Born-Oppenheimer EFT for quarkonium, hybrids tetraquarks, pentaquarks: spectra, transitions, production and in medium propagation

Nora Brambilla

Technische Universität München

Effective Field Theory
Lattice Gauge Theory

erc Advanced Grant EFT-XYZ

TUMQCD LATTICE Collaboration

IAS TUM Institute for Advanced Study

Quark Confinement and the Hadron Spectrum since 1994

MCQST

QWG

ORIGINS Excellence Cluster

MDSI Munich Data Science Institute

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They have the potential to give us information on the fundamental strong force

They represent a big challenge for theory

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Focus of the talk

We introduce a QCD derived nonrelativistic effective field theory, Born Oppenheimer EFT (BOEFT) that can address quarkonium, tetraquarks, pentaquarks, hybrids and doubly heavy baryons

The BOEFT is based on symmetries and factorization

-It allows for QCD perturbative calculations at short distance
-It factorizes long distance in few flavour independent correlators to be calculated on the lattice
-Factorization allows for model independent predictions

-It answers the question on the nature of the XYZ states

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In this same framework we can calculate quarkonium AND exotics XYZ:

Spectra, decays, transitions ✔

Production ✔

Medium propagation from the quarkonium example EFT+ open quantum system?

Talk based on these references:

N.B. , R. Bruschini, A Mohapatra, T. Scirpa, A. Vairo, F. Zheng, ‘Above threshold spectrum’

N.B. , A Mohapatra, T. Scirpa, A. Vairo, ‘Open flavor threshold effects on quarkonium’

N.B., M. Buteschoen, S. Hibler, A Mohapatra, A. Vairo, X. Wang ‘Production of X and penta’ in preparation

N.B. , A Mohapatra, A. Vairo, ‘Pentaquarks’ 2508.13050

N.B. , A Mohapatra, T. Scirpa, A. Vairo, ‘The nature of X(3827) and Tcc’ 2411.14306

M. Berwein, N.B. , A. Mohapatra, A. Vairo, **2408.04719** -> establish BOEFT for all cases

N.B. , G. Krein, J. Tarrus, A. Vairo, 1707.09647

M. Berwein, N.B. , J. Tarrus, A. Vairo, **1510.04299** -> establishes BOEFT for hybrids

N.B. , W.K. Lai, J. Segovia, J. Tarrus, A. Vairo, 1805.07713

N.B. , W.K. Lai, J. Segovia, J. Tarrus 1908.11699 -> spin corrections (hybrids)

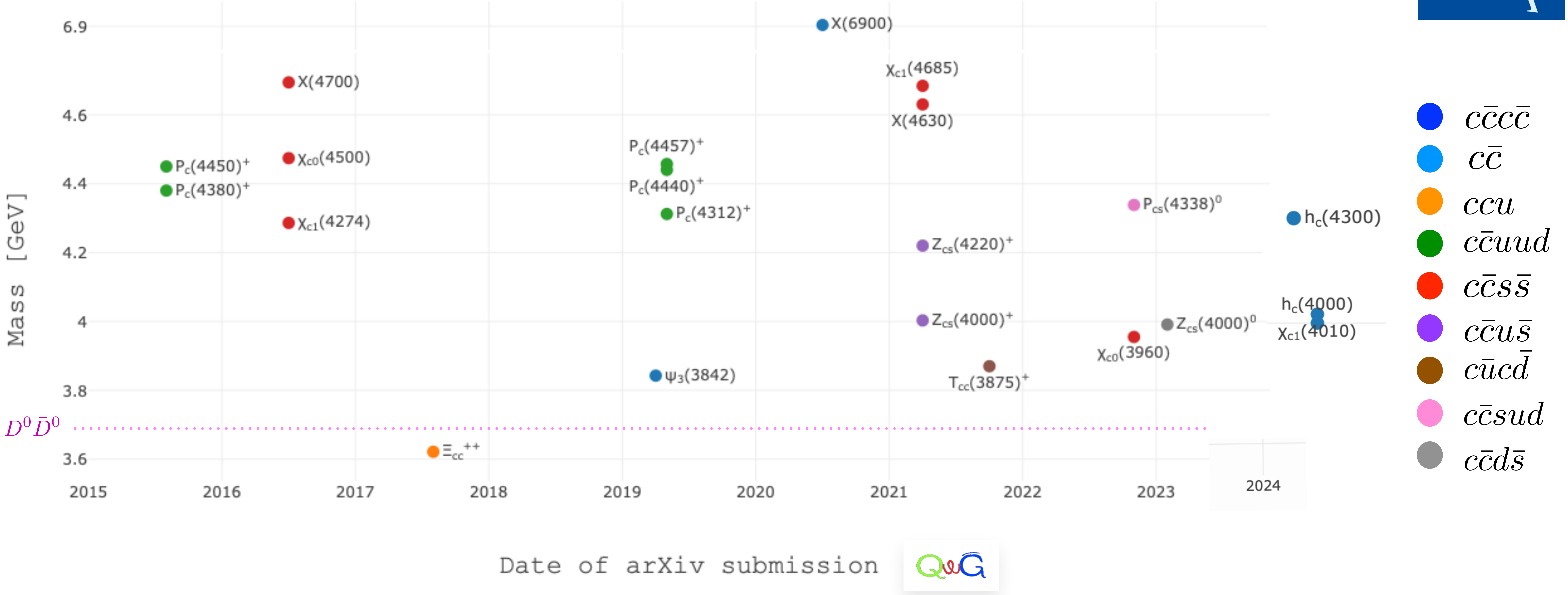
N.B. , W.K. Lai, A. Mohapatra, A. Vairo 2212.09187 -> semi-inclusive decays (hybrids)

N.B. , J. Soto, A. Pineda, A. Vairo hep-ph/9907240

N.B. , J. Soto, A. Pineda, A. Vairo hep-ph/0410047 -> quarkonium strongly coupled pNRQCD

N.B. , J. Soto, A. Pineda, A. Vairo hep-ph/0410047

REVOLUTION: Discovery in the Sector With Two Heavy Quarks



https://qwg.ph.nat.tum.de/exoticshub/

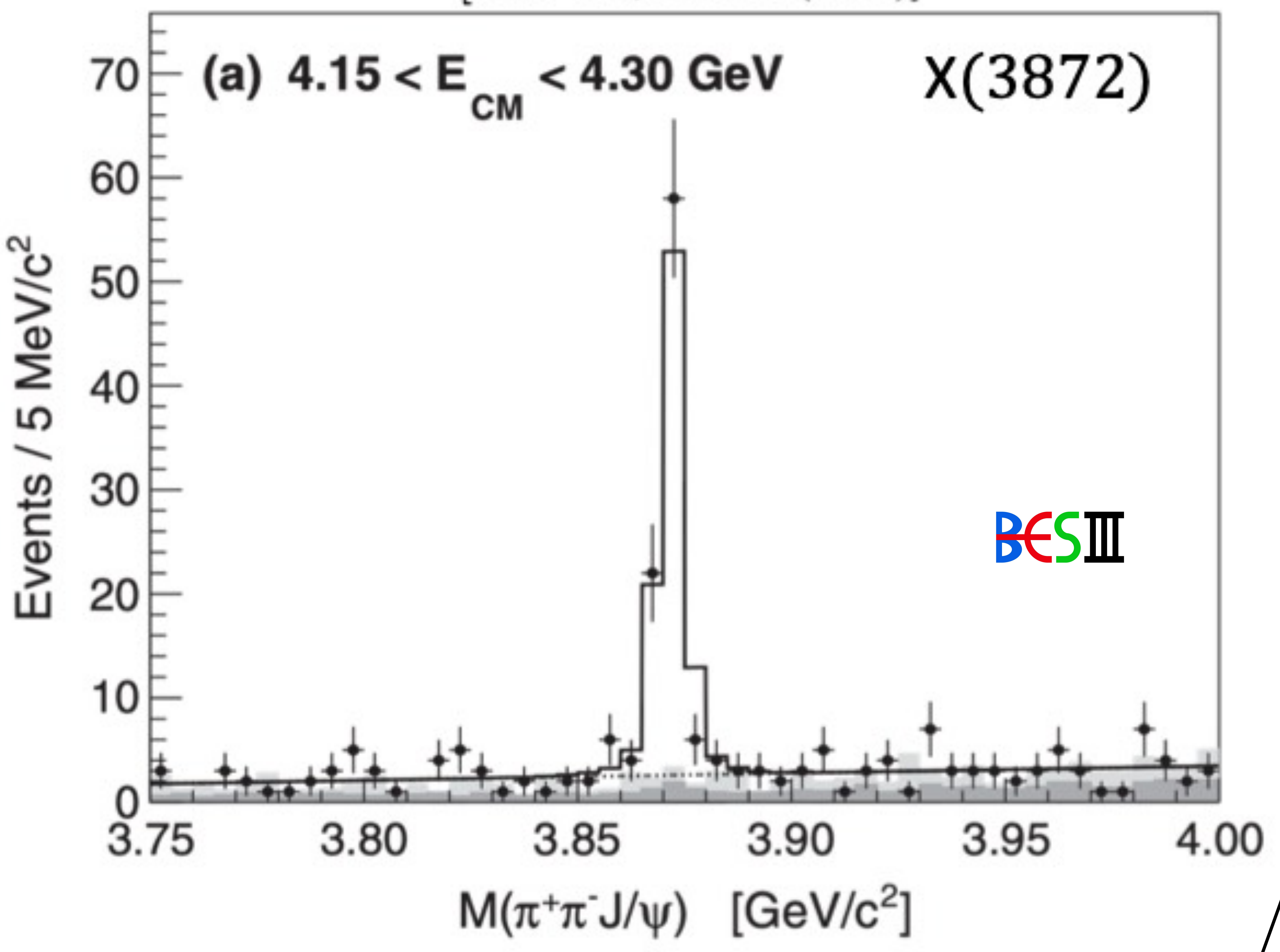
INTERPLAY AMONG MANY EXPERIMENTS:

UPCOMING EXPERIMENTS: STCF, Electron ion

X(3872) aka $\chi_{c1}(3872)$

Some surprisingly narrow states even if above/at strong decay thresholds

$e^+e^- \to \gamma X(3872)$; $X(3872) \to \pi^+\pi^- J/\psi$
[PRL 122, 232002 (2019)]



$$M_{X(3872)} - M_{D^0 D^{*0}} = 0.01 \pm 0.14 MeV$$

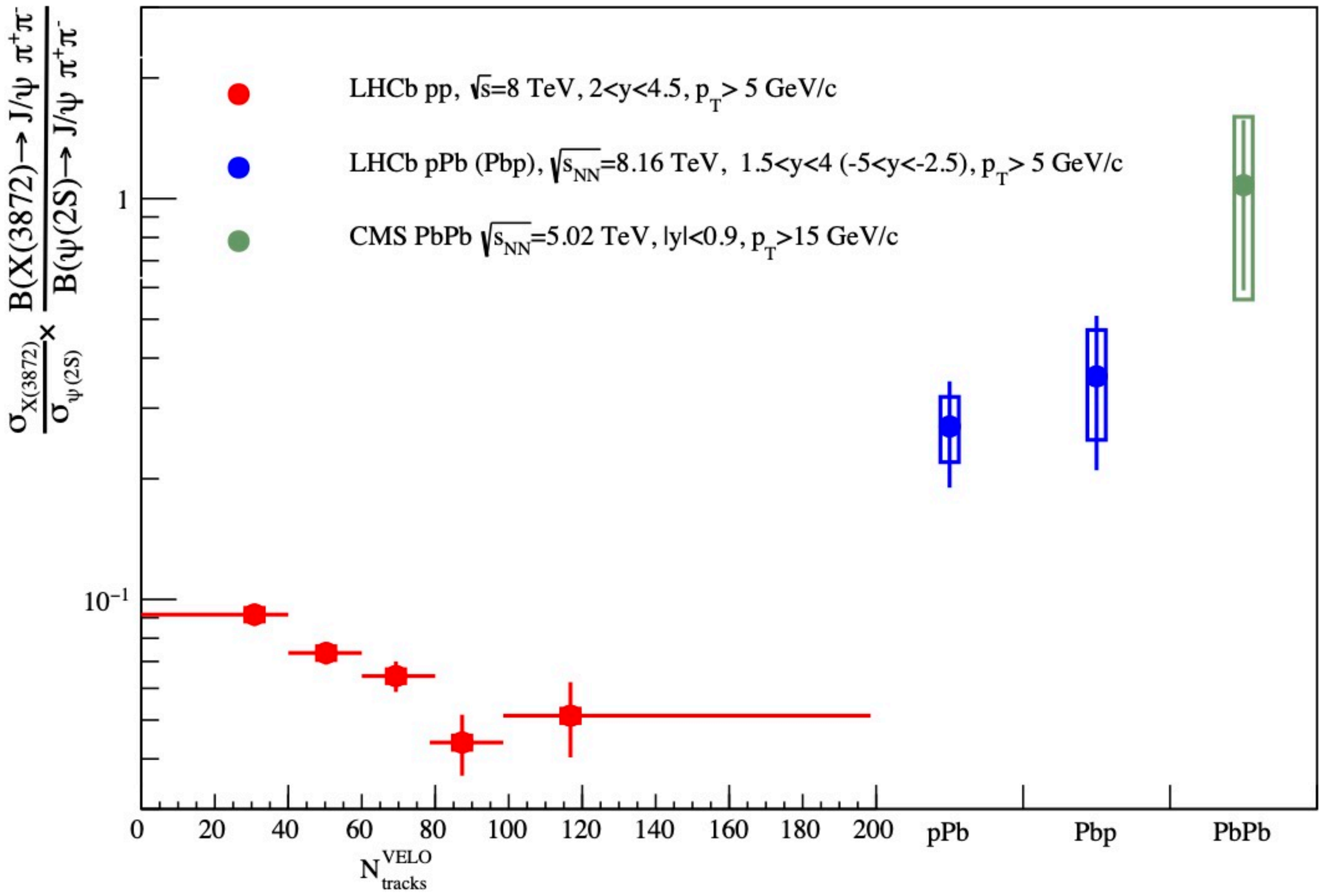
$$J^{PC} = 1^{++} \qquad I = 0$$

Observed in e+e-, B decays, hadroproduction (large cross section 30nb)

<-within 100 KeV of the threshold (**molecule?**)
width of 1 MeV! very small binding energy
Very large radius!

Compositeness, radiative decays, production suggest the presence of a **compact component**

X(3872) aka $\chi_{c1}(3872)$

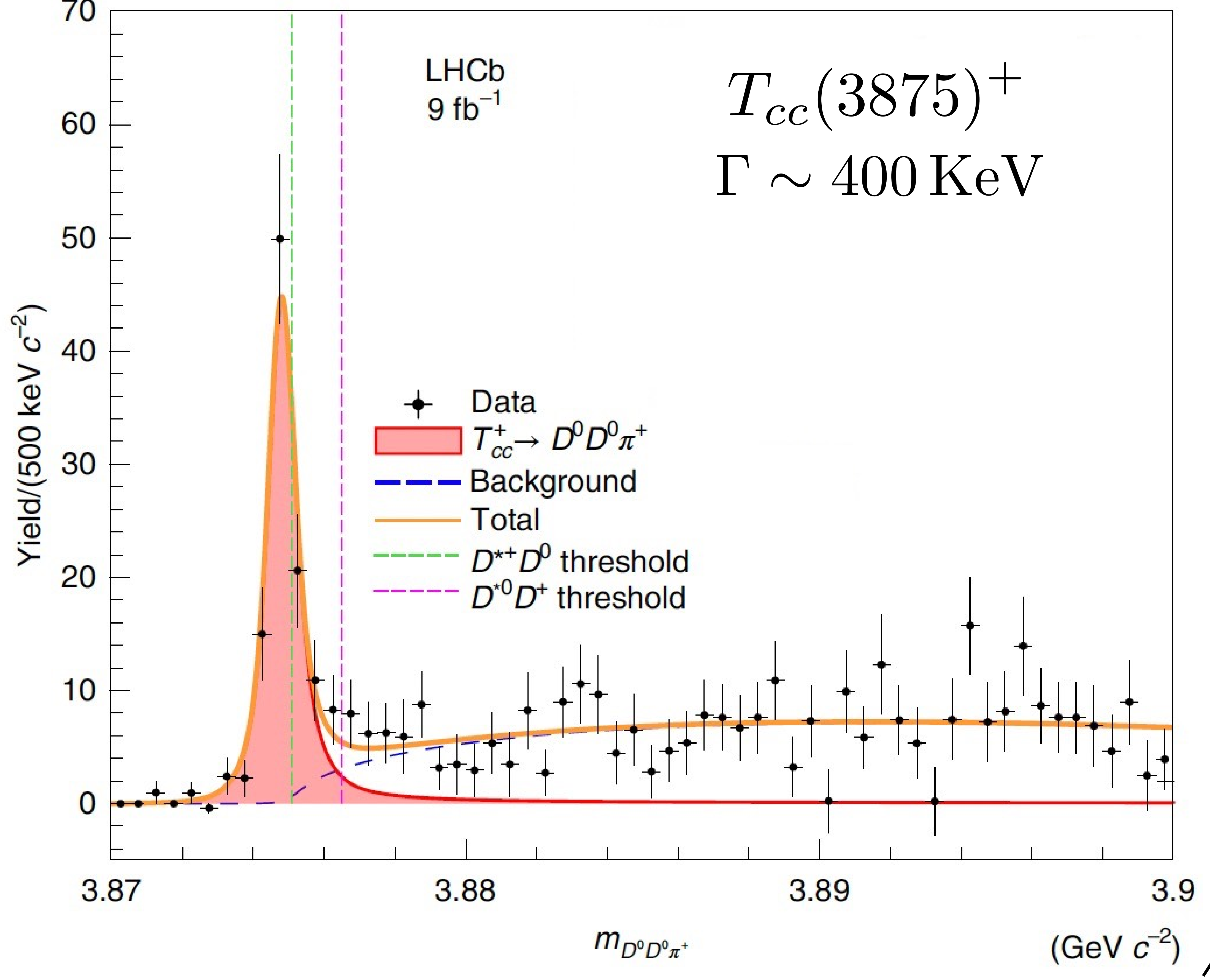


X Produced also in heavy ions where the deconfined strongly coupled QCD medium (Quark Gluon Plasma-QGP) is formed

New perspectives for XYZ studies!

$T_{cc}^{+}(3875)$

Nature Phys. 18 (2022) 7, 751-754



The longest lived exotic matter ever found!

$$J^P = 1^+ \qquad I = 0$$

<-within 300 KeV of the threshold (**molecule?**)

<-width of 48 KeV!

$$M_{T_{cc}(3875)^+} - (M_{D^{*+}} + M_{D^0}) = -0.27 \pm 0.06 MeV$$

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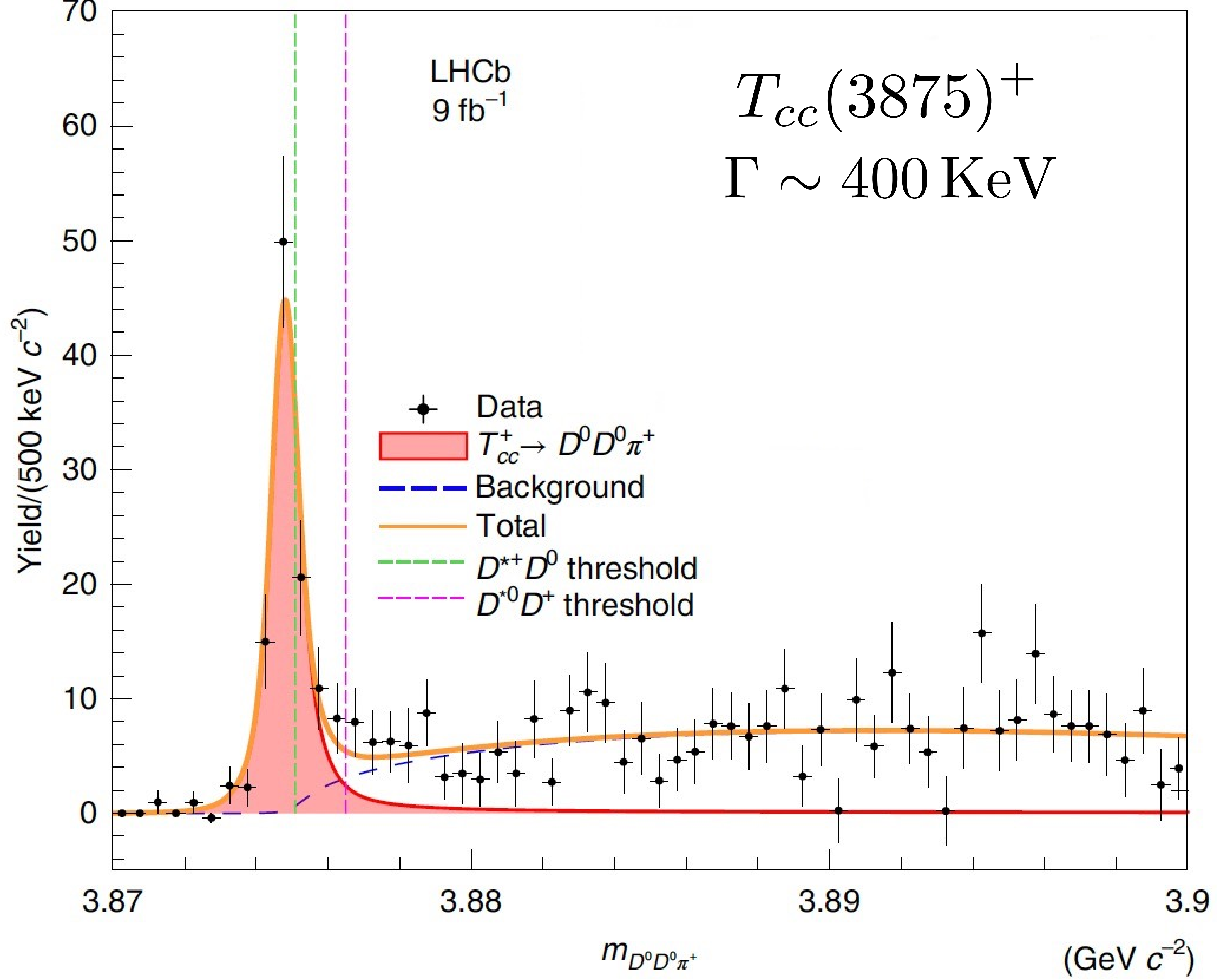

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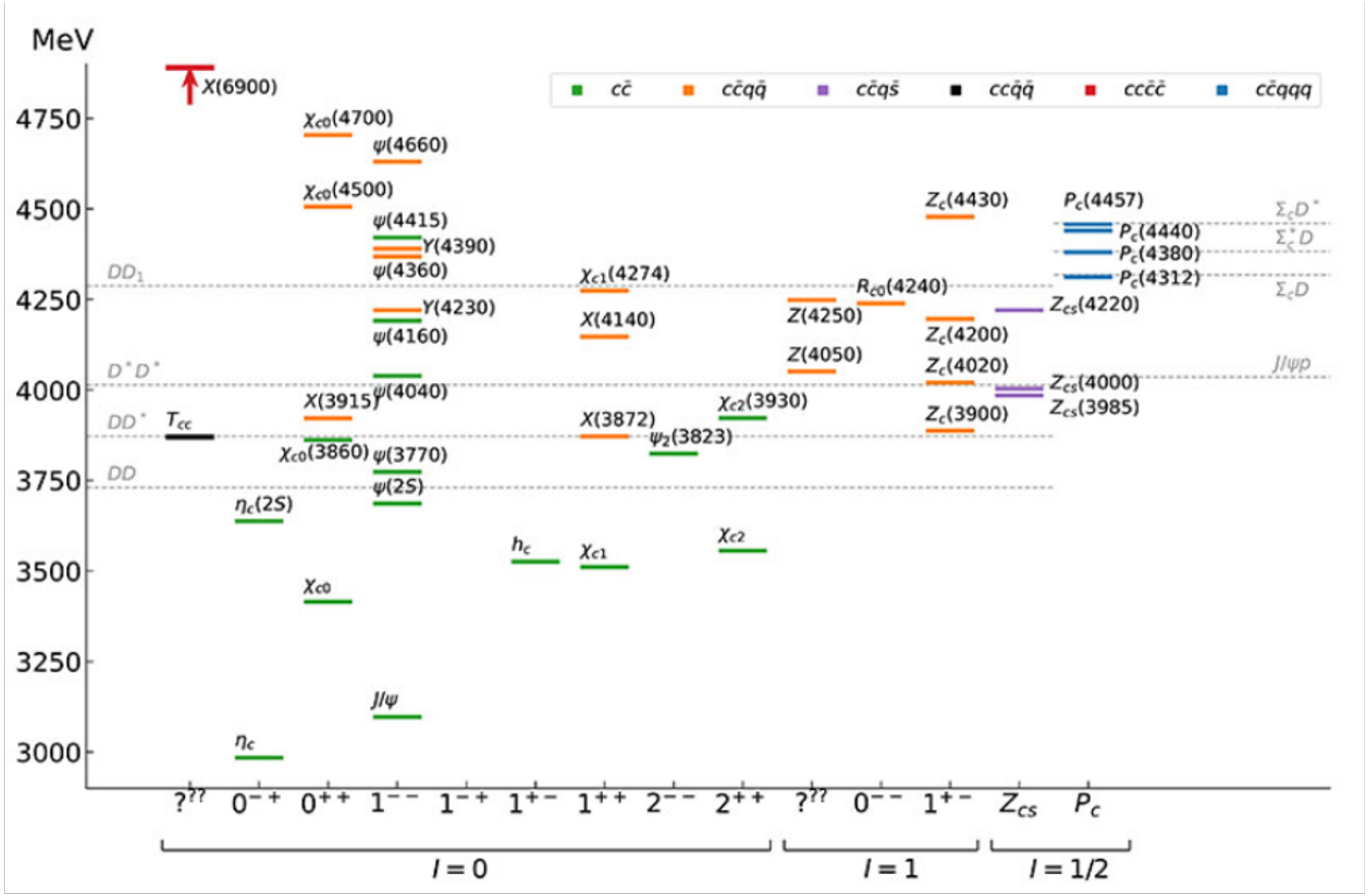
XYZs not merely composite particles, have unique properties can give us information about the strong force

—>Novel strongly correlated exotics systems

On the Other Hand We Should Understand All the States in a Given QQbar Sector...

Above threshold region

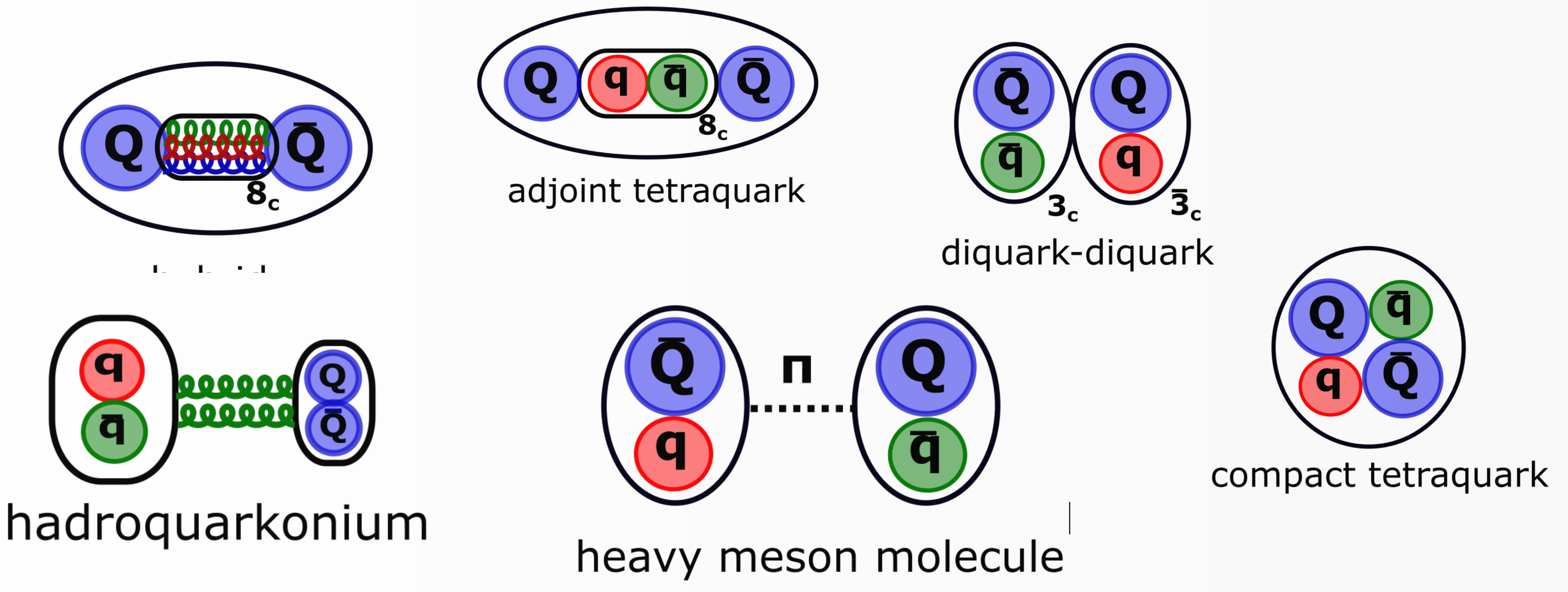
below threshold region



..the ones with exotics characteristics and the ones more standard, understand what drives the exotics features and these states mix

Close/above threshold new degrees of freedom like glue and light quarks are nonperturbative part in the binding.

- Multiquark states involve different configurations. Models assume some degrees of freedom in some configurations and a model interaction : molecular model, tetraquark model.. .



Where Is the Theory?

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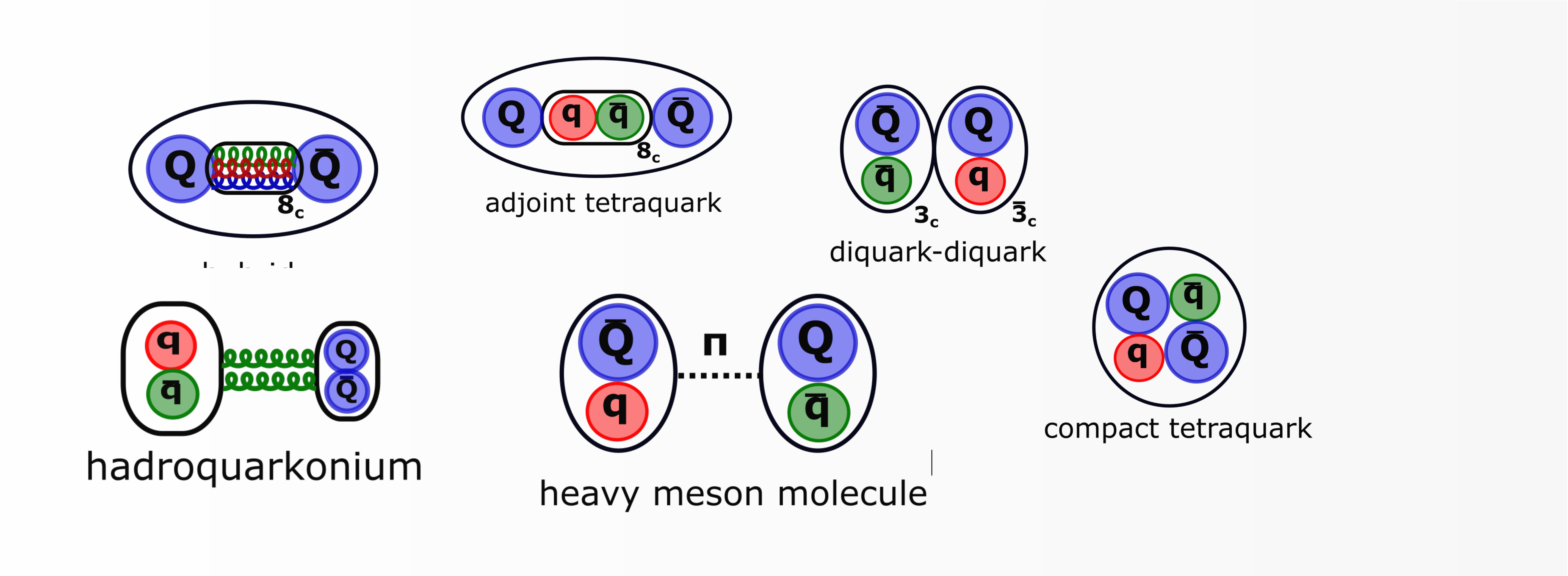
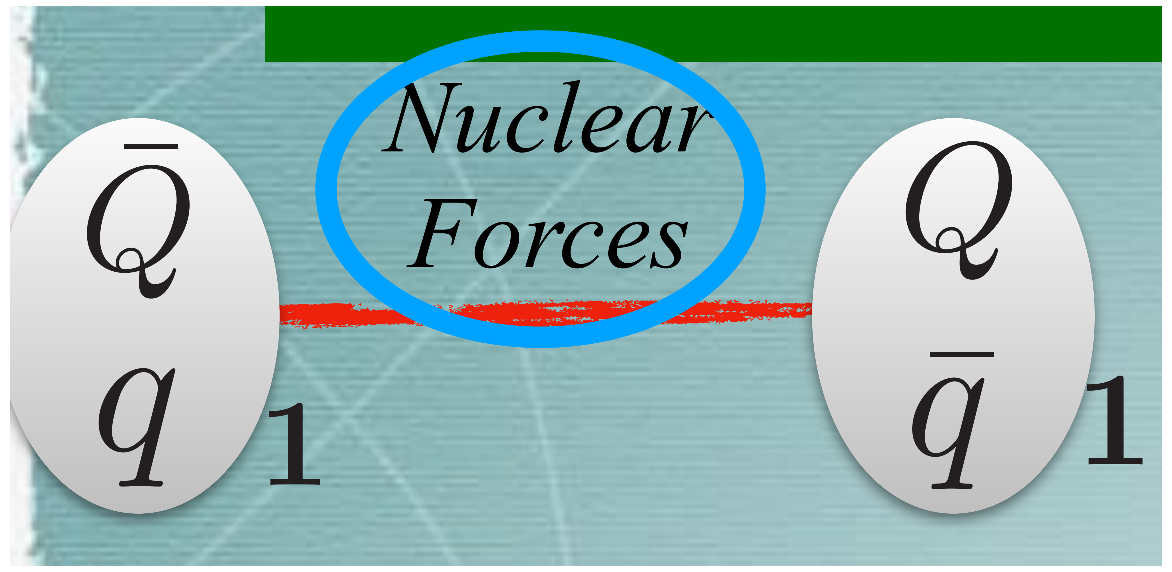


- On the nature of the X(3872) two models in particular compete: molecule versus compact tetraquark

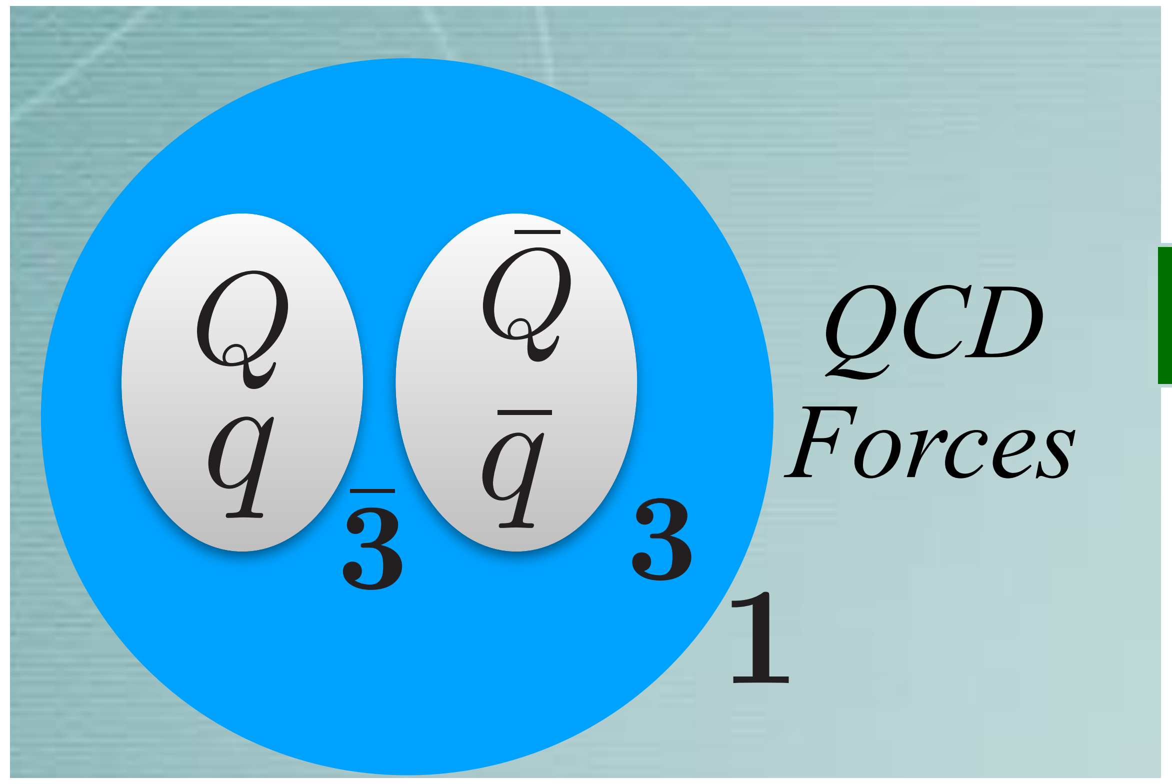


Hadron Molecule

- most of these states close to two-particle thresholds → molecular nature very natural!

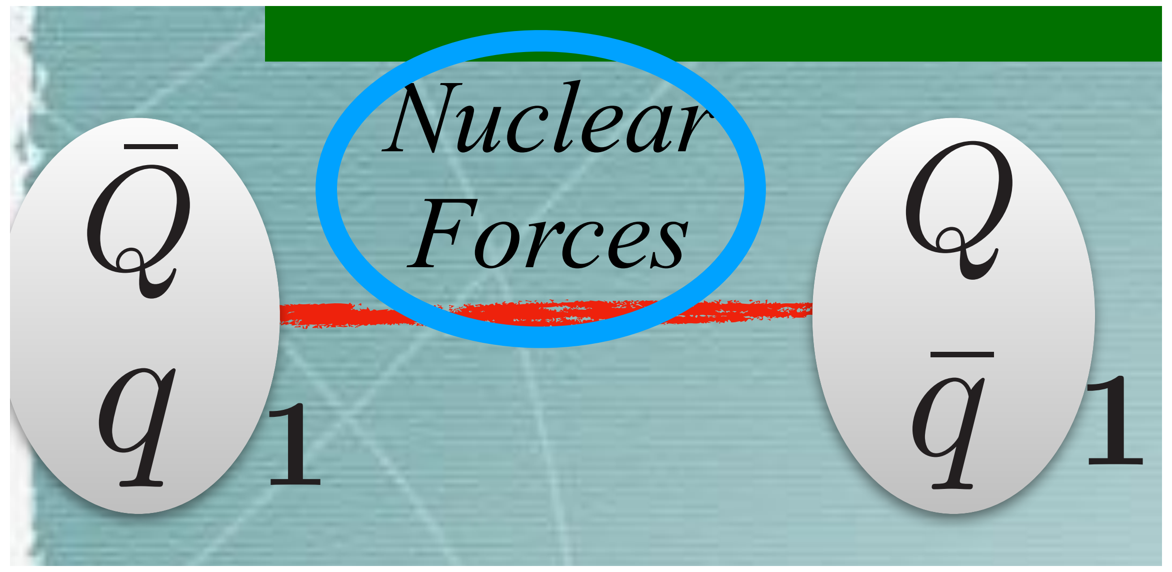
It can be developed in an EFT

Large radius



Compact Diquark-Antidiquark

Small radius



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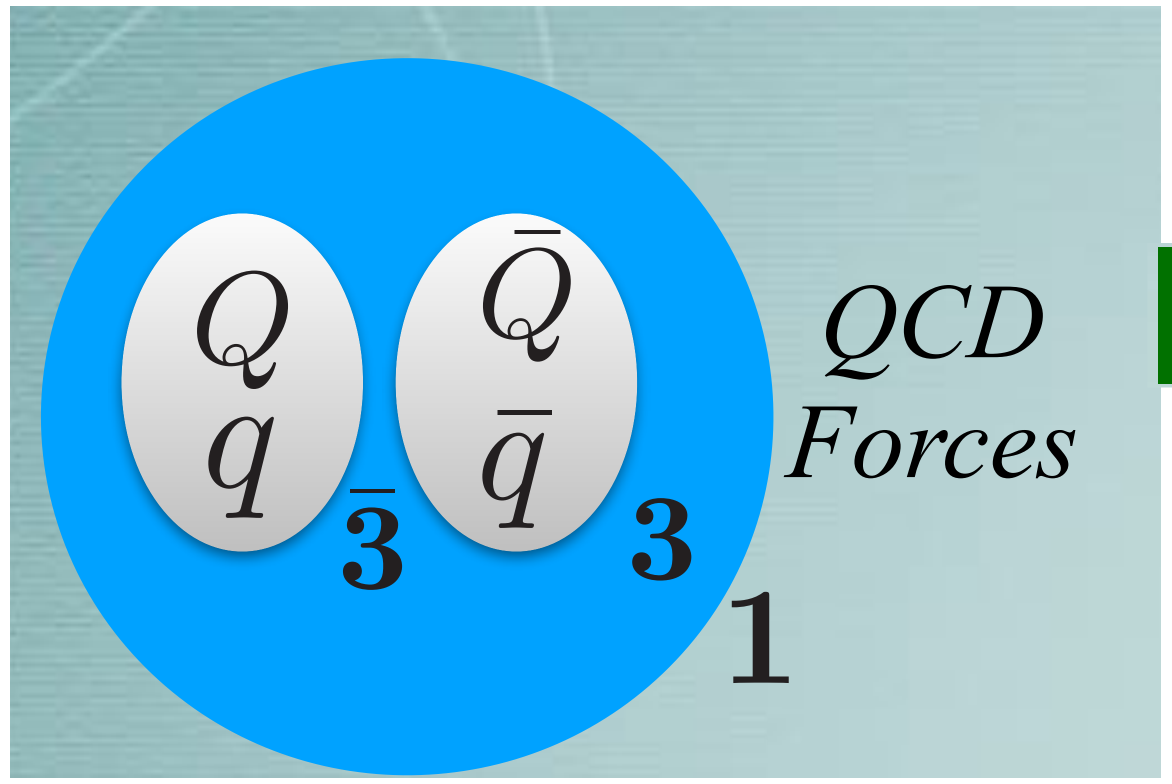

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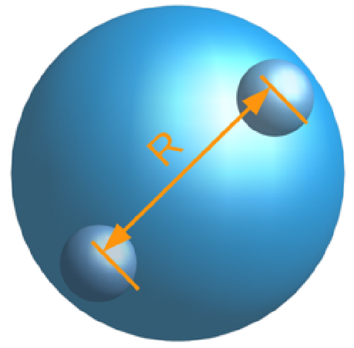
The two models confronted for decades, e.g. about hadroproduction

- **Two classes** of decay and production processes:
  - **long-distance processes**,
  - **short-distance processes**,

[hadroproduction of the $X(3872)$]

Compositeness $1 - Z$

Is that a meaningful parameter?l



$Z = 0$

$Z = 1$

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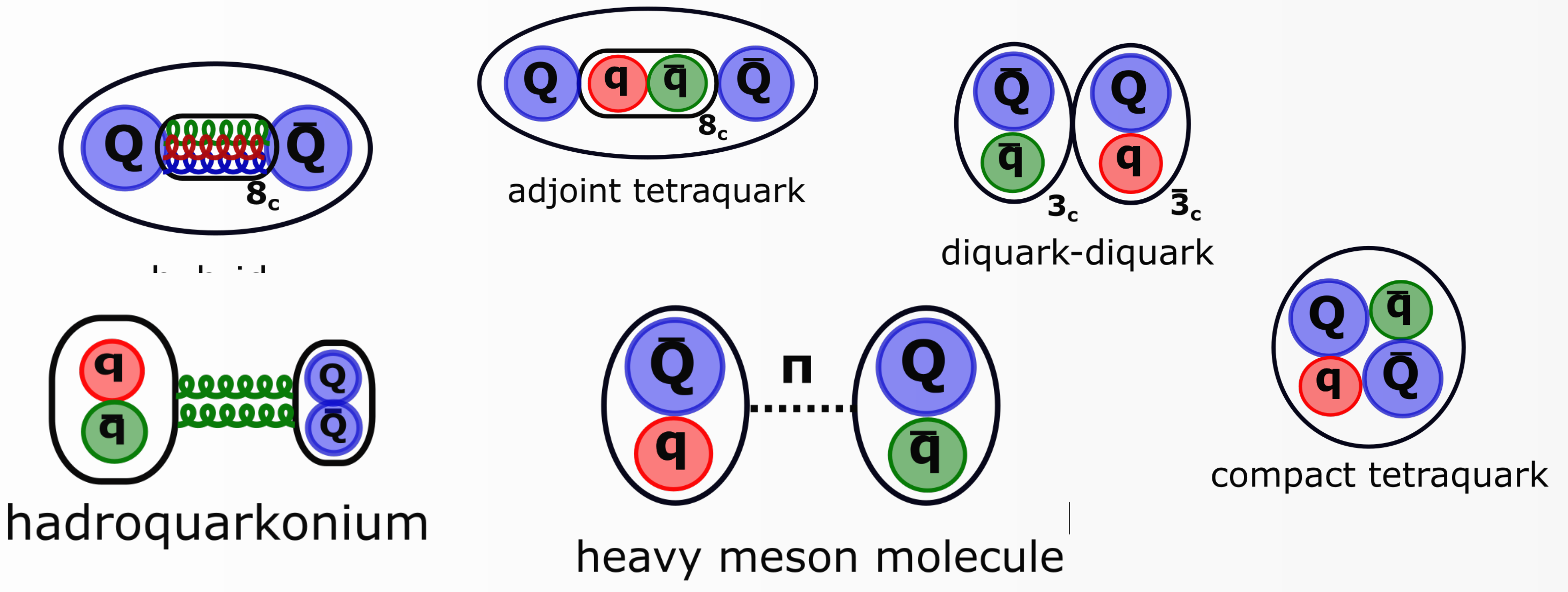

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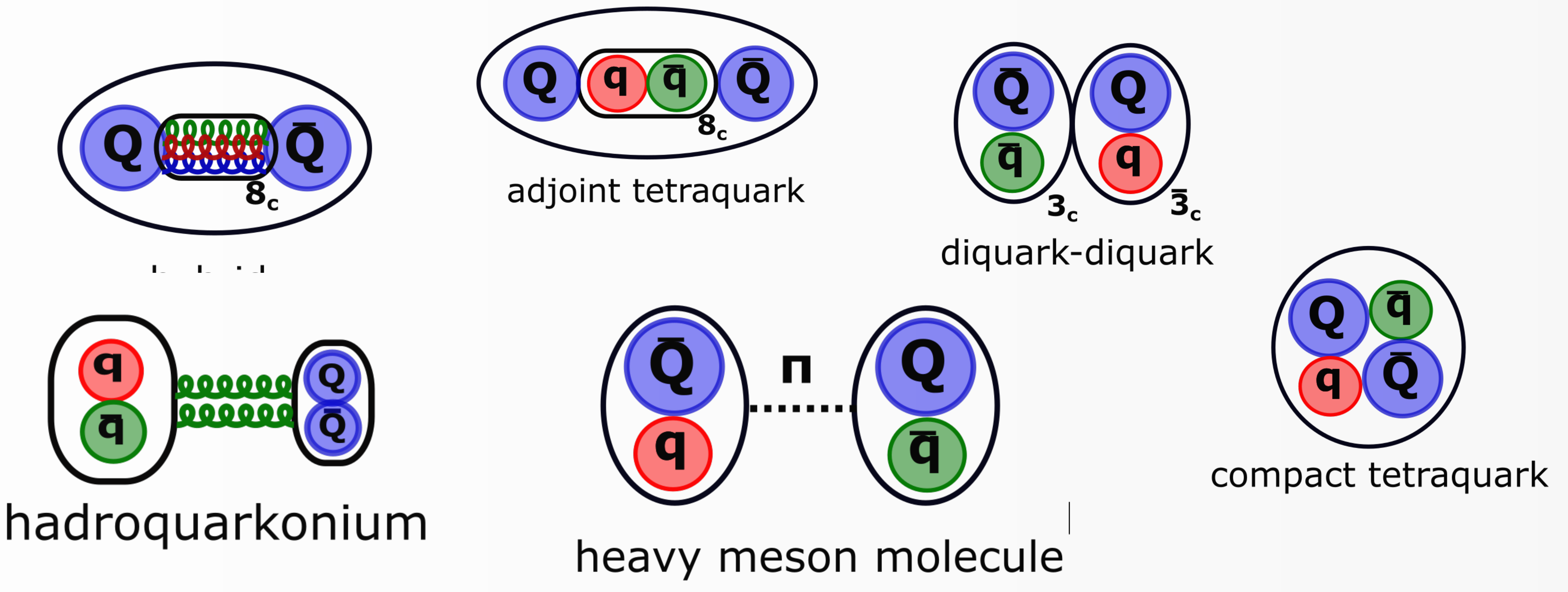

- Direct lattice calculations of exotics masses are limited by the large number of open decay modes and they are not suited for production or in medium studies

- the BOEFT is a flexible approach rooted in QCD that can address all properties of XYZ, spectra, transitions, production, propagation in medium. It is based on scales separation and requires few universal lattice QCD input. It allows to study the nature of the QCD force

A nonrelativistic effective field theory description is valid

The quarks being heavy guarantees

the hierarchy of non-relativistic energy scales $m_Q \gg p \sim 1/r \sim m_Q v \gg E \sim m_Q v^2$,

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Bound systems with a typical radius $\sim \Lambda_{\rm QCD}^{-1}$

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QCD —> NRQCD—> BOEFT

Born-Oppenheimer EFT for States With Two Heavy Quarks

construct a nonrelativistic EFT description on the basis of scale separations and symmetries

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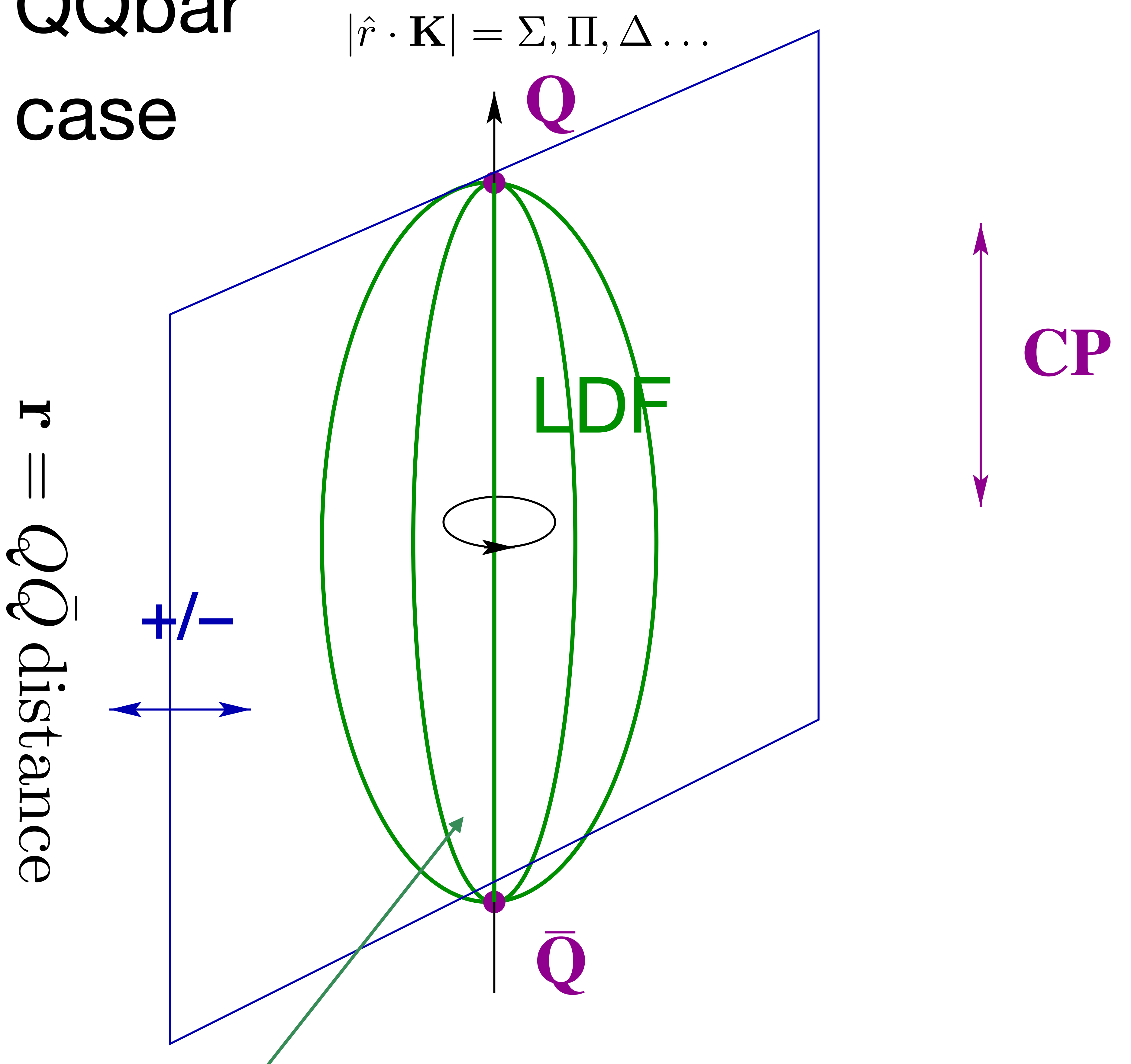
QQbar case



Nonperturbative light degrees of freedom:
any combination of glue and light quarks
to obtain a color singlet

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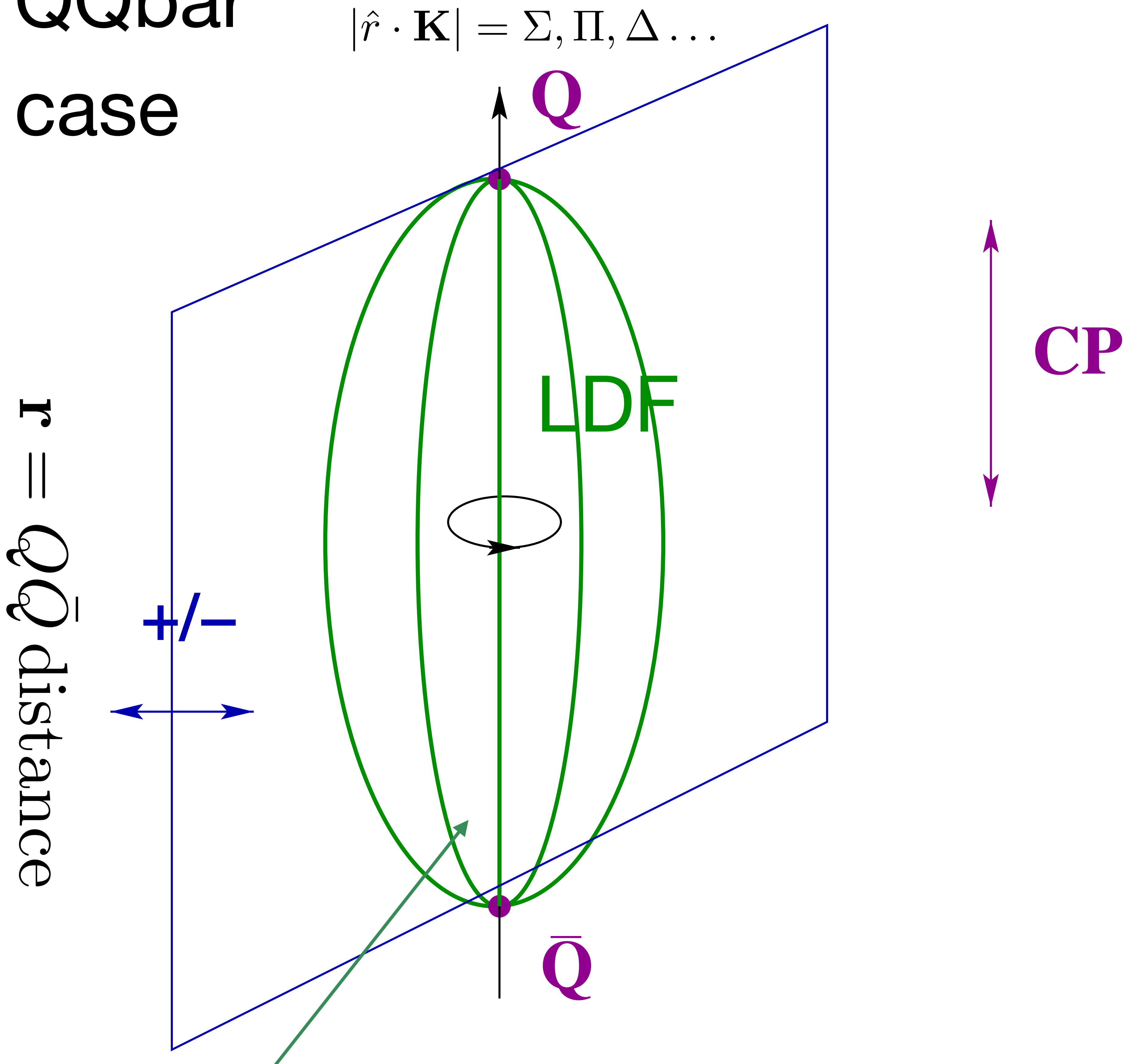

Nonperturbative light degrees of freedom:
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produce a hierarchy of NRQCD static energies identified by the quantum number of $D_{\infty h}$

Irreducible representations of $D_{\infty h}$

- $\boldsymbol{K}$: angular momentum of light d.o.f. $K^2 = k(k+1)$
  $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2, \pm 3, \ldots$
  $\Lambda = |\lambda| = 0, 1, 2, 3, \ldots \; (\Sigma, \Pi, \Delta, \Phi, \ldots)$
- Eigenvalue of CP: $\eta = +1\,(g), -1\,(u)$
- σ: eigenvalue of relfection about a plane containing $\hat{\boldsymbol{r}}$ (only for Σ states)

$$\Lambda_\eta^\sigma$$

Born-Oppenheimer EFT for States With Two Heavy Quarks

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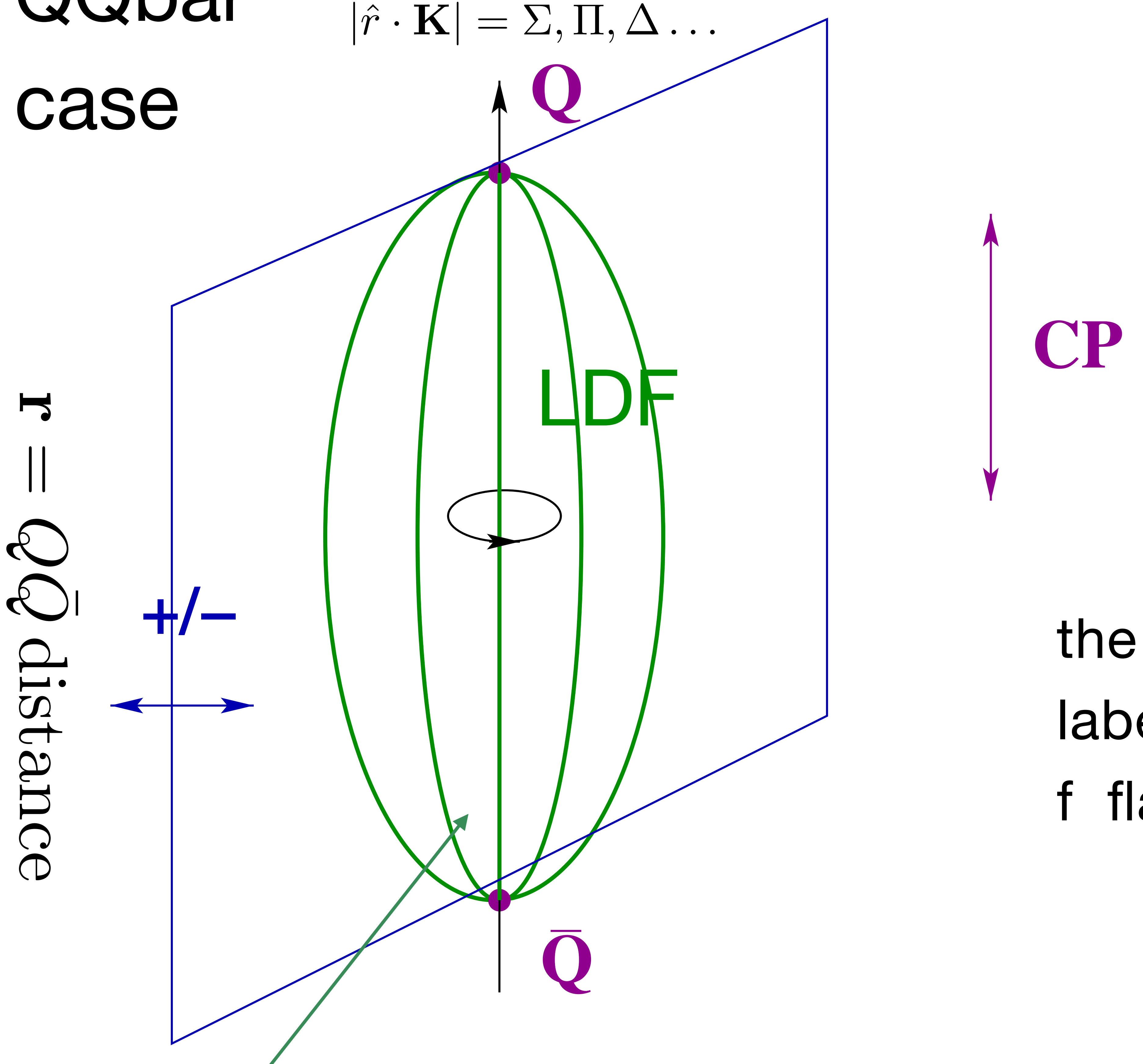

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the Light Degrees of Freedom (LDF)
labeled by $\kappa = \{k^{PC}, f\}$
f flavour quantum number

Notice that for r->0 the cylindrical symmetric becomes spherical (O(3) X C: several Λ_η^σ representations reduce to one single $k^{P(C)}$

k^{PC}	BO quantum #
0^{++}	Σ_g^+
0^{+-}	Σ_u^+
0^{-+}	Σ_u^-
1^{+-}	$\{\Sigma_u^-, \Pi_u\}$
1^{--}	$\{\Sigma_g^+, \Pi_g\}$
2^{--}	$\{\Sigma_g^-, \Pi_g, \Delta_g\}$

Nonperturbative light degrees of freedom: any combination of glue and light quarks to obtain a color singlet

Born-Oppenheimer EFT

Two heavy quarks with large mass $m \gg \Lambda_{\rm QCD}$ and residual scale separation $\Lambda_{\rm QCD} \gg E$

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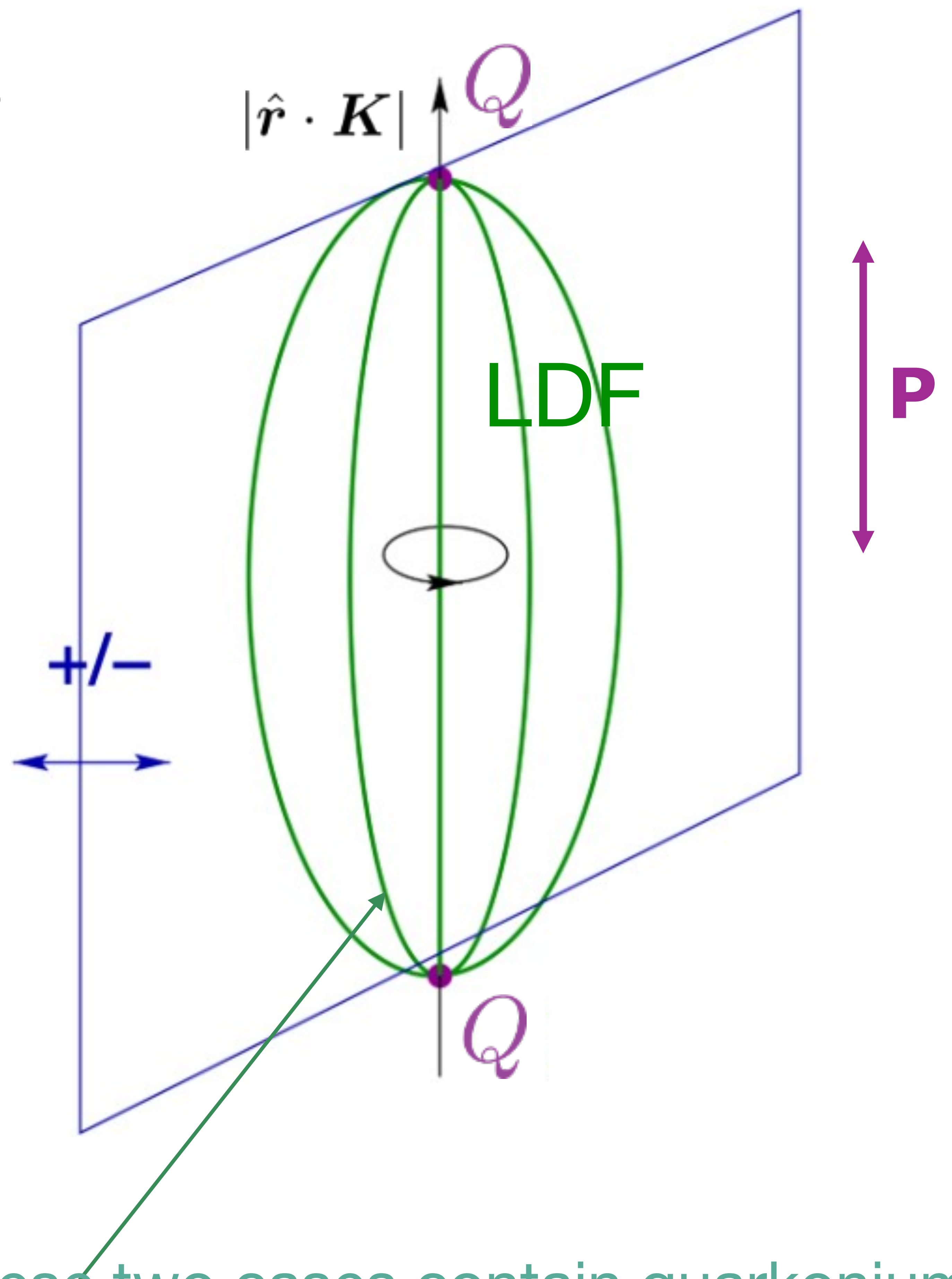

These two cases contain quarkonium, hybrids, tetraquarks, pentaquarks and doubly heavy baryons

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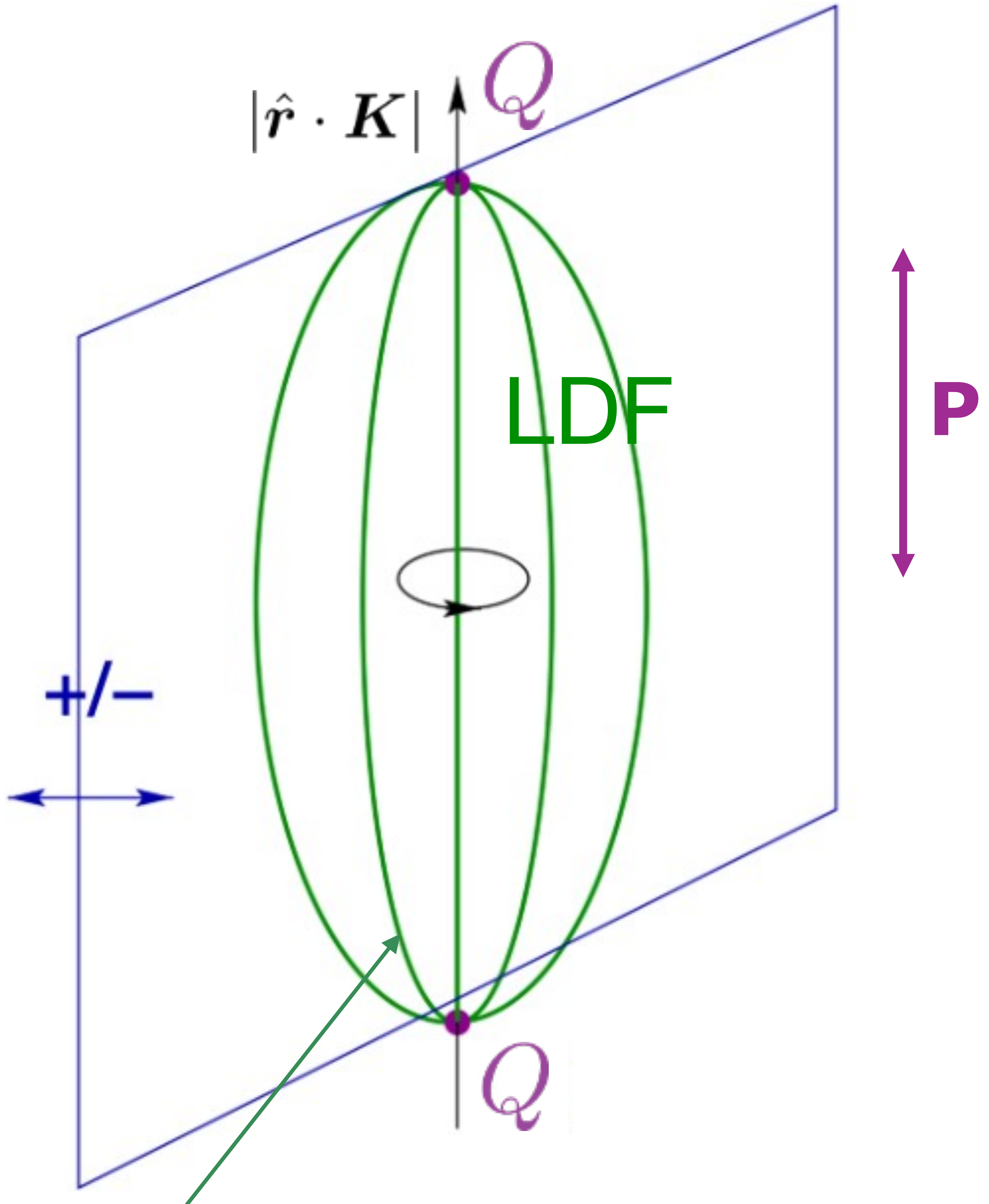

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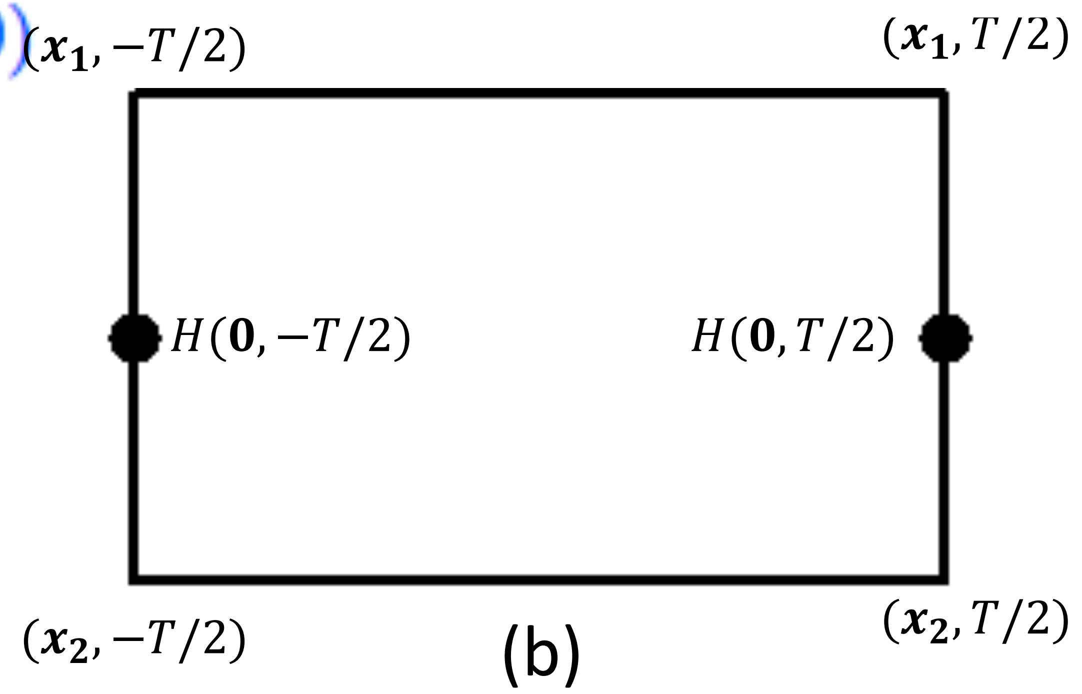
$$\mathcal{H}^{(0)} = \int d^3\mathbf{x}\, \frac{1}{2}\left(\mathbf{\Pi}^a\mathbf{\Pi}^a + \mathbf{B}^a\mathbf{B}^a\right) - \sum^{n_f} \bar{q}\, i\mathbf{D}\cdot\boldsymbol{\gamma}\, q \qquad n \equiv k, |\lambda|$$

$$\mathcal{H}^{(0)}|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1, \mathbf{x}_2)|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1)\chi(\mathbf{x}_2)|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$$E_n^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle X_n,\, T/2 | X_n,\, -T/2\rangle$$



(b)

Phi =Wilson lines and H= gluonic and light quarks produce a hierarchy of NRQCD static energies identified by the quantum number of $D_{\infty h}$

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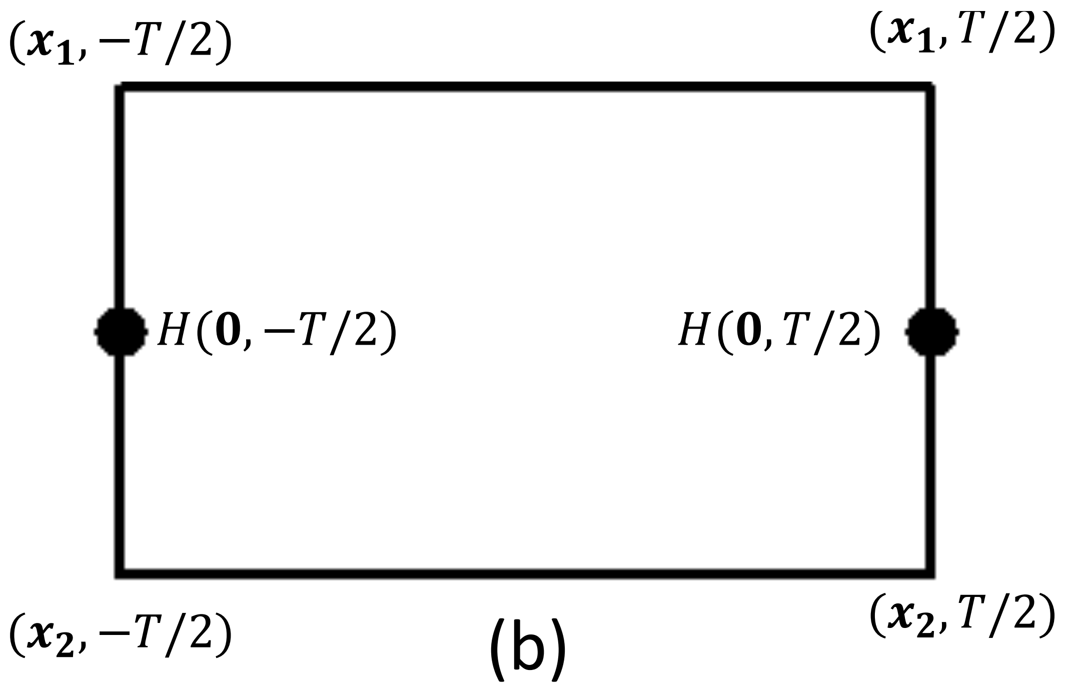
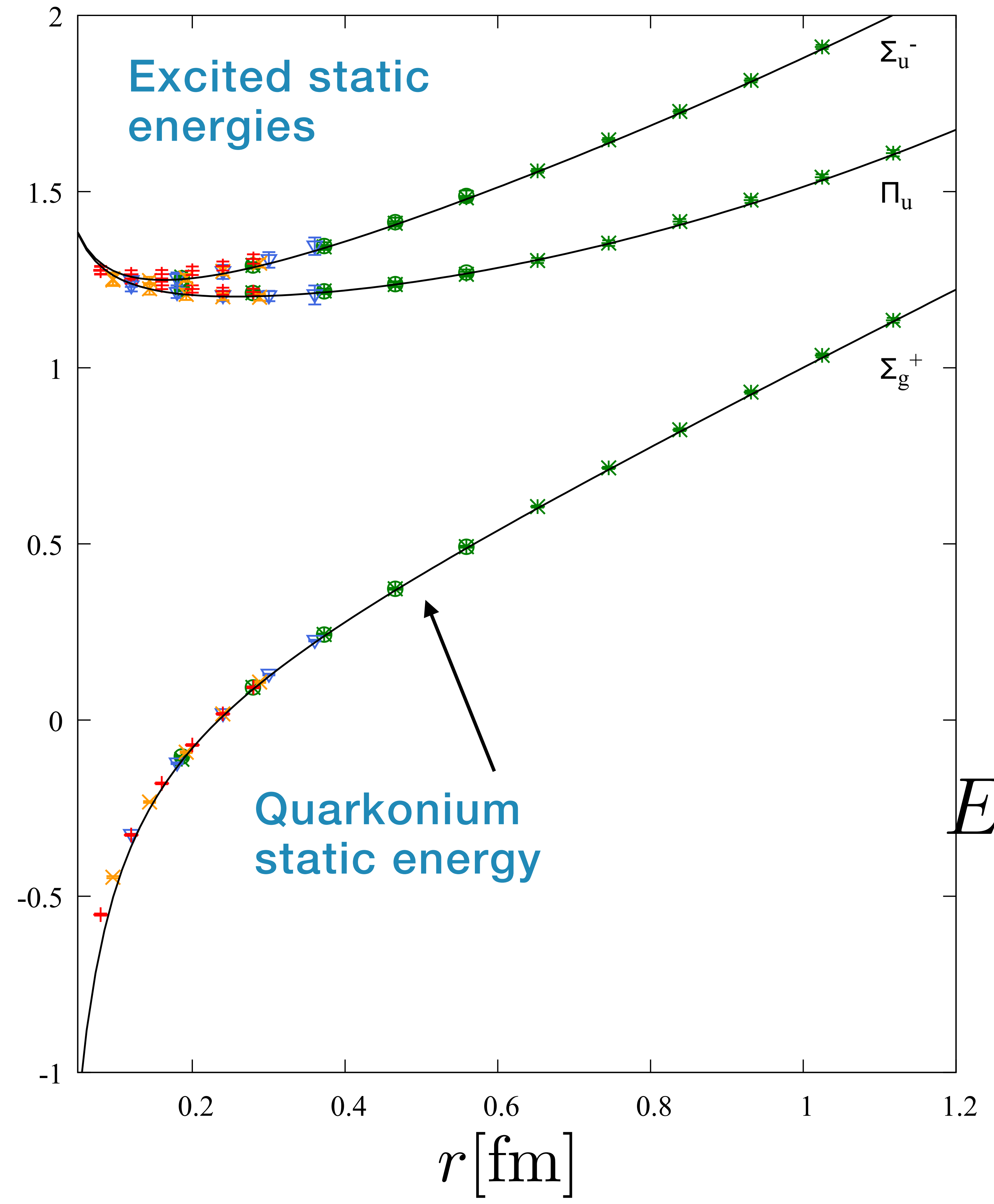


(b)

and:
the matching to BOEFT, the theory that has the Schroedinger as zero order problem

Born-Oppenheimer EFT for States With Two Heavy Quarks

$E^{(0)}_{\kappa,|\lambda|}(\mathbf{r})$



Mornigstar, Kuti from the 90' Wagner group

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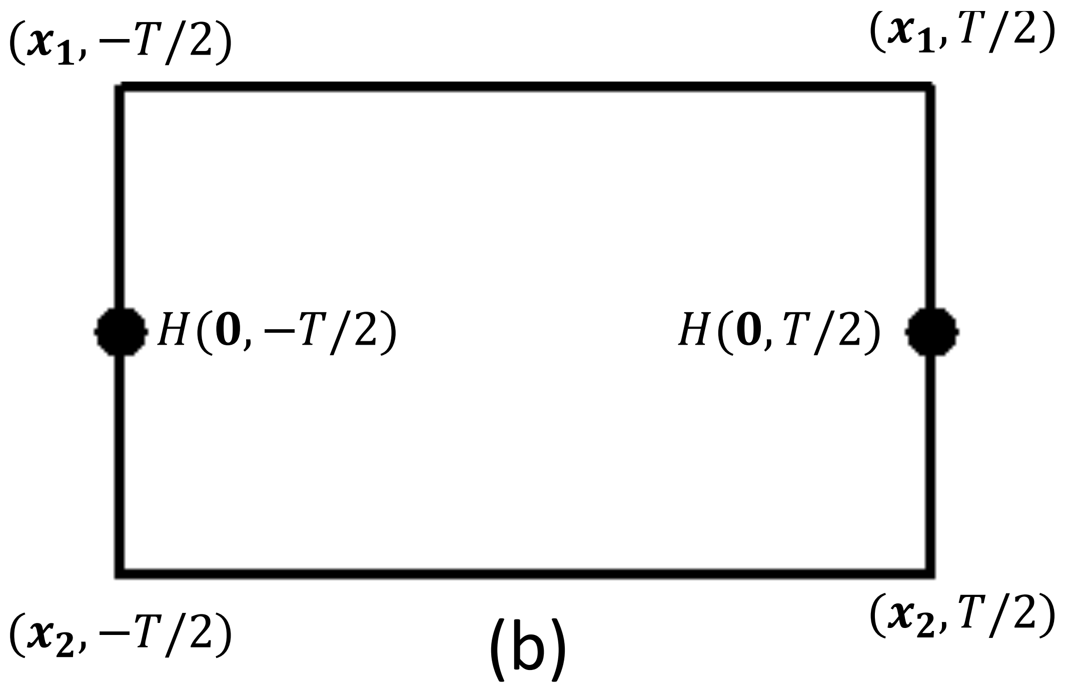

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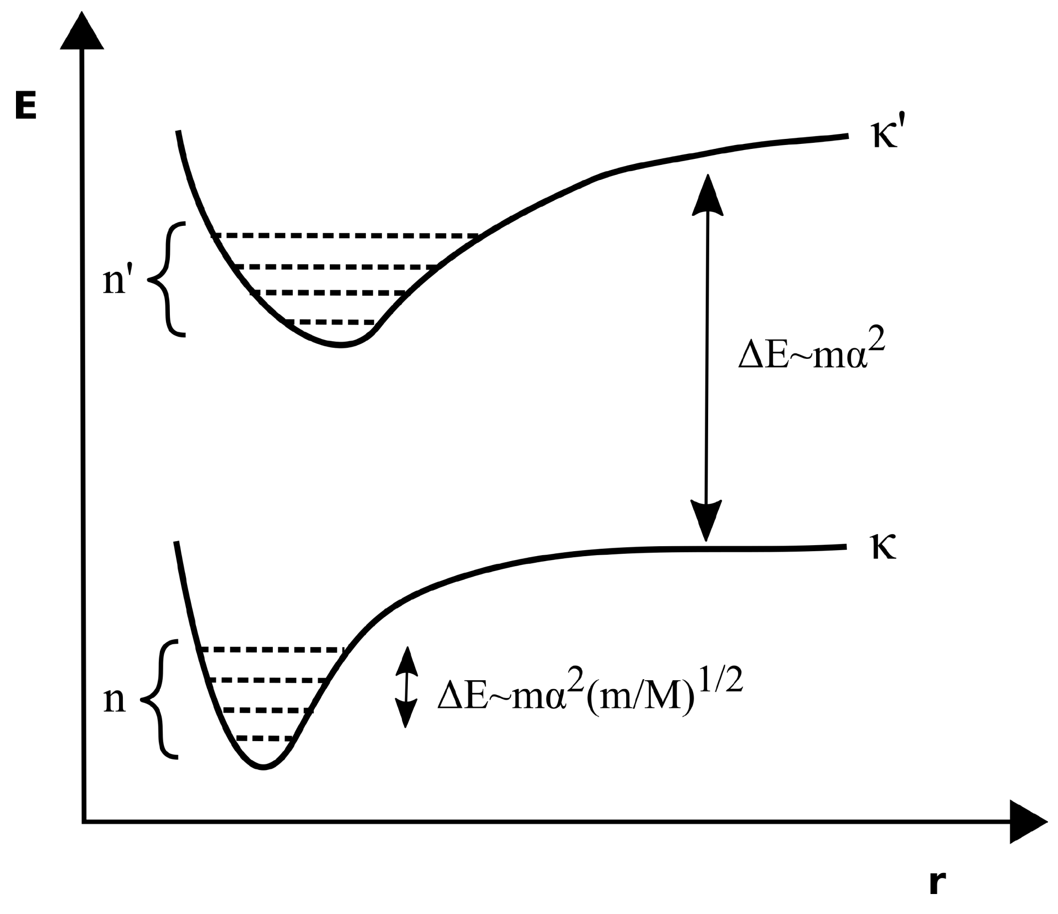
QED — Born Oppenheimer Description QCD



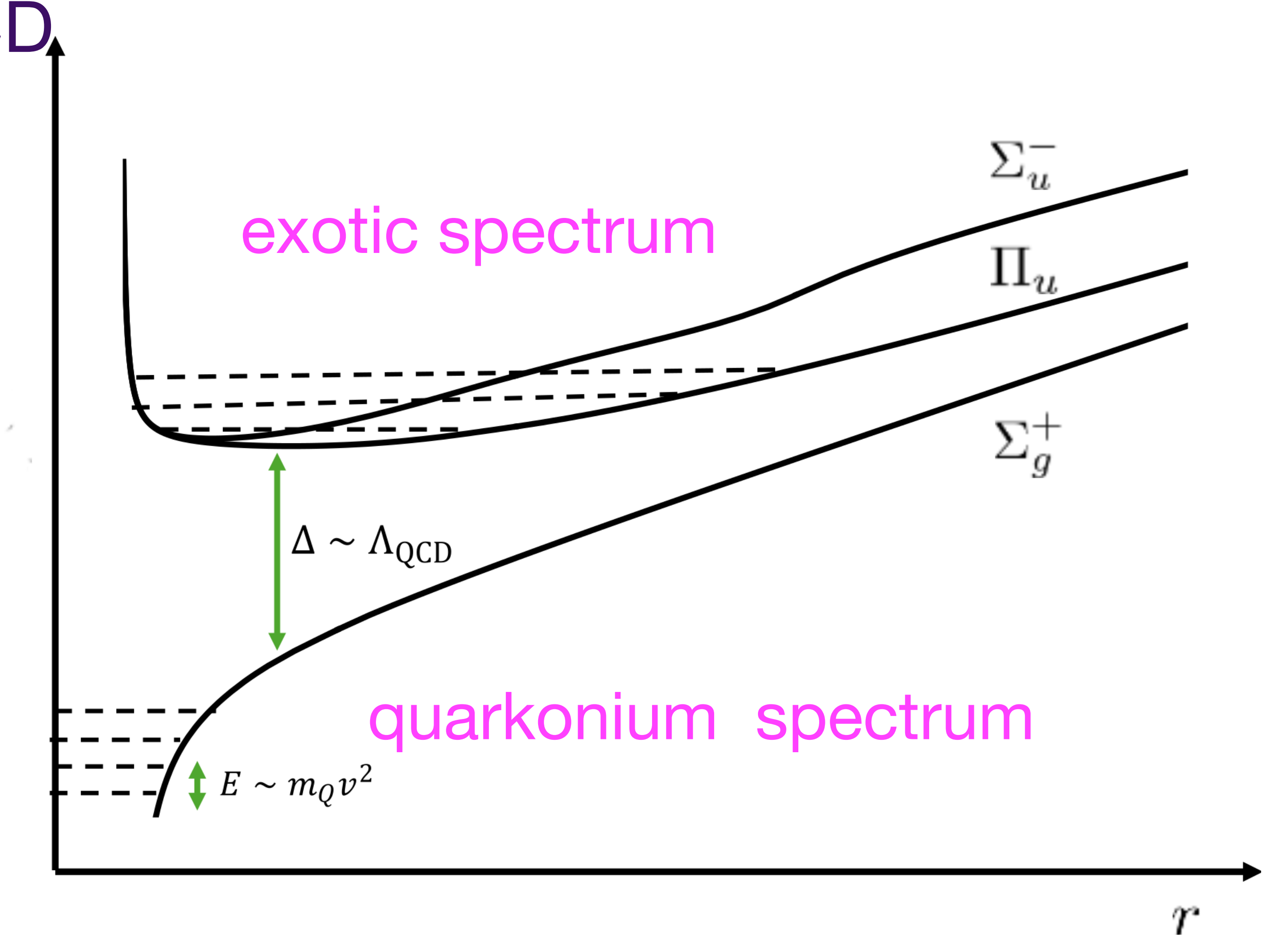
$$\Lambda_{QCD} > mv^2$$

fast (gluons, light quarks) and slow (heavy quarks)

like in molecular physics (fast-electrons, slow nuclei

Braaten PRL 111 (2013) 162003

Braaten Langmack Smith PRD 90 (2014) 014044



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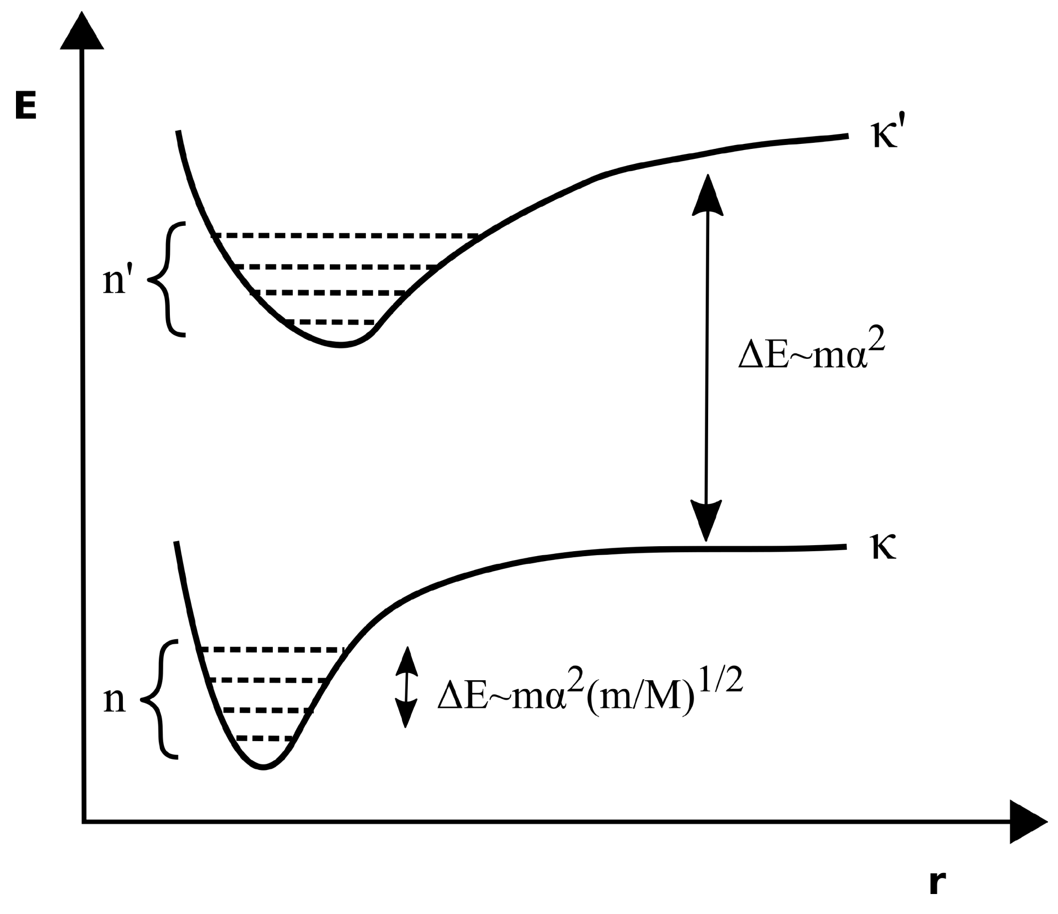

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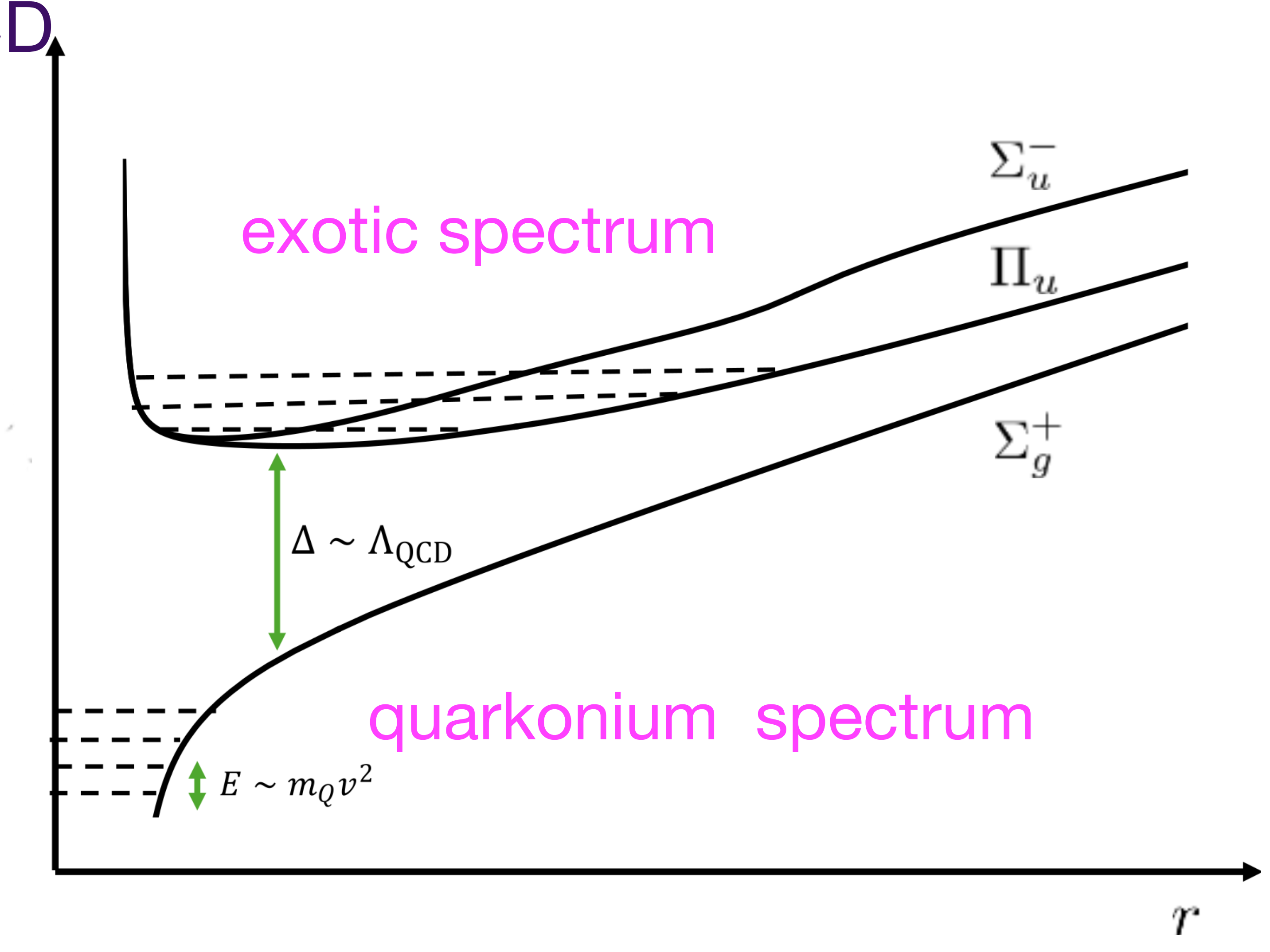
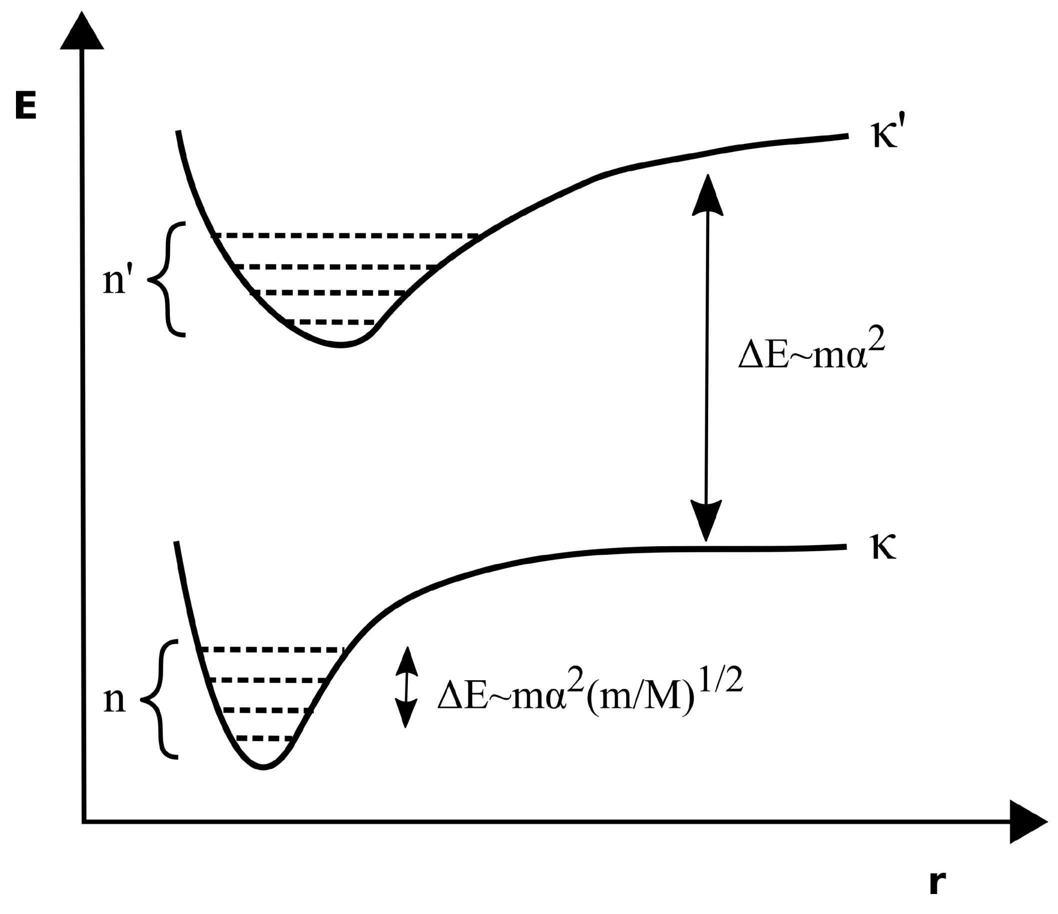


The heavy quarks move adiabatically in the presence of the light degrees of freedom, whose effect is encoded in a suitable set of potentials that depend on the distance r between the heavy quarks —->they encode the dynamics from the nonperturbative soft degrees of freedom

QED —— **Born Oppenheimer Description** QCD



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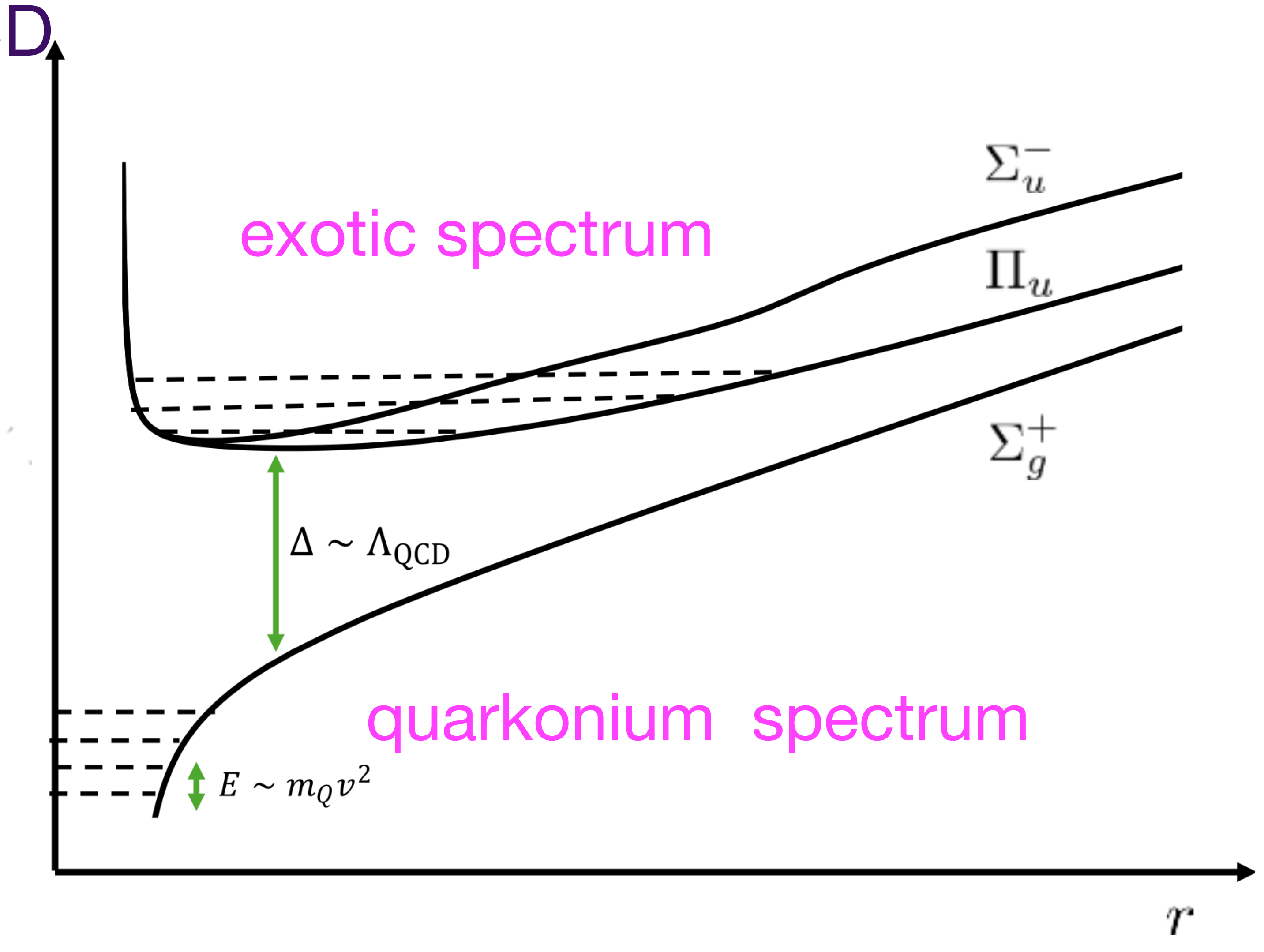
like in molecular physics (fast-electrons, slow nuclei

Braaten PRL 111 (2013) 162003

Braaten Langmack Smith PRD 90 (2014) 014044



The heavy quarks move adiabatically in the presence of the light degrees of freedom, whose effect is encoded in a suitable set of potentials that depend on the distance r between the heavy quarks —->they encode the dynamics from the nonperturbative soft degrees of freedom

Nonperturbative Matching to BOEFT

systematically

$$\langle H|\mathcal{H}|H\rangle = \langle nljs|\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}}{m^n}|nljs\rangle$$

expand quantomechanically NRQCD states and energies in 1/m around the zero order and identify the QCD potentials

$$|\underline{0};\mathbf{x}_1\mathbf{x}_2\rangle - > |(Q\bar{Q})_1\rangle \to$$ Quarkonium Singlet

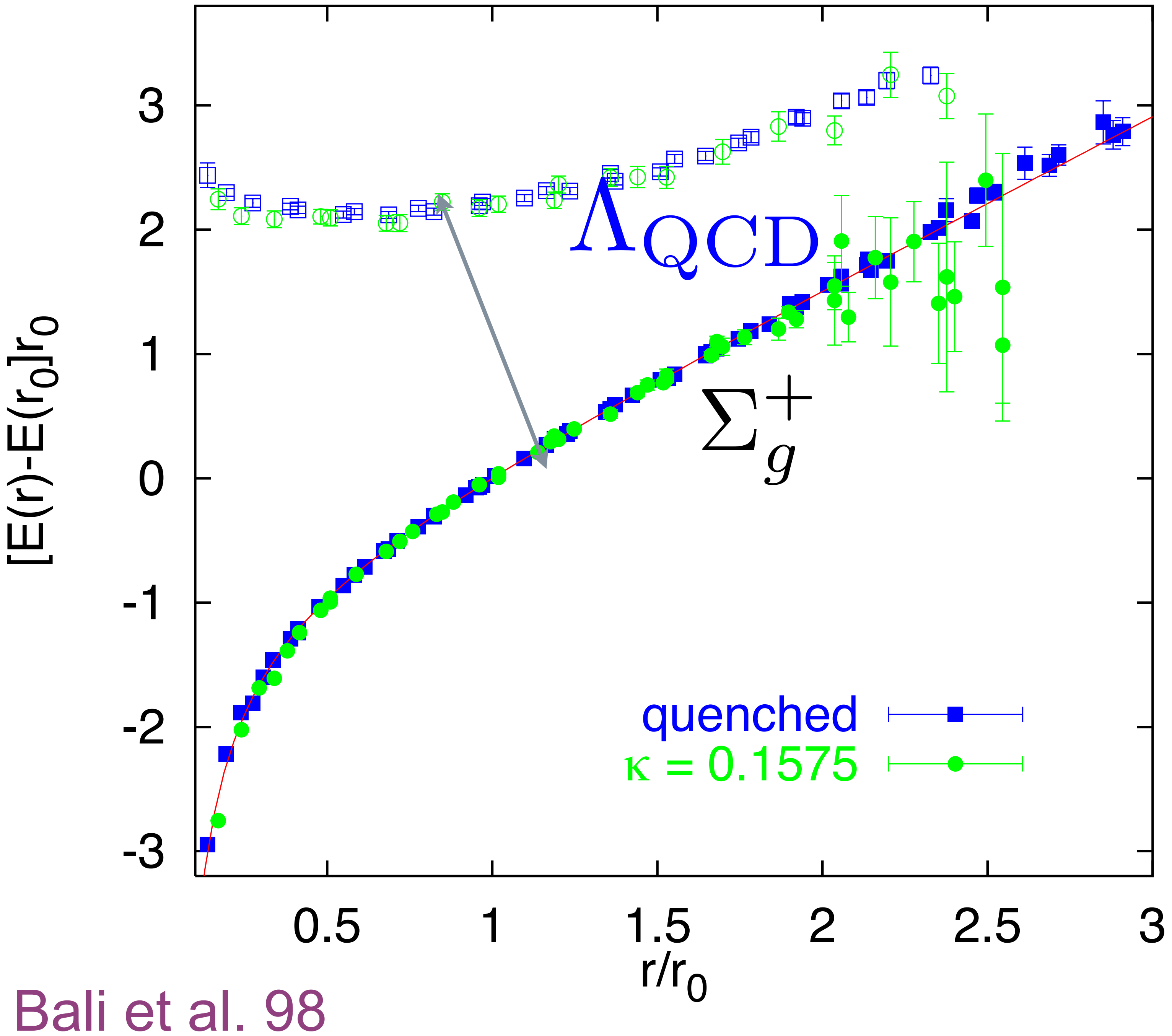
$$E_0(r) - > V_0(r)$$ pNRQCD

$$|\underline{n} > 0;\mathbf{x}_1\mathbf{x}_2\rangle - > |(Q\bar{Q})g^{(n)}\rangle \to$$ Higher Gluonic Excitations

$$|Q\bar{Q}q\bar{q}\rangle$$ Tetraquarks

$$E_n^{(0)}(r) - > V_n^{(0)}(r)$$ BOEFT

$\Sigma_g^+ \quad k^{PC} = 0^{++}$



Bali et al. 98

⇒ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

Brambilla Pineda Soto Vairo 00

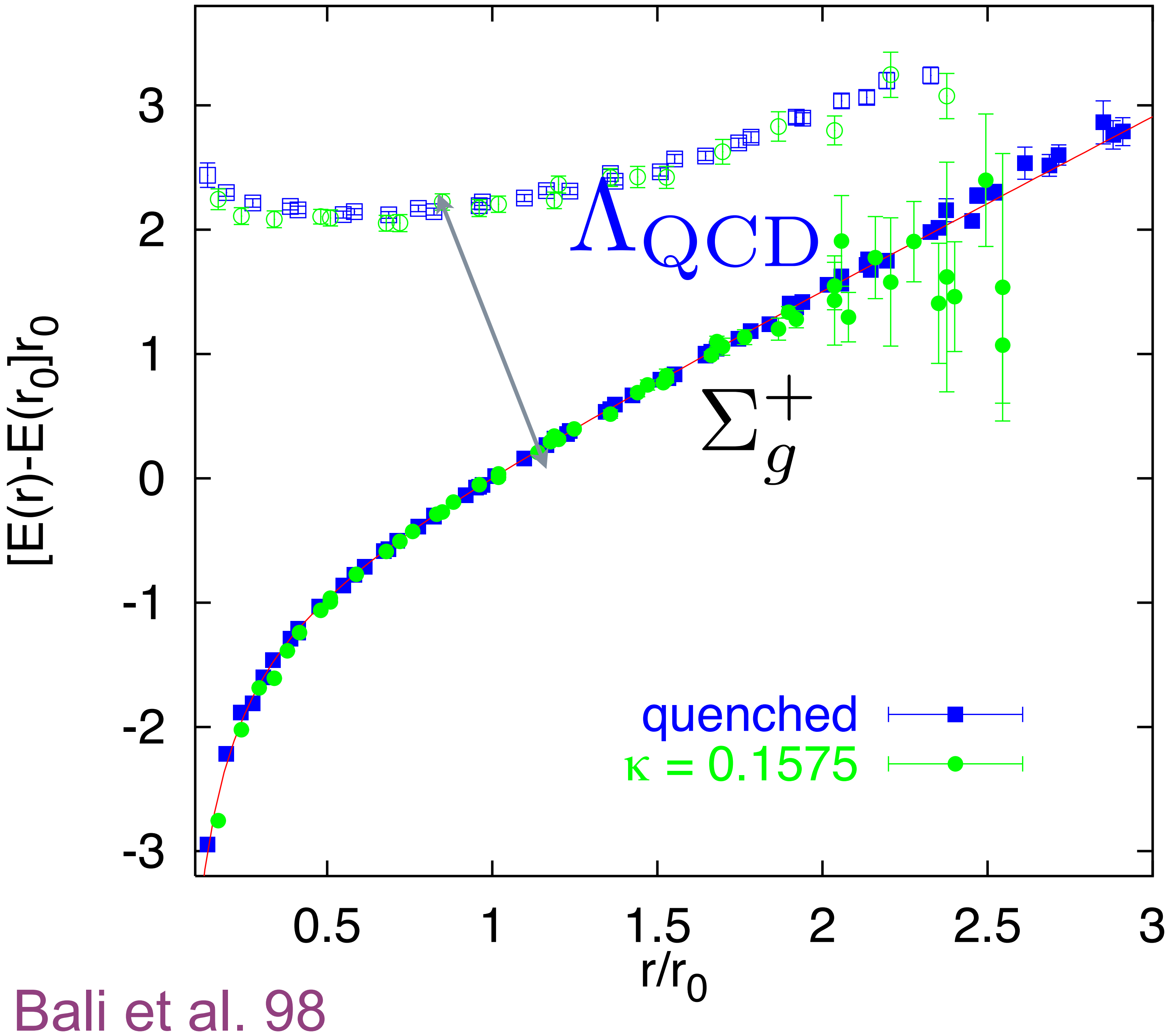
$$\mathcal{L} = \mathrm{Tr}\left\{ S^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) S \right\} + \Delta\mathcal{L}(\text{US light quarks})$$

$$V_S = V_0 + \frac{1}{m} V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

static — spin dependent — velocity dependent

BOEFT for Quarkonium

$$\Sigma_g^+ \quad k^{PC} = 0^{++}$$



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static — spin dependent — velocity dependent

- A pure potential description emerges from the EFT however this is not the constituent quark model, alphas and masses are the QCD fundamental parameters
- The potentials $V = \mathrm{Re}V + ImV$ from QCD in the matching: get spectra and decays
- We obtain the form of the nonperturbative potentials V in terms of generalized Wilson loops (stat that are low energy pure gluonic correlators: all the flavour dependence is pulled out

Applications regard: Spectrum, decays, production, studies of confinement

The singlet potential has the general structure

$$V = V_0 + \frac{1}{m} V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

static — spin dependent — velocity dependent

the fact that spin dependent corrections appear at order 1/m^2 is called Heavy Quark Spin Symmetry

$$E_0(r) = V_s^{(0)} = \lim_{T\to\infty} \frac{i}{T} \ln\langle W(r \times T)\rangle = \lim_{T\to\infty} \frac{i}{T} \ln \langle \square \rangle$$

$$W = \langle \exp\{ig \oint A^\mu dx_\mu\}\rangle$$

The QCD Potential

The singlet potential has the general structure

$$V = V_0 + \frac{1}{m} V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

static spin dependent velocity dependent

the fact that spin dependent corrections appear at order 1/m^2 is called Heavy Quark Spin Symmetry

$$E_0(r) = V_s^{(0)} = \lim_{T\to\infty} \frac{i}{T} \ln\langle W(r\times T)\rangle = \lim_{T\to\infty}\frac{i}{T}\ln\langle \square \rangle$$

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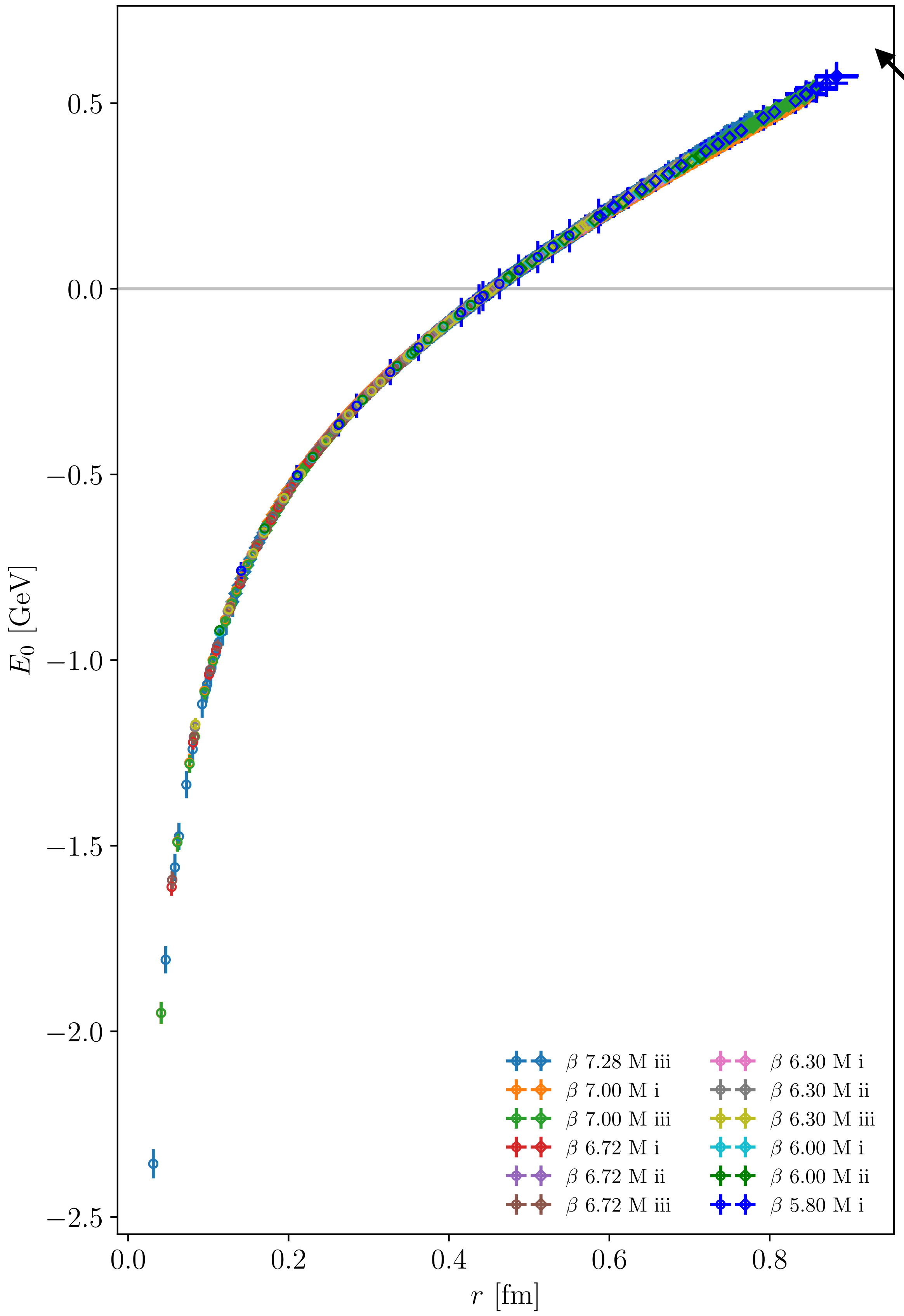

confinement behaviour

State of the art:
2+1+1 quarkonium
static energy $E_0(r)$

$$\Sigma_g^+$$

TUMQCD
N.B., Delgado, Kronfeld, Leino, Petreczky, Steinbeisser, Vairo, Weber *Phys.Rev.D* 107 (2023) 7. 074503 •

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static · spin dependent · velocity dependent

the fact that spin dependent corrections appear at order 1/m^2 is called Heavy Quark Spin Symmetry

$$V^{(1)} = -\frac{1}{2}\int_0^\infty dt\, t \left\langle \Box \right\rangle$$

E(t)

gauge invariant wilson loops can be calculated also in QCD vacuum model and large N

$$V^{(2)}_{\rm SD} = -\frac{r^k}{r^2} c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \langle \Box_{ij} \rangle\ \mathbf{L}_1 \cdot \mathbf{S}_2 + (1 \leftrightarrow 2)\ |V^{(2)}_{LS}$$

E(t) · B

$$-\frac{r^k}{r^2}\left(c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \langle \Box_{ij} \rangle - \frac{2c_F - 1}{2}\nabla^k V^{(0)}\right)\mathbf{L}_1 \cdot \mathbf{S}_1 + (1 \leftrightarrow 2)\ |V^{(1)}_{LS}$$

$$-c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\langle \Box_{ij} \rangle - \frac{\delta_{ij}}{3}\langle \Box \rangle \right)\left(\mathbf{S}_1 \cdot \mathbf{S}_2 - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S}_2 \cdot \hat{\mathbf{r}})\right)\ |V_T$$

$$+\left(\frac{2}{3}c_F^2 i \int_0^\infty dt \langle \Box \rangle - 4\left(d_{sv} + \frac{4}{3}d_{vv}\right)\delta^{(3)}(\mathbf{r})\right)\mathbf{S}_1 \cdot \mathbf{S}_2\ |V_S$$

c

Pineda Vairo PRD 63 (2001) 054007
Brambilla Pineda Soto Vairo PRD 63 (2001) 014023

The singlet potential has the general structure

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static — spin dependent — velocity dependent

the fact that spin dependent corrections appear at order 1/m^2 is called Heavy Quark Spin Symmetry

$$V^{(1)} = -\frac{1}{2}\int_0^\infty dt\, t \, \langle \Box_{E(t)\,E} \rangle$$

$$V_{\rm SD}^{(2)} = -\frac{r^k}{r^2} c_F \, \epsilon^{kij}\, i \int_0^\infty dt\, t \langle \Box_{E_i(t)\,B_j} \rangle \; \mathbf{L}_1 \cdot \mathbf{S}_2 + (1 \leftrightarrow 2) \; |V_{LS}^{(2)}$$

$$-\frac{r^k}{r^2}\left(c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \, \langle \Box_{i\,j} \rangle - \frac{2c_F - 1}{2} \nabla^k V^{(0)} \right) \mathbf{L}_1 \cdot \mathbf{S}_1 + (1 \leftrightarrow 2) \; |V_{LS}^{(1)}$$

$$-c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\langle \Box_{i\,j} \rangle - \frac{\delta_{ij}}{3} \langle \Box \rangle \right) \left(\mathbf{S}_1 \cdot \mathbf{S_2} - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S_2} \cdot \hat{\mathbf{r}}) \right) \; |V_T$$

$$+\left(\frac{2}{3} c_F^2 i \int_0^\infty dt \langle \Box \rangle - 4\left(d_{sv} + \frac{4}{3} d_{vv} \right) \delta^{(3)}(\mathbf{r}) \right) \mathbf{S}_1 \cdot \mathbf{S_2} \; |V_S$$

gauge invariant wilson loops can be calculated also in QCD vacuum model and large N

Pineda Vairo PRD 63 (2001) 054007
Brambilla Pineda Soto Vairo PRD 63 (2001) 014023

- the potentials contain the contribution of the scale m inherited from NRQCD matching coefficients —> they cancel any QM divergences, good UV behaviour
- the flavour dependent part is extracted in the NRQCD matching coefficients

the nonperturbative part is factorized and depends only on the glue —> only one lattice calculation to get the dynamics and the observables instead of an ab initio calculation of multiple Green functions

Tetraquarks, Hybrids, Pentaquarks

Berwein, N.B., Mohapatra, Vairo 2408.04719

1. We consider all the NRQCD static energies in presence of glue and light quarks:
QQbarg, QQbar qqbar, QQ qbar qbar, QQbar qqq, QQ qqqbar, QQq

2. We define the NRQCD static energies via gauge-invariant correlator of appropriate interpolating operators

3. We calculate the short distance behaviour: gluelumps, adjoint meson, triplet mesons, sextet mesons ….

4.BO quantum number is conserved: BO static energies evolve in heavy-light static energies with the same quantum numbers: allows to understand the form of the fundamental strong force

5.We consider separately NRQCD static energies separated by a gap Lambda_QCD

6.We match the NRQCD static energies to the corresponding potentials in BOEFT

Mixing appears:

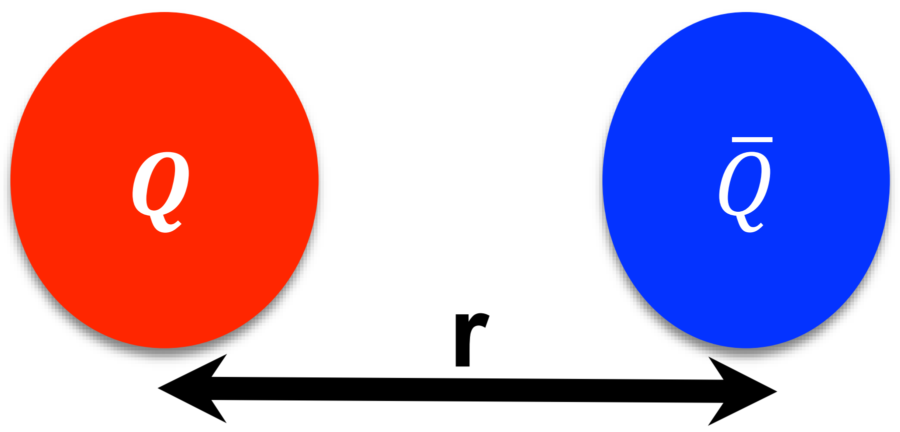
at short distance between static energies with same k—> **coupled Schr. Eqs.**

at large distance between static energies with same BO numbers that get close (avoided level crossing)—> **coupled Schr. Eqs.**

7. Solve the coupled Schroedinger eqs. To obtains X and the Tcc, hybrids, pentaquarks….

8. Consider spin relativistic corrections at order 1/m

$Q\bar{Q}X$



color: $3 \otimes \bar{3} = 1 \oplus 8$

X = gluon → **Hybrid**

$X = (q\bar{q})_8$ → **Tetraquark**

$X = (qqq)_8$ → **Pentaquark**

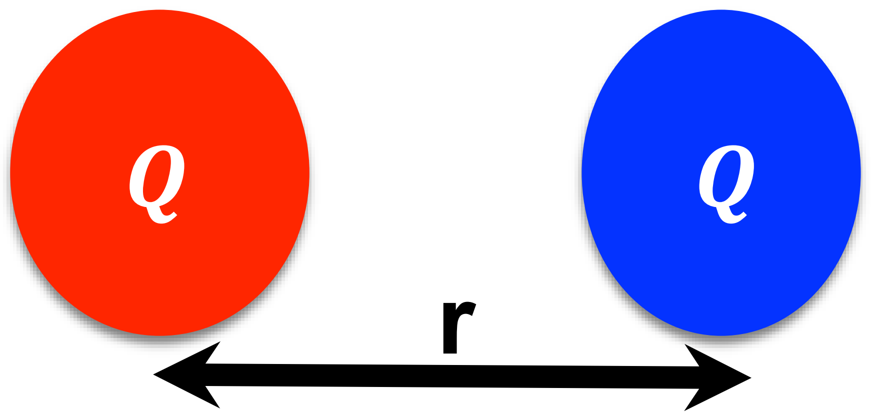
and so on

Total angular momentum of **$Q\bar{Q}$X** or **QQX** :

$$\mathbf{J} = \mathbf{L}_{Q\bar{Q}} + \mathbf{K} + \mathbf{S}_{Q\bar{Q}}$$

L

QQX



color: $3 \otimes 3 = \bar{3} + 6$

$X = q_3$ → **Double heavy baryon**

$X = (\bar{q}\bar{q})_3 \,/\, (\bar{q}\bar{q})_{\bar{6}}$ → **Tetraquark**

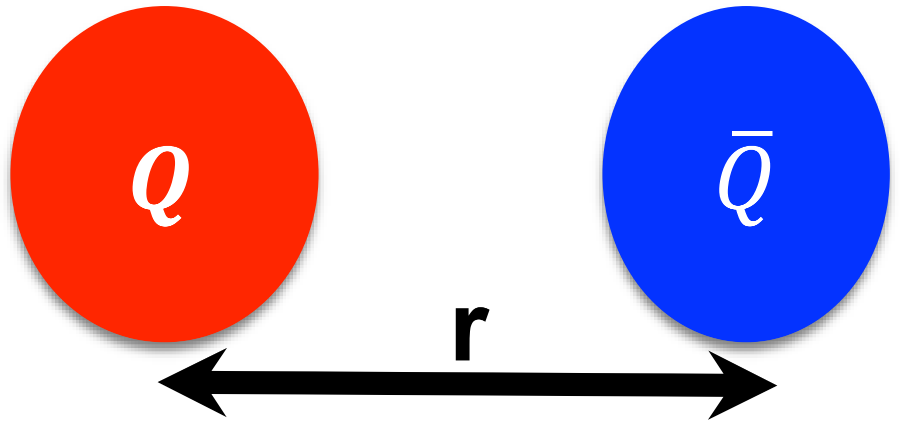
$X = (qq\bar{q})_3 \,/\, (qq\bar{q})_6$ → **Pentaquark**

and so on

X_8 : **Adjoint hadrons** (gluelump, adjoint meson, adjoint baryon….)

$X_{3/6}$: **Triplet or sextet hadrons** (meson, baryon….)

$Q\bar{Q}X$



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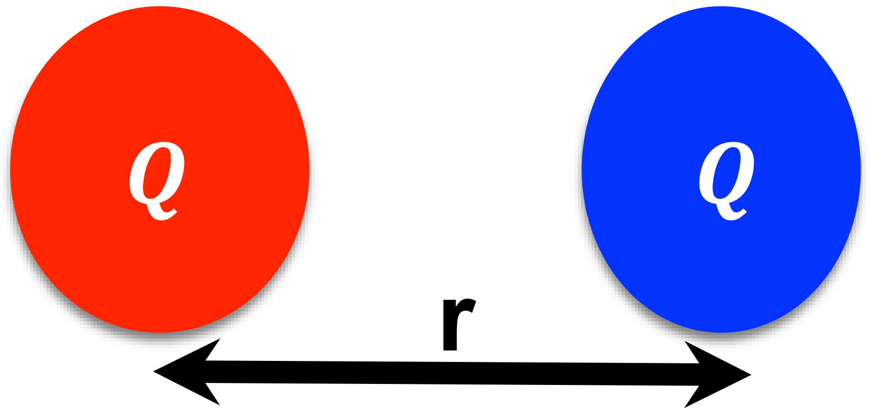
$$\mathbf{J} = \mathbf{L}_{\mathbf{Q\bar{Q}}} + \mathbf{K} + \mathbf{S}_{\mathbf{Q\bar{Q}}}$$

L

QQX



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and so on

X_8 : **Adjoint hadrons** (gluelump, adjoint meson, adjoint baryon….)

$X_{3/6}$: **Triplet or sextet hadrons** (meson, baryon….)

Coupled Schoedinger equations with Born-Oppenheimer (BO) potential:

$$V_{\kappa\lambda\lambda'}(r) = E^{(0)}_{\kappa,|\lambda|}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q} + \dots ,$$

Static Energy

Spin-dependent potentials

Notice: for exotics spin correction at order 1/mQ !!

Direct Outcomes Born-Oppenheimer EFT

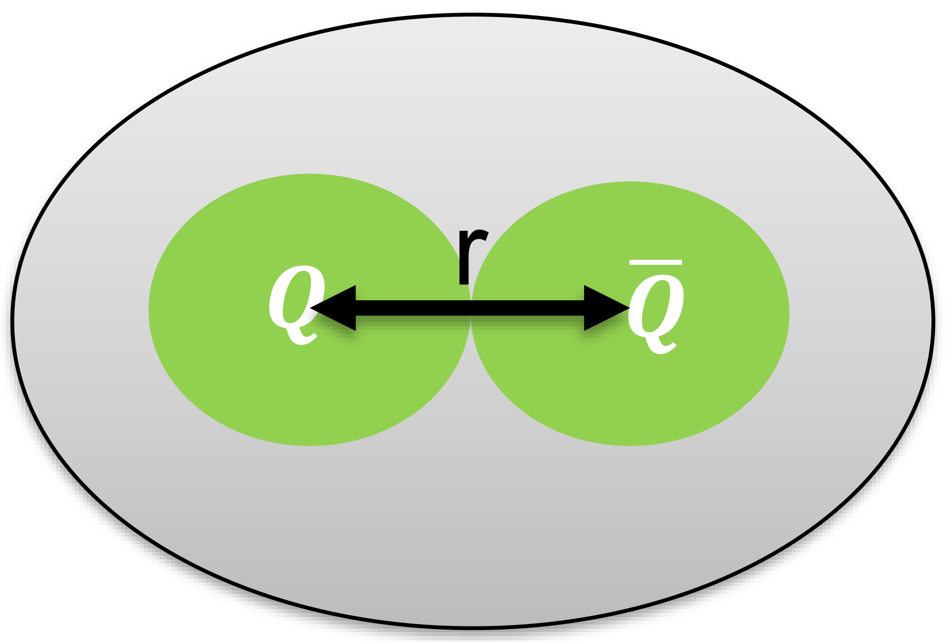
- Control of the short distance region

$$V_s(r) = -\frac{4\alpha_s}{3r}, \; V_o(r) = \frac{\alpha_s}{6r}$$

$$V_T(r) = -\frac{2\alpha_s}{3r}, \; V_\Sigma(r) = \frac{\alpha_s}{3r}$$

- Control of the short distance region

LDF-quantum #: $\kappa = \{K^{PC}, f\}$



Short-distance $(r \to 0)$

$Q\bar{Q}$: $E^{(0)}_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+} r^2 +$

$Q\bar{Q}X$: $E^{(0)}_{\Lambda^\sigma_\eta}(r) = V_o(r) + \Lambda_{H_\kappa} + b_{\Lambda^\sigma_\eta} r^2 + \cdots$

QQX: $E^{(0)}_{\Lambda^\sigma_\eta}(r) = V_l(r) + \Lambda_{H_\kappa, l} + b_{\kappa\lambda, l}\, r^2 + \cdots \; (l = T, \Sigma)$

$$\Lambda_{H_\kappa} = \lim_{T\to\infty} \frac{i}{T} \langle \mathrm{vac}| H^a_\kappa(T/2, \boldsymbol{R})\, \phi^{ab}\,(T/2, -T/2)\, H^{a\dagger}_\kappa(-T/2, \boldsymbol{R})|\mathrm{vac}\rangle$$

Field theory definition of gluelumps, adjoint mesons, triplet mesons.. masses

Triplet and sextet mesons operators give a gauge invariant field theory definition of (good and bad) diquarks

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Direct Outcomes Born-Oppenheimer EFT

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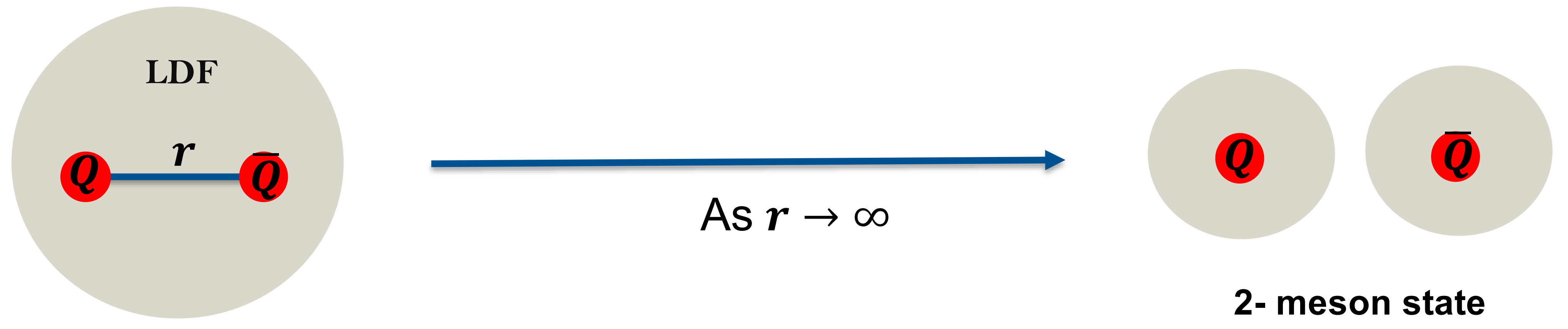
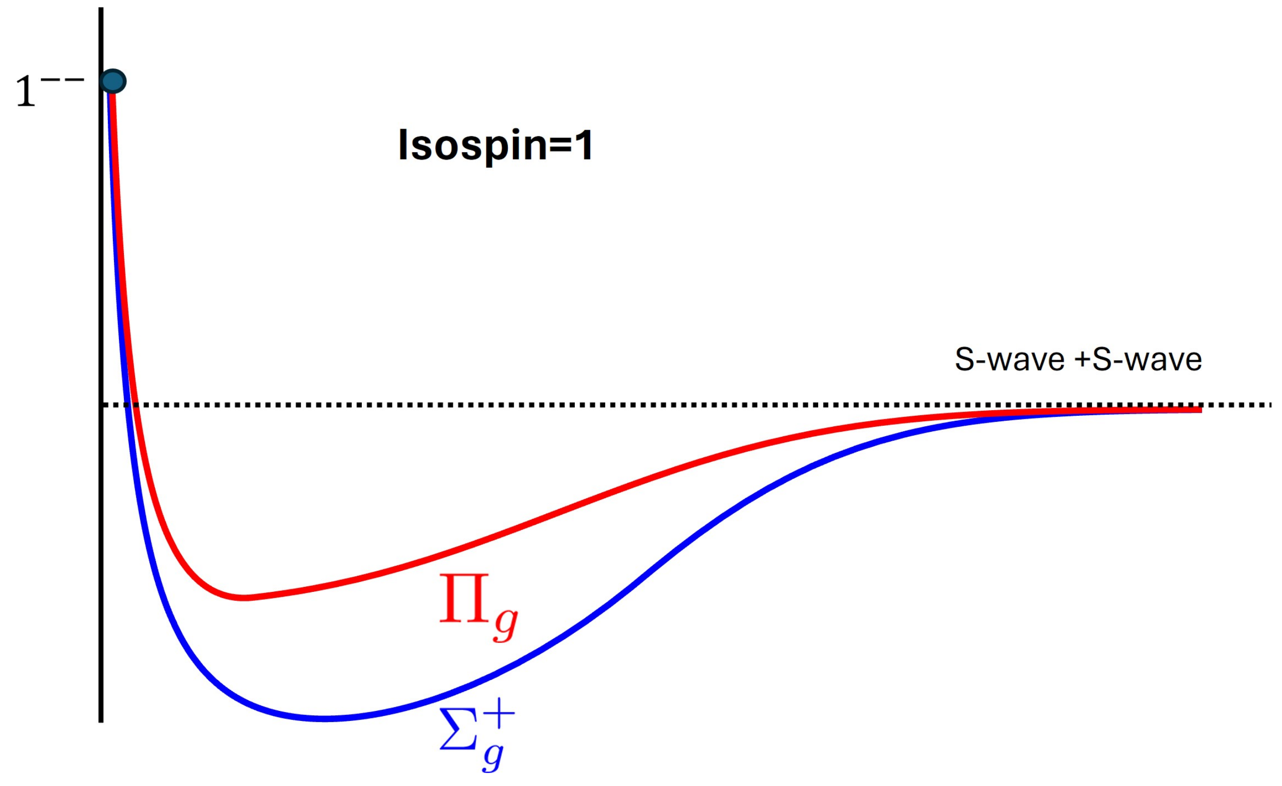
BO quantum number conserved

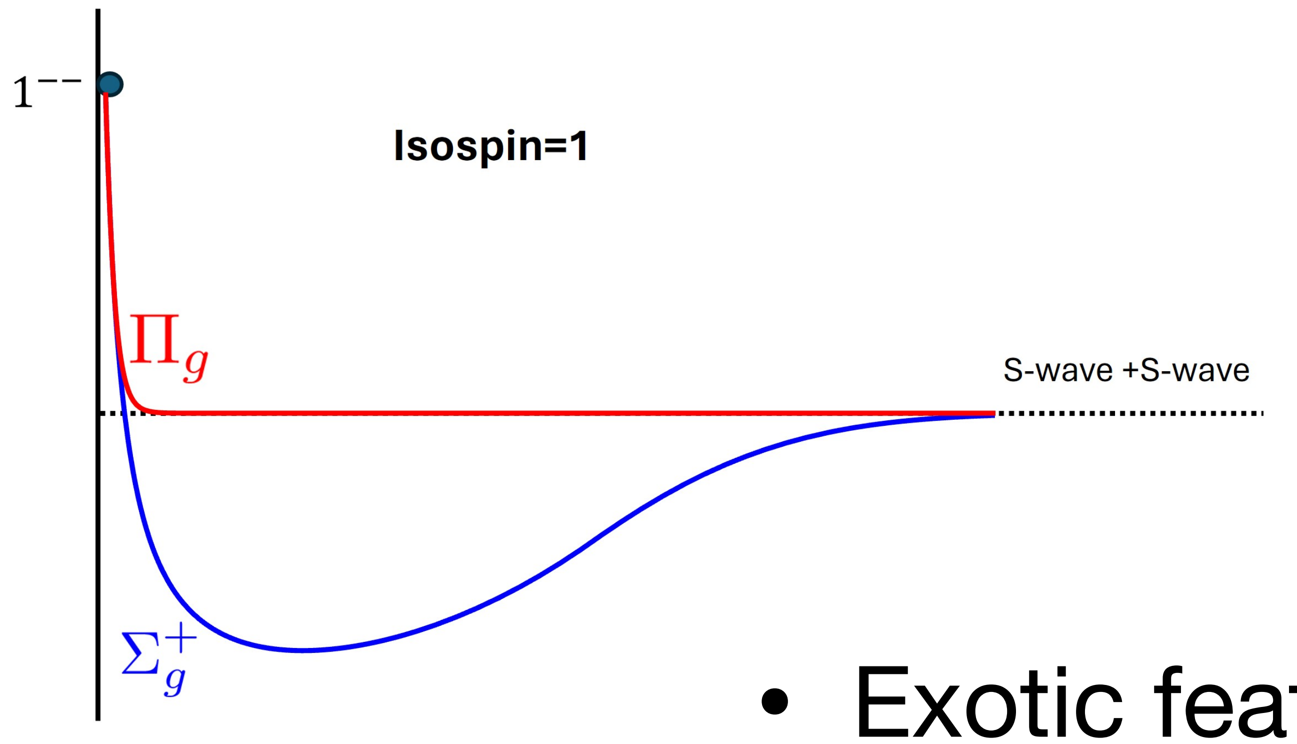


Consider $Q\bar{Q}q\bar{q}$ system:

BO-quantum # Λ_η^σ for adjoint meson:

BO-quantum # Λ_η^σ for meson-antimeson





The potentials should go to the heavy light static threshold—>consequences:

- Only a finite number of states possible
- Exotic features: e.g. the very large radius of the X(3872) comes from a 'fine tuned' value of the adjoint meson mass

Direct Outcomes Born-Oppenheimer EFT

- Control of the short distance region
- Control of the large distance region
- Same framework for quarkonium and exotics

Direct Outcomes Born-Oppenheimer EFT

- Control of the short distance region
- Control of the large distance region
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We fix the adjoint meson mass value on the X(3872) and with the same equations we can obtain all states in the I=0 QQbar sector below and above threshold

We calculate the open flavor threshold effects on quarkonium spectrum below threshold and the percentage of quarkonium and tetraquark in each state—> this has phenomenological consequneces

We can e.g, establish that 2p state at 4010 MeV above threshold is 90% quarkonium while the X(3872) is 90% tetraquark

Direct Outcomes Born-Oppenheimer EFT

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Direct Outcomes Born-Oppenheimer EFT

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We fix the adjoint meson mass on the X(3872) and we can predict everything in the bottom sector->
the Xb is below threshold and does not have exotic feature even if it 98% tetraquark!! It is not at
a heavy-light threshold!

Direct Outcomes Born-Oppenheimer EFT

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- We predict the avoided level crossing phenomenon but we need lattice input to know the strength of it

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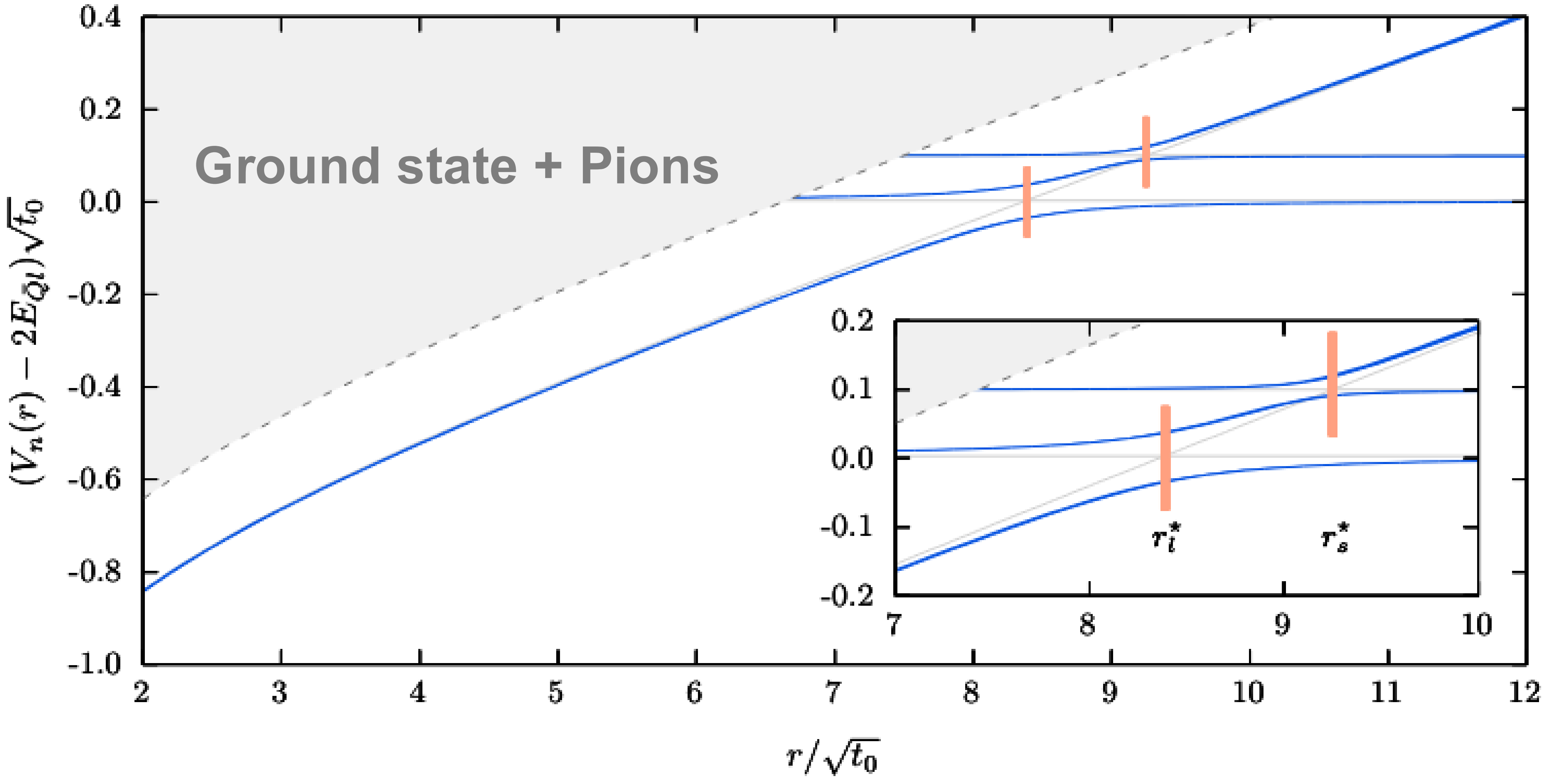


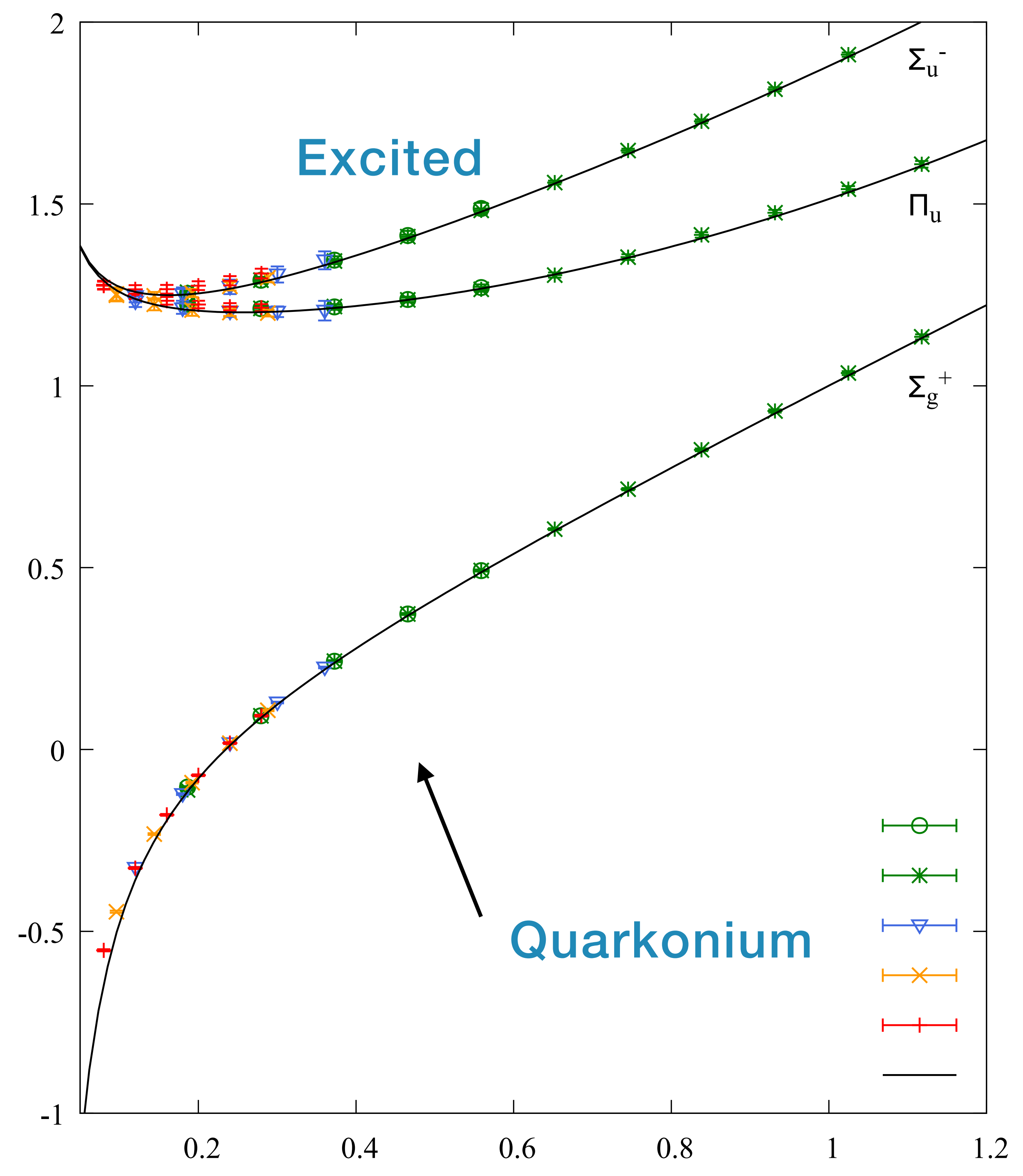
Bulava, Hoerz, Knechtli, Koch, Moir, Morningstar, Peardon, Phys. Lett. B. 793 (2019)

Bulava, Knechtli, Koch, Morningstar, Peardon, Phys. Lett. B. 854 (2024)

avoided level crossing

In adiabatic representation

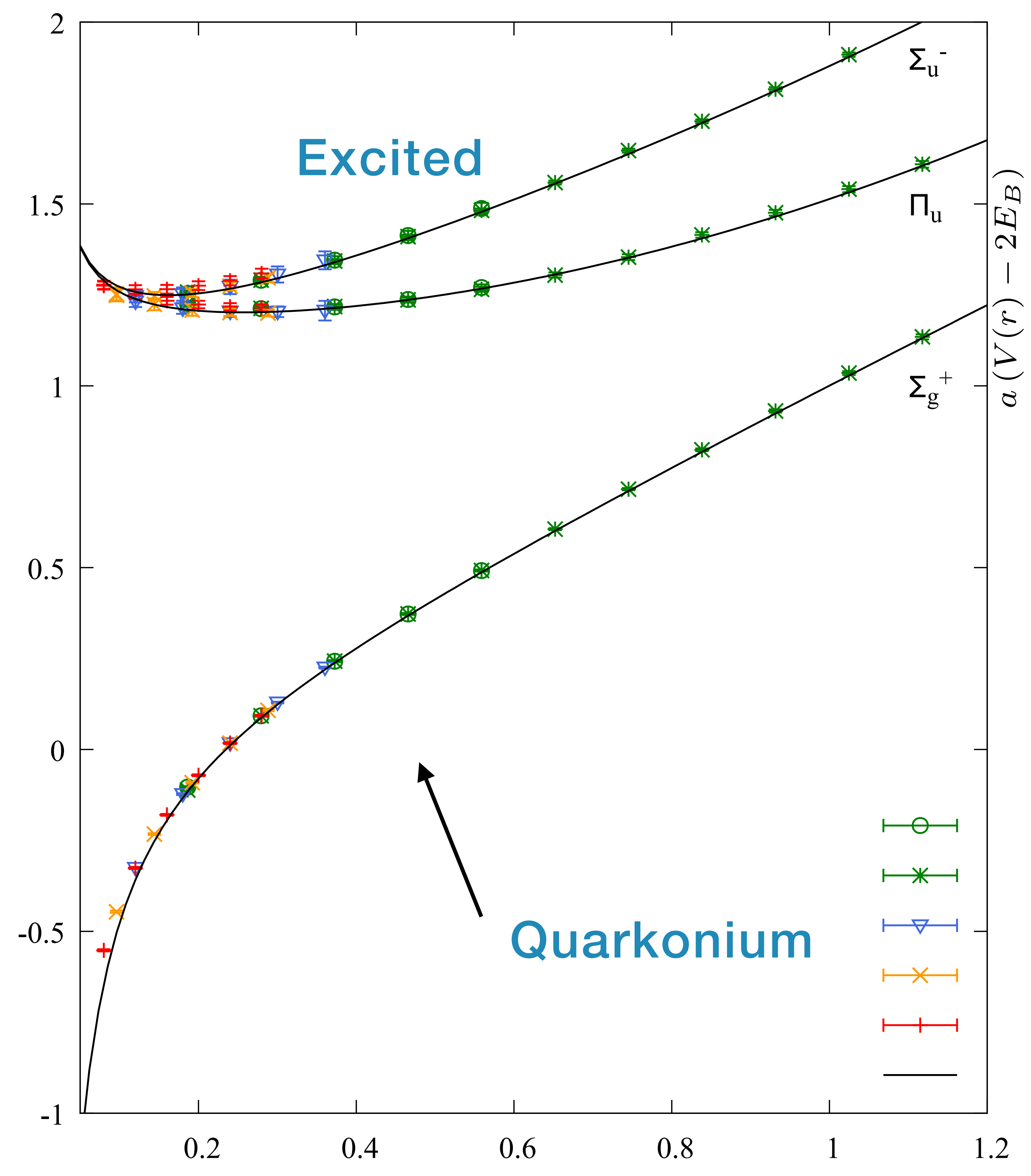
confinement force

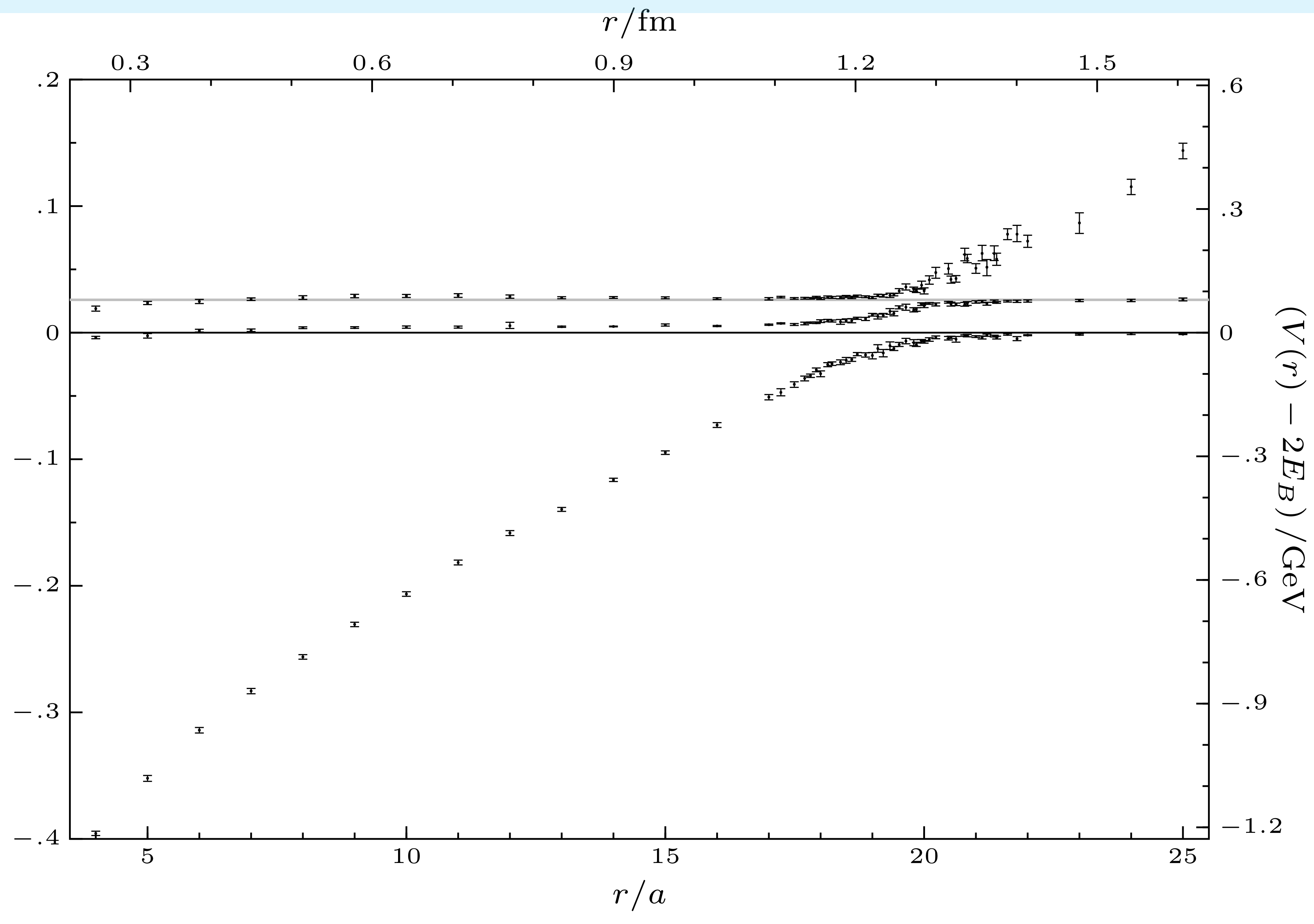


$r[\mathrm{fm}]$

Consequences on our Knowledge of the Strong Force

confinement force

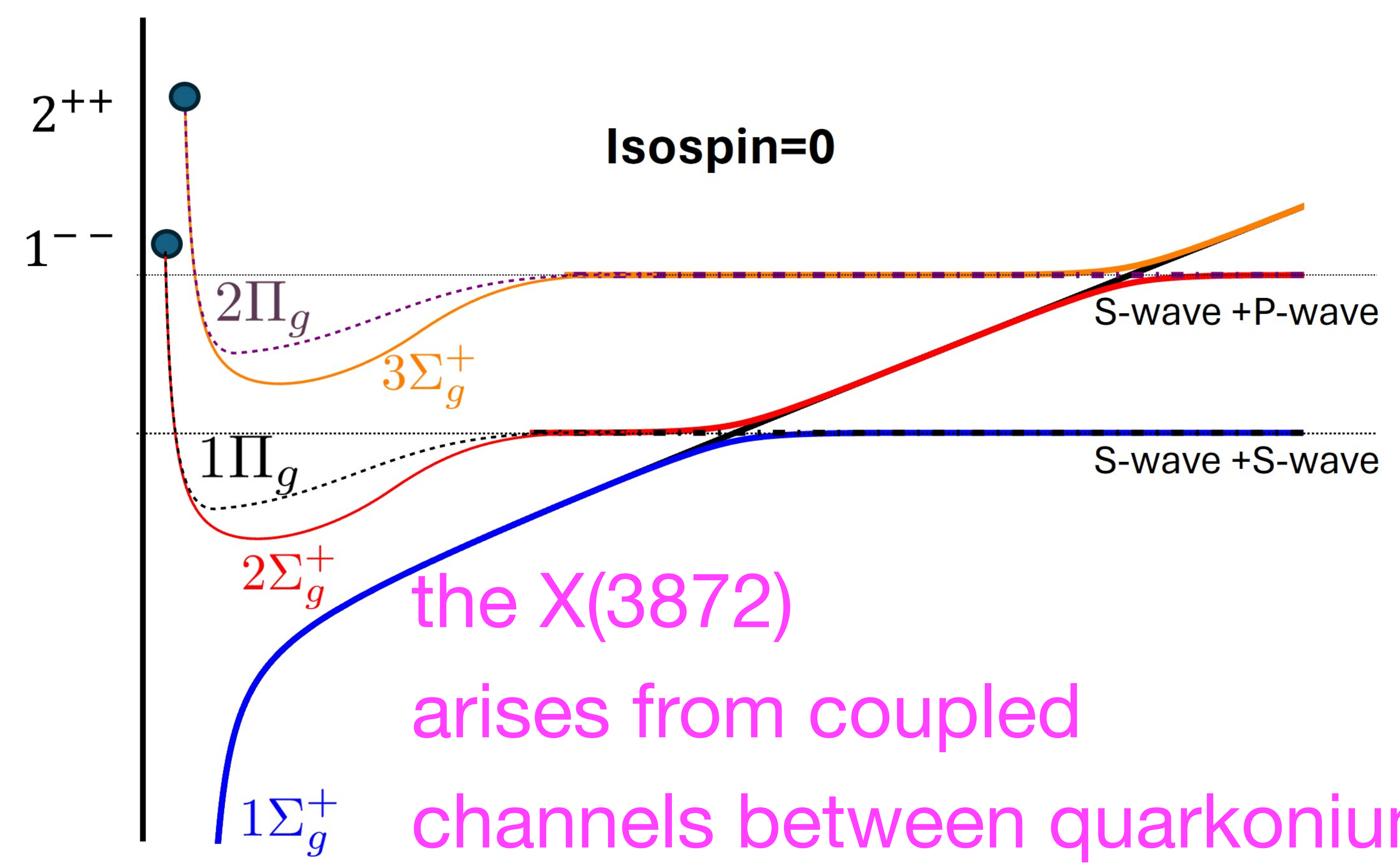




tetraquarks and heavy-light overlap at large distance

avoided level crossing between quarkonium and tetraquarks

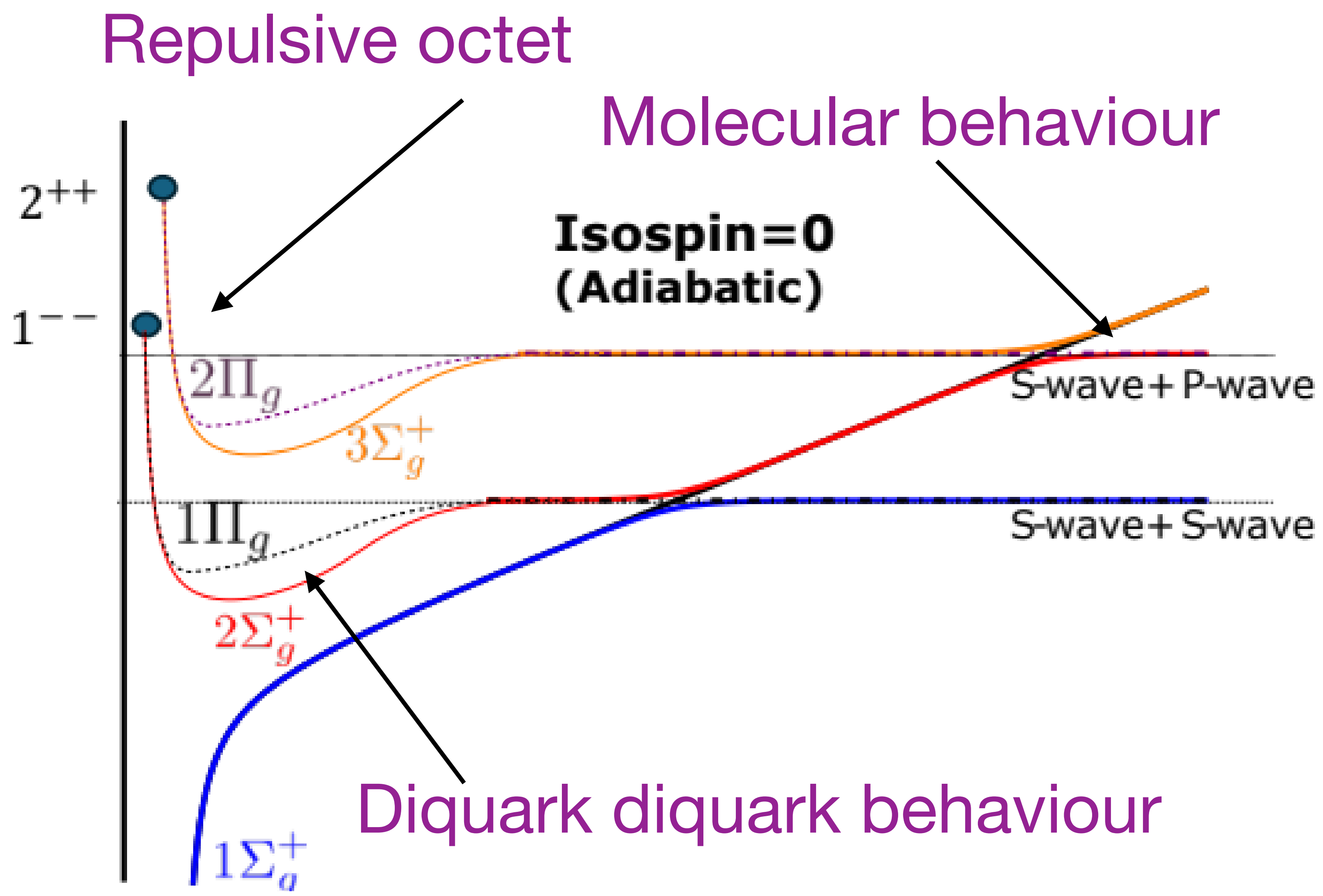
Quarkonium, Tetraquarks and heavy-light pairs



the X(3872) arises from coupled channels between quarkonium and the two first tetraquarks

Other Consequences

BOEFT Could subdue molecular and compact tetraquark pictures !

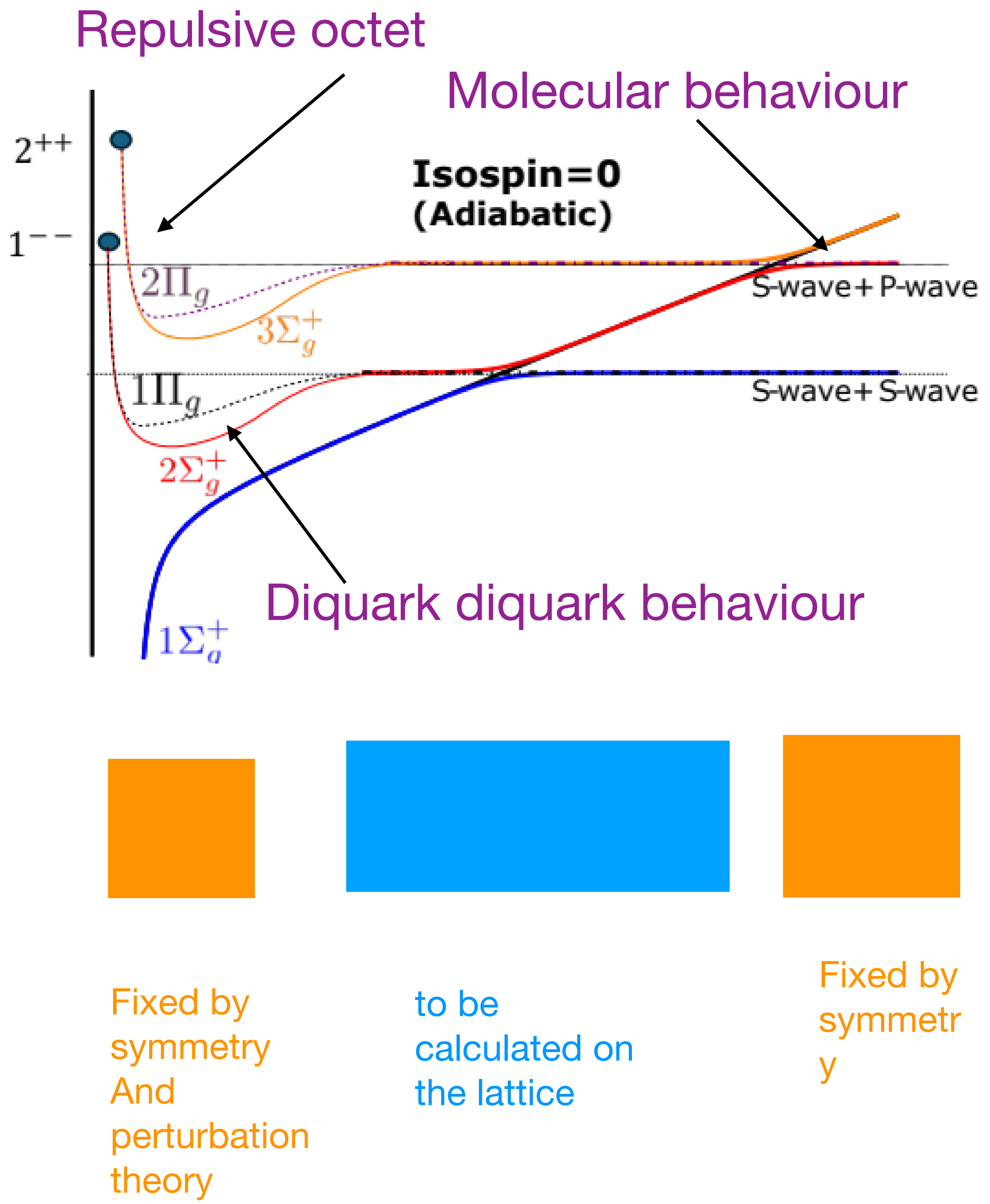


Fixed by symmetry And perturbation theory

to be calculated on the lattice

Fixed by symmetr y

BOEFT Could subdue molecular and compact tetraquark pictures !



On the nature of the states

X(3872)

emerges as composed dominantly of two tetraquarks contributions and a residual quarkonium part, while the $T_{cc}^+(3875)$ emerges as a single tetraquark. The value of the adjoint mass and the triplet mass together with the structure of the potential, which are constrained by the BOEFT, originate states with a very large radius, small binding energy and other properties compatible with experiments. Still, the states are neither simple molecules nor compact tetraquarks but result from a conspiracy between the short- and long-range behavior of potentials that are constrained by symmetry and computed in lattice qcd

Results on Spectroscopy (as From Abhishek Talk)

X(3872) : close to threshold, radius of 15 fm, 10% of quarkonium, 90% tetraquarks, spin multiplet, Radiative decays, compositeness ✔

Xb, mainly tetraquark, non exotic characteristics ✔

Tcc close to threshold, large radius, prediction of Tbb and Tbc in agreement with lattice ✔

Pentaquarks multiplets ✔

Zb and Zb' ✔

chi(2P) over threshold, quarkonium states below and above threshold ✔

What About Hadroproduction?

Important as there was a querelle for decades between the compact tetra and the molecular picture…

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N. Brambilla, H. S. Chung, A. Vairo and X. P. Wang
Inclusive production of J/ψ, $\psi(2S)$ and Υ states in pNRQCD
JHEP 03 (2023) 242 arXiv:2210.17345

N. Brambilla, H. S. Chung, A. Vairo and X. P. Wang
Production and polarization of S-wave quarkonia in potential Nonrelativistic QCD
Phys. Rev. D 105 (2022) 11, L111503 arXiv:2203.07778

N. Brambilla, H. S. Chung and A. Vairo
Inclusive production of heavy quarkonia in pNRQCD
JHEP 09 (2021) 032 arXiv:2106.09417

N. Brambilla, H. S. Chung and A. Vairo
Inclusive hadroproduction of P-wave heavy quarkonia in potential Nonrelativistic QCD
Phys. Rev. Lett. 126 (2021) 8 arXiv:2007.07613

Inclusive hadroproduction of X(3872),X_b and pentaquarks
N.B. Buetenschon, Hibler, Mohapatra, Vairo, Wang 2026

Chung, Lai 2505.06910

Inclusive Hadroproduction Cross Section in NRQCD

In NRQCD, the production cross sections for a quarkonium $\mathcal{Q}$ factorize

- in short distance coefficients, $\sigma_{Q\bar{Q}(N)}$, encoding contributions from energy scales of order m or larger,
- and in long distance matrix elements (LDMEs), $\langle\Omega|\mathcal{O}^{\mathcal{Q}}(N)|\Omega\rangle$, encoding contributions of order mv, mv^2 and $\Lambda_{\rm QCD}$,

perturbative

nonperturbative

$$N = {}^{2S+1}L_J^{1,8}$$

$$\sigma_{\mathcal{Q}+X} = \sum_N \sigma_{Q\bar{Q}(N)} \langle\Omega|\mathcal{O}^{\mathcal{Q}}(N)|\Omega\rangle.$$

SDC

LDMEs

Vacuum state

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perturbative

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- and in long distance matrix elements (LDMEs), $\langle\Omega|\mathcal{O}^{\mathcal{Q}}(N)|\Omega\rangle$, encoding contributions of order mv, mv^2 and Λ_{QCD},

$$\sigma_{\mathcal{Q}+X} = \sum_N \sigma_{Q\bar{Q}(N)} \langle\Omega|\mathcal{O}^{\mathcal{Q}}(N)|\Omega\rangle.$$

$N = {}^{2S+1}L_J^{1,8}$

SDC — LDMEs — Vacuum state

LDMEs depend on heavy quark flavour

different for charmonium and bottomonium, to be extracted from the data, cannot be calculated on the lattice

Color singlet and octet operators for hadroproduction of quarkonia have the form

$$\mathcal{O}^{\mathcal{Q}}(N_{\text{color singlet}}) = \chi^\dagger \mathcal{K}_N \psi \mathcal{P}_{\mathcal{Q}(\boldsymbol{P}=\boldsymbol{0})} \psi^\dagger \mathcal{K}'_N \chi$$

$$\mathcal{O}^{\mathcal{Q}}(N_{\text{color octet}}) = \chi^\dagger \mathcal{K}_N T^a \psi \Phi_\ell^{\dagger ab}(0) \mathcal{P}_{\mathcal{Q}(\boldsymbol{P}=\boldsymbol{0})} \Phi_\ell^{bc}(0) \psi^\dagger \mathcal{K}'_N T^c \chi$$

$\Phi_\ell(x)$ is a Wilson line along the direction ℓ in the adjoint representation

$\mathcal{P}_{\mathcal{Q}(\boldsymbol{P})}$ projects onto a state containing a heavy quarkonium $\mathcal{Q}$ with momentum $\boldsymbol{P}$.

Inclusive P Wave Hadroproduction Cross Section in NRQCD

$pp \to \chi_{QJ}(nP) + X$

$$\sigma_{\chi_{QJ}+X} = \sigma_{Q\bar{Q}({}^3P_J^{[1]})} \langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3P_J^{[1]})|\Omega\rangle + \sigma_{Q\bar{Q}({}^3S_1^{[8]})} \langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3S_1^{[8]})|\Omega\rangle$$

NRQCD

$$\langle\mathcal{O}^{\chi_{Q0}}({}^3P_0^{[1]})\rangle = \frac{1}{3} \langle\Omega|\, \chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma})\psi\, \mathcal{P}_{\chi_{Q0}}\, \psi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma})\chi\, |\Omega\rangle\,,$$

two Fock-states at LO in v: $n = \{{}^3P_J^{[1]}, {}^3S_1^{[8]}\}$

$$\langle\mathcal{O}^{\chi_{Q0}}({}^3S_1^{[8]})\rangle = \langle\Omega|\, \chi^\dagger\sigma^k T^A\psi\Phi_\ell^{\dagger AB}\, \mathcal{P}_{\chi_{Q0}}\, \Phi_\ell^{BC}\psi^\dagger\sigma^k T^C\chi\, |\Omega\rangle$$

$$\sigma_{\chi_{QJ}+X} = \sigma_{Q\bar{Q}(^3P_J^{[1]})} \langle \Omega | \mathcal{O}^{\chi_{QJ}}(^3P_J^{[1]}) | \Omega \rangle + \sigma_{Q\bar{Q}(^3S_1^{[8]})} \langle \Omega | \mathcal{O}^{\chi_{QJ}}(^3S_1^{[8]}) | \Omega \rangle$$

NRQCD

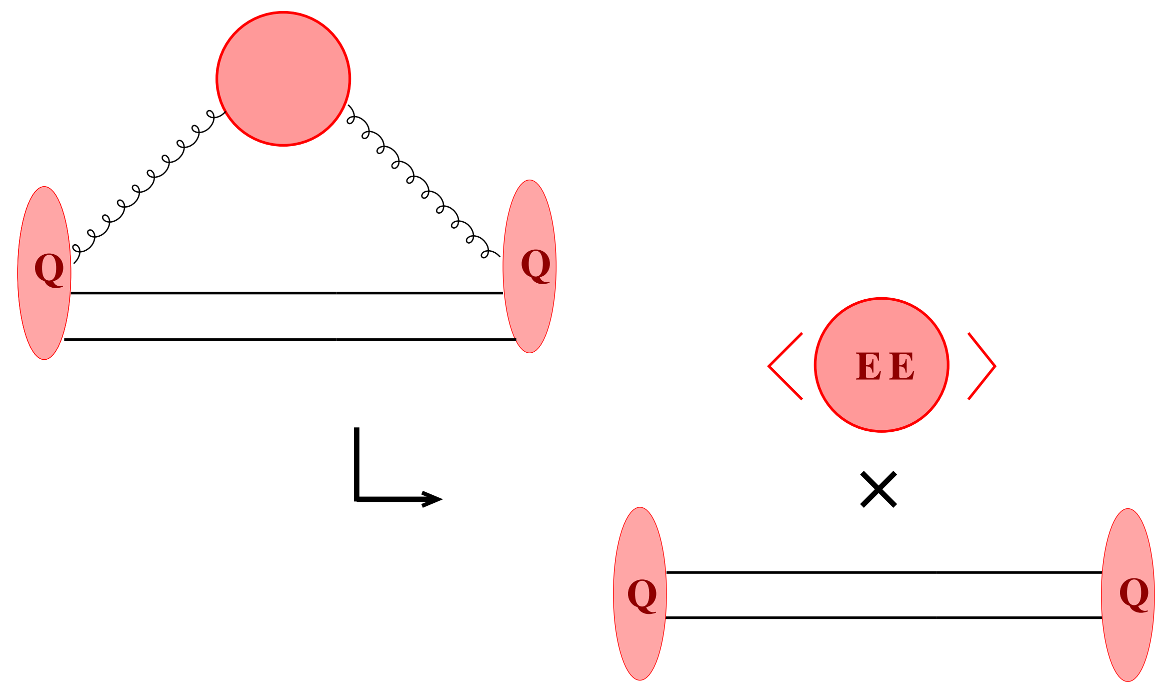
$$\langle \mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]}) \rangle = \frac{1}{3} \langle \Omega | \chi^\dagger (-\tfrac{i}{2} \overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma}) \psi \, \mathcal{P}_{\chi_{Q0}} \, \psi^\dagger (-\tfrac{i}{2} \overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma}) \chi \, | \Omega \rangle \, ,$$

$$\langle \mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]}) \rangle = \langle \Omega | \chi^\dagger \sigma^k T^A \psi \Phi_\ell^{\dagger AB} \, \mathcal{P}_{\chi_{Q0}} \, \Phi_\ell^{BC} \psi^\dagger \sigma^k T^C \chi \, | \Omega \rangle$$

two Fock-states at LO in v: $n = \{^3P_J^{[1]}, {}^3S_1^{[8]}\}$

Factorization in pNRQCD

(BOEFT for quarkonium)



$$\langle \mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]}) \rangle = \frac{3N_c}{2\pi} |\phi'_{\chi_{Q0}}(0)|^2,$$

$$\langle \mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]}) \rangle = \frac{3N_c}{2\pi} |\phi'_{\chi_{Q0}}(0)|^2 \frac{\mathcal{E}}{9N_c m_Q^2}$$

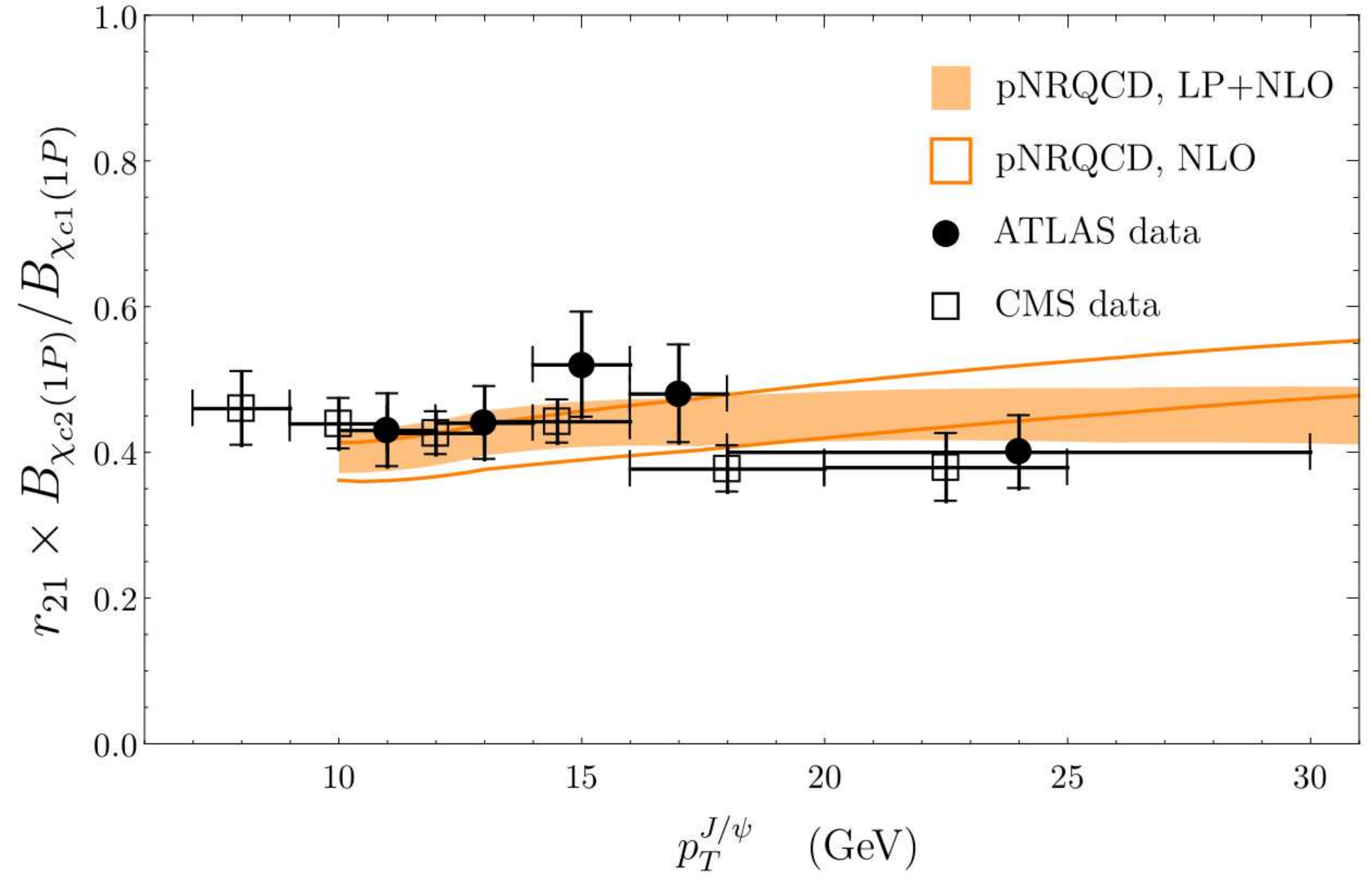
$$\mathcal{E} = \frac{3}{N_c} \int_0^\infty dt \, t \int_0^\infty dt' \, t' \, \langle \Omega | \Phi_\ell^{\dagger ab} \Phi^{\dagger ad}(0;t) g E^{d,i}(t) g E^{e,i}(t') \Phi^{ec}(0;t') \Phi_\ell^{bc} | \Omega \rangle$$

- reduced the number of unknown parameters
- all heavy quark dependence is in the wavefunction $\to$ correlators are universal and heavy-quark flavor independent

- The nonperturbative correlator can be calculated on the lattice

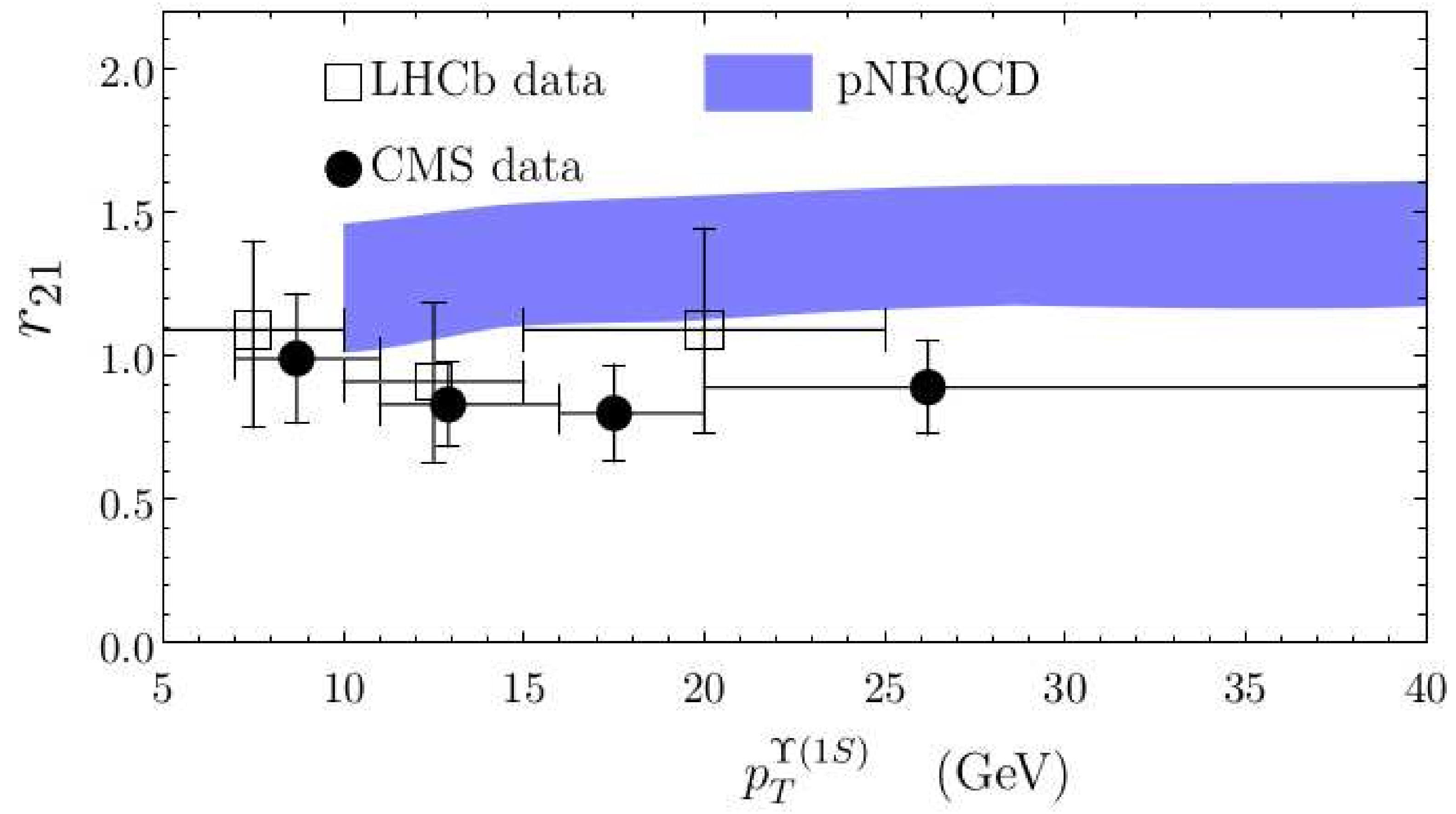
pNRQCD predictions in comparison to data

$(d\sigma_{\chi_{c2}(1P)}/dp_T)/(d\sigma_{\chi_{c1}(1P)}/dp_T)$

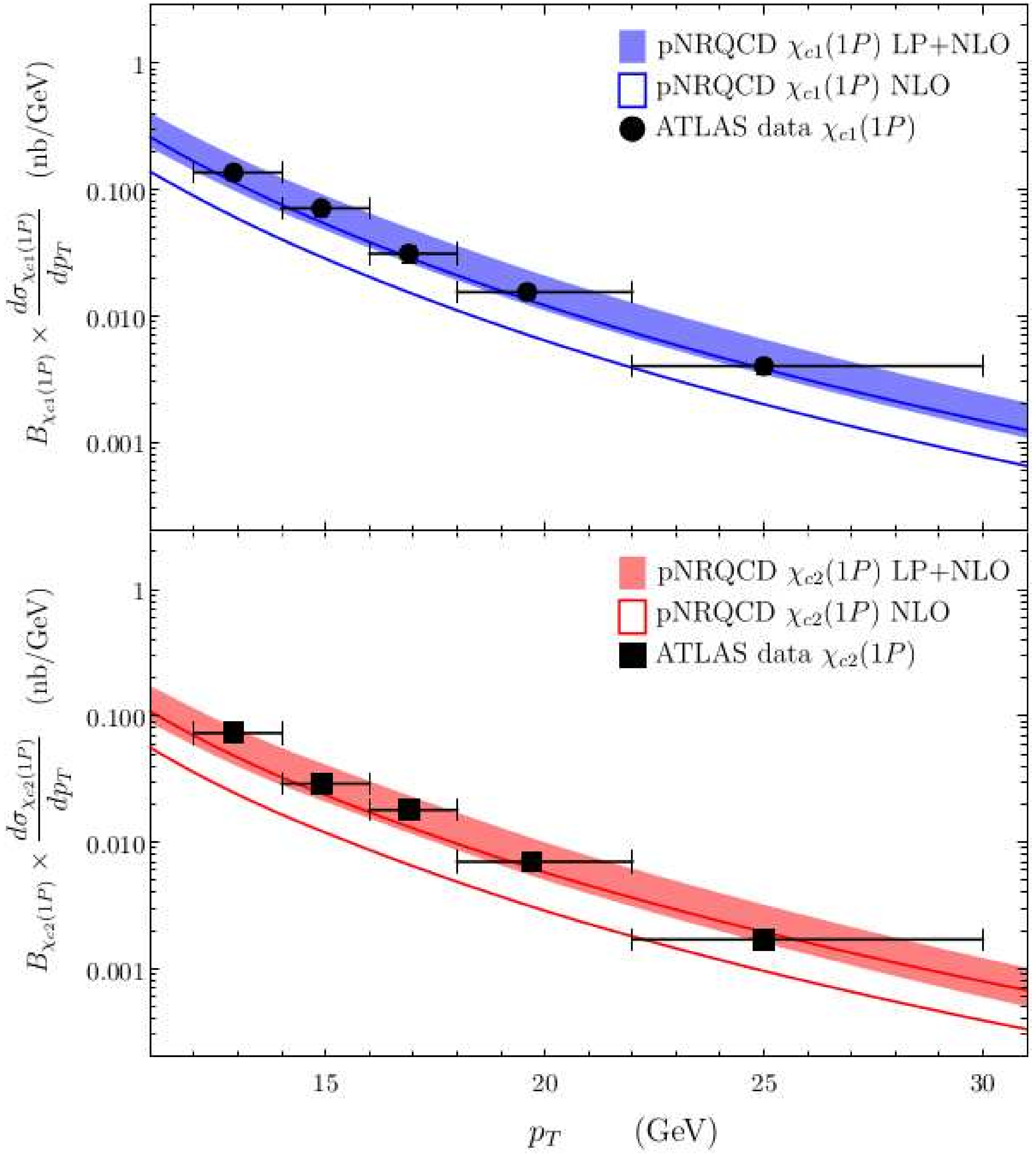


@ center of mass energy $\sqrt{s} = 7$ TeV and rapidity range $|y| < 0.75$.

$(d\sigma_{\chi_{b2}(1P)}/dp_T)/(d\sigma_{\chi_{b1}(1P)}/dp_T)$



$\sigma(pp \to \chi_{cJ}(1P) + X)$



A test of the universality of the pNRQCD factorization is provided by the ratio $(d\sigma_{\chi_{b2}(1P)}/dp_T)/(d\sigma_{\chi_{b1}(1P)}/dp_T)$ that depends only on $\mathcal{E}$ (at the scale of the b mass) and therefore is expected to be the same also for $2P$ and $3P$ bottomonium states.

Inclusive Hadroproduction Cross Section of the X(3872) in BOEFT

quarkonium production $\rightarrow \begin{cases} c\bar{c} \text{ in color singlet} \\ \langle \mathcal{O}^{\chi_{c0}}(^3S_1^{[8]}) \rangle = \mathcal{O}(v^2) \\ \langle \mathcal{O}^{\chi_{c0}}(^3P_0^{[1]}) \rangle = \mathcal{O}(v^2) \end{cases}$

Bodwin, Braaten, Lepage, Phys.Rev.D51:1125-1171,1995

exotics production $\rightarrow \begin{cases} c\bar{c} \text{ in color octet} \\ \langle \mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]}) \rangle = \mathcal{O}(v^0) \end{cases}$

Only one LDME

- $\rightarrow$ most dominant channel is $n = {}^3S_1^{[8]}$ for $\chi_{c1}(3872)$ production
- $\rightarrow$ quarkonium component is supressed by powers of v

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- $\rightarrow$ most dominant channel is $n = {}^3S_1^{[8]}$ for $\chi_{c1}(3872)$ production
- $\rightarrow$ quarkonium component is supressed by powers of v

- Production dominated by color-octet $(Q\bar{Q})_8$ matrix elements at leading power in v (velocity) expansion.

$$\sigma_{\chi_{c1}(3872)+X} = \sigma_{Q\bar{Q}(^3S_1^{[8]})} \langle\Omega| \mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]}) |\Omega\rangle \,,$$

Lai, Cheung, Phys. Rev. D 112 (2025), 054005

Octet LDME:

$$\langle\Omega| \mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]}) |\Omega\rangle = \langle\Omega| \chi^\dagger \sigma^i T^A \psi \Phi_\ell^{\dagger AB} \boxed{\mathcal{P}_{\chi_{c1}(3872)}} \Phi_\ell^{BC} \psi^\dagger \sigma^i T^C \chi |\Omega\rangle$$

Projection vector that projects onto all states that contain $\chi_{c1}(3872)$

- SDCs are the same as for χ_{cJ} quarkonium production and are known up to NLO in α_s

Butenschön, Kniehl, Nucl.Phys.B 950 (2020) 114843

LDME $\chi_{c1}(3872)$

$$\langle \mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]})\rangle = \frac{3}{2\pi}\,|\phi_S(\mathbf{0})|^2\,\mathcal{M}_S$$

BOEFT wave function for the X(3872)

- BOEFT factorization formally agrees with Lai, Chung, Phys.Rev.D 112 (2025) 5, 054005
- although disagrees on the exact value of $\mathcal{M}_S$
- $\mathcal{M}_S$ contains only LDFs present in $\chi_{c1}(3872)$

Summing over all states provides an upper bound for M_S (4/3) that constraints the LDME

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we fix LDME from B-Decay

$$\mathrm{Br}(B \to \chi_{c1}(3872) + X) = \mathrm{Br}(b \to c\bar{c}(^3S_1^{[8]}) + X)\,\langle \mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]})\rangle$$

Beneke, Maltoni, Rothstein, Phys.Rev.D 59 (1999) 054003

- experimental values taken from LHCb Collaboration, JHEP 01 (2022) 131, PDG, Phys.Rev.D 110 (2024) 3, 030001

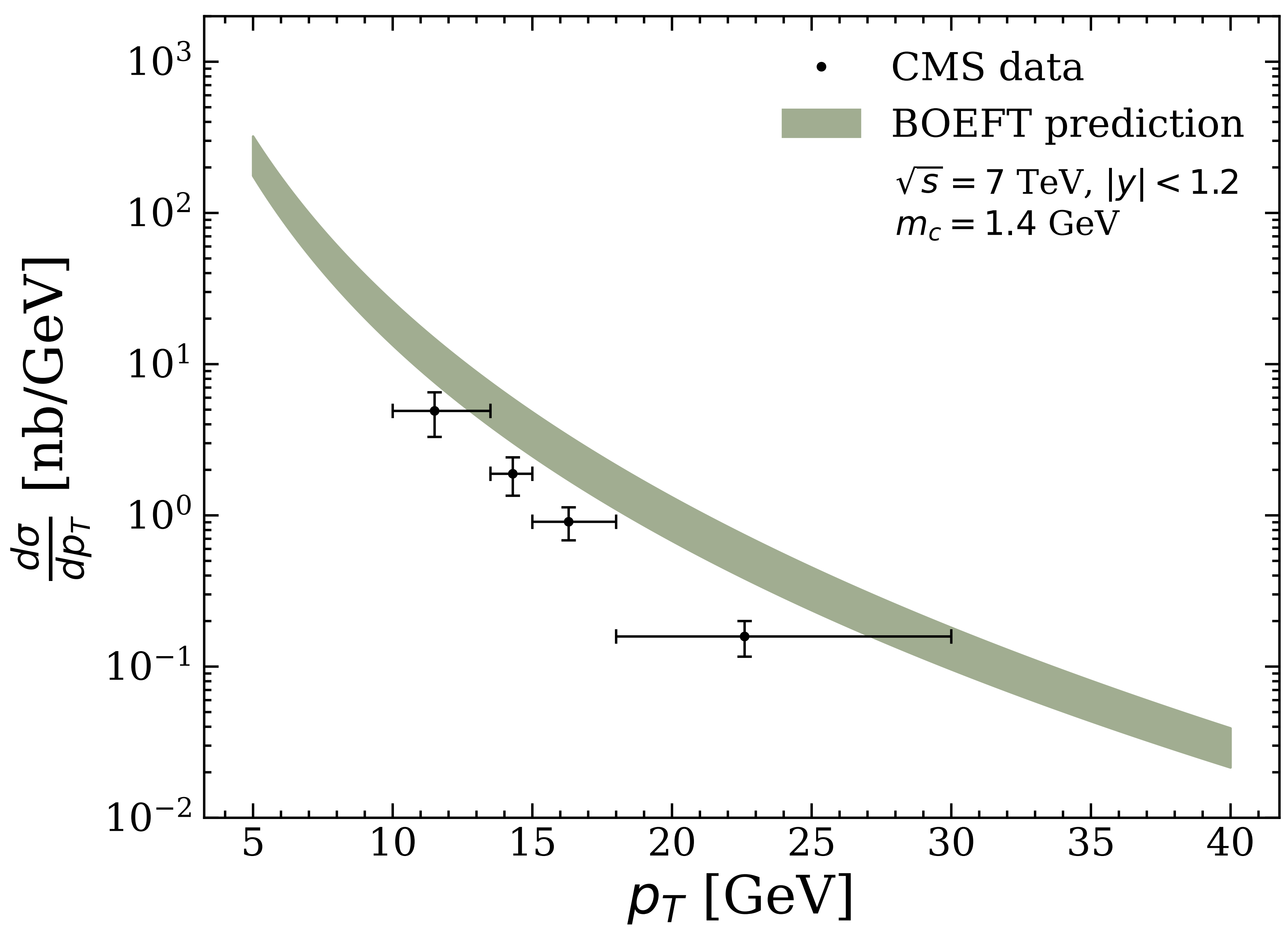
$$\langle\Omega|\,\mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]})\,|\Omega\rangle = 4.69^{+2.23}_{-2.16}\times 10^{-3}\,\mathrm{GeV}^3$$

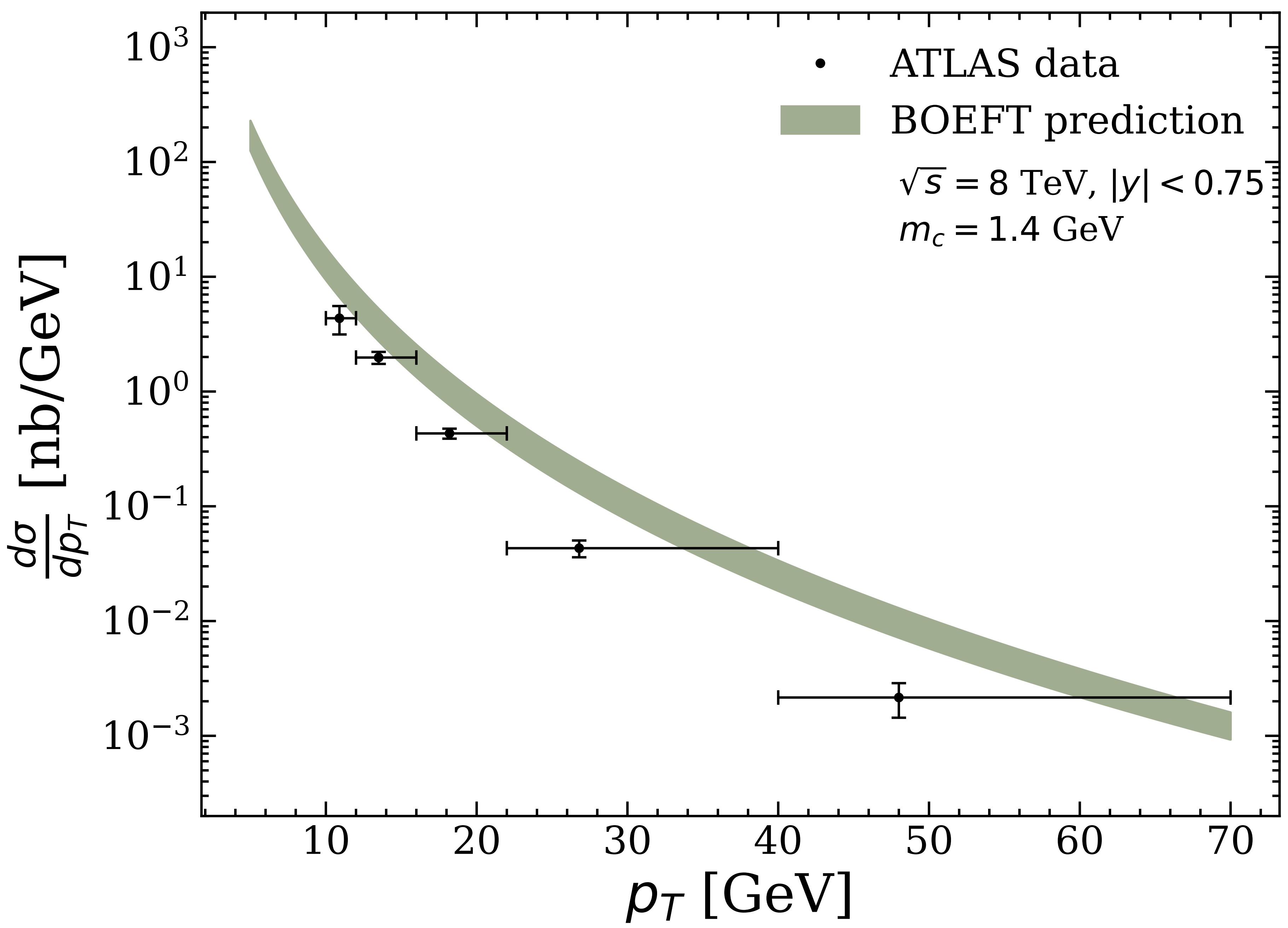
$$|\phi_{\Sigma_g^+}(0)|^2 \approx 0,$$
$$|\phi_S(0)|^2 = 5.78\times 10^{-3}\,\mathrm{GeV}^3$$
$$|\phi_D(0)|^2 \approx 0,$$

$$\mathcal{M}_S = 1.70^{+0.81}_{-0.78}\,.$$

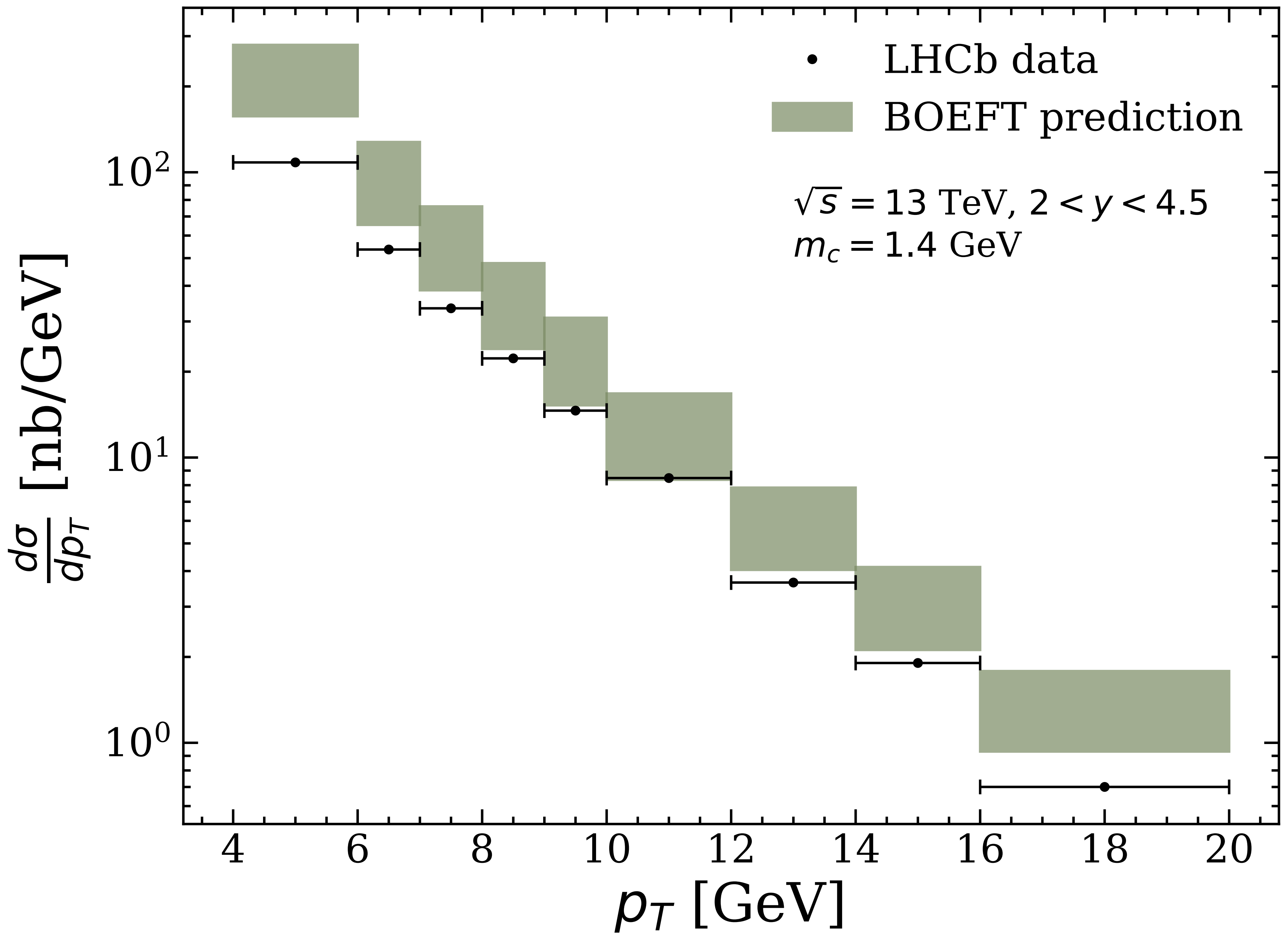
Cross-Sections

X(3872)

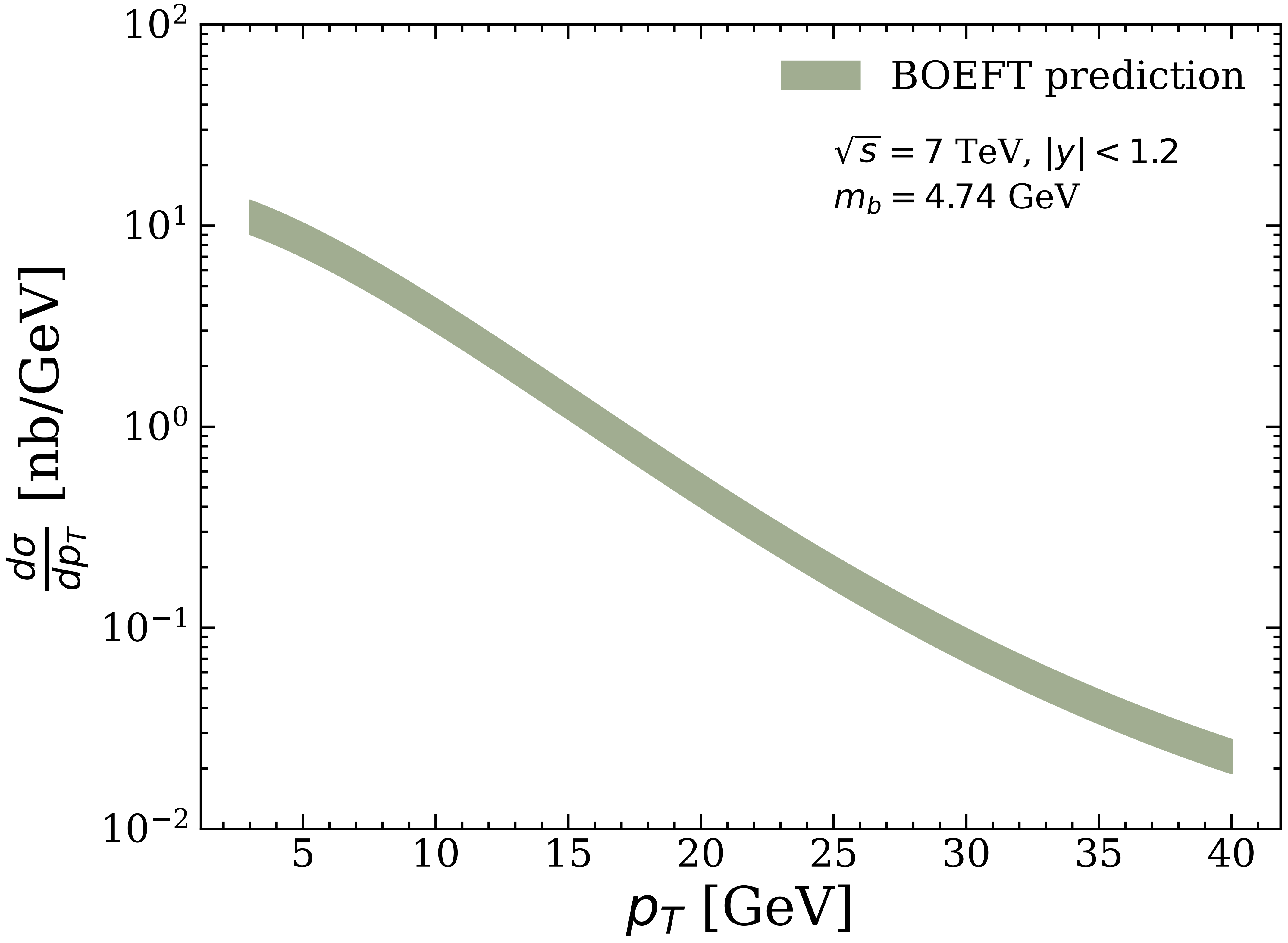




The upper bound significantly reduces uncertainties

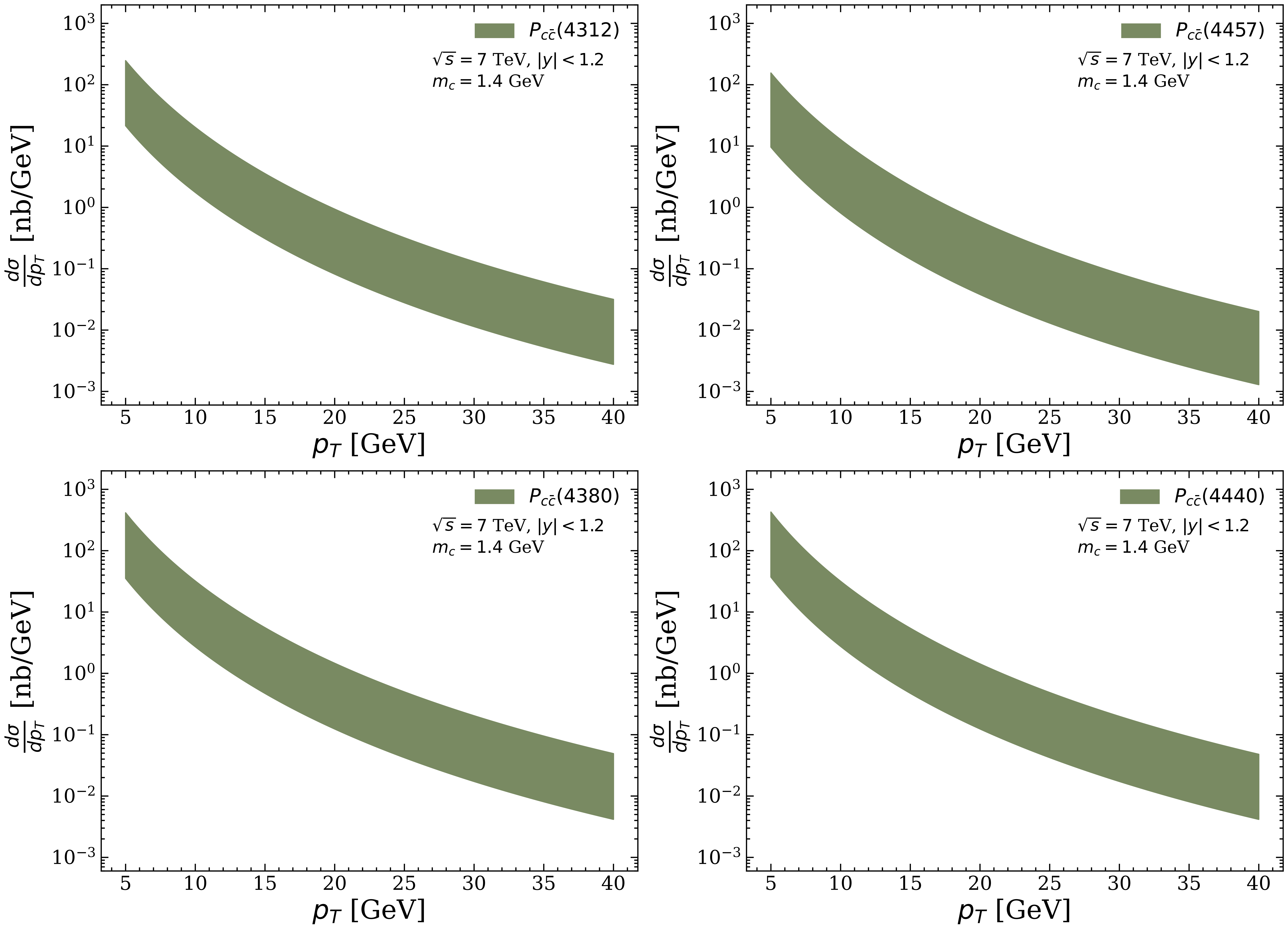


Cross-Sections Xb

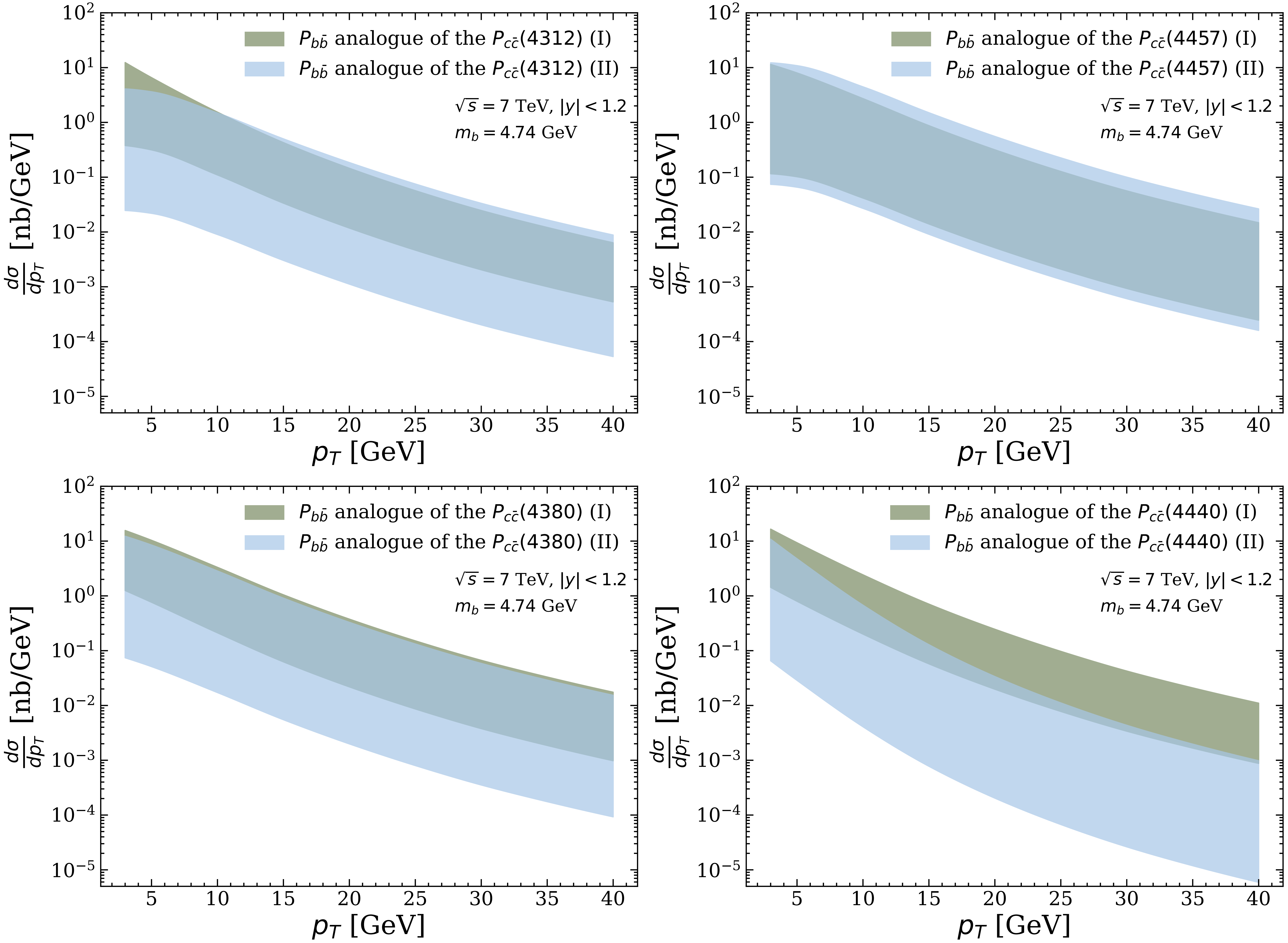


Pure prediction, everything fixed on charm sector, only the BOEFT wave function changes

Cross-Sections Pentaquarks for charm



Cross-Sections Pentaquarks for bottom



What About Evolution in Medium?

For bottomonium: EFT plus open quantum system gave us the nonequilibrium master equation for the evolution in a strongly coupled medium $T \sim gT$

The strongly QGP enters via transport coefficient defined in term of chromoelectric correlators at each T: the EFT acts as a layer that allows to use the lattice (equilibrium) input at each T

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$$m \gg 1/r \sim m\alpha_s \gg T \sim gT \gg E$$

quark-antiquark color singlet Hamiltonian $h_s = \frac{\mathbf{p}^2}{m} - \frac{4}{3}\frac{\alpha_s}{r}$

quark-antiquark color octet Hamiltonian $h_o = \frac{\mathbf{p}^2}{m} + \frac{1}{6}\frac{\alpha_s}{r}$

Open Quantum system

- Subsystem: heavy quarks/quarkonium
- Environment: quark gluon plasma

N.B., J. Soto, M. Escobedo, A. Vairo 2016, 2018 (1612.07248, 1711.04515)

We may define a density matrix in pNRQCD for the heavy quark-antiquark pair in a singlet and octet configuration:

$$\langle \mathbf{r}', \mathbf{R}' | \rho_s(t';t) | \mathbf{r}, \mathbf{R} \rangle \equiv \mathrm{Tr}\{\rho_{\mathrm{full}}(t_0)\, S^\dagger(t,\mathbf{r},\mathbf{R}) S(t',\mathbf{r}',\mathbf{R}')\}$$

$$\langle \mathbf{r}', \mathbf{R}' | \rho_o(t';t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{8} \equiv \mathrm{Tr}\{\rho_{\mathrm{full}}(t_0)\, O^{a\dagger}(t,\mathbf{r},\mathbf{R}) O^b(t',\mathbf{r}',\mathbf{R}')\}$$

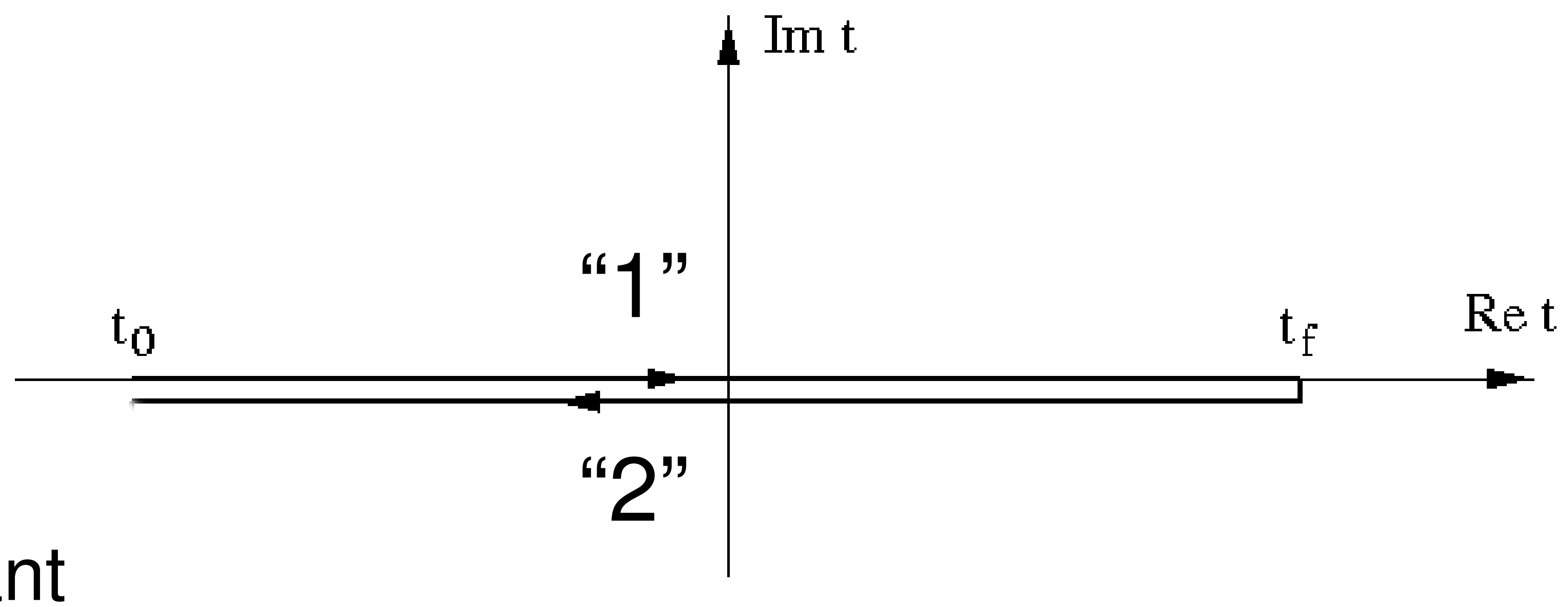
The system is in non-equilibrium because through interaction with the environment (quark gluon plasma) singlet and octet quark-antiquark states continuously transform in each other although the number of heavy quarks is conserved: $\mathrm{Tr}\{\rho_s\} + \mathrm{Tr}\{\rho_o\} = 1$.

Closed time path formalism

In the closed-time path formalism we can represent the density matrices as 12 propagators on a closed time path:

$$\langle \mathbf{r}', \mathbf{R}' | \rho_s(t';t) | \mathbf{r}, \mathbf{R} \rangle = \langle S_1(t', \mathbf{r}', \mathbf{R}') S_2^\dagger(t, \mathbf{r}, \mathbf{R}) \rangle$$

$$\langle \mathbf{r}', \mathbf{R}' | \rho_o(t';t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{8} = \langle O_1^b(t', \mathbf{r}', \mathbf{R}') O_2^{a\dagger}(t, \mathbf{r}, \mathbf{R}) \rangle$$



Differently from the thermal equilibrium case 12 propagators are relevant (in thermal equilibrium they are exponentially suppressed).

12 propagators are not time ordered, while 11 and 22 operators select the forward time direction $\propto \theta(t-t')$, $\theta(t'-t)$.

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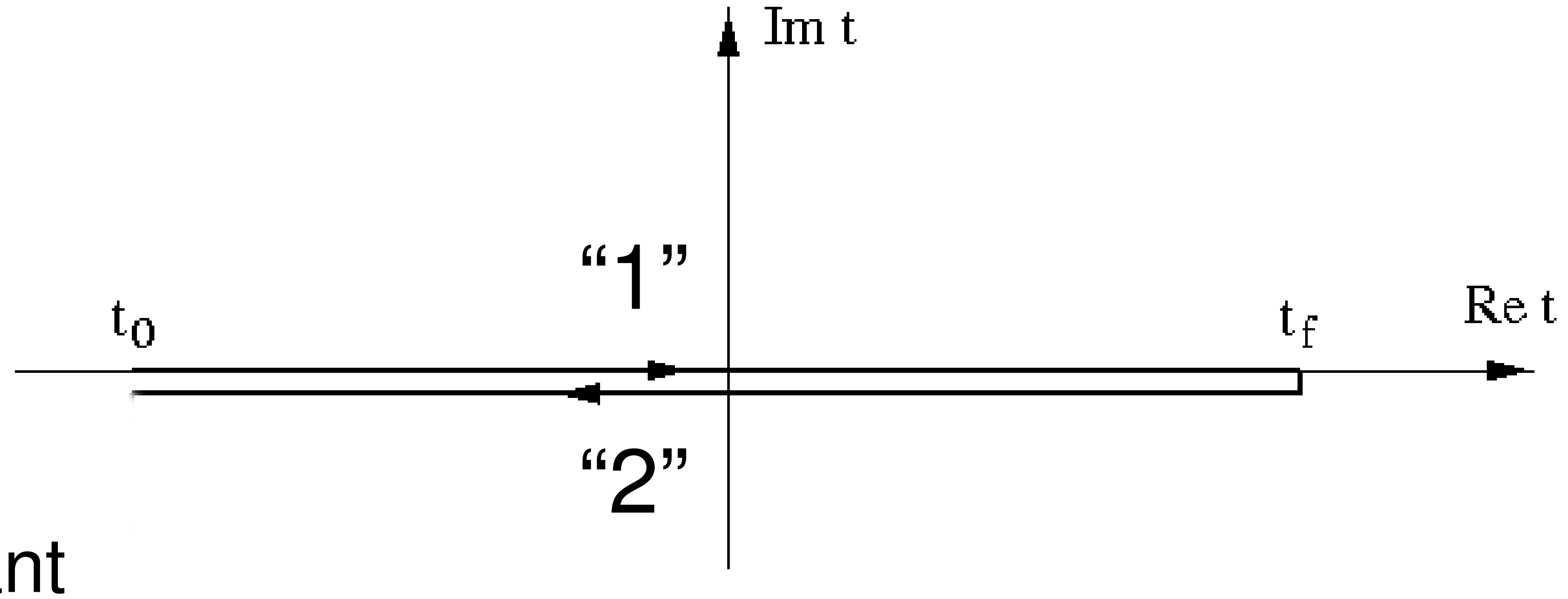

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Expansions

- The density of heavy quarks is much smaller than the one of light quarks: we expand at first order in the heavy quark-antiquark density.
- We consider T much smaller than the Bohr radius of the quarkonium: we expand up to order r^2 in the multipole expansion.

Density matrix evolution equationsl

$$\frac{d\rho_s(t;t)}{dt} = -i[h_s,\rho_s(t;t)] - \Sigma_s(t)\rho_s(t;t) - \rho_s(t;t)\Sigma_s^\dagger(t) + \Xi_{so}(\rho_o(t;t),t)$$

$$\frac{d\rho_o(t;t)}{dt} = -i[h_o,\rho_o(t;t)] - \Sigma_o(t)\rho_o(t;t) - \rho_o(t;t)\Sigma_o^\dagger(t) + \Xi_{os}(\rho_s(t;t),t) + \Xi_{oo}(\rho_o(t;t),t)$$

Using closed time path formalism and the expansions:

Self energies contain all thermal nonperturbative effects

$$\Sigma_s(t) = \frac{g^2}{2N_c}\int_{t_0}^{t} dt_2\, r^i\, e^{-ih_o(t-t_2)}\, r^j\, e^{ih_s(t-t_2)}\, \langle E^{a,i}(t,\mathbf{0})E^{a,j}(t_2,\mathbf{0})\rangle$$

$$\Xi_{so}(\rho_o(t_0;t_0),t) = \frac{g^2}{2N_c(N_c^2-1)}\int_{t_0}^{t} dt_2\, \Big[r^i\, e^{-ih_o(t-t_0)}\, \rho_o(t_0;t_0)\, e^{ih_o(t_2-t_0)} \times r^j\, e^{ih_s(t-t_2)}\, \langle E^{a,j}(t_2,\mathbf{0})E^{a,i}(t,\mathbf{0})\rangle + \text{H.c.}\Big]$$

A Wilson line in the adjoint representation is understood in the chromoelectric correlators.

contains everything :
screening,
singlet to octet, Landau damping

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and the expansions:

Physical interpretation

- The self energies Σ_s and Σ_o provide the in-medium induced mass shifts, $\delta m_{s,o}$, and widths, $\Gamma_{s,o}$, for the color-singlet and color-octet heavy quark-antiquark systems respectively:

$$-i\Sigma_{s,o}(t) + i\Sigma_{s,o}^\dagger(t) = 2\,\mathrm{Re}\,(-i\Sigma_{s,o}(t)) = 2\delta m_{s,o}(t)$$
$$\Sigma_{s,o}(t) + \Sigma_{s,o}^\dagger(t) = -2\,\mathrm{Im}\,(-i\Sigma_{s,o}(t)) = \Gamma_{s,o}(t)$$

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A Wilson line in the adjoint representation is understood in the chromoelectric correlators.

contains everything :
screening,
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- Ξ_{so} accounts for the production of singlets through the decay of octets, and Ξ_{os} and Ξ_{oo} account for the production of octets through the decays of singlets and octets respectively. There are two octet production mechanisms/octet chromoelectric dipole vertices in the pNRQCD Lagrangian.

Times Scales

Environment correlation time: $\tau_E \sim \frac{1}{T}$

Akamatsu PRD 91 (2015) 056002

System intrinsic time scale: $\tau_S \sim \frac{1}{E}$

System relaxation time: $\tau_R \sim \frac{1}{\text{self-energy}} \sim \frac{1}{\alpha_s a_0^2 \Lambda^3}$ $a_0 =$ Bohr radius, $\Lambda = T, E$

Then

$\tau_R \gg \tau_S, \tau_E$ $\tau_S \gg \tau_E$

and we have a Markovian, quantum Brownian system

We can separate the bound state from the medium contribution in the self-energy and we can obtain a Linblad equation

Linblad equation at LO in the E/T expansion

If $E \ll T \sim m_D$ the Lindblad equation for a strongly coupled plasma reads

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_i \left(C_i \rho C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho\}\right)$$

C_i collapse or jump operators: connect different internal states

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma(t) \begin{pmatrix} 1 & 0 \\ 0 & \frac{7}{16} \end{pmatrix},$$

$$C_i^0 = \sqrt{\frac{\kappa(t)}{8}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{8} & 0 \end{pmatrix}, \qquad C_i^1 = \sqrt{\frac{5\kappa(t)}{16}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

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the sQGP is characterised by two nonperturbative parameters (transport coefficients) kappa and gamma that must be calculated on the lattice

κ is the heavy-quark momentum diffusion coefficient:

$$\kappa = \frac{g^2}{18}\,\mathrm{Re}\int_{-\infty}^{+\infty} ds\, \langle \mathrm{T}\, E^{a,i}(s,\mathbf{0})\, \phi^{ab}(s,0)\, E^{b,i}(0,\mathbf{0})\rangle$$

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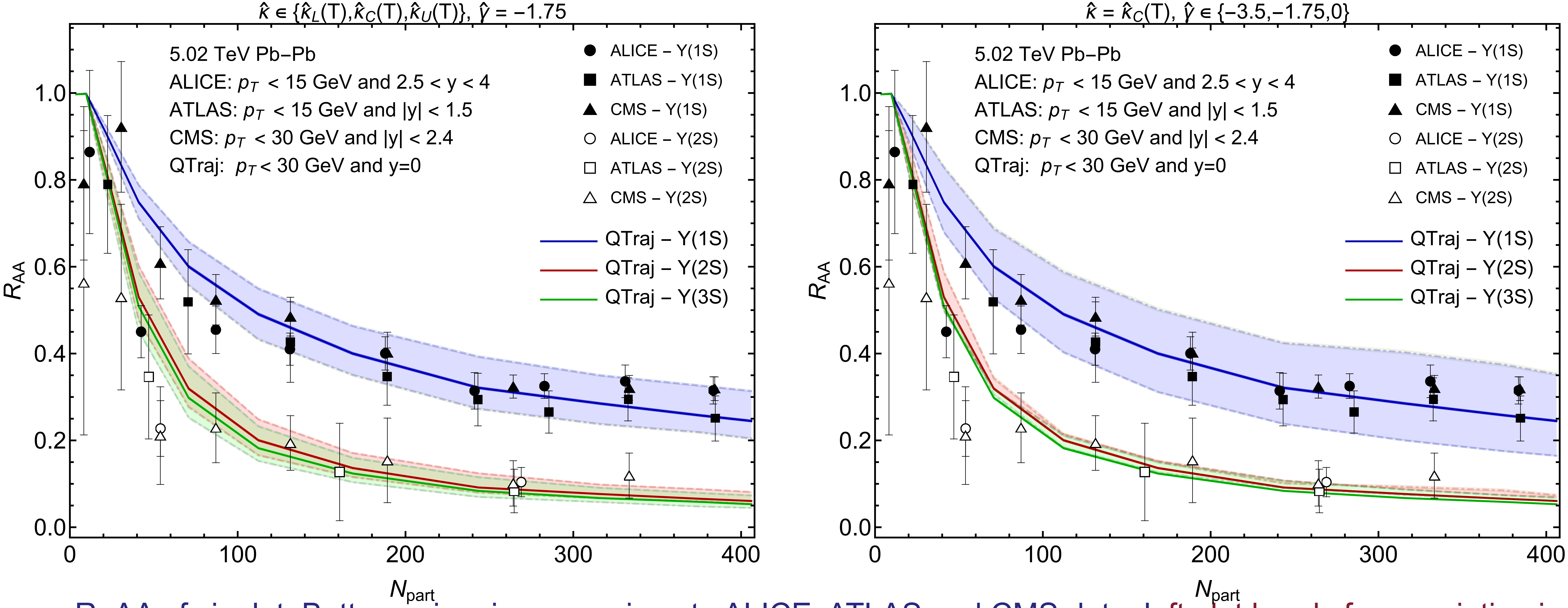
$$\gamma = \frac{g^2}{18} \mathrm{Im} \int_{-\infty}^{+\infty} ds \, \langle \mathrm{T}\, E^{a,i}(s,\mathbf{0})\, \phi^{ab}(s,0)\, E^{b,i}(0,\mathbf{0}) \rangle$$

the EFT allows to use lattice QCD equilibrium calculation to study the non equilibrium evolution! EFT is intermediate layer to non equilibrium

Nuclear modification factor R_AA (Linblad Leading order, t_F =250 MeV)

calculation with no free parameters, results depends on kappa function of T (calculated on the lattice) and gamma (extracted from the lattice)

$$R_{AA}(nS) = \frac{\langle n, \mathbf{q} | \rho_s(t_F; t_F) | n, \mathbf{q} \rangle}{\langle n, \mathbf{q} | \rho_s(0; 0) | n, \mathbf{q} \rangle}$$



R_AA of singlet Bottomonium in comparison to ALICE, ATLAS and CMS data, left plot bands from variation in kappa, right plot variation in gamma —> we can use R_AA to learn about the QGP!

N.B. Escobedo , Strickland, Vairo, Vander Griend, Weber, 2107.06222

N.B., M. Escobedo, M. Strickland, A. Vairo P. VanderGriend et al

arXiv:2302.11826

arXiv:2205.10289

What About Evolution in Medium?

Now we know how to obtain and solve the master equation without using the quantum brownian or the quantum optic limit: no hierarchy assumed between T and E !

N. B, A. Lin, T. Magorsch, A. Brambilla 2026

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Can we use this new technology and BOEFT plus open quantum system to describe the X(3872) evolution in QGP?

Outlook

- BOEFT aims at describing all states containing two heavy quarks in a QCD derived controlled framework

- It is based on symmetry and scales factorization

  at the short distance scale we have control of the perturbative calculation

  at the large distance scale the EFT predicts the heavy-light static energy behaviour

  we need lattice calculations of few gauge invariant universal correlators

the structure of the EFT allows for model independent predictions (in particular from the charm/bottom sector

- Once the lattice input is there the BOEFT allows applications to domain in general not directly accessible to a lattice calculation (decay, production, medium propagation)

- The results obtained on the X and the Tcc gives is an idea of their nature beyond the models and redefine our knowledge of the strong force

—>especially their nature may appear different at different scales

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This picture has the possibility to give a unified description to exotics and to leave the dynamics decide which configuration used by models will dominate in a given range

Combining BOEFT + open quantum systems one can attempt to study the X Y Z in heavy ion collisions