Simulating lattice gauge theories with hybrid qubit-qumode quantum computers

Open Quantum Systems: Dissipation and Decoherence from Subatomic to Cosmic Scales
Mainz, Germany

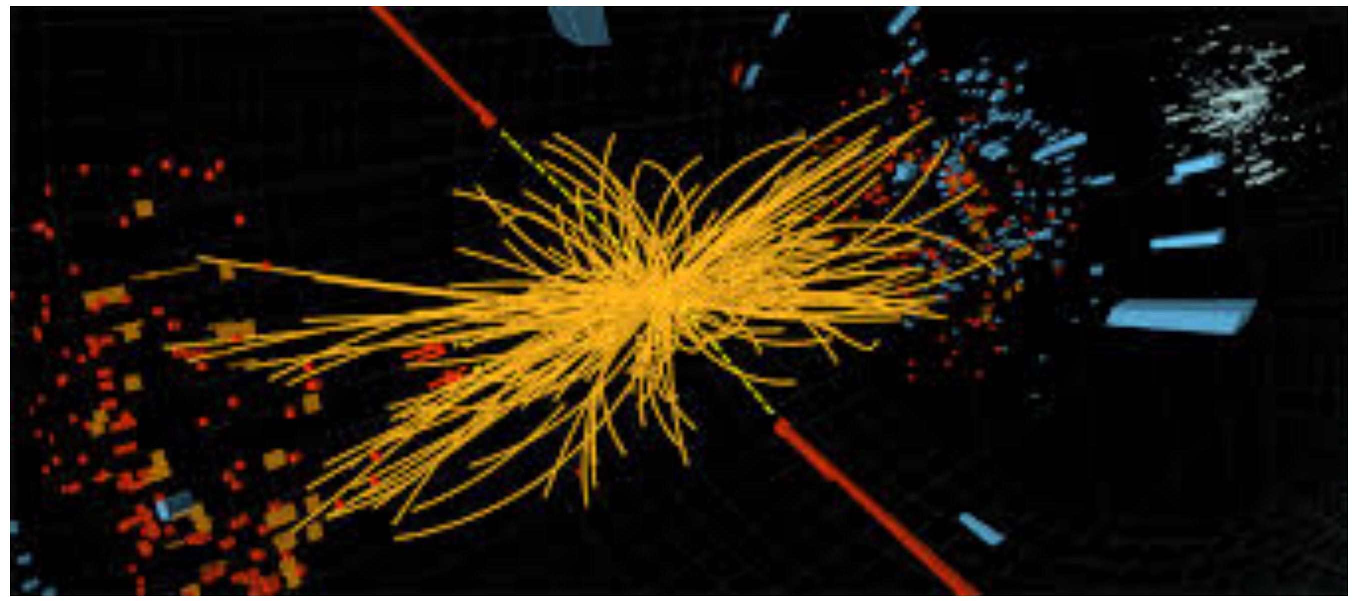
Tommaso Rainaldi - April 13, 2026

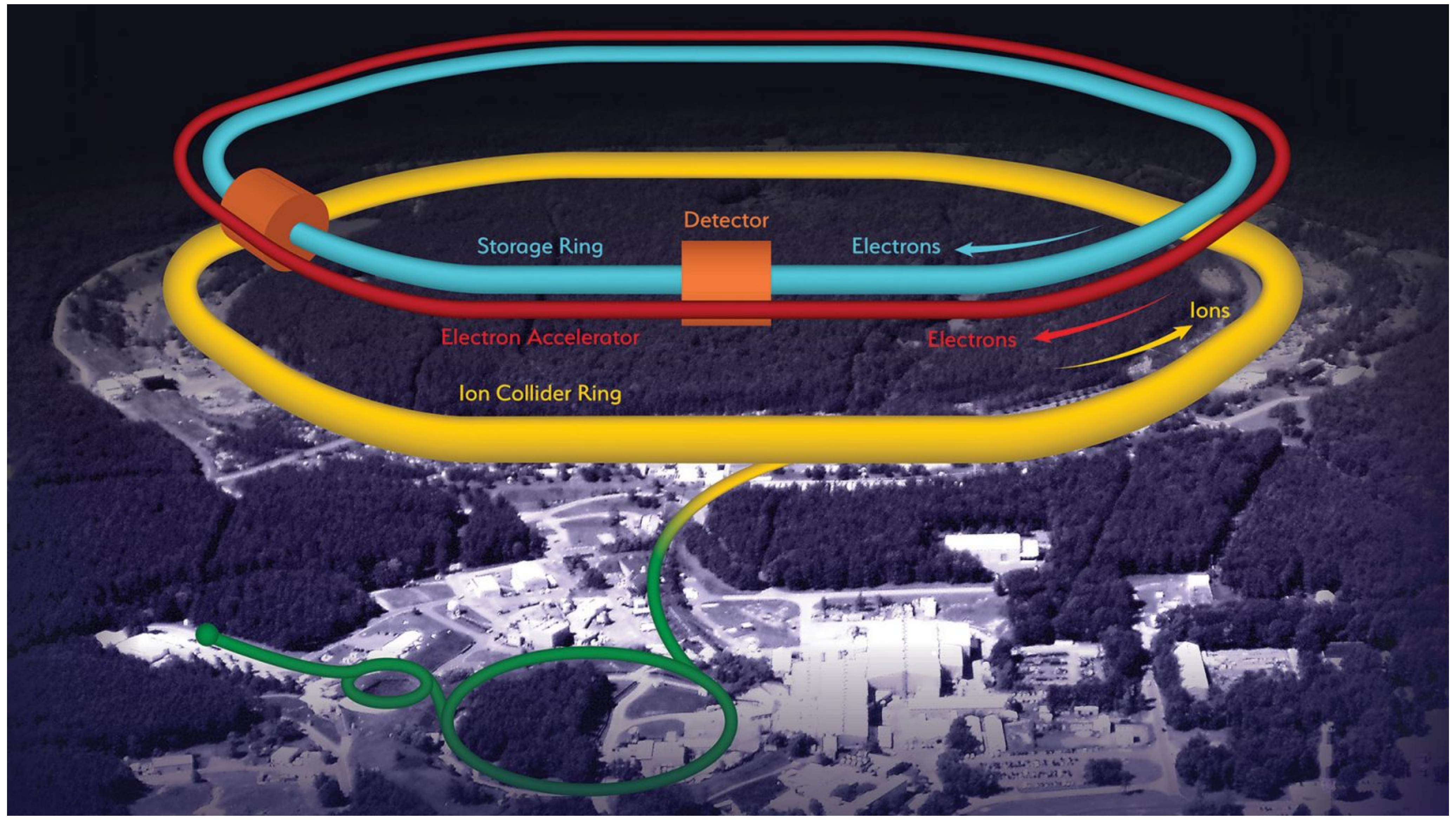
Stony Brook University

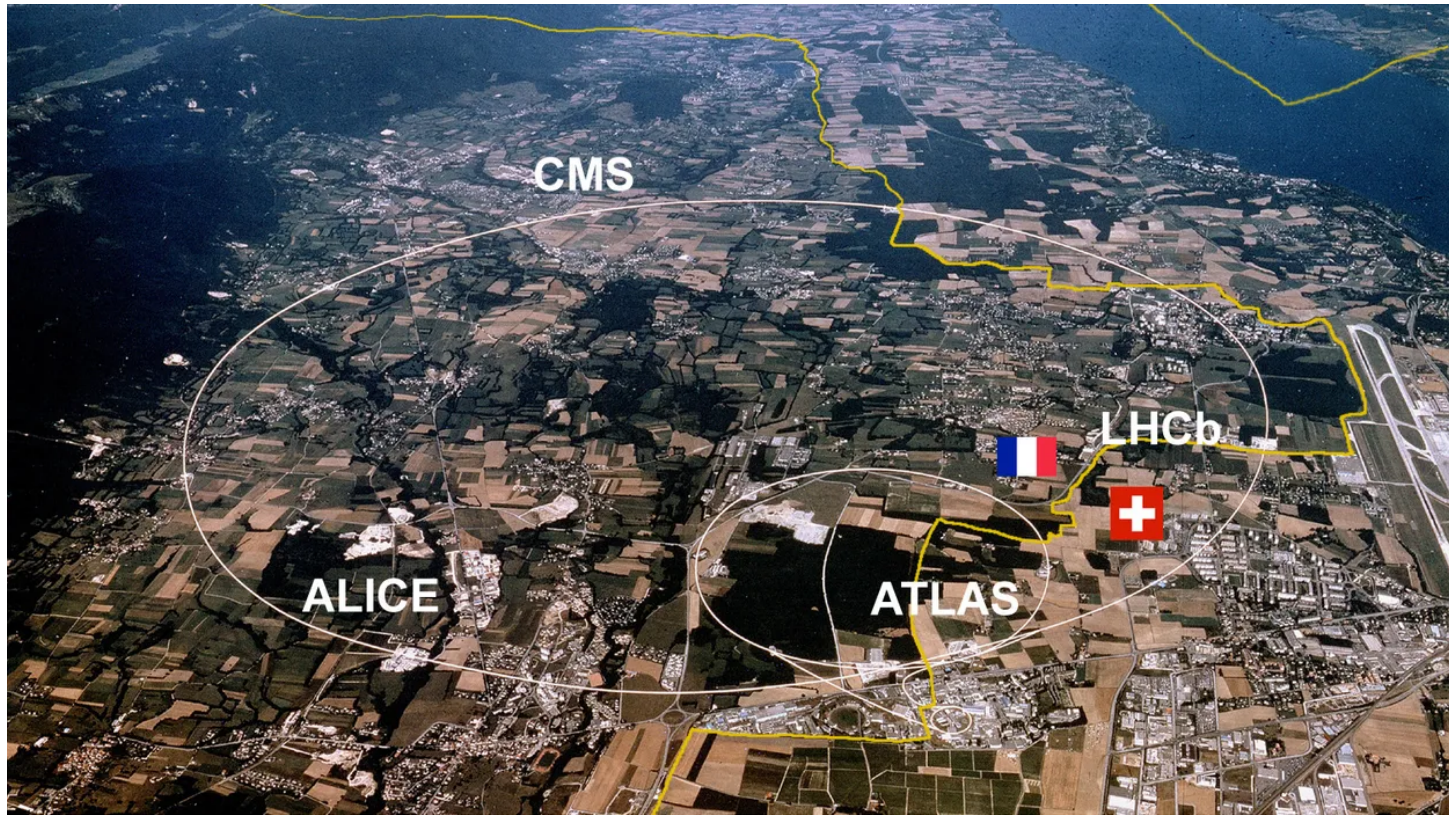
The big picture

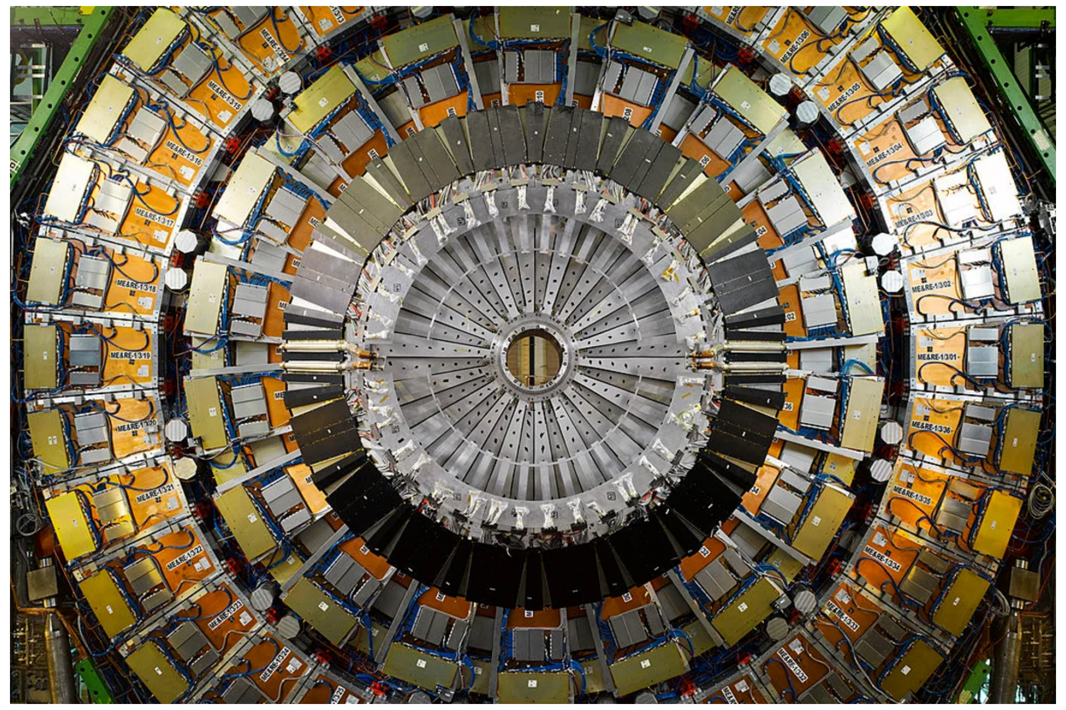
CERN

Large Hadron Collider







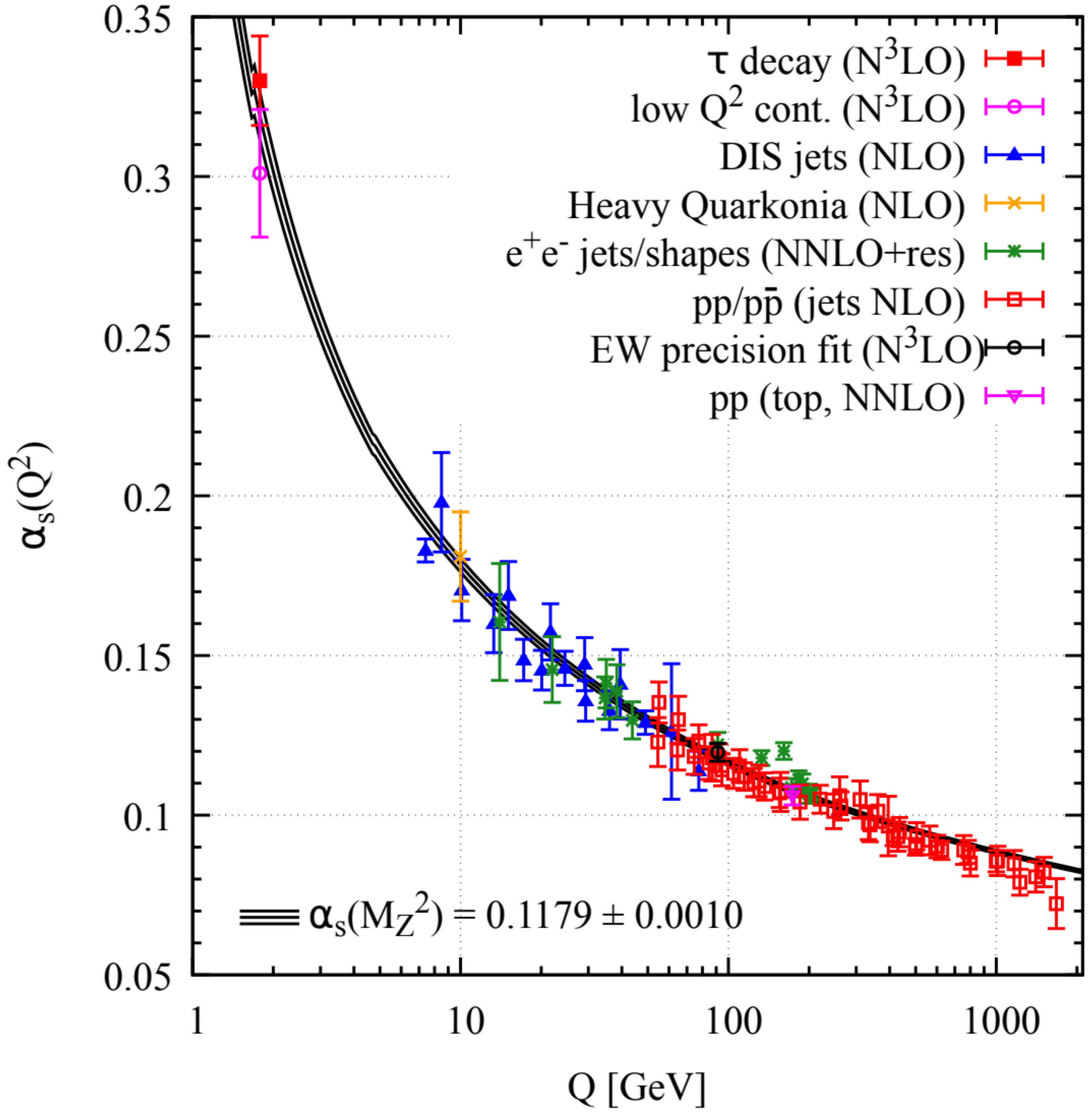


Brookhaven National Lab

Electron Ion Collider

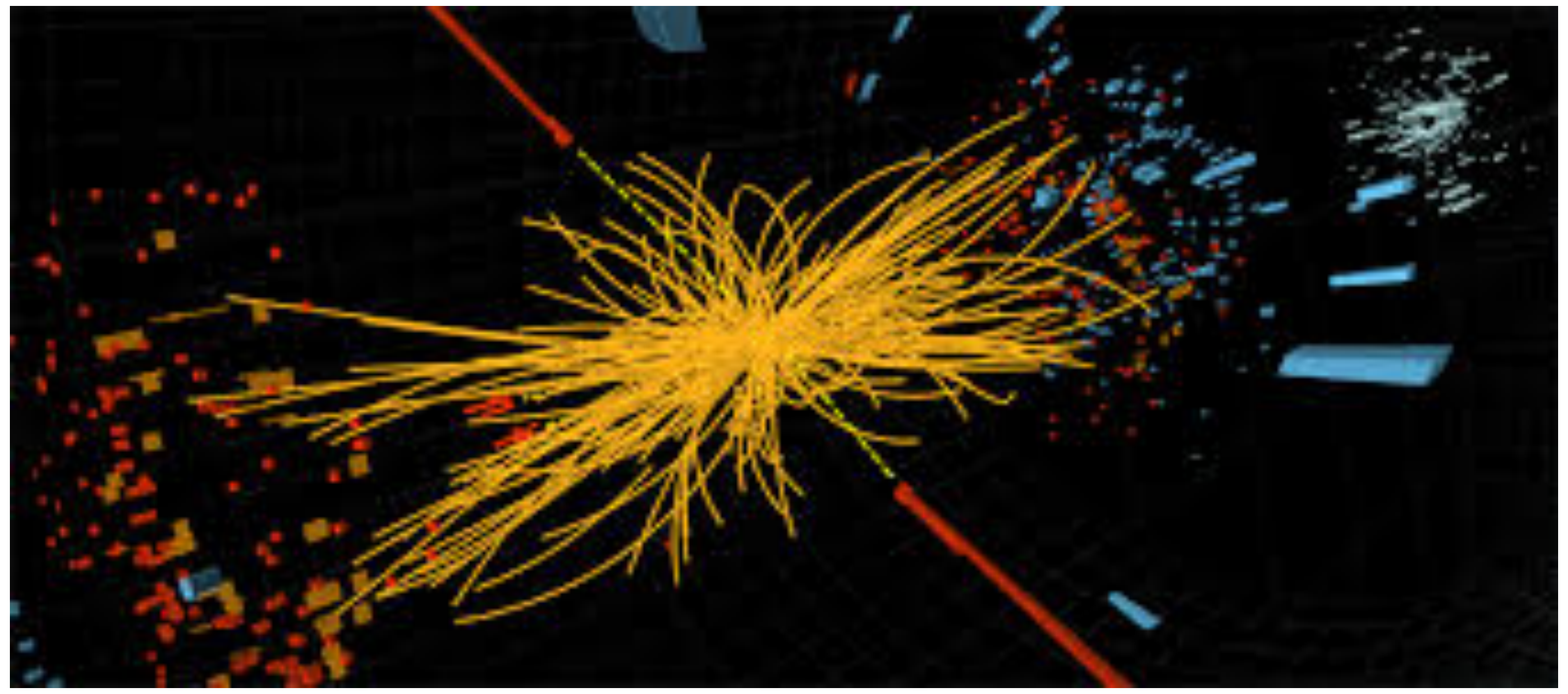
Non-perturbative physics

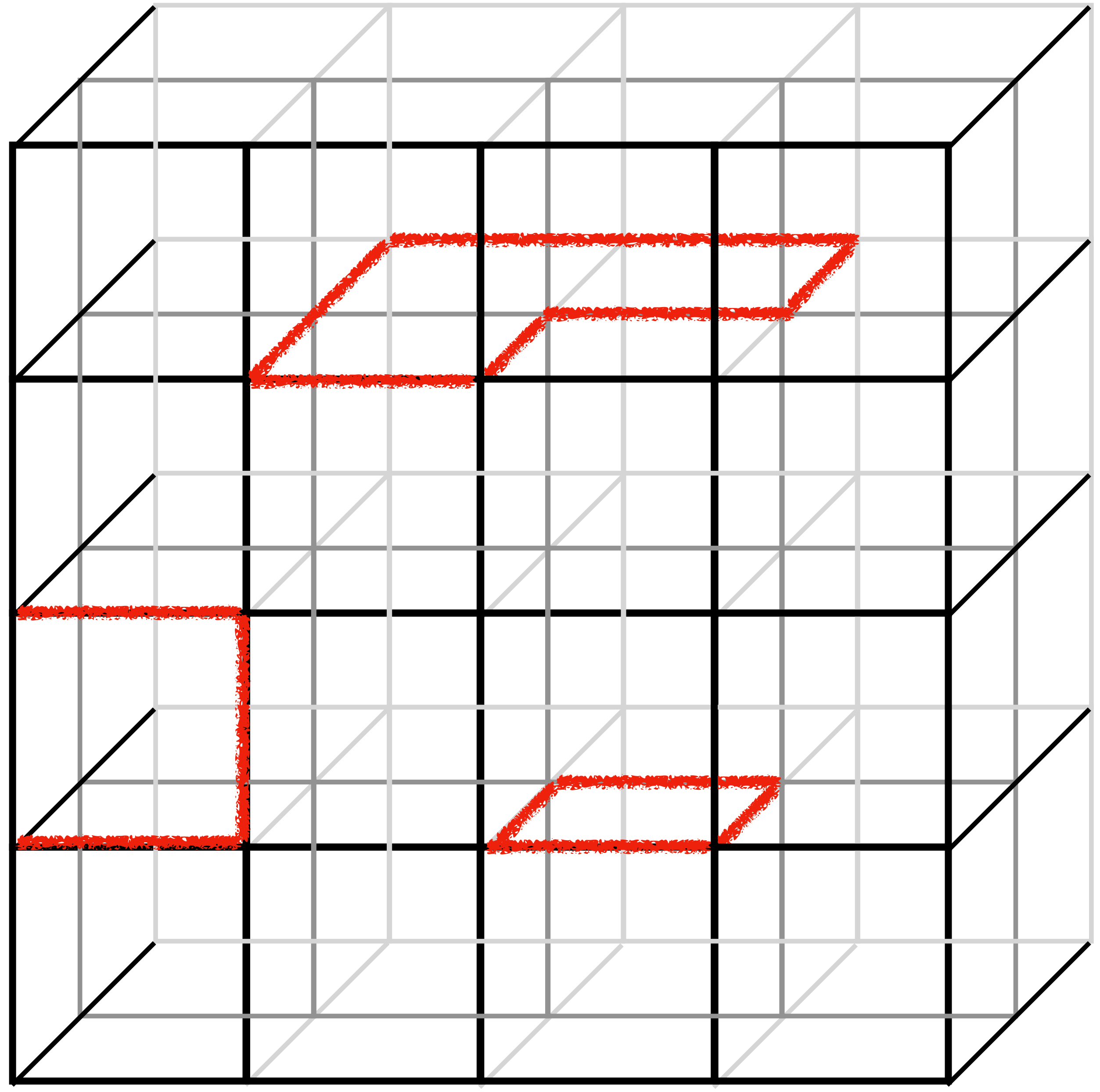
- Simulations of scattering experiments
- Nonpertubative physics and dynamics
- Lattice: Euclidean (path integral) and Hamiltonian



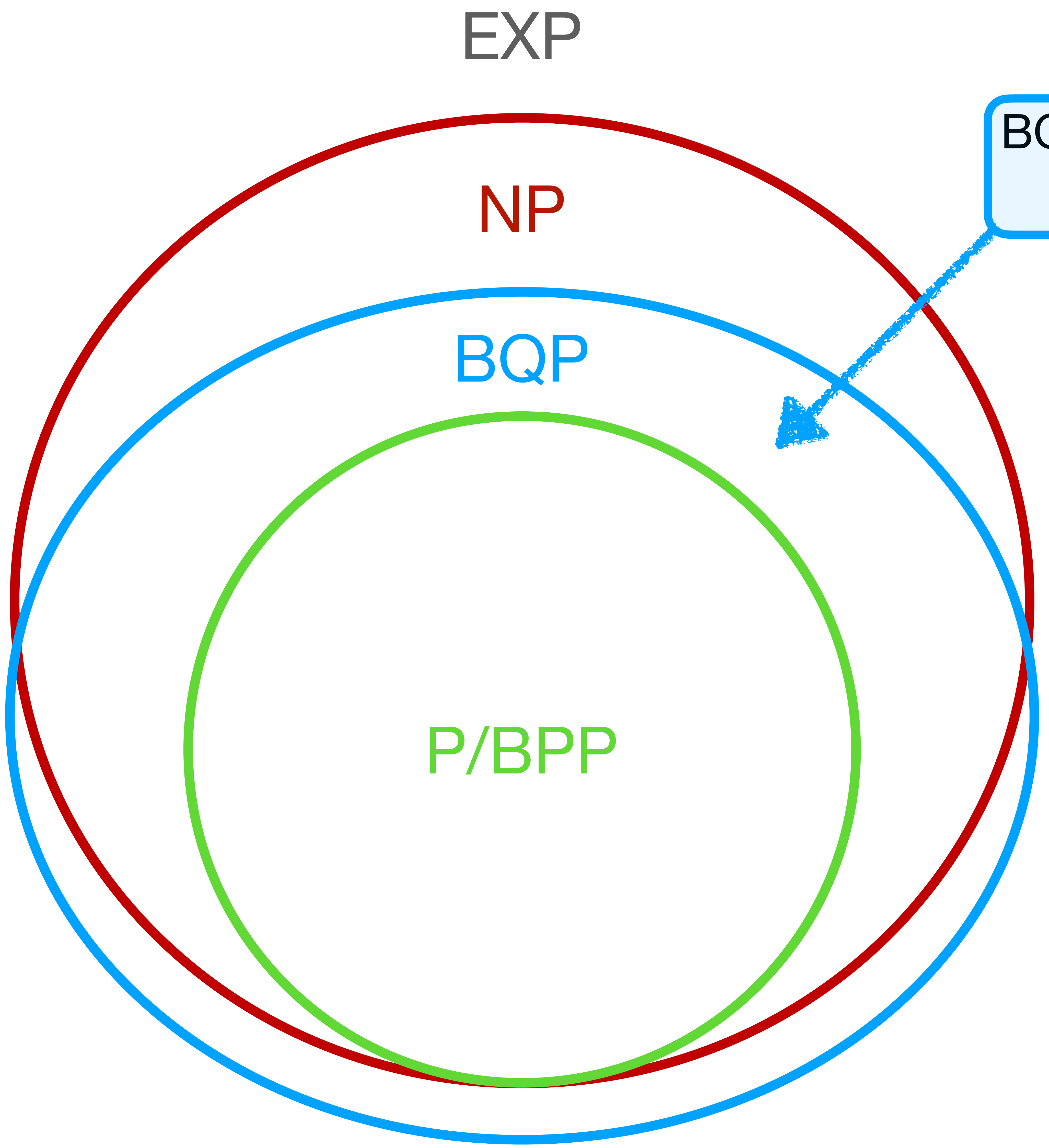
PDG

Wilson; Kogut, Susskind `70 s





How (computationally) difficult are QCD simulations?



BQP-completeness of scattering in scalar quantum field theory

Jordan, Krovi, Lee, Preskill `12-`17 (in 2+1 D)

What about QCD?

Verifiable in exponential time

Verifiable in polynomial time

Eg: Sudoku

Solvable quantumly in polynomial time

Eg: Shor's algorithm for factoring large numbers

Solvable in polynomial time

Eg: Multiplication of an $N \times N$ matrix is $(O)(N^3)$

Scattering process

Scalar field theory

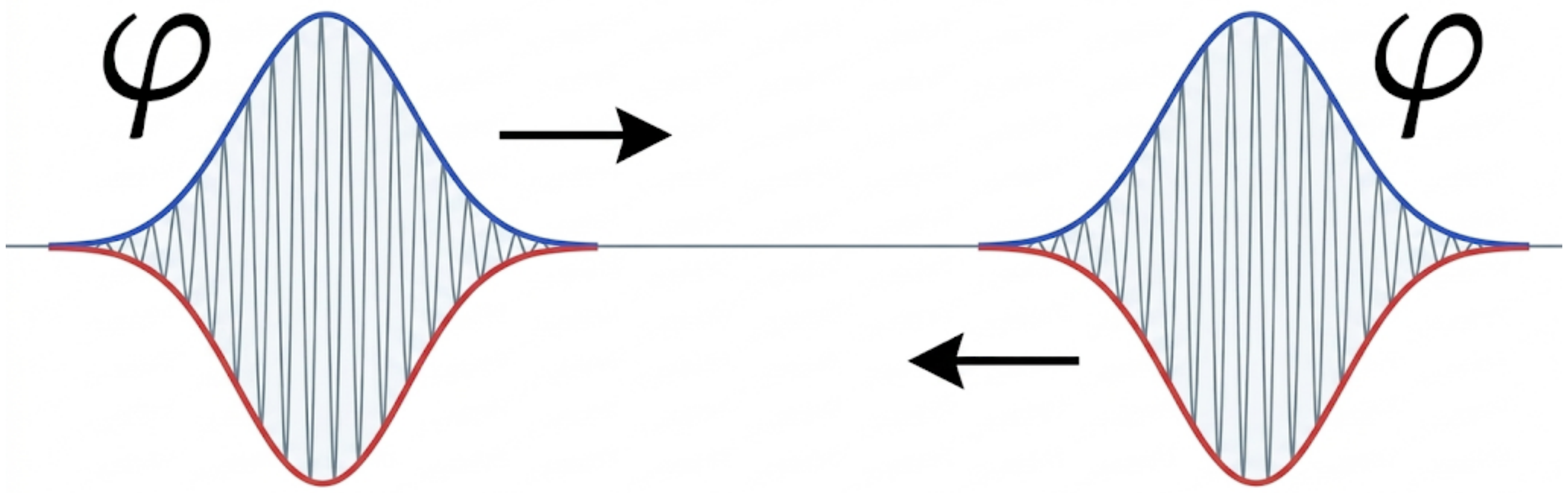
- Prepare wave packets
- Time evolve
- Measurement

BQP-completeness of scattering in scalar quantum field theory

Jordan, Krovi, Lee, Preskill `12-`17 (in 2+1 D)

Sample efficiently (BQP) from

$$|\langle X|U(t,t_0)|\varphi\varphi\rangle|^2$$

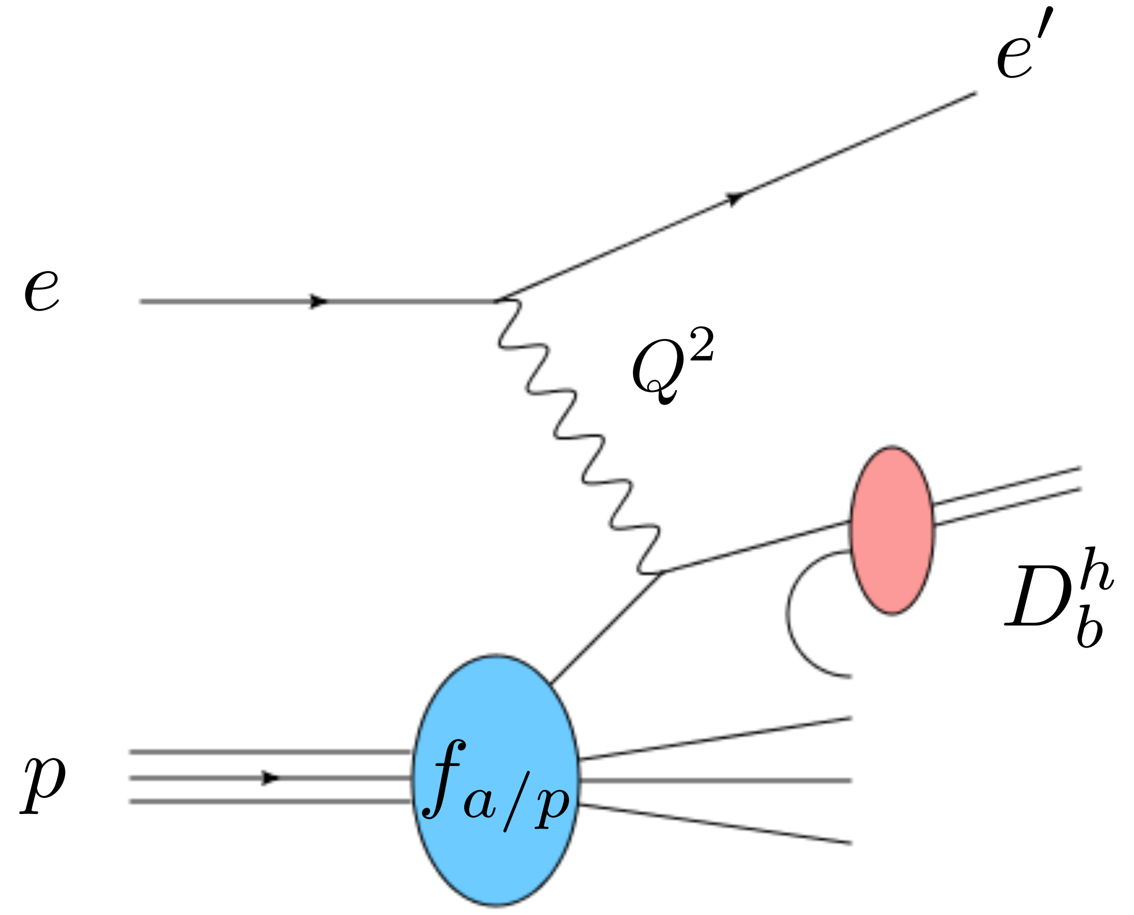


Factorization and hadronic structure

Perturbative

$$\mathrm{d}\sigma^{ep\to h+X} = \sum_{ab} f_{a/p} \otimes H_{ab} \otimes D_b^h$$

Non-perturbative

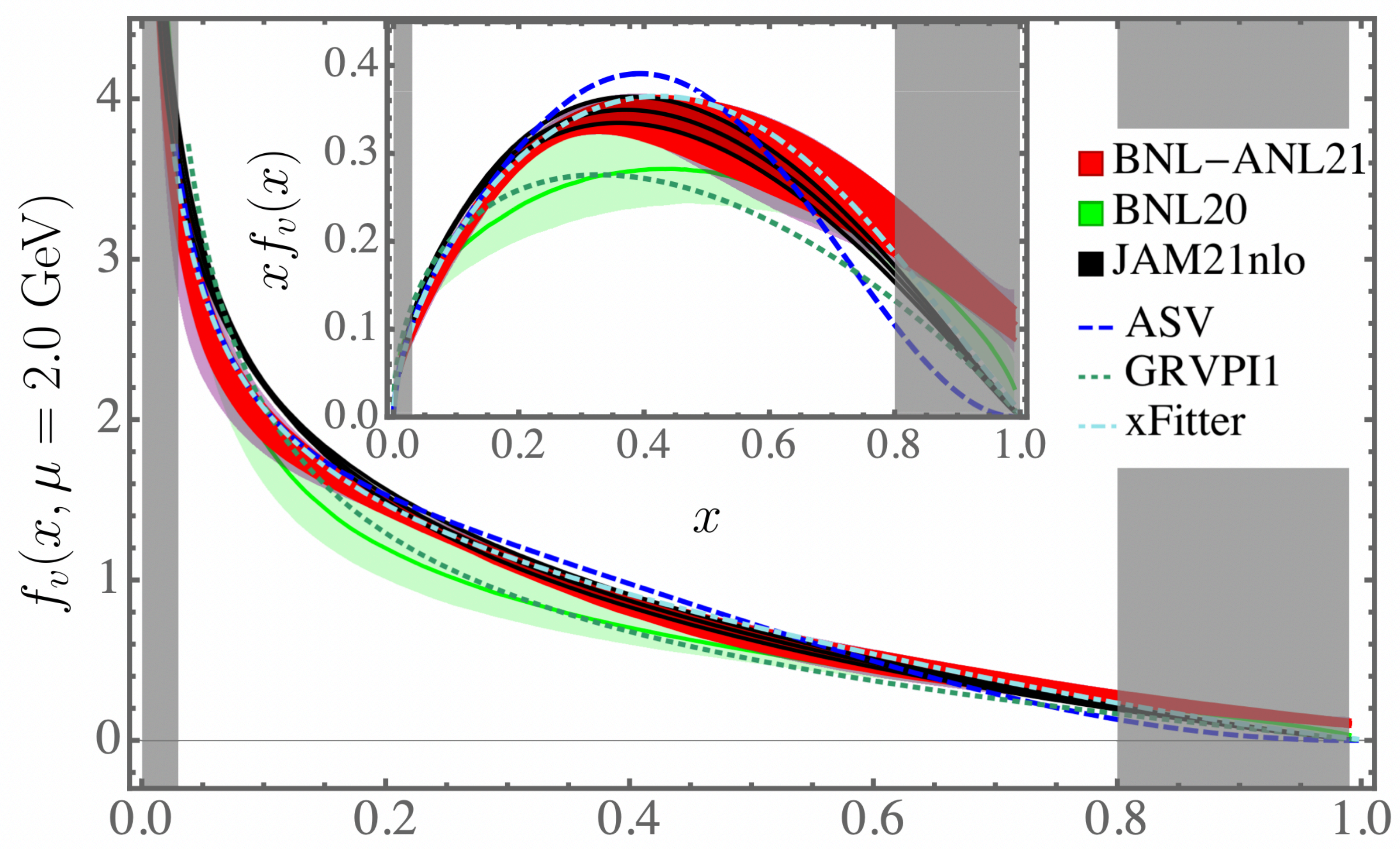


Collins, Soper, Sterman et al.

- Non-perturbative dynamics at low energies

$$\sim \mathrm{F.T.}\, \langle p|\mathcal{O}(t,\vec{x})\mathcal{O}(0)|p\rangle$$

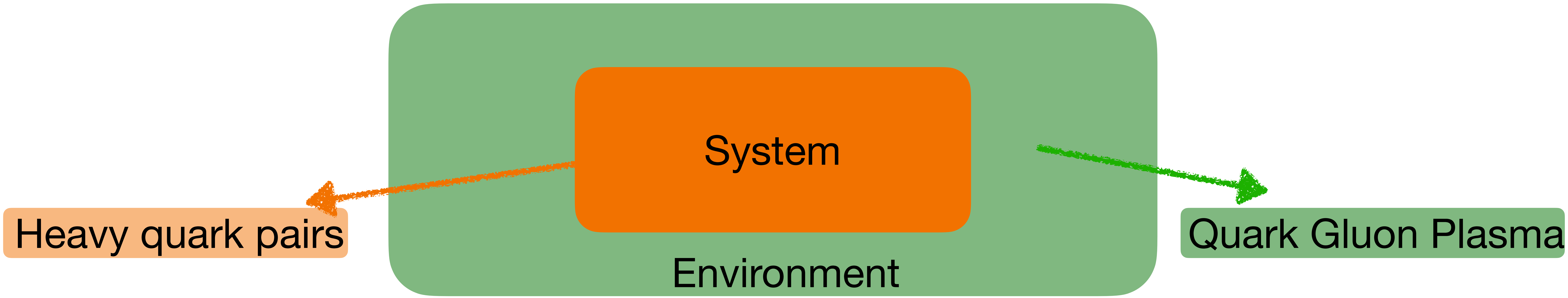
N-point lightcone correlation functions $t^2 - \vec{x}^{\,2} = 0$



Zhao et al. `22

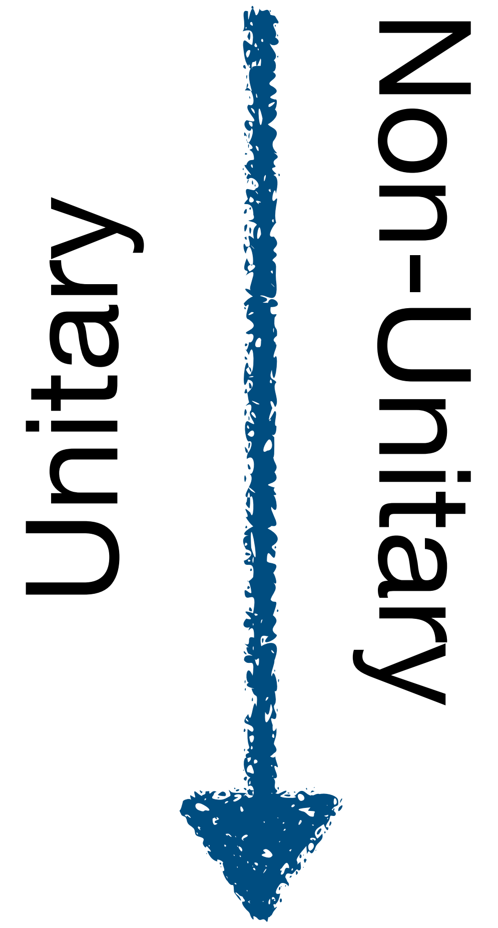
Open quantum systems

$$H_{total} = H_S \otimes I_E + I_S \otimes H_E + H_{int}$$



$$\rho(t=0) = \rho_S \otimes \rho_E$$

$$\mathrm{Tr}_E\left[\rho(t=0)\right] = \rho_S$$

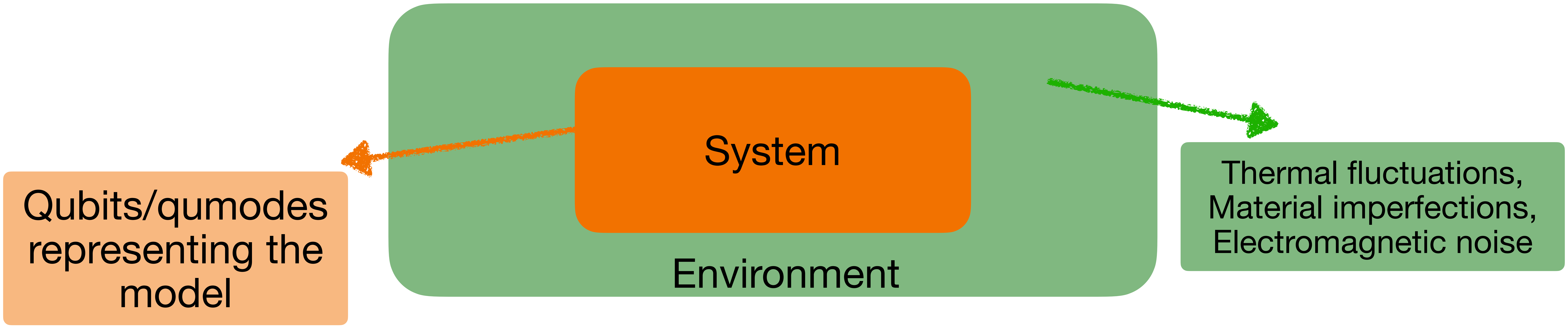


$$\rho(t) = U(t)\left(\rho_S \otimes \rho_E\right) U^\dagger(t)$$

$$\mathrm{Tr}_E\left[\rho(t)\right] = \mathrm{Tr}_E\left[U(t)\left(\rho_S \otimes \rho_E\right) U^\dagger(t)\right]$$

Open quantum systems

$$H_{total} = H_S \otimes I_E + I_S \otimes H_E + H_{int}$$



$$\rho(t=0) = \rho_S \otimes \rho_E$$

$$\text{Tr}_E\left[\rho(t=0)\right] = \rho_S$$

Unitary

Non-Unitary

$$\rho(t) = U(t)\left(\rho_S \otimes \rho_E\right) U^\dagger(t)$$

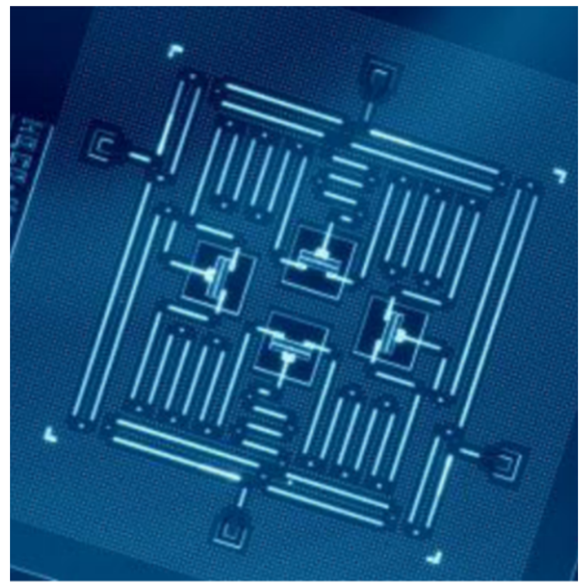
$$\text{Tr}_E\left[\rho(t)\right] = \text{Tr}_E\left[U(t)\left(\rho_S \otimes \rho_E\right) U^\dagger(t)\right]$$

See later for real hardware application

Technology and Research

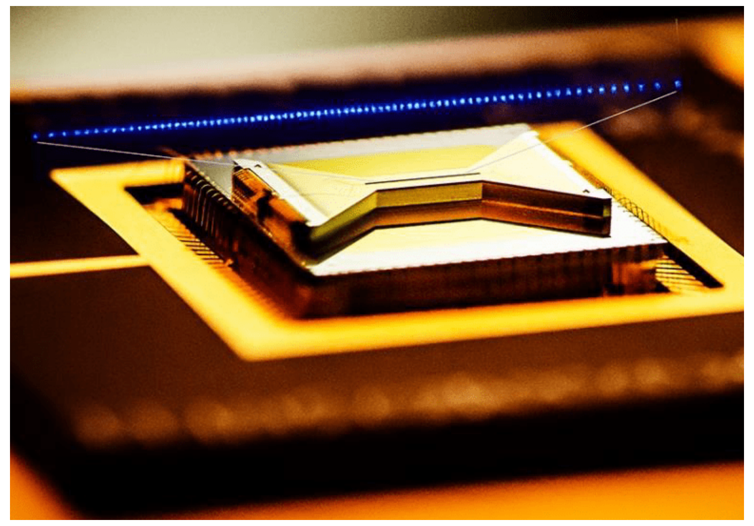
Industry level effort (some examples)

Superconducting qubits



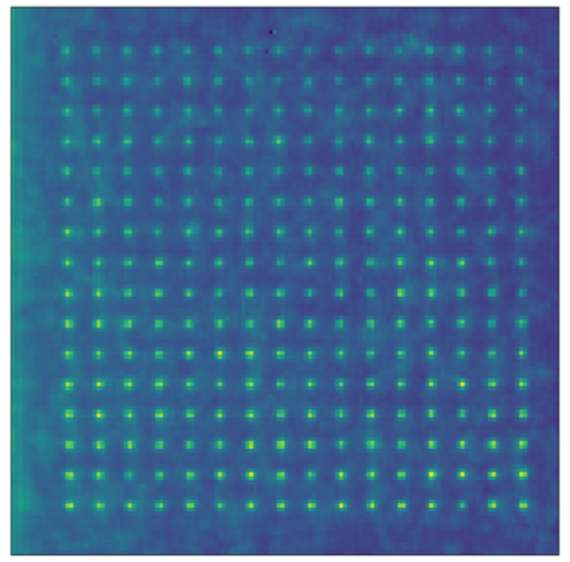
IBM
IQM
rigetti
Google AI Quantum
ALICE & BOB

Trapped ion qubits



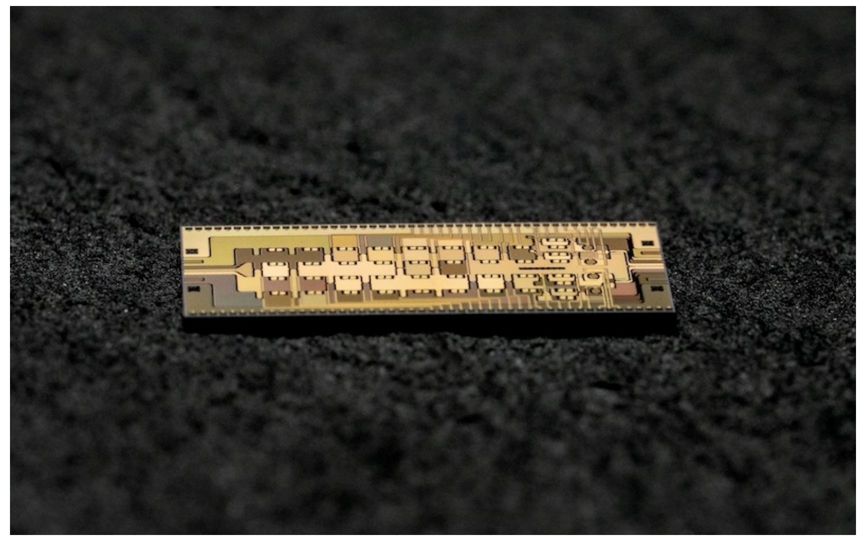
IONQ
oxford ionics
QUANTINUUM
AQT

Neutral atoms qubits



|QuEra>
Computing Inc.
Infleqtion
atom computing
Pasqal

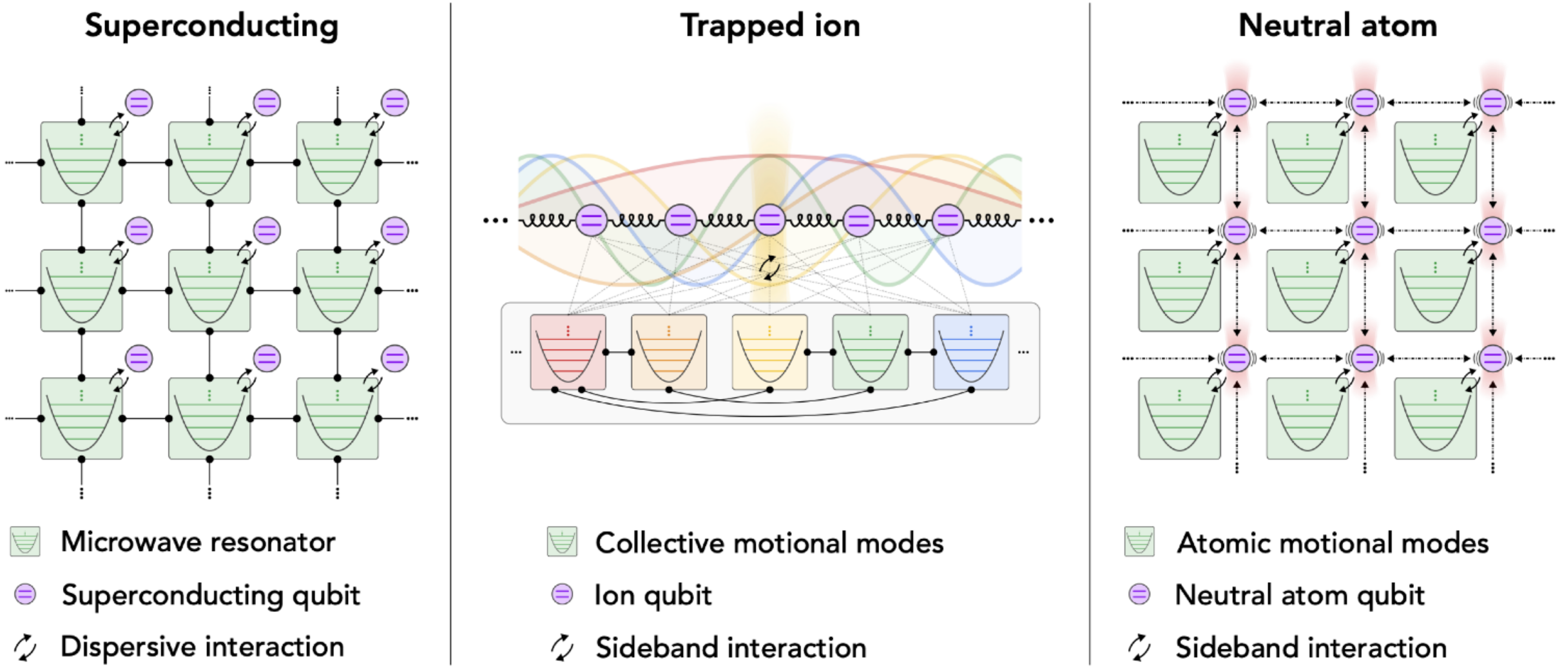
Photonics qubits



XANADU
PsiQuantum
QUIX QUANTUM

New paradigm: Hybrid architectures

Girvin, Wiebe et al `24



Platform dependence: Native gates, Coherence time, Connectivity, …

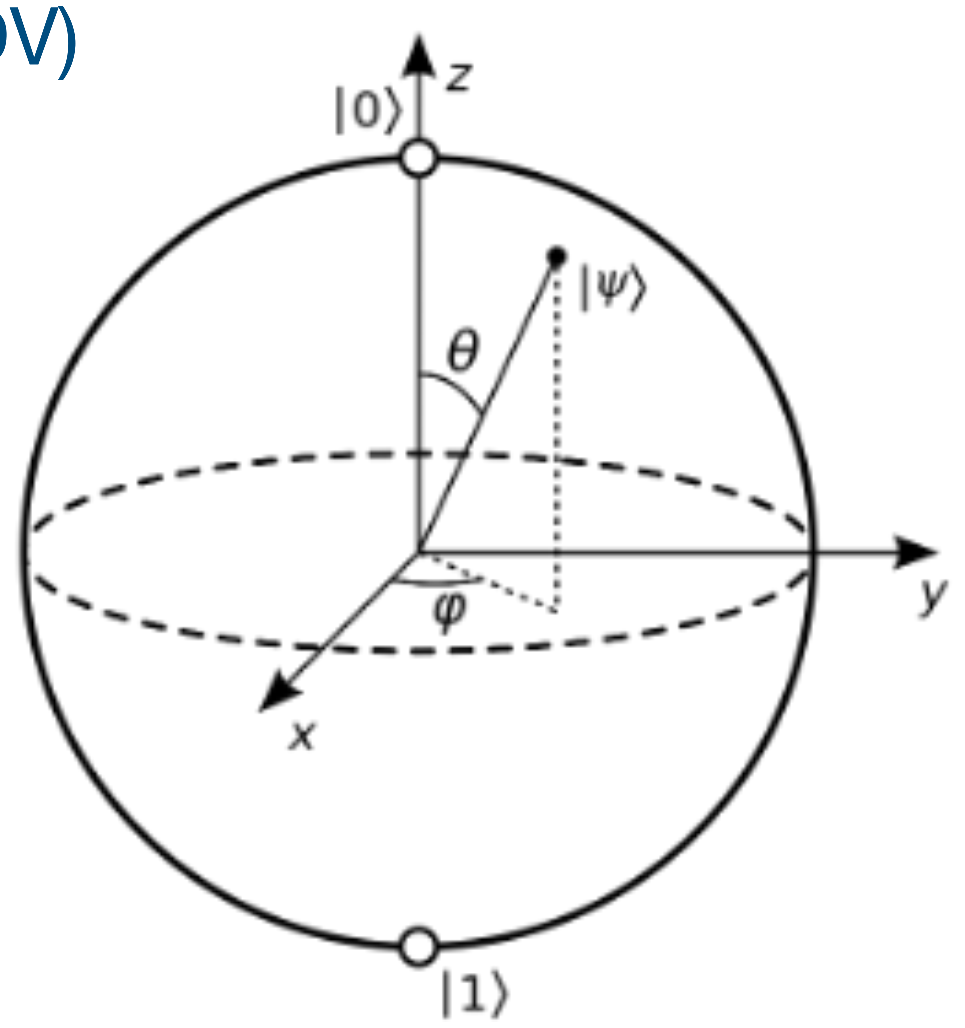
Quantum resources

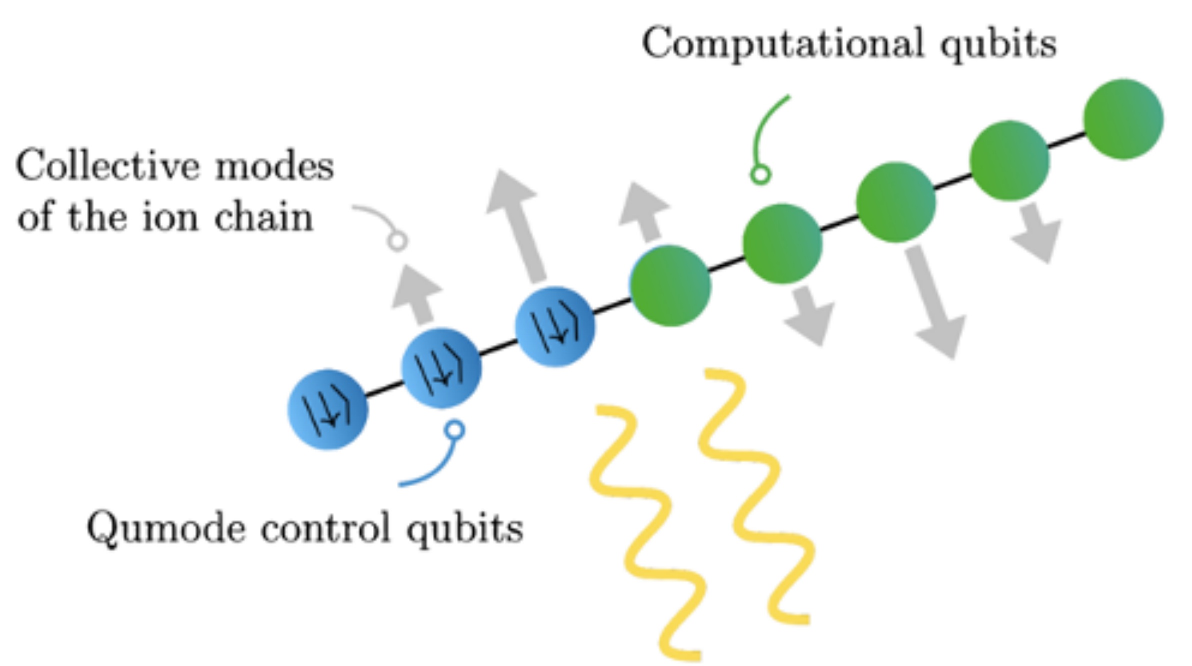
The elementary units for quantum computing

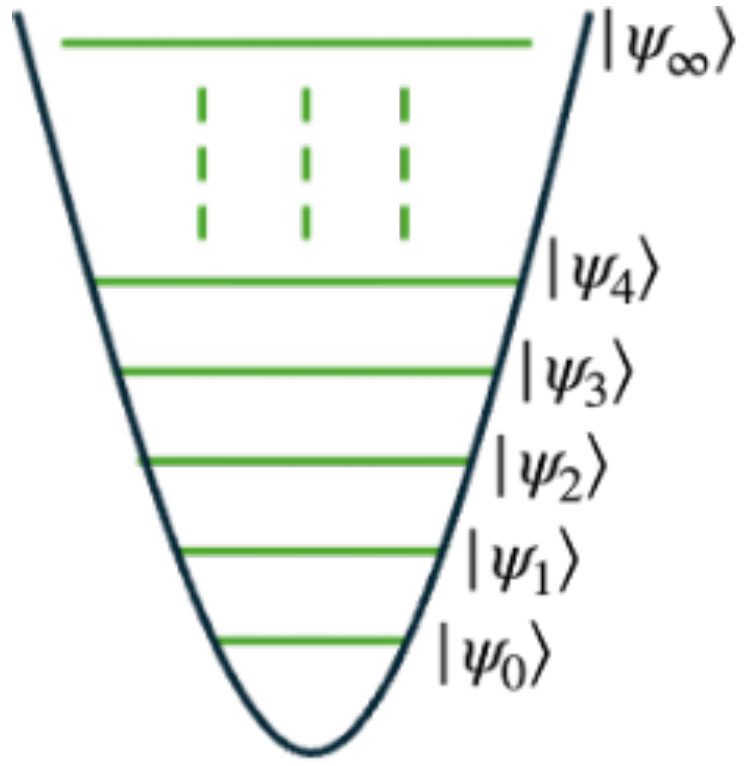
Gate based digital quantum computing

- Qubits (Qudits $2 \to d$)
  - Realization with: superconducting circuits, cold atoms, trapped ions, topological qubits,…
  - **Two-dimensional** Hilbert space per qubit
  - Digital gate-based computing with **discrete variables** (DV)

- Qumodes ($d \to \infty$)
  - Realization with: photonics, trapped ions, superconducting circuits,…
  - **Infinite-dimensional** Hilbert space per qumode
  - Gate based with **continuous variables** (CV)





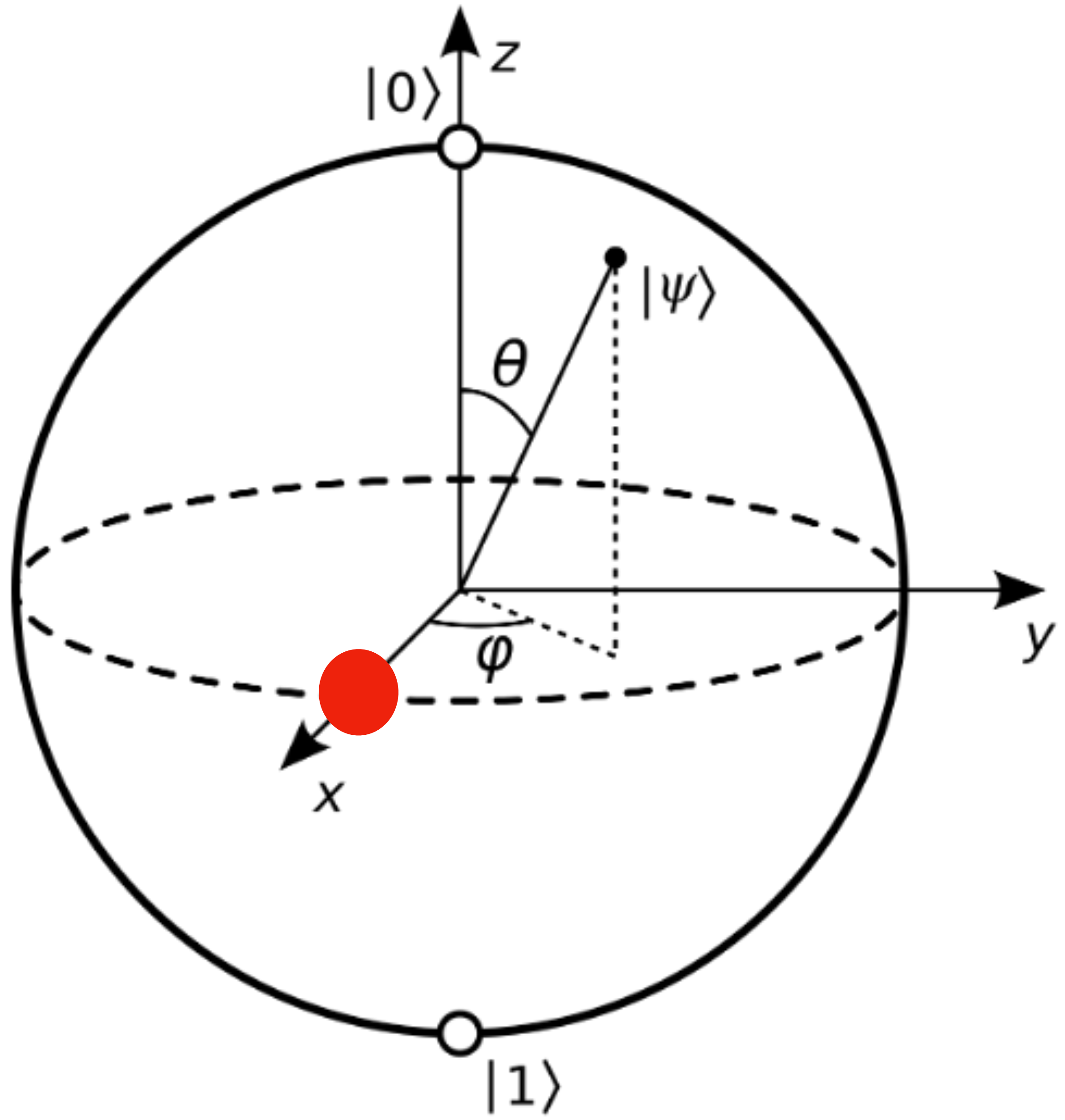


Araz, Grau, Montgomery, Ringer `24

Qubit states and operations

Gate type	Operation	Short	Operator
Qubit	Pauli operators		σ^i
	Rotation	$R_i(\theta)$	$e^{i\theta\sigma^i/2}$
	Controlled NOT	CNOT	$e^{i\frac{\pi}{4}(\mathbb{I}_1 - Z_1)(\mathbb{I}_2 - X_2)}$
Qumode	Rotation	$R(\theta)$	$e^{i\theta\hat{a}^\dagger\hat{a}}$
	Fourier	F	$e^{i\frac{\pi}{2}\hat{a}^\dagger\hat{a}}$
	Displacement	$D(z)$	$e^{z\hat{a}^\dagger - z^*\hat{a}}$
	Single-mode squeezing	$S(z)$	$e^{(z^*\hat{a}\hat{a} - z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Beam splitter	$BS(z)$	$e^{z\hat{a}^\dagger\hat{b} - z^*\hat{a}\hat{b}^\dagger}$
	Kerr	$K(z)$	$e^{iz(\hat{a}^\dagger\hat{a})^2}$
	Cross-Kerr	$CK(z)$	$e^{iz\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}}$
	Quadratic phase	$P(\theta)$	$e^{i\frac{\theta}{2}\hat{q}^2}$
	Cubic phase	$V(\theta)$	$e^{i\frac{\theta}{3}\hat{q}^3}$
Hybrid	Red sideband/Jaynes Cummings	$RSB(z)$	$e^{iz\hat{a}X^+ + iz^*\hat{a}^\dagger X^-}$
	Blue sideband/Anti-Jaynes Cummings	$BSB(z)$	$e^{iz\hat{a}^\dagger X^+ + iz^*\hat{a}X^-}$
	Controlled rotation	$CR(\theta)$	$e^{i\theta Z\hat{a}^\dagger\hat{a}}$
	Controlled displacement	$CD(z)$	$e^{Z(z\hat{a}^\dagger - z^*\hat{a})}$
	Controlled squeezing	$CS(z)$	$e^{Z(z^*\hat{a}\hat{a} - z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Controlled beam splitter	$CBS(z)$	$e^{Z(z\hat{a}^\dagger\hat{b} - z^*\hat{a}\hat{b}^\dagger)}$

Gate = Unitary operation



Superposition

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Entanglement

$$|\mathrm{Bell}\rangle = \frac{|00\rangle + |1\rangle}{\sqrt{2}}$$

Lloyd and Braunstein `99

Qumode states and operations

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	Rotation	$R_i(\theta)$	$e^{i\theta\sigma^i/2}$
	Controlled NOT	CNOT	$e^{i\frac{\pi}{4}(\mathbb{I}_1 - Z_1)(\mathbb{I}_2 - X_2)}$
Qumode	Rotation	$R(\theta)$	$e^{i\theta\hat{a}^\dagger\hat{a}}$
	Fourier	F	$e^{i\frac{\pi}{2}\hat{a}^\dagger\hat{a}}$
	Displacement	$D(z)$	$e^{z\hat{a}^\dagger - z^*\hat{a}}$
	Single-mode squeezing	$S(z)$	$e^{(z^*\hat{a}\hat{a} - z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Beam splitter	$BS(z)$	$e^{z\hat{a}^\dagger\hat{b} - z^*\hat{a}\hat{b}^\dagger}$
	Kerr	$K(z)$	$e^{iz(\hat{a}^\dagger\hat{a})^2}$
	Cross-Kerr	$CK(z)$	$e^{iz\hat{a}^\dagger\hat{a}\,\hat{b}^\dagger\hat{b}}$
	Quadratic phase	$P(\theta)$	$e^{i\frac{\theta}{2}\hat{q}^2}$
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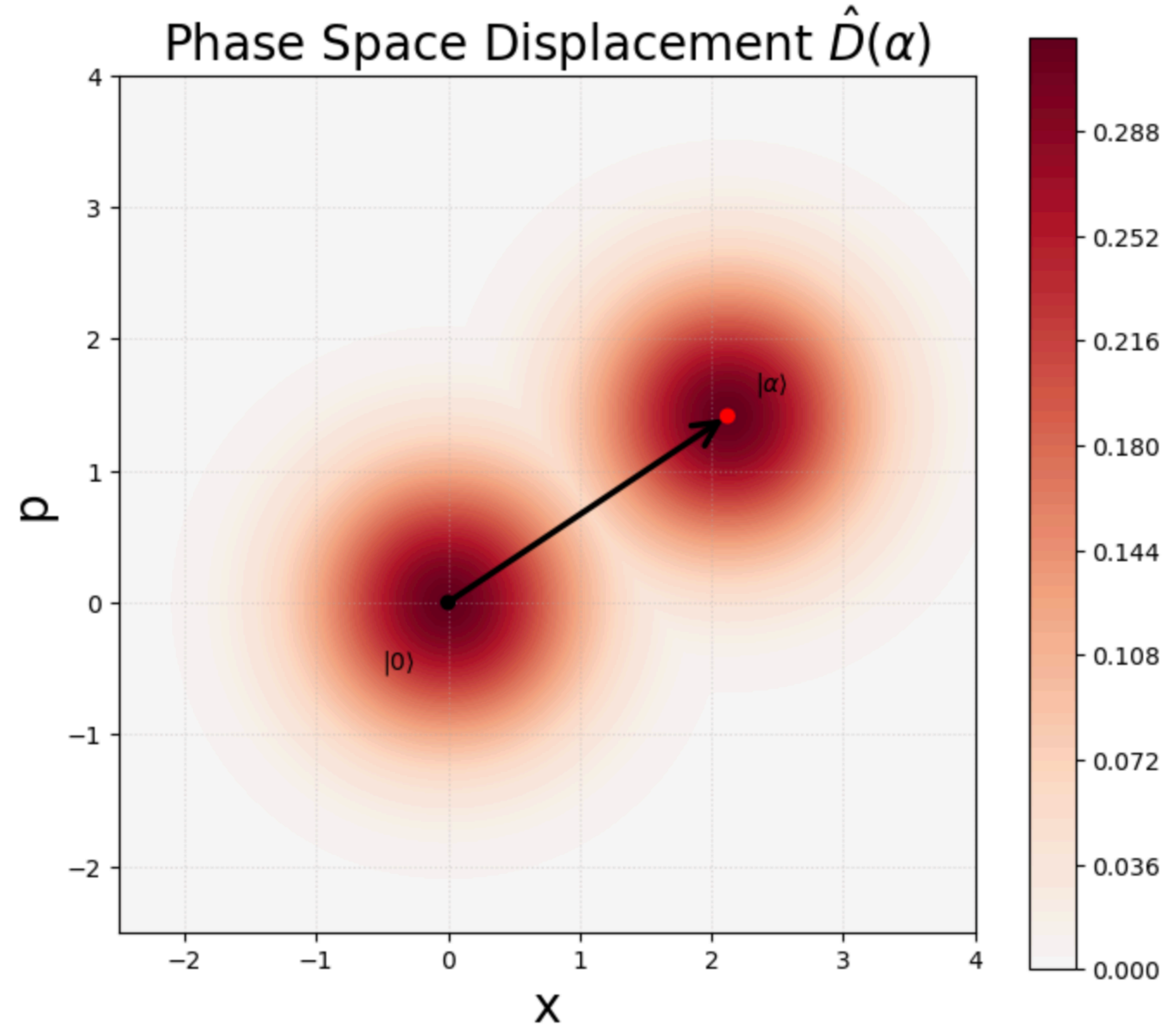
Wigner function



$$a, a^\dagger \iff \hat{q}, \hat{p}$$

Ladder operators **Quadratures**

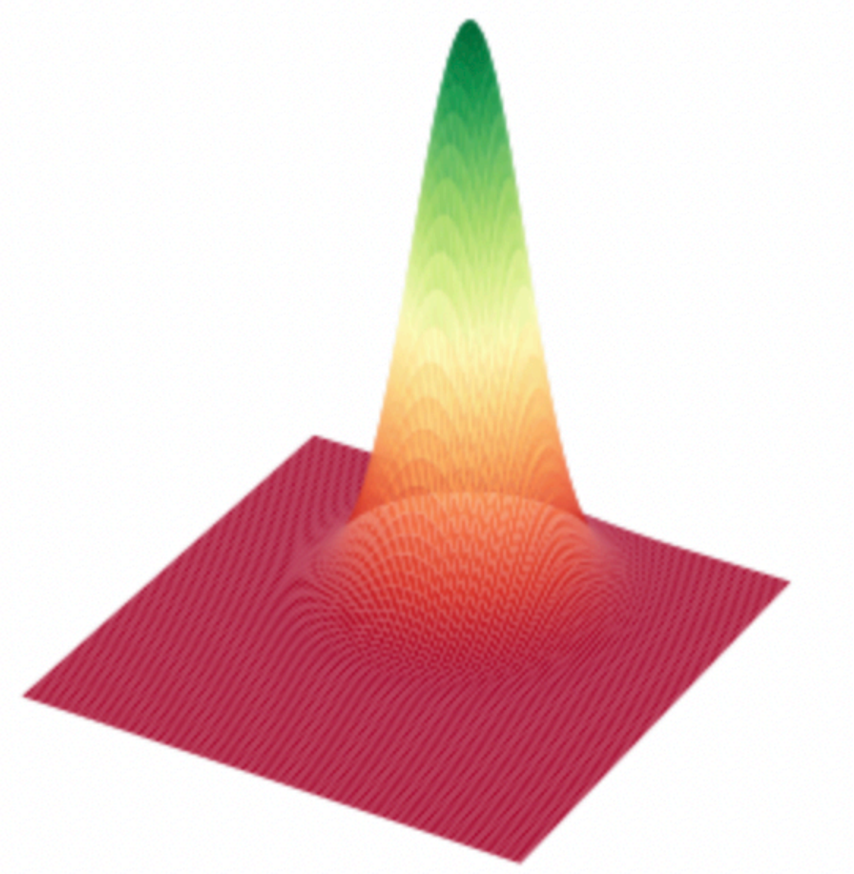
$$|n\rangle \propto \left(a^\dagger\right)^n |0\rangle$$

Universal gate set

- Displacement $D(z) = e^{z\hat{a}^\dagger - z^*\hat{a}}$

- Two-qumode beam splitter

$$U_{\rm bs}(z) = e^{z\hat{a}\hat{b}^\dagger - z^*\hat{a}^\dagger\hat{b}} \qquad z = \theta e^{i\phi}$$



$$W(q,p)$$

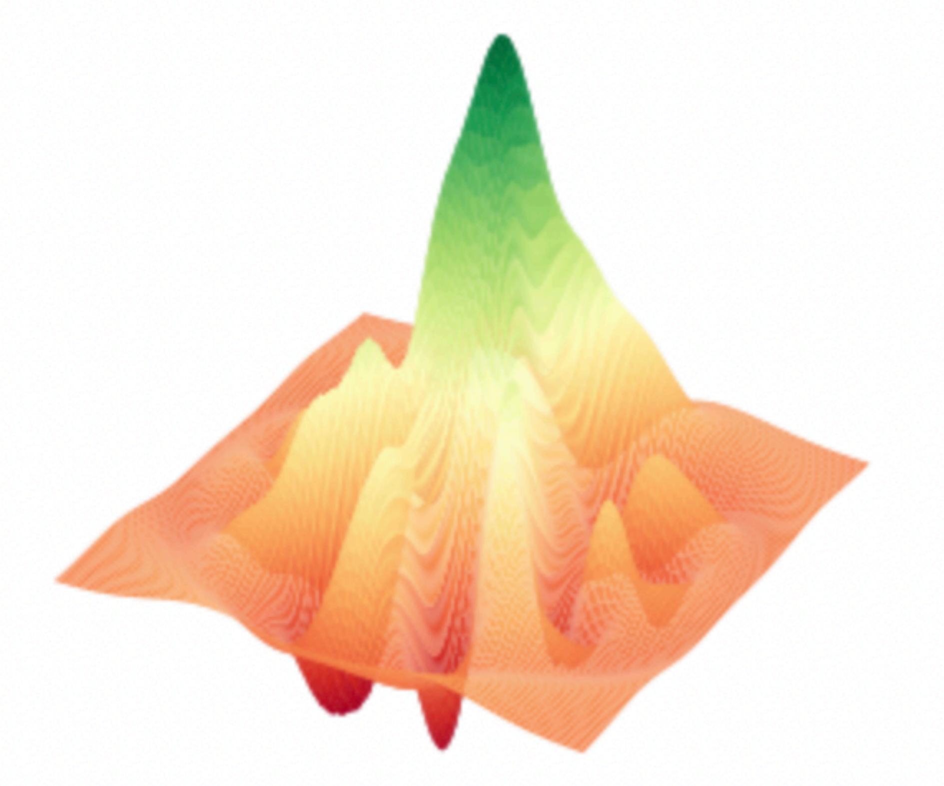
see Lloyd, Braunstein `99

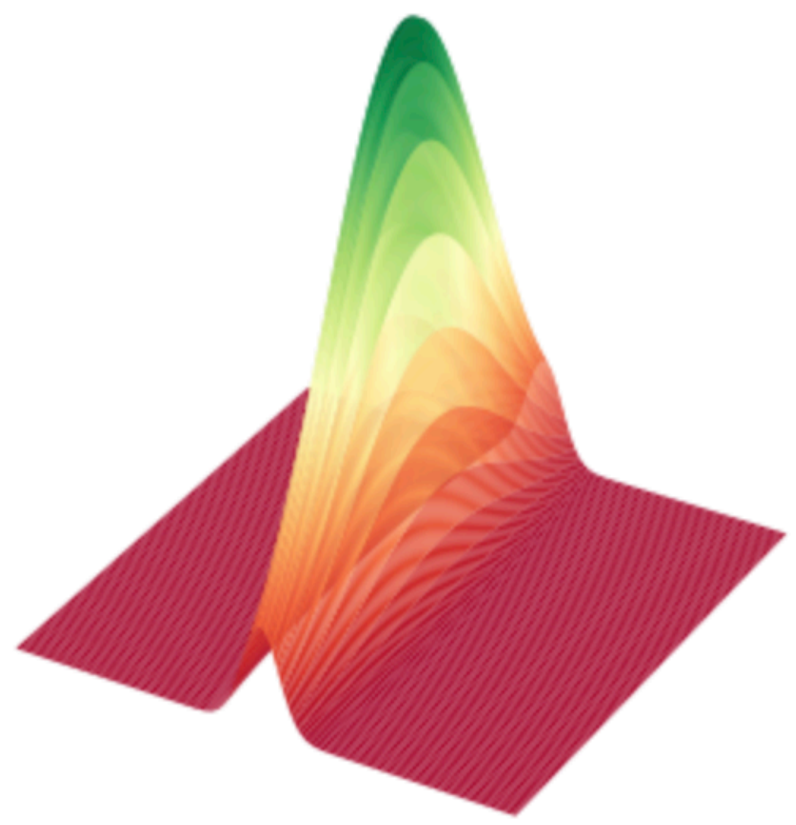
- Squeezing

$$S(z) = e^{\frac{1}{2}(z^*\hat{a}^2 - z\hat{a}^{\dagger 2})}$$

- Non-Gaussian operations

  e.g. Kerr gate or cubic gate





Hybrid states and operations

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	Rotation	$R_i(\theta)$	$e^{i\theta\sigma^i/2}$
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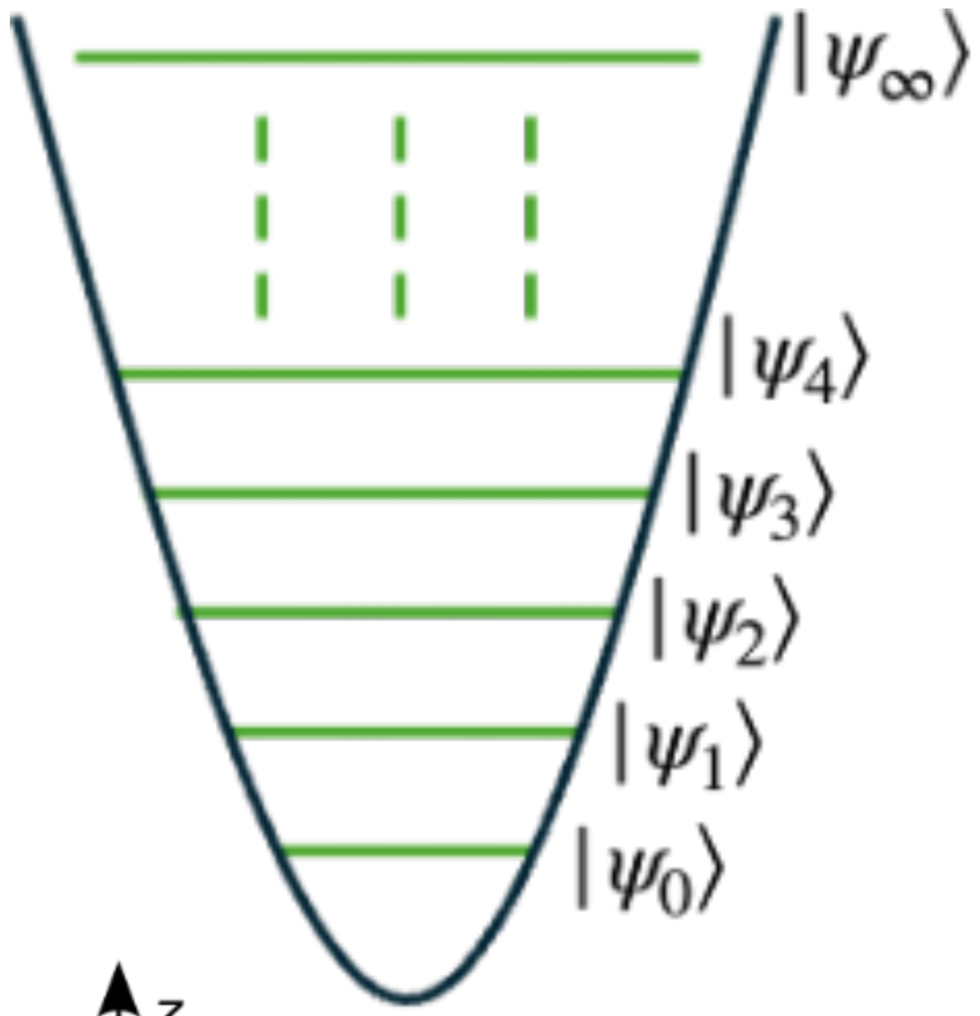
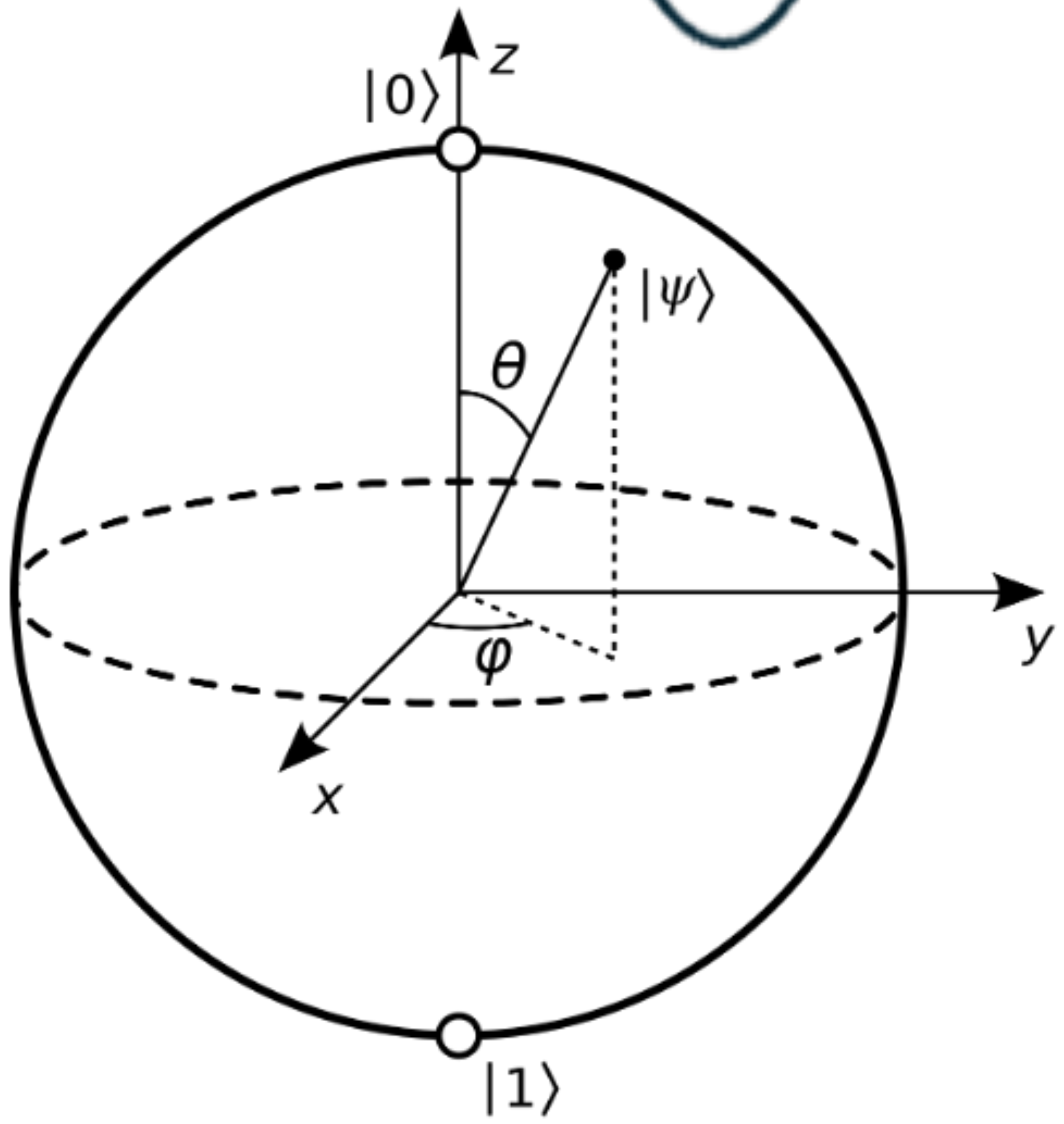
	Controlled beam splitter	$CBS(z)$	$e^{Z(z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger)}$

Lloyd and Braunstein `99





Qubit and qumode universality

There exists a minimal set of gates from which any other gate (unitary) can be built

Qubit only	Hybrid	Qumode only
Universal: Clifford $\{S = Z^{1/2}, H, \mathrm{CNOT}\}$; Magic $\{T = e^{-i\frac{\pi}{8}Z}$	Universal: $\mathrm{CD}(\alpha) = \mathrm{e}^{Z\otimes(\alpha \mathrm{a}^\dagger - \alpha^* \mathrm{a})}$, $R_{\sigma_\varphi}(\theta)$ (qubit rot.), $\mathrm{BS}(\theta, \phi)$ NEW $e^{-i\theta\cos(P(\hat{x},\hat{p}))\otimes\sigma_j}$	Universal: Gaussian $\{D(\alpha), R(\theta), S(\zeta), \mathrm{BS}(\theta,\phi)\}$; Non − Gaussian $\{V = e^{-i\theta\hat{q}^3}$
Pauli string σ_i	$\sigma_i \otimes P(\hat{x}, \hat{p})$	Polynomial $P(\hat{x}, \hat{p})$
Barenco et al, Gottesman, Knill	Girvin, Wiebe et al. \`24	Lloyd, Braunstein \`97

Non-polynomial (trigonometric) gates

RT, Rico, Ringer et al. \`25

Mapping physical models to quantum resources

Algebra representation with quantum resources

Bosonic CCR

* plus gauge group algebra

$$\left[\hat{\pi}_n, \hat{\phi}_{n'}\right] = - i\delta_{nn'}$$

There is no finite dimensional faithful representation of the Weyl/CCR algebra

Fermionic CAR

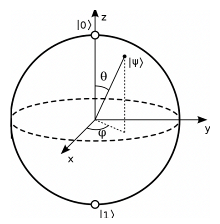
$$\left\{\psi_n, \psi^{\dagger}_{n'}\right\} = \delta_{nn'}$$

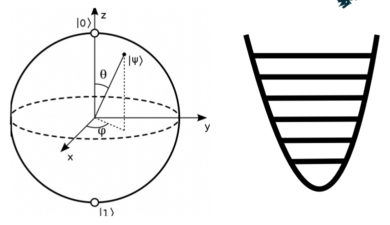
There exists finite dimensional faithful representations of the CAR algebra

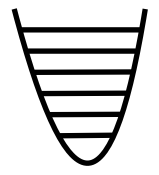
QCD with hybrid platforms

Simulate quarks and gluons with different quantum resources

QCD Lagrangian $\mathcal{L} = \bar{\psi}\left(i\partial^{\mu}\gamma_{\mu} - m\right)\psi + g_s\bar{\psi}\gamma^{\mu}T_a\psi A_{\mu}^{a} - \frac{1}{4}F_{a}^{\mu\nu}F_{\mu\nu}^{a}$

Qubits 





Qumodes

Qubit-qumode interaction

$\mathcal{H}_{\text{qumode}}^{m} \otimes \mathcal{H}_{\text{qubit}}^{n}$

Hybrid qubit/qumode approach

QED in 2+1 dimensions

Staggered QED in 2+1 dimensions

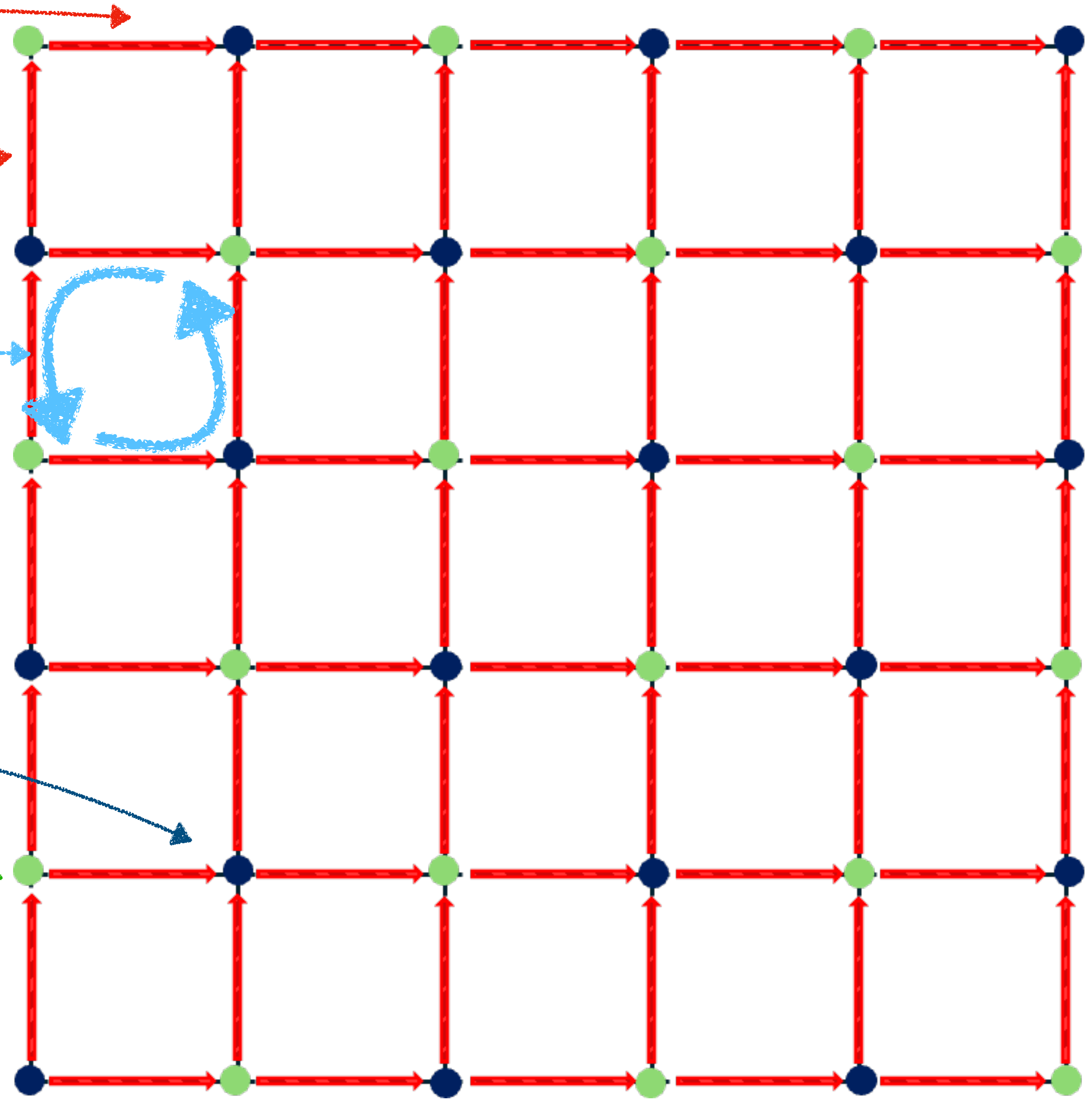
Hamiltonian for a general lattice

$$H_E = \frac{g^2}{2} \sum_{\mathrm{n}} \left(E^2_{\mathrm{n,e_x}} + E^2_{\mathrm{n,e_y}} \right)$$

$$H_B = -\frac{1}{2g^2a^2} \sum_n \left(P_n + P_n^\dagger - 2 \right)$$

$$H_M = m_0 \sum_n (-)^{n_x+n_y} \Psi_n^\dagger \Psi_n$$

$$H_K = \frac{1}{2} \sum_n \left[i\Psi_n^\dagger U_{n,e_x}^\dagger \Psi_{n+e_x} - (-)^{n_x+n_y} \Psi_n^\dagger U_{n,e_y}^\dagger \Psi_{n+e_y} \right] + \text{h.c.}$$



$$P_n \equiv U_{n,x} U_{n+e_x,y} U^\dagger_{n+e_y,x} U^\dagger_{n,y}$$

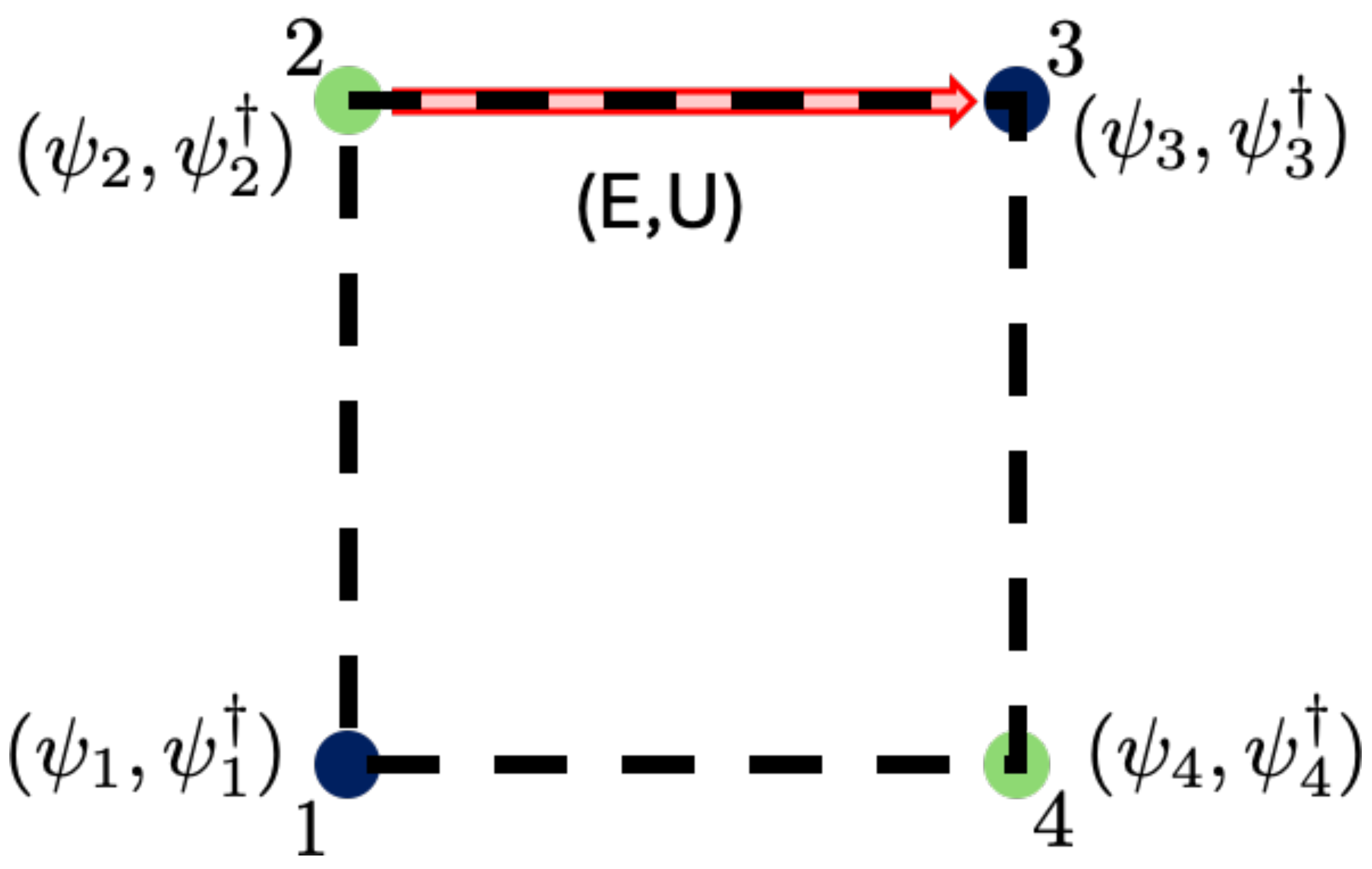
One plaquette world

$$H_E = \frac{g^2}{2}\left[E^2 + \left(E - Q_2\right)^2 + \left(E + Q_3\right)^2 + \left(E - Q_1 - Q_2\right)^2\right]$$

$$H_B = -\frac{1}{2g^2}\left(U + U^\dagger - 2\right)$$

$$H_M = m_0 \sum_{i=1}^{4} (-1)^{1+i} \Psi_i^\dagger \Psi_i$$

$$H_K = \frac{i}{2}\left(\Psi_1^\dagger \Psi_4 + \Psi_2^\dagger U^\dagger \Psi_3\right) - \frac{1}{2}\left(\Psi_1^\dagger \Psi_2 - \Psi_4^\dagger \Psi_3\right) + \text{h.c.}$$



$$U_n = e^{i\hat{\chi}_n} \in U(1)$$

$$\left[\hat{E}_n, \hat{\chi}_{n'}\right] = -i\delta_{nn'}$$

$$\left[\hat{E}_n, U_{n'}\right] = U_n \delta_{nn'}$$

$$\left\{\psi_n, \psi_{n'}^\dagger\right\} = \delta_{nn'}$$

Mapping non-compact bosons to Qumodes

One-to-one map for scalar theories

$$\left[\hat{\pi}_n, \hat{\phi}_{n'}\right] = -i\delta_{nn'}$$

$\Downarrow$ It is an isospectral representation

$$\left[\hat{p}_n, \hat{x}_{n'}\right] = -i\delta_{nn'}$$

$\text{Spec}(\hat{x}) = \mathbb{R}$, $\text{Spec}(\hat{p}) = \mathbb{R}$

Quantum electrodynamics in 2+1 D

$\text{Spec}(\hat{\chi}) = S^1$, $\text{Spec}(\hat{E}) = \mathbb{Z}$

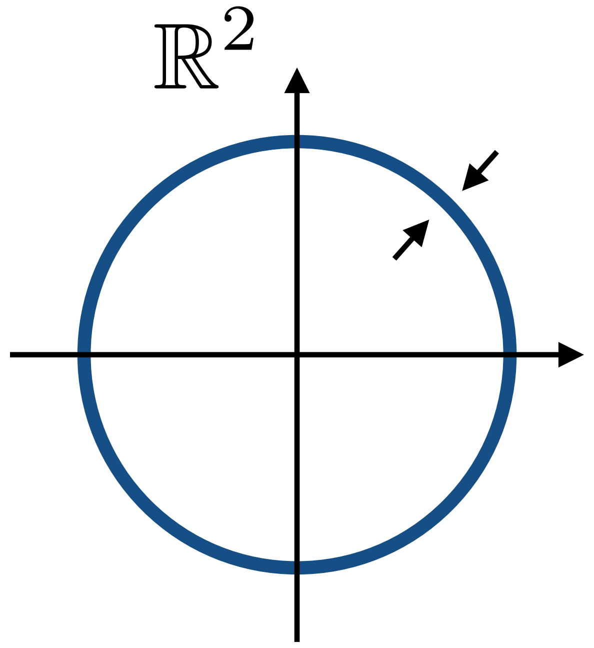
Compact gauge bosons

Solution: two-modes per link variable

$$U \mapsto \hat{q}^0 + i\hat{q}^1$$

$$E \mapsto J \equiv \hat{q}^0\hat{p}^1 - \hat{q}^1\hat{p}^0$$

With the operator constraint



$$(\hat{q}^0)^2 + (\hat{q}^1)^2 = \square$$

No discretization of the gauge group

No discretization of the algebra

Direct extension to SU(2) done

Ringer, Siopsis et al JHEP06(2025)084

Alternatively, use **non-polynomial (trigonometric) gates**

RT, Rico, Ringer et al `25

Experimental realization in preparation

Enforcing the constraint

$$\left[H, (\hat{q}^0_n)^2 + (\hat{q}^1_n)^2\right] = 0 \; \forall n$$

Enlarged phase space but
Gauge invariance is preserved
Physical sector is protected

Hamiltonian penalty term

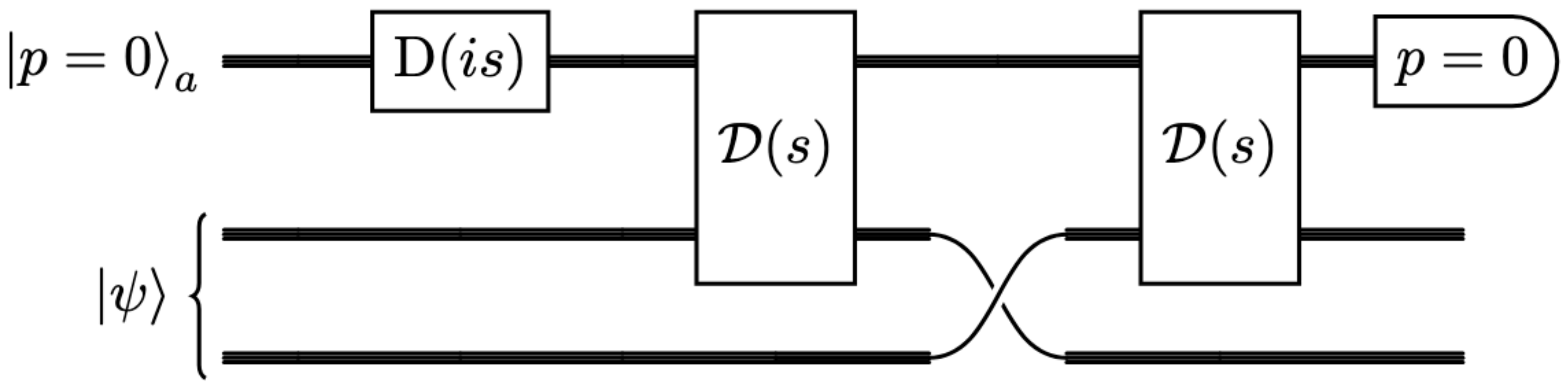
$$H \mapsto H + H_\mu$$

$$H_\mu = \frac{\mu}{2} \sum_{\mathbf{n},i} (\mathbf{q}^2_{\mathbf{n},i} - 1)^2$$

Disfavors non-physical configurations

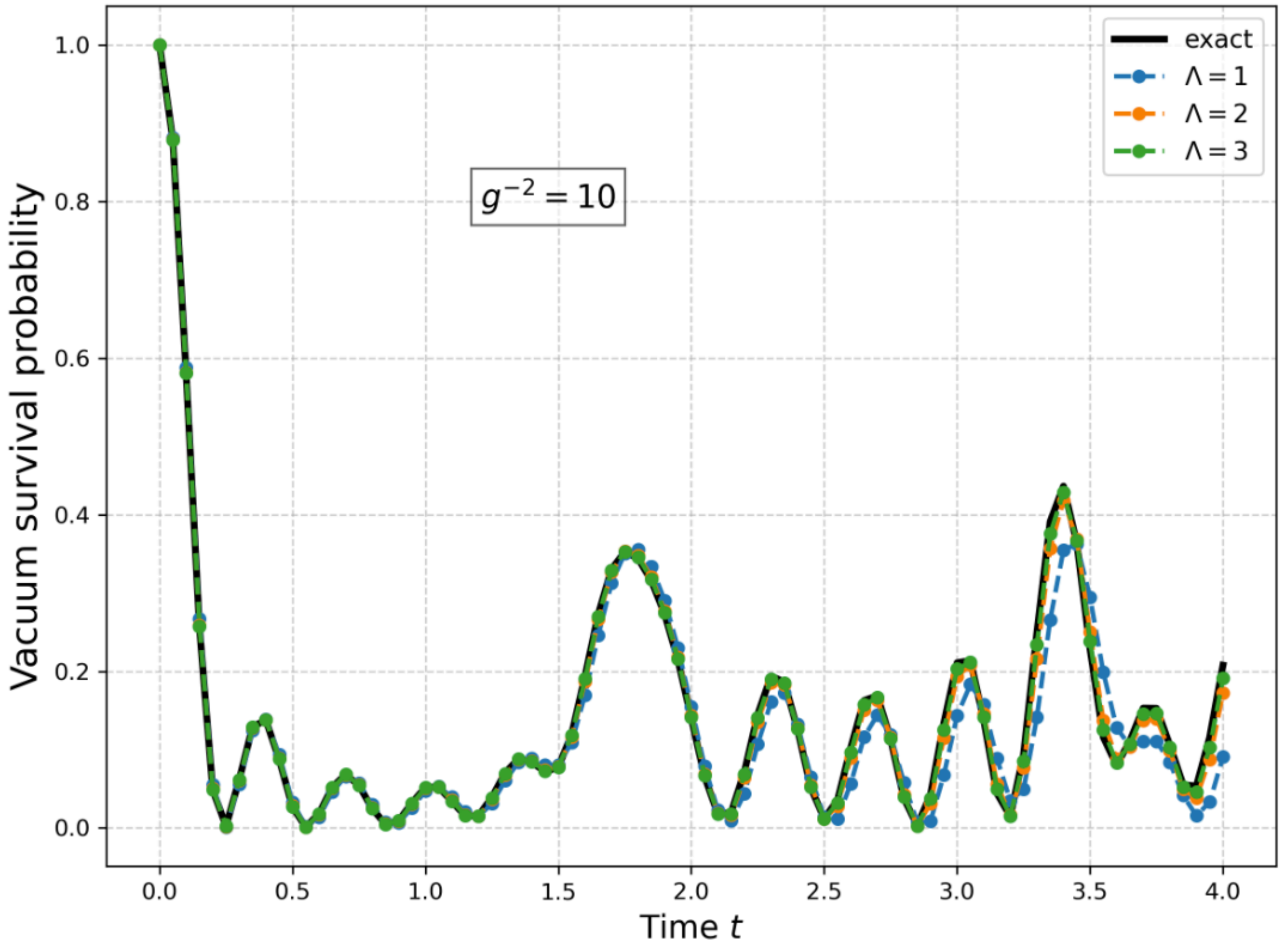
Inner product modification

$$|\psi\rangle \mapsto |\psi(\Lambda)\rangle \propto \int \mathrm{d}^2 q \, e^{-\frac{\Lambda^2}{g^2}(\mathbf{q}^2-1)^2} \psi(\mathbf{q}) \, |\mathbf{q}\rangle$$



$\mathscr{D}_\mu(s) = e^{-isq^2_\mu q_a}$ $\quad \Lambda = \frac{g}{2} s e^r$

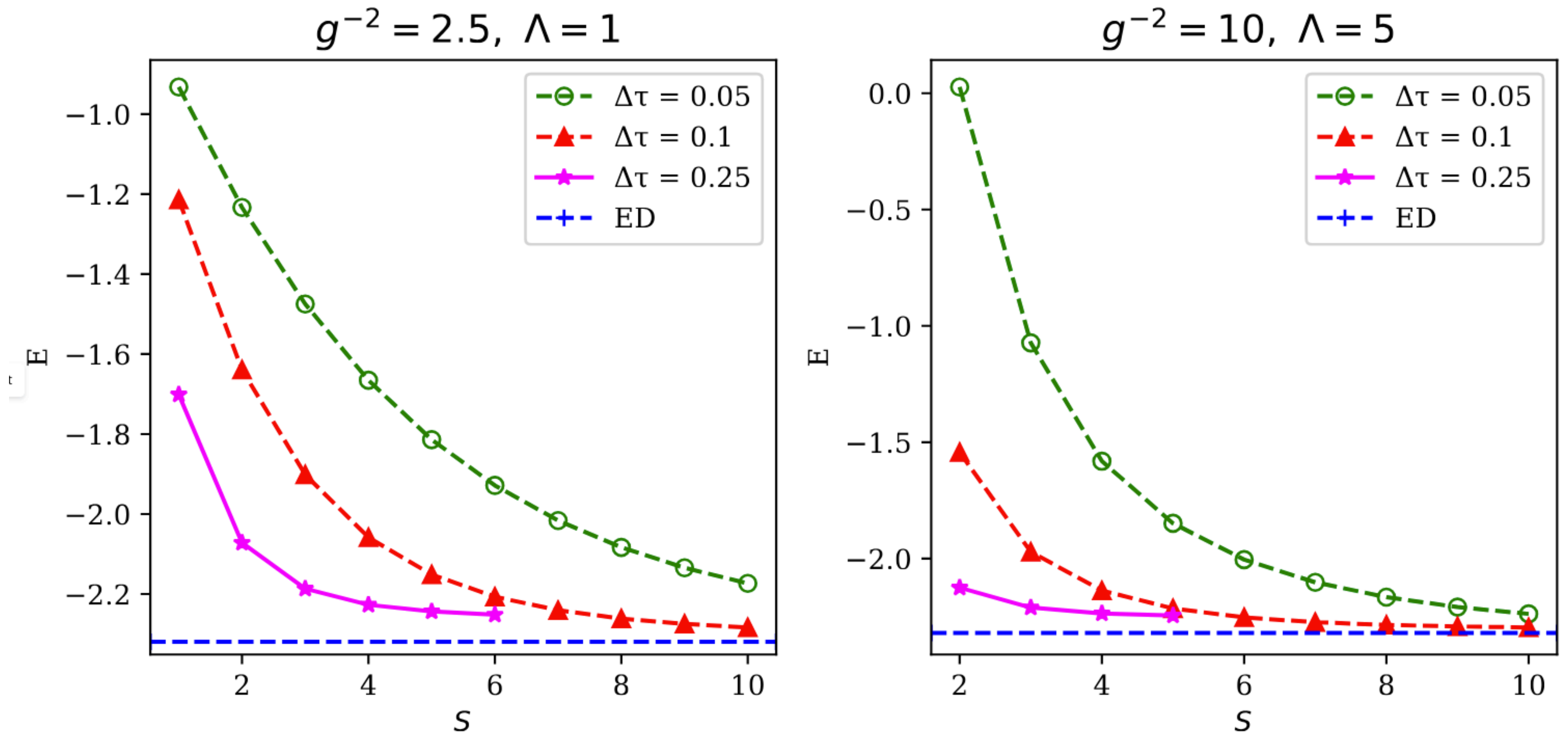
Testing quantum algorithms

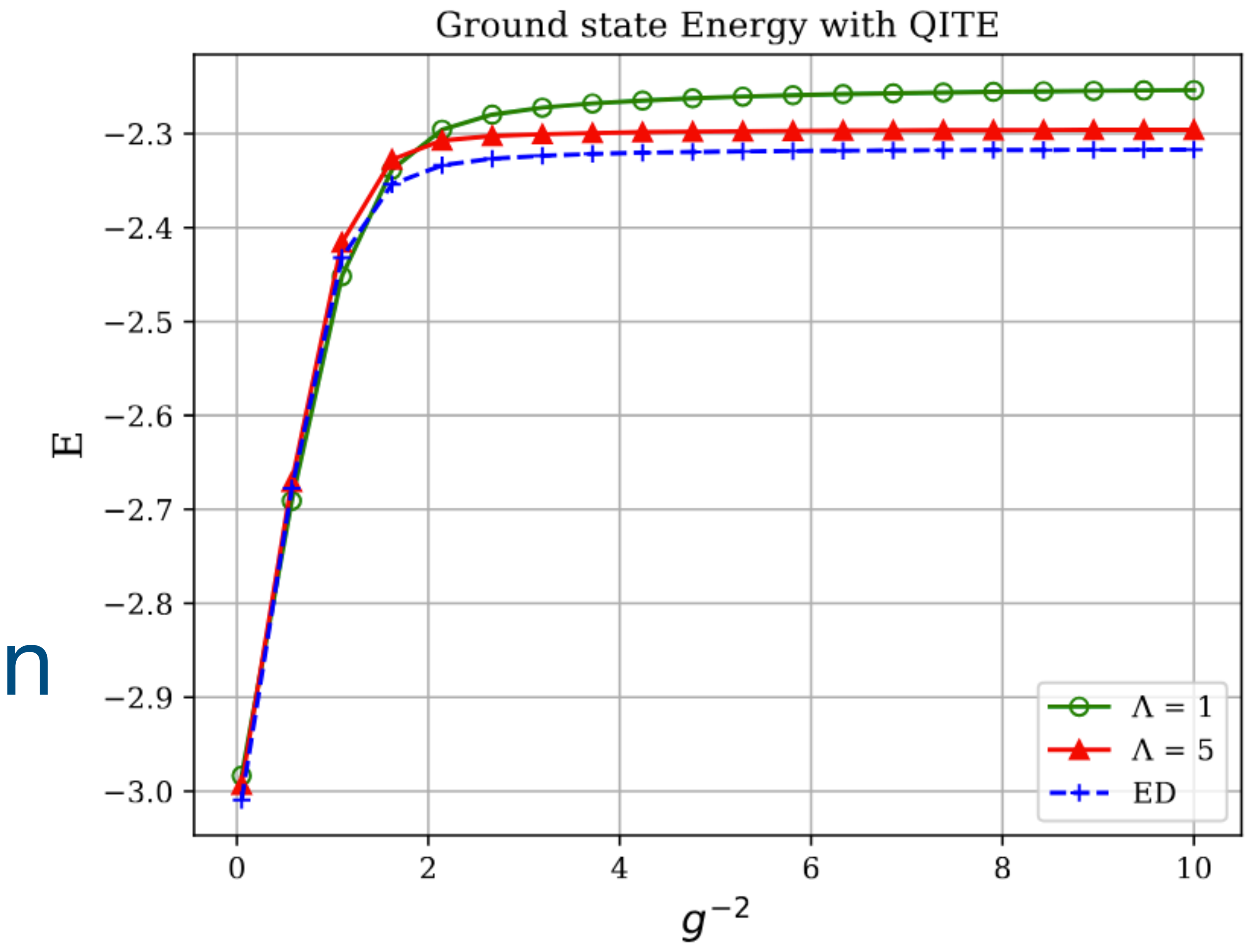


Time evolution

Ground state preparation

(QITE)





Ground state energy

The full Hamiltonian

Ale, RT, Rico, Ringer, Siopsis `25

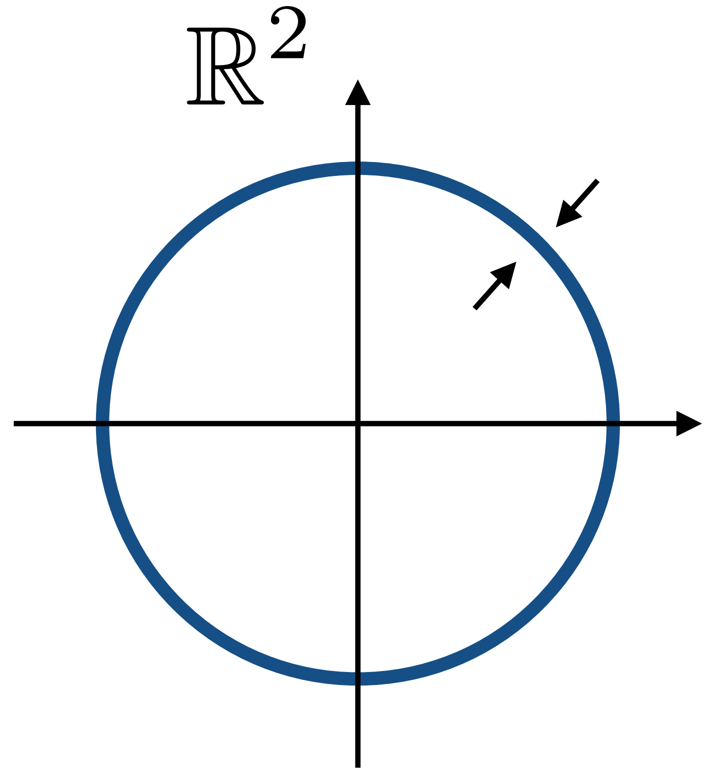
Due to Gauss's law

$$H_E^{(Q)} = g^2 \sum_{\boldsymbol{n},\boldsymbol{n}'} \left(\mathcal{H}^{(2)}_{\boldsymbol{n}\boldsymbol{n}'} J_{\boldsymbol{n}} J_{\boldsymbol{n}'} + \mathcal{H}^{(1)}_{\boldsymbol{n}\boldsymbol{n}'} Q_{\boldsymbol{n}} J_{\boldsymbol{n}'} + \mathcal{H}^{(0)}_{\boldsymbol{n}\boldsymbol{n}'} Q_{\boldsymbol{n}} Q_{\boldsymbol{n}'} \right),$$

$$H_B^{(Q)} = \frac{1}{2g^2} \sum_{\boldsymbol{n}} \left(\boldsymbol{q}_{\boldsymbol{n}+e_y} - \boldsymbol{q}_{\boldsymbol{n}} \right)^2,$$

$$H_M^{(Q)} = m_0 \sum_{\boldsymbol{n}} (-)^{n_x+n_y} Q_{\boldsymbol{n}},$$

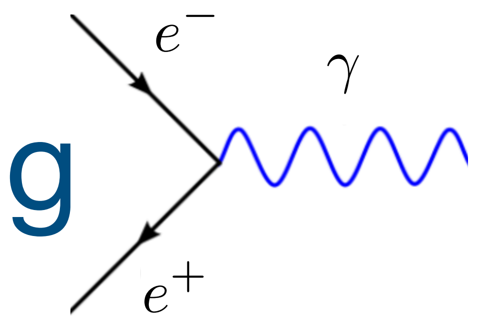
$$H_K^{(Q)} = \frac{1}{2} \sum_{\boldsymbol{n}} \left(q^0_{\boldsymbol{n}} - i q^1_{\boldsymbol{n}} \right) X^+_{\boldsymbol{n}} X^-_{\boldsymbol{n}+e_x} + \frac{1}{2} \sum_{\boldsymbol{n}} P_{L(\boldsymbol{n},\boldsymbol{n}+e_y)} X^+_{\boldsymbol{n}} X^-_{\boldsymbol{n}+e_y} + \text{h.c.}$$



$$Q_{\boldsymbol{n}} = \Psi^\dagger_{\boldsymbol{n}} \Psi_{\boldsymbol{n}} - \frac{1-(-)^{n_x+n_y}}{2} \mathbb{1}$$

$$J_{\boldsymbol{n}} = q^0_{\boldsymbol{n}} p^1_{\boldsymbol{n}} - q^1_{\boldsymbol{n}} p^0_{\boldsymbol{n}}$$

Electron/photon coupling



Qubit/qumode Qumode

An alternative approach * in preparation

Non-polynomial (trigonometric) gates

Complementing hybrid universality

Any hybrid gate $e^{-i\theta f(\hat{\mathbf{x}},\hat{\mathbf{p}})\otimes\prod_j \sigma_j}$

Taylor-like decomposition of the bosonic Hermitian exponent

$$f(\hat{\mathbf{x}},\hat{\mathbf{p}}) = \sum_{i,j}^{N}\sum_{n,m} c_{ij,nm}\hat{x}_j^n\hat{p}_j^m, \quad c_{ij,nm}\in\mathbb{R}$$

Fourier-like decomposition of the bosonic Hermitian exponent

$$f(\hat{\mathbf{x}},\hat{\mathbf{p}}) = \sum_{i,j}^{N}\sum_{n,m}\sum_{k,l} c_{ij,nm}^{(kl)}\cos^n(\hat{x}_j^k+\phi_{x,j})\sin^m(\hat{p}_j^l+\phi_{p,j}), \quad c_{ij,nm}^{(kl)}, \phi_{x,j}, \phi_{p,j}\in\mathbb{R}$$

Trigonometric gates representation

Encoding compactness in a mode and error correction

Inspired by Drell, Quinn et al `79

* in preparation

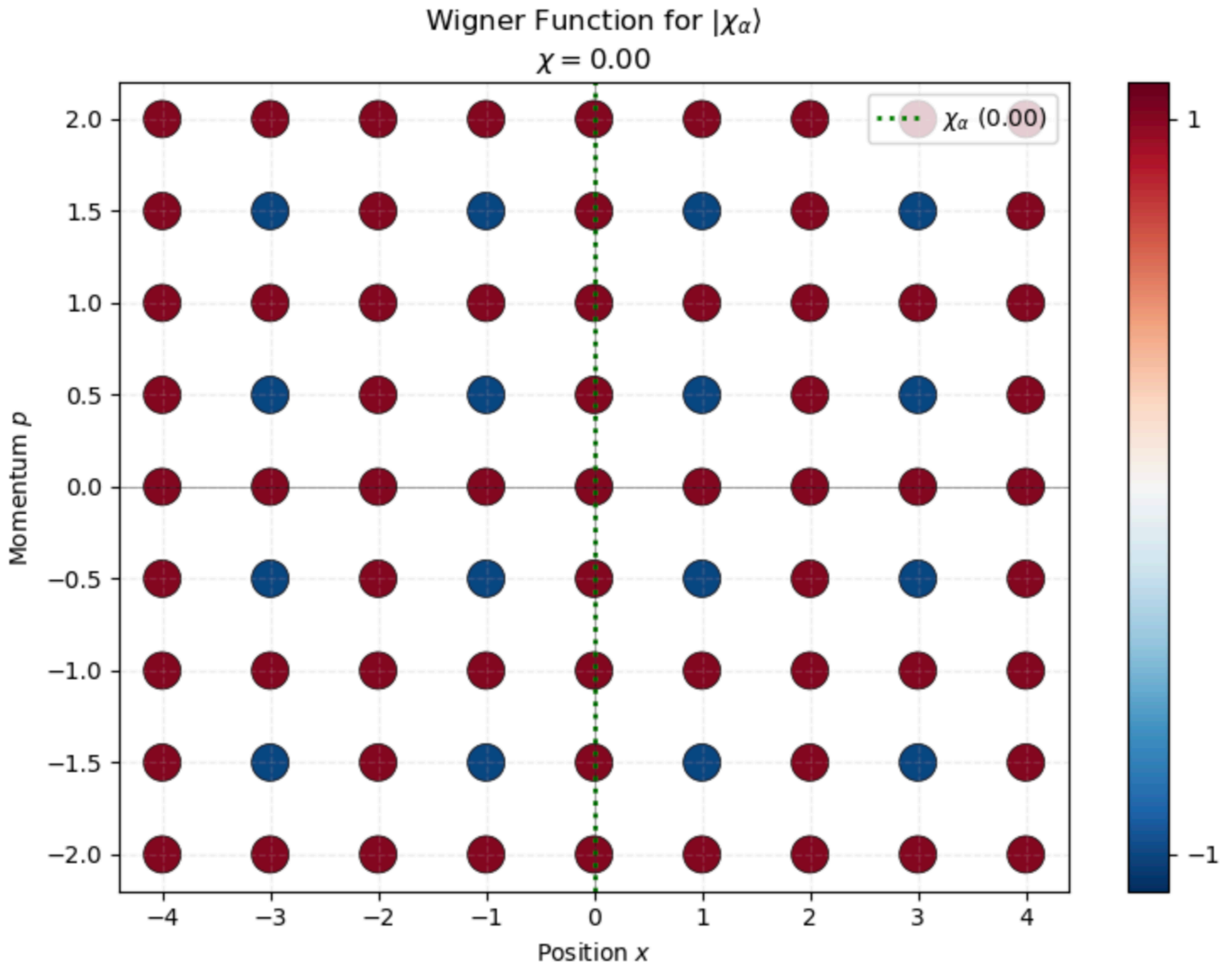
$L^2(\mathbb{R}) \to L^2(S^1)$ via stabilization operations

Natural error correction to protect the S^1 topology

$$H_{U(1)} = 2g^2\hat{E}^2 + \frac{1}{g^2}\left[1 - \cos(\hat{\chi})\right]$$

$$H_{U(1)} \mapsto 2g^2\left(\frac{\hat{p}}{\alpha}\right)^2 + \frac{1}{g^2}\left[1 - \cos(\alpha\hat{x})\right] - J \text{ Stabilizer}$$

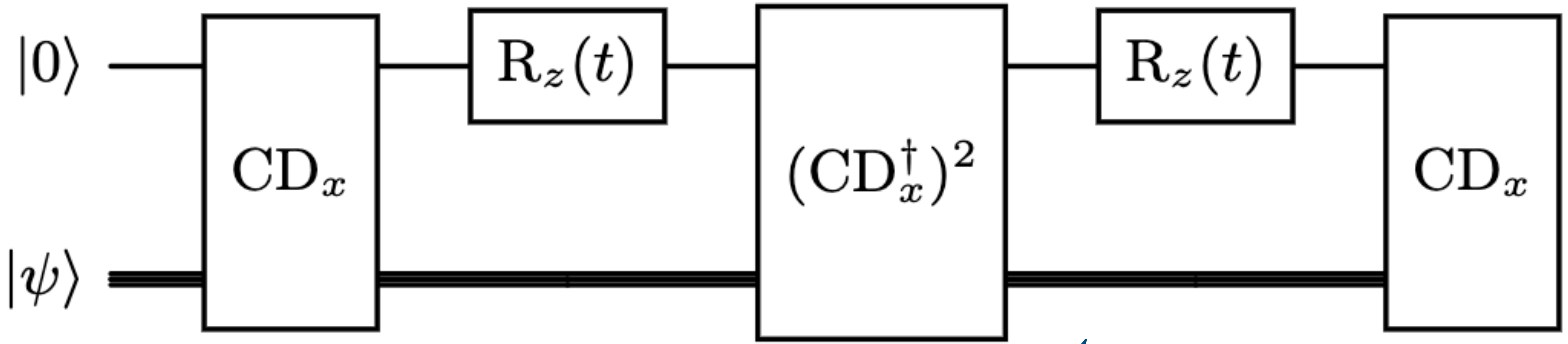
Only allowed (stabilized) states remain



Like a qunaught GKP state

Experimental realization

with QSCOUT at Sandia National Lab

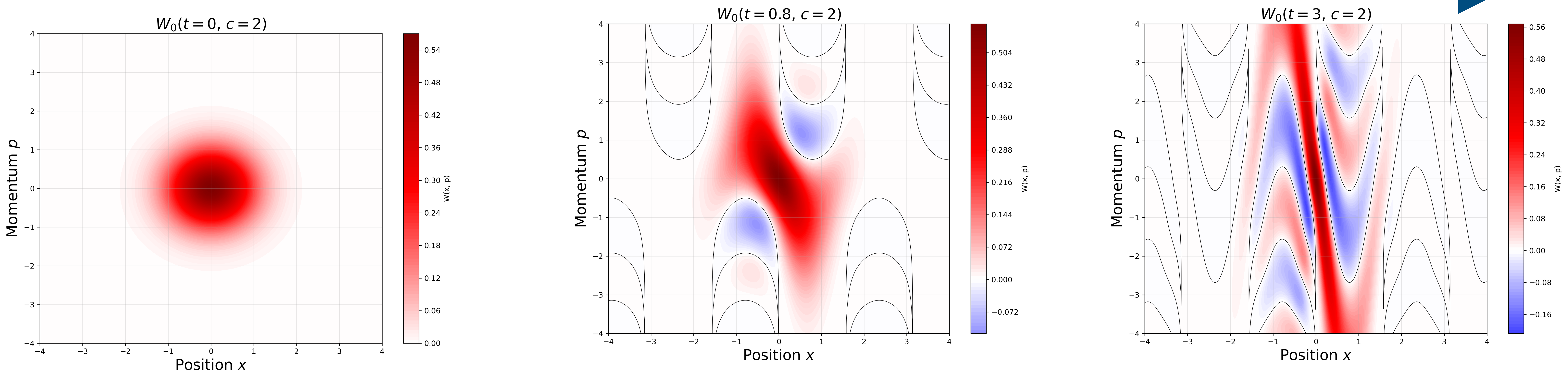


$$\approx e^{-it\cos(c\hat{x})}$$

RT, Ringer, Grau, Montgomery, S. Clark et al

In preparation

t



Error models with the OQS formalism

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H},\hat{\rho}] + \sum_i \left(\hat{C}_i \hat{\rho} \hat{C}_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \hat{\rho}\} \right)$$

$\hat{C} = \sqrt{\gamma}\hat{n}$ dephasing

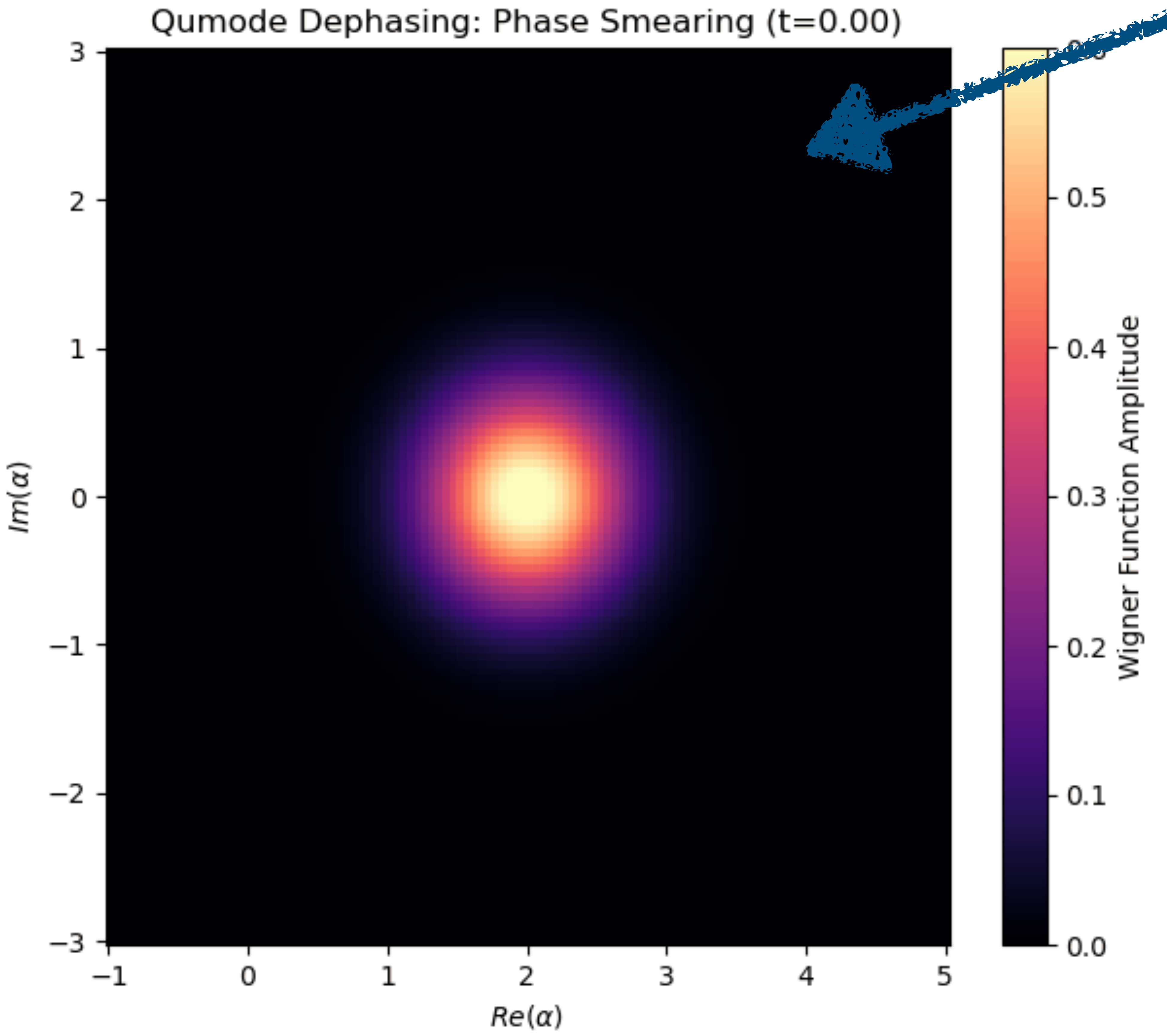
$$\langle n|\dot{\rho}|m\rangle = -\frac{\gamma}{2}(n-m)^2\rho_{nm}$$

$\hat{C} = \sqrt{\kappa}\hat{a}$ photon/phonon loss

$$\langle n|\dot{\rho}|m\rangle = \kappa\sqrt{(n+1)(m+1)}\rho_{n+1,m+1} - \frac{\kappa}{2}(n+m)\rho_{nm}$$

$\hat{C} = \sqrt{\gamma}\hat{a}^\dagger$ linear heating

$$\langle n|\dot{\rho}|m\rangle = \gamma\sqrt{nm}\rho_{n-1,m-1} - \frac{\gamma}{2}(n+m+2)\rho_{nm}$$



QSCOUT

Error models with the OQS formalism

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}] + \sum_i \left(\hat{C}_i \hat{\rho} \hat{C}_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \hat{\rho}\} \right)$$

$\hat{C} = \sqrt{\gamma}\hat{n}$ dephasing

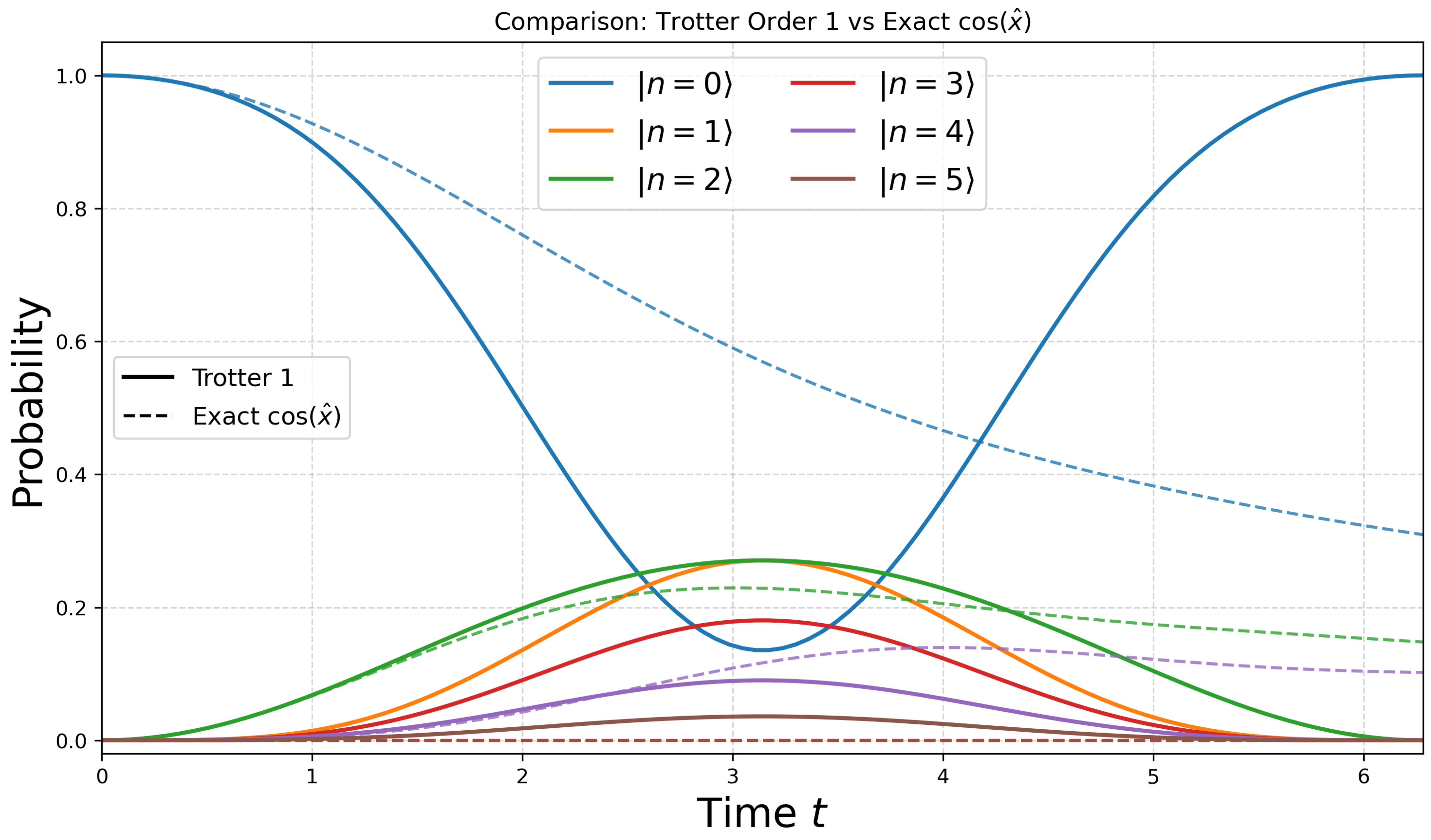
$$\langle n | \dot{\rho} | m \rangle = -\frac{\gamma}{2}(n-m)^2 \rho_{nm}$$

$\hat{C} = \sqrt{\kappa}\hat{a}$ photon/phonon loss

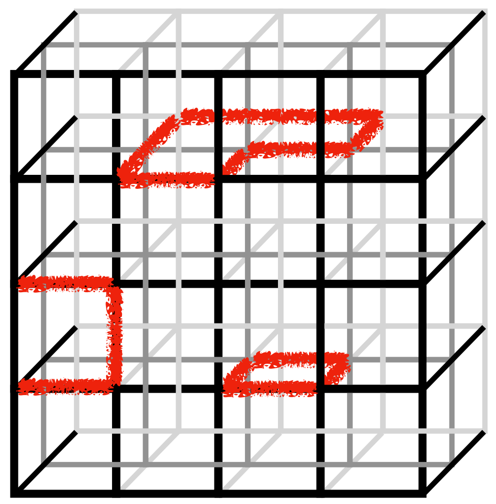
$$\langle n | \dot{\rho} | m \rangle = \kappa\sqrt{(n+1)(m+1)}\rho_{n+1,m+1} - \frac{\kappa}{2}(n+m)\rho_{nm}$$

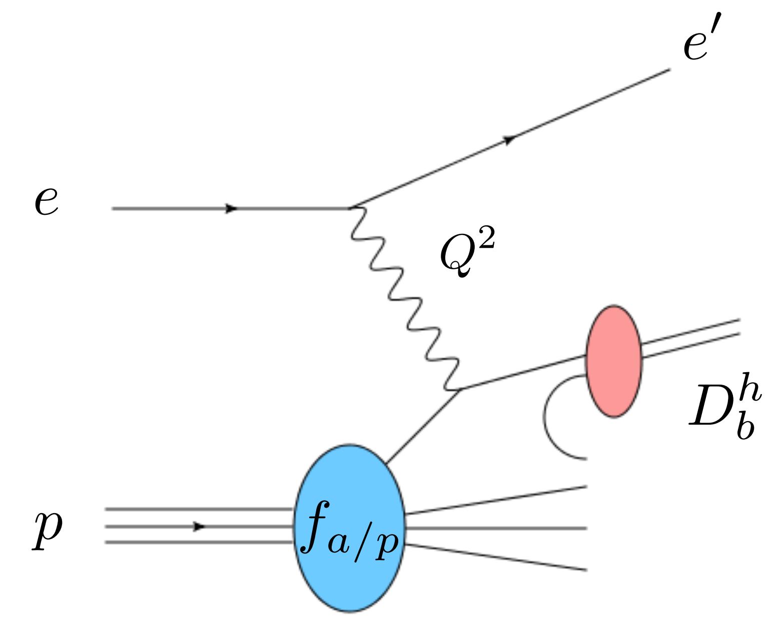
$\hat{C} = \sqrt{\gamma}\hat{a}^\dagger$ linear heating

$$\langle n | \dot{\rho} | m \rangle = \gamma\sqrt{nm}\rho_{n-1,m-1} - \frac{\gamma}{2}(n+m+2)\rho_{nm}$$

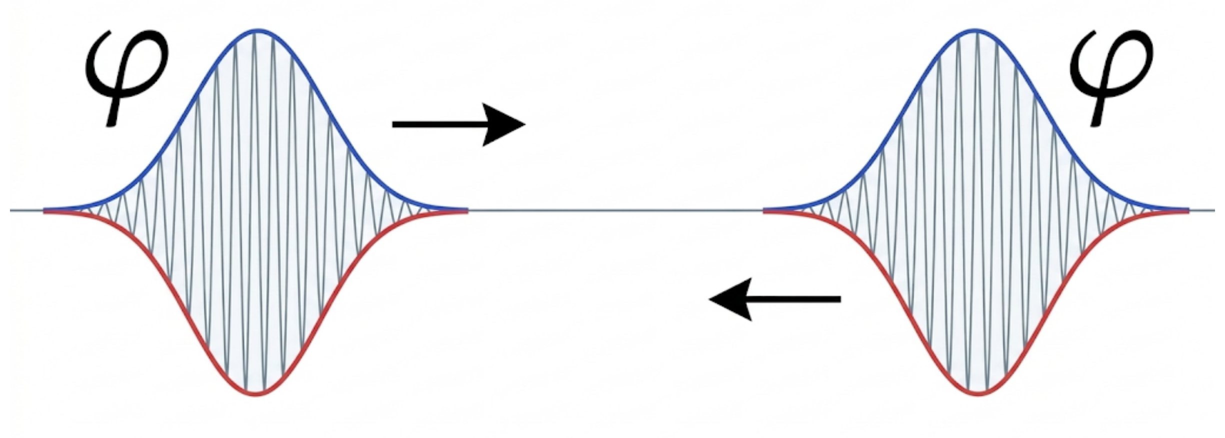


Summary





- **Hybrid formalism to map bosons to Qumodes and fermions to Qubits for lattice gauge theories**
    - Feasibility with different existing hybrid platforms
    - Ground state preparation, Time evolution, etc…
    - Scattering process, hadronization, structure, etc…

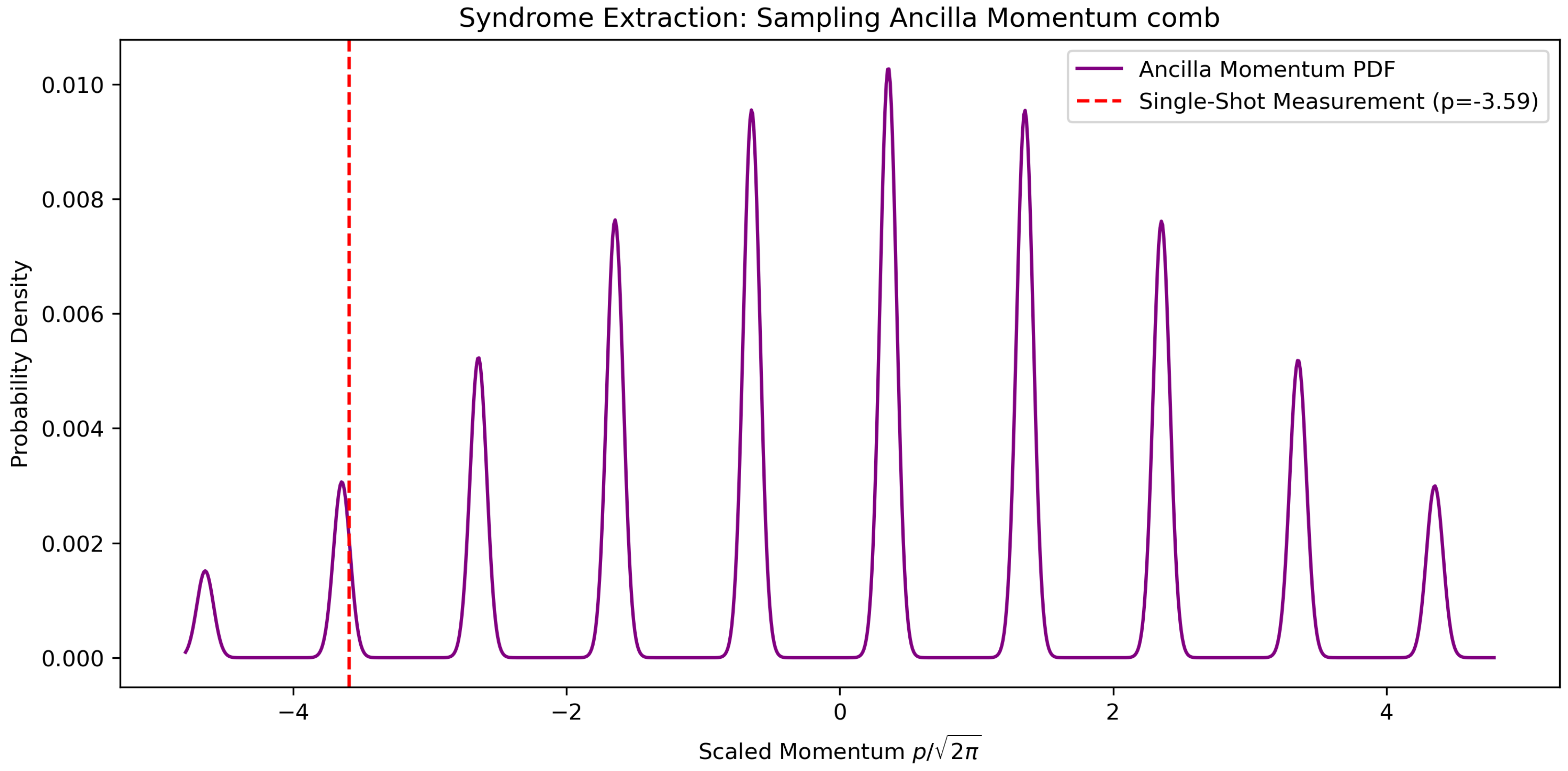


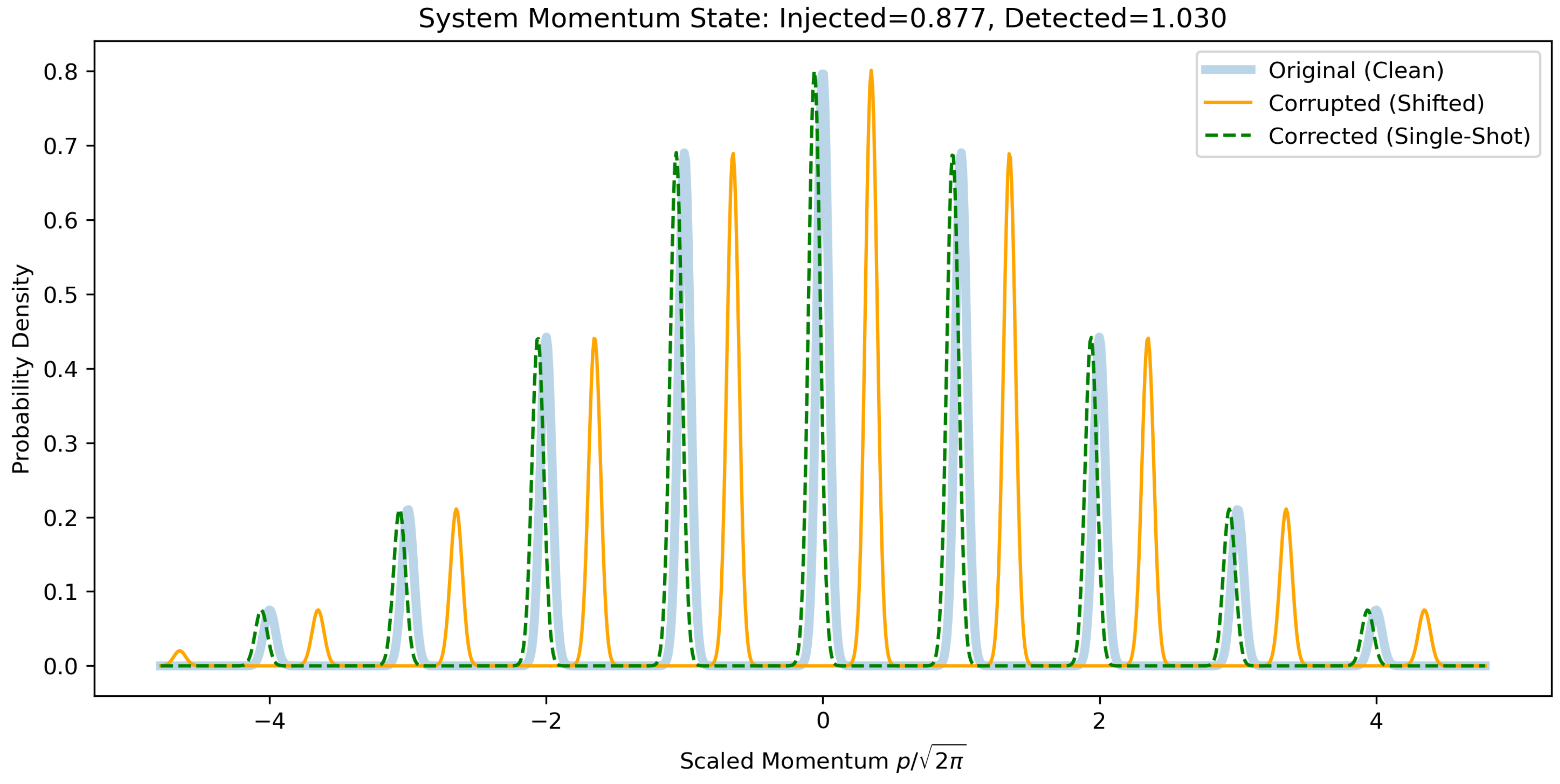
- **Explicit example for Abelian U(1) gauge theory with fermions in any dimension**
    - Extensions to non-Abelian theories are possible (SU(2) Ringer et al JHEP06(2025)084)
- **New class of gates: non-polynomial (trigonometric) gates (important for physics and much more)**
- **Open quantum systems: physical & error models**

Thank you

Extra slides

Error correction





Syndrome extraction with:

Ancilla qubit register or mode

Non demolition measurement

Syndrome correction

Peak recentered

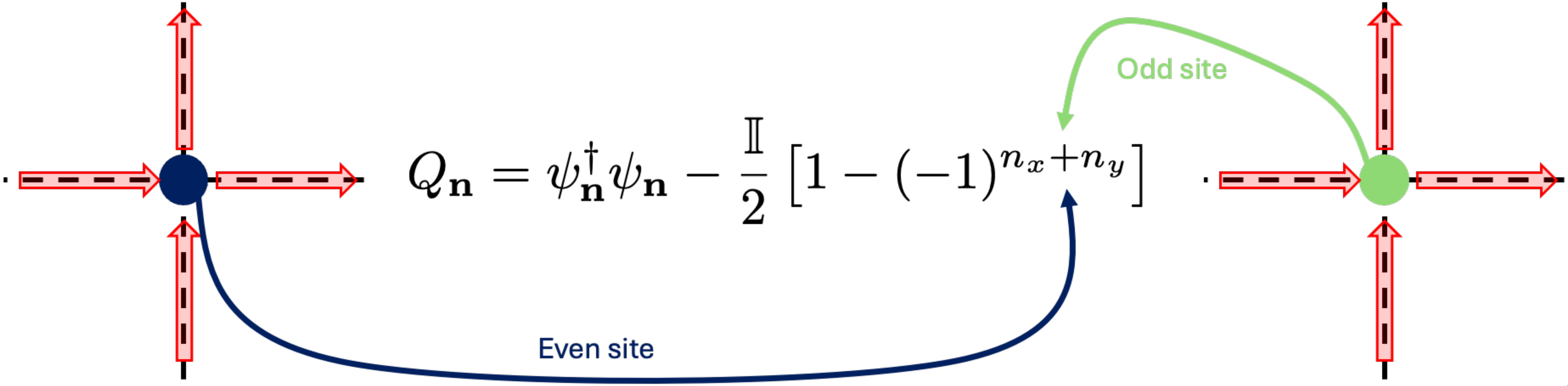
$$|n\sqrt{2\pi} + \delta\rangle_s \xrightarrow{\text{Feedback}} |n\sqrt{2\pi}\rangle_s$$

Gauss Law

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$E_{\mathbf{n},\hat{e}_x} + E_{\mathbf{n},\hat{e}_y} - E_{\mathbf{n},-\hat{e}_x} - E_{\mathbf{n},-\hat{e}_y} = Q_{\mathbf{n}}$$

$$Q_{\mathbf{n}} = \psi^{\dagger}_{\mathbf{n}}\psi_{\mathbf{n}} - \frac{\mathbb{I}}{2}\left[1 - (-1)^{n_x + n_y}\right]$$

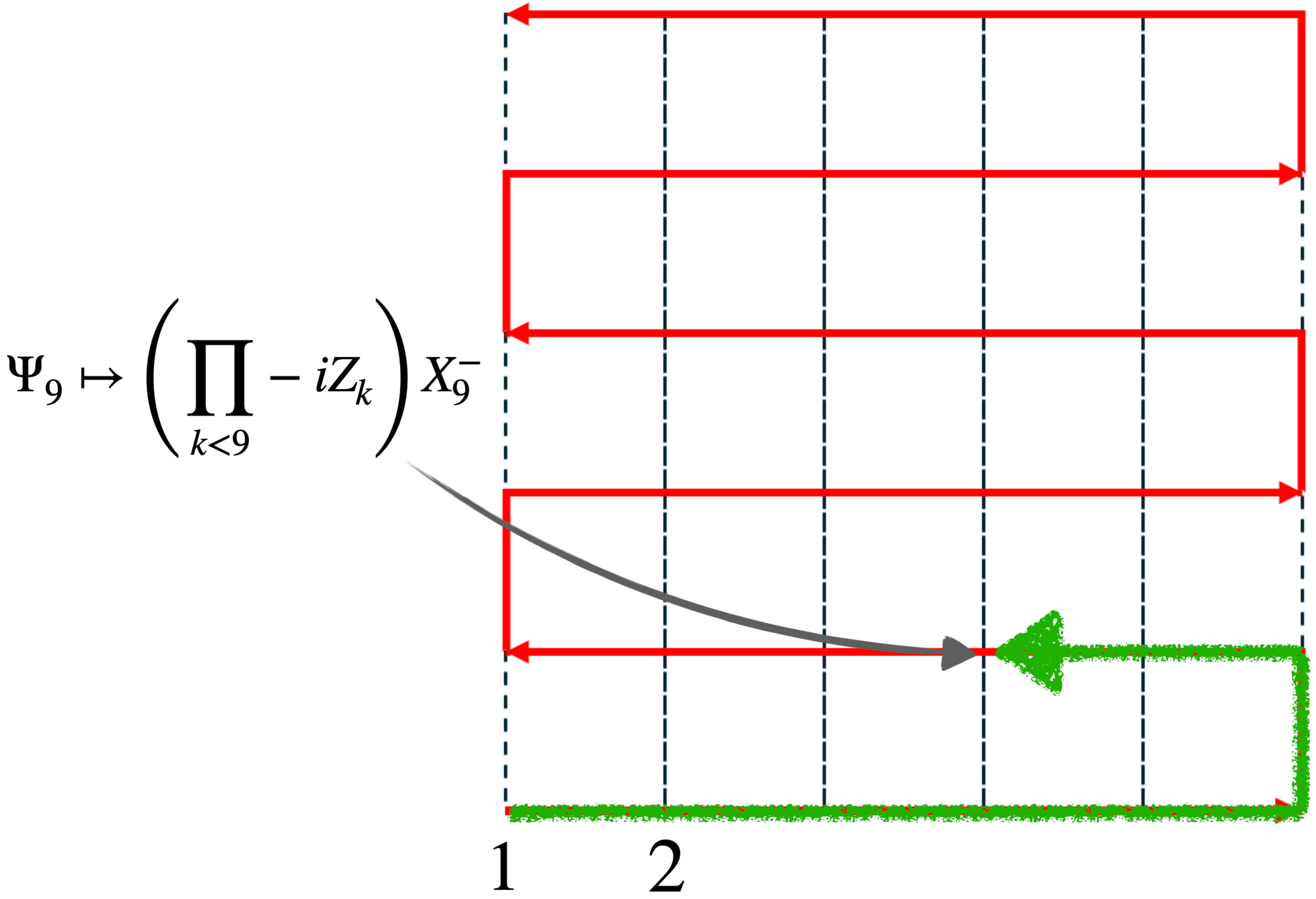


Mapping fermions to qubits

The Jordan-Wigner transform (not unique or the only one)

$$\psi_j \mapsto \left(\prod_{k<j} -i\,Z_k \right) \sigma_j^-$$

$$\psi_j^\dagger \mapsto \left(\prod_{k<j} +i\,Z_k \right) \sigma_j^+$$

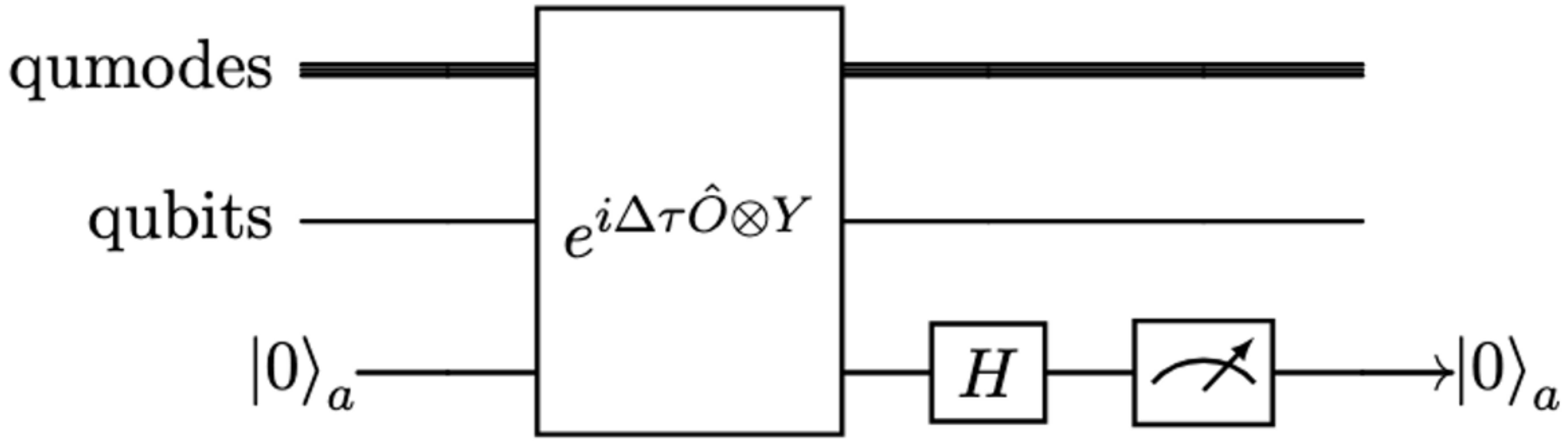


QITE method A

Zero qubit state

$$ {}_a\langle 0 | H_a e^{i\Delta\tau \hat{O} \otimes Y_a} | 0 \rangle_a = \frac{1}{\sqrt{2}} e^{-\Delta\tau \hat{O}} + \mathcal{O}((\Delta\tau)^2) $$

Need one ancilla (extra) qubit + postselection (measurement)



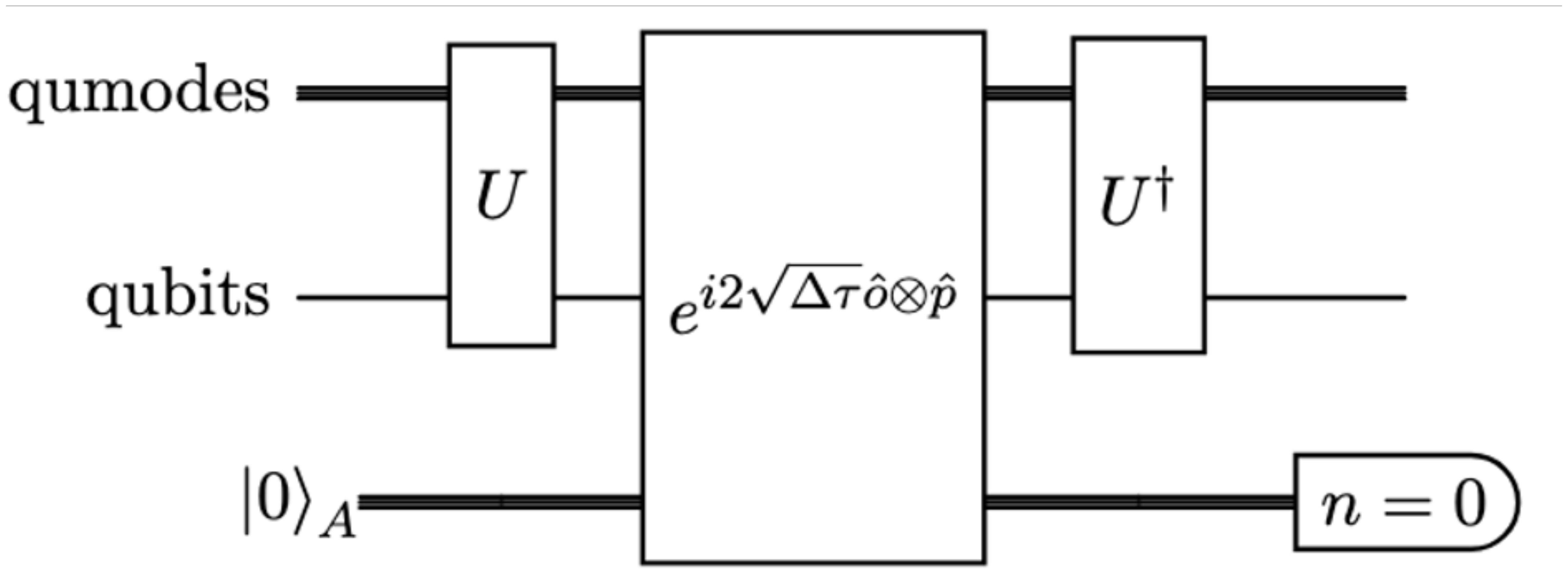
QITE method B

Zero Fock state

$$ {}_A\langle 0 | e^{i2\sqrt{\Delta\tau}\hat{o}\otimes\hat{p}_A} | 0 \rangle_A \propto e^{-\Delta\tau\hat{o}^2} $$

$$ U^\dagger e^{-\Delta\tau\hat{o}^2} U = e^{-\Delta\tau\hat{O}} $$

Need one ancilla (extra) qubit + postselection (measurement)

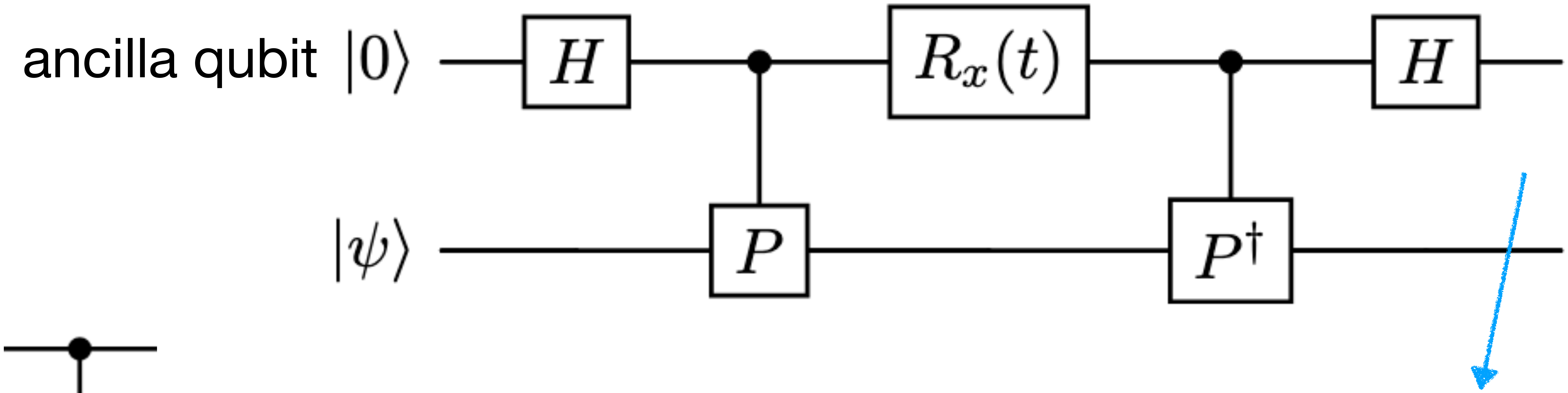


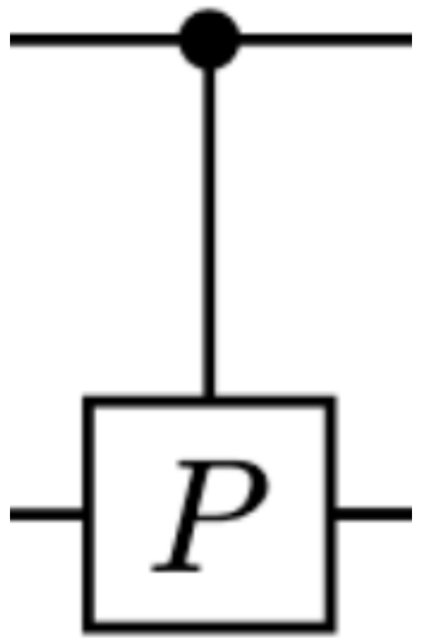
Building trigonometric gates

Learning from qubit exponentiation

Any Hermitian unitary operator P can be exponentiated using one ancilla qubit

!! It naively works only for qubits !!





$$= \mathbb{1} \otimes |+\rangle_a\langle+|_a + P \otimes |-\rangle_a\langle-|_a$$

$$e^{-i\frac{t}{2}P\otimes Z_a}|\psi\rangle|0\rangle_a = e^{-i\frac{t}{2}P}|\psi\rangle|0\rangle_a$$

Building trigonometric gates

Extending and expanding the logic to bosonic unitaries

Embed the bosonic unitary U into a larger Hilbert space
by entangling it with an ancilla qubit to make it Hermitian

We only need to be able to create controlled-U to make Σ

$$U = e^{i\hat{A}}$$

$$\Sigma = e^{i\hat{A}\otimes X} \cdot (I \otimes Z) = \frac{1}{2}\begin{pmatrix} U + U^\dagger & U^\dagger - U \\ U - U^\dagger & -U - U^\dagger \end{pmatrix} = \cos(\hat{A}) \otimes Z + \sin(\hat{A}) \otimes Y$$

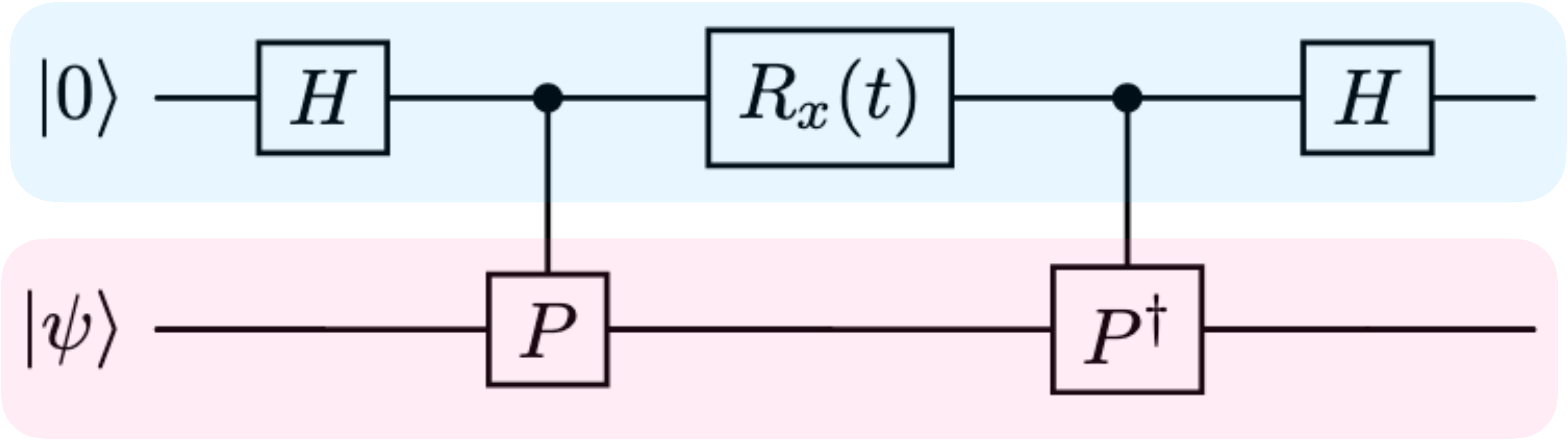
$$\bar{\Sigma} = (I \otimes Z) \cdot e^{i\hat{A}\otimes X} = \frac{1}{2}\begin{pmatrix} U + U^\dagger & -U^\dagger + U \\ -U + U^\dagger & -U - U^\dagger \end{pmatrix} = \cos(\hat{A}) \otimes Z - \sin(\hat{A}) \otimes Y$$

*This is just one example of many possible ones

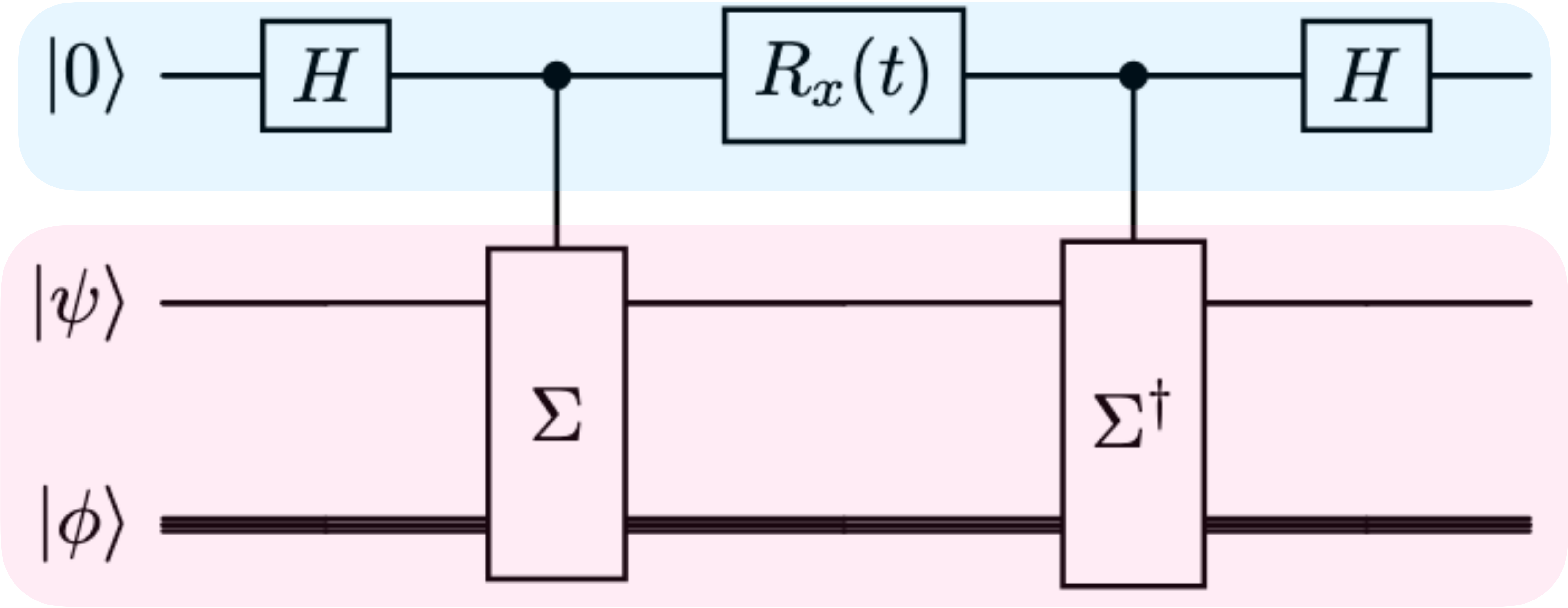
Building trigonometric gates

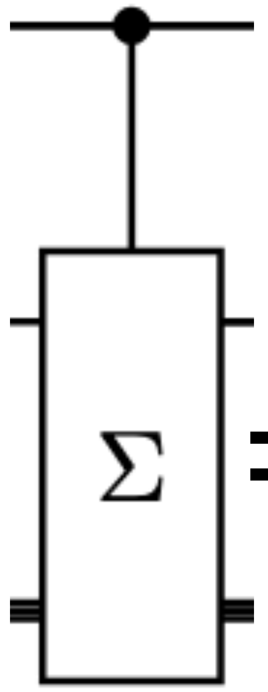
Now we can exponentiate Σ and/or $\overline{\Sigma}$ using the ancilla trick

Exponentiation of Pauli string



Exponentiation of hybrid Hermitian unitary



$C\Sigma =$ 

$= e^{i\hat{A}\otimes X_a\otimes\Pi_-^b}\left(\mathbb{I}\otimes CZ_{ab}\right)$

The circuit produces:

$$e^{it\Sigma\otimes Z}\,|\phi\rangle\,|\psi\rangle\,|0\rangle$$

Building trigonometric gates

Last step

Since it is $\Sigma + \overline{\Sigma} = 2\cos\hat{A} \otimes Z,$ $\Sigma - \overline{\Sigma} = 2\sin\hat{A} \otimes Y$

It follows that $$e^{i\frac{t}{2}\Sigma\otimes Z}e^{i\frac{t}{2}\overline{\Sigma}\otimes Z} = e^{it\cos(\hat{A})\otimes Z\otimes Z} + \mathcal{O}(t^2)$$

and $$e^{i\frac{t}{2}\Sigma\otimes Z}e^{-i\frac{t}{2}\overline{\Sigma}\otimes Z} = e^{it\sin(\hat{A})\otimes Y\otimes Z} + \mathcal{O}(t^2)$$