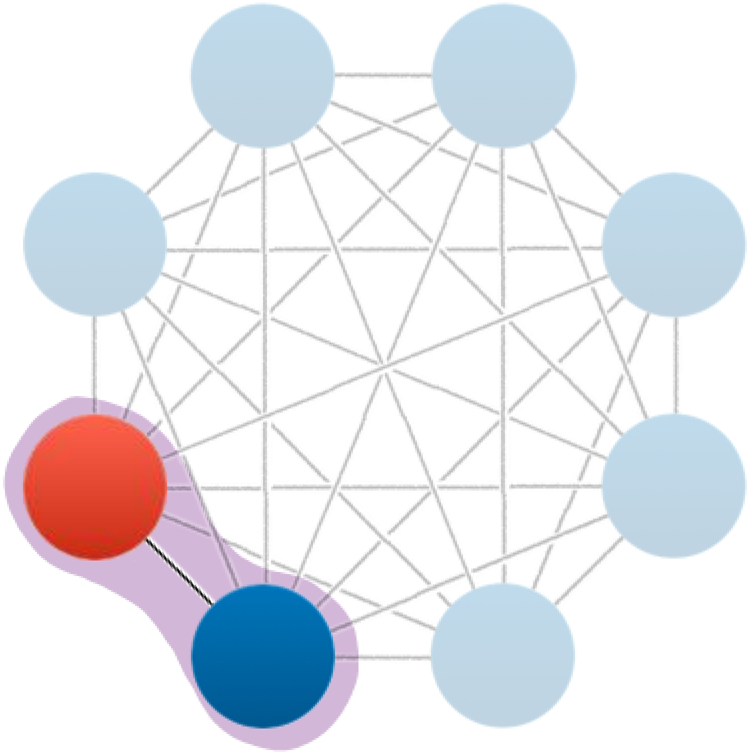
$\Phi[\hat{\rho}]$

Reduced Dynamics

$\mathcal{U}(\hat{H})[\hat{\rho}] = e^{-i\hat{H}t}\ \hat{\rho}\ e^{i\hat{H}t}$

Unitary

Non-Markovian Dynamics



K Qubits

Partial System

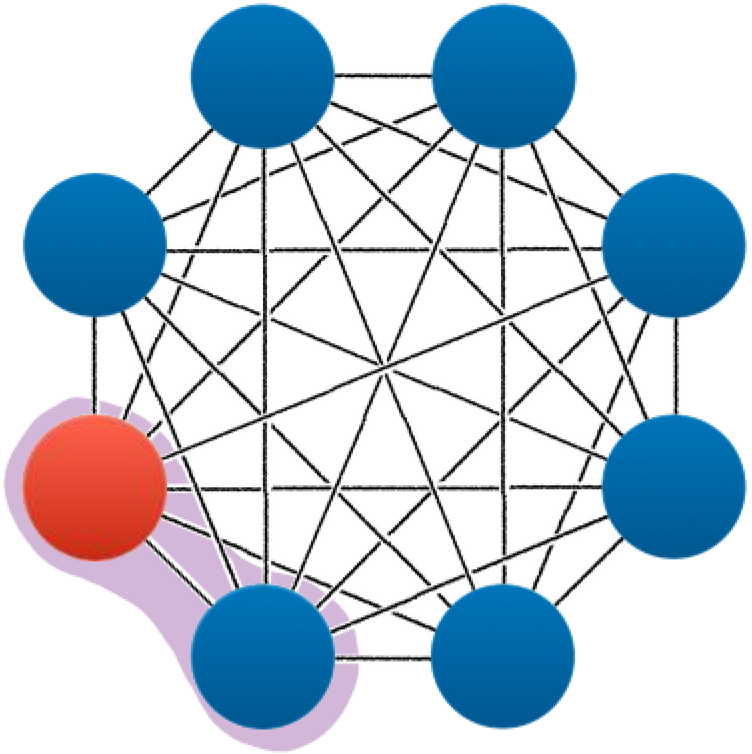
$\Phi(\hat{H} \qquad)[\hat{\rho}]$

Reduced Dynamics

$\mathcal{U}(\hat{H})[\hat{\rho}] = e^{-i\hat{H}t}\,\hat{\rho}\,e^{i\hat{H}t}$

Unitary

Non-Markovian Dynamics



K Qubits

Partial System

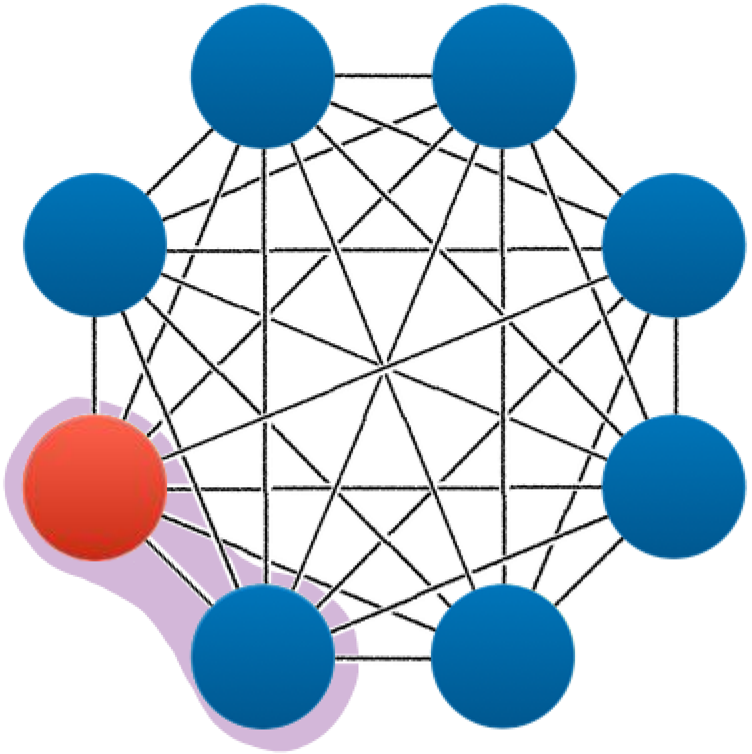
$\Phi(\hat{H}, \hat{\rho}_E \quad)[\hat{\rho}]$

Reduced Dynamics

$\mathcal{U}(\hat{H})[\hat{\rho}] = e^{-i\hat{H}t}\,\hat{\rho}\,e^{i\hat{H}t}$

Unitary

Non-Markovian Dynamics



K Qubits

Partial System

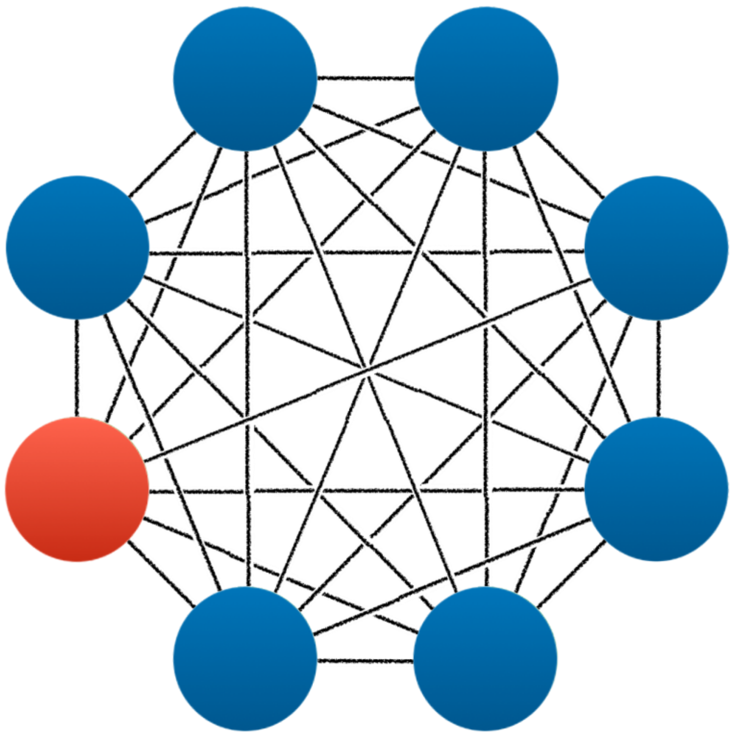
$\Phi(\hat{H}, \hat{\rho}_E, C_{SE})[\hat{\rho}]$

Reduced Dynamics

$\mathcal{U}(\hat{H})[\hat{\rho}] = e^{-i\hat{H}t}\ \hat{\rho}\ e^{i\hat{H}t}$

Unitary

Non-Markovian Dynamics



N Qubits

Large Autonomous System

$\Phi(\hat{H}, \hat{\rho}_E, C_{SE})[\hat{\rho}]$

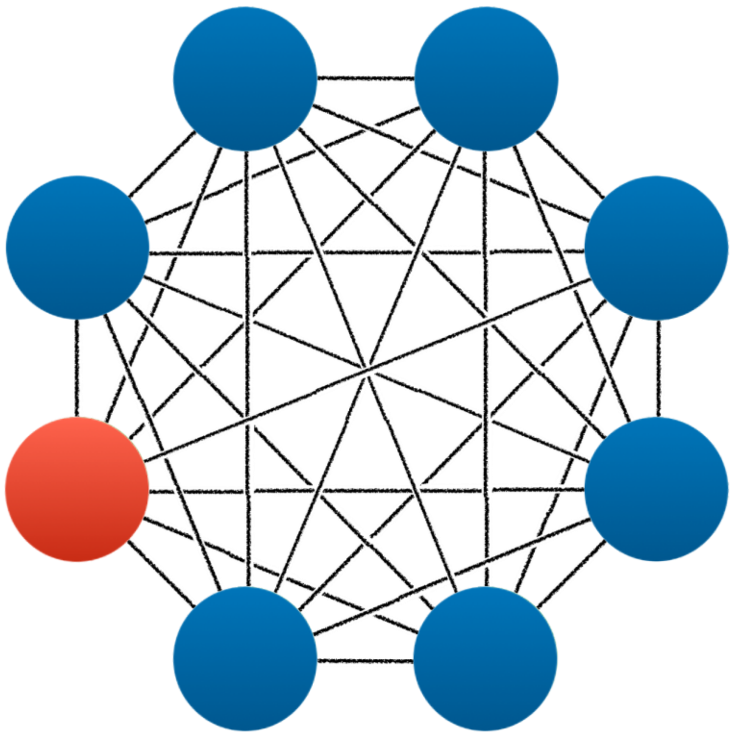
Reduced Dynamics

$\mathcal{U}(\hat{H})[\hat{\rho}] = e^{-i\hat{H}t}\ \hat{\rho}\ e^{i\hat{H}t}$

Unitary

$|\psi_{gen}\rangle\langle\psi_{gen}|$

Non-Markovian Dynamics



N Qubits

Large Autonomous System

$$\Phi\left(\hat{H}, \hat{\rho}_E, C_{SE}\right)[\hat{\rho}]$$

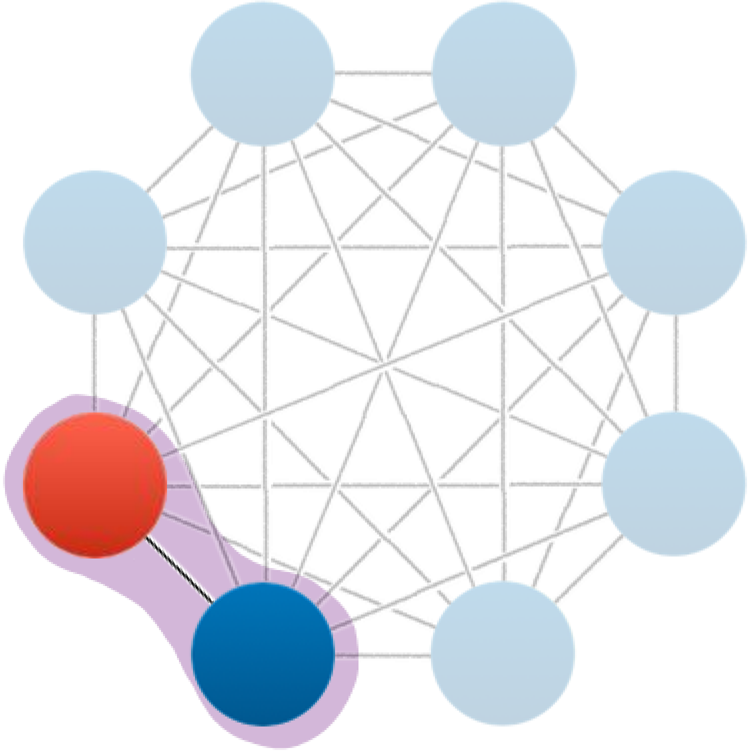
Reduced Dynamics

$$\mathcal{U}\left(\hat{H}\right)[\hat{\rho}] = e^{-i\hat{H}t}\,\hat{\rho}\,e^{i\hat{H}t}$$

Unitary

$$\hat{U}(t)\left|\psi_{gen}\right\rangle\!\left\langle\psi_{gen}\right|\hat{U}^{\dagger}(t)$$

Non-Markovian Dynamics



K Qubits

Partial System

$\Phi(\hat{H}, \hat{\rho}_E, C_{SE})[\hat{\rho}]$

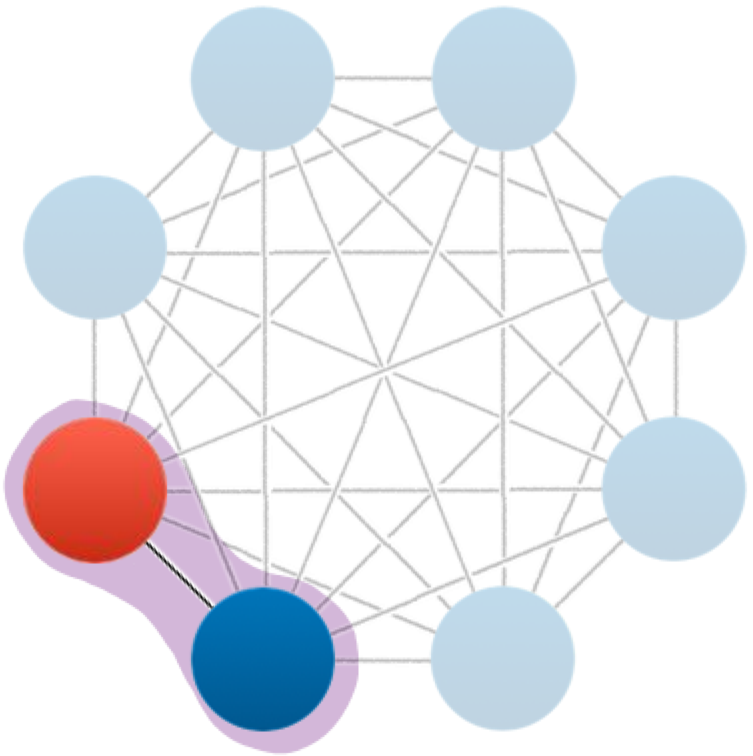
Reduced Dynamics

$\mathcal{U}(\hat{H})[\hat{\rho}] = e^{-i\hat{H}t}\, \hat{\rho}\, e^{i\hat{H}t}$

Unitary

$$\hat{\rho}_s(t) = \mathrm{Tr}_E\left[\hat{U}(t)\middle|\psi_{gen}\rangle\langle\psi_{gen}\middle|\hat{U}^\dagger(t)\right]$$

Non-Markovian Dynamics



K Qubits

Partial System

$$\Phi\left(\hat{H}, \hat{\rho}_E, C_{SE}\right)[\hat{\rho}]$$

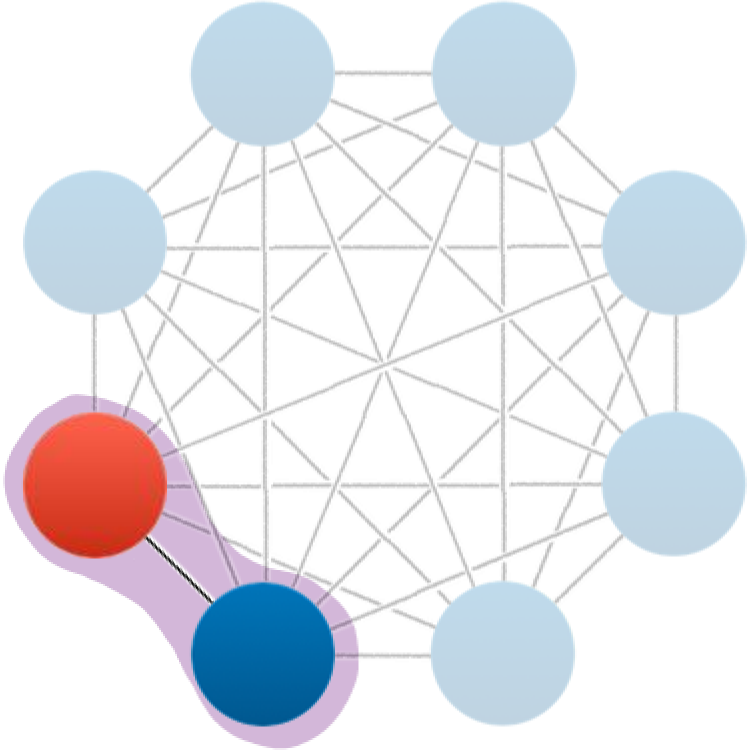
Reduced Dynamics

$$\mathcal{U}\left(\hat{H}\right)[\hat{\rho}] = e^{-i\hat{H}t}\,\hat{\rho}\,e^{i\hat{H}t}$$

Unitary

$$\begin{aligned}\hat{\rho}_s(t) &= \mathrm{Tr}_E\left[\hat{U}(t)\middle|\psi_{gen}\middle\rangle\middle\langle\psi_{gen}\middle|\hat{U}^\dagger(t)\right] \\ &= \Phi(0,t)[\hat{\rho}_s(0)]\end{aligned}$$

Non-Markovian Dynamics



K Qubits

Partial System

$\Phi(\hat{H}, \hat{\rho}_E, C_{SE})[\hat{\rho}]$

Reduced Dynamics

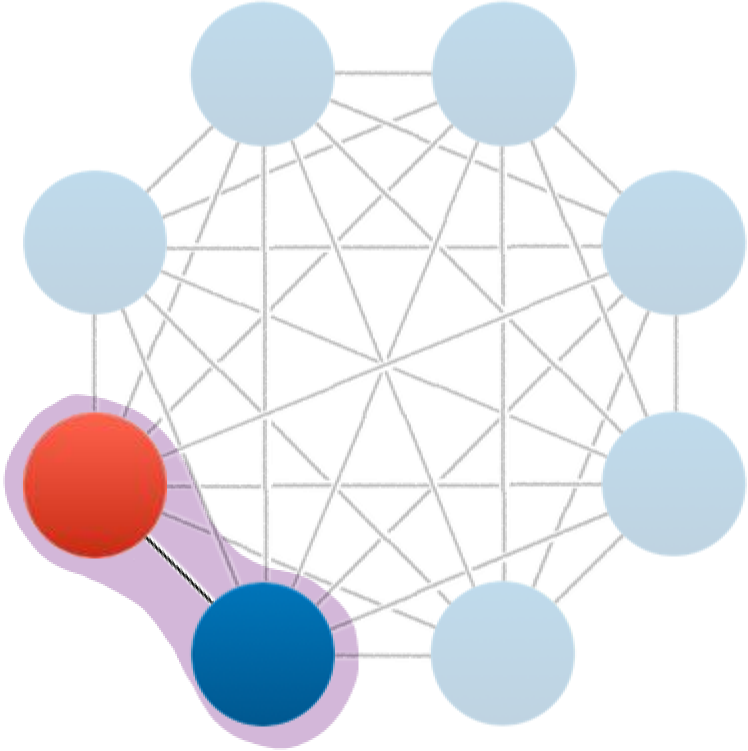
$\mathcal{U}(\hat{H})[\hat{\rho}] = e^{-i\hat{H}t}\, \hat{\rho}\, e^{i\hat{H}t}$

Unitary

$$\begin{aligned}\hat{\rho}_s(t) &= \mathrm{Tr}_E\left[\hat{U}(t)\middle|\psi_{gen}\rangle\langle\psi_{gen}\middle|\hat{U}^\dagger(t)\right] \\ &= \Phi(0,t)[\hat{\rho}_s(0)]\end{aligned}$$

$\hat{\rho}_S(0)$

Non-Markovian Dynamics



K Qubits

Partial System

$\Phi(\hat{H}, \hat{\rho}_E, C_{SE})[\hat{\rho}]$

Reduced Dynamics

$\mathcal{U}(\hat{H})[\hat{\rho}] = e^{-i\hat{H}t}\,\hat{\rho}\,e^{i\hat{H}t}$

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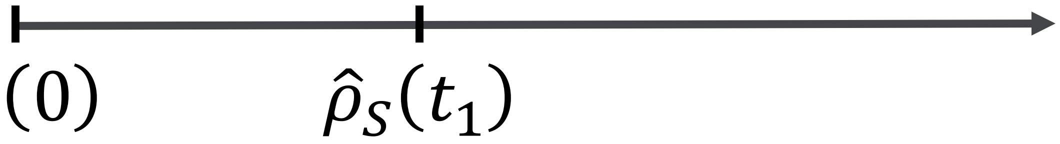
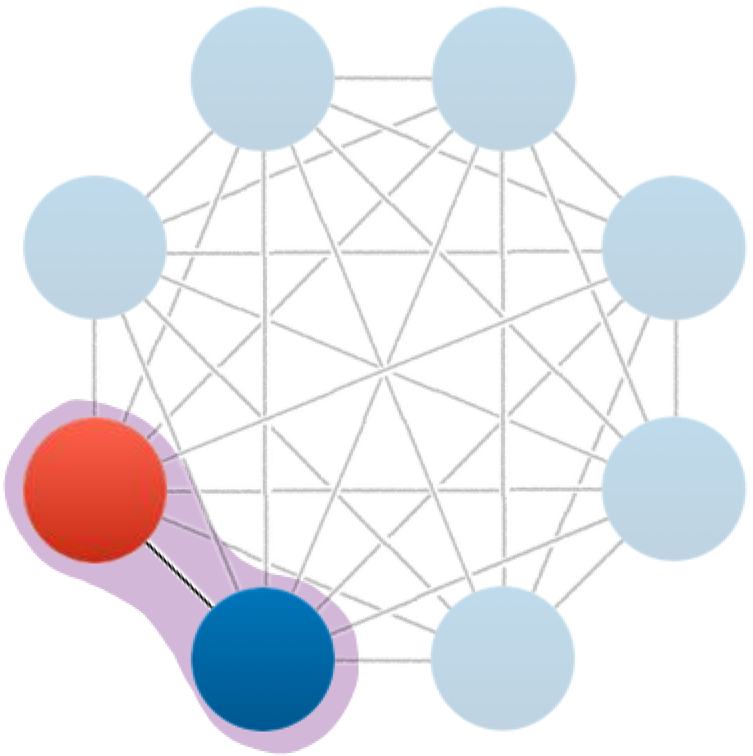


Non-Markovian Dynamics



K Qubits

Partial System

$$\Phi\left(\hat{H}, \hat{\rho}_E, C_{SE}\right)[\hat{\rho}]$$

Reduced Dynamics

$$\mathcal{U}\left(\hat{H}\right)[\hat{\rho}] = e^{-i\hat{H}t}\,\hat{\rho}\,e^{i\hat{H}t}$$

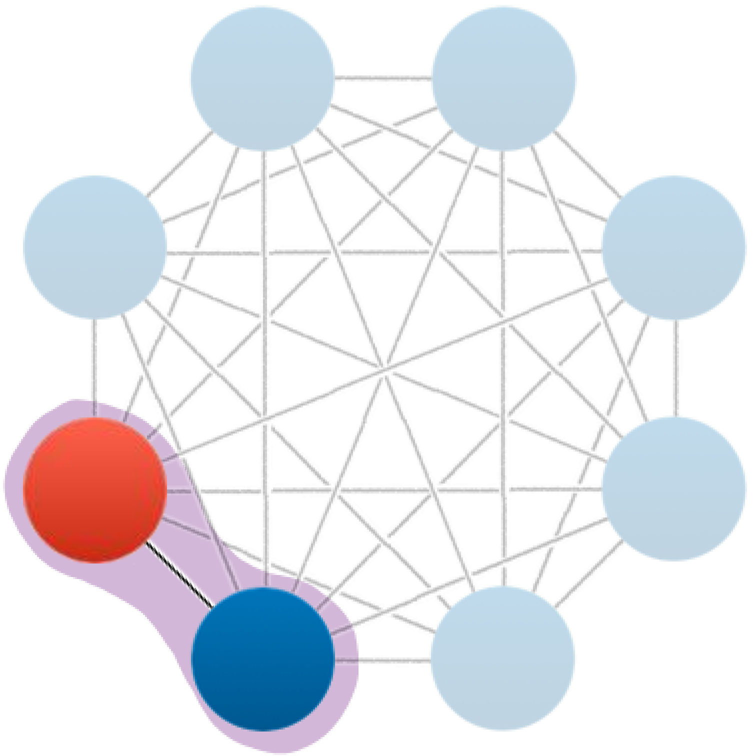
Unitary

$$\hat{\rho}_s(t) = \mathrm{Tr}_E\left[\hat{U}(t)\middle|\psi_{gen}\middle\rangle\middle\langle\psi_{gen}\middle|\hat{U}^\dagger(t)\right]$$

$$= \Phi(0,t)[\hat{\rho}_s(0)]$$

$\hat{\rho}_S(0)$ $\hat{\rho}_S(t_1)$ $\hat{\rho}_S(t_2)$

Non-Markovian Dynamics



K Qubits

Partial System

$$\Phi\left(\hat{H}, \hat{\rho}_E, C_{SE}\right)[\hat{\rho}]$$

Reduced Dynamics

$$\mathcal{U}\left(\hat{H}\right)[\hat{\rho}] = e^{-i\hat{H}t}\,\hat{\rho}\,e^{i\hat{H}t}$$

Unitary

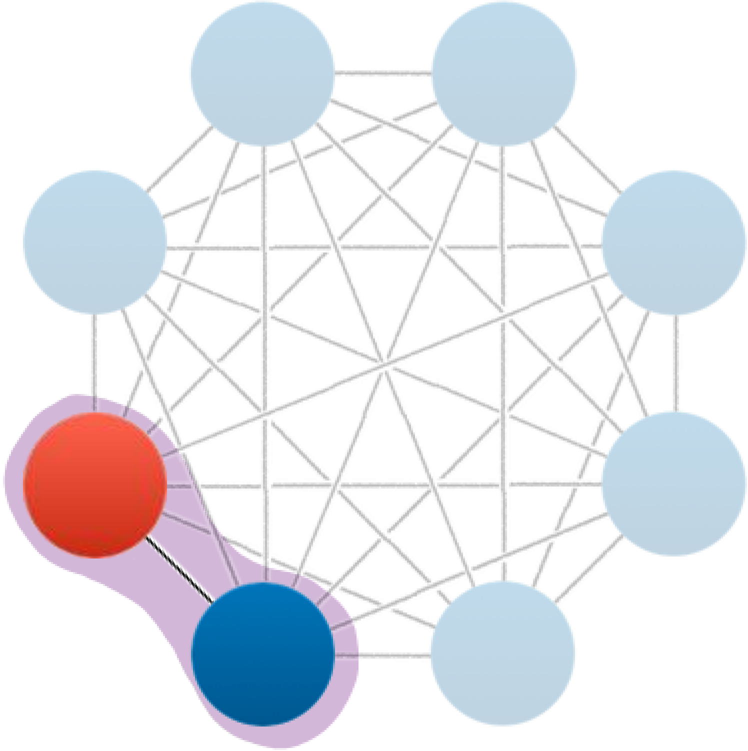
$$\begin{aligned}\hat{\rho}_s(t) &= \mathrm{Tr}_E\left[\hat{U}(t)\middle|\psi_{gen}\middle\rangle\middle\langle\psi_{gen}\middle|\hat{U}^\dagger(t)\right] \\ &= \Phi(0,t)[\hat{\rho}_s(0)]\end{aligned}$$

$\hat{\rho}_S(0)$ $\hat{\rho}_S(t_1)$ $\hat{\rho}_S(t_2)$

$$\Phi(t_1, t_2) = \Phi(0, t_2)\Phi^{-1}(0, t_1)$$

Non-Markovian Dynamics

$$\Phi(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}\,\hat{\rho}\,\hat{\phi}^\dagger + \sum_i \hat{\phi}_i\,\hat{\rho}\,\hat{\phi}_i^T$$



K Qubits

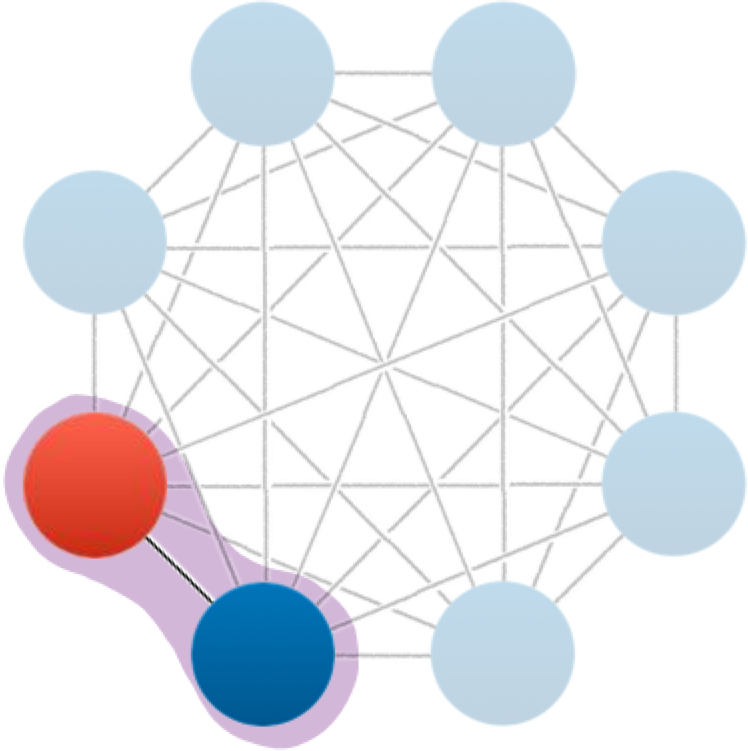
Partial System

$$\begin{aligned}\hat{\rho}_s(t) &= \mathrm{Tr}_E\left[\hat{U}(t)\left|\psi_{gen}\right\rangle\left\langle\psi_{gen}\right|\hat{U}^\dagger(t)\right] \\ &= \Phi(0,t)[\hat{\rho}_s(0)]\end{aligned}$$

$\hat{\rho}_S(0)$ $\hat{\rho}_S(t_1)$ $\hat{\rho}_S(t_2)$

$$\Phi(t_1, t_2) = \Phi(0, t_2)\Phi^{-1}(0, t_1)$$

Non-Markovian Dynamics



K Qubits

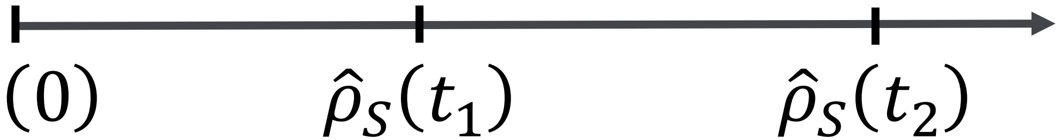
Partial System

$$\Phi(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}\,\hat{\rho}\,\hat{\phi}^\dagger + \sum_i \hat{\phi}_i\,\hat{\rho}\,\hat{\phi}_i^T$$

Trace Preserve: $\hat{\phi}^\dagger \hat{\phi} + \sum_i \hat{\phi}_i^T \hat{\phi}_i = \mathbb{1}$

$$\hat{\rho}_s(t) = \mathrm{Tr}_E\left[\hat{U}(t)\middle|\psi_{gen}\right\rangle\left\langle\psi_{gen}\middle|\hat{U}^\dagger(t)\right]$$

$$= \Phi(0, t)[\hat{\rho}_s(0)]$$



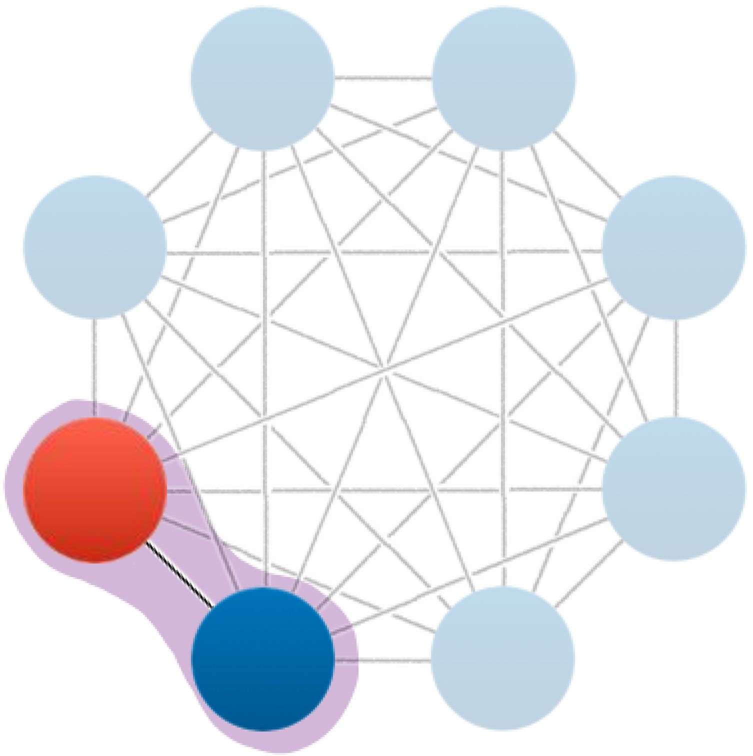
$$\Phi(t_1, t_2) = \Phi(0, t_2)\Phi^{-1}(0, t_1)$$

Non-Markovian Dynamics

No Assumptions

$$\Phi(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}\,\hat{\rho}\,\hat{\phi}^\dagger + \sum_i \hat{\phi}_i\,\hat{\rho}\,\hat{\phi}_i^T$$

Trace Preserve: $\hat{\phi}^\dagger\hat{\phi} + \sum_i \hat{\phi}_i^T\hat{\phi}_i = \hat{\mathbb{1}}$



K Qubits

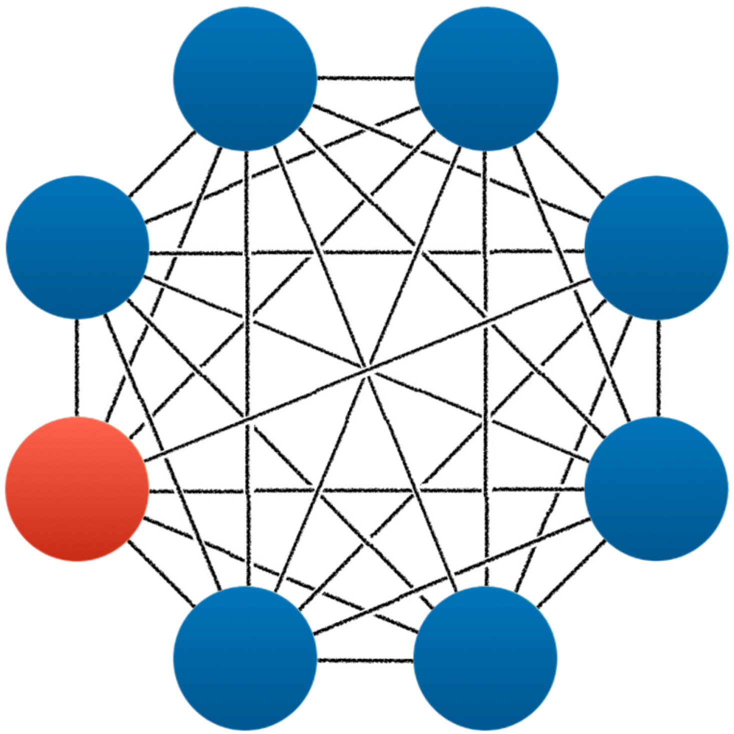
Partial System

$$\hat{\rho}_s(t) = \mathrm{Tr}_E\left[\hat{U}(t)\middle|\psi_{gen}\rangle\langle\psi_{gen}\middle|\hat{U}^\dagger(t)\right]$$
$$= \Phi(0,t)[\hat{\rho}_s(0)]$$

$\hat{\rho}_S(0)$ $\quad\quad$ $\hat{\rho}_S(t_1)$ $\quad\quad$ $\hat{\rho}_S(t_2)$

$$\Phi(t_1, t_2) = \Phi(0, t_2)\Phi^{-1}(0, t_1)$$

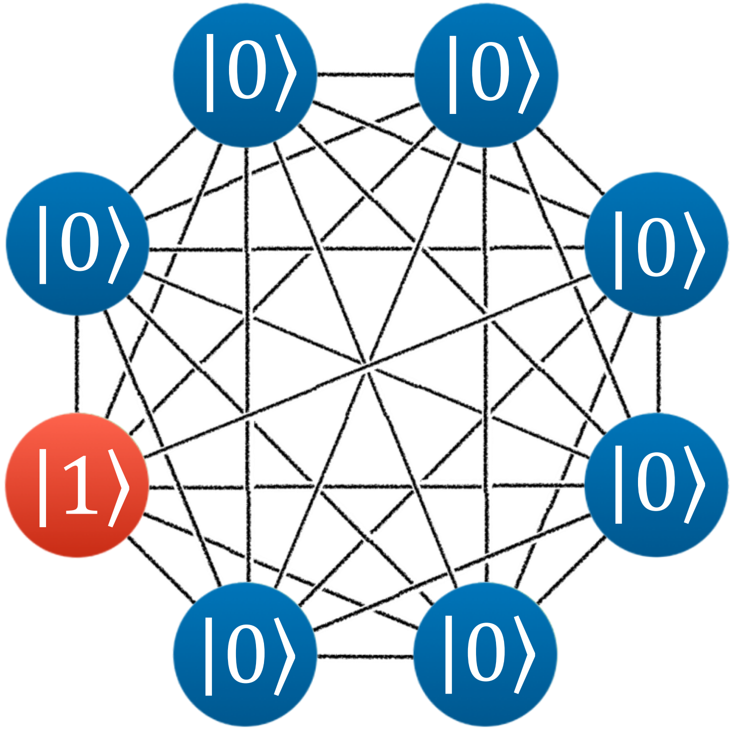
Non-Markovian Dynamics



N Qubits

Large Autonomous System

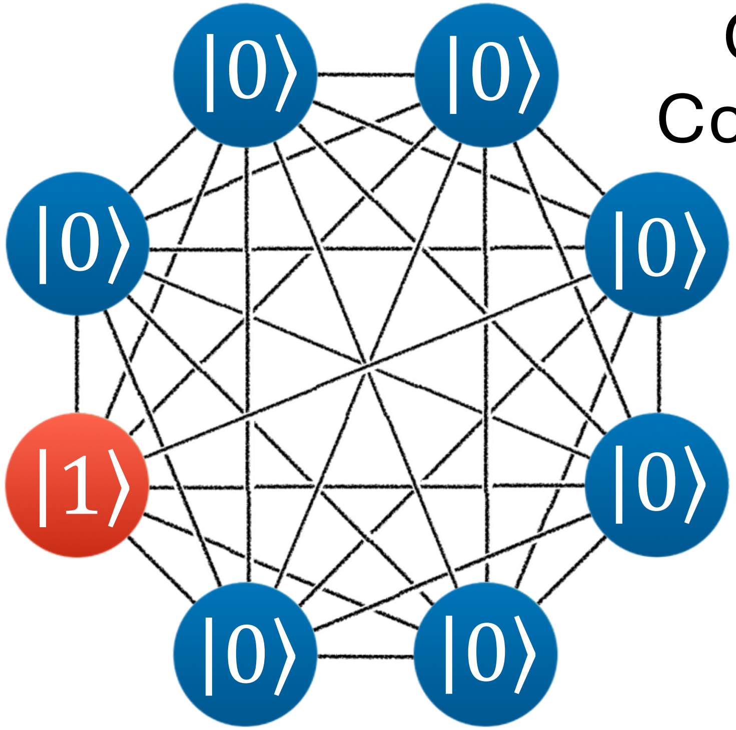
Non-Markovian Dynamics



$|10\cdots 0\rangle$

Large Autonomous System

Non-Markovian Dynamics



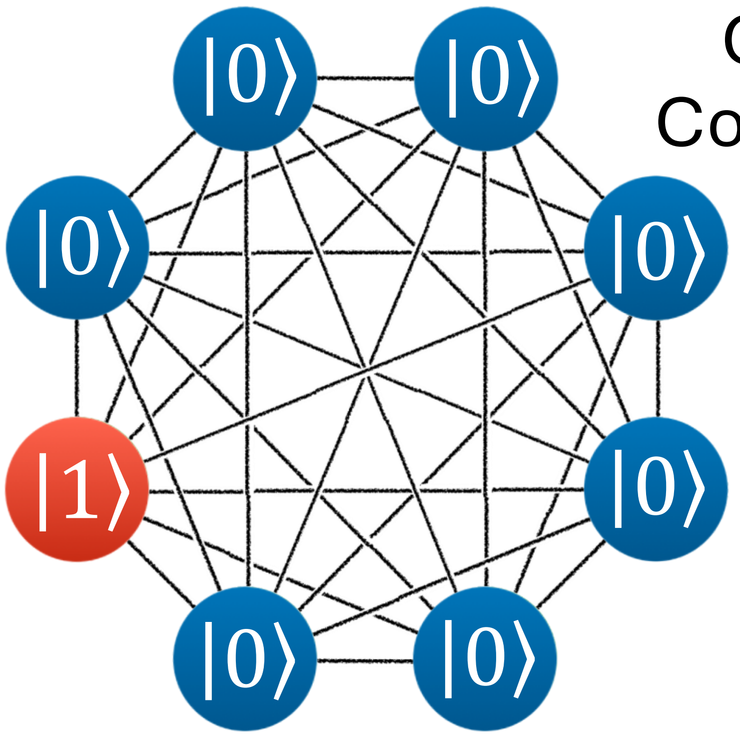
$|10\cdots0\rangle$

Large Autonomous System

Charge Conserving

$$\hat{U} = \begin{pmatrix} u_0 & & & & & & & \\ & u_s & u_d & u_d & \cdots & u_d & & \\ & u_d & u_s & u_d & \cdots & u_d & & \\ & u_d & u_d & u_s & \cdots & u_d & & \\ & \vdots & \vdots & \vdots & \ddots & \vdots & & \\ & u_d & u_d & u_d & \cdots & u_s & & \\ & & & & & & \ddots & \\ & & & & & & & \ddots \end{pmatrix}$$

Non-Markovian Dynamics



$|10\cdots 0\rangle$

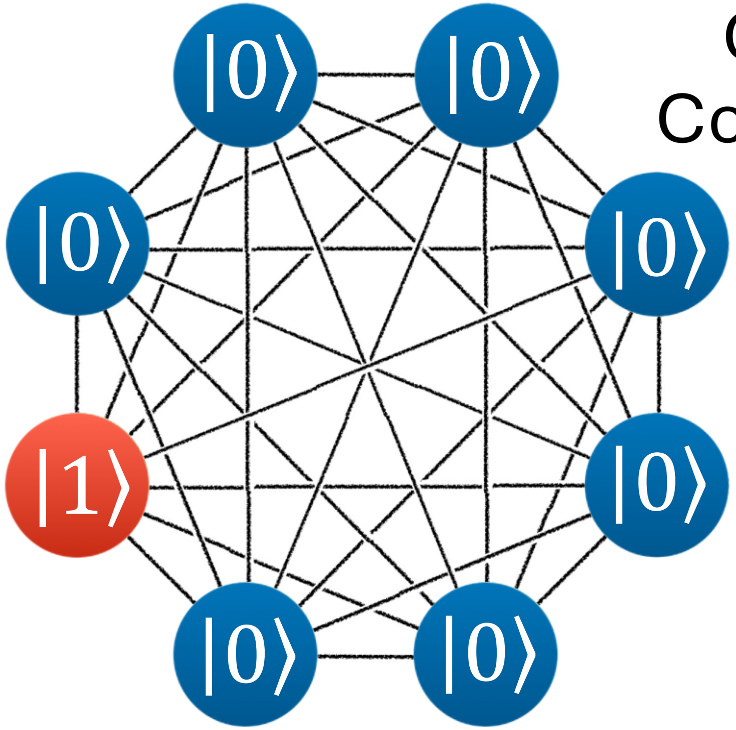
Large Autonomous System

Charge Conserving

$$\hat{U} = \begin{pmatrix} u_0 & & & \\ & \begin{matrix} u_s & u_d & u_d & \cdots & u_d \\ u_d & u_s & u_d & \cdots & u_d \\ u_d & u_d & u_s & \cdots & u_d \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_d & u_d & u_d & \cdots & u_s \end{matrix} & & \\ & & \ddots & \\ & & & \ddots \end{pmatrix}$$

$q = 1$

Non-Markovian Dynamics



$|\textcolor{red}{1}\textcolor{blue}{0\cdots 0}\rangle$

Large Autonomous System

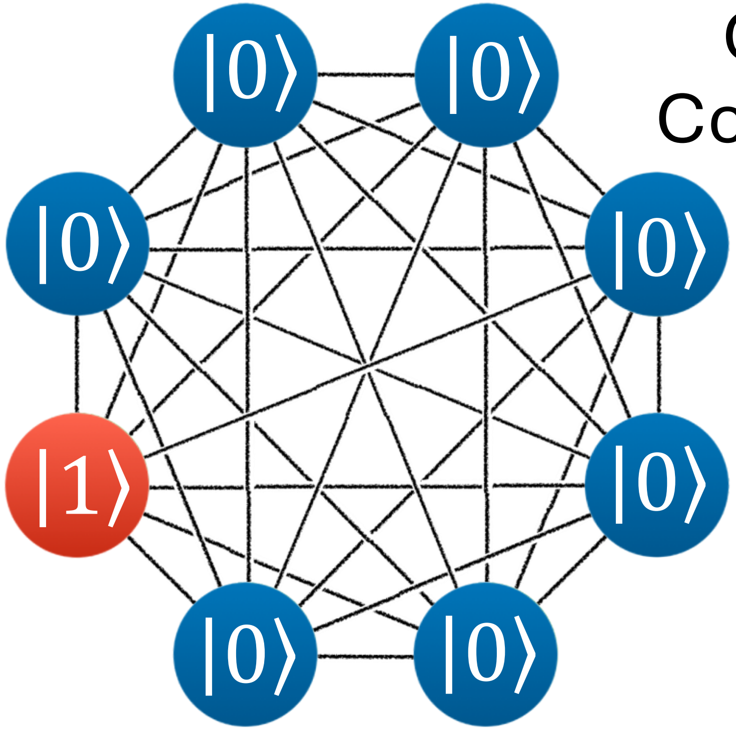
Charge Conserving

$$\hat{U} = \begin{pmatrix} u_0 & & \\ & \underbrace{\left.\begin{matrix} u_s & u_d & u_d & \cdots & u_d \\ u_d & u_s & u_d & \cdots & u_d \\ u_d & u_d & u_s & \cdots & u_d \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_d & u_d & u_d & \cdots & u_s \end{matrix}\right\} N}_{N} & \\ & & \ddots \end{pmatrix}$$

$$N|q = 1\rangle$$

$$|\textcolor{red}{1}\textcolor{blue}{0\cdots 0}\rangle, |\textcolor{blue}{0}\textcolor{red}{1}\textcolor{blue}{\cdots 0}\rangle, \cdots |\textcolor{blue}{00\cdots}\textcolor{red}{1}\rangle$$

Non-Markovian Dynamics



$|10\cdots0\rangle$

Large Autonomous System

Charge Conserving

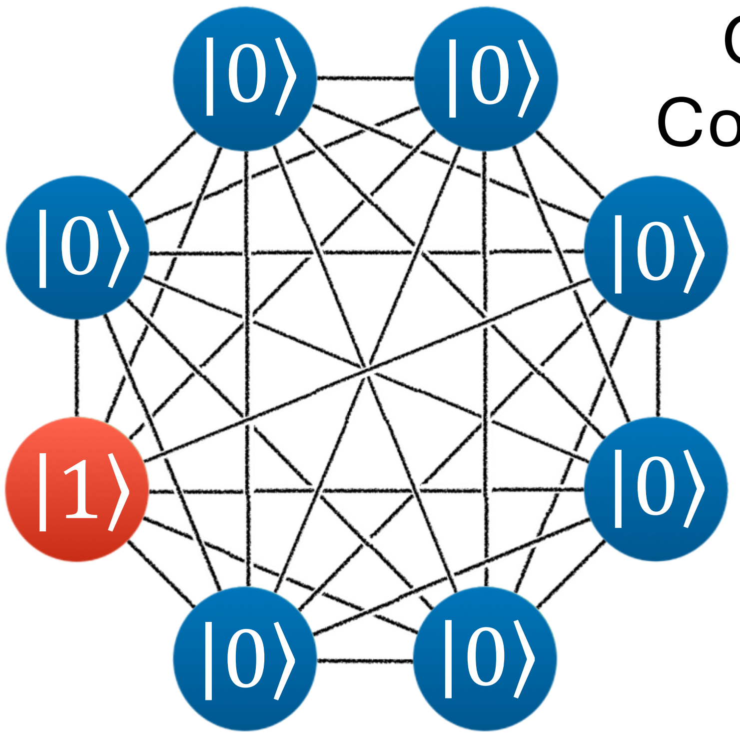
$$\hat{U} = \begin{pmatrix} u_0 & & \\ & \underbrace{\left.\begin{matrix} u_s & u_d & u_d & \cdots & u_d \\ u_d & u_s & u_d & \cdots & u_d \\ u_d & u_d & u_s & \cdots & u_d \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_d & u_d & u_d & \cdots & u_s \end{matrix}\right\} N}_{N} & \\ & & \ddots \\ & & & \ddots \end{pmatrix}$$

$N|q=1\rangle$

$|10\cdots0\rangle, |01\cdots0\rangle,$
$\cdots|00\cdots1\rangle$

u_s : Same State

Non-Markovian Dynamics



$|10\cdots0\rangle$

Large Autonomous System

Charge Conserving

$$\hat{U} = \begin{pmatrix} u_0 & & & \\ & \underbrace{\left.\begin{matrix} u_s & u_d & u_d & \cdots & u_d \\ u_d & u_s & u_d & \cdots & u_d \\ u_d & u_d & u_s & \cdots & u_d \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_d & u_d & u_d & \cdots & u_s \end{matrix}\right\} N}_{N} & & \\ & & \ddots & \\ & & & \ddots \end{pmatrix}$$

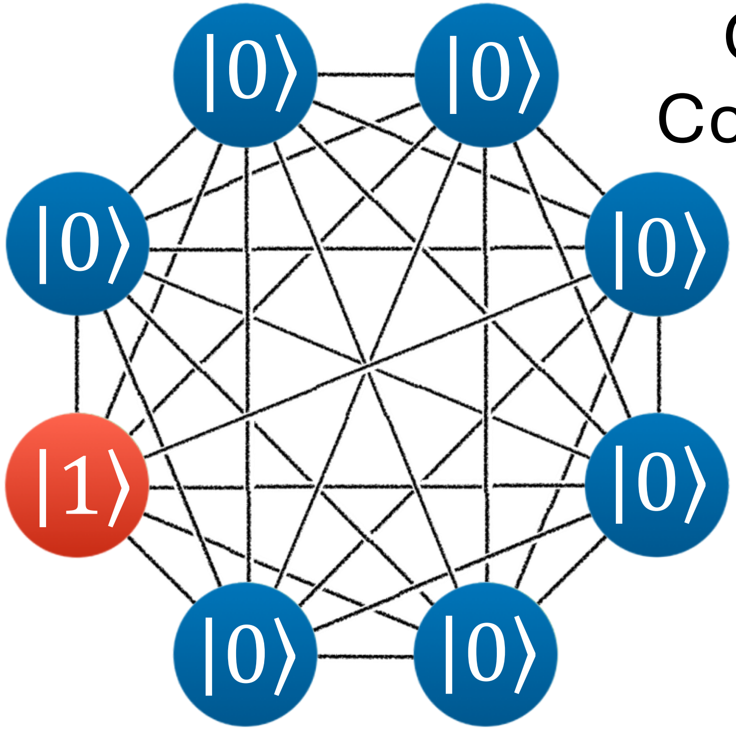
$N|q=1\rangle$

$|10\cdots0\rangle, |01\cdots0\rangle,$
$\cdots|00\cdots1\rangle$

u_s : Same State

u_d : Different State

Non-Markovian Dynamics



Charge Conserving

$$\hat{U} = \begin{pmatrix} u_0 & & & \\ & \underbrace{\left.\begin{matrix} u_s & u_d & u_d & \cdots & u_d \\ u_d & u_s & u_d & \cdots & u_d \\ u_d & u_d & u_s & \cdots & u_d \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_d & u_d & u_d & \cdots & u_s \end{matrix}\right\} N}_{N} & & \\ & & \ddots & \\ & & & \ddots \end{pmatrix}$$

$|10\cdots0\rangle$

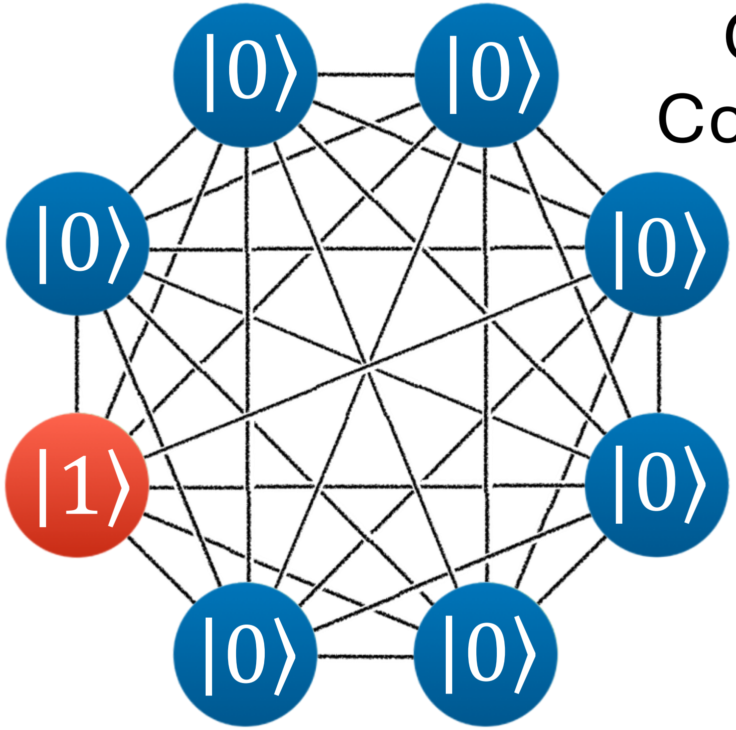
Large Autonomous System

$$|u_s|^2 + (N-1)|u_d|^2 = 1$$

u_s : Same State

u_d : Different State

Non-Markovian Dynamics



Charge Conserving

$$\hat{U} = \begin{pmatrix} u_0 & & & \\ & \underbrace{\left.\begin{matrix} u_s & u_d & u_d & \cdots & u_d \\ u_d & u_s & u_d & \cdots & u_d \\ u_d & u_d & u_s & \cdots & u_d \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_d & u_d & u_d & \cdots & u_s \end{matrix}\right\} N}_{N} & & \\ & & \ddots & \\ & & & \ddots \end{pmatrix}$$

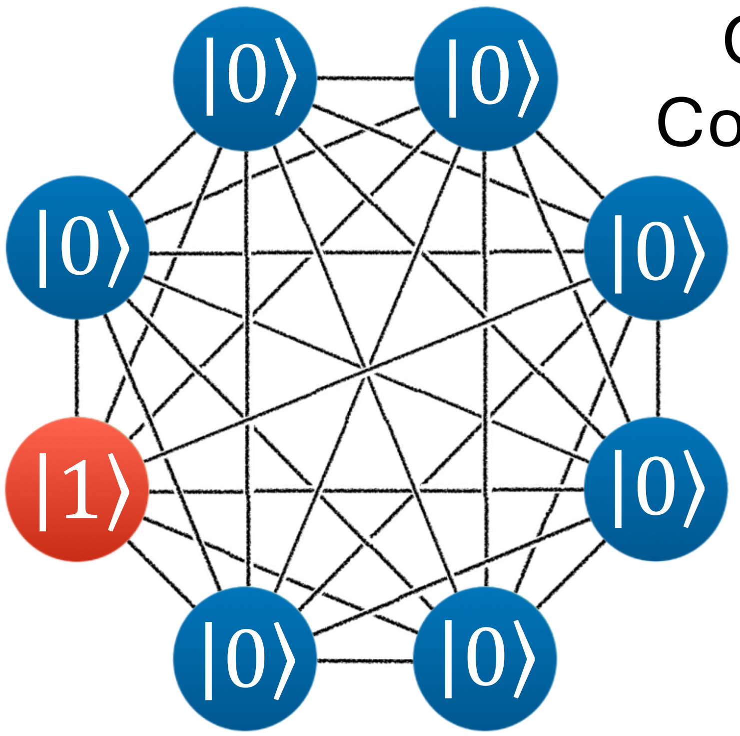
$|10 \cdots 0\rangle$

Large Autonomous System

$$|u_s|^2 + (N-1)|u_d|^2 = 1$$

u_d : Different State

Non-Markovian Dynamics



$|10\cdots0\rangle$

Large Autonomous System

Charge Conserving

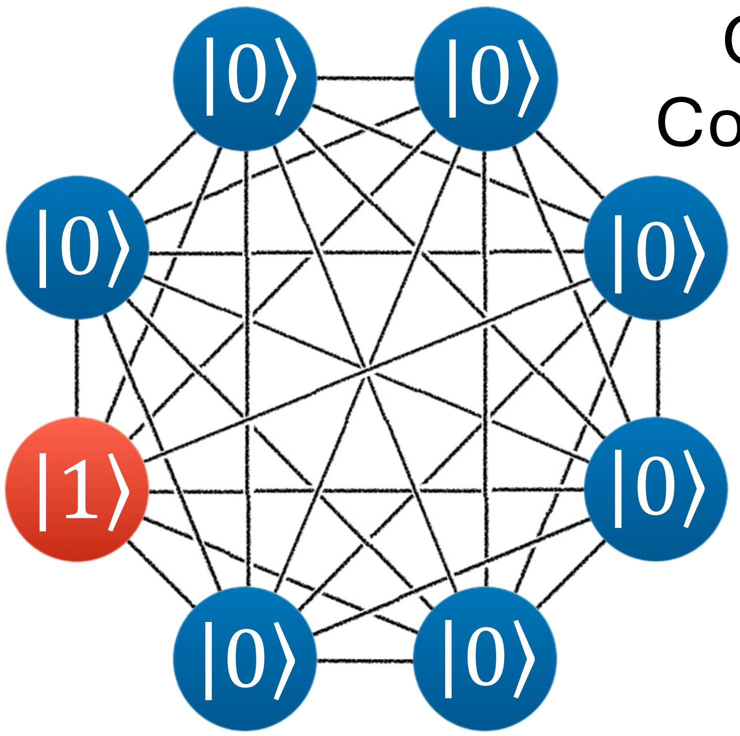
$$\hat{U} = \begin{pmatrix} u_0 & & \\ & \underbrace{\left.\begin{matrix} u_s & u_d & u_d & \cdots & u_d \\ u_d & u_s & u_d & \cdots & u_d \\ u_d & u_d & u_s & \cdots & u_d \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_d & u_d & u_d & \cdots & u_s \end{matrix}\right\} N}_{N} & \\ & & \ddots \end{pmatrix}$$

$$\hat{H} = \frac{1}{4}\sum_{i\neq j}^{N}\left[J\left(\hat{\sigma}_i^x\hat{\sigma}_j^x + \hat{\sigma}_i^y\hat{\sigma}_j^y\right) + J_z\,\hat{\sigma}_i^z\hat{\sigma}_j^z\right] + \sum_{i}^{N} h\,\hat{\sigma}_i^z$$

$|u_s|^2 + (N-1)|u_d|^2 = 1$

u_d : Different State

Non-Markovian Dynamics



$|10\cdots0\rangle$

Large Autonomous System

Charge Conserving

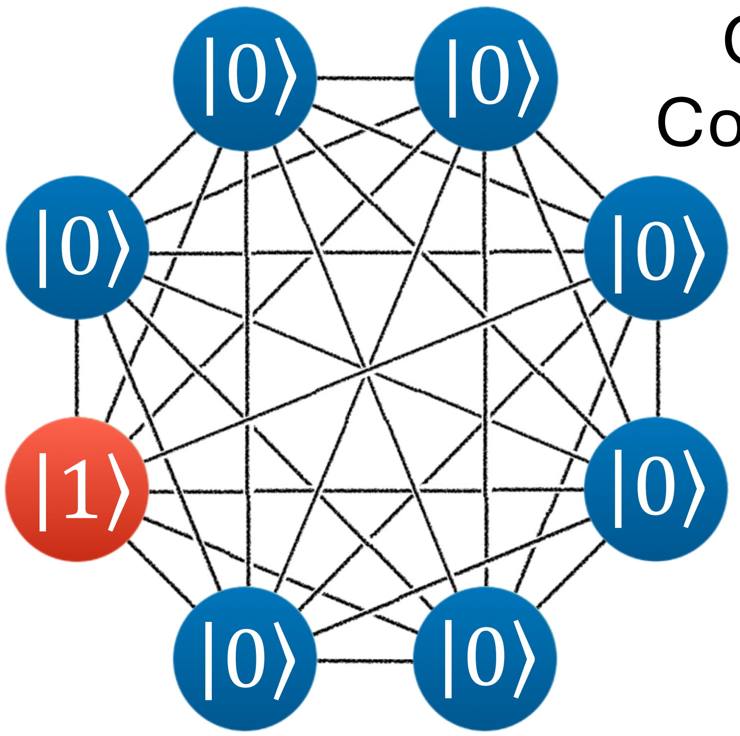
$$\hat{U} = \begin{pmatrix} u_0 & & \\ & \underbrace{\left.\begin{matrix} u_s & u_d & u_d & \cdots & u_d \\ u_d & u_s & u_d & \cdots & u_d \\ u_d & u_d & u_s & \cdots & u_d \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_d & u_d & u_d & \cdots & u_s \end{matrix}\right\} N}_{N} & \\ & & \ddots \end{pmatrix}$$

$$\hat{H} = \frac{1}{4}\sum_{i\neq j}^{N}\left[J\left(\hat{\sigma}_i^x\hat{\sigma}_j^x + \hat{\sigma}_i^y\hat{\sigma}_j^y\right) + J_z\,\hat{\sigma}_i^z\hat{\sigma}_j^z\right] + \sum_i^N h\,\hat{\sigma}_i^z$$

$$|u_s|^2 + (N-1)|u_d|^2 = 1$$

u_d : Different State

Non-Markovian Dynamics



$|10\cdots 0\rangle$

Large Autonomous System

Charge Conserving

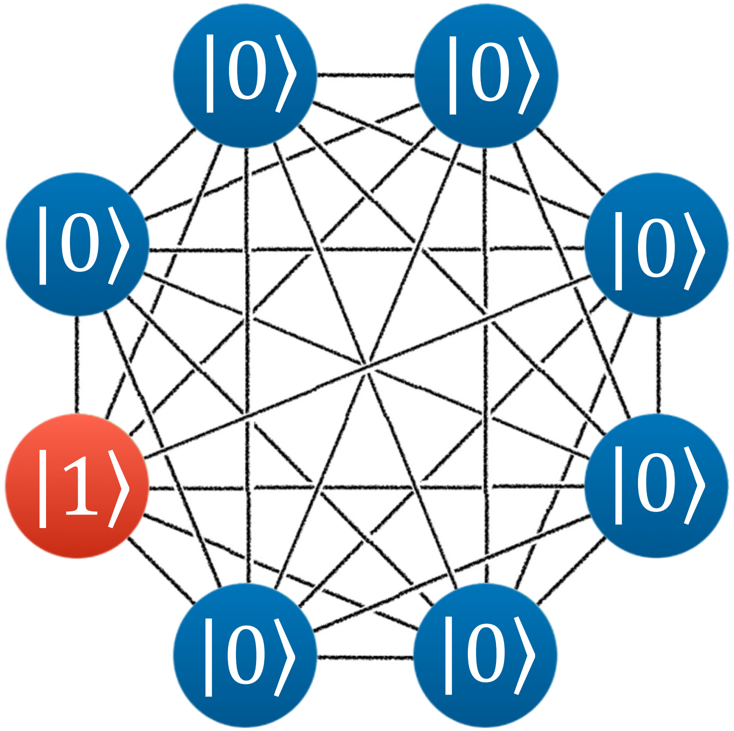
$$\hat{U} = \begin{pmatrix} u_0 & & \\ & \underbrace{\left.\begin{matrix} u_s & u_d & u_d & \cdots & u_d \\ u_d & u_s & u_d & \cdots & u_d \\ u_d & u_d & u_s & \cdots & u_d \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_d & u_d & u_d & \cdots & u_s \end{matrix}\right\} N}_{N} & \\ & & \ddots \end{pmatrix}$$

$$\hat{H} = \frac{1}{4}\sum_{i\neq j}^{N}\left[J\left(\hat{\sigma}_i^x\hat{\sigma}_j^x + \hat{\sigma}_i^y\hat{\sigma}_j^y\right) + J_z\,\hat{\sigma}_i^z\hat{\sigma}_j^z\right] + \sum_i^N h\,\hat{\sigma}_i^z$$

$$|u_s|^2 + (N-1)|u_d|^2 = 1$$

u_d : Different State

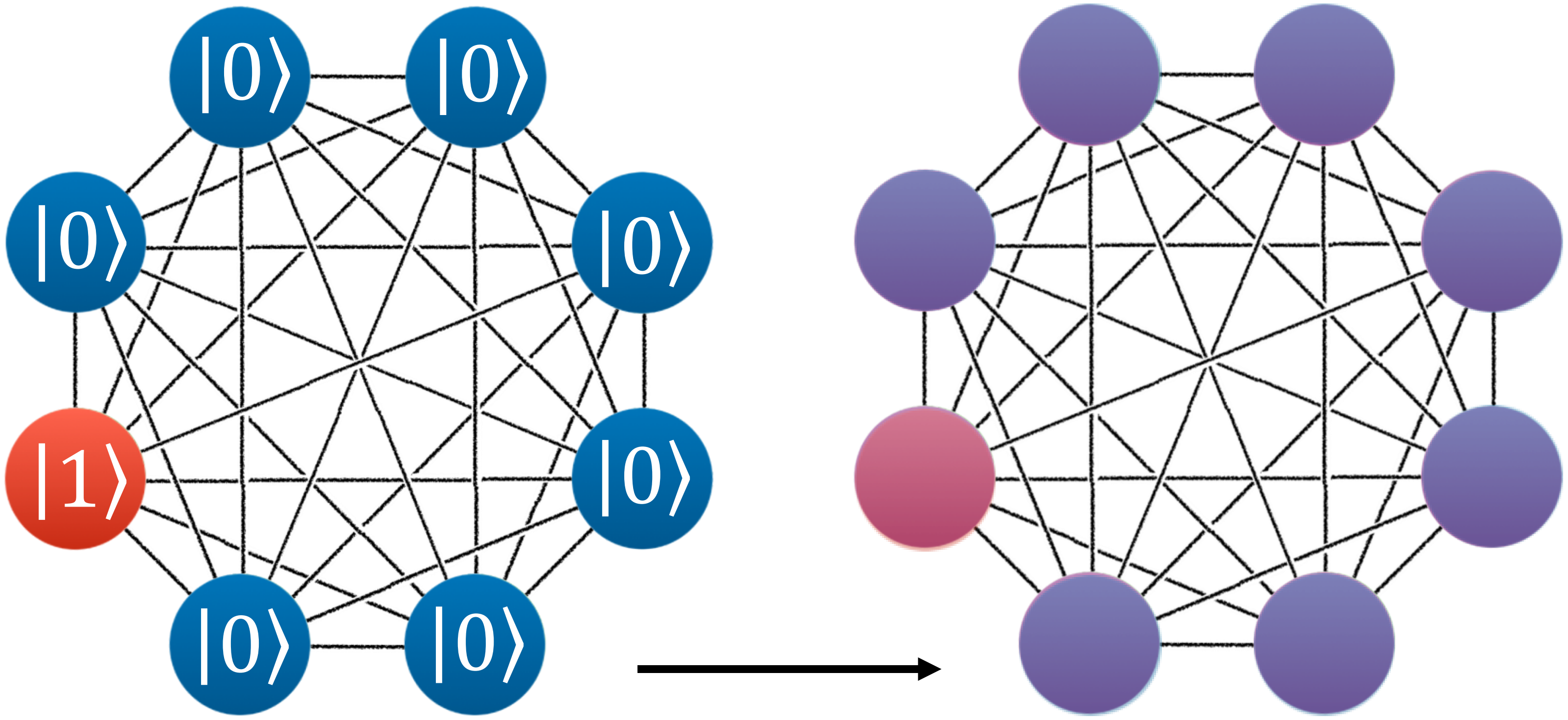
Non-Markovian Dynamics



$|10\cdots0\rangle$

Large Autonomous System

Non-Markovian Dynamics

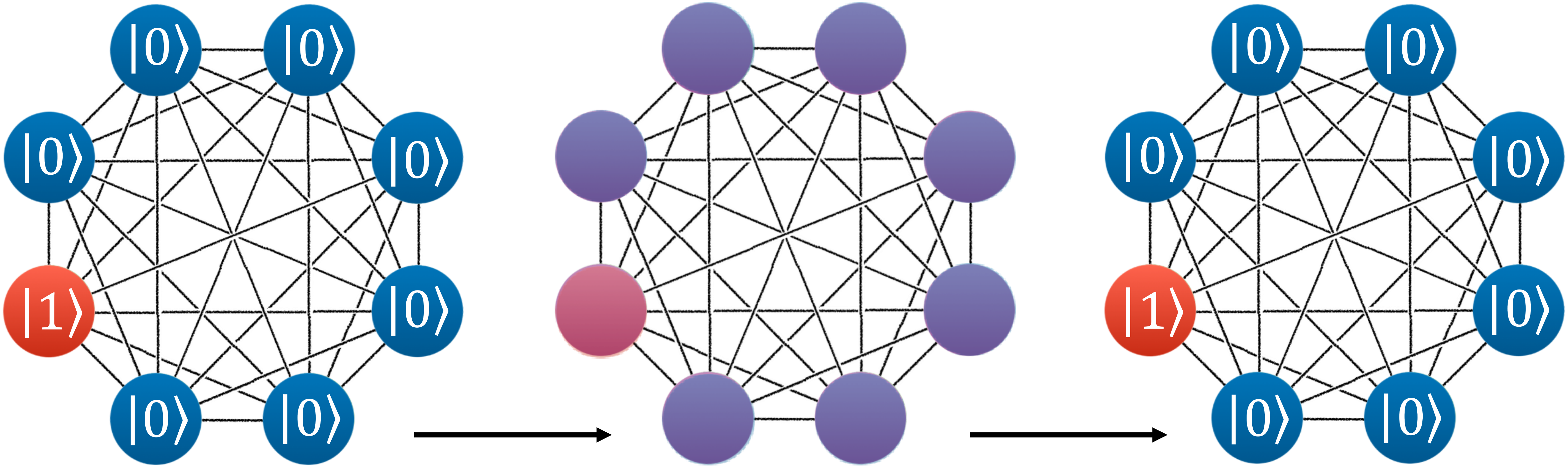


$|10\cdots0\rangle$

Large Autonomous System

$$u_s|10\cdots0\rangle + u_d|01\cdots0\rangle + \cdots + u_d|00\cdots1\rangle$$

Non-Markovian Dynamics



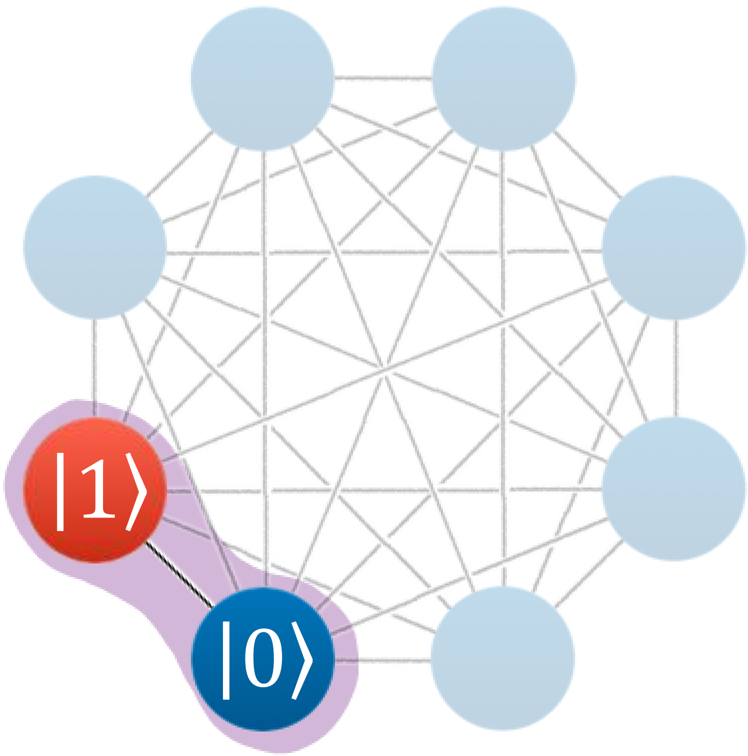
$|10\cdots0\rangle$

Large Autonomous System

$$u_s|10\cdots0\rangle + u_d|01\cdots0\rangle + \cdots + u_d|00\cdots1\rangle$$

$|10\cdots0\rangle$

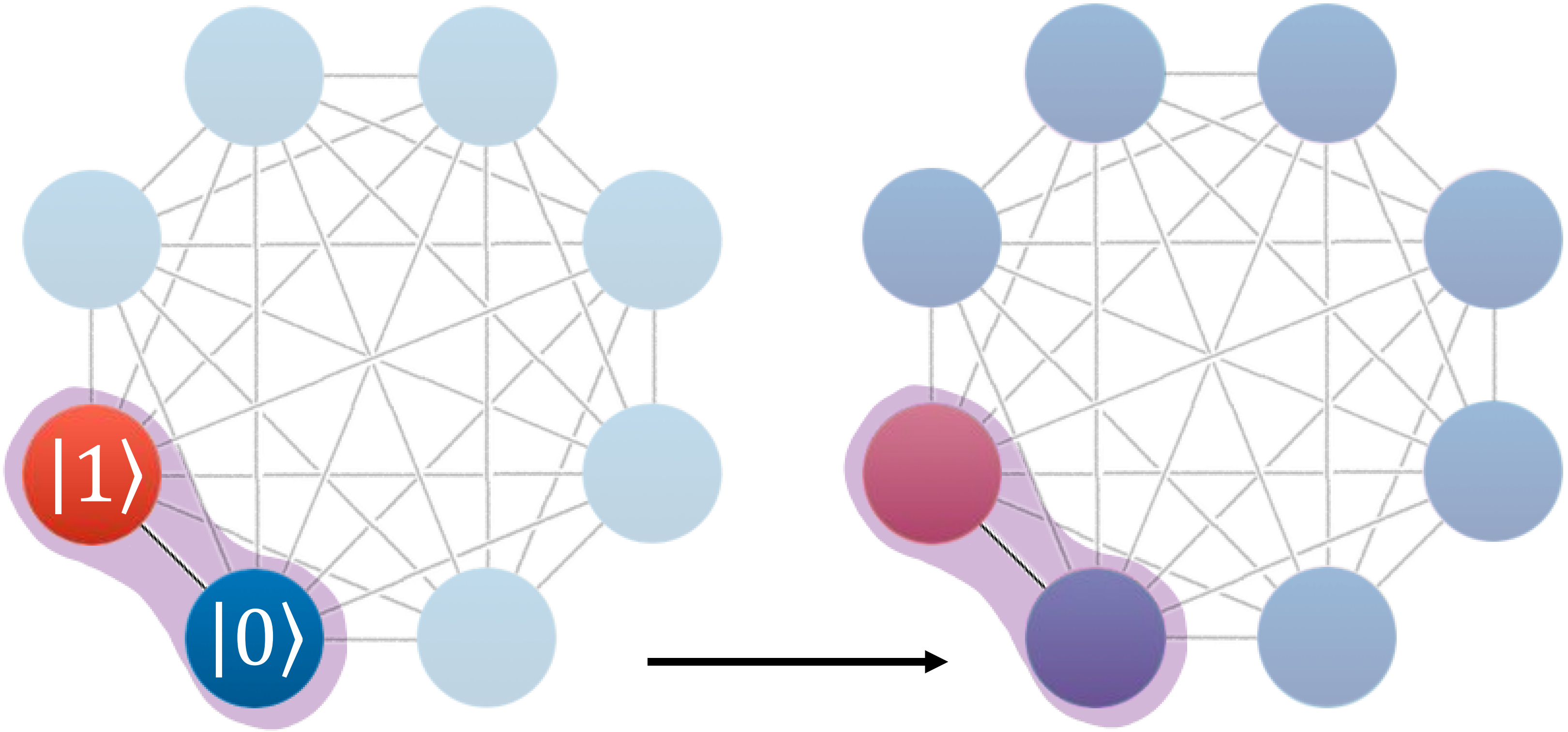
Non-Markovian Dynamics



$|10\cdots 0\rangle\langle 10\cdots 0|$

Partial System

Non-Markovian Dynamics

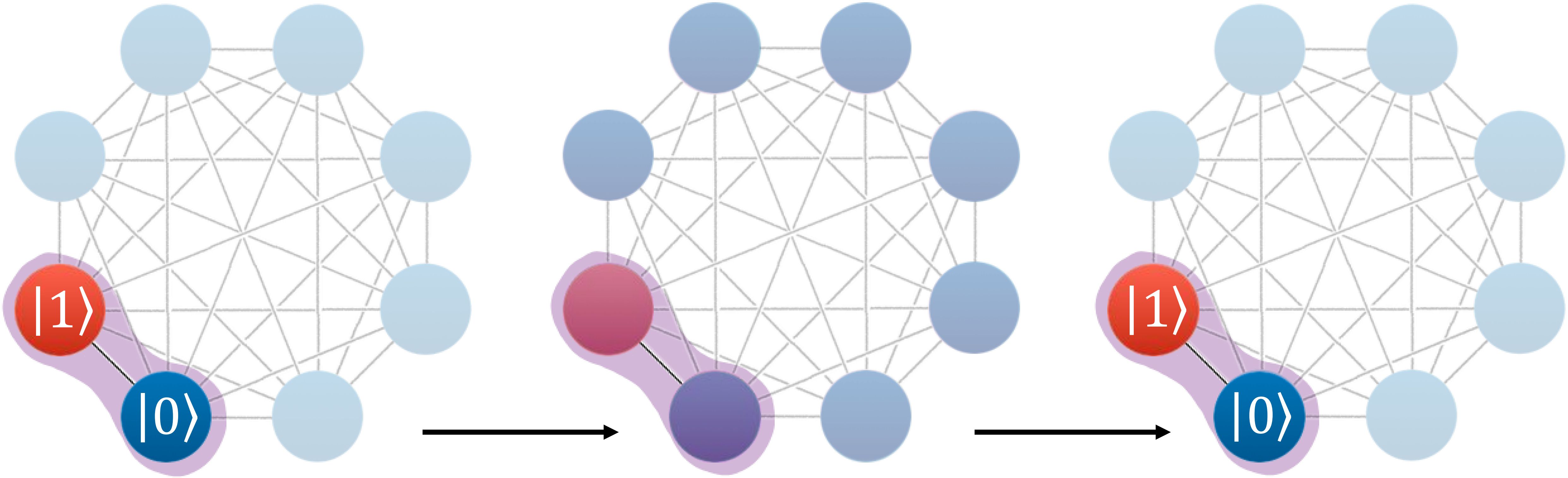


$|10\cdots0\rangle\langle10\cdots0|$

Partial System

$$(1-p^{1})|00\cdots0\rangle\langle00\cdots0| + p^{1}|\psi^{1}\rangle\langle\psi^{1}|$$

Non-Markovian Dynamics



$|10\cdots0\rangle\langle10\cdots0|$

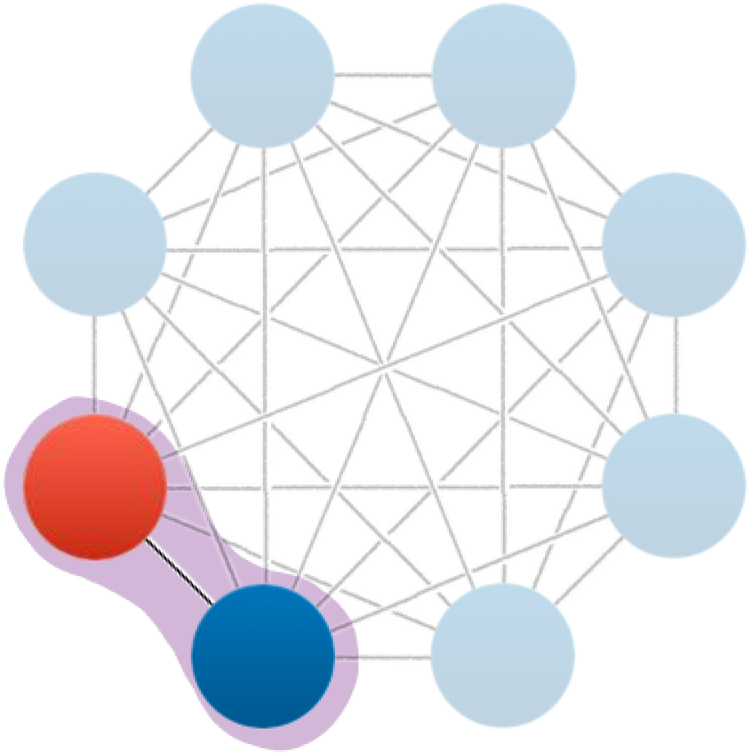
Partial System

$$(1-p^1)|00\cdots0\rangle\langle00\cdots0| + p^1|\psi^1\rangle\langle\psi^1|$$

$|10\cdots0\rangle\langle10\cdots0|$

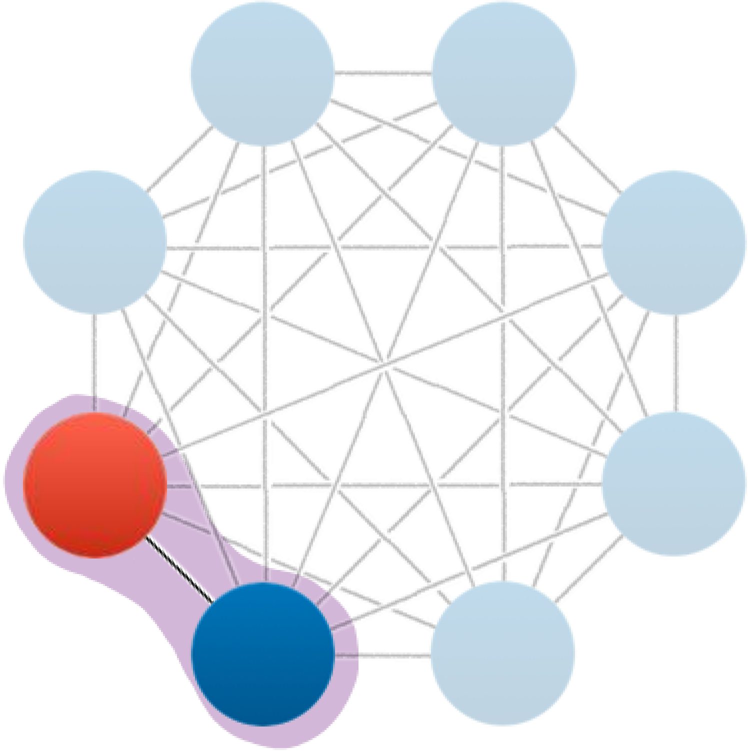
Non-Markovian Dynamics

$\Phi^{1}(t_1, t_2; K)[\hat{\rho}]$



Partial System

Non-Markovian Dynamics



Partial System

$$\Phi^{1}(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}^{1}\,\hat{\rho}\,\hat{\phi}^{1\dagger}$$

$$\begin{pmatrix} \varphi_0^1 & & & \\ & \underbrace{\left.\begin{matrix} \varphi_s^1 & \varphi_d^1 & \cdots & \varphi_d^1 \\ \varphi_d^1 & \varphi_s^1 & \cdots & \varphi_d^1 \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_d^1 & \varphi_d^1 & \cdots & \varphi_s^1 \end{matrix}\right\} K}_{K} & & \\ & & \ddots & \\ & & & \ddots \end{pmatrix}$$

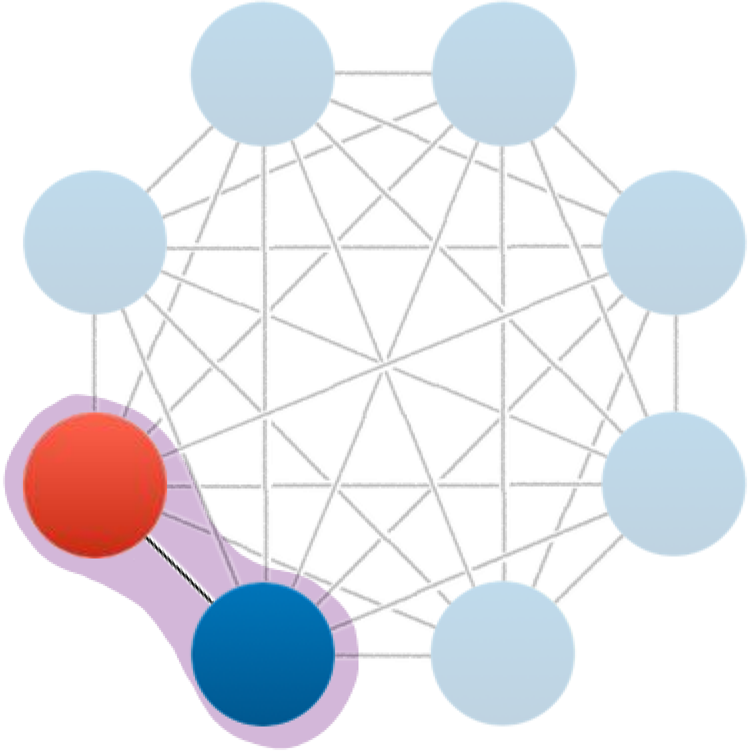
$$K|q = 1\rangle$$

φ_s^1 : Same State

φ_d^1 : Different State

Non-Markovian Dynamics

$$\Phi^{\mathbf{1}}(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}^{\mathbf{1}}\ \hat{\rho}\ \hat{\phi}^{\mathbf{1}\dagger} + \hat{\phi}^{\mathbf{1}}_{\tau}\ \hat{\rho}\ \hat{\phi}^{\mathbf{1}T}_{\tau}$$



Partial System

$$\begin{pmatrix} \varphi_0^{\mathbf{1}} & & \\ & \underbrace{\left.\begin{matrix} \varphi_s^{\mathbf{1}} & \varphi_d^{\mathbf{1}} & \cdots & \varphi_d^{\mathbf{1}} \\ \varphi_d^{\mathbf{1}} & \varphi_s^{\mathbf{1}} & \cdots & \varphi_d^{\mathbf{1}} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_d^{\mathbf{1}} & \varphi_d^{\mathbf{1}} & \cdots & \varphi_s^{\mathbf{1}} \end{matrix}\right\} K}_{K} & \\ & & \ddots \end{pmatrix} \begin{pmatrix} \underbrace{\sqrt{\varphi_\tau^{\mathbf{1}}}\ \ \sqrt{\varphi_\tau^{\mathbf{1}}}\ \ \cdots\ \ \sqrt{\varphi_\tau^{\mathbf{1}}}}_{K} \\ \\ \\ \end{pmatrix}$$

$$K|q = 1\rangle$$

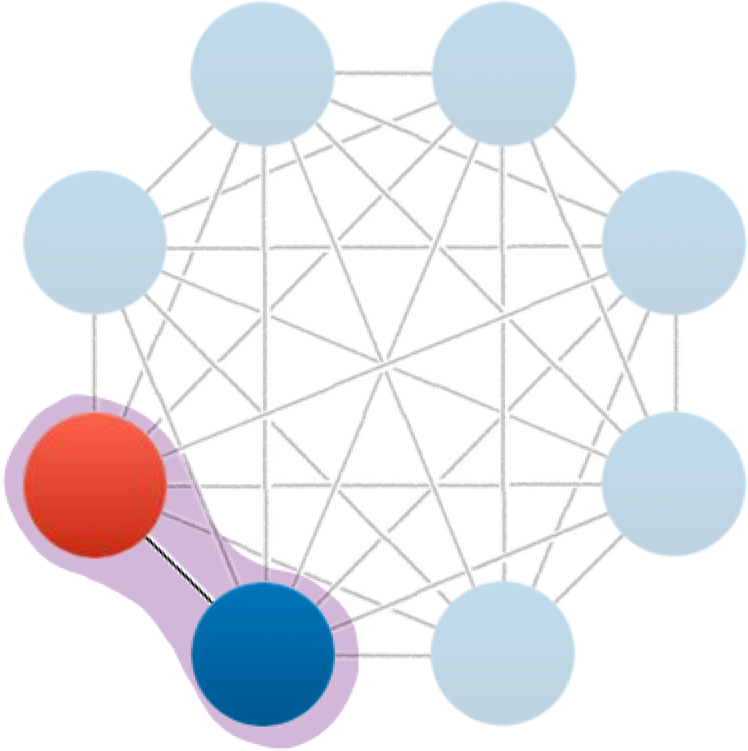
$\varphi_s^{\mathbf{1}}$: Same State

$\varphi_d^{\mathbf{1}}$: Different State

$$\sqrt{\varphi_\tau^{\mathbf{1}}} = \qquad \hat{\phi}_\tau^{\mathbf{1}}$$

Non-Markovian Dynamics

$$\Phi^{\mathbf{1}}(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}^{\mathbf{1}}\ \hat{\rho}\ \hat{\phi}^{\mathbf{1}\dagger} + \hat{\phi}^{\mathbf{1}}_{\tau}\ \hat{\rho}\ \hat{\phi}^{\mathbf{1}T}_{\tau}$$



$$\begin{pmatrix} \varphi_0^{\mathbf{1}} & & \\ & \underbrace{\left.\begin{matrix} \varphi_s^{\mathbf{1}} & \varphi_d^{\mathbf{1}} & \cdots & \varphi_d^{\mathbf{1}} \\ \varphi_d^{\mathbf{1}} & \varphi_s^{\mathbf{1}} & \cdots & \varphi_d^{\mathbf{1}} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_d^{\mathbf{1}} & \varphi_d^{\mathbf{1}} & \cdots & \varphi_s^{\mathbf{1}} \end{matrix}\right\rbrace K}_{K} & \\ & & \ddots \\ & & & \ddots \end{pmatrix} \begin{pmatrix} \underbrace{\sqrt{\varphi_\tau^{\mathbf{1}}} \quad \sqrt{\varphi_\tau^{\mathbf{1}}} \quad \cdots \quad \sqrt{\varphi_\tau^{\mathbf{1}}}}_{K} \\ \\ \end{pmatrix}$$

$K|q = 1\rangle$

$\varphi_s^{\mathbf{1}}$: Same State

$\varphi_d^{\mathbf{1}}$: Different State

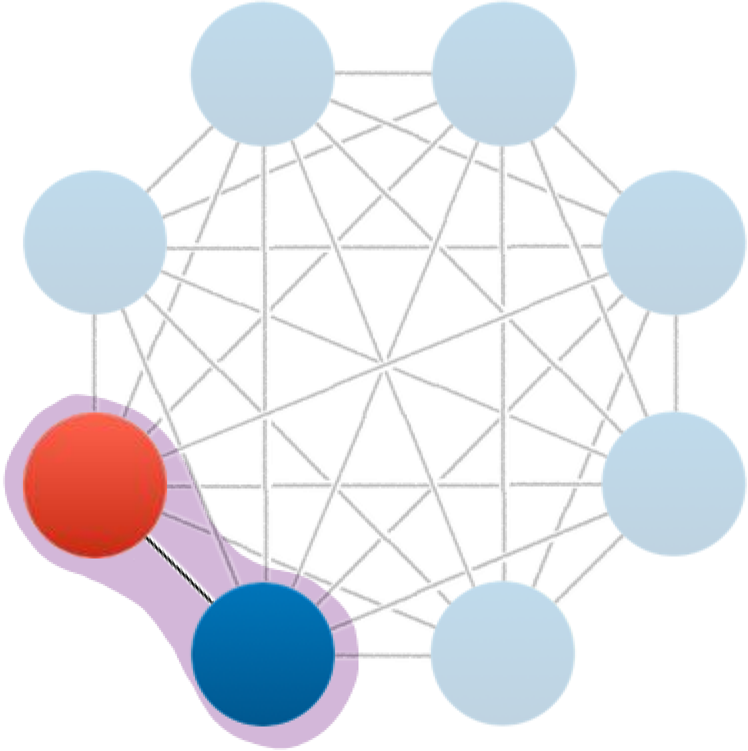
$\sqrt{\varphi_\tau^{\mathbf{1}}} =$ $\hat{\phi}_\tau^{\mathbf{1}}|q = 1\rangle$

Partial System

Non-Markovian Dynamics

$$\Phi^{\mathbf{1}}(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}^{\mathbf{1}}\ \hat{\rho}\ \hat{\phi}^{\mathbf{1}\dagger} + \hat{\phi}^{\mathbf{1}}_{\tau}\ \hat{\rho}\ \hat{\phi}^{\mathbf{1}T}_{\tau}$$

$$\begin{pmatrix} \varphi_0^{\mathbf{1}} & & \\ & \underbrace{\left.\begin{matrix} \varphi_s^{\mathbf{1}} & \varphi_d^{\mathbf{1}} & \cdots & \varphi_d^{\mathbf{1}} \\ \varphi_d^{\mathbf{1}} & \varphi_s^{\mathbf{1}} & \cdots & \varphi_d^{\mathbf{1}} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_d^{\mathbf{1}} & \varphi_d^{\mathbf{1}} & \cdots & \varphi_s^{\mathbf{1}} \end{matrix}\right\} K}_{K} & \\ & & \ddots \\ & & & \ddots \end{pmatrix} \begin{pmatrix} \underbrace{\sqrt{\varphi_\tau^{\mathbf{1}}}\quad \sqrt{\varphi_\tau^{\mathbf{1}}}\quad \cdots \quad \sqrt{\varphi_\tau^{\mathbf{1}}}}_{K} \\ \\ \\ \end{pmatrix}$$



$$K|q=1\rangle$$

Partial System

$\varphi_s^{\mathbf{1}}$: Same State

$\varphi_d^{\mathbf{1}}$: Different State

$$\sqrt{\varphi_\tau^{\mathbf{1}}} = \langle q=0|\hat{\phi}_\tau^{\mathbf{1}}|q=1\rangle$$

Non-Markovian Dynamics

$$\Phi^{\mathbf{1}}(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}^{\mathbf{1}}\ \hat{\rho}\ \hat{\phi}^{\mathbf{1}\dagger} + \hat{\phi}_\tau^{\mathbf{1}}\ \hat{\rho}\ \hat{\phi}_\tau^{\mathbf{1}T}$$

$$\begin{pmatrix} \varphi_0^{\mathbf{1}} & & & \\ & \underbrace{\left.\begin{matrix} \varphi_s^{\mathbf{1}} & \varphi_d^{\mathbf{1}} & \cdots & \varphi_d^{\mathbf{1}} \\ \varphi_d^{\mathbf{1}} & \varphi_s^{\mathbf{1}} & \cdots & \varphi_d^{\mathbf{1}} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_d^{\mathbf{1}} & \varphi_d^{\mathbf{1}} & \cdots & \varphi_s^{\mathbf{1}} \end{matrix}\right\} K}_{K} & & \\ & & \ddots & \\ & & & \ddots \end{pmatrix} \begin{pmatrix} & \underbrace{\sqrt{\varphi_\tau^{\mathbf{1}}} \quad \sqrt{\varphi_\tau^{\mathbf{1}}} \quad \cdots \quad \sqrt{\varphi_\tau^{\mathbf{1}}}}_{K} \\ & \\ & \end{pmatrix}$$

$$K|q = 1\rangle$$

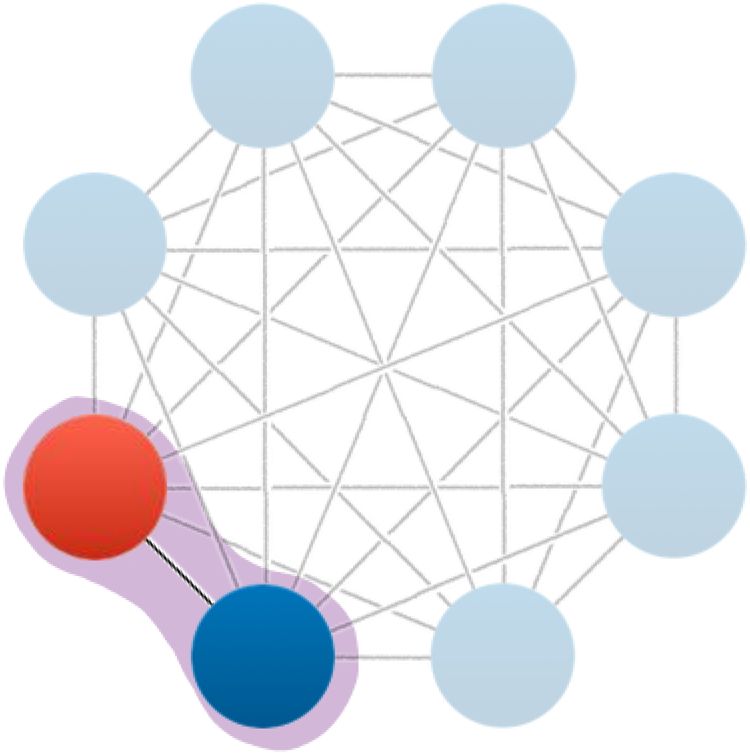
$$\left|\varphi_s^{\mathbf{1}}\right|^2 + (K-1)\left|\varphi_d^{\mathbf{1}}\right|^2 + \varphi_\tau^{\mathbf{1}} = 1$$

Trace Preserve

$\varphi_s^{\mathbf{1}}$: Same State

$\varphi_d^{\mathbf{1}}$: Different State

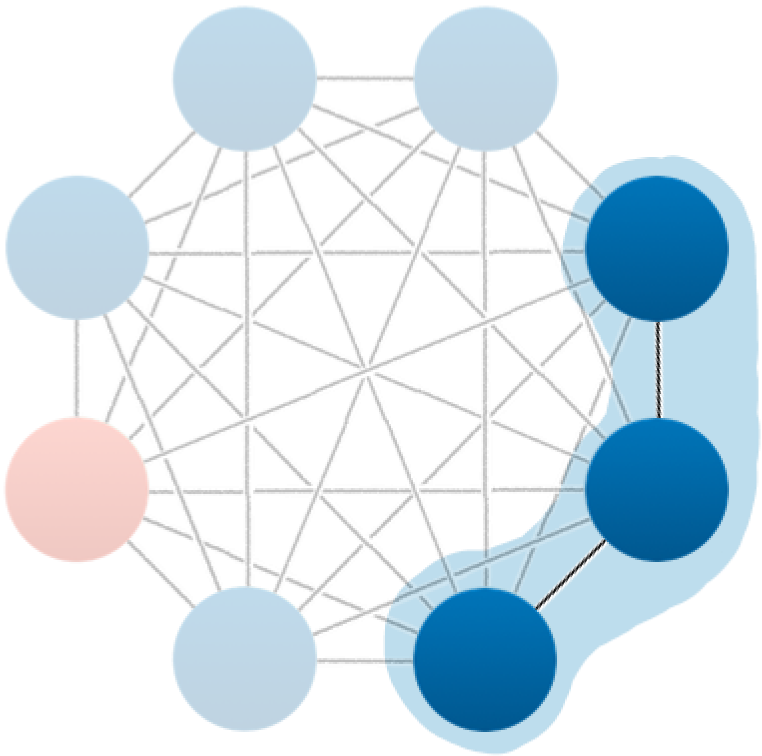
$$\sqrt{\varphi_\tau^{\mathbf{1}}} = \langle q = 0|\hat{\phi}_\tau^{\mathbf{1}}|q = 1\rangle$$



Partial System

Non-Markovian Dynamics

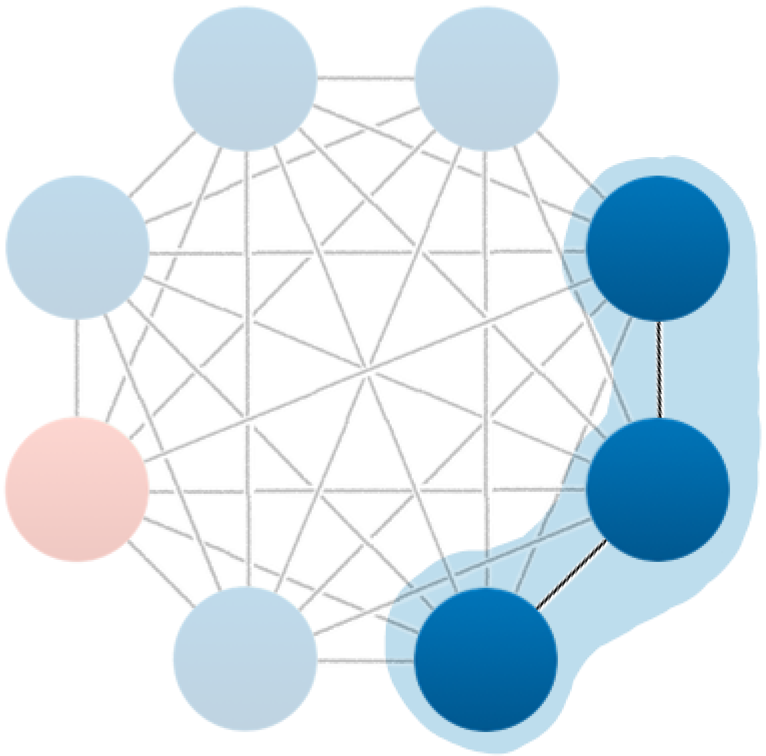
$\Phi^{\mathbf{0}}(t_1, t_2; K)[\hat{\rho}]$



Partial System

Non-Markovian Dynamics

$$\Phi^{\mathbf{0}}(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}^{\mathbf{0}}\ \hat{\rho}\ \hat{\phi}^{\mathbf{0}\dagger} + \hat{\phi}_0^{\mathbf{0}}\ \hat{\rho}\ \hat{\phi}_0^{\mathbf{0}T}$$

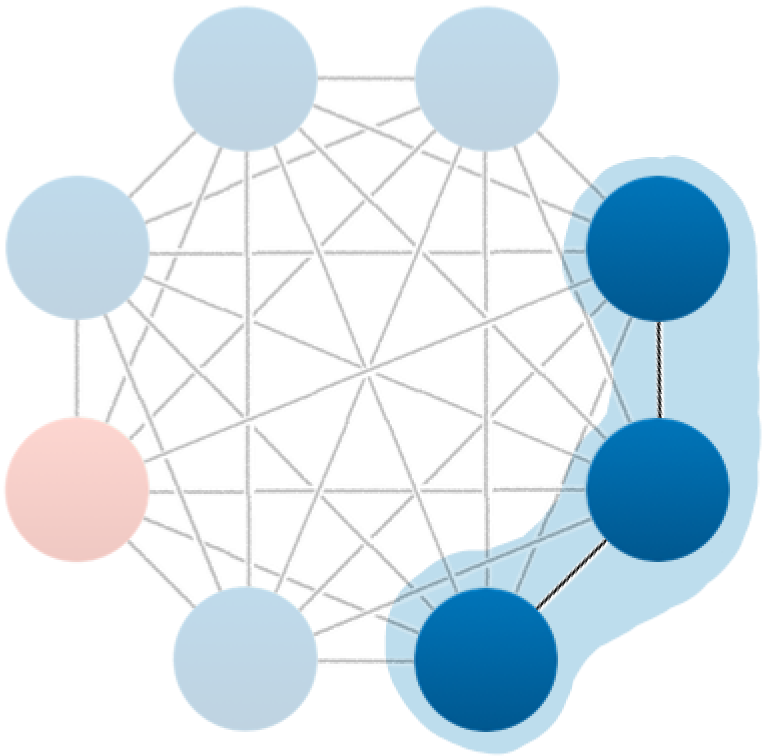


$$\begin{pmatrix} \varphi_s^{\mathbf{0}} & & & & & & \\ & \varphi_2^{\mathbf{0}} & & & & & \\ & & \varphi_2^{\mathbf{0}} & & & & \\ & & & \ddots & & & \\ & & & & \varphi_2^{\mathbf{0}} & & \\ & & & & & \ddots & \\ & & & & & & \ddots \end{pmatrix} \begin{pmatrix} \sqrt{\varphi_0^{\mathbf{0}} - \left|\varphi_s^{\mathbf{0}}\right|^2} & & \\ & & \\ & & \end{pmatrix}$$

Partial System

Non-Markovian Dynamics

$$\Phi^{\mathbf{0}}(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}^{\mathbf{0}}\,\hat{\rho}\,\hat{\phi}^{\mathbf{0}\dagger} + \hat{\phi}_0^{\mathbf{0}}\,\hat{\rho}\,\hat{\phi}_0^{\mathbf{0}T} + \hat{\phi}_\tau^{\mathbf{0}}\,\hat{\rho}\,\hat{\phi}_\tau^{\mathbf{0}T}$$



$$\begin{pmatrix} \varphi_s^{\mathbf{0}} & & & & & & \\ & \varphi_2^{\mathbf{0}} & & & & & \\ & & \varphi_2^{\mathbf{0}} & & & & \\ & & & \ddots & & & \\ & & & & \varphi_2^{\mathbf{0}} & & \\ & & & & & \ddots & \\ & & & & & & \ddots \end{pmatrix} \begin{pmatrix} \sqrt{\varphi_0^{\mathbf{0}} - |\varphi_s^{\mathbf{0}}|^2} & \\ & \\ & \end{pmatrix} \begin{pmatrix} \sqrt{\varphi_\tau^{\mathbf{0}}} & \\ \sqrt{\varphi_\tau^{\mathbf{0}}} & \\ \vdots & \\ \sqrt{\varphi_\tau^{\mathbf{0}}} & \end{pmatrix}$$

Partial System

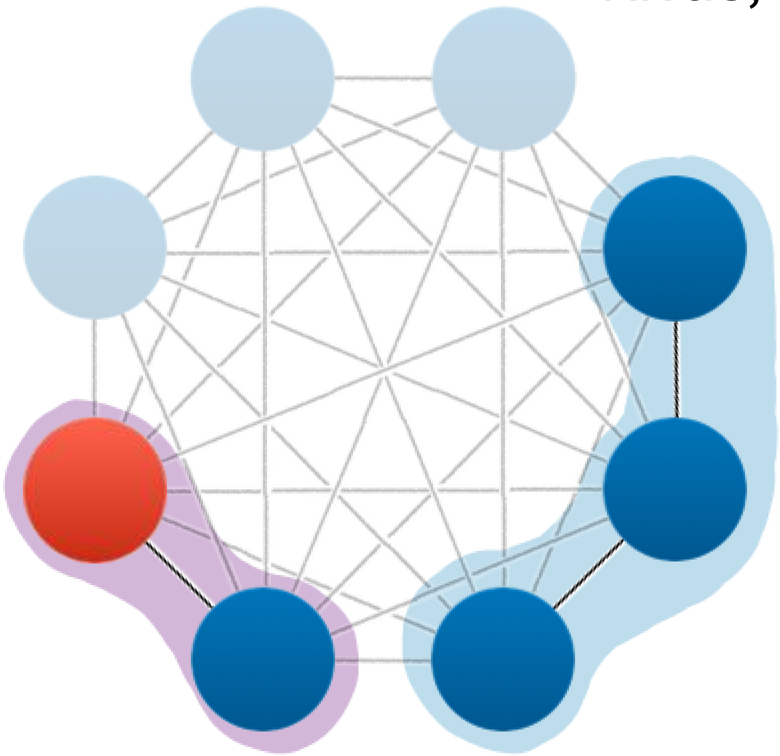
$$\sqrt{\varphi_\tau^{\mathbf{0}}} = \langle q = 1 | \hat{\phi}_\tau^{\mathbf{0}} | q = 0 \rangle$$

Non-Markovian Dynamics

Non-CP-Divisibility $\Longrightarrow$ Non-Markovianity

Rivas, Huelga, Plenio, PRL **105**, 050403 (2010)

$$\hat{\rho} = \sum_{\mu} p_{\mu} |\mu\rangle\langle\mu|$$



$\Phi(t_1, t_2)$ is P $\Longleftrightarrow$ $\Phi[|\mu\rangle\langle\mu|] \succcurlyeq 0$

Pure States

$\Phi(t_1, t_2)$ is CP $\Longleftrightarrow$ $\sum_{\mu,\nu} \Phi[|\mu\rangle\langle\nu|] \otimes |\mu\rangle\langle\nu| \succcurlyeq 0$

Max Entangled

Choi, Linear Algebra and its Applications **10**, 285 (1975)

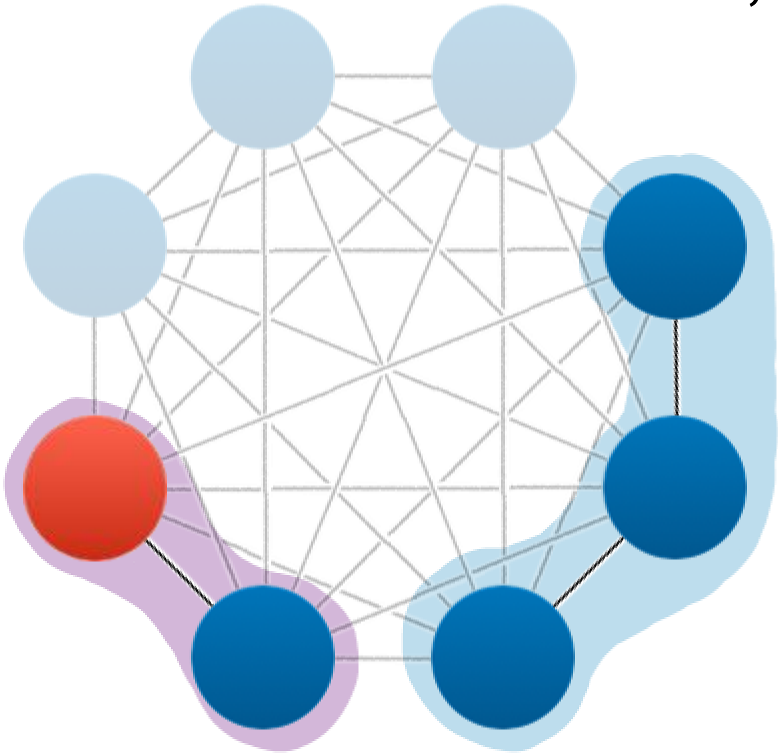
Partial Systems

Non-Markovian Dynamics

Non-CP-Divisibility $\Longrightarrow$ Non-Markovianity

Rivas, Huelga, Plenio, PRL **105**, 050403 (2010)

$$\hat{\rho} = \sum_{\mu} p_\mu |\mu\rangle\langle\mu|$$



$\Phi(t_1, t_2)$ is P $\quad\Longleftrightarrow\quad$ $\Phi[|\mu\rangle\langle\mu|] \succcurlyeq 0$

Pure States

$\Phi(t_1, t_2)$ is CP $\quad\Longleftrightarrow\quad$ $\sum_{\mu,\nu} \Phi[|\mu\rangle\langle\nu|] \otimes |\mu\rangle\langle\nu| \succcurlyeq 0$

Max Entangled

Choi, Linear Algebra and its Applications **10**, 285 (1975)

$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K)$

Eigenvalue can be negative

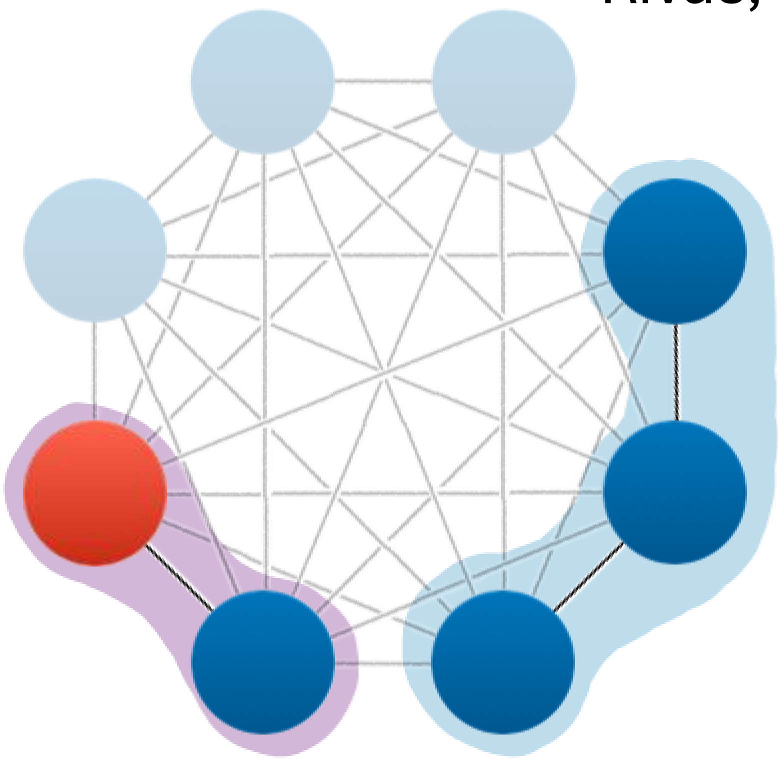
Partial Systems

Non-Markovian Dynamics

Non-CP-Divisibility $\Longrightarrow$ Non-Markovianity

Rivas, Huelga, Plenio, PRL **105**, 050403 (2010)

$$\hat{\rho} = \sum_{\mu} p_\mu |\mu\rangle\langle\mu|$$



$\Phi(t_1, t_2)$ is P $\Longleftrightarrow$ $\Phi[|\mu\rangle\langle\mu|] \succcurlyeq 0$

Pure States

$\Phi(t_1, t_2)$ is CP $\Longleftrightarrow$ $\sum_{\mu,\nu} \Phi[|\mu\rangle\langle\nu|] \otimes |\mu\rangle\langle\nu| \succcurlyeq 0$

Max Entangled

Choi, Linear Algebra and its Applications **10**, 285 (1975)

$\boxed{\Phi^{\boldsymbol{i}}(t_1, t_2; K) \text{ is NPNCP} \Longleftrightarrow \varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) < 0}$

Partial Systems

Eigenvalue can be negative

Excitation Flow

$$\Phi^{\mathbf{1}}(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}^{\mathbf{1}}\, \hat{\rho}\, \hat{\phi}^{\mathbf{1}\dagger} + \hat{\phi}_\tau^{\mathbf{1}}\, \hat{\rho}\, \hat{\phi}_\tau^{\mathbf{1}T}$$

$$\Phi^{\mathbf{0}}(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}^{\mathbf{0}}\, \hat{\rho}\, \hat{\phi}^{\mathbf{0}\dagger} + \hat{\phi}_0^{\mathbf{0}}\, \hat{\rho}\, \hat{\phi}_0^{\mathbf{0}T} + \hat{\phi}_\tau^{\mathbf{0}}\, \hat{\rho}\, \hat{\phi}_\tau^{\mathbf{0}T}$$

$$\sqrt{\varphi_\tau^{\mathbf{1}}} = \langle q = 0 | \hat{\phi}_\tau^{\mathbf{1}} | q = 1 \rangle$$

$$\sqrt{\varphi_\tau^{\mathbf{0}}} = \langle q = 1 | \hat{\phi}_\tau^{\mathbf{0}} | q = 0 \rangle$$

Excitation Flow

$$\Phi^{\mathbf{1}}(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}^{\mathbf{1}}\ \hat{\rho}\ \hat{\phi}^{\mathbf{1}\dagger} + \hat{\phi}_{\tau}^{\mathbf{1}}\ \hat{\rho}\ \hat{\phi}_{\tau}^{\mathbf{1}T}$$

$$\Phi^{\mathbf{0}}(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}^{\mathbf{0}}\ \hat{\rho}\ \hat{\phi}^{\mathbf{0}\dagger} + \hat{\phi}_0^{\mathbf{0}}\ \hat{\rho}\ \hat{\phi}_0^{\mathbf{0}T} + \hat{\phi}_{\tau}^{\mathbf{0}}\ \hat{\rho}\ \hat{\phi}_{\tau}^{\mathbf{0}T}$$

$$\sqrt{\varphi_{\tau}^{\mathbf{1}}} = \langle q = 0 | \hat{\phi}_{\tau}^{\mathbf{1}} | q = 1 \rangle$$

$$\varphi_{\tau}^{\mathbf{1}}(t, t + 0.05)$$

$$\sqrt{\varphi_{\tau}^{\mathbf{0}}} = \langle q = 1 | \hat{\phi}_{\tau}^{\mathbf{0}} | q = 0 \rangle$$

$$\varphi_{\tau}^{\mathbf{0}}(t, t + 0.05)$$

Excitation Flow

$$\Phi^{\mathbf{1}}(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}^{\mathbf{1}}\,\hat{\rho}\,\hat{\phi}^{\mathbf{1}\dagger} + \hat{\phi}_{\tau}^{\mathbf{1}}\,\hat{\rho}\,\hat{\phi}_{\tau}^{\mathbf{1}T}$$

$$\Phi^{\mathbf{0}}(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}^{\mathbf{0}}\,\hat{\rho}\,\hat{\phi}^{\mathbf{0}\dagger} + \hat{\phi}_{0}^{\mathbf{0}}\,\hat{\rho}\,\hat{\phi}_{0}^{\mathbf{0}T} + \hat{\phi}_{\tau}^{\mathbf{0}}\,\hat{\rho}\,\hat{\phi}_{\tau}^{\mathbf{0}T}$$

$$\sqrt{\varphi_{\tau}^{\mathbf{1}}} = \langle q = 0|\hat{\phi}_{\tau}^{\mathbf{1}}|q = 1\rangle$$

$$\varphi_{\tau}^{\mathbf{1}}(t, t + 0.05)$$

$$N = 5$$

$$\sqrt{\varphi_{\tau}^{\mathbf{0}}} = \langle q = 1|\hat{\phi}_{\tau}^{\mathbf{0}}|q = 0\rangle$$

$$\varphi_{\tau}^{\mathbf{0}}(t, t + 0.05)$$

Excitation Flow

$$\Phi^{\mathbf{1}}(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}^{\mathbf{1}}\,\hat{\rho}\,\hat{\phi}^{\mathbf{1}\dagger} + \hat{\phi}^{\mathbf{1}}_{\tau}\,\hat{\rho}\,\hat{\phi}^{\mathbf{1}T}_{\tau}$$

$$\Phi^{\mathbf{0}}(t_1, t_2; K)[\hat{\rho}] = \hat{\phi}^{\mathbf{0}}\,\hat{\rho}\,\hat{\phi}^{\mathbf{0}\dagger} + \hat{\phi}^{\mathbf{0}}_{0}\,\hat{\rho}\,\hat{\phi}^{\mathbf{0}T}_{0} + \hat{\phi}^{\mathbf{0}}_{\tau}\,\hat{\rho}\,\hat{\phi}^{\mathbf{0}T}_{\tau}$$

$$\sqrt{\varphi^{\mathbf{1}}_{\tau}} = \langle q = 0|\hat{\phi}^{\mathbf{1}}_{\tau}|q = 1\rangle$$

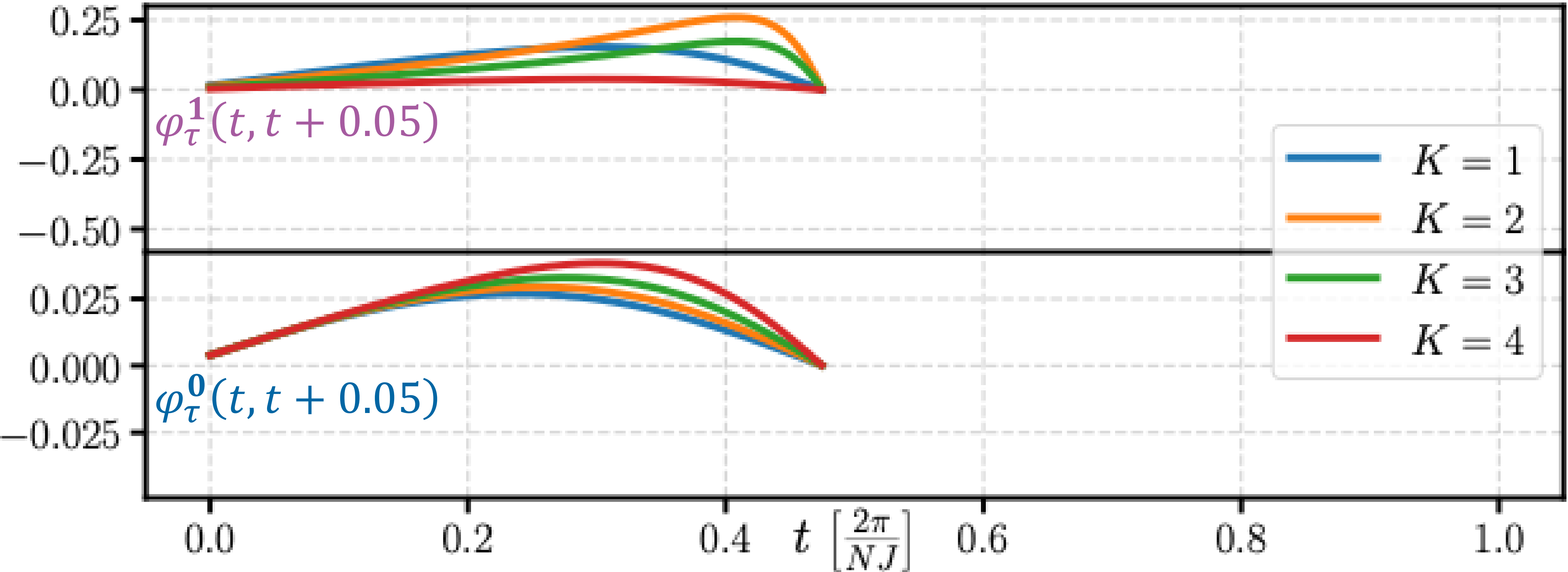
$$\varphi^{\mathbf{1}}_{\tau}(t, t + 0.05)$$

$$N = 5 \qquad K = 1, 2, 3, 4$$

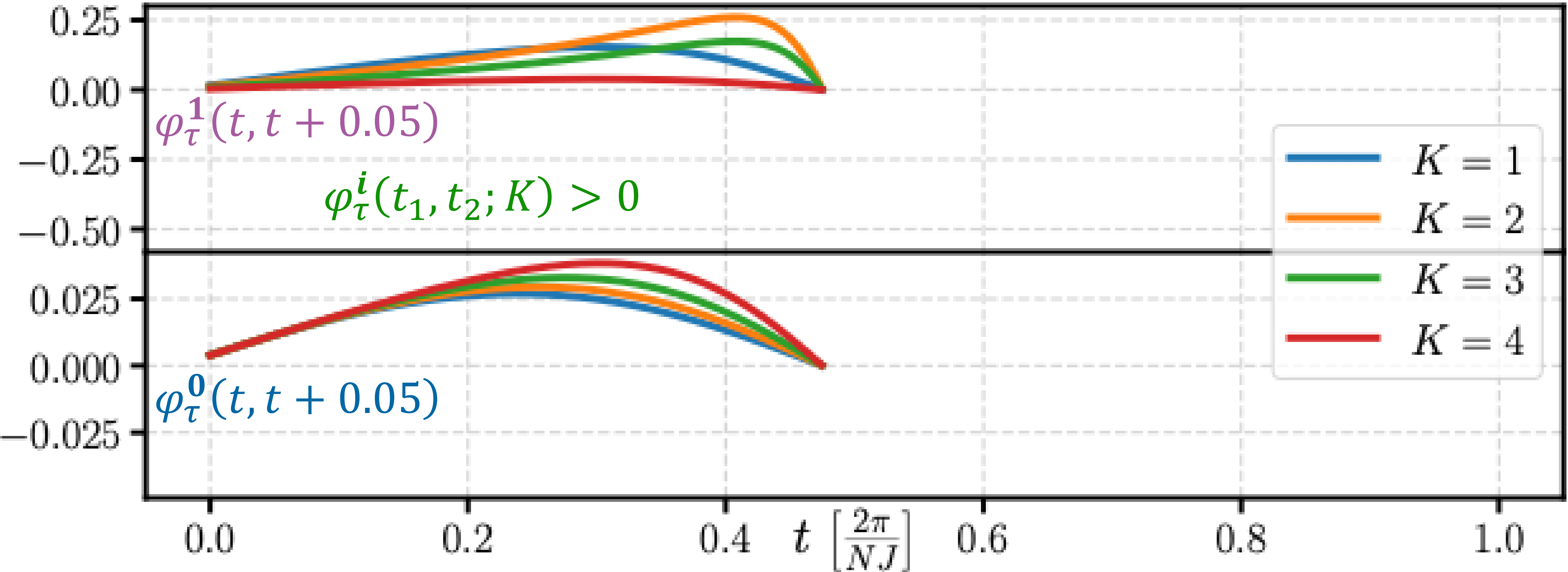
$$\sqrt{\varphi^{\mathbf{0}}_{\tau}} = \langle q = 1|\hat{\phi}^{\mathbf{0}}_{\tau}|q = 0\rangle$$

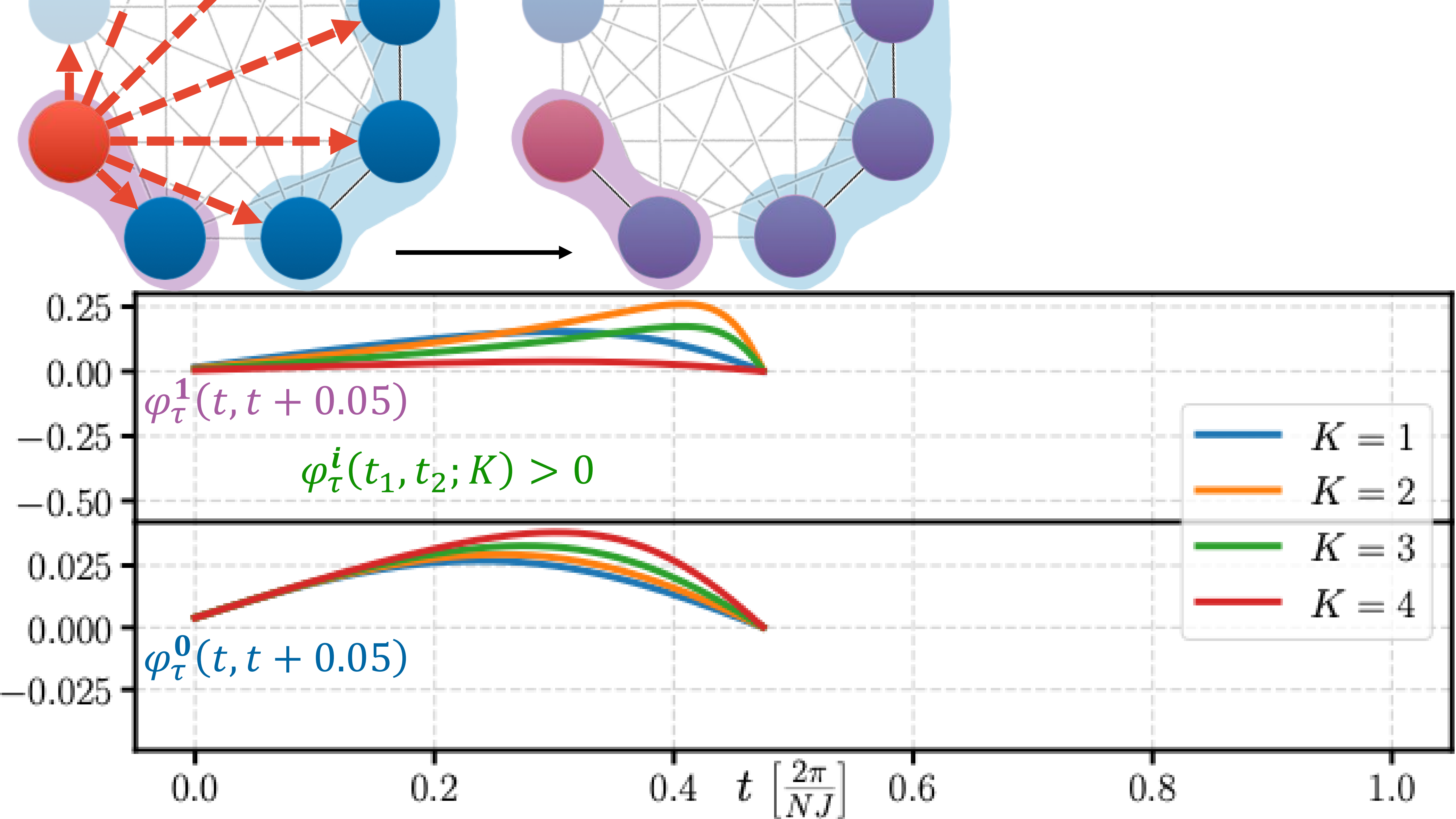
$$\varphi^{\mathbf{0}}_{\tau}(t, t + 0.05)$$

Excitation Flow

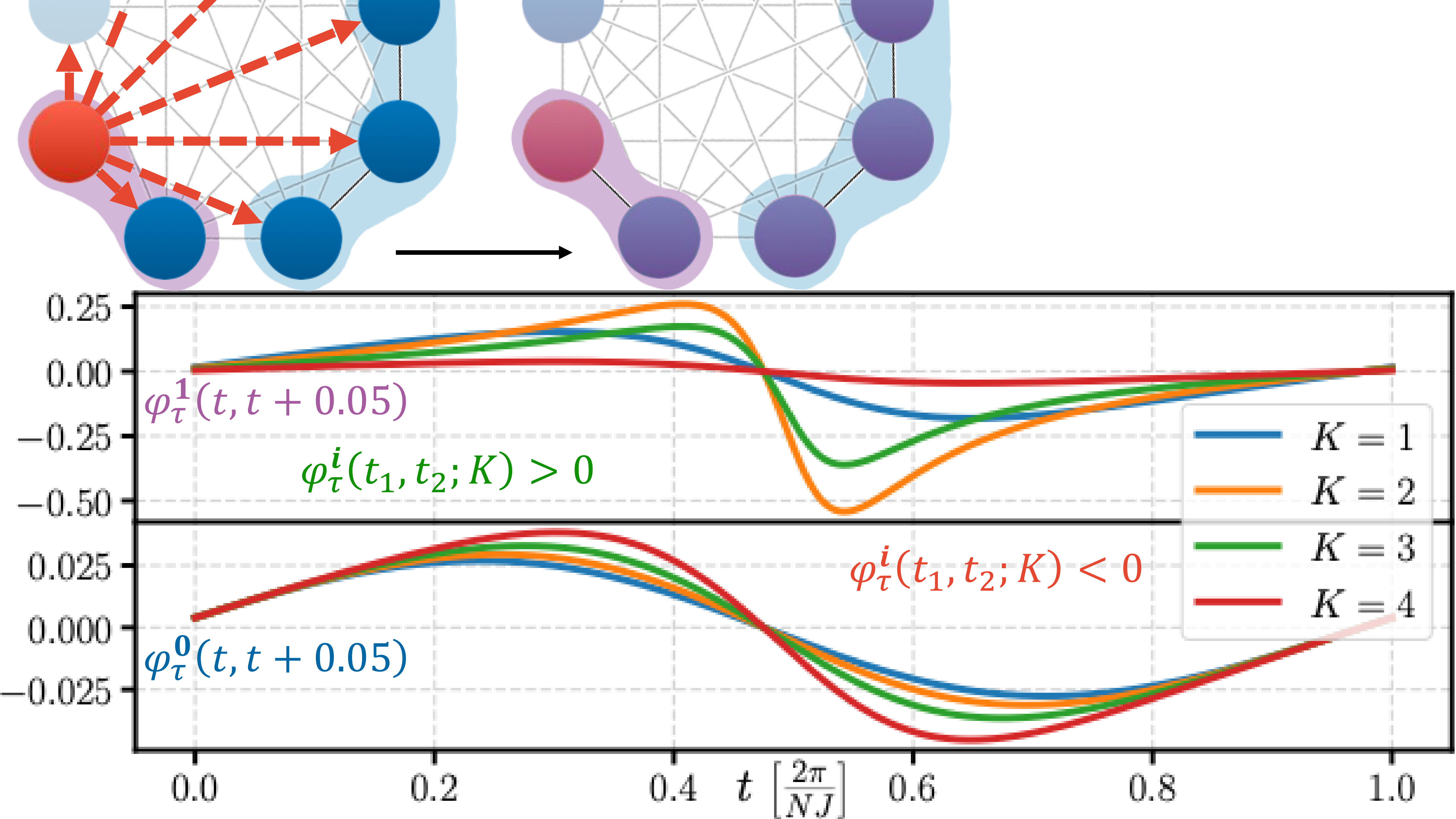


Excitation Flow

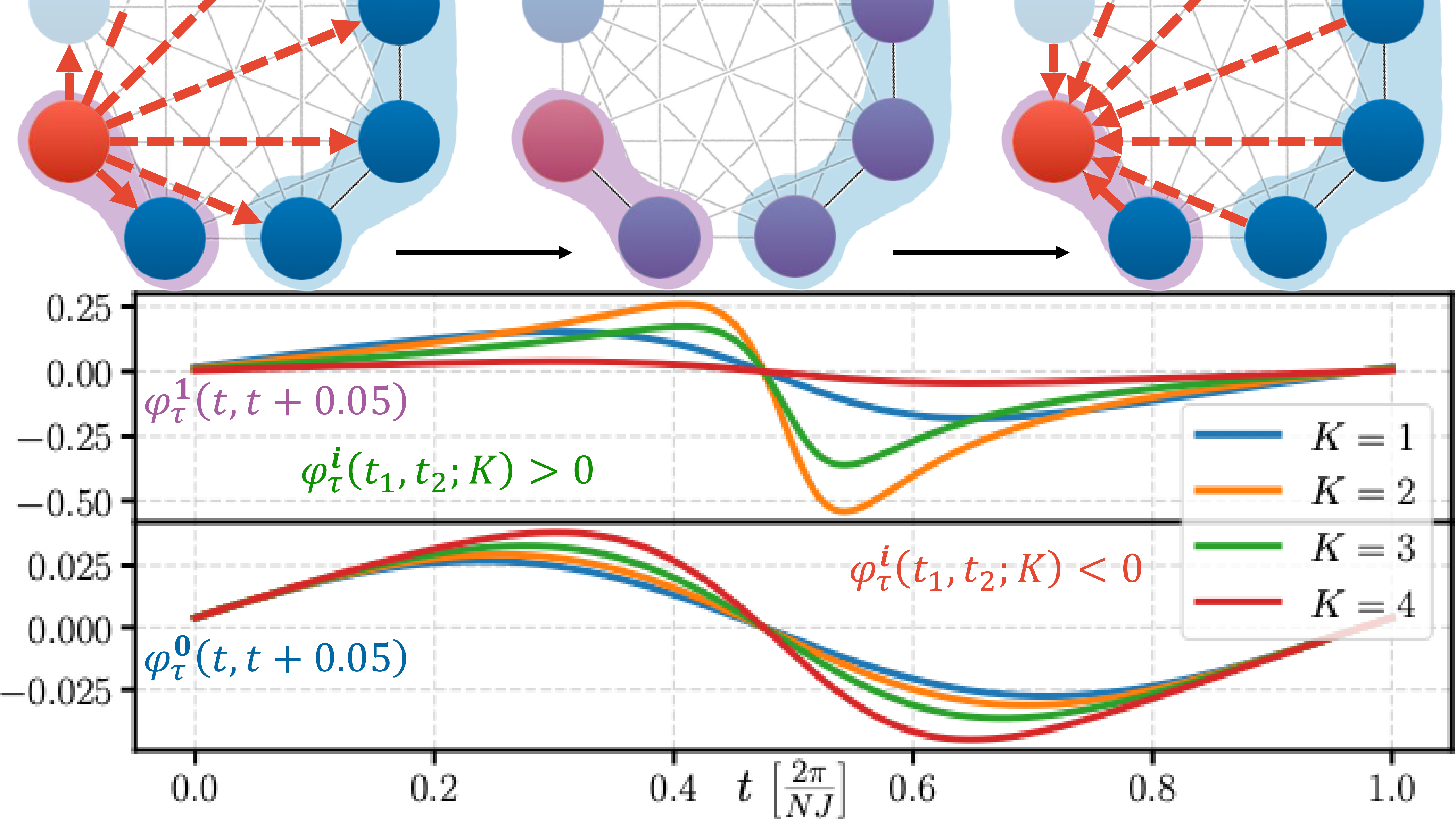




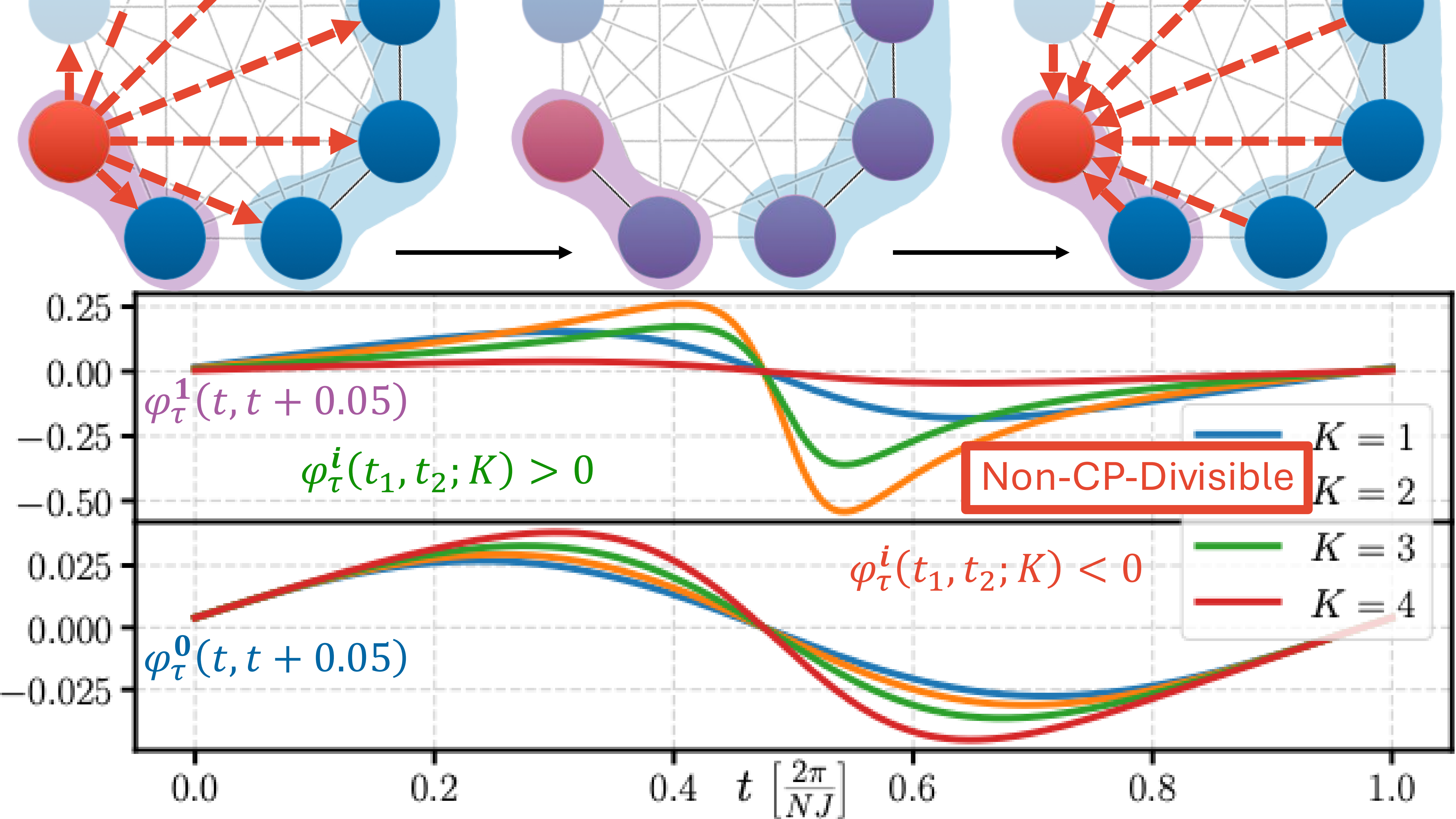
0.25
0.00
−0.25
−0.50
$\varphi_\tau^{\mathbf{1}}(t, t+0.05)$
$\varphi_\tau^{\mathbf{i}}(t_1, t_2; K) > 0$
$K = 1$
$K = 2$
$K = 3$
$K = 4$
0.025
0.000
−0.025
$\varphi_\tau^{\mathbf{0}}(t, t+0.05)$
0.0
0.2
0.4
$t\ \left[\frac{2\pi}{NJ}\right]$
0.6
0.8
1.0



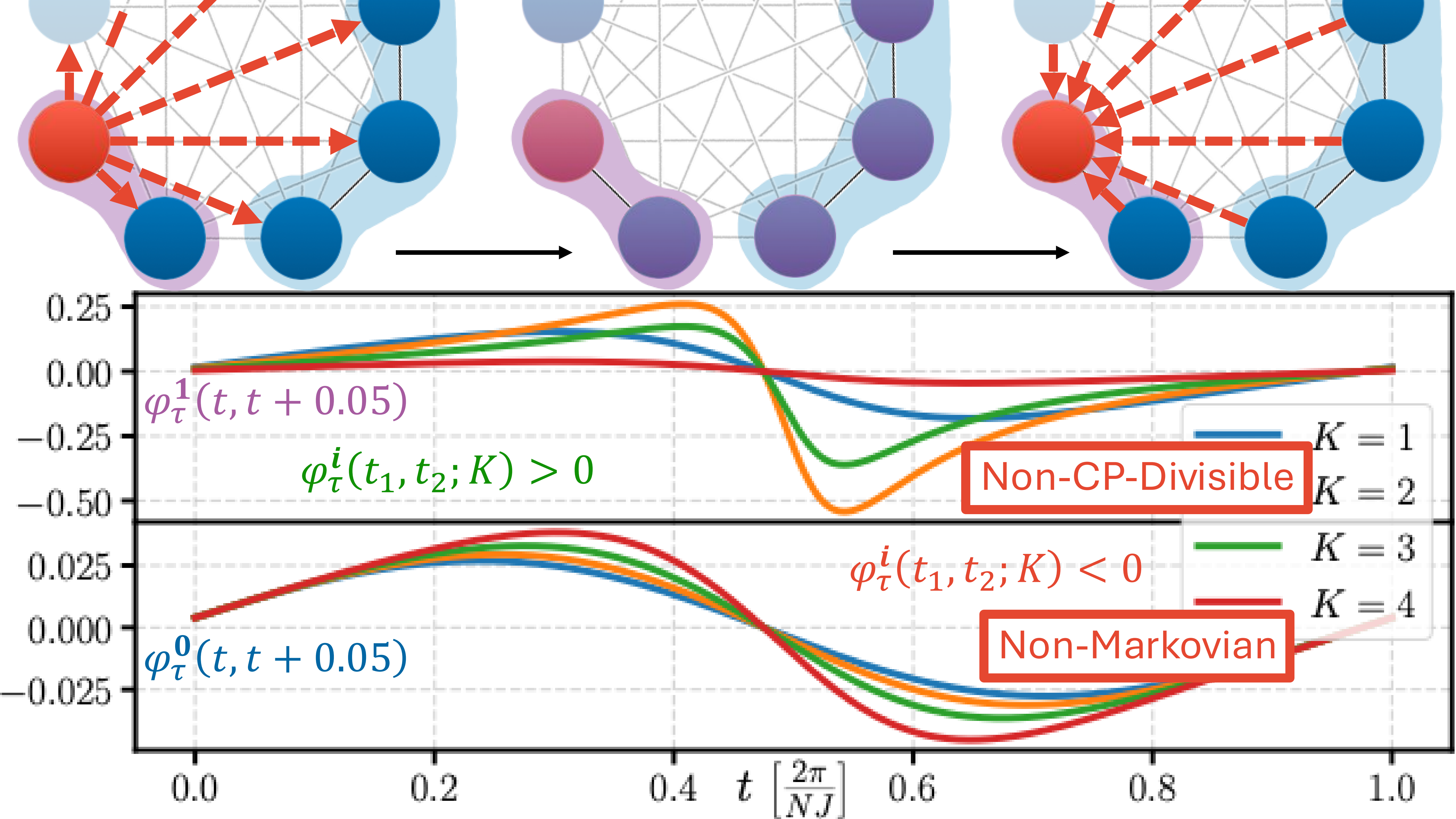
0.25
0.00
−0.25
−0.50
$\varphi_\tau^{\mathbf{1}}(t, t + 0.05)$
$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) > 0$
$K = 1$
$K = 2$
$K = 3$
$K = 4$
0.025
0.000
−0.025
$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) < 0$
$\varphi_\tau^{\mathbf{0}}(t, t + 0.05)$
0.0
0.2
0.4
$t\ \left[\frac{2\pi}{NJ}\right]$
0.6
0.8
1.0



$\varphi_\tau^{\mathbf{1}}(t, t + 0.05)$
$\varphi_\tau^{\mathbf{i}}(t_1, t_2; K) > 0$
$\varphi_\tau^{\mathbf{i}}(t_1, t_2; K) < 0$
$\varphi_\tau^{\mathbf{0}}(t, t + 0.05)$
$K = 1$
$K = 2$
$K = 3$
$K = 4$
0.25
0.00
−0.25
−0.50
0.025
0.000
−0.025
0.0
0.2
0.4
0.6
0.8
1.0
$t\ \left[\frac{2\pi}{NJ}\right]$



0.25
0.00
−0.25
−0.50
$\varphi_\tau^{\mathbf{1}}(t, t + 0.05)$
$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) > 0$
Non-CP-Divisible
$K = 1$
$K = 2$
$K = 3$
$K = 4$
0.025
0.000
−0.025
$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) < 0$
$\varphi_\tau^{\mathbf{0}}(t, t + 0.05)$
0.0
0.2
0.4
0.6
0.8
1.0
$t\ \left[\frac{2\pi}{NJ}\right]$



$\varphi_\tau^{\mathbf{1}}(t, t+0.05)$
$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) > 0$
Non-CP-Divisible
$K = 1$
$K = 2$
$K = 3$
$K = 4$
$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) < 0$
Non-Markovian
$\varphi_\tau^{\mathbf{0}}(t, t+0.05)$
0.25
0.00
−0.25
−0.50
0.025
0.000
−0.025
0.0
0.2
0.4
0.6
0.8
1.0
$t\ \left[\frac{2\pi}{NJ}\right]$

Pauli Basis ($K = 1$)

$$\hat{\rho} = \frac{1}{2}\left(\hat{\mathbb{I}} + b_x\, \hat{\sigma}^x + b_y\, \hat{\sigma}^y + b_z\, \hat{\sigma}^z \right)$$

Pauli Basis ($K = 1$)

$$\hat{\rho} = \frac{1}{2}\left(\hat{\mathbb{I}} + b_x\, \hat{\sigma}^x + b_y\, \hat{\sigma}^y + b_z\, \hat{\sigma}^z \right)$$

$$\vec{b} = \begin{pmatrix} 1 \\ b_x \\ b_y \\ b_z \end{pmatrix}$$

Pauli Basis $(K = 1)$

$$\hat{\rho} = \frac{1}{2}\left(\hat{\mathbb{I}} + b_x\, \hat{\sigma}^x + b_y\, \hat{\sigma}^y + b_z\, \hat{\sigma}^z \right)$$

$$\vec{b} = \begin{pmatrix} 1 \\ b_x \\ b_y \\ b_z \end{pmatrix} \qquad |\vec{b}| \leq 1$$

Pauli Basis ($K = 1$)

$$\hat{\rho} = \frac{1}{2}\left(\hat{\mathbb{I}} + b_x\, \hat{\sigma}^x + b_y\, \hat{\sigma}^y + b_z\, \hat{\sigma}^z \right)$$

$$\Lambda^{\mathbf{1}}(t_1, t_2)\vec{b} = \begin{pmatrix} 1 & & & \\ & |\varphi_s^{\mathbf{1}}| & & \\ & & |\varphi_s^{\mathbf{1}}| & \\ \varphi_\tau^{\mathbf{1}} & & & |\varphi_s^{\mathbf{1}}|^2 \end{pmatrix} \begin{pmatrix} 1 \\ b_x \\ b_y \\ b_z \end{pmatrix} \qquad |\vec{b}| \leq 1$$

$$\Lambda^{\mathbf{0}}(t_1, t_2)\vec{b} = \begin{pmatrix} 1 & & & \\ & |\varphi_s^{\mathbf{0}}| & & \\ & & |\varphi_s^{\mathbf{0}}| & \\ -\varphi_\tau^{\mathbf{0}} & & & \varphi_0^{\mathbf{0}} \end{pmatrix} \begin{pmatrix} 1 \\ b_x \\ b_y \\ b_z \end{pmatrix}$$

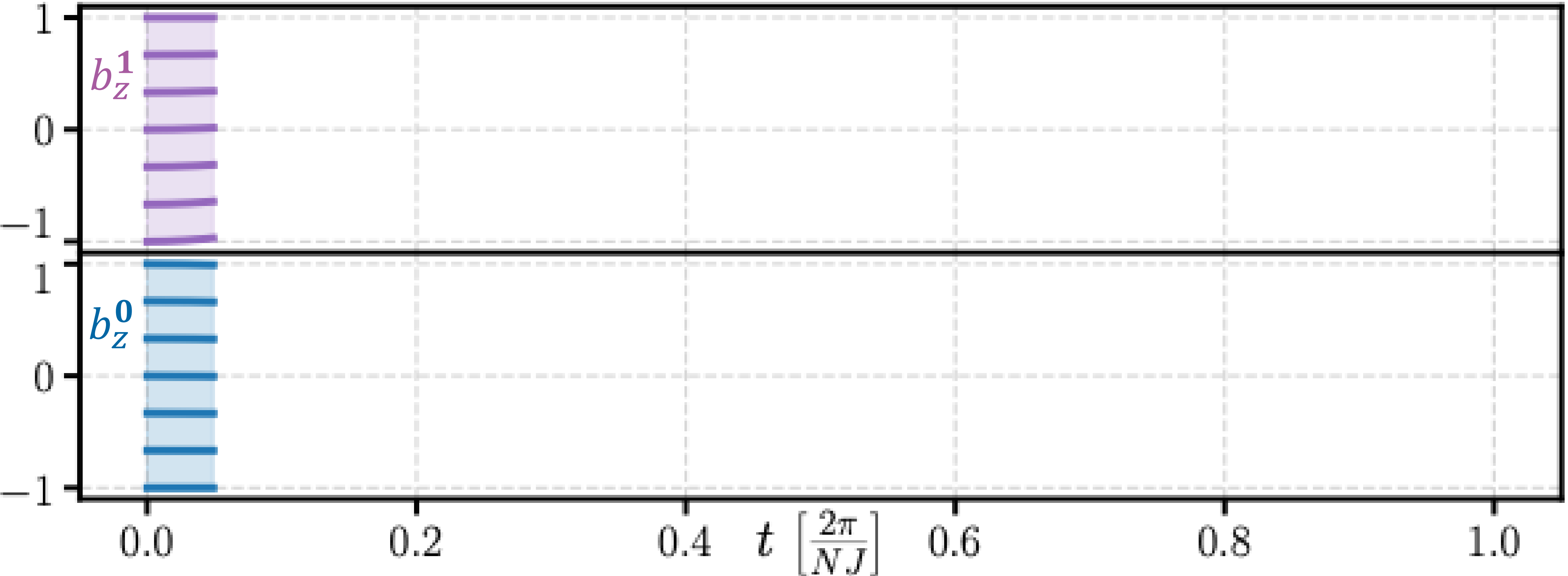
Pauli Basis ($K = 1$)

$$\hat{\rho} = \frac{1}{2}\left(\hat{\mathbb{I}} + b_x\,\hat{\sigma}^x + b_y\,\hat{\sigma}^y + b_z\,\hat{\sigma}^z \right)$$

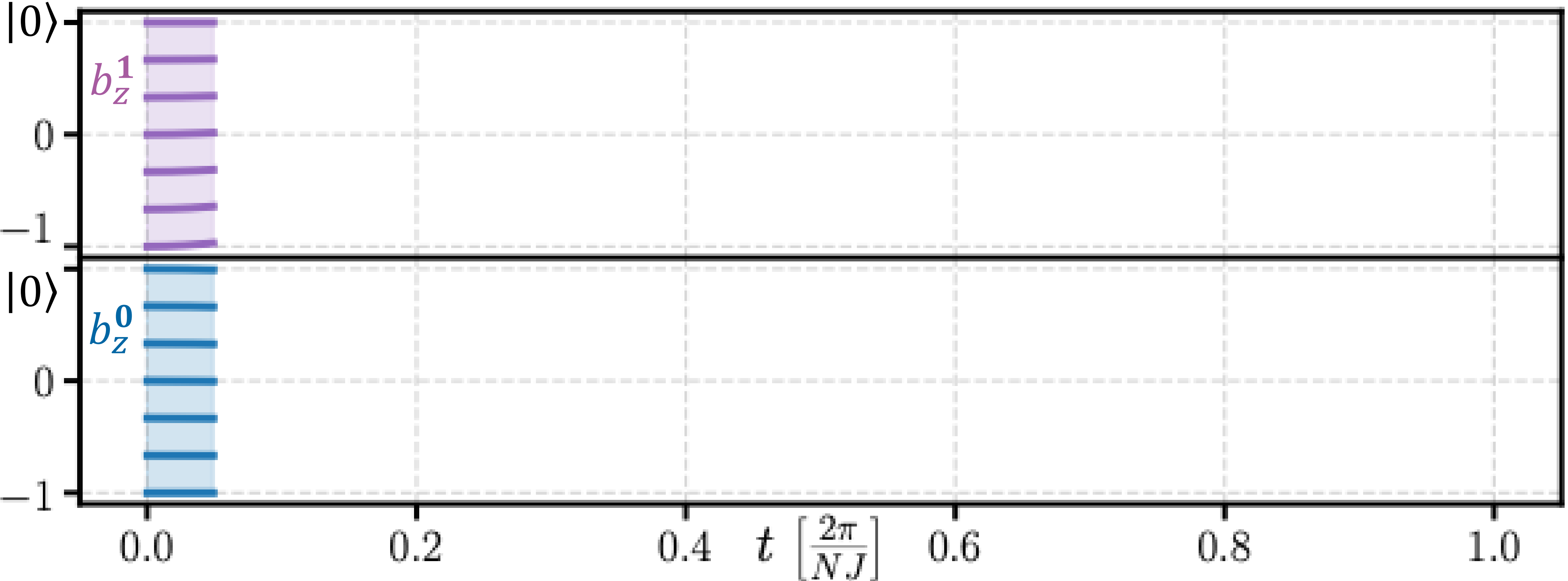
$$\Lambda^{\mathbf{1}}(0,t)\vec{b} = \begin{pmatrix} 1 & & & \\ & |\varphi_s^{\mathbf{1}}| & & \\ & & |\varphi_s^{\mathbf{1}}| & \\ \varphi_\tau^{\mathbf{1}} & & & |\varphi_s^{\mathbf{1}}|^2 \end{pmatrix} \begin{pmatrix} 1 \\ b_x \\ b_y \\ b_z \end{pmatrix} \qquad |\vec{b}| \leq 1$$

$$\Lambda^{\mathbf{0}}(0,t)\vec{b} = \begin{pmatrix} 1 & & & \\ & |\varphi_s^{\mathbf{0}}| & & \\ & & |\varphi_s^{\mathbf{0}}| & \\ -\varphi_\tau^{\mathbf{0}} & & & \varphi_0^{\mathbf{0}} \end{pmatrix} \begin{pmatrix} 1 \\ b_x \\ b_y \\ b_z \end{pmatrix}$$

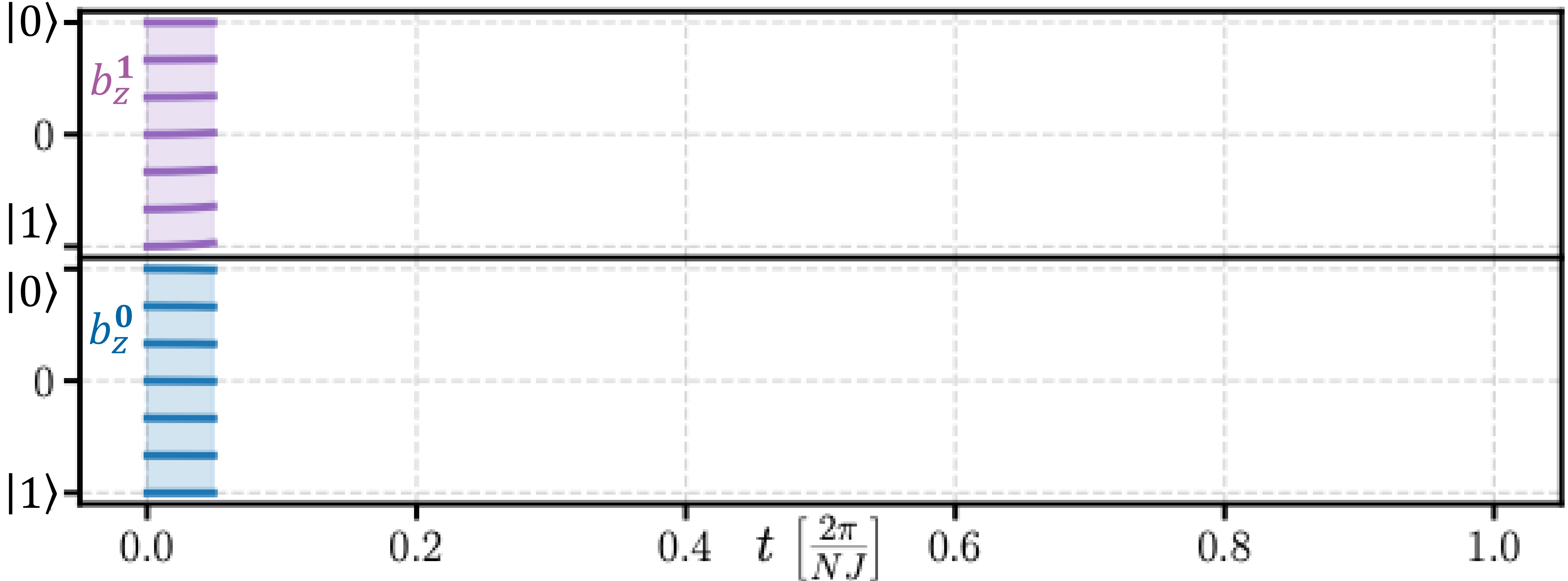
Pauli Basis ($K = 1$)



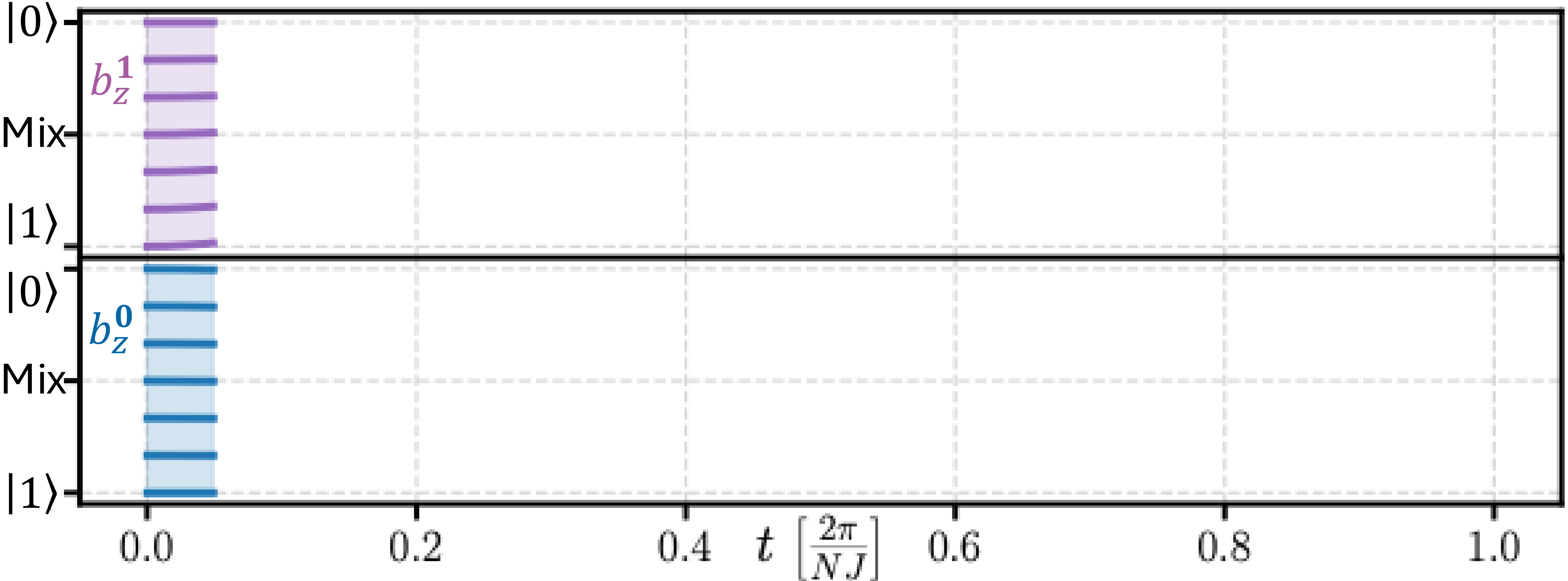
Pauli Basis ($K = 1$)

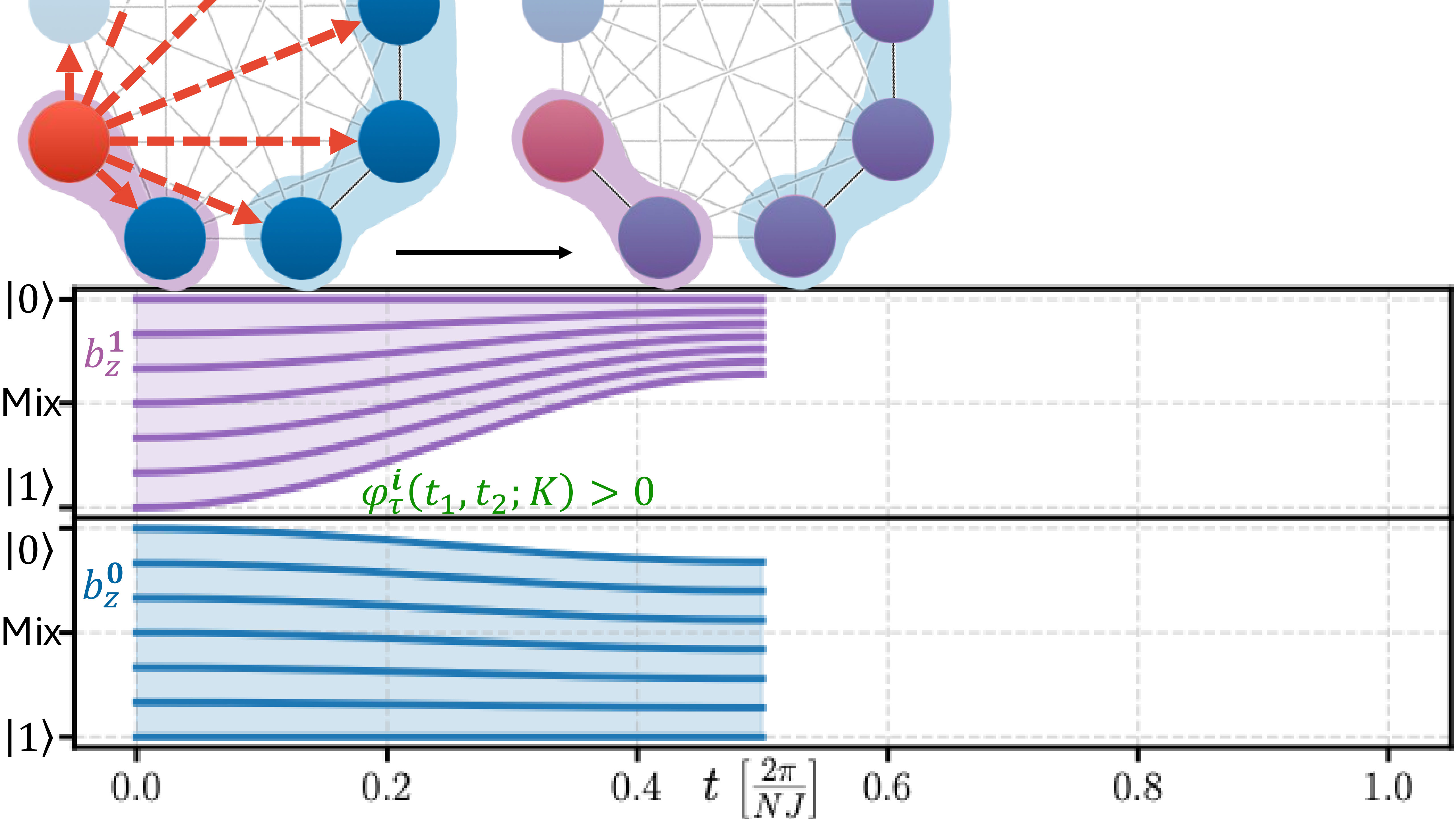


Pauli Basis ($K = 1$)

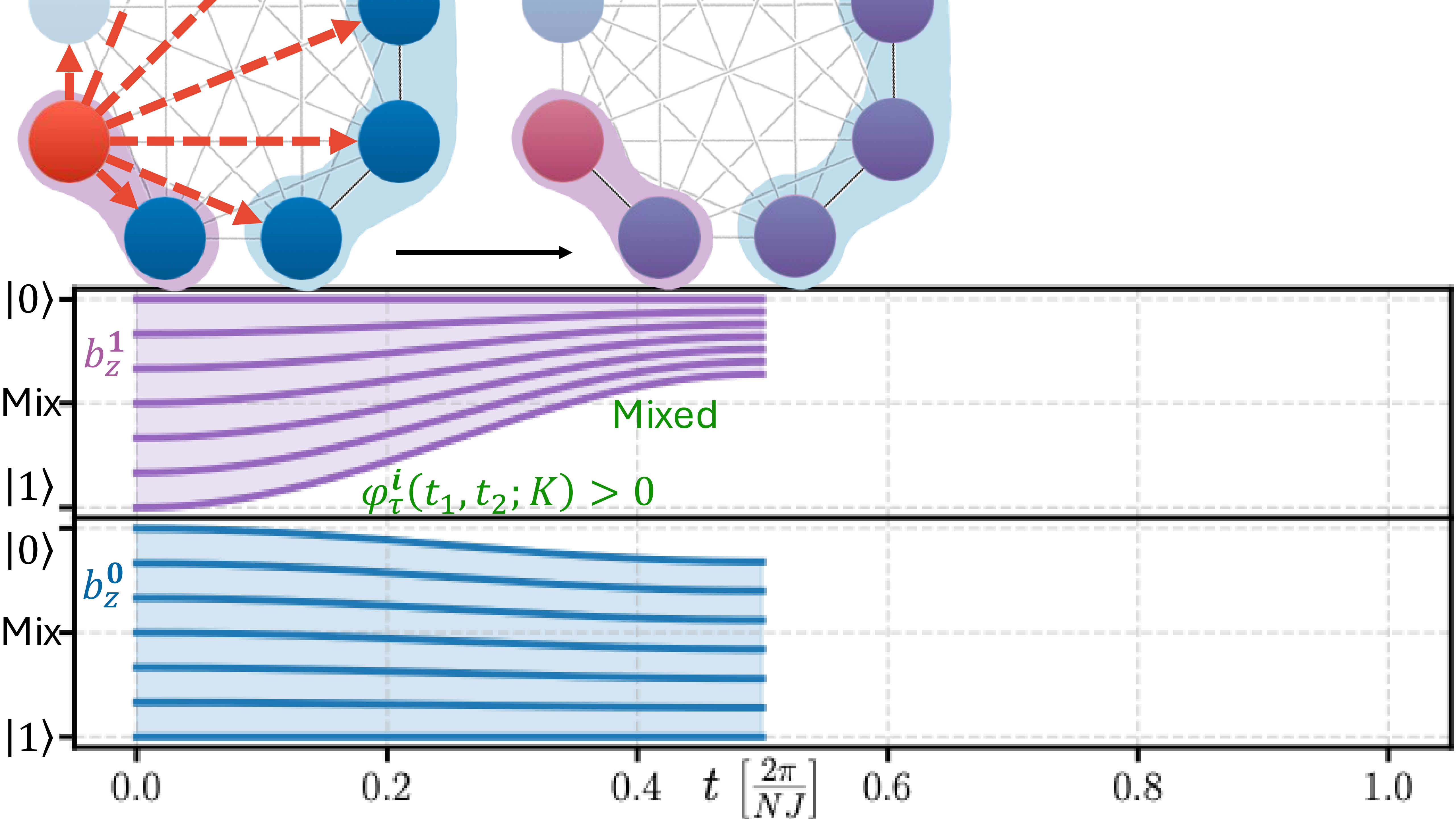


Pauli Basis ($K = 1$)

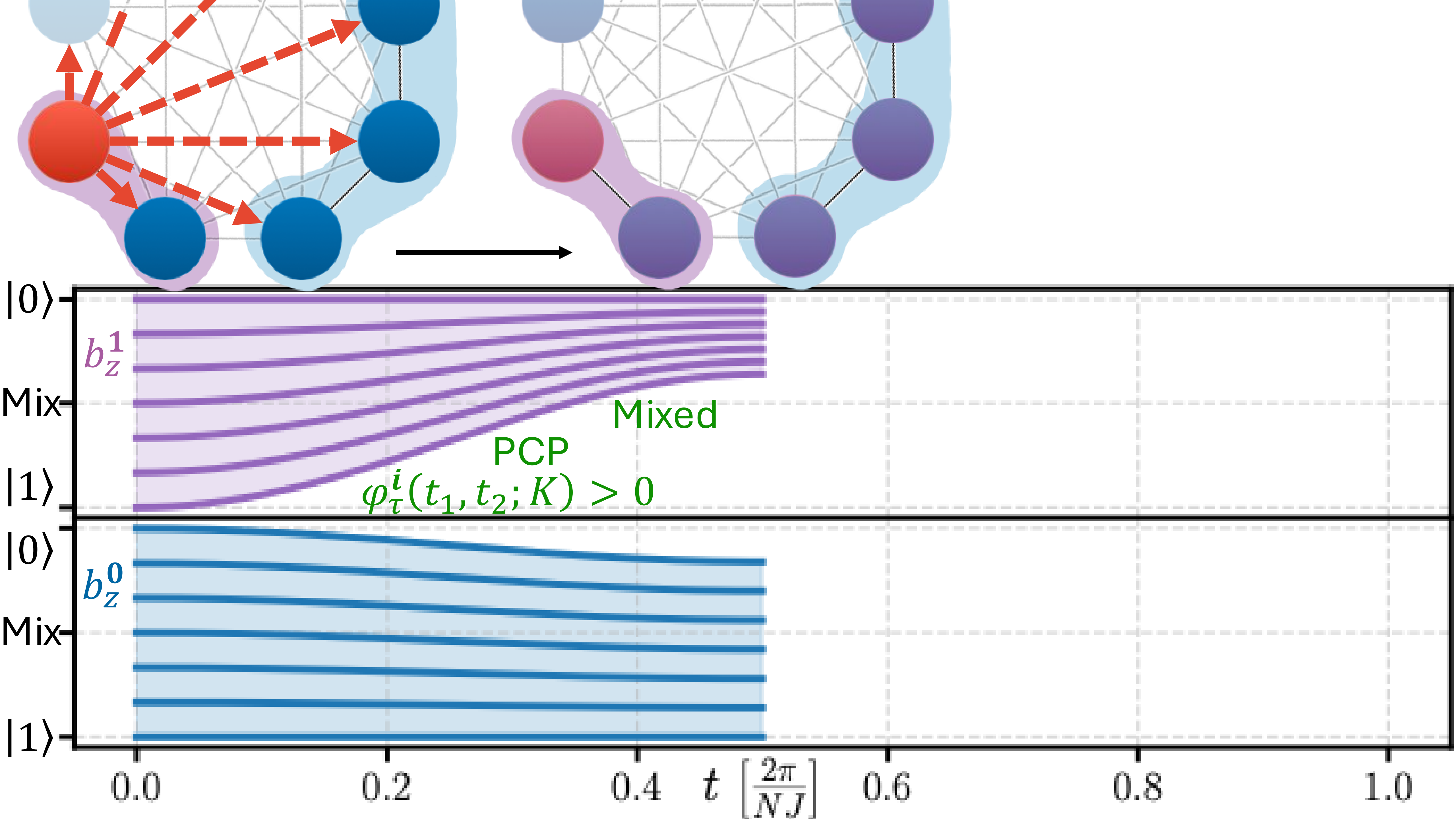




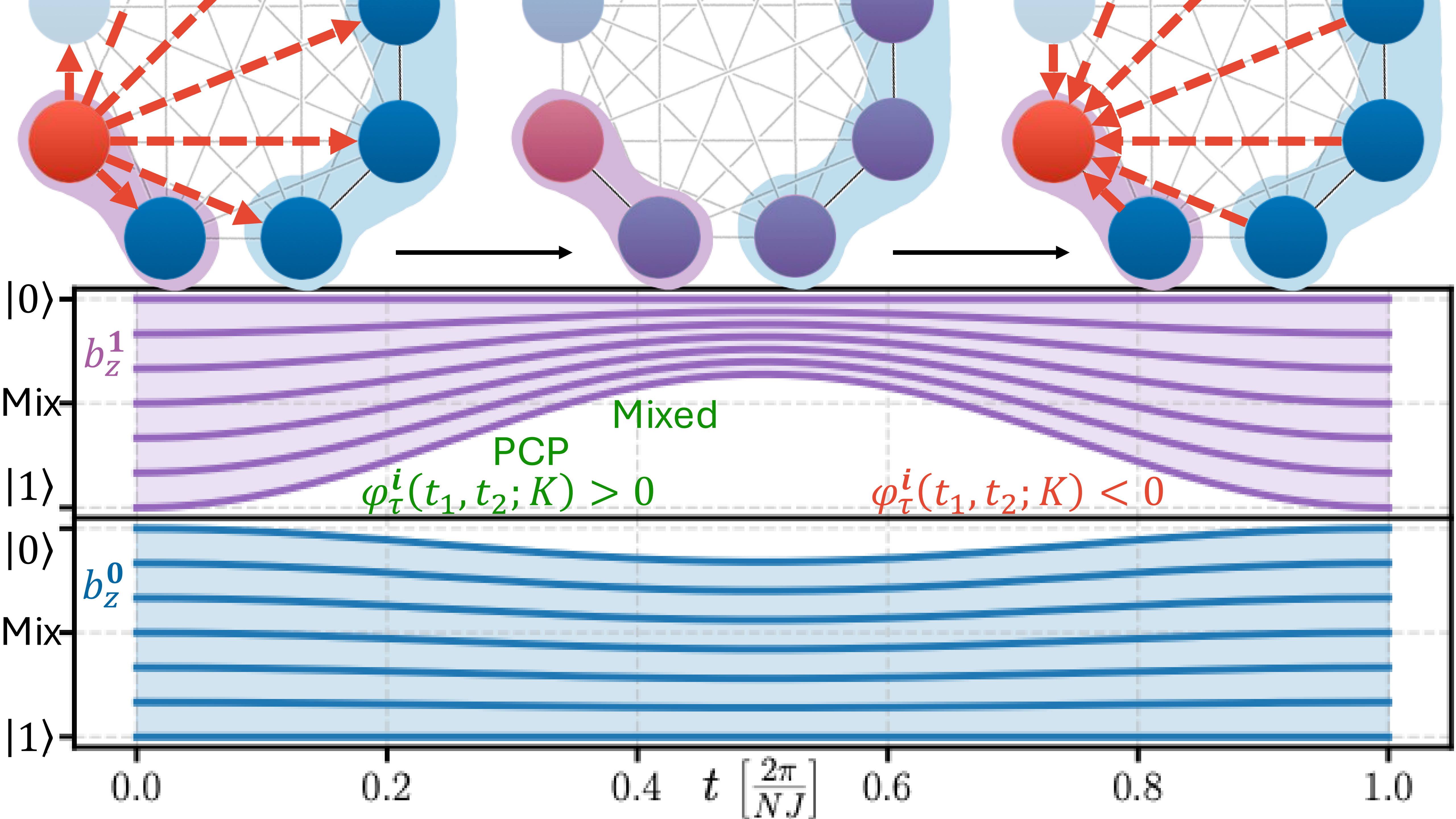
|0⟩
Mix
|1⟩
$b_z^{\mathbf{1}}$
$\varphi_\tau^{\mathbf{i}}(t_1, t_2; K) > 0$
|0⟩
Mix
|1⟩
$b_z^{\mathbf{0}}$
0.0
0.2
0.4
$t\ \left[\frac{2\pi}{NJ}\right]$
0.6
0.8
1.0



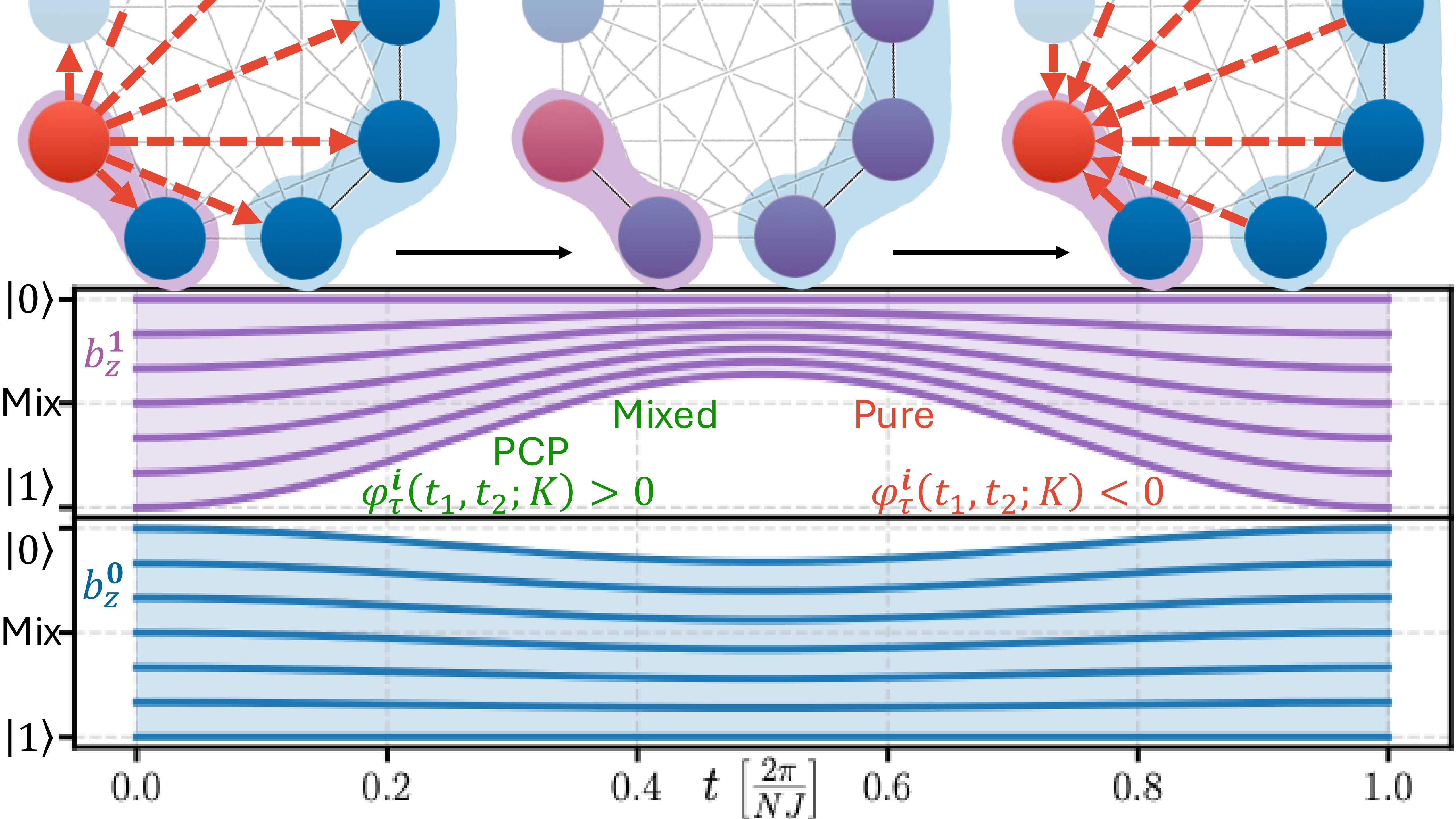
|0⟩
b_z^1
Mix
Mixed
|1⟩
$\varphi_\tau^i(t_1, t_2; K) > 0$
|0⟩
b_z^0
Mix
|1⟩
0.0
0.2
0.4
$t\ \left[\frac{2\pi}{NJ}\right]$
0.6
0.8
1.0



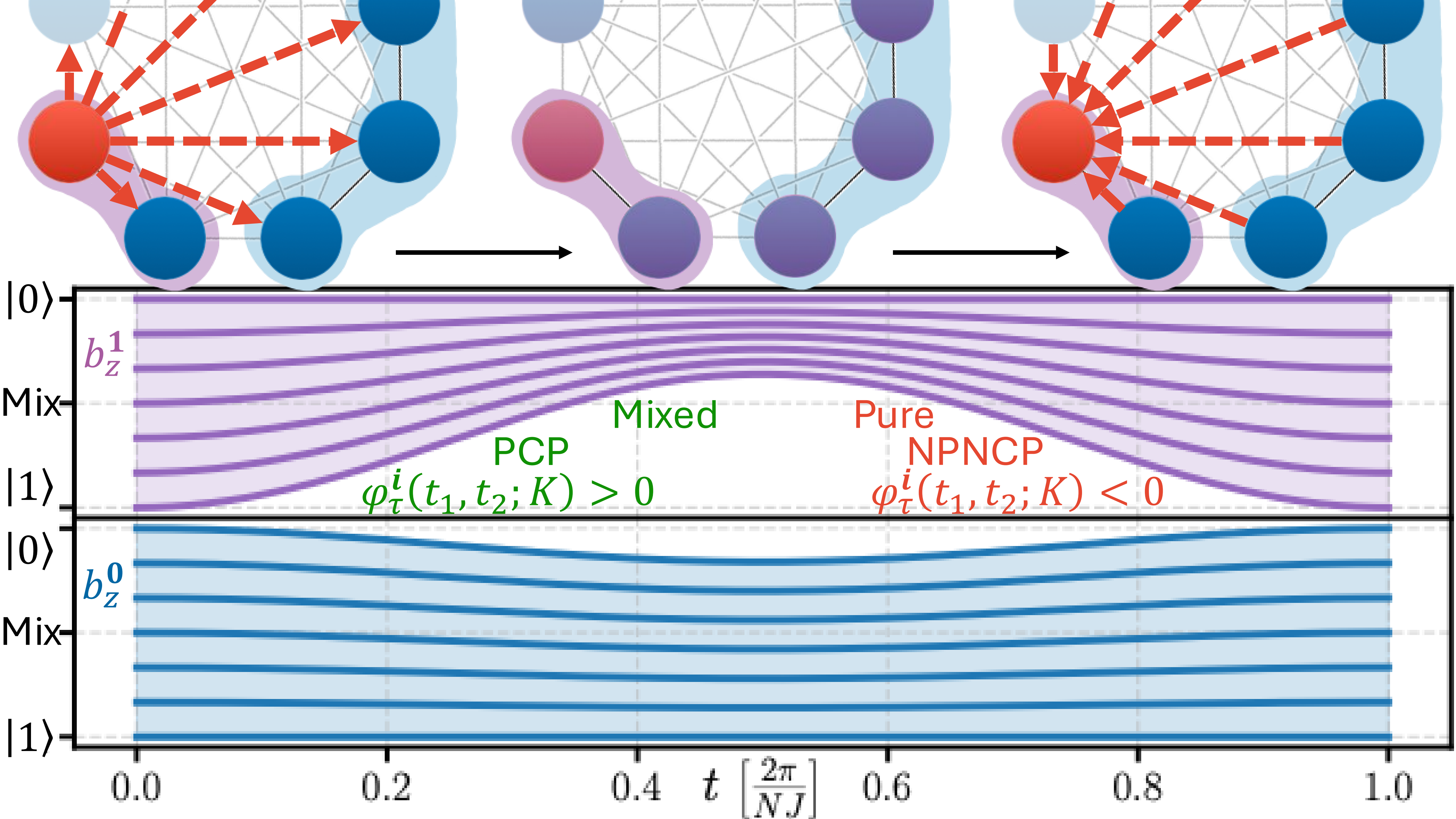
$|0\rangle$
b_z^1
Mix
$|1\rangle$
Mixed
PCP
$\varphi_\tau^i(t_1, t_2; K) > 0$
$|0\rangle$
b_z^0
Mix
$|1\rangle$
0.0
0.2
0.4
$t\ \left[\frac{2\pi}{NJ}\right]$
0.6
0.8
1.0



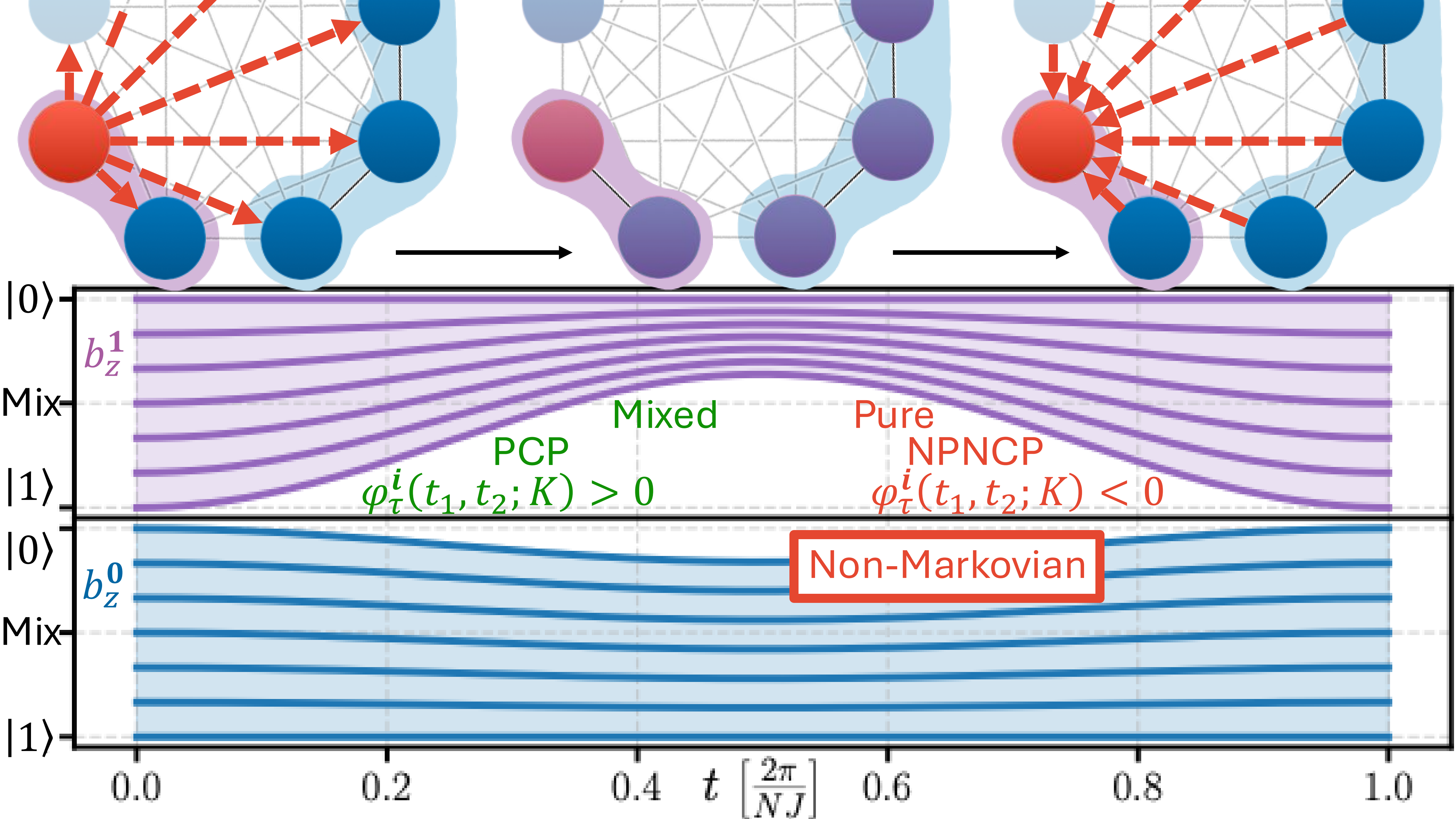
|0⟩
Mix
|1⟩
$b_z^{\mathbf{1}}$
Mixed
PCP
$\varphi_\tau^{\mathbf{i}}(t_1, t_2; K) > 0$
$\varphi_\tau^{\mathbf{i}}(t_1, t_2; K) < 0$
|0⟩
Mix
|1⟩
$b_z^{\mathbf{0}}$
0.0
0.2
0.4
$t\ \left[\frac{2\pi}{NJ}\right]$
0.6
0.8
1.0



|0⟩
Mix
|1⟩
$b_z^{\mathbf{1}}$
Mixed
Pure
PCP
$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) > 0$
$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) < 0$
|0⟩
Mix
|1⟩
$b_z^{\mathbf{0}}$
0.0
0.2
0.4
$t\ \left[\frac{2\pi}{NJ}\right]$
0.6
0.8
1.0



|0⟩
Mix
|1⟩
$b_z^{\mathbf{1}}$
Mixed
PCP
$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) > 0$
Pure
NPNCP
$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) < 0$
|0⟩
Mix
|1⟩
$b_z^{\mathbf{0}}$
0.0
0.2
0.4
$t\ \left[\frac{2\pi}{NJ}\right]$
0.6
0.8
1.0



|0⟩
Mix
|1⟩
$b_z^{\mathbf{1}}$
Mixed
PCP
$\varphi_\tau^{\mathbf{i}}(t_1, t_2; K) > 0$
Pure
NPNCP
$\varphi_\tau^{\mathbf{i}}(t_1, t_2; K) < 0$
|0⟩
Mix
|1⟩
$b_z^{\mathbf{0}}$
Non-Markovian
0.0
0.2
0.4
0.6
0.8
1.0
$t \left[\frac{2\pi}{NJ}\right]$

Domain of Positivity $(K = 1)$

$$\hat{\rho} = \frac{1}{2}\left(\hat{\mathbb{I}} + d_x\,\hat{\sigma}^x + d_y\,\hat{\sigma}^y + d_z\,\hat{\sigma}^z\right)$$

$$\Lambda^{\mathbf{1}}(t, t+\Delta t)\vec{d} = \begin{pmatrix} 1 & & & \\ & |\varphi_s^{\mathbf{1}}| & & \\ & & |\varphi_s^{\mathbf{1}}| & \\ \varphi_\tau^{\mathbf{1}} & & & |\varphi_s^{\mathbf{1}}|^2 \end{pmatrix} \begin{pmatrix} 1 \\ d_x \\ d_y \\ b_z \end{pmatrix}$$

$$\Lambda^{\mathbf{0}}(t, t+\Delta t)\vec{d} = \begin{pmatrix} 1 & & & \\ & |\varphi_s^{\mathbf{0}}| & & \\ & & |\varphi_s^{\mathbf{0}}| & \\ -\varphi_\tau^{\mathbf{0}} & & & \varphi_0^{\mathbf{0}} \end{pmatrix} \begin{pmatrix} 1 \\ d_x \\ d_y \\ d_z \end{pmatrix}$$

Domain of Positivity $(K = 1)$

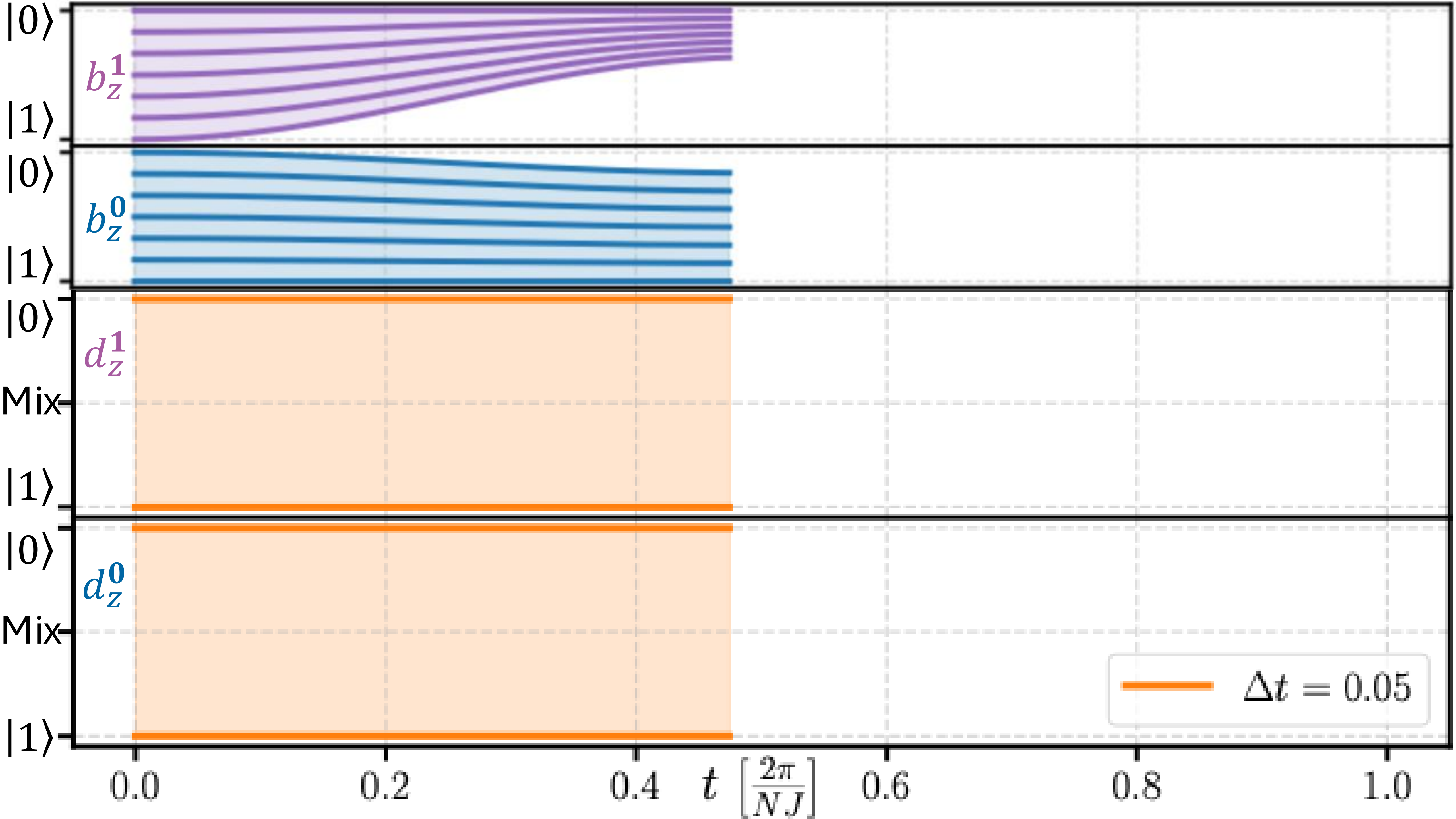
$$\hat{\rho} = \frac{1}{2}\left(\hat{\mathbb{I}} + d_x\, \hat{\sigma}^x + d_y\, \hat{\sigma}^y + d_z\, \hat{\sigma}^z \right)$$

$$\Lambda^{\mathbf{1}}(t, t+\Delta t)\vec{d} = \begin{pmatrix} 1 & & & \\ & |\varphi_s^{\mathbf{1}}| & & \\ & & |\varphi_s^{\mathbf{1}}| & \\ \varphi_\tau^{\mathbf{1}} & & & |\varphi_s^{\mathbf{1}}|^2 \end{pmatrix} \begin{pmatrix} 1 \\ d_x \\ d_y \\ b_z \end{pmatrix}$$

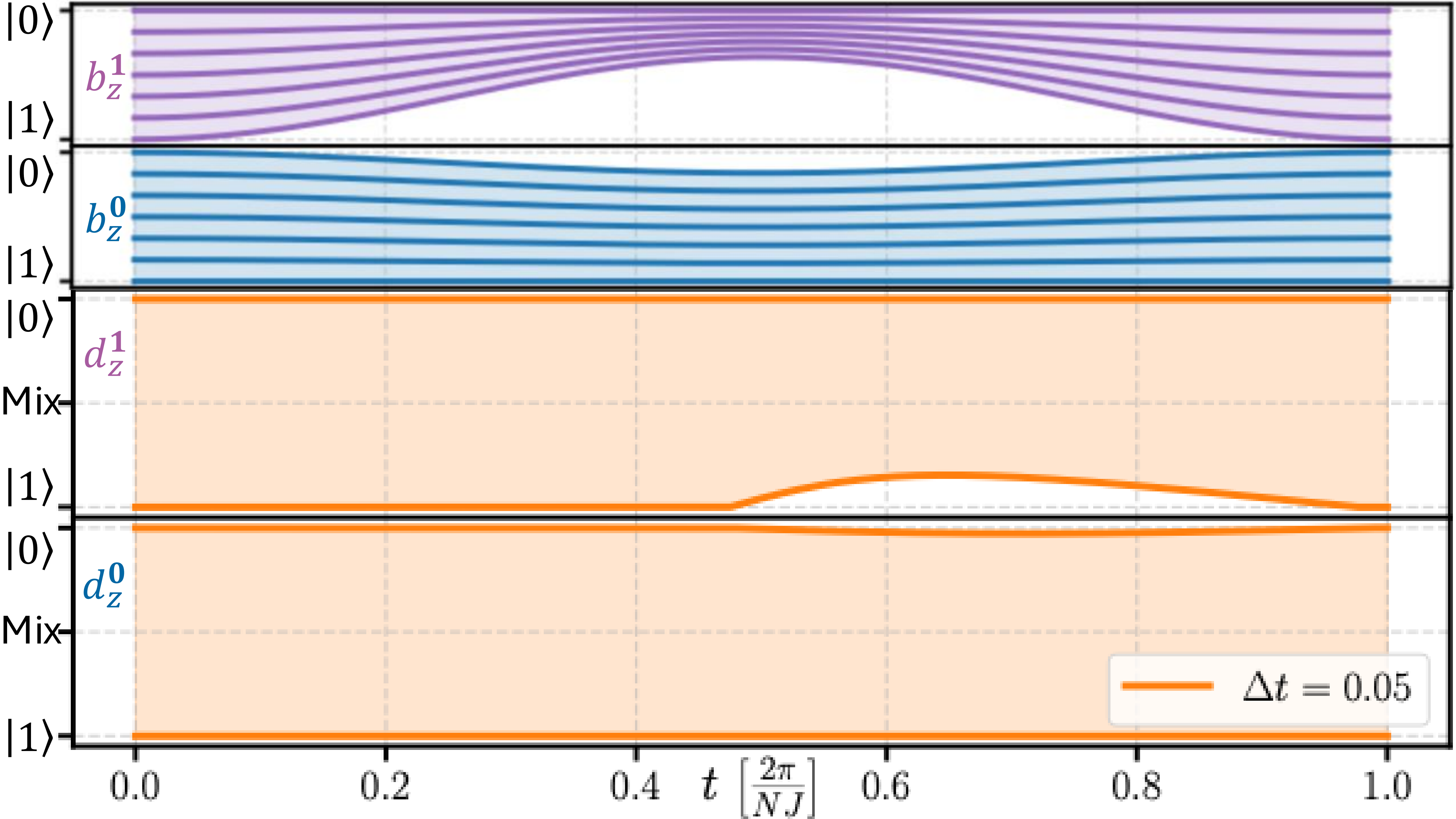
$$\Lambda^{\mathbf{0}}(t, t+\Delta t)\vec{d} = \begin{pmatrix} 1 & & & \\ & |\varphi_s^{\mathbf{0}}| & & \\ & & |\varphi_s^{\mathbf{0}}| & \\ -\varphi_\tau^{\mathbf{0}} & & & \varphi_0^{\mathbf{0}} \end{pmatrix} \begin{pmatrix} 1 \\ d_x \\ d_y \\ d_z \end{pmatrix}$$

$$|\, \Lambda^{\boldsymbol{i}}\, \vec{d}^{\boldsymbol{i}} \,| \leq 1$$

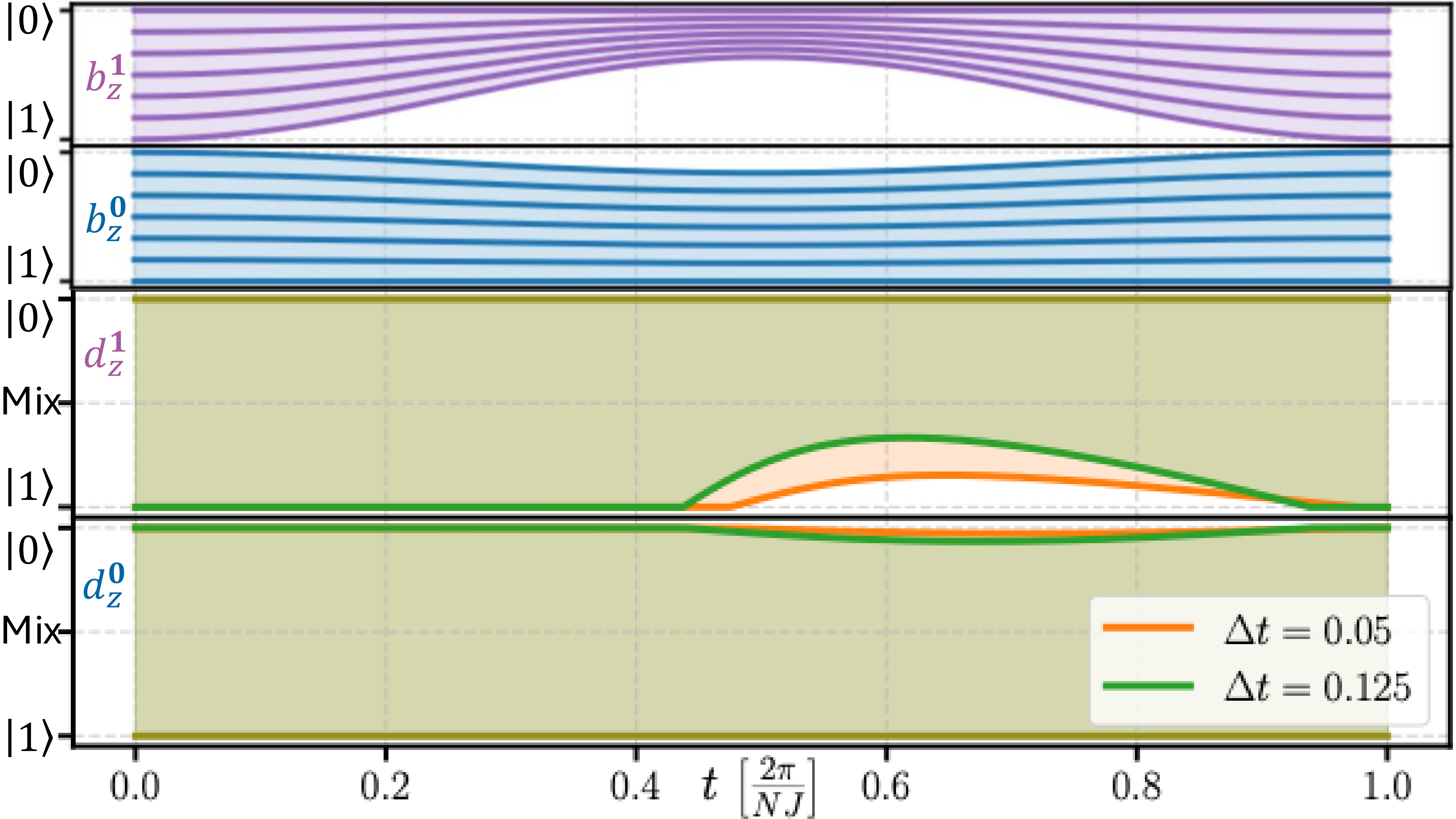
Domain Mapped
to Bloch Ball



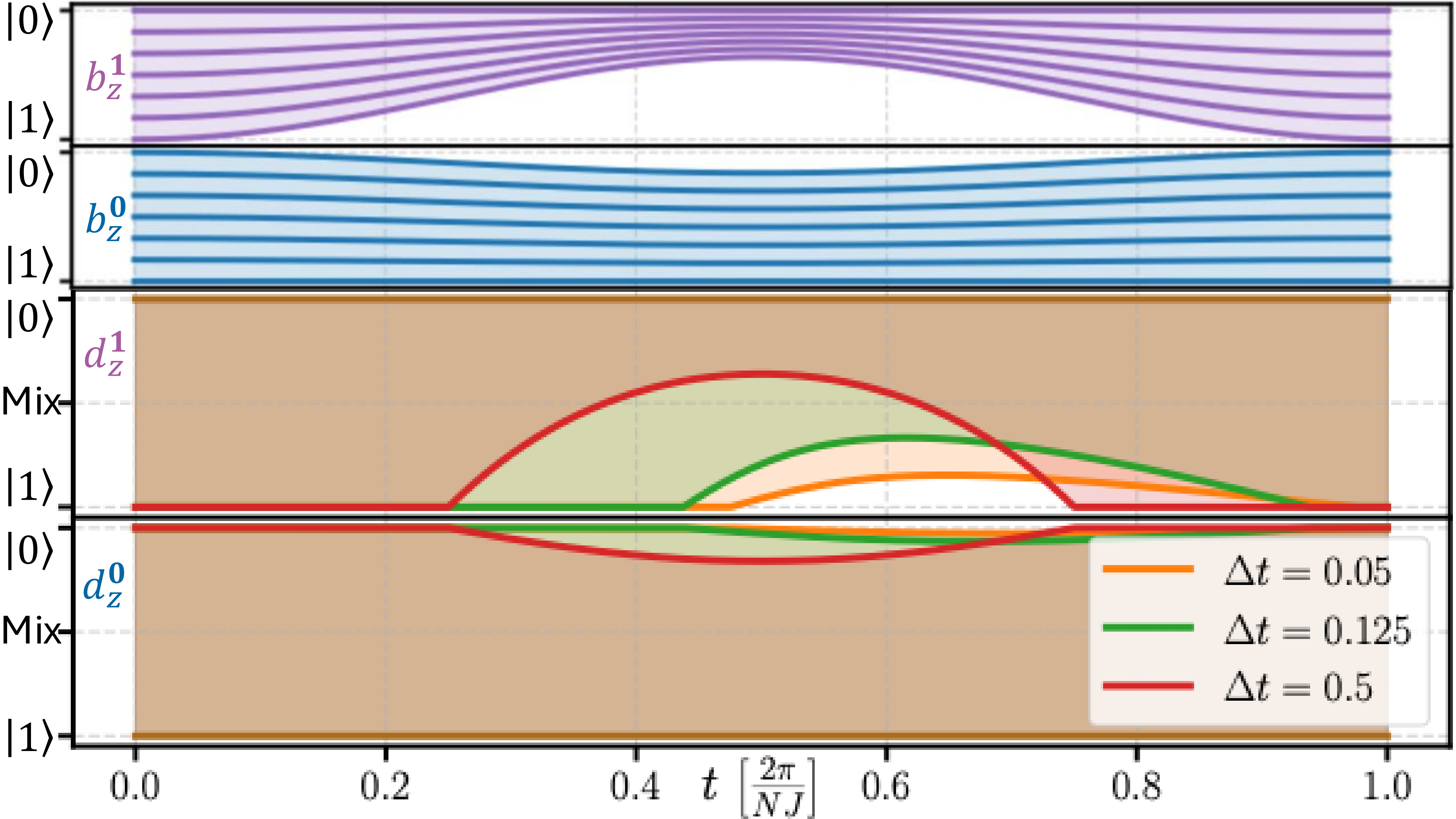
|0⟩
$b_z^{\mathbf{1}}$
|1⟩
|0⟩
$b_z^{\mathbf{0}}$
|1⟩
|0⟩
$d_z^{\mathbf{1}}$
Mix
|1⟩
|0⟩
$d_z^{\mathbf{0}}$
Mix
|1⟩
$\Delta t = 0.05$
0.0
0.2
0.4
$t\ \left[\frac{2\pi}{NJ}\right]$
0.6
0.8
1.0



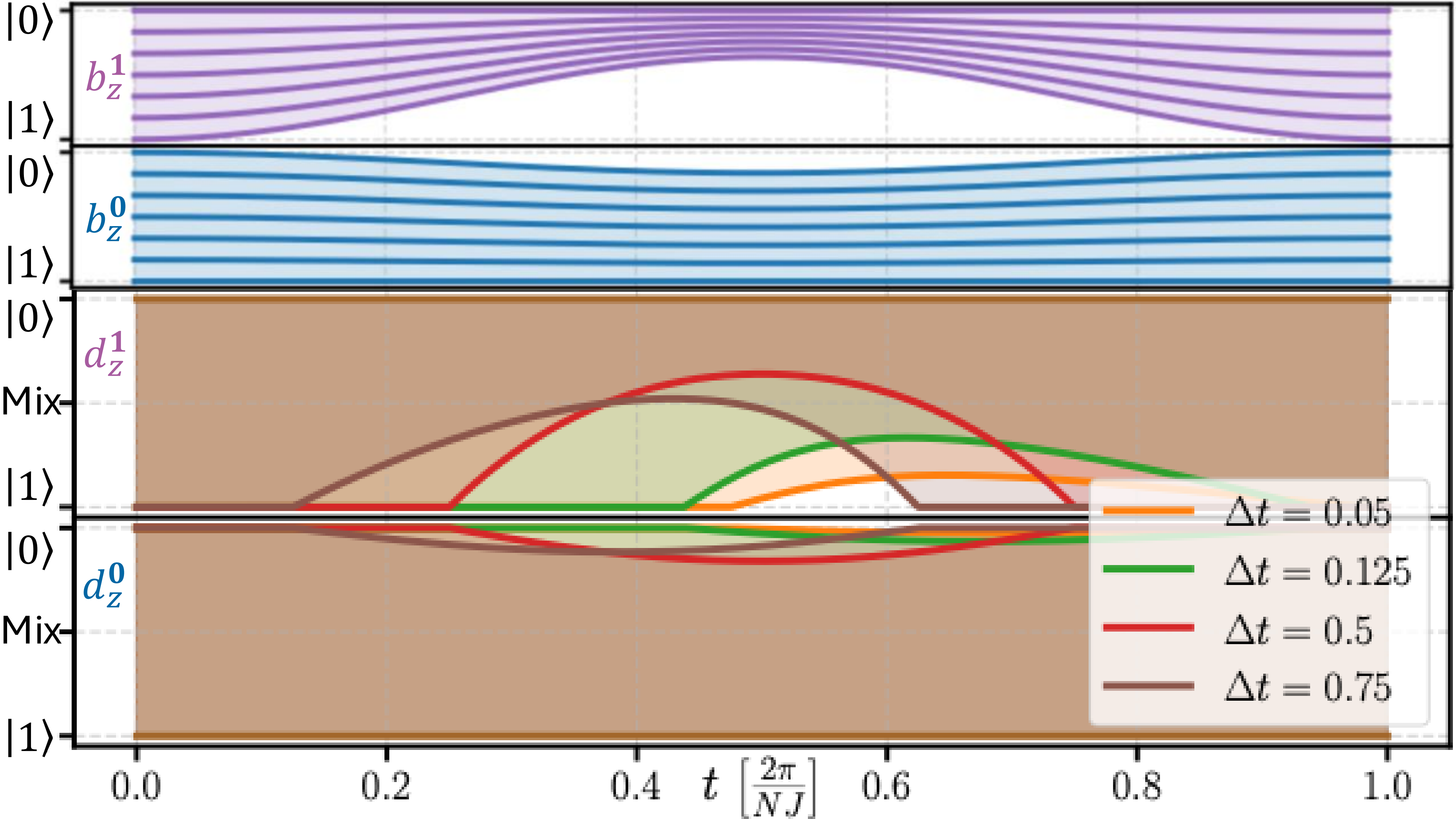
|0⟩
|1⟩
b_z^1
b_z^0
d_z^1
d_z^0
Mix
$\Delta t = 0.05$
0.0
0.2
0.4
0.6
0.8
1.0
$t\ \left[\frac{2\pi}{NJ}\right]$



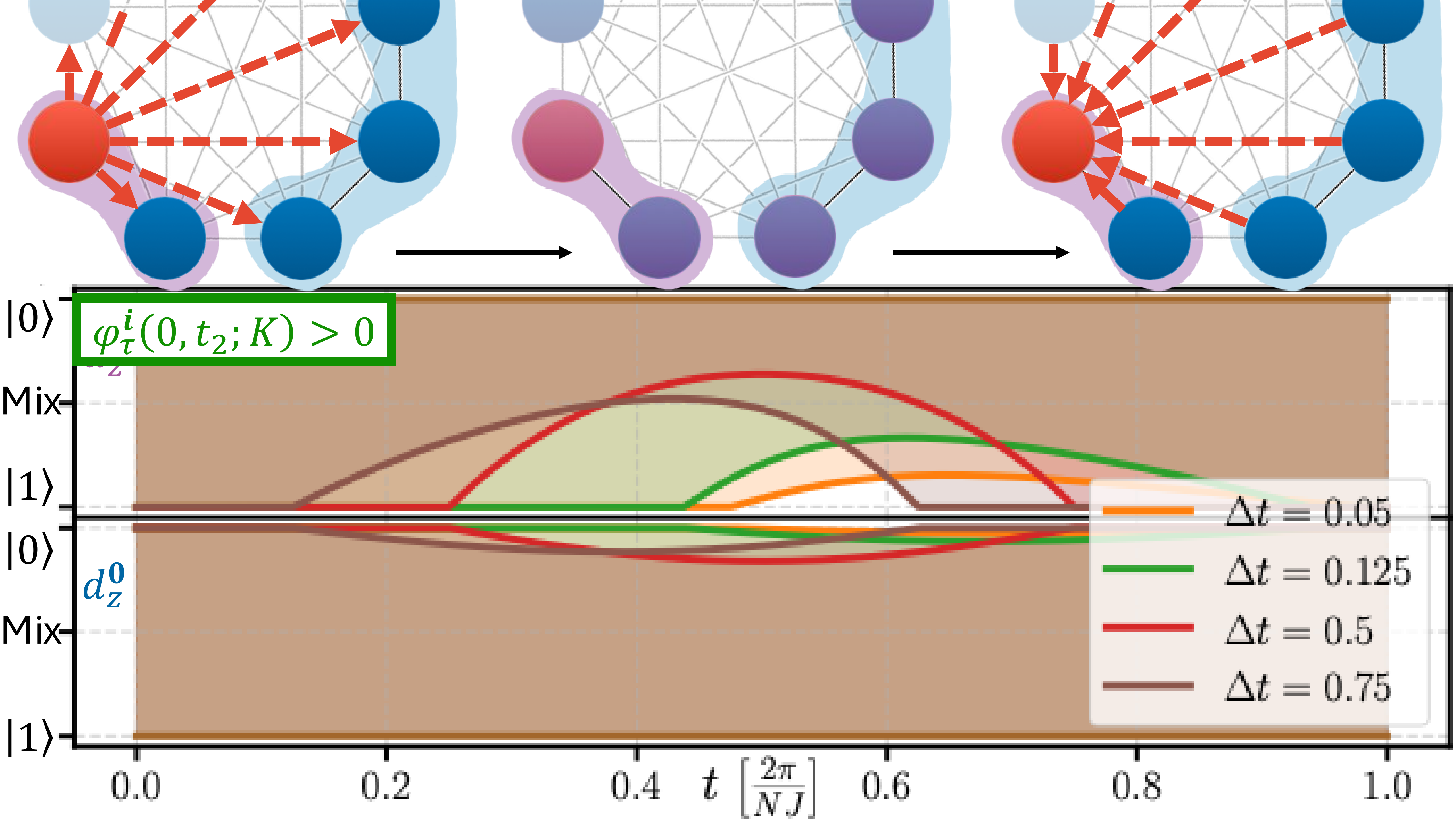
$|0\rangle$
b_z^1
$|1\rangle$
$|0\rangle$
b_z^0
$|1\rangle$
$|0\rangle$
d_z^1
Mix
$|1\rangle$
$|0\rangle$
d_z^0
Mix
$|1\rangle$
$\Delta t = 0.05$
$\Delta t = 0.125$
0.0
0.2
0.4
$t\ \left[\frac{2\pi}{NJ}\right]$
0.6
0.8
1.0



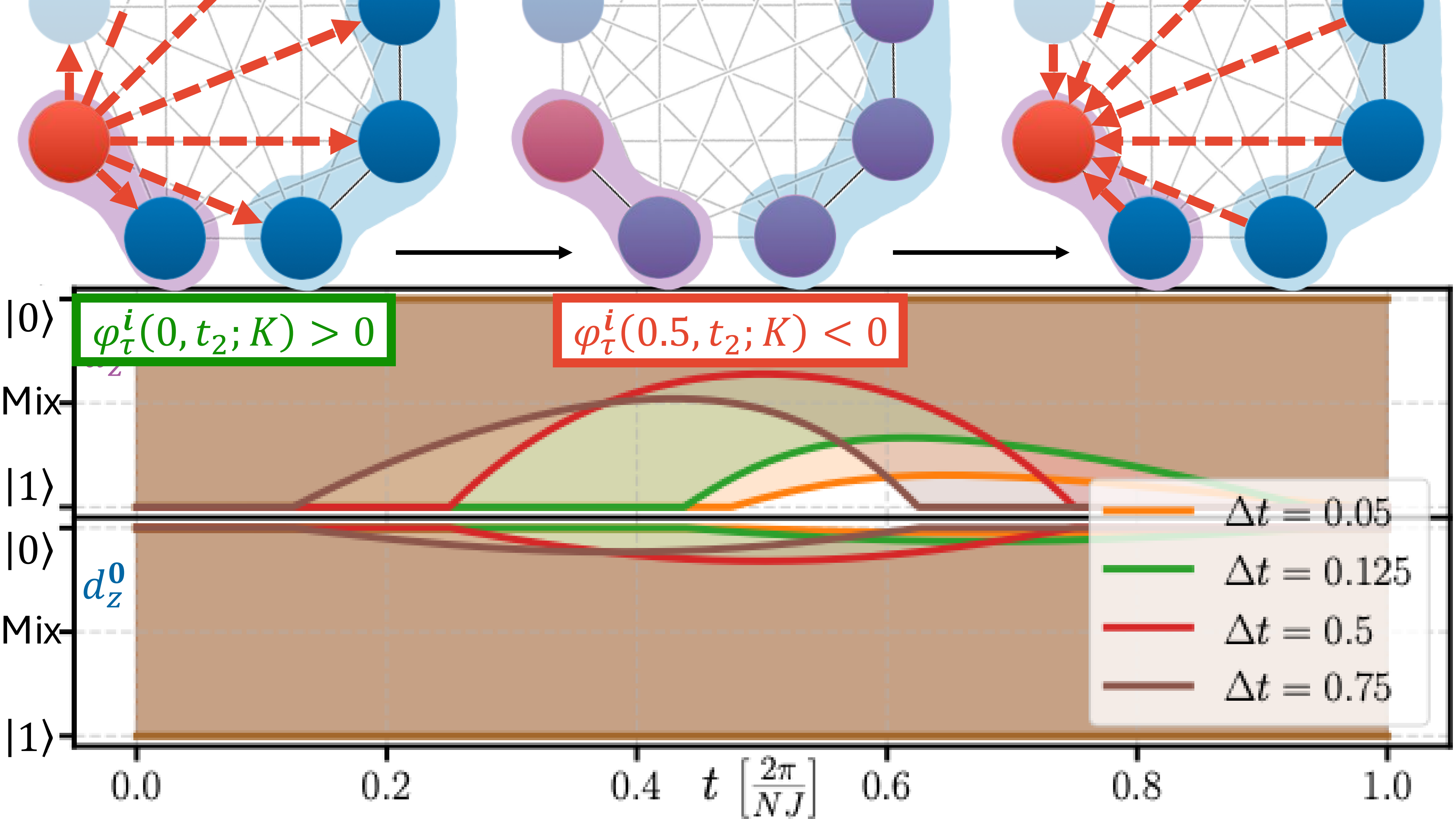
$|0\rangle$
$b_z^{\mathbf{1}}$
$|1\rangle$
$|0\rangle$
$b_z^{\mathbf{0}}$
$|1\rangle$
$|0\rangle$
$d_z^{\mathbf{1}}$
Mix
$|1\rangle$
$|0\rangle$
$d_z^{\mathbf{0}}$
Mix
$|1\rangle$
$\Delta t = 0.05$
$\Delta t = 0.125$
$\Delta t = 0.5$
0.0
0.2
0.4
$t\ \left[\frac{2\pi}{NJ}\right]$
0.6
0.8
1.0



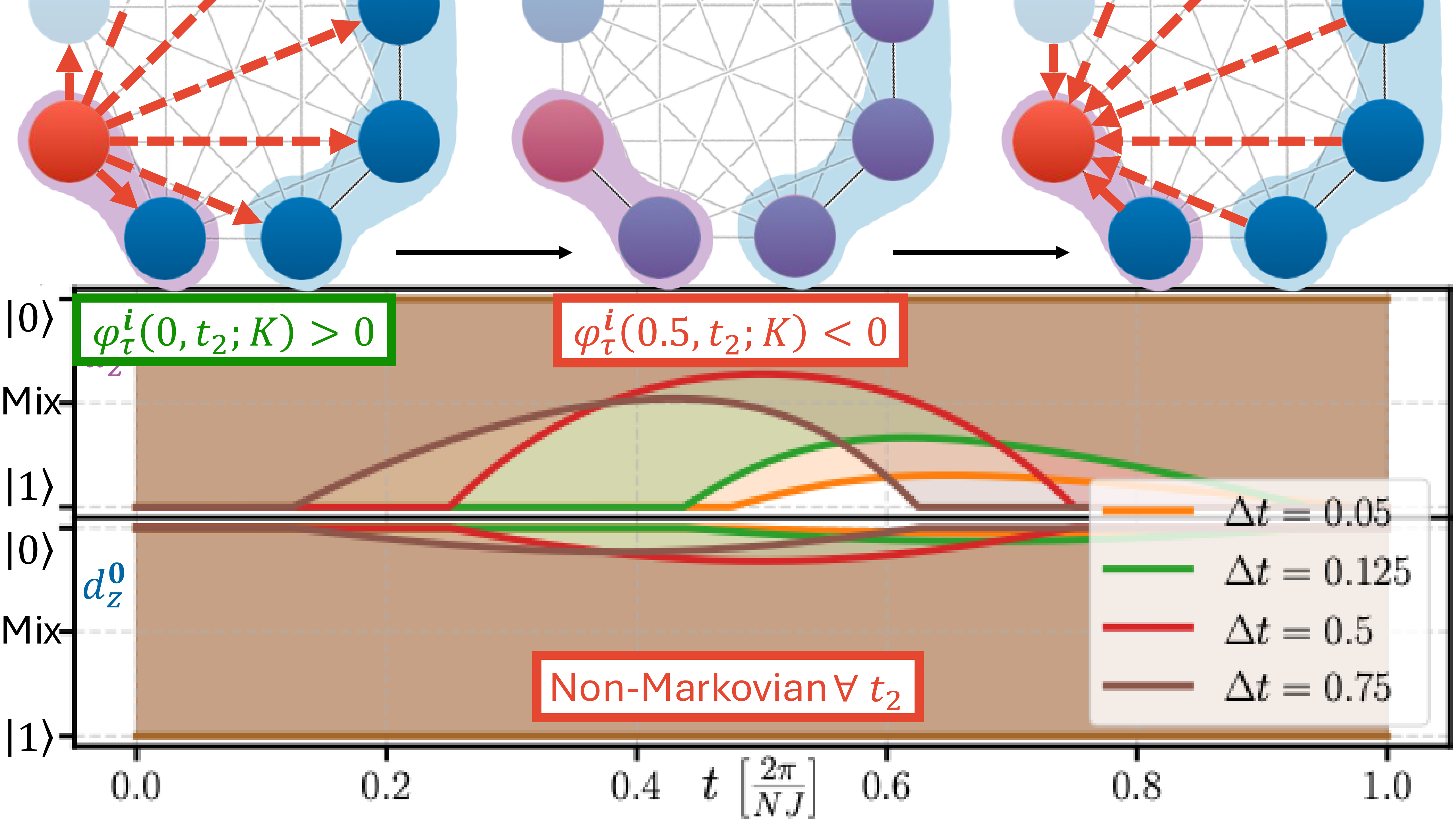
$|0\rangle$
$b_z^{\mathbf{1}}$
$|1\rangle$
$|0\rangle$
$b_z^{\mathbf{0}}$
$|1\rangle$
$|0\rangle$
$d_z^{\mathbf{1}}$
Mix
$|1\rangle$
$|0\rangle$
$d_z^{\mathbf{0}}$
Mix
$|1\rangle$
$\Delta t = 0.05$
$\Delta t = 0.125$
$\Delta t = 0.5$
$\Delta t = 0.75$
0.0
0.2
0.4
0.6
0.8
1.0
$t\ \left[\frac{2\pi}{NJ}\right]$



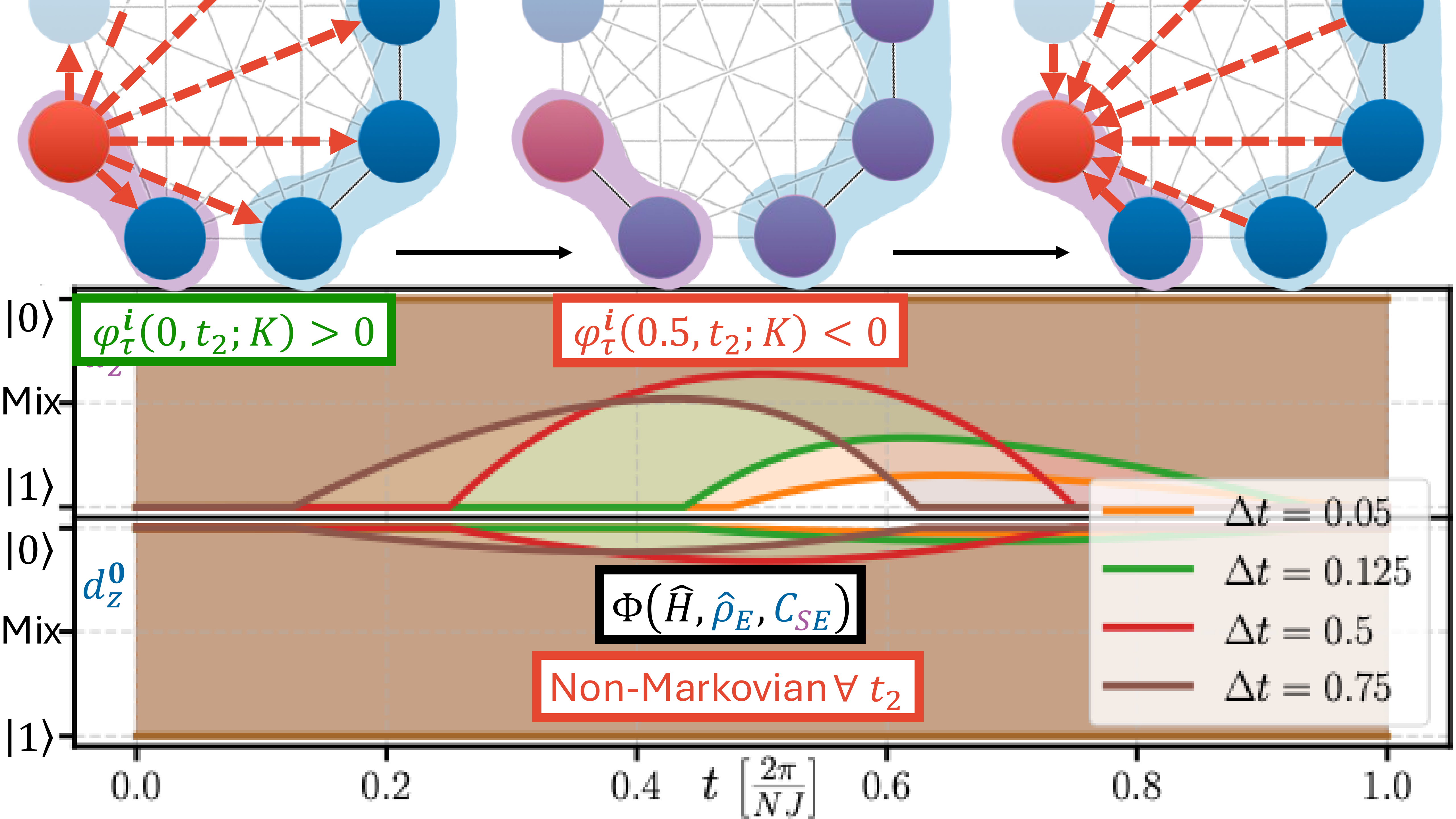
$\varphi_\tau^i(0, t_2; K) > 0$
$|0\rangle$
Mix
$|1\rangle$
d_z^0
$\Delta t = 0.05$
$\Delta t = 0.125$
$\Delta t = 0.5$
$\Delta t = 0.75$
0.0
0.2
0.4
0.6
0.8
1.0
$t\ \left[\frac{2\pi}{NJ}\right]$



$|0\rangle$
$\varphi_\tau^{i}(0,t_2;K) > 0$
$\varphi_\tau^{i}(0.5,t_2;K) < 0$
Mix
$|1\rangle$
$|0\rangle$
d_z^{0}
Mix
$|1\rangle$
$\Delta t = 0.05$
$\Delta t = 0.125$
$\Delta t = 0.5$
$\Delta t = 0.75$
0.0
0.2
0.4
$t\ \left[\frac{2\pi}{NJ}\right]$
0.6
0.8
1.0



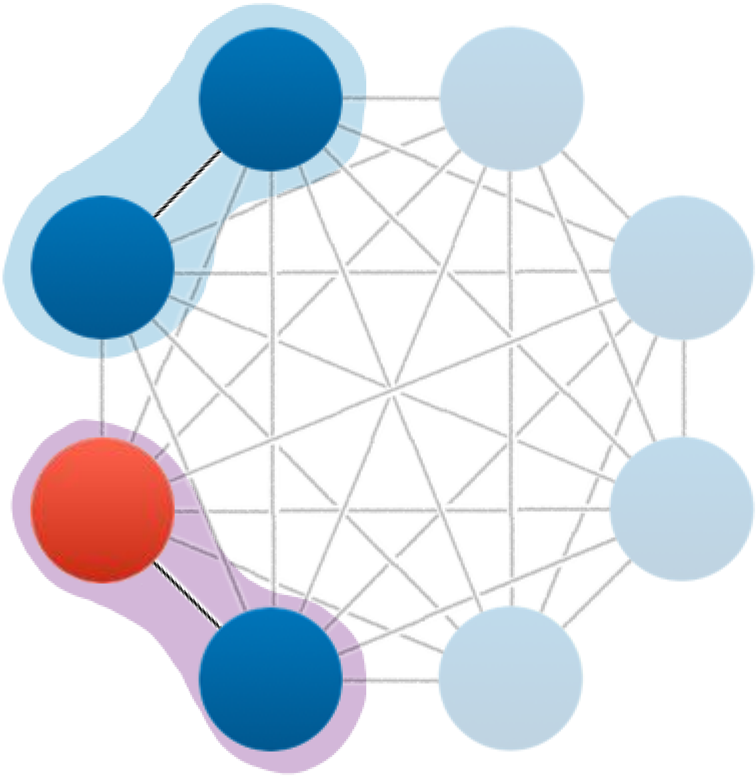
$|0\rangle$
$\varphi^{i}_{\tau}(0, t_2; K) > 0$
$\varphi^{i}_{\tau}(0.5, t_2; K) < 0$
Mix
$|1\rangle$
$|0\rangle$
$d_z^{\mathbf{0}}$
Mix
Non-Markovian $\forall\, t_2$
$|1\rangle$
$\Delta t = 0.05$
$\Delta t = 0.125$
$\Delta t = 0.5$
$\Delta t = 0.75$
0.0
0.2
0.4
$t\ \left[\frac{2\pi}{NJ}\right]$
0.6
0.8
1.0



$\varphi_\tau^{\boldsymbol{i}}(0,t_2;K)>0$
$\varphi_\tau^{\boldsymbol{i}}(0.5,t_2;K)<0$
|0⟩
Mix
|1⟩
|0⟩
$d_z^{\boldsymbol{0}}$
Mix
|1⟩
$\Phi\left(\widehat{H},\hat{\rho}_E,C_{SE}\right)$
Non-Markovian $\forall\ t_2$
$\Delta t=0.05$
$\Delta t=0.125$
$\Delta t=0.5$
$\Delta t=0.75$
0.0
0.2
0.4
$t\ \left[\frac{2\pi}{NJ}\right]$
0.6
0.8
1.0

Inference of Global Parameters

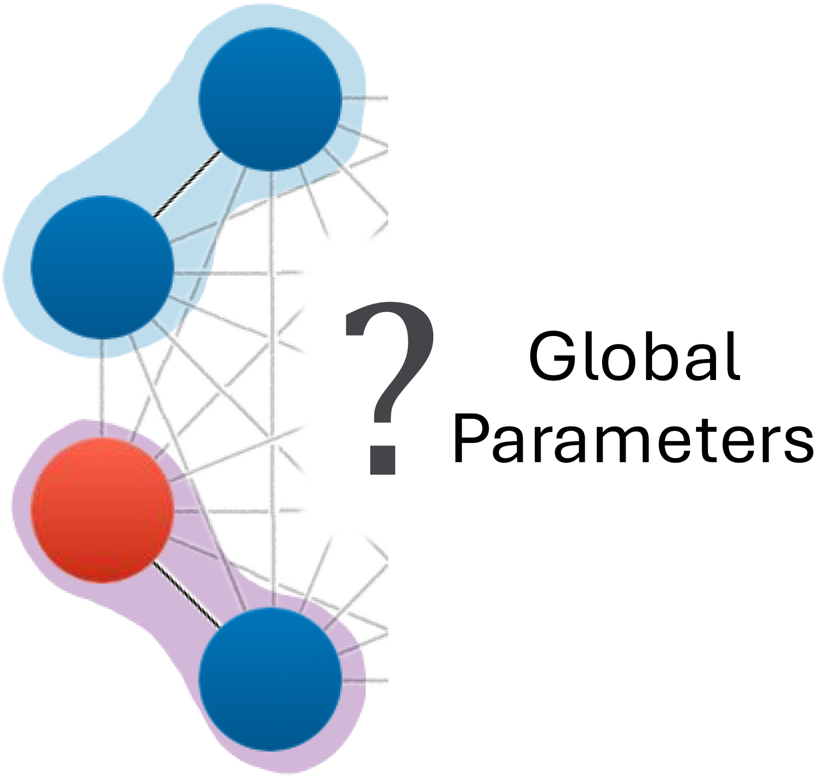
$\Phi^{\mathbf{0}}(t_1, t_2; K)$



$\Phi^{\mathbf{1}}(t_1, t_2; K)$

Inference of Global Parameters

$\Phi^{\mathbf{0}}(t_1, t_2; K)$

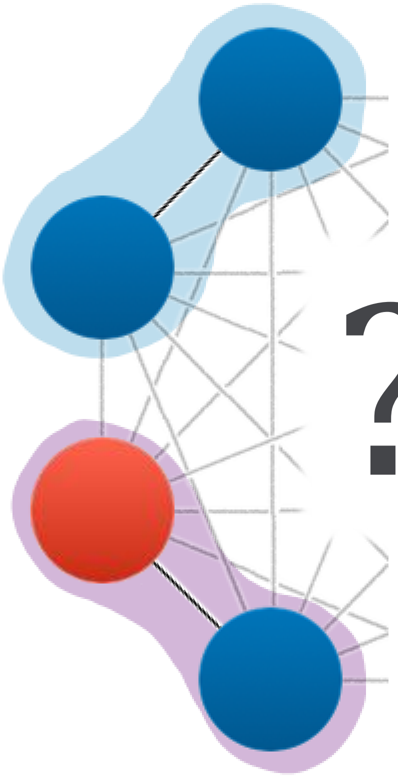


$\Phi^{\mathbf{1}}(t_1, t_2; K)$

Large Autonomous
System

Inference of Global Parameters

$\Phi^{\mathbf{0}}(t_1, t_2; K)$



$\Phi^{\mathbf{1}}(t_1, t_2; K)$

$\Theta \in \{N \quad \}$

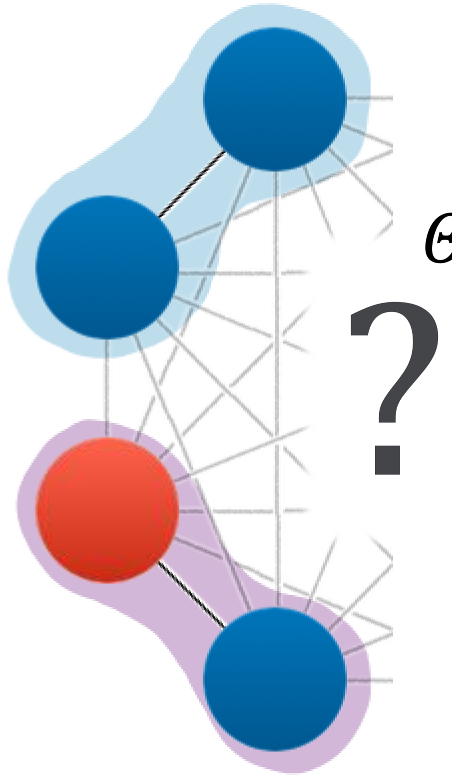
?

Global Parameters

Large Autonomous System

Inference of Global Parameters

$\Phi^{\mathbf{0}}(t_1, t_2; K)$



$\Phi^{\mathbf{1}}(t_1, t_2; K)$

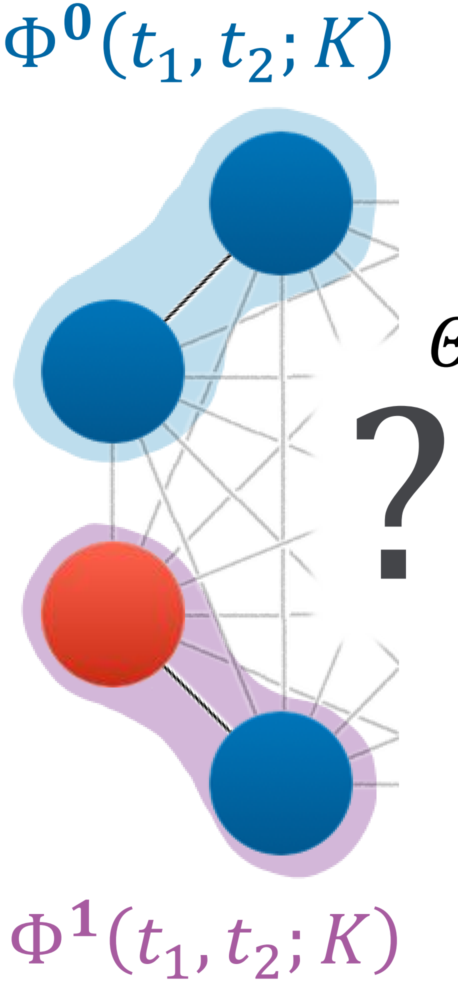
Large Autonomous System

$\Theta \in \{N, J\}$

Global Parameters

$$\hat{H} = \frac{1}{4}\sum_{i \neq j}^{N}\left[J\left(\hat{\sigma}_i^x \hat{\sigma}_j^x + \hat{\sigma}_i^y \hat{\sigma}_j^y\right) + J_z\, \hat{\sigma}_i^z \hat{\sigma}_j^z\right] + \sum_{i}^{N} h\, \hat{\sigma}_i^z$$

Inference of Global Parameters



$\Theta \in \{N, J\}$

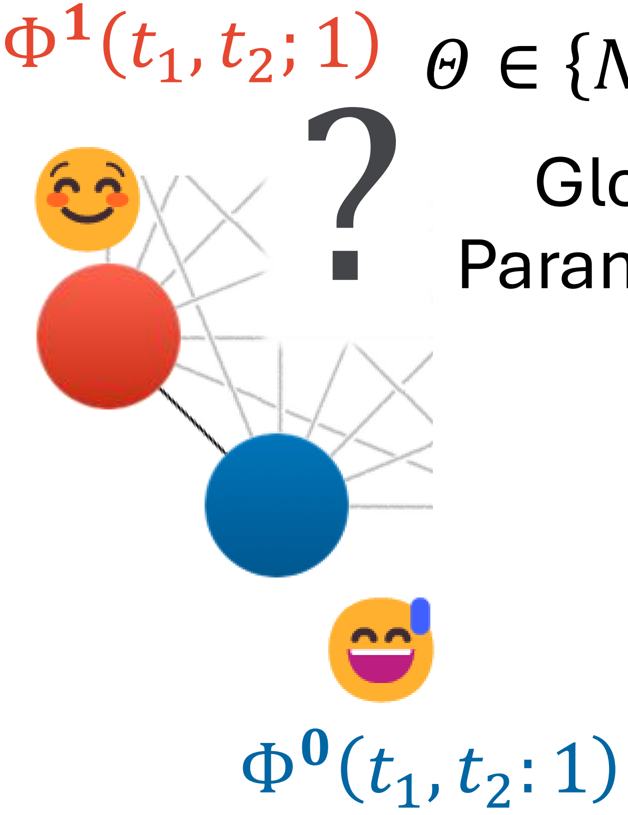
Global Parameters

$$\hat{H} = \frac{1}{4}\sum_{i\neq j}^{N}\left[J\left(\hat{\sigma}_i^x\hat{\sigma}_j^x + \hat{\sigma}_i^y\hat{\sigma}_j^y\right) + J_z\,\hat{\sigma}_i^z\hat{\sigma}_j^z\right] + \sum_{i}^{N} h\,\hat{\sigma}_i^z$$

$$\left(|\mathrm{u_d}(t_2)|^2 - |\mathrm{u_d}(t_1)|^2\right)\left(\frac{1}{\varphi_\tau^{\mathbf{0}}} - \frac{1}{\varphi_\tau^{\mathbf{1}}}\right) = 1 - \frac{1}{N-K}$$

Large Autonomous System

Inference of Global Parameters

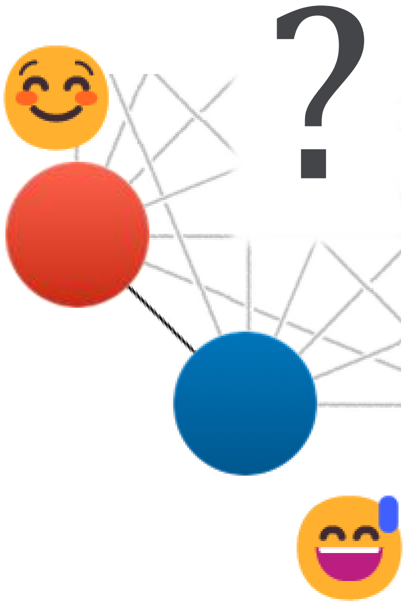


$\Theta \in \{N, J\}$

Global Parameters

Inference of Global Parameters

$\Phi^{1}(t_1, t_2; 1)$ $\Theta \in \{N, J\}$



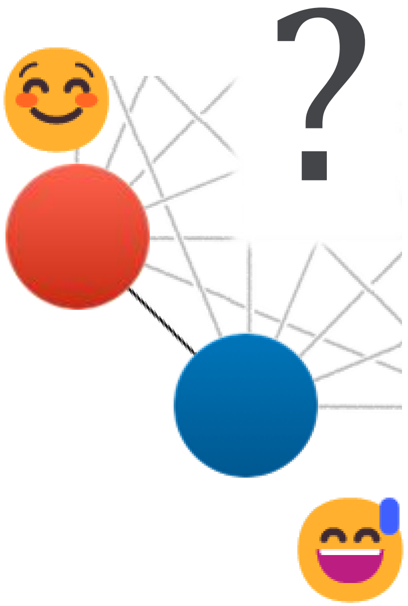
Global
Parameters

$\Phi^{0}(t_1, t_2 : 1)$

$\varphi_\tau^{0} = \varphi_\tau^{1}$

Inference of Global Parameters

$\Phi^{\mathbf{1}}(t_1, t_2; 1)$ $\Theta \in \{N, J\}$



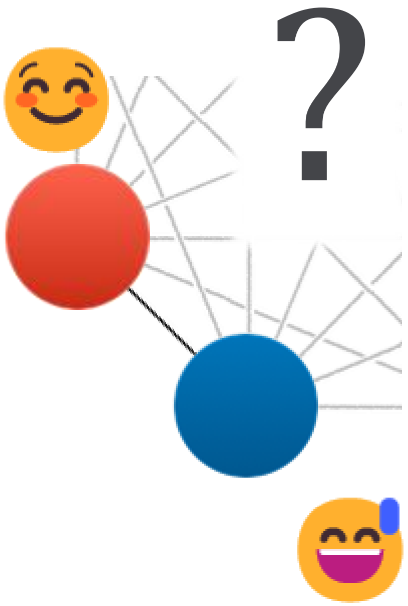
Global Parameters

$\Phi^{\mathbf{0}}(t_1, t_2 \colon 1)$

$$(|\mathrm{u_d}(t_2)|^2 - |\mathrm{u_d}(t_1)|^2)\left(\frac{1}{\varphi_\tau^{\mathbf{0}}} - \frac{1}{\varphi_\tau^{\mathbf{1}}}\right) = 1 - \frac{1}{N - K}$$

Inference of Global Parameters

$\Phi^{1}(t_1, t_2; 1)$ $\Theta \in \{N, J\}$



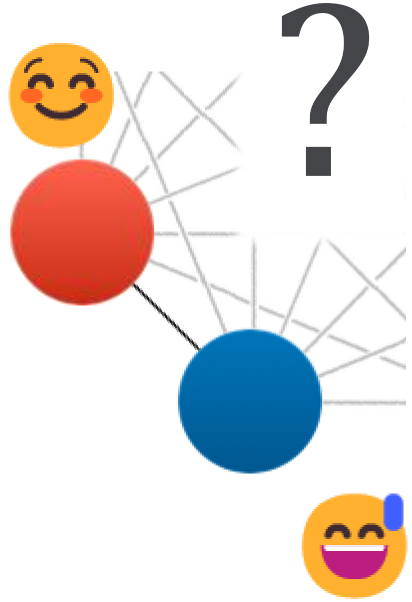
Global Parameters

$\Phi^{0}(t_1, t_2 : 1)$

$$\left(|\mathrm{u_d}(t_2)|^2 - |\mathrm{u_d}(t_1)|^2\right)\left(\frac{1}{\varphi_\tau^{\mathbf{0}}} - \frac{1}{\varphi_\tau^{\mathbf{1}}}\right) = 1 - \frac{1}{N - K}$$

Inference of Global Parameters

$\Phi^{\mathbf{1}}(t_1, t_2; 1)$ $\Theta \in \{N, J\}$



Global
Parameters

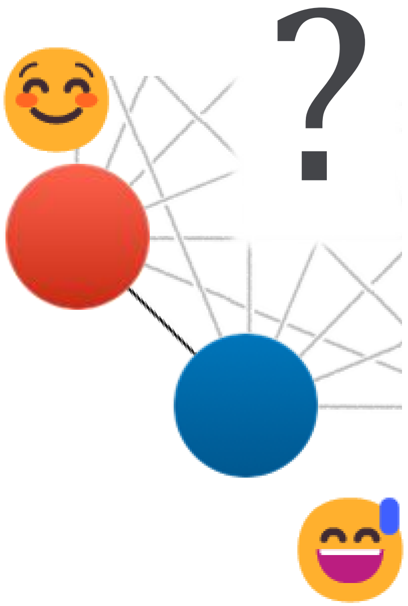
$\Phi^{\mathbf{0}}(t_1, t_2 : 1)$

$$T = \frac{2\pi}{NJ}$$

$$(|\mathrm{u_d}(t_2)|^2 - |\mathrm{u_d}(t_1)|^2)\left(\frac{1}{\varphi_\tau^{\mathbf{0}}} - \frac{1}{\varphi_\tau^{\mathbf{1}}}\right) = 1 - \frac{1}{N - K}$$

Inference of Global Parameters

$\Phi^{\mathbf{1}}(t_1, t_2; 1)$

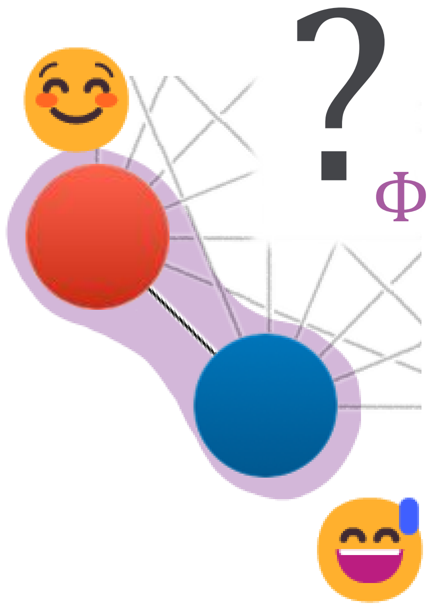


$\Phi^{\mathbf{0}}(t_1, t_2 : 1)$

$$\hat{\phi}^{\mathbf{1}} = \begin{pmatrix} \varphi_0^{\mathbf{1}} & \\ & \varphi_s^{\mathbf{1}} \end{pmatrix}$$

$$\hat{\phi}^{\mathbf{0}} = \begin{pmatrix} \varphi_s^{\mathbf{0}} & \\ & \varphi_2^{\mathbf{0}} \end{pmatrix}$$

Inference of Global Parameters



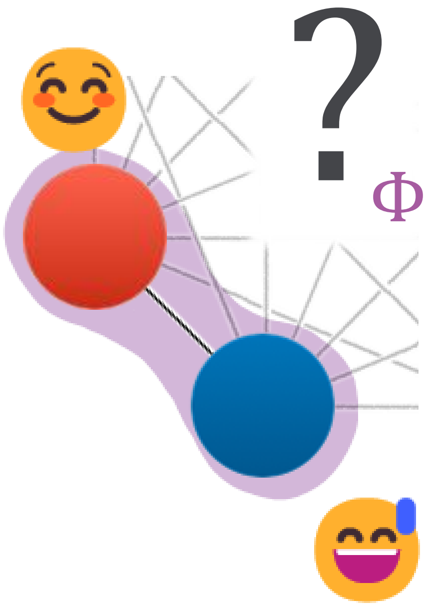
$$\hat{\phi}^{\mathbf{1}} = \begin{pmatrix} \varphi_0^{\mathbf{1}} & \\ & \varphi_s^{\mathbf{1}} \end{pmatrix}$$

$$\hat{\phi}^{\mathbf{0}} = \begin{pmatrix} \varphi_s^{\mathbf{0}} & \\ & \varphi_2^{\mathbf{0}} \end{pmatrix}$$

$$\hat{\phi}^{\mathbf{1}} = \begin{pmatrix} \varphi_0^{\mathbf{1}} & & & \\ & \varphi_s^{\mathbf{1}} & \varphi_d^{\mathbf{1}} & \\ & \varphi_d^{\mathbf{1}} & \varphi_s^{\mathbf{1}} & \\ & & & \varphi_2^{\mathbf{1}} \end{pmatrix}$$

Inference of Global Parameters

$\Phi^{\mathbf{1}}(t_1, t_2; 1)$



$\Phi^{\mathbf{1}}(t_1, t_2; 2)$

$\Phi^{\mathbf{0}}(t_1, t_2 : 1)$

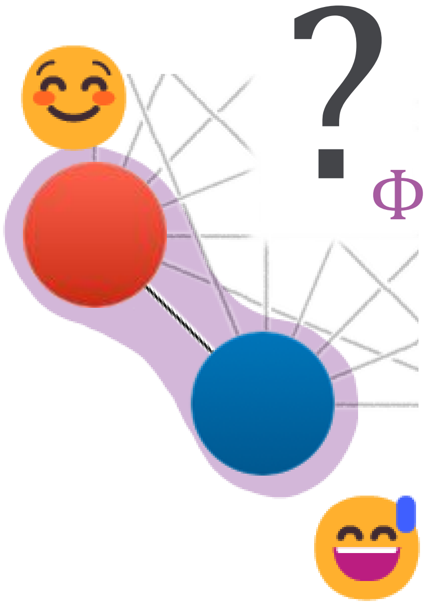
$$\hat{\phi}^1 = \begin{pmatrix} \varphi_0^{\mathbf{1}} & \\ & \varphi_s^{\mathbf{1}} \end{pmatrix}$$

$$\hat{\phi}^0 = \begin{pmatrix} \varphi_s^{\mathbf{0}} & \\ & \varphi_2^{\mathbf{0}} \end{pmatrix}$$

$$\hat{\phi}^1 = \begin{pmatrix} \varphi_0^{\mathbf{1}} & & & \\ & \varphi_s^{\mathbf{1}} & \varphi_d^1 & \\ & \varphi_d^1 & \varphi_s^{\mathbf{1}} & \\ & & & \varphi_2^{\mathbf{1}} \end{pmatrix}$$

Inference of Global Parameters

$\Phi^{\mathbf{1}}(t_1, t_2; 1)$



$\Phi^{\mathbf{1}}(t_1, t_2; 2)$

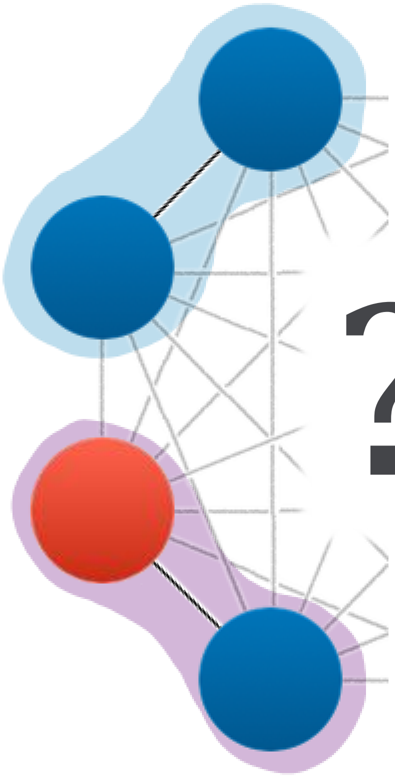
$\Phi^{\mathbf{0}}(t_1, t_2 : 1)$

$$\hat{\phi}^{\mathbf{1}} = \begin{pmatrix} \varphi_0^{\mathbf{1}} & \\ & \varphi_s^{\mathbf{1}} \end{pmatrix}$$

$$\hat{\phi}^{\mathbf{0}} = \begin{pmatrix} \varphi_s^{\mathbf{0}} & \\ & \varphi_2^{\mathbf{0}} \end{pmatrix}$$

$$\hat{\phi}^{\mathbf{1}} = \begin{pmatrix} \varphi_0^{\mathbf{1}} & & & \\ & \varphi_s^{\mathbf{1}} & \varphi_d^{\mathbf{1}} & \\ & \varphi_d^{\mathbf{1}} & \varphi_s^{\mathbf{1}} & \\ & & & \varphi_2^{\mathbf{1}} \end{pmatrix}$$

Fisher Information



$\Theta \in \{N, J\}$

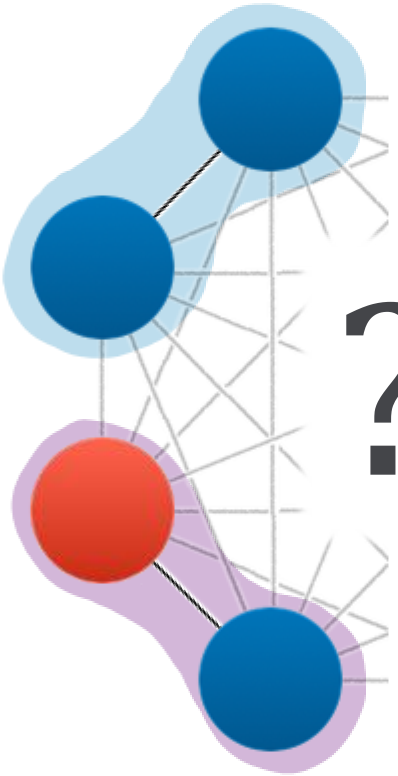
?

Global Parameters

Large Autonomous System

Fisher Information

$\hat{\rho}^{\mathbf{0}}(t, K, N, J)$



$\Theta \in \{N, J\}$

?

Global
Parameters

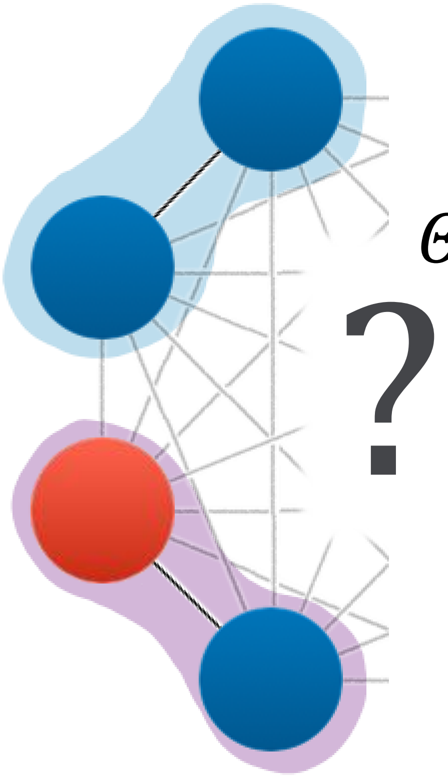
$\hat{\rho}^{\mathbf{1}}(t, K, N, J)$

Large Autonomous
System

Fisher Information

$$var(\Theta) \geq \frac{1}{\mathcal{F}_\Theta}$$

$\hat{\rho}^{\mathbf{0}}(t, K, N, J)$



$\Theta \in \{N, J\}$

Global Parameters

$\hat{\rho}^{\mathbf{1}}(t, K, N, J)$

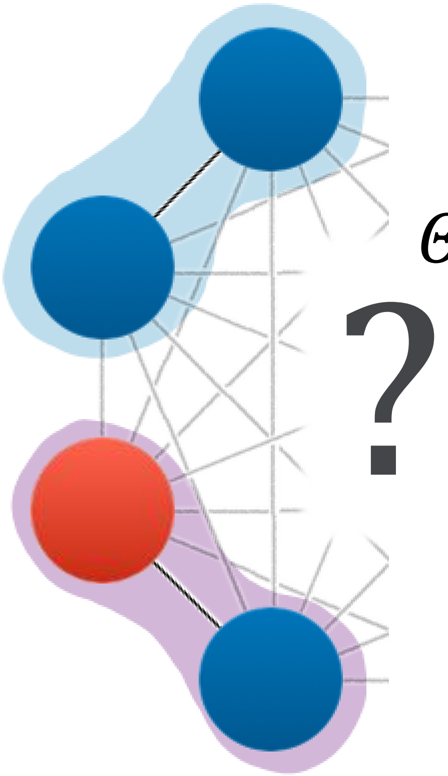
Large Autonomous System

Fisher Information

$$var(\Theta) \geq \frac{1}{\mathcal{F}_\Theta}$$

$\hat{\rho}^{\mathbf{0}}(t, K, N, J)$

$$\partial_\Theta p(x|\Theta)$$



$\Theta \in \{N, J\}$

Global Parameters

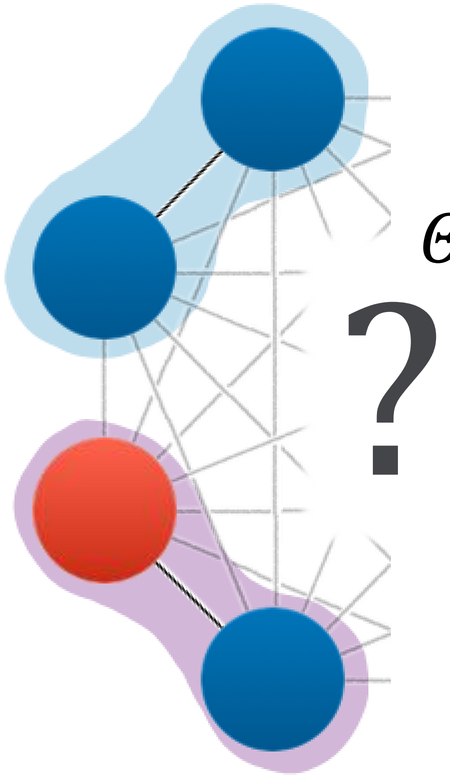
$\hat{\rho}^{\mathbf{1}}(t, K, N, J)$

Large Autonomous System

Fisher Information

$$var(\Theta) \geq \frac{1}{\mathcal{F}_{\Theta}}$$

$\hat{\rho}^{\mathbf{0}}(t, K, N, J)$



$\Theta \in \{N, J\}$

Global Parameters

$$\frac{\partial_{\Theta} p(x|\Theta)}{p(x|\Theta)}$$

$\hat{\rho}^{\mathbf{1}}(t, K, N, J)$

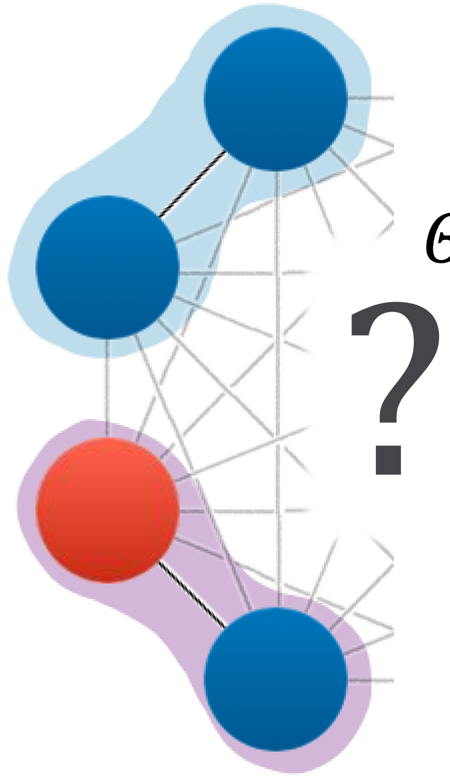
Large Autonomous System

Fisher Information

$$var(\Theta) \geq \frac{1}{\mathcal{F}_{\Theta}}$$

$$\frac{\partial_{\Theta} p(x|\Theta)}{p(x|\Theta)} = L_{\Theta}$$

$\hat{\rho}^{\mathbf{0}}(t, K, N, J)$



$\Theta \in \{N, J\}$

Global Parameters

$\hat{\rho}^{\mathbf{1}}(t, K, N, J)$

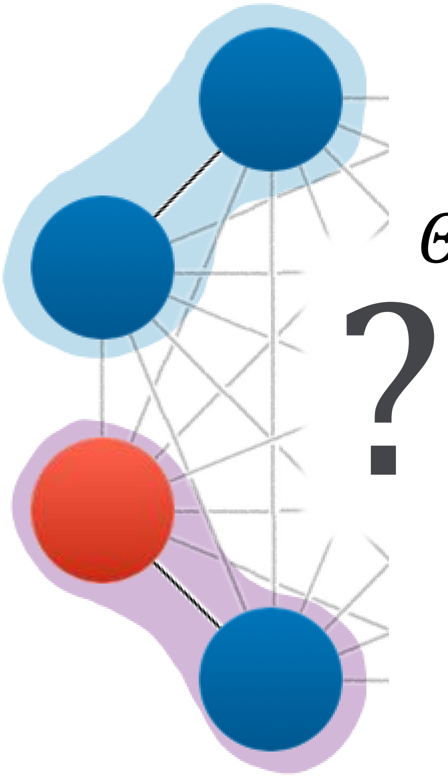
Large Autonomous System

Fisher Information

$$var(\Theta) \geq \frac{1}{\mathcal{F}_\Theta}$$

$$\frac{\partial_\Theta p(x|\Theta)}{p(x|\Theta)} = L_\Theta = \partial_\Theta \ln p(x|\Theta)$$

$\hat{\rho}^{\mathbf{0}}(t, K, N, J)$



$\Theta \in \{N, J\}$

Global Parameters

$\hat{\rho}^{\mathbf{1}}(t, K, N, J)$

Large Autonomous System

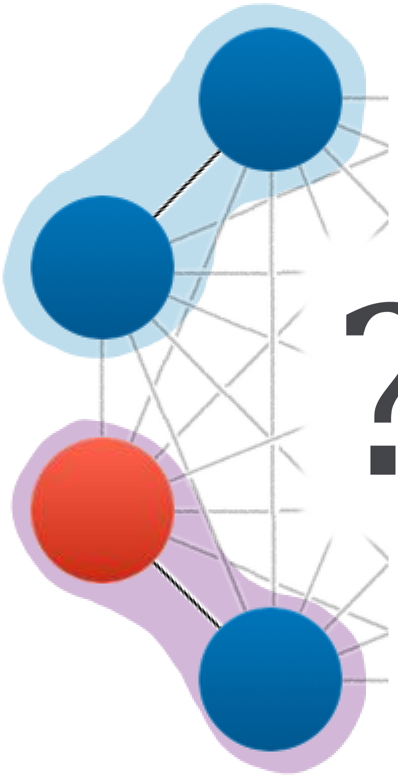
Fisher Information

$$var(\Theta) \geq \frac{1}{\mathcal{F}_\Theta}$$

$$\frac{\partial_\Theta p(x|\Theta)}{p(x|\Theta)} = L_\Theta = \partial_\Theta \, ln \, p(x|\Theta)$$

$$F_\Theta = var(L_\Theta)$$

$\hat{\rho}^{\mathbf{0}}(t, K, N, J)$



$\Theta \in \{N, J\}$

?

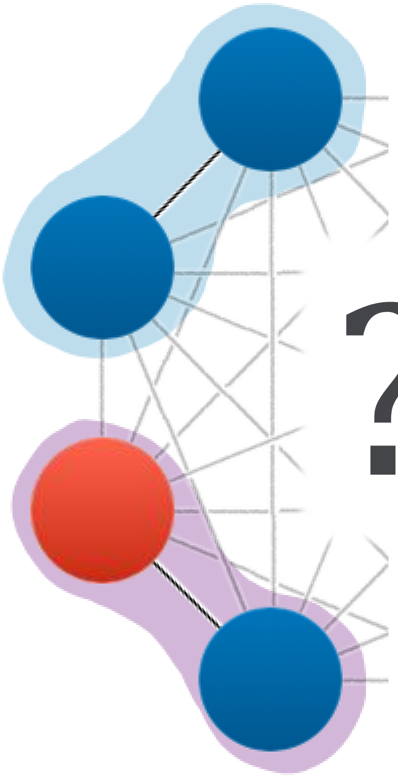
Global Parameters

$\hat{\rho}^{\mathbf{1}}(t, K, N, J)$

Large Autonomous System

Fisher Information

$\hat{\rho}^{\mathbf{0}}(t, K, N, J)$



$\Theta \in \{N, J\}$

?

Global Parameters

$\hat{\rho}^{\mathbf{1}}(t, K, N, J)$

Large Autonomous System

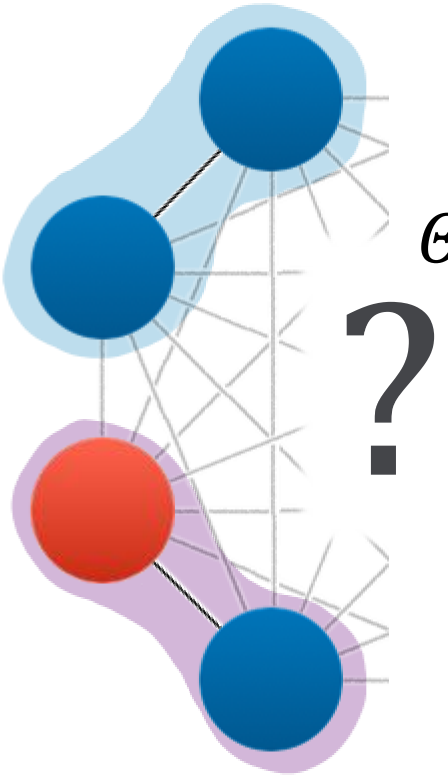
$$var(\Theta) \geq \frac{1}{\mathcal{F}_\Theta}$$

$$\frac{\partial_\Theta p(x|\Theta)}{p(x|\Theta)} = L_\Theta = \partial_\Theta \ln p(x|\Theta)$$

$$F_\Theta = var(L_\Theta) = \int dx \frac{(\partial_\Theta p)^2}{p}$$

Fisher Information

$\hat{\rho}^{\mathbf{0}}(t, K, N, J)$



$\Theta \in \{N, J\}$

Global Parameters

$\hat{\rho}^{\mathbf{1}}(t, K, N, J)$

Large Autonomous System

$$var(\Theta) \geq \frac{1}{\mathcal{F}_\Theta}$$

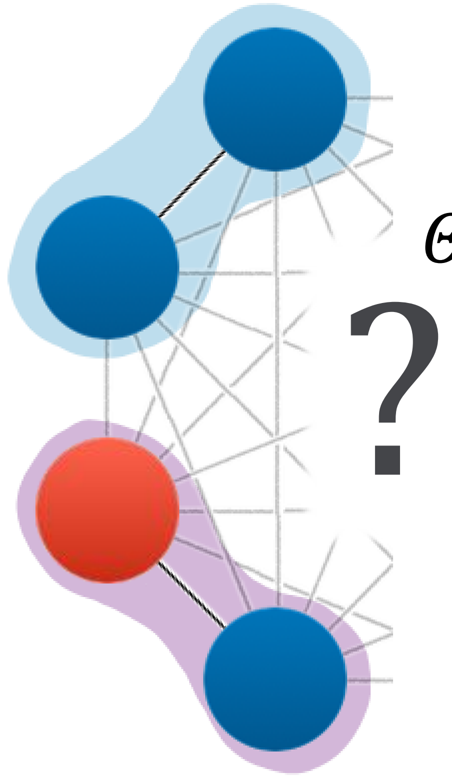
$$\frac{\partial_\Theta p(x|\Theta)}{p(x|\Theta)} = L_\Theta = \partial_\Theta \ln p(x|\Theta)$$

$$F_\Theta = var(L_\Theta) = \int dx \frac{(\partial_\Theta p)^2}{p}$$

$$\partial_\Theta \hat{\rho}(t, \Theta) = \quad \hat{\rho}(t, \Theta)\, \hat{L}_\Theta$$

Fisher Information

$\hat{\rho}^{\mathbf{0}}(t, K, N, J)$



$\hat{\rho}^{\mathbf{1}}(t, K, N, J)$

$\Theta \in \{N, J\}$

Global Parameters

Large Autonomous System

$$var(\Theta) \geq \frac{1}{\mathcal{F}_\Theta}$$

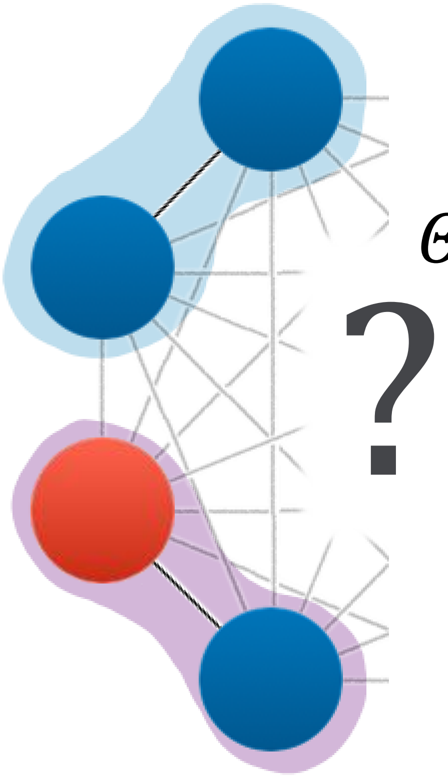
$$\frac{\partial_\Theta p(x|\Theta)}{p(x|\Theta)} = L_\Theta = \partial_\Theta \ln p(x|\Theta)$$

$$F_\Theta = var(L_\Theta) = \int dx \frac{(\partial_\Theta p)^2}{p}$$

$$\partial_\Theta \hat{\rho}(t, \Theta) = \frac{1}{2}\left[\, \hat{\rho}(t, \Theta)\, \hat{L}_\Theta + \hat{L}_\Theta\, \hat{\rho}(t, \Theta) \,\right]$$

Fisher Information

$\hat{\rho}^{\mathbf{0}}(t, K, N, J)$



$\Theta \in \{N, J\}$

Global Parameters

$\hat{\rho}^{\mathbf{1}}(t, K, N, J)$

Large Autonomous System

$$var(\Theta) \geq \frac{1}{\mathcal{F}_\Theta}$$

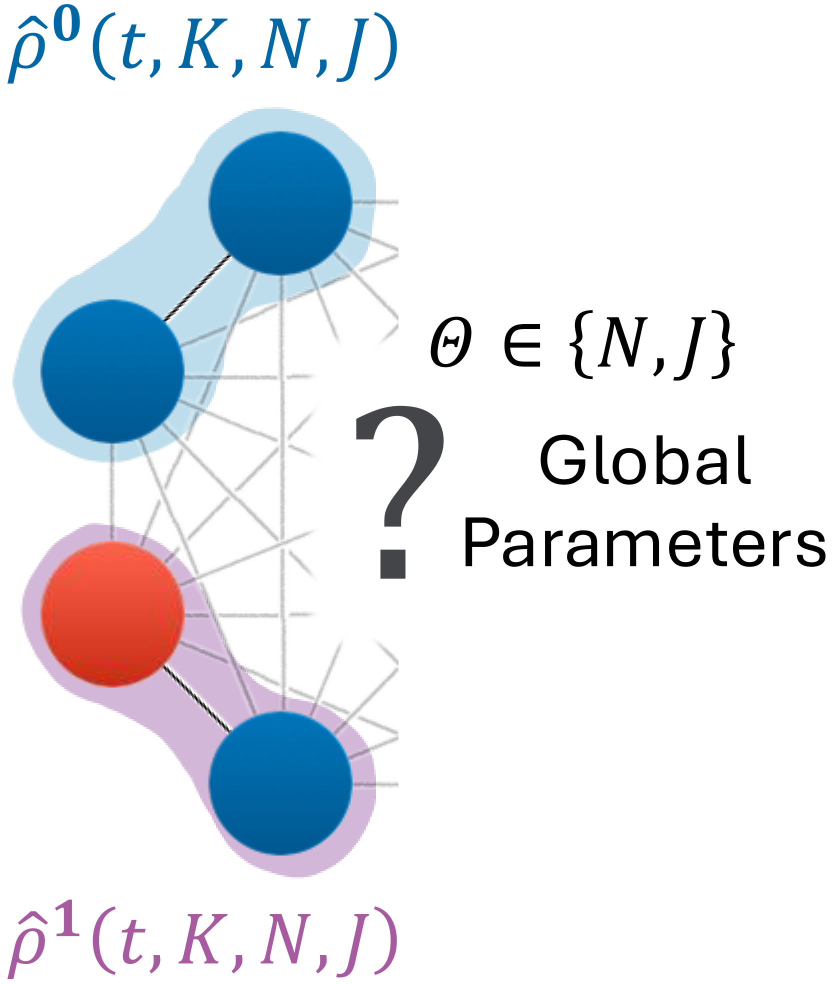
$$\frac{\partial_\Theta p(x|\Theta)}{p(x|\Theta)} = L_\Theta = \partial_\Theta \ln p(x|\Theta)$$

$$F_\Theta = var(L_\Theta) = \int dx \frac{(\partial_\Theta p)^2}{p}$$

$$\partial_\Theta \hat{\rho}(t, \Theta) = \frac{1}{2}\left[\, \hat{\rho}(t, \Theta)\, \hat{L}_\Theta + \hat{L}_\Theta\, \hat{\rho}(t, \Theta) \,\right]$$

$$\mathcal{F}_\Theta = \sum_i \frac{(\partial_\Theta p_i)^2}{p_i}$$

Fisher Information



Large Autonomous System

$$var(\Theta) \geq \frac{1}{\mathcal{F}_\Theta}$$

$$\frac{\partial_\Theta p(x|\Theta)}{p(x|\Theta)} = L_\Theta = \partial_\Theta \, ln\, p(x|\Theta)$$

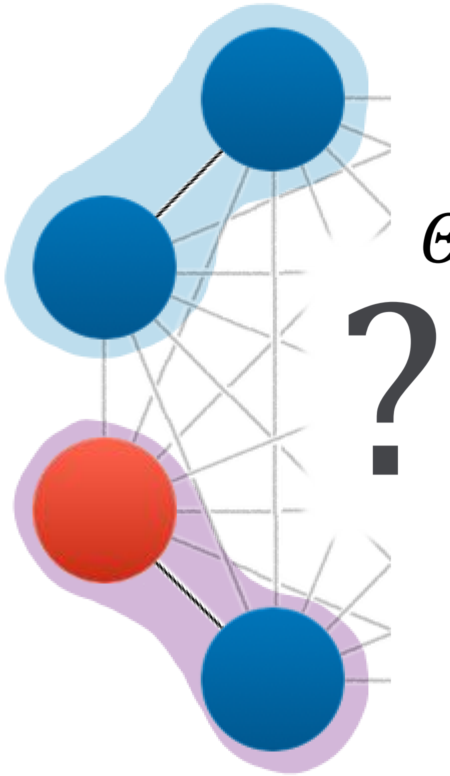
$$F_\Theta = var(L_\Theta) = \int dx \frac{(\partial_\Theta p)^2}{p}$$

$$\partial_\Theta \hat{\rho}(t, \Theta) = \frac{1}{2}\left[\, \hat{\rho}(t, \Theta)\, \hat{L}_\Theta + \hat{L}_\Theta\, \hat{\rho}(t, \Theta)\, \right]$$

$$\mathcal{F}_\Theta = \sum_i \frac{(\partial_\Theta p_i)^2}{p_i} + 2 \sum_{i,j} \frac{\left(p_i - p_j\right)^2}{p_i + p_j} \left|\langle \psi_i | \partial_\Theta \psi_j \rangle\right|^2$$

Fisher Information

$\hat{\rho}^{\mathbf{0}}(t,K,N,J)$



$\Theta \in \{N,J\}$

Global Parameters

$\hat{\rho}^{\mathbf{1}}(t,K,N,J)$

Large Autonomous System

$$var(\Theta) \geq \frac{1}{\mathcal{F}_\Theta}$$

$$\frac{\partial_\Theta p(x|\Theta)}{p(x|\Theta)} = L_\Theta = \partial_\Theta \ln p(x|\Theta)$$

$$F_\Theta = var(L_\Theta) = \int dx \frac{(\partial_\Theta p)^2}{p}$$

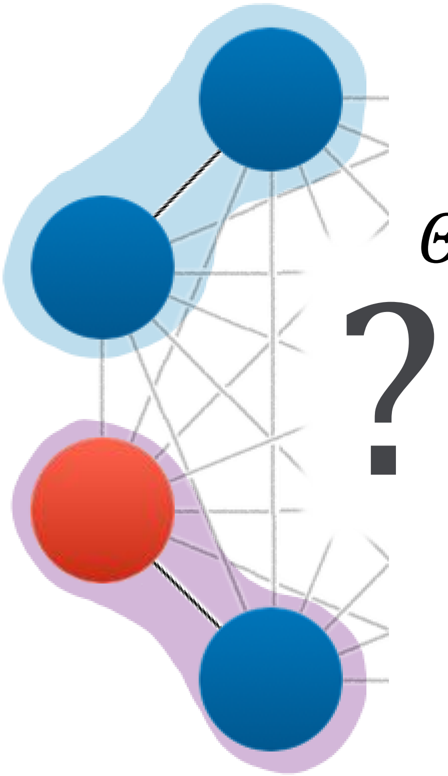
$$\partial_\Theta \hat{\rho}(t,\Theta) = \frac{1}{2}\left[\,\hat{\rho}(t,\Theta)\,\hat{L}_\Theta + \hat{L}_\Theta\,\hat{\rho}(t,\Theta)\,\right]$$

$$\mathcal{F}_\Theta = \sum_i \frac{(\partial_\Theta p_i)^2}{p_i} + 2\sum_{i,j} \frac{\left(p_i - p_j\right)^2}{p_i + p_j} \left|\langle \psi_i | \partial_\Theta \psi_j \rangle\right|^2$$

$\mathcal{F}_\Theta^C$ Classic

Fisher Information

$\hat{\rho}^{\mathbf{0}}(t, K, N, J)$



$\Theta \in \{N, J\}$

Global Parameters

$\hat{\rho}^{\mathbf{1}}(t, K, N, J)$

Large Autonomous System

$$var(\Theta) \geq \frac{1}{\mathcal{F}_\Theta}$$

$$\frac{\partial_\Theta p(x|\Theta)}{p(x|\Theta)} = L_\Theta = \partial_\Theta \ln p(x|\Theta)$$

$$F_\Theta = var(L_\Theta) = \int dx \frac{(\partial_\Theta p)^2}{p}$$

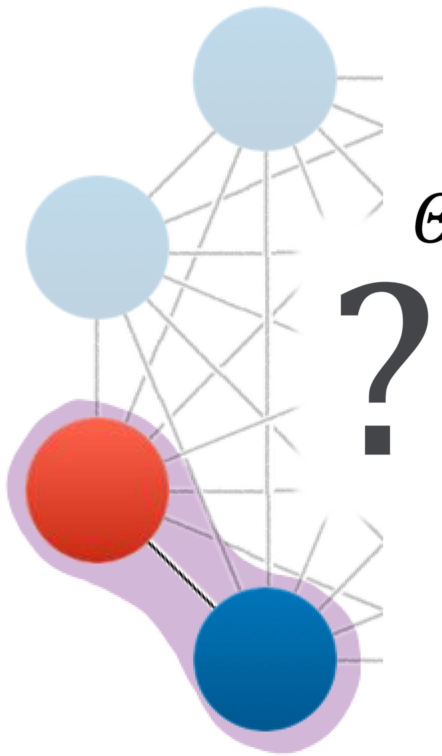
$$\partial_\Theta \hat{\rho}(t, \Theta) = \frac{1}{2}\left[\, \hat{\rho}(t, \Theta)\, \hat{L}_\Theta + \hat{L}_\Theta\, \hat{\rho}(t, \Theta) \,\right]$$

$$\mathcal{F}_\Theta = \sum_i \frac{(\partial_\Theta p_i)^2}{p_i} + 2 \sum_{i,j} \frac{\left(p_i - p_j\right)^2}{p_i + p_j} \left|\langle \psi_i | \partial_\Theta \psi_j \rangle\right|^2$$

$\mathcal{F}_\Theta^C$ Classic $\quad$ $\mathcal{F}_\Theta^Q$ Quantum

Fisher Information

$$\hat{\rho}^{\mathbf{1}}(t,K) = \left(1 - p^{\mathbf{1}}\right)|00\cdots0\rangle\langle00\cdots0| + p^{\mathbf{1}}\left|\psi^{\mathbf{1}}\right\rangle\left\langle\psi^{\mathbf{1}}\right|$$



$\Theta \in \{N, J\}$

Global Parameters

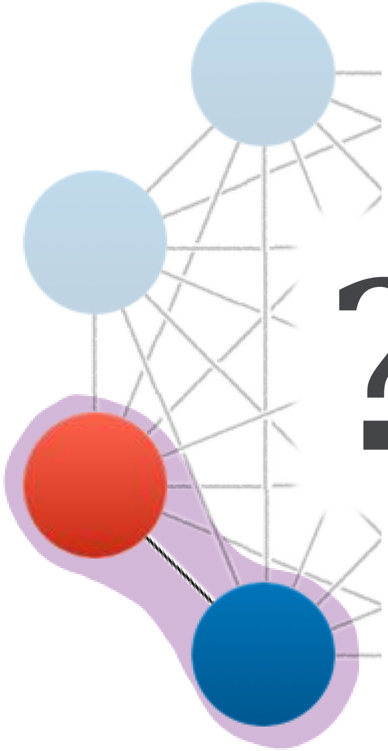
$$\left|\psi^{\mathbf{1}}\right\rangle = \frac{1}{\sqrt{p^{\mathbf{1}}}}\left[\begin{array}{c} u_s|10\cdots0\rangle + \\ u_d|01\cdots0\rangle + \cdots + u_d|00\cdots1\rangle \end{array}\right]$$

$\hat{\rho}^{\mathbf{1}}(t,K,N,J)$

Partial System

Fisher Information

$$\hat{\rho}^{1}(t,K) = \left(1 - p^{1}\right)|00\cdots0\rangle\langle00\cdots0| + p^{1}\left|\psi^{1}\right\rangle\left\langle\psi^{1}\right|$$



? $\Theta \in \{N, J\}$

Global Parameters

$\hat{\rho}^{1}(t,K,N,J)$

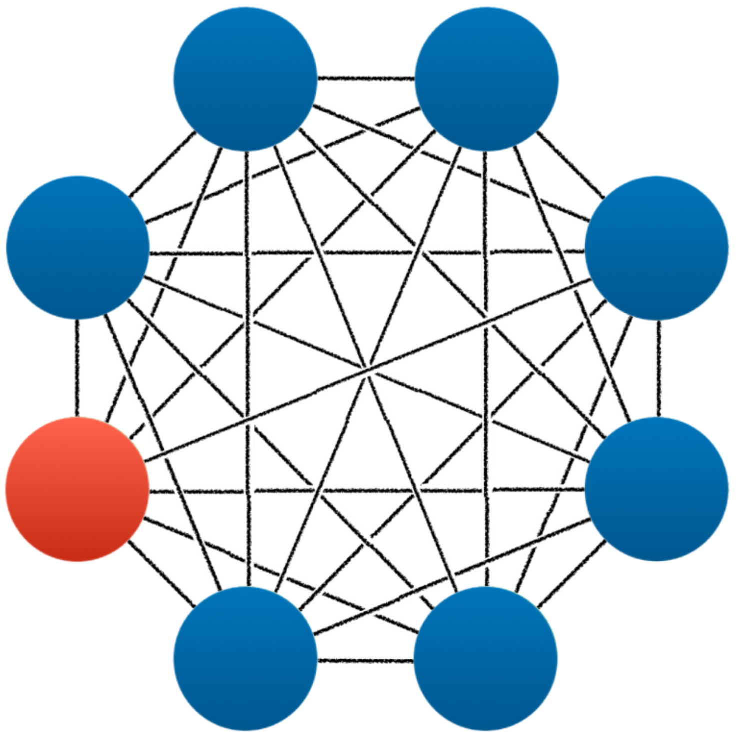
Partial System

$$\left|\psi^{1}\right\rangle = \frac{1}{\sqrt{p^{1}}}\left[\begin{matrix} u_s|10\cdots0\rangle + \\ u_d|01\cdots0\rangle + \cdots + u_d|00\cdots1\rangle \end{matrix}\right]$$

$$p^{1} = |u_s|^2 + (K-1)|u_d|^2$$

Fisher Information

$$\hat{\rho}^{\mathbf{1}}(t, K) = \left(1 - p^{\mathbf{1}}\right)|00\cdots0\rangle\langle00\cdots0| + p^{\mathbf{1}}|\psi^{\mathbf{1}}\rangle\langle\psi^{\mathbf{1}}|$$



$\hat{\rho}(t, N)$

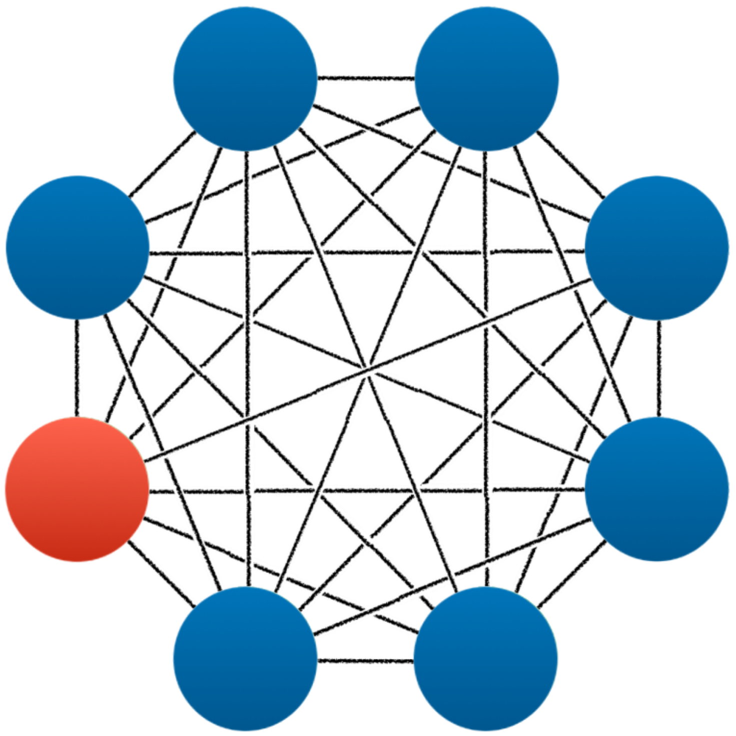
Large Autonomous System

$$|\psi^{\mathbf{1}}\rangle = \frac{1}{\sqrt{p^{\mathbf{1}}}}\left[\begin{array}{c} u_s|10\cdots0\rangle + \\ u_d|01\cdots0\rangle + \cdots + u_d|00\cdots1\rangle \end{array}\right]$$

$$p^{\mathbf{1}} = |u_s|^2 + (K-1)|u_d|^2$$

Fisher Information

$$\hat{\rho}^{\mathbf{1}}(t,K) = \left(1 - p^{\mathbf{1}}\right)|00\cdots 0\rangle\langle 00\cdots 0| + p^{\mathbf{1}}\left|\psi^{\mathbf{1}}\right\rangle\left\langle\psi^{\mathbf{1}}\right|$$



$$\left|\psi^{\mathbf{1}}\right\rangle = \frac{1}{\sqrt{p^{\mathbf{1}}}}\left[\begin{array}{c} u_s|10\cdots 0\rangle + \\ u_d|01\cdots 0\rangle + \cdots + u_d|00\cdots 1\rangle \end{array}\right]$$

$$p^{\mathbf{1}} = |u_s|^2 + (K-1)|u_d|^2$$

$$p^{\mathbf{1}}(N) = |u_s|^2 + (N-1)|u_d|^2$$

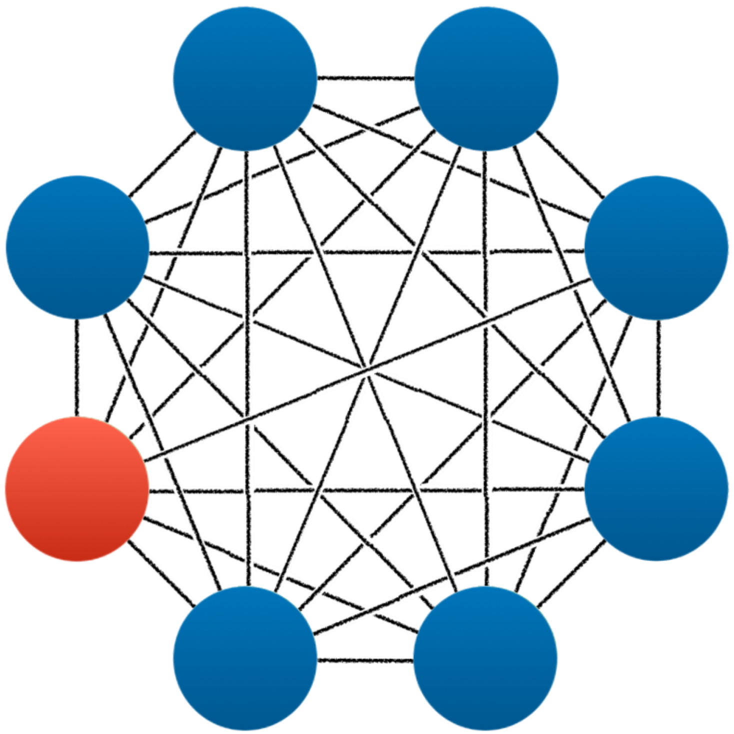
Unitary

$\hat{\rho}(t,N)$

Large Autonomous System

Fisher Information

$$\hat{\rho}^{\mathbf{1}}(t,K) = \left(1 - p^{\mathbf{1}}\right)|00\cdots0\rangle\langle00\cdots0| + p^{\mathbf{1}}\left|\psi^{\mathbf{1}}\right\rangle\left\langle\psi^{\mathbf{1}}\right|$$



$$\left|\psi^{\mathbf{1}}\right\rangle = \frac{1}{\sqrt{p^{\mathbf{1}}}}\left[\begin{array}{c} u_s|10\cdots0\rangle + \\ u_d|01\cdots0\rangle + \cdots + u_d|00\cdots1\rangle \end{array}\right]$$

$\hat{\rho}(t,N)$

Large Autonomous System

$$p^{\mathbf{1}} = |u_s|^2 + (K-1)|u_d|^2$$

$$p^{\mathbf{1}}(N) = |u_s|^2 + (N-1)|u_d|^2 = 1$$

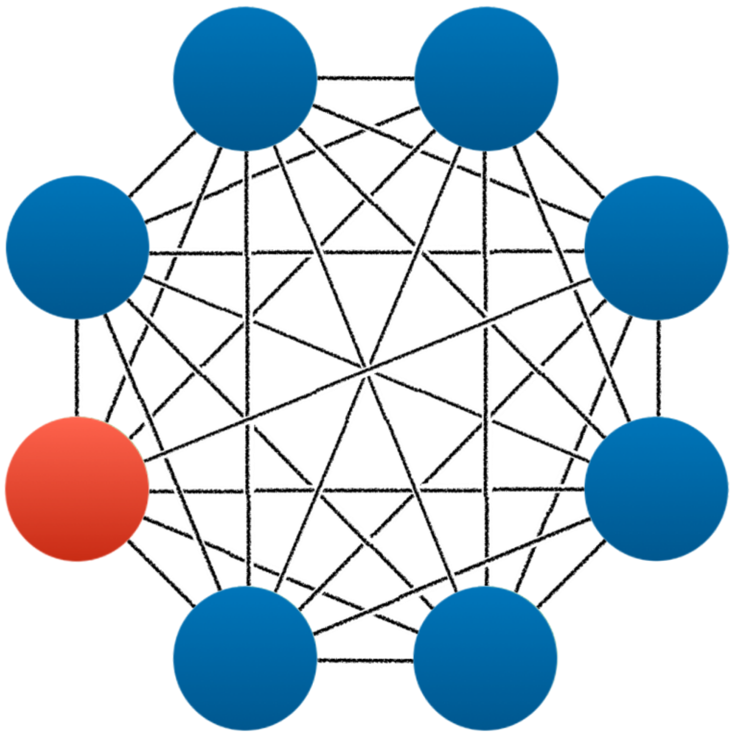
Unitary

Fisher Information

$$\hat{\rho}^{\mathbf{1}}(t,K) = \left(1 - p^{\mathbf{1}}\right)|00\cdots0\rangle\langle00\cdots0| + p^{\mathbf{1}}|\psi^{\mathbf{1}}\rangle\langle\psi^{\mathbf{1}}|$$

$$\hat{\rho}^{\mathbf{1}}(t,N) = |\psi^{\mathbf{1}}\rangle\langle\psi^{\mathbf{1}}|$$

$$|\psi^{\mathbf{1}}\rangle = \frac{1}{\sqrt{p^{\mathbf{1}}}}\left[\begin{array}{c} u_s|10\cdots0\rangle + \\ u_d|01\cdots0\rangle + \cdots + u_d|00\cdots1\rangle \end{array}\right]$$



$\hat{\rho}(t,N)$

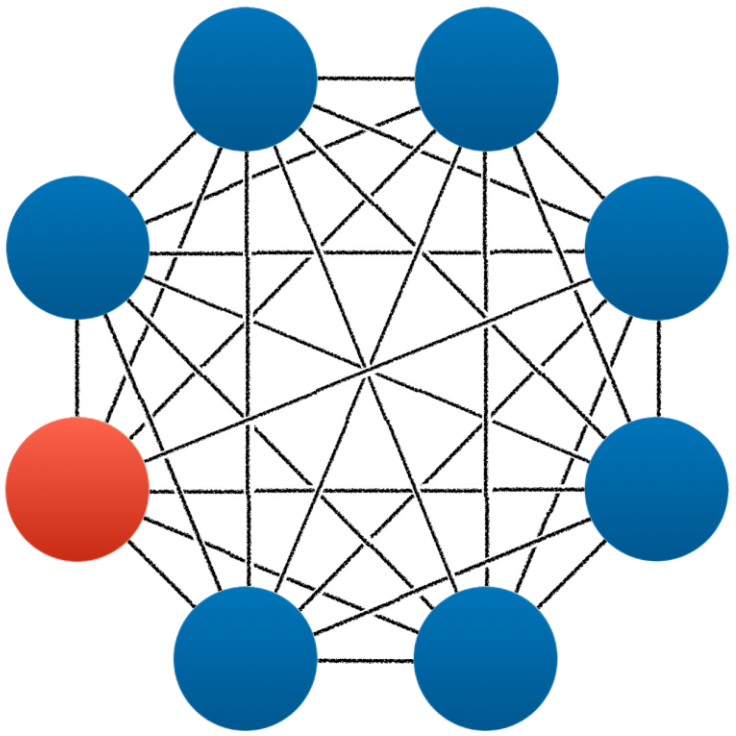
Large Autonomous System

$$p^{\mathbf{1}} = |u_s|^2 + (K-1)|u_d|^2$$

$$p^{\mathbf{1}}(N) = |u_s|^2 + (N-1)|u_d|^2 = 1$$

Unitary

Fisher Information



$\hat{\rho}(t, N)$

Large Autonomous System

$$\hat{\rho}^{\mathbf{1}}(t, K) = (1 - p^{\mathbf{1}})|00 \cdots 0\rangle\langle 00 \cdots 0| + p^{\mathbf{1}}|\psi^{\mathbf{1}}\rangle\langle\psi^{\mathbf{1}}|$$

$$\hat{\rho}^{\mathbf{1}}(t, N) = |\psi^{\mathbf{1}}\rangle\langle\psi^{\mathbf{1}}|$$

$$|\psi^{\mathbf{1}}\rangle = \frac{1}{\sqrt{p^{\mathbf{1}}}}\left[\begin{array}{c} u_s|10 \cdots 0\rangle + \\ u_d|01 \cdots 0\rangle + \cdots + u_d|00 \cdots 1\rangle \end{array}\right]$$

$$|\psi^{\mathbf{1}}(N)\rangle = \begin{array}{c} u_s|10 \cdots 0\rangle + \\ u_d|01 \cdots 0\rangle + \cdots + u_d|00 \cdots 1\rangle \end{array}$$

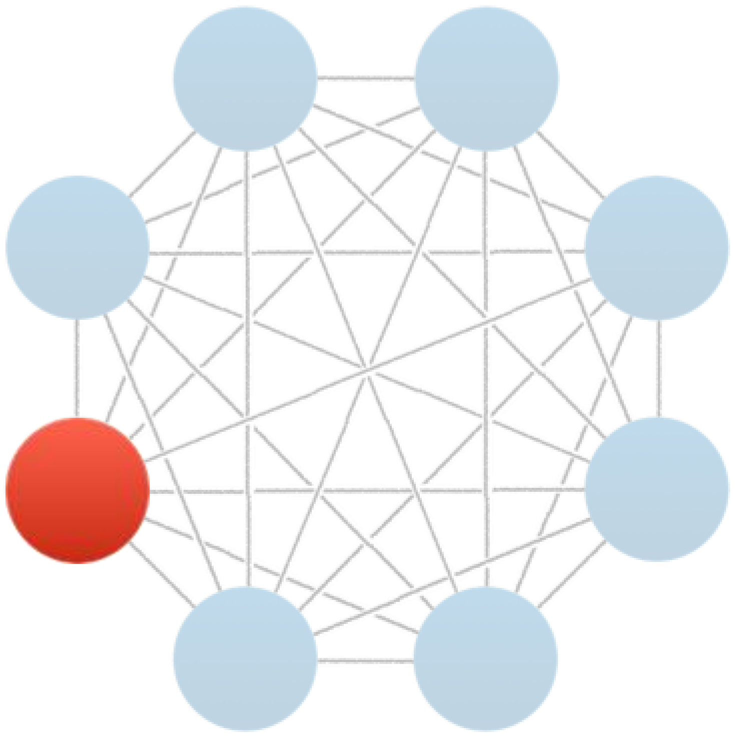
$$p^{\mathbf{1}} = |u_s|^2 + (K - 1)|u_d|^2$$

$$p^{\mathbf{1}}(N) = |u_s|^2 + (N - 1)|u_d|^2 = 1$$

Unitary

Fisher Information

$$\hat{\rho}^{\mathbf{1}}(t, K) = \left(1 - p^{\mathbf{1}}\right)|00\cdots0\rangle\langle00\cdots0| + p^{\mathbf{1}}\left|\psi^{\mathbf{1}}\right\rangle\left\langle\psi^{\mathbf{1}}\right|$$



$\hat{\rho}^{\mathbf{1}}(t, 1)$

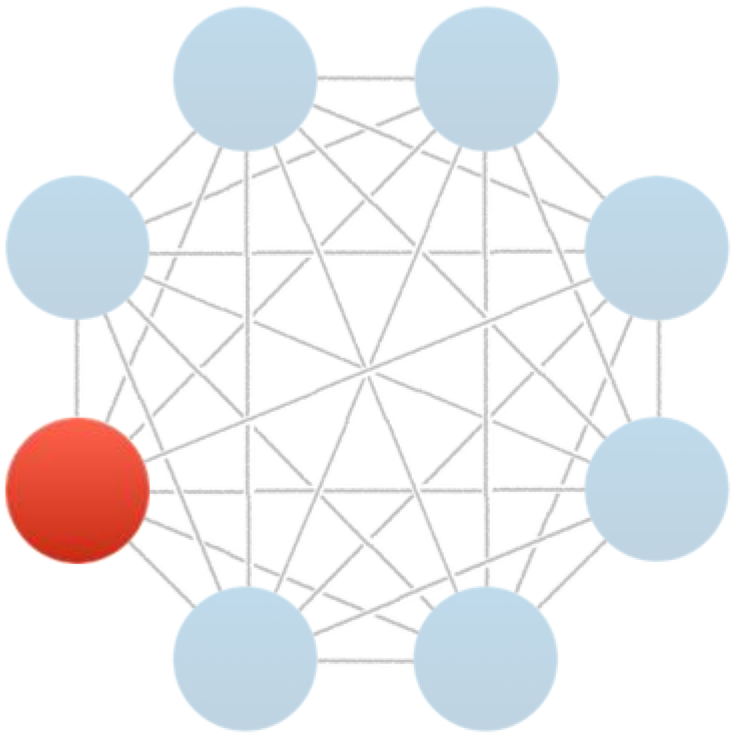
Single Qubit

$$\left|\psi^{\mathbf{1}}\right\rangle = \frac{1}{\sqrt{p^{\mathbf{1}}}}\begin{bmatrix} u_s|10\cdots0\rangle + \\ u_d|01\cdots0\rangle + \cdots + u_d|00\cdots1\rangle \end{bmatrix}$$

$$p^{\mathbf{1}} = |u_s|^2 + (K-1)|u_d|^2$$

Fisher Information

$$\hat{\rho}^{\mathbf{1}}(t,K) = \left(1 - p^{\mathbf{1}}\right)|00\cdots0\rangle\langle00\cdots0| + p^{\mathbf{1}}\left|\psi^{\mathbf{1}}\right\rangle\left\langle\psi^{\mathbf{1}}\right|$$



$\hat{\rho}^{\mathbf{1}}(t,1)$

Single Qubit

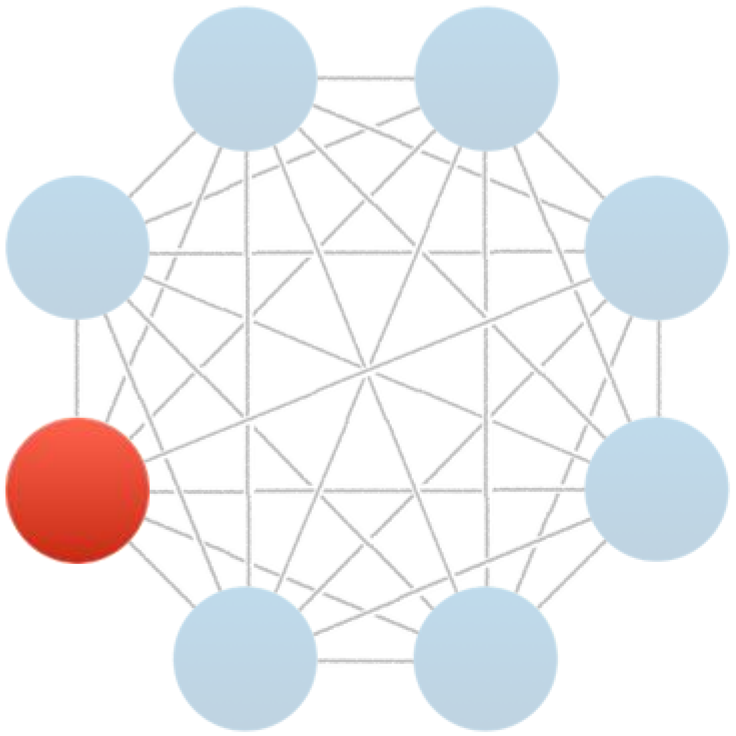
$$\left|\psi^{\mathbf{1}}\right\rangle = \frac{1}{\sqrt{p^{\mathbf{1}}}}\begin{bmatrix} u_s|10\cdots0\rangle + \\ u_d|01\cdots0\rangle + \cdots + u_d|00\cdots1\rangle \end{bmatrix}$$

$$p^{\mathbf{1}} = |u_s|^2 + (K-1)|u_d|^2$$

$$p^{\mathbf{1}}(1) = |u_s|^2$$

Fisher Information

$$\hat{\rho}^{\mathbf{1}}(t,K) = \left(1 - p^{\mathbf{1}}\right)|00\cdots0\rangle\langle00\cdots0| + p^{\mathbf{1}}\left|\psi^{\mathbf{1}}\right\rangle\!\left\langle\psi^{\mathbf{1}}\right|$$



$$\left|\psi^{\mathbf{1}}\right\rangle = \frac{1}{\sqrt{p^{\mathbf{1}}}}\left[\begin{matrix} u_s|10\cdots0\rangle + \\ u_d|01\cdots0\rangle + \cdots + u_d|00\cdots1\rangle \end{matrix}\right]$$

$$\left|\psi^{\mathbf{1}}(1)\right\rangle = |1\rangle$$

$\hat{\rho}^{\mathbf{1}}(t,1)$

Single Qubit

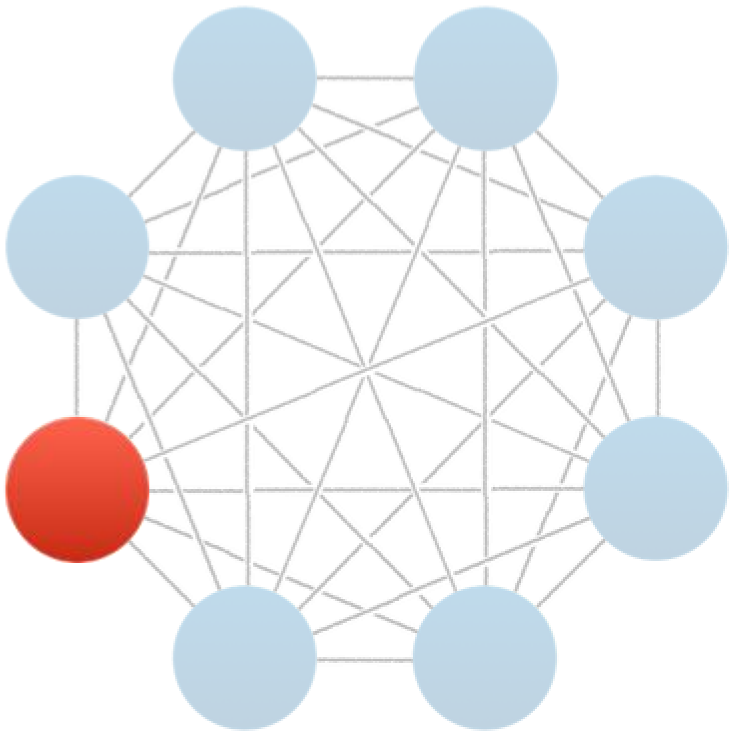
$$p^{\mathbf{1}} = |u_s|^2 + (K-1)|u_d|^2$$

$$p^{\mathbf{1}}(1) = |u_s|^2$$

Fisher Information

$$\hat{\rho}^{1}(t,K) = \left(1 - p^{1}\right)|00\cdots0\rangle\langle00\cdots0| + p^{1}\left|\psi^{1}\right\rangle\left\langle\psi^{1}\right|$$

$$\hat{\rho}^{1}(t,1) = (1 - |u_s|^2)|0\rangle\langle0| + |u_s|^2|1\rangle\langle1|$$



$$\left|\psi^{1}\right\rangle = \frac{1}{\sqrt{p^{1}}}\begin{bmatrix} u_s|10\cdots0\rangle + \\ u_d|01\cdots0\rangle + \cdots + u_d|00\cdots1\rangle \end{bmatrix}$$

$$\left|\psi^{1}(1)\right\rangle = |1\rangle$$

$\hat{\rho}^{1}(t,1)$

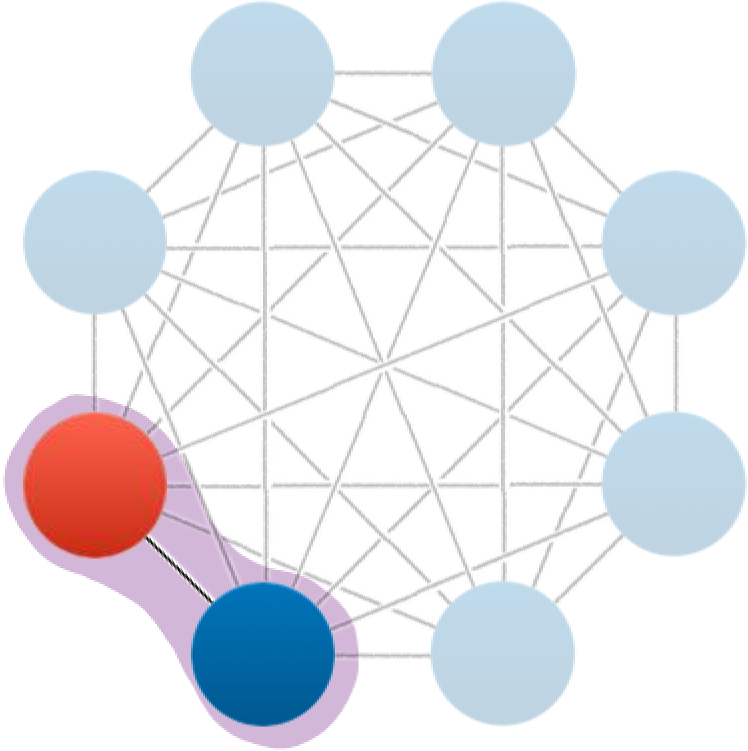
Single Qubit

$$p^{1} = |u_s|^2 + (K-1)|u_d|^2$$

$$p^{1}(1) = |u_s|^2$$

Fisher Information

$$\hat{\rho}^{\mathbf{1}}(t, K) = \left(1 - p^{\mathbf{1}}\right)|00 \cdots 0\rangle\langle 00 \cdots 0| + p^{\mathbf{1}}\left|\psi^{\mathbf{1}}\right\rangle\left\langle\psi^{\mathbf{1}}\right|$$



$\hat{\rho}^{\mathbf{1}}(t, K, N, J)$

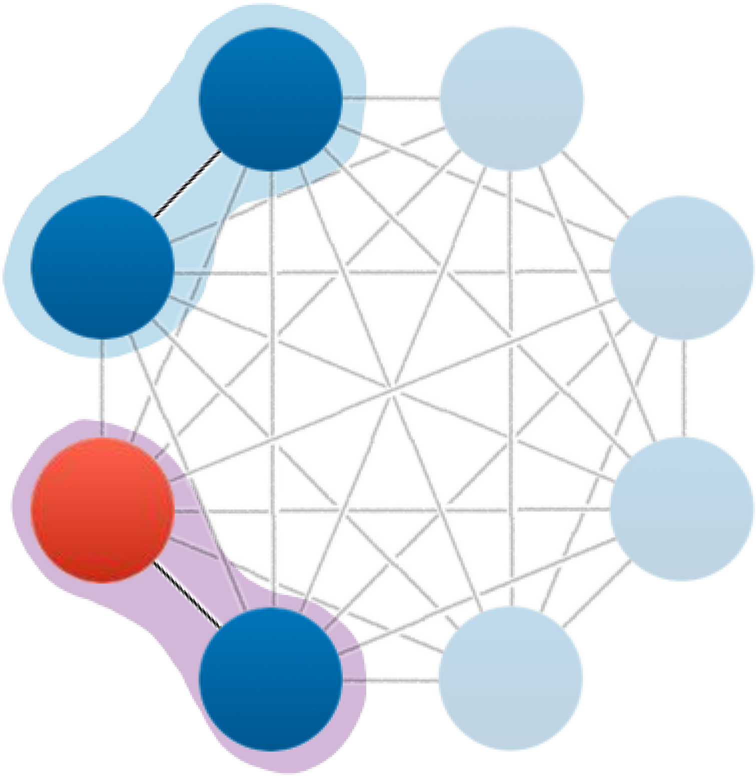
Partial System

$$\left|\psi^{\mathbf{1}}\right\rangle = \frac{1}{\sqrt{p^{\mathbf{1}}}}\begin{bmatrix} u_s|10 \cdots 0\rangle + \\ u_d|01 \cdots 0\rangle + \cdots + u_d|00 \cdots 1\rangle \end{bmatrix}$$

$$p^{\mathbf{1}} = |u_s|^2 + (K-1)|u_d|^2$$

Fisher Information

$\hat{\rho}^{\mathbf{0}}(t, K, N, J)$



$\hat{\rho}^{\mathbf{1}}(t, K, N, J)$

Partial Systems

$$\hat{\rho}^{\mathbf{1}}(t, K) = \left(1 - p^{\mathbf{1}}\right)|00\cdots 0\rangle\langle 00\cdots 0| + p^{\mathbf{1}}\left|\psi^{\mathbf{1}}\right\rangle\left\langle\psi^{\mathbf{1}}\right|$$

$$\hat{\rho}^{\mathbf{0}}(t, K) = \left(1 - p^{\mathbf{0}}\right)|00\cdots 0\rangle\langle 00\cdots 0| + p^{\mathbf{0}}\left|\psi^{\mathbf{0}}\right\rangle\left\langle\psi^{\mathbf{0}}\right|$$

$$\left|\psi^{\mathbf{1}}\right\rangle = \frac{1}{\sqrt{p^{\mathbf{1}}}}\begin{bmatrix} u_s|10\cdots 0\rangle + \\ u_d|01\cdots 0\rangle + \cdots + u_d|00\cdots 1\rangle \end{bmatrix}$$

$$\left|\psi^{\mathbf{0}}\right\rangle = \frac{1}{\sqrt{K}}\begin{bmatrix} |10\cdots 0\rangle + \\ |01\cdots 0\rangle + \cdots + |00\cdots 1\rangle \end{bmatrix}$$

$$p^{\mathbf{1}} = |u_s|^2 + (K-1)|u_d|^2$$

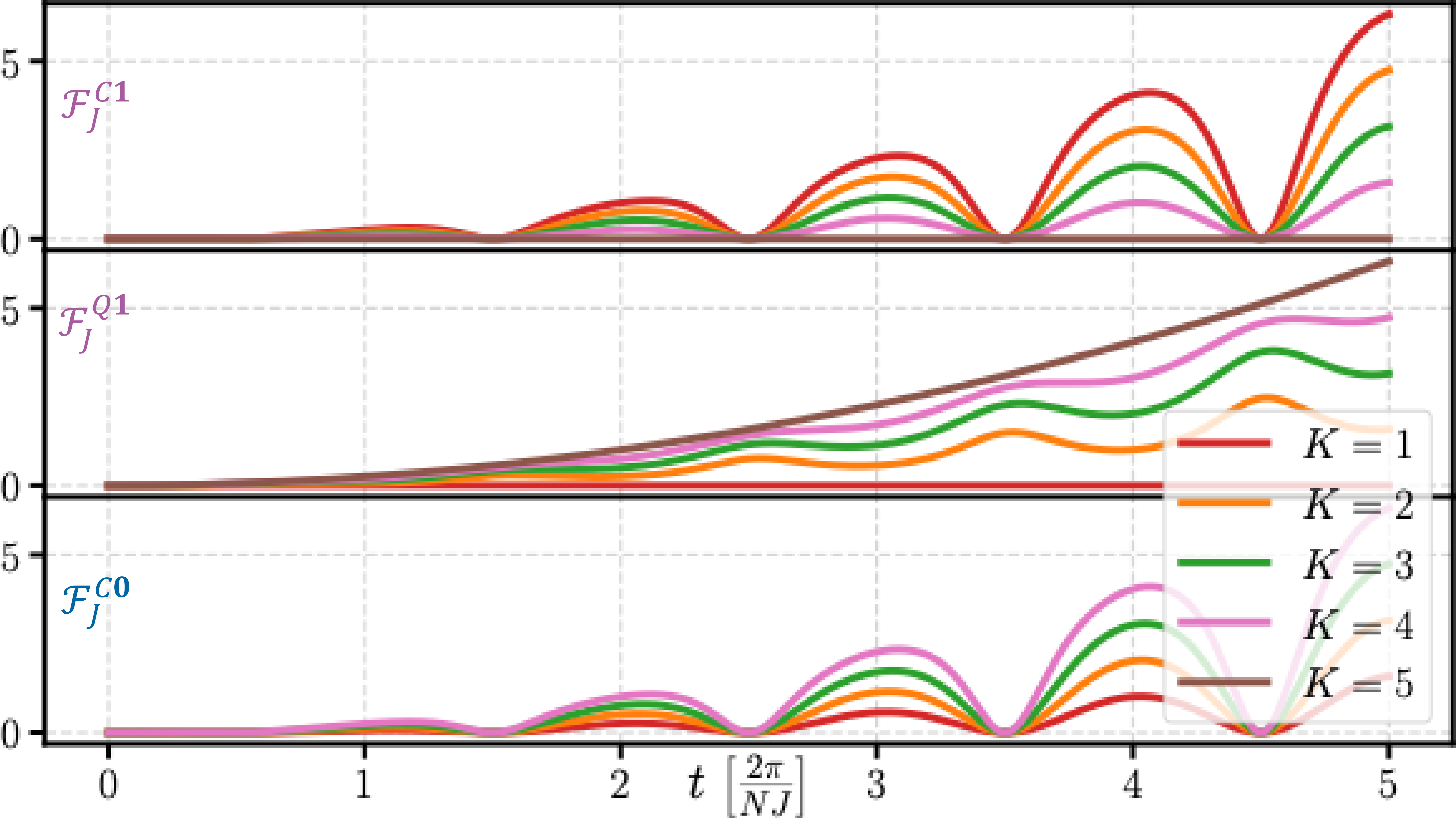
$$p^{\mathbf{0}} = K|u_d|^2$$

$\mathcal{F}_J^{C\mathbf{1}}$

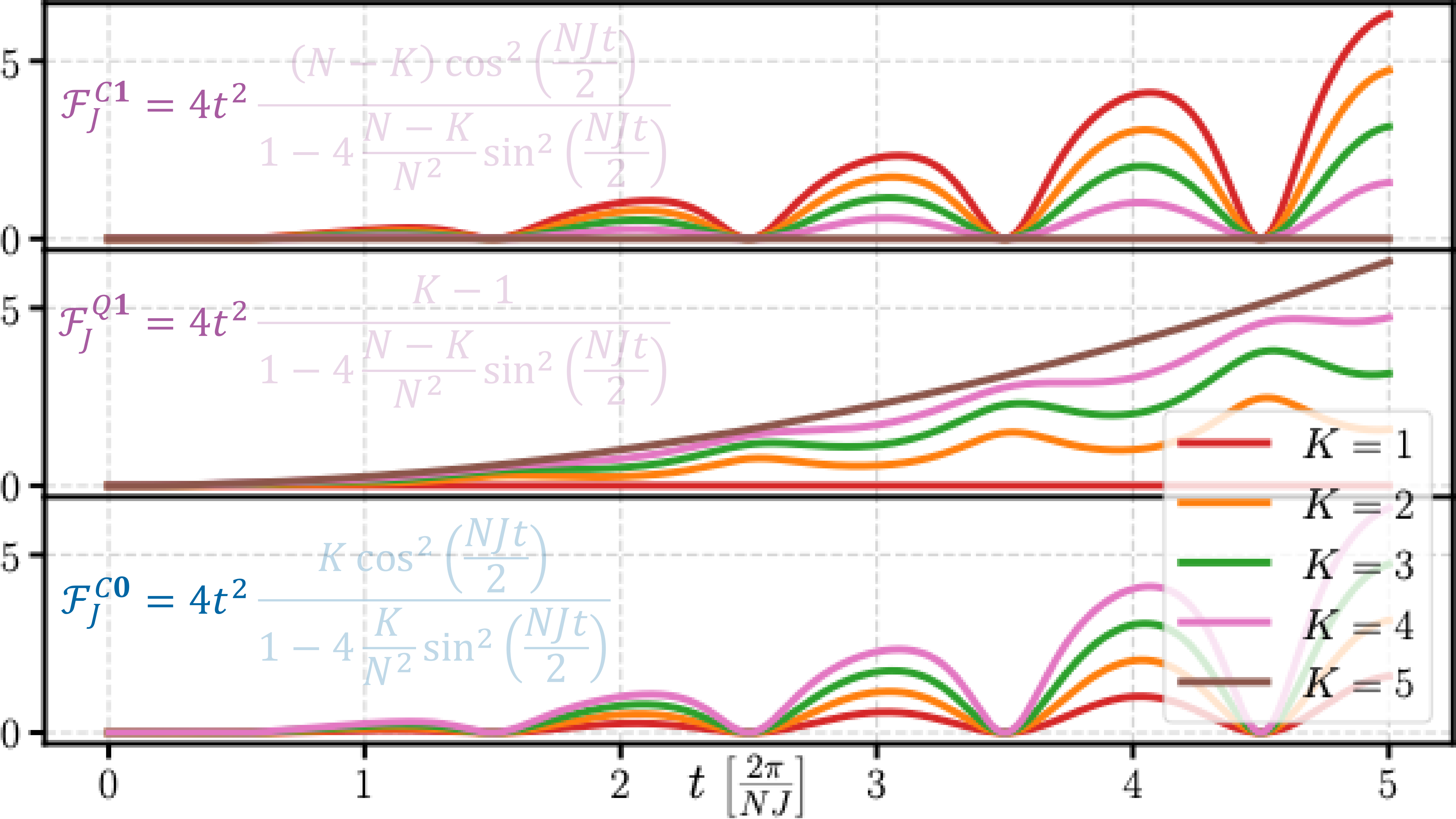
$\mathcal{F}_J^{Q\mathbf{1}}$

$\mathcal{F}_J^{C\mathbf{0}}$

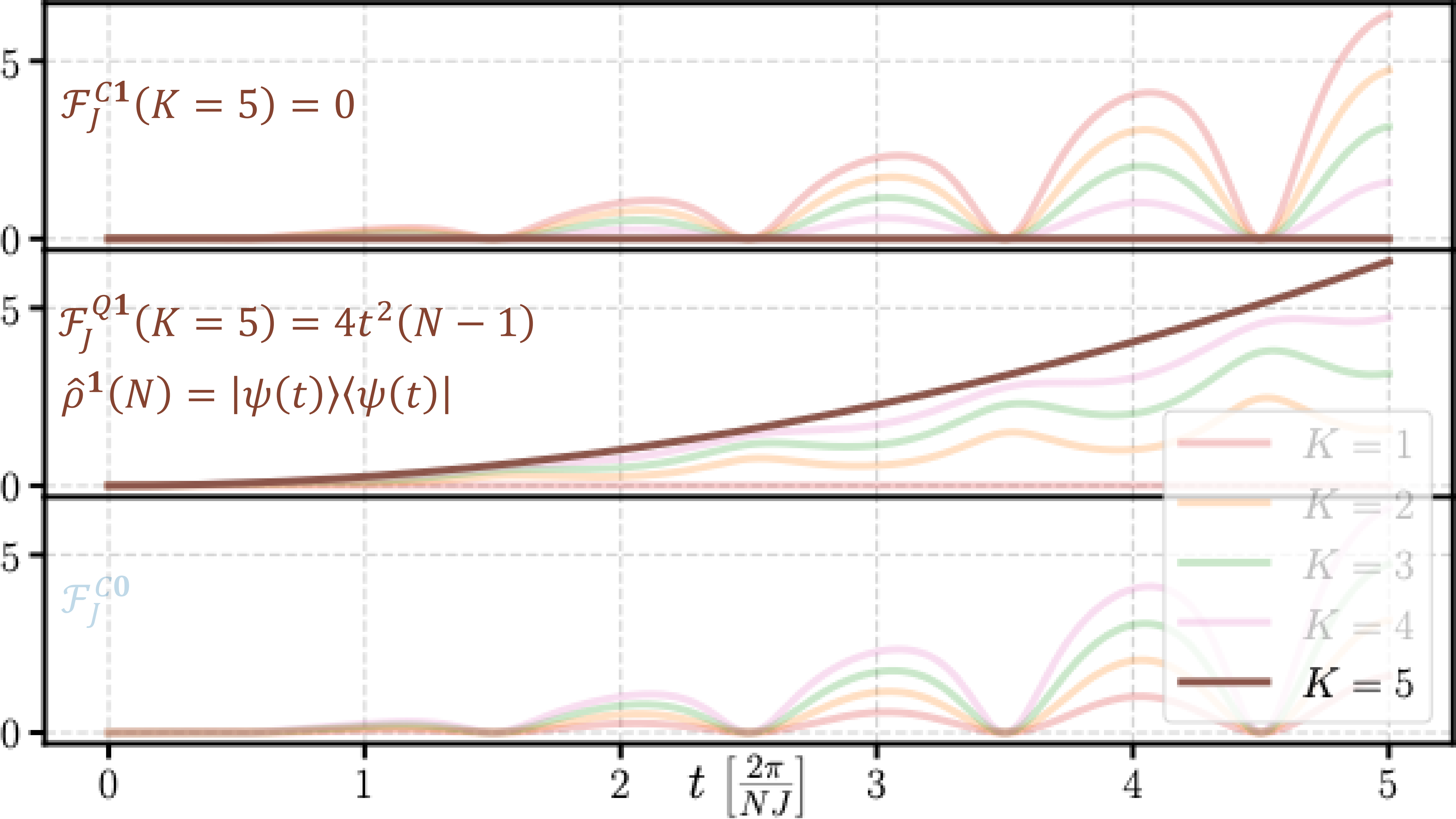
$\mathcal{F}_J^{Q\mathbf{0}} = 0$



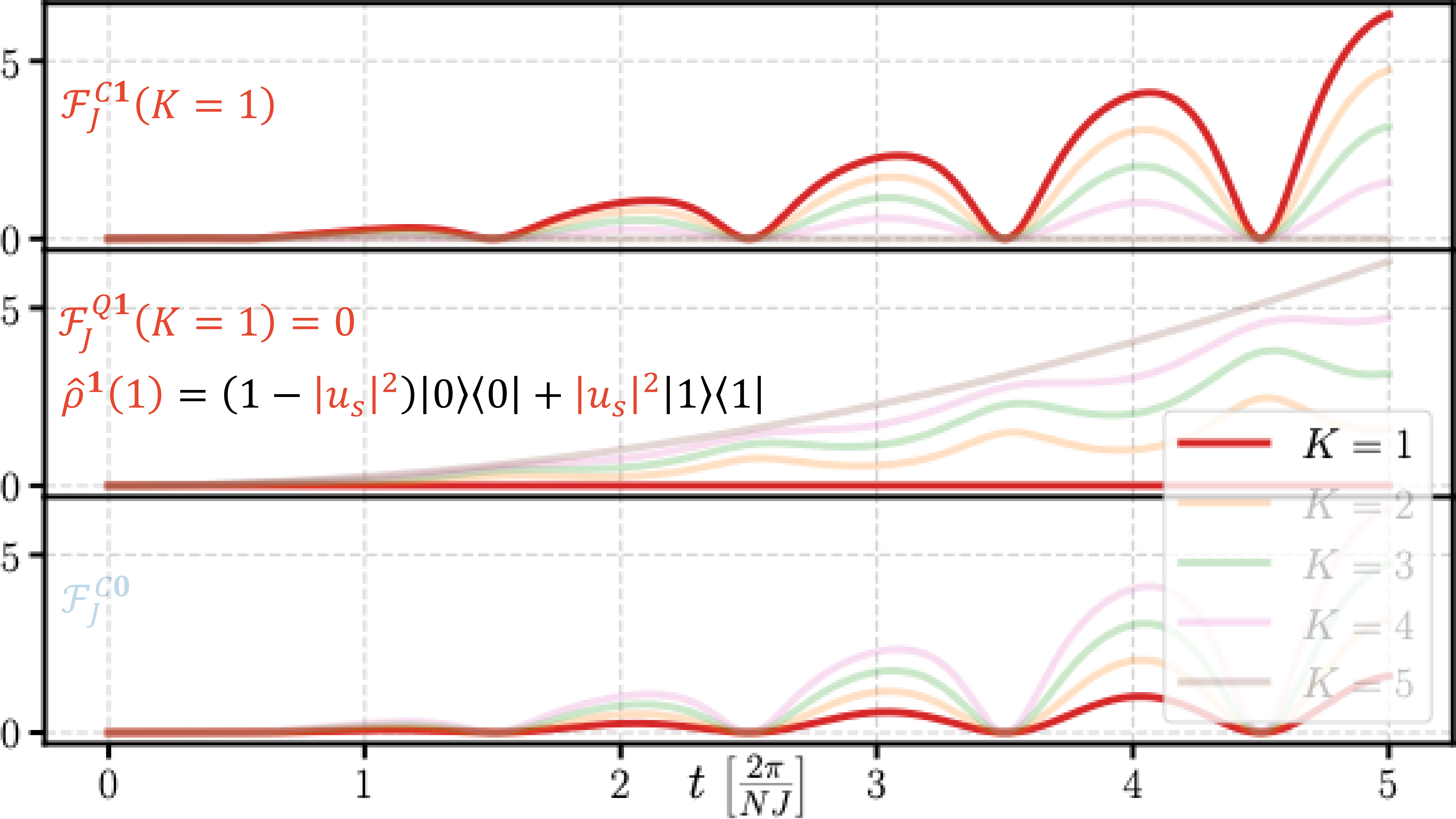
$\mathcal{F}_J^{C\mathbf{1}}$
$\mathcal{F}_J^{Q\mathbf{1}}$
$\mathcal{F}_J^{C\mathbf{0}}$
$K = 1$
$K = 2$
$K = 3$
$K = 4$
$K = 5$
$t \left[\frac{2\pi}{NJ}\right]$
0
5
0
1
2
3
4
5



$\mathcal{F}_J^{C1} = 4t^2 \frac{(N-K)\cos^2\left(\frac{NJt}{2}\right)}{1-4\frac{N-K}{N^2}\sin^2\left(\frac{NJt}{2}\right)}$
$\mathcal{F}_J^{Q1} = 4t^2 \frac{K-1}{1-4\frac{N-K}{N^2}\sin^2\left(\frac{NJt}{2}\right)}$
$\mathcal{F}_J^{C0} = 4t^2 \frac{K\cos^2\left(\frac{NJt}{2}\right)}{1-4\frac{K}{N^2}\sin^2\left(\frac{NJt}{2}\right)}$
0
5
0
1
2
3
4
5
$t\ \left[\frac{2\pi}{NJ}\right]$
$K=1$
$K=2$
$K=3$
$K=4$
$K=5$

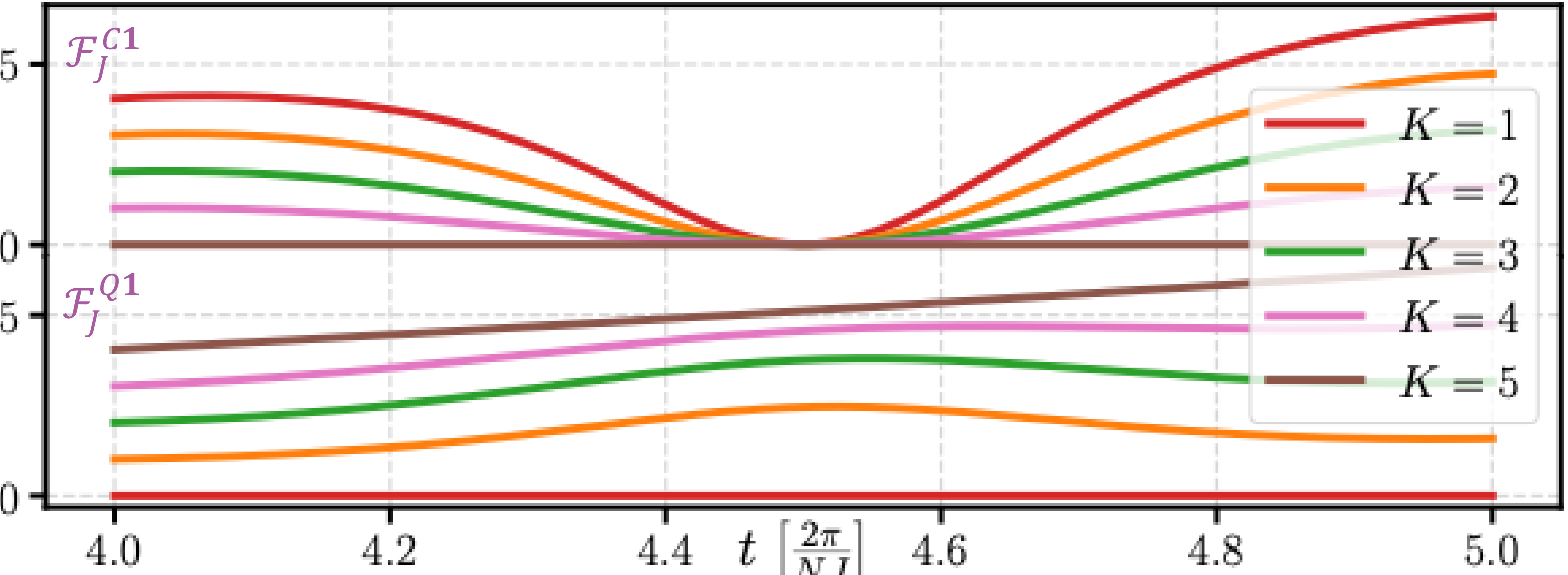


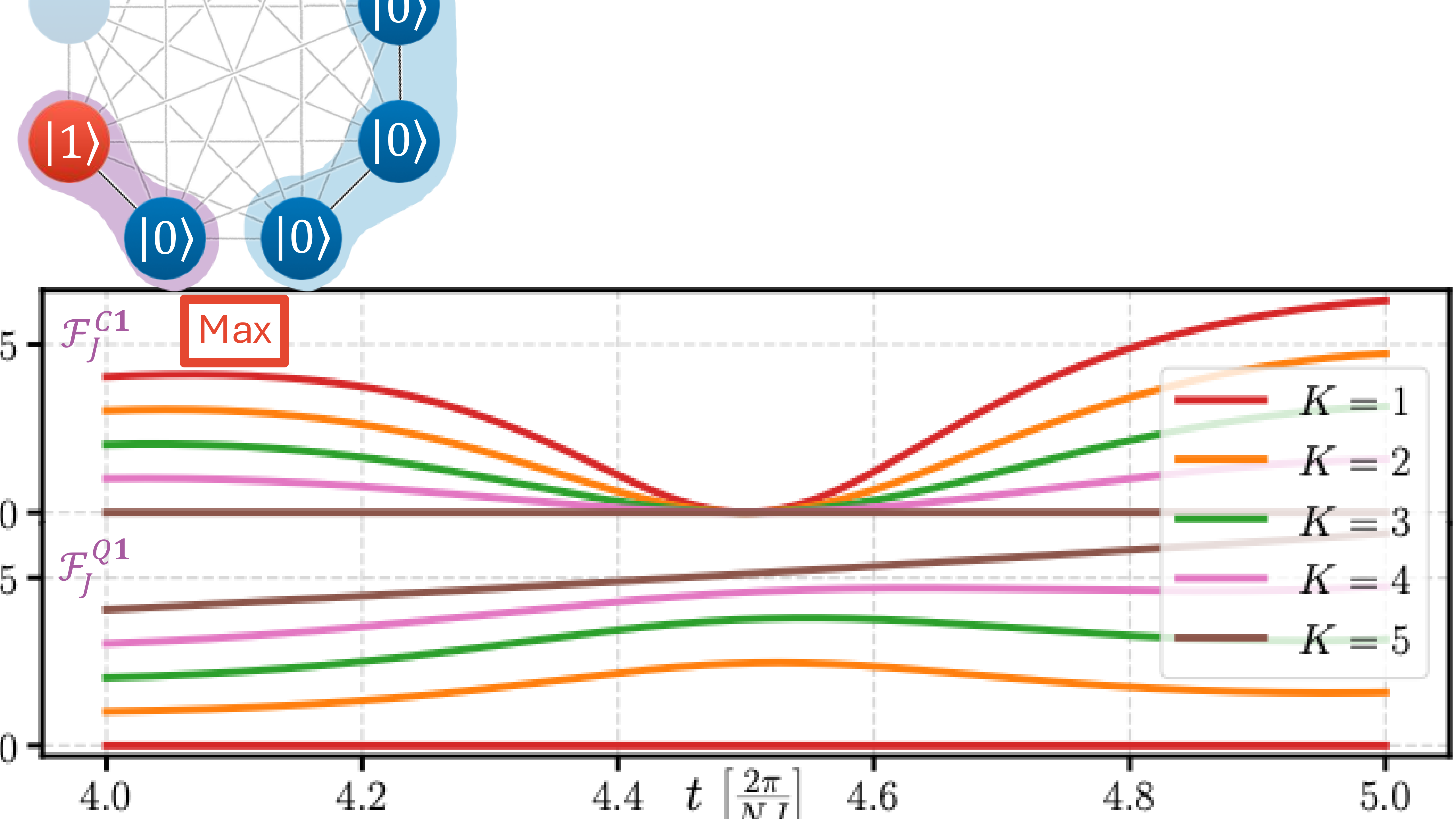
$\mathcal{F}_J^{C\mathbf{1}}(K=5)=0$
$\mathcal{F}_J^{Q\mathbf{1}}(K=5)=4t^2(N-1)$
$\hat{\rho}^{\mathbf{1}}(N)=|\psi(t)\rangle\langle\psi(t)|$
$\mathcal{F}_J^{C\mathbf{0}}$
$K=1$
$K=2$
$K=3$
$K=4$
$K=5$
0
5
0
1
2
3
4
5
$t\ \left[\frac{2\pi}{NJ}\right]$



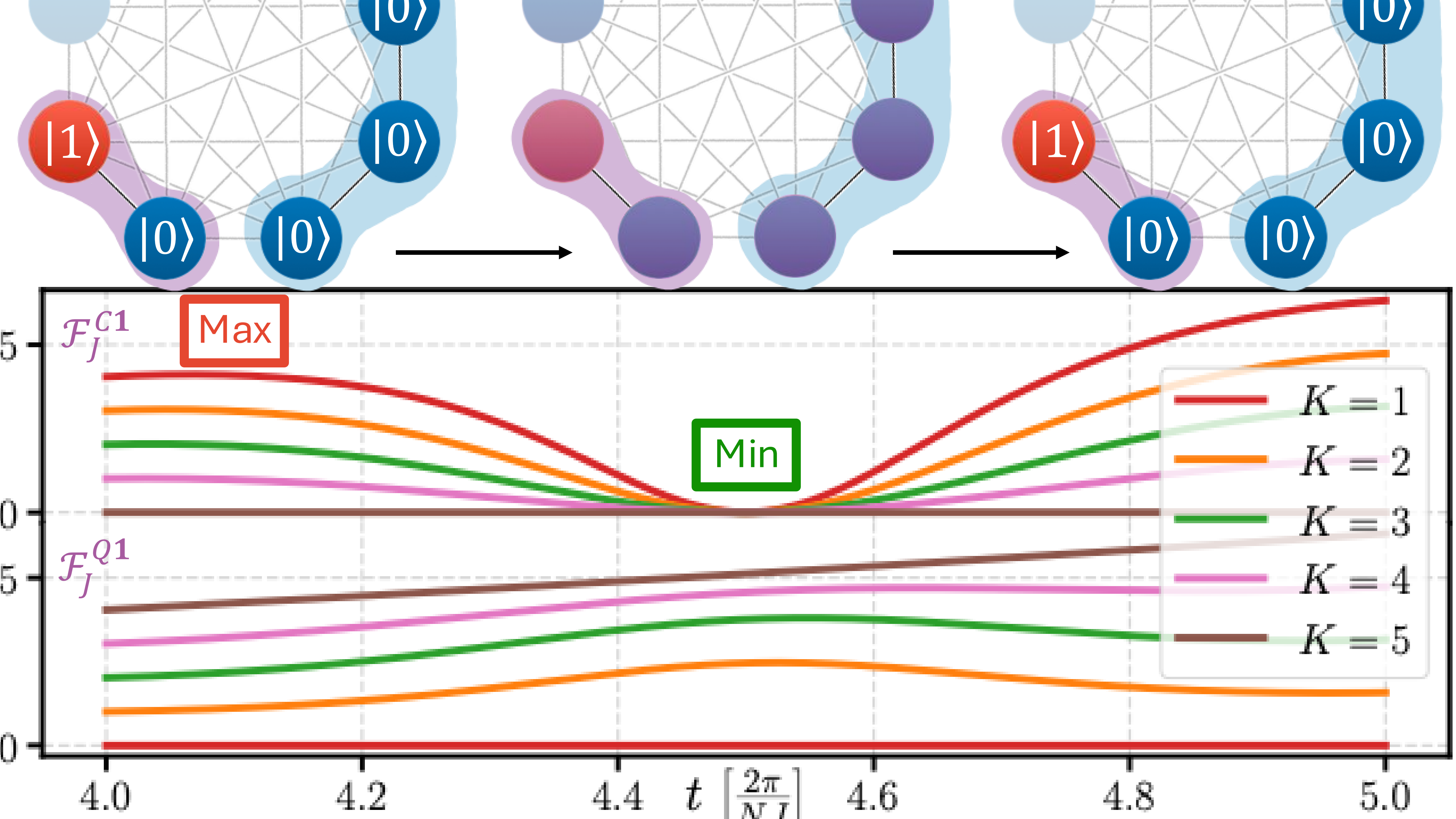
$\mathcal{F}_J^{C\mathbf{1}}(K=1)$
$\mathcal{F}_J^{Q\mathbf{1}}(K=1)=0$
$\hat{\rho}^{\mathbf{1}}(1)=(1-|u_s|^2)|0\rangle\langle 0|+|u_s|^2|1\rangle\langle 1|$
$\mathcal{F}_J^{C\mathbf{0}}$
$K=1$
$K=2$
$K=3$
$K=4$
$K=5$
$t\ \left[\frac{2\pi}{NJ}\right]$
0
1
2
3
4
5

Fisher Information

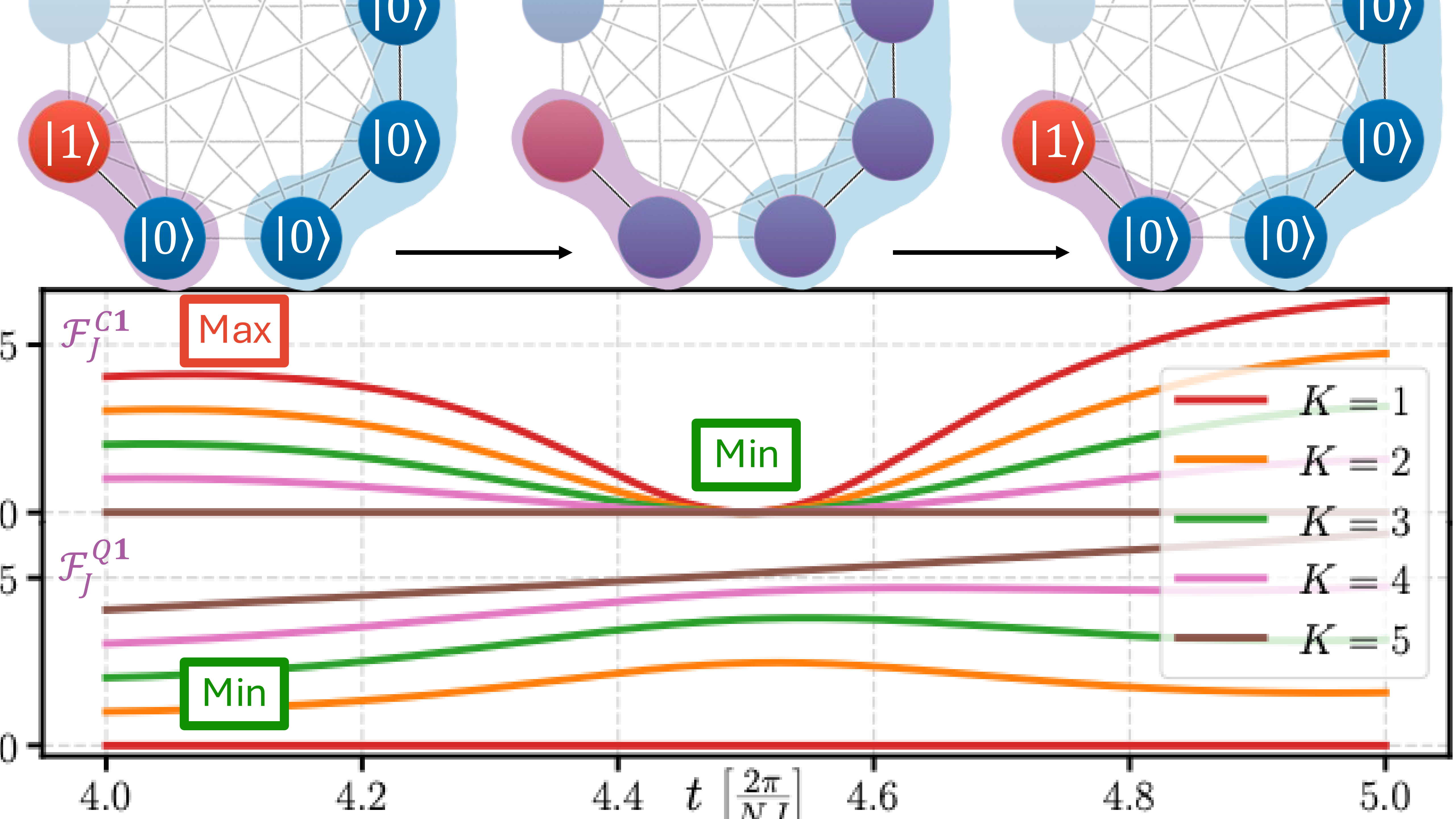




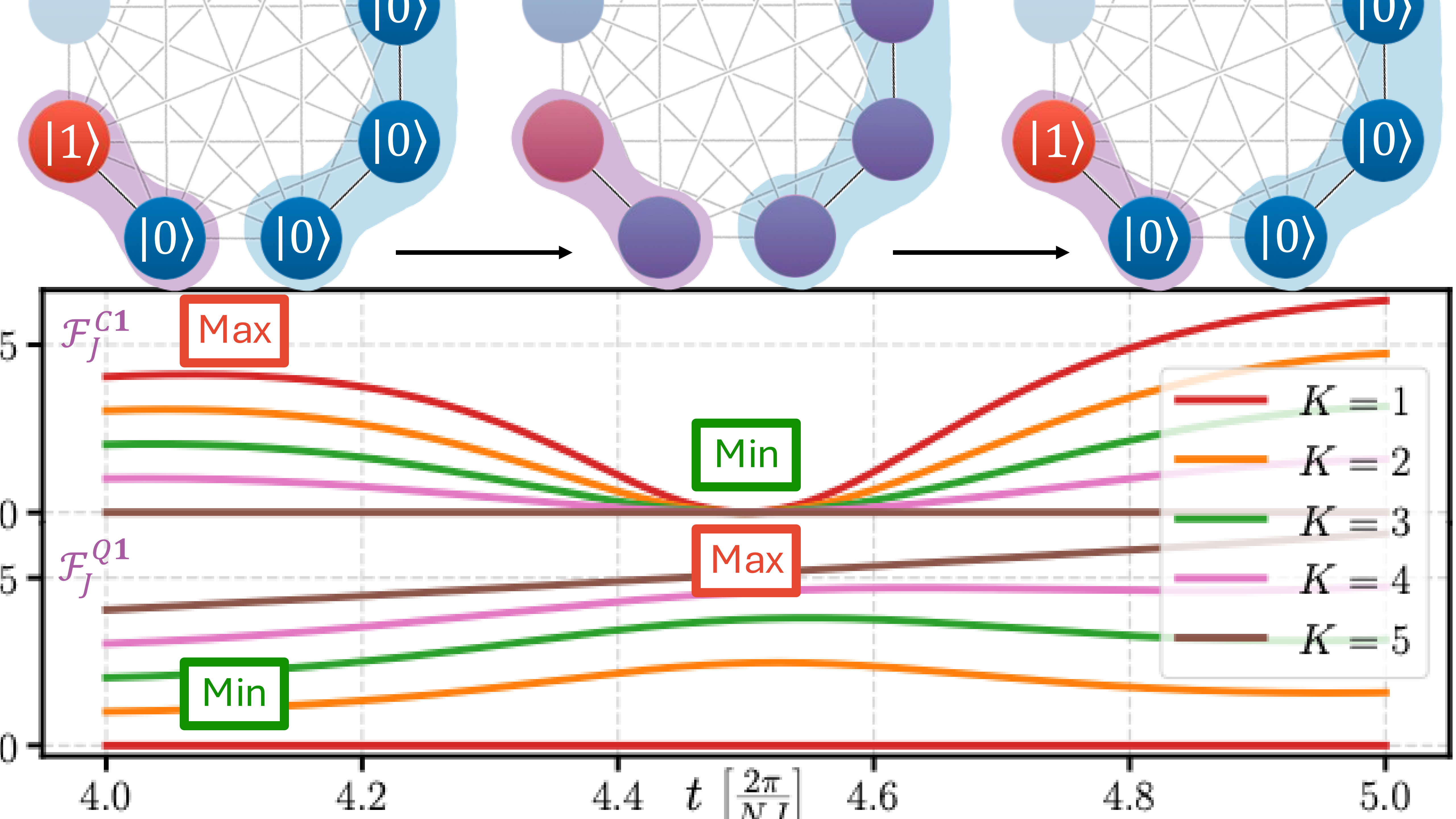
|1⟩
|0⟩
|0⟩
|0⟩
|0⟩
|0⟩
Max
$\mathcal{F}_J^{C1}$
$\mathcal{F}_J^{Q1}$
5
0
5
0
$K = 1$
$K = 2$
$K = 3$
$K = 4$
$K = 5$
4.0
4.2
4.4
4.6
4.8
5.0



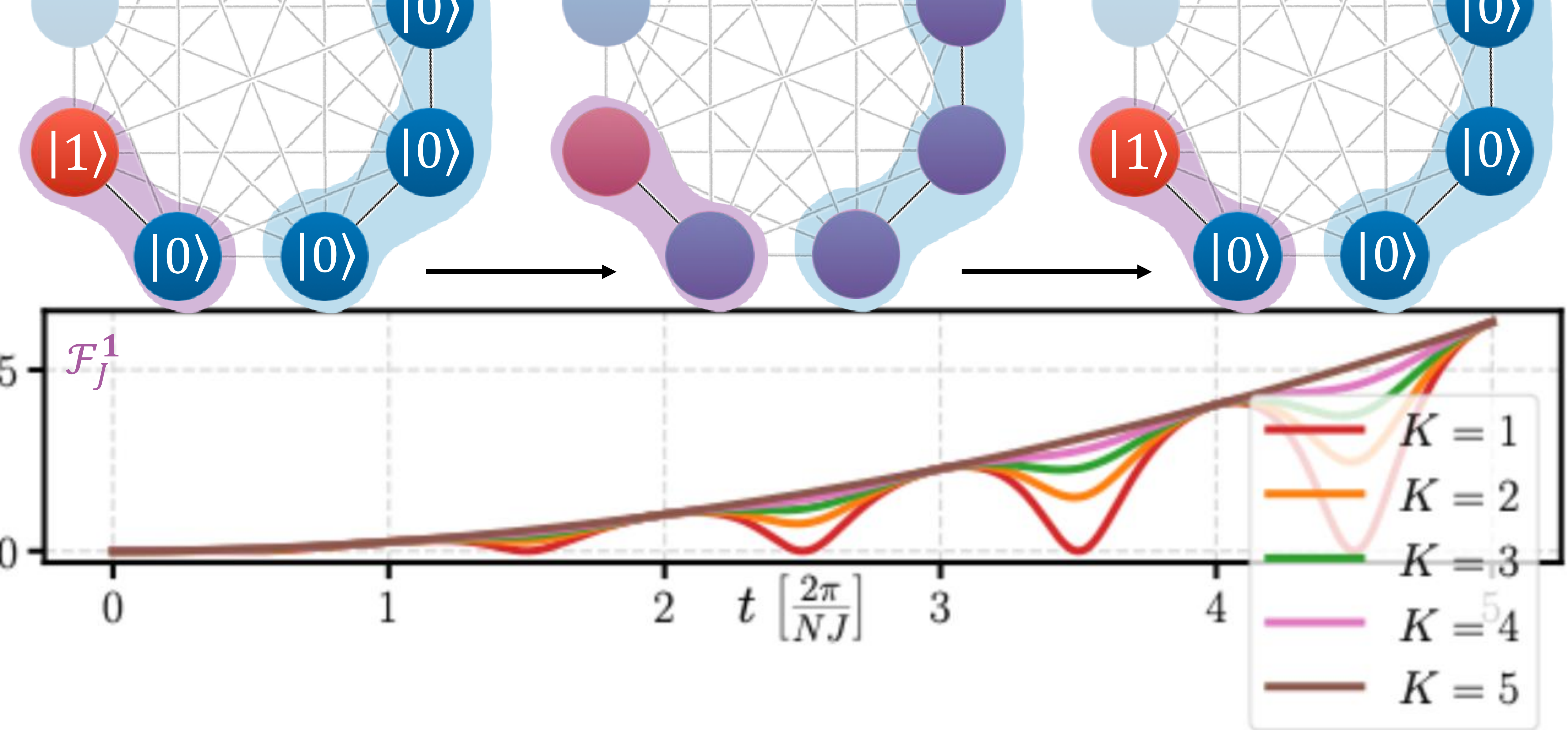
|1⟩
|0⟩
|0⟩
|0⟩
|0⟩
|1⟩
|0⟩
|0⟩
|0⟩
|0⟩
$\mathcal{F}_J^{C1}$
Max
Min
$\mathcal{F}_J^{Q1}$
$K=1$
$K=2$
$K=3$
$K=4$
$K=5$
5
0
5
0
4.0
4.2
4.4
4.6
4.8
5.0
$t\ \left[\frac{2\pi}{NJ}\right]$



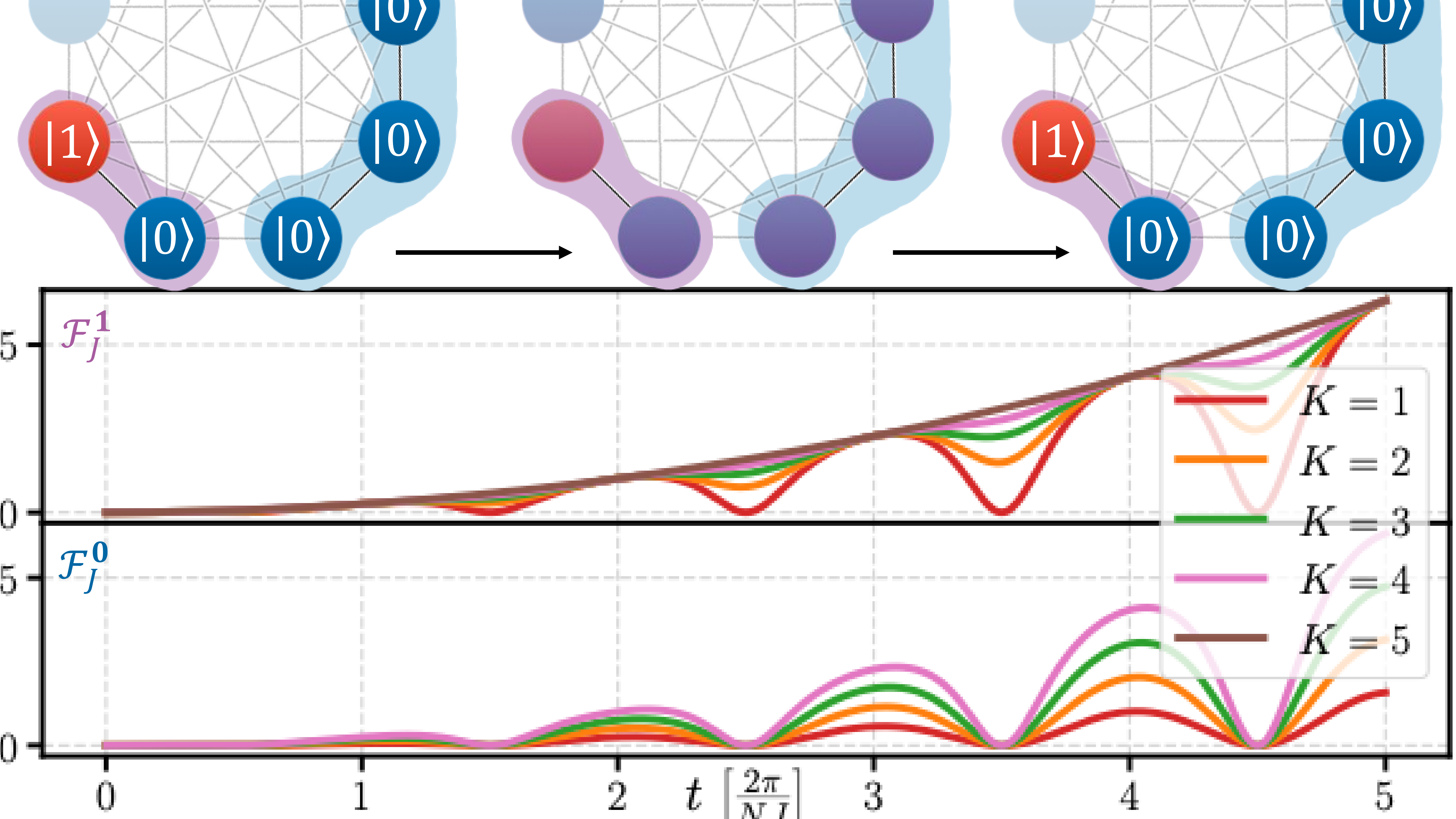
|1⟩
|0⟩
|0⟩
|0⟩
|0⟩
|1⟩
|0⟩
|0⟩
|0⟩
|0⟩
$\mathcal{F}_J^{C1}$
Max
Min
$\mathcal{F}_J^{Q1}$
Min
$K=1$
$K=2$
$K=3$
$K=4$
$K=5$
5
0
5
0
4.0
4.2
4.4
4.6
4.8
5.0



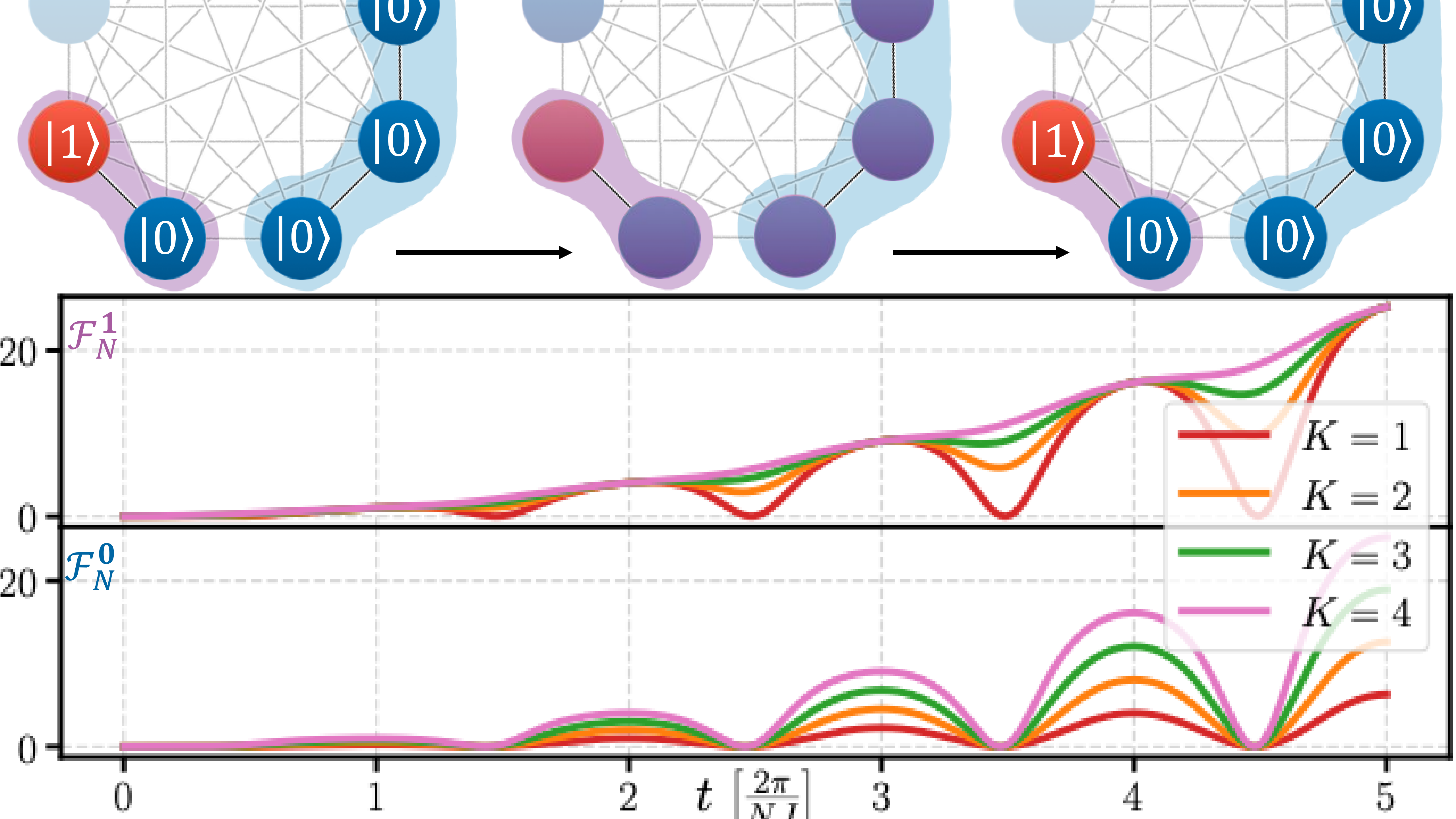
|1⟩
|0⟩
$\mathcal{F}_J^{C1}$
Max
Min
$\mathcal{F}_J^{Q1}$
Max
Min
5
0
5
0
$K=1$
$K=2$
$K=3$
$K=4$
$K=5$
4.0
4.2
4.4
$t\ \left[\frac{2\pi}{NJ}\right]$
4.6
4.8
5.0



|0⟩
|1⟩
|0⟩
|0⟩
|0⟩
|0⟩
|1⟩
|0⟩
|0⟩
|0⟩
$\mathcal{F}_J^{\mathbf{1}}$
5
0
0
1
2
$t\left[\frac{2\pi}{NJ}\right]$
3
4
5
$K=1$
$K=2$
$K=3$
$K=4$
$K=5$

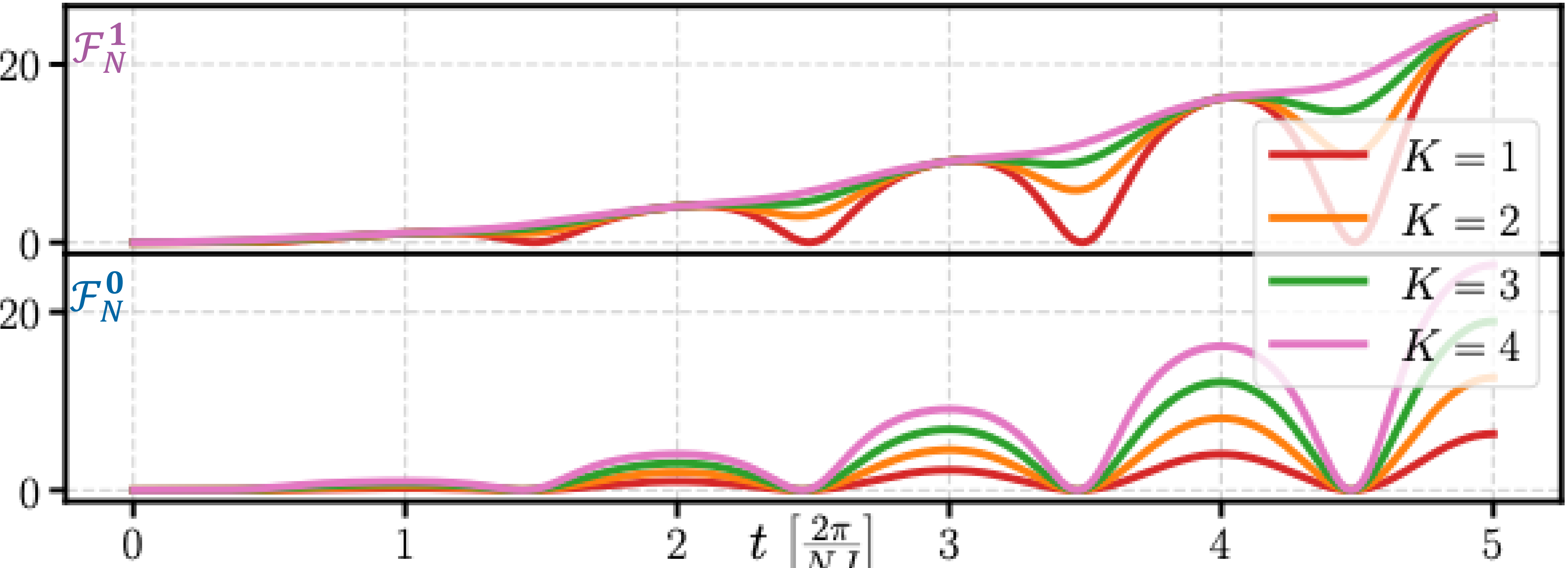


|1⟩
|0⟩
|0⟩
|0⟩
|0⟩
|0⟩
|1⟩
|0⟩
|0⟩
|0⟩
|0⟩
$\mathcal{F}_J^{\mathbf{1}}$
$\mathcal{F}_J^{\mathbf{0}}$
$K = 1$
$K = 2$
$K = 3$
$K = 4$
$K = 5$
$t\ \left[\frac{2\pi}{NJ}\right]$
0
5
0
5
0
1
2
3
4
5


|1⟩
|0⟩
|0⟩
|0⟩
|0⟩
$\mathcal{F}_N^{\mathbf{1}}$
$\mathcal{F}_N^{\mathbf{0}}$
20
0
$K = 1$
$K = 2$
$K = 3$
$K = 4$
0
1
2
3
4
5
$t\ \left[\frac{2\pi}{NJ}\right]$

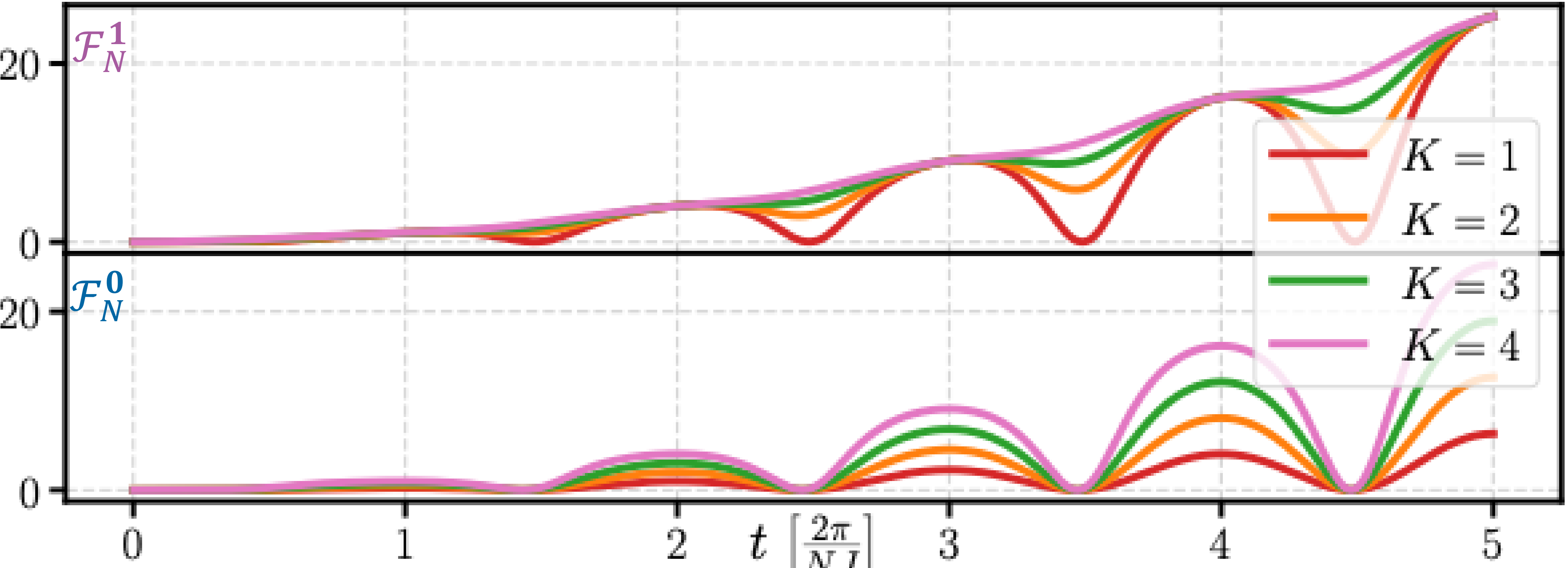
Fisher Information

$\mathcal{F}_{\Theta}^{\mathbf{1}}(K^{\mathbf{1}})$



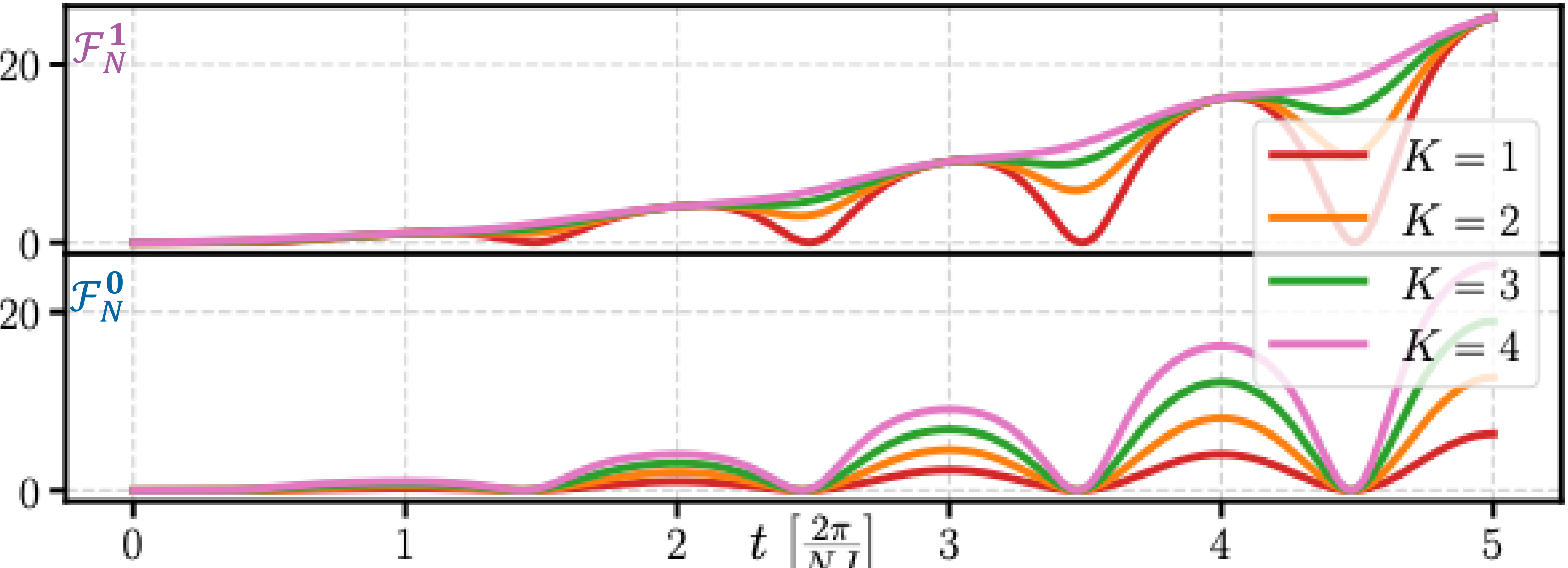
Fisher Information

$$\mathcal{F}_{\Theta}^{\mathbf{1}}(K^{\mathbf{1}}) + \mathcal{F}_{\Theta}^{\mathbf{0}}(K^{\mathbf{0}})$$



Fisher Information

$$\mathcal{F}_{\Theta}^{\mathbf{1}}(K^{\mathbf{1}}) + \mathcal{F}_{\Theta}^{\mathbf{0}}(K^{\mathbf{0}}) \neq \mathcal{F}_{\Theta}^{\mathbf{1}}(K^{\mathbf{1}} + K^{\mathbf{0}})$$



Fisher Information

$$\mathcal{F}_{\Theta} \propto |\partial_{\Theta}\hat{\rho}(t)|^2$$

Fisher Information

$$\mathcal{F}_{\Theta} \propto |\partial_{\Theta}\hat{\rho}(t)|^2$$

$$\partial_{\Theta}\hat{\rho}(t_2) = \partial_{\Theta}\{\Phi(t_1, t_2)[\hat{\rho}(t_1)]\}$$

Fisher Information

$$\mathcal{F}_\Theta \propto |\partial_\Theta \hat{\rho}(t)|^2$$

$$\begin{aligned}\partial_\Theta \hat{\rho}(t_2) &= \partial_\Theta\{\Phi(t_1, t_2)[\hat{\rho}(t_1)]\} \\ &= \{\partial_\Theta \Phi(t_1, t_2)\}[\hat{\rho}(t_1)]\end{aligned}$$

Dynamics

Fisher Information

$$\mathcal{F}_\Theta \propto |\partial_\Theta \hat{\rho}(t)|^2$$

$$\begin{aligned}\partial_\Theta \hat{\rho}(t_2) &= \partial_\Theta\{\Phi(t_1,t_2)[\hat{\rho}(t_1)]\} \\ &= \{\partial_\Theta \Phi(t_1,t_2)\}[\hat{\rho}(t_1)] + \Phi(t_1,t_2)[\partial_\Theta \hat{\rho}(t_1)]\end{aligned}$$

Dynamics State

Fisher Information

$$\mathcal{F}_\Theta \propto |\partial_\Theta \hat{\rho}(t)|^2$$

$$\propto |\{\partial_\Theta \Phi(t_1, t_2)\}[\hat{\rho}(t_1)]|^2$$

Dynamics

$$\partial_\Theta \hat{\rho}(t_2) = \partial_\Theta\{\Phi(t_1, t_2)[\hat{\rho}(t_1)]\}$$

$$= \{\partial_\Theta \Phi(t_1, t_2)\}[\hat{\rho}(t_1)] + \Phi(t_1, t_2)[\partial_\Theta \hat{\rho}(t_1)]$$

Dynamics State

Fisher Information

$$\mathcal{F}_\Theta \propto |\partial_\Theta \hat{\rho}(t)|^2$$

$$\propto |\{\partial_\Theta \Phi(t_1, t_2)\}[\hat{\rho}(t_1)]|^2 + |\Phi(t_1, t_2)[\partial_\Theta \hat{\rho}(t_1)]|^2$$

Dynamics State

$$\partial_\Theta \hat{\rho}(t_2) = \partial_\Theta \{\Phi(t_1, t_2)[\hat{\rho}(t_1)]\}$$

$$= \{\partial_\Theta \Phi(t_1, t_2)\}[\hat{\rho}(t_1)] + \Phi(t_1, t_2)[\partial_\Theta \hat{\rho}(t_1)]$$

Dynamics State

Fisher Information

$$\mathcal{F}_\Theta \propto |\partial_\Theta \hat{\rho}(t)|^2$$

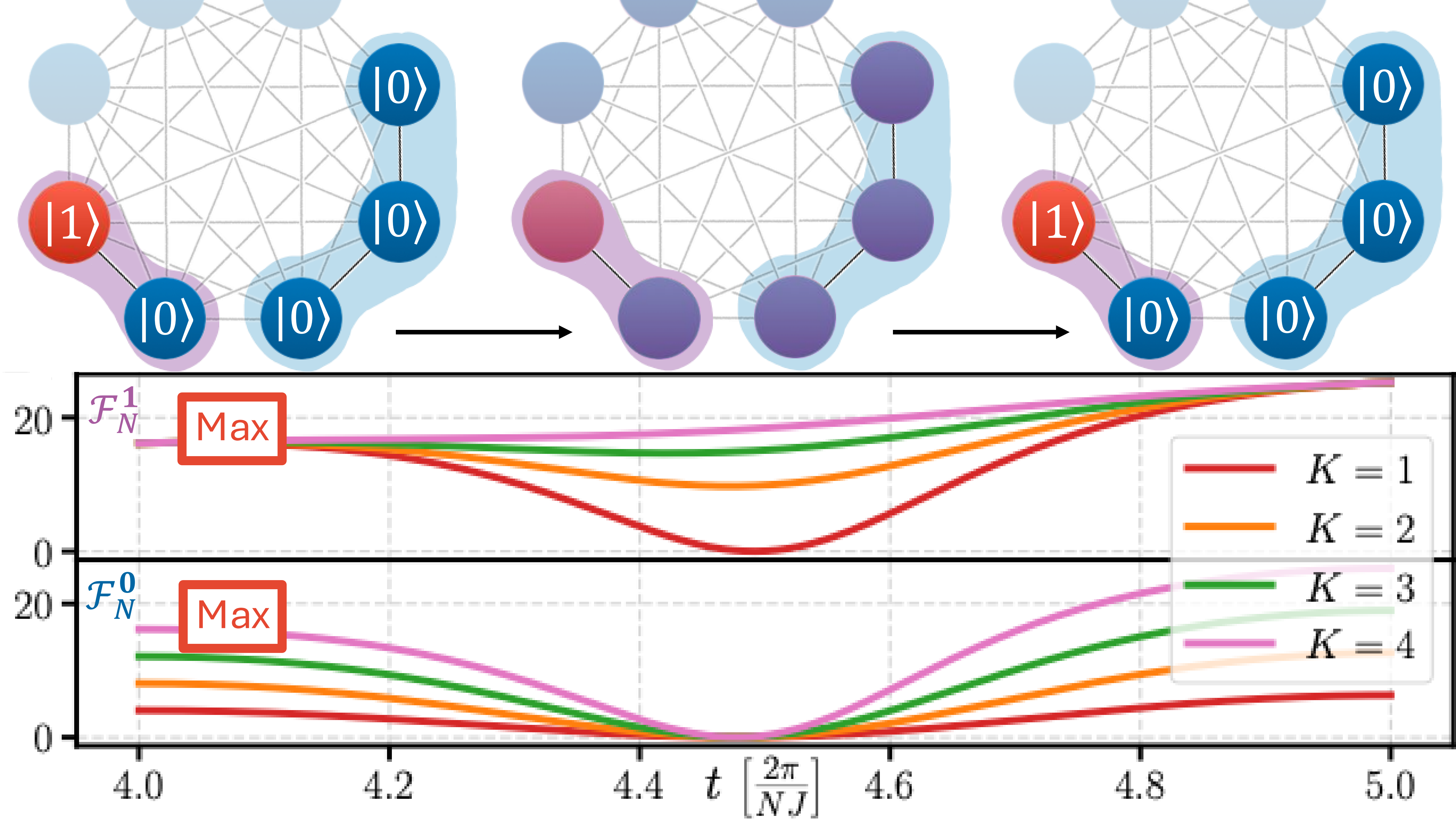
$$\propto |\{\partial_\Theta \Phi(t_1, t_2)\}[\hat{\rho}(t_1)]|^2 + |\Phi(t_1, t_2)[\partial_\Theta \hat{\rho}(t_1)]|^2 + \text{Cross Terms}$$

Dynamics State

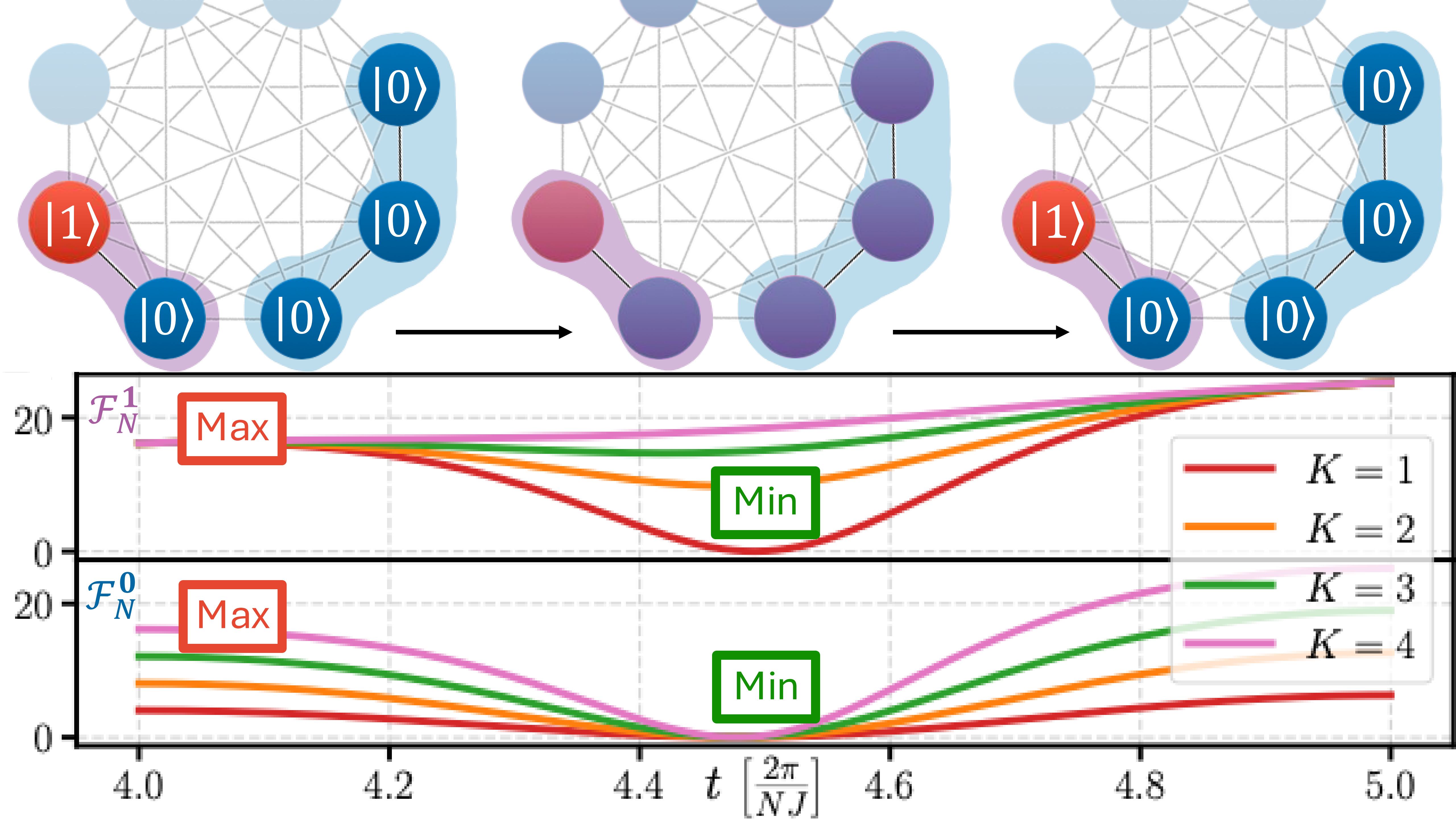
$$\partial_\Theta \hat{\rho}(t_2) = \partial_\Theta\{\Phi(t_1, t_2)[\hat{\rho}(t_1)]\}$$

$$= \{\partial_\Theta \Phi(t_1, t_2)\}[\hat{\rho}(t_1)] + \Phi(t_1, t_2)[\partial_\Theta \hat{\rho}(t_1)]$$

Dynamics State



|1⟩
|0⟩
|0⟩
|0⟩
|0⟩
|1⟩
|0⟩
|0⟩
|0⟩
|0⟩
$\mathcal{F}_N^{\mathbf{1}}$
Max
$\mathcal{F}_N^{\mathbf{0}}$
Max
20
0
20
0
$K=1$
$K=2$
$K=3$
$K=4$
4.0
4.2
4.4
4.6
4.8
5.0
$t\ \left[\frac{2\pi}{NJ}\right]$

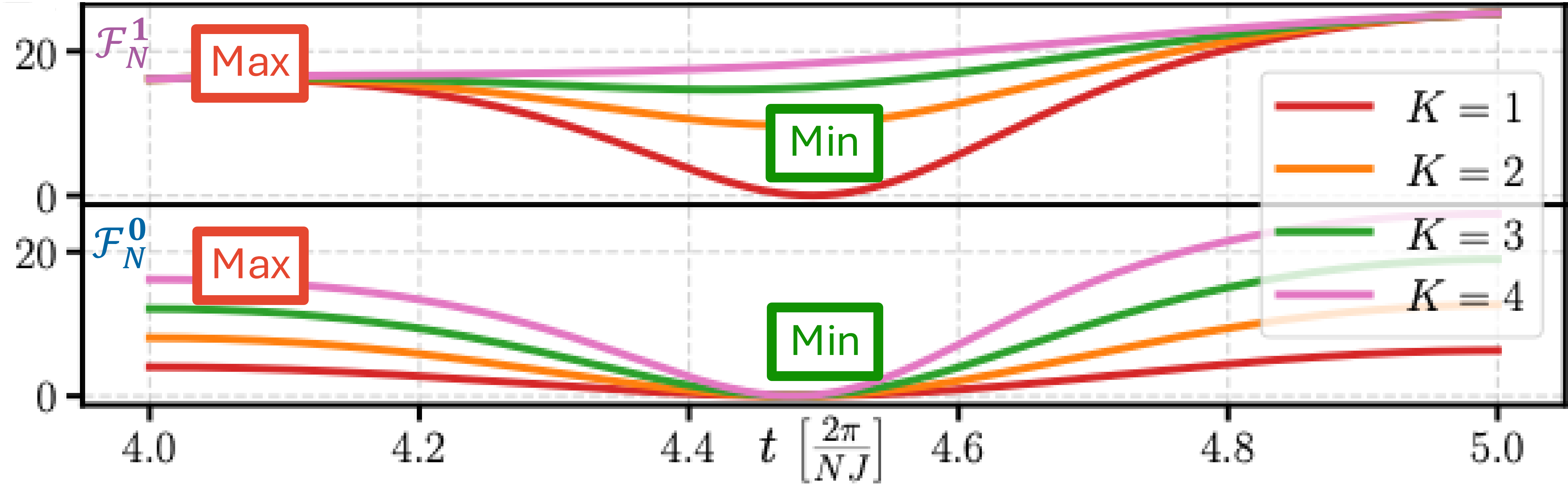


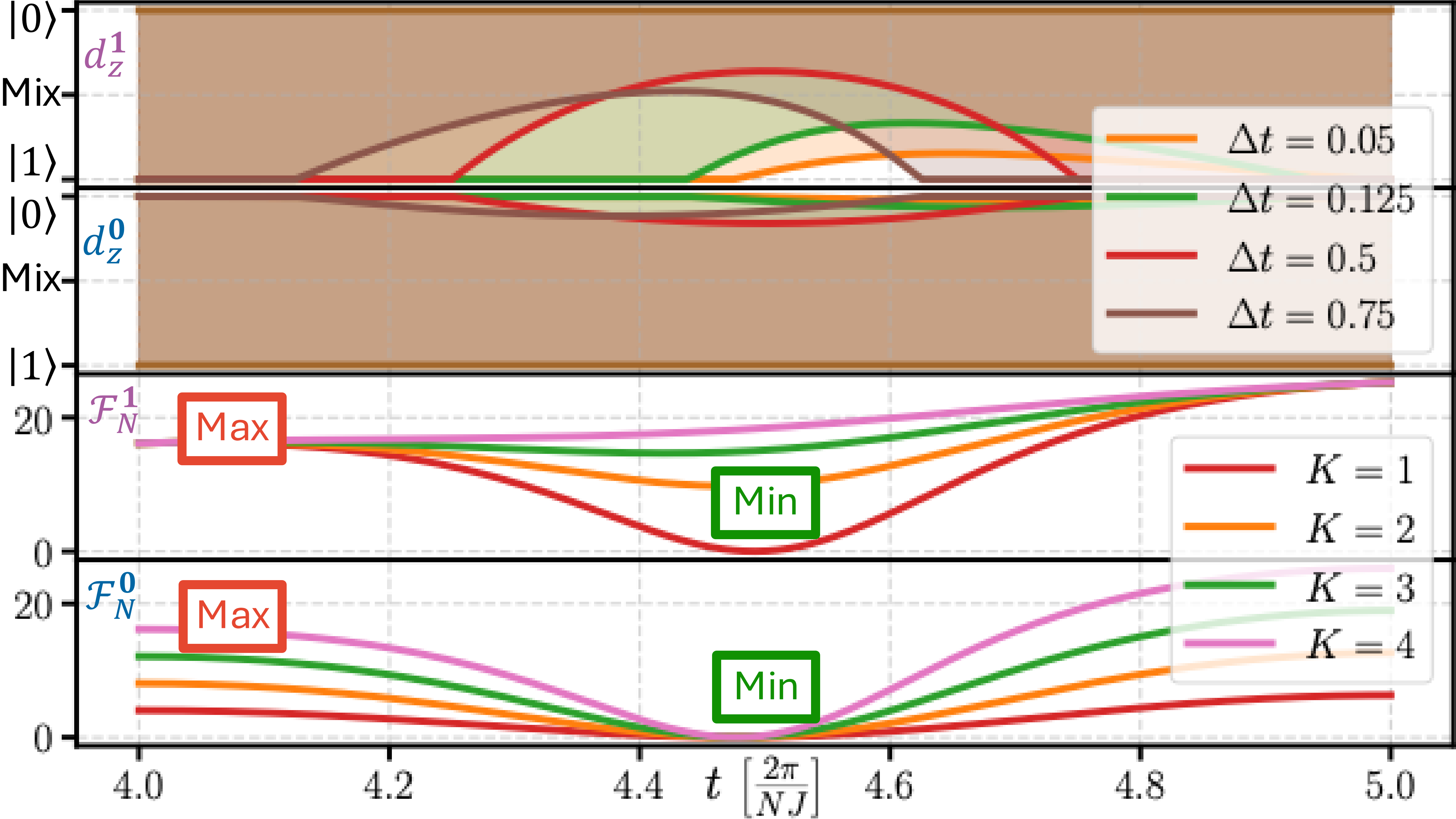
|1⟩
|0⟩
|0⟩
|0⟩
|0⟩
|1⟩
|0⟩
|0⟩
|0⟩
|0⟩
$\mathcal{F}_N^{\mathbf{1}}$
Max
Min
$\mathcal{F}_N^{\mathbf{0}}$
Max
Min
$K = 1$
$K = 2$
$K = 3$
$K = 4$
20
0
20
0
4.0
4.2
4.4
4.6
4.8
5.0
$t\ \left[\frac{2\pi}{NJ}\right]$

State v. Dynamics

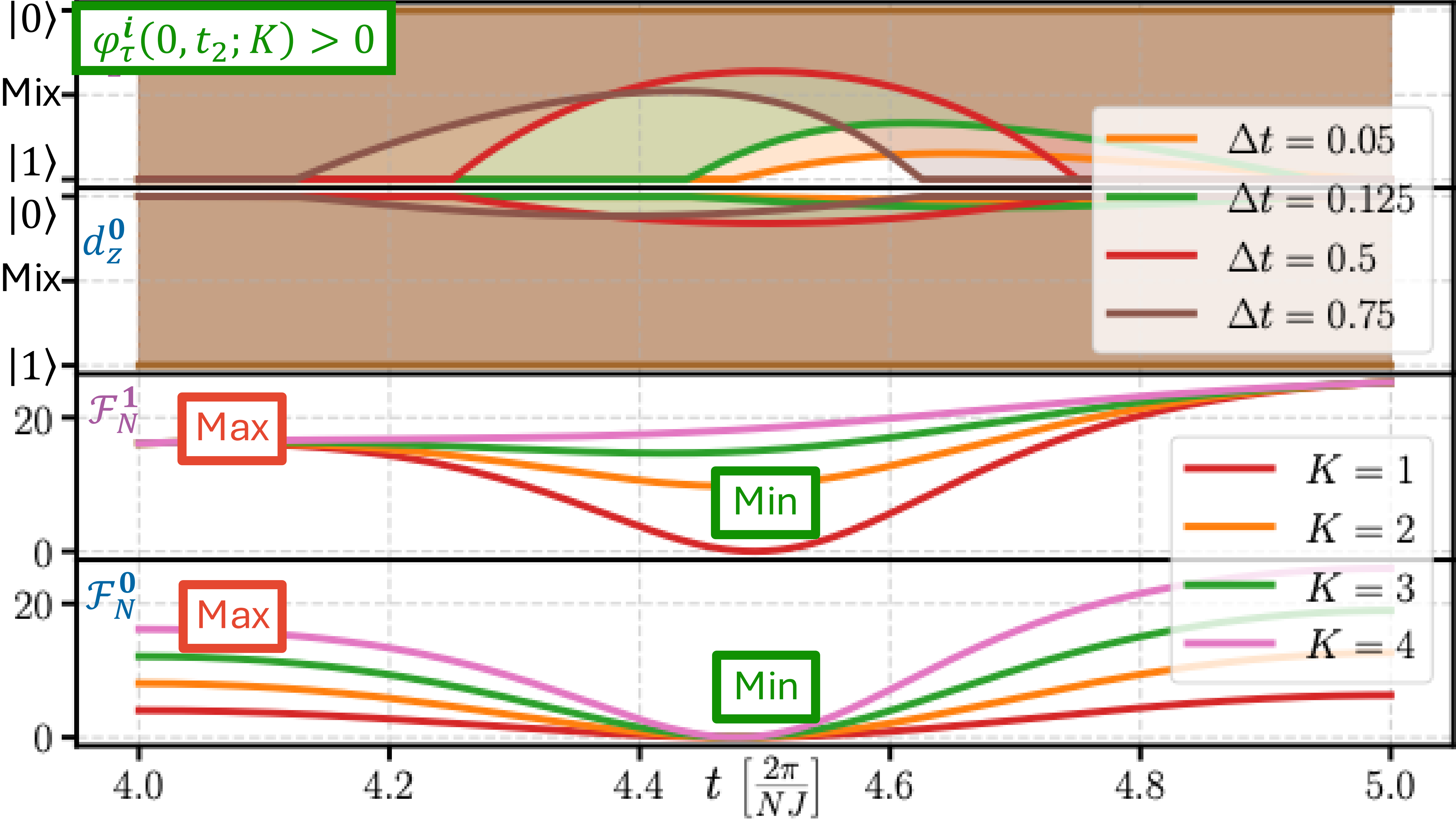
$$\left| \Lambda^{i}\, \vec{d}^{i} \right| \leq 1$$

Domain of Positivity

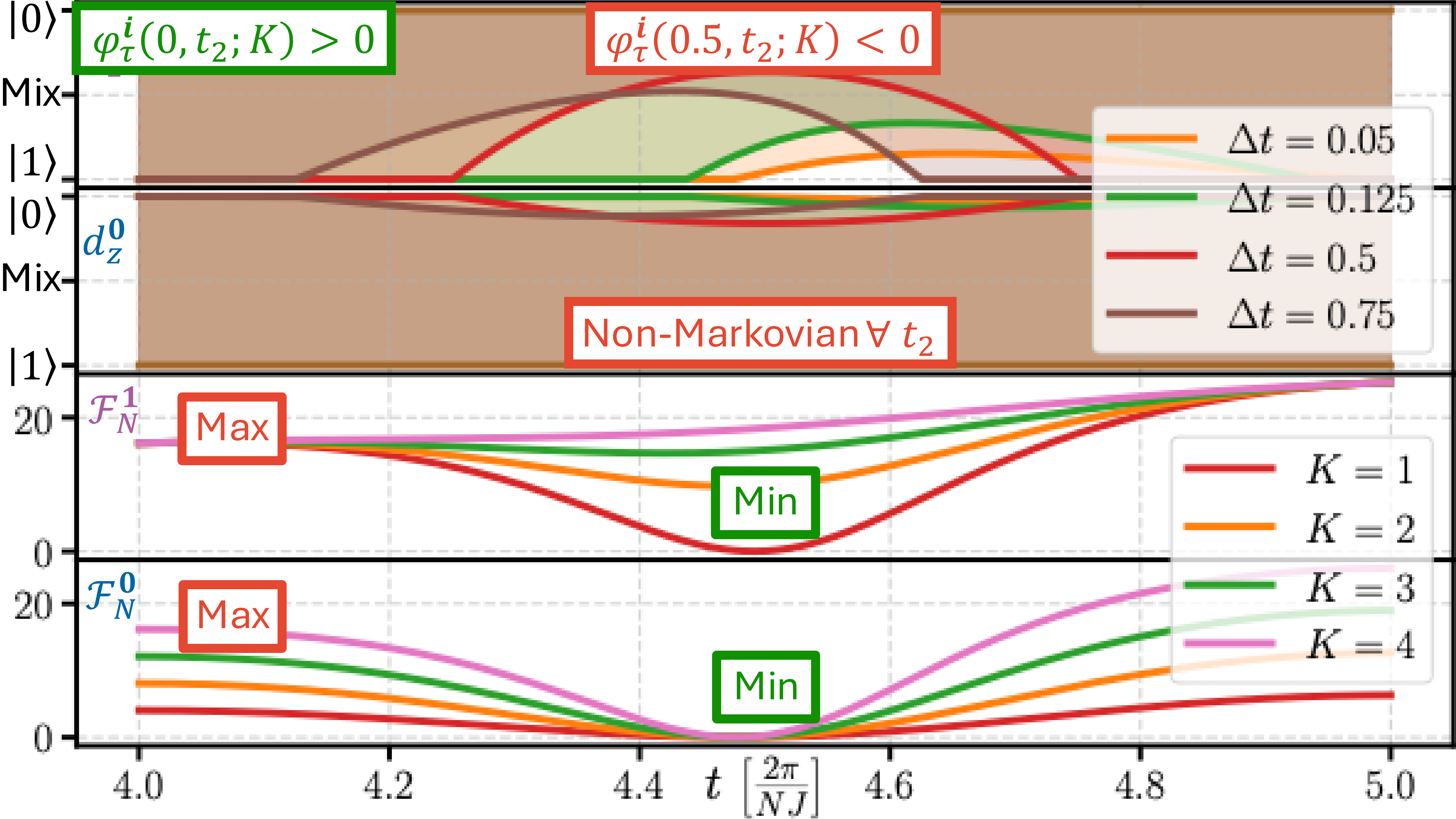




|0⟩
d_z^1
Mix
|1⟩
|0⟩
d_z^0
Mix
|1⟩
$\Delta t = 0.05$
$\Delta t = 0.125$
$\Delta t = 0.5$
$\Delta t = 0.75$
$\mathcal{F}_N^1$
20
0
Max
Min
$K = 1$
$K = 2$
$K = 3$
$K = 4$
$\mathcal{F}_N^0$
20
0
Max
Min
4.0
4.2
4.4
$t\ \left[\frac{2\pi}{NJ}\right]$
4.6
4.8
5.0



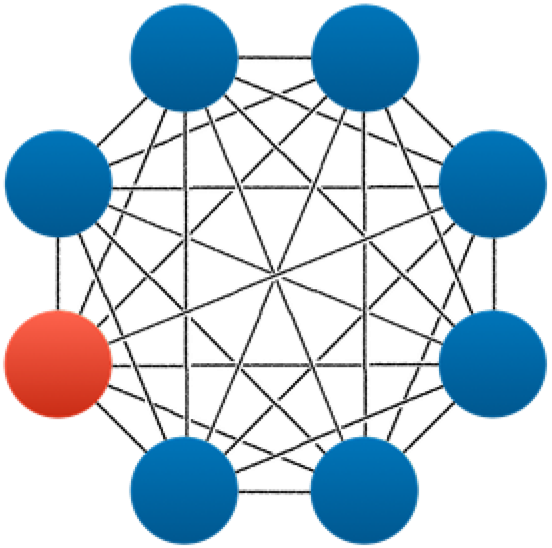
$\varphi_\tau^{\boldsymbol{i}}(0, t_2; K) > 0$
$|0\rangle$
Mix
$|1\rangle$
$d_z^{\boldsymbol{0}}$
$\Delta t = 0.05$
$\Delta t = 0.125$
$\Delta t = 0.5$
$\Delta t = 0.75$
$\mathcal{F}_N^{\boldsymbol{1}}$
Max
Min
$\mathcal{F}_N^{\boldsymbol{0}}$
$K = 1$
$K = 2$
$K = 3$
$K = 4$
0
20
4.0
4.2
4.4
4.6
4.8
5.0
$t \left[\frac{2\pi}{NJ}\right]$



$|0\rangle$
Mix
$|1\rangle$
$\varphi_\tau^{\boldsymbol{i}}(0, t_2; K) > 0$
$\varphi_\tau^{\boldsymbol{i}}(0.5, t_2; K) < 0$
$\Delta t = 0.05$
$\Delta t = 0.125$
$\Delta t = 0.5$
$\Delta t = 0.75$
$d_z^{\boldsymbol{0}}$
Non-Markovian $\forall\, t_2$
$\mathcal{F}_N^{\boldsymbol{1}}$
Max
Min
$K = 1$
$K = 2$
$K = 3$
$K = 4$
$\mathcal{F}_N^{\boldsymbol{0}}$
0
20
4.0
4.2
4.4
4.6
4.8
5.0
$t\ \left[\frac{2\pi}{NJ}\right]$

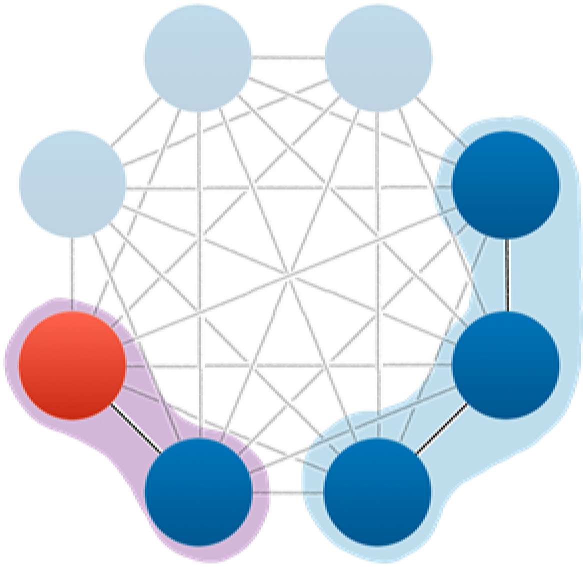
Summary

Summary



Summary

$$\left\{\Phi_{S_i}\left(t_1^{S_i}, t_2^{S_i}\right)\right\}_i$$

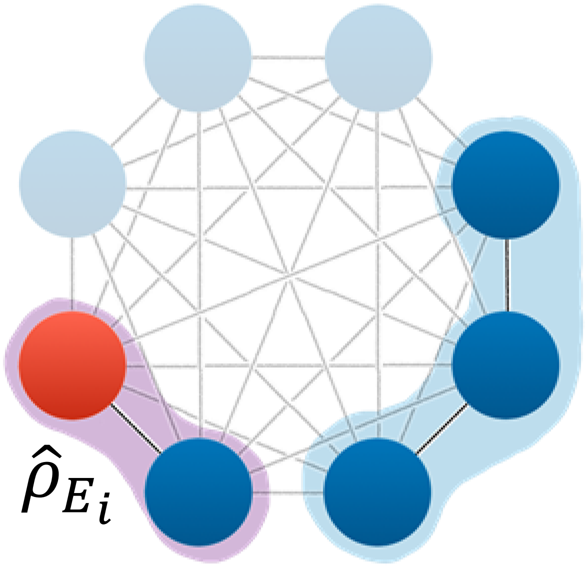


Ensemble of Correlated
Open Dynamics

Summary

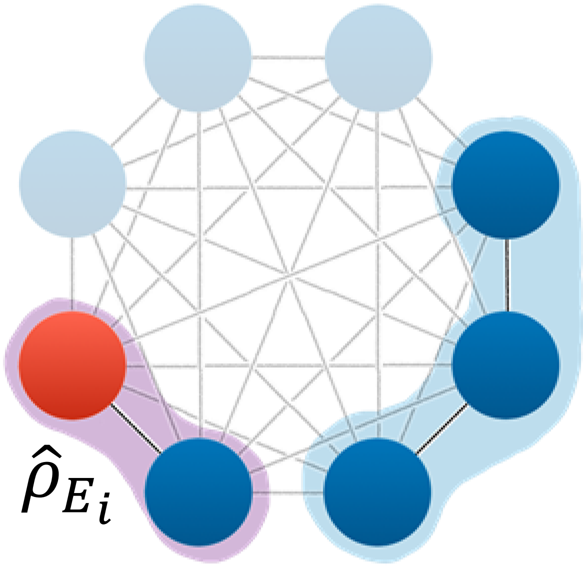
$$\left\{\Phi_{S_i}\left(t_1^{S_i}, t_2^{S_i}\right)\right\}_i$$

$$\hat{\rho}_{tot}(t_0) \neq \hat{\rho}_{S_i}(t_0) \otimes \hat{\rho}_{E_i}$$



Ensemble of Correlated Open Dynamics

Summary



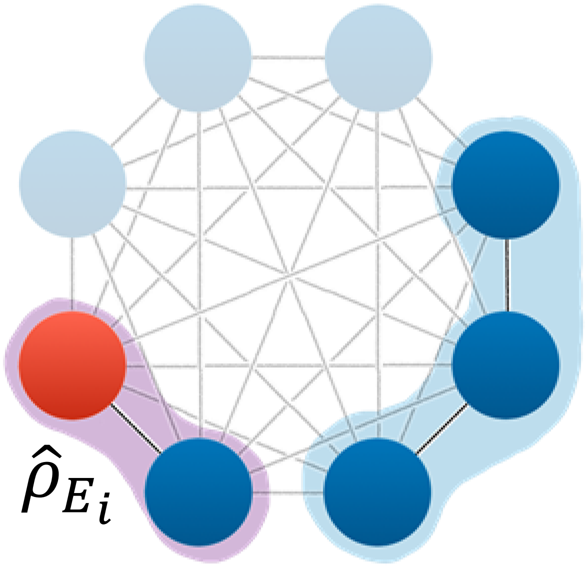
$\{\Phi_{S_i}(t_1^{S_i}, t_2^{S_i})\}_i$

$\hat{\rho}_{tot}(t_0) \neq \hat{\rho}_{S_i}(t_0) \otimes \hat{\rho}_{E_i}$

Ensemble of Correlated Open Dynamics

$\Phi^{i}(t_1, t_2; K)$ is NPNCP

Summary



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Ensemble of Correlated Open Dynamics

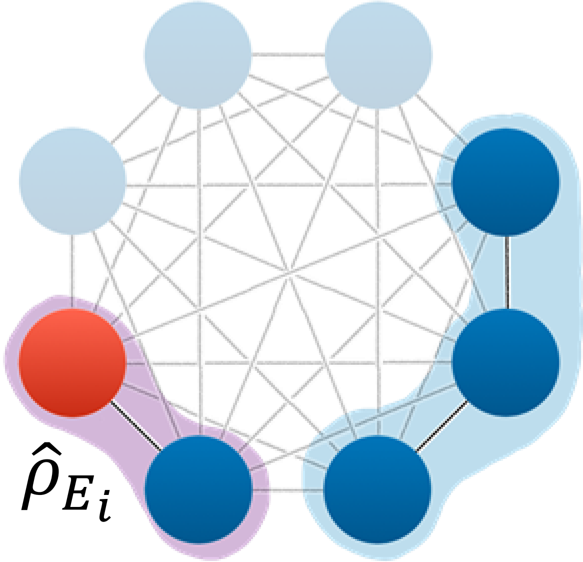
$$\Phi^{\boldsymbol{i}}(t_1, t_2; K)$$

is NPNCP

$$\Updownarrow$$

$$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) < 0$$

Summary



$$\{\Phi_{S_i}(t_1^{S_i}, t_2^{S_i})\}_i$$

$$\hat{\rho}_{tot}(t_0) \neq \hat{\rho}_{S_i}(t_0) \otimes \hat{\rho}_{E_i}$$

Ensemble of Correlated Open Dynamics

$\Phi^{\boldsymbol{i}}(t_1, t_2; K)$
is NPNCP

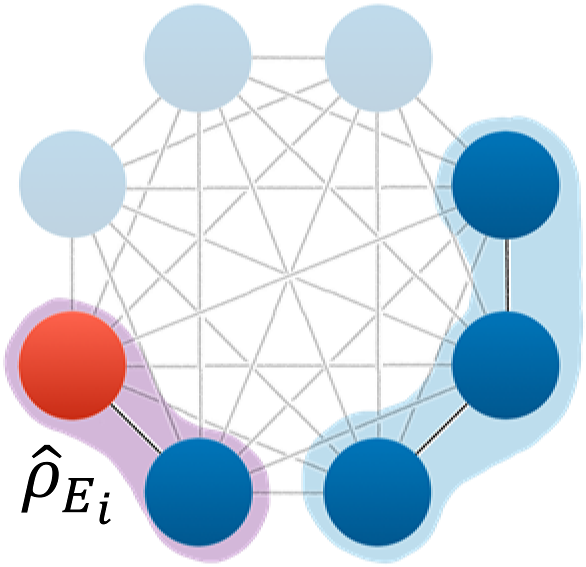
$$\Updownarrow$$

$$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) < 0$$

$$\Downarrow$$

Non-Markovian

Summary



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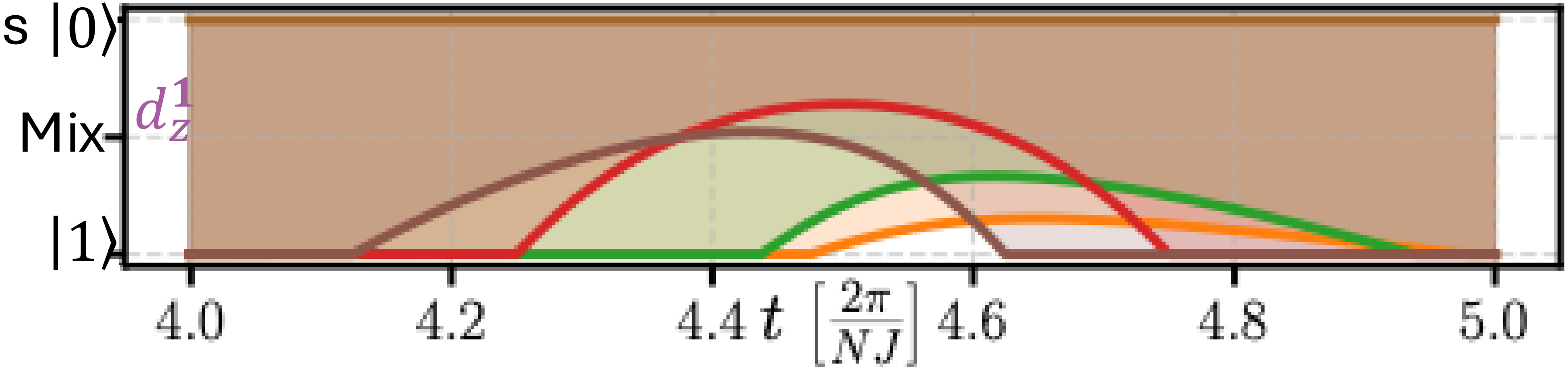
Ensemble of Correlated Open Dynamics



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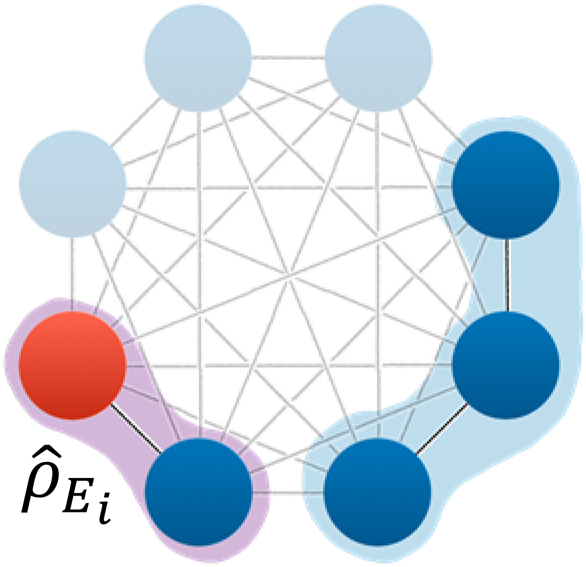

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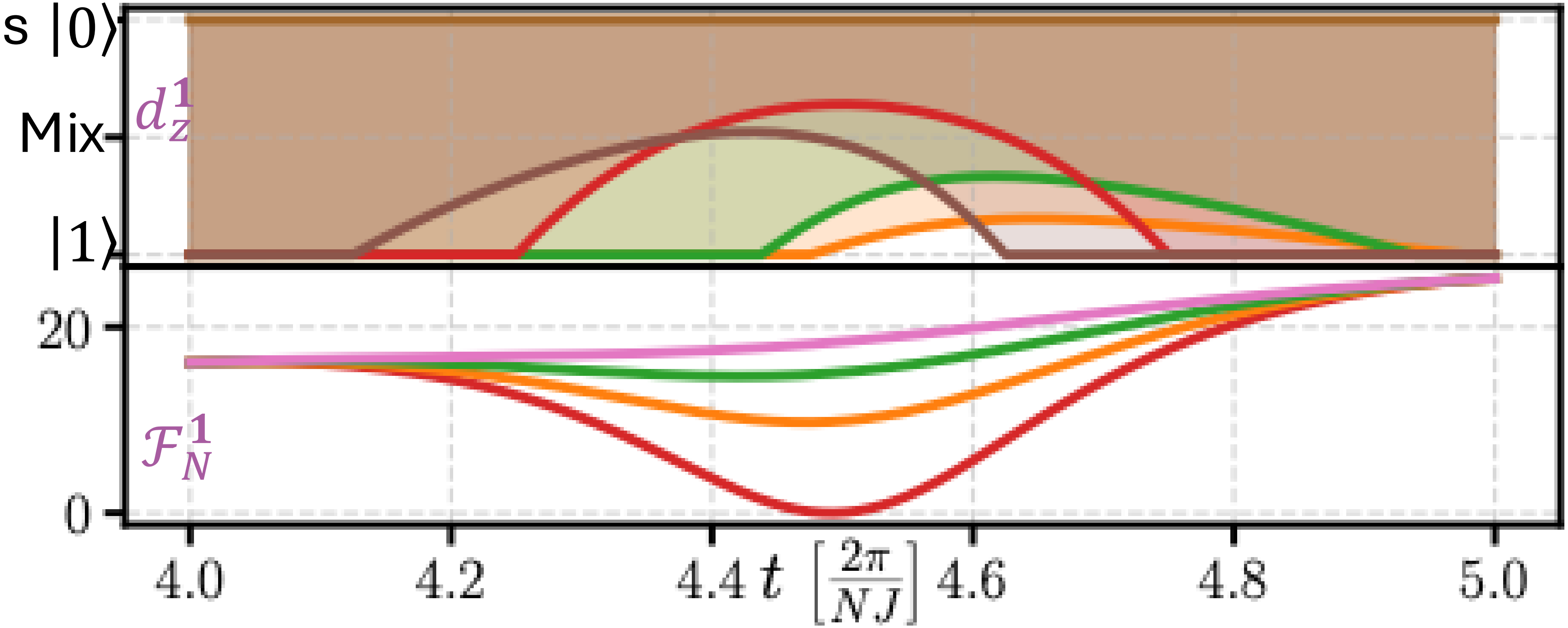

$\Phi^{\boldsymbol{i}}(t_1, t_2; K)$ is NPNCP

$\Updownarrow$

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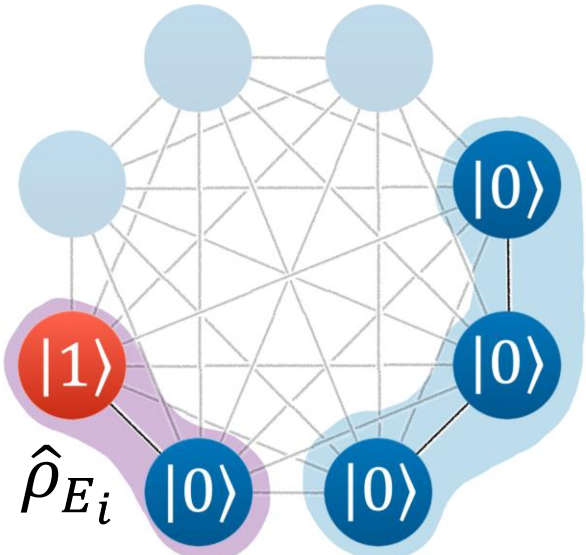
$\Downarrow$

Non-Markovian

Summary



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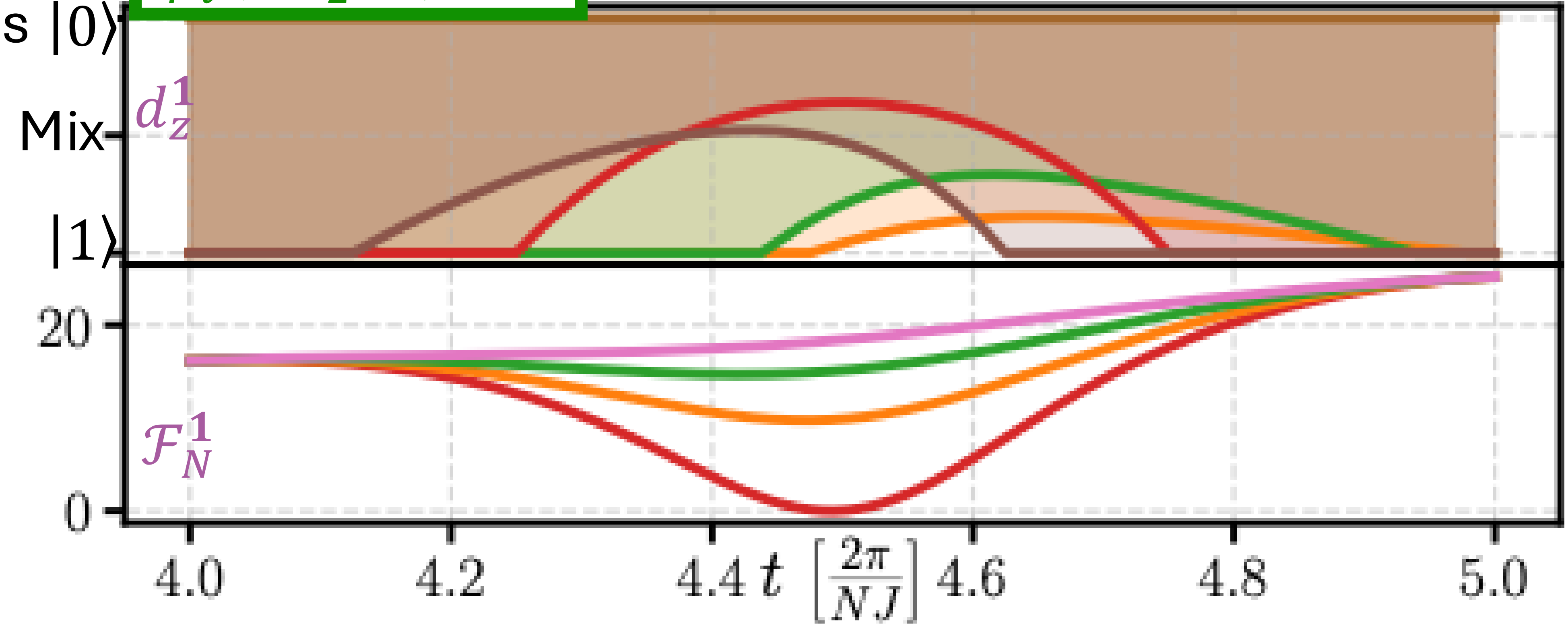
Ensemble of Correlated Open Dynamics

$\varphi_\tau^{\boldsymbol{i}}(0, t_2; K) > 0$



$\Phi^{\boldsymbol{i}}(t_1, t_2; K)$ is NPNCP

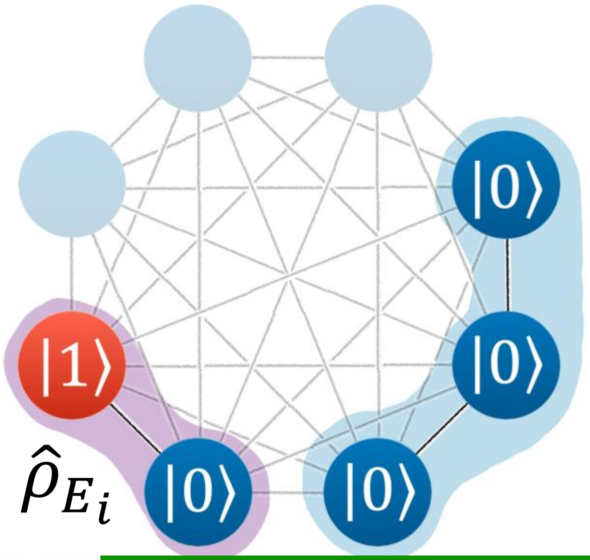
$\Updownarrow$

$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) < 0$

$\Downarrow$

Non-Markovian

Summary



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Ensemble of Correlated Open Dynamics

$\Phi^{\boldsymbol{i}}(t_1, t_2; K)$ is NPNCP

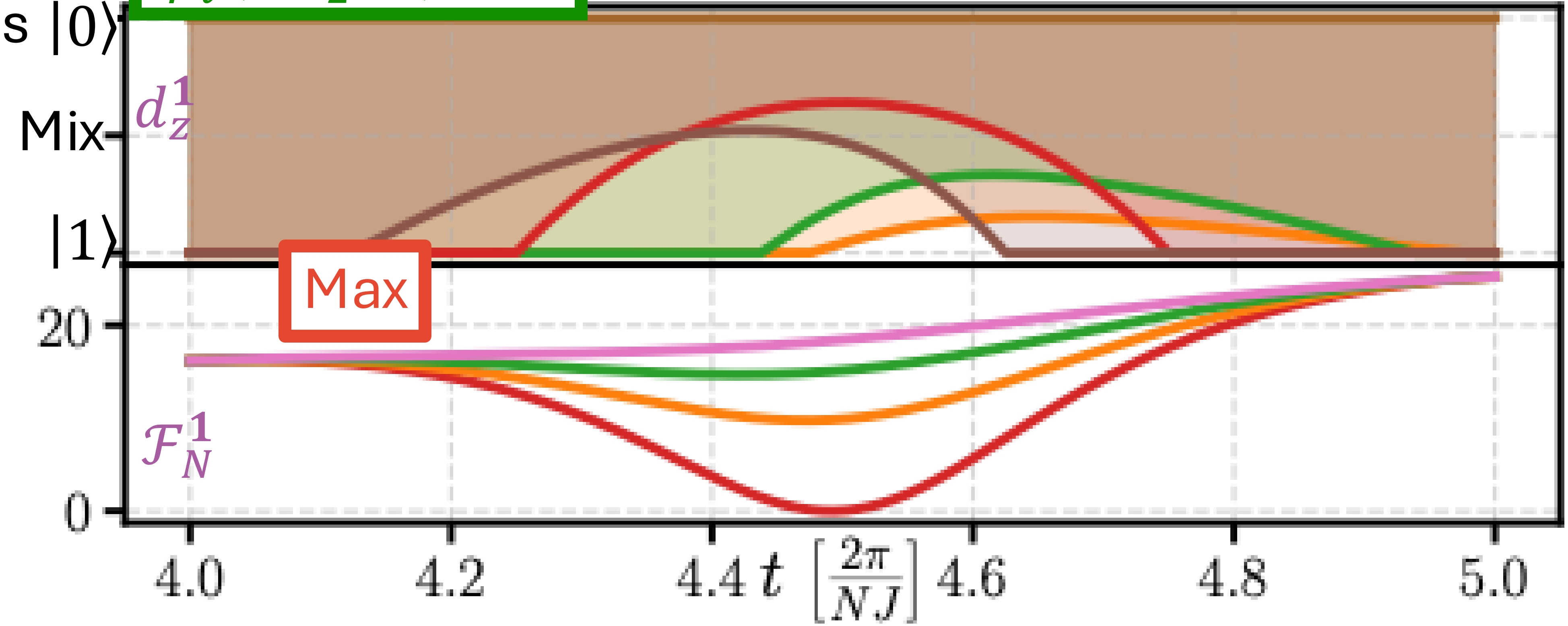
$\Updownarrow$

$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) < 0$

$\Downarrow$

Non-Markovian

$\varphi_\tau^{\boldsymbol{i}}(0, t_2; K) > 0$



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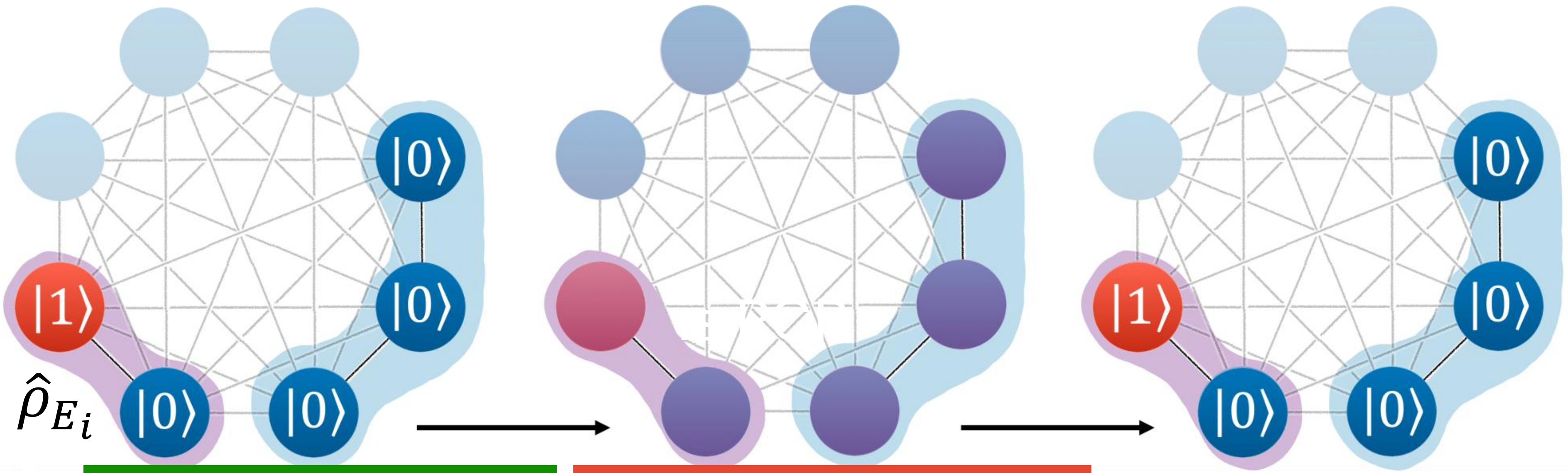
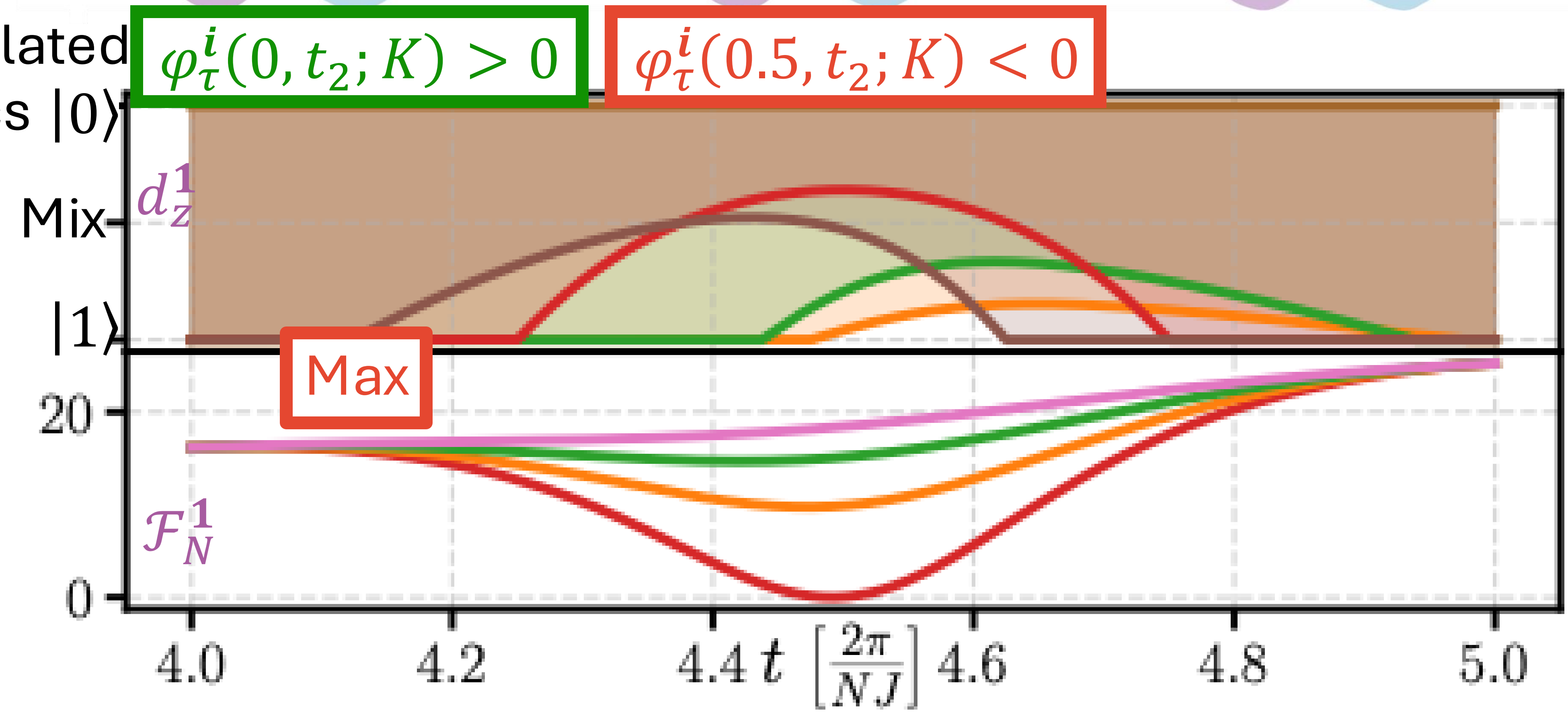


Ensemble of Correlated Open Dynamics



$\Phi^{\boldsymbol{i}}(t_1, t_2; K)$ is NPNCP

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$\varphi_\tau^{\boldsymbol{i}}(t_1, t_2; K) < 0$

$\Downarrow$

Non-Markovian

Summary

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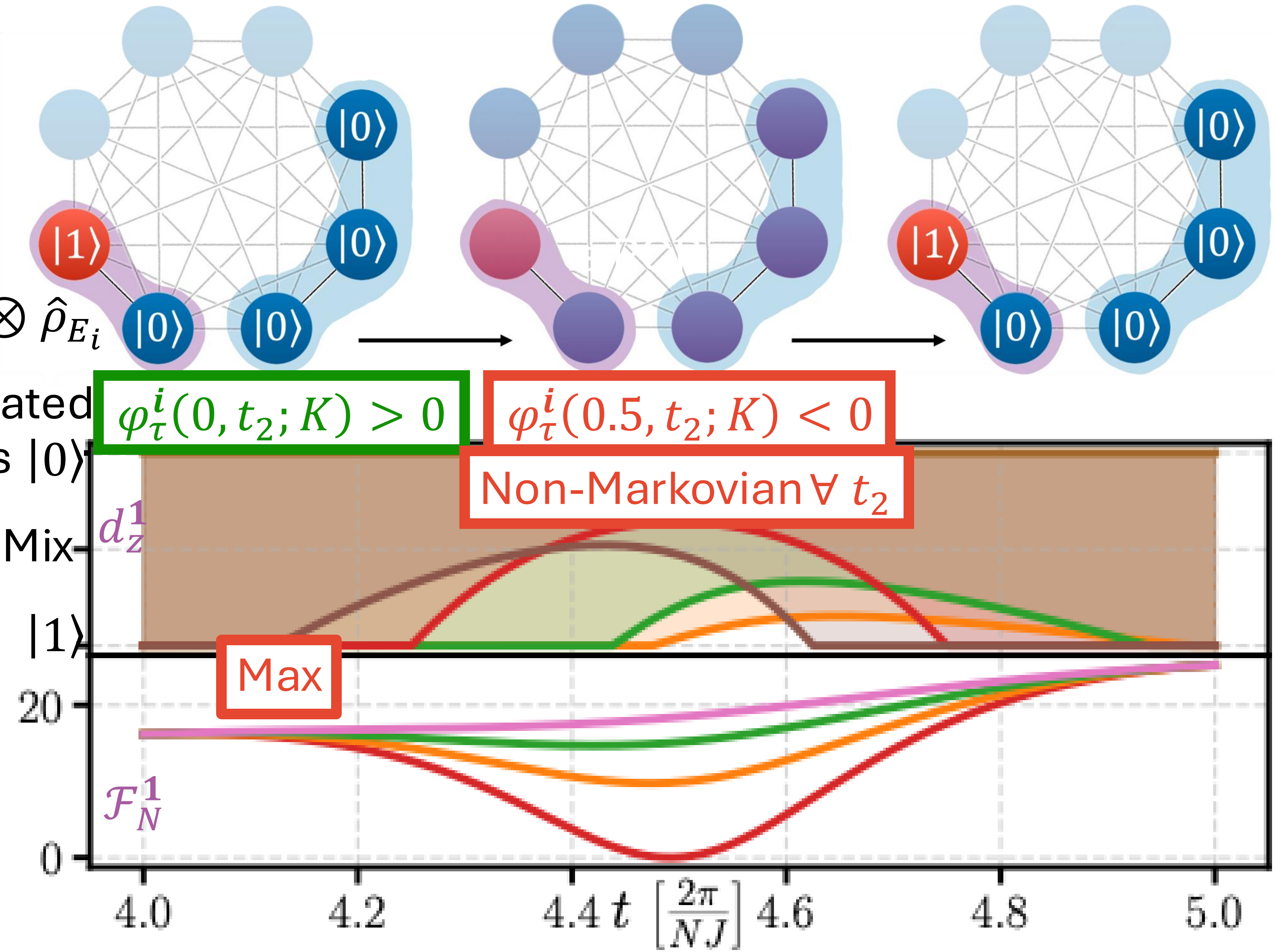
$\Downarrow$

Non-Markovian



Summary

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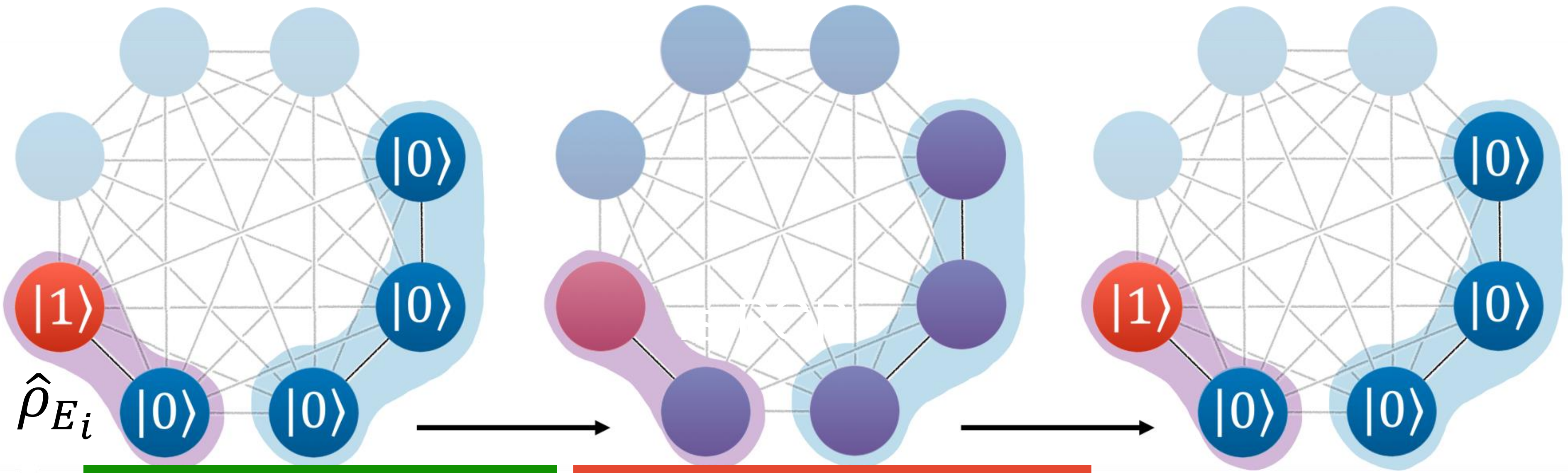
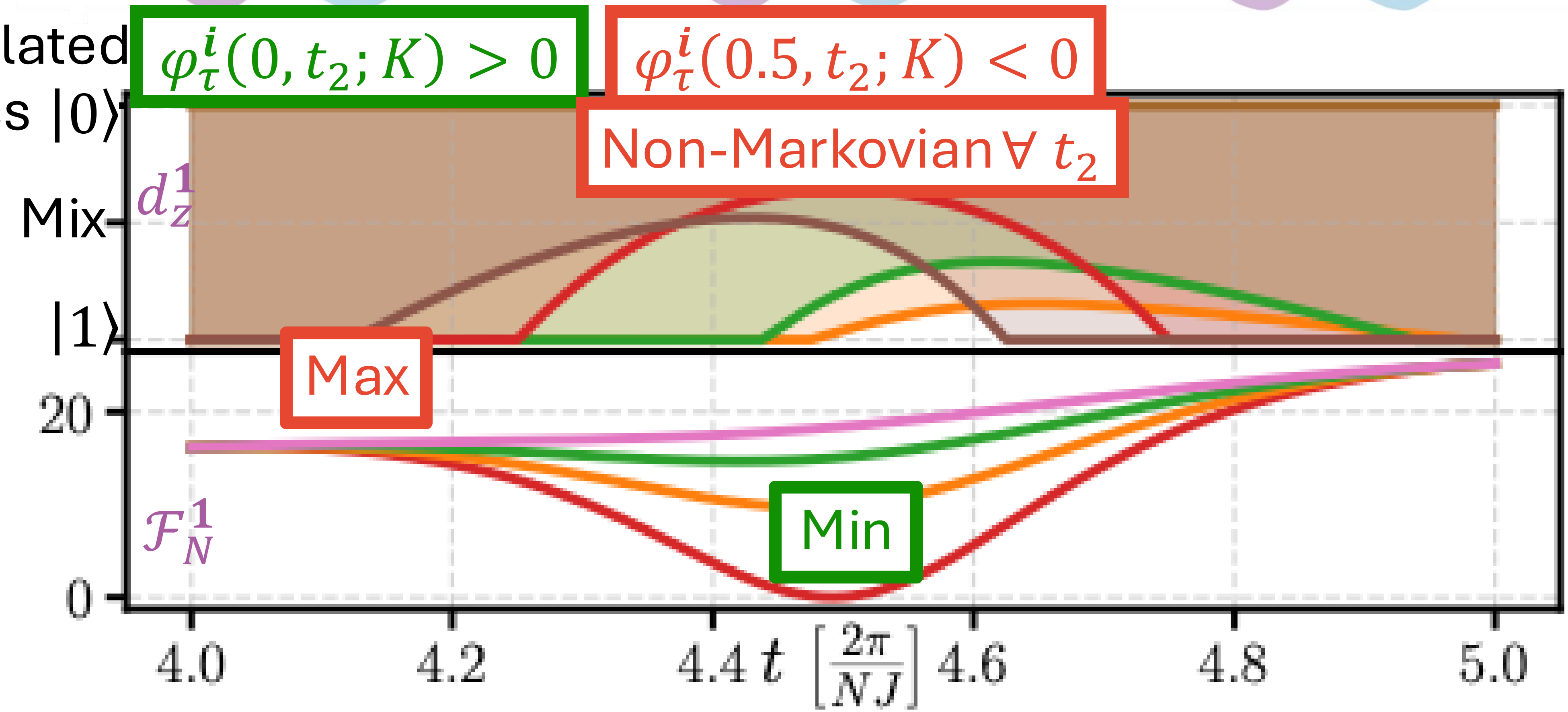


Ensemble of Correlated Open Dynamics



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Non-Markovian