Dipion transitions between Heavy Hybrids and Heavy Quarkonium

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Exotic Quarkonia in Heavy-ion Collisions

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JS, Sandra Tomàs Valls, 2602.xxxxx

Heavy Flavors

- Heavy quarks: $Q = c, b, t,\ m_Q \gg \Lambda_{QCD}$
- Heavy hadrons: hadrons containing at least a heavy quark: $Q = b, c$
- In the hadron rest frame the heavy quark moves slowly $\Longrightarrow$ use a non-relativistic approximation
- A universal way to encode it together with relativistic correction is using Effective Field Theories
- NRQCD/HQET are the suitable ones
- They imply heavy quark spin symmetry at leading order.

NRQCD

W.E. Caswell and G.P. Lepage, Phys. Lett. **167B**, 437 (1986)
G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51** (1995) 1125

$$m_Q \quad >> \quad m_Q v \quad , \quad m_Q v^2 \quad , \quad \Lambda_{QCD}$$

$$\begin{aligned}\mathcal{L}_\psi \quad = \quad & \psi^\dagger \Bigg\{ iD_0 + \frac{1}{2m_Q}\mathbf{D}^2 + \frac{1}{8m_Q^3}\mathbf{D}^4 + \frac{c_F}{2m_Q}\boldsymbol{\sigma}.g\mathbf{B} + \\ & + \frac{c_D}{8m_Q^2}\left(\mathbf{D}.g\mathbf{E} - g\mathbf{E}.\mathbf{D}\right) + i\,\frac{c_S}{8m_Q^2}\boldsymbol{\sigma}.\left(\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times\mathbf{D}\right)\Bigg\}\psi\end{aligned}$$

c_F, c_D and c_S are short distance matching coefficients calculable from QCD in powers of $\alpha_{\rm s}$. They depend on m_Q and μ (factorization scale) but not on the lower energy scales.

Hadrons with two heavy quarks

$$Q = b,\ c \quad , \quad q = u,\ d,\ s$$

- $QQ+$ light quarks and gluons
  - Double Heavy Baryons: QQq
  - Tetraquarks: $QQ\bar{q}\bar{q}$
  - Pentaquarks: $QQqq\bar{q}$
  - Hybrids: $QQqg$
  - ...

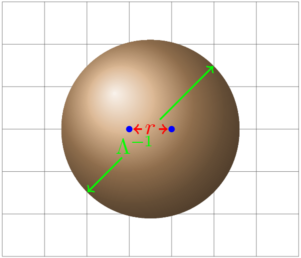
- $Q\bar{Q}+$ light quarks and gluons
  - Heavy Quarkonium: $Q\bar{Q}$
  - Hybrids: $Q\bar{Q}g$
  - Tetraquarks: $Q\bar{Q}q\bar{q}$
  - Pentaquarks: $Q\bar{Q}qqq$
  - ...

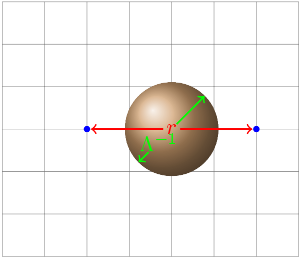
Heavy Quarkonium

$Q\bar{Q}$ bound state , $m_Q >> \Lambda_{QCD}$, $\alpha_s(m_Q) << 1$

- Heavy quarks move slowly $v << 1$
- Non-relativistic system $\rightarrow$ multiscale problem
  - $m_Q >> m_Q v$ (Relative momentum)
  - $m_Q v >> m_Q v^2$ (Binding energy)
  - $m_Q >> \Lambda_{QCD}$
- EFTs are useful (N. Brambilla, A. Pineda, JS and A. Vairo, Rev. Mod. Phys. 77, 1423 (2005))
  - NRQCD: $m_Q \gg m_Q v$, $m_Q v^2$, Λ_{QCD} (W.E. Caswell and G.P. Lepage, Phys. Lett. **167B**, 437 (1986))
  - pNRQCD (weak coupling): $m_Q v \gg m_Q v^2$, Λ_{QCD} (A. Pineda, JS, Nucl.Phys.Proc.Suppl.64:428-432,1998)
  - pNRQCD (strong coupling): $m_Q v$, $\Lambda_{QCD} \gg m_Q v^2$ (N. Brambilla, A. Pineda, JS, A. Vairo, Nucl.Phys.B566:275,2000)

How does the hadron look like ?





$m_Q v \sim 1/r \gg m_Q v^2 \gtrsim \Lambda_{QCD}$

weak coupling pNRQCD

$m_Q v \sim 1/r \gtrsim \Lambda_{QCD} \gg m_Q v^2$

strong coupling pNRQCD

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Born-Oppenheimer EFT

Figures: Najjar, Bali, 2009

pNRQCD weak coupling regime $\Lambda_{QCD} \lesssim m_Q v^2 \ll m_Q v$

$$\mathcal{L}_{\rm pNRQCD} = \int d^3\mathbf{r}\, {\rm Tr}\left\{ {\rm S}^\dagger \left(i\partial_0 - h_s(\mathbf{r},\mathbf{p},\mathbf{P_R},\mathbf{S}_1,\mathbf{S}_2,\mu)\right) {\rm S} + \right.$$

$$\left. + {\rm O}^\dagger \left(iD_0 - h_o(\mathbf{r},\mathbf{p},\mathbf{P_R},\mathbf{S}_1,\mathbf{S}_2,\mu)\right) {\rm O} \right\}$$

$$+ V_A(r,\mu) {\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\, {\rm S} + {\rm S}^\dagger \mathbf{r}\cdot g\mathbf{E}\, {\rm O} \right\} +$$

$$+ \frac{V_B(r,\mu)}{2} {\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\, {\rm O} + {\rm O}^\dagger {\rm O} \mathbf{r}\cdot g\mathbf{E} \right\} + \mathcal{O}(\mathbf{r}^2, \frac{1}{m_Q})$$

- $h_{s,o} = \frac{\mathbf{p}^2}{m_Q} + V_{s,o}(r,\mu) + \mathcal{O}(\frac{1}{m_Q})$, quantum mechanical Hamiltonians with scale dependent potentials calculable in pertubation theory in $\alpha_{\rm s}(m_Q v)$ and $1/m_Q$ ($V_s \simeq -4\alpha_{\rm s}/3r$, $V_o \simeq \alpha_{\rm s}/6r$)
- Spin symmetry holds in $h_{s,o}$ up to $\mathcal{O}(\frac{1}{m_Q^2})$
- S=S($\mathbf{r}$, $\mathbf{R}$, t), O=O($\mathbf{r}$, $\mathbf{R}$, t) are the color singlet/octet wave function fields
- $\mathbf{E}=\mathbf{E}(\mathbf{R}, t)$ is the chromoelectric field

Born-Oppenheimer EFT $m_Q v^2 \ll \quad \Lambda_{QCD} \lesssim mv$

$$L_{\rm BOEFT} = \int d^3\mathbf{x}_1 \int d^3\mathbf{x}_2 \; S^\dagger \left(i\partial_0 - h_s(\mathbf{x}_1 - \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2) \right) S,$$

$$h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2) = \frac{\mathbf{p}_1^2}{2m_Q} + \frac{\mathbf{p}_2^2}{2m_Q} + V_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2),$$

$$V_s = V_s^{(0)} + \frac{V_s^{(1)}}{m_Q} + \frac{V_s^{(2)}}{m_Q^2} + \cdots ,$$

All V_ss can be, and most of them have been, calculated on the lattice

- $V_s^{(0)}$ and $V_s^{(1)}$ are central Spin Symmetry holds
- $V_s^{(2)}$ contains spin and velocity dependent terms

Born-Oppenheimer EFT at LO

- Matching to NRQCD in the static limit $\Rightarrow$ $V_s^{(0)}$ is the ground state energy of two static color sources separated at a distance r
- Can be extracted from lattice calculations of the Wilson loop

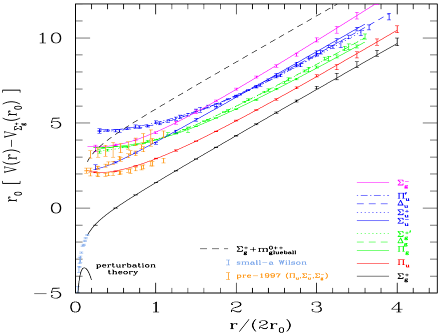


Figure: Meyer, Swanson, 2015

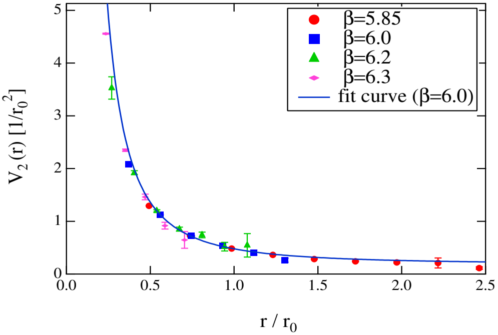
- Well fitted by the Cornell potential

$$V_s^{(0)} = V_{\Sigma_g^+}(r) \approx -\frac{k_g}{r} + \kappa r + E_g^{Q\bar{Q}} \quad , \; k_g = 0.489 \quad , \kappa = 0.187\,GeV^2$$

BOEFT beyond LO

Ex. at $\mathcal{O}(1/m_Q^2)$: $V_{L_2S_1}^{(1,1)}$ spin-orbit potential (Eichten, Feinberg, 79)

$$V_{L_2S_1}^{(1,1)}(r) = -\frac{c_F}{r^2} i\mathbf{r} \cdot \lim_{T\to\infty} \int_0^T dt\, t \, \langle\langle g\mathbf{B}(t, \mathbf{r}/2) \times g\mathbf{E}(0, -\mathbf{r}/2)\rangle\rangle$$



$V_2 = V_{L_2S_1}^{(1,1)}/c_F$, Koma, Koma, 09

pNRQCD strong coupling regime beyond LO

An example at $\mathcal{O}(1/m_Q^2)$: the $V_{L_2S_1}^{(1,1)}$ spin-orbit potential

- Short distance constraint: it must coincide with the perturbative evaluation,

$$V_{L_2S_1}^{(1,1)}(r) \sim c_F \frac{C_f \alpha_{\rm s}}{r^3} \quad , \; r \to 0$$

(Gupta, Radford, 81)

- Long distance constraint: it must coincide with the QCD effective string theory result

$$V_{L_2S_1}^{(1,1)}(r) = -\frac{c_F g^2 \Lambda^2 \Lambda'}{\kappa \, r^2} \quad , \; r \to \infty$$

(Perez-Nadal, JS, 08)

$QQ/Q\bar{Q}$ + light quarks and gluons ($m_Q v$, $\Lambda_{QCD} \gg m_Q v^2$)

(JS, Tarrús Castellà, 20; see Abhishek's talk today and Nora's on Tue)

$$\mathcal{L}_{\rm BOEFT} = \sum_{\kappa^p} \Psi^\dagger_{\kappa^p} \left[i\partial_t - h_{\kappa^p} \right] \Psi_{\kappa^p}$$

$$h_{\kappa^p} = \frac{\boldsymbol{p}^2}{m_Q} + \frac{\boldsymbol{P}^2}{4m_Q} + V^{(0)}_{\kappa^p}(\boldsymbol{r}) + \frac{1}{m_Q} V^{(1)}_{\kappa^p}(\boldsymbol{r}, \boldsymbol{p}) + \mathcal{O}\left(\frac{1}{m_Q^2}\right)$$

- LDF$\equiv$ light quarks + gluons, characterized by their quantum numbers (κ, p ...)
  - $\kappa \equiv$ total angular momentum, $p \equiv$ parity (P)
  - Quantum numbers not explicitely displayed: C-parity, baryon number (B), isospin (I), strangeness (S), principal quantum number
- $V^{(0)}_{\kappa^p}$, $V^{(1)}_{\kappa^p}$, ... must be calculated non-perturbatively
- A truncation of $\mathcal{L}_{\rm HEH}$ needed for practical calculations $\Longrightarrow$ keep a limited number of lower lying κ^p

- $V^{(0)}_{\kappa^p}$ is a $(2\kappa+1)\times(2\kappa+1)\times\mathbb{I}_{2\,Q_1}\times\mathbb{I}_{2\,Q_2}$ matrix, which can be decomposed into irreducible representations of $D_{\infty\,h}$, the symmetry group of a diatomic molecule

$$V^{(0)}_{\kappa^p}(\boldsymbol{r})=\sum_{\Lambda}V^{(0)}_{\kappa^p\Lambda}(r)\mathcal{P}_{\kappa\Lambda}$$

$\mathcal{P}_{\kappa\Lambda}$ projects onto LDF angular momenta $\pm\Lambda$ in the direction joining the two heavy quarks, $\Lambda=\kappa,\ \kappa-1,\ \dots,\ \kappa-[\kappa]$

$$\begin{aligned}
\mathcal{P}_{\frac{1}{2}\frac{1}{2}}&=\mathbb{I}_2^{\rm lq}\\
\mathcal{P}_{\frac{3}{2}\frac{1}{2}}&=\frac{9}{8}\mathbb{I}_4^{\rm lq}-\frac{1}{2}\left(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_{3/2}\right)^2\\
\mathcal{P}_{\frac{3}{2}\frac{3}{2}}&=-\frac{1}{8}\mathbb{I}_4^{\rm lq}+\frac{1}{2}\left(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_{3/2}\right)^2\\
\mathcal{P}_{10}&=\mathbb{I}_3^{\rm lq}-(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_1)^2\\
\mathcal{P}_{11}&=(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_1)^2\\
&\dots
\end{aligned}$$

- $V^{(1)}_{\kappa^p} = V^{(1)}_{\kappa^p\mathrm{SI}} + V^{(1)}_{\kappa^p\mathrm{SD}}$
- $V^{(1)}_{\kappa^p\mathrm{SI}}$ does not depend on the spin or orbital angular momentum of the heavy quarks $\Longrightarrow$ admits the same decomposition as $V^{(0)}_{\kappa^p}$
- $V^{(1)}_{\kappa^p\mathrm{SD}}$ depends on the spin and orbital angular momentum of the heavy quarks

$$V^{(1)}_{\kappa^p\mathrm{SD}}(\boldsymbol{r}) = \sum_{\Lambda\Lambda'} \mathcal{P}_{\kappa\Lambda} \left[V^{sa}_{\kappa^p\Lambda\Lambda'}(\boldsymbol{r}) \boldsymbol{S}_{QQ} \cdot (\mathcal{P}_{10} \cdot \boldsymbol{S}_\kappa) + V^{sb}_{\kappa^p\Lambda\Lambda'}(\boldsymbol{r}) \boldsymbol{S}_{QQ} \cdot (\mathcal{P}_{11} \cdot \boldsymbol{S}_\kappa) \right.$$
$$\left. + V^{l}_{\kappa^p\Lambda\Lambda'}(\boldsymbol{r}) \left(\boldsymbol{L}_{QQ} \cdot \boldsymbol{S}_\kappa \right) \right] \mathcal{P}_{\kappa\Lambda'}$$

$$2\boldsymbol{S}_{QQ} = \boldsymbol{\sigma}_{QQ} = \boldsymbol{\sigma}_{Q_1} \mathbb{I}_{2\,Q_2} + \mathbb{I}_{2\,Q_1} \boldsymbol{\sigma}_{Q_2} \quad , \quad \mathcal{P}^{ij}_{10} = \hat{\boldsymbol{r}}^i \hat{\boldsymbol{r}}^j \quad , \quad \mathcal{P}^{ij}_{11} = \delta^{ij} - \hat{\boldsymbol{r}}^i \hat{\boldsymbol{r}}^j$$

Matching to NRQCD

- Build an NRQCD operator with the quantum numbers of Ψ_{κ^p}

$$\mathcal{O}^{Q\bar{Q}}_{\kappa^p}(t,\, \boldsymbol{r},\, \boldsymbol{R}) = \chi_c^\top(t,\, \boldsymbol{x}_2)\phi(t,\, \boldsymbol{x}_2,\, \boldsymbol{R})\mathcal{Q}_{Q\bar{Q}\kappa^p}(t,\, \boldsymbol{R})\phi(t,\, \boldsymbol{R},\, \boldsymbol{x}_1)\psi(t,\, \boldsymbol{x}_1)$$
$$\mathcal{O}^{QQ}_{\kappa^p}(t,\, \boldsymbol{r},\, \boldsymbol{R}) = \psi^\top(t,\, \boldsymbol{x}_2)\phi^\top(t,\, \boldsymbol{R},\, \boldsymbol{x}_2)\mathcal{Q}_{QQ\kappa^p}(t,\, \boldsymbol{R})\phi(t,\, \boldsymbol{R},\, \boldsymbol{x}_1)\psi(t,\, \boldsymbol{x}_1)$$

- Examples:
  - Hybrid

$$\mathcal{Q}^{\alpha}_{Q\bar{Q}\,1^{+-}}(t,\boldsymbol{x}) = \left(\boldsymbol{e}^\dagger_\alpha \cdot \boldsymbol{B}(t,\boldsymbol{x})\right)$$

  - $Q\bar{Q}q\bar{q}$ tetraquark

$$\mathcal{Q}_{Q\bar{Q}\,0^{++}}(t,\boldsymbol{x}) = \left[\bar{q}(t,\boldsymbol{x})T^a q(t,\boldsymbol{x})\right] T^a$$

  - Doubly heavy baryons

$$\mathcal{Q}^{\alpha}_{QQ\,(1/2)^{+}}(t,\boldsymbol{x}) = \underline{T}^l \left[P_+ q^l(t,\boldsymbol{x})\right]^\alpha$$

  - $QQ\bar{q}\bar{q}$ tetraquark

$$\mathcal{Q}_{QQ\,0^{-}}(t,\boldsymbol{x}) = \left[\bar{q}(t,\boldsymbol{x})\underline{T}^l\gamma^2 q^*(t,\boldsymbol{x})\right] \underline{T}^l$$

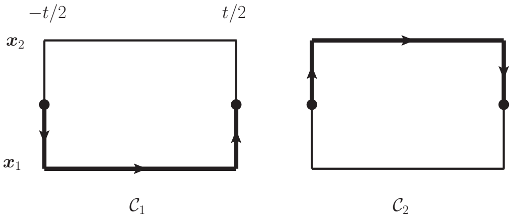
Matching to NRQCD

- Impose $\mathcal{O}^h_{\kappa^p}(t,\, \boldsymbol{r},\, \boldsymbol{R}) = \sqrt{Z_{h\kappa^p}}\Psi_{h\kappa^p}(t,\, \boldsymbol{r},\, \boldsymbol{R})\,, \quad h = QQ\,, Q\bar{Q}.$

$$\langle 0|\, T\{\mathcal{O}^h_{\kappa^p}(t/2)\mathcal{O}^{h\dagger}_{\kappa^p}(-t/2)\}|0\rangle = \sqrt{Z_{h\kappa^p}}\langle 0|\, T\{\Psi_{h\kappa^p}(t/2)\Psi^\dagger_{h\kappa^p}(-t/2)\}|0\rangle\sqrt{Z^\dagger_{h\kappa^p}}$$

  - Then at $\mathcal{O}(1)$

$$V^{(0)}_{h\kappa^p\Lambda}(\boldsymbol{r}) = \lim_{t\to\infty}\frac{i}{t}\log\left(\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda}\langle 1\rangle^{h\kappa^p}_{\Box}\right]\right)$$



- At $\mathcal{O}\left(\frac{1}{m_Q}\right)$, for instance,

$$V^{sb}_{\kappa^p\Lambda\Lambda'} = -\frac{c_F}{2}\lim_{t\to\infty}\sqrt{\frac{\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda}\right]\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda'}\right]}{\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda}\langle 1\rangle^{\kappa^p}_{\Box}\right]\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda'}\langle 1\rangle^{\kappa^p}_{\Box}\right]}}$$

$$\times\frac{V^{(0)}_{\kappa^p\Lambda} - V^{(0)}_{\kappa^p\Lambda'}}{\sin\left[\left(V^{(0)}_{\kappa^p\Lambda} - V^{(0)}_{\kappa^p\Lambda'}\right)\frac{t}{2}\right]}$$

$$\times\int_{-t/2}^{t/2} dt'\,\frac{\mathrm{Tr}\left[\left(\boldsymbol{S}_\kappa\cdot\mathcal{P}^{\mathrm{c.r.}}_{11}\right)\cdot\left(\mathcal{P}_{\kappa\Lambda}\langle g\boldsymbol{B}(t',\boldsymbol{x}_1)\rangle^{\kappa^p}_{\Box}\mathcal{P}_{\kappa\Lambda'}\right)\right]}{\mathrm{Tr}\left[\left(\boldsymbol{S}_\kappa\cdot\mathcal{P}^{\mathrm{c.r.}}_{11}\right)\cdot\left(\mathcal{P}_{\kappa\Lambda}\boldsymbol{S}_\kappa\mathcal{P}_{\kappa\Lambda'}\right)\right]}$$

Applications

- Doubly Heavy Baryons: QQq (JS, Tarrús Castellà, 20, 21)
- Hyperfine splittings of Heavy Quarkonium Hybrids: $Q\bar{Q}g$ (JS, Tomàs Valls, 23; see Sandra's talk on Tue)

Disclamer:

- Interactions with heavy-light meson/baryon pairs neglected
- They have been addressed in the BOEFT for heavy quarkonium (Tarrús Castellà, 22; Bruschini, 23)
- It has been recently generalized to double heavy exotics (Tarrús Castellà, 24; Bruschini, 24; see Abhishek's talk today and Nora's on Tue)

Heavy Quarkonium Hybrids

- Spin average spectrum (BO approximation; Braaten, Langmack, Hudson Smith, 14; Berwin, Brambilla, Tarrús Castellà, Vairo, 15; Oncala, JS, 17)
  - Based on lattice data (Juge, Kuti, Morningstar, 02; Bali, Pineda, 03)
  - More recent and accurate lattice data available (Capitani, Philipsen, Reisinger, Riehl, Wagner, 18; Schlosser, Wagner, 21; Höllwieser, Knechtli, Korzec, Peardon, Urrea-Niño, 23)
- Inclusive decay width to heavy quarkonium (Oncala, JS, 17)
  - Revisited in (Brambilla, Lai, Mohapatra, Vairo, 22)
    - ★ Improved the $\Delta S = 0$ transitions $\mathcal{O}(1/m_Q^0)$
    - ★ Calculated the $\Delta S = 1$ transitions $\mathcal{O}(1/m_Q^2)$
  - Updated recently (Oncala, JS, 25)
    - ★ Use potential based on recent lattice data (Alasiri, Braaten, Mohapatra, 24)
    - ★ Refined error analysis

Heavy Quarkonium Hybrids

- Mixing with heavy quarkonium ($\mathcal{O}(1/m_Q)$, spin-dependent; Oncala, JS, 17)
  - Important effects when a quarkonium state and a hybrid state with the same quantum numbers have similar masses
  - Leads to violations of spin conservation
  - The mixing potentials estimated by interpolating the known short distance form pNRQCD and the long distance form estimated with EST
  - In a large mixing scenario, $\psi(4230)$, $\psi(4360)$, $\psi(4660)$ and $\Upsilon(10860)$ are sizable mixtures of spin 0 hybrids and spin 1 quarkonium
  - Recent lattice data on the mixing potentials have recently appeared that need to be incorporated (Schlosser, Wagner, 25)
- Selection rules for exclusive decays (Braaten, Langmack, Hudson Smith, 14; Bruschini, 23; Braaten, Bruschini, 24)

LO results κ^{PC}=1^{+-} for charm

(Oncala,JS,25)

nL_J	w-f	$M_{c\bar{c}}$	$M_{c\bar{c}g}$	$\mathbb{S}=0$ $\mathcal{J}^{PC}$	$\mathbb{S}=1$ $\mathcal{J}^{PC}$	Λ_η^ϵ
$1s$	S	3068		0^{-+}	1^{--}	Σ_g^+
$2s$	S	3674		0^{-+}	1^{--}	Σ_g^+
$3s$	S	4149		0^{-+}	1^{--}	Σ_g^+
$1p_0$	P^+		4455	0^{++}	1^{+-}	Σ_u^-
$4s$	S	4562		0^{-+}	1^{--}	Σ_g^+
$2p_0$	P^+		4917	0^{++}	1^{+-}	Σ_u^-
$5s$	S	4937		0^{-+}	1^{--}	Σ_g^+
$3p_0$	P^+		5315	0^{++}	1^{+-}	Σ_u^-
$4p_0$	P^+		5650	0^{++}	1^{+-}	Σ_u^-
$1p$	S	3457		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$2p$	S	3958		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$1(s/d)_1$	P^{+-}		4028	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$1p_1$	P^0		4171	1^{++}	$(0,1,2)^{+-}$	Π_u
$2(s/d)_1$	P^{+-}		4394	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$3p$	S	4388		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$2p_1$	P^0		4556	1^{++}	$(0,1,2)^{+-}$	Π_u
$3(s/d)_1$	P^{+-}		4678	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$4(s/d)_1$	P^{+-}		4755	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$4p$	S	4774		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$3p_1$	P^0		4912	1^{++}	$(0,1,2)^{+-}$	Π_u
$5(s/d)_1$	P^{+-}		5087	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$5p$	S	5130		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$1d$	S	3762		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$2d$	S	4209		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$1(p/f)_2$	P^{+-}		4245	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$1d_2$	P^0		4369	2^{--}	$(1,2,3)^{-+}$	Π_u
$2(p/f)_2$	P^{+-}		4601	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$3d$	S	4608		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$2d_2$	P^0		4739	2^{--}	$(1,2,3)^{-+}$	Π_u
$3(p/f)_2$	P^{+-}		4892	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$4d$	S	4974		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$4(p/f)_2$	P^{+-}		4945	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$

LO results $\kappa^{PC}=1^{+-}$ for bottom

(Oncala,JS,25)

nL_J	w-f	$M_{b\bar{b}}$	$M_{b\bar{b}g}$	$\mathbb{S}=0$ $\mathcal{J}^{PC}$	$\mathbb{S}=1$ $\mathcal{J}^{PC}$	Λ_η^ϵ
$1s$	S	9551		0^{-+}	1^{--}	Σ_g^+
$2s$	S	10017		0^{-+}	1^{--}	Σ_g^+
$3s$	S	10355		0^{-+}	1^{--}	Σ_g^+
$4s$	S	10643		0^{-+}	1^{--}	Σ_g^+
$5s$	S	10901		0^{-+}	1^{--}	Σ_g^+
$1p_0$	P^+		10977	0^{++}	1^{+-}	Σ_u^-
$6s$	S	11139		0^{-+}	1^{--}	Σ_g^+
$2p_0$	P^+		11261	0^{++}	1^{+-}	Σ_u^-
$3p_0$	P^+		11525	0^{++}	1^{+-}	Σ_u^-
$4p_0$	P^+		11782	0^{++}	1^{+-}	Σ_u^-
$1p$	S	9879		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$2p$	S	10234		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$3p$	S	10532		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$1(s/d)_1$	P^{+-}		10704	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$1p_1$	P^0		10772	1^{++}	$(0,1,2)^{+-}$	Π_u
$4p$	S	10798		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$2(s/d)_1$	P^{+-}		10905	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$2p_1$	P^0		10995	1^{++}	$(0,1,2)^{+-}$	Π_u
$5p$	S	11041		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$3(s/d)_1$	P^{+-}		11103	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$4(s/d)_1$	P^{+-}		11131	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$3p_1$	P^0		11209	1^{++}	$(0,1,2)^{+-}$	Π_u
$6p$	S	11268		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$5(s/d)_1$	P^{+-}		11318	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$1d$	S	10106		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$2d$	S	10416		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$3d$	S	10689		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$1(p/f)_2$	P^{+-}		10823	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$1d_2$	P^0		10880	2^{--}	$(1,2,3)^{-+}$	Π_u
$4d$	S	10939		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$2(p/f)_2$	P^{+-}		11023	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$

$Q\bar{Q} \to Q\bar{Q} + \pi\pi$ in BOEFT

- At LO in $1/m_Q$ and the chiral expansion

$$\mathrm{L}_{\mathrm{int}} = \int d^3\mathbf{R} \int d^3\mathbf{r}\, \mathrm{tr}\Big[S^\dagger(\mathbf{R},\mathbf{r},t)\left(g_0(\mathbf{r})\partial_0 U^\dagger \partial^0 U + g_1(\mathbf{r})\partial_i U^\dagger \partial^i U + \right.$$

$$\left. + g_2(\mathbf{r}) r^i r^j \partial_i U^\dagger \partial_j U + g_3(\mathbf{r})\left(U^\dagger \mathcal{M} + \mathcal{M}^\dagger U\right)\right) S(\mathbf{R},\mathbf{r},t)\Big]$$

$$U = e^{\frac{i\vec{\pi}\vec{\tau}}{f_\pi}} \quad , \quad \vec{\pi}\vec{\tau} = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

  - $\vec{\pi} = \vec{\pi}(\mathbf{R}, t)$, $\mathcal{M} = m_q \mathbb{I}$ (light quark mass), $m_\pi^2 = 2 m_q B_0$, $B_0 \sim \Lambda_{QCD}$
  - It implicitly assumes that $1/r, \Lambda_{QCD} \gg \Delta E$
  - For $r \ll 1/\Lambda_{QCD}$, $g_k(\mathbf{r})$, $k = 0, 1, 2, 3$, are analytic in $\mathbf{r}$, $g_k(\mathbf{r}) \sim r^2$ (multipole expansion: Gottfried, 78; Voloshin 79)
  - For $r \gg 1/\Lambda_{QCD}$, one should be able to estimate them in the EST
    - ⋆ We need an interaction Lagrangian of pions with the QCD string

$Q\bar{Q}g(1^{+-}) \to Q\bar{Q} + \pi\pi$ in BOEFT

- At LO in $1/m_Q$ and the chiral expansion

$$\mathrm{L}_{\mathrm{int}} = \int d^3\mathbf{R} \int d^3\mathbf{r}\, \mathrm{tr}\left[S^\dagger(\mathbf{R},\mathbf{r},t)\left(\epsilon_{ijk} r^j g_4(\mathbf{r})\partial_0 U^\dagger \partial^k U\right) H^i(\mathbf{R},\mathbf{r},t)\right] + \mathrm{H.c.}$$

  - It implicitly assumes that $1/r, \Lambda_{QCD} \gg \Delta E$
  - For $r \ll 1/\Lambda_{QCD}$, $g_4(\mathbf{r})$ is analytic in $\mathbf{r}$, $g_4(\mathbf{r}) \sim 1$ (multipole expansion: Pineda, Tarrús Castellà, 19; Tarrús Castellà, Passemar, 21)
  - For $r \gg 1/\Lambda_{QCD}$, one should be able to estimate them in the EST
    - ⋆ We need an interaction Lagrangian of pions with the QCD string

The interaction of pions with the QCD string

- Pions: Lorentz invariance, chiral symmetry
- String: Lorentz invariance, reparameterization invariance
- Pions+String: Locality $\Longrightarrow$ Embed the string in the Chiral Lagrangian

$$S_{\rm int} = \int d^2\xi \sqrt{-\mathrm{det}(\partial_a x^\mu \partial_b x_\mu)} \left(\mathcal{L}_{\mathrm{Ch}} + \eta \partial_a x^\mu \partial^a x^\nu \mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\nu U\right)\right)$$

$$\mathcal{L}_{\mathrm{Ch}}^{LO} = \lambda \mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \lambda' \mathrm{Tr}(U^\dagger \mathcal{M} + \mathcal{M}^\dagger U)$$

$$\mathcal{L}_{\mathrm{Ch}}^{NLO} = \lambda''' \mathrm{Tr}(\mathcal{M}^\dagger \mathcal{M}) + \lambda'''' \mathrm{Tr}(U^\dagger \mathcal{M} U^\dagger \mathcal{M} + \mathrm{h.c.}) + \cdots$$

- $U = U(x(\xi))$
- We fix the frame such that $\xi = (x^0, x^3) = (t, z)$, then $x^i = x^i(t,z)$, $i = 1, 2$
- We assume $1/r \sim m_\pi$ and count $x^i \sim 1/\Lambda_{QCD}$ and $\partial_\mu \sim 1/r$ (on x^i) $\sim m_\pi$ (on U)

Quark mass dependence of the string tension

- It implies the following light quark mass (m_q) dependence $(m_\pi^2 = 2m_q B_0$, $B_0 \sim \Lambda_{QCD})$ of the string tension

$$\sigma \to \sigma - \Bigg(4\lambda' m_q + \left[\frac{3B_0}{4\pi^2 f_\pi^2} \left(2B_0 \left(\lambda + \frac{\eta}{2} \right) - \lambda' \right) \times \left(\ln \frac{2B_0 m_q}{\mu^2} - 1 \right) + 2\lambda'' \right] m_q^2 \Bigg)$$

$$\lambda'' = \lambda''' + 2\lambda'''' - 3B_0\eta/(4\pi f_\pi)^2$$

Scattering

- Pion scattering off the string
  - String states are characterized by the representations of $D_{\infty\, h} \subset O(3)$, $|L_z|^h_{CP}$. $|L_z| = 0(\Sigma), 1(\Pi), 2(\Delta), \ldots,\ CP = +(g), -(u),\ h = +, -$

$$0^{++} \supset \Sigma_g^+ \quad , \quad 1^{+-} \supset \Sigma_u^- \, , \Pi_u \quad , \quad 2^{--} \supset \Sigma_g^- \, , \Pi_g \, , \Delta_g \quad , \quad \ldots$$

  - $\pi(q)\Sigma_g^+ \to \pi(q')\Sigma_g^+$

$$\mathcal{A} = \frac{8}{f_\pi^2}\left(\lambda q_\mu q'^\mu - \frac{\lambda' m_\pi^2}{2B_0} + \eta\left(q_0 q_0' - q_z q_z'\right)\right) \frac{\sin\left[(q_z - q_z')\frac{r}{2}\right]}{(q_z - q_z')}$$

$$\underset{r|q_z - q_z'| \ll 1}{\simeq} \frac{8}{f_\pi^2}\left(\lambda q_\mu q'^\mu - \frac{\lambda' m_\pi^2}{2B_0} + \eta\left(q_0 q_0' - q_z q_z'\right)\right) \frac{r}{2}$$

Decay

- Decay of string excitations by pion emission
  - $\Sigma_u^- \to \Sigma_g^+ \pi(q)\pi(q')$

$$\mathcal{A} = 0$$

  - $\Pi_u^{L/R} \to \Sigma_g^+ \pi(q)\pi(q')$

$$\mathcal{A}^R = \mathcal{A}^{L^*} = \frac{8i\sqrt{\pi}}{f_\pi^2\sqrt{\sigma}r}\frac{\cos\left[(q_z+{q'}_z)\frac{r}{2}\right]}{\frac{\pi^2}{r^2}-(q_z+{q'}_z)^2}\{[\mathrm{q}+\mathrm{q}']\times \left(\lambda q_\mu {q'}^\mu + \frac{\lambda' m_\pi^2}{2B_0} + \eta(q_0 q_0' - q_z q_z')\right) +\eta\left[\frac{\pi}{r}\left(E_q \mathrm{q}' + E_{q'}\mathrm{q}\right) + (q_z+q_z')(q_z\mathrm{q}+q_z'\mathrm{q})\right]\right\}$$

$$\underset{r|q_z+q_z'|\ll 1}{\simeq} \frac{8\eta i}{f_\pi^2\sqrt{\pi\sigma}}\left(E_q\mathrm{q}' + E_{q'}\mathrm{q}\right)$$

$$\mathrm{q} = \frac{1}{\sqrt{2}}\left(q^1 + iq^2\right) \quad , \quad \mathrm{q}' = \frac{1}{\sqrt{2}}\left({q'}^1 + i{q'}^2\right)$$

Quarkonium dipion transitions

- For large principal quantum number, the long distance form of $g_k(\mathbf{r})$, $k = 0, 1, 2, 3$, is expected to dominate
- The following interactions match the result of EST in the static limit
  - $\Sigma_g^+ \to \Sigma_g^+ \pi(q)\pi(q')$

$$\mathrm{L_{int}} = \int d^3\mathbf{R} \int d^3\mathbf{r}\, \mathrm{tr}\Big[S^\dagger(\mathbf{R},\mathbf{r},t) \int_{-r/2}^{r/2} dz\, g(r,z)\Big(\mathcal{L}_{\mathrm{Ch}} + \\ + \eta \mathrm{tr}\left(\partial_0 U^\dagger \partial_0 U - \hat{r}^i \hat{r}^j \partial_i U^\dagger \partial_j U\right)\Big) S(\mathbf{R},\mathbf{r},t)\Big]$$

$$U = U(t, \mathbf{R} + z\hat{\mathbf{r}})$$

  - For $r\Delta E \ll 1$ the expected form in the BOEFT is recovered: $g_0(\mathbf{r}) = = \int_{-r/2}^{r/2} dz\, g(r,z)(\lambda+\eta) = (\lambda+\eta) r$, $g_1(\mathbf{r}) = \int_{-r/2}^{r/2} dz\, g(r,z)\lambda = \lambda r$, $g_2(\mathbf{r}) = \int_{-r/2}^{r/2} dz\, g(r,z)(-\eta) = -\eta r$, $g_3(\mathbf{r}) = \int_{-r/2}^{r/2} dz\, g(r,z)\lambda' = \lambda' r$
  - Recall that in the multipole expansion ($r\Lambda_{QCD} \ll 1$), $g_k(r) \sim r^2$, $k = 0, 1, 2, 3$.

Hybrid to quarkonium dipion transitions

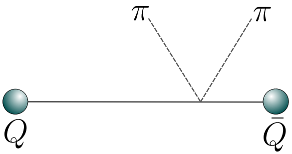
- The following interactions match the result of EST in the static limit
  - $\Sigma_u^-\,,\Pi_u^{L/R}\to\Sigma_g^+\,\pi(q)\pi(q')$

$$
\begin{aligned}
\mathrm{L}_{\mathrm{int}} &= \int d^3\mathbf{R}\int d^3\mathbf{r}\,\mathrm{tr}\left[S^\dagger(\mathbf{R},\mathbf{r},t)H^i(\mathbf{R},\mathbf{r},t)\right]\int_{-r/2}^{r/2}dz\,h(r,z)\epsilon^{ijk}r^j \\
&\quad \left[\partial_k\,\mathcal{L}_{\mathrm{Ch}}+2\eta\left(\frac{i\pi}{r}\mathrm{tr}\left(\partial_0U^\dagger\partial_kU\right)+\hat{r}^l\hat{r}^s\partial_s\mathrm{tr}\left(\partial_lU^\dagger\partial_kU\right)\right)\right]
\end{aligned}
$$

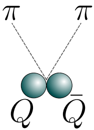
$$U=U(t,\mathbf{R}+z\hat{\mathbf{r}})$$

  - For $r\Delta E\ll 1$ the expected form in the BOEFT is recovered with $g_4(\mathbf{r})=\int_{-r/2}^{r/2}dz\,h(r,z)2\eta i\pi/r=8i\eta/\sqrt{\pi\sigma}\,r$
  - Recall that in the multipole expansion ($r\Lambda_{QCD}\ll 1$), $g_4(r)\sim 1$

String picture



Multipole expansion



Extracting the long distance parameters

- The same 3 parameters appear both in the quarkonium and in the hybrid to quarkonium transitions

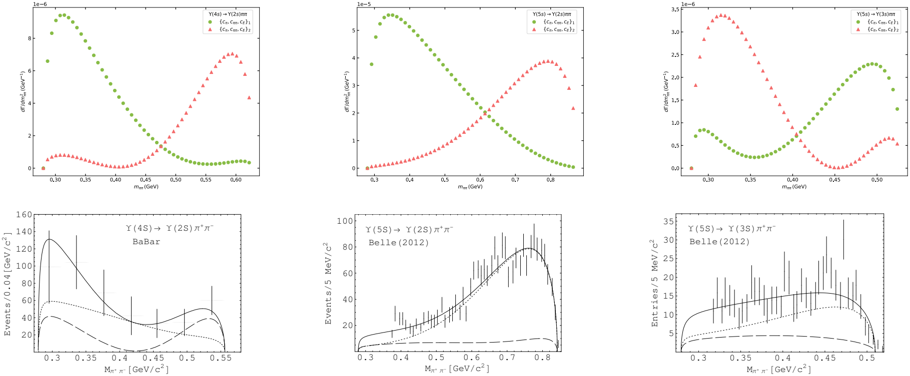
$$c_{\pi\pi} \equiv \frac{1}{2}\left(\lambda + \frac{\eta}{2}\right), \quad c_{\pi} \equiv -\lambda - \eta + \frac{\lambda'}{2B_0}, \quad c_E \equiv \eta/4$$

- We calculate the dipion invariant mass distribution and the decay width for quarkonium and hybrid to quarkonium transitions in terms of these parameters
- We extract them from $\Gamma(\Upsilon(4s) \to \Upsilon(2s)\pi^+\pi^-)$ (Guido, Mussa, Tamponi *et al.*, 17 [Belle]), $\Gamma(\Upsilon(3s) \to \Upsilon(2s)\pi^+\pi^-)$ (PDG) and $\Gamma(\Upsilon(10860) \to \Upsilon(2s)\pi^+\pi^-)$ (Bondar, Garmash, Kuzmin, 13)

$$\left\{\begin{array}{c} c_{\pi} = \pm 0.5160 \pm 0.0182 \\ c_{\pi\pi} = \pm 0.0076 \pm 0.0042 \\ c_E = \mp 0.0608 \pm 0.0014 \end{array}\right\}_1 \qquad \left\{\begin{array}{c} c_{\pi} = \pm 0.5184 \pm 0.0178 \\ c_{\pi\pi} = \mp 0.0341 \pm 0.0027 \\ c_E = \mp 0.0196 \pm 0.0057 \end{array}\right\}_2$$

Which is the right set? Green or Red?

(Surovtsev, Bydzovsky, Gutsche, Kaminski, Lyubovitskij, Nagy, 15)

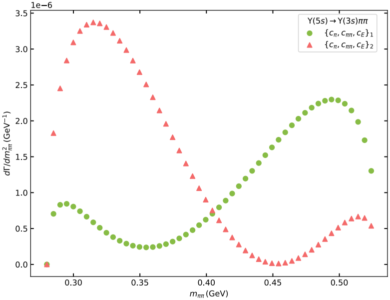


An Application

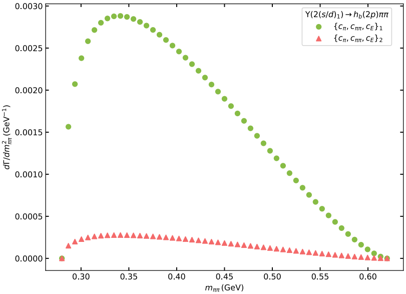
- Once we have extracted the parameters we can calculate the remaning transitions
- The green set is slightly favored
- Let us focus on $\Upsilon(10860)$
  - Dipion decays both to $\Upsilon(nS)$, $n = 1, 2, 3$, and to $h_b(nS)$, $n = 1, 2$ have been measured and have the same size
  - It could be a mixture of a quarkonium 5s state and a hybrid $2(s/d)_1$ (H_1'), (Oncala, JS, 17; 25)

$\Upsilon(10860)$

$\Upsilon(5s) \to \Upsilon(3s)\pi\pi$



$\Upsilon(2(s/d)_1) \equiv H_1' \to h_b(2p)\pi\pi$



Conclusions

- General:
  - The BOEFT provides a QCD based framework to address doubly-heavy exotics systematically
  - It requires non-perturbative potentials as an input
  - When those potentials are not available, a Cornell-like approach of interpolating between the short distance QCD (pNRQCD) calculation and a long distance EST calculation appears to be promising
- Specific:
  - We propose interaction Lagrangians for dipion transitions in the BOEFT
  - We have estimated the long distance behavior of the transition form factors $(g_k(r))$by introducing an interaction Lagrangian of pions with the QCD string
    - ★ Provides the light-quark mass dependence of the string tension
  - We have calculated the dipion invariant mass spectrum for several transitions, assuming that they are long distance dominated.
- It may help to elucidate the nature of some charmonium and bottomonium states (quarkonium vs hybrid)

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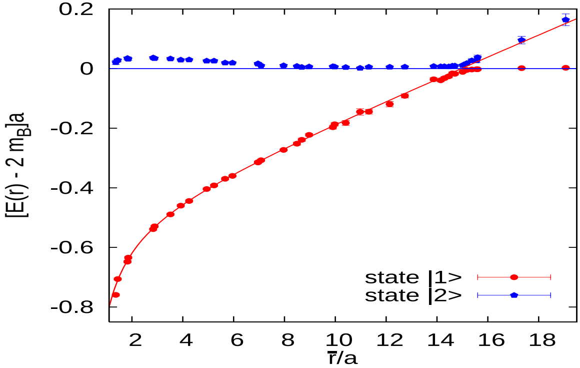
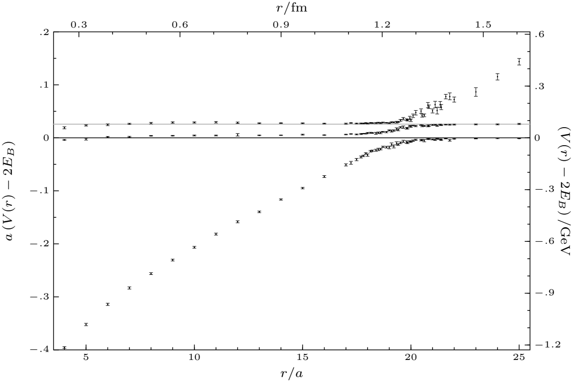
String breaking



Bali, Neff, Duessel, Lippert, Schilling, 2005

String breaking



Bulava, Hörz, Knechtli, Koch, Moir, Morningstar, Peardon, 2019