TOWARDS OPTIMAL QUANTUM METROLOGY FOR HEAVY-QUARKONIUM

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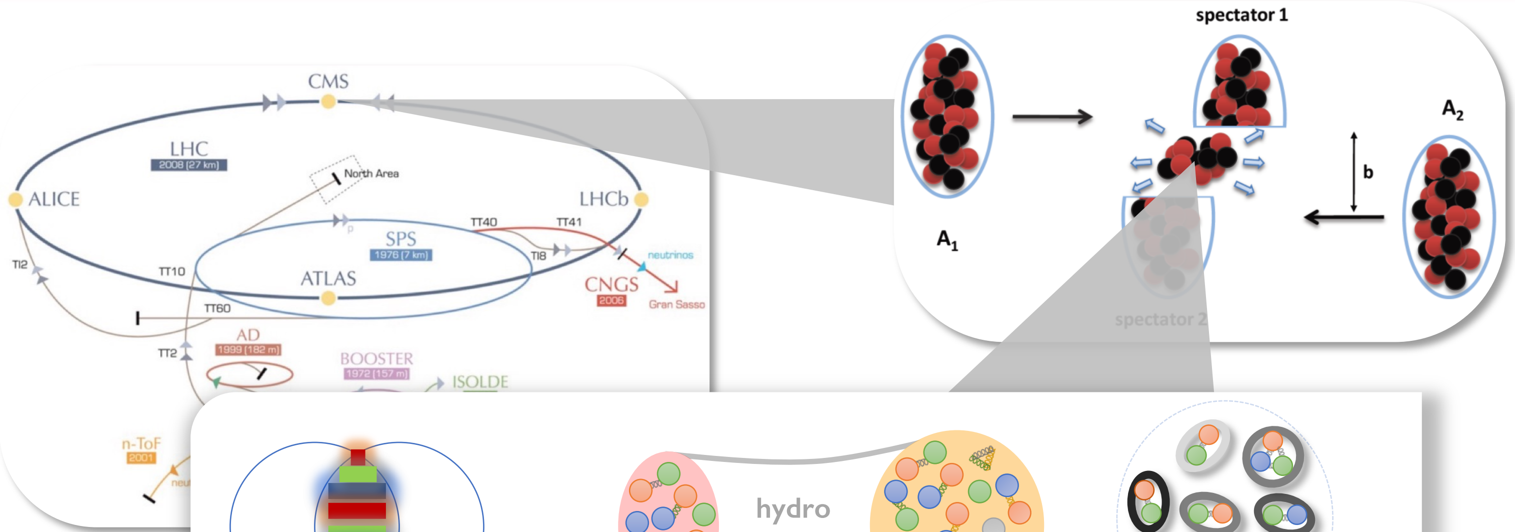
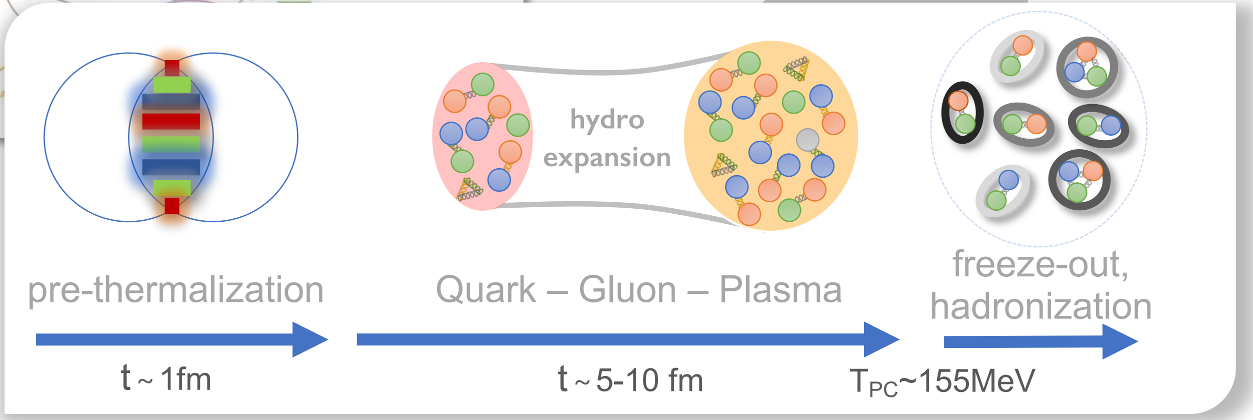
Based on collaboration with
V. López-Pardo, A.R. arXiv:2506.23600 & (in preparation)

Outline

- Motivation: exploring the properties of hot nuclear matter
- The quarkonium OQS ecosystem in heavy-ion collisions
- Towards optimal quantum metrology for in-medium heavy quarkonium
- Conclusion and Outlook

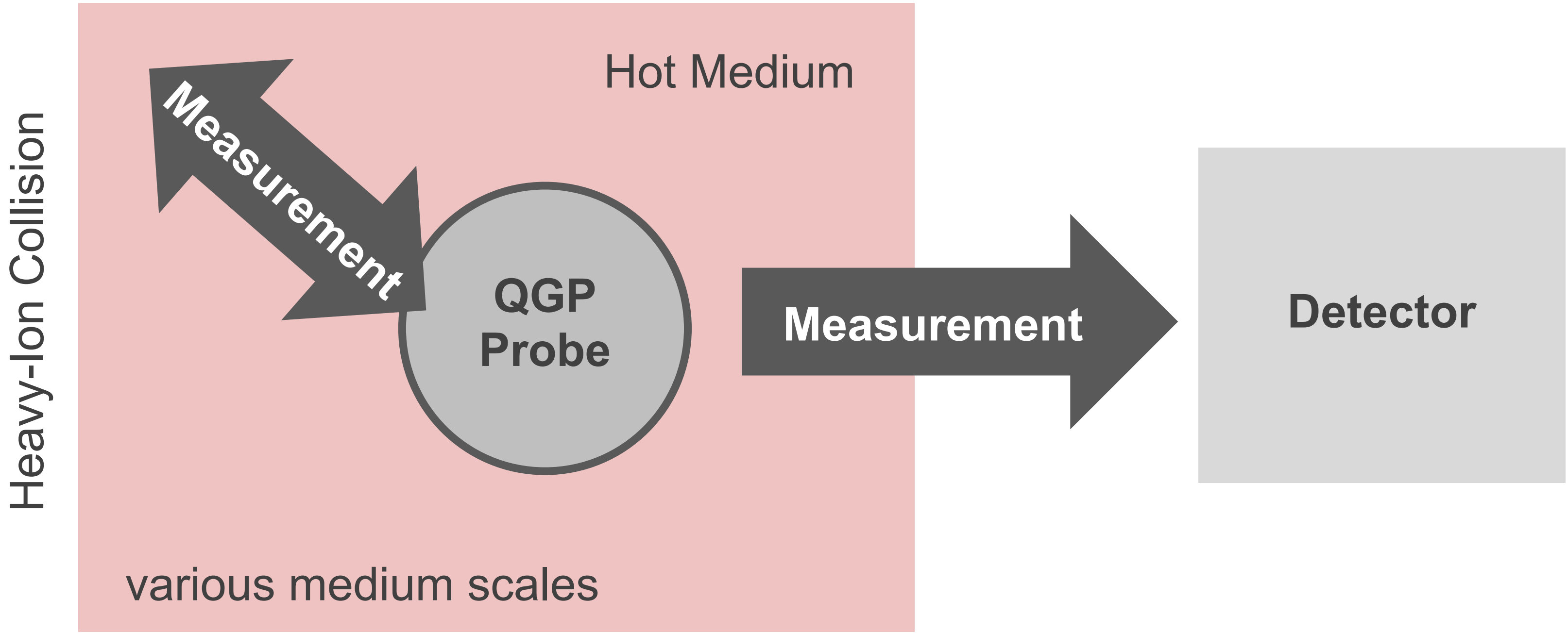
Heavy-ion collisions





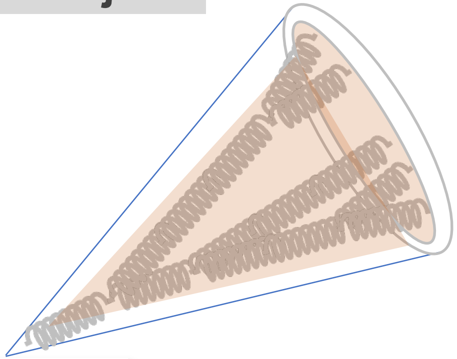
1fm ≡ 3x10^{-24} s in units where $c=k_B=h=1$

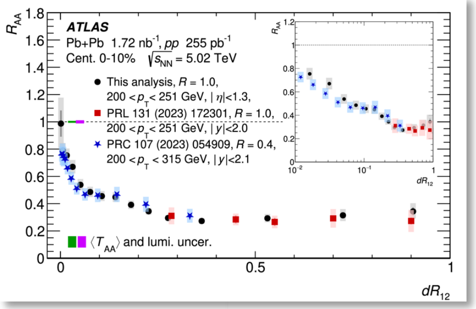
A tale of two measurements

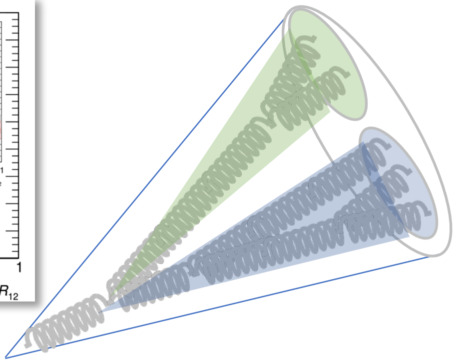


Probing & being probed by the medium

in-medium jets



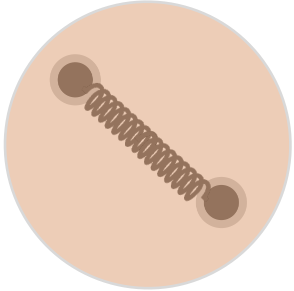


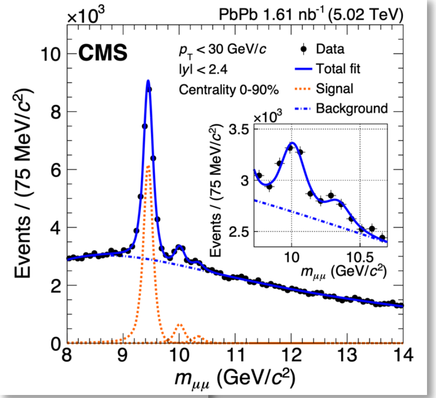


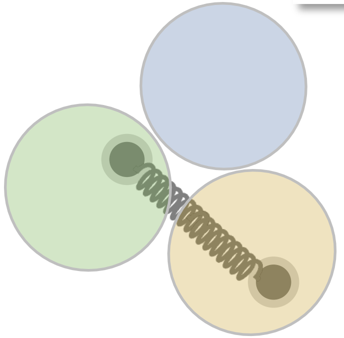
[ATLAS collaboration]
PLB 871 (2025) 139929

for recent theory results see e.g. Mehtar-Tani et.al. PLB 869 (2025) 139827, JHEP 02 (2026) 048 and Vaidya arXiv:2603.00238

in-medium QQ







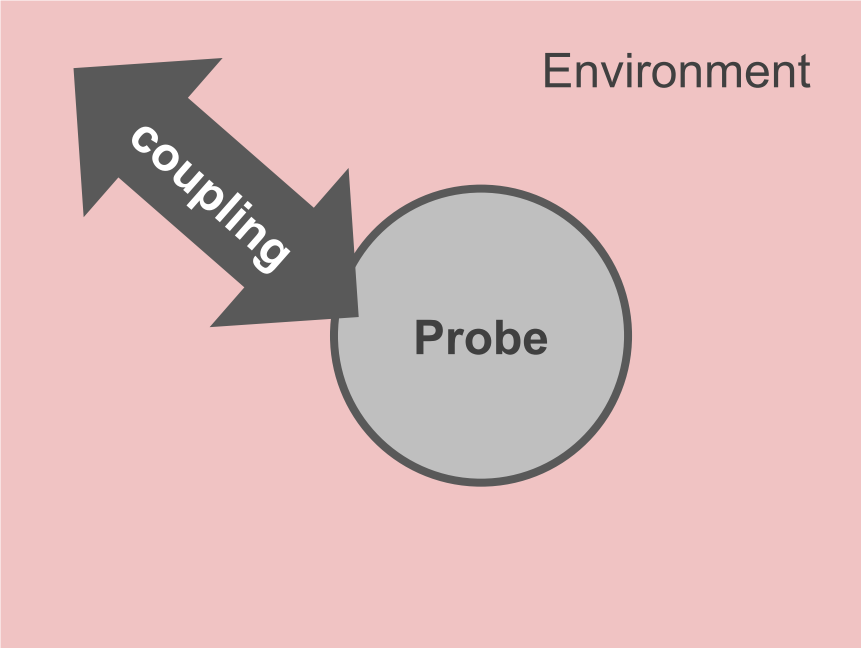
[CMS collaboration]
PRL 133 (2024) 022302

For reviews see: A.R. Phys. Rept. 858, 1 (2020), R. Sharma EPJ. ST 230, 3, 697 (2021), X. Yao IJMP A 36, 20, 2130010 (2021), Y. Akamatsu PPNP 123, 103932 (2022), for recent lattice results see [HotQCD] PRD 109 (2024) 7, 074504

Open Quantum Systems

- Demystifying the measurement process: coupling to environment

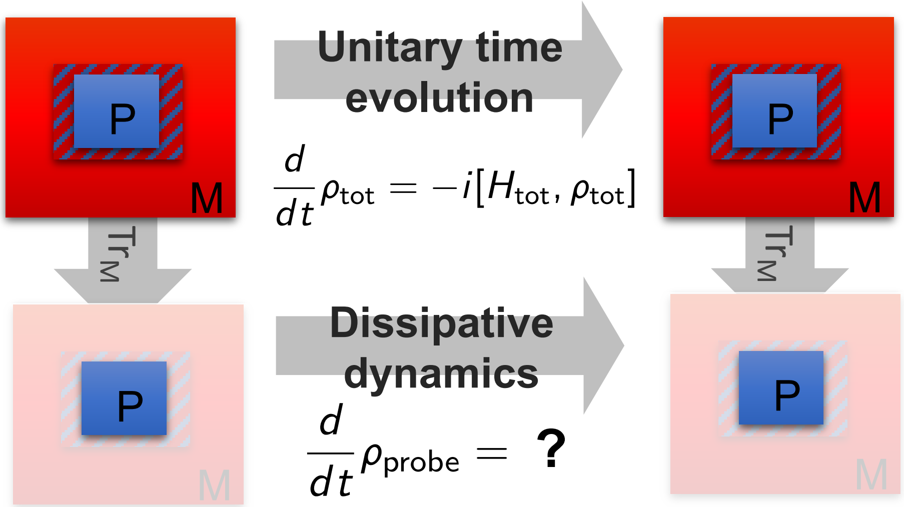
for a textbook see Breuer, Pettrucione The Theory of Open Quantum Systems



- Measurement is a **dynamical process**: focus on real-time evolution
- In presence of **separation of scales**: simplification (close relation to EFTs)

The Open Quantum System framework

$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm M} + I_{\rm probe} \otimes H_{\rm M} + H_{\rm int} = H_{\rm tot}^\dagger$$



$$\frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

$$\frac{d}{dt}\rho_{\rm probe} = \textbf{?}$$

$$\rho_{\rm probe} = {\rm Tr}_{\rm M}[\rho_{\rm tot}]$$

reduced density matrix

Separation of time-scales determines nature of e.o.m. :

Environment relaxation scale τ_E :

$$\langle \Xi_m(t)\Xi_m(0)\rangle \sim e^{-t/\tau_E}$$

probe system scale τ_S :

$$\tau_S \sim 1/|\omega - \omega'|$$

probe relaxation scale $\tau_{\rm rel}$:

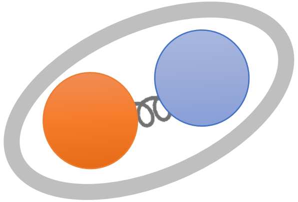
$$\langle p(t)\rangle \propto e^{-t/\tau_{\rm rel}}$$

- In case of Markovian time evolution ($\tau_E \ll \tau_{\rm rel}$) leads to a **Lindblad equation:**

$$\frac{d}{dt}\rho_{Q\bar Q} = -i[\tilde H_{Q\bar Q}, \rho_{Q\bar Q}] + \sum_k \gamma_k \left(L_k \rho_{Q\bar Q} L_k^\dagger - \frac{1}{2} L_k^\dagger L_k \rho_{Q\bar Q} - \frac{1}{2}\rho_{Q\bar Q} L_k^\dagger L_k \right)$$

$$\langle n|\rho_{Q\bar Q}|n\rangle > 0, \forall n, \quad \rho_{Q\bar Q}^\dagger = \rho_{Q\bar Q}, \quad {\rm Tr}[\rho_{Q\bar Q}] = 1$$

In-medium heavy quarkonium



Quarkonium separation of scales: $M_Q > \Lambda_{QCD}$, $M_Q > T_{med}$

In vacuum: m^{Υ}=9.460GeV, Γ^{Υ} = 54(1)keV; $m^{J/\psi}$=3.096 GeV, $\Gamma^{J/\psi}$ = 93(3)keV

- Different Quarkonium species form an ideal set of standard candles

	Υ(1S)	χ_b(1P)	J/ψ(1S)	χ_c(1P)	ψ(2S)
E_{bind}	1.1GeV	630MeV	640MeV	220MeV	60MeV

estimated temperature range realized in LHC collisions

degree of thermalization

non-equilibrium theory,
resolves history of collision

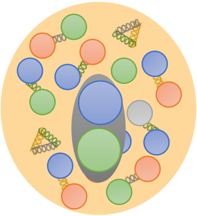
equilibrium theory,
informs of late stages

The EFT approach to heavy quarkonium

- Separation of time-scales mirrored in a separation of energy scales
- Exploit $\frac{T}{m_Q} \ll 1,\ \frac{\Lambda_{QCD}}{m_Q} \ll 1$ to treat heavy quarks non-relativistically

see Brambilla et. al. Rev.Mod.Phys. 77 (2005) 142

Relativistic T>0 field theory



$E \sim m_Q$ — **QCD** — **Dirac fields**

$Q(x), \bar{Q}(x)$

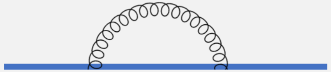
$E \sim m_Q v$ — **NRQCD** — **Pauli fields**

$\psi, \psi^\dagger, \chi, \chi^\dagger$

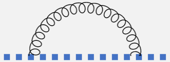
$E \sim m_Q v^2$ — **pNRQCD** — **Color singlet and octet wavefunctions**

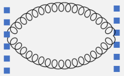
$\psi_S(R,t), \psi_O(R,t)$

$$\mathcal{L} = \bar{Q}\big(i\gamma^\mu(\partial_\mu - igA_\mu) + m\big)Q$$

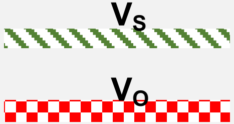


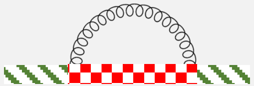
$$\mathcal{L}_\psi = \psi^\dagger\big(i(\partial_0 - igA_0) + c_1(\nabla - ig\mathbf{A})^2/2m_Q + c_2\boldsymbol{\sigma}\mathbf{B}/2m_Q + c_3(\nabla - ig\mathbf{A})^4/8m_Q^3 + \ldots\big)\psi$$





$$\mathcal{L}_{SO} = S^\dagger\Big(i\partial_0 + d_1\frac{\nabla^2}{2\mu} - V_S^{(0)} + \ldots\Big)S + V_A\mathrm{Tr}[O^\dagger\mathbf{r}\cdot\mathbf{E}S]$$
$$O^\dagger\Big(i\partial_0 + f_1\frac{\nabla^2}{2\mu} - V_O^{(0)} + \ldots\Big)O + V_B\mathrm{Tr}[O^\dagger\mathbf{r}\cdot\mathbf{E}O] + \ldots$$





The Quarkonium OQS ecosystem

QCD

multipole expansion

pNRQCD

small dipole approximation

Lindblad equation

Dynamics in terms of non-perturbative **transport coefficients**

N. Brambilla et.al. PRD99 (2019) 096028, PRD 104 (2021) 9, 094049, PRD 108 (2023) 1, L011502 & JHEP 04 (2026) 007

beyond E/T

weak coupling

Lindblad equation

"quantum optical limit"

Boltzmann equation

X. Yao, T. Mehen PRD99 (2019) 096028
X. Yao et.al. JHEP 01 (2021) 046

$$\frac{\partial}{\partial t} f_{nl}(\mathbf{X},\mathbf{K},t) + \mathbf{v}\cdot\nabla_{\mathbf{X}} f_{nl}(\mathbf{X},\mathbf{K},t) = \mathcal{C}^{(rec)}_{nl}(\mathbf{X},\mathbf{K},t) - \mathcal{C}^{(diss)}_{nl}(\mathbf{X},\mathbf{K},t)$$

time scale separation
close to equilibrium

Rate equation

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\left[N_\psi - N^{eq}_\psi(T)\right]$$

non-relativistic quarks

NRQCD

weak coupling
"quantum Brownian motion limit"

Y. Akamatsu PRD91 (2015) 056002
T. Miura et.al. PRD 106 (2022) 7, 074001

Lindblad equation

color singlet projection
quantum state diffusion

T. Miura et.al. PRD101 (2020) 034011

Non-linear stochastic Schrödinger

fast decoherence

Semi-classical Langevin

J.P. Blaizot, M. Escobedo JHEP 1806 (2018) 034
A. Daddi-Hammou et.al. PRD 113 (2026) 1, 014017

dissipationless limit

Y. Akamatsu, A.R. PRD85 (2012) 105011

Stochastic Schrödinger

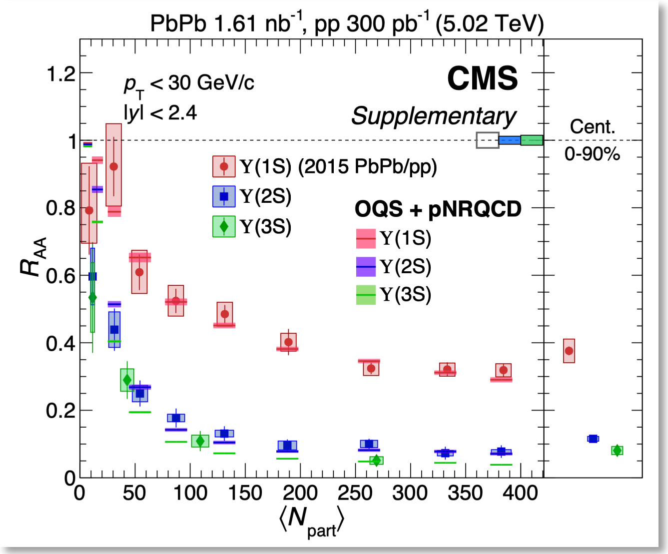
adiabatic approximation

Deterministic Schrödinger

$$i\partial_t\psi = \left[\frac{-\nabla^2}{2m_Q} + Re[V](R,T) - iIm[V](R,T)\right]\psi$$

OQS for discovery in heavy-ion collisions

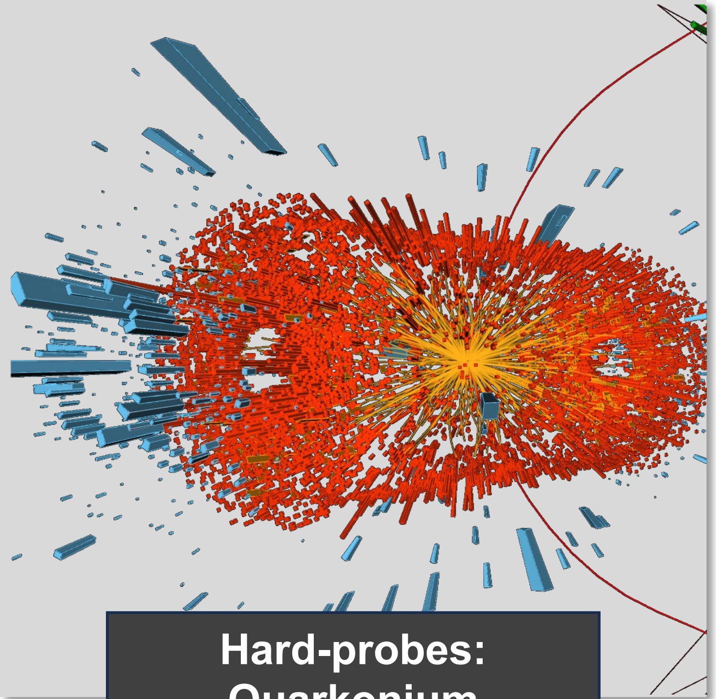
- Need to translate **methods development** into **physics opportunities**



- Key challenge I: **scarcity** of existing observables
  - How to exploit optimally existing data
- Key challenge II: QGP properties are **not direct observables**
  - How to exploit OQS approach to achieve a (thermo-), (viscosi-), (diffusi-) meter?
- Key challenge III: **limited** resources
  - How to decide which observable provides best ROI?

A fruitful analog: impurity physics

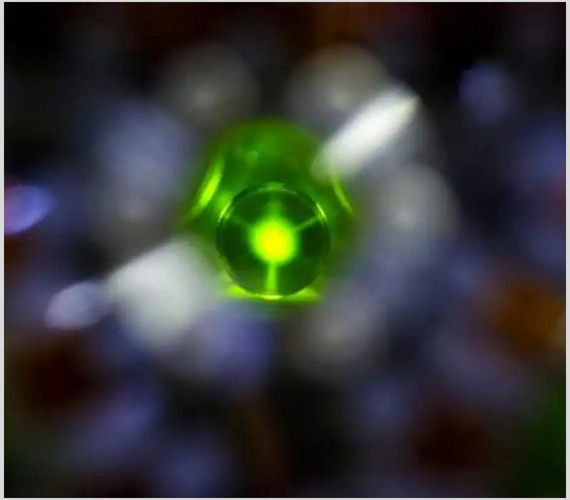
Heavy-ion Collisions



[CMS Collaboration]

Hard-probes: Quarkonium

Ultracold atoms



Joint Quantum Institute, UMD

Bosonic or Fermionic Polarons

Quantum Metrology

- The study of optimizing the measurement process exploiting quantum features

see e.g. M. Mehboudi et.al. J. Phys. A52 (2019) 303001

error in T measurement — spread in impurity property

$$\delta T[\hat{O}] = \frac{\langle \hat{O}^2 \rangle - \langle \hat{O} \rangle^2}{\sqrt{N \chi_T^2[\hat{O}]}}$$

classically random var. $\sim \frac{1}{\sqrt{N}}$ quantum Cramer-Rao bound $\geq \frac{1}{\sqrt{N\mathcal{F}[T]}}$

# of measurements — how sensitive is O to T — Fisher information

Optimum sensitivity $\chi_T[\hat{O}] = \partial_\xi \mathrm{Tr}[\rho_{\rm probe}(\xi)\hat{O}]_{\xi=T}$ reached via unique quantity $\hat{\Lambda}_T$:

$$\hat{\Lambda}_T \hat{\rho}_{\rm probe} + \hat{\rho}_{\rm probe}\hat{\Lambda}_T = 2\partial_T \hat{\rho}_{\rm probe}$$

“symmetric logarithmic derivative” (SLD)

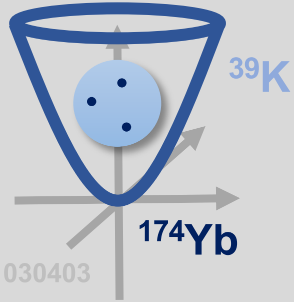
- Fisher information $\mathcal{F}_\xi = \mathrm{Tr}[\hat{\rho}(\xi)\hat{\Lambda}_\xi^2]$ (“how much can we learn about ξ”)

State-of-the-art – SLD in thermometry

$$\hat{\Lambda}_T \hat{\rho}_{\text{probe}} + \hat{\rho}_{\text{probe}} \hat{\Lambda}_T = 2\partial_T \hat{\rho}_{\text{probe}}$$

Equilibrium Thermometry

Laser cool **majority gas ^{39}K** together with **minority gas ^{174}Yb**



M. Mehboudi et.al. PRL122 (2019) 030403

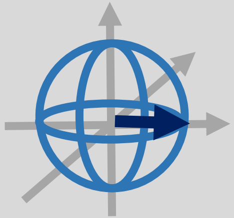
$$\hat{\Lambda}_T = C_x \left(\hat{x}^2 - \langle x^2 \rangle\right) + C_p \left(\hat{p}^2 - \langle p^2 \rangle\right)$$ ✅

$$C_x = \frac{4\langle p^2\rangle^2 \chi_T(\hat{x}^2) + \hbar^2 \chi_T(\hat{p}^2)}{8\langle x^2\rangle^2 \langle p^2\rangle^2 - \hbar^4/2}$$ ✅

$$\chi_\theta(\hat{O}) = \partial_\theta \mathrm{Tr}[\hat{\rho}(\theta)\hat{O}]$$ ❌

Non-equilibrium Thermometry

Kicks from environment **decohere two-level** system **impurity.**



M. T. Mitchison et.al. PRL 125, 080402 (2020)

$$\hat{\Lambda}_T \propto \cos(\varphi)\,\hat{\sigma}_{\parallel} + \sin(\varphi)\,\hat{\sigma}_{\perp}$$
$$\tan(\varphi) = \frac{|v|(1-|v|)^2\,\partial_T \phi}{\partial_T |v|}.$$ ✅

$\partial_T |v|$ $\partial_T \phi$ ❌

- Challenge: construction of SLD requires **explicit solution** of Lindblad equation

SLD from the master equation

master equation

$$\partial_t \hat\rho(t,\theta) = \hat{\mathcal{L}}[\theta]\hat\rho(t,\theta)$$

definition of SLD

$$\partial_\theta \hat\rho(\theta) = \frac{1}{2}\left(\hat\Lambda_\theta \hat\rho(\theta) + \hat\rho(\theta)\hat\Lambda_\theta\right) - \hat\rho(\theta)\langle\hat\Lambda_\theta\rangle$$

$$\partial_\theta\left(\partial_t \hat\rho(t,\theta)\right) = \partial_t\left(\partial_\theta \hat\rho(t,\theta)\right)$$

Schwarz' theorem: symmetry of second partial derivatives

V. López-Pardo, A.R. arXiv:2506.23600

choice of experimentally accessible operators

$$\hat\Lambda_\theta = \sum_i c_\theta^{(i)} \hat A_i$$

$$\mathrm{Tr}\left[\partial_\theta \partial_t \hat\rho(t,\theta)\,\hat A_j\right] = \mathrm{Tr}\left[\partial_t \partial_\theta \hat\rho(t,\theta)\,\hat A_j\right]$$

$$\sum_i M_{ji}\, c_\theta^{(i)} = D_j$$

Explicit construction of SLD via solution of linear system of equations

Quantum Brownian motion (equilibrium)

- SLD for **temperature T** from the Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\rm probe} = -\frac{i}{\hbar}[\hat{H}_{\rm probe}, \hat{\rho}_{\rm probe}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}_{\rm probe}]] - \frac{i\gamma}{\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}_{\rm probe}\}]$$

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

coherent dynamics　　fluctuations　　dissipation

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

$$D_{\rm T} = \left[0, 0, 0, -2\frac{\gamma}{b}, 0\right]$$

$$M = \begin{bmatrix} 0 & -2b\langle p^2\rangle & 0 & 0 & 0 \\ 2c\langle x^2\rangle & 2\gamma\langle p^2\rangle - \frac{2\gamma T}{b} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\gamma & -8b\langle p^2\rangle\langle x^2\rangle - 2b \\ 0 & 0 & 0 & 8\gamma\langle p^2\rangle^2 - \frac{8\gamma T\langle p^2\rangle}{b} & 8c\langle p^2\rangle\langle x^2\rangle + 2c \\ 0 & 0 & 2b + 8c\langle x^2\rangle^2 & -8b\langle p^2\rangle^2 - 2c & -\frac{8\gamma T\langle x^2\rangle}{b} - 2\gamma + 8\gamma\langle p^2\rangle\langle x^2\rangle \end{bmatrix}$$

$$c_T^{(x)} = 0, \quad c_T^{(p)} = 0,$$

$$c_T^{(x^2)} = \frac{4c\left(a^4b^2 + 4(bc+\gamma^2)\right)}{a^8b^4 - 16b^2c^2}, \quad c_T^{(p^2)} = \frac{4}{a^4b - 4c}, \quad c_T^{(\{x,p\})} = \frac{16c\gamma}{a^8b^3 - 16bc^2}.$$

reproduces the known result from explicit density matrix (T in equilibrium)

V. López-Pardo, A.R. arXiv:2506.23600

need to include the {x,p} operator to obtain a well-posed linear system even though $\langle\{x,p\}\rangle_{eq} = 0$

Quantum Brownian motion (squeezed Gaussian)

- SLD for **temperature T & relaxation rate γ** from Caldeira-Leggett master equation

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

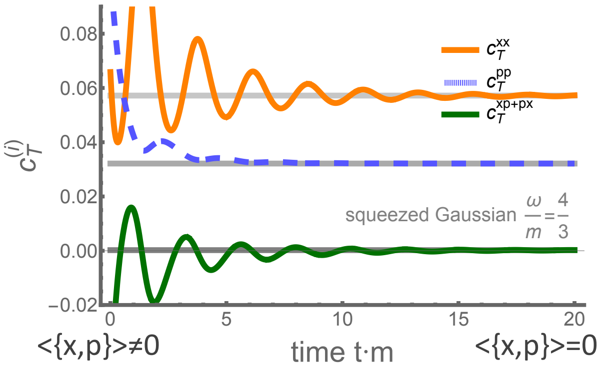
T → γ →

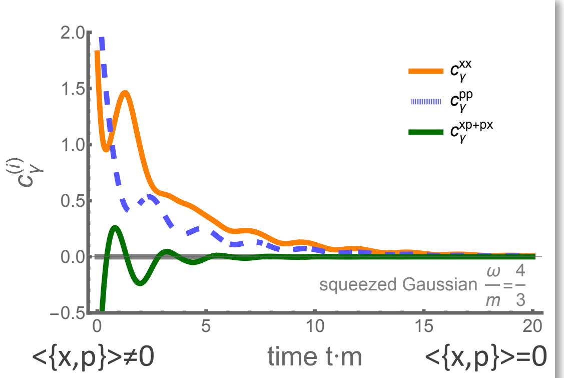
$$D_T = \left[0, 0, 0, -2\frac{\gamma}{b}, 0\right]$$

$$D_\gamma = \left[0, 0, 0, -2\frac{T}{b} + 4\left\langle p^2\right\rangle, 2\left\langle\{\hat{x}, \hat{p}\}\right\rangle\right]$$

$$M_T = M_\gamma$$

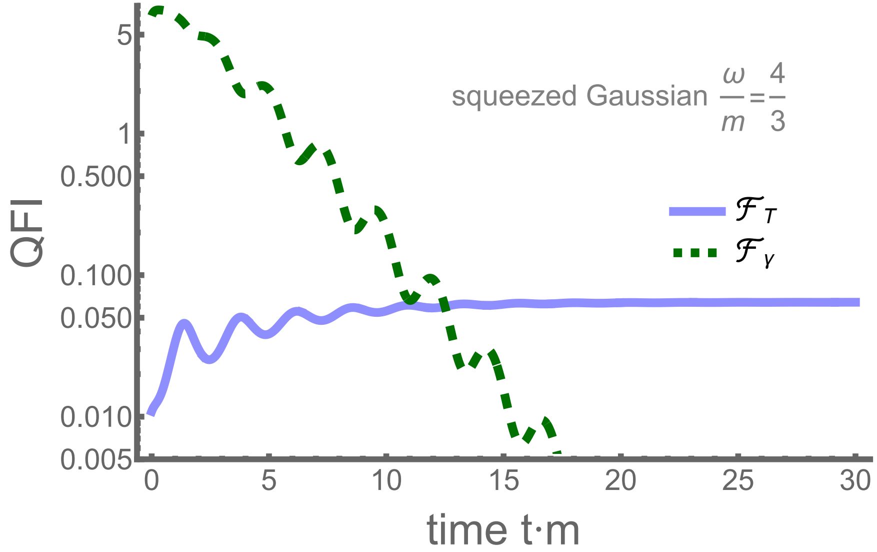
$$\begin{bmatrix} -b\langle\{x,p\}\rangle & -2b\langle p^2\rangle & 0 & 0 & 0 \\ 2c\langle x^2\rangle + \gamma\langle\{x,p\}\rangle & 2\gamma\langle p^2\rangle - \frac{2\gamma T}{b} + c\langle\{x,p\}\rangle & 0 & 0 & 0 \\ 0 & 0 & -4b\langle x^2\rangle\langle\{x,p\}\rangle & -4b\langle p^2\rangle\langle\{x,p\}\rangle + 2\gamma & -8b\langle x^2\rangle\langle p^2\rangle - 2b - 2b\langle\{x,p\}\rangle^2 \\ 0 & 0 & 4c\langle x^2\rangle\langle\{x,p\}\rangle + 2\gamma\langle\{x,p\}\rangle^2 & 8\gamma\langle p^2\rangle^2 - \frac{8\gamma T}{b}\langle p^2\rangle + 4c\langle p^2\rangle\langle\{x,p\}\rangle & 8c\langle x^2\rangle\langle p^2\rangle + 2c + 2c\langle\{x,p\}\rangle^2 + 8\gamma\langle p^2\rangle\langle\{x,p\}\rangle - \frac{4\gamma T}{b}\langle\{x,p\}\rangle \\ 0 & 0 & 8c\langle x^2\rangle^2 + 2b + 4\gamma\langle x^2\rangle\langle\{x,p\}\rangle & -8b\langle p^2\rangle^2 - 2c + 4\gamma\langle p^2\rangle\langle\{x,p\}\rangle + 2c\langle\{x,p\}\rangle^2 - \frac{4\gamma T}{b}\langle\{x,p\}\rangle & 8\gamma\langle x^2\rangle\langle p^2\rangle - \frac{8\gamma T}{b}\langle x^2\rangle - 2\gamma + 8c\langle x^2\rangle\langle\{x,p\}\rangle - 8b\langle p^2\rangle\langle\{x,p\}\rangle + 2\gamma\langle\{x,p\}\rangle^2 \end{bmatrix}$$





Crosscheck QFI:

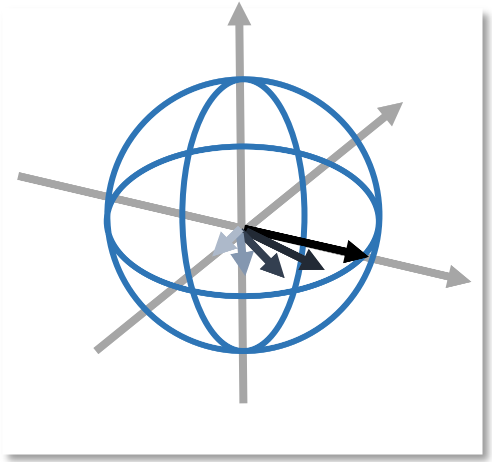
- As expected: temperature sensitivity highest after full thermalization, relaxation rate becomes inaccessible since equilibrium time translation invariant



squeezed state <{x,p}>≠0 → equilibrium <{x,p}>=0

A direct link to heavy quarkonium

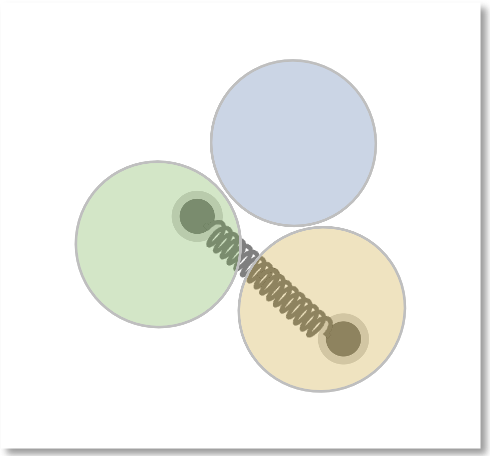
T=10^{-9}K



decoherence of a qubit
in a quantum gas

M. T. Mitchison et.al.
PRL 125, 080402 (2020)

T=10^{12}K

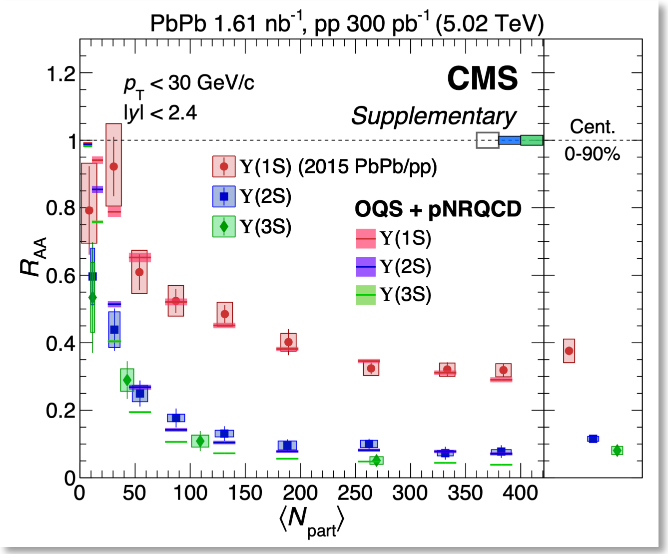


color decoherence of
heavy quarkonium in HIC

see e.g. S. Kajimoto, Y.Akamatsu,
M. Asakawa, A.R., PRD97 (2018), 014003

Adressing observable scarcity

- Goal: can we find an SLD based on the available data?



- Only make reference to lowest lying states ϒ(1S), ϒ(2S), ϒ(3S), χ_{b012}(1P)
- Operator basis for SLD is set of projection operators { |n><n| }.
- End result: determine linear combination of R_{AA}s that exhibits most sensitivity to certain medium property (e.g. T)

Application to Quarkonium – pNRQCD OQS

- As a first step: optimal estimation of local properties – transport coefficients

pNRQCD & strongly coupled medium

Non-perturbative medium but Coulombic bound states

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_O \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

$$L_i^{S\leftrightarrow O} = \sqrt{\frac{\kappa}{N_c^2-1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix} \quad L_i^{O\leftrightarrow O} = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

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governed by two (static) transport coefficients:

$$\kappa \propto \frac{1}{6N_c}\int_0^\infty dt \left\langle \{E^{a,i}(t,\mathbf{0}), E^{a,i}(t,\mathbf{0})\}\right\rangle$$ heavy quarkonium diffusion constant

$$\gamma \propto -\frac{i}{6N_c}\int_0^\infty dt \left\langle [E^{a,i}(t,\mathbf{0}), E^{a,i}(t,\mathbf{0})]\right\rangle$$ potential correction

$$\frac{d}{dt}\rho_{Q\bar{Q}} = -i[\tilde{H}_{Q\bar{Q}}, \rho_{Q\bar{Q}}] + \sum_k \gamma_k \left(L_k \rho_{Q\bar{Q}} L_k^\dagger - \frac{1}{2} L_k^\dagger L_k \rho_{Q\bar{Q}} - \frac{1}{2}\rho_{Q\bar{Q}} L_k^\dagger L_k \right)$$

$$\mathrm{Tr}\left[\partial_\theta \partial_t \hat{\rho}(t,\theta)\, \hat{A}_j\right] = \mathrm{Tr}\left[\partial_t \partial_\theta \hat{\rho}(t,\theta)\, \hat{A}_j\right]$$

$$\begin{aligned} M_{ij} =& +\tfrac{i}{2}\left\langle \{\hat{A}_j, [\hat{H}, \hat{A}_i]\}\right\rangle \\ &+ \tfrac{1}{2}\sum_n \left(\left\langle \hat{C}_n^+ \hat{A}_j\,, [\hat{A}_i, \hat{C}_n]\right\rangle + \left\langle [\hat{C}_n^+, \hat{A}_i]\, \hat{A}_j \hat{C}_n \right\rangle - \tfrac{1}{2}\left\langle [\hat{A}_j, [\hat{A}_i, \hat{\Gamma}_n]]\right\rangle \right) \\ &- \langle \hat{A}_j \rangle \left(-i\left\langle \{\hat{A}_i, \hat{H}\}\right\rangle + \sum_n \left(\left\langle \hat{C}_n^+ \hat{A}_i \hat{C}_n \right\rangle - \tfrac{1}{2}\left\langle \{\hat{A}_i, \hat{\Gamma}_n\}\right\rangle \right) \right) \end{aligned}$$

$$D_j = -\tfrac{i}{2}\begin{pmatrix} \left\langle [\hat{A}_j^s, \hat{r}^{\,2}]\right\rangle & 0 \\ 0 & \left\langle [\hat{A}_j^o, \hat{r}^{\,2}]\right\rangle \Omega_o \end{pmatrix}$$

$$\hat{A}_i = \{|\Upsilon(1S)\rangle\langle\Upsilon(1S)|, |\Upsilon(2S)\rangle\langle\Upsilon(2S)|, |\Upsilon(3S)\rangle\langle\Upsilon(3S)|, |\chi_i(1P)\rangle\langle\chi_i(1P)|\}$$

First PRELIMINARY insights

- Using the assumptions of singlet dominance and no off-diagonal contributions
- Highly restricted basis: Υ(1S) Υ(2S) Υ(3S) χ_{b012}(1P): system matrix Mij degenerate

SLD for transport coefficient γ

{{cY1S → 0. – 0.00348522 cC10 – 0.00348522 cC11 – 0.00348522 cC12 – 0.261354 cY2S – 0.216845 cY3S}}

SLD for transport coefficient κ

{{cY1S → 20.4013 – 0.00348522 cC10 – 0.00348522 cC11 – 0.00348522 cC12 – 0.261354 cY2S – 0.216845 cY3S}}

- Υ(1S) survival sensitive to κ but determination of γ requires excited states survival

Conclusion & Outlook

- Quarkonium is a powerful QCD laboratory with potential as precision QGP probe
- Open quantum systems ecosystem for quarkonium mature & extensive
- Quantum Metrology: Optimal observables via SLD – but requires analytic solution
- Recent progress:Exploit symmetry of mixed partial derivatives to construct SLD from master equation directly. V. López-Pardo, A.R. arXiv:2506.23600
- Proof of principle: explicit form of SLD from Caldeira-Leggett master equation for environment temperature & relaxation rate (non-equilibrium).
- Work in progress: application to pNRQCD quarkonium master equation

Thank you for your attention