Exotic Quarkonia in Heavy-ion Collisions (2-6 Feb 2026, Mainz, Germany)

Recent progress in theoretical treatment of quarkonia production in AA collisions

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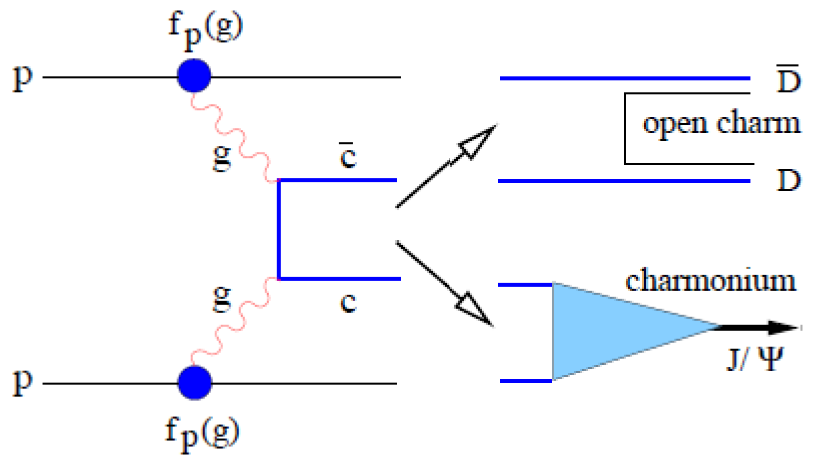
and Pays de la Loire

Common semi classical assumption in the quarkonia community

Early decoupling between various states



Equilibrium
(detailed balance)

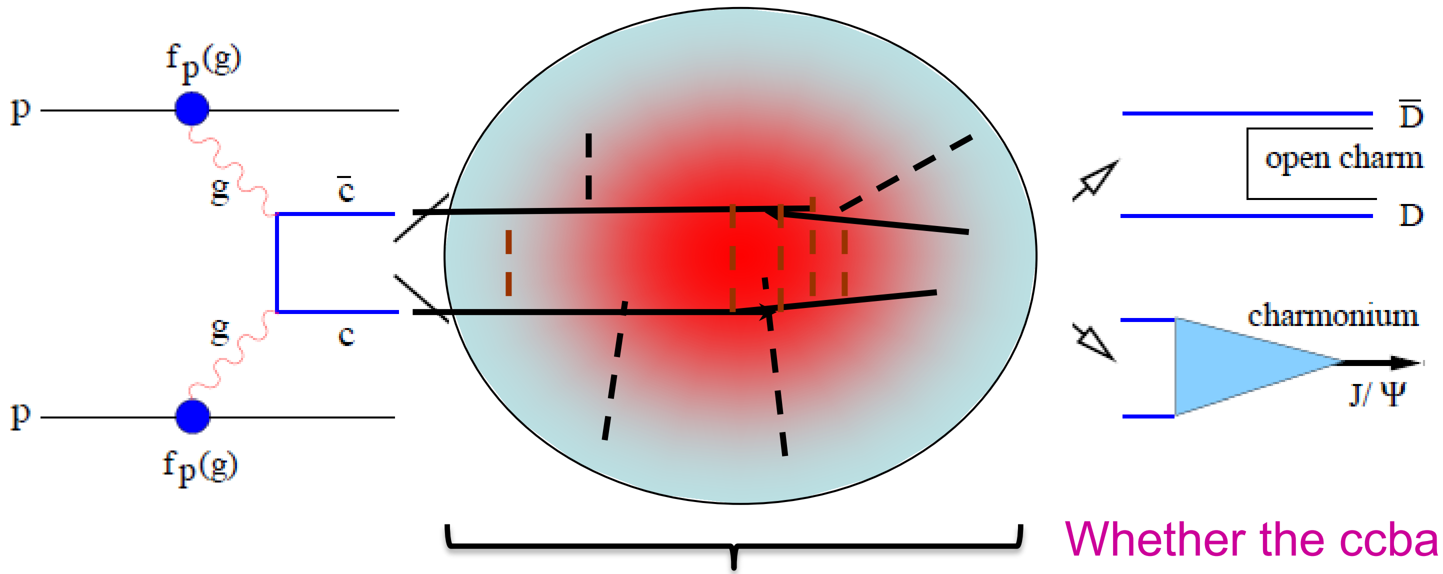
And then, “simple” rate equations :

$$\frac{dN_\Phi(t)}{dt} = -\Gamma(T(t))\,(N_\Phi(t) - N_\Phi^{\rm eq}(T(t)))$$

Appears to be sufficient for most of the phenomenology !

Beyond the semi classical assumption

Reality ?



Very complicated QFT

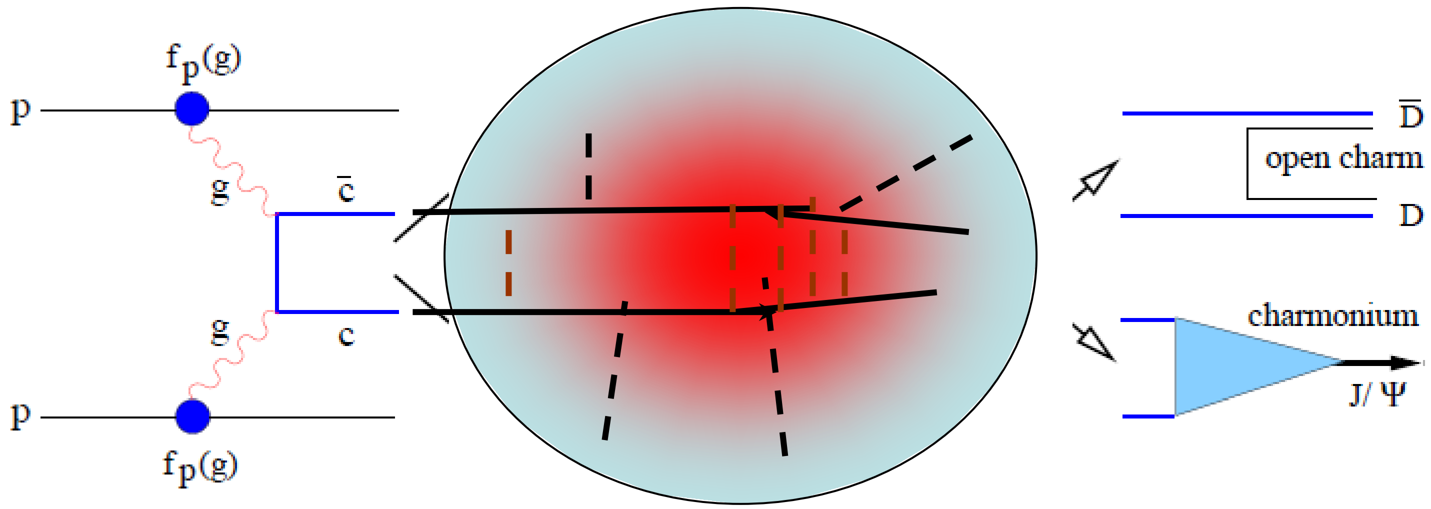
problem at finiteT(t) !!!

Whether the ccbar pair emerges as a bound quarkonia or as DDbar pair is only resolved at the end of the evolution

But one should aim at solving it, especially as the quarkonia content of a QQbar quantum state is at most of the order of a few % (continuous transitions under external perturbations)

Beyond the semi classical assumption

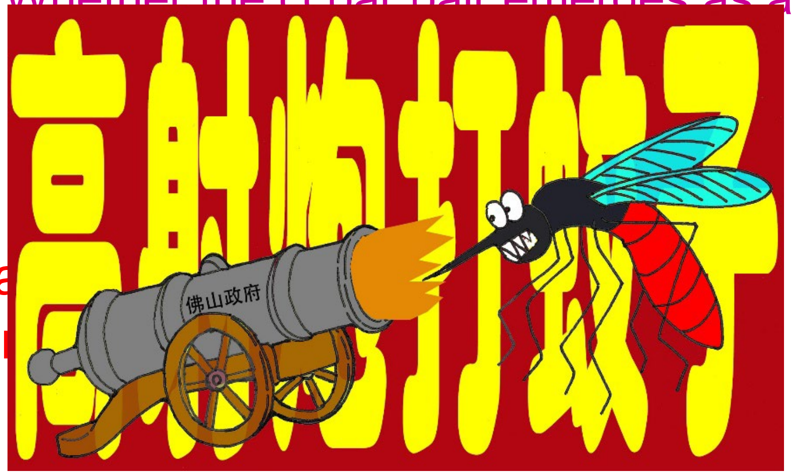
Reality ?



Very complicated QFT

problem at finiteT(t) !!!

Whether the ccbar pair emerges as a

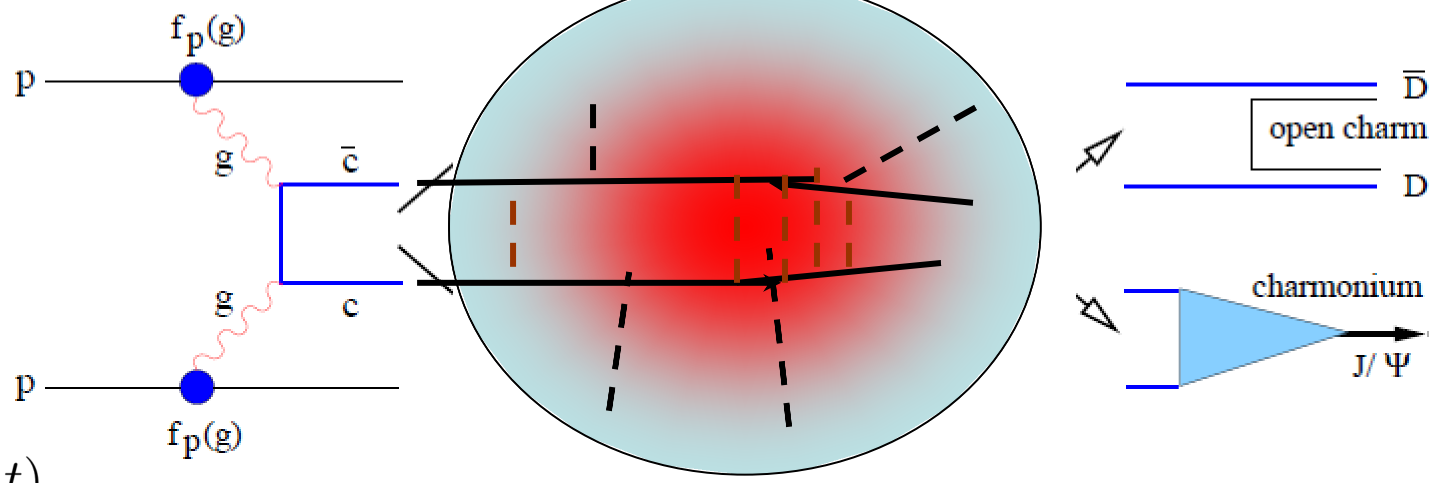


But one should aim at solving it, especially as the qua
quantum state is at most of the order of a few % (co
external perturbations)

Beyond the semi classical assumption

Reality ?

Textbook physics



$$\frac{\partial \rho_S(\mathbf{x},\mathbf{x}',t)}{\partial t} = -F(\mathbf{x}-\mathbf{x}')\rho_S(\mathbf{x},\mathbf{x}',t)$$

Decoherence factor:

Short wave length (λ=1/T<< Δx=1/mQ)

$$F(\mathbf{x}-\mathbf{x}') = \Gamma_{\rm tot}$$

Total collision rate

Long wave length (1/T >> Δx=1/mQ)

$$F \approx \int dq\rho(q)v(q)q^2\sigma_{\rm transp}(q) \times (\mathbf{x}-\mathbf{x}')^2 \approx \kappa(\mathbf{x}-\mathbf{x}')^2 \approx \frac{\kappa}{m_Q^2}$$

$$\tau_{\rm decoh} \approx \frac{m_Q^2}{\kappa}$$ A couple of fm… of the order of the QGP size

$$\kappa \approx 1{\rm GeV}^2/{\rm fm}$$

A special Quantum Master Equation: The Lindblad Equation

There are many different QME… a special one :

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = \boxed{-i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right]} + \sum_i \boxed{\gamma_i}\left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2}\left\{L_i L_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

γ_i Characterize the coupling of the system (Q-Qbar) with the environment

$H_{Q\bar{Q}}$: $\{Q, \bar{Q}\}$ $\underbrace{\text{kinetics + Vacuum potential V}}_{\hat{H}^{(0)}_{Q\bar{Q}}}$ + Lamb shift / screening (every unitary term that is generated by tracing out the environment ⇔ Von Neumann)

L_i : Collapse (or Lindblad) operators, depend on the properties of the medium

3 important conservation properties :

$\rho^\dagger_{Q\bar{Q}} = \rho_{Q\bar{Q}}$	$\mathrm{Tr}[\rho_{Q\bar{Q}}] = 1$	$\langle\varphi\|\rho_{Q\bar{Q}}\|\varphi\rangle > 0, \forall\|\varphi\rangle$
(Hermiticity)	(Norm)	(Positivity)

… but in general, non unitary !!! (relaxation)

A special Quantum Master Equation: The Lindblad Equation

Non unitary / dissipative evolution ≡ decoherence

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)] + \sum_i \gamma_i \left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2}\left\{ L_i L_i^\dagger, \rho_{Q\bar{Q}}(t) \right\} \right]$$

Genuine transitions :

- ✓ Singlet <-> octet
- ✓ Octet <-> octet

Can be reshuffled into non Hermitic effective hamiltonian

$$\hat{H}_{Q\bar{Q},\mathrm{eff}} = \hat{H}_{Q\bar{Q}} - i\sum_j \gamma_j \frac{L_j L_j^\dagger}{2}$$

≡ Dissociation width

Indeed, starting from a singlet density matrix : $\rho^{\mathrm{s}}_{Q\bar{Q}} = |s\rangle\langle s|$, one generates an octet composant :

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle o|\rho_{Q\bar{Q}}|o\rangle = \sum_i \gamma_i \langle o|L_i|s\rangle\langle s|L_i^\dagger|o\rangle = \sum_i \gamma_i |\langle o|L_i|s\rangle|^2$$

Usual transition rate

Nice feature : Can be brought to the form of a stochastic Schroedinger equation (quantum jump method : QTRAJ) => much more effective numerically

A special QME: The Lindblad Equation

Starting from generic $\hat{H} = \hat{H}^{(0)}_{Q\bar{Q}} + \hat{H}_E + \hat{H}_{\rm int}$ with $\hat{H}_{\rm int} = \sum A_{\rm QGP} \times S$ how to obtain a Linblad equation ?

QGP operator (field) Q-Qbar operator (charge)

Also named n^a in the rest of the talk

Inspired from Yao Int. J. of Mod. Phys. A, Vol. 36, No. 20, 2130010 (2021)

Pictorial summary

τ_E: environment autocorrelation time **τ_R: system (quarkonium) relaxation time**

Subsystem + environment: von Neumann equation

↓ **Iterate Von Neumann eq. + Trace out environment**

Subsystem: non-unitary, time-irreversible evolution

↓ **Weak syst-environment coupling + Markovian limit $\tau_E \ll \tau_R$** Ok QCD : $\tau_R \propto m_Q^2$

Redfield equation

A-A correlator

$$\frac{\partial}{\partial t}\rho_I(t) = -\frac{1}{\hbar^2}\sum_{m,n}\int_0^\infty d\tau\left(C_{mn}(\tau)\left[S_{m,I}(t), S_{n,I}(t-\tau)\rho_I(t)\right] - C^*_{mn}(\tau)\left[S_{m,I}(t), \rho_I(t)S_{n,I}(t-\tau)\right]\right)$$

Some similitude with the Lindblad equation but with time delay effects => **Not Lindbladian**

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right] + \sum_i \gamma_i\left[L_i\rho_{Q\bar{Q}}(t)L_i^\dagger - \frac{1}{2}\left\{L_iL_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

QCD time scales

τ_E: environment autocorrelation time

$$\tau_E \approx \frac{1}{m_D} \approx \frac{1}{CT} \approx \frac{1}{T}$$ (C taken as close to unity)

τ_S: system intrinsic time scale

$$\tau_S \approx \underbrace{\frac{1}{\Delta E}} \approx \frac{1}{m_Q v^2}$$ with $v \approx \alpha_S$ ... at the beginning of the evolution

Difference between energy levels

τ_R: system relaxation time

$$\Gamma = \tau_R^{-1} \sim 2\langle\psi|W\psi\rangle \approx \alpha_s T \times \Phi(m_D r) \approx \alpha_s T \times \Phi(\frac{CT}{m_Q \alpha_s})$$

At "small" T $\left(T \lesssim \frac{m_Q \alpha_S}{C}\right)$: dipole approximation : $\Gamma = \tau_R^{-1} \approx \frac{C^2 T^3}{\alpha_s m_Q^2}$

$$\frac{\tau_R}{\tau_E} = \frac{\alpha_s m_Q^2}{CT^2} \gg 1$$ And $$\frac{\tau_R}{\tau_S} = \frac{\alpha_s^3 m_Q^3}{C^2 T^3} \gg 1$$ for $T \lesssim m_Q \frac{\alpha_S}{C^{2/3}}$

Fine with the Markovian assumption (vanilla calculations; more to come)

Inspired from Yao Int. J. of Mod. Phys. A, Vol. 36, No. 20, 2130010 (2021)

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Redfield equation

Smallest time scales wins it all !

$\tau_S \ll \tau_R$

Quantum Optical Regime

Davies secular equation

Lindblad equation

Eigenstates of the HQ Hamiltonian

$\omega \approx E_n - E_m$

Not the same basis !

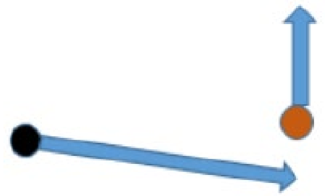
$\tau_E \ll \tau_S$

Quantum Brownian Motion

Lindblad equation

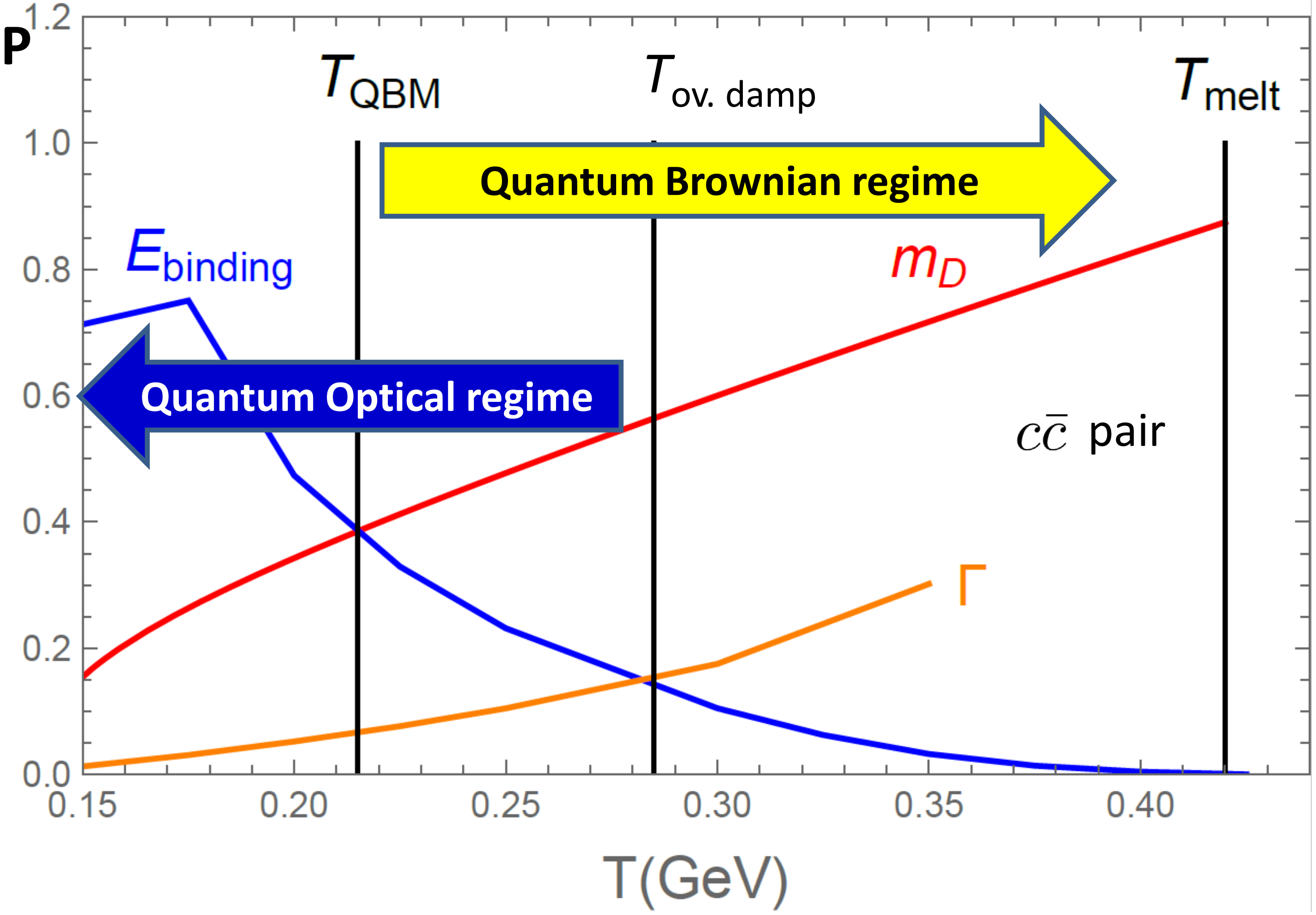
Caldeira Leggett

Phase space densities

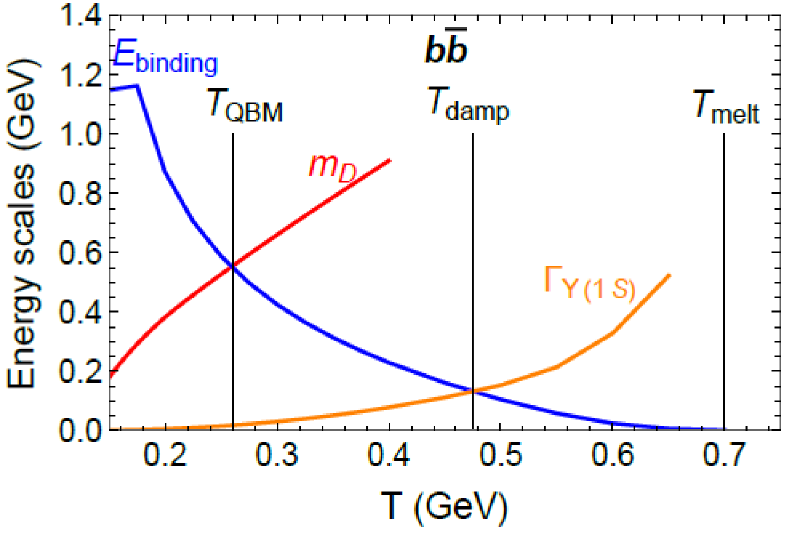


Two types of dynamical modelling

SLOW QGP

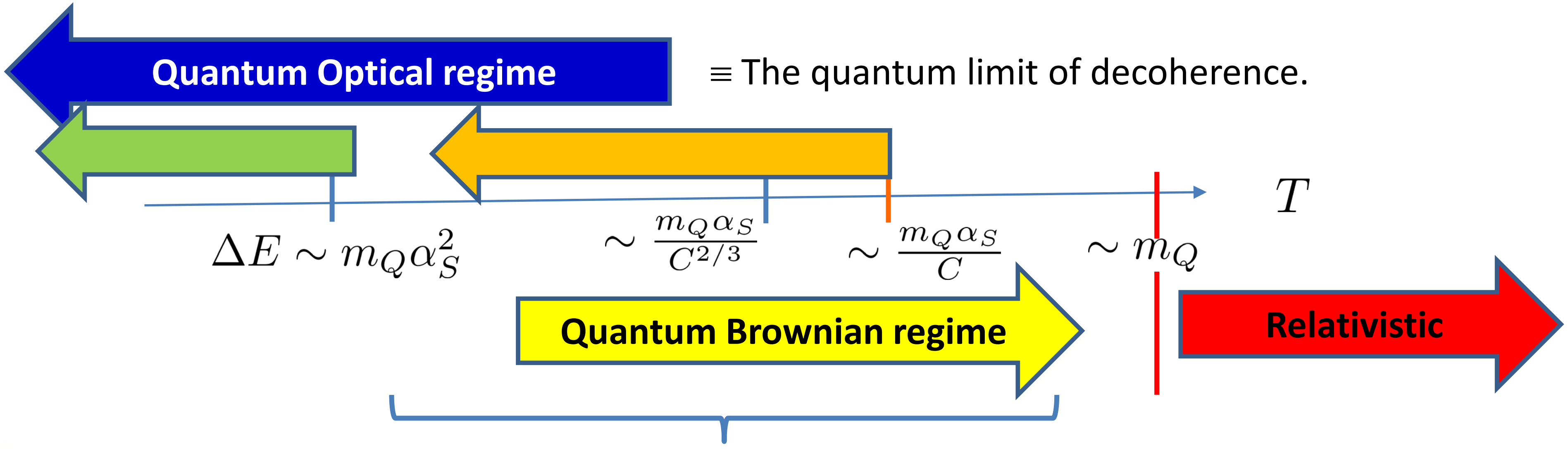


FAST QGP



Numbers extracted from a specific potential model : Katz et al, Phys. Rev. D 101, 056010 (2020)

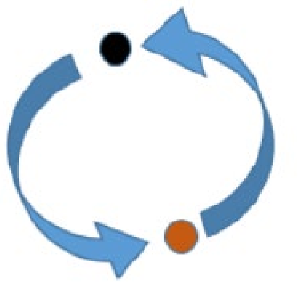
QCD Temperature scales



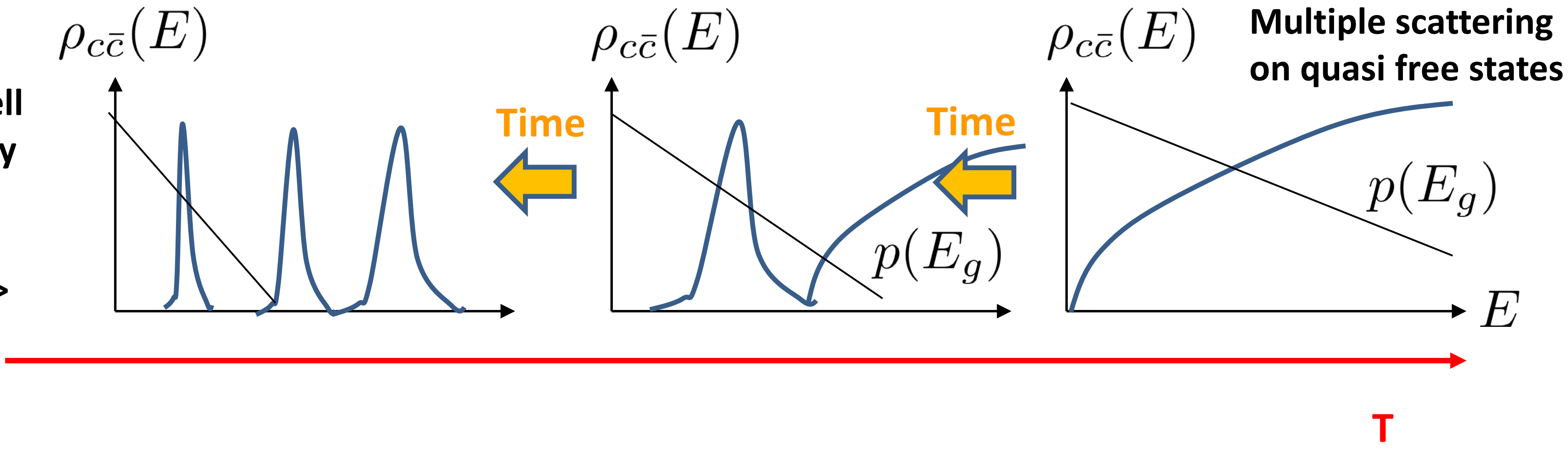
≡ The quantum limit of decoherence.

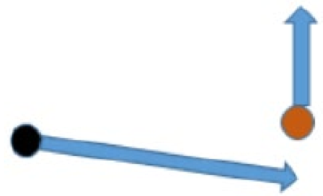
For these « large » temperatures, the Q-Qbar gain enough energy to overwhelm the real binding potential => larger distance => larger decoherence ….

≡ The intermediary regime.



dissociation of well identified levels by scarce "high-energy" modes (dilute medium => cross section ok)





On the menu

- A short tour in the quantum Brownian regime (S.Delorme et al. JHEP 06 (2024) 060) : lessons and issues
- One step beyond (Universal Lindblad Equation) : in preparation
- ~~The semi classical approximation~~ Phys. Rev. D 113 (2026), 014017: see talk by JPB

Non abelian Quantum Master Equation for Qs (Nantes Saclay)

$$i\frac{d\mathcal{D}}{dt} = [\mathcal{H}, \mathcal{D}]$$

Interaction representation →

$$i\frac{d\mathcal{D}^I(t)}{dt} = [\mathcal{H}_1(t), \mathcal{D}^I(t)]$$

$$\mathcal{H} = \mathcal{H}_Q + \mathcal{H}_1 + \mathcal{H}_{pl}$$

Coulomb gauge

Free Quark Hamiltonian

Quark-Plasma Interactions…

Plasma Hamiltonian

$$H_1 = -g \int_{\boldsymbol{r}} A_0^a(\boldsymbol{r}) n^a(\boldsymbol{r})$$

No magnetic term (NR)

color charge density of the heavy particles

… treated as a perturbation

$$\mathcal{D}^I(t) = \mathcal{U}_I(t, t_0)\mathcal{D}(t_0)\mathcal{U}_I^\dagger(t, t_0)$$

Average over plasma d.o.f + rapid environment hypothesis

Generic Linblad – like QME on $\mathcal{D}_Q$

$$\frac{d\mathcal{D}_Q^I(t)}{dt} = -g^2 \int_{t_0}^{t} dt' \int_{\boldsymbol{x}\boldsymbol{x}'} \left([n^A(t,\boldsymbol{x}), n^A(t',\boldsymbol{x}')\mathcal{D}_Q^I(t')]\Delta^>(t-t',\boldsymbol{x}-\boldsymbol{x}') + [\mathcal{D}_Q^I(t')n^A(t',\boldsymbol{x}'), n^A(t,\boldsymbol{x})]\Delta^<(t-t',\boldsymbol{x}-\boldsymbol{x}') \right)$$

← $\Delta^>, \Delta^<$ Time ordered (HTL) gluon propagators ordered on the KS contour

Non abelian Quantum Master Equation for Qs (Nantes Saclay)

Series expansion in τ_E/τ_S

Compact form: $$\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\,\mathcal{D}_Q$$ with $$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$$

$$\mathcal{L}_0\,\mathcal{D}_Q \equiv -i[H_Q, \mathcal{D}_Q],$$

$$\mathcal{L}_1\,\mathcal{D}_Q \equiv -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q],$$

Mean field hamiltonian

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2 n^a_{\boldsymbol{x}} \mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

Fluctuations => decohence

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}]\right)$$

Dissipation

N.B. : Friction is NOT of the Linbladian form => the evolution breaks positivity.

Positivity and Linblad form can be restored at the price of extra subleading terms* :

$$\left\{\left(n^a_{\mathbf{x}} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\left(n^a_{\mathbf{x}'} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right), \mathcal{D}_{Q\bar{Q}}\right\} - 2\left(n^a_{\mathbf{x}} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\mathcal{D}_{Q\bar{Q}}\left(n^a_{\mathbf{x}'} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right)$$

$\mathcal{L}_2$

* As well as another time discretization

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$\mathcal{L}_3$

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Mean field hamiltonian

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2 n^a_{\boldsymbol{x}} \mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

Fluctua
decohe

External “ingredients” : complex potential V + I W

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}]\right)$$

Dissipation

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$\mathcal{L}_4$

Application to QED-like and QCD for both cases of 1 body and 2 body densities…

Non abelian Quantum Master Equation for a QQbar pair (Nantes Saclay)

singlet density matrix

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s\\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{r}_{rel},\mathbf{r}'_{rel},t)\\ \mathcal{D}_o(\mathbf{r}_{rel},\mathbf{r}'_{rel},t)\end{pmatrix}$$

octet density matrix

$$\mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so}\\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$

singlet-octet transitions

$$\hat{\rho}_S = \mathcal{D}_{\rm s}|1\rangle\langle 1| + \mathcal{D}_{\rm o}\sum_a |o_a\rangle\langle o_a|$$

2 coupled color representations (singlet octet)

The Linblad Operator contains various terms representing several aspects of HQ physics

Unitary dynamics:

- $\mathcal{L}_0$: kinetic term
- $\mathcal{L}_1$: (screened) real potential term

Imaginary potential W → Non-Unitary dynamics:

- $\mathcal{L}_2$: fluctuations => heating and decoherence
- $\mathcal{L}_3$: dissipation
- $\mathcal{L}_4$: mandatory to preserve positivity (but sub-dominant)

J-P.Blaizot & M.A. Escobedo, JHEP 2018.6 (2018)

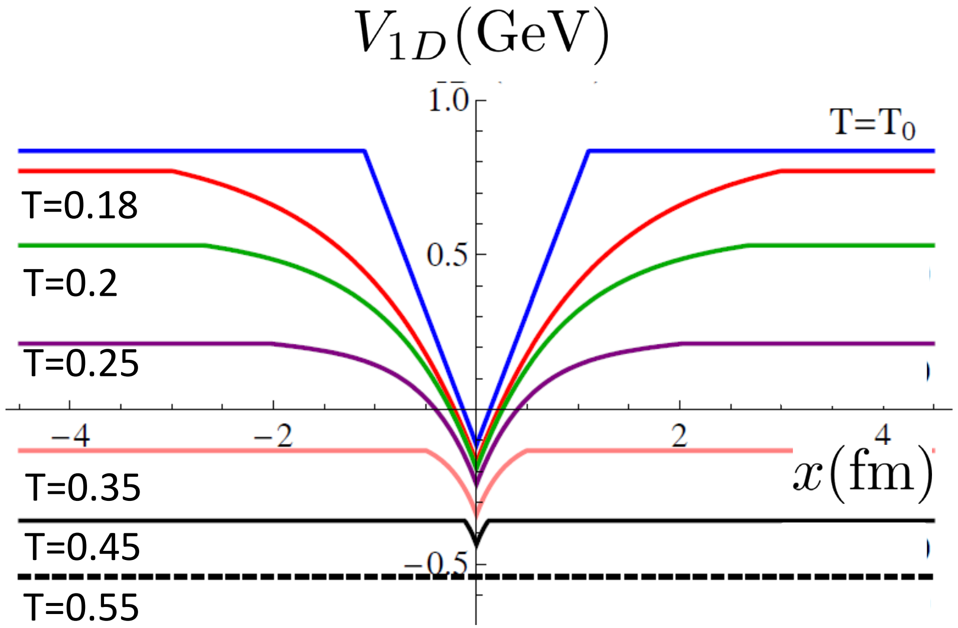
Further implementation features

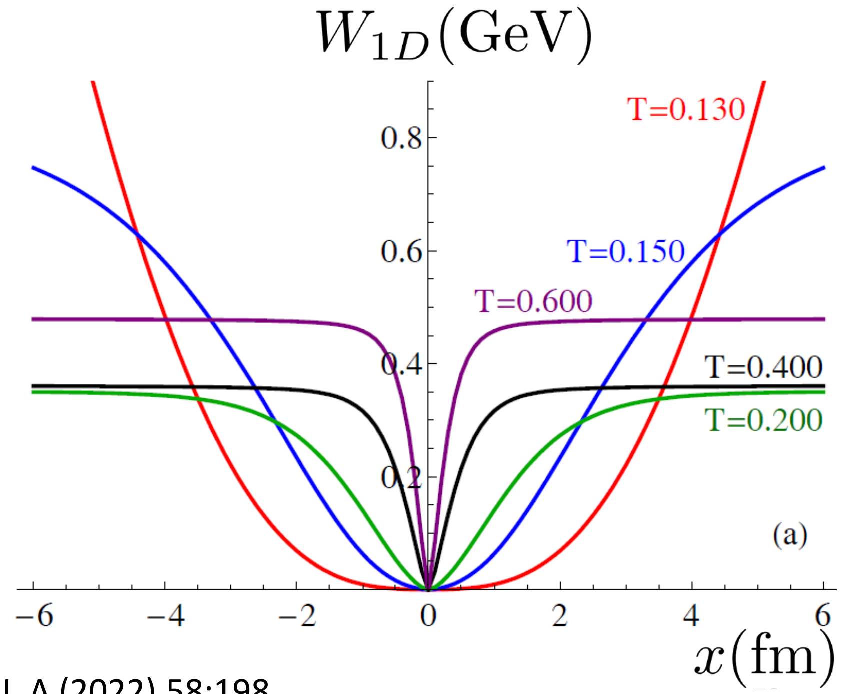
- **1D grid** for both $s \in [-s_{\max}, +s_{\max}]$ and $s' \in [-s_{\max}, +s_{\max}]$

!!! Not the radial decomposition of $\mathcal{D}_{c\bar{c}}(\vec{s}, \vec{s}\,')$ which is more cumbersome

Even states will be considered as « S like » while odd states will be considered as « P like » states

Need to design a realistic 1D bona fide potential V + i W (based on 3D lQCD results, tuned to reproduce 3D mass spectra and decay widths)



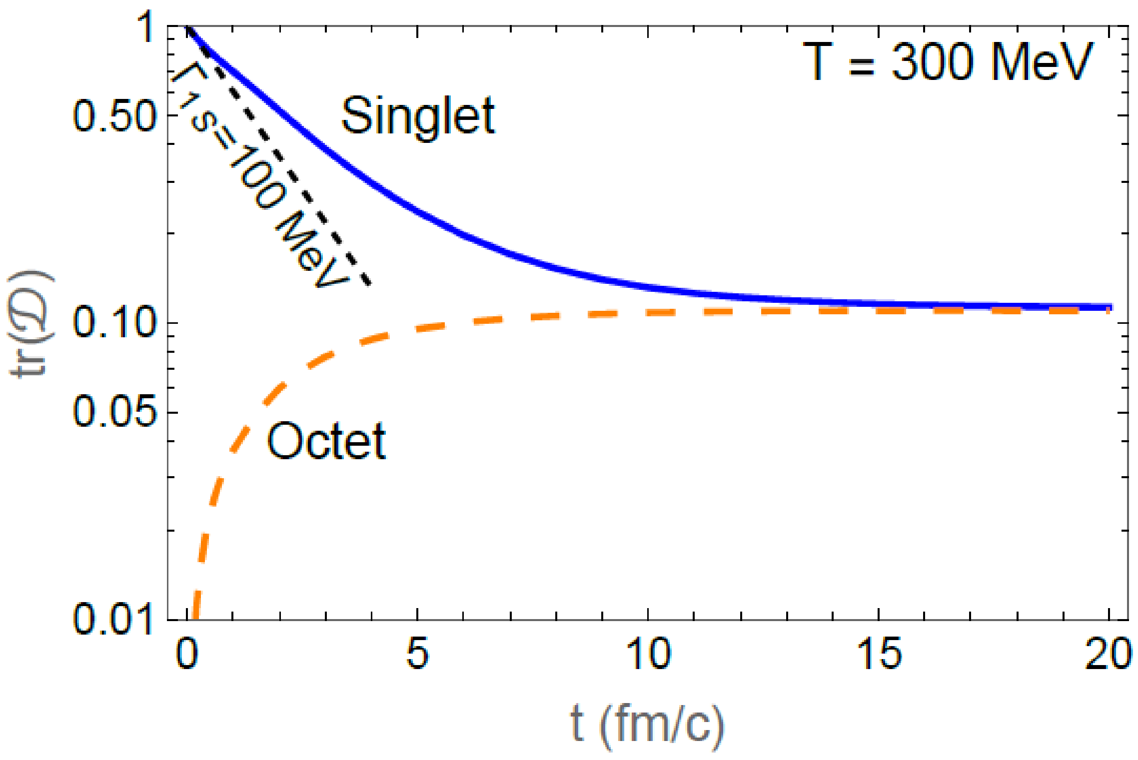


1D potential from R. Katz, S. Delorme & PBG, Eur. Phys. J. A (2022) 58:198

Some selected results for 1 c-cbar pair

Color Dynamics : Singlet – octet probabilities:

➢ Starting from a singlet 1S-like, one expects some equilibration / thermalisation -> asymptotic values : $D_{\rm s}^{\rm eq} = D_{\rm o}^{\rm eq} = \frac{1}{9}$ $\quad (1+8) \times \frac{1}{9}$

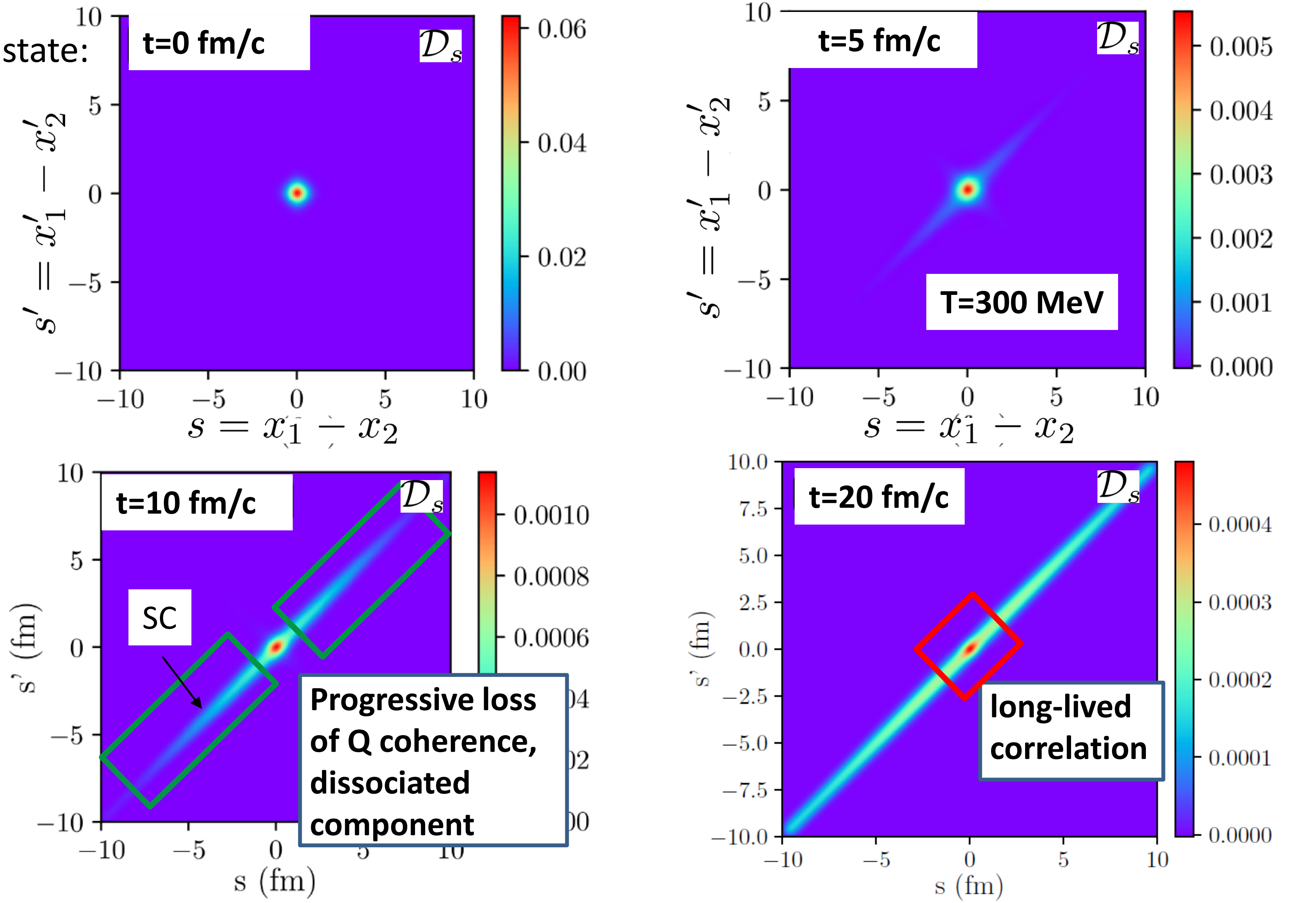


- At early times : Quasi exponential behaviour exp(-t/τ), with thermalisation time $\tau_o < \tau_s \approx 2$ fm/c
- Color appears to thermalize on time scales < QGP life time, but not instantaneoulsy.
- c-cbar can interact with the surrounding QGP as an octet => energy loss

S.Delorme et al. JHEP 06 (2024) 060

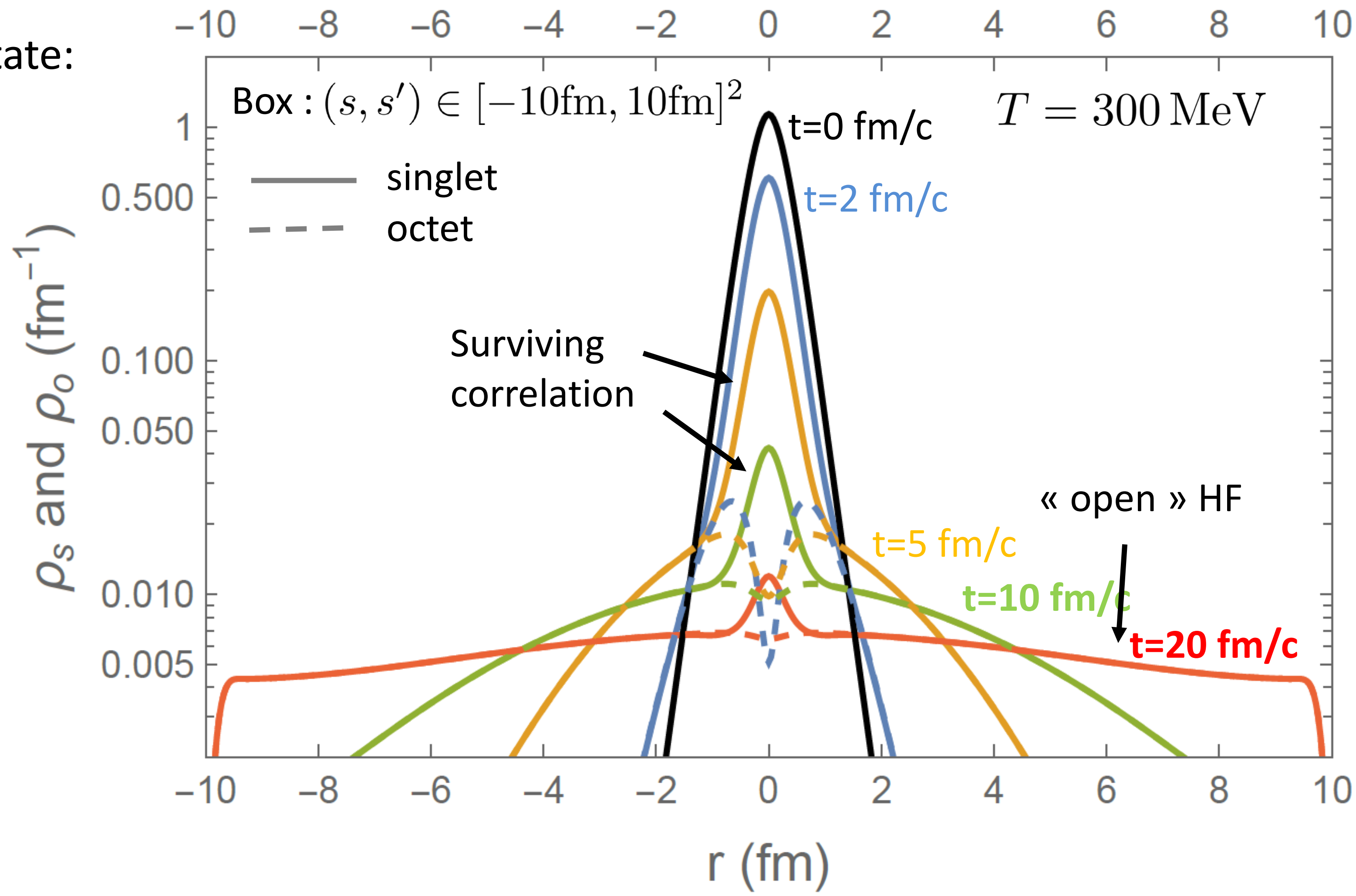
Evolution of the Density matrix

1S singlet initial state:



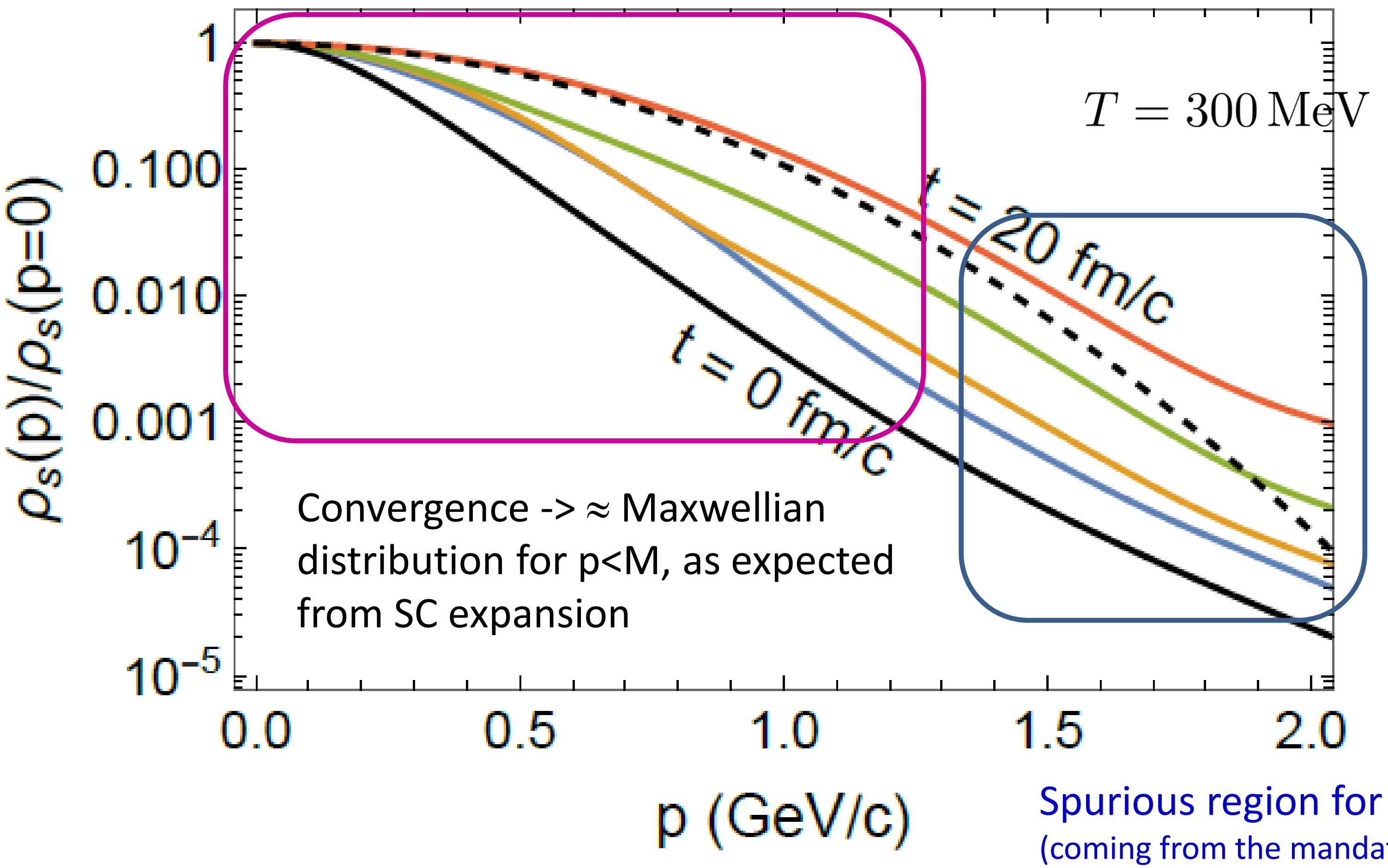
Evolution of the spatial density

1S singlet initial state:



Some c-cbar stay at intermediate distance ("recombination") … remaining peak in the asymptotic distribution

Evolution of the momentum density



Spurious region for p>M
(coming from the mandatory regularization of the imaginary potential…room for improvement)

Mostly sensitive to the distribution at large relative distance (individual c quarks)

Asymptotic distributions: Abelian (QED-like)

- Semi classical approximation : $W(r,p) \propto e^{-\frac{p^2}{m_Q T} - \frac{V(r)}{T}}$ (Wigner representation)

2 lines calculation :

$$\mathcal{L}_2 + \mathcal{L}_3 \to \frac{\mathcal{H}(r)+\mathcal{H}(0)}{2}\partial_p \left[\frac{\partial_p}{2} + \frac{p}{m_Q T}\right] W(r,p) \Rightarrow W(r,p) \propto e^{-\frac{p^2}{m_Q T}}$$

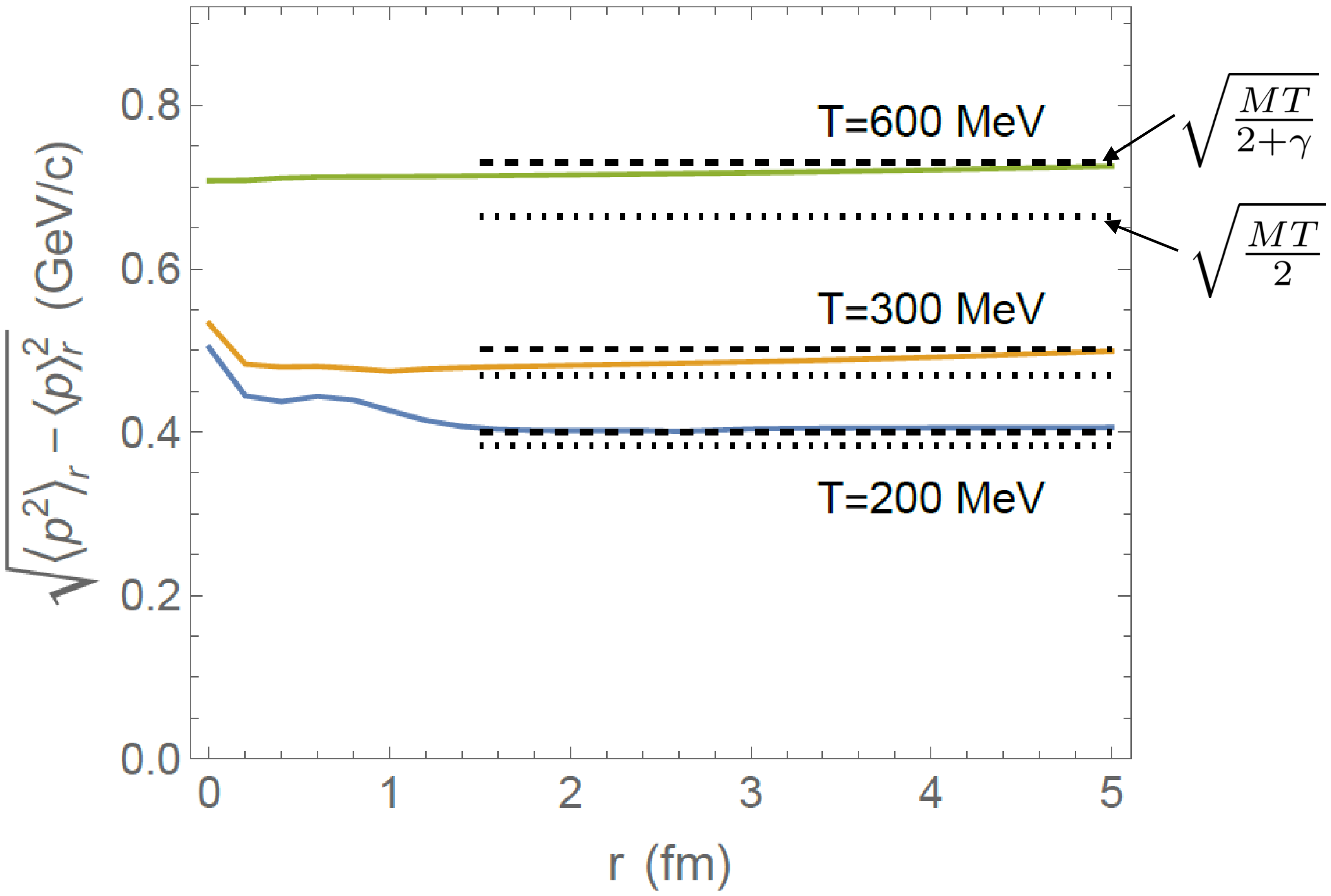
$$\mathcal{L}_0 + \mathcal{L}_1 \to \left(-\frac{p\,\partial_r}{m_Q} + \partial_r V(r)\partial_p\right) \Rightarrow W(r,p) \propto e^{-\frac{p^2}{m_Q T} - \frac{V(r)}{T}}$$

Introducing $\mathcal{L}_4$ seems to spoil the late – time Gibbs –Boltzmann !!!

A bit more subtle, though

Asymptotic distribution: QCD

- Rms relative momentum as a function of relative distance.

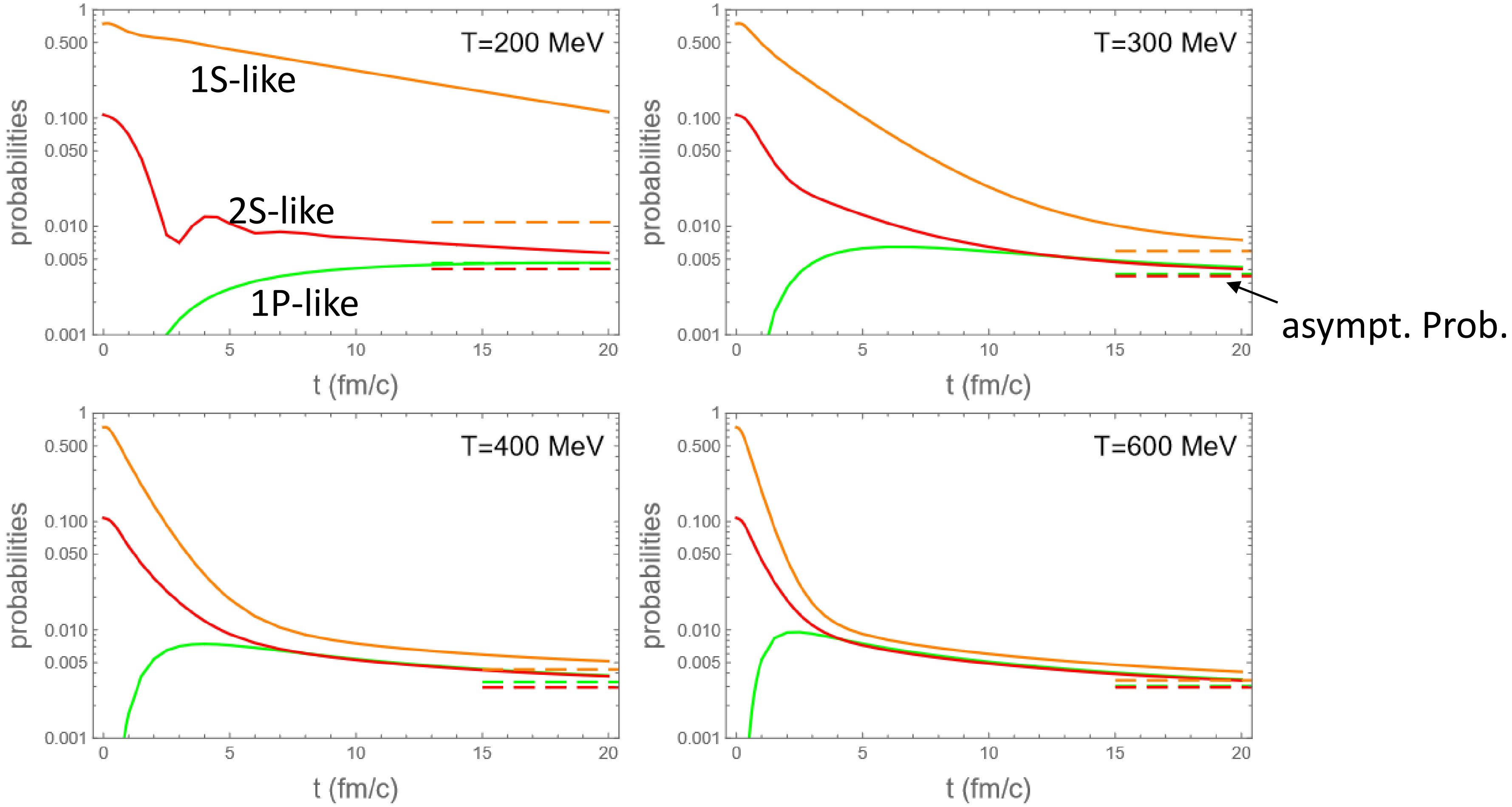


Slight modification of the effective temperature due to the L_4 term

$$\gamma = \frac{\tilde{W}^{(4)}(0)}{16MT\,\tilde{W}''(0)}$$

Results for projection on vacuum states

Starting from a compact S-like state : $\psi(s) \propto e^{-\frac{s^2}{2\sigma^2}}$ $\sigma = 0.165\,\mathrm{fm}$ $p_\Phi = \mathrm{tr}(\mathcal{D}_s D_\Phi)$

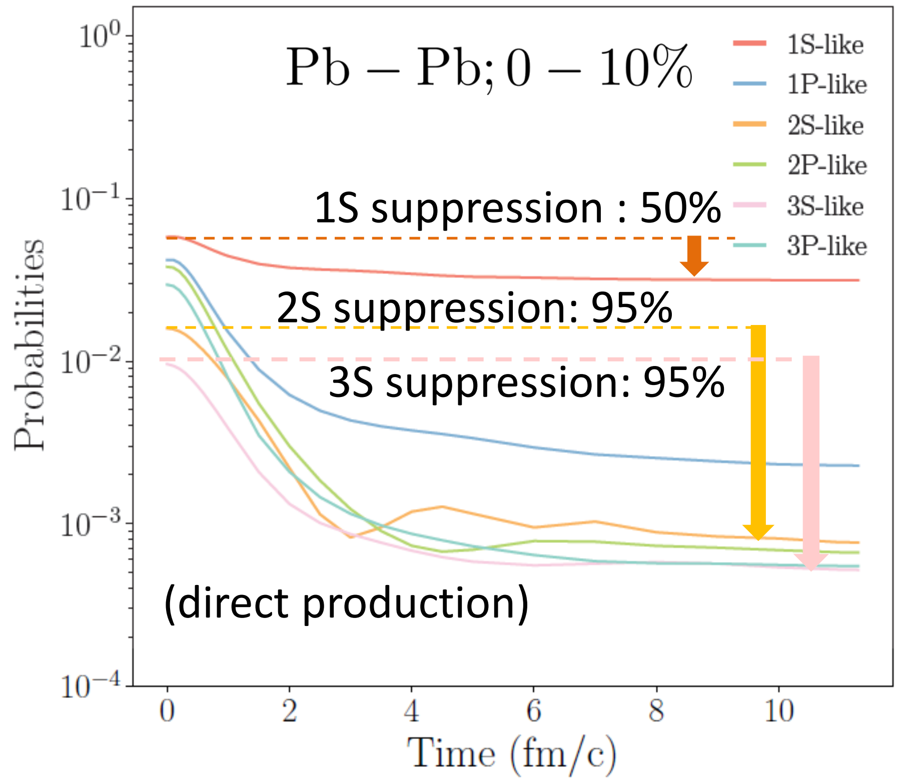


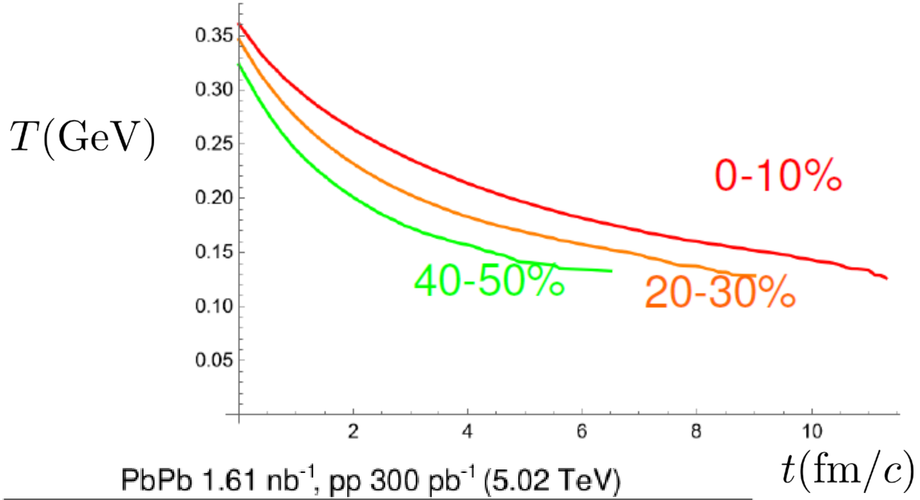
- Natural evolution for 1S-like suppression, from low to high T
- 2S state do not decay $\alpha e^{-\Gamma_{2S}t}$ at early time… partly driven by the ground state at later time.

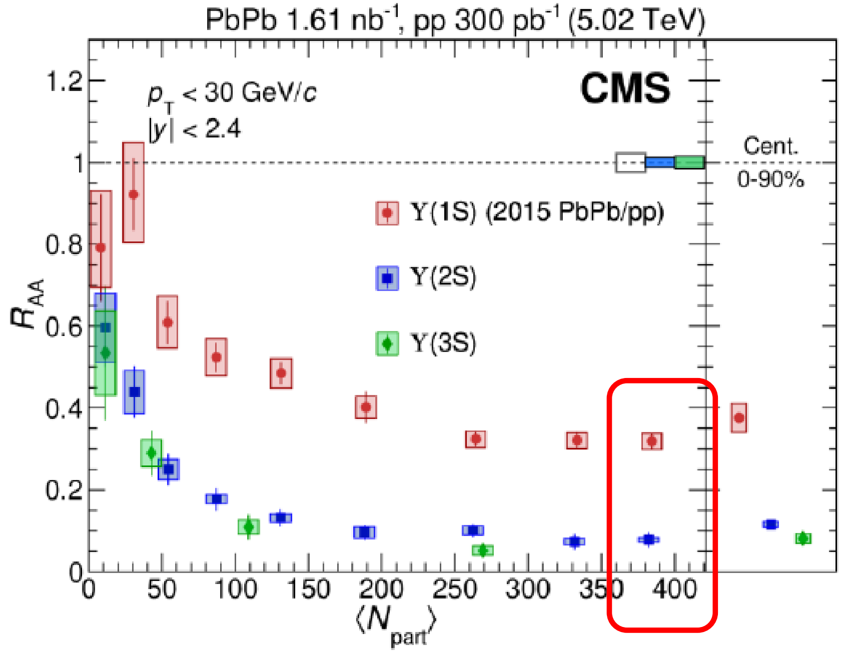
Contact with experiment ($b\bar{b}$)

- Bottomonia yield using the QME with EPOS4 (T,v) profiles and starting from a compact $b\bar{b}$ state.

$$\psi(s) \propto e^{-\frac{s^2}{2\sigma^2}} \left(1 + a_{\mathrm{odd}}\frac{s}{\sigma}\right) \quad \sigma = 0.045\,\mathrm{fm}\ a_{\mathrm{odd}} = 3.5$$

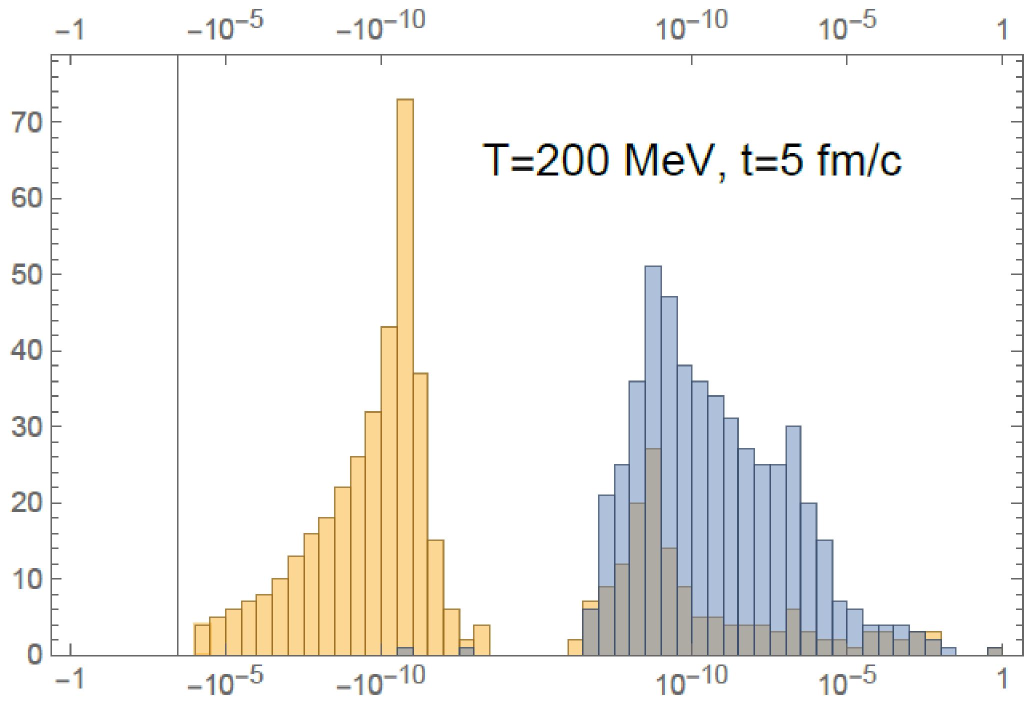






- See Stephane Delorme's talk at SQM24 for more details.

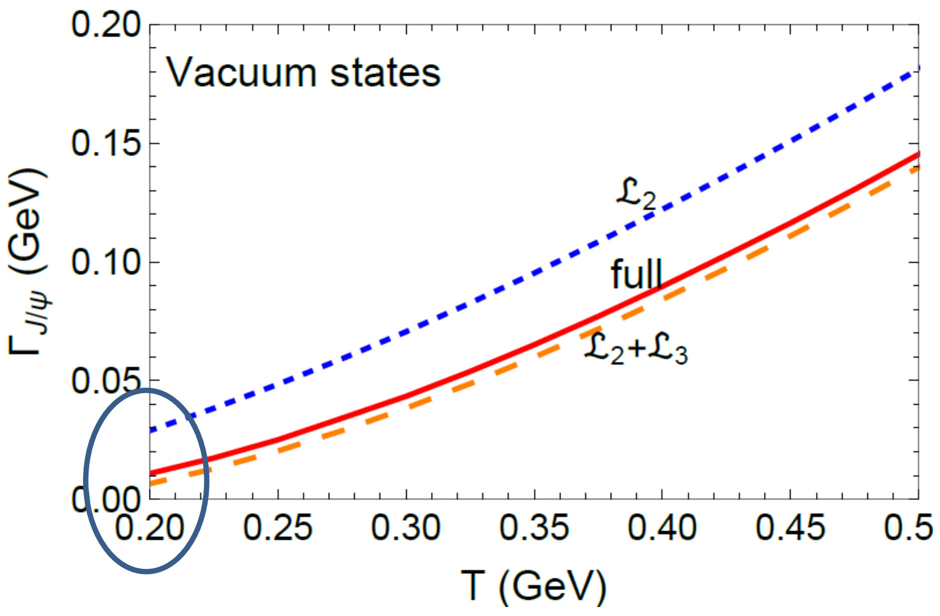
The role of positivity...

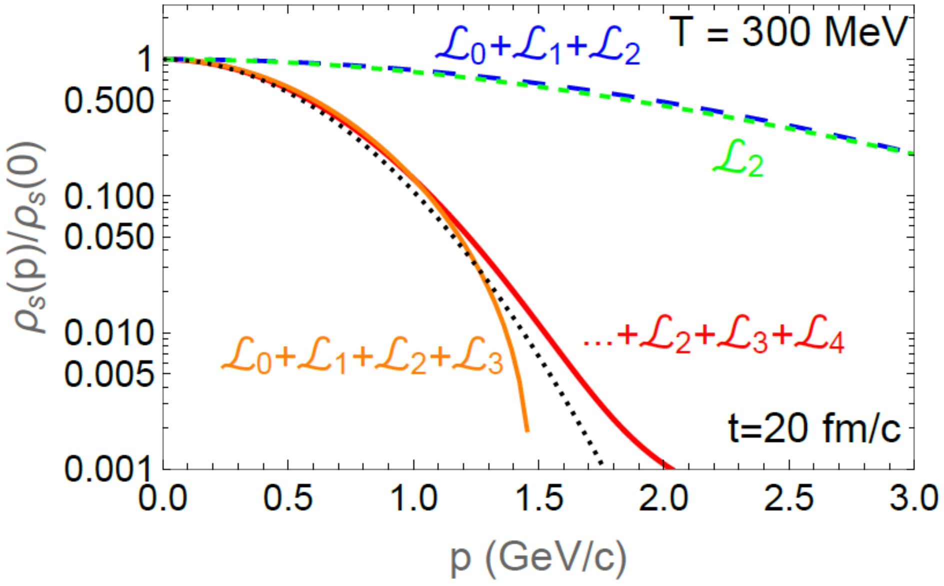


Distribution of singlet's density matrix eigenvalues

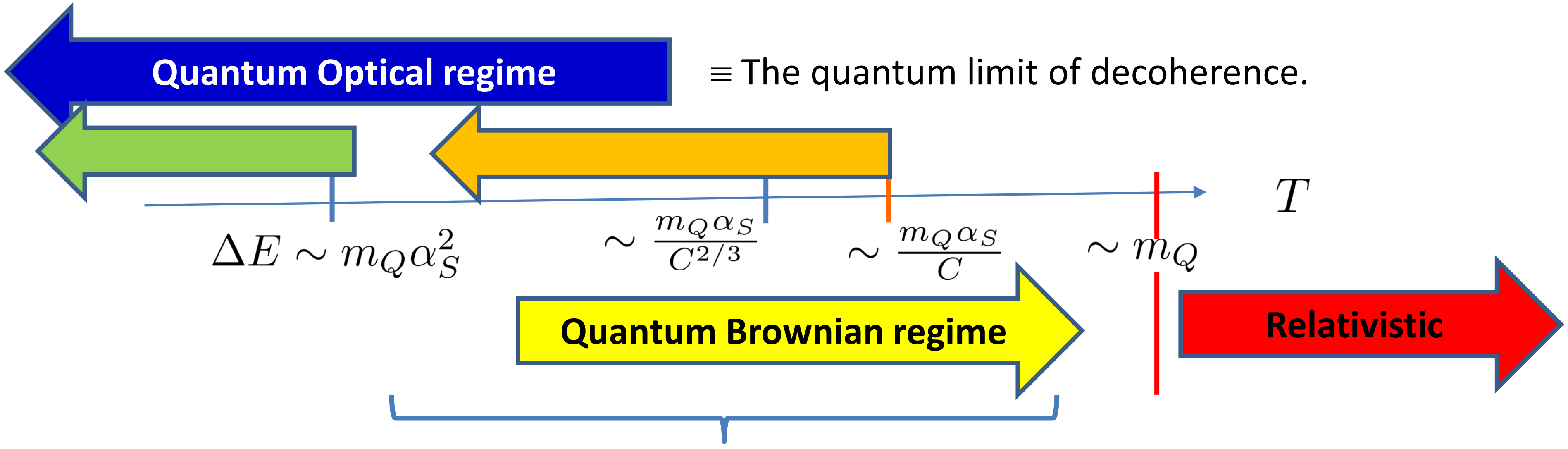
Without L_4

with L_4





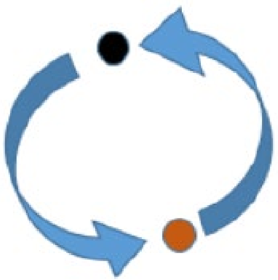
QCD Temperature scales



≡ The quantum limit of decoherence.

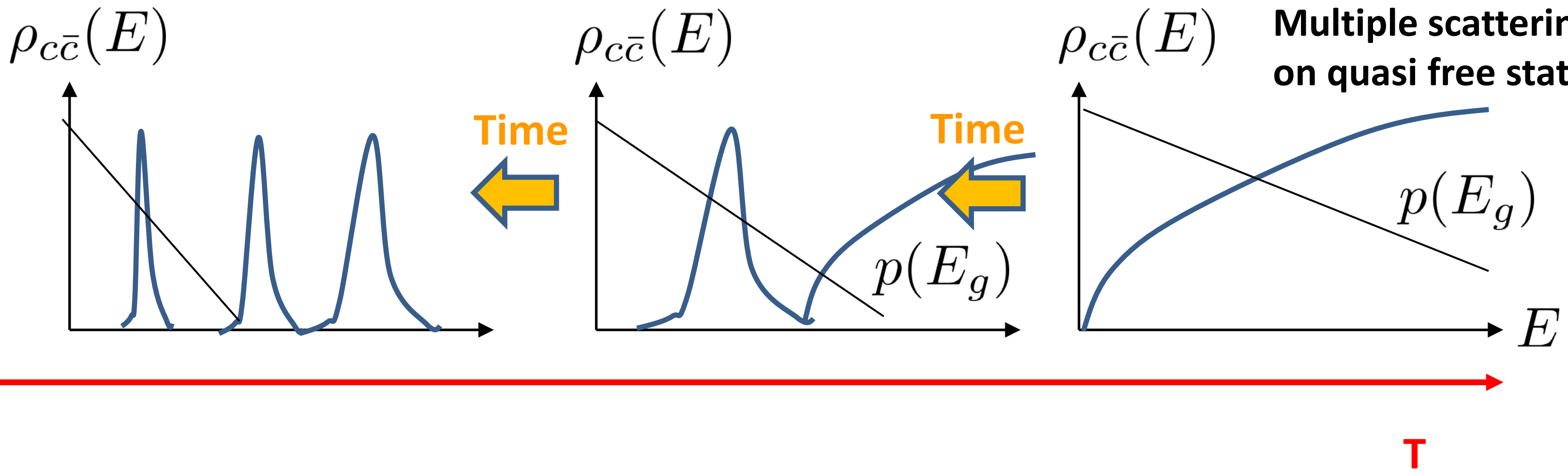
For these « large » temperatures, the Q-Qbar gain enough energy to overwhelm the real binding potential => larger distance => larger decoherence ....

≡ The intermediary regime.

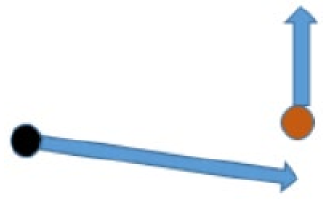


$\omega \approx E_n - E_m$

dissociation of well identified levels by scarce “high-energy” modes (dilute medium => cross section ok)



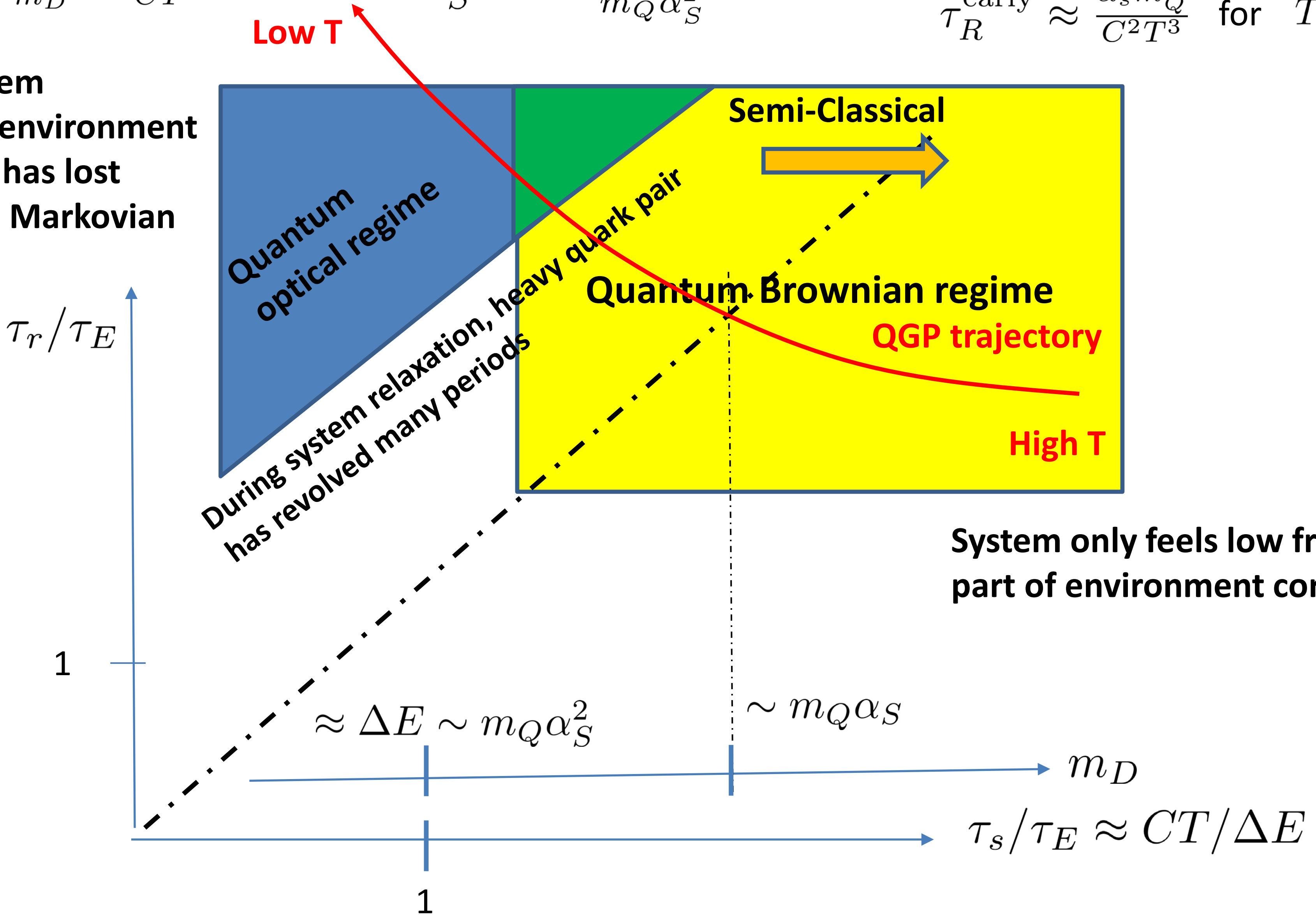
Multiple scattering on quasi free states



QCD time scales

$\tau_E \approx \frac{1}{m_D} = \frac{1}{CT}$ $\tau_S^{\text{early}} \approx \frac{1}{m_Q \alpha_S^2}$ $\tau_R^{\text{early}} \approx \frac{\alpha_s m_Q^2}{C^2 T^3}$ for $T \lesssim \frac{m_Q \alpha_S}{C}$

During system relaxation, environment correlation has lost memory => Markovian process



System only feels low frequency part of environment correlation

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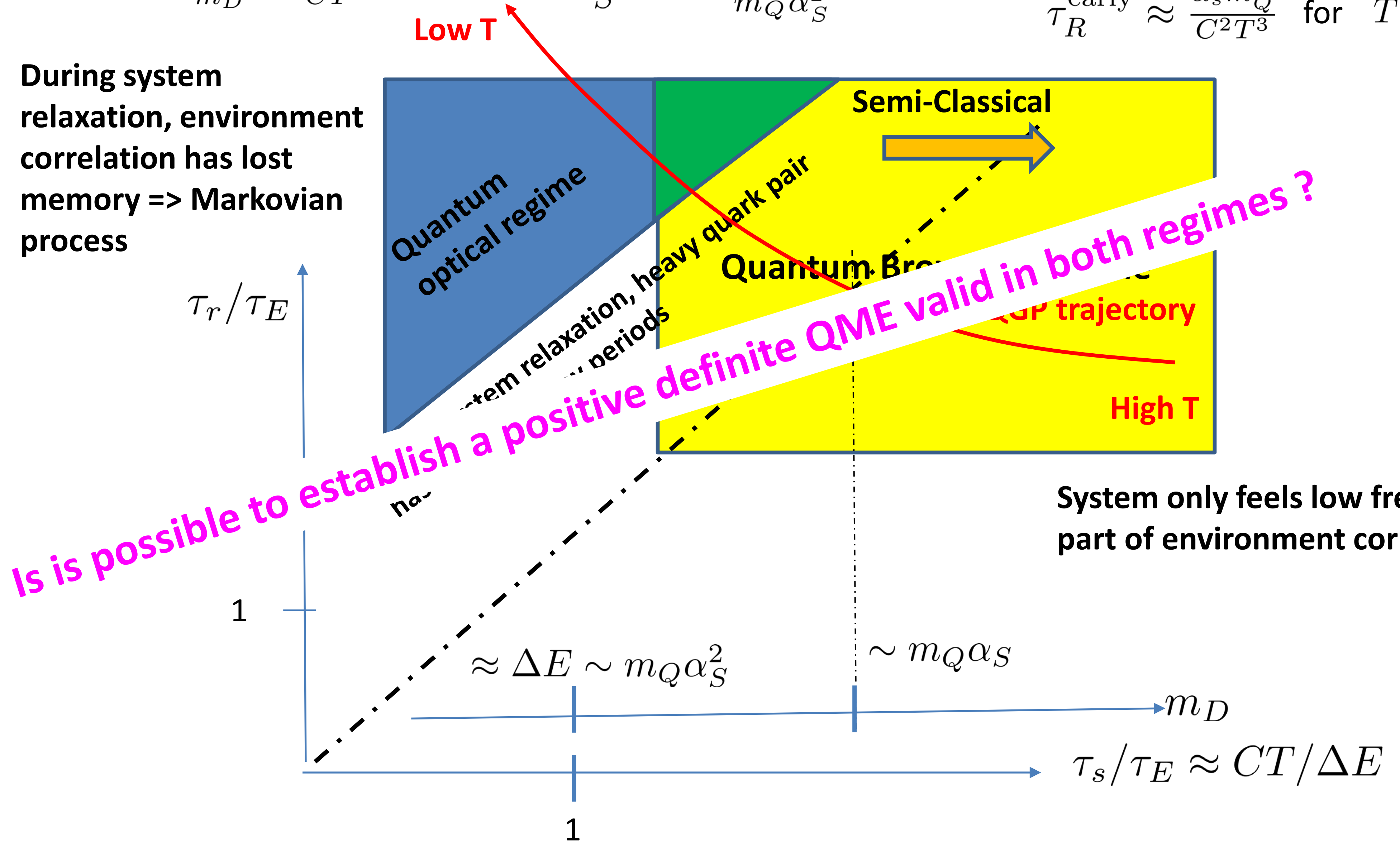


System only feels low frequency part of environment correlation

The core of the ULE derivation

Lindblad structure $\frac{d\rho(t)}{dt} = -i\left[H_Q + H_{LS}, \rho(t)\right] + \sum_n \gamma_n \left(L_n \rho(t) L_n^\dagger - \frac{1}{2}\left\{ L_n^\dagger L_n, \rho(t) \right\} \right)$

Starting from the NRQCD Hamiltonian $H_{\rm tot} = \left(\frac{\boldsymbol{p}_Q^2}{2M} + \frac{\boldsymbol{p}_{\bar{Q}}^2}{2M} \right) \otimes I_{\rm QGP} + I_{Q\bar{Q}} \otimes H_{\rm QGP} + \int_{\boldsymbol{x}} n_{\boldsymbol{x}}^a \otimes g A_0^a(\boldsymbol{x})$

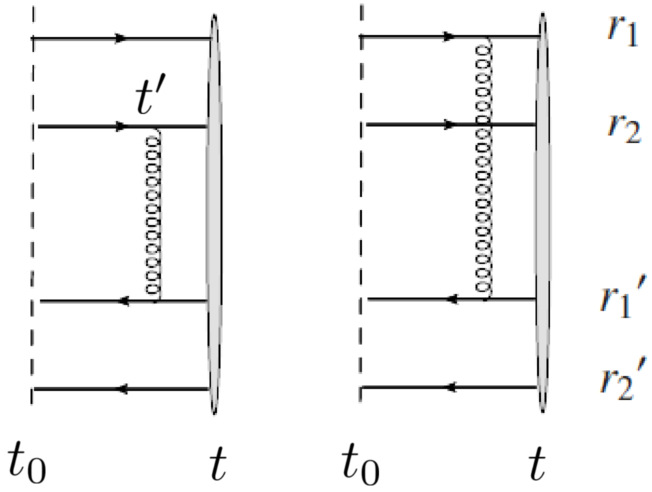
Iterating the Von Neumann equation, tracing out the QGP d.o.f. and assuming a slow evolution of ρ^I in the interaction representation (Born - Markov), one arrives at the Redfield equation

$$\frac{d\rho^I(t)}{dt} = -\int_{t_0}^{t} dt' \int_{xx'} \left[n^a(t,x), n^a(t',x') \rho^I(t) \right] \Delta^>(t-t', x-x') + h.c.$$

Where the gluon propagator is

$$\delta^{ab} \Delta^>(t_1 - t_2, \boldsymbol{x} - \boldsymbol{x}') = g^2 \left\langle T_C \left[A_0^a(t_1, \boldsymbol{x}) A_0^b(t_2, \boldsymbol{x}') \right] \right\rangle_0$$

IMPORTANT : localized on $t_1 - t_2 \lesssim \tau_E \sim \frac{1}{T}$



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Jumps -> other states

Redfield equation : $$\frac{d\rho^I(t)}{dt} = -\int_{t_0}^{t} dt' \int_{xx'} \left[n^a(t,x), n^a(t',x')\rho^I(t) \right] \Delta^>(t-t', x-x') + h.c.$$

$$\delta^{ab}\Delta^>(t_1 - t_2, \boldsymbol{x} - \boldsymbol{x}') = g^2 \left\langle T_C \left[A_0^a(t_1, \boldsymbol{x}) A_0^b(t_2, \boldsymbol{x}') \right] \right\rangle_0$$

Jump contribution : $$\frac{d\rho^I(t)}{dt} = \int_{t_0}^{t} dt' \int_{xx'} \Delta^>(t-t', x-x') n^a(t',x')\rho^I(t) n^a(t,x) + h.c. + \cdots$$

No symetrized form ! => not Linblad structure

For t << τ_R:

(slow evolution of ρ)

$$\rho^I(t) - \rho^I(t_0) = \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \int_{xx'} \Delta^>(t'-t'', x-x') n^a(t'',x')\rho^I(t) n^a(t',x) + h.c. + \cdots$$

$$= \int_{t_0}^{t} dt' \int_{t_0}^{+\infty} dt'' \theta(t'-t'') \int_{xx'} \Delta^>(t'-t'', x-x') n^a(t'',x')\rho^I(t) n^a(t',x) + h.c. + \cdots$$

$$\theta(t'-t'') = \frac{1}{2} + \frac{1}{2}\mathrm{sign}(t'-t'')$$

« Lamb shift »

The core of the ULE derivation

Lindblad structure $\frac{d\rho(t)}{dt} = -i\left[H_Q + H_{LS}, \rho(t)\right] + \sum_n \gamma_n \left(L_n \rho(t) L_n^\dagger - \frac{1}{2}\left\{L_n^\dagger L_n, \rho(t)\right\}\right)$

$$\rho^I(t) - \rho^I(t_0) = \frac{1}{2}\int_{t_0}^{t} dt' \int_{t_0}^{+\infty} dt'' \int_{x'x''} \Delta^{>}(t'-t'', x'-x'') n^a(t'', x'') \rho^I(t) n^a(t', x') + h.c. + \cdots$$

t' and t'' still intricated

Trick : $$\Delta^{>}(t'-t'', x'-x'') = \int_{-\infty}^{+\infty} dv \int_y g(v-t'', y-x'') g(t'-v, x'-y)$$ g : jump correlators

with $g(t'-v, x'-y) = g^\star(v-t', y-x')$

$$\rho^I(t) = \underbrace{\frac{1}{2}\int_y \int dv}_{\sim \sum_n \gamma_n} \int_{t_0}^{t} dt' \int_{t_0}^{+\infty} dt'' \int_{x''} g(v-t'', y-x'') \hat{n}^a(t'', x'') \rho^I(t) \int_{x'} g^\star(v-t', y-x') \hat{n}^a(t', x') + \cdots$$

$$= \underbrace{\frac{1}{2}\int_y \int dv}_{\sim \sum_n \gamma_n} \underbrace{\int_{t_0}^{+\infty} dt'' \int_{x''} g(v-t'', y-x'') \hat{n}^a(t'', x'')}_{\equiv L_n ???} \rho^I(t) \underbrace{\int_{t_0}^{\mathbf{t}} dt' \int_{x'} g^\star(v-t', y-x') \hat{n}^a(t', x')}_{\equiv L_n^\dagger ???} + h.c. + \cdots$$

Not quite ! As time derivation wrt t ``obviously'' breaks this structure !!! **Redfield : NOT LINDBLAD**

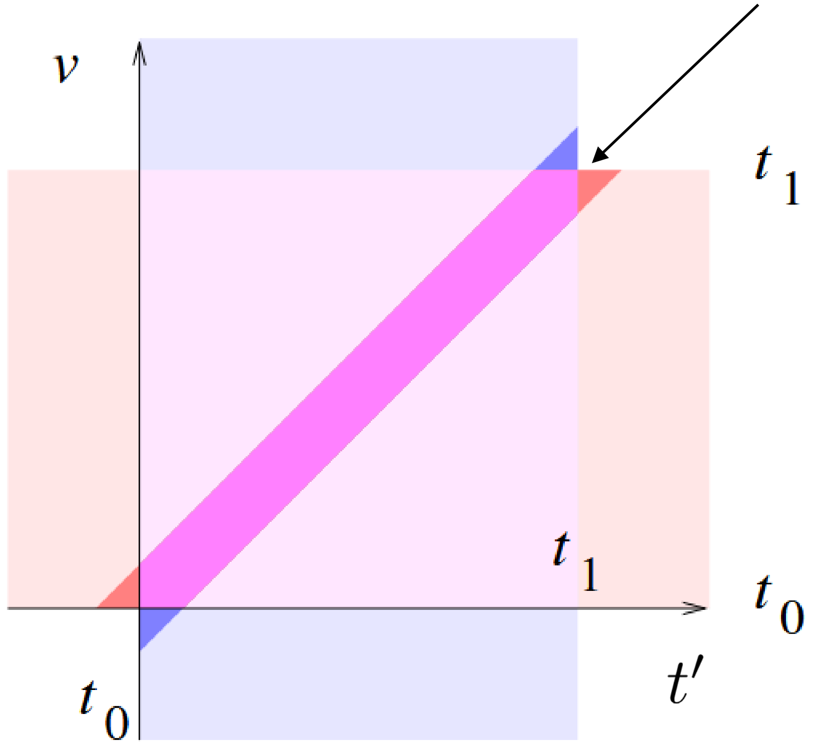
The core of the ULE derivation

Lindblad structure $\frac{d\rho(t)}{dt} = -i\left[H_Q + H_{LS}, \rho(t)\right] + \sum_n \gamma_n \left(L_n \rho(t) L_n^\dagger - \frac{1}{2}\left\{ L_n^\dagger L_n, \rho(t) \right\} \right)$

$$\rho^I(t) = \frac{1}{2}\int_y \int_{-\infty}^{+\infty} dv \int_{t_0}^{t} dt' \int_{t_0}^{+\infty} dt'' \int_{x''} g(v-t'', y-x'')\hat{n}^a(t'', x'')\rho^I(t) \int_{x'} g^\star(v-t', y-x')\hat{n}^a(t', x') + \cdots$$

How to proceed without introducing further approximations (than $\tau_E << \tau_R \approx t-t_0$) ? **In fact, there is an {} of equivalent QME**

Nathan and Rudner (2020): In significant contributions to the integrals stem from regions where the 3 times v, t' and t'' are separated by τ_E at most => one can **permutate the boundaries on v and t'** (at the price of a small correction in τ_E / τ_R)



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$$\rho^I(t) = \frac{1}{2}\int_y \int_{t_0}^{t} dv \int_{-\infty}^{+\infty} dt' \int_{t_0}^{+\infty} dt'' \int_{x''} g(v-t'', y-x'')\hat{n}^a(t'', x'')\rho^I(t) \int_{x'} g^\star(v-t', y-x')\hat{n}^a(t', x') + \cdots$$

Then proceeding with time derivative:

$$\frac{d\rho^I(t)}{dt} = \frac{1}{2}\int_y \int_{-\infty}^{+\infty} dt' \int_{-\infty}^{+\infty} dt'' \int_{x''} g(t-t'', y-x'')\hat{n}^a(t'', x'')\rho^I(t) \int_{x'} g^\star(t-t', y-x')\hat{n}^a(t', x') + h.c. + \cdots$$

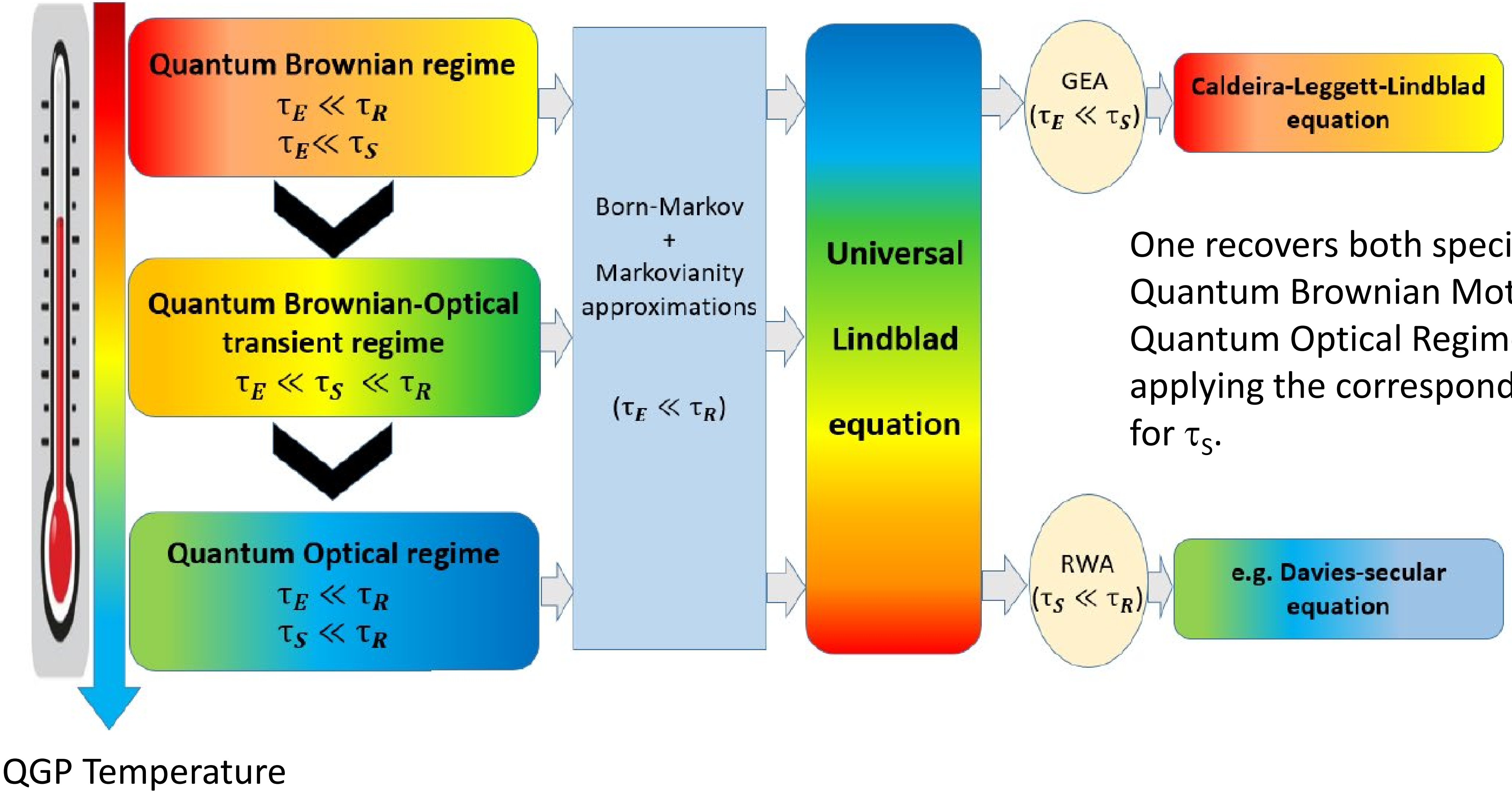
Markovianity

$$= \frac{1}{2}\int_y \int_{-\infty}^{+\infty} dt'' \int_{x''} g(t-t'', y-x'')\hat{n}^a(t'', x'')\rho^I(t) \int_{-\infty}^{+\infty} dt' \int_{x'} g^\star(t-t', y-x')\hat{n}^a(t', x') + h.c. + \cdots$$

Linblad structure provided one defines $\hat{L}_y = \int_{-\infty}^{+\infty} dt'' \int_{x''} g(t-t'', y-x'')\hat{n}^a(t'', x'')$ Contains all spectral information of the QGP heat bath !

N.B.: Same structure in the Schroedinger representation

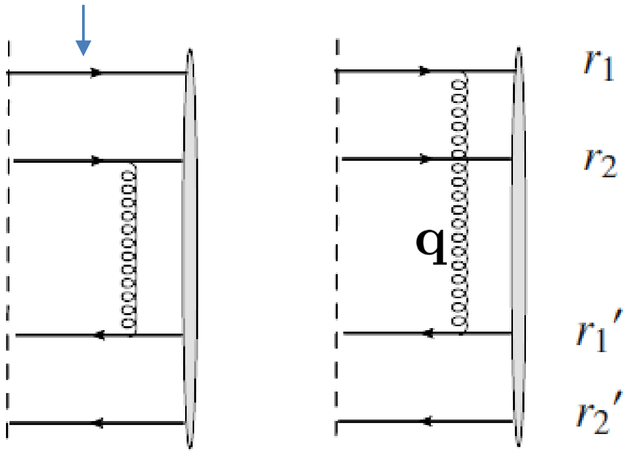
Schematic applicability of the ULE :



One recovers both specific QME in the Quantum Brownian Motion and in the Quantum Optical Regimes when applying the corresponding hierarchies for τ_S.

Illustrations for a coupled singlet-octet universal Lindblad equations

Q-Qbar pair



- Scattering from gluons change the color representation : o <-> s

$$\mathcal{D}_Q = \begin{pmatrix} \mathcal{D}_s \\ \mathcal{D}_o \end{pmatrix}$$

- One assumes some binding Poschl-Teller potential in the singlet representation => in-QGP Bound states (|n>, |m>,…)

- For the octet representation, no binding potential => diffusion states |**k**> => « large » energy gap.

- Illustration for the singlet -> octet (center of mass integrated)

$$\frac{d\rho}{dt} = \cdots + \int_y \hat{L}_{s\to o}(y)\rho\hat{L}^\dagger_{s\to o}(y) + \cdots$$

with $$\hat{L}_{s\to o}(y) = \int d\mathbf{k}\sum_n \tilde{L}_1(\mathbf{k},n,y)|\mathbf{k}\rangle\langle n|$$

Transition s -> o

$$\tilde{L}_{s\to o}(\mathbf{k},n,y) = -i\sqrt{\frac{1}{2N_c}}\int d\mathbf{q}\, e^{i\mathbf{q}\cdot\mathbf{y}}\tilde{g}\left(q^0 = E_n - \frac{k^2}{M},\mathbf{q}\right)\langle\mathbf{k}|\sin\frac{\mathbf{q}\cdot\mathbf{s}}{2}|n\rangle$$

Fourier transform of *g*, sqrt of the QGP correlator $\Delta^>$

Dipolar transition matrix element

Illustrations of transition rates

- Starting form a singlet state : $\hat{\rho}(0) = |n\rangle\langle n|$
- The transition rate towards octet state |k> (⇔ differential dissociation rate) is $d\Gamma/d\mathbf{k} = \frac{d}{dt}\langle\mathbf{k}|\hat{\rho}(t)|\mathbf{k}\rangle$

Probability to find the Q-Qbar in an octet state

- With previous expressions :

$$d\Gamma/d\mathbf{k} = \frac{N_c^2-1}{2N_c}\int d\mathbf{q}\,\underbrace{\tilde{g}^2\left(q^0 = E_n - \frac{k^2}{M}, \mathbf{q}\right)}_{\text{QGP spectral density}} |\langle\mathbf{k}|\sin\frac{\mathbf{q}\cdot\mathbf{s}}{2}|n\rangle|^2$$

QGP spectral density

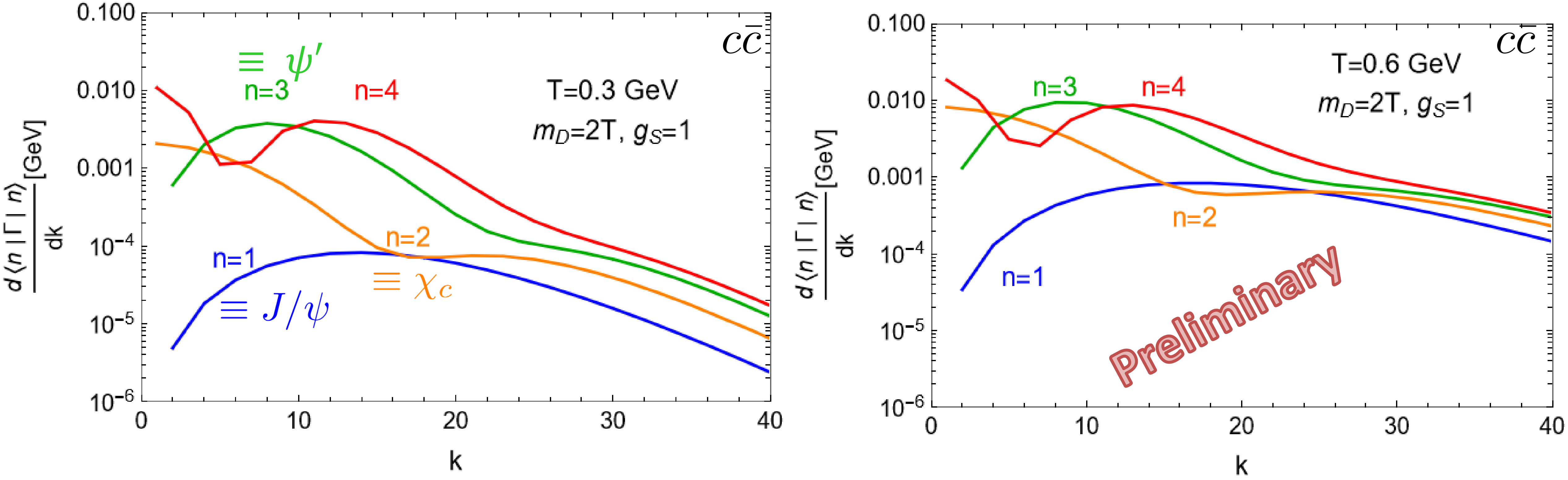
Dipolar transition matrix element

Result quite similar to the Fermi Golden rule

- But also genuine quantum coherences: $\frac{d}{dt}\langle\mathbf{k}'|\hat{\rho}(t)|\mathbf{k}\rangle \neq 0$

Illustrations of differential transition rates (1D case)

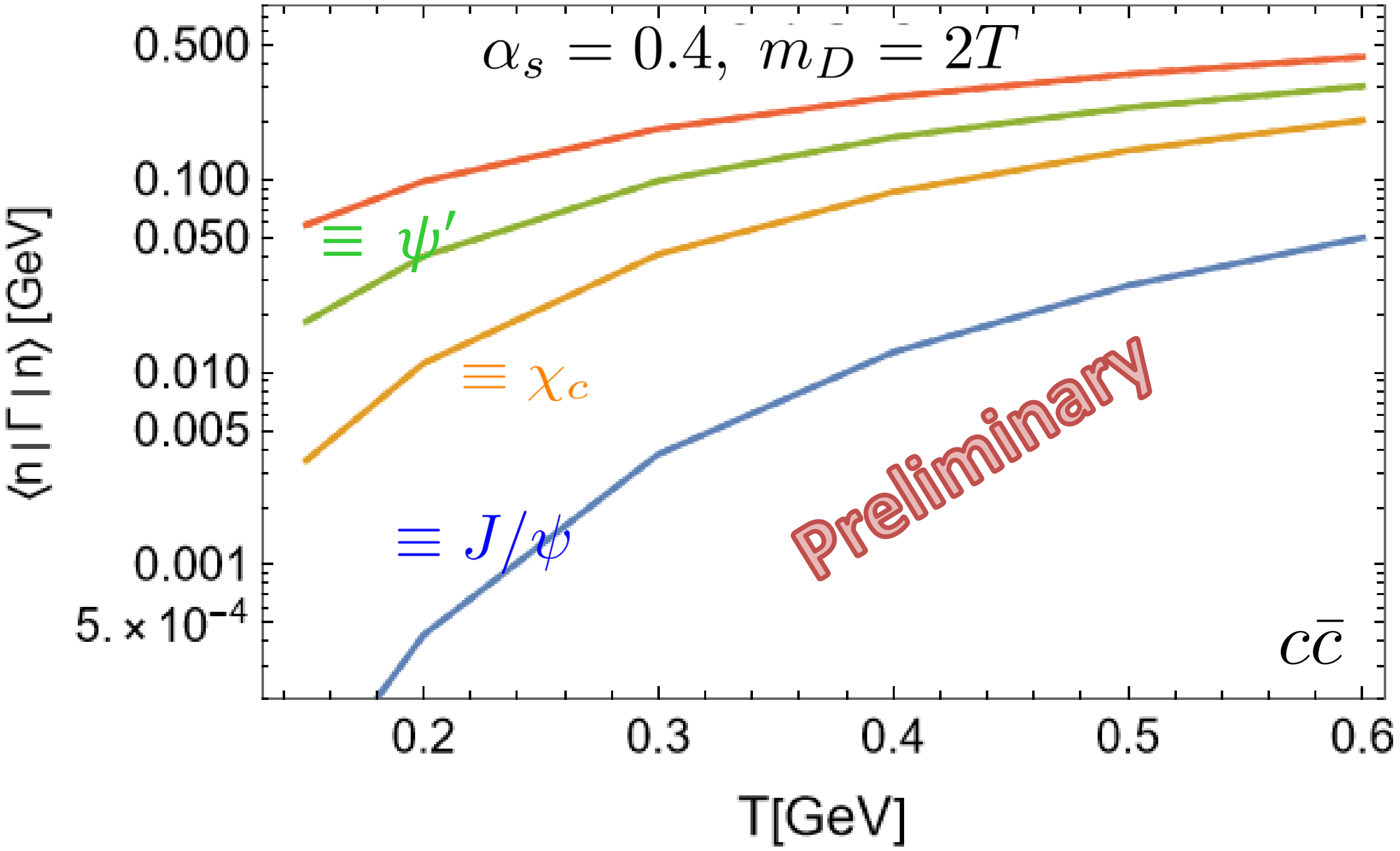
$$d\Gamma_{s\to o}(n)/d\mathbf{k} = \frac{N_c^2-1}{2N_c}\int d\mathbf{q}\,\tilde{g}^2\left(q^0 = E_n - \frac{k^2}{M}, \mathbf{q}\right)|\langle \mathbf{k}|\sin\frac{\mathbf{q}\cdot\mathbf{s}}{2}|n\rangle|^2$$



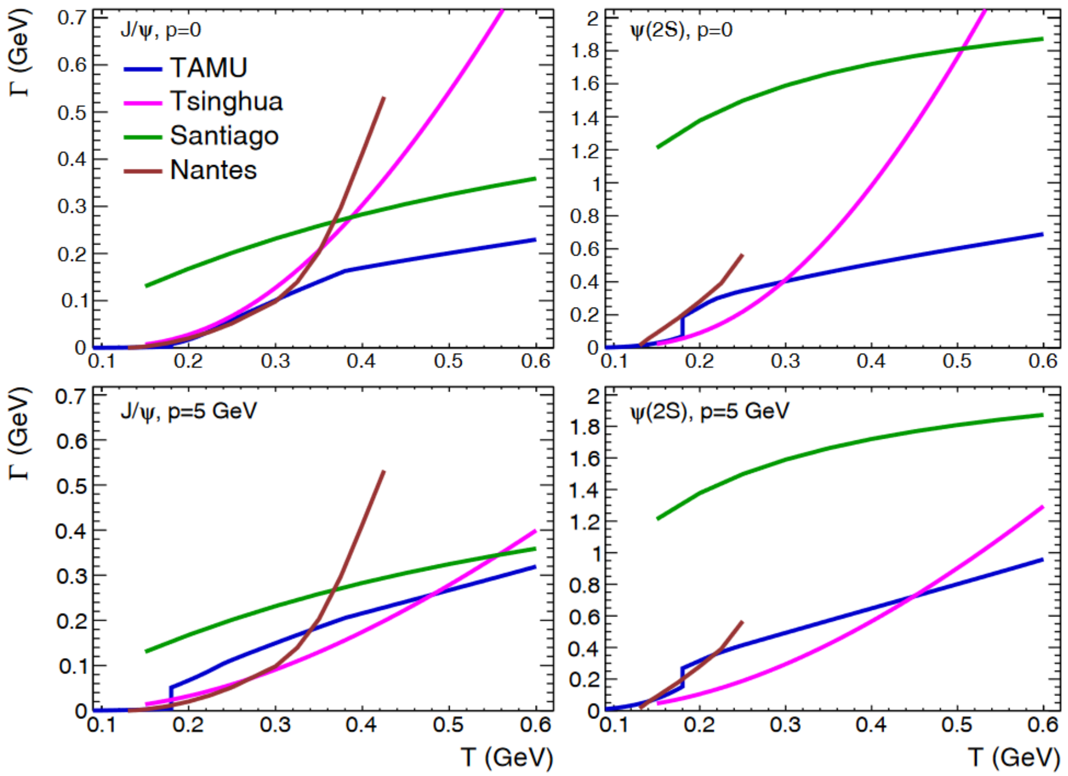
- Hierarchy from ground state -> excited state and continuity from low to high T
- For k close to 0 : selection rule for even bound states $\langle \mathbf{k}=0|\sin\frac{\mathbf{q}\cdot\mathbf{s}}{2}|n_{\text{even}}\rangle = 0$

Illustrations of transition rates (1D case)

Total transition rate : $\Gamma_{\mathrm{s}\to\mathrm{o}}(n) = \int d\mathbf{k} d\Gamma_{\mathrm{s}\to\mathrm{o}}(n)/d\mathbf{k}$



A. Andronic et al, *Eur.Phys.J.A* 60 (2024) 4, 88



Compatible with recent compilation of dissociation rates

Illustrations of spectral densities for charmonia

Lindblad equation : $\frac{d\rho(t)}{dt} = -i\left[H_Q + H_{LS}, \rho(t)\right] + \sum_n \gamma_n \left(L_n \rho(t) L_n^\dagger - \frac{1}{2}\left\{ L_n^\dagger L_n, \rho(t) \right\} \right)$

: Von Neumann equation with

$$H_{\mathrm{eff}} = H_Q + H_{\mathrm{LS}} - i \sum_n \frac{\gamma_n}{2} L_n^\dagger L_n$$

Non Hermitic

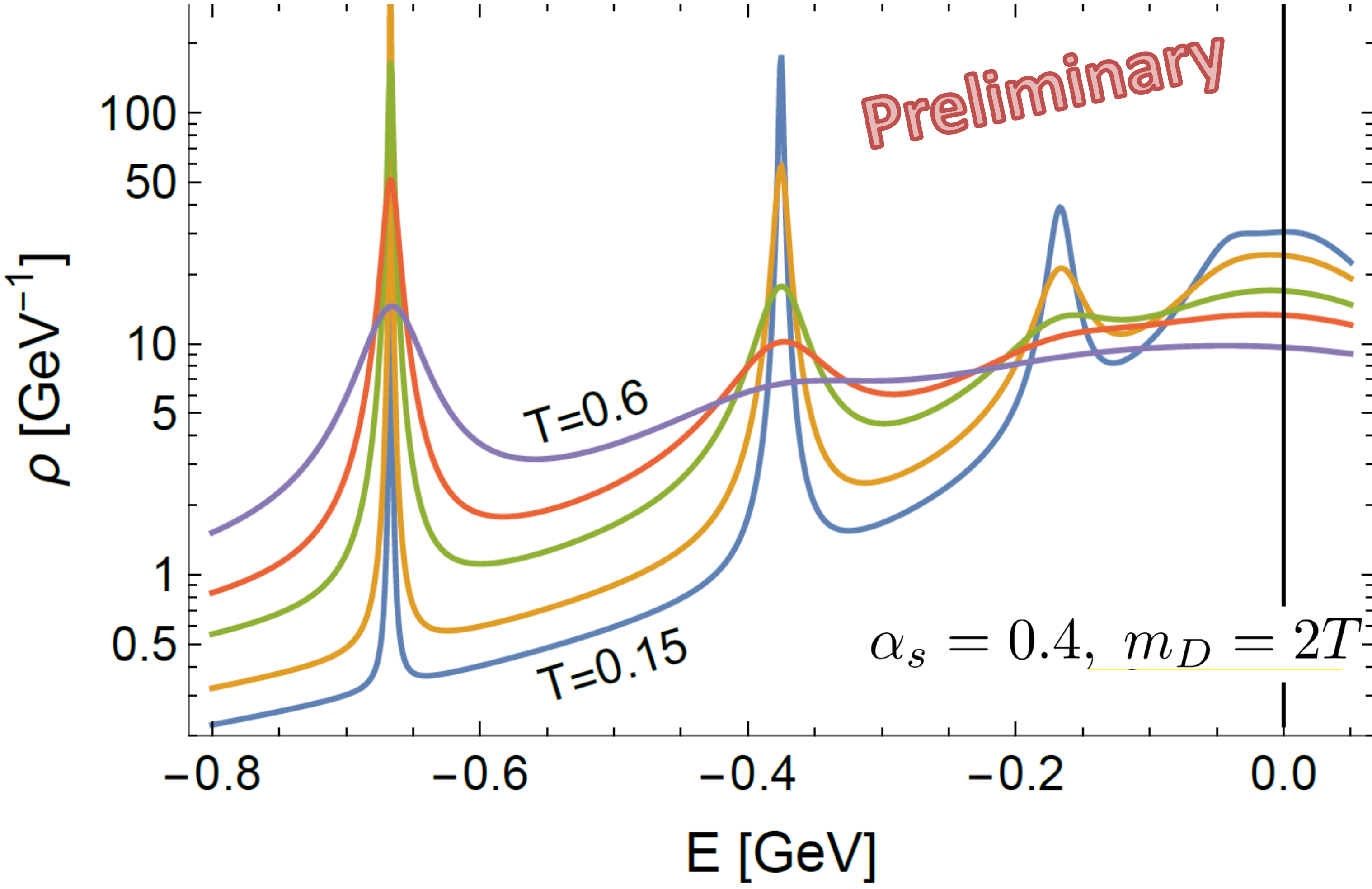
Definition of the spectral density :

$$\rho(E) = \frac{1}{\pi} \Im \operatorname{tr} \frac{1}{E - H_{\mathrm{eff}} - i\epsilon}$$

At T=0, a sum of Dirac peaks

Main result: fast melting of the χ_c state, progressive melting of the J/ψ state beyond Tc

… correct interpolation between the Quantum Optical regime at low T and the Quantum Brownian Motion regime at high T



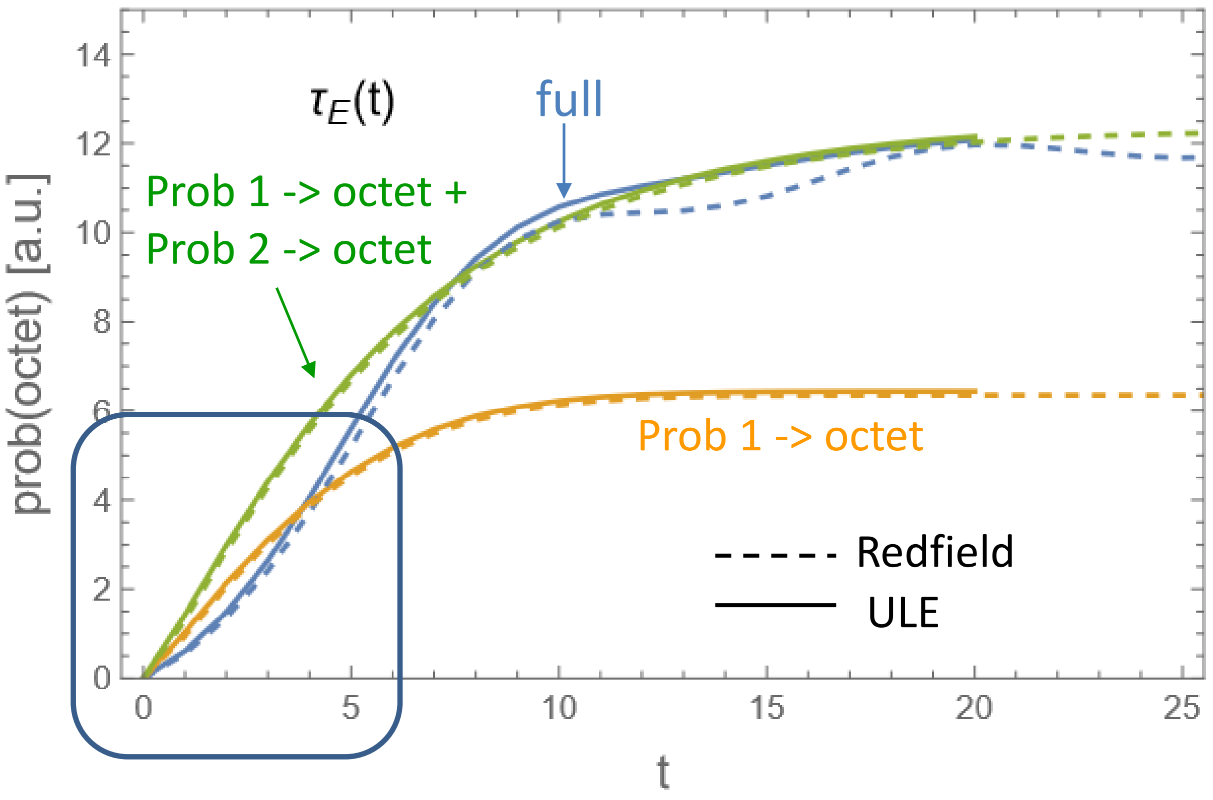
A first dynamical calculation

Cooling medium : $\tau_E(t) = 0.2 \times (1+t)$

Initial state : $\cos\theta|1\rangle + \sin\theta|2\rangle$ with θ chosen ⇔ minimal size

$E_1^{\text{binding}} = 1$

$E_2^{\text{binding}} = 1/2$



« Early » evolution : strong signature of quantum coherence

Good overall agreement between conventional Redfield eq. And ULE

Conclusions

With time, errors and confusions, discussions with colleagues and the impetus of young researchers, the field makes some progress and we hope to contribute to it by :

- **Identifying QME suitable for quarkonium formation in QGP**
- **Solving them when possible**
- **Going the dirty semiclassical way with controlled approximations**

Simplification for loosely bound states (exotic quarkonia ?) : deep QBM regime may apply