Born-Oppenheimer Effective Theory for XYZ Exotics

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Phys. Rev. D 110, 094040 (2024) (Editors Suggestion), Phys. Rev. Lett 135, 121902 (2025),
Phys. Rev. D 112, 114037 (2025)

TUM TECHNISCHE UNIVERSITÄT MÜNCHEN

DFG Deutsche Forschungsgemeinschaft

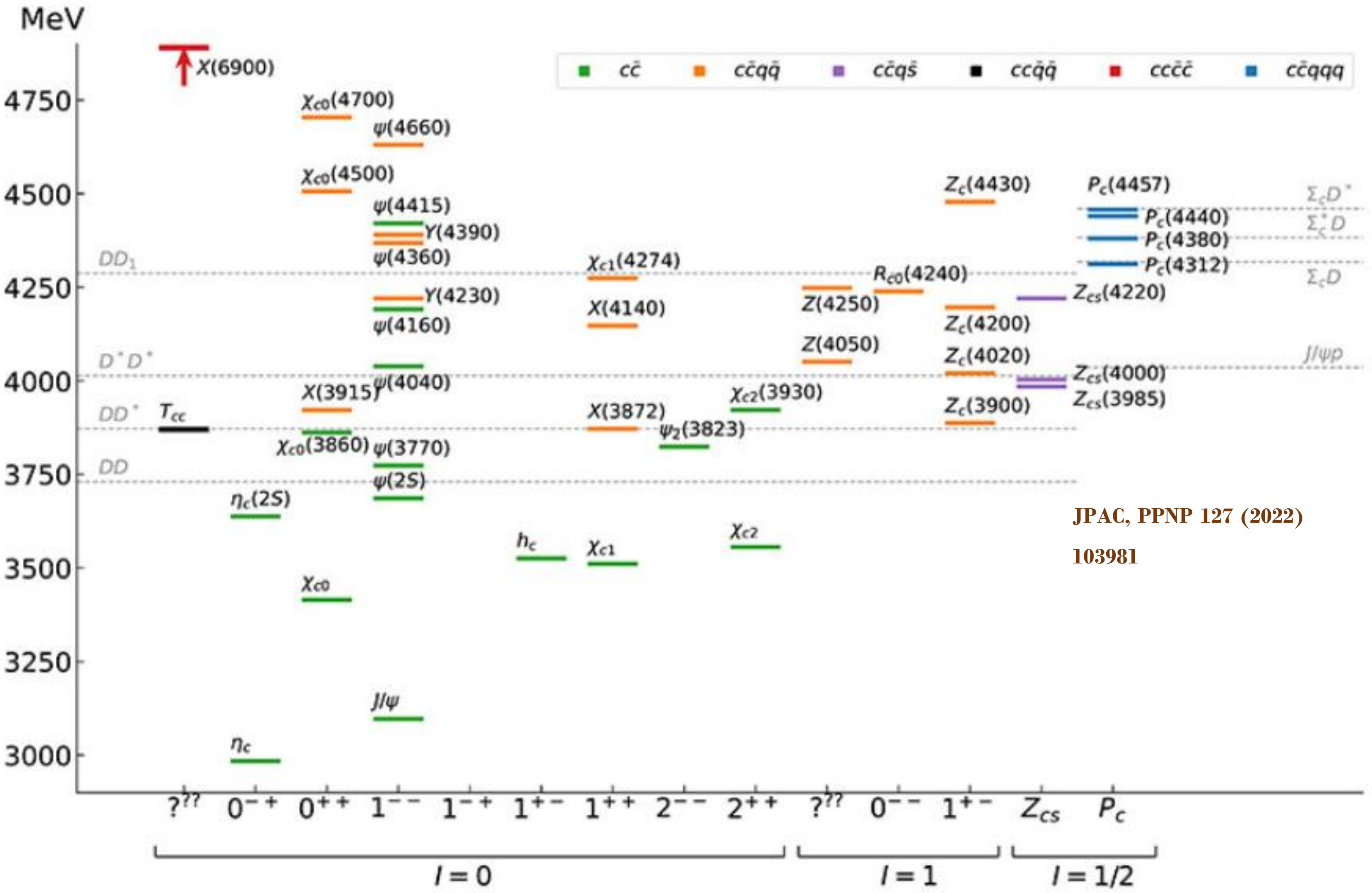
erc Advanced Grant EFT-XYZ

Exotic Hadron

- **XYZ** Exotics: States with at-least 2-heavy quarks.

- Recent count including both **tetraquarks and pentaquarks**:
  - 54 in $c\bar{c}$ sector
  - 5 in $b\bar{b}$ sector
  - 4 with all c and $\bar{c}$
  - 1 with two charm quarks.

Lebed, arXiv 2308.00781 (2023)



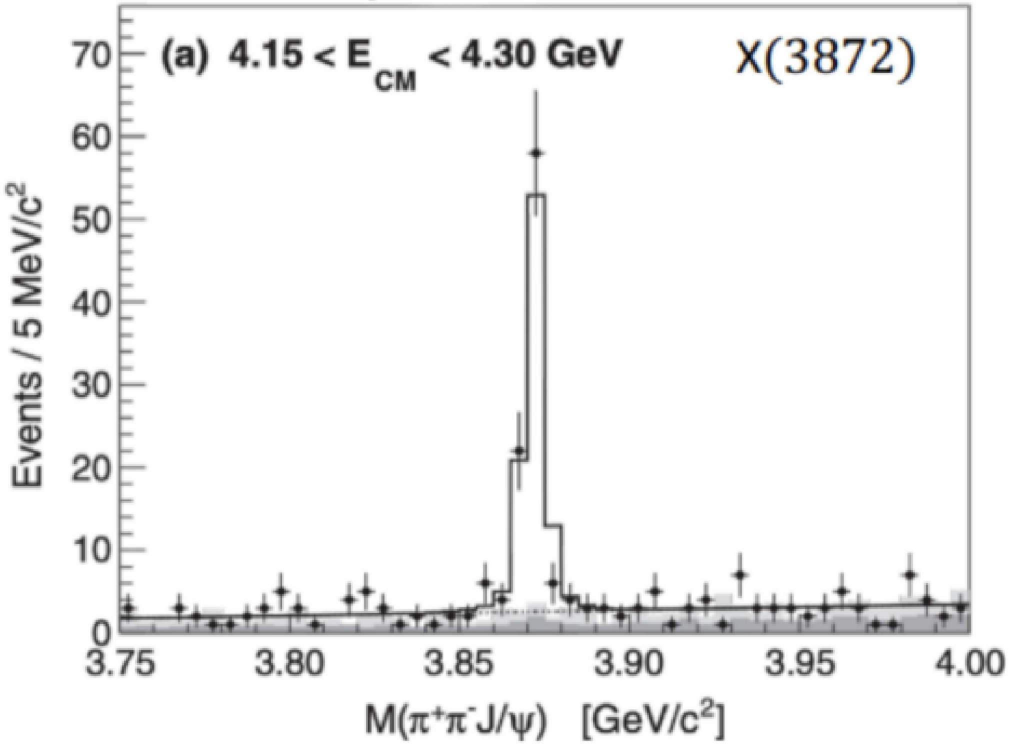
JPAC, PPNP 127 (2022) 103981

Tetraquarks or pentaquarks:
Non-zero isospin states.
New theoretical predictions on static potentials based on Born-Oppenheimer EFT

$\chi_{c1}(3872)$

$e^+e^- \to \gamma X(3872)$; $X(3872) \to \pi^+\pi^- J/\psi$

BesIII coll. PRL 122 (2019) 202001

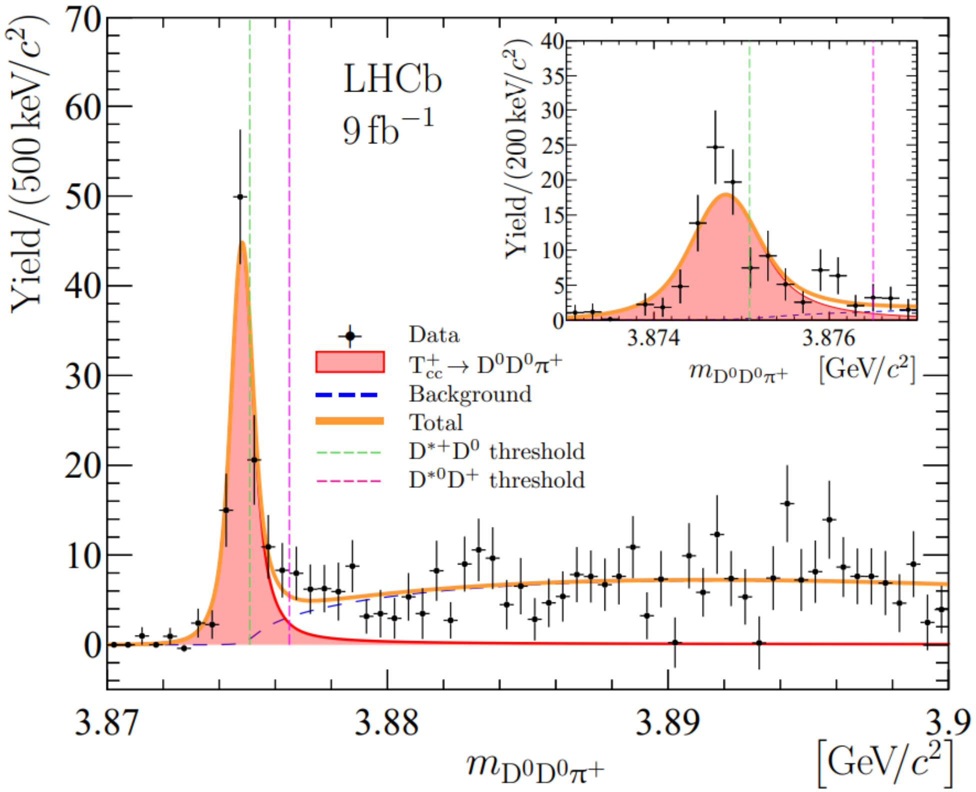


$$m_{\chi_{c1}(3872)} - (m_{D^{*0}} + m_{\bar{D}^0}) = -0.07 \pm 0.12 \text{ MeV}.$$

LHCb, JHEP 08 (2020) 123

- **Quantum numbers**: **$J^{PC}=1^{++}$ (Isospin=0)**

$T_{cc}^+(3875)$

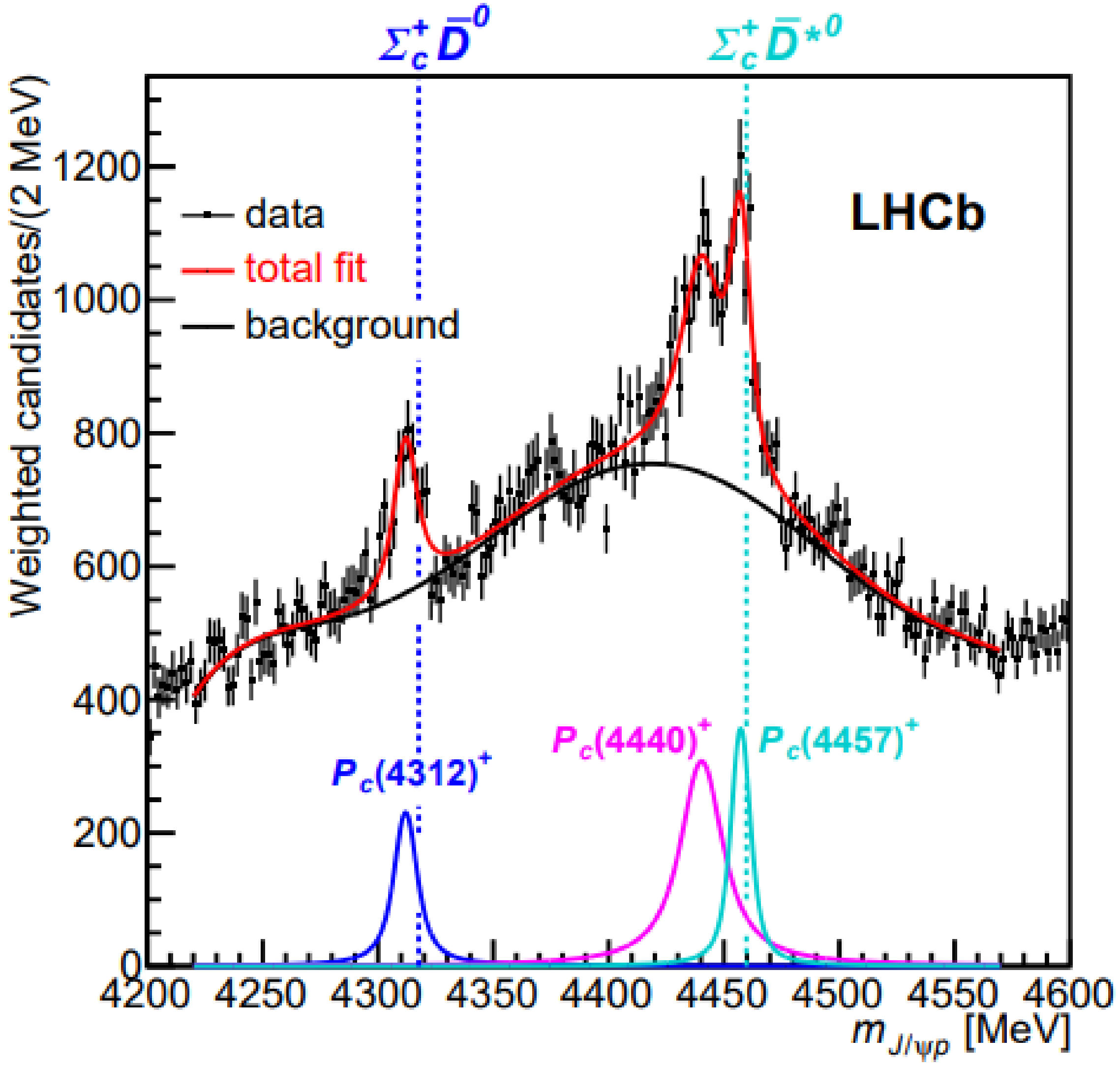


LHCb (Nature Phys. 18 (2022) 7, 751; Nature Comm. 13 (2022) 3351)

$$m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0}) = -0.27 \pm 0.06 \text{ MeV}.$$

- Consistent with **isoscalar** with **$J^P=1^+$**
- **Longest lived** Exotic particle: $\Gamma \sim 50$ keV

Pentaquark



LHCb, Phys. Rev. Lett. 122, (2019), 222001

Observed states

- 4 states with isospin $I = 1/2$:

$P_{c\bar{c}}(4312)^+$, $P_{c\bar{c}}(4380)^+$, $P_{c\bar{c}}(4440)^+$, $P_{c\bar{c}}(4457)^+$

J^P quantum numbers not established

All pentaquark states seen by LHCb experiment

PDG 2025

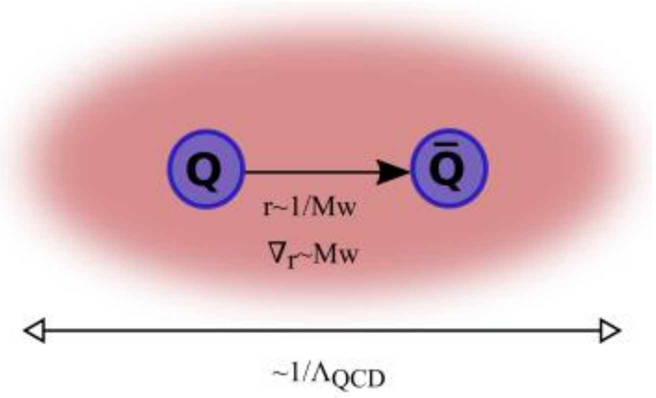
Born-Oppenheimer EFT

BOEFT: Exotic Hadron

- Exotic hadron $(Q\bar{Q}X, QQX, \ldots\ldots)$, X: any combination of light quark and gluons (LDF) for color singlet.

- Multiple scales in Exotics:

Extended objects:



  - Mass of heavy quark: m
  - Energy scale for LDF: Λ_{QCD}
  - Relative momentum between heavy quarks: $mv \sim 1/r$
  - Heavy Quark kinetic energy scale: mv^2

$$m \gg mv \gtrsim \Lambda_{QCD} \gg mv^2$$

Heavy quark: slow-degrees of freedom   ***X***: fast-degrees of freedom

R. Oncala, J. Soto, Phys. Rev. D96 (2017)

Berwein, Brambilla, Castellà, Vairo Phys. Rev. D. 92 (2015)

Brambilla, Krein, Castellà, Vairo Phys. Rev. D. 97, (2018)

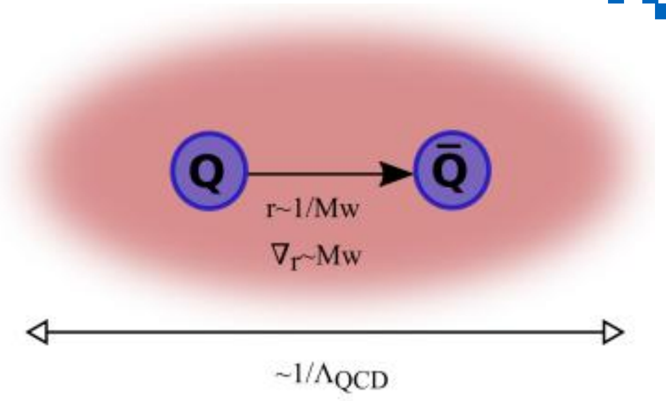
Castellà, Soto Phys. Rev. D. 102 (2020)

Berwein, Brambilla, AM, Vairo, arXiv 2408.04719

BOEFT: Exotic Hadron

- Hierarchy of scales:

$$m \gg mv \gtrsim \Lambda_{QCD} \gg mv^2$$



- Time-scale for dynamics of $Q\bar{Q}$: $\sim \dfrac{1}{mv2} \gg \dfrac{1}{\Lambda_{QCD}}$

Heavy quarks **static** with respect to light quarks or gluons

Born-Oppenheimer (BO) Approximation

Braaten, Langmack, Smith Phys. Rev. D. 90, 014044 (2014)

Juge, Kutti, Morningstar, Phys. Rev. Lett. 90, 161601 (2003)

- **Bound-state dynamics** at energy scale mv^2 ! Integrate out all energy scales above mv^2.

$$\text{QCD} \rightarrow \text{NRQCD} \rightarrow \text{pNRQCD/BOEFT}$$

R. Oncala, J. Soto, Phys. Rev. D96 (2017)

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92 (2015)

Brambilla, Krein, Castellà , Vairo Phys. Rev. D. 97, (2018)

Castellà , Soto Phys. Rev. D. 102 (2020)

Berwein, Brambilla, AM, Vairo, arXiv 2408.04719

BOEFT: Quantum #'s

- **BO-quantum number** (**r ≠ 0**): heavy quarks static, Cylindrical symmetry group $D_{\infty h}$

Labelling LDF static energies:

✓ Absolute value of component of **LDF** **angular momentum** $\boldsymbol{K}$

$$|\boldsymbol{r} \cdot \boldsymbol{K}| \equiv \boldsymbol{\Lambda} = \mathbf{0}, \mathbf{1}, \mathbf{2}, \ldots \ldots .. (\text{or } \boldsymbol{\Sigma}, \boldsymbol{\Pi}, \boldsymbol{\Delta}, \boldsymbol{\Phi}, \ldots ..)$$

✓ Product of charge conjugation and parity ($\boldsymbol{CP}$):

$$\boldsymbol{\eta} = +\mathbf{1}\ (\mathbf{g}),\ -\mathbf{1}\ (\boldsymbol{u})$$

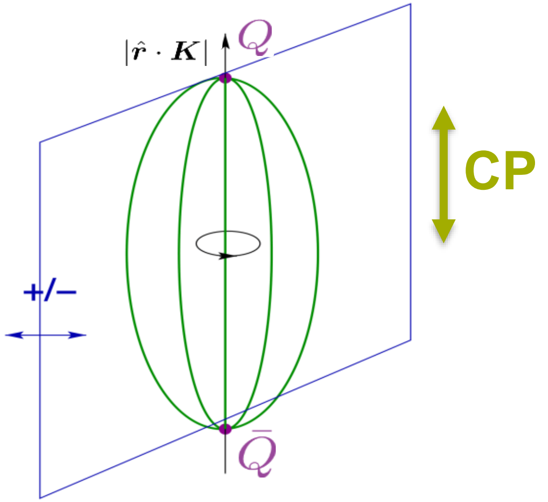
✓ $\boldsymbol{\sigma}$: Eigenvalue of reflection about a plane containing static sources.

$$\sigma = P\,(-1)^{K_{\text{light}}} = \pm 1$$

Born, Oppenheimer, Annalen der Physik 389 (1927)

Landau, Lifshitz & Pitaevskii, QM book

$$\Lambda_\eta^\sigma$$



Examples:

K^{PC}	Λ_η^σ
0^{++}	Σ_g^+
0^{+-}	Σ_u^+
1^{+-}	$\{\Sigma_u^-, \Pi_u\}$
2^{--}	$\{\Sigma_g^-, \Pi_g, \Delta_g\}$

- **Spherical symmetry** restored in **r → 0** limit: Labelled by LDF quantum #'s:

$$\kappa = \{K^{PC}, f\}$$

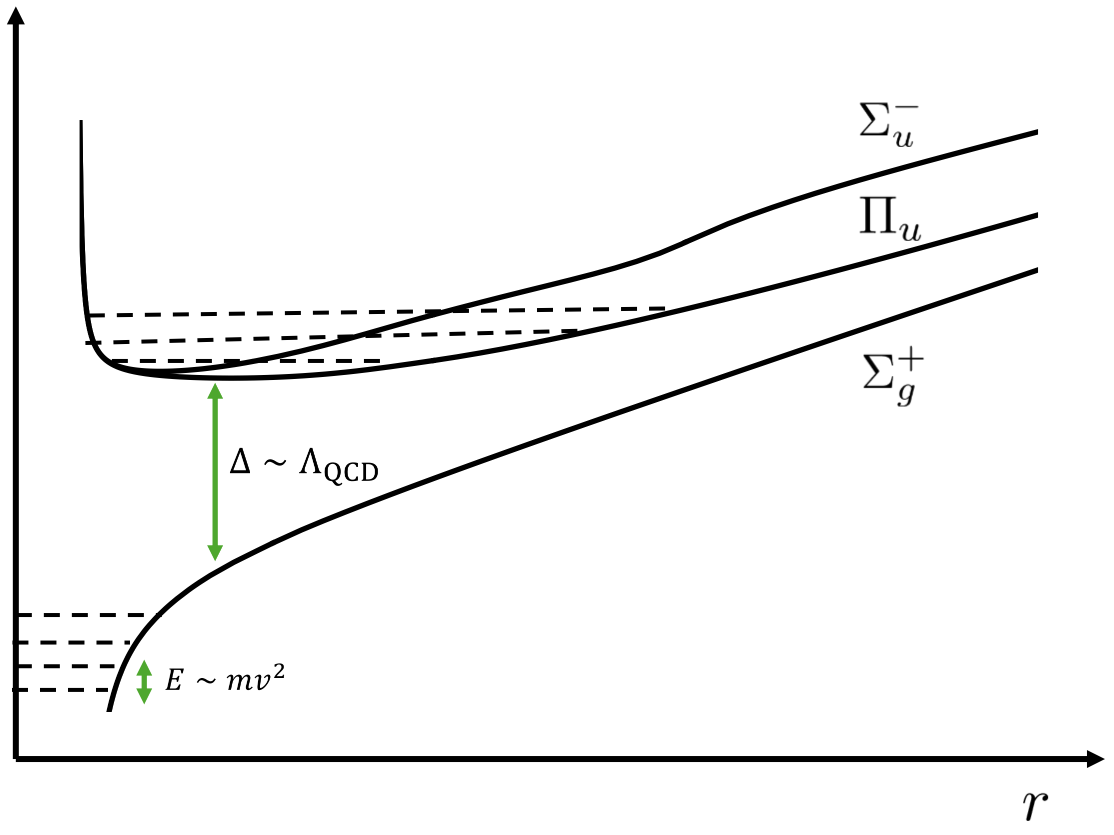
BOEFT

- BOEFT Lagrangian (**See Joan Soto talk on Mon !**):

$$L_{\text{BOEFT}} = L_{Q\bar{Q}} + L_{Q\bar{Q}g} + L_{Q\bar{Q}q\bar{q}} + L_{Q\bar{Q}qqq} + L_{\text{mixing}} + \cdots$$

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040 Castellà , Soto Phys. Rev. D. 102, 014012 (2020)

Brambilla, Krein, Castellà , Vairo Phys. Rev. D. 97, (2018)



Σ_g^+, Σ_u^-, Π_u: represent different potentials between heavy quark-antiquark

- Gap $\Lambda_{\text{QCD}} \gg mv^2$ between low-lying states: quarkonium, hybrid, tetraquark etc.

- L_{mixing}: Mixing due to similar masses and same quantum-numbers.

  - Mixing at static potential level (avoided level crossing)
    Ex. Tetraquark-quarkonium mixing

  - Mixing between states suppressed by $O(1/m_Q)$
    Ex. Hybrid-quarkonium mixing

    See Marc Wagner talk on Mon !

R. Oncala, J. Soto, Phys. Rev. D96 014004 (2017)

BOEFT

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

- BOEFT Lagrangian (**See Joan Soto talk on Mon !**):

$$L_{\mathrm{BOEFT}} = \int d^3\boldsymbol{R} \int d^3\boldsymbol{r} \sum_{\kappa\lambda\lambda'} \mathrm{Tr}\Big\{ \Psi^{\dagger}_{\kappa\lambda}(\mathbf{r},\,\mathbf{R},\,t) \Big[i\partial_t\,\delta_{\lambda\lambda'} - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda}(\theta,\phi)\,\frac{\boldsymbol{\nabla}^2_r}{m_Q} P^{i}_{\kappa\lambda'}(\theta,\phi) \Big] \Psi_{\kappa\lambda'}(\mathbf{r},\,\mathbf{R},\,t) \Big\}$$

LDF-quantum #: $\kappa = \{K^{PC}, f\}$ BO-quantum #: Λ^{σ}_{η} $\lambda = \pm\Lambda$

Projection vectors for $D_{\infty h}$: $P^{i}_{K\lambda}(\theta,\varphi) = D^{\lambda *}_{Ki}(0,\theta,\varphi)$

- **BO potentials: Potential between Q & $\bar{Q}$** due to LDF (light quarks, gluons).

Born-Oppenheimer (BO) potential:

$$V_{\kappa\lambda\lambda'}(r) = \boxed{E^{(0)}_{\kappa,|\lambda|}(r)}\,\delta_{\lambda\lambda'} + \boxed{\frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q}} + \ldots\,,$$

Static Energy

Spin-dependent potentials

Brambilla, Lai, Segovia, Castellà, Phys. Rev. D. 101, (2020)

See Marc Wagner talk on Mon !

- Static potentials with same LDF-quantum # κ degenerate in $r \to 0$ limit.

BOEFT

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

- **Good** quantum numbers:
  - BO-orbital momentum: $\boldsymbol{L} = \boldsymbol{L}_Q + \boldsymbol{K}$
  - Heavy quark Spin: $\boldsymbol{S}_Q$ (HQSS limit)
  - Total angular momentum: $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}_Q$

$\boldsymbol{K}$: LDF angular-momentum or spin

$\boldsymbol{L_Q}$: orbital-angular momentum of QQ or $Q\bar{Q}$ pair.

- Radial Coupled-Channel Schrödinger equation:

$$\sum_{\lambda}\left[-\frac{1}{m_Q r^2}\partial_r\, r^2\partial_r + \frac{1}{m_Q r^2} \boxed{M_{\lambda'\lambda}} + E^{(0)}_{\kappa,|\lambda|}(r)\delta_{\lambda\lambda'}\right]\psi^{(N)}_{\kappa\lambda}(r) = \mathcal{E}_N\, \psi^{(N)}_{\kappa\lambda'}(r)$$

Mixing term $M_{\lambda'\lambda}$: Mixing static potentials with different BO-quantum numbers Λ_η^σ corresponding to same LDF-quantum # κ

Mixing term $M_{\lambda'\lambda}$: Angular momentum L^2 spherically symmetric but static states have cylindrical symmetry

- BO potentials are inputs into Schroedinger equations.

BOEFT

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Castellà , Soto Phys. Rev. D. 102, 014012 (2020)

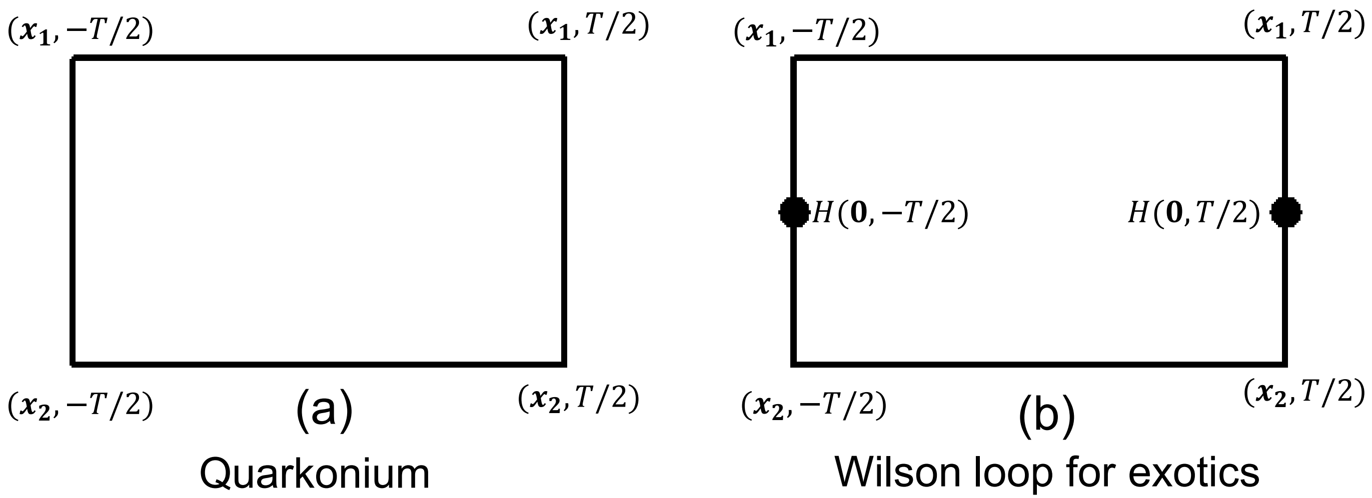
NRQCD operator (gauge invariant) for exotic hadron $Q\bar{Q}X$ or QQX :

$$\mathcal{O}_{\kappa,\lambda}(t,\boldsymbol{r}) = \chi^\dagger(t,\boldsymbol{r}/2)\,\phi(t;\boldsymbol{r}/2,\boldsymbol{0})\,P^{\alpha\dagger}_{\kappa,\lambda}H^{\alpha}_{\kappa}(t,\boldsymbol{0})\,\phi(t;\boldsymbol{0},-\boldsymbol{r}/2)\,\psi(t,-\boldsymbol{r}/2)$$

H^{α}_{κ} : LDF (gluon or light-quarks) operator characterizing X based on quantum # κ (isospin, color etc..)

$P^{\alpha}_{\kappa,\lambda}$: Projection vectors for projecting onto cyclindrical symmetry $D_{\infty h}$ representations.

$$E^{(0)}_{\kappa,|\lambda|}(r) = \lim_{T\to\infty}\frac{i}{T}\log\left[\langle \text{vac}|\mathcal{O}_{\kappa,\lambda}(T/2,\,\boldsymbol{r},\,\boldsymbol{R})\,\mathcal{O}^\dagger_{\kappa,\lambda}(-T/2,\,\boldsymbol{r},\,\boldsymbol{R})|\text{vac}\rangle\right]$$

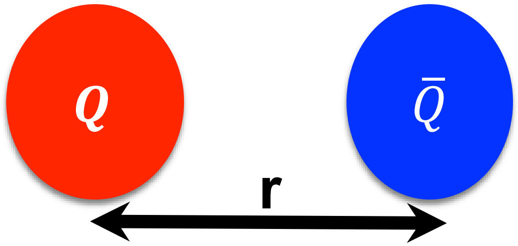


See Joan Soto and
Marc Wagner talk on Mon !

Exotic Hadron

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

$Q\bar{Q}X$



color: $3 \otimes \bar{3} = 1 \oplus 8$

X_8 = gluon → **Hybrid**

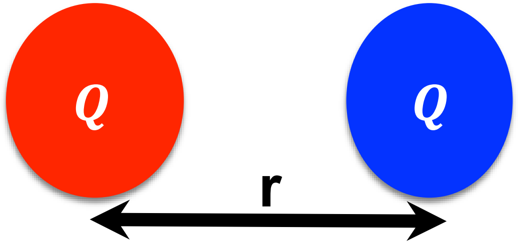
$X_8 = q\bar{q}$ → **Tetraquark / Molecule**

$X_8 = qqq$ → **Pentaquark / Molecule** and so on

Total angular momentum of **QQ̄X** or **QQX** :

$$\mathbf{J} = \mathbf{L}_{\mathbf{Q\bar{Q}}} + \mathbf{K} + \mathbf{S}_{\mathbf{Q\bar{Q}}}$$

QQX



color: $3 \otimes \bar{3} = \bar{3} \oplus 6$

X= q → **Double heavy baryon**

$X = \bar{q}\bar{q}$ → **Tetraquark**

$X = q\bar{q}q$ → **Pentaquark** and so on

BO static potentials $E^{(0)}_{\kappa,|\lambda|}$: LDF (light quarks, gluons) energies **in presence of two heavy quarks**

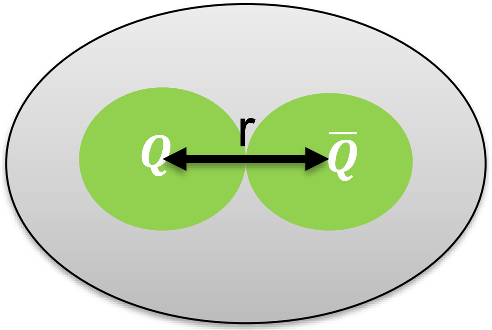
BOEFT can address all these states with inputs from Lattice QCD on BO potentials !

BOEFT: Potentials

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

LDF-quantum #: $\kappa = \{K^{PC}, f\}$

BO-quantum #: Λ_η^σ



Short-distance ($r \to 0$)

$$(Q\bar{Q})_1 : \; E^{(0)}_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+} r^2 + \cdots$$

$$(Q\bar{Q})_8 X_8 : \; E^{(0)}_{\Lambda_\eta^\sigma}(r) = V_o(r) + \Lambda_{H_\kappa} + b_{\Lambda_\eta^\sigma} r^2 + \cdots$$

$$(QQ)_l X_l : \; E^{(0)}_{\Lambda_\eta^\sigma}(r) = V_l(r) + \Lambda_{H_\kappa, l} + b_{\kappa\lambda, l}\, r^2 + \cdots \quad (l = T, \Sigma)$$

$$V_s(r) = -\frac{4\alpha_s}{3r}, \; V_o(r) = \frac{\alpha_s}{6r}$$

$$V_T(r) = -\frac{2\alpha_s}{3r}, \; V_\Sigma(r) = \frac{\alpha_s}{3r}$$

Λ_{H_κ}: **Adjoint hadron** mass for **$Q\bar{Q}X$** states

: **Triplet or sextet hadron** mass for **QQX** states

BOEFT: Potentials

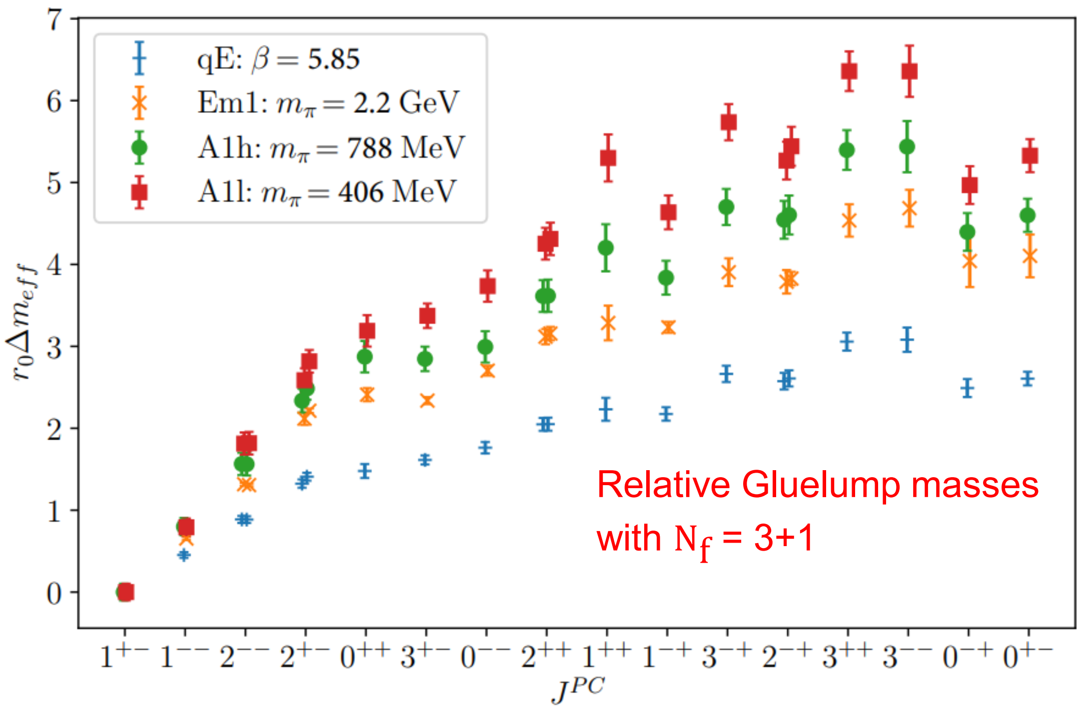
Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

$$\Lambda_{H_\kappa} = \lim_{T\to\infty} \frac{i}{T} \langle \text{vac} | H^a_\kappa(T/2,\, \boldsymbol{R})\, \phi^{ab}\,(T/2, -T/2)\, H^{a\dagger}_\kappa(-T/2,\, \boldsymbol{R}) | \text{vac} \rangle$$

Foster, Michael (UKQCD) Phys. Rev. D 59 (1999)

Campbell, Jorysz, Michael Phys. Lett. B 167 (1986)

- Gluelump / adjoint meson or baryon mass for **$Q\bar{Q}X$** states
- Triplet meson or baryon / Sextet meson or baryon mass for **QQX** states



Höllwieser, Knechtli, Korzec, Peardon, Struckmeier, Andres Urea-Nino
PoS LATTICE2024 (2025) 102

- Adjoint meson (1^{--} & 0^{-+}) : quenched with valence quarks

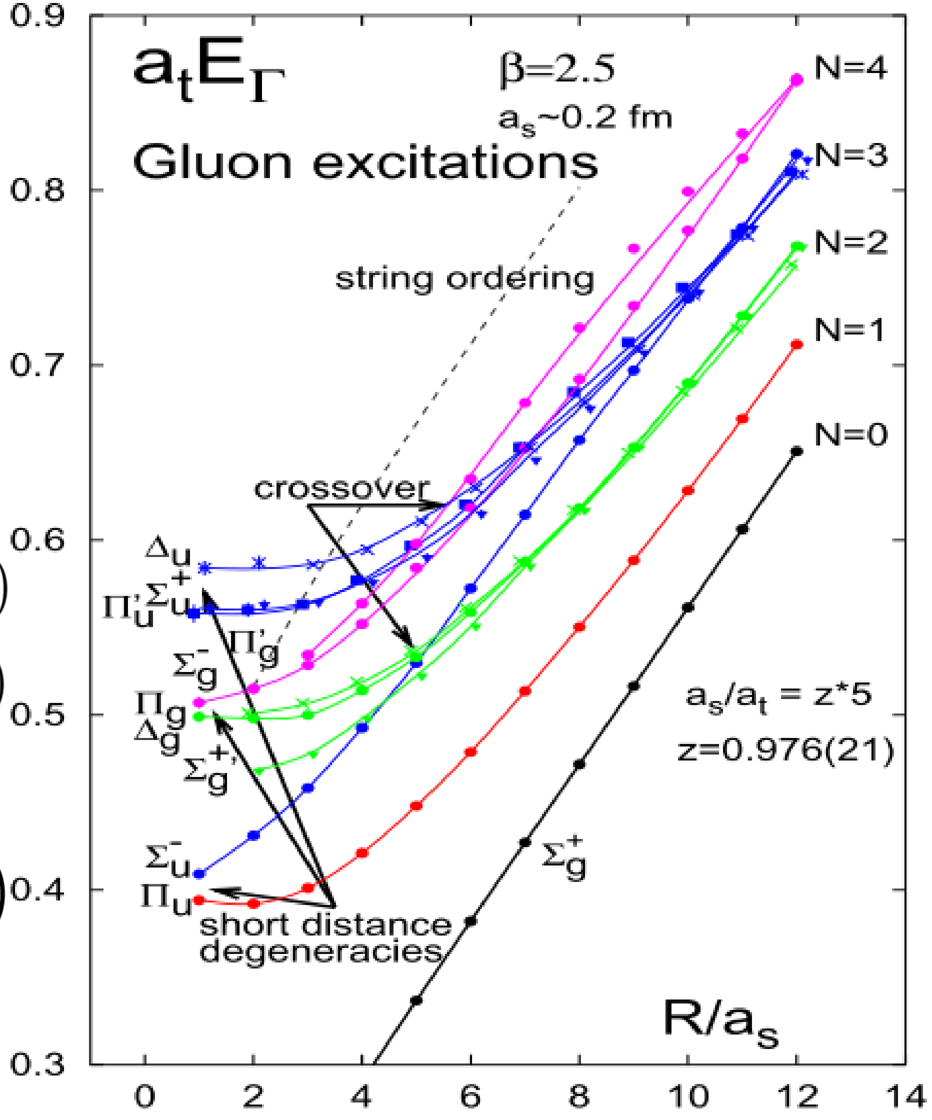
  Foster, Michael (UKQCD) Phys. Rev. D 59 (1999)

$$m_A\left(1^{--}\right) - m_G\left(1^{+-}\right) = -10(103)\ \text{MeV}$$
$$m_A\left(0^{-+}\right) - m_G\left(1^{+-}\right) = 34(161)\ \text{MeV}$$

New results on **isospin=1** adjoint mesons **!!!**

- In context of **QQX states**, emergence of **diquark**: BOEFT operator for triplet meson coincides with good diquark but directly gauge-invariant !

BO potentials : Quenched



0 ← r → ∞

Λ_η^σ corresponding to **gluelump** quantum # K^{PC}

$$2^{+-}\ (\Sigma_u^+, \Pi'_u, \Delta_u)$$
$$2^{--}\ (\Delta_g, \Sigma_g^-, \Pi'_g)$$
$$1^{--}\ (\Sigma_g^{+\prime}, \Pi_g)$$
$$1^{+-}\ (\Pi_u, \Sigma_u^-)$$

$$N = 3\,(\Sigma_u^-, \Sigma_u^+, \Pi'_u, \Delta_u, \cdots)$$
$$N = 2\,(\Sigma_g^{+\prime}, \Pi_g, \Delta_g)$$
$$N = 1\,(\Pi_u)$$
$$N = 0\,(\Sigma_g^+)$$

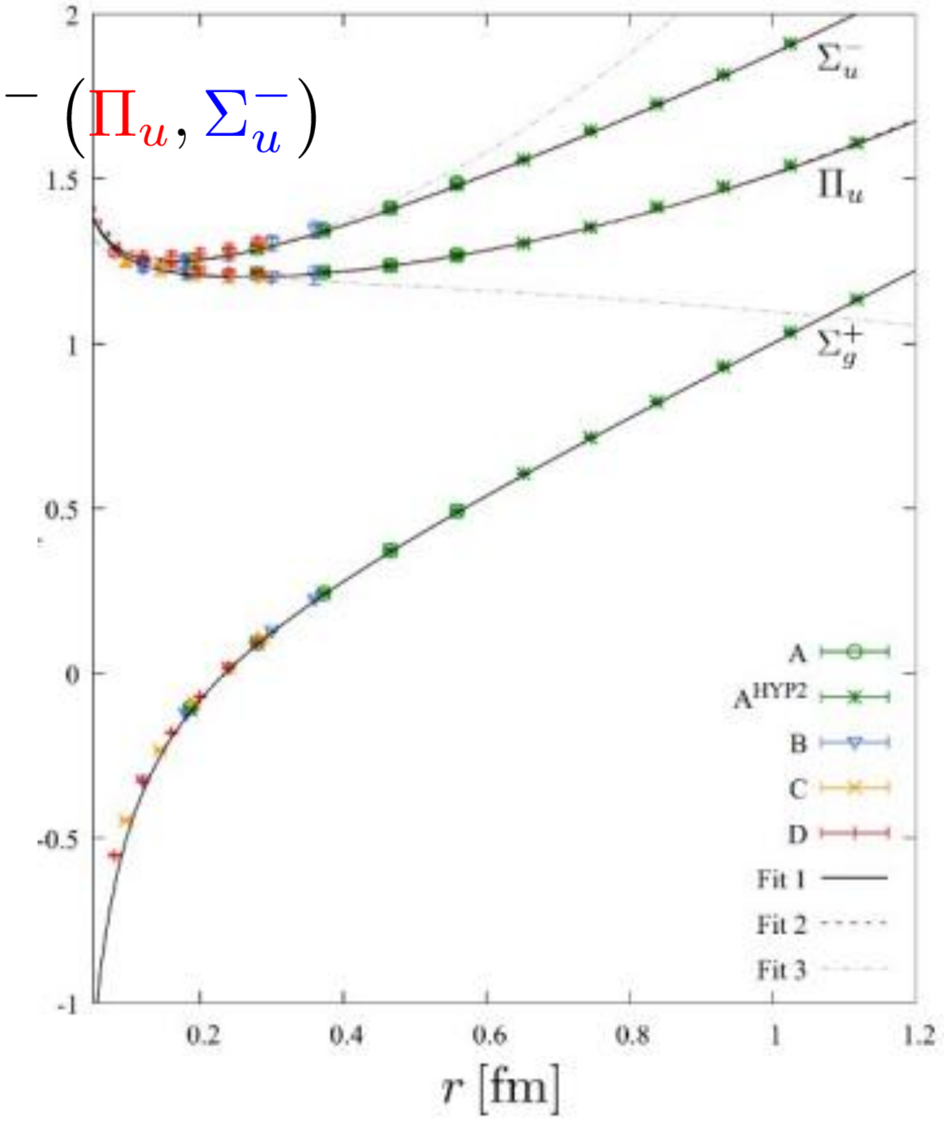
Observation:
BO-quantum # Λ_η^σ **conserved** at all values of r

K. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)

BO potentials: Quenched

gluelump
quantum # K^{PC}

$1^{+-}\ (\Pi_u, \Sigma_u^-)$



Σ_g^+: Quarkonium Potential

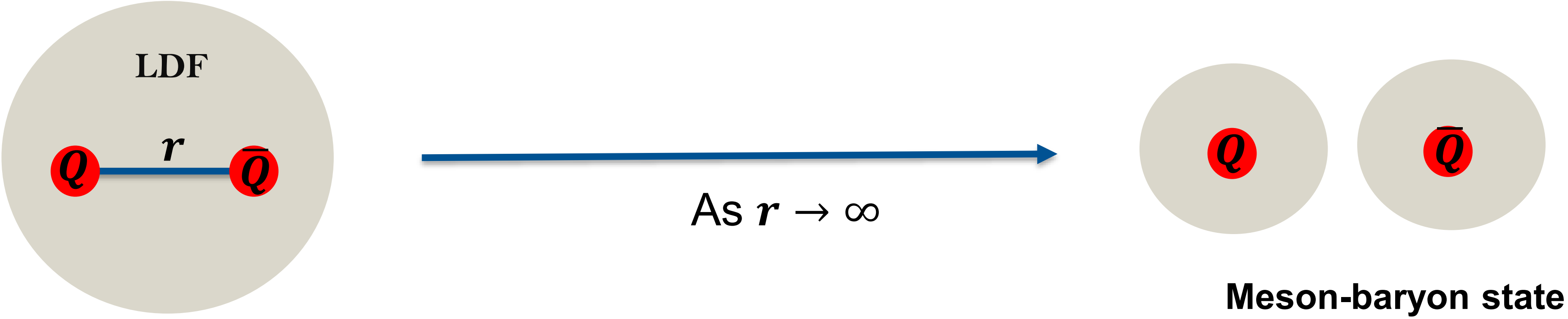
(Σ_u^-, Π_u): Quarkonium Hybrid potentials

Observation:
BO-quantum # Λ_η^σ **conserved** at all values of r

Schlosser and Wagner
Phys. Rev. D. 105, (2022)

Pentaquarks

BO potentials: Pentaquarks



Consider $Q\bar{Q}qqq$ system:

BO-quantum # Λ_η^σ as $r \to 0$:

$Q\bar{Q}$ color state	Light spin k^P	BO quantum # $D_{\infty h}$
Octet 8	$(1/2)^+$	$(1/2)_g$
	$(3/2)^+$	$\{(1/2)'_g, (3/2)_g\}$

There are **two** $k^P = (1/2)^+$ states

BO-quantum # Λ_η^σ for meson-baryon as $r \to \infty$

$k^P_{qq} \otimes k^P_q$	k^P	BO quantum # $D_{\infty h}$
$0^+ \otimes (1/2)^+$	$(1/2)^+$	$(1/2)_g$
$1^+ \otimes (1/2)^+$	$(1/2)^+$	$(1/2)_g$
	$(3/2)^+$	$\{(1/2)'_g, (3/2)_g\}$

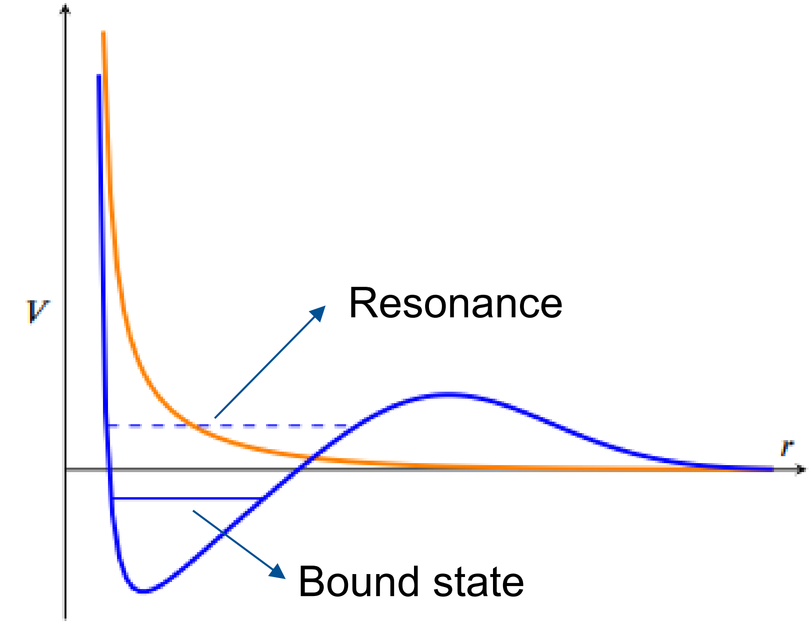
$\Lambda_c - \bar{D}$ threshold

$\Sigma_c - \bar{D}$ threshold

BO-quantum # Λ_η^σ conservation: Pentaquark potentials at small r should smoothly connect to meson-baryon threshold at large r !

Brambilla, AM, Vairo, arXiv 2508.13050

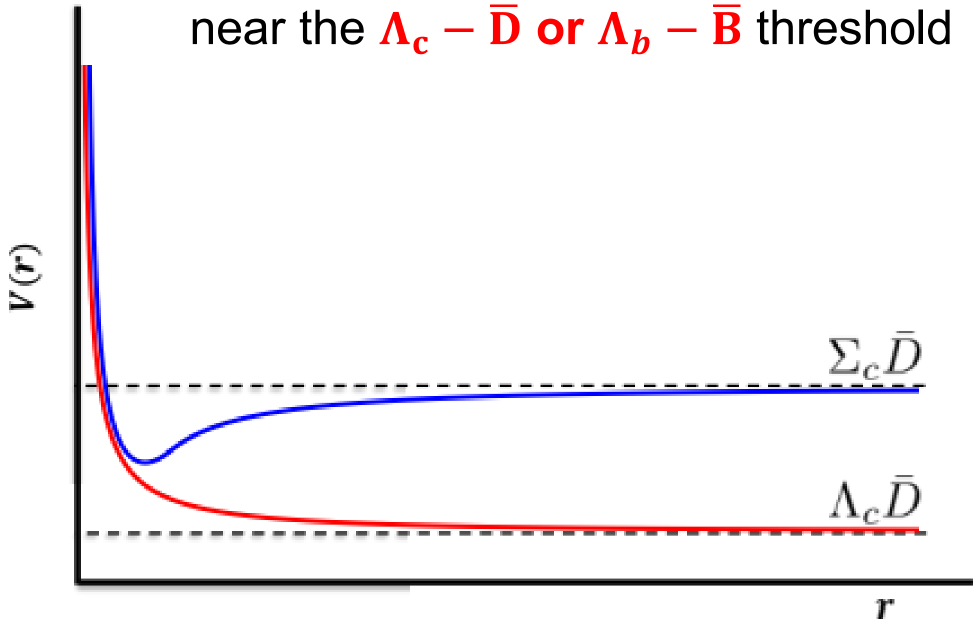
BO potentials: Pentaquarks



Braaten, Bruschini

Phys. Lett. B 863 (2025) 139386

No pentaquark states observed near the $\boldsymbol{\Lambda_c - \bar{D}}$ **or** $\boldsymbol{\Lambda_b - \bar{B}}$ threshold



Intermediate region dictates if there are bound states or resonance !.

Pentaquark

Brambilla, AM, Vairo, arXiv 2508.13050

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Lowest pentaquark multiplets:

$Q\bar{Q}$ color state	Light spin k^P	BO quantum # $D_{\infty h}$	l	J^P $\{S_Q=0, S_Q=1\}$
Octet	$(1/2)^+$	$(1/2)_g$	$1/2$	$\{1/2^-, (1/2, 3/2)^-\}$
$\mathbf{8}$	$(3/2)^+$	$\{(1/2)'_g, (3/2)_g\}$	$3/2$	$\{3/2^-, (1/2, 3/2, 5/2)^-\}$

No lattice QCD results on adjoint baryon mass: $\Lambda_{(1/2)^+}$ and $\Lambda_{(3/2)^+}$

Treat $\Lambda_{(1/2)^+}$ and $\Lambda_{(3/2)^+}$ as **free parameter** to fix on $P_{c\bar{c}}$ spectrum.

Coupled-channel Equations $k^P = (1/2)^+$:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\,\partial_r + \frac{(l-1/2)(l+1/2)}{m_Q r^2} + V_{(1/2)_g}\right]\psi^{(N)}_{(1/2)^+} = \mathcal{E}_{1/2}\,\psi^{(N)}_{(1/2)^+}$$

$l = 1/2$

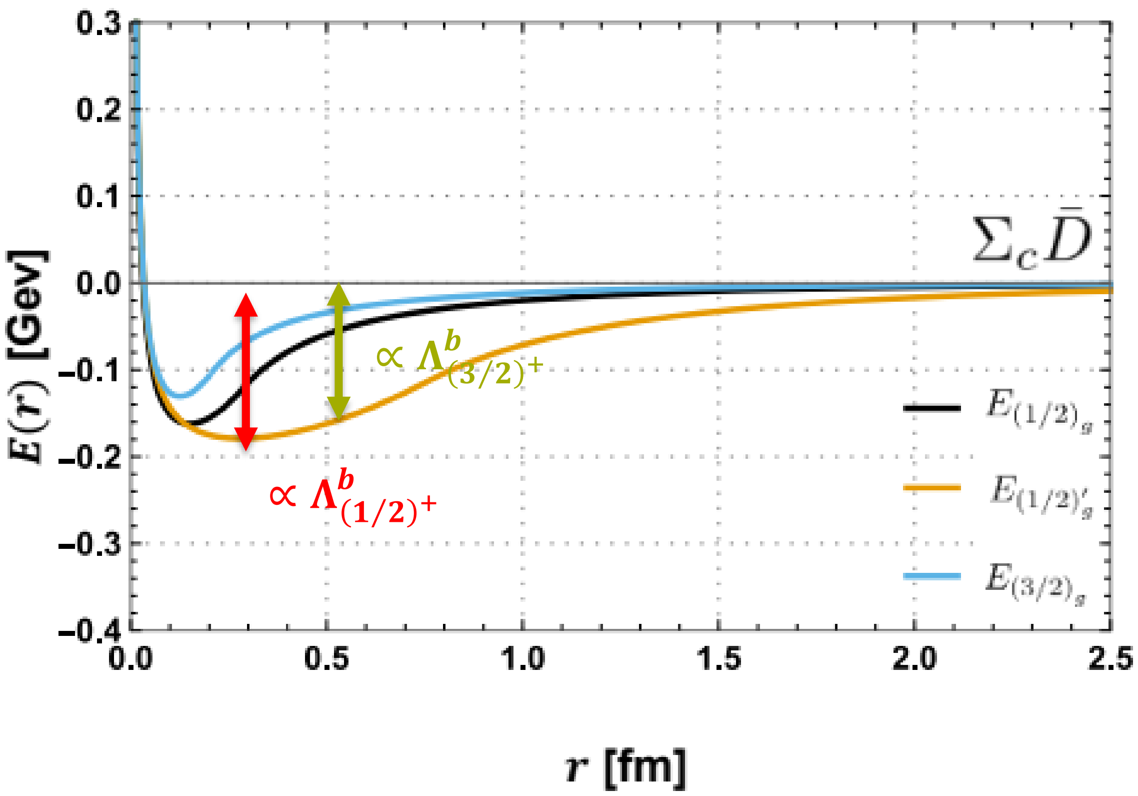
Coupled-channel Equations $k^P = (3/2)^+$:

$$\left[-\frac{1}{m_Q r^2}\partial_r r^2\partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l-1)-\frac{9}{4} & -\sqrt{3l(l+1)-\frac{9}{4}} \\ -\sqrt{3l(l+1)-\frac{9}{4}} & l(l+1)-\frac{3}{4}\end{pmatrix} + \begin{pmatrix} V_{(1/2)'_g} & 0 \\ 0 & V_{(3/2)_g}\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{1/2} \\ \psi^{(N)}_{3/2}\end{pmatrix} = \mathcal{E}_{3/2}\begin{pmatrix}\psi^{(N)}_{1/2} \\ \psi^{(N)}_{3/2}\end{pmatrix}$$

$l = 3/2$

Brambilla, AM, Vairo, arXiv 2508.13050

Pentaquark: $P_{c\bar{c}}$



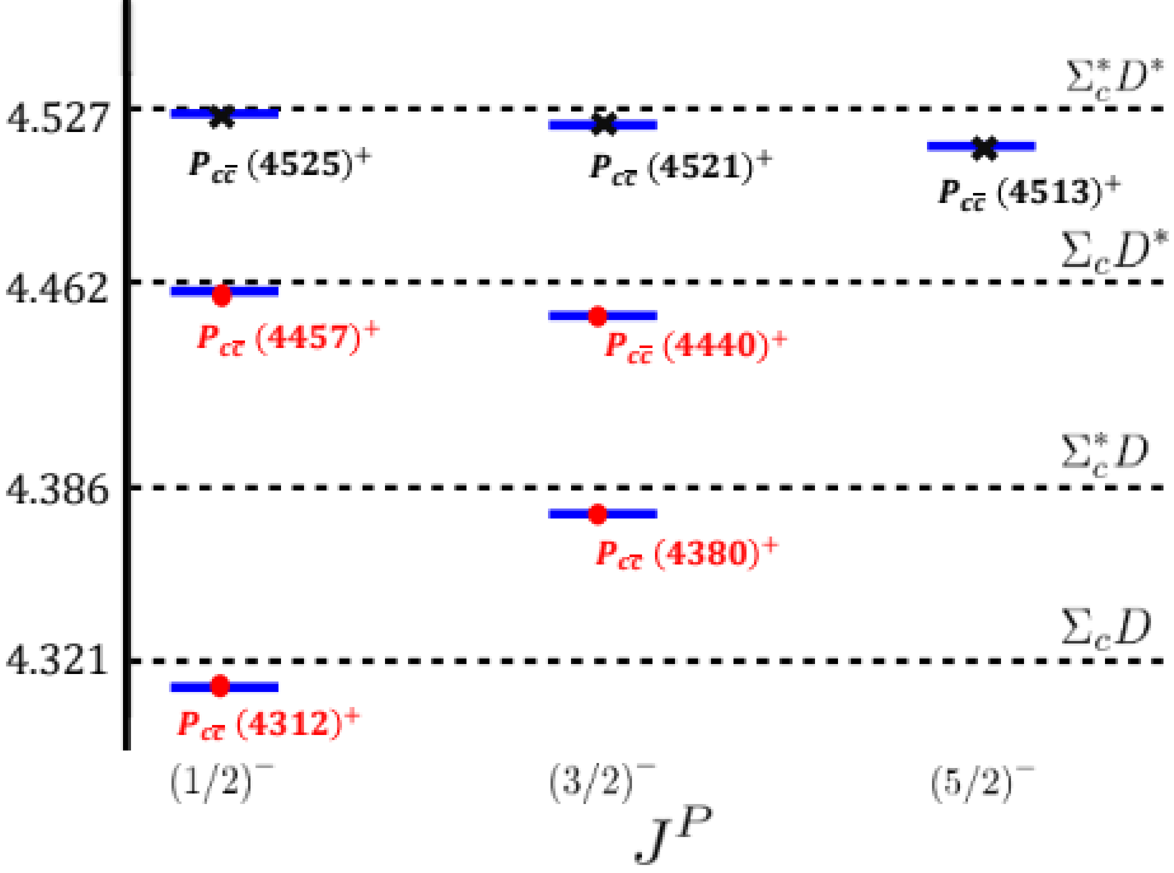
Adjoint baryon $(qqq)_8$ energy:

$\Lambda^b_{(1/2)^+} \approx$ **1125** MeV, $\Lambda^b_{(3/2)^+} \approx$ **1152** MeV (RS-scheme)

Spin-splitting in meson - baryon :

$$V_{SS} = \frac{2\Delta_1^Q}{3} \boldsymbol{S}_1 \cdot \boldsymbol{K}_1 + \Delta_2^Q \boldsymbol{S}_2 \cdot \boldsymbol{K}_2$$

First-order perturbation theory in Vss:



- ✓ Compact pentaquark model: 10 states (some with parity = +)

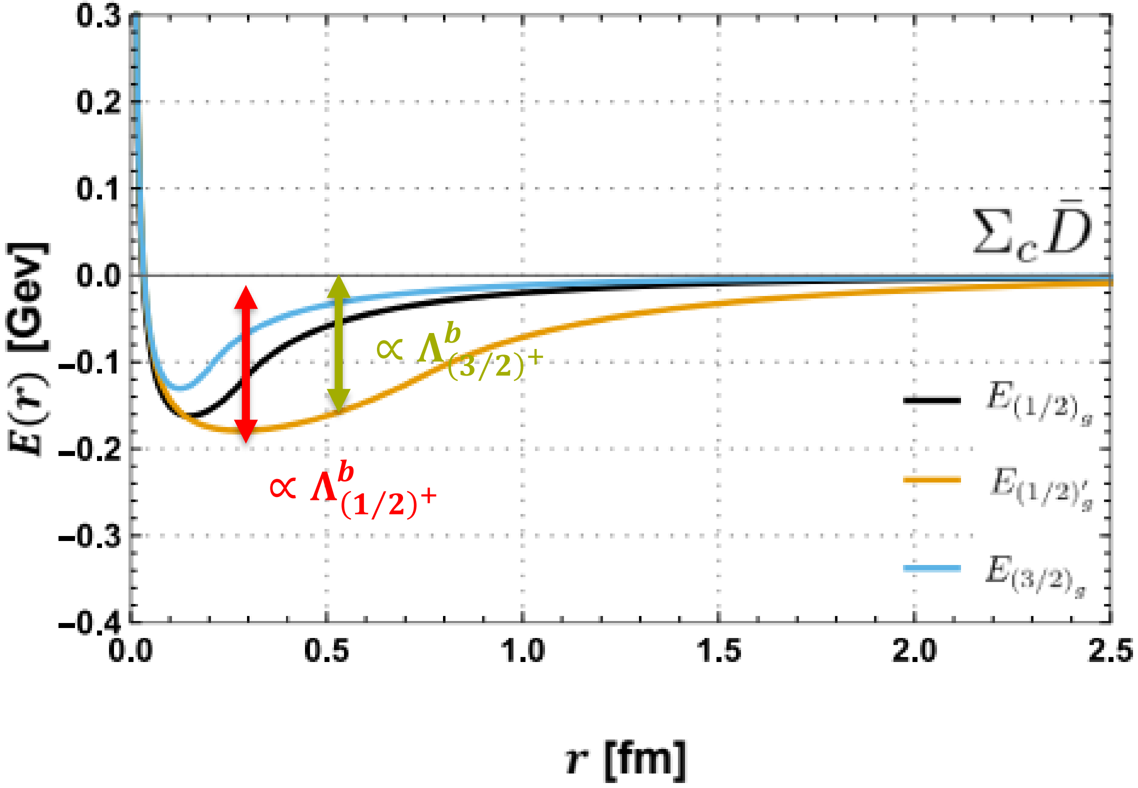
Maiani, Polosa, Riquer Phys. Lett. B, 749 (2015) Ali, Parkhomenko, Phys. Lett. B, 793 (2019)

- ✓ Molecular model: 7 states

Du, Baru, Guo, Hanhart, Meissner, Oller, Wang, JHEP 08, 157, Phys. Rev. Lett. 124, 072001 (2020)

Brambilla, AM, Vairo, arXiv 2508.13050

Pentaquark: $P_{b\bar{b}}$



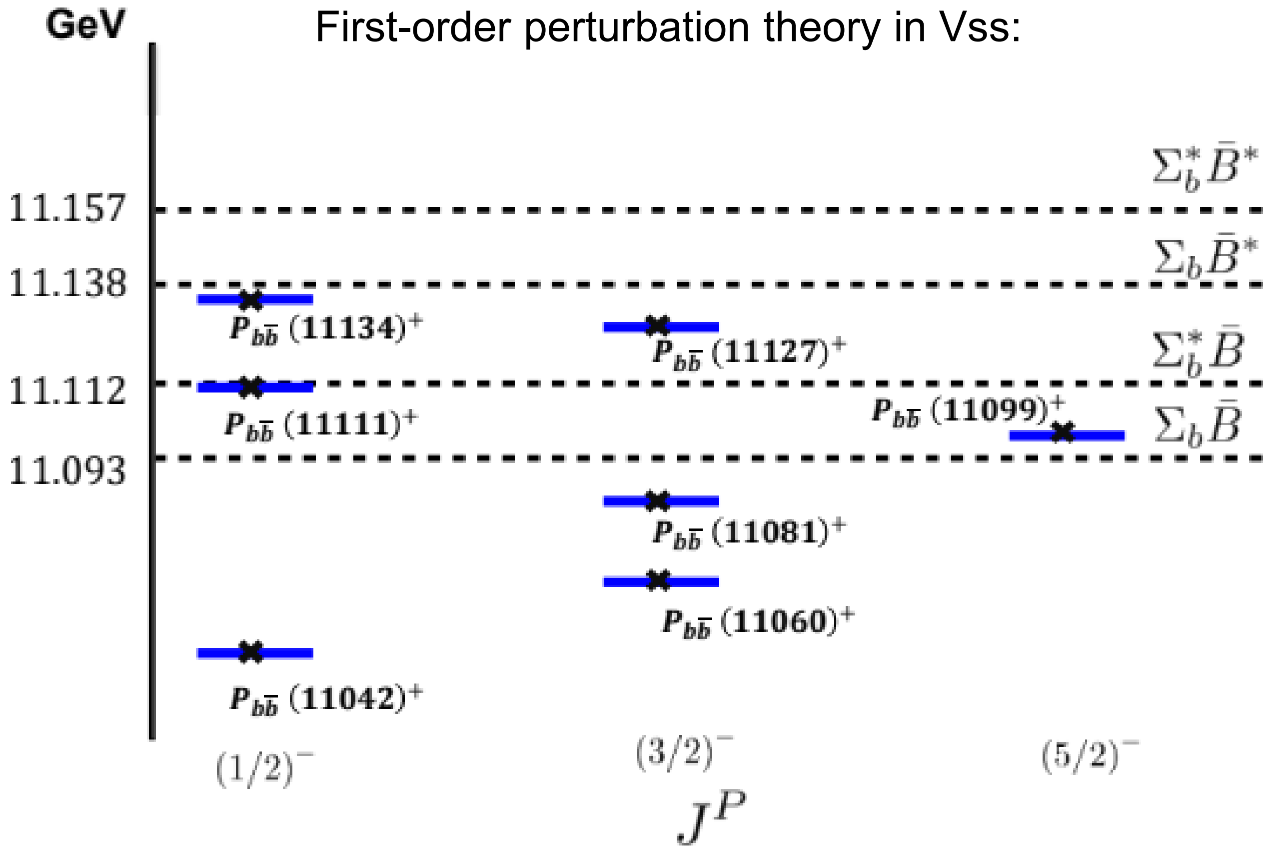
Adjoint baryon $(qqq)_8$ energy:

$\Lambda^b_{(1/2)^+} \approx \mathbf{1125}$ MeV, $\Lambda^b_{(3/2)^+} \approx \mathbf{1152}$ MeV (RS-scheme)

Spin-splitting in meson - baryon :

$$V_{SS} = \frac{2\Delta_1^Q}{3} \boldsymbol{S}_1 \cdot \boldsymbol{K}_1 + \Delta_2^Q \boldsymbol{S}_2 \cdot \boldsymbol{K}_2$$

First-order perturbation theory in Vss:



Pentaquark: Decays

- Semi-inclusive decays:

$$P_{c\bar{c}} \to Q_n + Y$$

Q_n: low-lying quarkonium Y: light hadrons

- ✓ ΔE: Large energy difference ⇒ $\Delta E \equiv E_{P_{c\bar{c}}} - E_n \gtrsim 1$ **GeV**.
- ✓ Decays to J/ψ through **spin-flipping M1 transitions.**
- ✓ Hierarchy of scales: $\Delta E \gg \Lambda_{QCD} \gg mv^2$ (Perturbative computation, for $\Delta E \gg \Lambda_{QCD}$)

Reported in PDG 2024:

$P_{c\bar{c}}$	Γ (total width)
$P_{c\bar{c}}(4312)$	10 ± 5 MeV
$P_{c\bar{c}}(4380)$	210 ± 90 MeV
$P_{c\bar{c}}(4440)$	21^{+10}_{-11} MeV
$P_{c\bar{c}}(4457)$	$6.4^{+6}_{-2.8}$ MeV

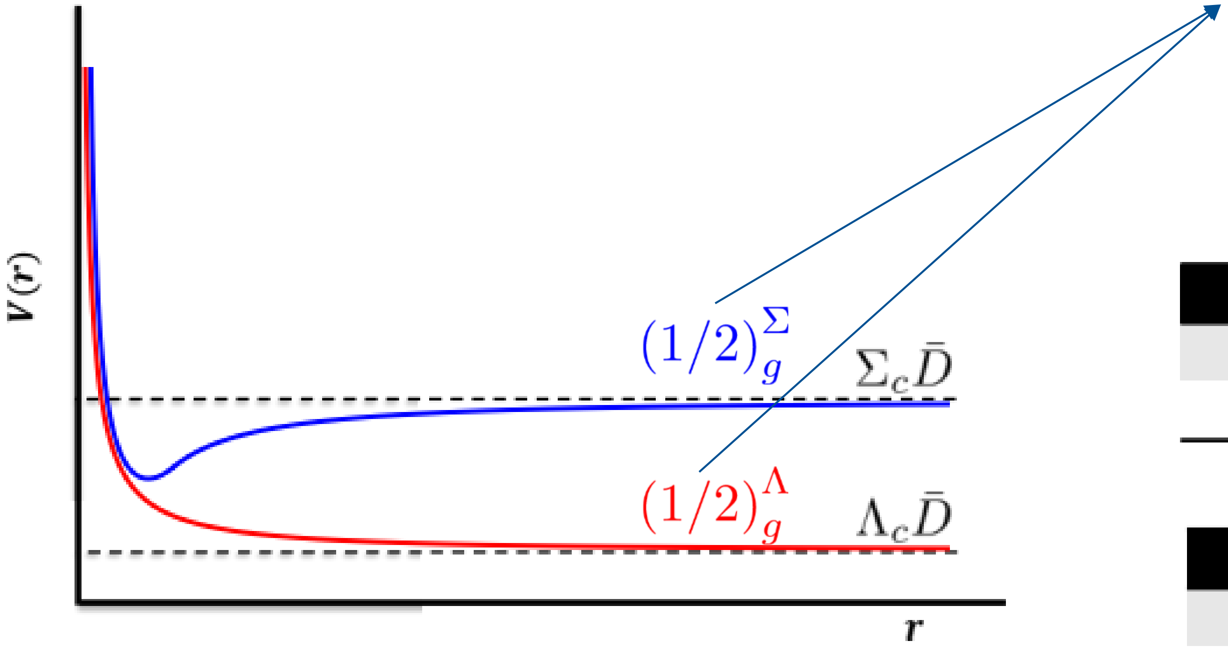
Semi-inclusive decays:

$P_{c\bar{c}}$	J^P	$\Gamma\,(P_{c\bar{c}} \to J/\psi + \cdots)$
$P_{c\bar{c}}(4312)$	$(1/2)^-$	$2^{+1}_{-1}{}^{+1}_{-1}$ MeV
$P_{c\bar{c}}(4457)$	$(1/2)^-$	$1^{+0.5}_{-0.3}{}^{+0.4}_{-0.3}$ MeV
$P_{c\bar{c}}(4380)$	$(3/2)^-$	$14^{+6}_{-3}{}^{+5}_{-4}$ MeV
$P_{c\bar{c}}(4440)$	$(3/2)^-$	$20^{+9}_{-5}{}^{+8}_{-6}$ MeV

➢ $P_{c\bar{c}}\left[J^P = (5/2)^-\right]$ only decays to η_c !!!

Pentaquark: Decays

- **Ratio of decays** to $\Lambda_c - \bar{D}$ and $\Lambda_c - \overline{D^*}$: $\frac{\Gamma(P_{c\bar{c}} \to \Lambda_c - \bar{D})}{\Gamma(P_{c\bar{c}} \to \Lambda_c - \overline{D^*})}$



- Decay happens through **transition amplitude $g^{\Sigma-\Lambda}_{(1/2)}$**
- $g^{\Sigma-\Lambda}_{(1/2)}$ currently unknown can be **determined by lattice QCD**

$J^P = (1/2)^-$

	$P_{c\bar{c}}(4312)^+$	$P_{c\bar{c}}(4440)^+$	$P_{c\bar{c}}(4507)^+$
$\Lambda_c \bar{D}$	0	3.52	5.48
$\Lambda_c \bar{D}^*$	3.59	3.24	2.17

$J^P = (3/2)^-$

	$P_{c\bar{c}}(4380)^+$	$P_{c\bar{c}}(4457)^+$	$P_{c\bar{c}}(4515)^+$
$\Lambda_c \bar{D}$	0	0	0
$\Lambda_c \bar{D}^*$	3.74	1.17	4.09

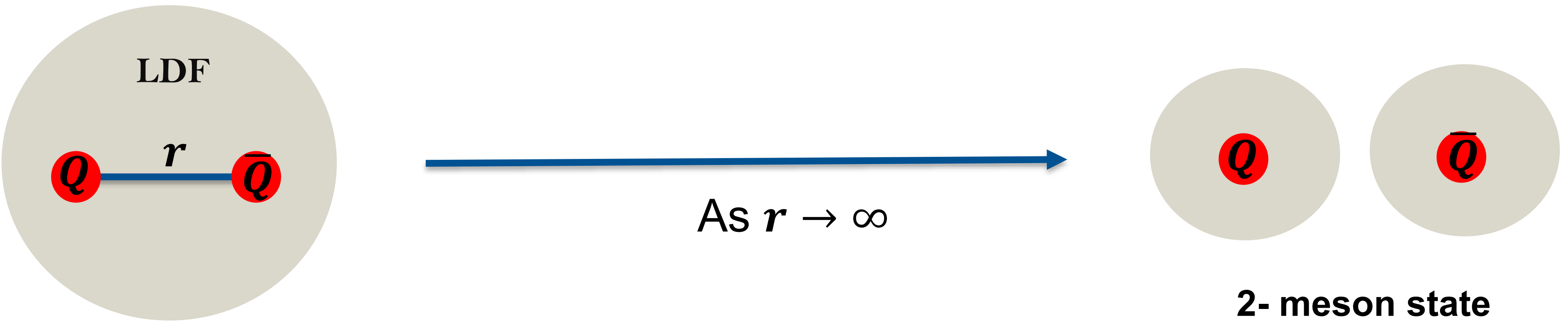
$J^P = (5/2)^-$

	$P_{c\bar{c}}(4526)^+$
$\Lambda_c \bar{D}$	0
$\Lambda_c \bar{D}^*$	0

- $P_{c\bar{c}}(4380)$, narrower state, compared to experimental broad width
- $P_{c\bar{c}}\left[J^P = (5/2)^-\right]$ decays to $\Lambda_c - \bar{D}$ and $\Lambda_c - \overline{D^*}$ in d-wave !!!

$\chi_{c1}(3872)$ & $T_{cc}^{+}(3875)$

BO potentials: Tetraquarks



Consider $Q\bar{Q}q\bar{q}$ system:

BO-quantum # Λ_η^σ as $r \to 0$:

$Q\bar{Q}$ (color)	Light Spin K^{PC}	Λ_η^σ $(D_{\infty h})$
Octet	0^{-+}	Σ_u^-
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$

BO-quantum # Λ_η^σ for meson-antimeson as $r \to \infty$

$K_{\bar{q}}^P \otimes K_q^P$	K^{PC}	Static energies $D_{\infty h}$
$(1/2)^- \otimes (1/2)^+$	0^{-+}	$\{\Sigma_u^-\}$
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$

s-wave+s-wave
Ex. **$D\bar{D}$** threshold

BO-quantum # Λ_η^σ conservation: Tetraquark potentials at small r should smoothly connect to meson pair threshold at large r !

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Braaten, Bruschini Phys. Lett. B 863 (2025) 139386

BO potential: Unquenched

String breaking: quarkonium & meson-antimeson

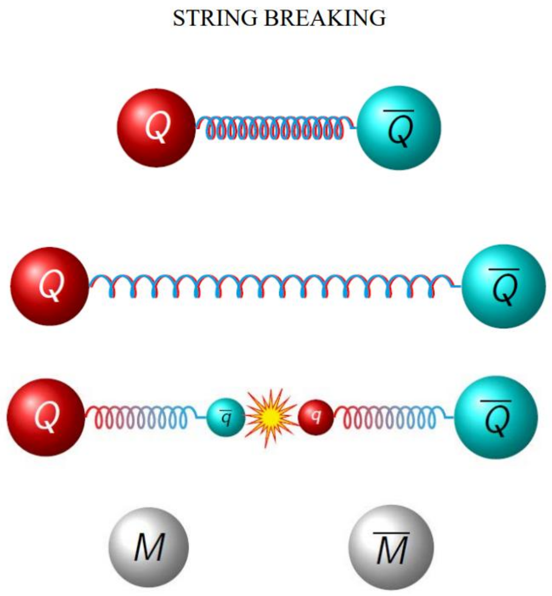
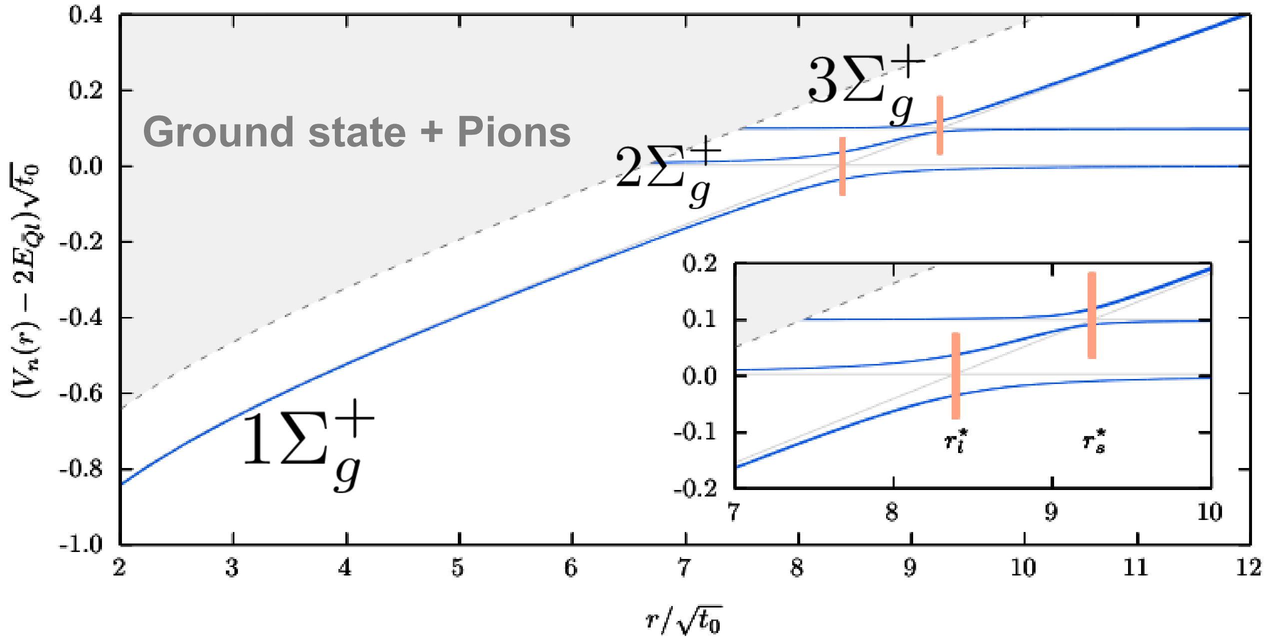


Figure from R. Bruschini talk

Bulava, Hoerz, Knechtli, Koch, Moir, Morningstar, Peardon, Phys. Lett. B. 793 (2019)

Bulava, Knechtli, Koch, Morningstar, Peardon, Phys. Lett. B. 854 (2024)

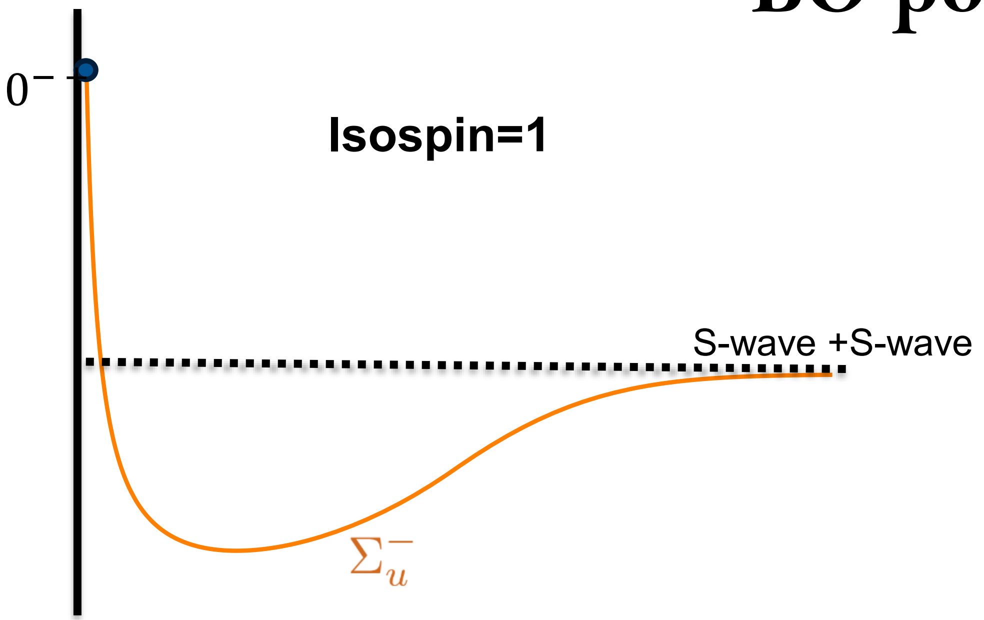
$$m_\pi \approx 200 - 340 \text{ MeV} \quad m_K \approx 440 - 480 \text{ MeV}$$



String breaking radius ≈ 1.22 fm

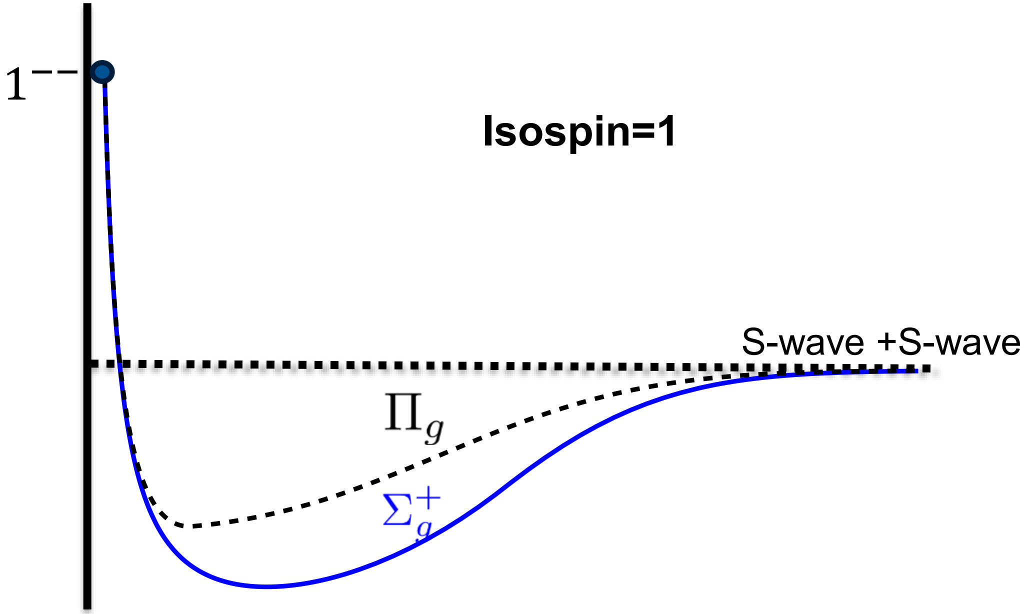
Avoided crossing between **degenerate** static potentials with **same** BO-quantum # Σ_g^+

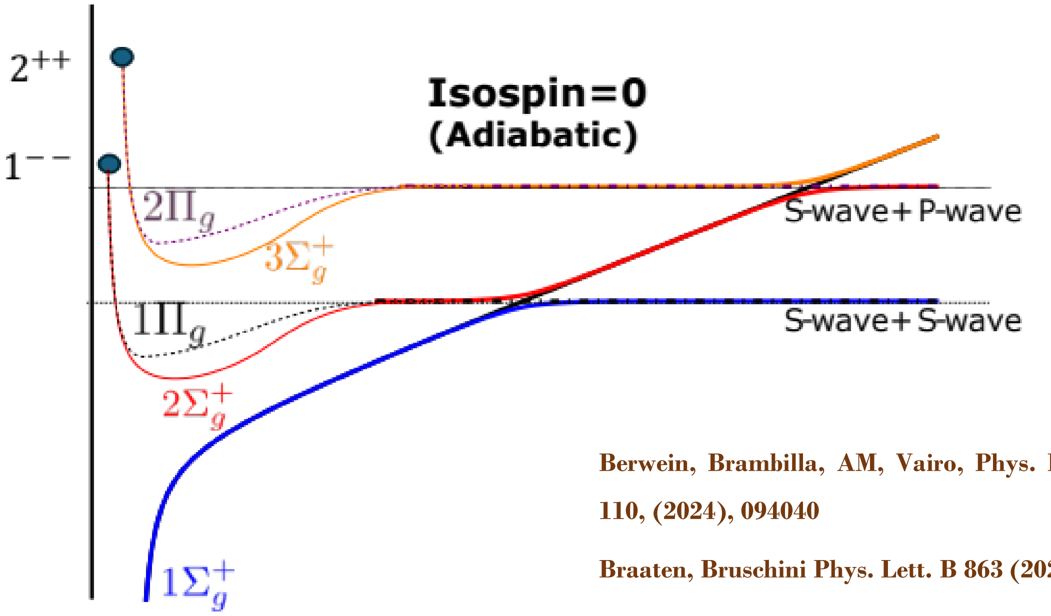
BO potentials: Tetraquarks



Key Takeaways: Tetraquark static potential behavior

- Repulsive behavior at small r due to adjoint color ($r \to 0$)
- Heavy meson pair threshold at large r ($r \to \infty$)
- Avoided crossing with quarkonium static energy (Isospin=0)





Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Braaten, Bruschini Phys. Lett. B 863 (2025) 139386

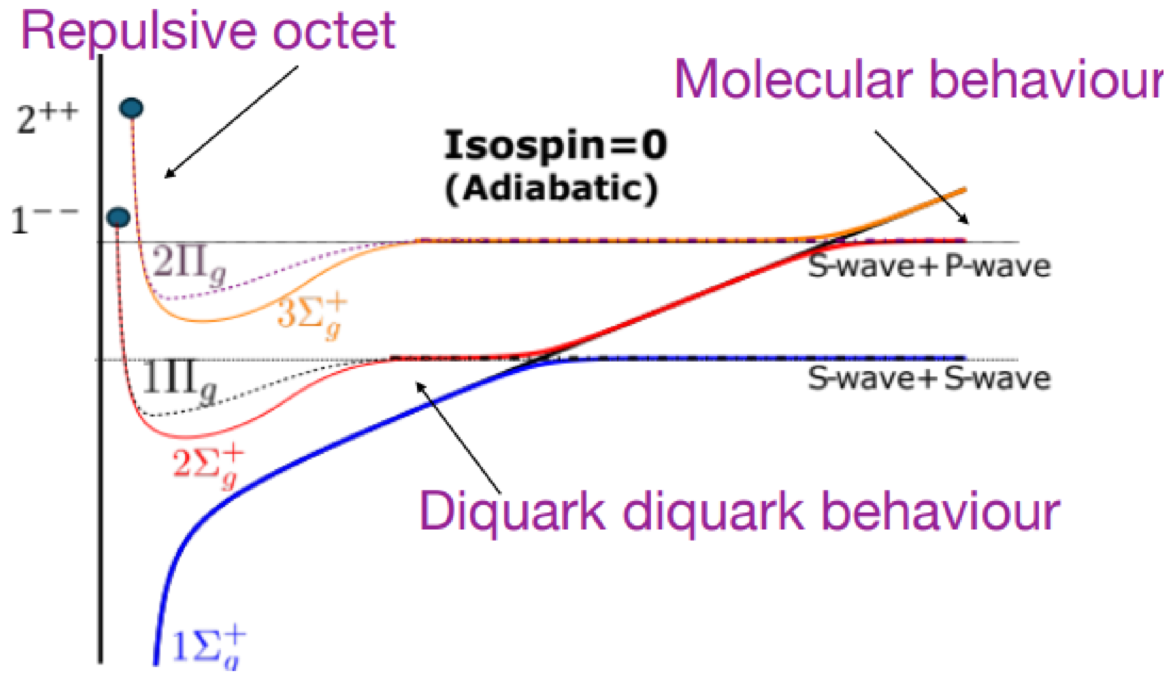
BO potentials

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

The BOEFT

Behavior of tetraquark static energy:

- Adjoint meson behavior at small r $(r \rightarrow 0)$
- Heavy meson pair threshold at large r $(r \rightarrow \infty)$



Fixed by symmetry And perturbation theory

to be calculated on the lattice

Fixed by symmetry

BOEFT Could subdue molecular and compact tetraquark pictures

BOEFT: $Q\bar{Q}q\bar{q}$ multiplets

Berwein, Brambilla, AM, Vairo,

Phys. Rev. D. 110, (2024), 094040

Isospin-0

$q\bar{q}$ spin k^{PC}	BO quantum # Λ_η^σ	l	J^{PC} $\{S_Q=0, S_Q=1\}$
1^{--}	$\Sigma_g^{+\prime}, \Pi_g$	1	$\{1^{+-}, (0,1,2)^{++}\}$
	$\Sigma_g^{+\prime}$	0	$\{0^{-+}, 1^{--}\}$
	Π_g	1	$\{1^{-+}, (0,1,2)^{--}\}$
	$\Sigma_g^{+\prime}, \Pi_g$	2	$\{2^{-+}, (1,2,3)^{--}\}$

Lowest multiplet with $\chi_{c1}(3872)$ **$J^{PC}=1^{++}$**

1^{--} Adjoint meson mass unknown.

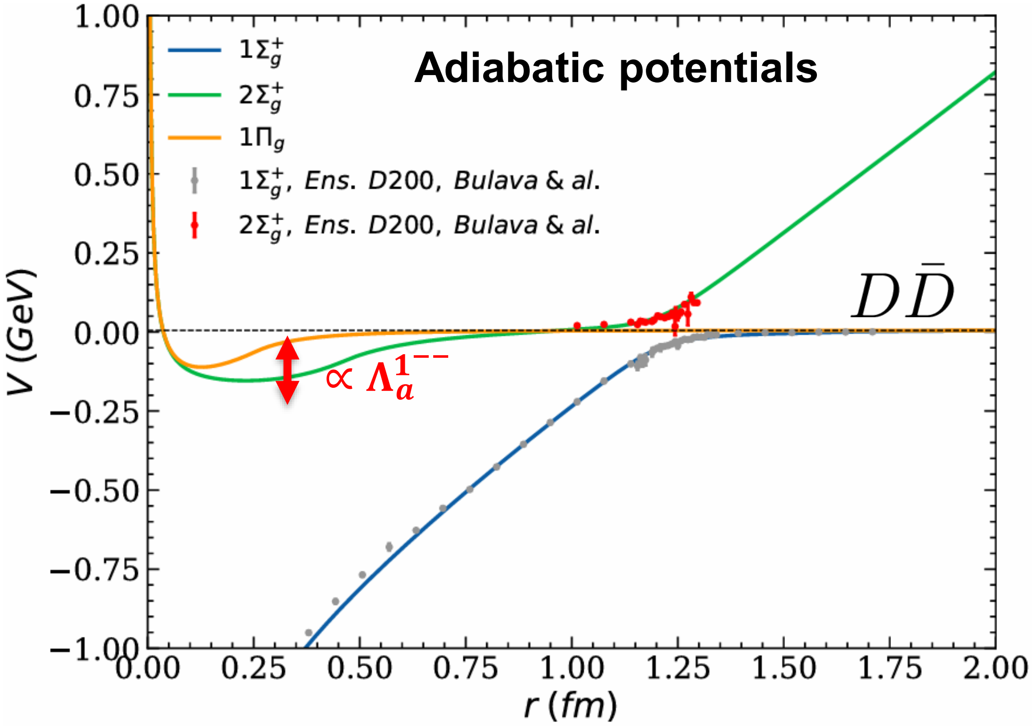
Fix it to reproduce $\chi_{c1}(3872)$.

Coupled-channel Equations:

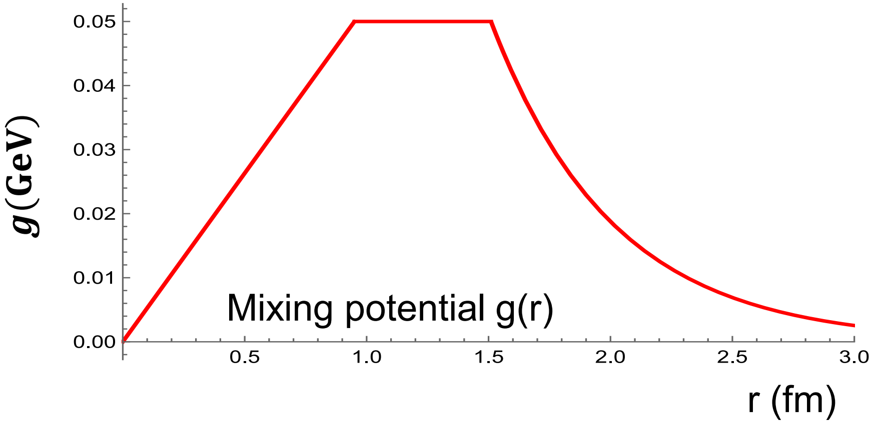
$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1) & 0 & 0 \\ 0 & l(l+1)+2 & -2\sqrt{l(l+1)} \\ 0 & -2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_{\Sigma_g^+}(r) & g(r) & 0 \\ g(r) & E_{\Sigma_g^{+\prime}}(r) & 0 \\ 0 & 0 & E_{\Pi_g}(r) \end{pmatrix}\right]\begin{pmatrix}\psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi\end{pmatrix} = \mathcal{E}\begin{pmatrix}\psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi\end{pmatrix}$$

Brambilla, AM, Scirpa, Vairo

Phys. Rev. Lett. 135 (2025), 131902



Bulava et al Phys. Lett. B. 854 (2024)



$\chi_{c1}(3872)$

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Brambilla, AM, Scirpa, Vairo Phys. Rev. Lett. 135 (2025), 131902

Coupled-channel Equations:

$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1) & 0 & 0 \\ 0 & l(l+1)+2 & -2\sqrt{l(l+1)} \\ 0 & -2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_{\Sigma_g^+}(r) & g(r) & 0 \\ g(r) & E_{\Sigma_g^{+\prime}}(r) & 0 \\ 0 & 0 & E_{\Pi_g}(r) \end{pmatrix}\right]\begin{pmatrix}\psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi\end{pmatrix} = \mathcal{E}\begin{pmatrix}\psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi\end{pmatrix}$$

$l = 1$

Results:

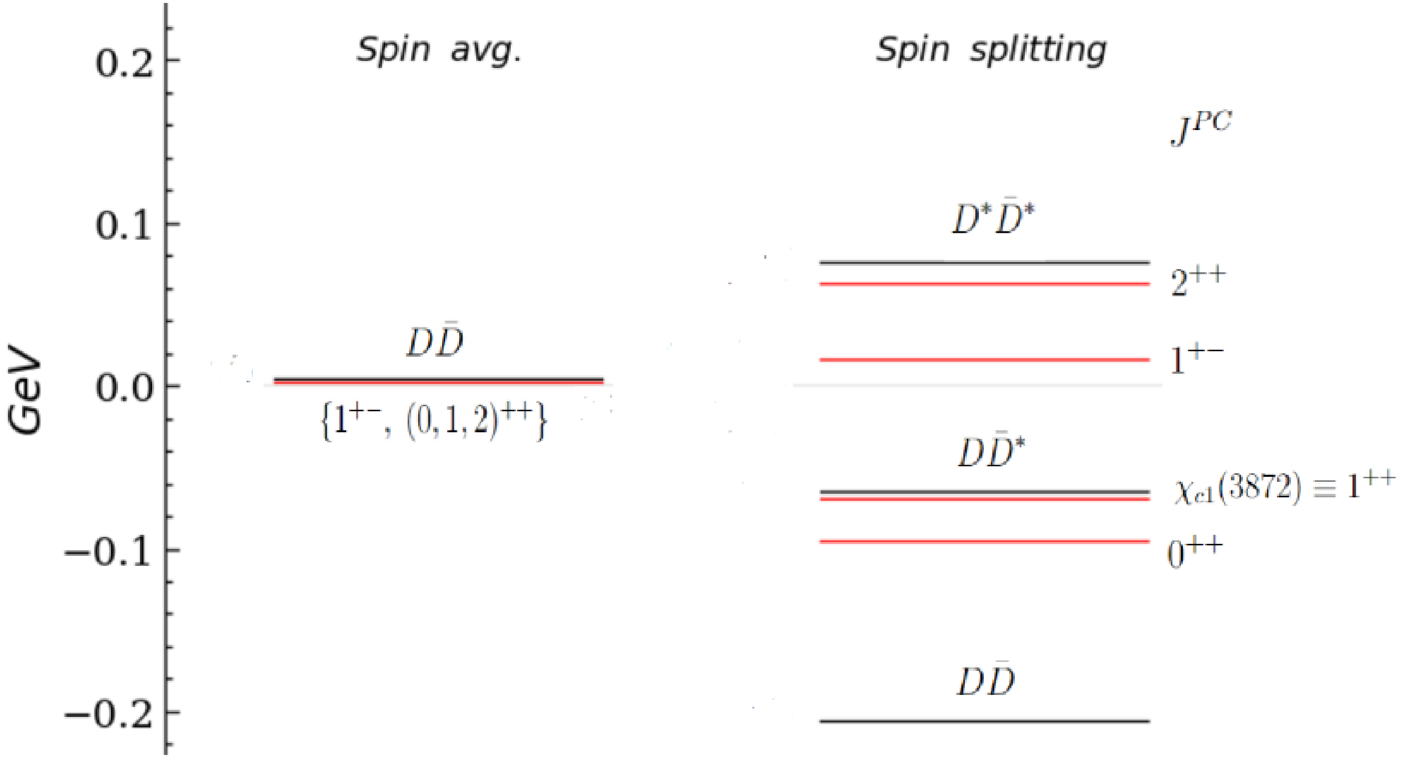
1) 2P Quarkonium percentage: $|\psi_\Sigma|^2 \approx 8\ \%$
2) Tetraquark percentage: $|\psi_{\Sigma'}|^2 \approx 38\ \%$, $|\psi_\Pi|^2 \approx 54\ \%$
3) Radius $\sim$ 15 fm.
4) Deeper state in bottom sector: 15 MeV below spin-isospin averaged $B\bar{B}$ threshold.

Adjoint $(q\bar{q})_8$: $\Lambda_a^{1^{--}} \approx$ **915** MeV (pole mass scheme)

Using lattice QCD spin-splitting results for hybrids ($Q\bar{Q}g$)

$\chi_{c1}(3872)$

Brambilla, AM, Scirpa, Vairo
Phys. Rev. Lett. 135 (2025), 131902



Adjoint $(q\bar{q})_8$ energy: $\Lambda_a^{1^{--}} \approx$ **914** MeV :
No bound states in higher multiplets T_2^1, T_3^1, T_4^1 ...

Note:
$\chi_{c1}(3872)$ and $\chi_{c1}(2P)$ both with **$J^{PC}=1^{++}$** are two distinct states.

1^{++} state: Identified with $\chi_{c1}(3872)$

1^{+-} state: Mass around 3.957 (11) GeV. Identified with X(3940) ?

2^{++} state: Mass around 4.004 (14) GeV.

0^{++} state: Mass around 3.846 (11) GeV.

$\chi_{c1}(3872)$

Brambilla, AM, Scirpa, Vairo 2411.14306

1) Quarkonium percentage: $|\psi_\Sigma|^2 \approx 8\,\%$
2) Tetraquark percentage: $|\psi_{\Sigma'}|^2 \approx 38\,\%$, $|\psi_\Pi|^2 \approx 54\,\%$
3) Radius > 15 fm.

We **naturally** get $8-13\,\%$ quarkonium component in $\chi_{c1}(3872)$ due to **avoided level crossing**

Radiative decays:

$$\mathcal{R}_{\gamma\psi} = \frac{\Gamma_{\chi_{c1}(3872)\to\gamma\psi(2s)}}{\Gamma_{\chi_{c1}(3872)\to\gamma J\psi}},$$

Our estimate: $\boldsymbol{R_{\gamma\psi} = 2.99 \pm 2.36}$ (assuming only through $\chi_{c1}(2P)$ **component**)

LHCb: $\boldsymbol{R_{\gamma\psi} = 1.67 \pm 0.25}$ Aaij et al arXiv: 2406.17006

Compositeness:

BES III: $\boldsymbol{Z = 0.18^{+0.20}_{-0.23}}$

Ablikim et al. Phys. Rev. Lett 132, 151903 (2024)

EMPPR: $0.052 < Z < 0.14$

Esposito, Maiani, Pilloni, Polosa, Riquer Phys. Rev. D 105, L031503 (2022)

Agreement with our $8-13\,\%$ quarkonium (compact) component

Lattice QCD:

$c\bar{c}$ operator along with $D\bar{D}^*$ relevant for $\chi_{c1}(3872)$ signal

Padmanath, Lang, Prelovsek Phys. Rev. D 92, 034501 (2015) Prelovsek and Leskovec Phys. Rev. Lett 111, 192001 (2013)

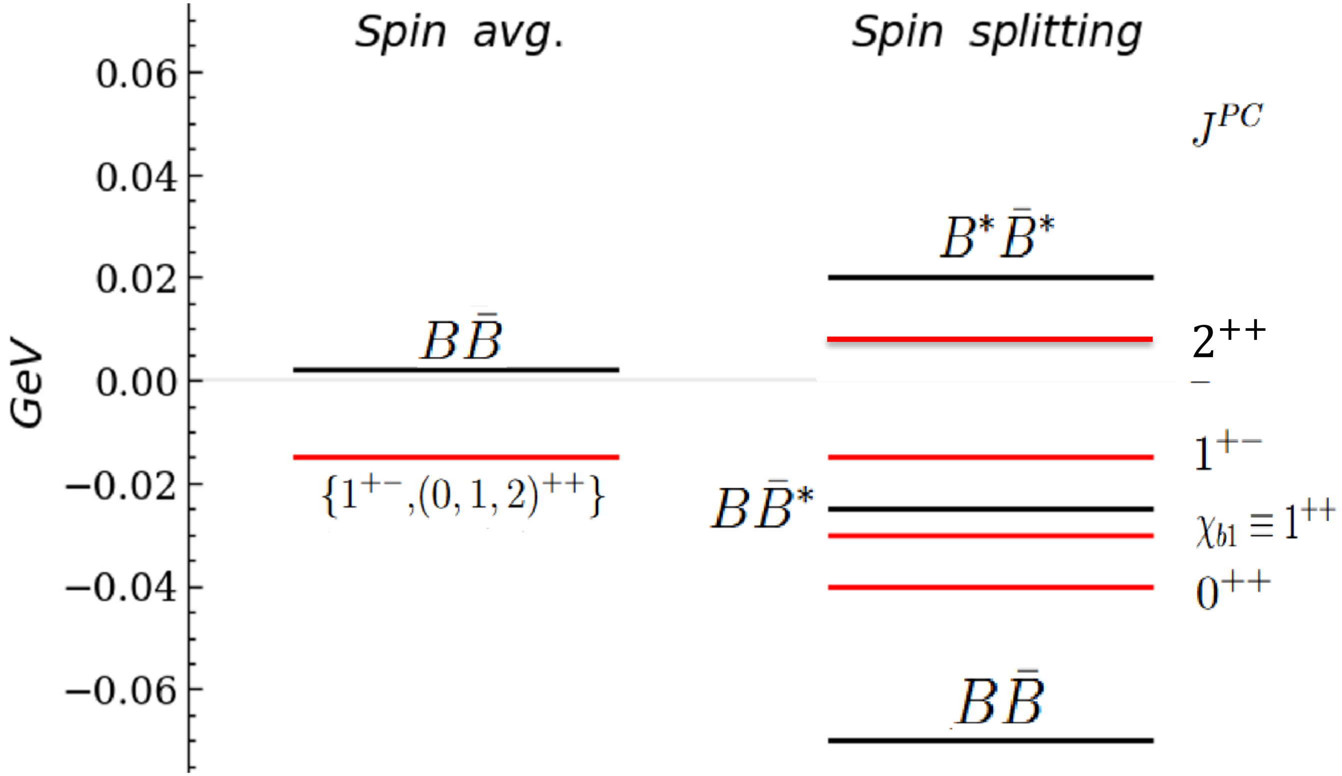
X_b

Brambilla, AM, Scirpa, Vairo
Phys. Rev. Lett. 135 (2025), 131902

Results with
adjoint meson energy $\approx$ **915** MeV

Using lattice QCD spin-splitting results for hybrids ($Q\bar{Q}g$)

1) Quarkonium percentage: $|\psi_\Sigma|^2 \approx 1.5\ \%$
2) Tetraquark percentage:
   $|\psi_{\Sigma'}|^2 \approx 45.4\ \%,\ |\psi_\Pi|^2 \approx\ 53.1\ \%$



Multiplet T_1^1: $\{1^{+-}, (0,1,2)^{++}\}$

1^{++}: identified with X_b: Mass around 10.595 GeV

1^{+-} state: Mass around 10.612 GeV.

2^{++} state: Mass around 10.635 GeV.

0^{++} state: Mass around 10.576 GeV.

BOEFT: $QQ\bar{q}\bar{q}$ multiplets

Berwein, Brambilla, AM, Vairo,
Phys. Rev. D. 110, (2024), 094040

light antiquarks

q q'

{qq'}, 1^+ [qq'], 0^+

"Bad diquark" **"Good diquark"**

Defines the Born-Oppenheimer static potentials $\Sigma_g^+, \{\Sigma_g^-, \Pi_g\}$

Bad diquark – Good diquark
≈ 200 MeV

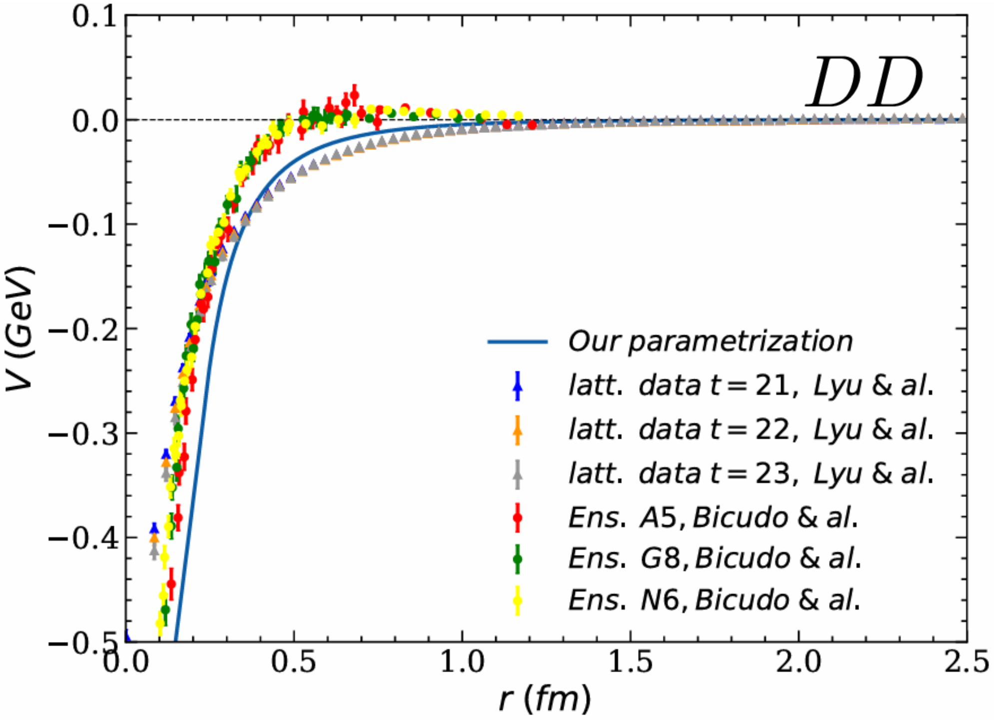
QQ color state	Light spin K^{PC}	Static energies	Isospin I	l	J^{PC} $S_Q = 0$	J^{PC} $S_Q = 1$
anti-triplet $\bar{3}$	0^+	$\{\Sigma_g^+\}$	0	0	—	1^+
				1	1^-	—
	1^+	$\{\Sigma_g^-, \Pi_g\}$	1	0	0^-	—
				1	1^-	$(0,1,2)^+$

J^P for T_{cc}^+

$T_{cc}^{+}(3875)$

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Brambilla, AM, Scirpa, Vairo Phys. Rev. Lett. 135 (2025), 131902



Good diquark : $\Lambda_t^{0^+} \approx \mathbf{664}$ MeV (pole mass scheme)

Lyu, Aoki, Doi, Hatsuda, Ikeda, Meng, Phys. Rev. Lett. 131, 161901 (2023)

Bicudo, Marinkovic, Mueller, Wagner, arXiv 2409.10786

Schrödinger Equation:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\,\partial_r + \frac{l(l+1)}{m_Q r^2} + V_{\Sigma_g^+}\right]\psi_{\Sigma_g^+} = \mathcal{E}_N\,\psi_{\Sigma_g^+}\,.$$

$l = 0$

Results:

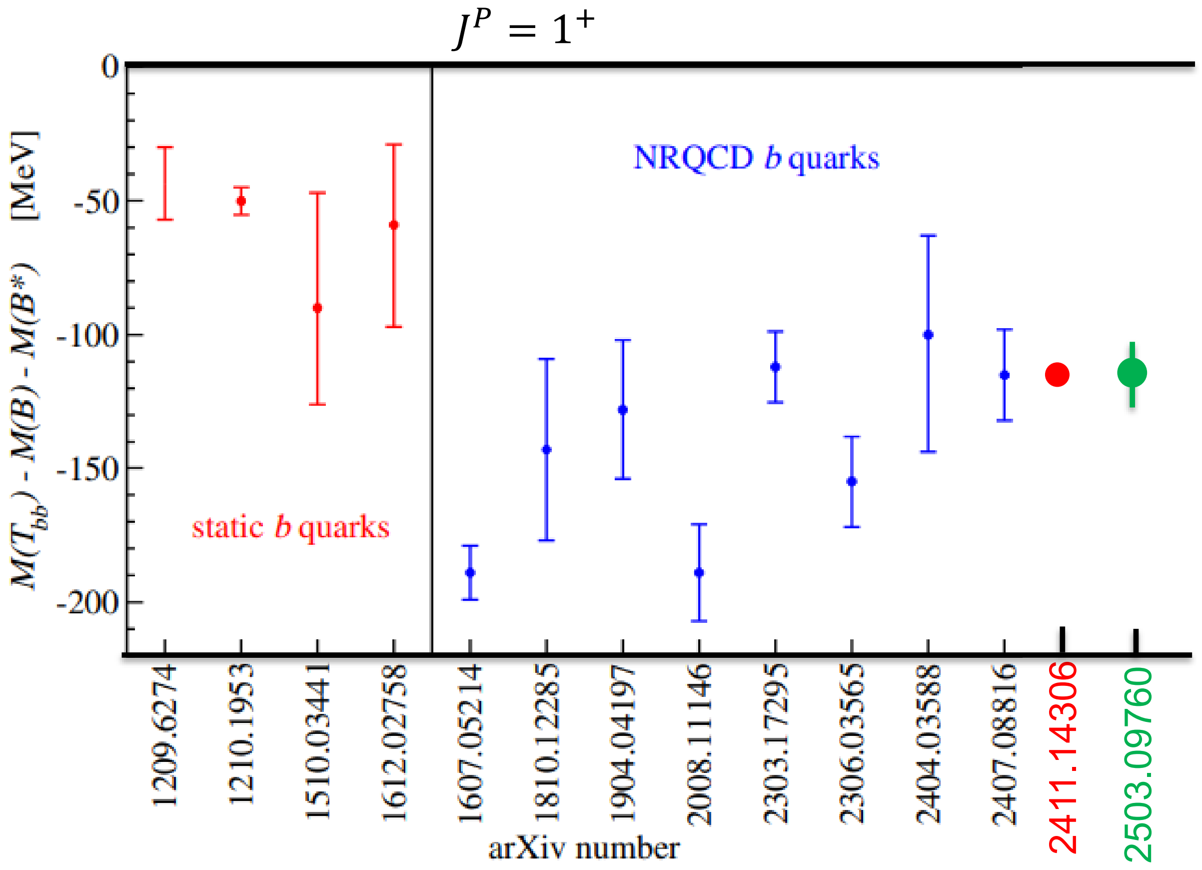
1) T_{cc} state : 320 keV below DD threshold
2) Radius ~ 8 fm or larger.
3) Deeper bound state in bb sector: T_{bb} 116 MeV below BB threshold.
4) Deeper bound state in bc sector: T_{bc} 25 MeV below DB threshold.

Good diquark energy $\Lambda_t^{0^+}$ can be confirmed by lattice QCD !!

T_{bb} & T_{bc}

Brambilla, AM, Scirpa, Vairo
Phys. Rev. Lett. 135 (2025), 131902

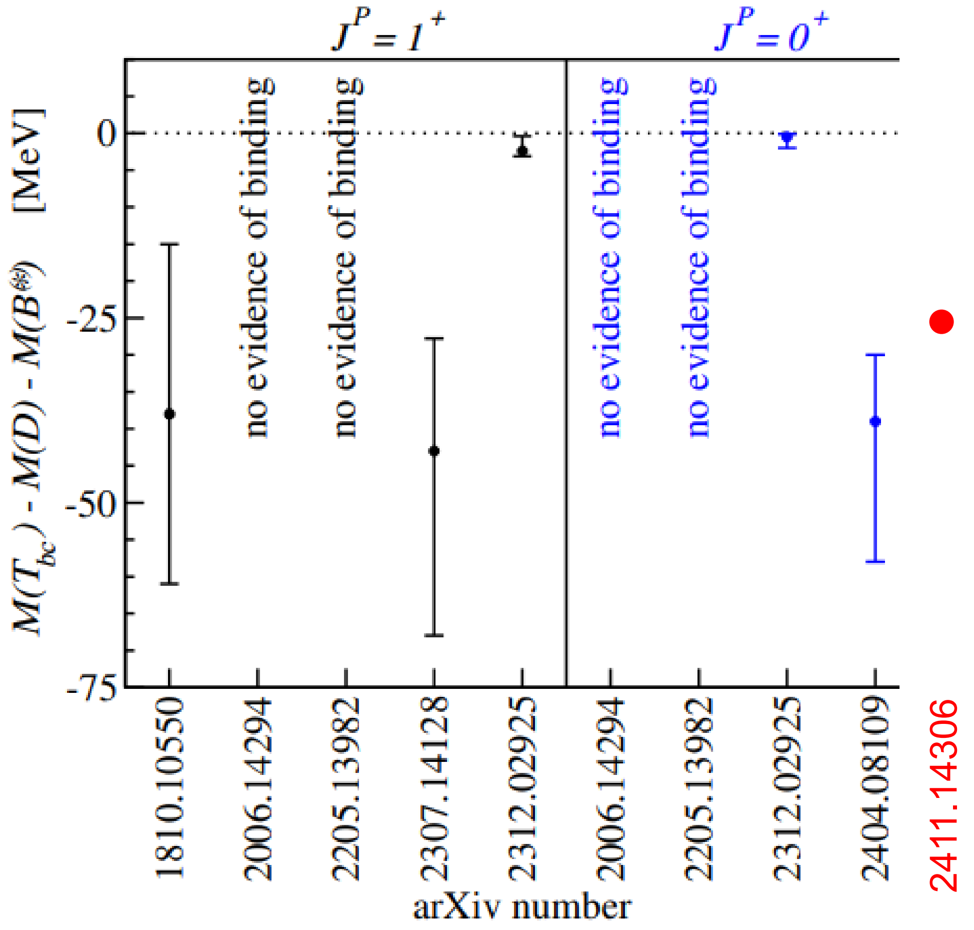
T_{bb} binding energy comparison:



EFT and heavy-quark-diquark symmetry prediction for T_{bb}: 133 ± 25 MeV

Braaten, He, AM, Phys. Rev. D. 103, 016001 (2021)

T_{bc} binding energy comparison:



Our result 25 MeV for both $J^P = \{0^+, 1^+\}$

Figures from Randy Lewis talk: LepageFest 2024

Isospin =1
Adjoint meson spectrum

Z_b & Z_c

$Q\bar{Q}$ color state	$q\bar{q}$ spin k^{PC}	BO quantum # Λ_η^σ	l	J^{PC} $\{S=0, S=1\}$
Octet **8**	0^{-+}	Σ_u^-	0	$\{0^{++}, 1^{+-}\}$
			1	$\{1^{--}, (0,1,2)^{-+}\}$
	1^{--}	$\Sigma_g^{+\prime}, \Pi_g$	1	$\{1^{+-}, (0,1,2)^{++}\}$
		$\Sigma_g^{+\prime}$	0	$\{0^{-+}, 1^{--}\}$
		Π_g	1	$\{1^{-+}, (0,1,2)^{--}\}$

Isospin-1 channel: $Z_c\,(3900), Z_c\,(4200), Z_b(10610),\ Z_b(10610)$

Mixing between adjoint mesons $K^{PC} = 0^{-+}$ and $K^{PC} = 1^{--}$

Light-quark spin-symmetry !!

Berwein, Brambilla, AM, Vairo,
Phys. Rev. D. 110, (2024), 094040

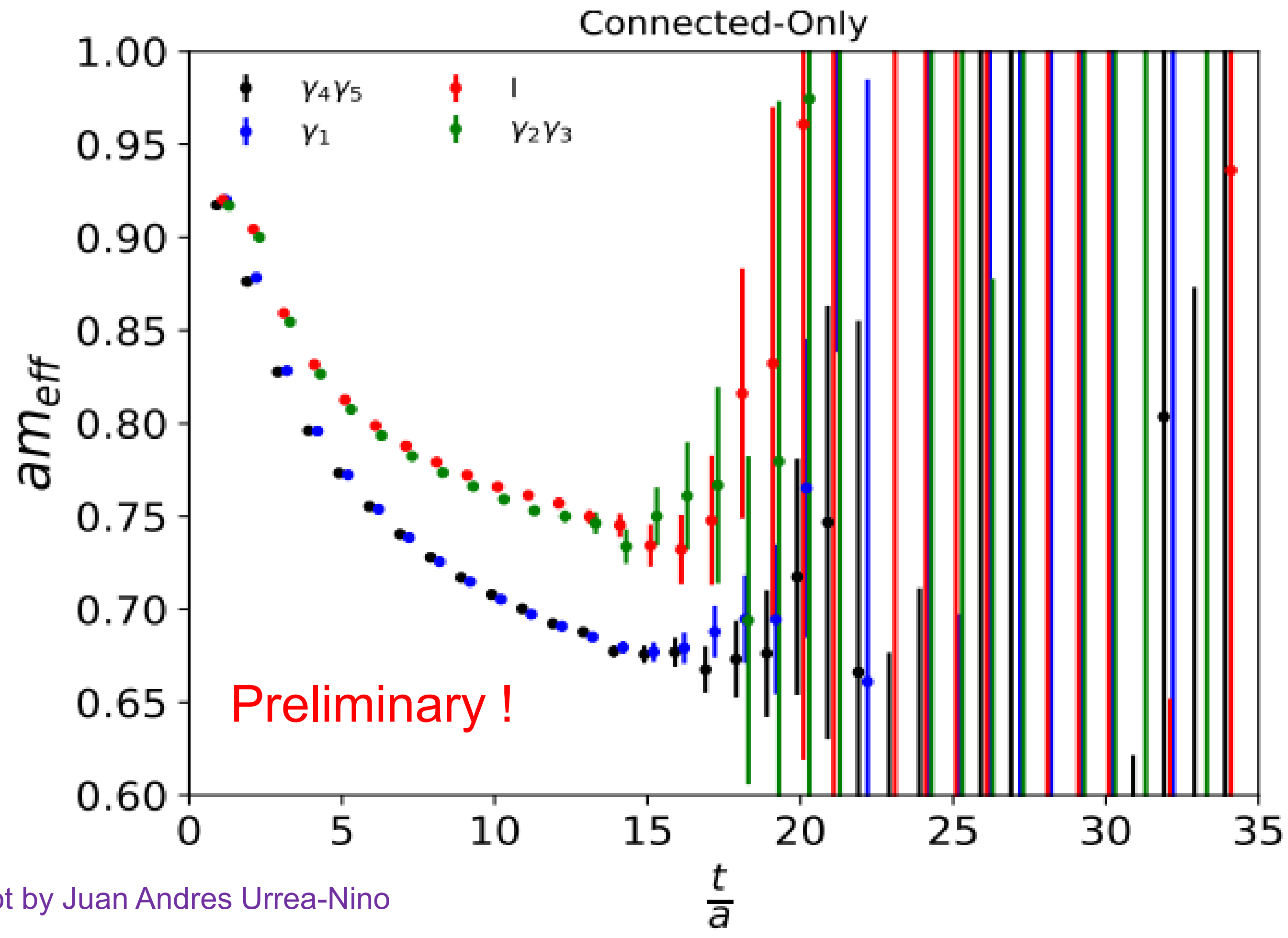
Voloshin, Phys. Rev. D. 93, 074011 (2016)

Braaten, Bruschini Phys. Lett. B 863 (2025) 139386

Isospin=1 adjoint spectrum

Slide: Sipaz Sharma talk, Lattice 2025, Mumbai.

$$am^{i}_{eff}(t,t+1) = \log \frac{C_{ii}(t)}{C_{ii}(t+1)}; \; C_{ij}(t) = \sum_{n=1}^{\infty} e^{-E_n t} \langle \hat{O}_i | n \rangle \langle n | \hat{O}_j \rangle$$



Plot by Juan Andres Urrea-Nino

$N_f = 3+1$ **ensemble,** $m_\pi = 406$ **MeV,** $a = 0.05359$ **fm**

$\gamma_1 : 1^{--}$ $\quad \gamma_4\gamma_5 : 0^{-+}$

- ❑ 0^{-+} and 1^{--} adjoint mesons are degenerate as predicted by BOEFT + experimental input.

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$1 : 0^{++}$ $\quad \gamma_2\gamma_3 : 1^{+-}$

- ❑ 0^{++} and 1^{+-} adjoint mesons are degenerate
- ❑ There are 6 adjoint mesons including 0^{++} and 1^{+-} associated with s-wave + p-wave meson-pair threshold.

Summary

- Born-Oppenheimer EFT (BOEFT):
  - Theoretical framework based on scale separation, symmetries with some inputs from lattice QCD.
  - Quarkonium and Exotic XYZ mesons can be described in the same framework.

- Tetraquark / pentaquark static potentials features based on BOEFT:
  - $Q\bar{Q}$ **systems: Repulsive** at small r $(r \to 0)$ due to octet (adjoint) color
  - QQ **systems: Attractive (**Triplet) or repulsive (sextet) at small r $(r \to 0)$**.**
  - Heavy meson pair or heavy meson baryon threshold at large r $(r \to \infty)$**.**
  - $Q\bar{Q}$ systems: Avoided crossing between tetraquark and quarkonium static potential (Isospin=0).

- Exotic dynamics governed by Schrödinger equations. Universal non-perturbative parameters same for both charm and bottom.

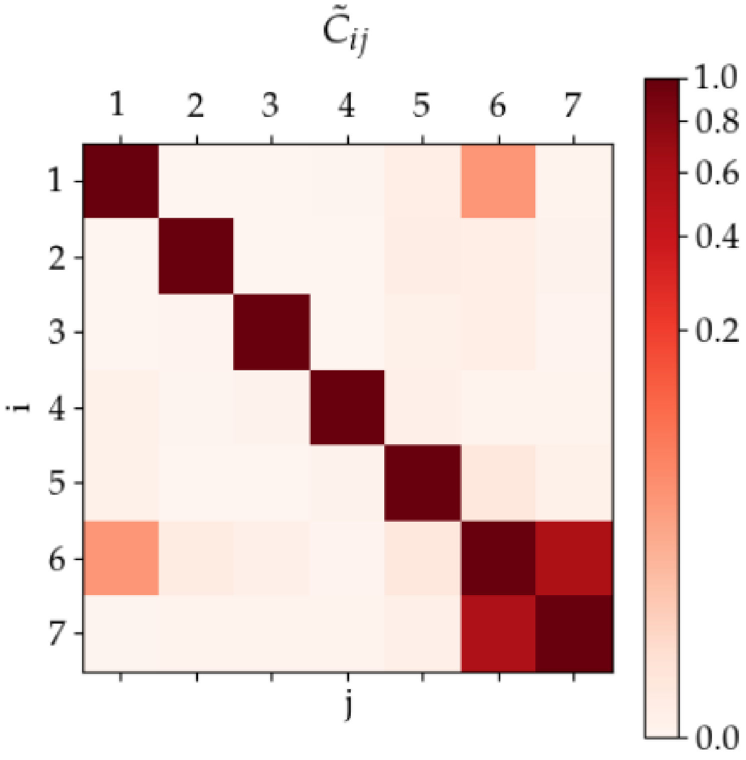
- **BOEFT + lattice QCD**, enable quantitative predictions of spectra, decays, and even production rates and in-medium studies of exotics, offering key insights into their underlying nature.

See Nora Brambilla talk on Tues !

Backup Slides

Preliminary steps towards extracting Z_b static energies

- We constructed a 7×7 correlation matrix, with
  - Op1: $\Upsilon\pi(0)$
  - Op2,3,4: $\Upsilon\pi(1,2,3)$
  - Op5: $\Upsilon b_1(0)$
  - Op6: $B\bar{B}^*$
  - Op7: O_{BO} with 0^{-+} LDF configuration (Σ_u^-)
- $\tilde{C}_{ij}(r) = \dfrac{|\sum_{t=1}^{n_t} C_{ij}(r,t)|}{\sqrt{|\sum_{t=1}^{n_t} C_{ii}(r,t)||\sum_{t=1}^{n_t} C_{jj}(r,t)|}}$
- Op6 has a visible overlap with Op1 but Op7 does not have a visible overlap with Op1.



$r/a = 1$

Slide: Sipaz Sharma talk, Lattice 2025, Mumbai.