

Isospin breaking corrections to the hadronic vacuum polarization contribution to the muon $g-2$

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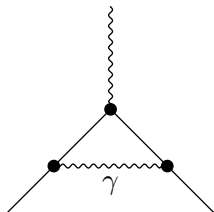
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UNIVERSITÄT MAINZ

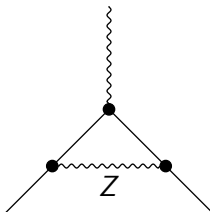


The anomalous magnetic moment of the muon ($g-2$)

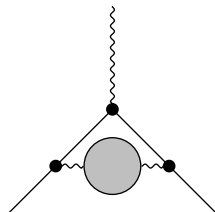
- classical result from Dirac equation predicts exactly two
- correction from higher order terms



QED corrections

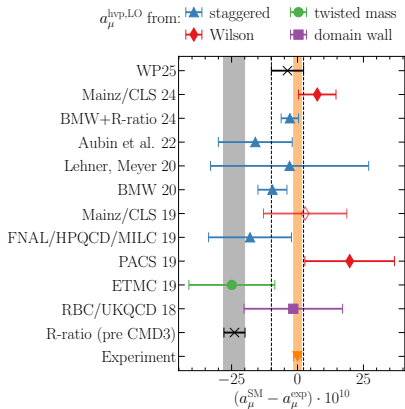


Electroweak corrections



Hadronic corrections

Current status of g-2 lattice calculations



- experimental data based R-Ratio methods differ by 5σ from experimental measurements of muon g-2
- latest lattice results agree with experimental measurements^a
- lattice community wide effort to reduce error to below $\sim \mathcal{O}(1\%)$

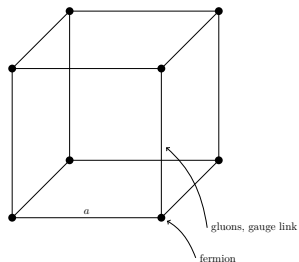
^aarXiv: 2505.21476 (hep-ph).

Overview plot of recent g-2 results (updated from^a)

^aarXiv: 2312.13753.

Lattice QCD

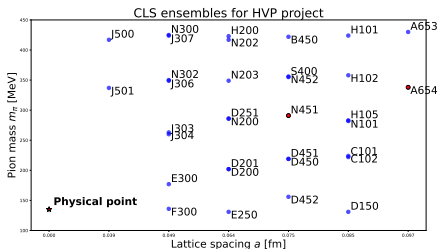
- large coupling \Rightarrow Lattice QCD as non-perturbative tool
- lattice QCD is an euclidean and discretized quantum field theory in finite volume
- lattice spacing as ultraviolet cut-off and finite volume as infrared cut-off
- use Monte-Carlo techniques: generate an ensemble with many configurations via a Markov chain



- average configurations of a single ensemble, then extrapolate to the physical point

CLS ensembles

- tree-level Symanzik improved Lüscher-Weisz gauge action and non-perturbatively $\mathcal{O}(a)$ improved Wilson fermions^a
- large range of pion masses down to the physical point
- six different lattice spacings [0.097 fm, 0.085 fm, 0.075 fm, 0.064 fm, 0.049 fm, 0.039 fm]
- A654 and N451 as test ensembles for new setup: periodic boundary conditions, $L^3 \times T = 24^3 \times 48, 48^3 \times 128$



^aarXiv: 1411.3982 (hep-lat).

Getting the HVP from the lattice

- compute correlation function on the lattice
- time momentum representation: integral over two-point correlation function with a kernel¹

$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt \tilde{K}(t; m_\mu) G(t)$$

$$G(t) = \frac{1}{3} \sum_{k=1}^3 \langle J_k(t) J_k(0) \rangle$$

$$J_\mu(t) = \sum_{f, \vec{x}} \bar{q}_f(t, \vec{x}) \gamma_\mu q_f(t, \vec{x})$$

¹arXiv: 1107.4388 (hep-lat).

²arXiv: 2505.24344 (hep-lat).

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- other lattice methods exists, e.g. the CCS-method
- ⇒ done in Mainz by Dominik Erb, Cornelia Tölle, Christoph Schröder and Harvey Meyer, e.g.²

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Euclidean windows

- to deal with specific lattice problems, it is suitable to construct "window observables"¹
- introduce Θ function in the integrand:
$$\Theta(t, t', \Delta) = \frac{1}{2}(1 + \tanh[(t - t')\Delta])$$

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²arXiv: 2401.11895 (hep-lat).

³arXiv: 2206.06582 (hep-lat).

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$$\Theta(t, t', \Delta) = \frac{1}{2}(1 + \tanh[(t - t')\Delta])$$
- short distance window: deals with discretization effects while neglecting signal-to-noise problems etc.²
- intermediate distance window: nearly free of lattice artifacts, cleanest signal³
- long distance window: suffers from signal-to-noise problem; mitigated by bounding method, low-mode averaging and spectral reconstruction⁴

¹arXiv: 1801.07224 (hep-lat).

²arXiv: 2401.11895 (hep-lat).

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Isospin breaking effects via the RM123 method

- isoQCD ensembles ($m_u = m_d = m_{ud}$ and no QED effects)
- introduces systematic error of $\mathcal{O}(1\%)$

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Isospin breaking effects via the RM123 method

- isoQCD ensembles ($m_u = m_d = m_{ud}$ and no QED effects)
- introduces systematic error of $\mathcal{O}(1\%)$
- treat these additional effects as small perturbation and expand in $\Delta m_d = m_d - m_{ud}$, $\Delta m_u = m_u - m_{ud}$ and in e^{12}

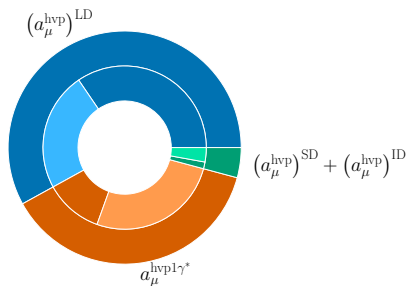
$$\langle O \rangle = \frac{\int D\phi O e^{-S_{full}}}{\int D\phi e^{-S_{full}}} = \frac{\int D\phi O (1 + \sum_i \Delta\epsilon_i S_i) e^{-S_0}}{\int D\phi (1 + \sum_i \Delta\epsilon_i S_i) e^{-S_0}}$$

- this expansion leads to new operators and subsequently to new diagrams ($\mathcal{O}(20)$ diagrams in total)

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Error contributions of the latest Mainz result

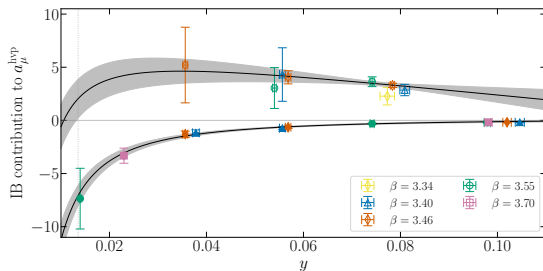


Contribution to the total variance^a

^aarXiv: 2411.07969 (hep-lat).

- short and intermediate distance windows are well under control
- major error contributions are the long distance window and the isospin breaking corrections
 - 1 long distance window: statistical error slightly dominates
 - 2 isospin breaking corrections: systematic error dominates

Current status of isospin breaking computations



Chiral extrapolation of the isospin breaking corrections¹

- connected part not well constrained, in particular in the physical pion mass region
- missing disconnected mass insertion diagram and self-energy
- methods for con. and disc. QED diagrams differ substantially

¹arXiv: 2411.07969 (hep-lat).

Diagrams of interest



Diagram 1: Disconnected mass insertion for the valence quark



Diagram 2: Connected mass insertion diagram for the valence quark



Diagram 3: Photon self-energy diagrams for the valence quark



Diagram 4: Photon exchange diagram for the valence quark



Diagram 5: Disconnected photon self-energy diagrams

We are also interested in other photon disconnected diagrams.

Classical setup vs. new setup

- current setup produces larger errors at small pion mass
- current setup of computation: placing a three dimensional stochastic source at the source time of the diagram
- new setup: different source setup (different stochastic estimator) could improve signal quality
- new setup: 4D volume sources at the operator insertion
- this is particularly useful in this case as the operator insertion is summed over \Rightarrow stochastic all-to-all propagator
- improvement in the signal quality for the mass insertion diagrams
- no improvement for photon involved diagrams
- get disconnected diagrams for "free"

Improvement mass insertion correlation function

- reduction of statistical noise \Rightarrow reduced errors
- signal "stays" longer

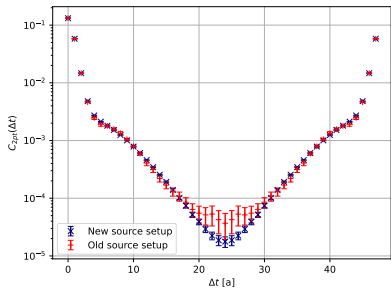
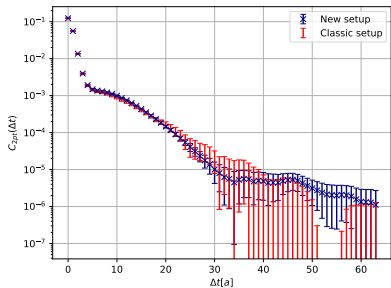


Diagram 1: Connected mass insertion diagram for the valence quark

Improvement mass insertion correlation function

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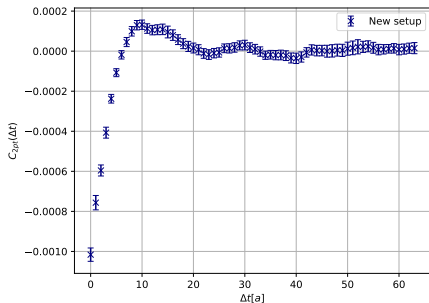
Two-point connected correlation function for N451 (122 configs)



Diagram 1: Connected mass insertion diagram for the valence quark

Disconnected diagrams on N451

- due to the better estimator, the disconnected diagram has a signal
- for large pion masses, the disconnected diagram is subdominant, expected to be larger for smaller pion masses



Two-point disconnected mass insertion correlation function
for N451 (122 configs)



Diagram 1: Disconnected mass insertion for the valence quark

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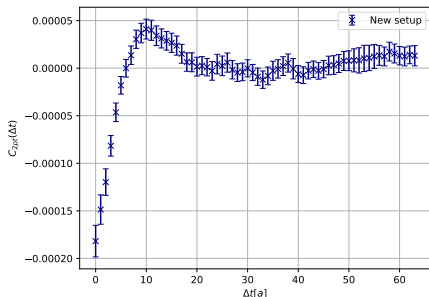


Diagram 1: Disconnected photon self-energy diagrams

Two-point disconnected photon self-energy correlation
function for N451 (122 configs)

Analysis procedure I

- rewrite $J_\mu = J_\mu^3 + \frac{1}{\sqrt{3}} J_\mu^{81}$
- $\Rightarrow G^{i,j} = \langle J_\mu^i J_\mu^j \rangle$, with $i, j \in [0, 3, 8]$
- build $G^{i,j}$ for every expansion parameter from the single correlator, two options:
 - use mass insertion and photon self-energy from the new setup, take over photon exchange diagram from the old setup
 - use mass insertion from the new setup and get the photon corrections by taking ratios of correlators
 - second option much better in terms of signal quality \rightarrow maximizes effect of correlations

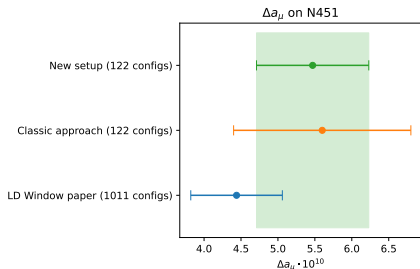
¹arXiv: 2206.06582 (hep-lat).

Analysis procedure II

- operator mixing between G^{ij} , with $i, j \in [0, 3, 8]$ ¹
- $\Rightarrow G_R^{EM} = \sum_{i,j} \left(Z_{3,i} + \frac{1}{\sqrt{3}} Z_{8,i} \right) \left(Z_{3,j} + \frac{1}{\sqrt{3}} Z_{8,j} \right) G_B^{ij}$
- multiply G_R^{EM} by kernel and then integrate up (in practice a sum which needs to be cut at some point due to signal-to-noise problem)
 - sum up contributions to get final result
 - divergencies of self energy diagram are canceled by the mass insertion diagrams via expansion coefficients

¹arXiv: 2112.00878 (hep-lat).

Preliminary results on connected pieces N451



- results compatible within 1σ
- new setup reduces error bar (roughly $\mathcal{O}(1/3)$)
- longer signal \Rightarrow reduces ambiguity of cutting the correlator \Rightarrow allows for more robust integration

differences in analysis with LD window paper¹ result:

- error and correlation of renormalization coefficients taken into account
- correlator replaced by fit of lowest state at long euclidean times
- bounding method applied to reduce uncertainties in the tail

¹arXiv: 2411.07969 (hep-lat).

Inclusion of disconnected diagrams

- expected correlation between disconnected mass insertion and photon self-energy with connected diagrams
- updating of expansion coefficients and renormalization parameters
- current statistic not sufficient for reliable extraction of the full HVP
- error of disconnected in gauge noise \Rightarrow improvement only possible by extending number of configurations

Summary and Outlook

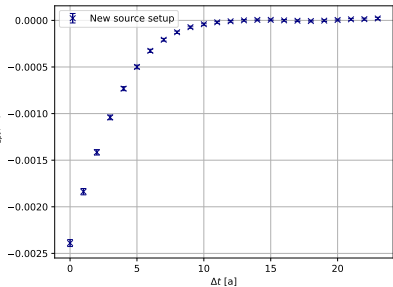
Summary:

- ① isospin breaking corrections are essential for precision calculations
- ② new source setup shows promising approach to improve accuracy
- ③ disconnected contributions currently under study

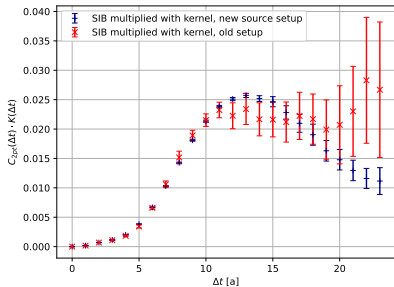
Outlook:

- ① include disconnected diagrams
- ② improve statistics on existing ensembles
- ③ extend ensemble space (computer time proposal submitted, waiting for approval)

Backup: Correlators on A654

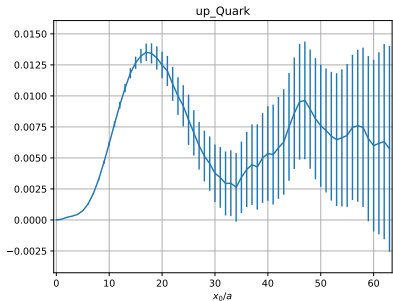


Two-point disconnected correlation function for A654 (5068 configs)

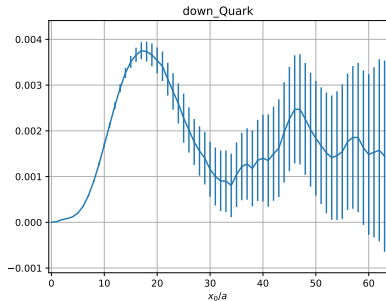


Integrand for mass insertion for A654 (5068 configs)

Backup: Integrands for N451

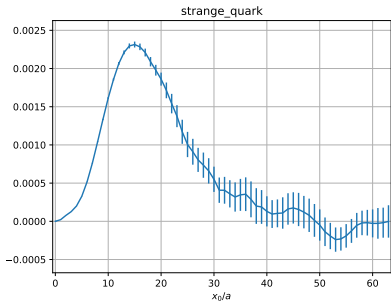


Integrand for up quark contribution

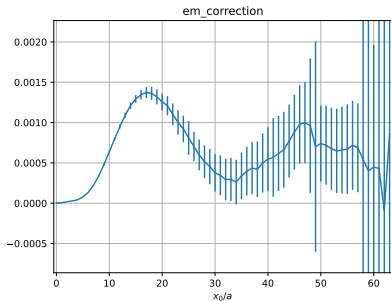


Integrand for down quark contribution

Backup: Integrands for N451



Integrand for strange quark contribution



Integrand for electromagnetic contribution