

The Compact X and Z and their *Invisible* Molecular Partners

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in collaboration with

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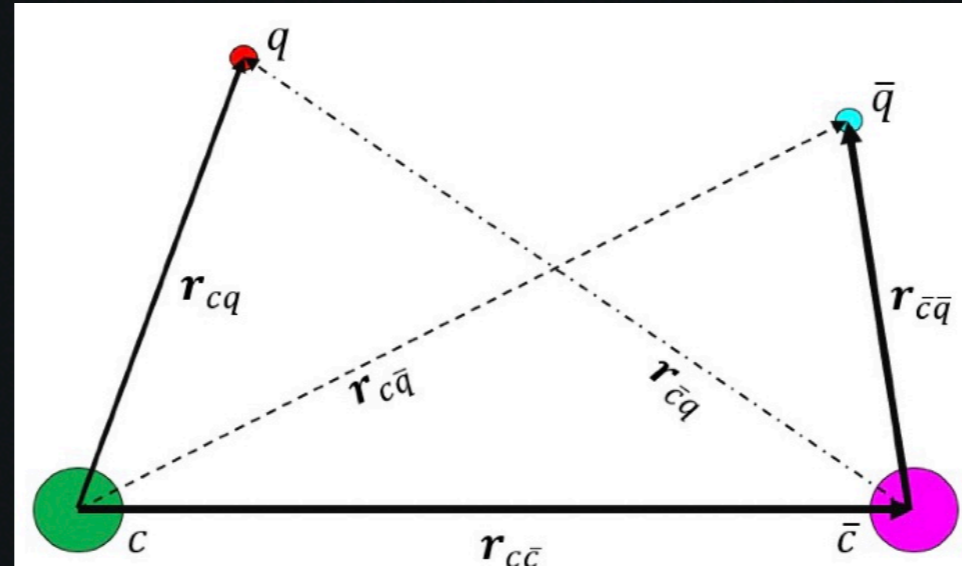
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[arXiv: 2512.21538](https://arxiv.org/abs/2512.21538)

The Born-Oppenheimer Tetraquark*

A "Toy Model"



$$T \equiv |(c\bar{c})_8 (q\bar{q})_8\rangle_1$$

$$T_D = \alpha |(cq)_{\bar{3}}(\bar{c}\bar{q})_3\rangle + \beta |(cq)_6(\bar{c}\bar{q})_{\bar{6}}\rangle$$

$$T_M = \alpha' |(c\bar{q})_1(\bar{c}q)_1\rangle + \beta' |(c\bar{q})_8(\bar{c}q)_8\rangle$$

*L. Maiani, A.D.P., V. Riquer, PRD100 (2019) 1, 014002

*B. Grinstein, D. Germani, A.D.P., JHEP 04 (2025) 004

E. Braaten, R. Bruschini, Phys.Lett.B 863 (2025) 139386

N. Brambilla, A. Mohapatra, T. Scirpa, A. Vairo, Phys.Rev.Lett. 135 (2025) 13, 13

The Born-Oppenheimer Tetraquark

$$\Psi_T(\mathbf{r}_{cq}, \mathbf{r}_{\bar{c}\bar{q}}, \mathbf{R}_{c\bar{c}}) = \Psi_{c\bar{c}}(\mathbf{R}_{c\bar{c}}) \cdot \Psi_{q\bar{q}}(\mathbf{r}_{cq}, \mathbf{r}_{\bar{c}\bar{q}}, \mathbf{R}_{c\bar{c}})$$

$$\Psi_{q\bar{q}}(\mathbf{r}_{cq}, \mathbf{r}_{\bar{c}\bar{q}}, \mathbf{R}_{c\bar{c}}) = a_D(\mathbf{R}_{c\bar{c}}) \underbrace{\psi_D(\mathbf{r}_{cq})\psi_D(\mathbf{r}_{\bar{c}\bar{q}})}_{T_D} + a_M(\mathbf{R}_{c\bar{c}}) \underbrace{\psi_M(\mathbf{r}_{c\bar{q}})\psi_M(\mathbf{r}_{\bar{c}q})}_{T_M}$$

"Diquarks" $V_D(\zeta) = -\frac{1}{3} \frac{\alpha_s}{\zeta} + k\zeta$ where $\zeta = \mathbf{r}_{cq}, \mathbf{r}_{\bar{c}\bar{q}}$

"Mesons" $V_M(\zeta) = -\frac{7}{6} \frac{\alpha_s}{\zeta} + k\zeta$ where $\zeta = \mathbf{r}_{c\bar{q}}, \mathbf{r}_{\bar{c}q}$

$$\psi_{\mathcal{C}} = \sqrt{\frac{\mathcal{C}^3}{\pi}} \exp(-\mathcal{C}\zeta) \quad \text{where } \mathcal{C} = D, M$$

THE $(c\bar{c})_8$ CONFIGURATION: CHROMO-HYDROGEN

In the one-gluon-exchange model, we can deduce the couplings

$$\text{For example } \lambda_{cq} = \lambda_{\bar{c}\bar{q}} = \underbrace{\frac{2}{3}}_{|\alpha|^2} \underbrace{\frac{1}{2}}_{C(6)} \left(-\frac{4}{3}\right) + \underbrace{\frac{1}{3}}_{|\beta|^2} \underbrace{\frac{1}{2}}_{C(6)} \left(\frac{10}{3} - \frac{8}{3}\right) = -\frac{1}{3}$$

Couplings are extracted by the diagonal matrix $\bigoplus_i \frac{1}{2}(C(S_i) - C(R_1) - C(R_2)) I_i$

$$\begin{aligned} \lambda_{c\bar{c}} &= \lambda_{q\bar{q}} = +\frac{1}{6}\alpha_s \\ \lambda_{cq} &= \lambda_{\bar{c}\bar{q}} = -\frac{1}{3}\alpha_s \\ \lambda_{c\bar{q}} &= \lambda_{\bar{c}q} = -\frac{7}{6}\alpha_s \end{aligned}$$

The B.O. Potentials ΔE_{\pm}

$$\begin{pmatrix} H_D - \Delta E & H_{DM} - S_{DM}^2 \Delta E \\ H_{DM} - S_{DM}^2 \Delta E & H_M - \Delta E \end{pmatrix} \begin{pmatrix} a_D \\ a_M \end{pmatrix} = 0 \quad \rightarrow \quad \Delta E_{\pm}$$

$$H_D = \langle \psi_D \psi_D | H_{q\bar{q}} | \psi_D \psi_D \rangle \quad \dots \quad S_{DM} = \int_{r_{cq}} \psi_D(r_{cq}) \psi_M(r_{cq} - \mathbf{R}_{c\bar{c}})$$

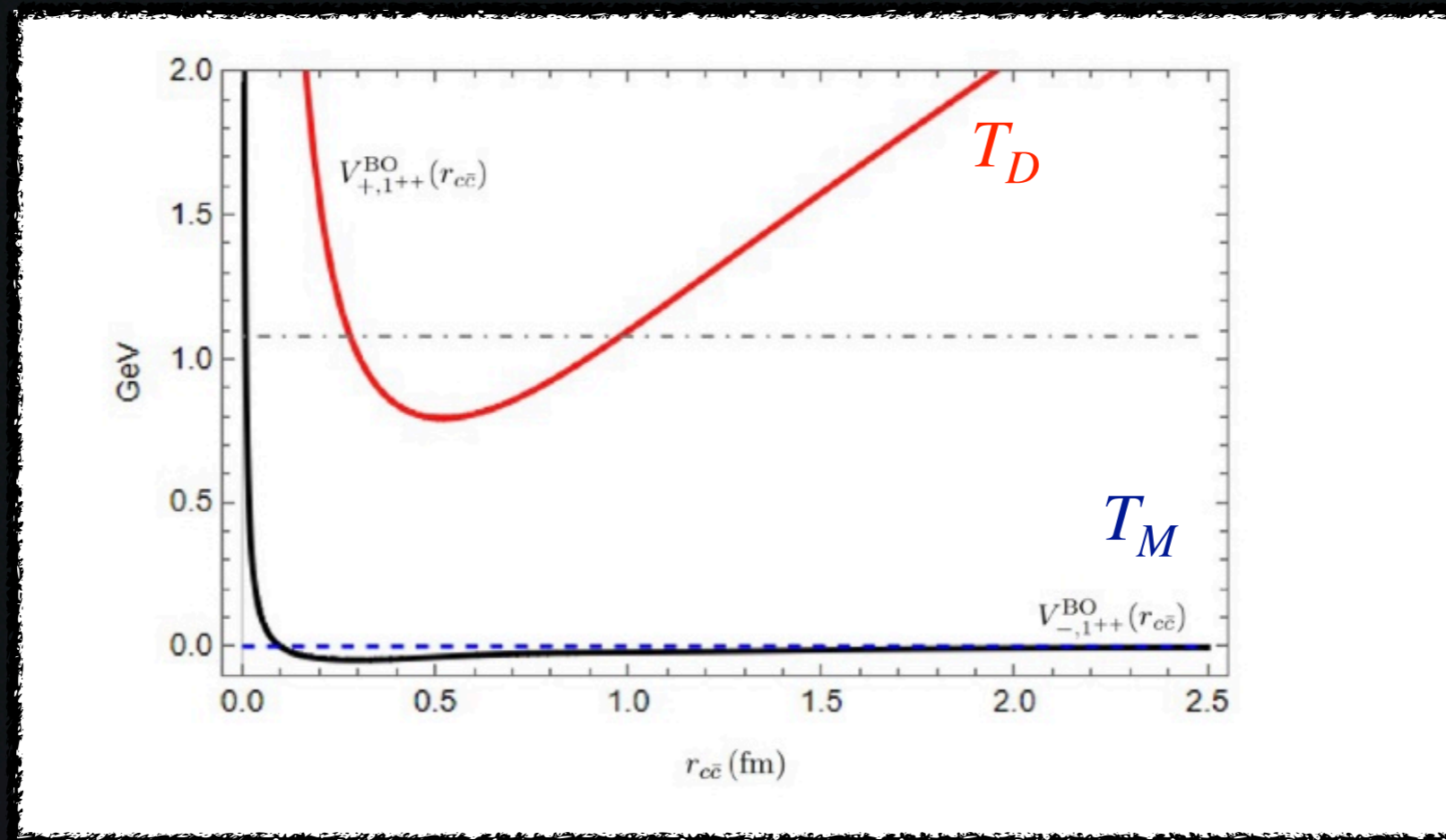
$$H_M = \langle \psi_M \psi_M | H_{q\bar{q}} | \psi_M \psi_M \rangle$$

$$H_{DM} = \langle \psi_D \psi_M | H_{q\bar{q}} | \psi_D \psi_M \rangle \quad \dots \quad H_{q\bar{q}} = -\frac{\Delta r_{cq}}{2m_{cq}} - \frac{\Delta r_{\bar{c}\bar{q}}}{2m_{\bar{c}\bar{q}}} + V_{\text{Coul}} + V_{\text{Conf}} + V_{\text{Spin}}$$

$$\Delta E_{\pm}(r_{c\bar{c}}) = \frac{1}{2(1 - S_{DM}^4)} \left(H_D + H_M - 2S_{DM}^2 H_{DM} \pm \sqrt{(H_D + H_M - 2S_{DM}^2 H_{DM})^2 - 4(1 - S_{DM}^4)(H_D H_M - H_{DM}^2)} \right)$$

$$(H_{c\bar{c}} + \Delta E_{\pm}(R_{c\bar{c}})) \Psi_{c\bar{c}}(R_{c\bar{c}}) = E \Psi_{c\bar{c}}(R_{c\bar{c}})$$

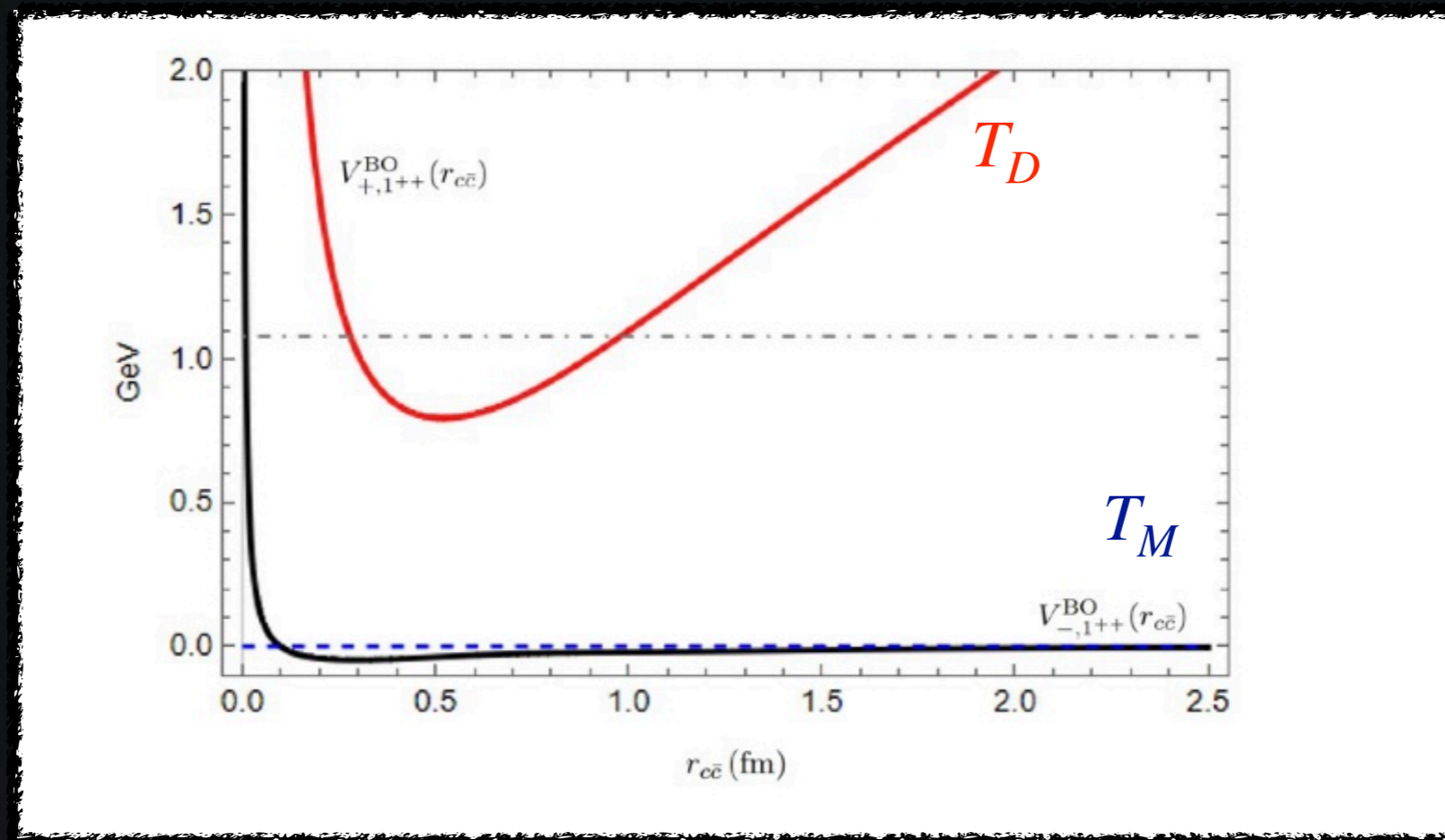
The B.O. Potentials



$$V_{-}(R_{c\bar{c}}) = \frac{1}{6} \frac{\alpha_s}{R_{c\bar{c}}} + \Delta E_{-}$$

$$V_{+}(R_{c\bar{c}}) = \frac{1}{6} \frac{\alpha_s}{R_{c\bar{c}}} + \Delta E_{+} + k R_{c\bar{c}}$$

The B.O. Potentials



Shallow, **compact**, bound states, superpositions of color $(\mathbf{1}, \mathbf{1}) + (\mathbf{8}, \mathbf{8})$ with a **size closer to the typical hadron size**. Distinct from true molecules since defined by color forces and spin-interactions.

Their wavefunctions do not match the universal form predicted for a shallow molecular bound states.

Compact and molecular states

Together with the B.O. state, in the lower potential, there could be a *truly molecular state* — which is *not* contained in the B.O. description.

We assume that this molecular state belongs to a isotriplet so that:

X_S^0 (**1, 1 + 8, 8**) isosinglet/compact

$X_T^{0,\pm}$ (**1, 1**) isotriplet/molecular/no B.O.

X_S (= X_S^0) can mix with X_T (= X_T^0)

X_S and X_T

We argue that $X(3872) = X_S$

The loosely bound isotriplet is **not efficiently produced** at colliders: production rates are very much suppressed in high energy prompt collisions and even in B decays.

X_S can oscillate in the invisible X_T

We assume that:

i) X_T quasi-degenerate with X_S

ii) X_T is a shallow bound state with $B \ll M$

C. Bignamini, B. Grinstein, F. Piccinini, A.D.P., C. Sabelli *Phys.Rev.Lett.* 103 (2009) 162001

A. Carducci, D. Germani, B. Grinstein, A.D.P., arXiv: [2512.21538](https://arxiv.org/abs/2512.21538)

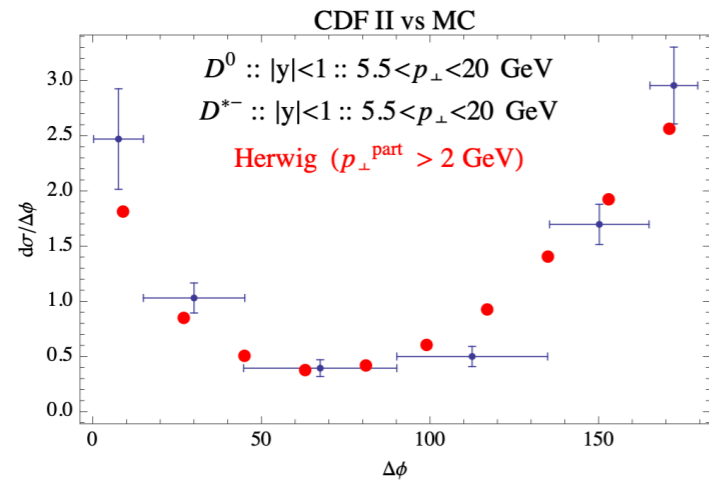


FIG. 1: The $D^0 D^{*-}$ pair cross section as function of $\Delta\phi$ at CDF Run II. The transverse momentum, p_\perp , and rapidity, y , ranges are indicated. Data points with error bars, are compared to the leading order event generator Herwig. The cuts on parton generation are $p_\perp^{\text{part}} > 2$ GeV and $|y^{\text{part}}| < 6$. We have checked that the dependency on these cuts is not significant. We find that we have to rescale the Herwig cross section values by a factor $K_{\text{Herwig}} \simeq 1.8$ to best fit the data on open charm production.

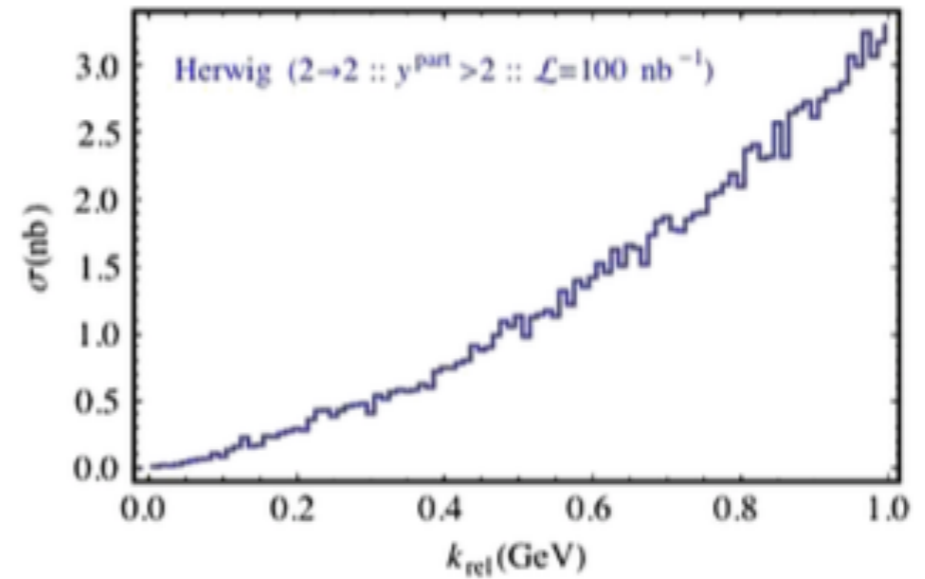
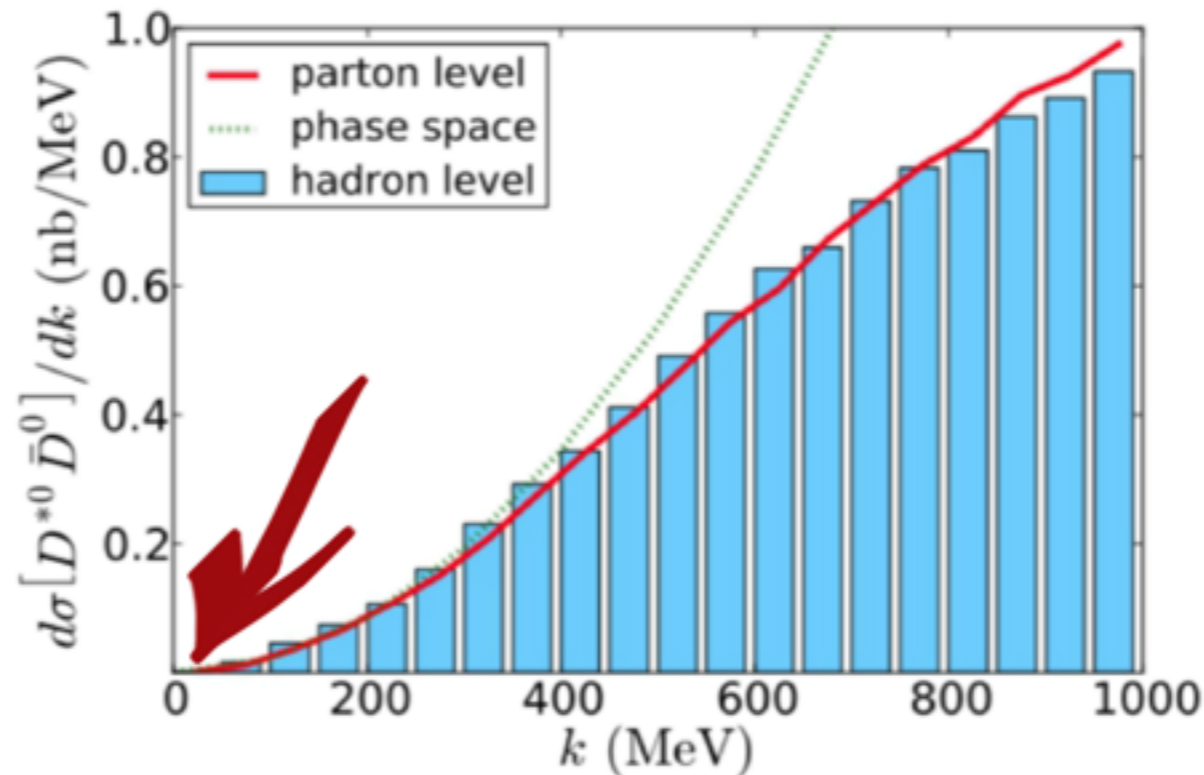


FIG. 3 (color online). The integrated cross section obtained with HERWIG as a function of the center of mass relative momentum of the mesons in the $D^0 \bar{D}^{*0}$ molecule. This plot is obtained after the generation of 55×10^9 events with parton cuts $p_\perp^{\text{part}} > 2$ GeV and $|y^{\text{part}}| < 6$. The cuts on the final D mesons are such that the molecule produced has a $p_\perp > 5$ GeV and $|y| < 0.6$.

Bignamini, Grinstein, Piccinini, ADP, Sabelli, PRL103 (2009) 162001

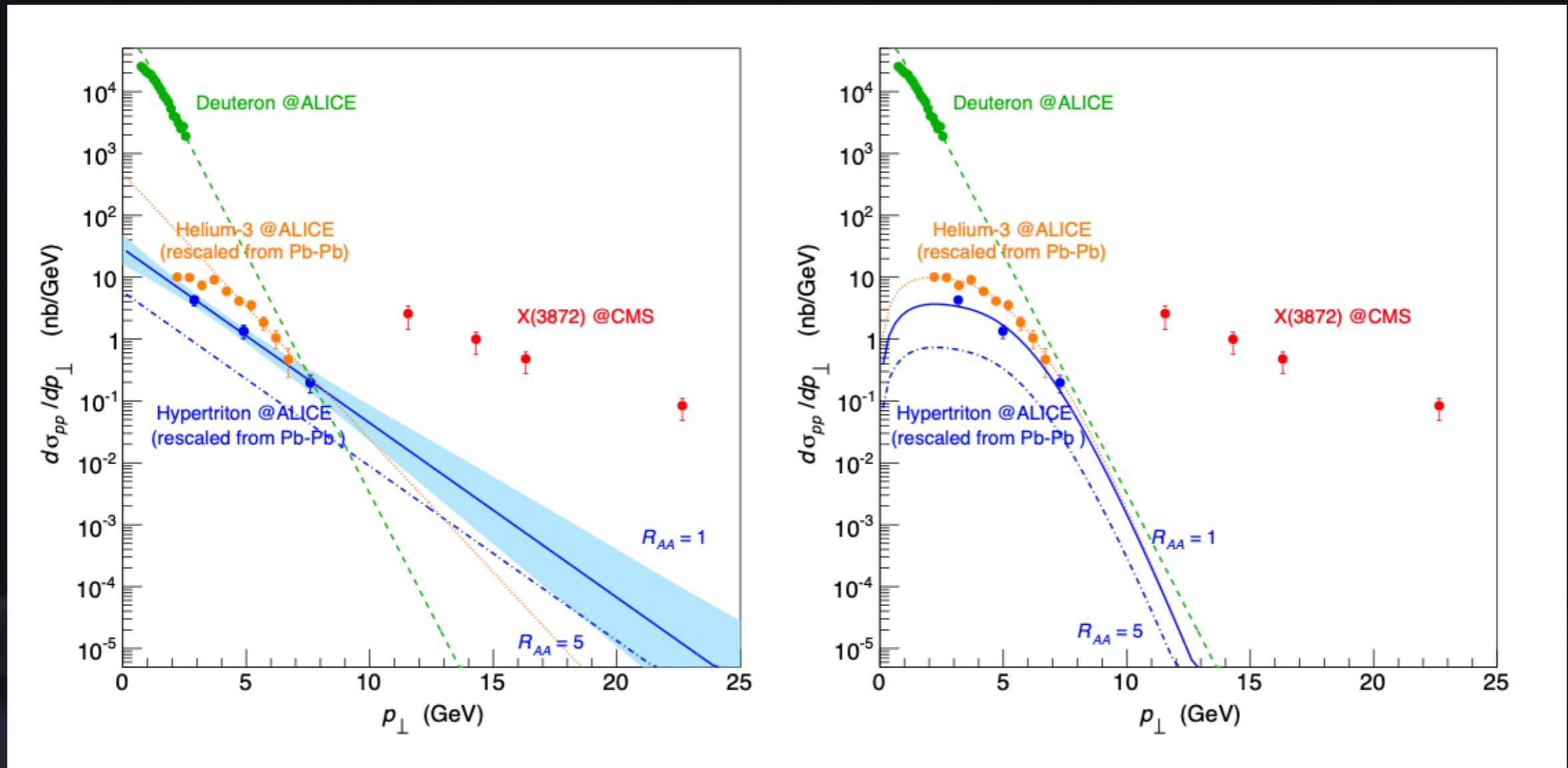
$$p \sim \Delta p \sim \sqrt{2mB} \sim \sim 14 \text{ MeV}$$



Braaten and Artoisenet, PRD81103 (2010) 114018

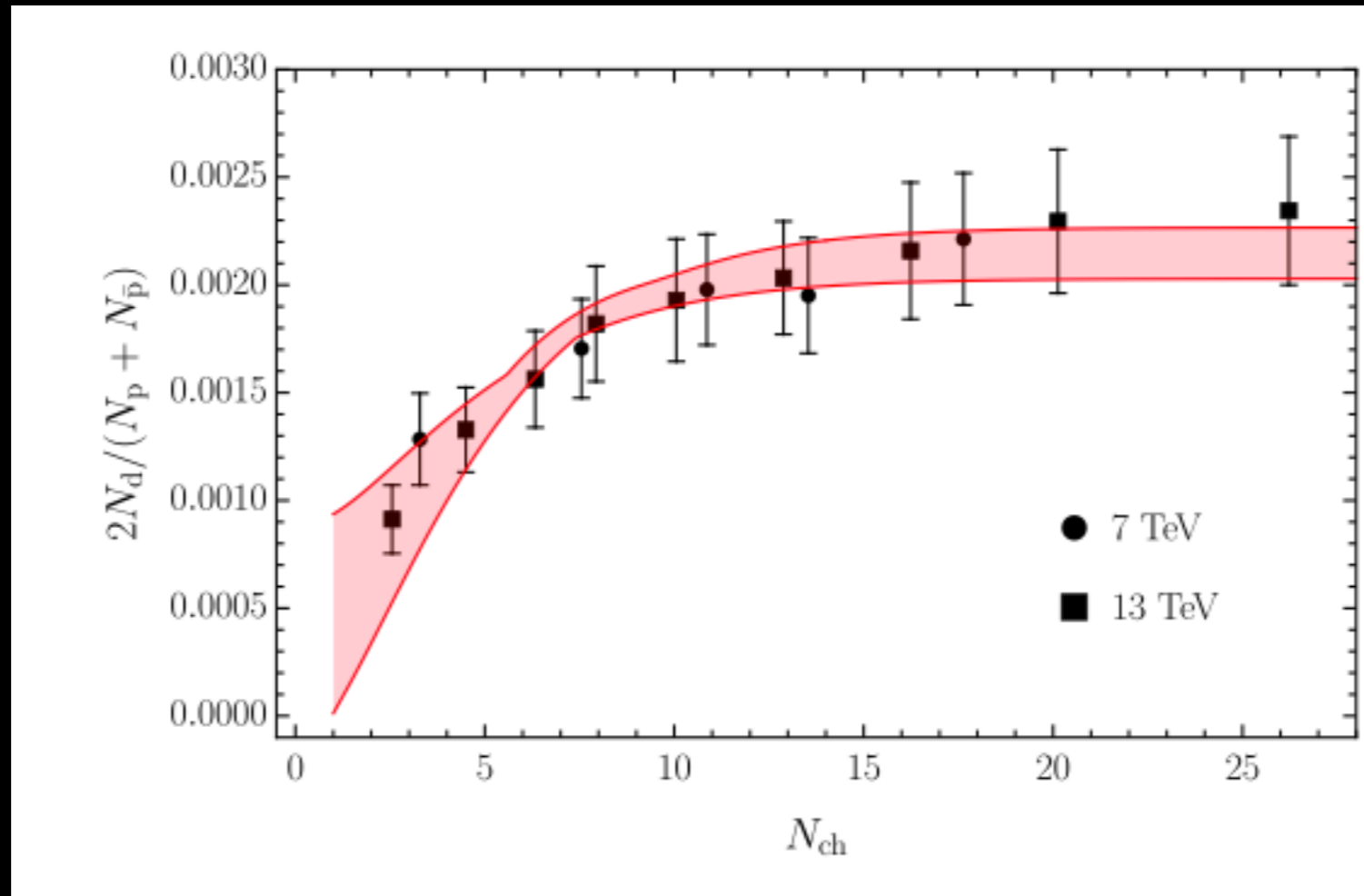
The most fine tuned bound state

...ever seen. Too weakly bound to be a final state in pp collisions.



DOES THE X(3872) BEHAVE AS THE DEUTERON?

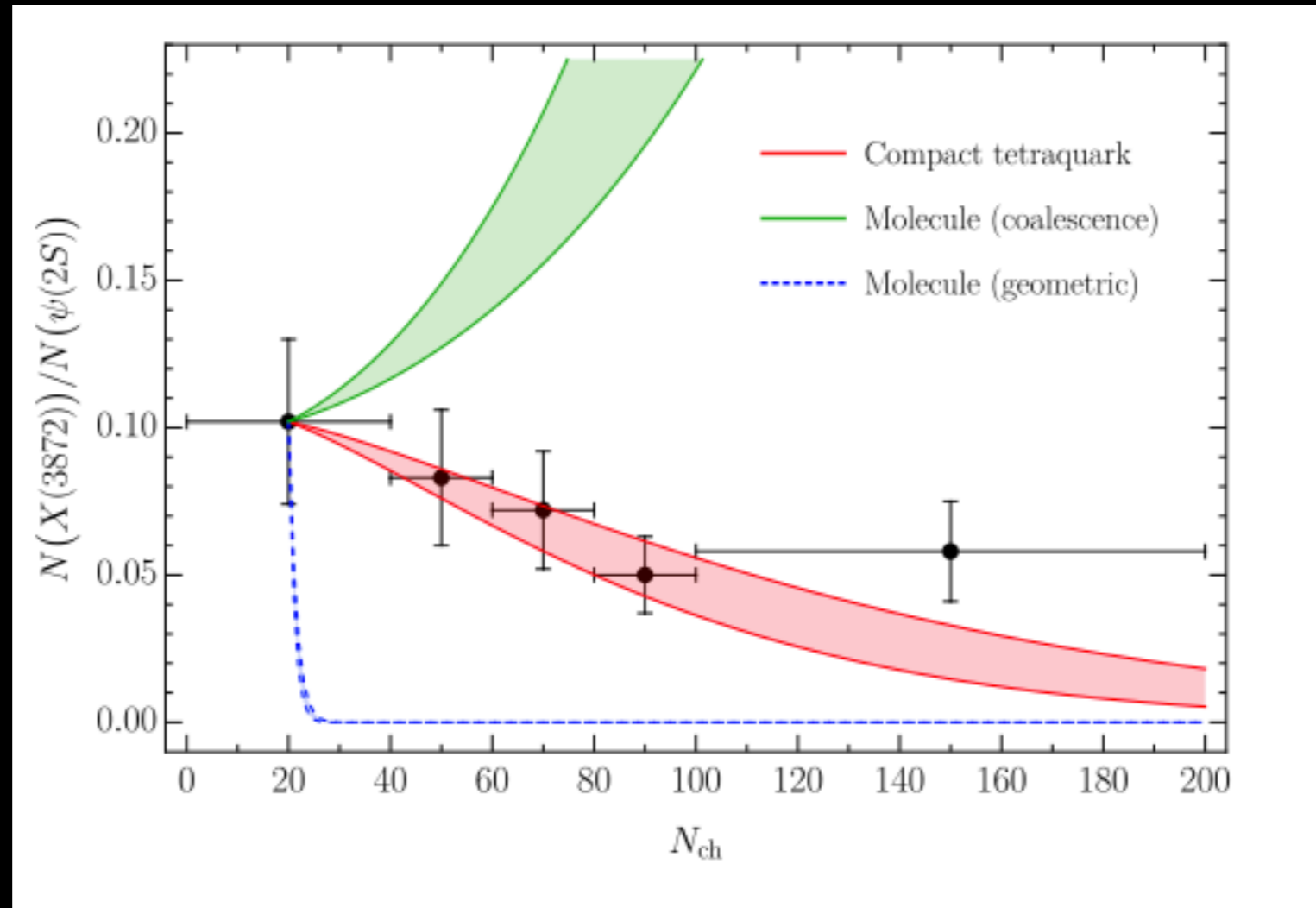
ALICE: 1902.09290; 2003.03184



Esposito, Ferreiro, Pilloni, ADP, Salgado *Eur. Phys. J. C* 81 (2021) 669

Number of deuterons as a function of the multiplicity computed with kinetic theory (coalescence momentum varied between $50 < k < 250$ MeV – up to 360 MeV for X)

DOES THE X(3872) BEHAVE AS THE DEUTERON?



Esposito, Ferreiro, Pilloni, ADP, Salgado *Eur. Phys. J. C* 81 (2021) 669

The coalescence picture predicts a behavior (green band) qualitatively different from data – see talk by M Durham

X_S and X_T

problems potentially solved within this scheme

1) Reconcile with the sizable isospin violations observed

$$\mathcal{F} = \frac{\text{Br}(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\text{Br}(X \rightarrow J/\psi \pi^+ \pi^-)} = \begin{cases} 1.0 \pm 0.4 \text{ (stat)} \pm 0.3 \text{ (syst)} & [\text{Belle}] \\ 0.8 \pm 0.3 & [\text{BABAR}] \\ 1.43^{+0.28}_{-0.23} & [\text{BESIII}] \end{cases}$$

2) As of today *no charged partners* of the $X(3872)$ have been discovered *nor extra neutral states*: in the quark model we should have two neutral X particles (within a few MeV).

3) A triplet of states $Z^{0,\pm}$, slightly heavier, is observed with *opposite charge conjugation* with respect to the X . These three particles are all **compact** quark states. *No isospin violations are expected in this system.*

The mixing mechanism

Both X_S and X_T couple to $D\bar{D}^*$

$$\begin{aligned}
 (-ig_{\text{mix}}) &= \begin{array}{c} D^0 \\ \text{---} X_S \text{---} \bigcirc \text{---} X_T \text{---} \\ \bar{D}^{0*} \end{array} + \begin{array}{c} D^+ \\ \text{---} X_S \text{---} \bigcirc \text{---} X_T \text{---} \\ D^{-*} \end{array} = \\
 &= (-ig_S)(ig_T)i\frac{m}{\pi^2} \left(-\Lambda + \sqrt{-2mE} \arctan \left(\frac{\Lambda}{\sqrt{-2mE}} \right) \right) + \\
 &+ (ig_S)(ig_T)i\frac{m_+}{\pi^2} \left(-\Lambda + \sqrt{-2m_+(E - \Delta)} \arctan \left(\frac{\Lambda}{\sqrt{-2m_+(E - \Delta)}} \right) \right)
 \end{aligned}$$

where $E = m_X - m_D - m_{D^*} \neq B$.

Notice that $g_{\text{mix}} = 0$ in the limit $\Delta \rightarrow 0$, where

$$\Delta = m_{D^{*+}} + m_{D^+} - m_{D^{*0}} - m_{D^0} \simeq 8 \text{ MeV}.$$

The reduced mass of the charged mesons is $m_+ \simeq m$.

The mixing mechanism

Both X_S and X_T couple to $D\bar{D}^*$

$$\begin{aligned}
 (-ig_{\text{mix}}) &= \begin{array}{c} D^0 \\ \text{---} X_S \text{---} \text{---} \text{---} X_T \text{---} \\ \text{---} \bar{D}^{0*} \text{---} \end{array} + \begin{array}{c} D^+ \\ \text{---} X_S \text{---} \text{---} \text{---} X_T \text{---} \\ \text{---} D^{-*} \text{---} \end{array} = \\
 &= (-ig_S)(ig_T)i\frac{m}{\pi^2} \left(-\Lambda + \sqrt{-2mE} \arctan \left(\frac{\Lambda}{\sqrt{-2mE}} \right) \right) + \\
 &+ (ig_S)(ig_T)i\frac{m_+}{\pi^2} \left(-\Lambda + \sqrt{-2m_+(E - \Delta)} \arctan \left(\frac{\Lambda}{\sqrt{-2m_+(E - \Delta)}} \right) \right)
 \end{aligned}$$

Studying the Flatte` lineshape of the $X = X_S$ at the LHCb allows to measure the coupling g_S .

As for g_T : it can be estimated theoretically — we will use an argument due to Landau and Weinberg.

The mixing mechanism

Both X_S and X_T couple to $D\bar{D}^*$

$$\begin{aligned}
 (-ig_{\text{mix}}) &= \begin{array}{c} D^0 \\ \text{---} X_S \text{---} \bigcirc \text{---} X_T \text{---} \\ \bar{D}^{0*} \end{array} + \begin{array}{c} D^+ \\ \text{---} X_S \text{---} \bigcirc \text{---} X_T \text{---} \\ D^{-*} \end{array} = \\
 &= (-ig_S)(ig_T)i\frac{m}{\pi^2} \left(-\Lambda + \sqrt{-2mE} \arctan \left(\frac{\Lambda}{\sqrt{-2mE}} \right) \right) + \\
 &+ (ig_S)(ig_T)i\frac{m_+}{\pi^2} \left(-\Lambda + \sqrt{-2m_+(E - \Delta)} \arctan \left(\frac{\Lambda}{\sqrt{-2m_+(E - \Delta)}} \right) \right)
 \end{aligned}$$

We have to introduce a physical cutoff Λ in this calculation if we want to use the Landau* coupling g_T , which is defined for meson pairs recoiling at very low energy (this introduces a dependency of g_{mix} on B)

$$g_T^2 = \frac{2\pi}{m} \sqrt{\frac{2B}{m}}$$

*Esposito, Germani, Glioti, ADP, [2502.02505](#) (review); and ADP *Phys.Lett.B* 746 (2015) 248-250

The mixing mechanism

Both X_S and X_T couple to $D\bar{D}^*$

$$\begin{aligned}
 (-ig_{\text{mix}}) &= \begin{array}{c} D^0 \\ \text{---} X_S \text{---} \bigcirc \text{---} X_T \text{---} \\ \bar{D}^{0*} \end{array} + \begin{array}{c} D^+ \\ \text{---} X_S \text{---} \bigcirc \text{---} X_T \text{---} \\ D^{-*} \end{array} = \\
 &= (-ig_S)(ig_T)i\frac{m}{\pi^2} \left(-\Lambda + \sqrt{-2mE} \arctan \left(\frac{\Lambda}{\sqrt{-2mE}} \right) \right) + \\
 &+ (ig_S)(ig_T)i\frac{m_+}{\pi^2} \left(-\Lambda + \sqrt{-2m_+(E - \Delta)} \arctan \left(\frac{\Lambda}{\sqrt{-2m_+(E - \Delta)}} \right) \right)
 \end{aligned}$$

As for Λ we expect

$$\Lambda \simeq k_{\text{max}} \sim \langle k \rangle + \Delta k = \sqrt{2mB} + \Delta k = \frac{3}{2}\sqrt{2mB}$$

We will compare this with the what obtained from the loop calculation and $g_{\text{mix}}(B)$

Isospin violating decays

The role of X_S and X_T

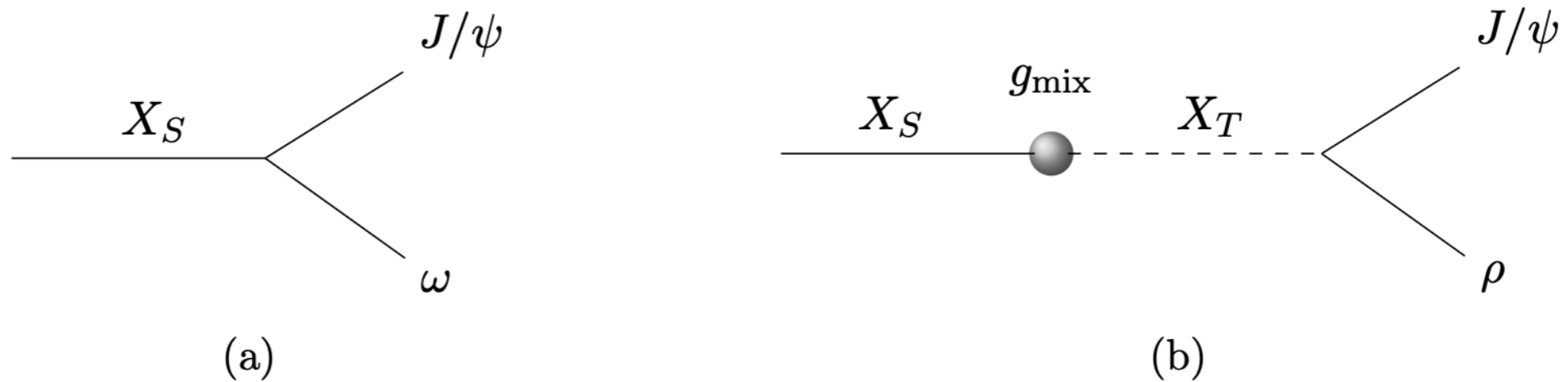


Figure 1: Decay mechanisms for $X(3872) \rightarrow J/\psi V$ with $V = \rho, \omega$. (a) Isospin-conserving decay. (b) Isospin-violating decay induced by mixing with the isotriplet.

$$\mathcal{F} = \frac{\Gamma(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\Gamma(X \rightarrow J/\psi \pi^+ \pi^-)} = \left| \frac{g_{\text{mix}}}{E_T + i \frac{\Gamma_T}{2}} \right|^{-2} \frac{\alpha |G_\omega|^2}{\Gamma(X_T \rightarrow J/\psi \pi^+ \pi^-)}$$

$$E_T = m_{X_S} - m_{X_T} \equiv m_X - (m_D + m_D^* - B) = (m_X - m_D - m_D^*) + B \equiv E + B$$

$\alpha = 3\text{-body } (\omega \rightarrow 3\pi) \text{ phase space factor}$

Determination of g_{mix}

The role of X_S and X_T

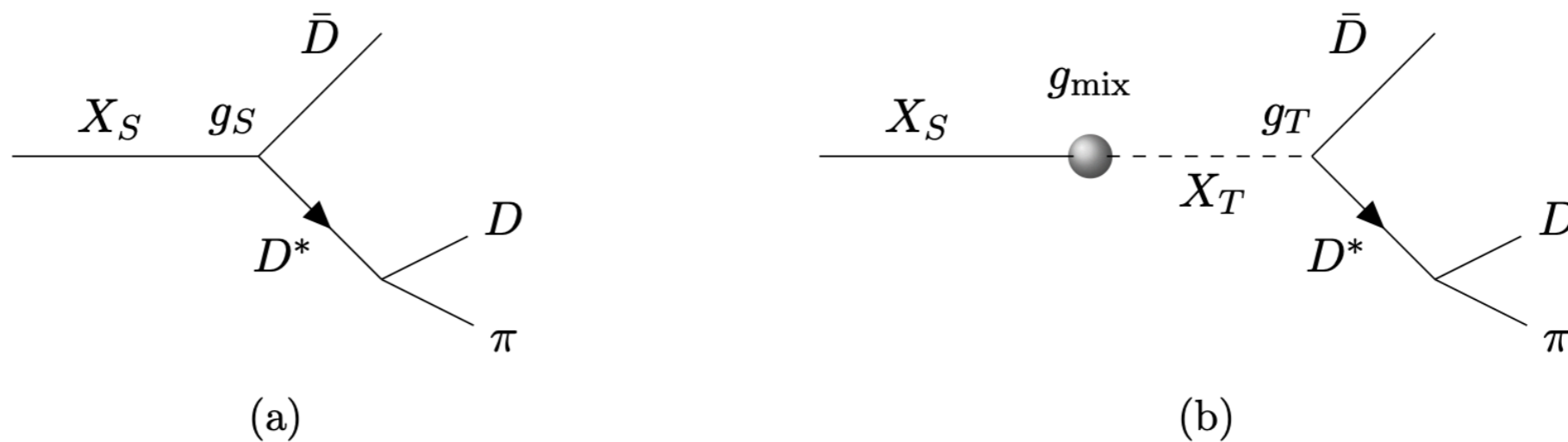
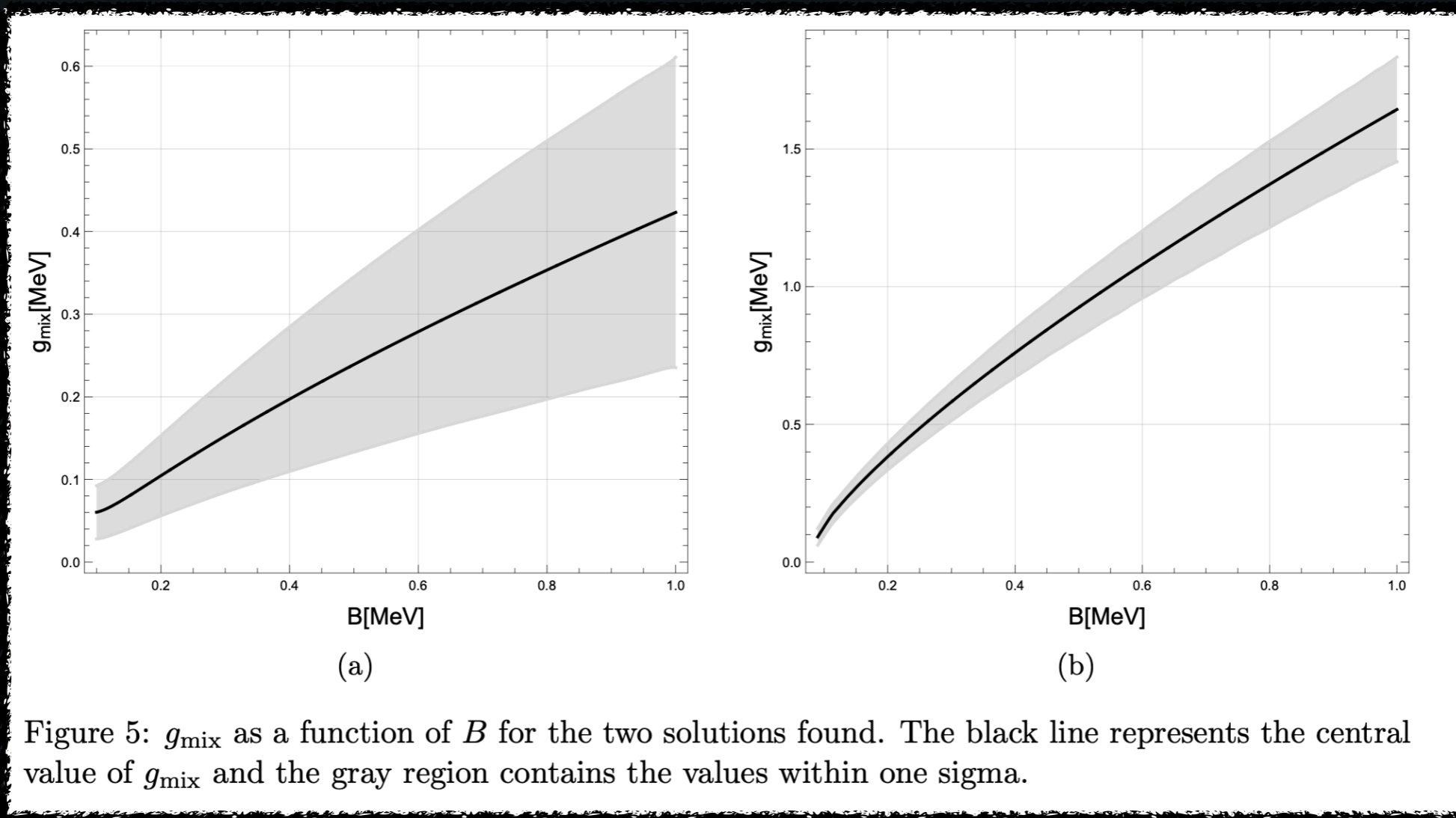


Figure 4: Decay diagrams for $X_S \rightarrow D\bar{D}\pi$. (a) Direct decay. (b) Decay via isospin mixing through X_T .

$$g^2 \propto \left| g_S - \frac{g_{\text{mix}} g_T}{E + B + i\Gamma_T/2} \right|^2$$

Two solutions for $g_{\text{mix}}(B)$



Two solutions for g_{mix} , as a function of the unknown parameter B , are possible from $X \rightarrow DD\pi$ — we will use the r.h.s. $g_{\text{mix}}(B)$.

Two solutions for $g_{\text{mix}}(B)$

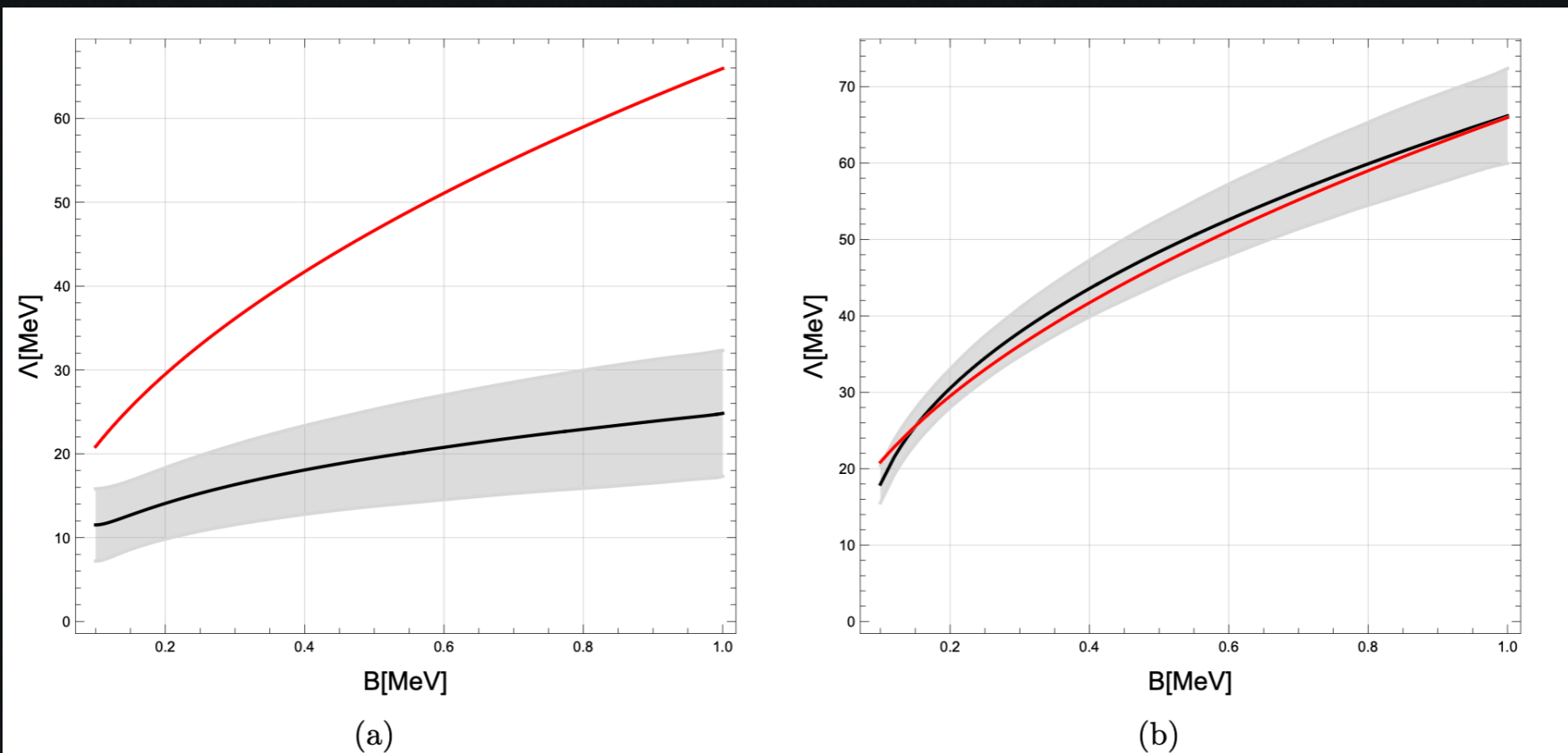


Figure 10: Λ as a function of B for the two solutions of g_{mix} . The black line represents the central value of Λ and the gray region contains the values within one sigma. The red line marks the values obtained through the estimate in Eq. (42).

(red line) $\Lambda \simeq = \frac{3}{2} \sqrt{2mB}$

Determination of \mathcal{F}

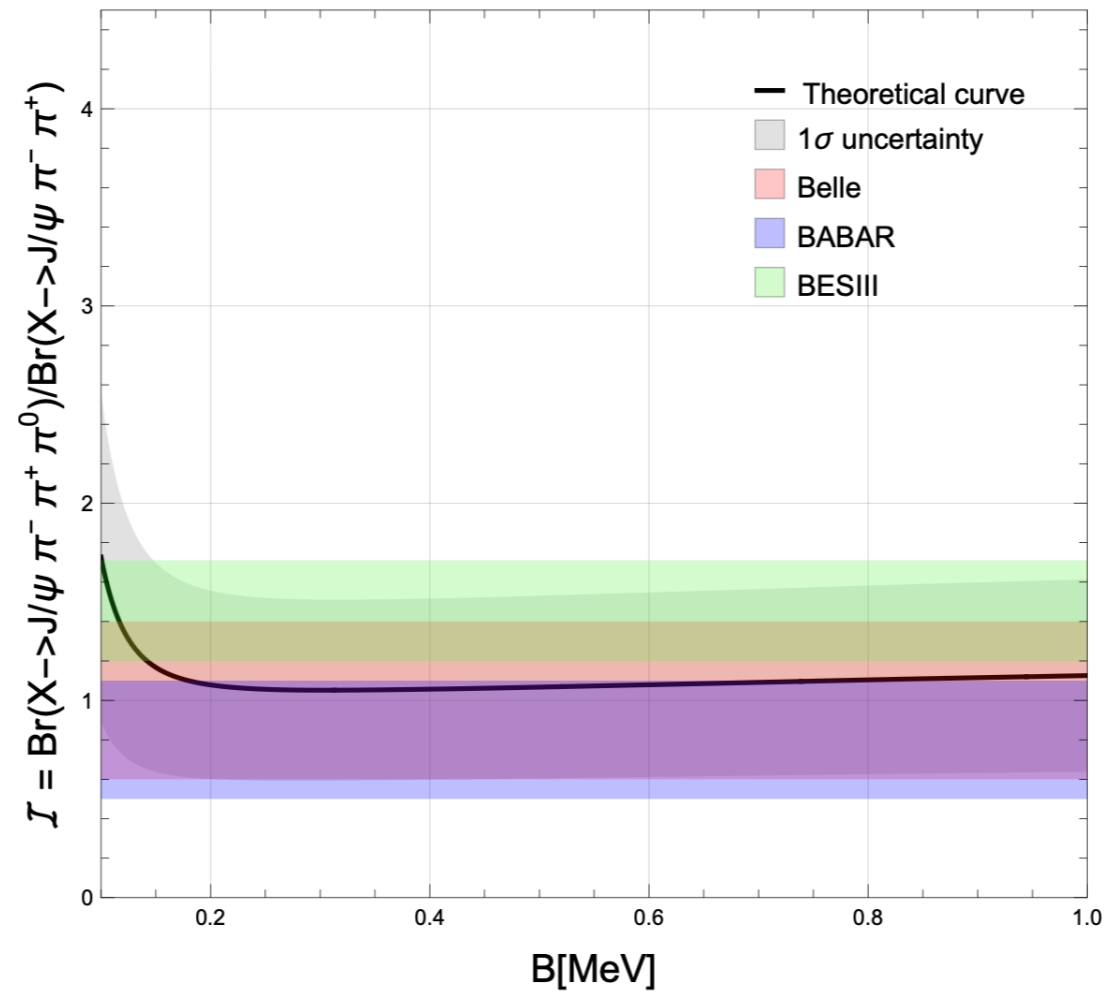


Figure 6: \mathcal{F} as a function of B for the solution g_{mix} shown in Fig. 9b. The black line represents the central value of \mathcal{F} obtained from Eq. (38), with the 1σ uncertainty band shown in gray. The horizontal bands correspond to the experimental values in Eq. (1).

One of the two $g_{\text{mix}}^{\text{II}}$ works well to explain the \mathcal{F} ratio as in figure.

The $X_T \rightarrow J/\psi\rho$ decay

There is **no data on the X_T decay**, because this particle is not produced at colliders (if not through the mixing).

We model the decay as dominantly given by the light quark-antiquark annihilation into a virtual γ which subsequently converts into ρ^0 by VMD.

We apply the same method we used to describe the $X \rightarrow \psi\gamma$ decays*

*L. Maiani, B. Grinstein, A.D.P., *Phys.Rev.D* 109 (2024) 7, 074009

The Z particles reversed pattern

In the case of the X system we have that:

X_S is a compact tetraquark state whereas X_T is a loosely bound molecule together with two charge partners — unobservable.

In the case of the Z system

Z_T are compact tetraquarks (all visible) and Z_S is a singlet molecule — unobservable.

What is the role of C in all this ?

Isospin violations in Z decays

The Z s' feature a **reversed pattern** with respect to the X

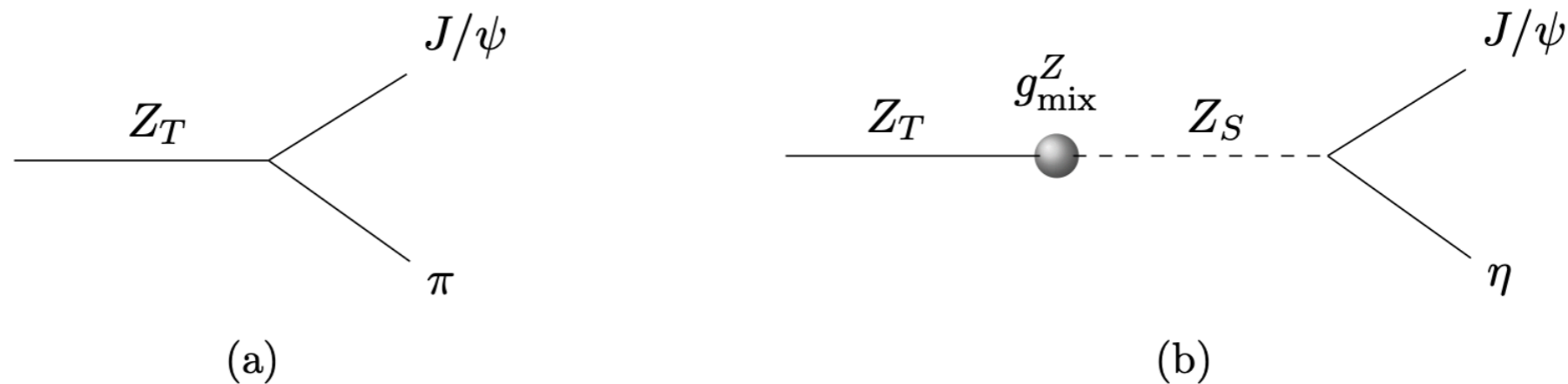


Figure 13: Decay mechanisms for $Z(3900) \rightarrow J/\psi P$ with $P = \pi, \eta$. (a) Isospin-conserving decay. (b) Isospin-violating decay induced by mixing with the isosinglet.

$$\mathcal{F}_Z = \frac{\text{Br}(Z(3900) \rightarrow J/\psi \eta)}{\text{Br}(Z(3900) \rightarrow J/\psi \pi)} < 0.15 \quad [\text{BESIII}]$$

Isospin violations in Z decays

The Z 's feature a **reversed pattern** with respect to the X

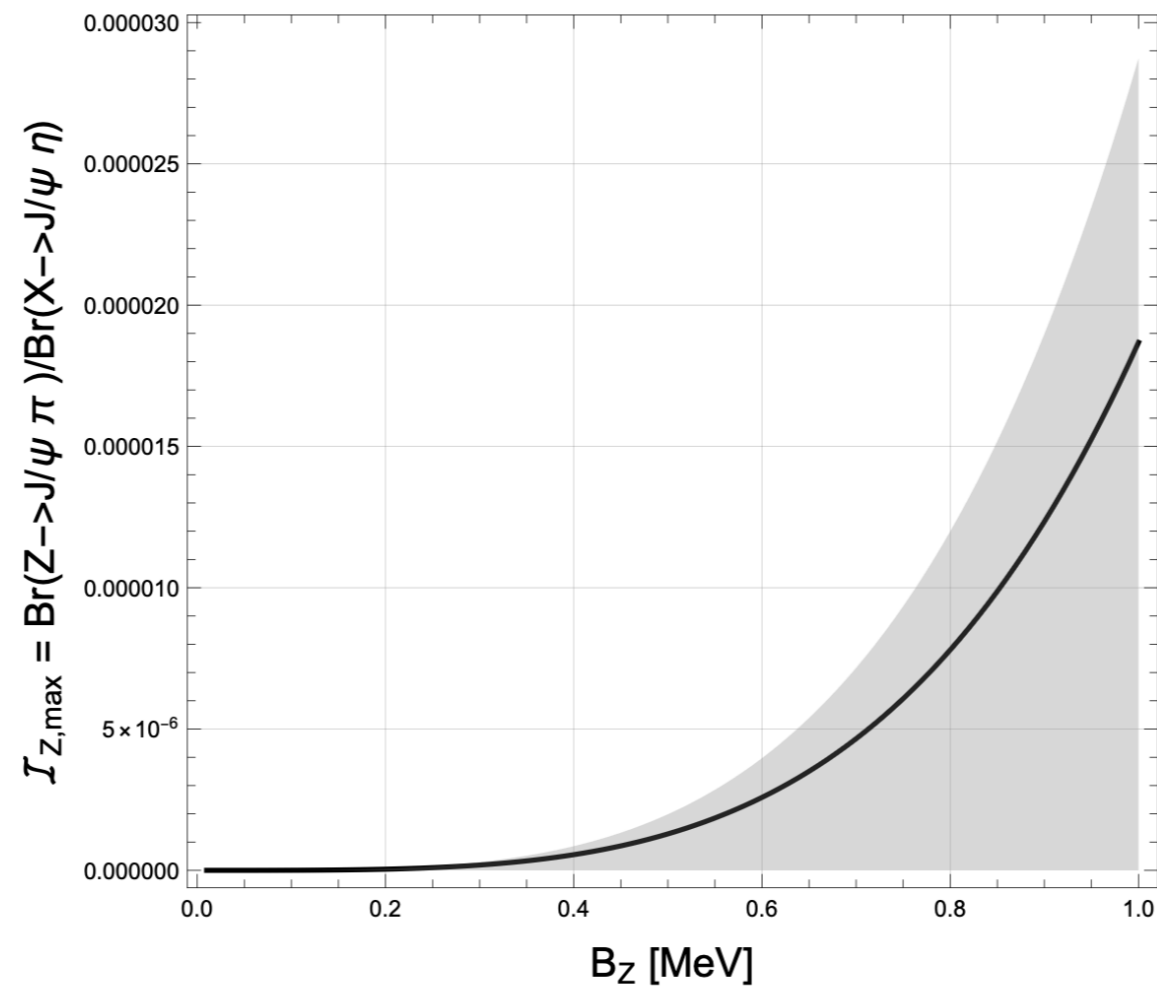


Figure 14: $\mathcal{I}_{Z,\max}$ as a function of B_Z . The black line represents the central value of $\mathcal{I}_{Z,\max}$ and the gray region contains the values within one sigma.

Conclusions

The mirroring patterns

- Loosely bound molecules are not efficiently produced at high energy colliders — unobservable at high energy — *what about heavy ion collisions?*
- *$X(3872)$ corresponds to an X_S compact iso-singlet.* In the iso-triplet configurations quarks arrange into loosely bound color neutral states. *The neutral component of the molecular isotriplet corresponds to the X_T .*
- $X_S \rightarrow X_T$ oscillations explain the isospin violation observed. Consistency with $X_S \rightarrow D\bar{D}\pi$ decays is found.
- *In the case of Z particles the pattern is reversed.* The unobservable molecular component is the iso-singlet Z_S .
- *No isospin violation expected in the Z system.*
- This analysis can be extended to the known exotic spectroscopy.

Backup

Isospin violations in Z decays

The Z s' feature a **reversed pattern** with respect to the X

$$g_{\text{mix}}^Z = \frac{Z_T}{D^-} \text{---} \text{---} \text{---} \frac{Z_S}{D^{+*}} = g_T^Z g_S^Z \frac{m_+}{\pi^2} \left(\Lambda_Z - \sqrt{-2m_+ E_Z} \arctan \frac{\Lambda_Z}{\sqrt{-2m_+ E_Z}} \right)$$

$$E_Z = m_Z - m_{D^{+*}} - m_{D^-} = 7.3 \pm 2.6 \text{ MeV}$$

The close charged threshold (from below!)

$$g_{\text{mix}}^Z = g_T^Z g_S^Z \frac{m_+}{\pi^2} \left(\Lambda_Z - \sqrt{2m_+ E_Z} \operatorname{arctanh} \frac{\Lambda_Z}{\sqrt{2m_+ E_Z}} \right).$$

THE BORN-OPPENHEIMER APPROXIMATION AND – COMPACT – TETRAQUARKS

BORN-OPPENHEIMER FOR MOLECULES

$$(T_e(p) + T_N(P) + V(x, X)) \Psi = E \Psi$$

Neglect T_N . Discrete spectrum, normalizable states $(\Psi, \Psi) = 1$.

Switching off T_N converts Ψ into Φ_X^n with continuum normalization.

$$(T_e(p) + V(x, X)) \Phi_X^n = \mathcal{E}_n(X) \Phi_X^n$$

$$(\Phi_X^n, \Phi_Y^m) = \delta^{nm} \prod_{N,i} \delta(X_{N,i} - Y_{N,i})$$

Eventually the solution Ψ can be expressed in terms of the Φ_X^n **complete set**, where all the electron dynamics is contained

$$\Psi = \int dX \Psi_n(X) \Phi_X^n$$

BORN-OPPENHEIMER FOR MOLECULES

The Φ_X^n can be written in turn as

$$\Phi_X^n = \int dx \psi_n(x, X) \Phi_{x,X}$$

so as the equation to solve to construct the Φ_X^n is

$$(T_e(-i\nabla_x) + V(x, X)) \psi_n(x, X) = \mathcal{E}_n(X) \psi_n(x, X)$$

$\mathcal{E}_n(X)$ can be obtained by a variational principle, i.e. minimizing $\langle T_e + V(x, X) \rangle_{\psi_n}$, while holding X fixed. The $\mathcal{E}_n(X)$ obtained is then inserted back into the nuclear part **in place of T_e and V**

$$\left(T_N(-i\nabla_X) + \mathcal{E}_n(X) \right) \Psi_n(X) = E \Psi_n(X)$$

BORN-OPPENHEIMER FOR MOLECULES

Deducing the latter equation requires to apply T_N on Φ_X^n and in turn on $\psi_n(x, X)$ and $\Phi_{x, X}$ (where $P_{Ni} \Phi_{x, X} = i\hbar \frac{\partial}{\partial X_{Ni}} \Phi_{x, X}$)

The BO approximation consists in dropping the gradient

$$\nabla_{X_{Ni}} \psi(x, X) \simeq 0$$

which is motivated by

$$\Psi_n(X) \nabla_{X_{Ni}} \psi(x, X) \sim \left(\frac{m}{M} \right)^{1/4} \psi(x, X) \nabla_{X_{Ni}} \Psi_n(X)$$

BORN-OPPENHEIMER FOR MOLECULES

The BO approximation consists in dropping the gradient

$$\nabla_X \psi(x, X) \simeq 0$$

which is motivated by

$$\Psi(X) \nabla_{X_{Ni}} \psi(x, X) \sim \left(\frac{m}{M} \right)^{1/4} \psi(x, X) \nabla_{X_{Ni}} \Psi(X)$$

used to obtain

$$\left(T_N(-i\nabla_X) + \mathcal{E}(X) \right) \Psi(X) = E \Psi(X)$$

The B.O. Tetraquark spectrum

