Structure of Exotic States and Their Production in Heavy Ion Collisions

Su Houng Lee

YONSEI UNIVERSITY

- Why are Exotics interesting
- Quark model: two-body
- Quark model: three-body
- Implications to exotics
- Few thoughts on production

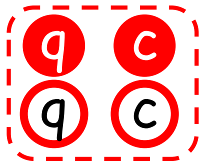
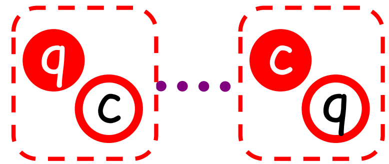
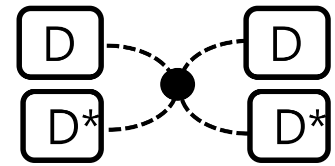
Acknowledgments:

Previous works+

S. Noh, A. Park, H. Yun, S. Cho, SHL: PLB 862(2025)139278

Jongheon Baek, A. Park, S. Noh, H. Yun, K. Han, SHL, in preparation

Types of Exotic particles

	Compact multiquark	Molecule	Resonance
Picture			
Size Threshold width	$\langle r \rangle <\ 0.6$ fm Near threshold or other small	$\langle r \rangle > 2$ fm Near threshold small	$\langle r \rangle \sim 1$ fm Above threshold or other large
Typical model used	Quark Model	Meson exchange models	Unitary approach Quark model
	Effective field theory: constants QCD sum rules: uncertainty		

Here, I ask "does quark model predict compact multiquark configuration?"

Why are exotics interesting?

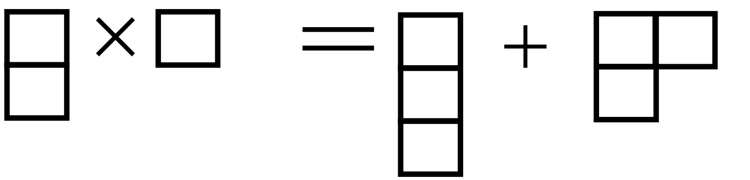
- A New color configuration

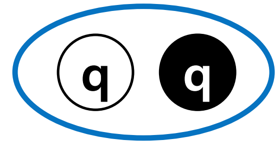
- Gateway to multi-quark states and high-density

So far only $(q\bar{q})_{C=1}$, $(qq)_{C=\bar{3}}$ color states are seen

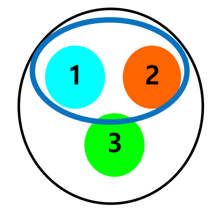
- Color state of Meson: $\bar{3}\times 3 = 1+8$

$(\bar{q})_{C=\bar{3}} \times (q)_{C=3}$ $\quad (q\bar{q})_{C=1}$ $\quad (q\bar{q})_{C=8}$





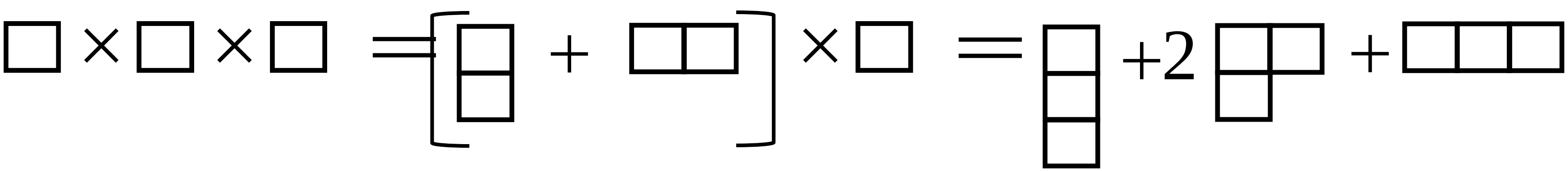
$(q\bar{q})_{C=1}$



$(qq)_{C=\bar{3}}$

- Color state of Baryon: $3\times 3\times 3 = \left(\bar{3}+6\right)\times 3 = 1+2\cdot 8+10$

$(q)_{C=3} \times (q)_{C=3} \times (q)_{C=3}$ $\quad (qq)_{C=\bar{3}}$ $\quad (qq)_{C=6}$ $\quad (qq)_{C=\bar{3}}$



- But, Color states $(q\bar{q})_{C=8}$ and $(qq)_{C=6}$ are attractive in the Spin=1 channel

A new color configuration of SU(3)

☞ Usual ground state hadron $(q\bar{q})_{C=1}$ $(qq)_{C=\bar{3}}$ or $(\overline{qq})_{C=3}$

☞ But Exotics contain additional color configurations with higher degeneracy
For example: Tetraquark state

$$3\times 3\times \bar{3}\times \bar{3} = (\bar{3}+6)\times(3+\bar{6}) = 3\times\bar{3}+6\times\bar{6}+\ldots$$

$$(qq)_{C=\bar{3}}\otimes(\overline{qq})_{C=3} \quad \text{and} \quad (qq)_{C=6}\otimes(\overline{qq})_{C=\bar{6}}$$

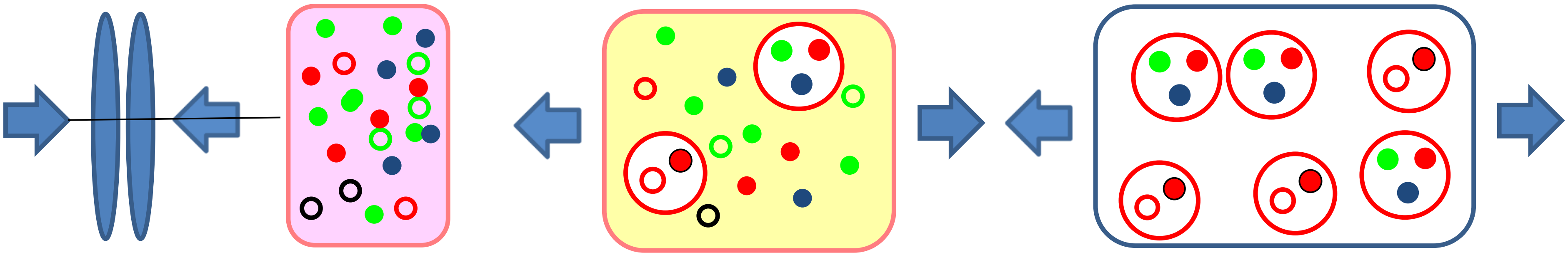
degeneracy: 3×3 and 6×6

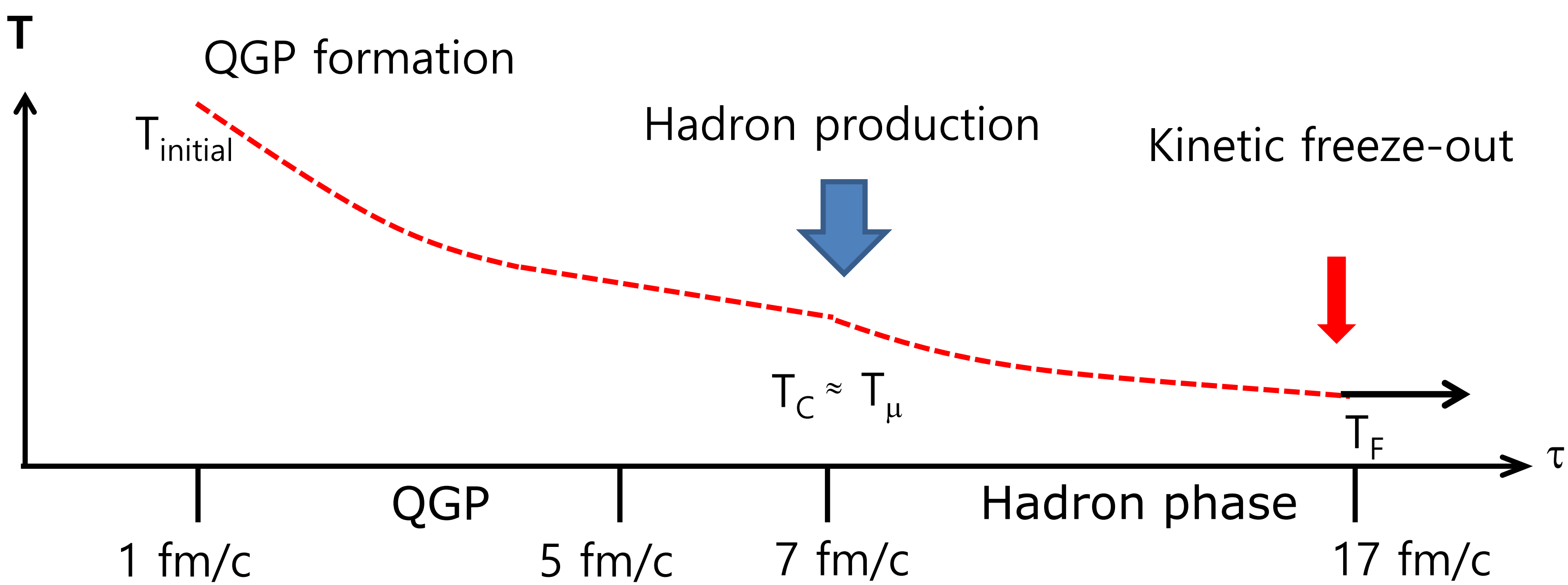
$$3\times\bar{3}\times 3\times\bar{3} = (1+8)\times(1+8) = 1\times 1+8\times 8+\ldots$$

$$(q\bar{q})_{C=1}\otimes(q\bar{q})_{C=1} \quad \text{and} \quad (q\bar{q})_{C=8}\otimes(q\bar{q})_{C=8}$$

degeneracy: 1×1 and 8×8

QGP contains all configurations and correlations → Exotics





Quark Model: Two body

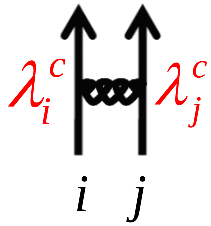
- Two-body quark force: color-color and color-spin interaction

- Insight into exotics

Quark Model: two-body interaction

- Color-Color interaction:

$$H=\sum_{i=1}^{n}\left(m_i+\frac{p_i^2}{2m_i}\right)-\sum_{i<j}^{n}\left(\lambda_i^c\lambda_j^c\right)V_{ij}^{C}\left(r_{ij}\right)-\sum_{i<j}^{n}\frac{\left(\lambda_i^c\lambda_j^c\right)\left(\sigma_i\sigma_j\right)}{m_i m_j}V_{ij}^{SS}\left(r_{ij}\right)$$



$$V_{ij}^{C}\left(r_{ij}\right)=\left(-\frac{\kappa}{r_{ij}}+\frac{r_{ij}}{a_0}-D\right)$$

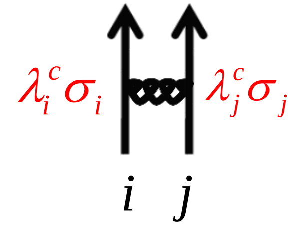
☞ Color-Color factors: important only within a color singlet configuration

$$\sum_{i<j}^{N}\left(\lambda_i^c\lambda_j^c\right)=\frac{1}{2}\left[\left(\lambda_1^c+\ldots+\lambda_N^c\right)^2-\lambda_1^2-\ldots\lambda_N^2\right]=0-\frac{8}{3}\left(N_{B_1}+N_{B_2}\right)=\sum_{i<j}^{N_{B_1}}\left(\lambda_i^c\lambda_j^c\right)+\sum_{i<j}^{N_{B_2}}\left(\lambda_i^c\lambda_j^c\right)$$

$$N=N_{B_1}+N_{B_2}$$

❑ Color-Spin:

$$H = \sum_{i=1}^{n}\left(m_i + \frac{p_i^2}{2m_i}\right) - \sum_{i<j}^{n}\left(\lambda_i^c \lambda_j^c\right) V_{ij}^C\left(r_{ij}\right) - \sum_{i<j}^{n} \frac{\left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i \sigma_j\right)}{\underline{m_i m_j}} V_{ij}^{CS}\left(r_{ij}\right)$$



$$V_{ij}^{CS} = \frac{\hbar^2 \kappa'_{ij}}{m_i m_j c^2 r_{0ij}} \frac{\exp\{-(r_{ij}/r_{0ij})^2\}}{r_{ij}} F_i^c F_j^c \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, \quad r_{0ij} = \left(\alpha + \frac{\beta\, m_i\, m_j}{m_i + m_j}\right)^{-1}, \quad \kappa'_{ij} = \kappa_0 \left(1 + \frac{\gamma\, m_i\, m_j}{m_i + m_j}\right)$$

☞ Color-spin factors

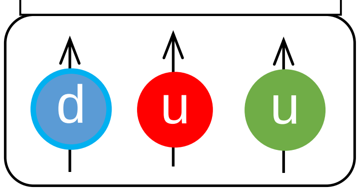
	Q–Q				Q–$\bar{Q}$			
Color	$\underline{3}$	6	$\underline{3}$	6	1	8	1	8
Flavor	A	A	S	S				
Spin	A(0)	S(1)	S(1)	A(0)	0	0	1	1
$K = -\left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i^s \sigma_j^s\right)$	-8	-4/3	8/3	4	-16	2	16/3	-2/3

$K < 0$ attraction; $K > 0$ repulsion

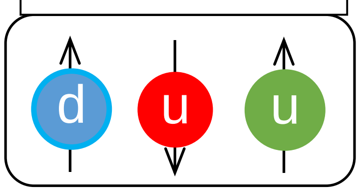
$$M_\Delta - M_P \approx [+8 - (-8)] \times \left[\int \frac{dx_{12}}{m_1 m_2} V_{12}^{SS}\left(x_{12}\right)\left|\psi\left(x_{12}\right)\right|^2\right] \approx 290 \text{ MeV}$$

$K = 1 \rightarrow 18$ MeV

Delta $K = +8$



Proton $K = -8$



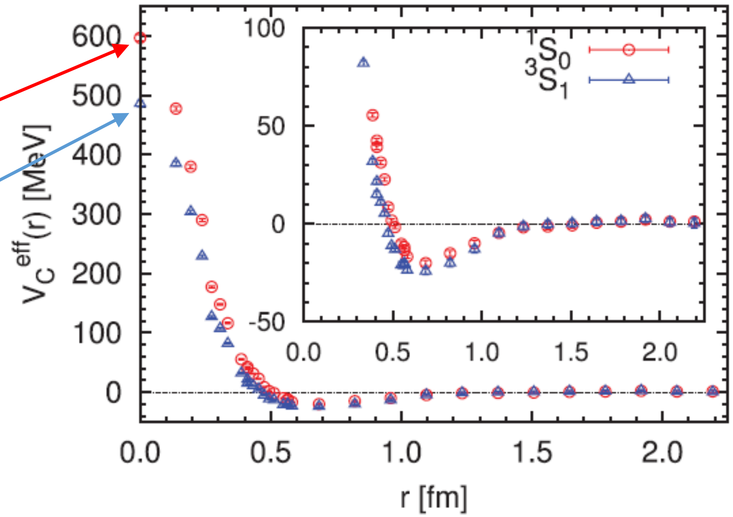
Quark Model vs Lattice comparison : A.Park, Lee, Inoue, Hatsuda, EPJA 56(2020)3,93

☞ NN force in SU(2) spin 1 vs spin 0 channel: comparison to lattice

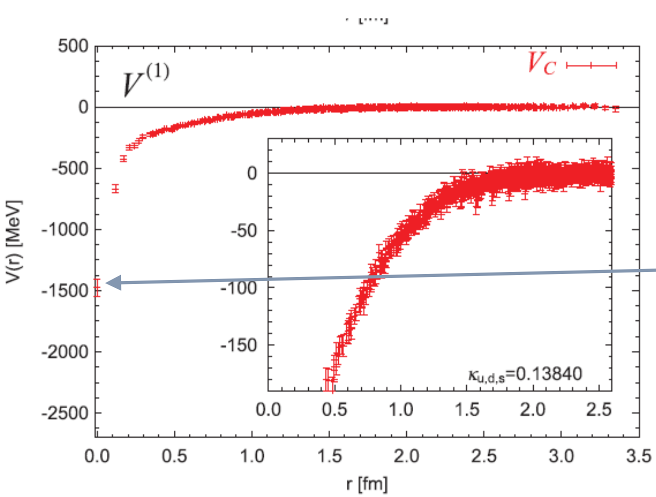
$$K_{2-N} = K_{6-quark} - \left(K_{1N} + K_{1N} \right)$$

$$\frac{K_{2-N}^{S=0}}{K_{2-N}^{S=1}} = 1.29$$ → comparison

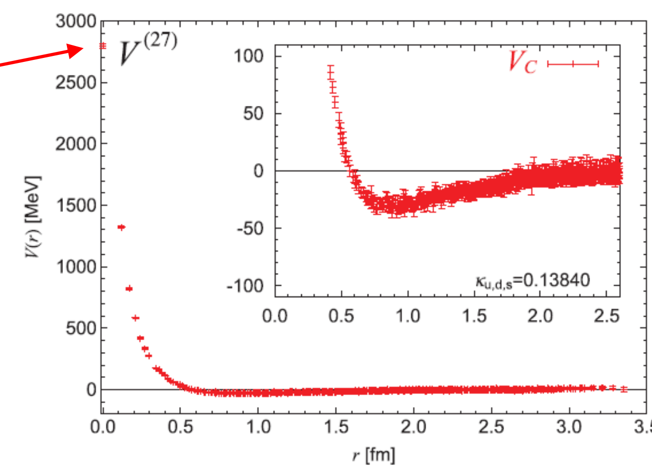
QCD HAL collaboration



☞ H dibaryon channel: Flavor 1 vs Flavor 27



$$\frac{K_{2-N}^{F=27}}{K_{2-N}^{F=1}} = -3$$

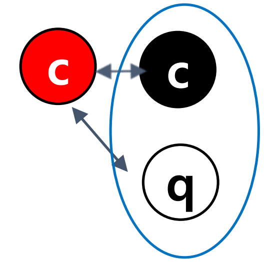


(HAL QCD Collaboration)

Why Heavy quarks are needed for multiquark configuration

☞ Color-color interaction becomes stronger (Karliner Rosner)

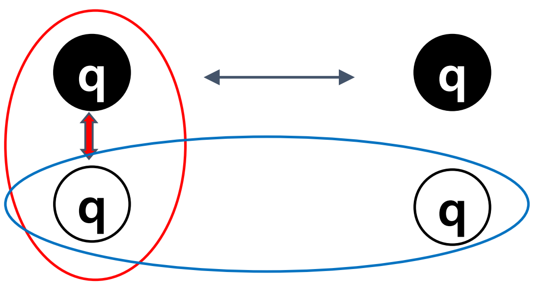
$$H_{cc} = \ldots + \lambda_i^c \lambda_j^c \left(\frac{g^2}{r_{ij}} \right) + \ldots . \qquad r \approx \frac{1}{mg^2}, \qquad E_C \approx -mg^4$$



☞ Color-spin interaction becomes weaker with heavy quarks

When all light quarks
Fall apart into two mesons

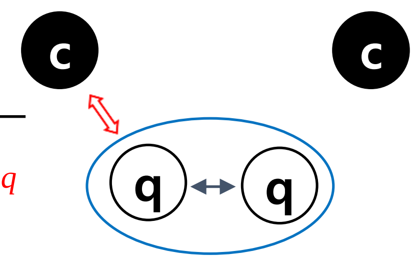
$$-16 \frac{1}{m_q m_q}$$



$$-8 \frac{1}{m_q m_q}$$

When heavy quarks,
could be compact (Tcc)

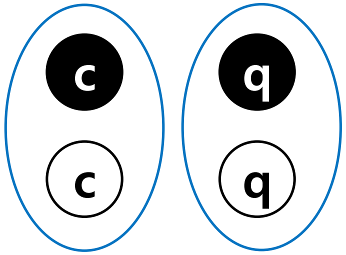
$$-16 \frac{1}{m_c m_q}$$



$$-8 \frac{1}{m_q m_q}$$

X(3872)

$$I^G\left(J^{PC}\right)=0^+\left(1^{++}\right)$$



☞ Color-spin $\left(\text{C=color, S=spin}\right)$

$\sim +140$ MeV

$$K_{X(3872)}-K_D-K_{D^*}=\begin{pmatrix} \frac{16}{3}\frac{1}{m_c^2}+\frac{16}{3}\frac{1}{m_q^2}+\frac{32}{3}\frac{1}{m_c m_q} & 0 \\ 0 & -\frac{2}{3}\frac{1}{m_c^2}-\frac{2}{3}\frac{1}{m_q^2}-\frac{4}{3}\frac{1}{m_c m_q} \end{pmatrix}$$

$\left(c\bar{c}\right)_{S=1}^{C=1}\otimes\left(q\bar{q}\right)_{S=1}^{C=1}$

$\left(c\bar{c}\right)_{S=1}^{C=8}\otimes\left(q\bar{q}\right)_{S=1}^{C=8}$

~ -20 MeV

☞ Color-color interaction of $\left(c\bar{c}\right)_{S=1}^{C=8}\otimes\left(q\bar{q}\right)_{S=1}^{C=8}$ is repulsive

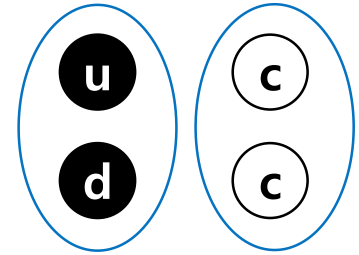
To overcome additional kinetic term attraction has to be >100 MeV

Full quark model calculation → Fall apart to two mesons

(W. Park, SHL, NPA925 (2014) 161)

Tcc(3875)

$$I^G\left(J^P\right)=0^+\left(1^+\right)$$



☞ Color-spin

~ -100 MeV

$$K_{T_{cc}(3875)}-K_D-K_{D^*}=\begin{pmatrix} -8\frac{1}{m_q^2}+\frac{8}{3}\frac{1}{m_c^2}+\frac{32}{3}\frac{1}{m_c m_q} & -8\sqrt{2}\frac{1}{m_c m_q} \\ -8\sqrt{2}\frac{1}{m_c m_q} & -\frac{4}{3}\frac{1}{m_q^2}+4\frac{1}{m_c^2}+\frac{32}{3}\frac{1}{m_c m_q} \end{pmatrix} \begin{matrix} (ud)_{S=0}^{C=\bar{3}}\otimes(\overline{cc})_{S=1}^{C=3} \\ (ud)_{S=1}^{C=6}\otimes(\overline{cc})_{S=0}^{C=\bar{6}} \end{matrix}$$

$\sim +17$ MeV

☞ Color-color interaction of $(ud)_{S=0}^{C=\bar{3}}\otimes(\overline{cc})_{S=1}^{C=3}$ is attractive

Full quark model calculation → Could be compact

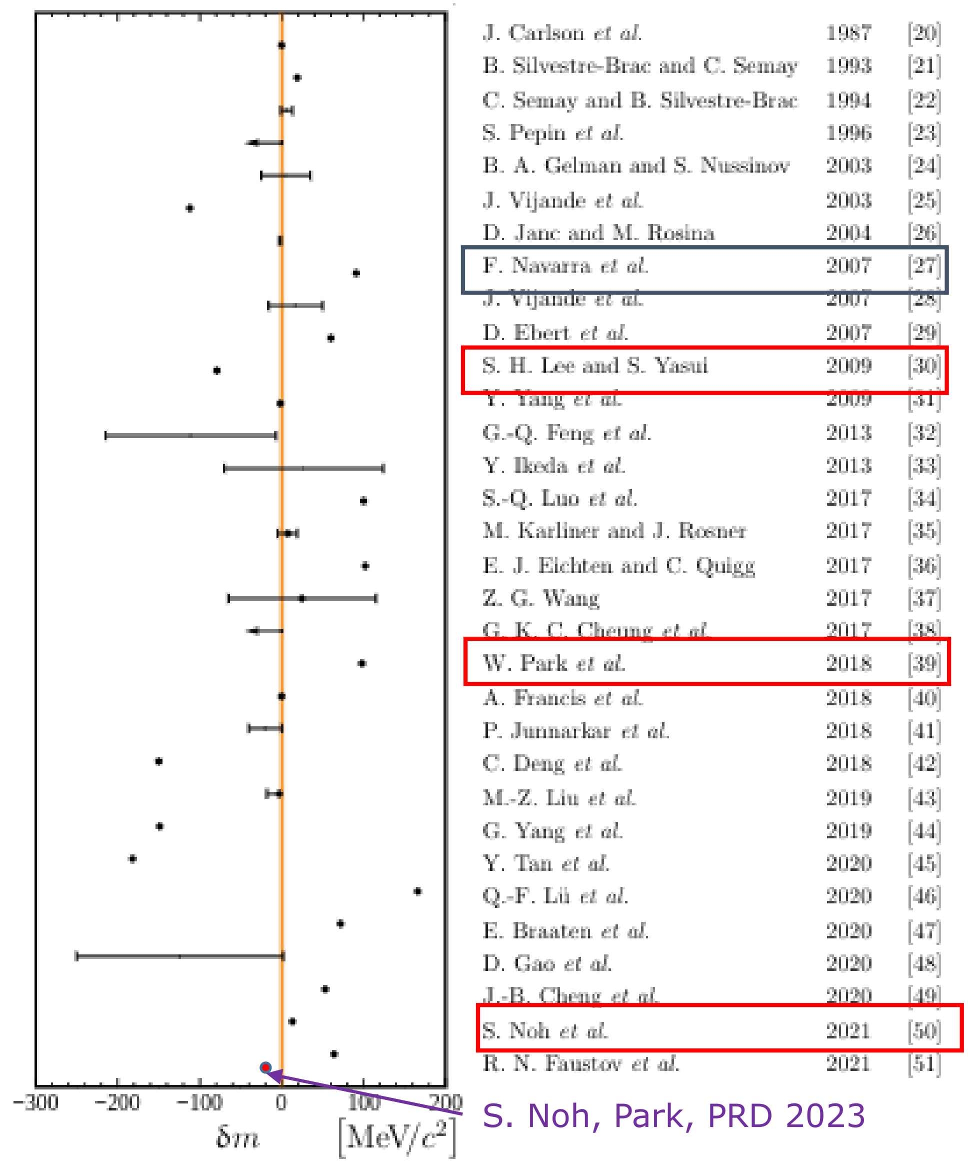
Full Quark Model calculation suggests: ex S.Noh, W.Park, Lee, PRD10(2021)114009

-2021- Tcc(3875) LHCb coll.

☞ There is a strong short range attraction for Tcc → Could be compact, but depends sensitively on parameters:

☞ The short range attraction for X(3872) is very weak

→ Can not be compact



Most stable color-spin interaction

$I=0,\ S=1/2$	$udcc\bar{s}$ ($\Delta E=-131$)
H_{hyp}	$11/4C_{cc}$-$11/2C_{c\bar{s}}$-$25/2C_{uc}$-$33/2C_{u\bar{s}}$-$17/4C_{uu}$
$\Xi_{cc}K$	$8/3C_{cc}$-$32/3C_{uc}$-$16C_{u\bar{s}}$
$I=0,\ S=1/2$	$udsc\bar{c}$ ($\Delta E=-122$)
H_{hyp}	-$22/3C_{s\bar{c}}$-$44/3C_{u\bar{c}}$-$28/3C_{us}$-$14/3C_{uu}$
$\Lambda\eta_c$	-$8C_{uu}$-$16C_{c\bar{c}}$
$I=0,\ S=1/2$	$udss\bar{c}$ ($\Delta E=-113$)
H_{hyp}	$11/4C_{ss}$-$11/2C_{s\bar{c}}$-$25/2C_{us}$-$33/2C_{u\bar{c}}$-$17/4C_{uu}$
ΛD_s	-$8C_{uu}$-$16C_{s\bar{c}}$
$I=1/2,\ S=1/2$	$uuds\bar{c}$ ($\Delta E=-92$)
H_{hyp}	-$33/4C_{s\bar{c}}$-$55/4C_{u\bar{c}}$-$21/2C_{us}$-$7/2C_{uu}$
PD_s	-$8C_{uu}$-$16C_{s\bar{c}}$

TABLE IV. The binding energy of the pentaquark. (unit ; MeV)

Pentaquark	$I=0$, $S=1/2$	Threshold, multiplicity	$I=0$, $S=3/2$	Threshold, multiplicity	$I=1$, $S=1/2$	Threshold, multiplicity	$I=1$, $S=3/2$	Threshold, multiplicity
$udsc\bar{c}$	ΔE=-124	$\Lambda\eta_c$, 7	ΔE=-43	$\Lambda J/\psi$, 5	ΔE=-46	$\Sigma\eta_c$, 8	ΔE=-31	$\Sigma J/\psi$, 7
$udss\bar{c}$	ΔE=-117	ΛD_s, 4	ΔE=-62	ΛD_s^*, 3	ΔE=54	ΣD_s, 4	ΔE=1	ΣD_s^*, 4
$udcc\bar{s}$	ΔE=-135	$\Xi_{cc}K$, 4	ΔE=-94	Ξ_{cc}^*K, 3	ΔE=133	$\Xi_{cc}K$, 4	ΔE=85	Ξ_{cc}^*K, 4
$udcc\bar{c}$	ΔE=-38	$\Lambda_c\eta_c$, 4	ΔE=-43	$\Lambda_c J/\psi$, 3	ΔE=14	$\Sigma_c\eta_c$, 4	ΔE=-31	$\Sigma_c^*\eta_c$, 4
$udss\bar{b}$	ΔE=-92	ΛB_s, 4	ΔE=-67	ΛB_s^*, 3	ΔE=24	ΣB_s, 4	ΔE=20	ΣB_s^*, 4

taquark	$I=0$, $S=1/2$	Threshold, multiplicity	$I=0$, $S=3/2$	Threshold, multiplicity	Pentaquark	$I=1$, $S=1/2$	Threshold, multiplicity	$I=1$, $S=3/2$	Threshold, multiplicity
$uudd\bar{s}$	ΔE=98	PK, 1	ΔE=74	PK^*, 1	$uud\bar{s}$	ΔE=337	PK, 2	ΔE=-74	ΔK, 2
$uudd\bar{c}$	ΔE=66	PD, 1	ΔE=58	PD^*, 1	$uud\bar{c}$	ΔE=223	PD, 2	ΔE=79	PD^*, 2
$uudd\bar{b}$	ΔE=54	PB, 1	ΔE=52	PB^*, 1	$uud\bar{b}$	ΔE=175	PB, 2	ΔE=172	PB^*, 2

taquark	$I=1/2$, $S=1/2$	Threshold, multiplicity	$I=1/2$, $S=3/2$	Threshold, multiplicity	Pentaquark	$I=1/2$, $S=1/2$	Threshold, multiplicity	$I=1/2$, $S=3/2$	Threshold, multiplicity
$uuds\bar{b}$	ΔE=-77	PB_s, 5	ΔE=-45	PB_s^*, 4	$uuds\bar{c}$	ΔE=-99	PD_s, 5	ΔE=-39	PD_s^*, 4
$uudc\bar{s}$	ΔE=17	$\Lambda_c K$, 5	ΔE=-88	$\Sigma_c^* K$ 4	$uudc\bar{c}$	ΔE=-34	$P\eta_c$, 5	ΔE=-15	PJ/ψ, 4
$sssu\bar{c}$	ΔE=133	ΞD_s, 3	ΔE=-17	ΞD_s^*, 3	$sssu\bar{b}$	ΔE=87	ΞB_s, 3	ΔE=73	ΞB_s^*, 3

Pentaquark	$I=3/2$, $S=1/2$	Threshold, multiplicity	$I=3/2$, $S=3/2$	Threshold, multiplicity	$I=3/2$, $S=5/2$	Threshold, multiplicity
$uuus\bar{c}$	ΔE=214	ΣD, 3	ΔE=-42	ΔD_s, 3	ΔE=0	ΔD_s^*, 1
$uuus\bar{b}$	ΔE=170	ΣB, 3	ΔE=142	ΣB^*, 3	ΔE=0	ΔB_s^*, 1
$uuuc\bar{s}$	ΔE=274	$\Sigma_c K$, 3	ΔE=186	$\Sigma_c^* K$, 3	ΔE=0	ΔD_s^*, 1
$uuuc\bar{c}$	ΔE=191	$\Sigma_c D$, 3	ΔE=-20	$\Delta\eta_c$, 3	ΔE=0	$\Delta J/\psi$, 1

Pentaquak	$udsc\bar{c}$	$udss\bar{c}$	$udcc\bar{s}$	$udcc\bar{c}$	$udss\bar{b}$	$uuud\bar{s}$	$uuud\bar{c}$	$uuud\bar{b}$
$I=1,\ S=5/2$	ΔE=-44	ΔE=-17	ΔE=6	ΔE=0	ΔE=-17	ΔE=0	ΔE=0	ΔE=0
Threshold, multiplicity	$\Sigma^* J/\psi$, 2	$\Sigma^* D_s^*$, 1	$\Xi_{cc}^* K^*$, 1	$\Sigma_c^* J/\psi$, 1	$\Sigma^* B_s^*$, 1	ΔK^*, 1	ΔD^*, 1	ΔB^*, 1

Pentaquak	$udsc\bar{c}$	$udss\bar{c}$	$udcc\bar{s}$	$udcc\bar{c}$	$udss\bar{b}$
$I=0,\ S=5/2$	ΔE=-12	ΔE=-7	ΔE=-3	ΔE=-17	ΔE=-7
Threshold, multiplicity	$\Xi_c^* D^*$, 1	$\Xi^* D^*$, 1	$\Xi_{cc}^* K^*$, 1	$\Xi_{cc}^* D^*$, 1	$\Xi^* B^*$, 1

Pentaquak	$uuds\bar{c}$	$uudc\bar{s}$	$uudc\bar{c}$	$uuds\bar{b}$	$sssu\bar{c}$	$sssu\bar{b}$
$I=1/2,\ S=5/2$	ΔE=-4	ΔE=-1	ΔE=-9	ΔE=-4	ΔE=-34	ΔE=-34
Threshold, multiplicity	$\Sigma^* D^*$, 1	$\Sigma_c^* K^*$, 1	$\Sigma_c^* D^*$, 1	$\Sigma^* B^*$, 1	$\Xi^* D_s^*$, 1	$\Xi^* B_s^*$, 1

Quark Model – Three-body force

- Intrinsic uncertainty

Noh et al. PLB862 (2025) 139278, Jeongheon Baek (PPNP in preparation)

Use this quark model to Fit the meson spectrum

- Importance of exact wave function (J. Baek et al)

Meson ($J^{P(C)}$)	$\eta_c\,(0^{-+})$	$J/\psi\,(1^{--})$	$D\,(0^-)$	$D^*\,(1^-)$	$\pi\,(0^{-+})$	$\rho\,(1^{--})$	$K\,(0^-)$	$K^*\,(1^-)$
Experimental Value (MeV)	2983.6	3096.9	1864.8	2010.3	139.57	775.11	493.68	891.66
1 Gaussian ($\sigma = 5.86$ MeV)	2996.9	3089.6	1864.1	2010.7	139.39	775.49	494.62	888.82
Deviation ($M - M_{\text{exp}}$ in MeV)	+13.3	−7.3	−0.7	+0.4	−0.18	+0.38	+0.94	−2.84
30 Gaussians ($\sigma = 1.36$ MeV)	2984.0	3095.8	1863.5	2013.3	139.58	774.17	493.77	891.45
Deviation ($M - M_{\text{exp}}$ in MeV)	+0.4	−1.1	−1.3	+3.0	+0.01	−0.94	+0.09	−0.21

$$\sigma = \left(\frac{1}{N-1} \sum_i \left(M_i^{Cal} - M_i^{Exp} \right)^2 \right)^{1/2}$$

$\sigma_{1-\text{Gaussian}} = 5.86$ MeV $\rightarrow$ $\sigma_{30-\text{Gaussian}} = 1.36$ MeV

Quark model parameters from meson fit

$\kappa = 85.2189$ MeV fm $\quad a_0 = 0.0298683\ (\text{fm/MeV})^{1/2} \quad D = 1082.19$ MeV $\quad \kappa_0 = 207.473$ MeV

$\alpha = 0.911659\ \text{fm}^{-1} \quad \beta = 0.00090649\ (\text{MeV fm})^{-1} \quad \gamma = 0.00120125\ \text{MeV}^{-1}$

$m_{ud} = 316.551$ MeV $\quad m_s = 595.328$ MeV $\quad m_c = 1882.96$ MeV

When using parameters from meson fit to calculate Baryons

- Fails even when exact wave functions is used

Baryon ($\mathcal{J}^P$)	$p\,(1/2^+)$	$\Delta\,(3/2^+)$	$\Sigma\,(1/2^+)$	$\Lambda\,(1/2^+)$	$\Sigma_c\,(1/2^+)$	$\Lambda_c\,(1/2^+)$	$\Xi_{cc}\,(1/2^+)$	$\Sigma^*\,(3/2^+)$	$\Sigma_c^*\,(3/2^+)$
Experimental Mass (MeV)	938.27	1232.00	1197.45	1115.68	2453.97	2286.46	3621.46	1383.70	2518.48
M_S (MeV, $\sigma = 73.34$)	1036.27	1348.85	1255.90	1168.64	2495.92	2315.78	3630.22	1462.16	2587.03
Deviation ($M_S - M_{\text{exp}}$)	+98.00	+116.85	+58.45	+52.96	+41.95	+29.32	+8.76	+78.46	+68.55
$M_{S,P,D}^{\text{GEM}}$ (MeV, $\sigma = 46.28$)	997.56	1313.42	1231.40	1124.22	2476.24	2294.00	3619.29	1433.87	2570.33
Deviation ($M_{S,P,D}^{\text{GEM}} - M_{\text{exp}}$)	+59.29	+81.42	+33.95	+8.54	+22.27	+7.54	−2.17	+50.17	+51.85
Improvement ΔM (MeV)	−38.71	−35.43	−24.50	−44.42	−19.68	−21.78	−10.93	−28.28	−16.70

$$\sigma_{\text{Meson}} = 1.36 \text{ MeV} \rightarrow \sigma_{\text{Baryon}} = 46.28 \text{ MeV}$$

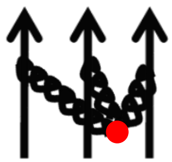
☞ It is an old problem in quark model. When using the same parameters,
There seems to be an inherent problem when going from 2 to 3 quark system

But

- Nuclear Physics lesson: from Deuteron to Triton need three-body force
- Three-body quark force: Lessons from Nuclear Physics
- Resolution of the Failure

- V. Dmitrasinovic, PLB499(2001)135: S. Pepin, Fl Stancu, PRD65 (2002)

$$C_{123} = \begin{cases} d^{abc} F_1^a F_2^b F_3^c \\ if^{abc} F_1^a F_2^b F_3^c \end{cases}, \quad \text{where} \quad F_i^a = \frac{\lambda_i^a}{2} \text{ for quark } i$$



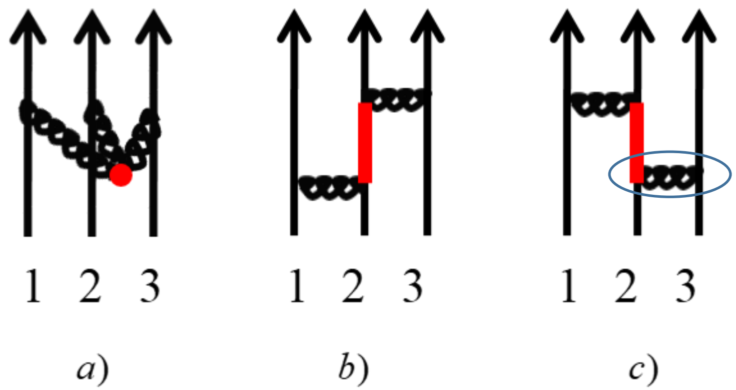
$$\left[F^a, F^b\right] = if^{abc} F^c \quad \text{and} \quad \left\{F^a, F^b\right\} = \frac{1}{3}\delta^{ab} + d^{abc} F^c$$

- For a Baryon, $C_{123} = \begin{cases} d^{abc} F_1^a F_2^b F_3^c = \frac{10}{9} \\ if^{abc} F_1^a F_2^b F_3^c = 0 \end{cases}$, does not improve Δ-N mass splitting

- When anti-quark are involved, $\tilde{C}_{123} = \begin{cases} -d^{abc} F_1^a F_2^b \bar{F}_3^c \\ d^{abc} F_1^a \bar{F}_2^b \bar{F}_3^c \\ -d^{abc} \bar{F}_1^a \bar{F}_2^b \bar{F}_3^c \end{cases}$, '-' put in by hand

$$\left[\bar{F}^a, \bar{F}^b\right] = if^{abc} \bar{F}^c \quad \text{and} \quad \left\{\bar{F}^a, \bar{F}^b\right\} = \frac{1}{3}\delta^{ab} - d^{abc} \bar{F}^c$$

- Origin could be similar to Nuclear-Three-body force



$$L_Y^{3Q} = \sum_{i<j<k} \left(AL_{ijk}^{C-C} + BL_{ijk}^{S-S} + CL_{ijk}^{C-S} \right) \times f_Y(r_{ij}, r_{jk}, r_{ki}),$$

$$C \to (\lambda_2^c \lambda_3^c)$$

$$S \to \frac{1}{m_i m_j} (\lambda_2^c \lambda_3^c)(\sigma_2 \sigma_3)$$

$$f_Y(r_{ij}, r_{jk}, r_{ki}) = \exp\left[-\frac{c}{\hbar}(m_{ij}r_{ij} + m_{jk}r_{jk} + m_{ki}r_{ki}) \right]$$

Three-body spatial profile		(a) $\delta^{(3)}(\mathbf{r})$ approximation		(b) Mass-dep. Yukawa		(c) Mass-dep. Gaussian	
Observed mass ($M_{\text{exp}}, \mathcal{J}^P$)	Two-body mass ($\sigma = 46.28$)	with L^{3Q} ($\sigma = 12.71$)	M^{3Q} (MeV)	with L_Y^{3Q} ($\sigma = 3.67$)	M^{3Q} (MeV)	with L_G^{3Q} ($\sigma = 6.41$)[a]	M^{3Q} (MeV)
$p(938.27, 1/2^+)$	997.56(+59.29)	958.00(+19.73)	−39.56	940.31(+2.04)	−57.25	947.08(+8.81)	−50.48
$\Delta(1232, 3/2^+)$	1313.42(+81.42)	1243.64(+11.64)	−69.78	1231.28(−0.72)	−82.14	1232.82(+0.82)	−80.60
$\Sigma(1197.45, 1/2^+)$	1231.40(+33.95)	1187.48(−9.97)	−43.92	1192.9(−4.55)	−38.50	1189.16(−8.29)	−42.24
$\Lambda(1115.68, 1/2^+)$	1124.22(+8.54)	1111.78(−3.90)	−12.44	1110.94(−4.74)	−13.28	1110.88(−4.8)	−13.34
$\Sigma_c(2453.97, 1/2^+)$	2476.24(+22.27)	2441.86(−12.11)	−34.38	2455.27(+1.30)	−20.97	2447.52(−6.45)	−28.72
$\Lambda_c(2286.46, 1/2^+)$	2294.00(+7.54)	2290.70(+4.23)	−3.31	2292.53(+6.07)	−1.47	2292.70(+6.24)	−1.30
$\Xi_{cc}(3621.46, 1/2^+)$	3619.29(−2.17)	3599.99(−21.47)	−19.30	3618.58(−2.88)	−0.71	3619.27(−2.19)	−0.02
$\Sigma^*(1383.7, 3/2^+)$	1433.87(+50.17)	1378.37(−5.33)	−55.50	1383.11(−0.59)	−50.76	1381.71(−1.99)	−52.16
$\Sigma_c^*(2518.48, 3/2^+)$	2570.33(+51.85)	2518.59(+0.11)	−51.74	2522.11(+3.63)	−48.22	2526.82(+8.27)	−43.58

[a] The Gaussian-type three-body strengths are $A = -(30.43\text{ MeV})^2$, $B = (126.79\text{ MeV})^6$, $C = -(77.28\text{ MeV})^4$.

$$\sigma_{\text{Meson}} = 1.36\text{ MeV} \to \sigma_{\text{Baryon}}(\text{no three-body}) = 46.28\text{ MeV} \to \sigma_{\text{Baryon}}(\text{with three-body}) = 3.67\text{ MeV}$$

→ With three-body force, the Baryon spectrum is well reproduced with the parameters from the meson fit

☞ Effect of 3-quark-interaction on Tetraquarks : Repulsive

$$\mathrm{H}_{\mathrm{Total}} = \mathrm{H}_{\text{2-body}} + A \cdot L^{C-C} + B \cdot L^{S-S} + C \cdot L^{C-S}$$

Particle	Measured mass (MeV)	$\sum_{i<j<k} L_{ijk}^{C-C}$	$\sum_{i<j<k} L_{ijk}^{S-S}$	$\sum_{i<j<k} L_{ijk}^{C-S}$
T_{cc}	3875	-4.84236	0.0319013	20.9444
$X(3872)$	3872	19.3694	0.0427164	-1.36541

Implication for Exotics from quark model

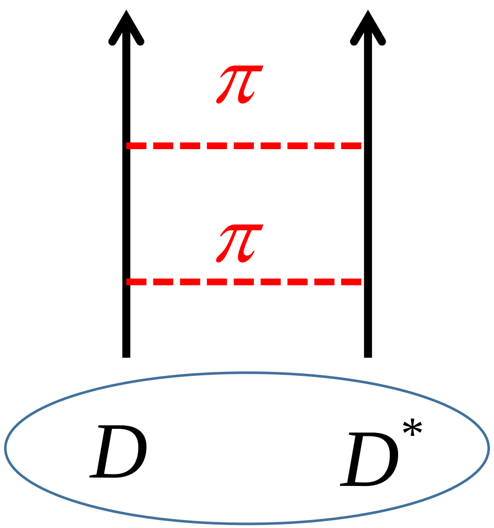
- Both X(3872) and T_{cc} will probably not be compact

- T_{bb} will definitely be compact

- For Pentaquarks probably not P_c but there are other possibilities

Can X(3872) be D-$\bar{D}^*$ and Tcc be D-D^* Molecules?

Perspectives from the π-exchange



$$M\left(J_M, I_M\right)$$

Especially important when

$J_M \neq 0$ Mixing with D-wave

and

$I_M < \left(I_D + I_{D^*}\right)$ Mixing is strong

Few Thoughts on production:

1. If X(3872) is a $\bar{D}\,D^*$ S-wave molecule (with H. Yun, K. Han,S. Noh, A. Park)

2. Compact sizes (with C.M. Ko, Sungtae Cho)

Some remarks: in 2008,

IOPscience

Heavy-ion collisions at the LHC—Last call for predictions

N Armesto[1], N Borghini[2], S Jeon[3], U A Wiedemann[4], S Abreu[5], S V Akkelin[6], J Alam[7], J L Albacete[8], A Andronic[9], D Antonov[10] + Show full author list

Published 18 April 2008 • 2008 IOP Publishing Ltd

Journal of Physics G: Nuclear and Particle Physics, Volume 35, Number 5

Citation N Armesto *et al* 2008 *J. Phys. G: Nucl. Part. Phys.* **35** 054001

10.3. Charmed exotics from heavy-ion collision

Our contribution to the volume

S H Lee, S Yasui, W Liu and C M Ko

We discuss why charmed multiquark hadrons are likely to exist and explore the possibility of observing such states in heavy-ion reactions at the LHC.

Multiquark hadronic states are usually unstable as their quark configurations are energetically above those of combined meson and/or baryon states. However, constituent quark model calculations suggest that multiquark states might become stable when some of the light quarks are replaced by heavy quarks. Two possible states that could be realistically observed in heavy-ion collisions at LHC are the tetraquark $T_{cc}(ud\bar{c}\bar{c})$ [385] and the pentaquark

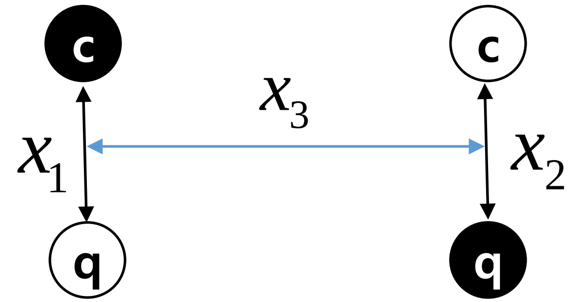
nature physics

Observation of an exotic narrow doubly charmed tetraquark

LHCb Collaboration*

Quark Spatial wave function of X(3872) : $D\bar{D}^*$ molecule

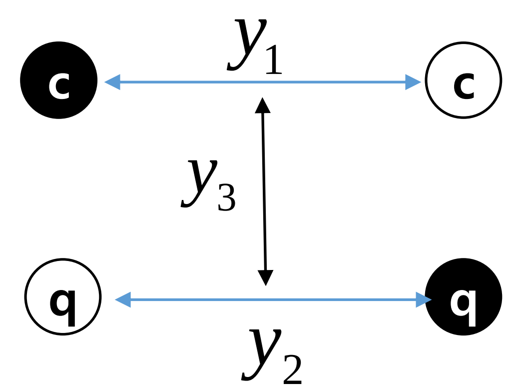
☞ S-wave in $(q\bar{c}),(c\bar{q})$ basis $D-\bar{D}^*$



$$\psi_1^{Spatial} \propto \exp\left[-a_1x_1^2 - a_2x_2^2 - a_3x_3^2\right]$$

$$R_{D\text{ or }D^*} \sim 0.55\text{ fm}, \quad R_{D-D^*} \sim 4\text{ fm}$$

☞ Transformation into $(c\bar{c}),(q\bar{q})$ basis



$$\psi_1^{Spatial} \propto \exp\left[-b_1y_1^2 - b_{12}y_1\cdot y_2 - b_2y_2^2 - b_3y_3^2\right]$$

$$R_{(c\bar{c})} \sim 4.01\text{ fm}, \quad R_{(q\bar{q})} \sim 4.06\text{ fm}, \quad R_{(c\bar{c})-(q\bar{q})} \sim 0.394\text{ fm}$$

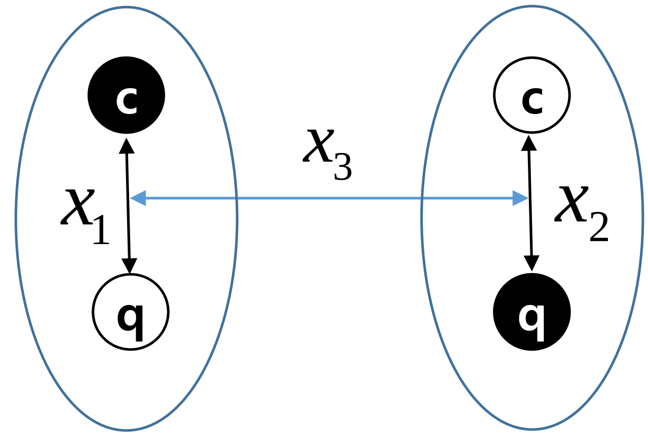
Quark Color-spin wave function of X(3872): $D\bar{D}^*$ molecule

$I^G\left(J^{PC}\right)=0^+\left(1^{++}\right)$

☞ In $(q\bar{c}),(c\bar{q})$ basis

$$|1'\rangle=(q\bar{c})^{C=1}_{S=0}\otimes(c\bar{q})^{C=1}_{S=1} \quad \rightarrow D-\bar{D}^*$$

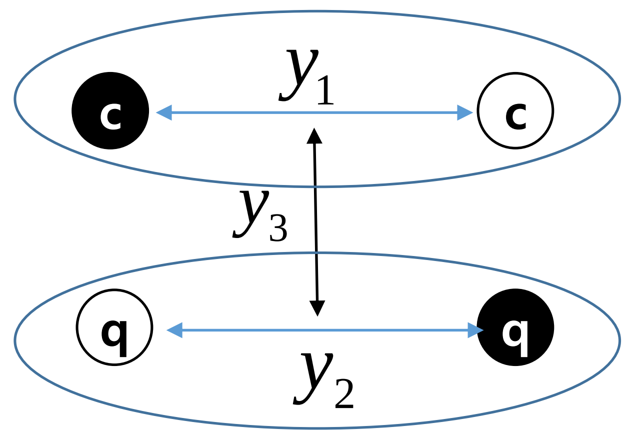
$$|2'\rangle=(q\bar{c})^{C=8}_{S=0}\otimes(c\bar{q})^{C=8}_{S=1}$$



☞ Transformation into $(c\bar{c}),(q\bar{q})$ basis

$$|1\rangle=(c\bar{c})^{C=8}_{S=1}\otimes(q\bar{q})^{C=8}_{S=1}$$

$$|2\rangle=(c\bar{c})^{C=1}_{S=1}\otimes(q\bar{q})^{C=1}_{S=1}$$



$$|1'\rangle=\frac{2\sqrt{2}}{3}|1\rangle+\frac{1}{3}|2\rangle$$

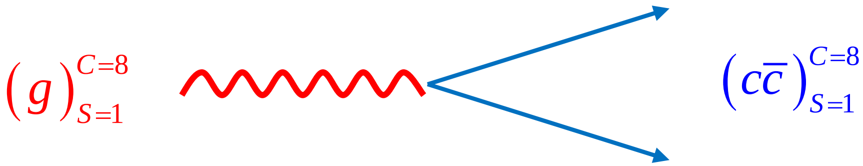
$$|2'\rangle=-\frac{1}{3}|1\rangle+\frac{2\sqrt{2}}{3}|2\rangle$$

→ $D\bar{D}^*$ is mostly composed of $(c\bar{c})^{C=8}_{S=1}\otimes(q\bar{q})^{C=8}_{S=1}$

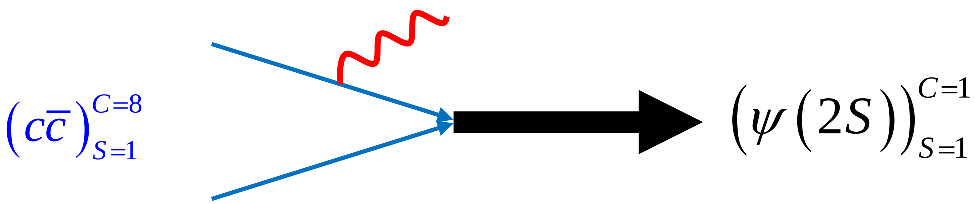
Possible explanation of abundant production of X(3872)

☞ $D\bar{D}^*$ is mostly composed of $(c\bar{c})^{C=8}_{S=1} \otimes (q\bar{q})^{C=8}_{S=1}$

☞ Gluon (spin 1 color octet) jet at high Pt to leading order decays to $(c\bar{c})^{C=8}_{S=1}$



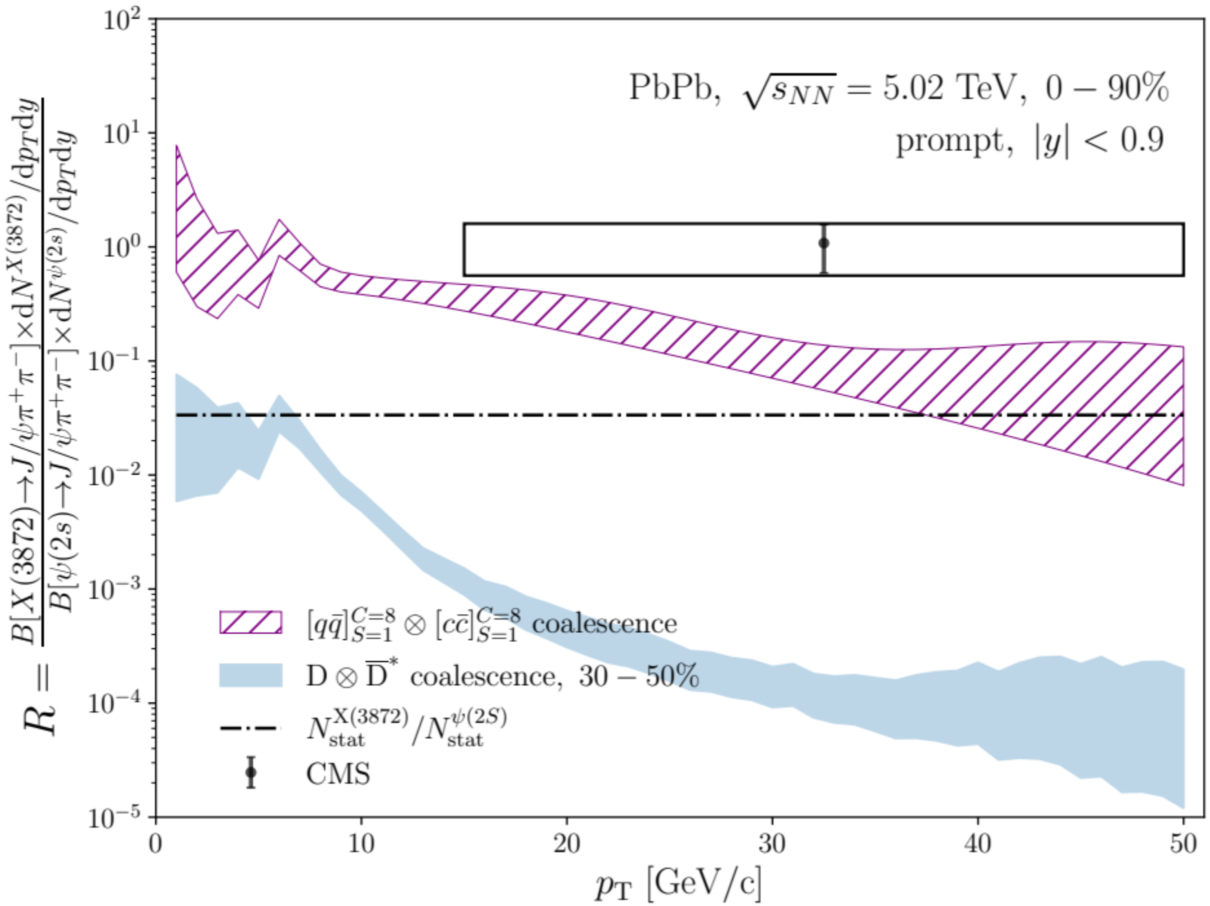
☞ ψ(2S) production at high Pt is dominated by $(c\bar{c})^{C=8}_{S=1}$ but has to be multiplied by a small overlap into color singlets: NRQCD



☞ Use pp data+ Pert QCD+ NRQCD $$\frac{d\sigma^{[c\bar{c}(^3S_1^{[8]})]}}{d^2p_T} = \frac{1}{\langle \mathcal{O}^{\psi(2S)}_{[c\bar{c}(^3S_1^{[8]})]} \rangle} \frac{d\sigma^{[c\bar{c}(^3S_1^{[8]})] \to \psi(2S)}}{d^2p_T}.$$

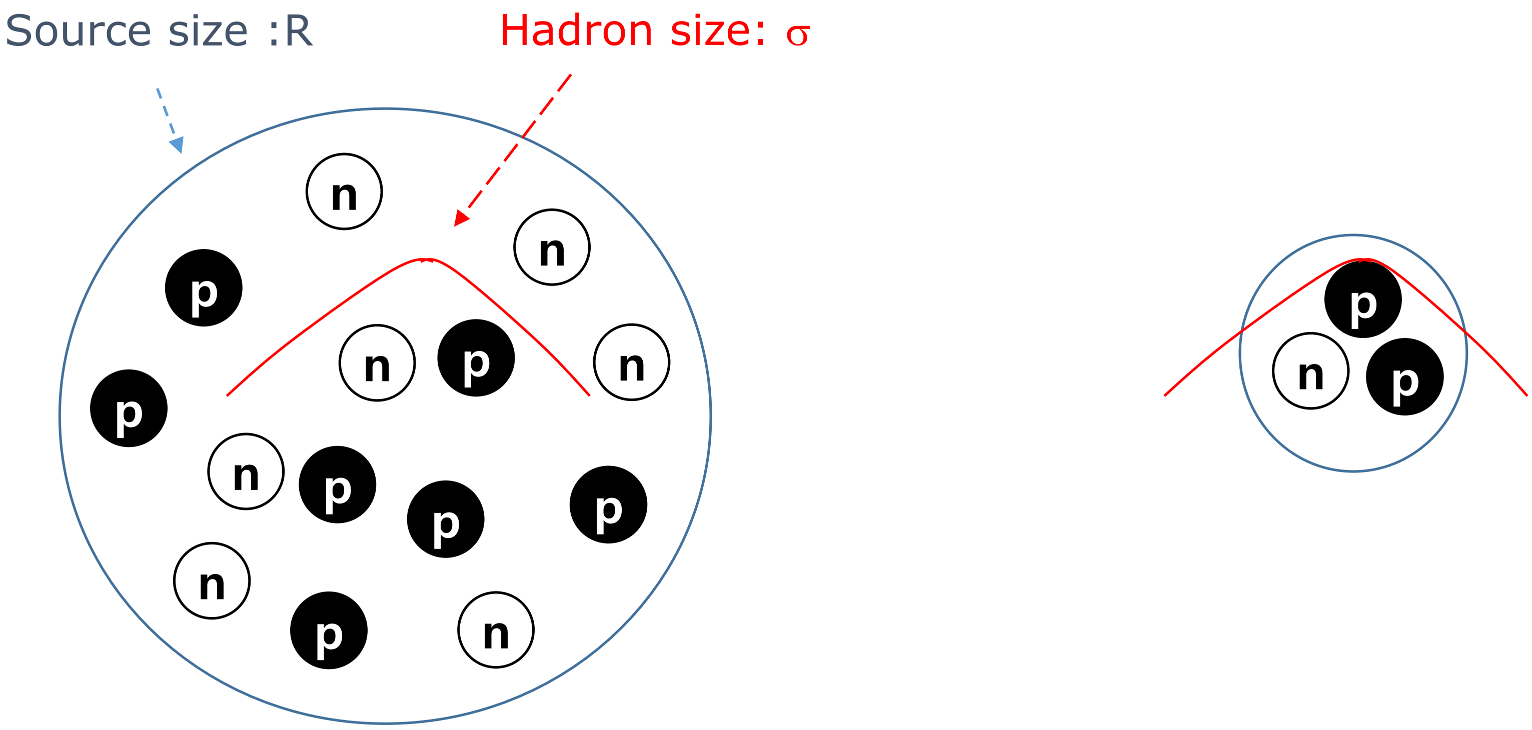
Possible explanation of abundant production of X(3872) (Hyeongock Yun)

☞ Use coalescence model on $(c\bar{c})^{C=8}_{S=1} + (q+\bar{q})^{C=8}_{S=1}$



Source size vs final hadron size

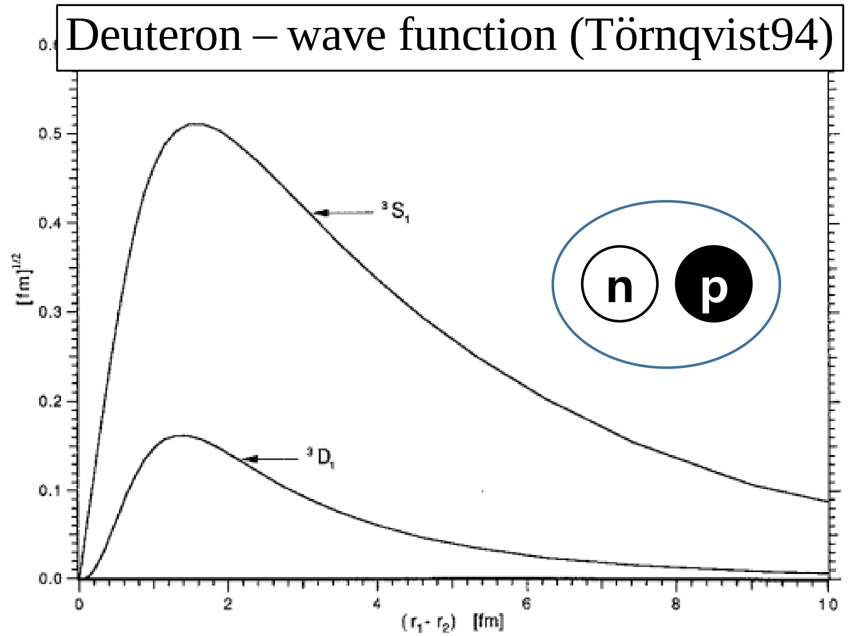
- Deuteron production from small system (K. Sun, C. Ko, B. Donigus PLB792(2019)

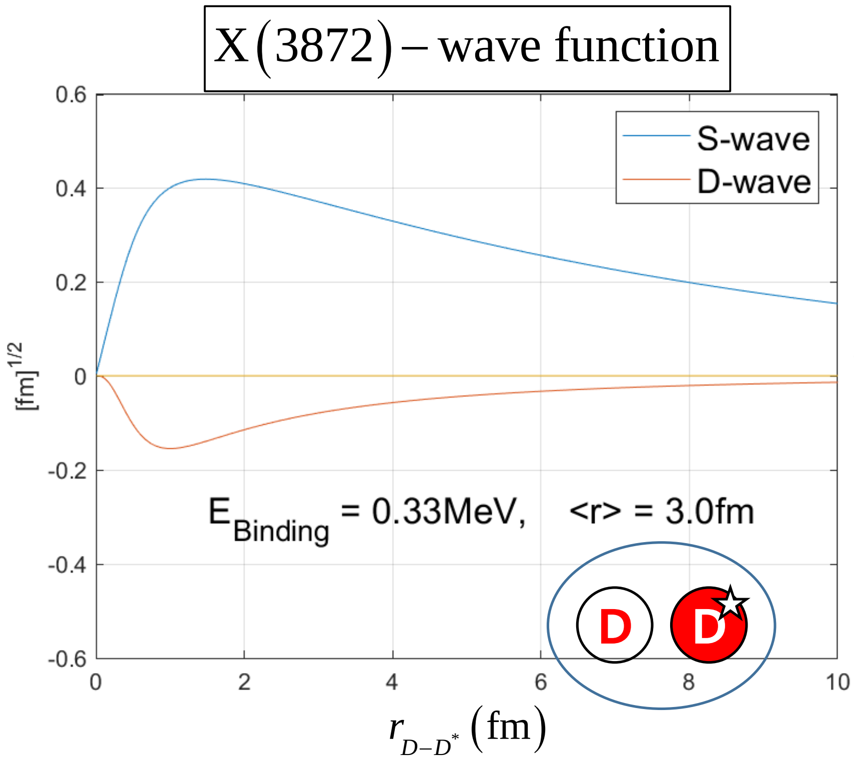


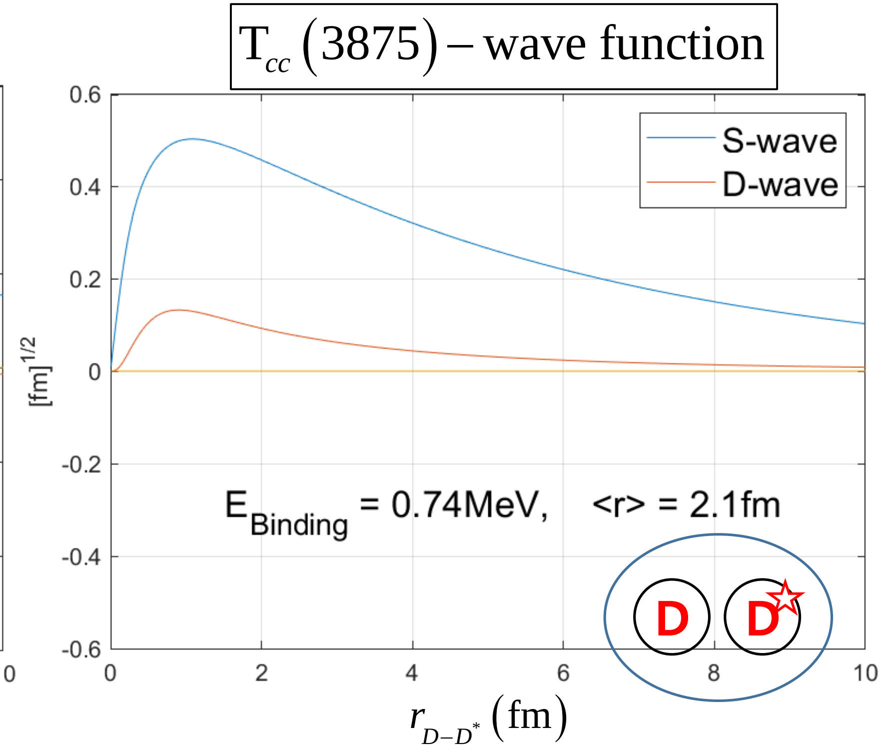
3 dimension coalescence model $$\frac{N_{\text{Deuteron}}}{N_{\text{Proton}}} \propto \frac{1}{\left[1+\frac{1}{4}\left(\frac{\sigma}{R}\right)^2\right]^{3/2}}$$

Yun et al. (PRC107 (2023))

☞ Meson exchange model:
X(3872), Tcc(3875) wave function
similar to Deuteron wave function

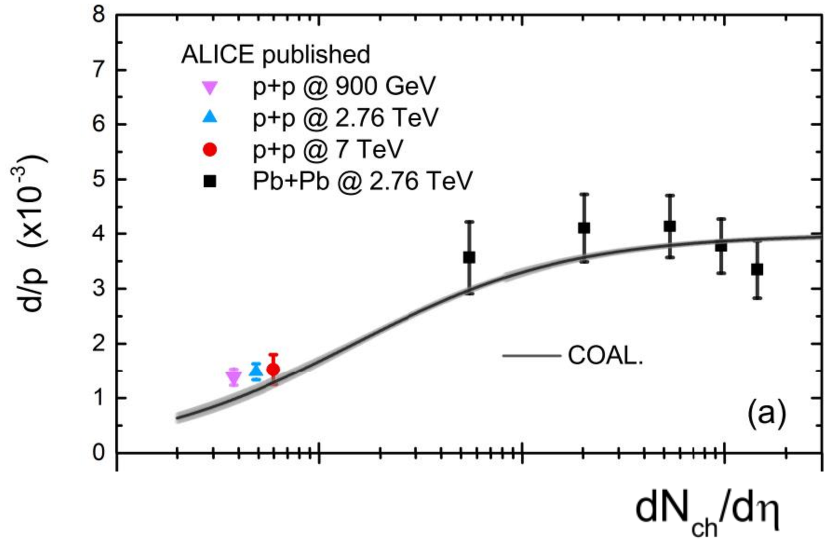






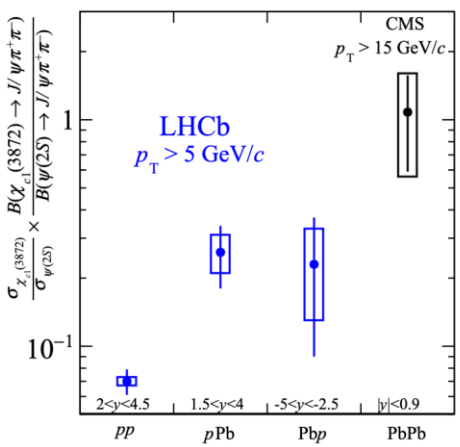
☞ Deuteron data

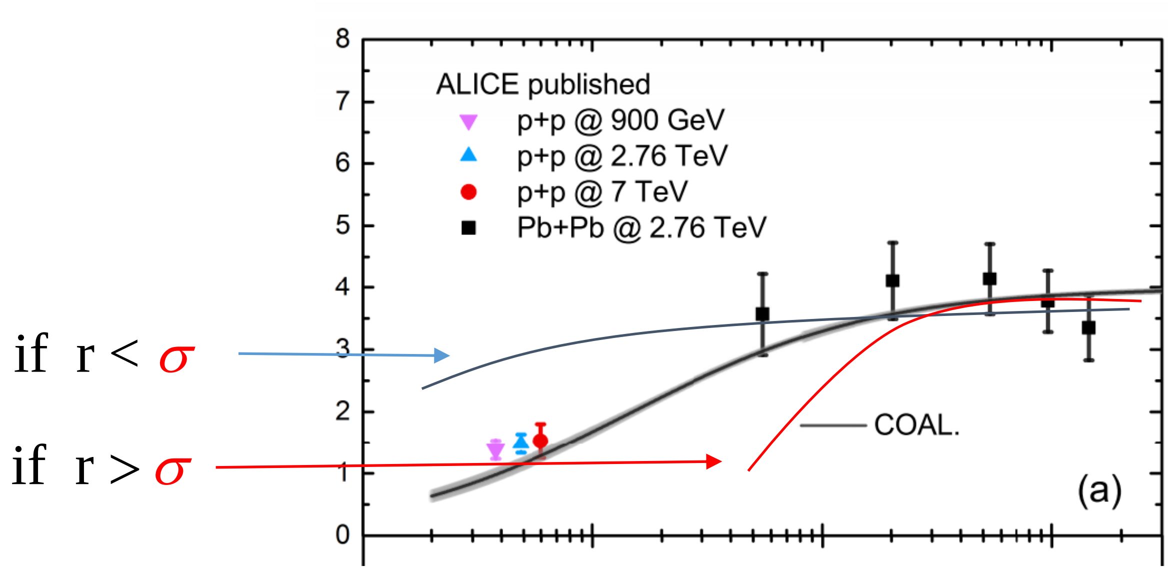
$\sigma \sim$ 2 fm



☞ If X(3872) is a D D* molecule with size r.

Hence, X(3872)/D ratio as a function of multiplicity can give us a hint on r





Summary

- Most exotics must have multiple heavy quark: HIC is an excellent factory

- Quark model analysis suggests:

  X(3872) most likely a broad molecule; Tcc not conclusive.

  Also, Most of the observed Pentaquark can not be compact.

  But Tbb should be compact multiquark state

- Discriminating Crypto-exotic states:

  f0(980) does not follow strangeness enhancement (ALICE)

  v2 measurements, exotic/hadron as a function of PT, ……

- Discriminating compact vs molecular exotic states:

  Look for anomalous production: Exotics have large color factors

  Study production as a function of multiplicity

  …..