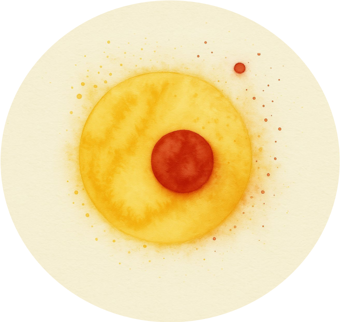
Quarkonium beyond the E/T expansion: Non-Lindblad master equations



Tom Magorsch

in collaboration with
Nora Brambilla, Arthur Lin and Antonio Vairo

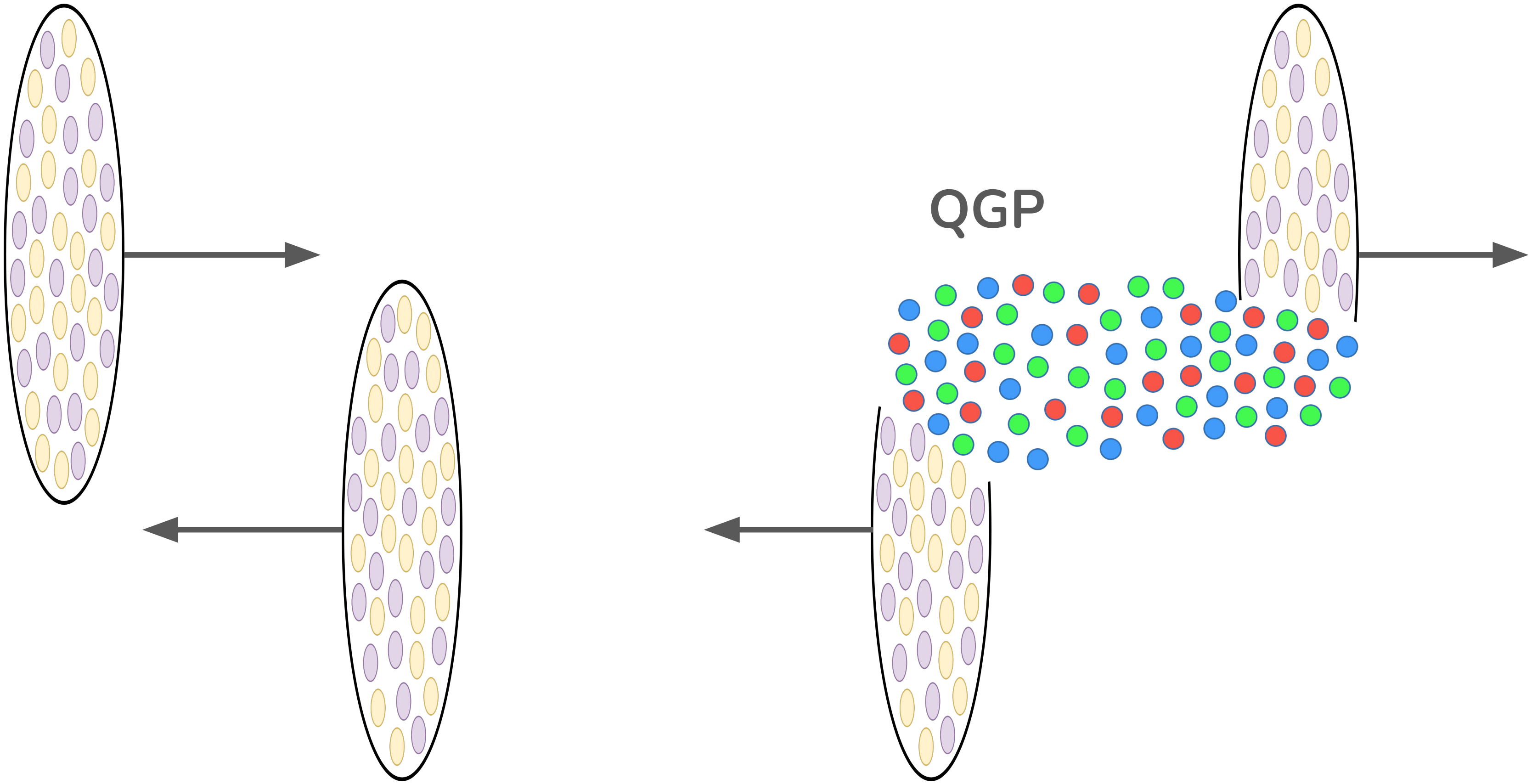
Open Quantum Systems: Dissipation and Decoherence from Subatomic to Cosmic Scales

21.04.26

TUM mitp Mainz Institute for Theoretical Physics

LMU TUM ESO lrz ORIGINS Excellence Cluster

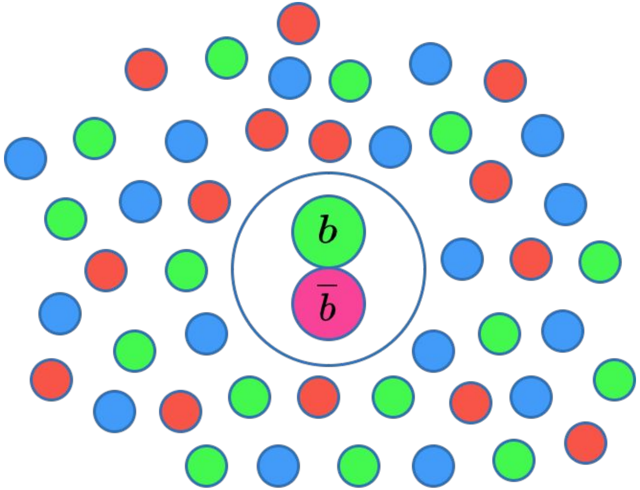
Heavy Ion Collisions



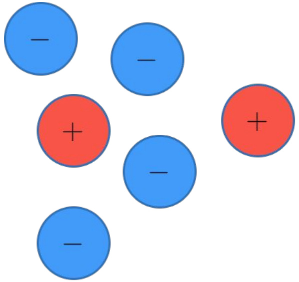
Quarkonium suppression

Veres, Gábor I. "Heavy ion physics at CMS and ATLAS: hard probes." *arXiv preprint arXiv:1905.10461* (2019).

Propagation through QGP



T. Matsui, H. Satz, Phys. Lett. B 178 (1986) 416

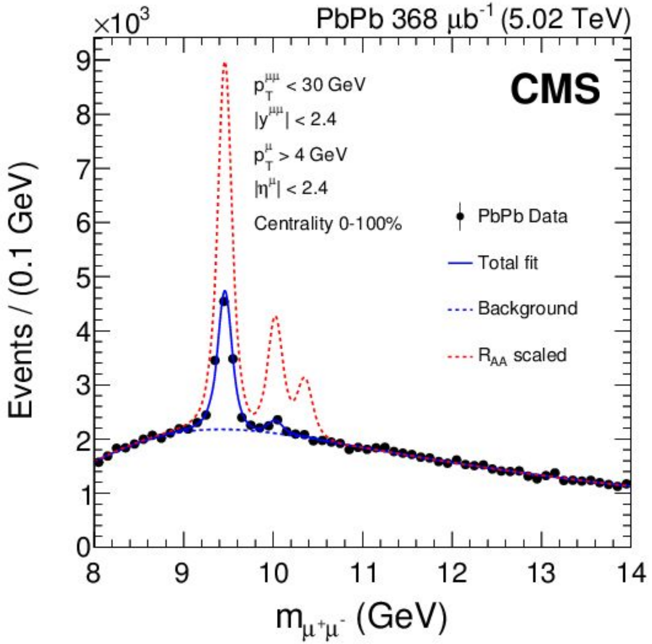


$$V(r) = -\frac{\alpha}{r}$$

$$\downarrow$$

$$V(r) = -\frac{\alpha}{r}\mathrm{e}^{-m_D r}$$

Debye-screening in medium

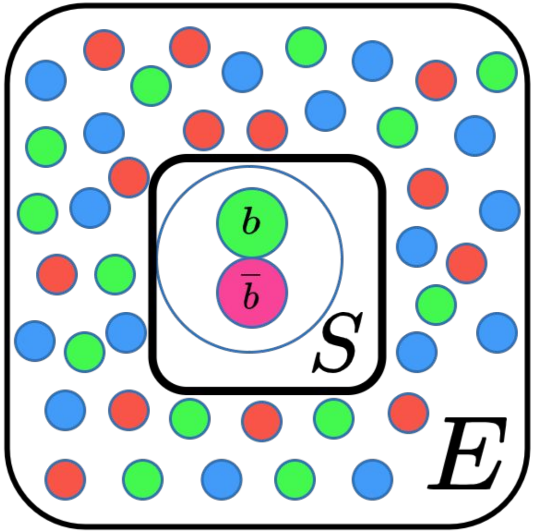


Quarkonium suppression

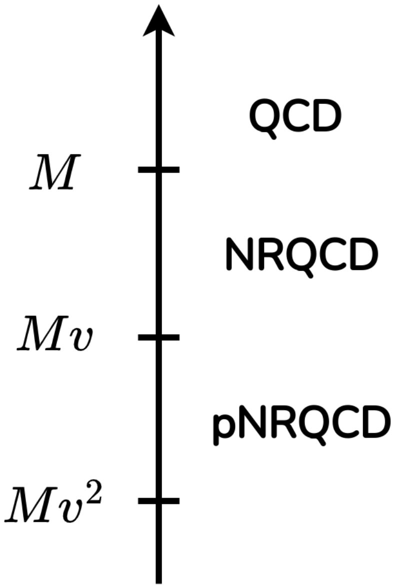
Yukinao Akamatsu, Prog.Part.Nucl.Phys. 123 (2022), 103932
Xiaojun Yao, Int.J.Mod.Phys.A 36 (2021) 20, 2130010

See week 1 talks of
Antonio Vairo
Rishi Sharma
Aoumeur Hammou

See week 2 talk of
Alexander Rothkopf

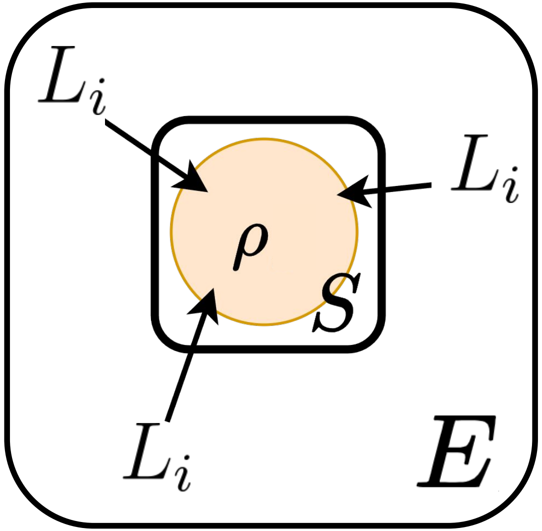


+



Brambilla, Escobedo, Soto, Vairo: Phys. Rev. D 97 (2018) 7, 074009

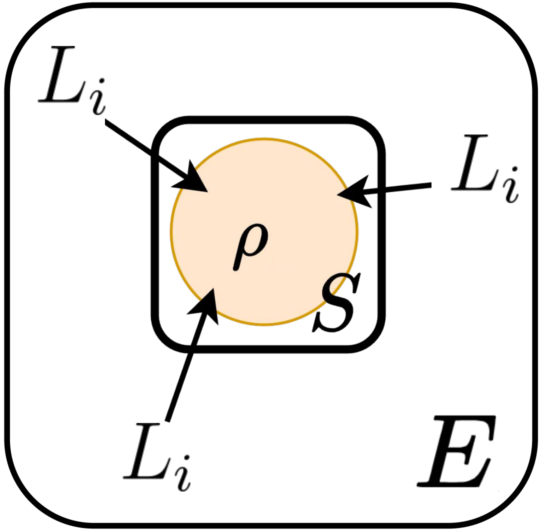
Quantum master equation



$$\frac{d\rho(t)}{dt} = -i\left[H, \rho(t)\right] + \sum_{nm} h_{nm}\left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2}\left\{L_i^{m\dagger} L_i^n, \rho(t)\right\}\right)$$

Brambilla, Escobedo, Soto, Vairo: Phys. Rev. D 97 (2018) 7, 074009

Quarkonium suppression



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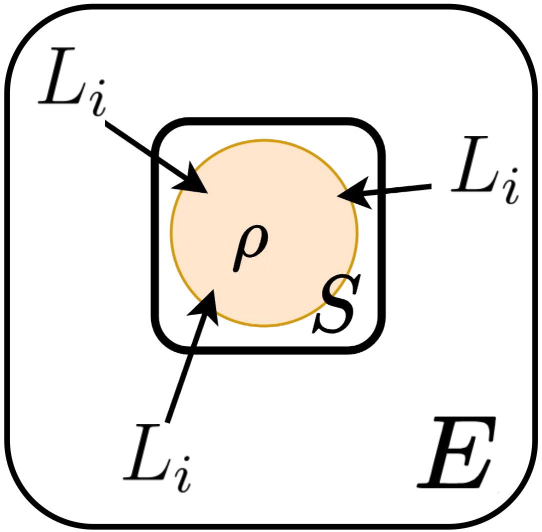
$$h = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$L_i^0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} r_i, \qquad L_i^1 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{N_c^2-4}{2(N_c^2-1)} A_i^{oo\dagger} \end{pmatrix}$$

$$L_i^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} r_i, \qquad L_i^3 = \begin{pmatrix} 0 & \frac{1}{N_c^2-1} A_i^{os\dagger} \\ A_i^{so\dagger} & 0 \end{pmatrix}$$

Brambilla, Escobedo, Soto, Vairo: Phys. Rev. D 97 (2018) 7, 074009

Quarkonium suppression



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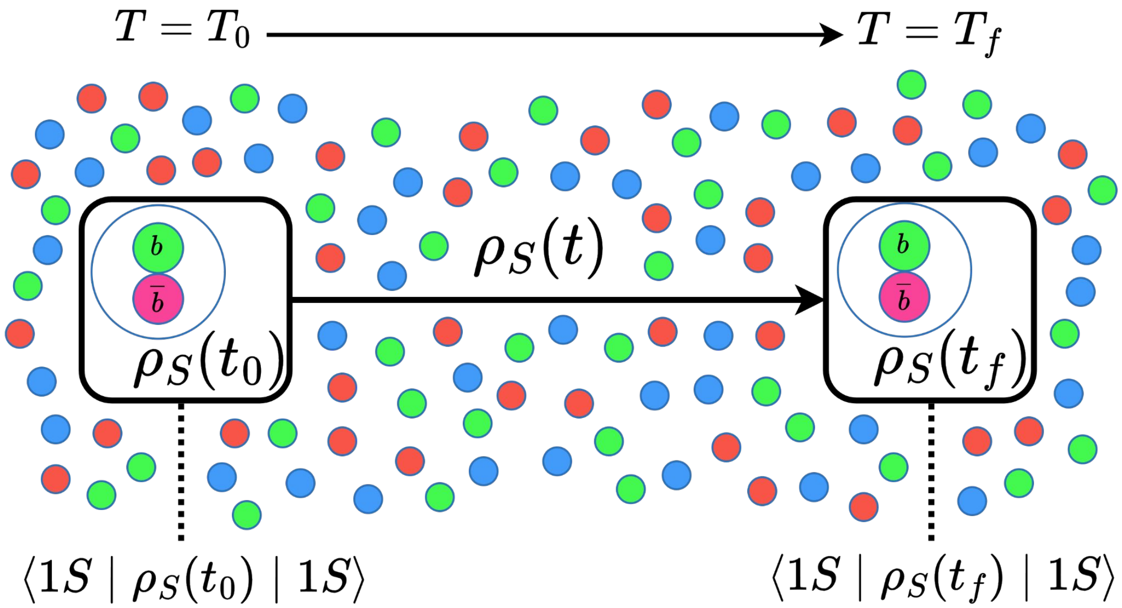
$$A_i^{uv} = \frac{g^2}{6N_c} \int_0^\infty \mathrm{d}s e^{-ih_u s} r_i e^{ih_v s} \langle E_j^a(s) \Omega(s,0)^{ab} E_j^b(0) \rangle$$

Brambilla, Escobedo, Soto, Vairo: Phys. Rev. D 97 (2018) 7, 074009

Solving the master equation gives the evolution of the bottomonium



$$P_{\text{survival}}(1S) = \frac{\langle 1S|\rho(t_f)|1S\rangle}{\langle 1S|\rho(t_0)|1S\rangle}$$

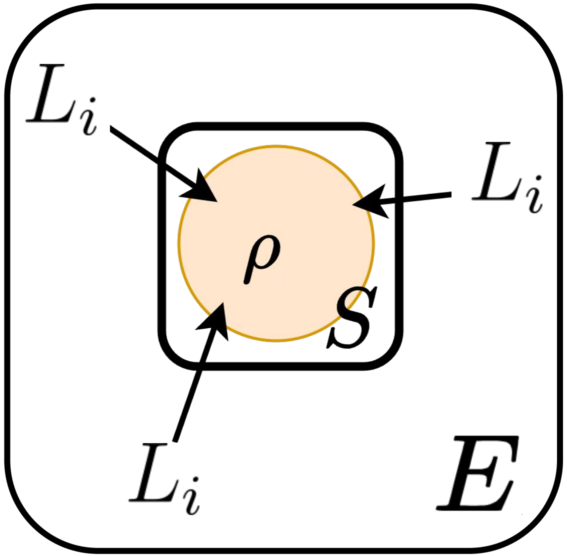
To simplify the equation one can expand in E/T

$$A_i^{uv} = \int_0^\infty \mathrm{d}s e^{-ih_u s} r_i e^{ih_v s} \langle E_j^a(s)\Omega^{ab}(s,0)E_j^b(0)\rangle$$

$$1 - ih_u s + \mathcal{O}(E^2/T^2) \qquad 1 - ih_v s + \mathcal{O}(E^2/T^2)$$

Next-to-Leading order in E/T

Leading order in E/T



Brambilla, N., Escobedo, M. Á., Islam, A., Strickland, M., Tiwari, A., Vairo, A., & Vander Griend, P. (2022). *Journal of High Energy Physics*, *2022*(8), 1-39.

To simplify the equation one can expand in E/T

Transport coefficients

$$A_i^{uv} = \frac{r_i}{2}(\kappa - i\gamma) + \kappa\left(-\frac{ip_i}{2MT} + \frac{\Delta V_{uv}}{4T}r_i\right)$$

LO

NLO

Brambilla, N., Escobedo, M. Á., Islam, A., Strickland, M., Tiwari, A., Vairo, A., & Vander Griend, P. (2022). *Journal of High Energy Physics*, *2022*(8), 1-39.

The master equation is not positive

$$\frac{d\rho(t)}{dt} = -i\,[H, \rho(t)] + \sum_{nm} h_{nm} \left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2}\left\{ L_i^{m\dagger} L_i^n, \rho(t) \right\} \right)$$

$$L_i \longrightarrow L_i'$$

$$h = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \longrightarrow h' = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$\mathcal{O}(E^2/T^2)$

Neglect negative terms as they are suppressed!

Projection on spherical harmonics

$$\frac{d\rho(t)}{dt} = -i[H, \rho(t)] + \sum_{n=0}^{\cancel{1}\ 6} \left(L_{\cancel{i}}^{n} \rho(t) L_{\cancel{i}}^{n\dagger} - \frac{1}{2}\{L_{\cancel{i}}^{n\dagger} L_{\cancel{i}}^{n}, \rho(t)\} \right)$$

Hilbert space:

$$H_c \otimes H_l \otimes H_r$$

c = s,o

l = 0,1,2,...

Radial wavefunction

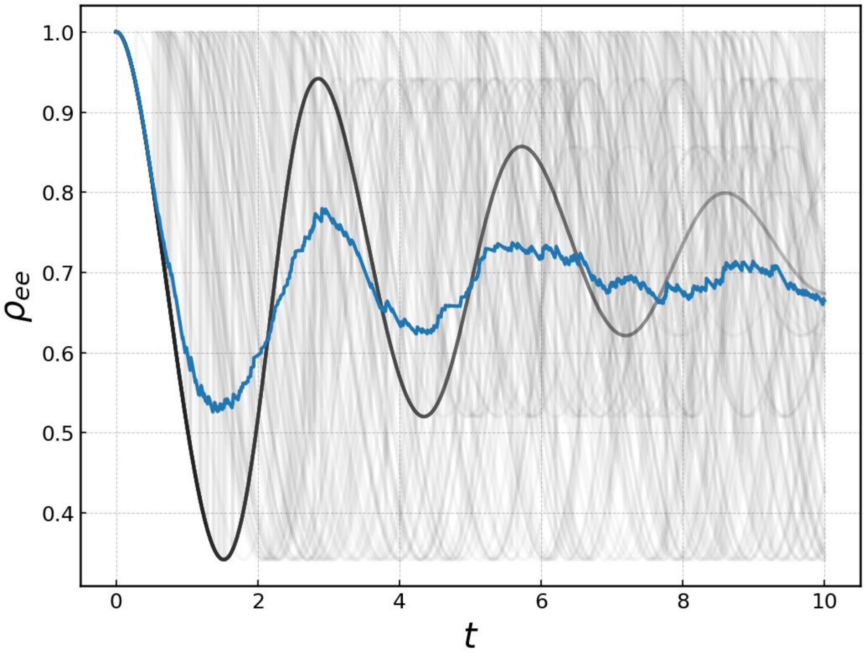
$$\rho = \begin{pmatrix} \rho_s^0 & 0 & \dots & 0 & 0 & \dots \\ 0 & \rho_s^1 & \dots & 0 & 0 & \dots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \\ 0 & 0 & \dots & \rho_o^0 & 0 & \dots \\ 0 & 0 & \dots & 0 & \rho_o^1 & \dots \\ \vdots & \vdots & & \vdots & \vdots & \ddots \end{pmatrix}$$

Quantum trajectory algorithm

J. Dalibard, Y. Castin, and K. Mølmer, Wave-function approach to dissipative processes in quantum optics, Phys. Rev. Lett. 68 (1992), pp. 580–583.

- Idea:
  1. Evolve individual trajectories $|\psi(t)\rangle$ **stochastically**
  2. Calculate observables by averaging over trajectories

$$\langle O\rangle = \mathbb{E}[\langle\psi(t)|O|\psi(t)\rangle]$$



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- Idea:
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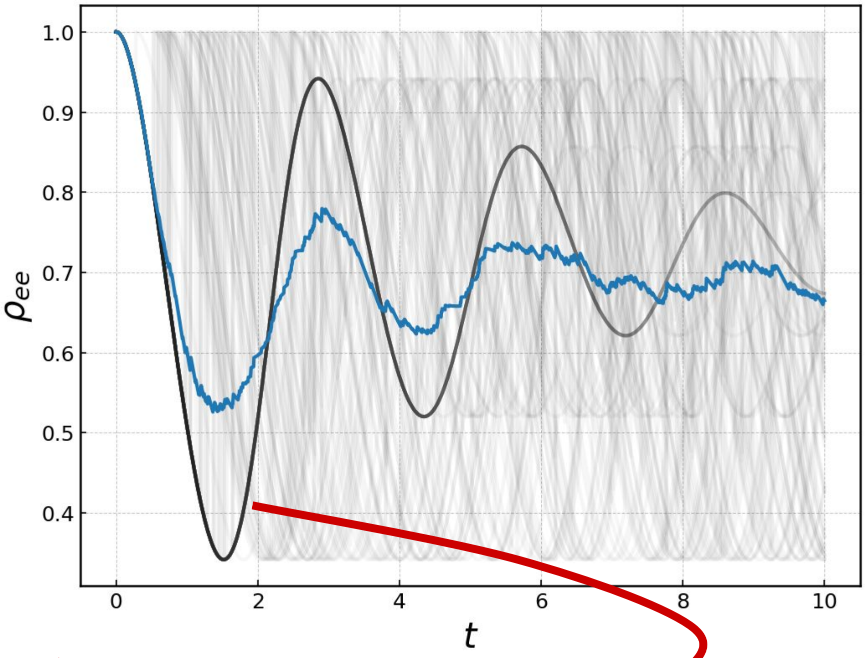


“H_{eff}-evolution“

$$\frac{d\rho(t)}{dt} = -i[H, \rho(t)] + \sum_{n=0}^{1} \left(L_i^n \rho(t) L_i^{n\dagger} - \frac{1}{2}\{L_i^{n\dagger} L_i^n, \rho(t)\} \right)$$

Quantum trajectory algorithm

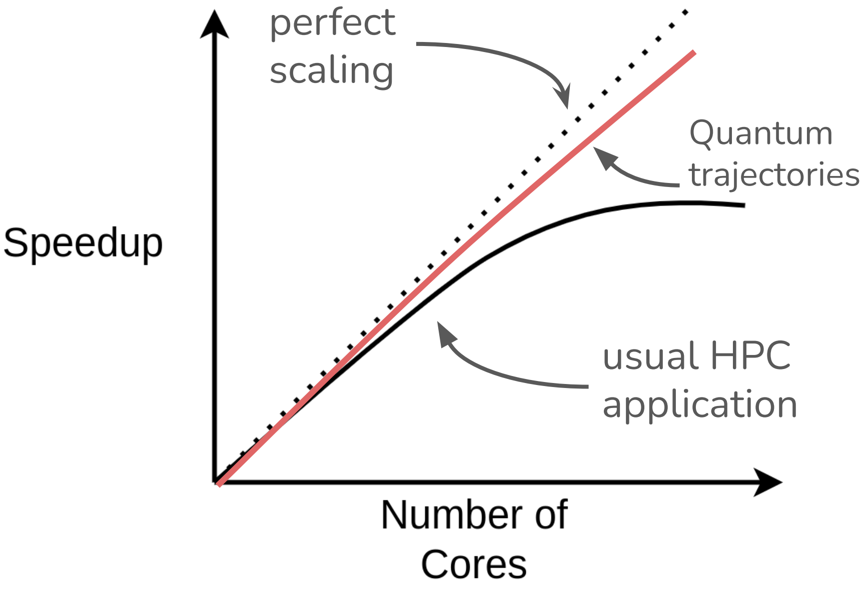
J. Dalibard, Y. Castin, and K. Mølmer, Wave-function approach to dissipative processes in quantum optics, Phys. Rev. Lett. 68 (1992), pp. 580–583.

- Idea:
  1. Evolve individual trajectories **stochastically**
  2. Calculate observables by averaging over trajectories

$$\langle O \rangle = \mathbb{E}[\langle \psi(t)|O|\psi(t)\rangle]$$

Advantages:

- Evolve vector of size N_H instead N_H^2 density matrix
- Simulation of individual trajectories is **embarrassingly parallel**

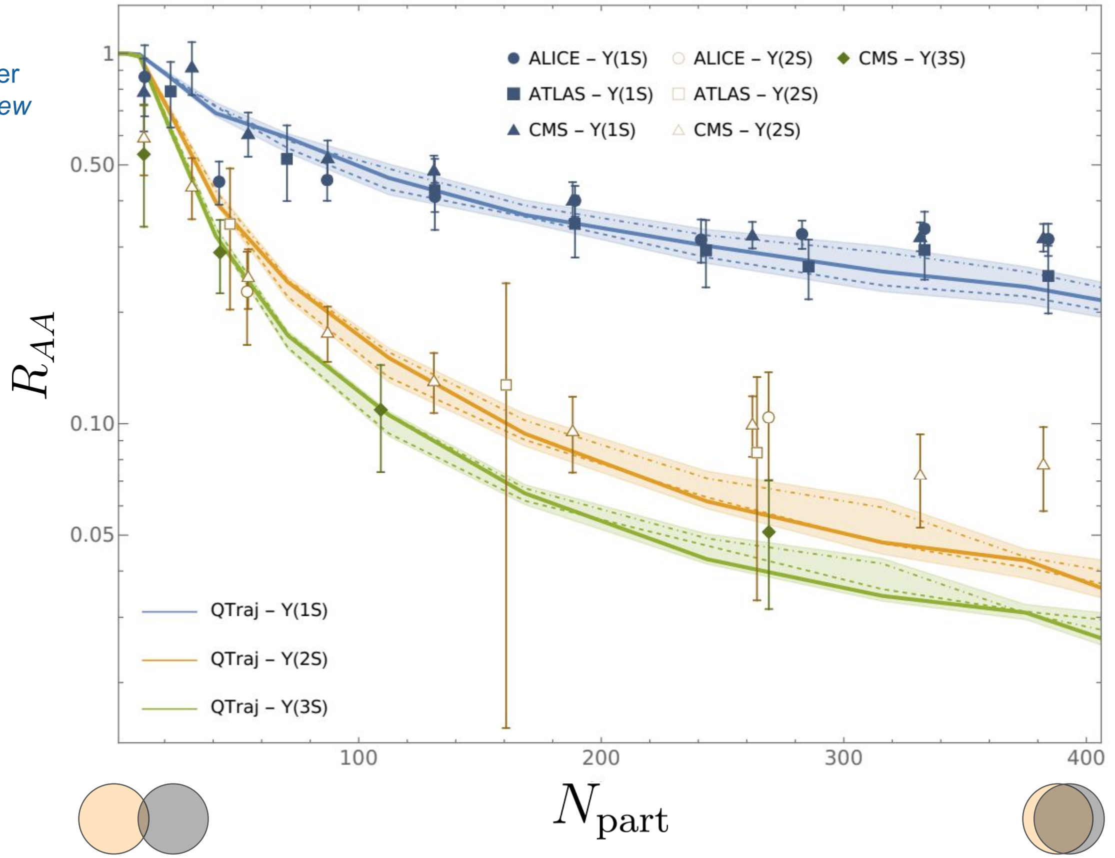


QTraj

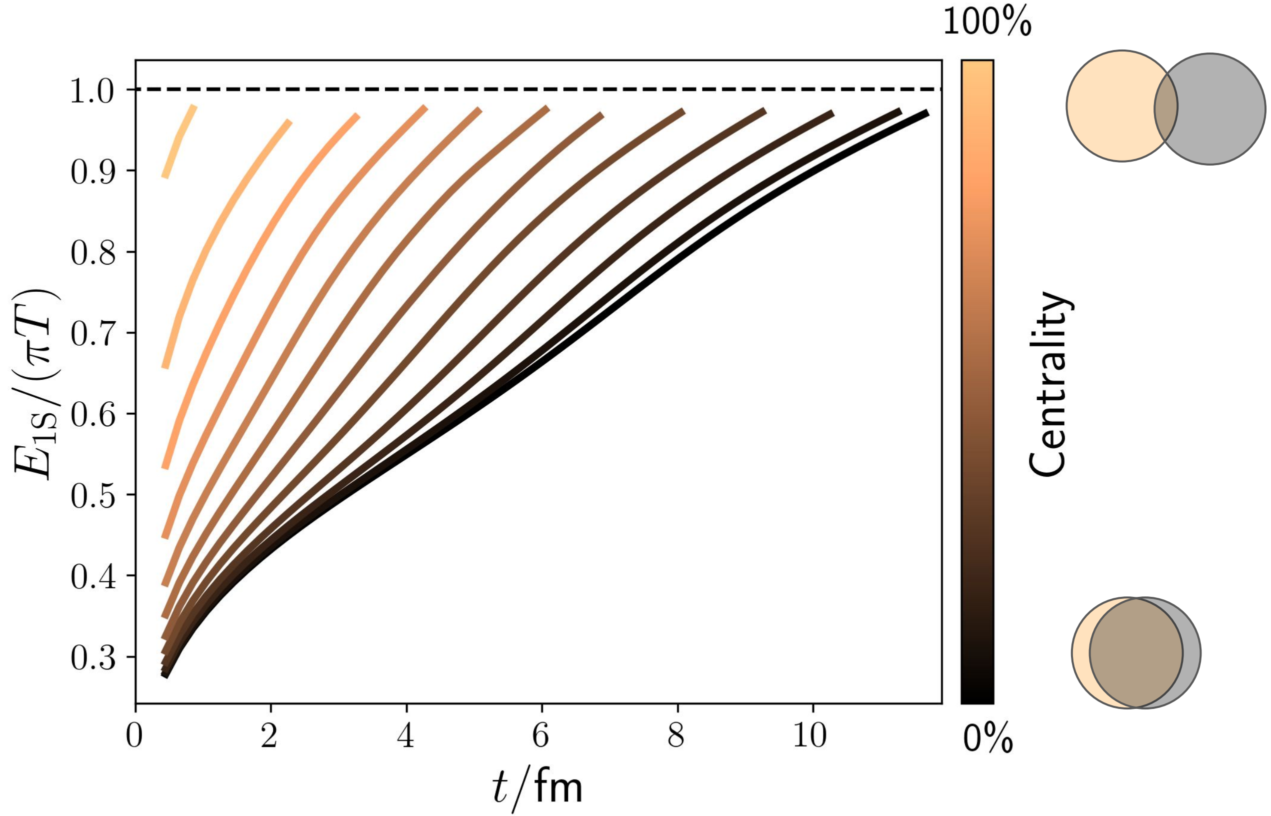
Omar, H. B., Escobedo, M. Á., Islam, A., Strickland, M., Thapa, S., Vander Griend, P., & Weber, J. H. (2022). *Computer Physics Communications*, *273*, 108266.

Overlaps lead to phenomenological predictions

Brambilla, N., Magorsch, T., Strickland, M., Vairo, A., & Vander Griend, P. (2024). *Physical Review D*, *109*(11), 114016.



At low temperatures the E/T expansion converges slowly



The original master equation is not positive

$$\frac{d\rho(t)}{dt} = -i\left[H, \rho(t)\right] + \sum_{nm} h_{nm}\left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2}\left\{L_i^{m\dagger} L_i^n, \rho(t)\right\}\right)$$

$$h = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad L_i^n \propto \frac{g^2}{6N_c}\int_0^\infty \mathrm{d}s e^{-ih_u s} r_i e^{ih_v s}\langle E_j^a(s)\Omega(s,0)^{ab}E_j^b(0)\rangle$$

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?

$$h' = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad L_i^n \propto \frac{g^2}{6N_c} \int_0^\infty \mathrm{d}s e^{-ih_u s} r_i e^{ih_v s} \langle E_j^a(s) \Omega(s,0)^{ab} E_j^b(0) \rangle$$

Non-positive master equation

Unraveling of general master equations

$$\sigma_n = \pm 1$$

$$\frac{d\rho(t)}{dt} = -i\left[H, \rho(t)\right] + \sum_n \sigma_n \left(L_i^n \rho(t) L_i^{n\dagger} - \frac{1}{2}\left\{ L_i^{n\dagger} L_i^n, \rho(t) \right\} \right)$$

Hilbert Space Extension

Rishi Sharma talk

Breuer, H. P., Kappler, B., & Petruccione, F. (1999). *Physical Review A*, *59*(2), 1633.

Kondov, I., Kleinekathöfer, U., & Schreiber, M. (2003). *The Journal of chemical physics*, *119*(13), 6635-6646.

$$\mathcal{H}_S \oplus \mathcal{H}_S$$

Weighted averaging

Becker, T., Netzer, C., & Eckardt, A. (2023). *Physical Review Letters*, *131*(16), 160401.

Donvil, B., & Muratore-Ginanneschi, P. *Nature Commun.* 13 (2022) 4140

$$\mathbb{E}[\mu(t)\langle\psi(t)|O|\psi(t)\rangle]$$

Unraveling of general master equations

Becker, T., Netzer, C., & Eckardt, A. (2023). *Physical Review Letters*, *131*(16), 160401.

$$\rho(t) = \mathbb{E}\left[s(t)|\psi(t)\rangle\langle\psi(t)|\right] \quad s(t) = \pm 1$$

Challenges:

1. **Sign problem:** $\mathbb{E}[s(t)] = \exp\left[-2\int_0^t \sum_i r_{i,-}(s)ds\right]$

2. **Tailed distribution:** $\mathrm{Var}[\langle\psi(t)|O|\psi(t)\rangle]$ large

Works for small lattices, i.e. $N_r \sim 150$

Operator optimizations

Becker, T., & Eckardt, A. (2025). *Physical Review E*, *111*(3), 034132.

Becker, T., Netzer, C., & Eckardt, A. (2023). *Physical Review Letters*, *131*(16), 160401.

$$\frac{d}{dt}\rho(t) = -i[H,\rho(t)] + \left(L_+\rho L_+^\dagger - \frac{1}{2}\{L_+^\dagger L_+, \rho(t)\}\right) - \left(L_-\rho L_-^\dagger - \frac{1}{2}\{L_-^\dagger L_-, \rho(t)\}\right)$$

Optimal form of time-local non-Lindblad master equations

Tobias Becker[1,*] and André Eckardt[1,†]

[1] *Institut für Theoretische Physik, Technische Universität Berlin, Hardenbergstrasse 36, 10623 Berlin, Germany*

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$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \left(L_+ \rho L_+^\dagger - \frac{1}{2}\{L_+^\dagger L_+, \rho(t)\}\right) - \left(L_- \rho L_-^\dagger - \frac{1}{2}\{L_-^\dagger L_-, \rho(t)\}\right)$$

$$\begin{pmatrix} L'_+ \\ L'_- \end{pmatrix} = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix} \begin{pmatrix} L_+ \\ L_- \end{pmatrix}$$

Operator optimizations

Becker, T., & Eckardt, A. (2025). *Physical Review E*, *111*(3), 034132.

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$$L'_+(w, \phi) = e^{i\phi/2}\cosh(w)L_+ + e^{-i\phi/2}\sinh(w)L_-$$

$$L'_-(w, \phi) = e^{i\phi/2}\sinh(w)L_+ + e^{-\phi/2}\cosh(w)L_-$$

Operator optimizations

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$$\underset{\omega,\phi}{\operatorname{argmin}} \left|\left|L'_-\right|\right|^2$$

Operator optimizations

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$$L'_-(w,\phi) = e^{i\phi/2}\sinh(w)L_+ + e^{-\phi/2}\cosh(w)L_-$$

$$\underset{\omega,\phi}{\operatorname{argmin}} ||L'_-||^2$$

$$||L_\sigma||^2 = \mathrm{Tr}[\rho_{\mathrm{proj}} L_\sigma^\dagger L_\sigma]$$

Operator optimizations

Becker, T., & Eckardt, A. (2025). *Physical Review E*, *111*(3), 034132.

Becker, T., Netzer, C., & Eckardt, A. (2023). *Physical Review Letters*, *131*(16), 160401.

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$$\underset{\omega,\,\phi}{\operatorname{argmin}} \left|\left|L'_-\right|\right|^2$$

$$\phi = \pi - \arg\left[\mathrm{Tr}\left(\rho_{\mathrm{proj}} L_-^\dagger L_+\right)\right]$$

$$w = \frac{1}{2}\tanh^{-1}\left(\frac{2\mathrm{Tr}\left(\rho_{\mathrm{proj}} L_-^\dagger L_+\right)}{||L_+||^2 + ||L_-||^2}\right)$$

Truncation

$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \left(L_+ \rho L_+^\dagger - \frac{1}{2}\{L_+^\dagger L_+, \rho(t)\}\right) - \left(L_- \rho L_-^\dagger - \frac{1}{2}\{L_-^\dagger L_-, \rho(t)\}\right)$$

Truncation

$$\underbrace{\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \left(L_+\rho L_+^\dagger - \frac{1}{2}\{L_+^\dagger L_+, \rho(t)\}\right)}_{\text{“Lindblad equation”}} - \underbrace{\left(L_-\rho L_-^\dagger - \frac{1}{2}\{L_-^\dagger L_-, \rho(t)\}\right)}_{\mathcal{O}(E^2/T^2)}$$

Truncation

$$\underset{\omega, \phi}{\mathrm{argmin}} \left|\left|L'_-\right|\right|^2 \qquad \begin{pmatrix} L'_+ \\ L'_- \end{pmatrix} = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix} \begin{pmatrix} L_+ \\ L_- \end{pmatrix}$$

$$\underbrace{\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \left(L'_+ \rho L'^{\dagger}_+ - \frac{1}{2}\{L'^{\dagger}_+ L'_+, \rho(t)\} \right)}_{\text{“Lindblad equation”}} - \underbrace{\left(L'_- \rho L'^{\dagger}_- - \frac{1}{2}\{L'^{\dagger}_- L'_-, \rho(t)\} \right)}_{\text{minimal}}$$

Chromoelectric correlator

$$L_i^n \propto \frac{g^2}{6N_c} \int_0^\infty \mathrm{d}s e^{-ih_u s} r_i e^{ih_v s} \langle E_j^a(s) \Omega(s,0)^{ab} E_j^b(0) \rangle$$

Relation to transport coefficients

$$\int_0^\infty \mathrm{d}s \langle E_j^a(s) \Omega^{ab}(s,0) E_j^b(0) \rangle = \frac{\kappa + \mathrm{i}\gamma}{2}$$

$$\mathrm{i} \int_0^\infty \mathrm{d}s \, s \, \langle E_j^a(s) \Omega^{ab}(s,0) E_j^b(0) \rangle = \frac{\kappa}{4T}$$

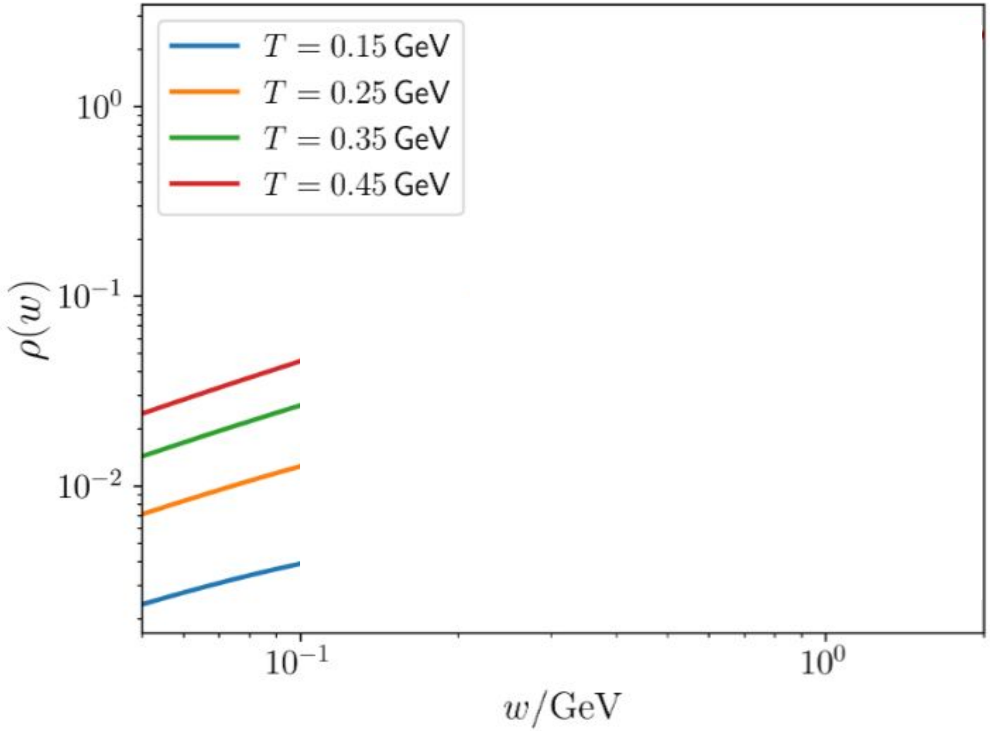
Chromoelectric correlator

$$L_i^n \propto \frac{g^2}{6N_c}\int_0^\infty \mathrm{d}s e^{-ih_u s} r_i e^{ih_v s}\langle E_j^a(s)\Omega(s,0)^{ab}E_j^b(0)\rangle$$

Definition in terms of spectral function $\rho(w)$

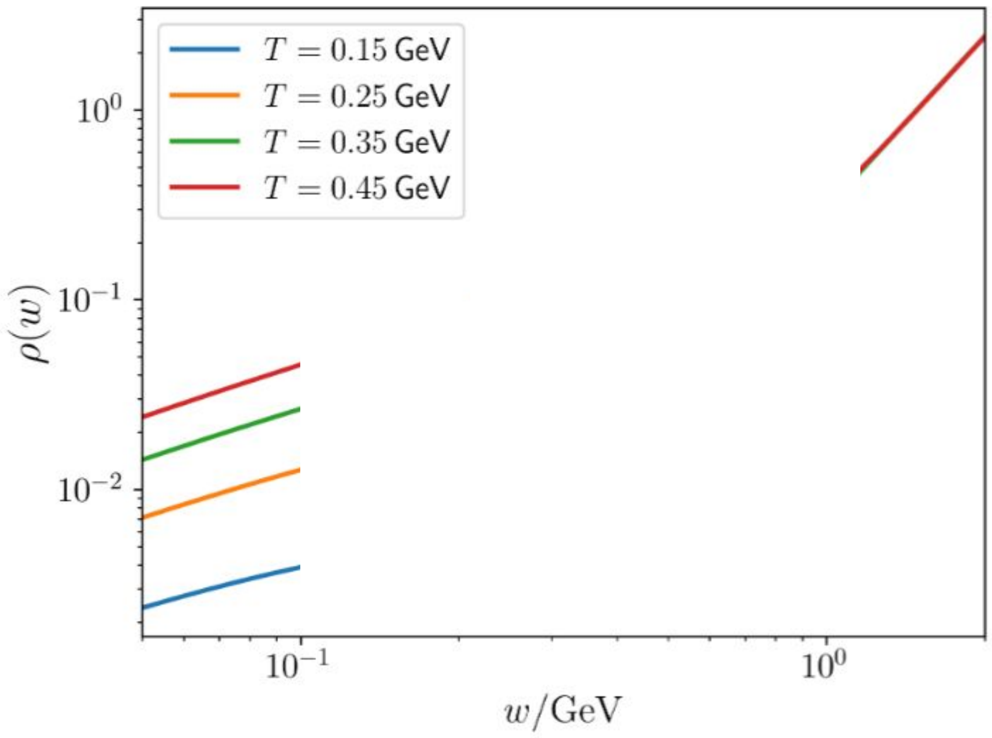
$$\langle E_j^a(s)\Omega^{ab}(s,0)E_j^b(0)\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}w e^{-iws}\rho(w)(1+n_B(w))$$

Correlator model



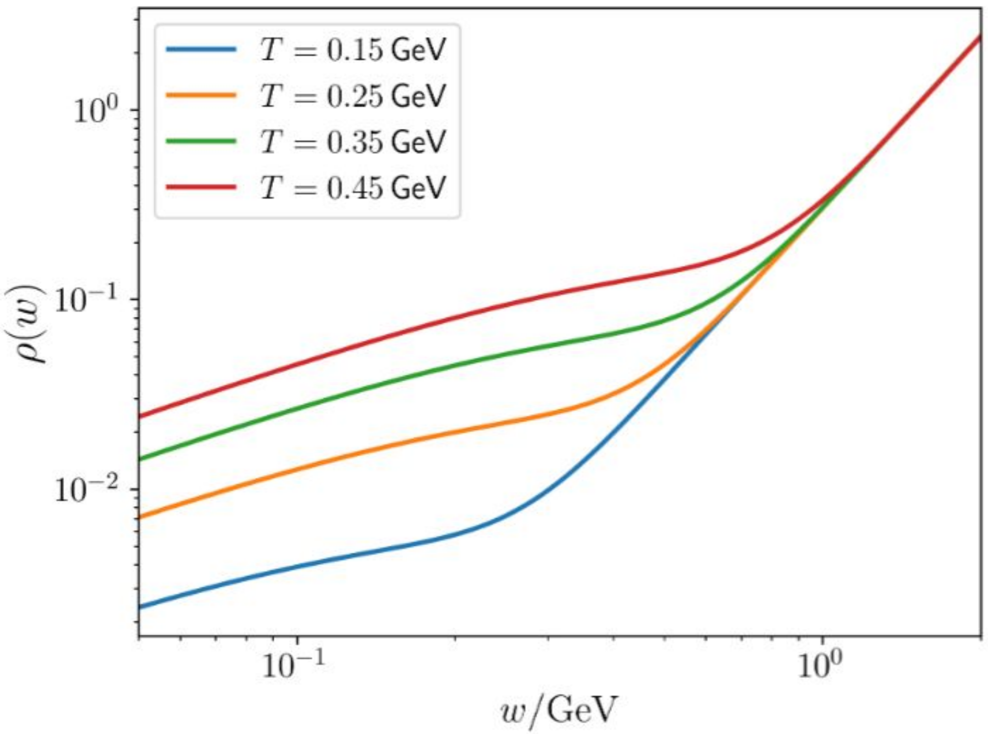
$$\rho_{\mathrm{IR}}(w) \propto \hat{\kappa} T^2 w$$

Correlator model



$$\rho_{\mathrm{UV}}(w) \propto w^3$$

Correlator model

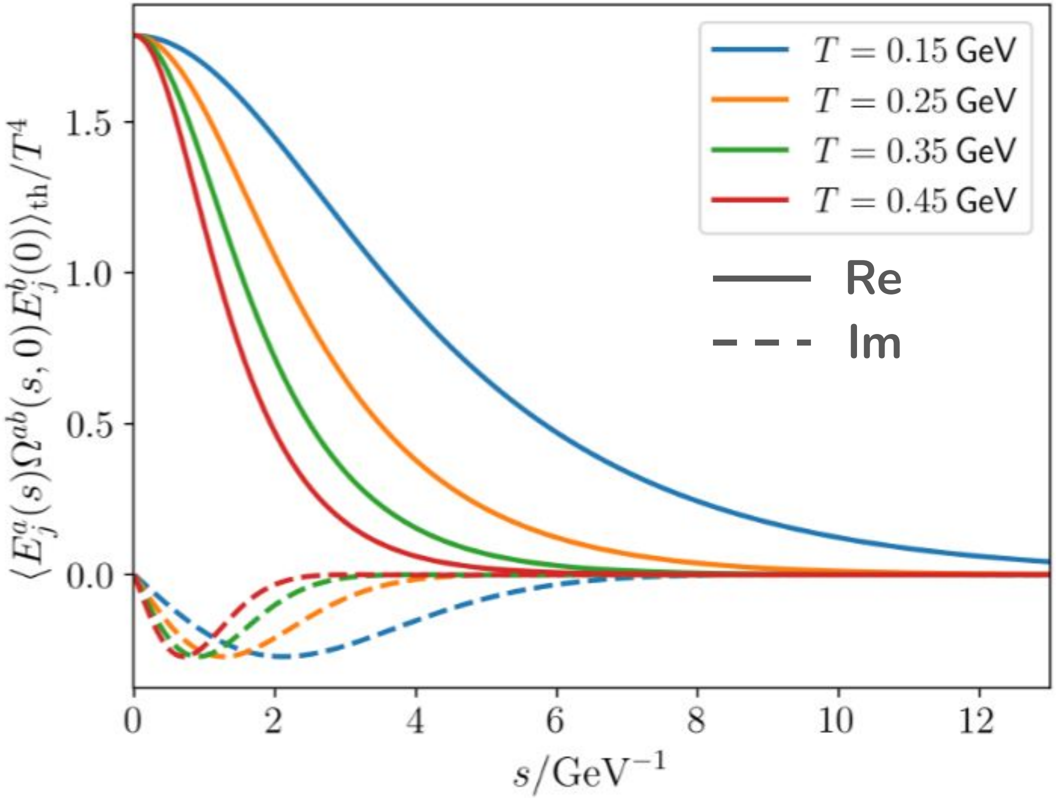


$$\lim_{w \to 0} s(w) = 0$$

$$\lim_{w \to \infty} s(w) = 1$$

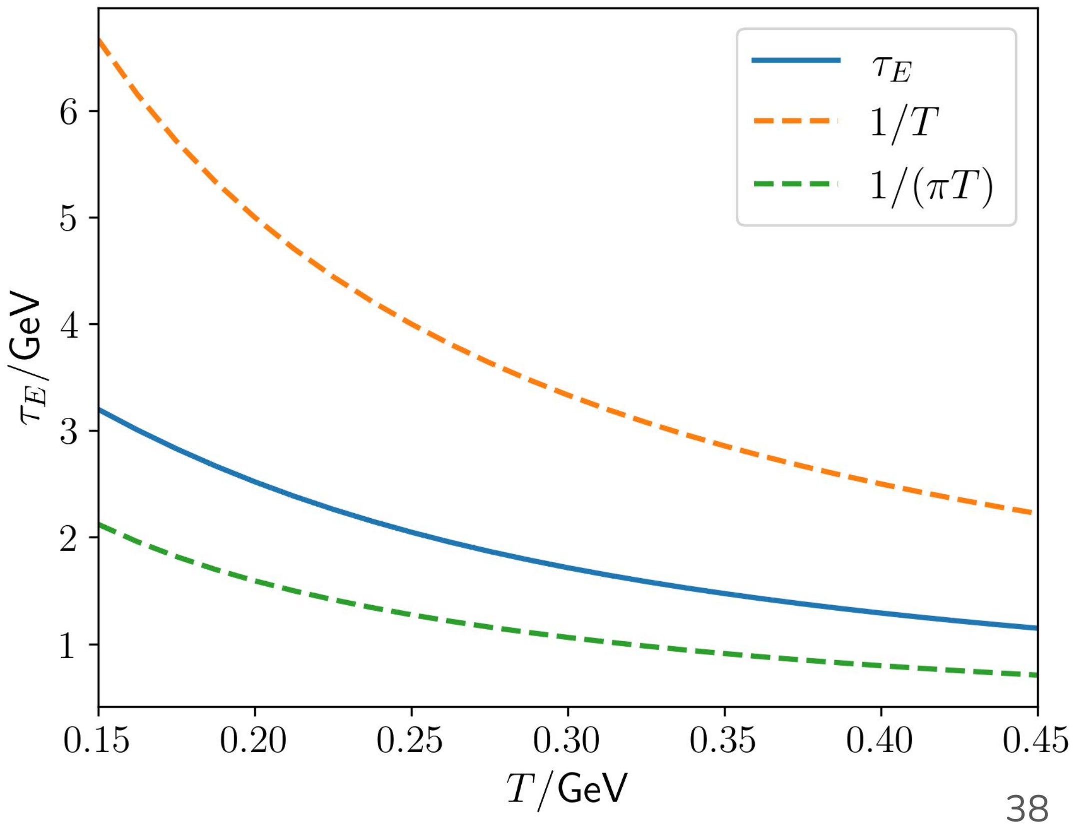
$$\rho(w) = (1 - s(w))\rho_{\text{IR}}(w) + s(w)\rho_{\text{UV}}(w)$$

Thermal correlator



Chromoelectric memory timescale

$$\tau_E = \frac{\int \mathrm{d}s\ s\left|\langle E_j^a(s)\Omega^{ab}(s,0)E_j^b(0)\rangle_{\mathrm{th}}\right|}{\int \mathrm{d}s\left|\langle E_j^a(s)\Omega^{ab}(s,0)E_j^b(0)\rangle_{\mathrm{th}}\right|}$$

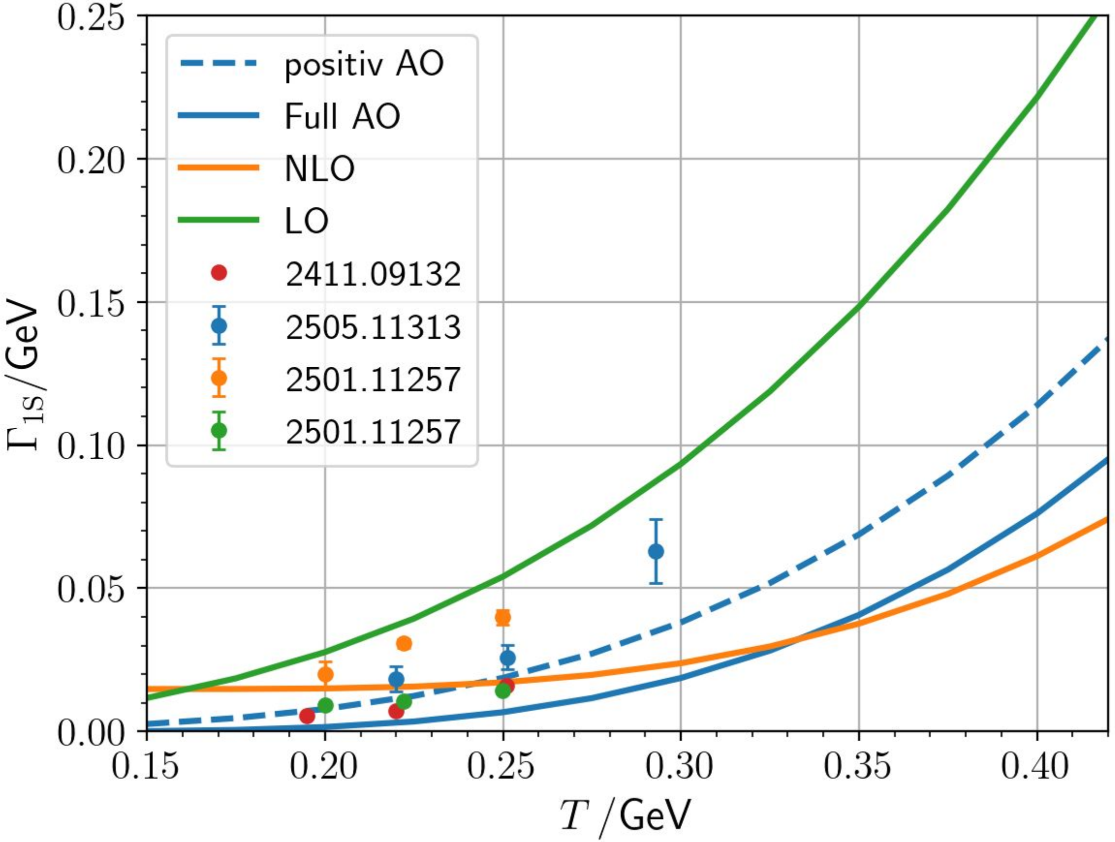


In-medium width

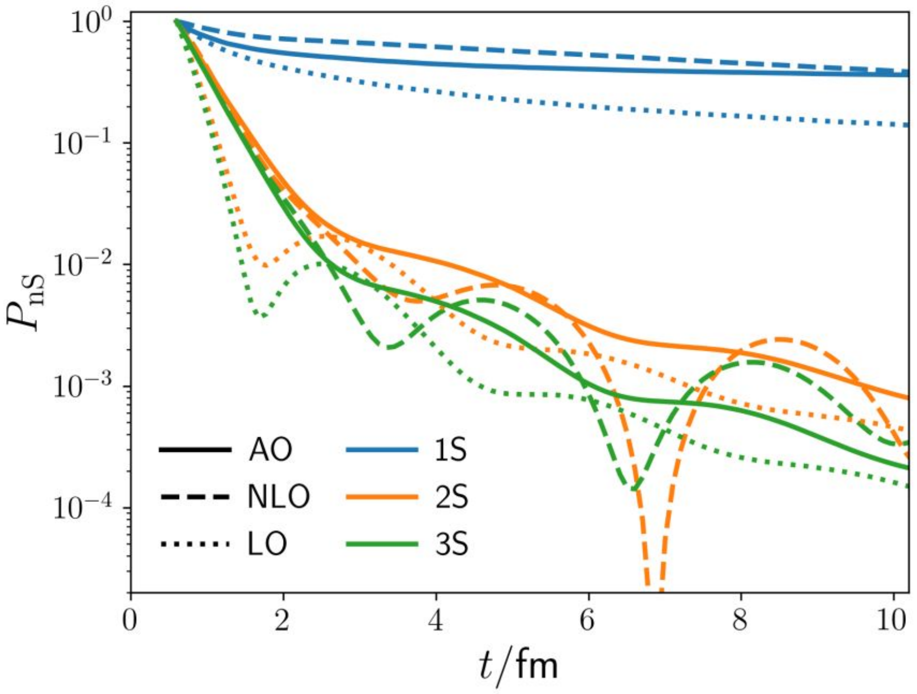
full AO

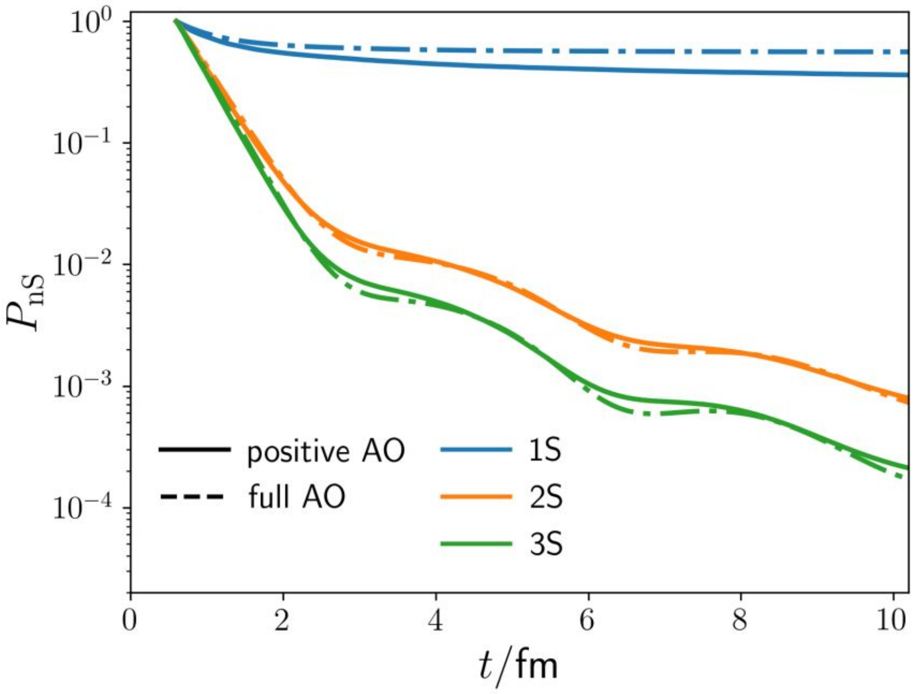
$$\Gamma_{s\to o} = \underbrace{{L^{\uparrow,+}_{s\to o}}^{\dagger} L^{\uparrow,+}_{s\to o}}_{\text{positive AO}} - {L^{\uparrow,-}_{s\to o}}^{\dagger} L^{\uparrow,-}_{s\to o}$$

positive AO



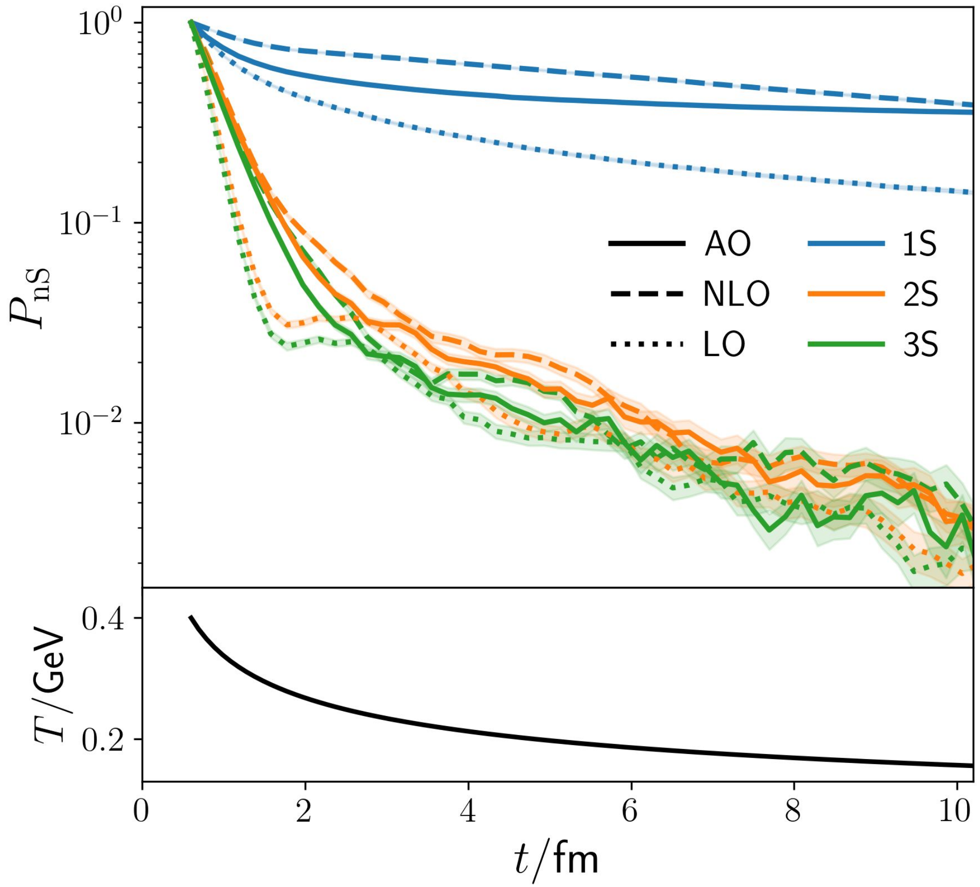
H_{eff} simulation



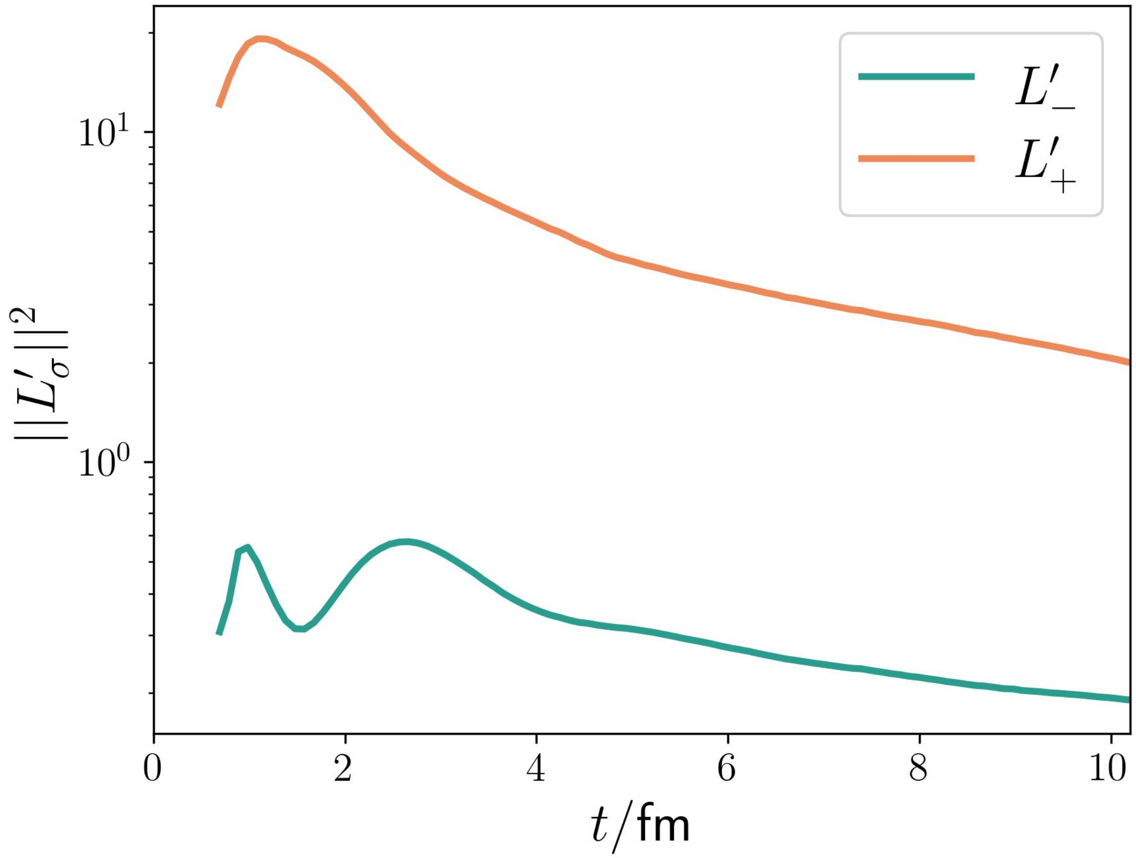


$$H_{\text{eff}} = H - \frac{\mathrm{i}}{2}\Gamma_{s\to o}$$

Jump simulation

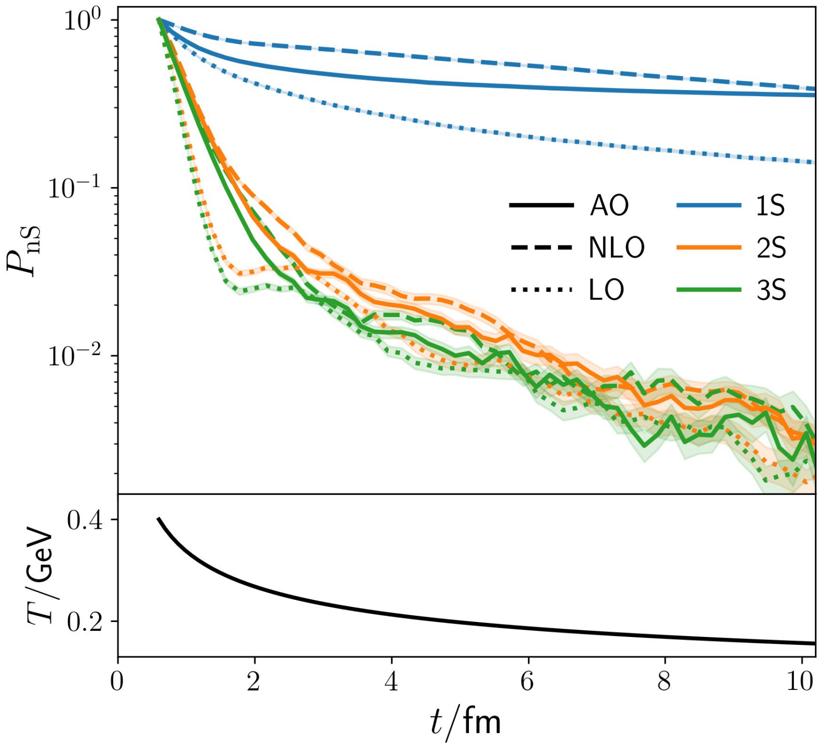


Operator norm



Summary

- At low temperatures our master equation is not positive
- Operator optimizations lead to efficient Lindblad truncation
- Efficient simulation of quarkonium dynamics at low temperatures



Outlook: - **Correlator study**
- **Pheno with Hydro**

Backup

Quantum Trajectories

e.g.: $U = 1 - iH_{\text{eff}}\delta t$

$$\psi_0$$

$$\downarrow$$

1. Evolve state $|\psi(t)\rangle$ with $H_{\text{eff}} = H - \frac{i}{2}C^\dagger C$

$$|\psi'(t+\delta t)\rangle = U|\psi(t)\rangle$$

Quantum Trajectories

e.g.: $U = 1 - iH_{\text{eff}}\delta t$

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2. Compute norm

$$\langle\psi'(t+\delta t)|\psi'(t+\delta t)\rangle = 1 - \delta p < 1$$

Quantum Trajectories

e.g.: $U = 1 - iH_{\text{eff}}\delta t$

ψ_0 ↓

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$$\langle\psi'(t+\delta t)|\psi'(t+\delta t)\rangle = 1 - \delta p < 1$$

3. Apply jump operator C with probability δp

$$P\left(|\psi(t+\delta t)\rangle = C|\psi(t)\rangle\right) = \delta p$$
$$P\left(|\psi(t+\delta t)\rangle = |\psi'(t+\delta t)\rangle\right) = 1 - \delta p$$

Quantum Trajectories

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4. Normalize $|\psi(t+\delta t)\rangle$

Pseudo Lindblad Quantum Trajectories

Becker, T., Netzer, C., & Eckardt, A. (2023). *Physical Review Letters*, *131*(16), 160401.

$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \left(L_+\rho L_+^\dagger - \frac{1}{2}\{L_+^\dagger L_+, \rho(t)\}\right) - \left(L_-\rho L_-^\dagger - \frac{1}{2}\{L_-^\dagger L_-, \rho(t)\}\right)$$

Pseudo Lindblad Quantum Trajectories

Becker, T., Netzer, C., & Eckardt, A. (2023). *Physical Review Letters*, *131*(16), 160401.

two jump operators

$$L_+, L_-$$

$$\psi_0 \quad s(0) = 1$$

1. Calculate rates r_σ

$$r_\sigma = \frac{||L_\sigma|\psi(t)\rangle||^2}{|||\psi(t)\rangle||^2}$$

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1. Calculate rates r_σ

$$r_\sigma = \frac{||L_\sigma|\psi(t)\rangle||^2}{|||\psi(t)\rangle||^2}$$

2. With probability $r_+\delta t$

$$|\psi(t+\delta t)\rangle = \frac{L_+|\psi(t)\rangle}{\sqrt{r_+(t)}} \qquad s(t+\delta t) = s(t)$$

With probability $r_-\delta t$

$$|\psi(t+\delta t)\rangle = \frac{L_-|\psi(t)\rangle}{\sqrt{r_-(t)}} \qquad s(t+\delta t) = -s(t)$$

Pseudo Lindblad Quantum Trajectories

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$$|\psi(t+\delta t)\rangle = \frac{L_-|\psi(t)\rangle}{\sqrt{r_-(t)}} \qquad s(t+\delta t) = -s(t)$$

With probability $1 - \sum_\sigma r_\sigma \delta t$

$$|\psi(t+\delta t)\rangle = \frac{(1 - i\delta t H_{\mathrm{Heff}})|\psi(t)\rangle}{\sqrt{1 - \sum_\sigma r_\sigma(t)\delta t}} \qquad s(t+\delta t) = s(t)$$

Pseudo Lindblad Quantum Trajectories

Becker, T., Netzer, C., & Eckardt, A. (2023). *Physical Review Letters*, *131*(16), 160401.

$$\rho(t) = \mathbb{E}\left[s(t)|\psi(t)\rangle\langle\psi(t)|\right]$$

Pseudo Lindblad Quantum Trajectories

$$\psi_0 \quad s(0) = 1$$

1. Draw $p_1 \in [0,1]$. While $R(t) > p_1$ evolve

$$R(t') = \exp\left(-\int_t^{t'} \sum_i r_i(s)ds\right) R(t) \qquad |\psi(t')\rangle = \frac{\exp\left(-i\int_t^{t'} H_{\text{eff}}(s)ds\right)}{\sqrt{\exp\left(-\int_t^{t'} \sum_i r_i(s)ds\right)}}$$

$$r_i(t) = \frac{||L_i|\psi(t)\rangle||^2}{||\,|\psi(t)\rangle||^2} \qquad s(t') = s(t)$$

2. Draw Jump operator i with probability $\propto r_i(t)$ and perform jump

$$|\psi(t)\rangle \leftarrow \frac{L_i|\psi(t)\rangle}{\sqrt{r_i(t)}}$$

3. Flip the sign bit **if the applied jump operator is a negative one**

$$s(t) \leftarrow -s(t)$$

Pseudo Lindblad Quantum Trajectories

$$\rho(t) = \mathbb{E}\left[s(t)|\psi(t)\rangle\langle\psi(t)|\right]$$

$\bar{s}(t) \to 0$ **Sign problem**

$$\overline{s}(t) = \exp\left[-2\int_0^t \sum_i r_{i,-}(s)ds\right]$$

Minimize negative rates

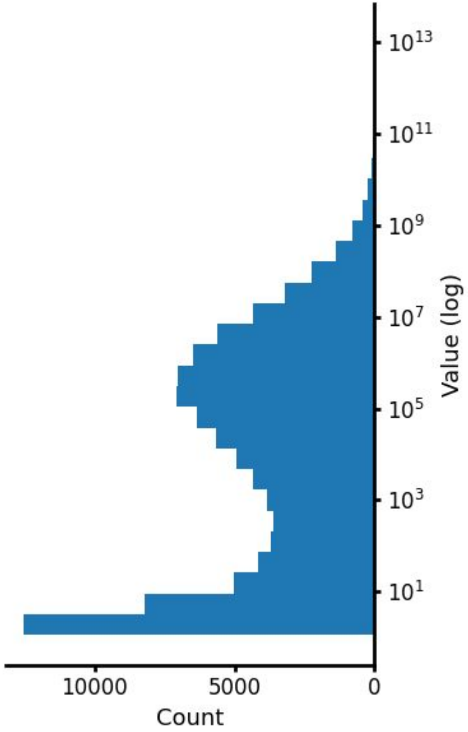
$$r_i(t) = \frac{||L_i|\psi(t)\rangle||^2}{||\,|\psi(t)\rangle||^2}$$

Because trajectories can have large norm, we are sampling a heavy tailed distribution

$$|\psi(t')\rangle = \frac{\exp\left(-i\int_t^{t'} H_{\text{eff}}(s)ds\right)}{\sqrt{\exp\left(-\int_t^{t'} \sum_i r_i(s)ds\right)}}$$

Norm can grow

$$\mathrm{Tr}[A\rho(t)] = \mathbb{E}\left[s(t)\langle\psi(t)|A|\psi(t)\rangle\right]$$



Tailed distribution: Takes many samples to converge

Operator optimizations

Becker, T., & Eckardt, A. (2025). *Physical Review E*, *111*(3), 034132.

Becker, T., Netzer, C., & Eckardt, A. (2023). *Physical Review Letters*, *131*(16), 160401.

Global optimization

$$\underset{\omega,\phi}{\operatorname{argmin}} \left|\left|L'_-\right|\right|^2$$

$$\phi = \pi - \arg\left[\mathrm{Tr}(L_+ L_-^\dagger)\right]$$

$$w = \frac{1}{2}\operatorname{arctanh}\left(\frac{2\,\mathrm{Tr}(L_+ L_-^\dagger)}{\mathrm{Tr}(L_+ L_+^\dagger) + \mathrm{Tr}(L_- L_-^\dagger)}\right)$$

Local optimization

$$\underset{\omega,\phi}{\operatorname{argmin}} \langle\psi|L_-'^\dagger L'_-|\psi\rangle$$

$$\phi = \pi - \arg\left[\langle\psi|L_-^\dagger L_+|\psi\rangle\right]$$

$$w = \frac{1}{2}\operatorname{arctanh}\left(\frac{2\left|\langle\psi|L_-^\dagger L_+|\psi\rangle\right|}{||L_+|\psi\rangle||^2 + ||L_-|\psi\rangle||^2}\right)$$

Connecting to phenomenology

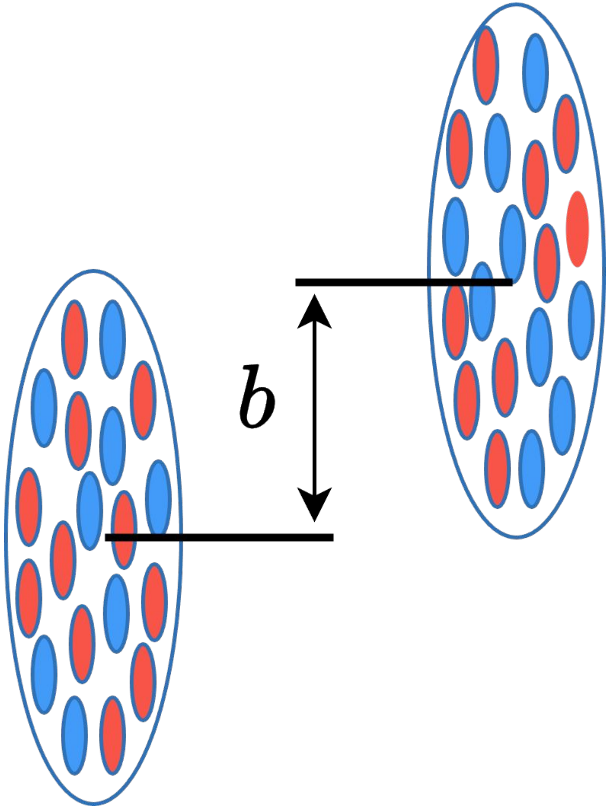
$$R^i_{AA}(c) = \frac{\left(F \cdot S(c) \cdot \vec{\sigma}_{\text{direct}}\right)^i}{\sigma^i_{\text{exp}}}$$

$$S_{ii} = P_{\text{survival}}(i) = \frac{\langle i|\rho(t_f)|i\rangle}{\langle i|\rho(t_0)|i\rangle}$$

Connecting to phenomenology

centrality

$$R^i_{AA}(c) = \frac{\left(F \cdot S(c) \cdot \vec{\sigma}_{\text{direct}}\right)^i}{\sigma^i_{\text{exp}}}$$



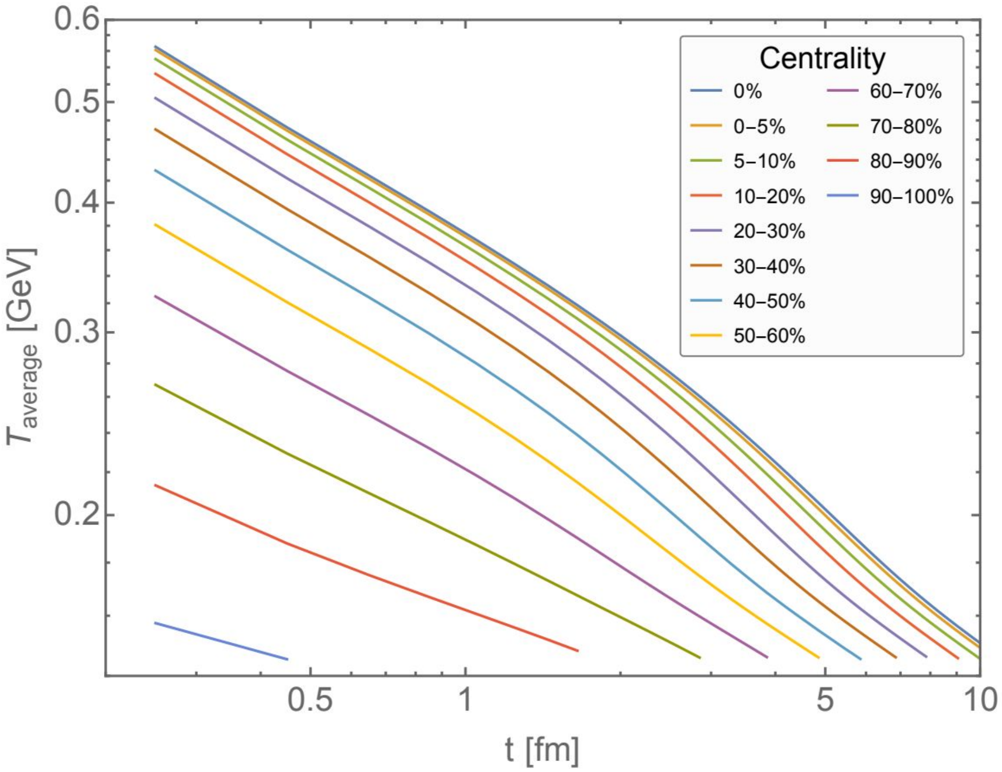
b: Impact parameter

Connecting to phenomenology

Brambilla, Nora, et al. *Journal of High Energy Physics* 2021.5 (2021): 1-47.

centrality

$$R^i_{AA}(c) = \frac{(F \cdot S(c) \cdot \vec{\sigma}_{\mathrm{direct}})^i}{\sigma^i_{\mathrm{exp}}}$$



Connecting to phenomenology

Feeddown matrix

$$R^i_{AA}(c) = \frac{\left(F \cdot S(c) \cdot \vec{\sigma}_{\text{direct}}\right)^i}{\sigma^i_{\text{exp}}}$$

$$\vec{\sigma}_{\text{direct}} = F^{-1}\vec{\sigma}_{\text{exp}}$$