Off-Equilibrium Effects on Heavy Flavor and Quarkonia in the QCD medium

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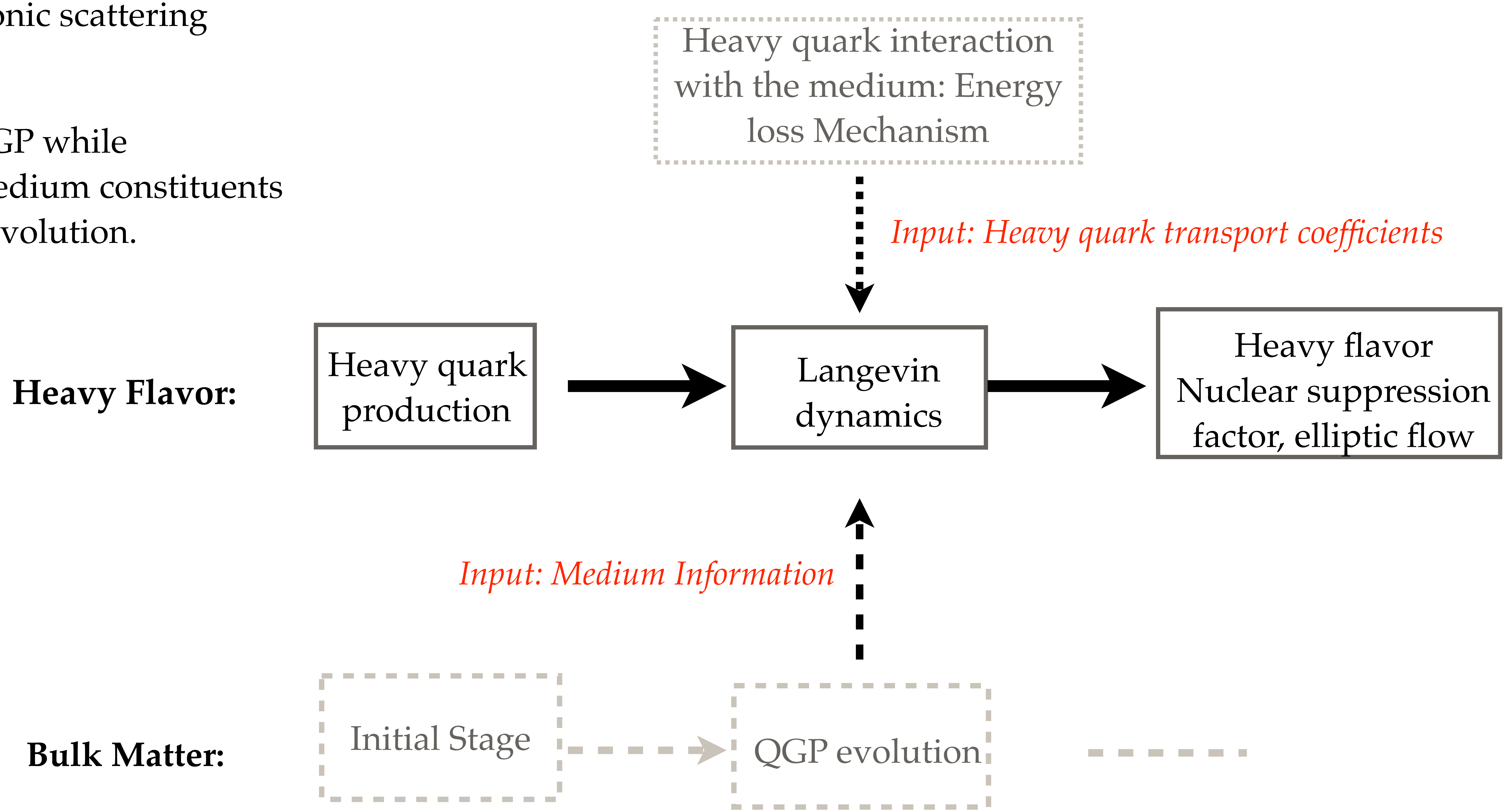
EQHC 2026

Heavy quarks as effective probes of hot nuclear matter

- Mostly produced at the very early stage of the collision in partonic scattering processes.

- Transverse through QGP while interacting with the medium constituents and witness the QGP evolution.



Heavy Quark transport

- Transport coefficients: Drag and momentum Diffusion
- Simplified Boltzmann Equation
- Soft momentum transfer

$$\frac{\partial f_{HQ}}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(\mathbf{p})\, f_{HQ} + \frac{\partial}{\partial p_j}\left[B_{ij}(\mathbf{p})\, f_{HQ}\right]\right]$$

For the process $HQ(P) + g/q(Q) \rightarrow HQ(P^{'}) + g/q(Q^{'})$

$$A_i = \frac{1}{\gamma_c}\frac{1}{2P^0}\int\frac{d^3\mathbf{q}}{(2\pi)^3 2Q^0}\int\frac{d^3\mathbf{p}^{'}}{(2\pi)^3 2P^{'0}}\int\frac{d^3\mathbf{q}^{'}}{(2\pi)^3 2Q^{'0}}(2\pi)^4\delta^4(P+Q-P^{'}-Q^{'})\sum|\mathcal{M}_{HQ,g/q}|^2 f_{g/q}(Q)\left(1 \pm f_{g/q}(Q^{'})\right)\left(\mathbf{p}-\mathbf{p}^{'}\right)_i$$

$$\equiv \langle\langle \left(\mathbf{p}-\mathbf{p}'\right)_i \rangle\rangle$$

HQ drag → thermal average of the momentum transfer

$$A_i = p_i A(p^2, T)$$

$$\mathrm{B}_{ij} = \langle\langle \left(\mathbf{p}-\mathbf{p}^{'}\right)_i \left(\mathbf{p}-\mathbf{p}^{'}\right)_j \rangle\rangle$$

Momentum diffusion → square of the momentum transfer

$$B_{ij} = \left(\delta_{ij} - \frac{p_i p_j}{p^2}\right)B_0(p^2,T) + \frac{p_i p_j}{p^2}B_1(p^2,T)$$

Charm quark drag in an expanding QGP

MK, M. Singh, V.Chandra, S. Jeon, C. Gale, PRC 102 (2020), 044907

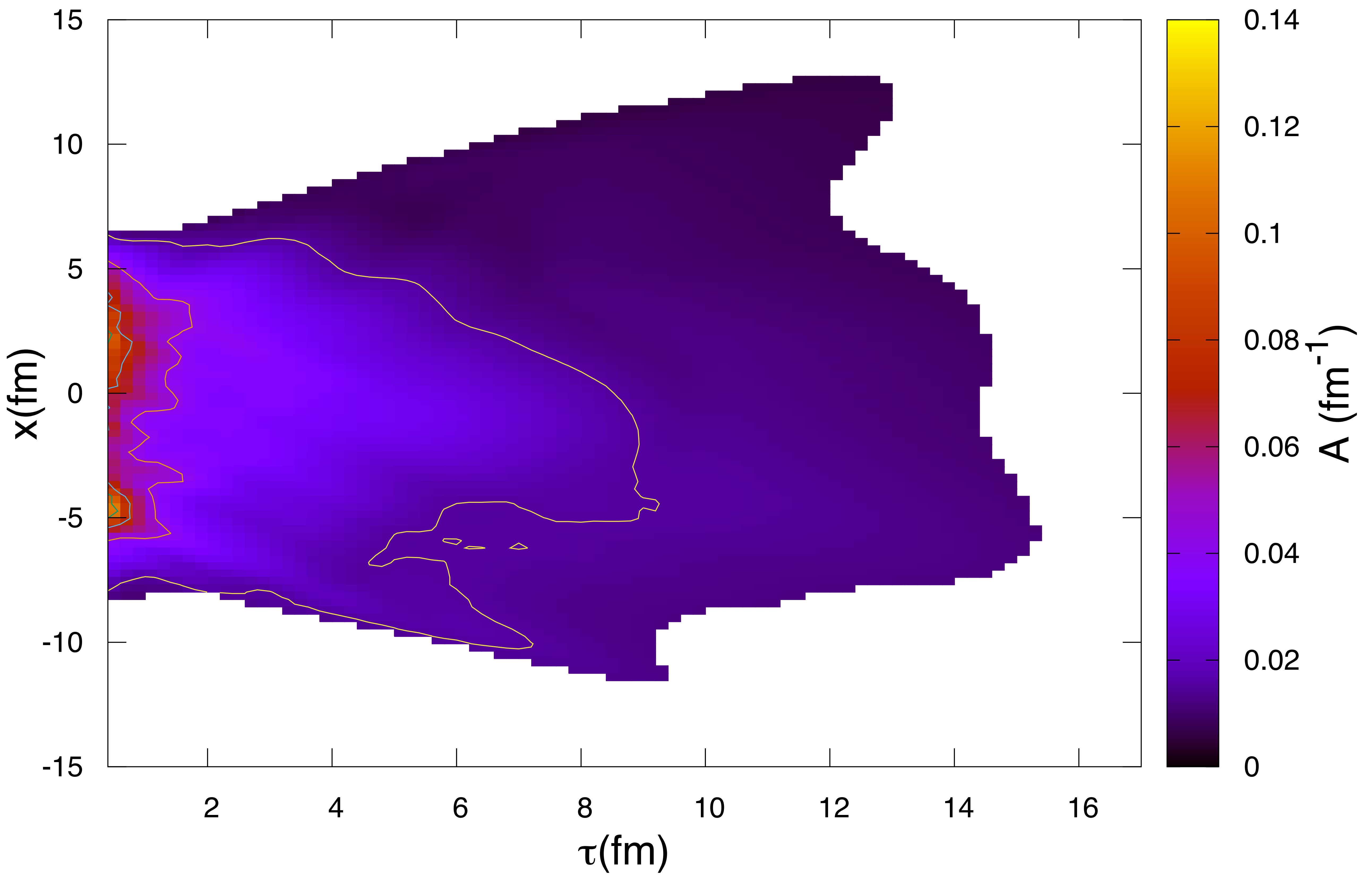
- Viscous hydrodynamics approach

  B. Schenke, S. Jeon, C. Gale, PRC 2010

- Fluctuating Initial state + MUSIC

  C. Gale, S. Jeon,B. Schenke, P. Tribedy, Venugopalan PRL 2012

- Pb+Pb collision at 2.76 TeV

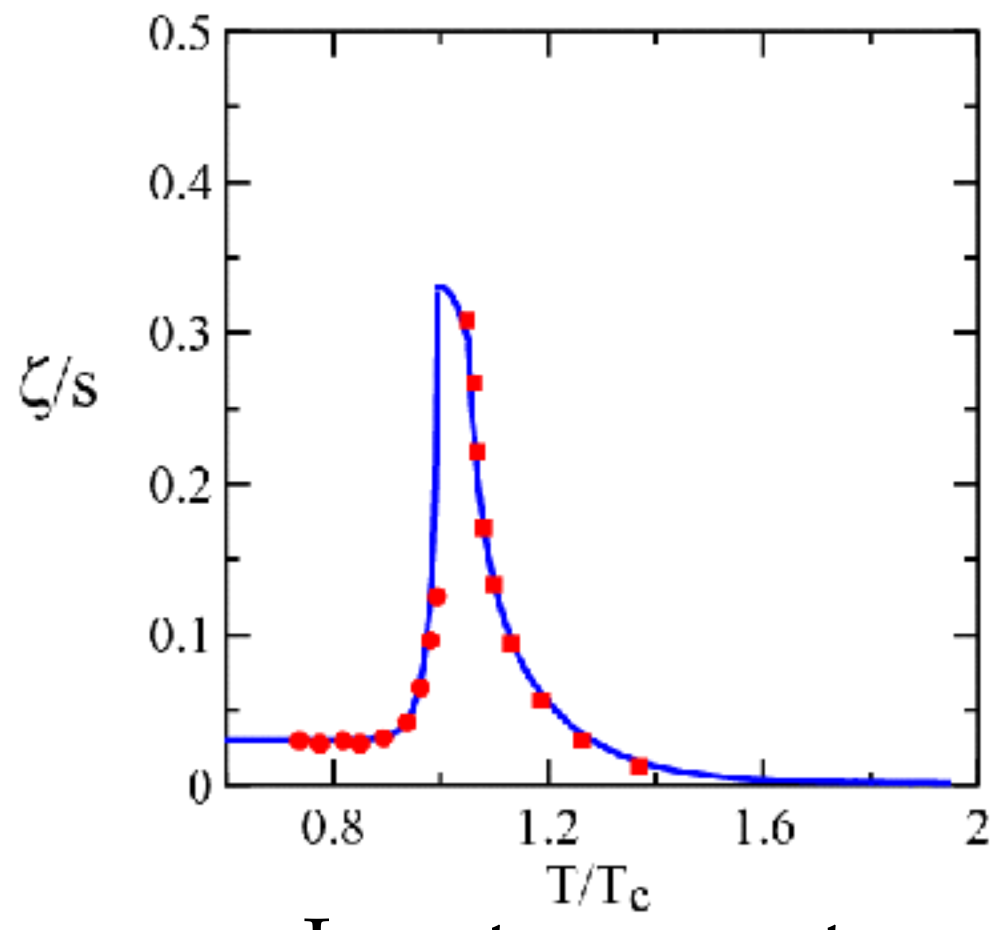


Drag coefficient of a charm quark with momentum p = 5 GeV at different space-time points. Curved lines indicate constant drag coefficient contours.

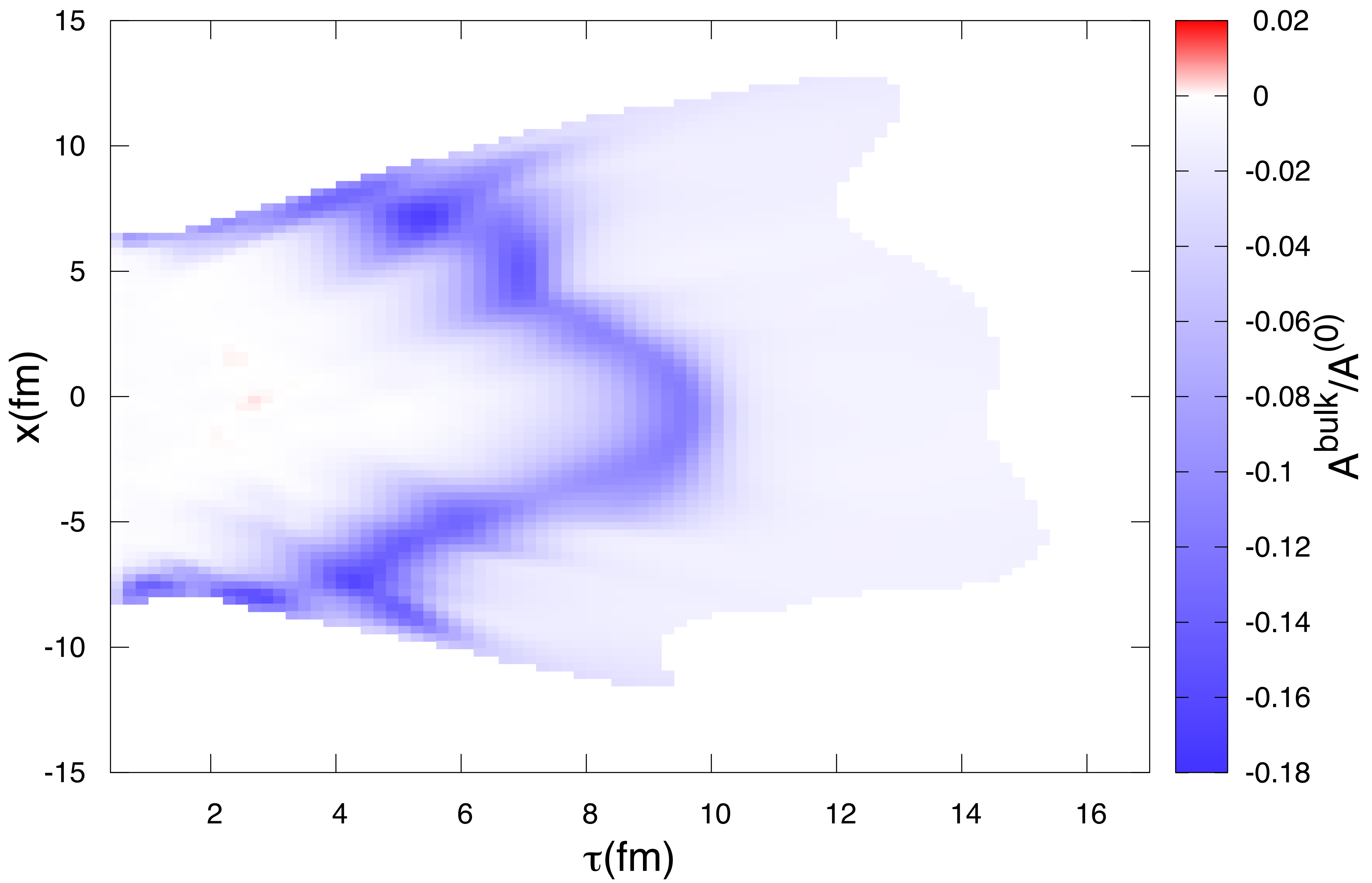
Bulk viscous corrections: $A_i \simeq A_i^{(0)} + A_i^{bulk}$

Choice of bulk viscosity

(Taken from *S. Ryu et.al., PRL 115 (2015), 132301*)



Input parameter

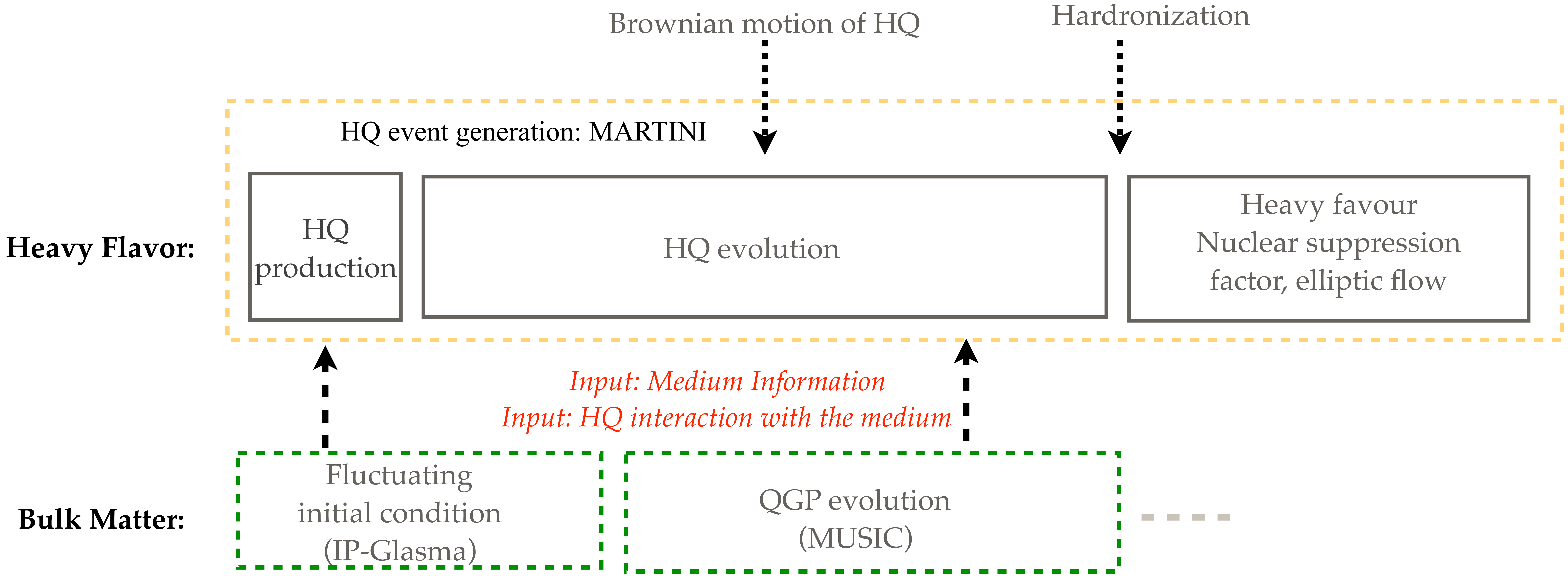


- Effects of viscous dynamics to the HQ drag and diffusion depends on the choice of the temperature behaviour of viscous coefficients.
- These viscous corrections are essential to maintain consistency in the theoretical description of HQ dynamics in expanding QGP medium.
- Impact of these corrections on heavy flavour phenomenology?

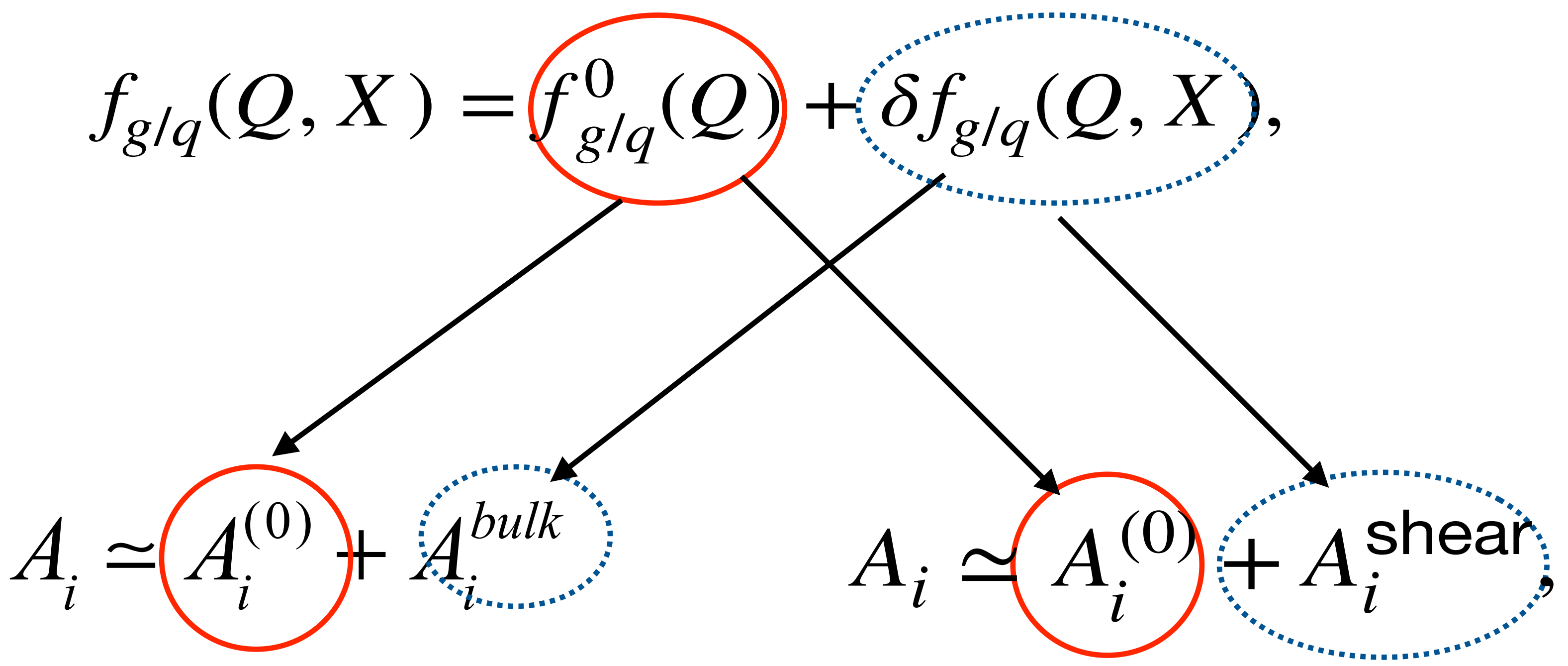
Phenomenological Model

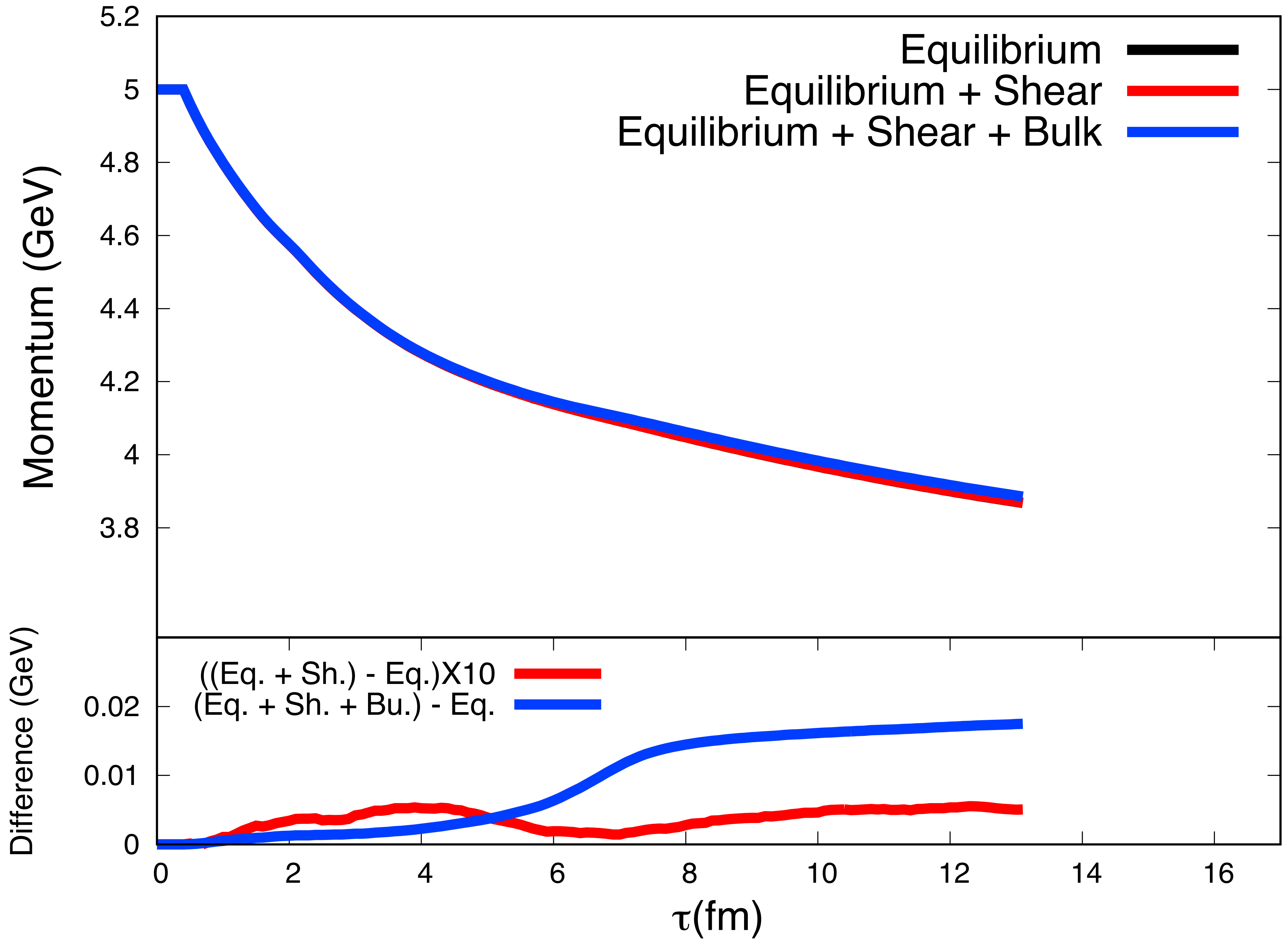
__MK__, M. Singh, S. Jeon, and C. Gale, PRC 102, 044907 (2020)
M. Singh, __MK__, S. Jeon, and C. Gale, PRC 108, 054901 (2023)

Our framework: MARTINI+MUSIC



Viscous effects on charm quark momentum evolution





- Charm quark momentum evolution in the viscous QGP medium
- The initial momentum is taken as p=5 GeV
- Equilibrium and equilibrium + shear curves almost overlap as the effect of shear correction on energy loss is negligible.

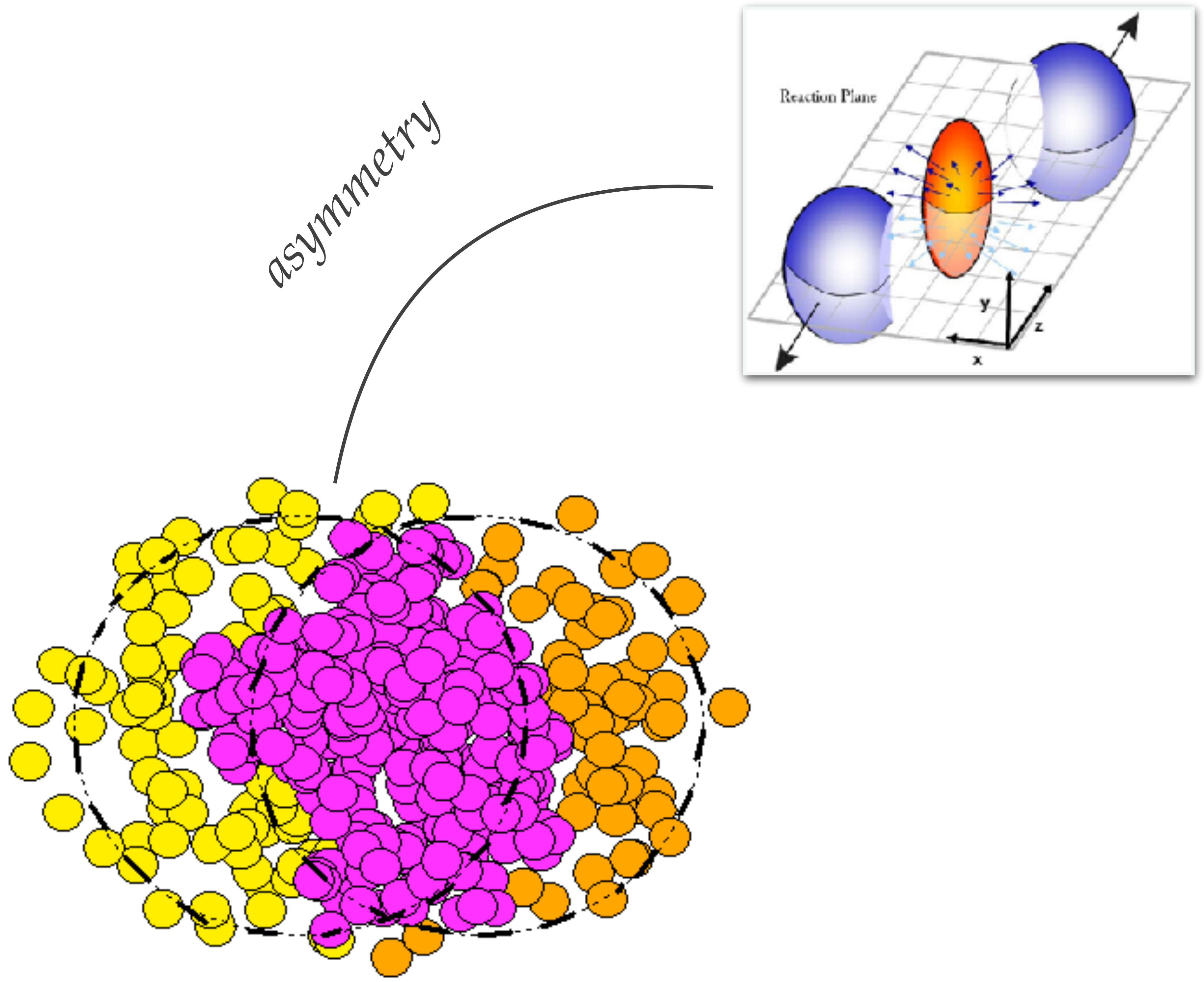
Two key observables

Nuclear suppression factor R_{AA}

$$R_{AA} = \frac{\text{Heavy flavor spectra \textbf{after} going through QGP}}{\text{Heavy flavor spectra \textbf{before} going through QGP}}$$

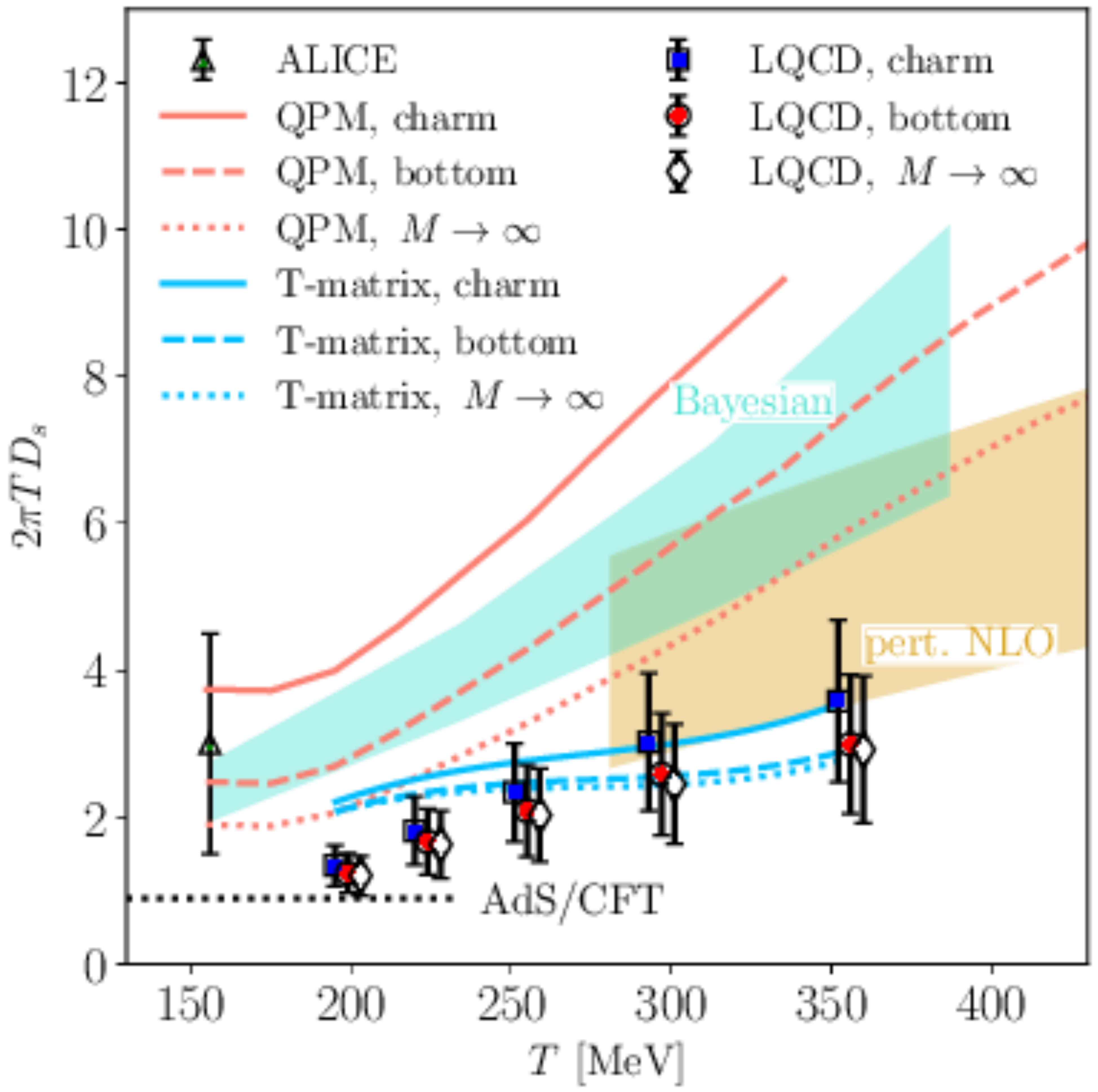
$$= \frac{\text{Heavy flavor spectra in \textbf{heavy-ion + heavy-ion} collision}}{\text{Heavy flavor spectra in \textbf{proton+proton} collision (scaled)}}$$

Elliptic flow v_2



Heavy quark interaction: recent updates

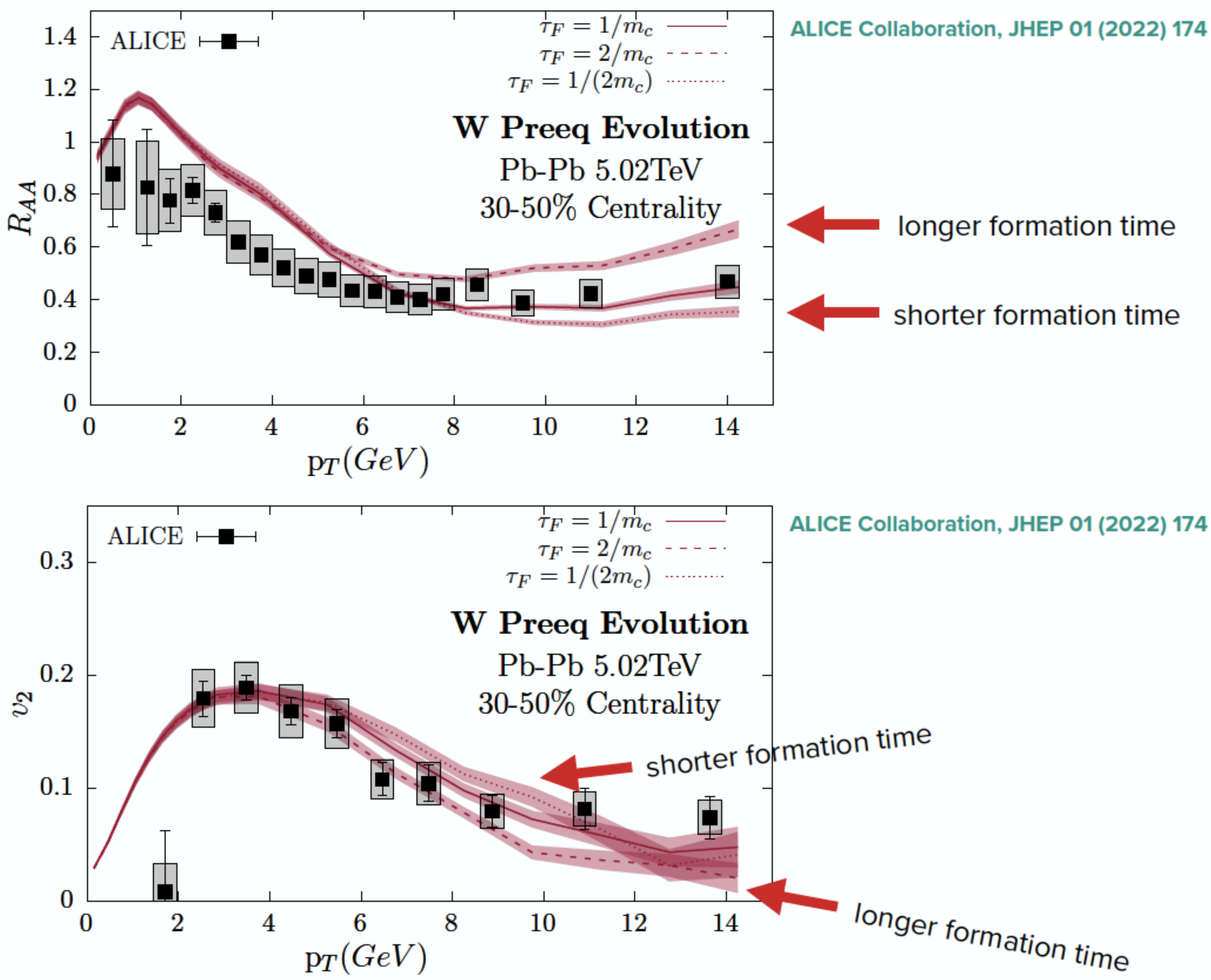
- Spatial diffusion coefficient
  - Total energy loss= Collisional +Radiative *S.Cao, G. Qin, S. Bass, PRC 92, 024907),*
  - Non- perturbative effects: Quasiparticle model (*F. Scardina, et.al., PRC, 2017*), T-Matrix estimation (*H. van Hees, M. Mannarelli, V. Greco, R. Rapp (2008)*; *Z. Tang, S. Mukherjee, P. Petreczky, R. Rapp, EPJA 60, 92 (2024)*
  - Model-to-data comparison: *Y. Xu et.al., PRC 97, 014907 (2018)*
- Lattice results: (*D. Banerjee, et.al., PRD, 2012*)
  First lattice data (2+1 flavor):
  [HotQCD Collaboration] PRL 130, 231902 (2023)
  [HotQCD Collaboration] PRL 132, 051902 (2024)



[HotQCD Collaboration] PRL 132, 051902 (2024)

D meson observables

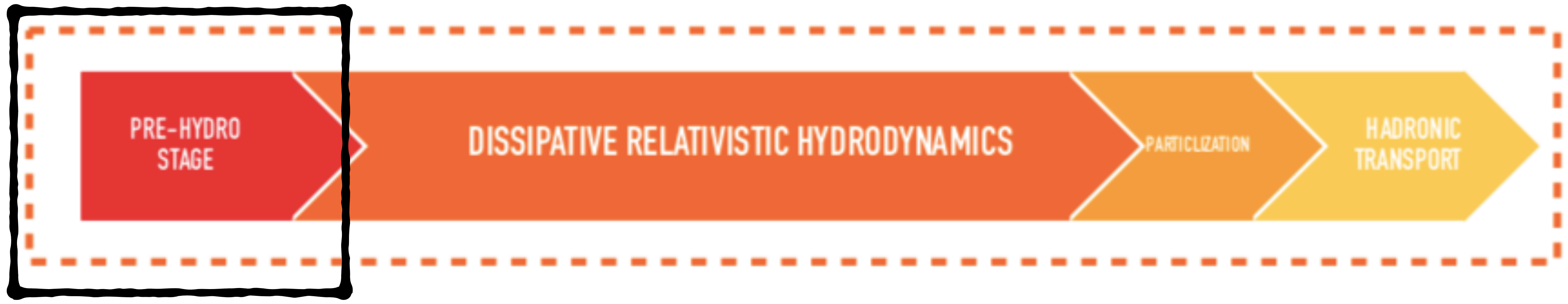
We can extract information about the system from these observables
(eg, formation time of heavy quark)

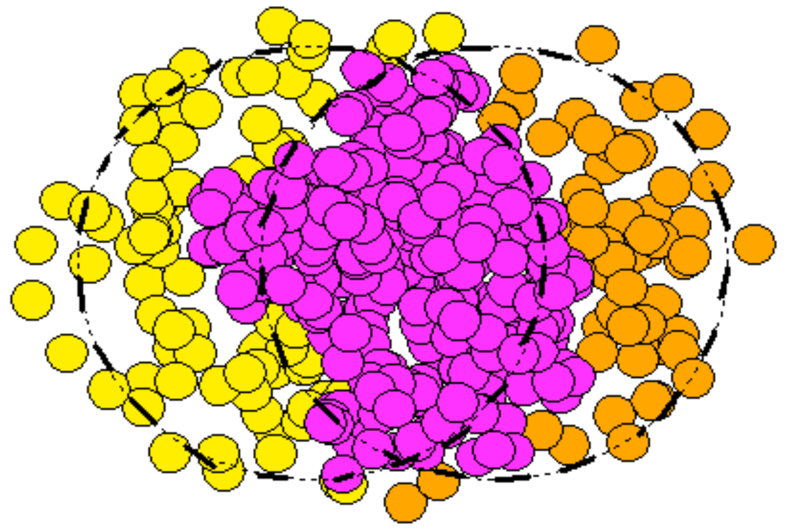


Question I. Can heavy quarks be an efficient probe of initial stage of HIC?

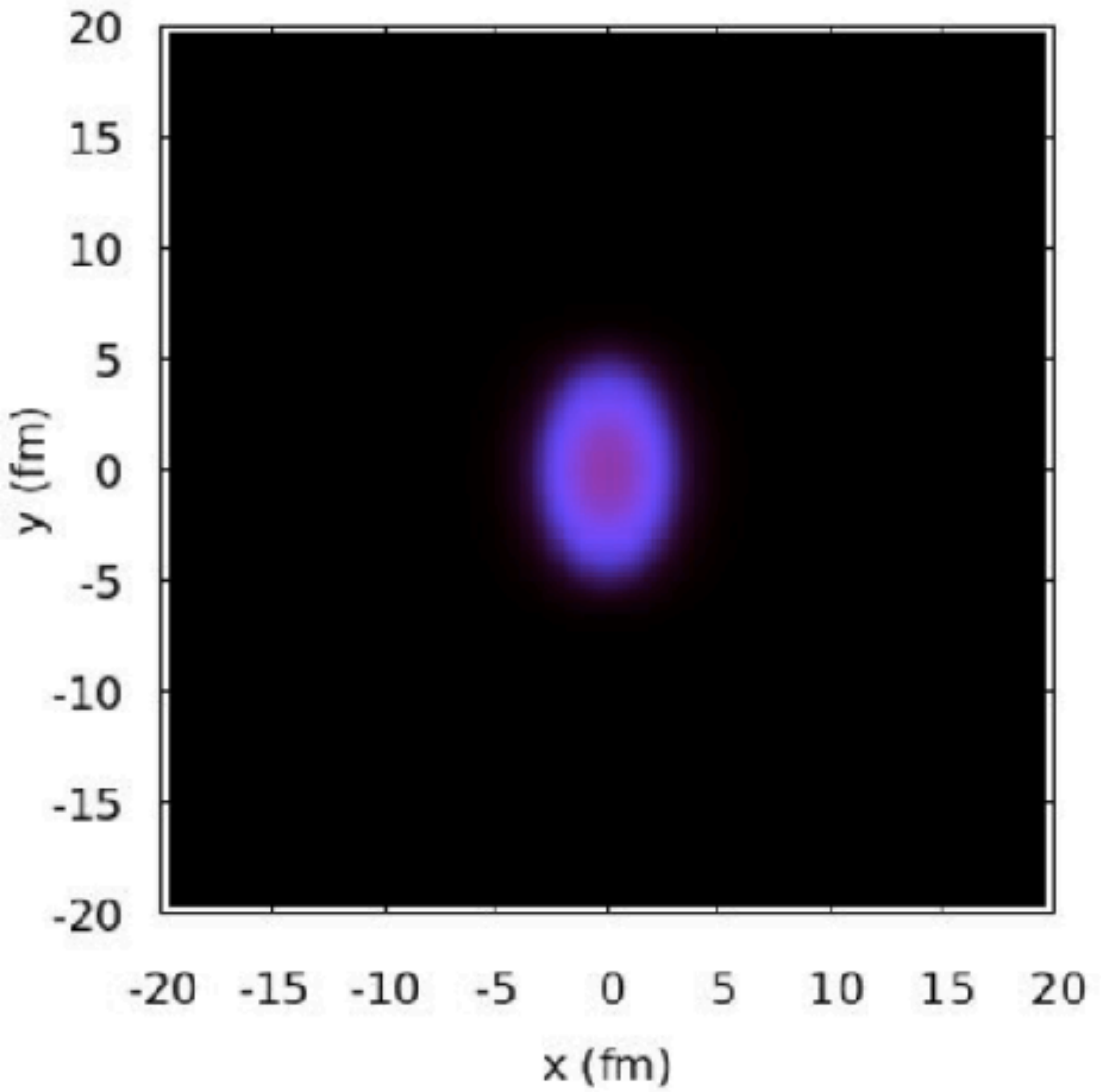
- The initial stage of collision, which sets the initial conditions for the hydrodynamical evolution of the QGP

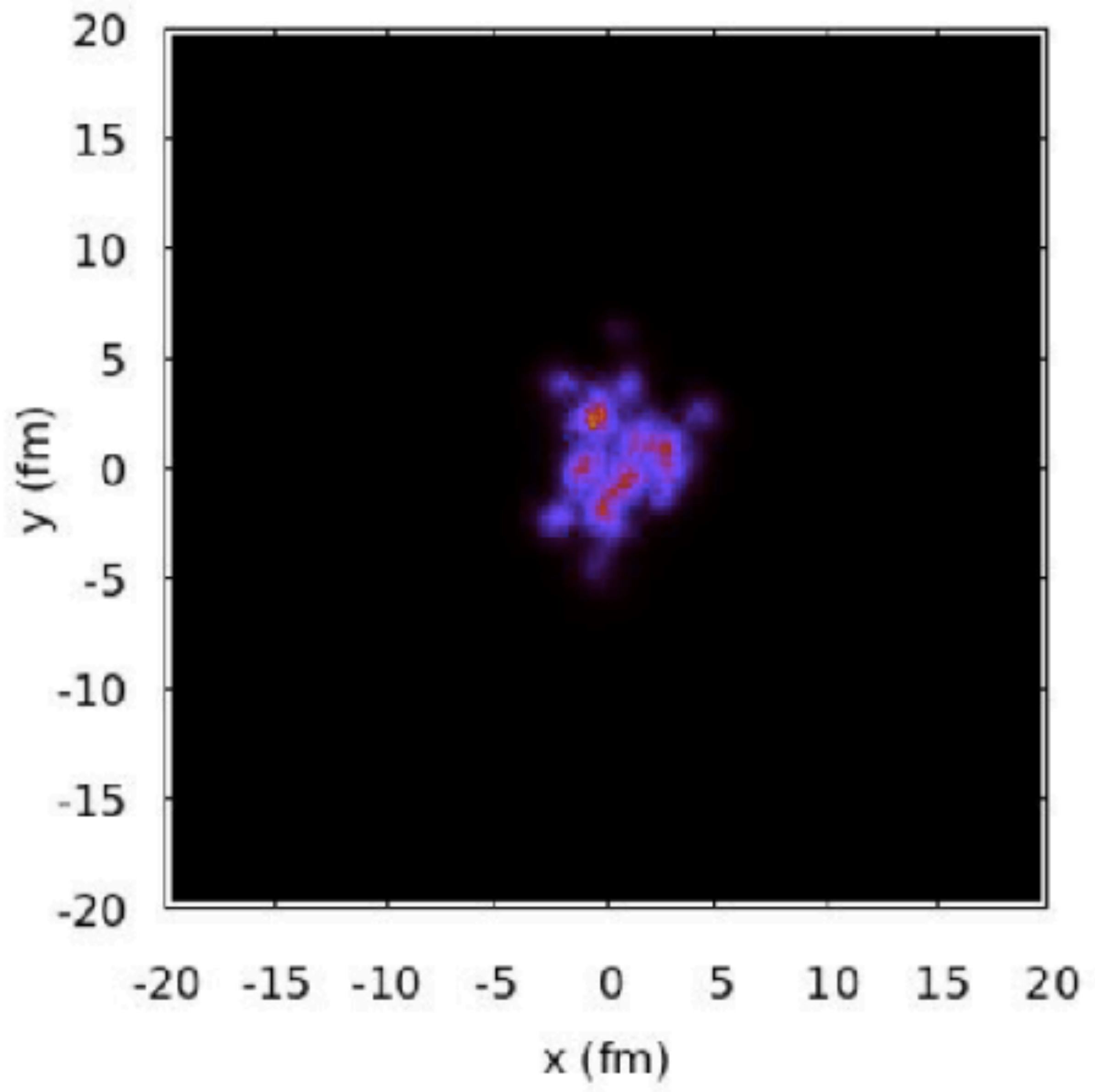
- As heavy quark is mostly created in the very early stage of the collision event, they can retain imprints of the initially produced matter in nucleus-nucleus collisions.





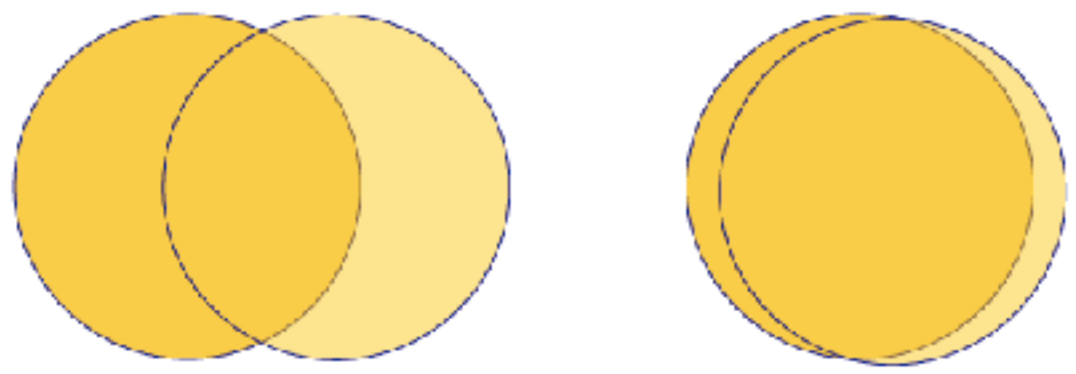
- Smooth background
- Fluctuating realistic Condition (*results from IP-GLASMA*)

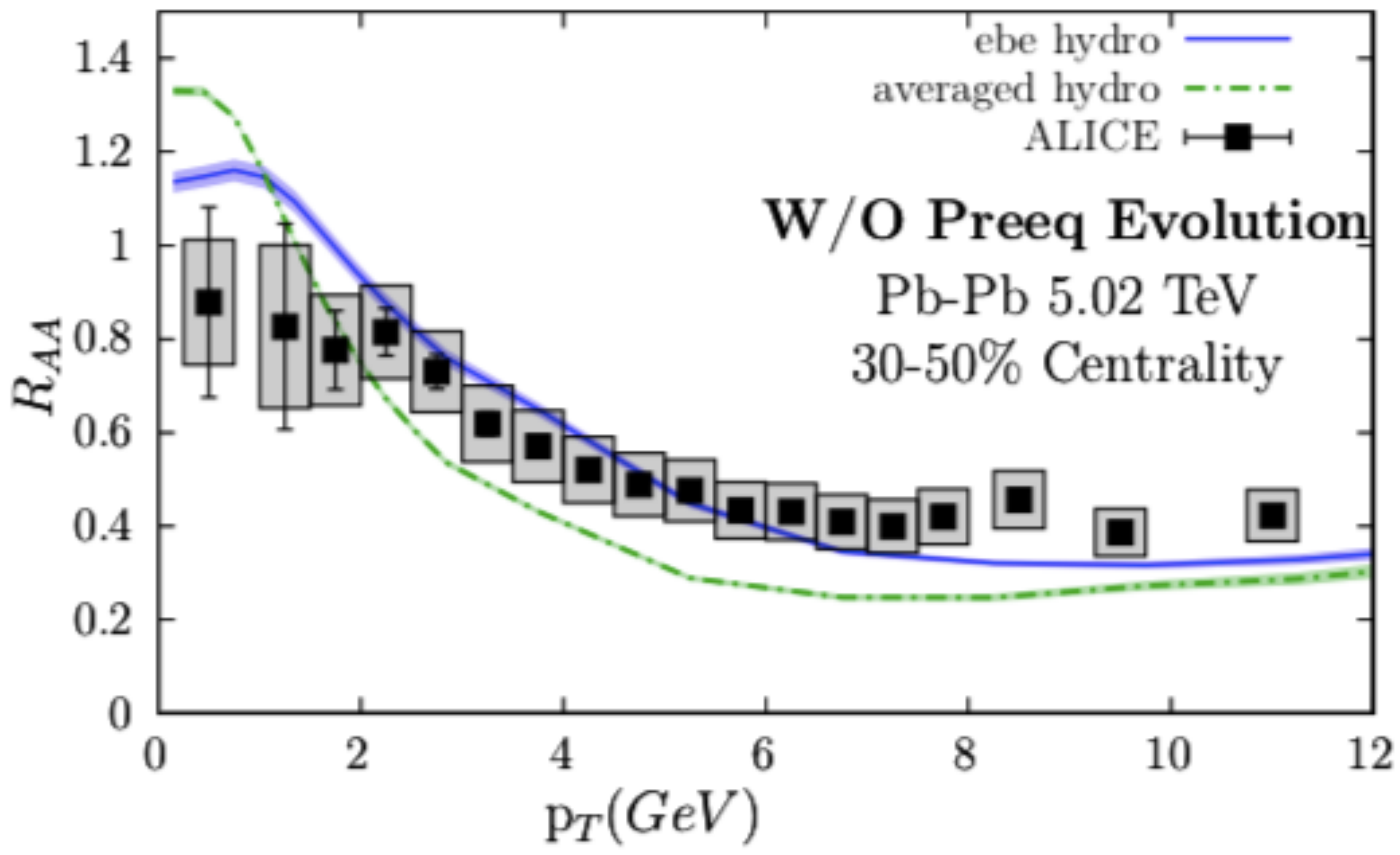


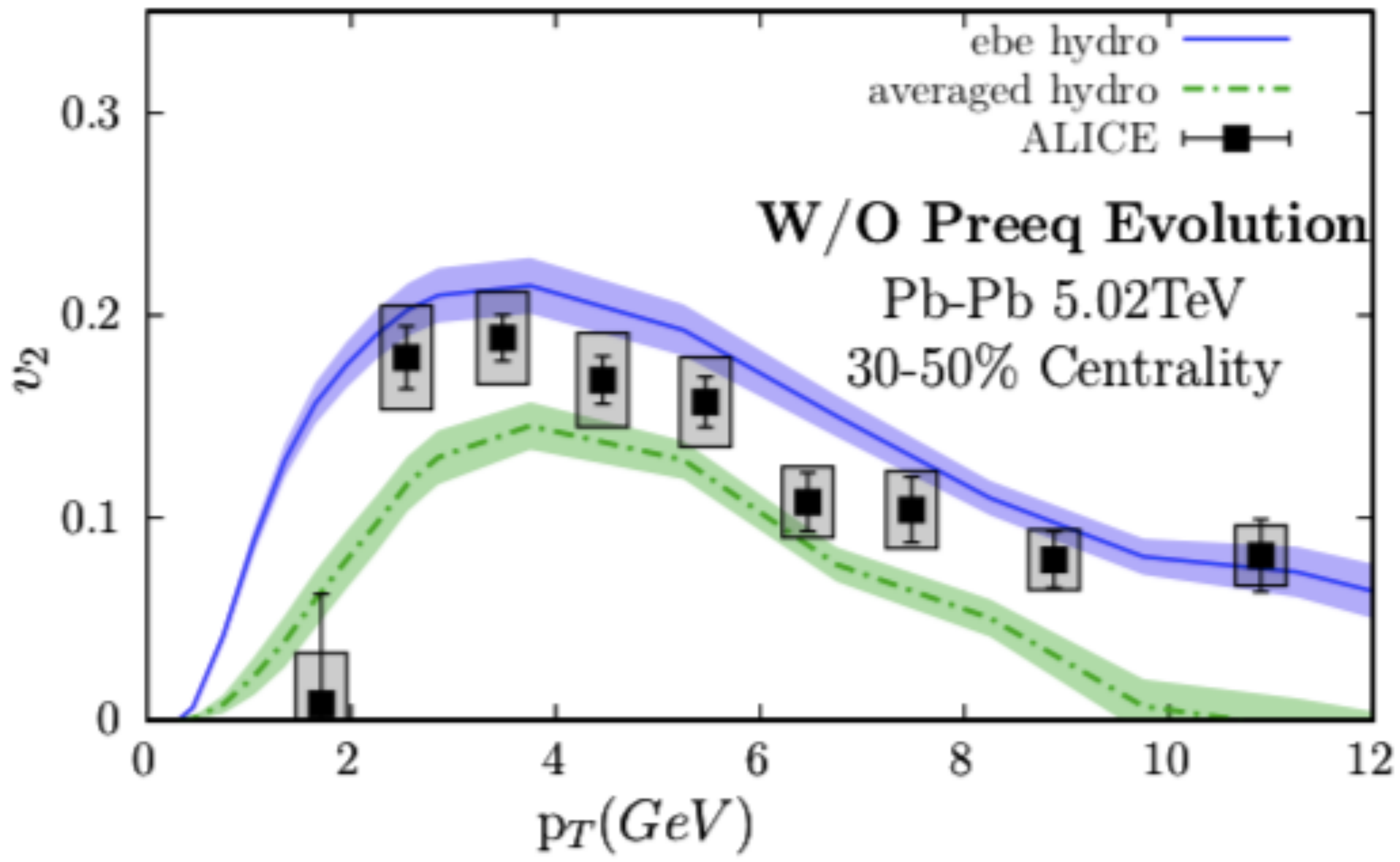


Smooth v/s fluctuating initial stage

M. Singh, MK, B. Schenke, S. Jeon, and C. Gale, PRC (2026)





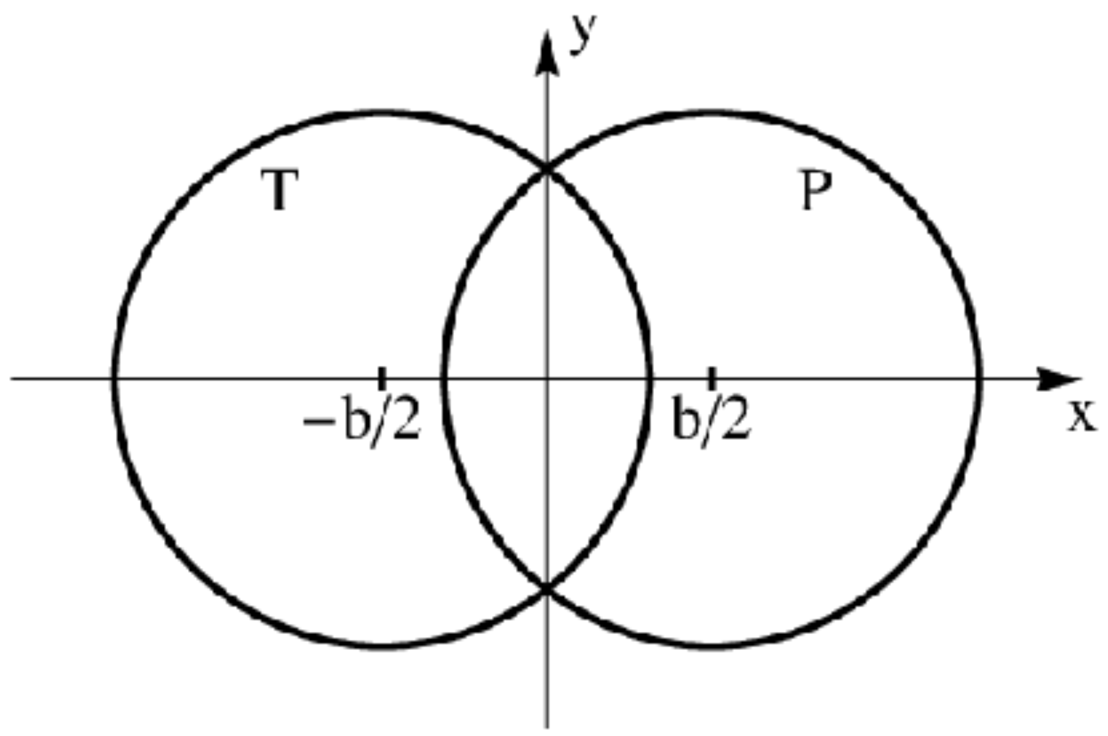


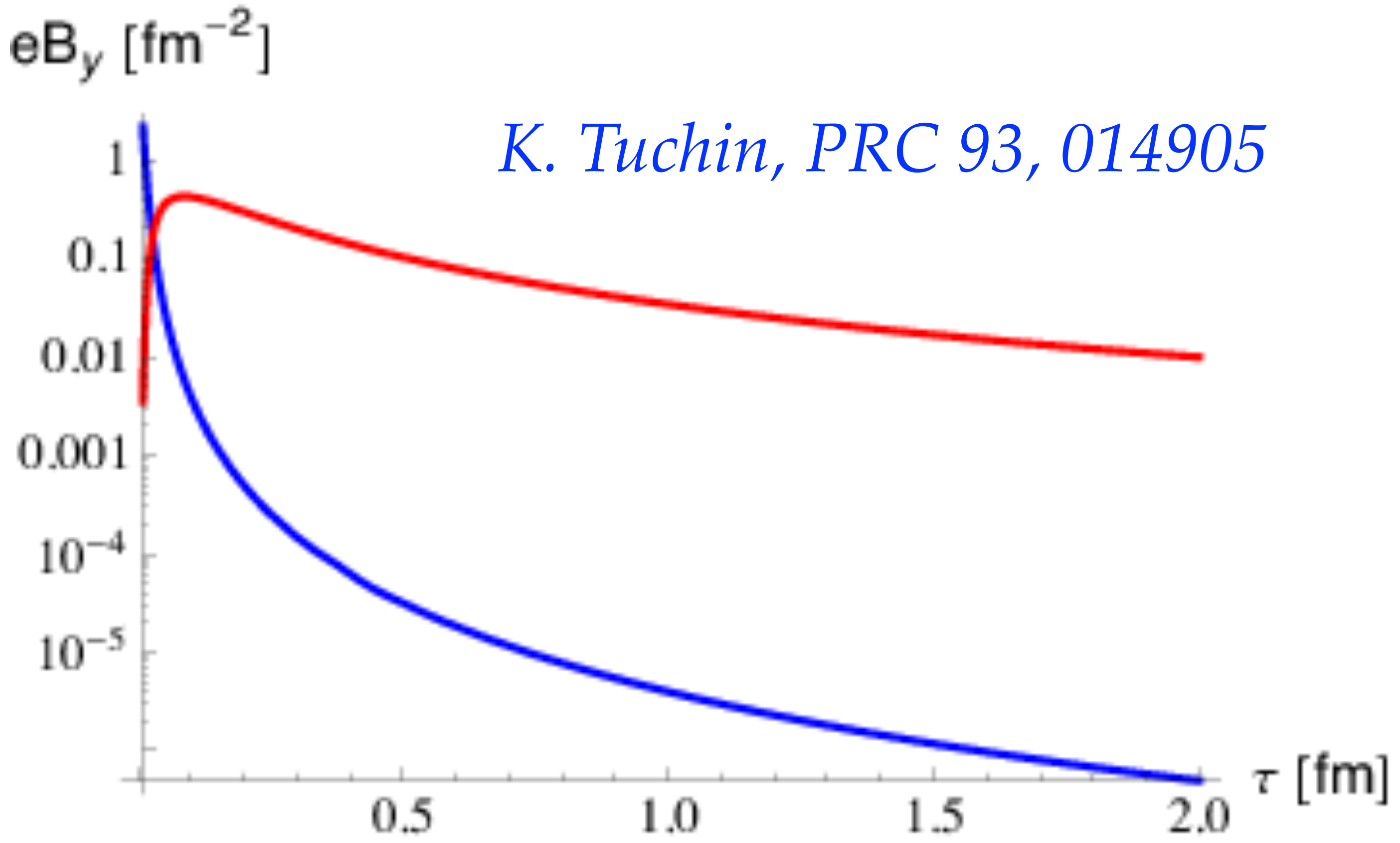
Note: So far, we didn't consider heavy quark energy loss in the Pre-equilibrium stage

One possible way to study the evolution of charm quark in non-Abelian background is to consider them as classical colored particles with Wong's equations as the governing equation of classical particles in the Glasma

Pooja, Santosh K. Das, Vincenzo Greco, Marco Ruggieri, Eur.Phys.J.Plus 138, 313 (2023)

Question II. Can heavy quarks be an efficient probe of magnetic field in HIC?





- Strong magnetic field may be generated in heavy-ion collisions *D. E. Kharzeev et.al. PRD 78, 074033 (2008)*

  The generated magnetic field can be in the order of $eB \approx (1-15)m_\pi^2$ *V. Skokov, et.al Int. J. Mod. Phys. A 24, 5925 (2009)*

  This can be the largest magnetic field ever created in the laboratory!!

How the fermions behaves when the magnetic field provides the largest energy scale in the system?

- The evolution and lifetime of the magnetic field created in the initial stages of heavy-ion collisions is one of the challenging question in HIC physics

- Heavy quarks, created in the very early stage, may witness the strongest magnetic field generated in HIC

Heavy flavor directed flow as a sign of magnetic field

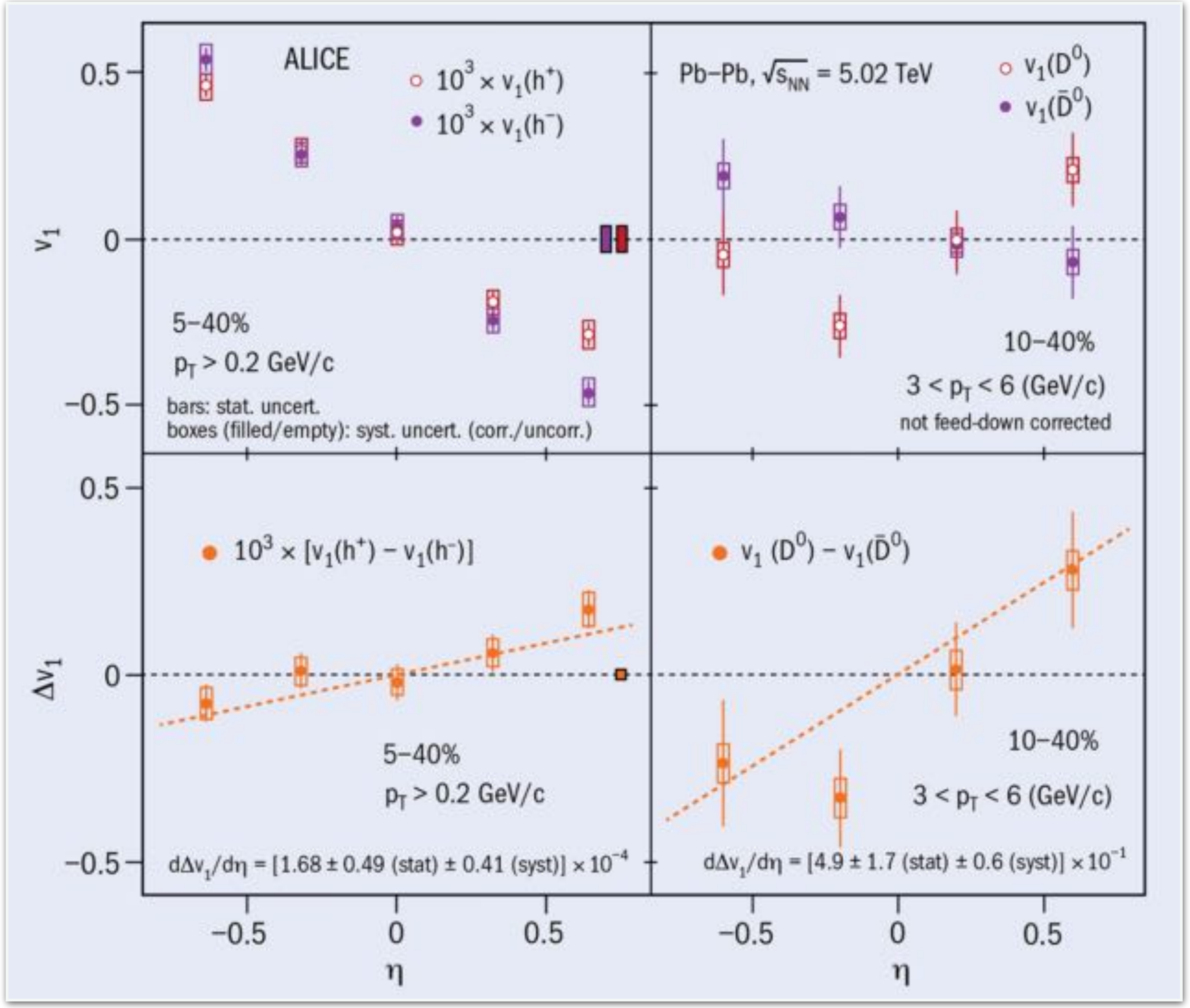
Measurements on heavy quark directed flow provide sign of the large magnetic fields *[ALICE Collaboration], PRL 125, 022301 (2020).*

Similar observation at the RHIC: *J. Adam et. al. [STAR Collaboration], PRL 123,162301 (2019).*

Heavy mesons directed flow is observed to be three orders of magnitude higher than that of the charged hadron

Splitting of heavy mesons directed flow is observed to be three orders of magnitude higher than that of the charged hadron

"Directed flow of charm quarks as a witness of the initial strong magnetic field in ultra-relativistic heavy ion collisions" S. K.Das, S.Plumari, S.Chatterjee, J.Alam, F.Scardina, V.Greco, PLB 768, 260-264 (2017).



Heavy flavor directed flow as a sign of magnetic field

Measurem

PHYSICAL REVIEW LETTERS 120, 192301 (2018)

Large Directed Flow of Open Charm Mesons Probes the Three-Dimensional Distribution of Matter in Heavy-Ion Collisions

Sandeep Chatterjee* and Piotr Bożek†

AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, Aleja Mickiewicza 30, 30-059 Krakow, Poland

adam et. al. [STAR

123,162301 (2019).

Heavy mesons directed fl

Directed flow of charm quarks as a witness of the initial strong magnetic field in ultra-relativistic heavy ion collisions

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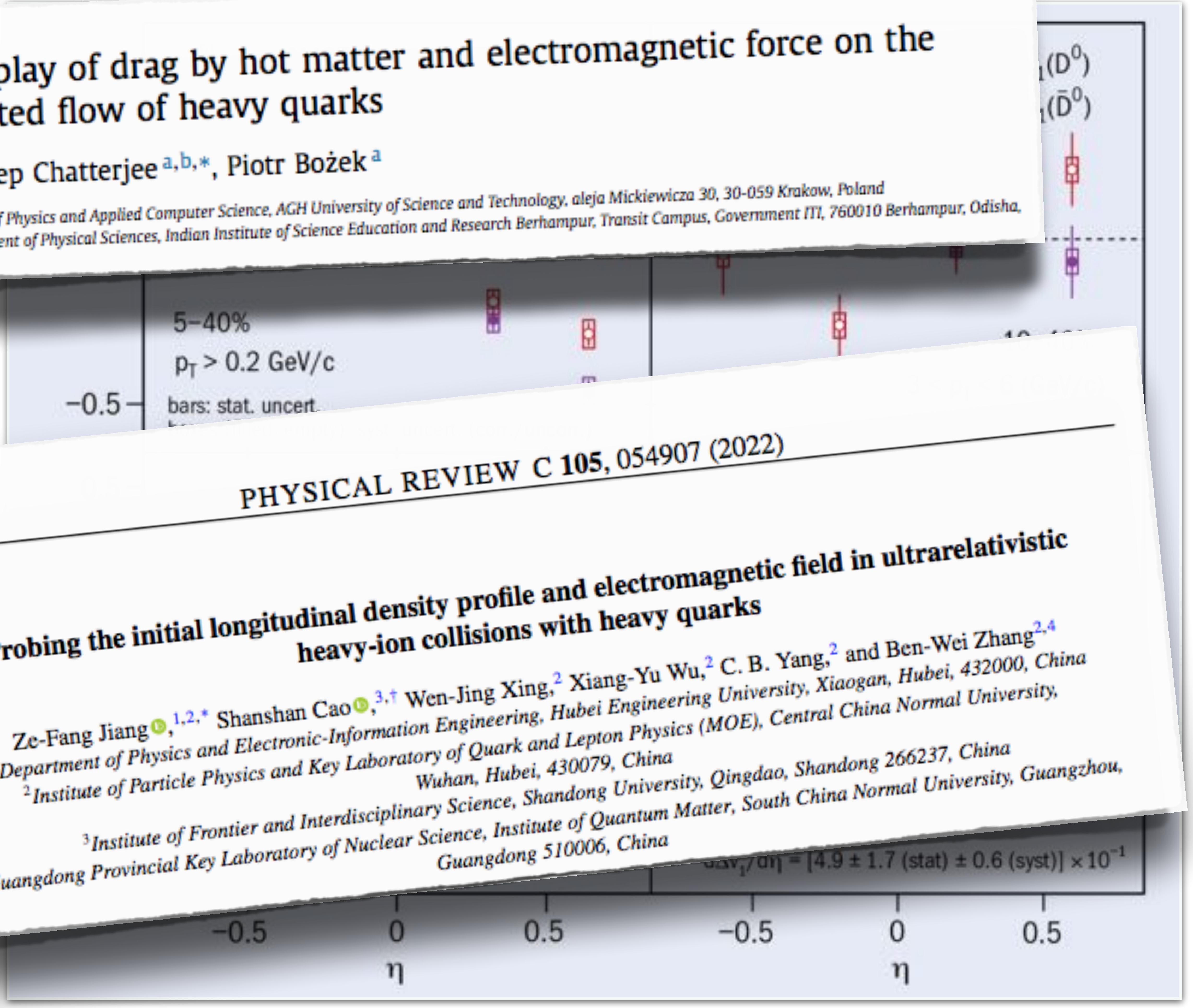
"Directed flow of charm quarks as a witness of the initial strong magnetic field in ultra-relativistic heavy ion collisions" S. K.Das, S.Plumari, S.Chatterjee, J.Alam, F.Scardina, V.Greco, PLB 768, 260-264 (2017).



Interplay of drag by hot matter and electromagnetic force on the directed flow of heavy quarks

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[b] Department of Physical Sciences, Indian Institute of Science Education and Research Berhampur, Transit Campus, Government ITI, 760010 Berhampur, Odisha, India

PHYSICAL REVIEW C 105, 054907 (2022)

Probing the initial longitudinal density profile and electromagnetic field in ultrarelativistic heavy-ion collisions with heavy quarks

Ze-Fang Jiang[1,2,*] Shanshan Cao[3,†] Wen-Jing Xing,[2] Xiang-Yu Wu,[2] C. B. Yang,[2] and Ben-Wei Zhang[2,4]

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[3] Institute of Frontier and Interdisciplinary Science, Shandong University, Qingdao, Shandong 266237, China
[4] Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou, Guangdong 510006, China

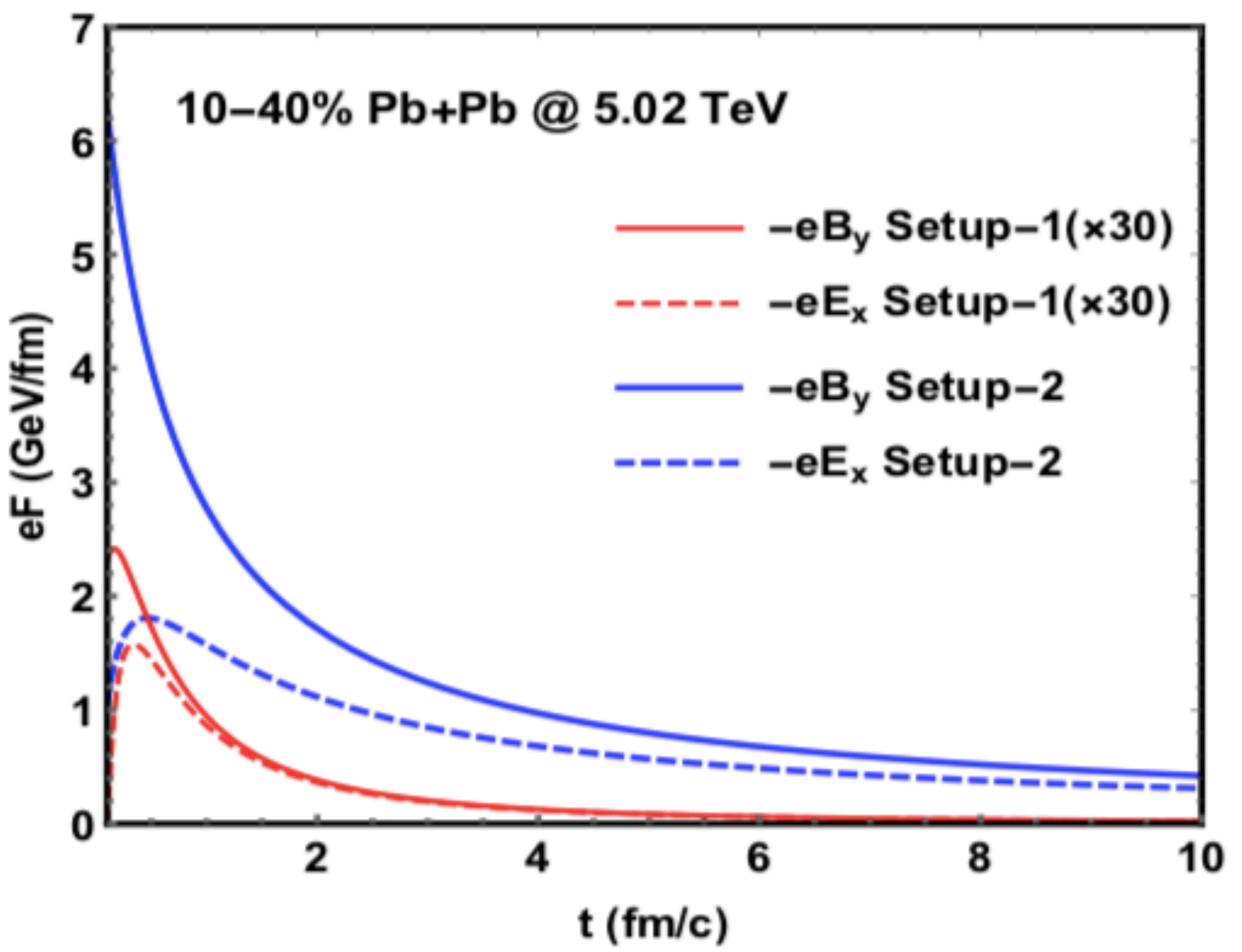
Heavy quark+magnetic field

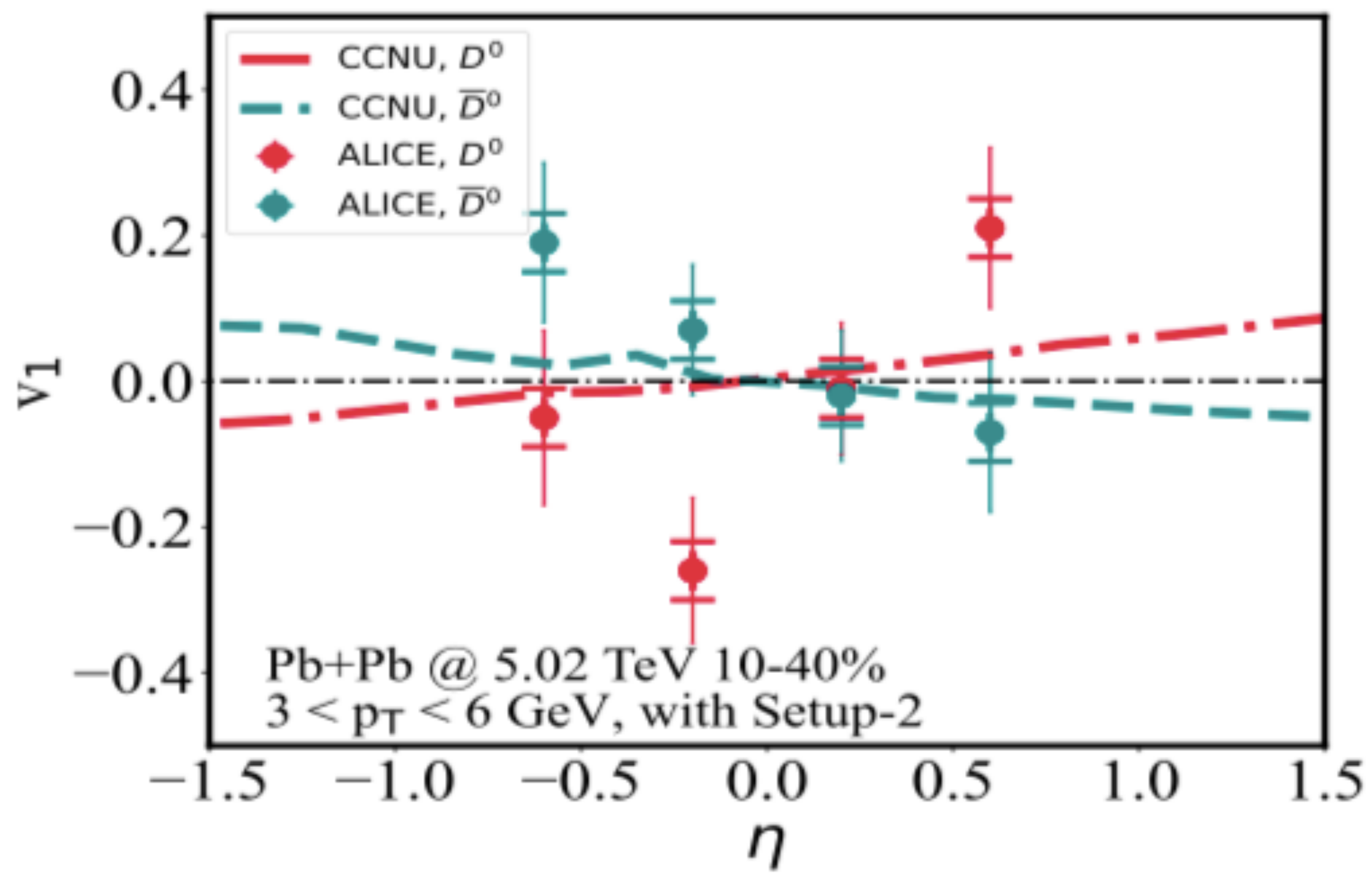
Ze-Fang Jinag et.al., PRC 105, 054907 (2022)

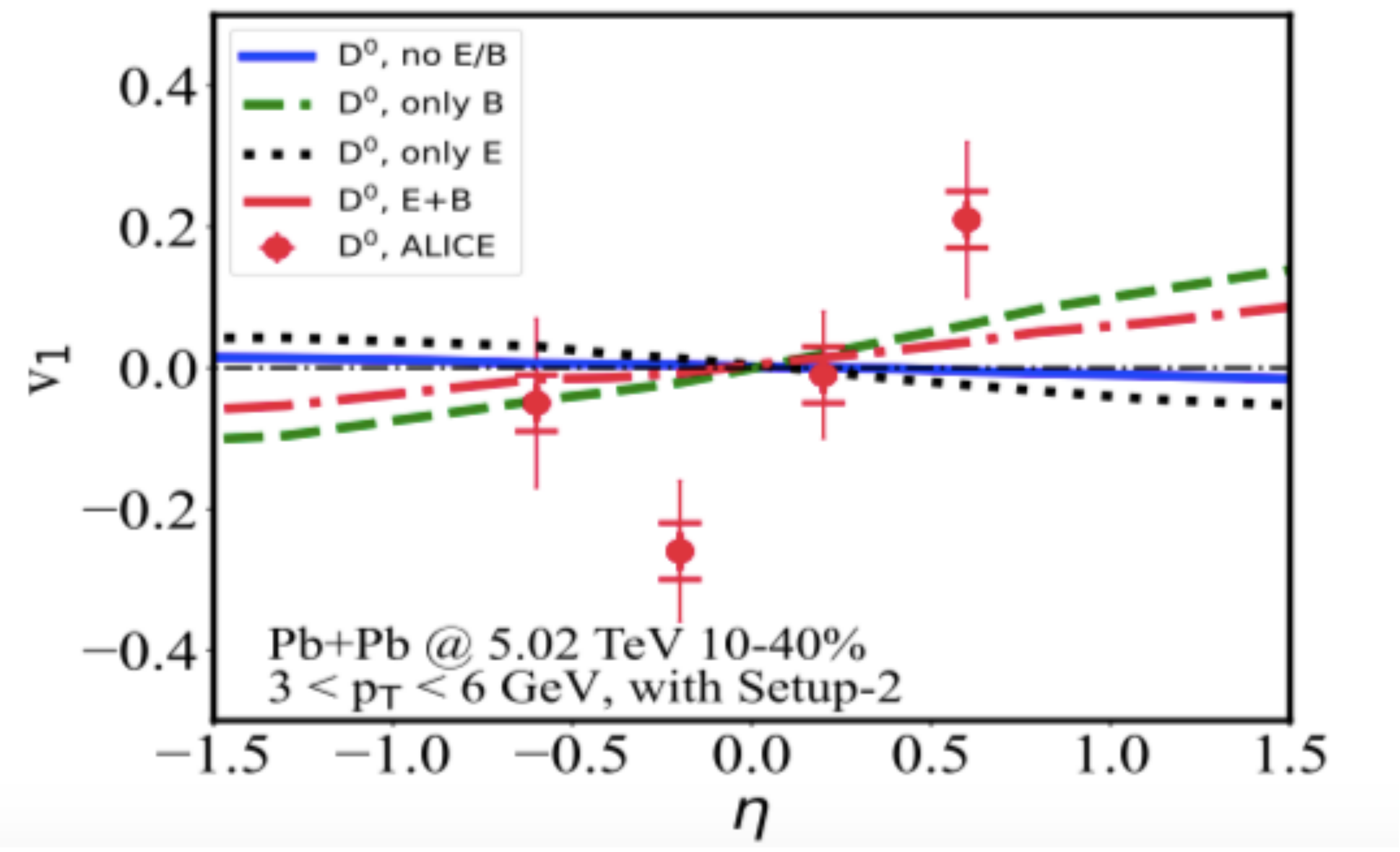
$$\frac{d\vec{p}}{dt} = -\eta_D(\vec{p})\vec{p} + \vec{\xi} + \vec{f}_g + q(\vec{E} + \vec{v} \times \vec{B}).$$

Drag force and thermal random force on heavy quarks inside a thermal medium.

Lorentz Force







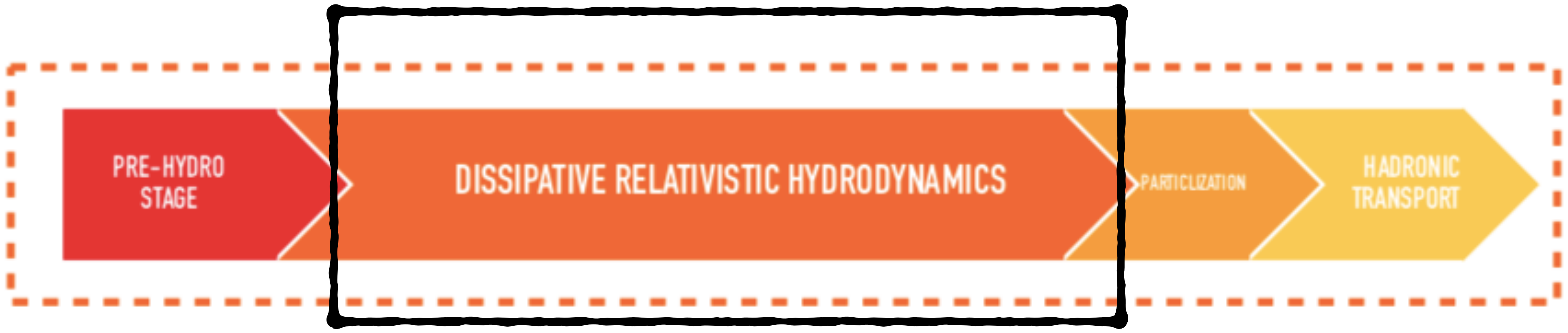
Topics for discussion

Goal

- *Address the gap in theoretical studies by incorporating electromagnetic field effects into heavy-quark physical observables.*
- *Understand the strength and evolution of the magnetic field in collision physics*

Theoretical formulation on the Brownian motion of heavy quark transport in magnetized nuclear medium

- **Brownian motion**: Langevin equation in the presence of the magnetic field
- Diffusion coefficient of heavy quark in the presence of magnetic fields
- Radiative energy loss in magnetic field
- **Magnetized medium**: Setting up a magnetohydrodynamical description of the nuclear matter



HQ diffusion in magnetized medium

__MK__, S. K. Das and V. Chandra, PRD 100,074003 (2019)

Solving Dirac equation in the presence of magnetic field, we obtain Landau levels as energy eigenvalues

$$E_l \equiv E = \sqrt{p_z^2 + m_f^2 + 2l\,|\,q_f B\,|}$$

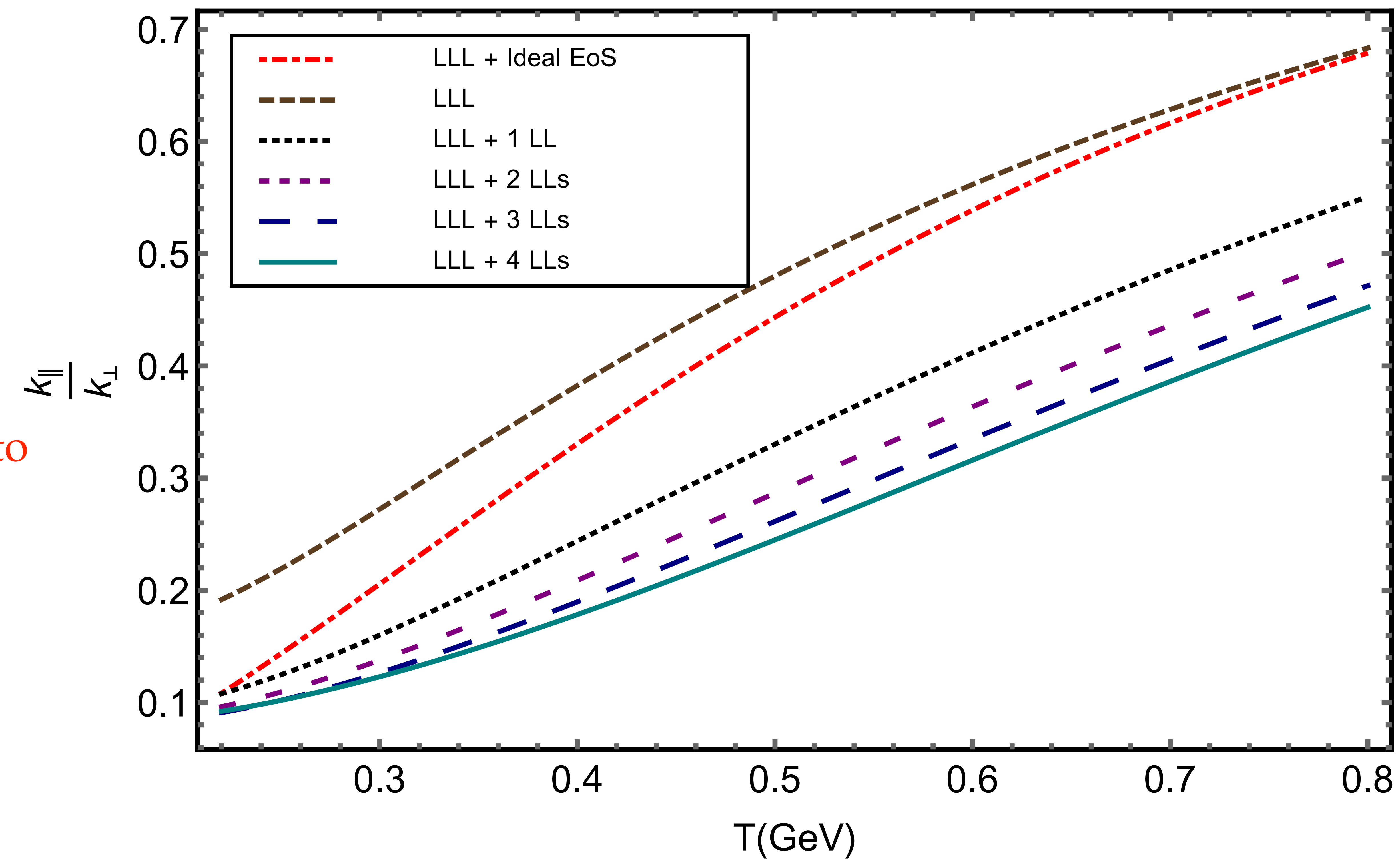
Consider the process, $p + k \rightarrow p' + k'$

In the static limit $\boxed{B_{ij} \rightarrow K\delta_{ij}}$

The magnetic field provides the preferred spatial direction and one need to consider the heavy quark motion parallel and perpendicular to the field.

$$K_{\parallel} = \langle\langle q_z^2 \rangle\rangle \qquad K_{\perp} = \frac{1}{2}\langle\langle q_{\perp}^2 \rangle\rangle$$

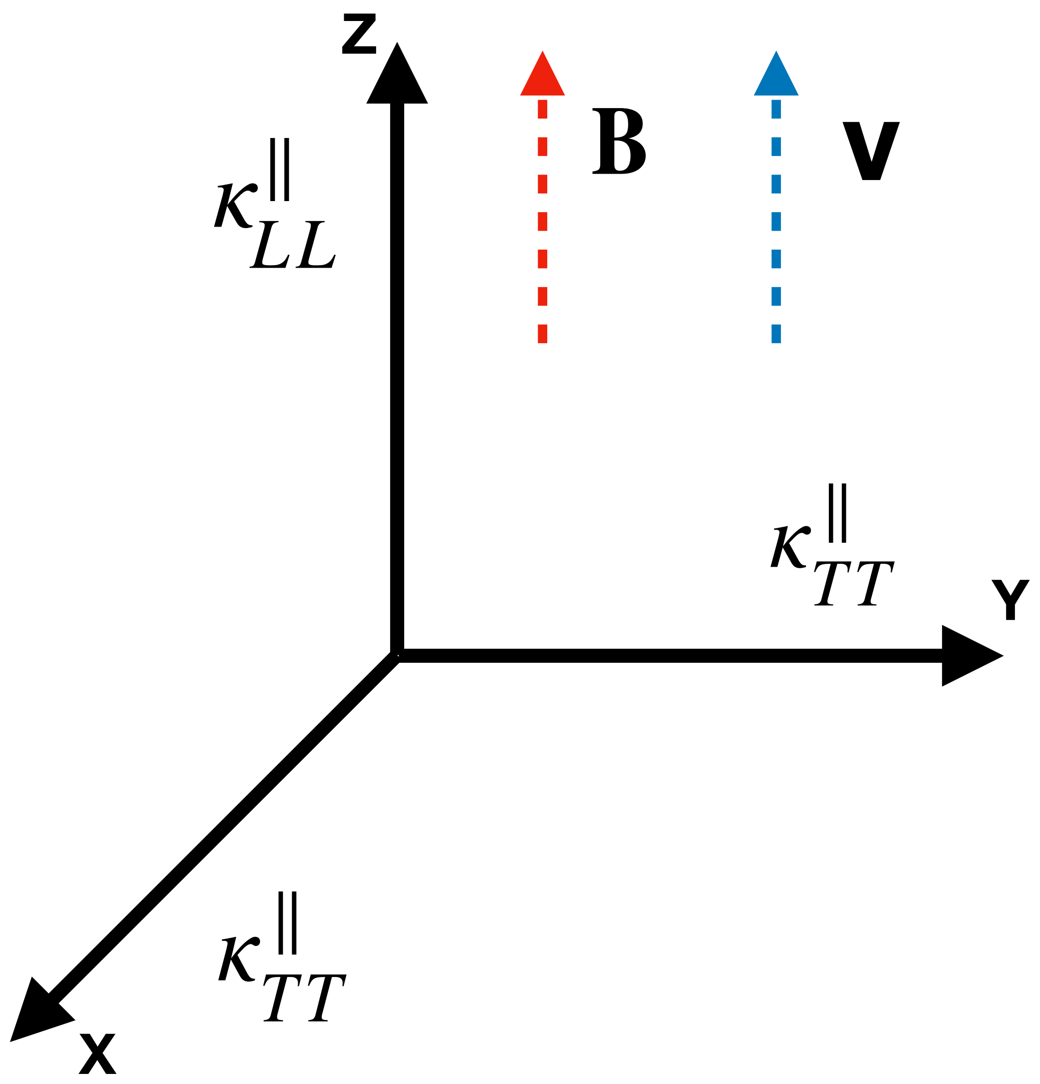
Matrix element in the magnetized QGP *K. Fukushima, K. Hattori, H. U. Yee and Y. Yin, PRD (2016)*



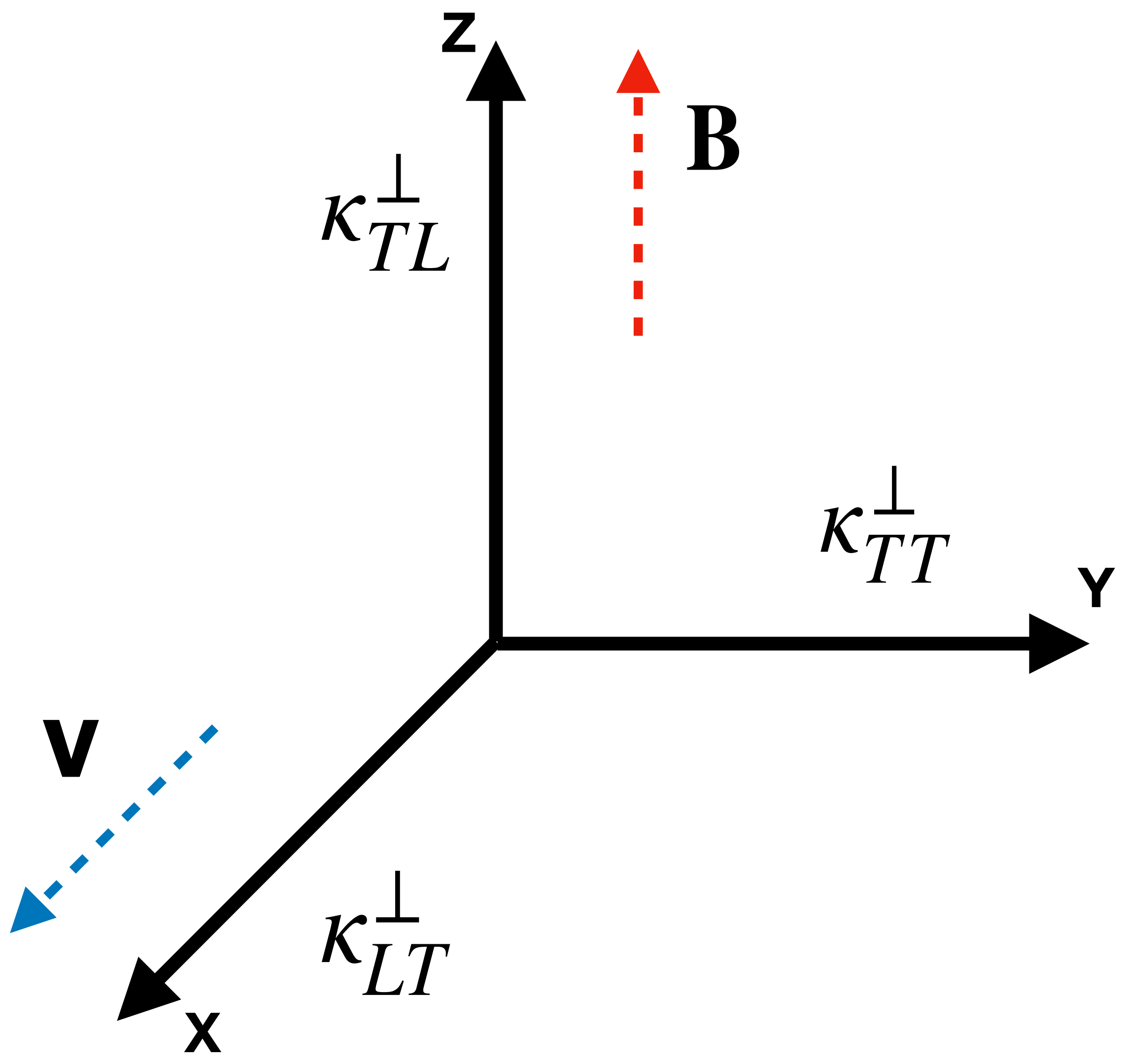
Magnetic field induced anisotropy in the heavy quark momentum diffusion

HQ diffusion beyond the static limit at finite B

MK, S. K. Das and V. Chandra, PRD 100,074003 (2019)
A. Bandyopadhyay, PRD 109, 034013 (2024)



HQ motion parallel to **B**



HQ motion transverse to **B**

Quantifying the impact of magnetic field on the heavy quark observables ?

Quarkonium potential

- While the Cornell potential, comprising of a short-range Coulomb interaction and a long- range linear confinement term, captures quarkonium properties in vacuum, its extension to a thermal medium is far more complex.

$$V(r) = \int \frac{d^3p}{(2\pi)^3} \left(e^{i\boldsymbol{p}\cdot\boldsymbol{r}} - 1\right) \frac{V_{\text{Cornell}}(p)}{\epsilon(p)}$$

encodes the screening effects of the thermal medium

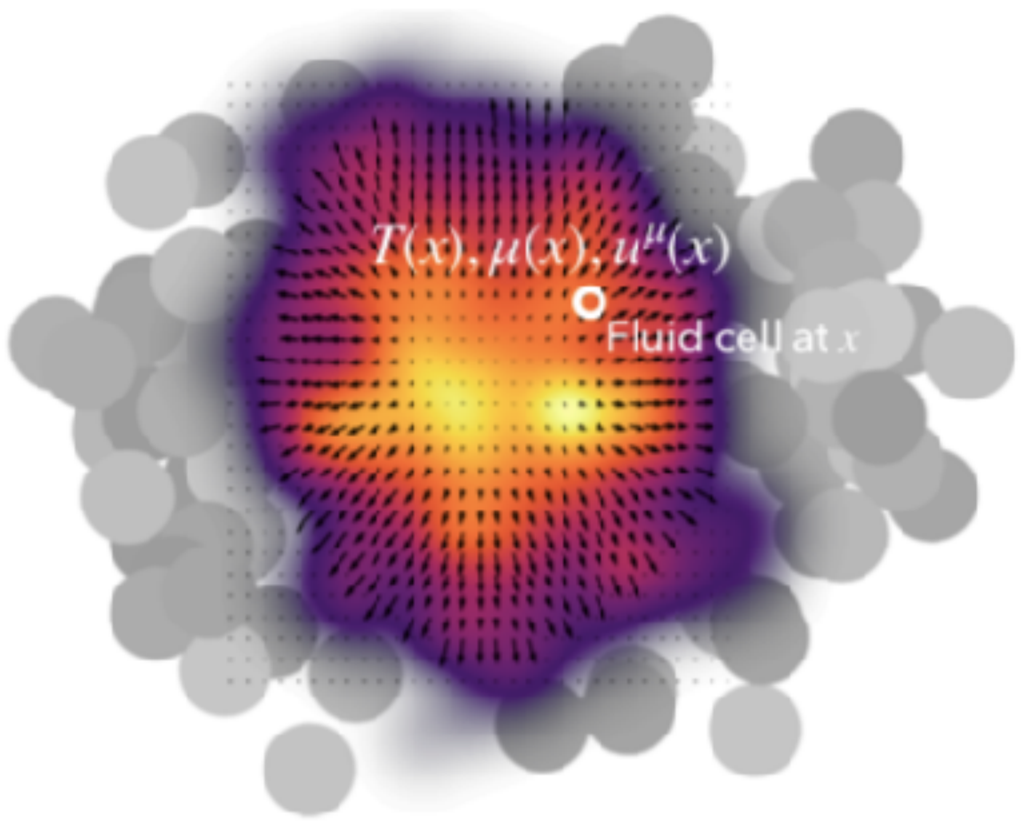
- Dielectric permittivity can be obtained from gluon-self energy by applying the Hard Thermal Loop (HTL) approximation,

$$\epsilon^{-1}(p) = \frac{p^2}{p^2 + m_{D,R}^2} - i\,\frac{\pi T p, m_{D,R}^2}{(p^2 + m_{D,R}^2)^2}$$

medium-induced screening

accounts for the Landau damping

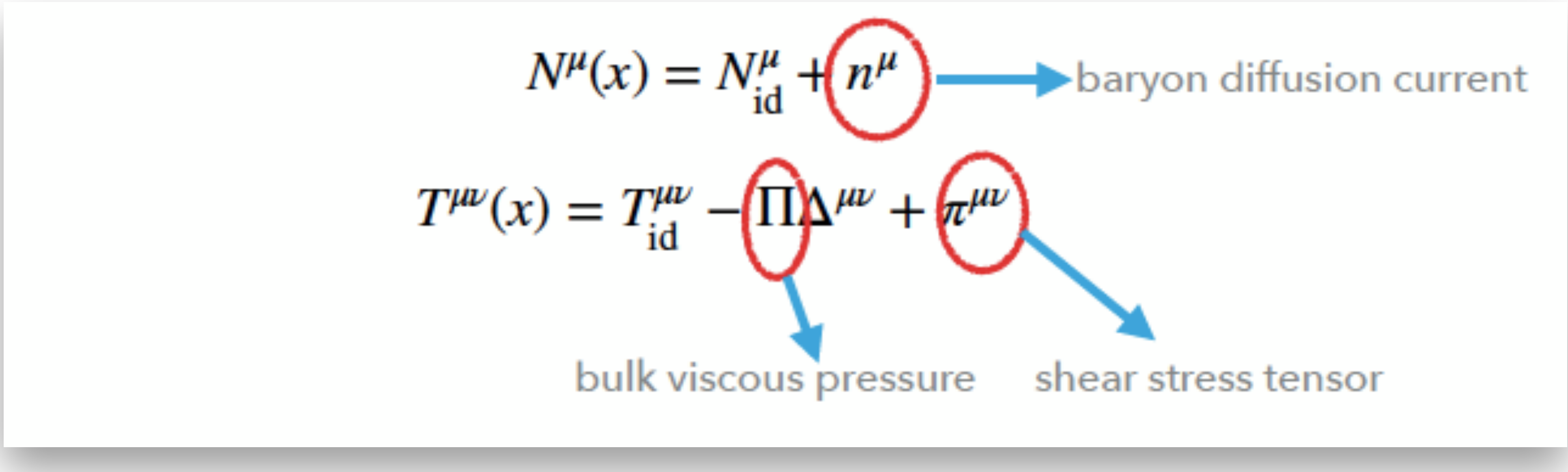
Equilibrium and non-equilibrium part of fluid



Equilibrium part : Fluid cell with collective expansion, thermodynamic properties $T(x)$, $\mu(x)$ and flow velocity

$$f_0(x,p) = \frac{1}{\exp\left[\beta\,(u\cdot p) + \alpha\right] + a}$$

Non-equilibrium physics : $f(x,p) = f_0(x,p) + \delta f(x,p)$



- Estimation of Non-equilibrium distribution function: Transport theory

$$p^\mu \partial_\mu f = -\frac{(u\cdot p)}{\tau_{\rm R}}\,(f - f_0)$$

RTA approximation

$$\delta f_{(1)} = -\frac{\tau_R}{(u\cdot p)}\,p^\mu \partial_\mu f$$

When the relaxation time is momentum-dependent, the conservation laws can no longer be guarantee

$$\delta f_{(1)} = -\frac{\tau_R(x,p)}{(u\cdot p)}\,p^\mu \partial_\mu f + \delta f^*_{(1)}$$

Extended RTA approximation

D. Dash, S. Bhadury, S. Jaiswal, A. Jaiswal, PLB 831, 137202 (2022)

Relativistic hydrodynamics within ERTA

D. Dash, S. Bhadury, S. Jaiswal, A. Jaiswal, PRC 108, 064913 (2023)

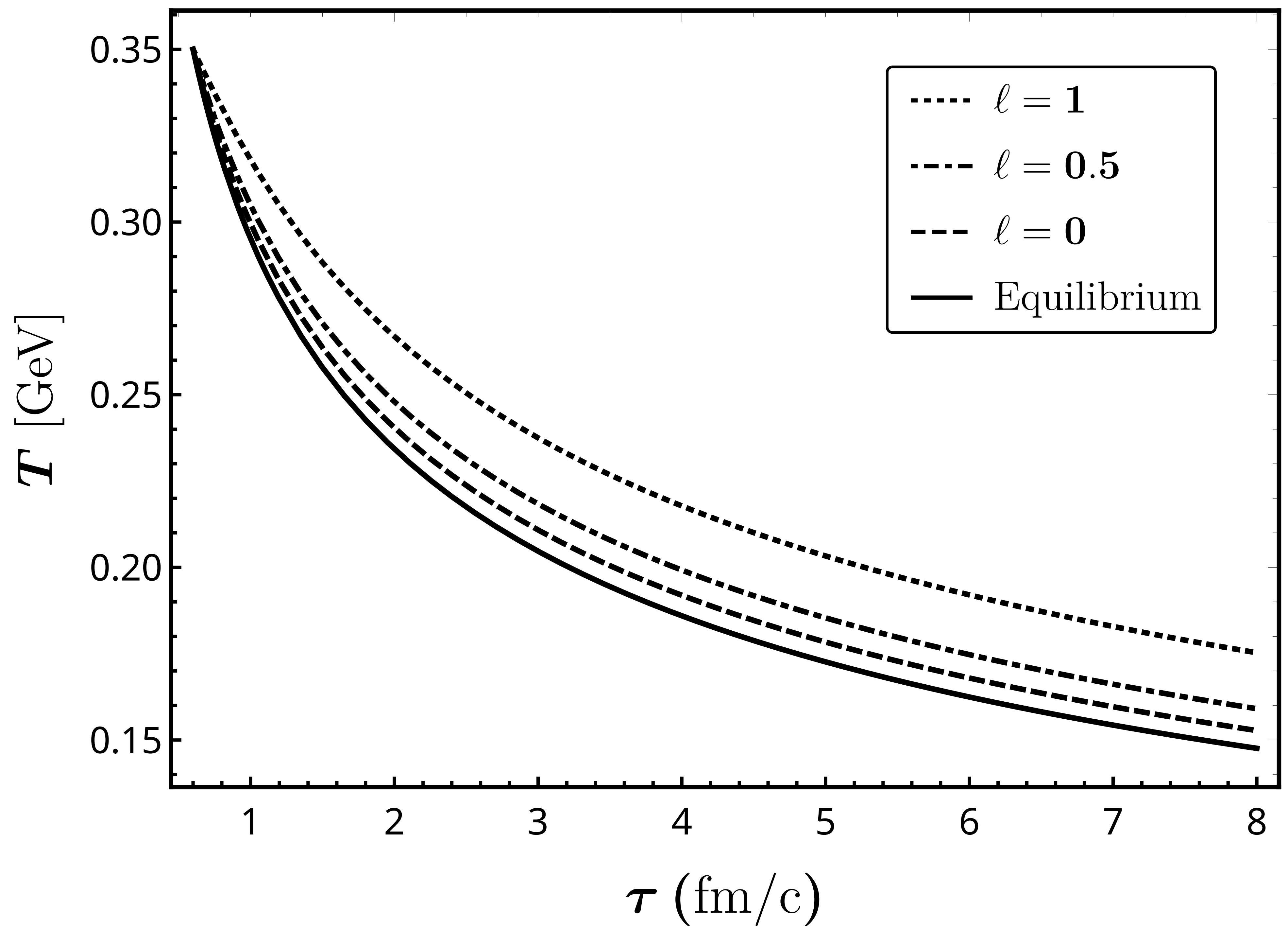
- Formulation of relativistic hydrodynamics from Extended RTA (ERTA) approach, incorporating an energy-dependent relaxation time

- Employed a power law parametrization to describe the energy dependence of the relaxation time

$$\tau_{\rm R}(x,p) = \tau_{\rm eq}(x)\left(\frac{u\cdot p}{T}\right)^{\ell}$$

- The resulting transport coefficients are found to be sensitive to the energy dependence of the relaxation time

Temperature evolution for boost invariant fluid



$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \tau_{\pi\pi}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} - \tau_{\pi n}n^{\langle\mu}\dot{u}^{\nu\rangle} + \lambda_{\pi n}n^{\langle\mu}\nabla^{\nu\rangle}\alpha + l_{\pi n}\nabla^{\langle\mu}n^{\nu\rangle}.$$

- Using the derived hydrodynamic equations, the evolution of a fluid can be studied

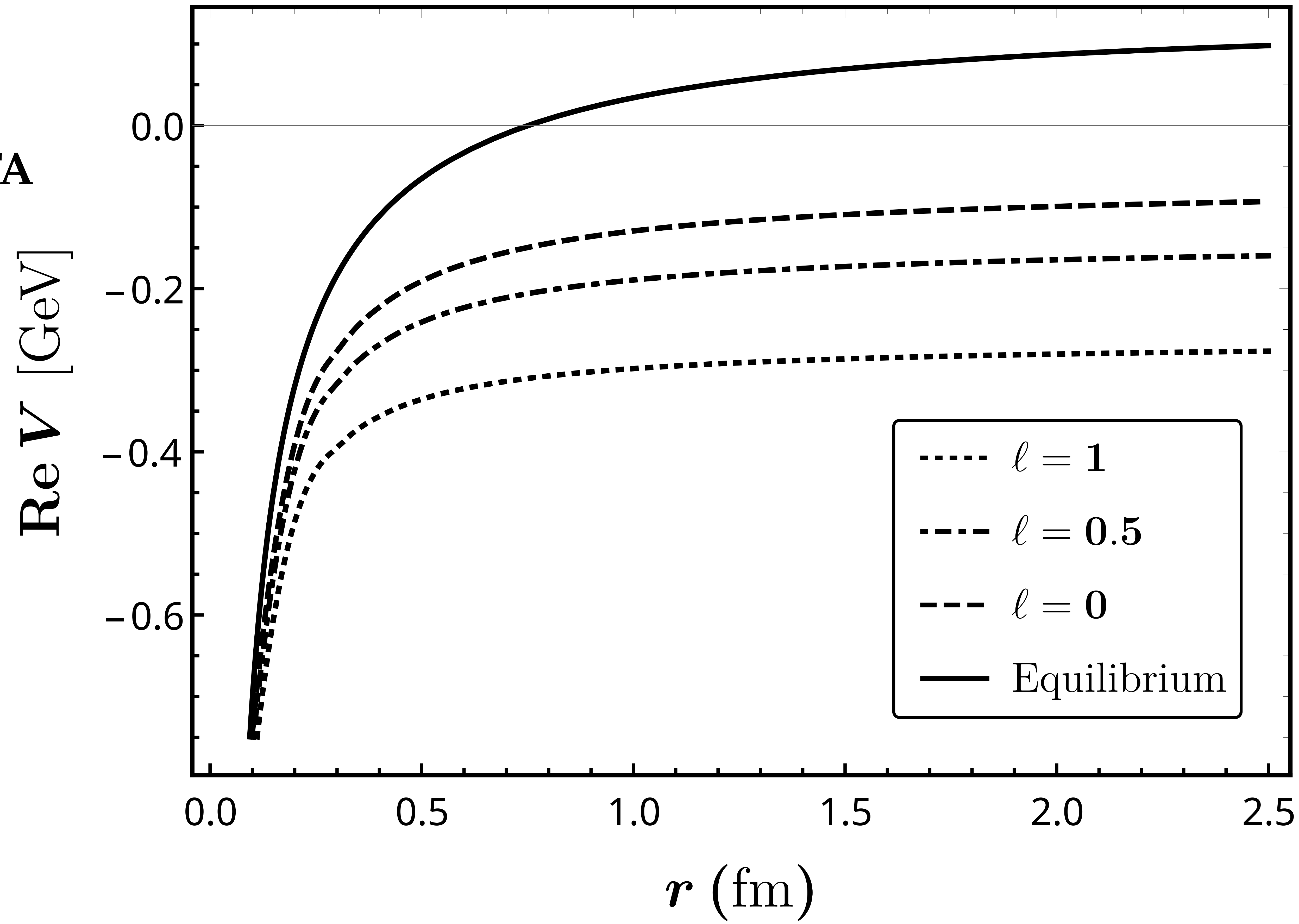
Modified potential within the ERTA

S. Singh, S. Bhadury, R.Ghosh, MK, PRD 109, 094056 (2025)

<u>Step1</u>: Non-equilibrium correction to distribution function

<u>Step II</u>: Modified self energy and screening within the ERTA

<u>Step III</u>: Modified quarkoium potential



- The ERTA framework introduces substantial modifications to the RTA-based corrections
- As the value of ℓ increases, the potential exhibits enhanced screening, indicating stronger medium modifications driven by non-equilibrium effects

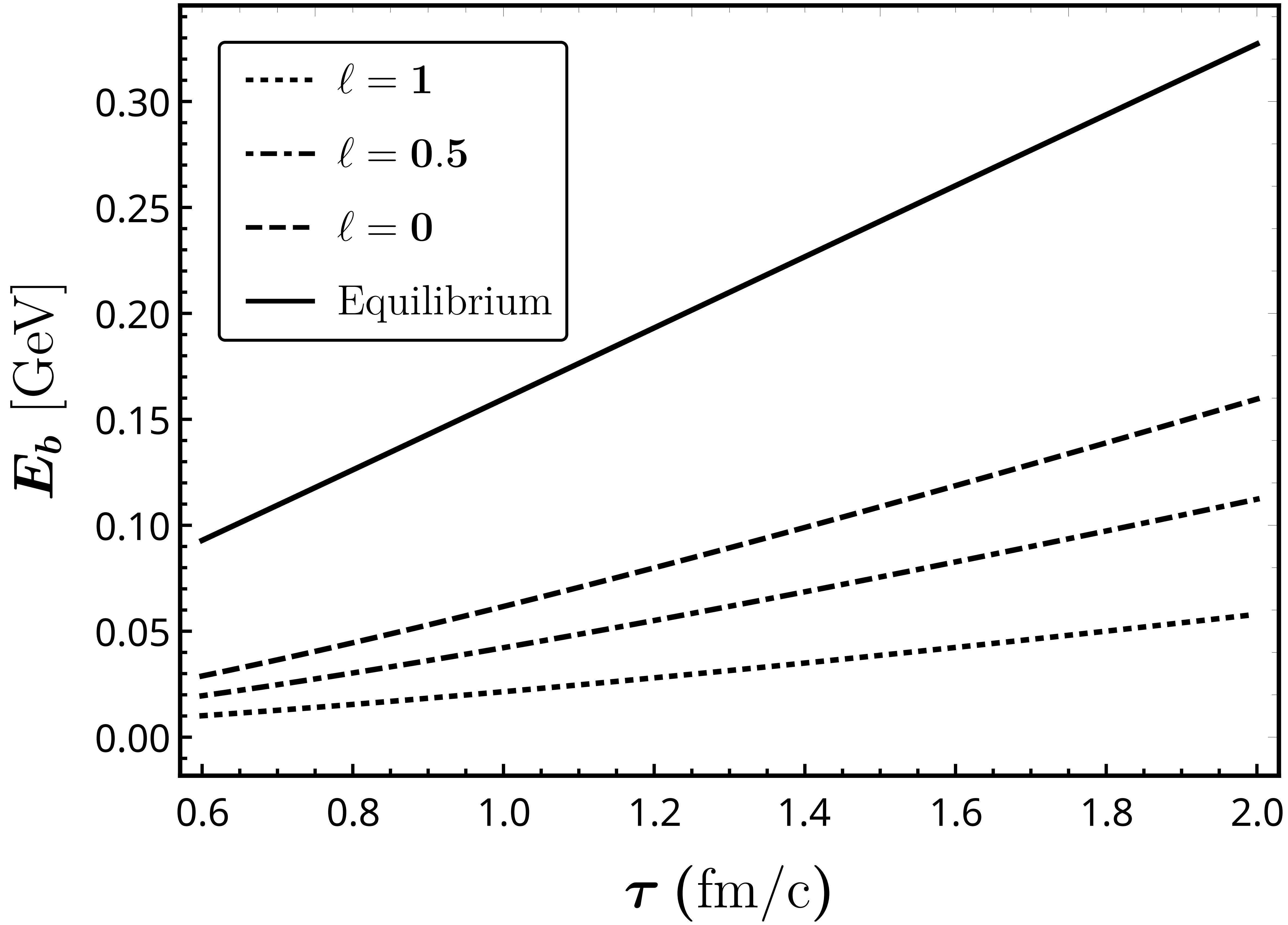
Binding energy of quarkonium

S. Singh, S. Bhadury, R.Ghosh, MK, PRD 109, 094056 (2025)

- Solving the Schrodinger equation with this potential yields the quantized energy levels of quarkonium systems such as charmonium

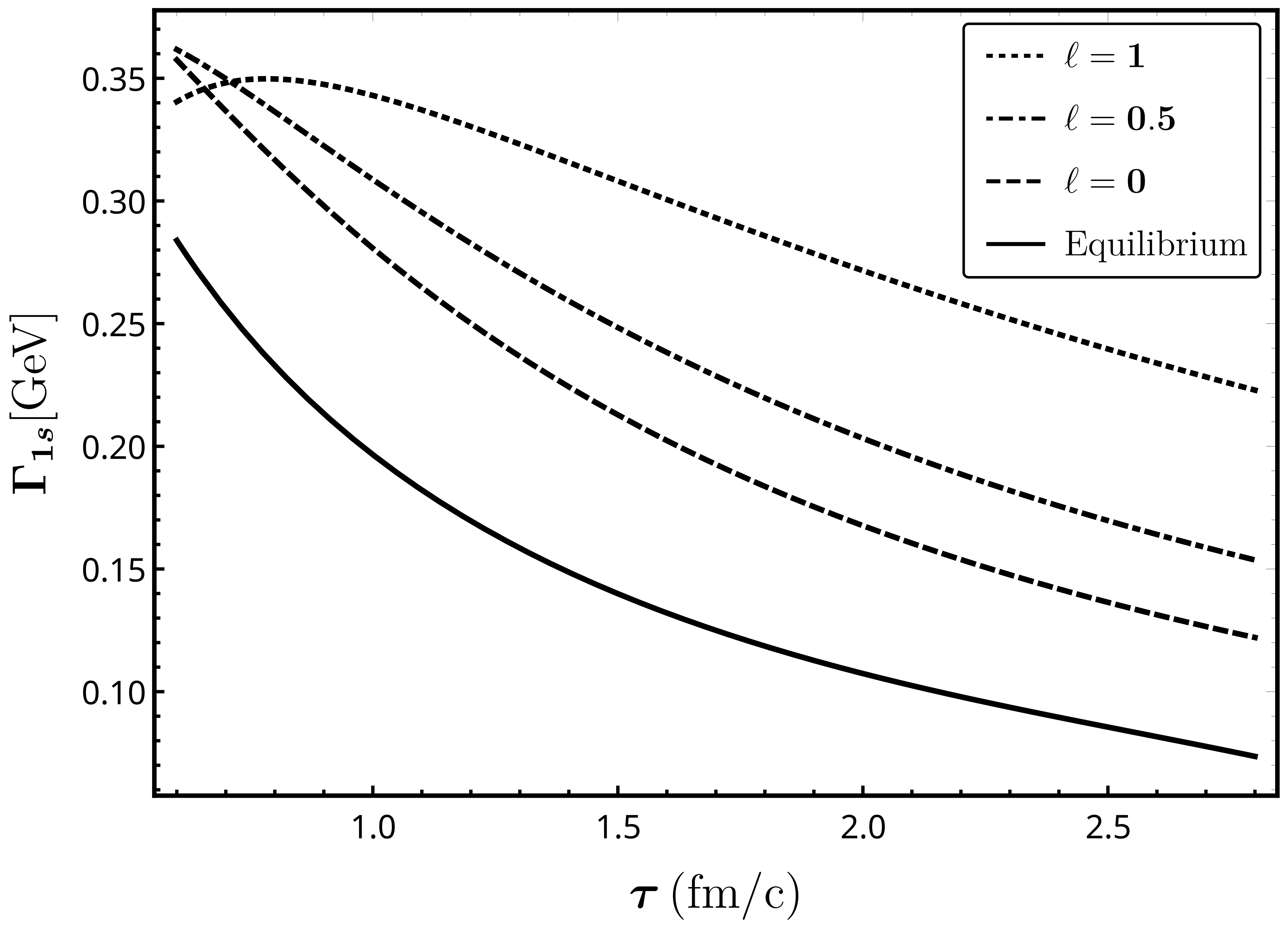
- The black solid line represents the equilibrium result, corresponding to the case without any viscous corrections

- As the system evolves and cools, the quarkonium states become more strongly bound due to reduced thermal screening in the medium



Thermal width of quarkonium

$$\Gamma(T) = -\int d^3\mathbf{r}\, |\Psi(\mathbf{r})|^2\, \mathrm{Im}\, V(\mathbf{r};T)$$



- These estimations are in simple system
- A realistic medium expansion is required
- As in the case of heavy quark, magnetic field may have an important role in the properties of quarkonium.

Summary and Outlook

Heavy quarks:

- We have studied the heavy quark transport in an 3+1-D expanding QCD medium while including the pre-equilibrium corrections
- Heavy quark transport coefficients act as the input parameters for the Langevin simulation: Impact of bulk evolution on heavy flavour observables
- Charm quark observables are sensitive to the initial state of the heavy-ion collision
- Magnetic field introduces anisotropy in the heavy quark transport

Heavy quarkonium

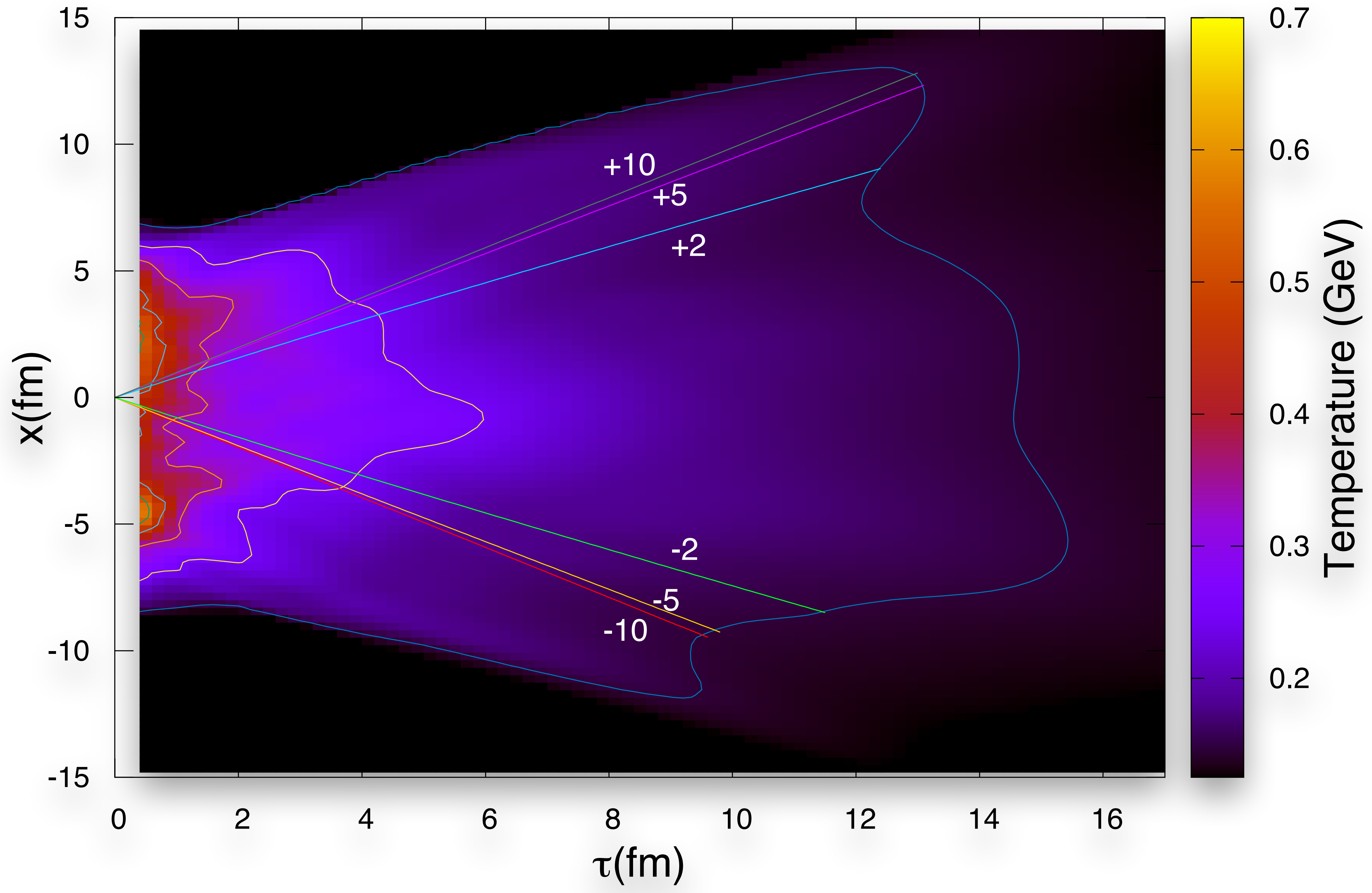
- Quarkonium potential within the Extended RTA approach
- The impact of magnetic field on the potential and the observables

Thank you !

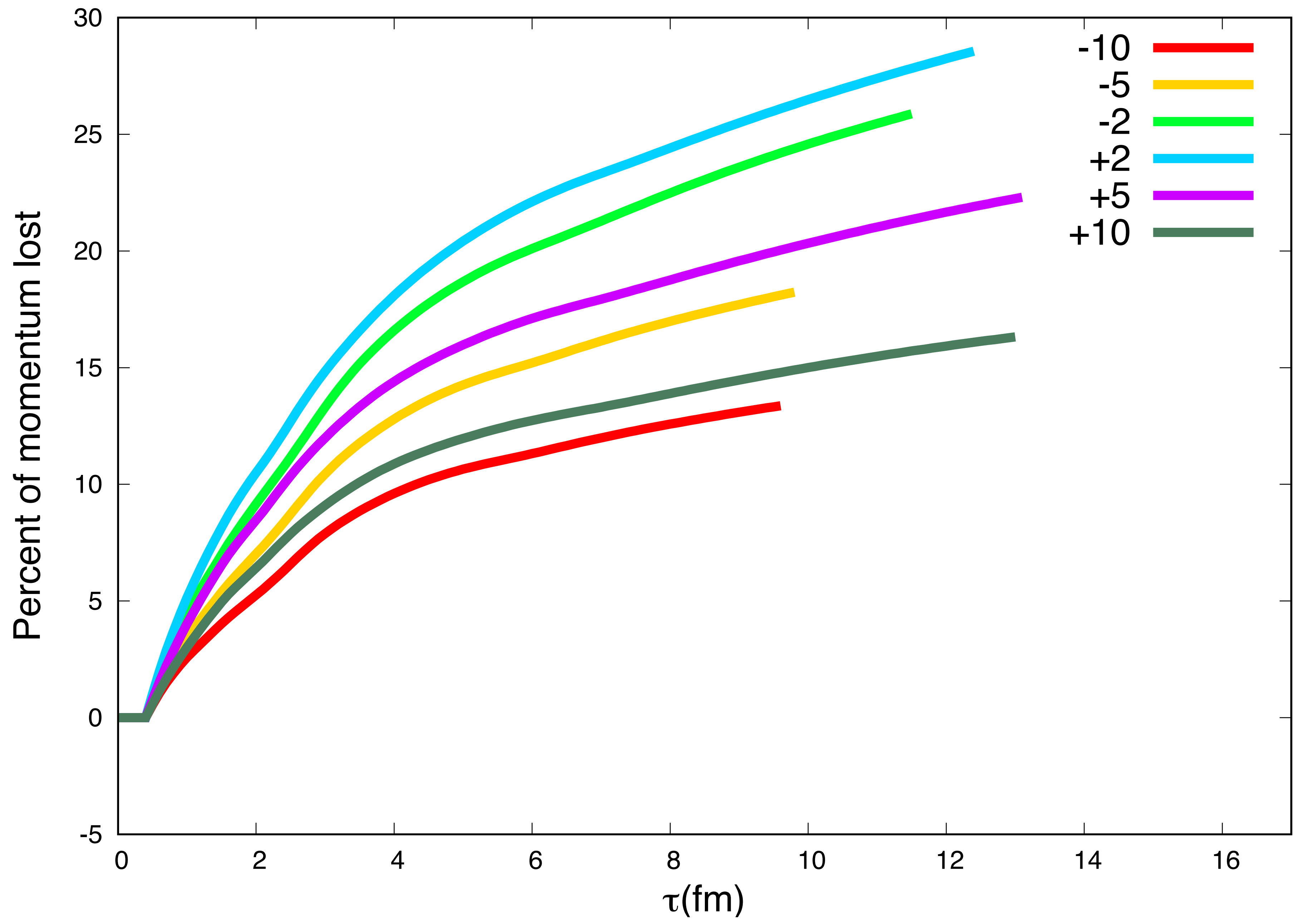
Collisional energy loss

The drag force which accounts for the resistance to the heavy quark motion, leads to its energy loss in the QGP medium.

The collisional energy loss of the heavy quark in the QGP is related to the drag coefficient as, $-\frac{dE}{dL} = A(p^2, T)p$



Trajectory of charm quark motion in the space-time with initial momentum



The charm quark percentage of momentum lost with proper time in the viscous medium at LHC for the trajectory.

Bulk viscous correction: Heavy quark drag

Step1: Bulk correction to distribution function

$$\delta f_{g/q}(Q,X) = \Pi B_X(X) B_M(Q,T), \qquad B_X(X) = \frac{1}{15\left(\frac{1}{3}-c_s^2\right)\left(\epsilon+\mathscr{P}\right)}, \qquad B_M(Q,T) = \frac{1}{T} f^0_{g/q}(Q)\left(1 \pm f^0_{g/q}(Q)\right)\left(E_q - \frac{m^2_{g/q}}{E_q}\right).$$

Step II: Correction to the general integral

$$\langle\langle F(p^{'})\rangle\rangle^{\mathrm{bulk}} = \frac{\Pi B_X(X)}{512\pi^4\gamma_c}\frac{1}{E_p}\Big[\Lambda_1(p,T) \pm \Lambda_2(p,T)\Big],$$

Step III: Bulk correction to HQ drag

$$A_i \simeq A_i^{(0)} + A_i^{bulk}$$

$$\Lambda_1 = \int_0^\infty \frac{q^2}{E_q}dq\int_{-1}^{1} d\cos\chi \,\frac{\sqrt{(s+m_c^2-m_{g/q}^2)^2-4sm_c^2}}{s} B_M(Q,T)\int_{-1}^{1} d\cos\theta_{cm}\sum|\mathscr{M}_{HQ,g/q}|^2\int_0^{2\pi} d\phi_{cm} e^{\beta E_{q^{'}}} f_{g/q}(E_{q^{'}})F(p^{'}),$$

$$\Lambda_2 = \int_0^\infty \frac{q^2}{E_q}dq\int_{-1}^{1} d\cos\chi \,\frac{\sqrt{(s+m_c^2-m_{g/q}^2)^2-4sm_c^2}}{s} f^0_{g/q}(E_q)\int_{-1}^{1} d\cos\theta_{cm}\sum|\mathscr{M}_{HQ,g/q}|^2\int_0^{2\pi} d\phi_{cm}\, B_M(Q^{'},T)F(p^{'}).$$

MARTINI+MUSIC

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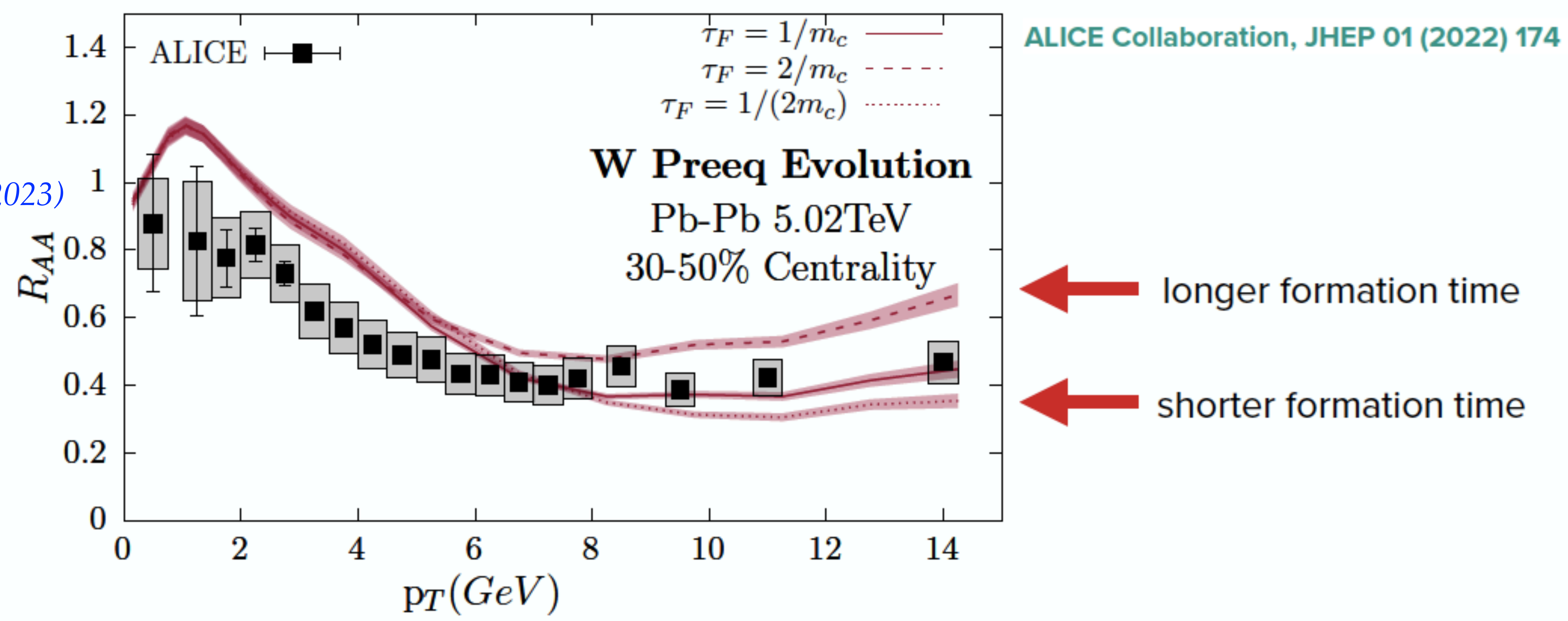

We can extract information about the system from these observables
(eg, formation time of heavy quark)