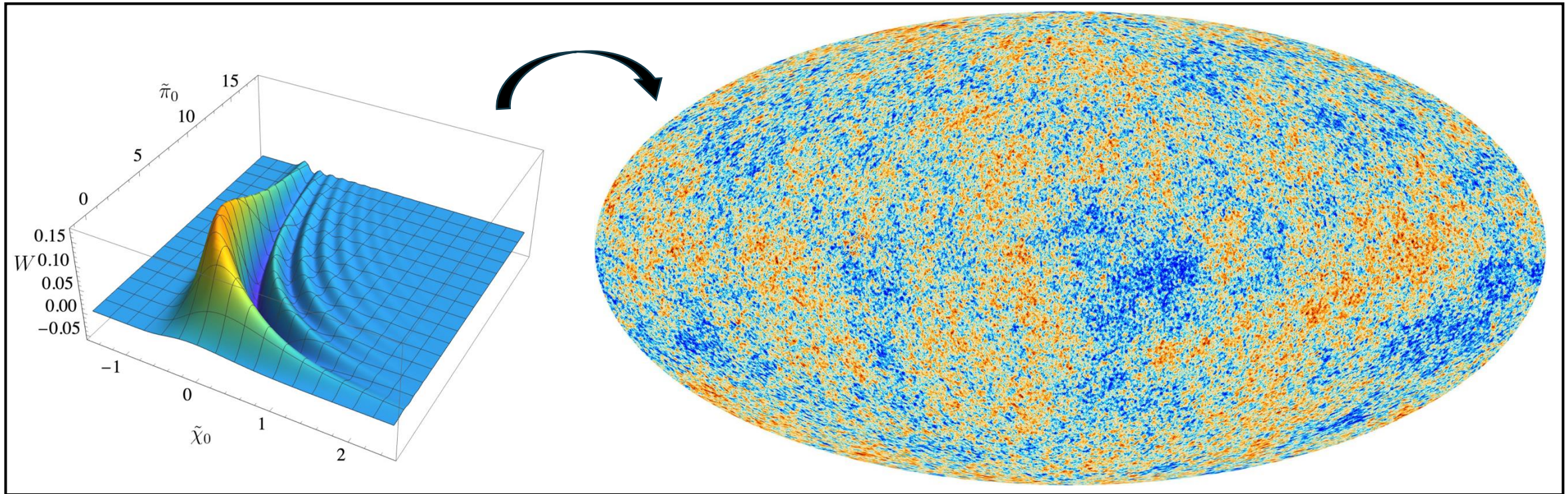


When Inflationary Perturbations Refuse to Classicalise: the role of non-Gaussianity in Wigner negativity

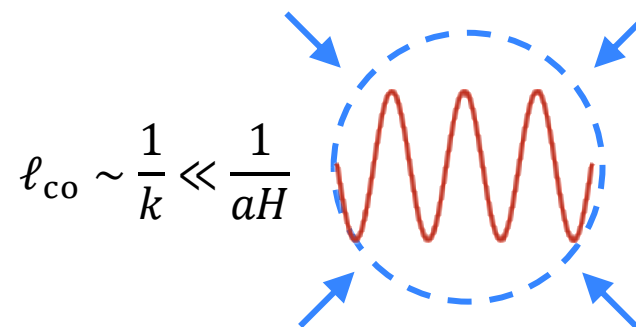
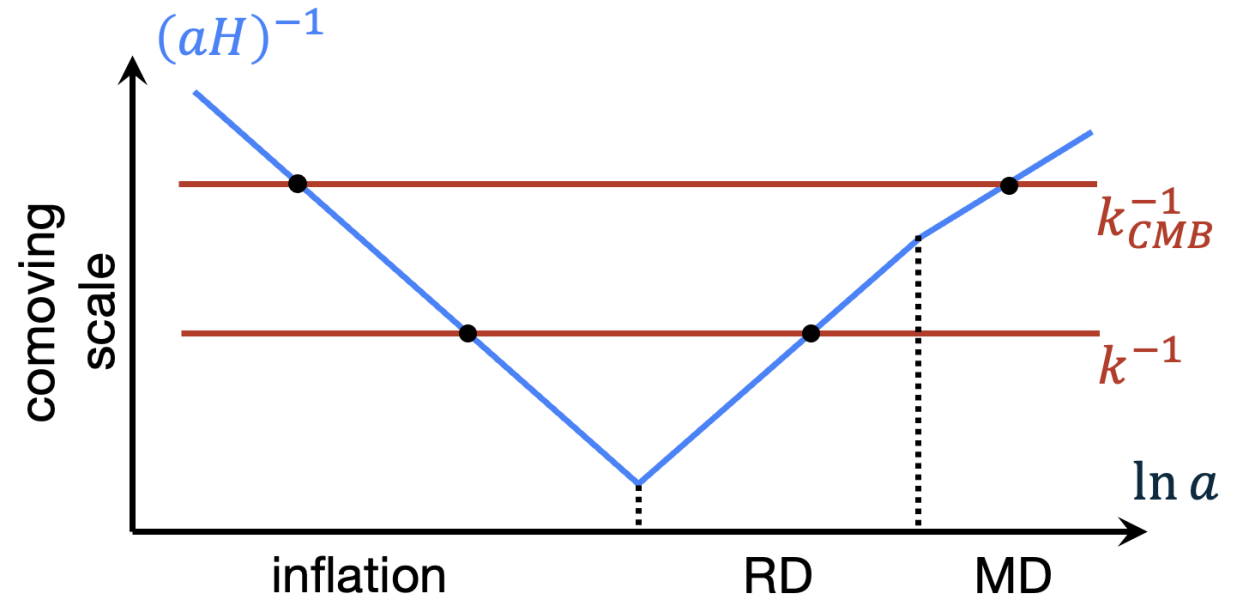


Motivation

- Origin of cosmic structure
- Quantum fluctuations produced during inflation ($\ddot{a} > 0$)

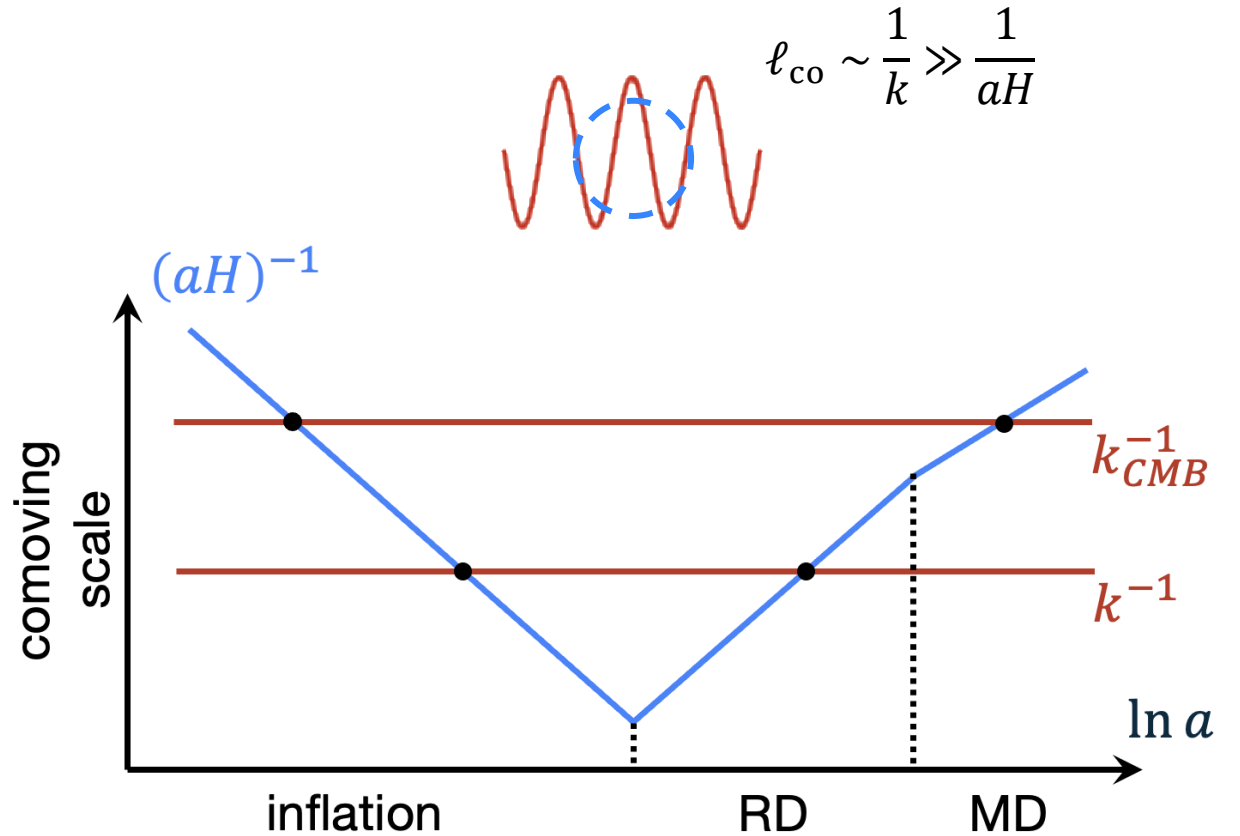
$$ds^2 = -dt^2 + a^2(t)dx^2$$

$a(t)$ = scale factor
 $H = \dot{a}/a$ = Hubble



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 - Stretched to superhorizon scales ($k \ll aH$), where they “freeze out”



$$\ell_{co} \sim \frac{1}{k} \gg \frac{1}{aH}$$

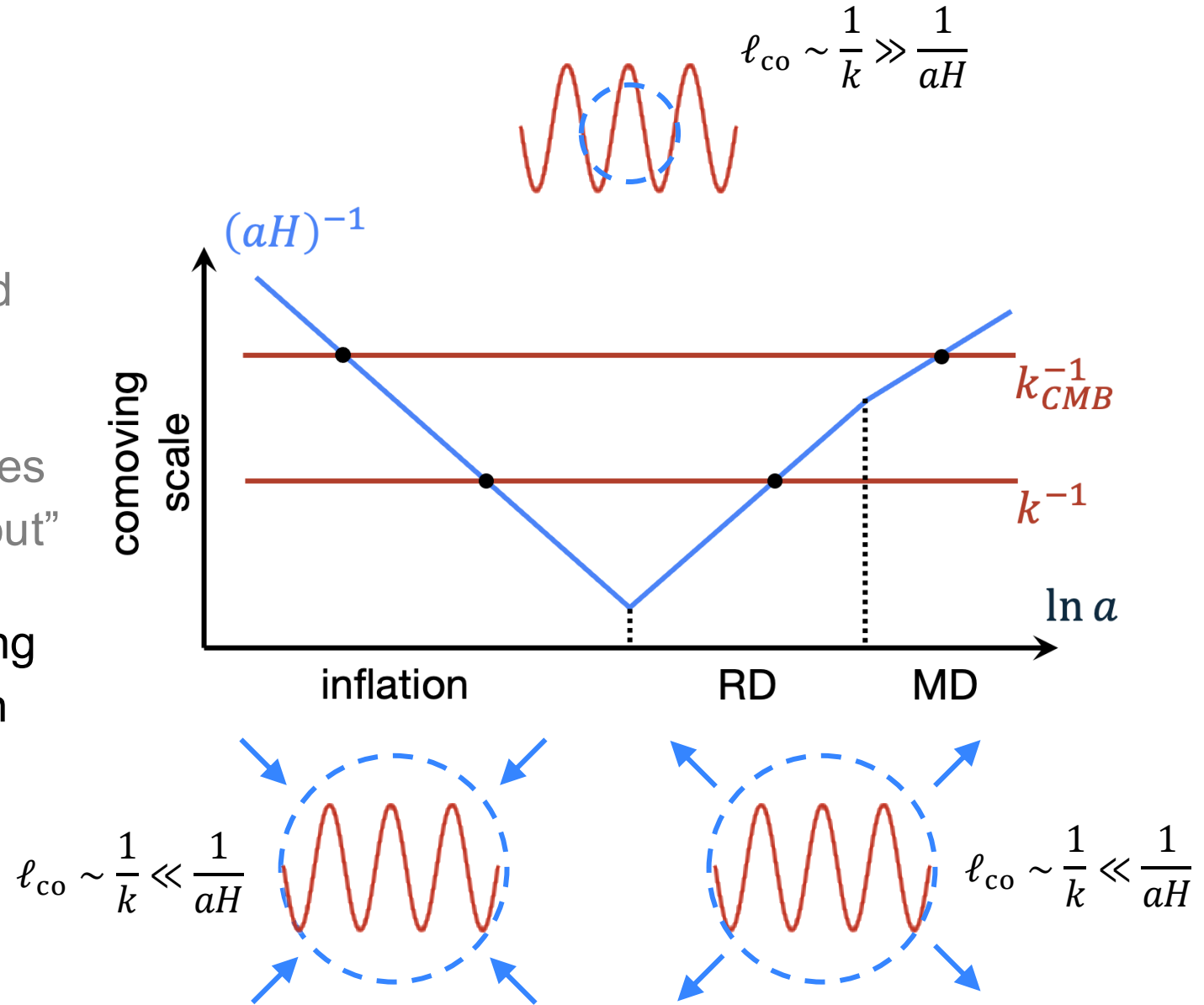
A diagram showing a red sinusoidal wave with a long wavelength. A dashed blue circle is drawn around one full cycle of the wave, indicating its comoving length ℓ_{co} .

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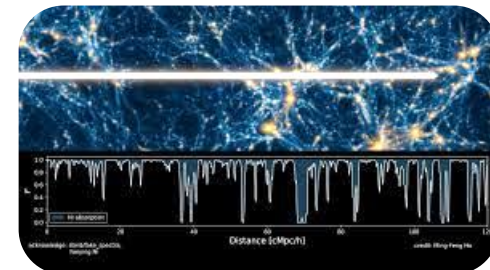
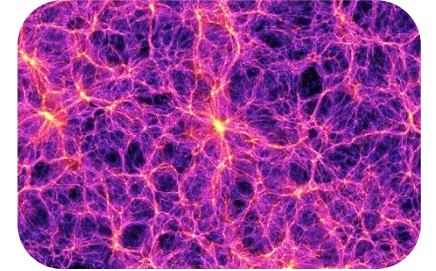
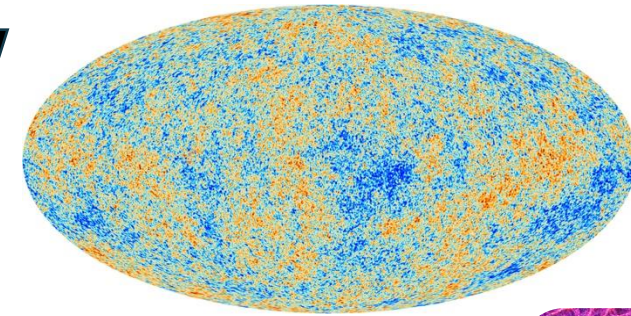
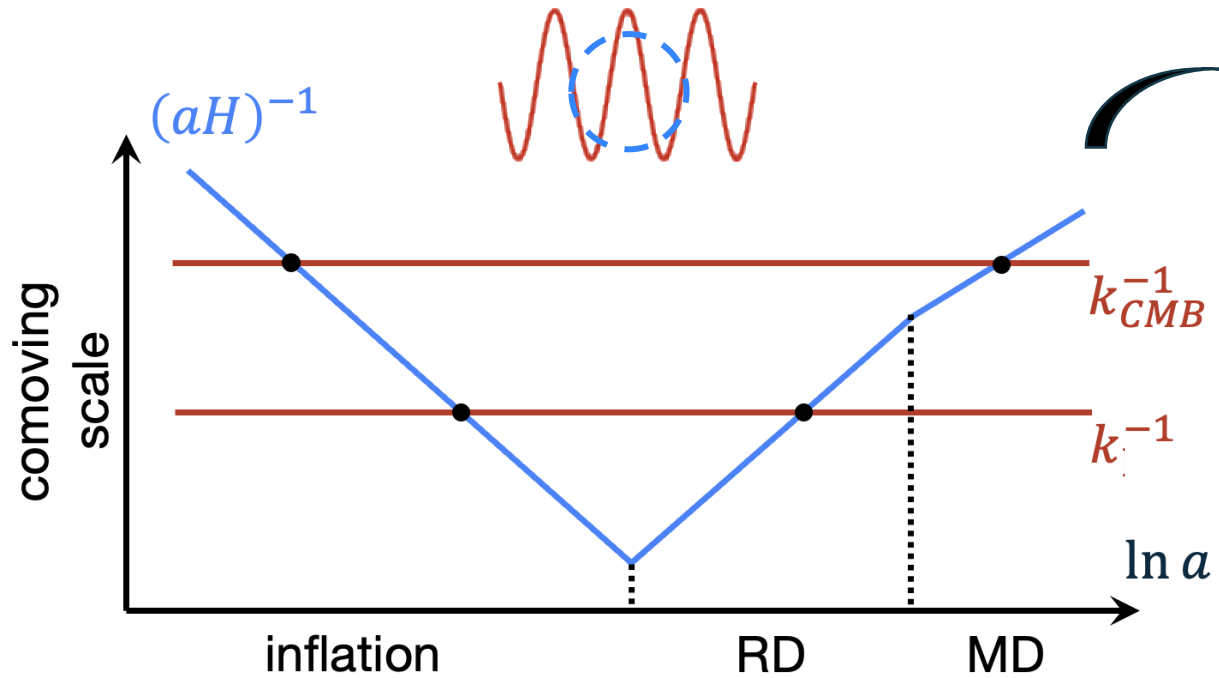
A diagram showing a red sinusoidal wave with a short wavelength. A dashed blue circle is drawn around one full cycle of the wave, indicating its comoving length ℓ_{co} . Four blue arrows point inward from the corners of the dashed circle towards the wave, representing the horizon scale $1/aH$.

Motivation

- Origin of cosmic structure
 - Quantum fluctuations produced during inflation ($\ddot{a} > 0$)
 - Stretched to superhorizon scales ($k \ll aH$), where they “freeze out”
 - Reenter post-inflation, becoming the density perturbations which seed cosmic structure



Motivation



Motivation

what inflation predicts

- Inflationary perturbations = **quantum**, $\Psi = | \text{img} \rangle + | \text{img} \rangle + \dots$
- In practice, treat as **classical** stochastic fields

what we do

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 - Practical: Not guaranteed to capture higher-point statistics
 - Fundamental: Quantum measurement problem

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- **Quantum-to-classical* transition** on super-horizon scales
 - Closed system: Squeezing/suppression of decaying mode
 - Open system: Decoherence

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*We adopt an operational definition of “classical”

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what we do

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$$ds^2 = -dt^2 + a^2(t)e^{2\mathcal{R}(t,\mathbf{x})} d\mathbf{x}^2$$

\mathcal{R} = curvature perturbation

Motivation

Closed System – “Decoherence without Decoherence”

- On **super-horizon** scales during **slow roll**, commutator effectively hidden:

$$\frac{[\hat{\mathcal{R}}, \hat{\pi}]}{\langle\{\hat{\mathcal{R}}, \hat{\pi}\}\rangle} \sim \frac{i\hbar}{a\hbar} \ll 1$$

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- Schrödinger picture: Large squeezing parameter,

$$e^{-2r_k} \ll 1$$

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- Justifies $\langle\hat{\mathcal{O}}\rangle \rightarrow \langle\mathcal{O}\rangle_{\text{stoch}}$

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- Curvature perturbation & conjugate momentum

$$\begin{aligned}\hat{\mathcal{R}}_{\mathbf{k}}(\eta) &= \mathcal{R}_{\mathbf{k}}(\eta)\hat{a}_{\mathbf{k}} + \mathcal{R}_{\mathbf{k}}^*(\eta)\hat{a}_{-\mathbf{k}}^\dagger \\ \hat{\pi}_{\mathbf{k}}(\eta) &= z^2\hat{\mathcal{R}}'_{\mathbf{k}}(\eta)\end{aligned}$$

$$\begin{aligned}[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] &= \delta(\mathbf{k} - \mathbf{k}') \\ z^2 &= 2m_{\text{Pl}}^2 a^2 \epsilon_1\end{aligned}$$

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$$z^2 = 2m_{\text{Pl}}^2 a^2 \epsilon_1$$

- Assume slow roll + Bunch Davies vacuum:

$$\mathcal{R}_{\mathbf{k}}(\eta) = \frac{H}{m_{\text{Pl}}\sqrt{2\epsilon_1}} \frac{(-k\eta)}{\sqrt{2k^3}} \left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}$$

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Closed System – “Decoherence without Decoherence”

- Super-horizon limit $(-k\eta) \ll 1$:

$$\mathcal{R}_k = \mathcal{R}_k^{\text{Re}} + i\mathcal{R}_k^{\text{Im}} \rightarrow \mathcal{D}_k + \mathcal{G}_k$$

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$$\hat{\mathcal{R}}_k = \sqrt{2}(\mathcal{R}_k^{\text{Re}} \hat{x}_k - \mathcal{R}_k^{\text{Im}} \hat{p}_k)$$
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- Approx. commutation:

$$\boxed{[\hat{\mathcal{R}}_{\text{cl}}, \hat{\pi}_{\text{cl}}] \simeq 0}$$

Motivation

Limitations:

- **Applies *only* during slow roll, or at the linear level of description**
 - ↳ Beyond linear order, non-Gaussianities
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 - ↳ Does not reflect fundamental properties of state
- **Squeezed states are very “quantum”!**
 - ↳ Just appears classical for late-time, coarse-grained observables
 - ↳ Paradoxical; same dynamics also produce strong Fourier-space entanglement

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- **Wigner function** $W(x, p)$
- Weyl transform of density operator

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- Necessary condition for “classicality”:

W is everywhere **positive** and normalized to 1

Motivation

Wigner *negativity* reflects quantum interferences

Illustration: $|\psi\rangle = \frac{1}{\sqrt{2}}(|\psi_+\rangle + |\psi_-\rangle)$ with $\psi_{\pm}(x) = \mathcal{N}e^{-(x \mp \bar{x})^2/2\sigma^2}$

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- Density matrix:

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- Wigner:

$$W = W_+ + W_- + W_{\text{int}}$$

$$W_{\pm}(x, p) \sim e^{-(x \mp \bar{x})^2 / \sigma^2} e^{-\sigma^2 p^2}, \quad W_{\text{int}}(x, p) \sim e^{-x^2 / \sigma^2} e^{-\sigma^2 p^2} \cos(2\bar{x}p)$$

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positive

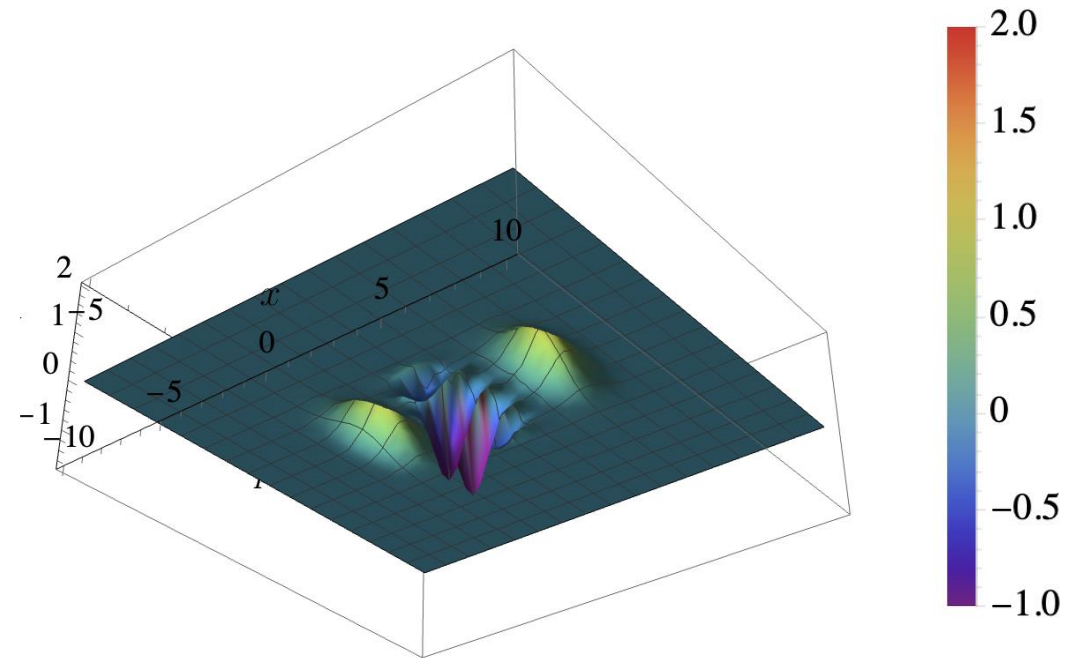
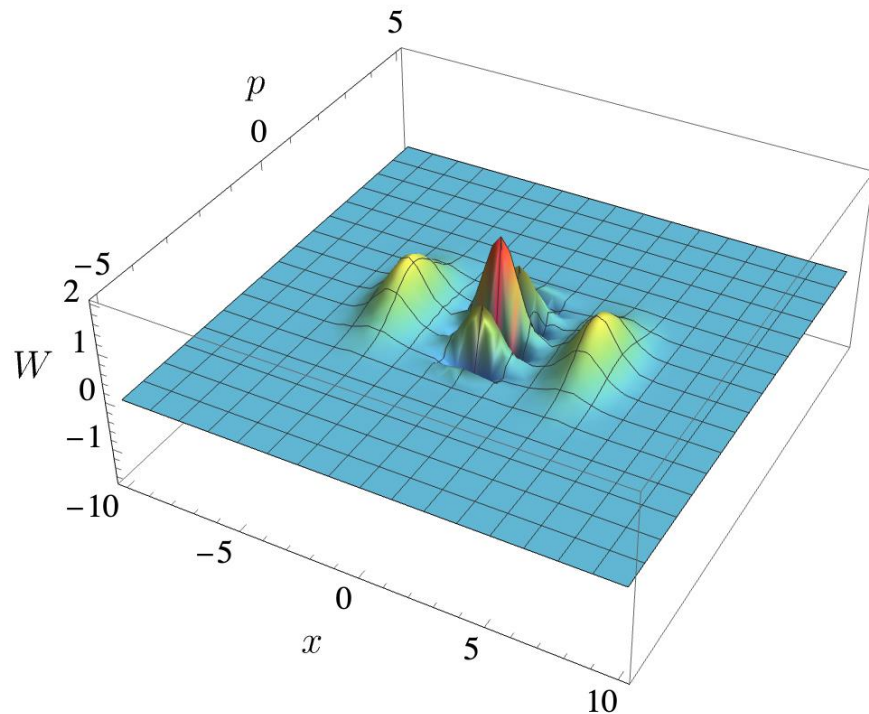
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oscillatory

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- In phase space:



Under Fourier transform, displacement in $x \Rightarrow$ oscillations in p

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Hudson's theorem:

A pure state has a non-negative Wigner function if and only if it is globally Gaussian

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Goal: Compute W for $\mathcal{R}, \pi_{\mathcal{R}}$ in non-slow-roll background

EFT of Inflation

- To construct $W(\mathcal{R}, \pi_{\mathcal{R}})$ that captures any potential **non-Gaussian** features, need to go beyond linear perturbation theory
- In **linear theory**, Hamiltonian is **quadratic** \Rightarrow Wigner function is **Gaussian**
 \hookrightarrow Hudson's theorem
- Effective field theory (EFT) of inflation offers a non-perturbative scheme

Cheung et al., [0709.0293]

EFT of Inflation

- Consider single field inflation with homogeneous background $\bar{\phi}(t)$
- Background defines preferred time-slicing
⇒ spontaneously breaks time-translation invariance
- Inflaton perturbations $\delta\phi(t, \vec{x})$ transform non-trivially w.r.t. time diffs

$$t \rightarrow t + \xi^0(t, \vec{x}), \quad \delta\phi(t, \vec{x}) \rightarrow \delta\phi(t, \vec{x}) + \dot{\bar{\phi}}(t)\xi^0(t, \vec{x})$$

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To construct action:

- 1) Go to unitary gauge, $\delta\phi = 0$ ← all perturbations encoded in metric, $\delta\phi$ is “eaten”
- 2) Write down all ops respecting remaining spatial diffs
- 3) Restore full diff invariance by promoting $\xi^0(x) \rightarrow \chi(x)$

EFT of Inflation

- **Goldstone** $\chi(x)$
 - Restores full diff invariance by non-linearly realizing time translation symmetry
$$t \rightarrow t + \xi^0(x), \quad \chi(x) \rightarrow \chi(x) - \xi^0(x)$$
 - Describes scalar perturbations around FLRW (related to \mathcal{R})

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$$t \rightarrow t + \xi^0(x), \quad \chi(x) \rightarrow \chi(x) - \xi^0(x)$$
 - Describes scalar perturbations around FLRW (related to \mathcal{R})
- Advantage of re-introducing Goldstone
 - At “high energies”, scalar & tensor sectors decouple
 - Mixing b/w χ and metric fluctuations suppressed by powers of ϵ_1 or $1/m_{\text{Pl}}^2$
 - In **decoupling limit**, can study physics of χ in unperturbed background

EFT of Inflation

- Decoupled action for χ :

$$S = -m_{\text{Pl}}^2 \int d^4x a^3(t) \left[3H^2(t + \chi) + 2(1 + \dot{\chi}) \dot{H}(t + \chi) + \dot{H}(t + \chi) \left(\dot{\chi}^2 - \frac{1}{a^2} (\partial\chi)^2 \right) \right]$$

$(t + \chi)$ transforms linearly under diffs



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- To leading order in ϵ_1 :

$$S = m_{\text{Pl}}^2 \int d^4x a^3 H^2 \epsilon_1(t + \chi) \left(\dot{\chi}^2 - \frac{1}{a^2} (\partial\chi)^2 \right)$$



all interactions encoded in χ -dependence of background dynamics

EFT of Inflation

Relating χ to \mathcal{R}

- If \mathcal{R} is conserved, then in the decoupling limit so too is χ , and

$$\mathcal{R} = -H\chi$$

- Otherwise, relationship between \mathcal{R} and χ remains non-linear

$$\mathcal{R} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} (H\chi^n)^{(n-1)}$$

- Nevertheless, χ is no less fundamental than \mathcal{R}

Matching Prescription

- Tracking quantum state of $\chi(x)$ on sub-horizon scales = hard
↳ must solve Schrödinger eq for a wavefunctional

Matching Prescription

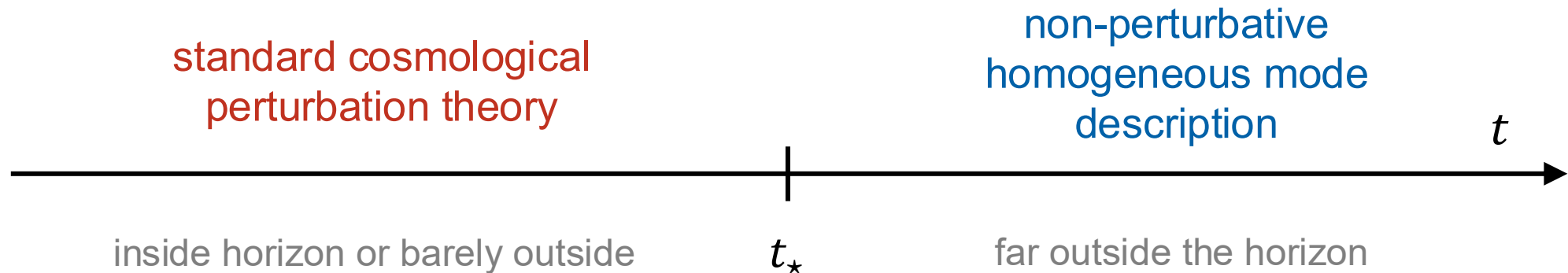
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- Adopt same strategy as δN formalism:
 - On small scales – perturbation theory
 - On large scales – separate universe approach
 - ↳ i.e. focus on **homogeneous mode** $\chi_0(t)$

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- Adopt same strategy as δN formalism:
 - On small scales – perturbation theory
 - On large scales – separate universe approach
 - ↪ i.e. focus on **homogeneous mode** $\chi_0(t)$
- Results only account for non-linearities on large scales
 - Nevertheless, the most relevant ones
 - Inside horizon, perturbations remain close to vacuum Gaussian state
 - Outside horizon, parametric amplification

Matching Prescription

- Introduce coarse-graining parameter $\sigma \lesssim 1$
- For modes $k > \sigma aH$, use **cosmological perturbation theory**
- For modes $k < \sigma aH$, use **homogeneous non-perturbative description**
- Match descriptions at time t_* , defined s.t. $k = \sigma aH|_{t_*}$



Wavefunction

Hubble flow parameters:

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_{i+1} = \frac{1}{H} \frac{d \ln \epsilon_i}{dt}$$

- Constant roll background: $\epsilon_2 = \text{constant}$
 - Then: $\epsilon_1(t) = \bar{\epsilon}_1 e^{\epsilon_2 H t}$
 - Encompasses slow roll and ultra-slow roll

Wavefunction

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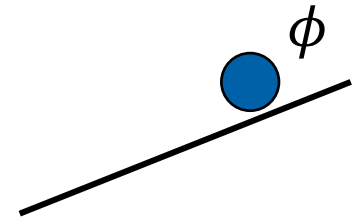
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 - Then: $\epsilon_1(t) = \bar{\epsilon}_1 e^{\epsilon_2 H t}$
 - Encompasses slow roll and ultra-slow roll

- **Slow roll (SR)**

- $|\epsilon_2| \simeq 0$
- Acceleration term in KG eq negligible, $3H\dot{\phi} + V' \simeq 0$

friction term locked
with slope term



Wavefunction

Hubble flow parameters:

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_{i+1} = \frac{1}{H} \frac{d \ln \epsilon_i}{dt}$$

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- **Slow roll (SR)**

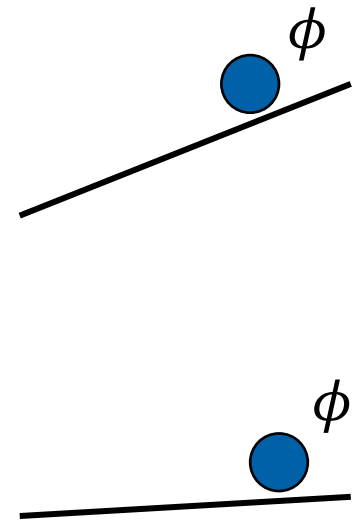
- $|\epsilon_2| \simeq 0$
- Acceleration term in KG eq negligible, $3H\dot{\phi} + V' \simeq 0$

friction term locked
with slope term

- **Ultra-slow roll (USR)**

- $\epsilon_2 \simeq -6$
- Slope term in KG eq negligible, $\ddot{\phi} + 3H\dot{\phi} \simeq 0$

friction term locked
with acceleration



Wavefunction

$$\kappa^2 = 2\mathcal{V}m_{\text{Pl}}^2 H^2 \bar{\epsilon}_1$$

- Lagrangian:

$$L = \frac{1}{2} \kappa^2 e^{(3+\epsilon_2)Ht} e^{\epsilon_2 H \chi_0} \dot{\chi}_0^2$$

- Field transformation:

$$Y = Y_b \left(1 - e^{\epsilon_2 H \chi_0 / 2}\right), \text{ with } Y_b = -\frac{2\kappa}{\epsilon_2 H}$$

- Schrödinger eq:

$$i \frac{\partial \psi_Y}{\partial t} = -\frac{1}{2} e^{-(3+\epsilon_2)Ht} \frac{\partial^2 \psi_Y}{\partial Y^2}, \text{ with } \psi_Y(Y_b) = 0$$

Wavefunction

- Solution:

$$\psi_Y = N(t) \left(e^{-\beta(t)Y^2} - e^{-\beta(t)(Y-2Y_b)^2} \right) \Theta(Y_b - Y)$$

with

$$\dot{\beta} = -2ie^{-(3+\epsilon_2)Ht}\beta^2, \quad \dot{N}/N = -ie^{-(3+\epsilon_2)Ht}\beta$$

- Set $\beta(t_*)$ to match the 2-point function of perturbation theory
- Transform back to original variables (χ_0, π_0)

$$\psi_0(\chi_0) = \sqrt{Y'(\chi_0)} \psi_Y(Y(\chi_0))$$

Wigner Function

- For a pure state:

$$W(\chi_0, \pi_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} du e^{-i\pi_0 u} \psi^* \left(\chi_0 - \frac{u}{2} \right) \psi \left(\chi_0 + \frac{u}{2} \right)$$

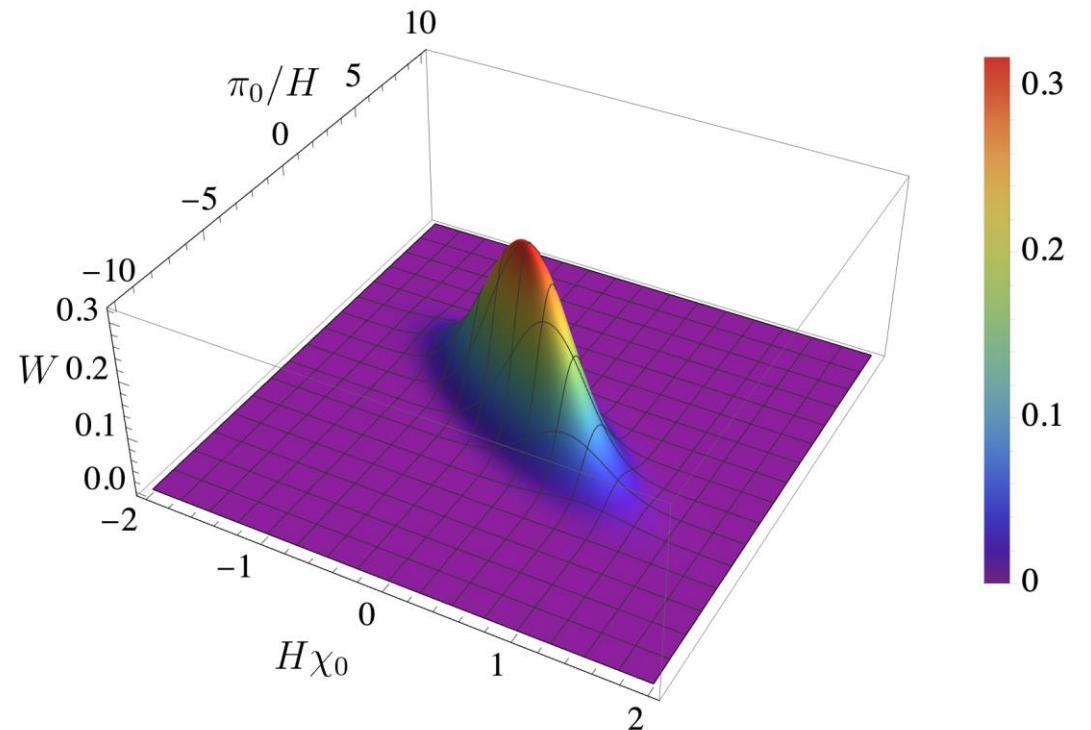
- Recall:

Wigner **negativity** \Rightarrow quantum interference

Wigner Function

Slow Roll:

$$W_{\text{SR}} = \frac{1}{\pi} \exp \left[-2\kappa^2 \beta_R \chi_0^2 - \frac{1}{2\kappa^2 \beta_R} (\pi_0 + 2\kappa^2 \beta_I \chi_0)^2 \right]$$

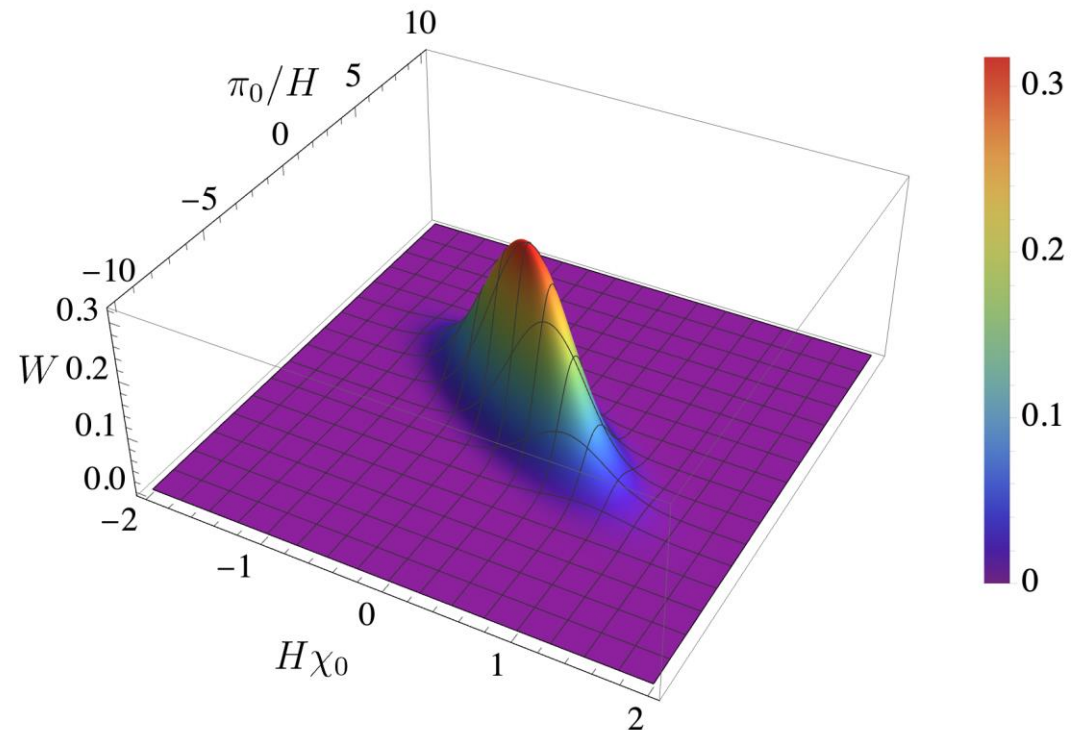


Wigner Function

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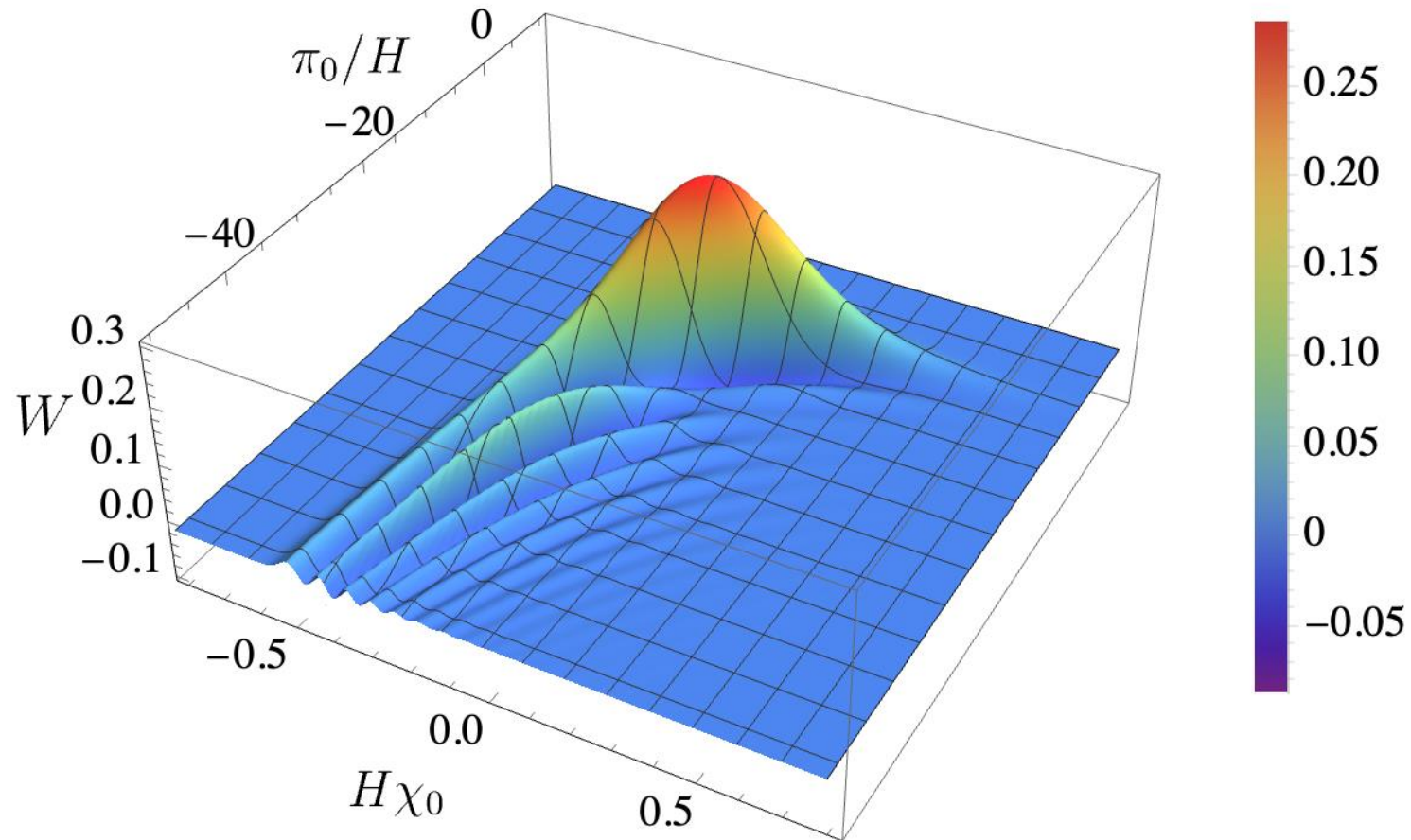
- Multivariate Gaussian
- Minimum uncertainty state:
 $\sigma_{\chi\chi}^2 \sigma_{\pi\pi}^2 - \sigma_{\chi\pi}^2 = 1/4$
- Identical to linear theory prediction



$$(\bar{\mathcal{P}}_{\mathcal{R}}, \sigma) = (0.14, 1)$$

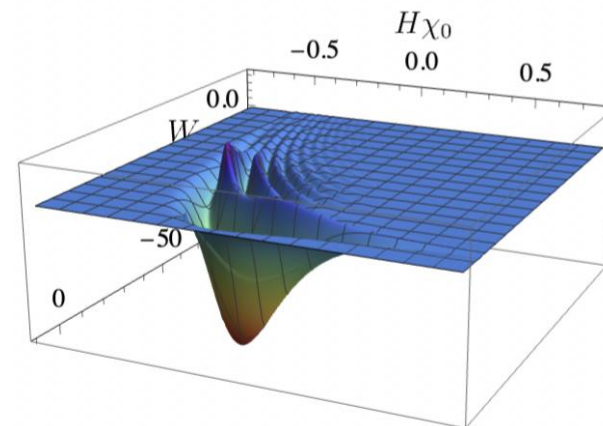
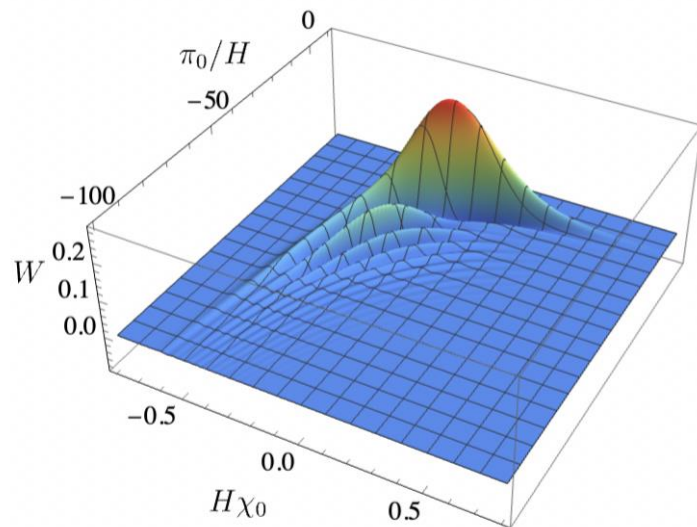
Wigner Function

Ultra-Slow Roll:

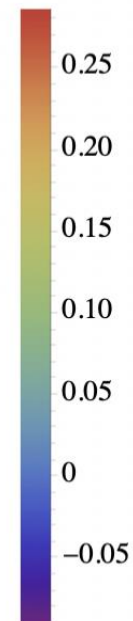
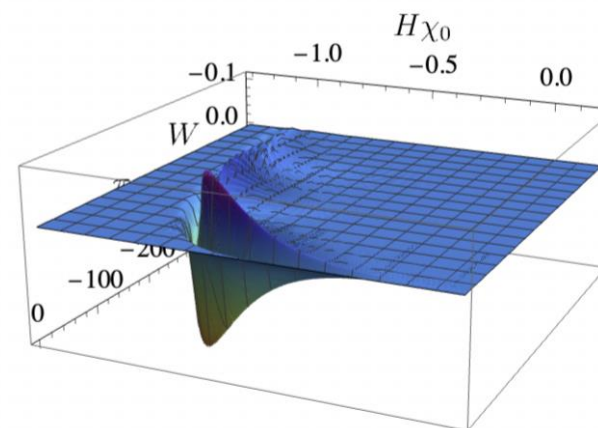
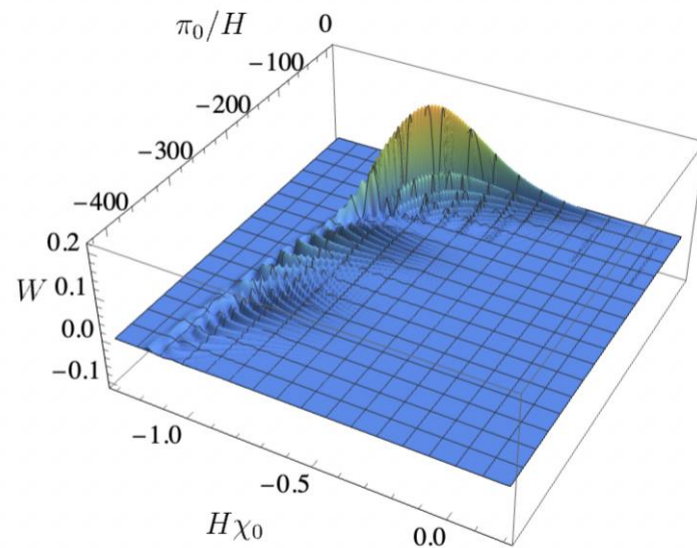


$$(\bar{\mathcal{P}}_{\mathcal{R}}, \sigma) = (0.14, 1)$$

$$\Delta N_{\star} = 0$$



$$\Delta N_{\star} = 0.72$$



Wigner Function

Contributions:

1) Interference with reflected Gaussian

↪ In (Y, π_Y) , similar to cat state $|\psi_{\text{cat}}\rangle \propto |\alpha\rangle + e^{i\theta} |-\alpha\rangle$, but with truncated domain

2) Non-linear transformation

↪ Gaussianity in $(Y, \pi_Y) \Rightarrow$ negativity in (χ_0, π_0) (Hudson's theorem)

Wigner Function

$$x = 3H\chi_0, \quad p = \pi_0/3H$$

$$\mathcal{A} = 2Y_b^2|\beta|, \quad \beta = |\beta|e^{i\phi_\beta}$$

Contributions:

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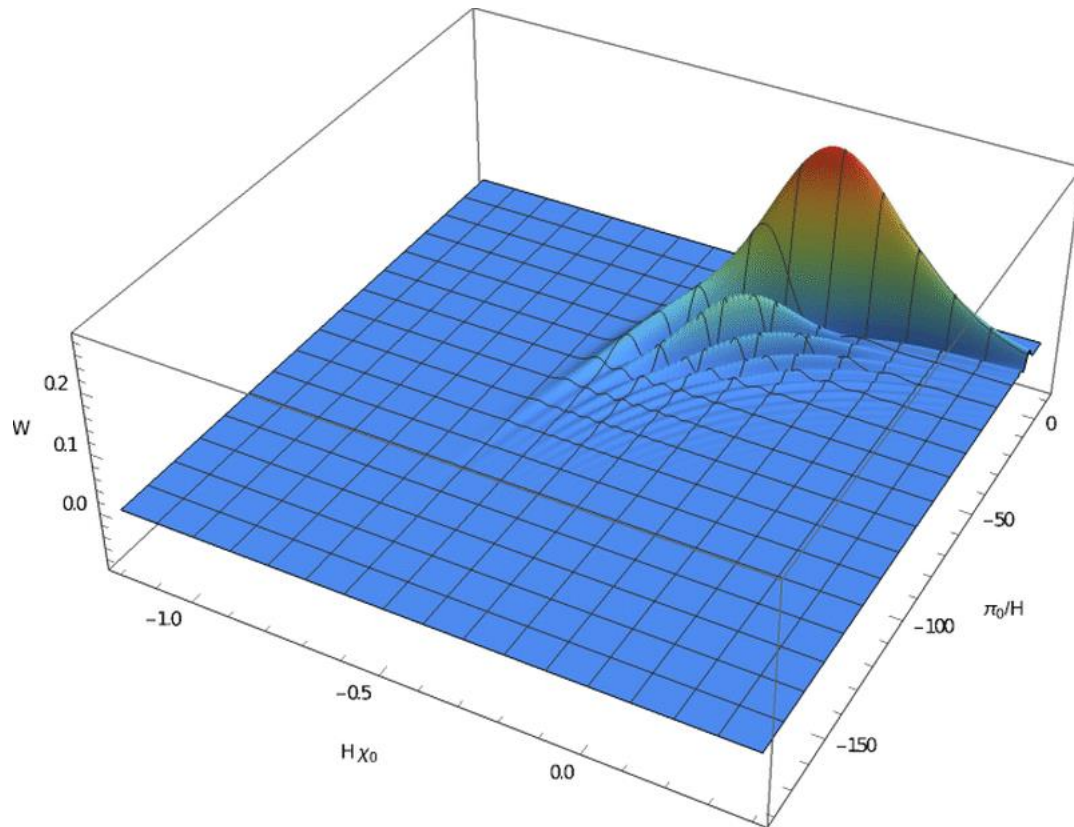
↪ Gaussianity in $(Y, \pi_Y) \Rightarrow$ negativity in (χ_0, π_0) (Hudson's theorem)

Schematically:

$$W_{\text{USR}} = \mathcal{M}e^{-x} \sum_{n=-\infty}^{\infty} \overbrace{[I_{2n}(2\mathcal{A}e^{-x}) - J_{2n}(2\mathcal{A}e^{-x})]}^{\geq 0} \underbrace{e^{i\phi_\beta(n-ip)}}_{e^{i\phi_\beta n} e^{i\phi_\beta p}} \overbrace{K_{n+ip}(\mathcal{A}e^{-2x})}^{\text{dynamical source of negativity}}$$

Wigner Function

Time evolution:



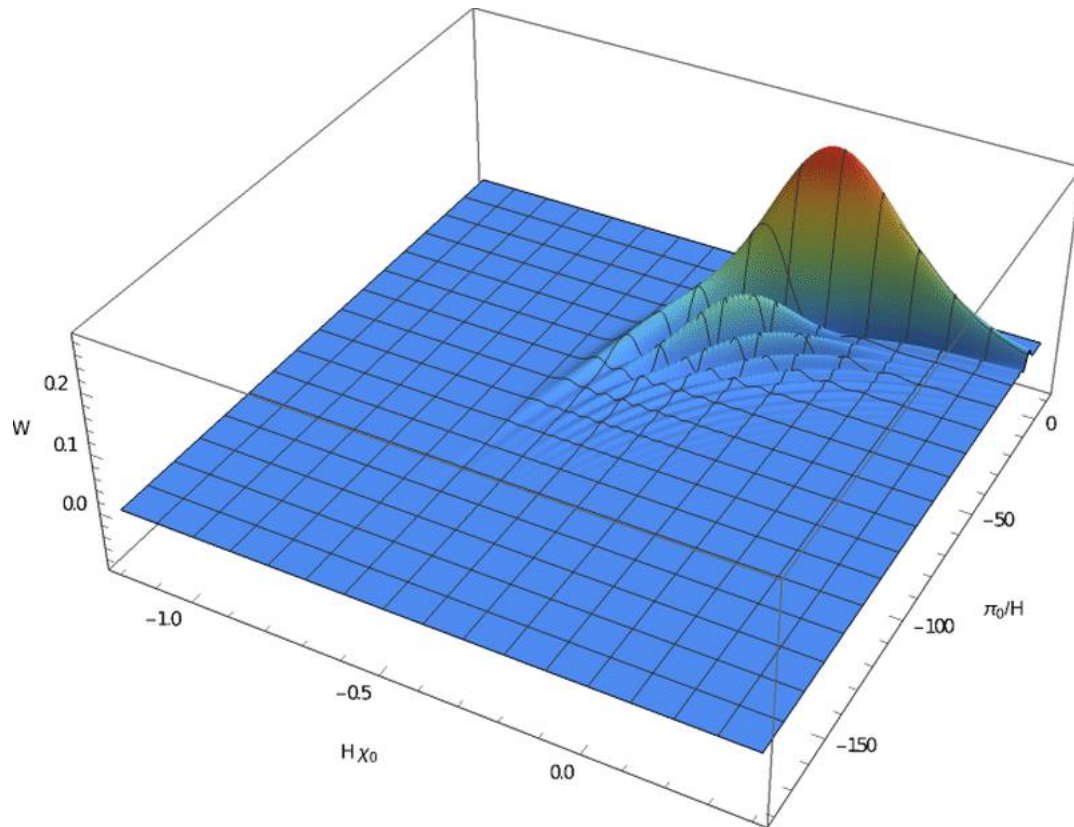
Increasing t leads to:

- Squeezing

$$\Delta N_{\star} = 0 - 2.81$$

Wigner Function

Time evolution:



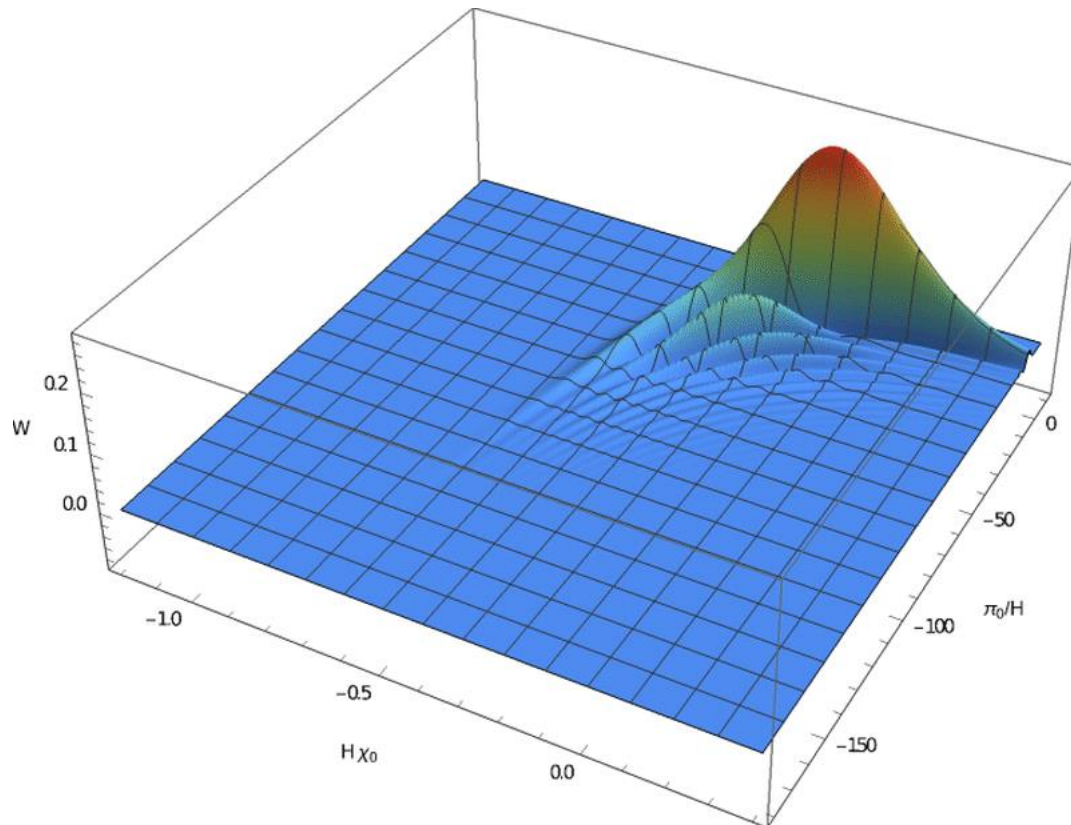
$$\Delta N_{\star} = 0 - 2.81$$

Increasing t leads to:

- Squeezing
- Smaller $\mathcal{A} \propto |\beta|$
- Higher frequency, smaller amplitude oscillations in K_{ν}
- New zeroes \leftrightarrow higher values of n in the sum

Wigner Function

Time evolution:



$$\Delta N_{\star} = 0 - 2.81$$

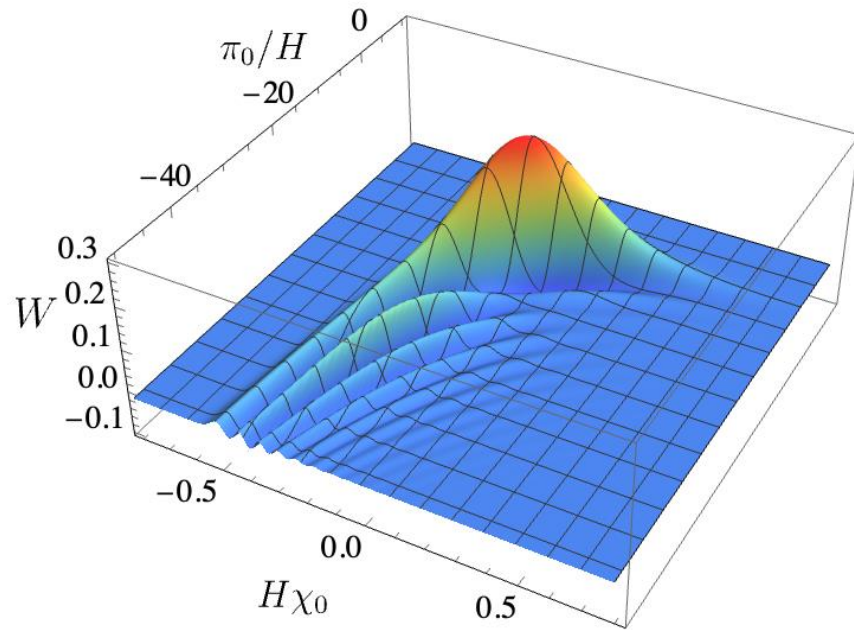
Increasing t leads to:

- Squeezing
- Smaller $\mathcal{A} \propto |\beta|$
- Higher frequency, smaller amplitude oscillations in K_{ν}
- New zeroes \leftrightarrow higher values of n in the sum
- More negative ϕ_{β}
- Oscillations pushed to larger $\pi_0 \propto p$ due to $e^{\phi_{\beta} p}$ factor

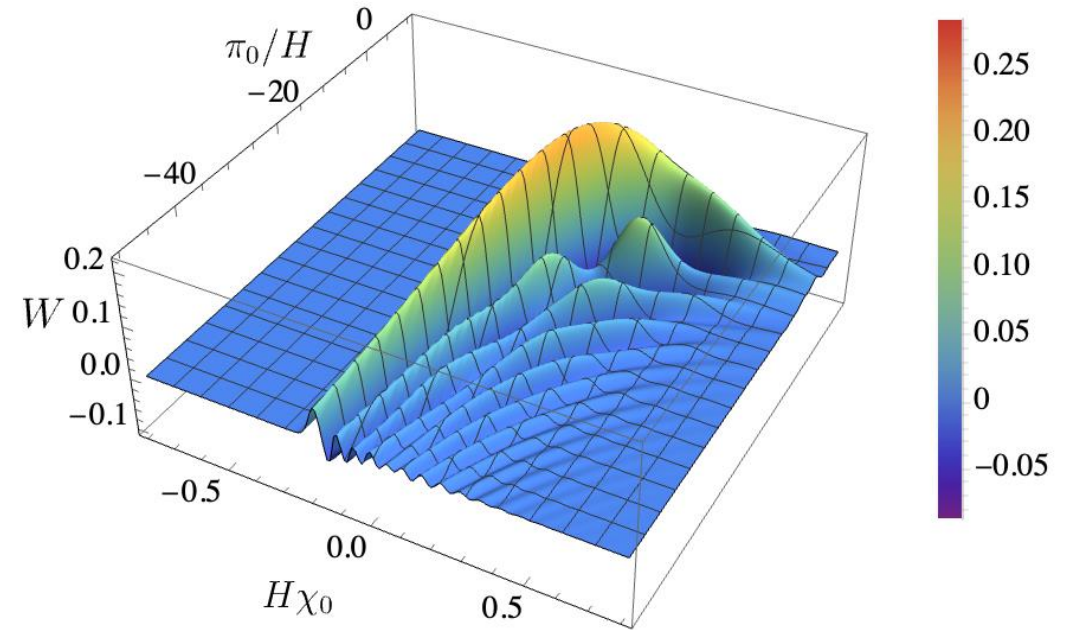
$$\Delta N_* = 0$$

Wigner Function

Changing parameters:



$$(\bar{\mathcal{P}}_{\mathcal{R}}, \sigma) = (0.14, 1)$$

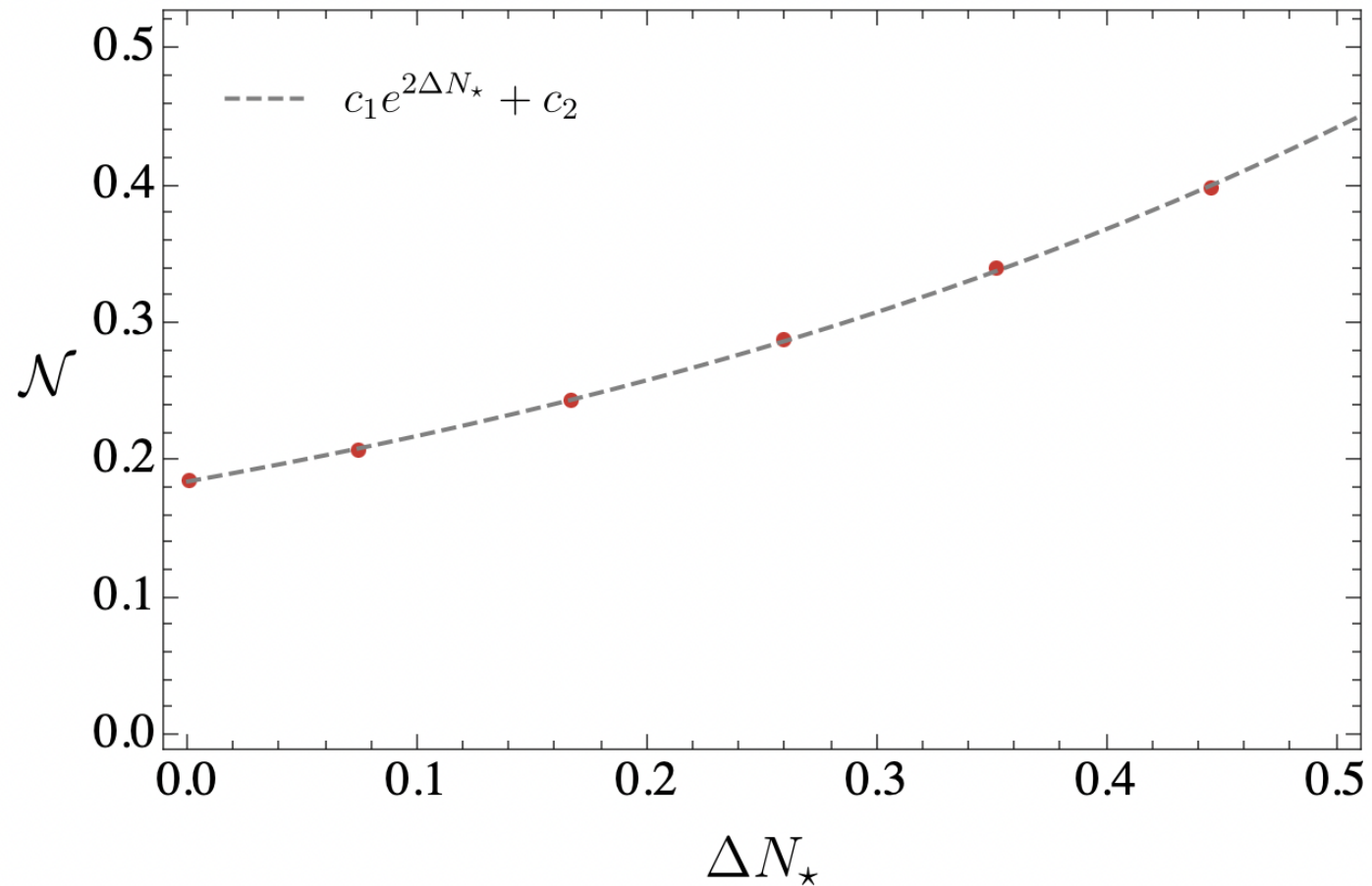


$$(\bar{\mathcal{P}}_{\mathcal{R}}, \sigma) = (0.014, 1)$$

Results

Negativity:

$$\mathcal{N} = \frac{1}{2} \left(\int d\chi_0 d\pi_0 |W(\chi_0, \pi_0)| - 1 \right)$$



$$\mathcal{N} \propto a^2$$

Lessons

1) Quantum effects = important

- W is not globally positive \Rightarrow cannot be interpreted as true prob. distribution
- May still be able to reproduce some observables
- Observables sensitive to Wigner negativity?

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- Squeezed state is still a quantum state
- Just appears classical under certain coarse-grained observables
- Squeezing amplitude can be eliminated by canonical transformation

Lessons

3) USR = qualitatively different

- Following squeezing argument: Classicality happens when one mode dominates over the other; USR equally classical as SR \Rightarrow *Not so!*
- From Wands duality, expect symmetry between SR & USR [Wands, 1998]

Lessons

3) USR = qualitatively different

- Following squeezing argument: Classicality happens when one mode dominates over the other; USR equally classical as SR \Rightarrow *Not so!*
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4) Can we prove cosmic structures have QM origin?

- Many attempts
 - \hookrightarrow Entanglement measures (quantum discord) [Martin & Vennin, 2016]
 - \hookrightarrow Cosmic Bell, Leggett-Garg inequalities [Maldecena, 2015]
 - \hookrightarrow Features in primordial bispectrum [Green & Porto, 2020]
- Hard; entanglement in Fourier space \neq real space
- Wigner negativity suggests should \exists observables sensitive to these interferences

Next steps

- Open-system effects
 - ↳ Noise, dissipation
 - ↳ Does Wigner negativity survive decoherence?
- Gradient corrections
 - ↳ Allow embedding USR in more realistic scenario
- Predictions for observables
 - ↳ Quantities sensitive to Wigner negativity?
 - ↳ Rare processes in tail of distribution?

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Decoherence

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- Long-wavelength sector
 - Super-horizon during inflation
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 - Oscillate rapidly on sub-horizon scales
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 - Super-horizon during inflation
 - Origin of present-day observables (CMB, LSS) \Rightarrow *system*
- Short-wavelength sector
 - Oscillate rapidly on sub-horizon scales
 - Practically unobservable \Rightarrow *environment*
- Cosmological horizon provides a natural system-environment split
- Non-linear nature of GR unavoidably couples the two sectors

Decoherence

- Open system nature \Rightarrow *decoherence*
 - \hookrightarrow Suppression of off-diagonal elements of $\hat{\rho}_{\text{red}}$ in pointer basis
 - \hookrightarrow Erasure of Wigner negativity

Decoherence

- Open system nature \Rightarrow **decoherence**
 - \hookrightarrow Suppression of off-diagonal elements of $\hat{\rho}_{\text{red}}$ in pointer basis
 - \hookrightarrow Erasure of Wigner negativity

Illustration: $|\psi\rangle = \frac{1}{\sqrt{2}}(|\psi_+\rangle + |\psi_-\rangle)$ with $\psi_{\pm}(x) = \mathcal{N}e^{-(x \mp \bar{x})^2/2\sigma^2}$

- Reduced density matrix: $\hat{\rho}_{\text{red}} \simeq \frac{1}{2}(|\psi_+\rangle\langle\psi_+| + |\psi_-\rangle\langle\psi_-|)$
- Wigner: $W_{\text{red}} \simeq \frac{1}{2}(W_1 + W_2)$
- Sum of two Gaussians \Rightarrow everywhere positive

Decoherence

Decoherence will erase some degree of Wigner negativity, but how much?

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- Decoherence **rate** vs negativity generation **rate**?

Decoherence

Decoherence will erase some degree of Wigner negativity, but how much?

- Decoherence **rate** vs negativity generation **rate**?
- Estimates in the literature: Often quantified with purity
 - Not strictly a criterion of classicality
 - Depend on coarse graining
 - No straightforward connection to observables
 - Wigner function \Rightarrow more directly connects to observables

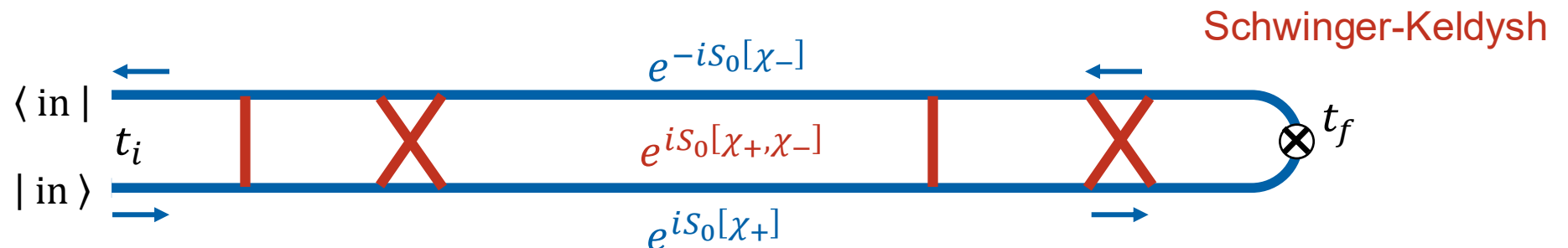
Decoherence

- Investigate using open EFT of inflation Agui Salcedo, Colas, & Pajer [2404.15416]
- Open effective functional:

$$S_{\text{eff}}[\chi_+, \chi_-] = \underbrace{S_0[\chi_+]}_{\text{system}} - \underbrace{S_0[\chi_-]}_{\text{environment}} + S_{\text{IF}}[\chi_+, \chi_-]$$

- Influence functional encodes:

- Dissipation, $S_{\text{IF}} \subset -\gamma(t + \chi_r)\dot{\chi}_r\chi_a + \dots$
 - Noise, $S_{\text{IF}} \subset -i\beta(t + \chi_r)\chi_a^2 + \dots$
- with $\chi_r = \frac{1}{2}(\chi_+ + \chi_-)$, $\chi_a = \chi_+ - \chi_-$

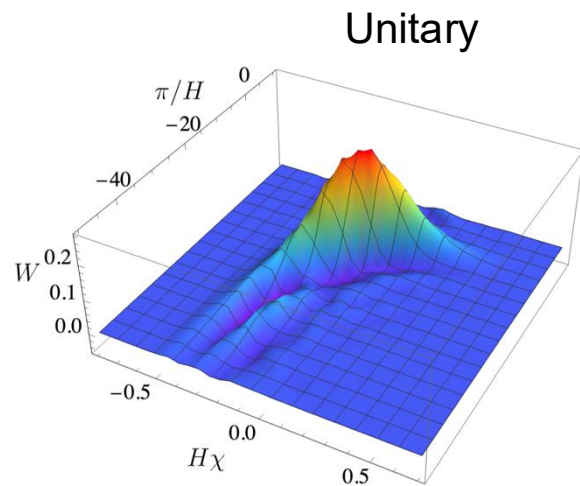


Sneak peak: Decoherence

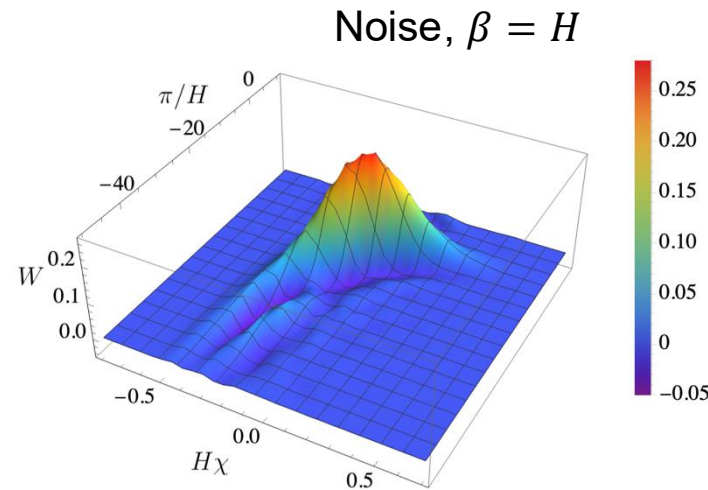
- Master equation: $\partial_t \hat{\rho} = \mathcal{L}_{\text{eff}} \hat{\rho}$

$$\partial_t W(\chi, \pi, t) = -i H_{\text{eff}}(\chi_r = \chi, \chi_a = i\partial_\pi, \pi_r = \pi, \pi_a = -i\partial_\chi, t) W(\chi, \pi, t)$$

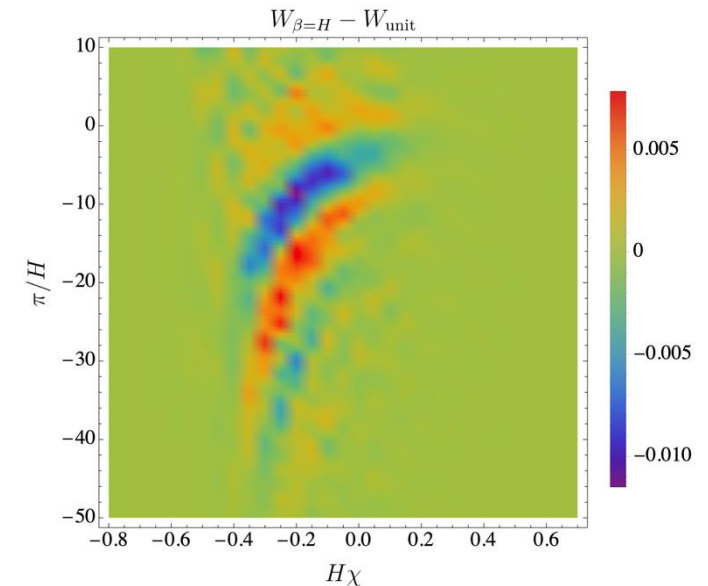
- Solving...



$\mathcal{N} = 0.15$



$\mathcal{N} = 0.13$



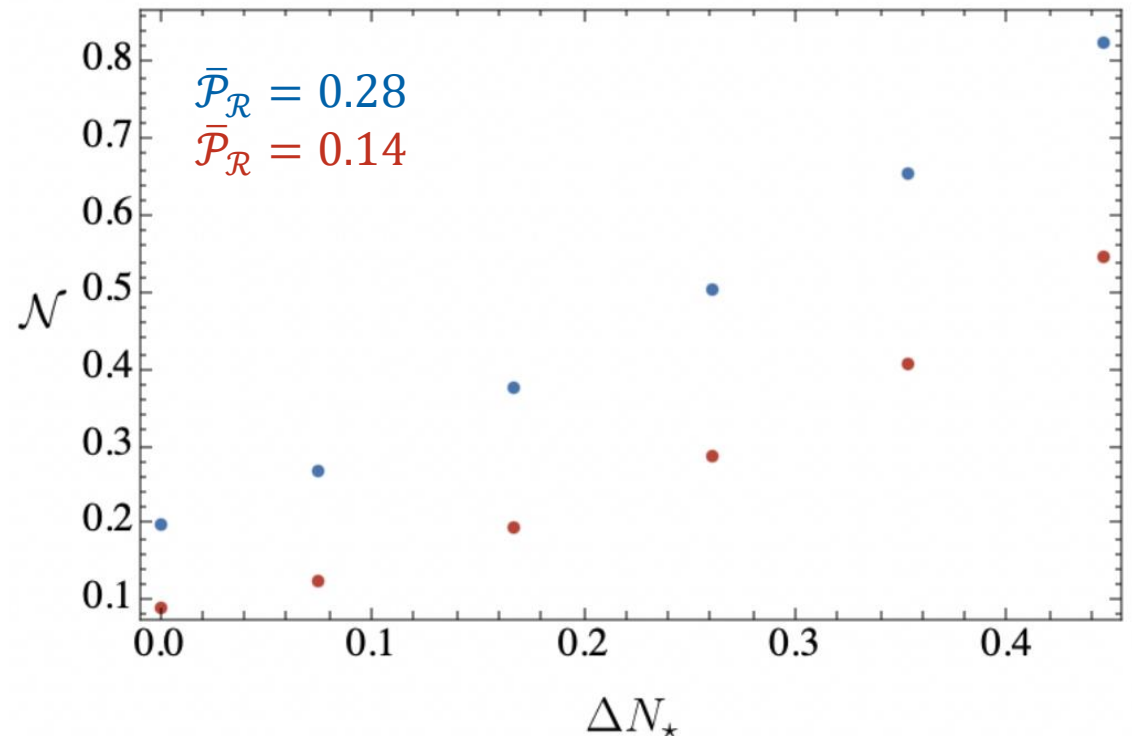
Back-up slides: Questions

Isn't $\sigma = 1$ rather large?

- Yes
- $\sigma = e^{-N_*} = 1$ corresponds to matching at horizon crossing
- In separate universe approach, physical observables are expected to be independent of σ provided it is taken sufficiently small
- We have verified that our results are robust to moderate variations in σ
- Taking $\sigma \ll 1$ underestimates the impact of non-linearities in our setup by decreasing the interval during which they are incorporated

Why smaller $\bar{\mathcal{P}}_{\mathcal{R}} \Rightarrow$ more oscillations?

- Naively, one would expect smaller $\bar{\mathcal{P}}_{\mathcal{R}} \Rightarrow$ more perturbative \Rightarrow less oscillations/negativity
- Why do we see the opposite?
- For fluctuations to be perturbative, need $|\epsilon_2 H \chi_0| < 1$
- Upon restricting to this phase space region, we see that indeed smaller $\bar{\mathcal{P}}_{\mathcal{R}} \Rightarrow$ less oscillations/negativity



What can I measure?

- I don't know!

What can I measure?

- I don't know! (We're working on it :))
- The phase space structure of W provides guidance for where features might appear
- Ex. If oscillatory features are confined to the tails of the distribution, expect first observable consequences to arise in rare events probing these tails
- Recall: $\langle \mathcal{O} \rangle_{\text{stoch}} = \int dx dp \mathcal{O}(x, p) W(x, p)$
- If $\mathcal{O}(x, p)$ is the Weyl transform of $\hat{\mathcal{O}}$, then $\langle \mathcal{O} \rangle_{\text{stoch}} = \langle \hat{\mathcal{O}} \rangle$
- By studying how the Weyl transform of an operator overlaps with regions of Wigner negativity, one can construct targeted observables

Lessons: Extended cut

1) Availability of stochastic description?

To have a stochastic description:

$$\vec{q} = (\chi_0, \pi_0)^T$$

$$\langle \hat{\mathcal{O}} \rangle = \langle \mathcal{O} \rangle_{\text{stoch}}, \quad \text{where} \quad \langle \mathcal{O} \rangle_{\text{stoch}} \equiv \int d^2q \mathcal{O}(\vec{q}) W(\vec{q})$$

Requirements:

- 1) W is everywhere positive and normalized to 1
- 2) W obeys classical e.o.m.
- 3) Above relation is true for relevant observables

regardless of squeezing!



(1), (2) automatically satisfied in linear theory (quadratic H , Gaussian state);

(3) depends on \mathcal{O}

- Satisfied for all 2-pt functions of Hermitian observables
- More generally, iff Weyl transform equals classical counterpart, $\tilde{\mathcal{O}} = \mathcal{O}$

1) Availability of stochastic description?

- We have shown that (1), (2) fail in certain regions of parameter space, where W goes **negative**
- W can then no longer be interpreted as a true probability distribution
- Since it still is positive in certain parts of parameter space, one may still be able to reproduce **some observables** with the stochastic description
- Finding which observables are affected is left to future work

2) Squeezing doesn't imply classicality

Standard lore:

- Squeezing of state/decay of decaying mode on superhorizon scales during SR
- Quantum exp. values \rightarrow stochastic exp. Values
- Approx. commutation $[\hat{\mathcal{R}}_{\text{cl}}, \hat{\pi}_{\text{cl}}] \simeq 0$
- Non-commuting components suppressed as $|\mathcal{R}_k^{\text{Re}} / \mathcal{R}_k^{\text{Im}}| \sim e^{-2r_k}$

Following this argument, one would conclude that “classicality” happens when one mode dominates over the other

\Rightarrow one would reason that USR is equally as “classical” as SR

de Putter&Doré,
[1905.01394]

not so!

2) Squeezing doesn't imply classicality

- As we've just noted, success of a stochastic description for most observables is actually attributable to being in linear theory
- Squeezing alone does **not** imply classicality
 - ↳ Squeezed state is still a pure quantum state
 - ↳ Just appears classical under certain coarse-grained observables
- Squeezing amplitude
 - ↳ Can be **eliminated** via canonical transformation ← no reason it should track intrinsic properties of the state
- Wigner negativity
- Non-linearities are actually **amplified** by squeezing

3) SR & USR are qualitatively different

- It is perhaps surprising that the USR Wigner function should be so dramatically non-Gaussian given the **Wands duality**
- Wands duality
 - Relates spectra of perturbations in cosmological backgrounds with the same Mukhanov-Sasaki mass term z''/z ,

$$\frac{z''}{z} = (aH)^2 \left(2 - \epsilon_1 + \frac{3}{2}\epsilon_2 - \frac{1}{2}\epsilon_1\epsilon_2 + \frac{1}{4}\epsilon_2^2 + \frac{1}{2}\epsilon_2\epsilon_3 \right)$$

Wands,
[gr-qc/9809062]

- For both SR & USR, $z''/z \simeq 2(aH)^2$
- V, ν , etc take the same form in SR & USR due to WD; nevertheless, different $W(\chi_0, \pi_0)$ due to different non-linear mapping
- This duality is a **linear theory** result

4) Cosmological quantum signature?

- Goal: Show that the CMB anisotropies (or other cosmological observables) had **quantum** origin
- *Hard*; few known examples
 - Absence of pole in folded limit of nG correlators [Green & Porto, 2001.09149]
 - Construct observables that can violate Bell inequalities [Maldecena, 1508.01082]
- Usual counterargument: **Squeezing**
- As we've just seen, though, *squeezing is not inherently meaningful*
- Immediate questions:
 - *What observables are sensitive to the Wigner negativity?*
 - *Implications for non-Gaussianities?*
- **Decoherence**

Back-up slides: Relating χ to \mathcal{R}

EFT of Inflation

$$H^{(n)} \equiv \frac{d^n}{dt^n} H$$

Relating χ to \mathcal{R}

- Comoving gauge:

$$\chi(t) = 0, \quad h_{ij}(t) = a^2 e^{2\mathcal{R}} \delta_{ij}$$

- Spatially flat gauge:

$$\chi(\tilde{t}) \neq 0, \quad h_{ij}(\tilde{t}) = a^2 \delta_{ij}$$

- Coord. systems related by time diff:

$$\tilde{t} = t + \xi_0$$

- Equating metrics, solve for:

$$\mathcal{R} = \ln \left(\frac{a(t+\xi_0)}{a(t)} \right) = \int_t^{t+\xi_0} dt' H(t') \quad \Rightarrow \quad \mathcal{R} = \sum_{n=1}^{\infty} \frac{1}{n!} H^{(n-1)}(t) \xi_0^n$$

EFT of Inflation

$$f^{(n)} \equiv \frac{d^n}{dt^n} f$$

Relating χ to \mathcal{R}

- Meanwhile, in going from flat to comoving gauge,

$$\chi(\tilde{t}) \rightarrow \chi(\tilde{t}) + \xi_0 = \chi(t + \xi_0) + \xi_0 = \chi(t) = 0$$

- Formal solution:

$$\xi_0(t) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} (\chi^n)^{(n-1)}$$

- Combining with $\mathcal{R}(\xi_0)$,

$$\mathcal{R} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} (H\chi^n)^{(n-1)}$$

Back-up slides: Matching conditions

Matching Conditions

- Recall: Homogeneous-mode description applicable only for times $t > t_*$
- Set initial conditions at t_* with linear perturbation theory
- Concretely: Compute 2-pt functions with both descriptions; demand equivalence at t_*

Homogeneous-mode description:

$$\langle \hat{Y}^2 \rangle = \frac{1}{4\beta_R}, \quad \langle \hat{\pi}_X^2 \rangle = \frac{|\beta|^2}{\beta_R}, \quad \langle \hat{Y} \hat{\pi}_Y \rangle = \frac{i}{2} - \frac{\beta_I}{2\beta_R}$$

$$\beta = -\frac{i}{2} \frac{\langle \hat{Y} \hat{\pi}_Y \rangle}{\langle \hat{Y}^2 \rangle}$$

Matching Conditions

Linear perturbation theory:

- At linear order:

$$Y = \kappa \chi_0, \quad \pi_Y = \pi_0 / \kappa$$

- At matching, identify:

$$\langle \hat{\chi}_0^2 \rangle \rightarrow \mathcal{P}_{\chi\chi}(t_*, k), \quad \langle \hat{\chi}_0 \hat{\pi}_0 \rangle \rightarrow \mathcal{V} \mathcal{P}_{\chi\pi}(t_*, k)$$

- Also:

$$\mathcal{P}_{\chi\chi}(k) = \frac{1}{H^2} \mathcal{P}_{\mathcal{R}\mathcal{R}}(k), \quad \mathcal{P}_{\chi\pi}(k) = \frac{\kappa^2}{\mathcal{V} H^2} e^{-(3+\epsilon_2)Ht} \mathcal{P}_{\mathcal{R}\dot{\mathcal{R}}}(k)$$

$$\Rightarrow \beta_* = -\frac{i \mathcal{P}_{\mathcal{R}\dot{\mathcal{R}}}(t_*, k)}{2 \mathcal{P}_{\mathcal{R}\mathcal{R}}(t_*, k)} e^{(3+\epsilon_2)Ht_*}$$

Matching Conditions

$$\nu = |3 + \epsilon_2|/2$$

- For constant ϵ_2 :

$$\mathcal{R}_k(\eta) = \sqrt{\frac{H^2}{2m_{pl}^2\epsilon_1}} \frac{(-k\eta)^{3/2}}{\sqrt{2k^3}} \left(A_k \sqrt{\frac{\pi}{2}} H_\nu^{(1)}(-k\eta) + B_k \sqrt{\frac{\pi}{2}} H_\nu^{(2)}(-k\eta) \right)$$

- Bogoliubov coefficients:

- Presume SR \rightarrow CR at some $\eta_t < \eta_*$
 - While perturbations begin in Bunch Davies vacuum during initial SR phase, transition may **excite** the state
 - From hom. mode perspective, “initial” conditions look like excited vacuum state
 - For modes that were deeply subhorizon at η_t , recover BD
- $\Rightarrow (A_k, B_k) = (1, 0)$

$$(A_k, B_k) = (1, 0)$$

$$B_k \neq 0$$

Briaud et al.,
[2501.14681]

Matching Conditions

- In general:

$$\beta_{\star} = \frac{iH}{2} \left[\left(\sigma \frac{A_k^* H_{\nu-1}^{(2)}(\sigma) + B_k^* H_{\nu-1}^{(1)}(\sigma)}{A_k^* H_{\nu}^{(2)}(\sigma) + B_k^* H_{\nu}^{(1)}(\sigma)} \right) + \left(\frac{3 + \epsilon_2}{2} - \nu \right) \right] e^{(3+\epsilon_2)Ht_{\star}}$$

- For SR and USR $\nu = 3/2$, and further taking $(A_k, B_k) = (1, 0)$:

$$\beta_{\star} = \frac{iH}{2} \left(\frac{\sigma^2}{1 + i\sigma} + \frac{\epsilon_2}{2} \right) e^{(3+\epsilon_2)Ht_{\star}}$$

$$\beta_{\star}^R \sim \sigma^3$$

$$\beta_{\star}^I \sim \text{constant} + \sigma^2$$

Back-up slides: Wavefunction

Wavefunction

$$Y_b = -\frac{2\kappa}{\epsilon_2 H}$$

- Constant roll ($\epsilon_2 = \text{constant}$):

$$\psi_0(\chi_0) = \left(\frac{2\kappa^2 \beta_R}{\pi}\right)^{1/4} \left(1 - \exp\left[2\frac{|\beta|^2}{\beta_R} Y_b^2\right]\right)^{-1/2} e^{\epsilon_2 H \chi_0 / 4} \\ \times \left(\exp\left[-\beta Y_b^2 \left(\exp\left[\frac{1}{2}\epsilon_2 H \chi_0\right] - 1\right)^2\right] - \exp\left[-\beta Y_b^2 \left(\exp\left[\frac{1}{2}\epsilon_2 H \chi_0\right] + 1\right)^2\right]\right)$$

Wavefunction

$$Y_b = -\frac{2\kappa}{\epsilon_2 H}$$

- Constant roll ($\epsilon_2 = \text{constant}$):

$$\psi_0(\chi_0) = \left(\frac{2\kappa^2 \beta_R}{\pi}\right)^{1/4} \left(1 - \exp\left[2\frac{|\beta|^2}{\beta_R} Y_b^2\right]\right)^{-1/2} e^{\epsilon_2 H \chi_0 / 4} \\ \times \left(\exp\left[-\beta Y_b^2 \left(\exp\left[\frac{1}{2}\epsilon_2 H \chi_0\right] - 1\right)^2\right] - \exp\left[-\beta Y_b^2 \left(\exp\left[\frac{1}{2}\epsilon_2 H \chi_0\right] + 1\right)^2\right]\right)$$

- Slow roll ($\epsilon_2 = 0$):

$$\psi_0^{\text{SR}}(\chi_0) = \left(\frac{2\kappa^2 \beta_R}{\pi}\right)^{1/4} \exp[-\beta \kappa^2 \chi_0^2]$$