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TWO-PHOTON EXCHANGE EFFECTS IN MUONIC HYDROGEN

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WHY THIS MATTERS

- Lamb shift of muonic hydrogen → Amplified hadronic uncertainties
- Main culprit: **two-photon exchange**
- Catch up to experiments (proposed 5-times improvement!)
- Proton structure model with the latest data
- Analytic parametrization **over the whole energy region**



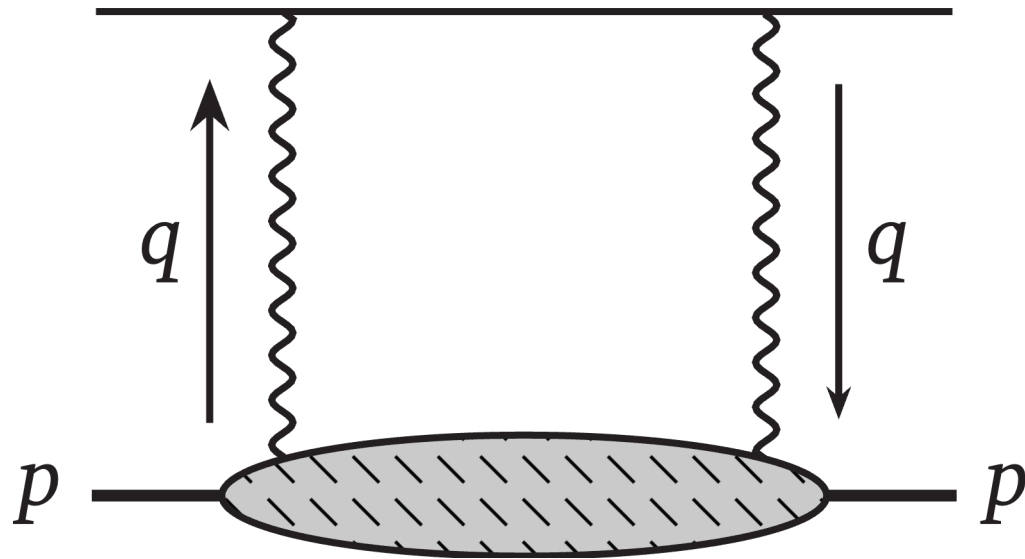
Theory



Experiment

WHAT IS THERE TO DO?

Forward two-photon exchange (TPE)



- Parametrize proton structure:
 $F_1(\nu, Q^2), F_2(\nu, Q^2)$
- Contribution of intermediate proton \rightarrow Born terms
- Contribution of intermediate hadronic **resonances**
- Some **Non-resonant** processes to consider

HOW TO DO IT

Resonant Contribution

- Consider individual nucleon resonances
- Collect exclusive data from experiments (e.g. CLAS)
- Combine for inclusive **prediction**

High Energy Regime

- Model non-resonant behaviour using physical arguments
- Consider known limits and restrictions
- **Fit** model to inclusive data in a consistent way

$$\text{Resonant} + \text{Non-resonant} = \text{Total}$$

The Precision Physics Times

Mainz correspondent: Panagiotis Kalamidas

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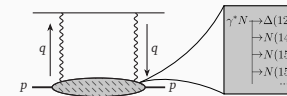
TWO-PHOTON EXCHANGE EFFECTS IN MUONIC HYDROGEN

MOTIVATION

- Lamb shift of muonic hydrogen:
 $\Delta E_{LS}^{\mu\text{H}}(\mu\text{H}) = [206.0344(3) - 5.2259 r_p^2/\text{fm}^2 + 0.0289(25)] \text{ meV}$
 $\Delta E_{LS}^{\text{exp}}(\mu\text{H}) = 202.3706(23) \text{ meV}$
 where r_p is the proton charge radius
- Largest uncertainties are hadronic and dominated by two-photon exchange (TPE) effects
- Catch up to experiments (proposed 5-times improvement!)
- Update parametrization of structure functions with the latest data
- Analytic parametrization over the whole energy region

THE THEORY BEHIND

Forward TPE diagram



Formulate with unpolarized Compton tensor

$$T^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2) + \frac{1}{M_N^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu\right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu\right) T_2(\nu, Q^2)$$

Connect to structure functions

$$\text{Im } T_1(\nu, Q^2) = \frac{e^2}{4M_N} F_1(x, Q^2)$$

$$\text{Im } T_2(\nu, Q^2) = \frac{e^2}{4Q^2} F_2(x, Q^2)$$

Parametrize structure functions and fit data

Dispersion relations

$$\text{Re } T_1(\nu, Q^2) = \text{Re } T_1^{\text{Born}}(\nu, Q^2) + T_1^{\text{sub}}(0, Q^2) + \frac{2\nu^2}{\pi} \mathcal{P} \int_{\nu_0}^{\infty} d\nu' \frac{\text{Im } T_1(\nu', Q^2)}{\nu'(\nu'^2 - \nu^2)}$$

$$\text{Re } T_2(\nu, Q^2) = \text{Re } T_2^{\text{Born}}(\nu, Q^2) + \frac{2}{\pi} \mathcal{P} \int_{\nu_0}^{\infty} d\nu' \frac{\nu'}{\nu'^2 - \nu^2} \text{Im } T_2(\nu', Q^2)$$

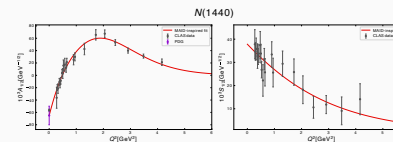
- Subtraction function $T_1^{\text{sub}}(0, Q^2)$ needed
- Low energy expansion:

$$T_1^{\text{sub}}(0, Q^2) \rightarrow \beta_M Q^2 + \mathcal{O}(Q^4)$$

where is the β_M magnetic polarizability

EXPLORING THE RESONANCE REGION

- Include current 4* resonances
- Fit $\gamma^* N \rightarrow N^*$ helicity amplitudes ($A_{1/2}, A_{3/2}, S_{1/2}$) derived in electroexcitation experiments ($\pi N, \eta N, \pi\pi N$)



- Include latest CLAS data for low Q^2
- Employ Breit-Wigner parametrization to get the longitudinal and transversal cross sections
- Get $F_2^{\text{Born}}(W^2, Q^2)$ from exclusive data no fit to inclusive needed!

COVERING THE HIGH-ENERGY REGIME

- Well-known high energy behavior (Donnachie, A. and Landshoff, P.V., 2004)
- Only part fitted to inclusive data

For inclusion readers only!

- Parametrization for non-resonant part, smoothly connects to DL fit:

$$F_2^{\text{Born}}(x, Q^2) = x \left\{ f_N(Q^2) (2M\nu)^{\alpha_N} \left(1 - \frac{\nu_0}{\nu}\right)^{\alpha_N + \alpha_N(Q^2)} \left(1 + \frac{\nu_0}{\nu}\right)^{\alpha_N(Q^2)} + (P_2 \text{ term}) + (P_1 \text{ term}) \right\}$$

$$F_L^{\text{Born}}(x, Q^2) = \frac{A_{NL} Q^4}{(1 + Q^2/Q_{NL}^2)^{\alpha_{NL} + 1}} (2M\nu)^{\alpha_{NL}} \left(1 - \frac{\nu_0}{\nu}\right)^{\alpha_{NL} + \alpha_{NL}(Q^2)}$$

- Additional pion-loop terms for the low-energy behaviour
- Connection to $F_1(x, Q^2)$:

$$F_1(x, Q^2) = \frac{1}{2x} \left\{ (1 + Q^2/\nu^2) F_2(x, Q^2) - F_L(x, Q^2) \right\}$$

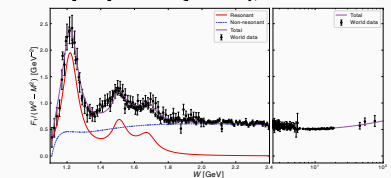
- From superconvergence of $T_1(\nu, Q^2) - T_1^{\text{Born}}(\nu, Q^2)$:

$$T_1^{\text{sub}}(0, Q^2) = \frac{1}{4\pi M_N} F_2^{\text{Born}}(0, Q^2) + T_1^{\text{Born}}(0, Q^2) + \frac{e^2}{2\pi M_N} \int_{\nu_0}^{\infty} d\nu' \frac{\nu'}{Q^2 - \nu'^2} \text{Im } T_1(\nu', Q^2)$$

- Parametrize $T_1^{\text{sub}}(\nu, Q^2)$

RESULTS ... SO FAR

- Fit at photoproduction (preliminary)



- Separate fit at electroproduction for:
 → Relatively scarce longitudinal data
 → Abundant $F_2(x, Q^2)$ data

OUTLOOK REPORT

- We use exclusive data for the resonance contribution and fit the non-resonant background \neq previous inclusive fits
- New fit will include latest Jefferson Lab data lacking from previous ones
- Application for new dispersive estimate of the TPE contribution to the Lamb shift in muonic hydrogen
- Planned extension to update hyperfine splitting prediction

Some details were harmed during the making of this 3-minute presentation!

If you're interested in the full-picture, come by my poster!



Questions? Ideas? Suggestions?
 Contact me at pekalam1@uni-mainz.de

