Super-Leading Logarithms in $t\bar{t}$-Production

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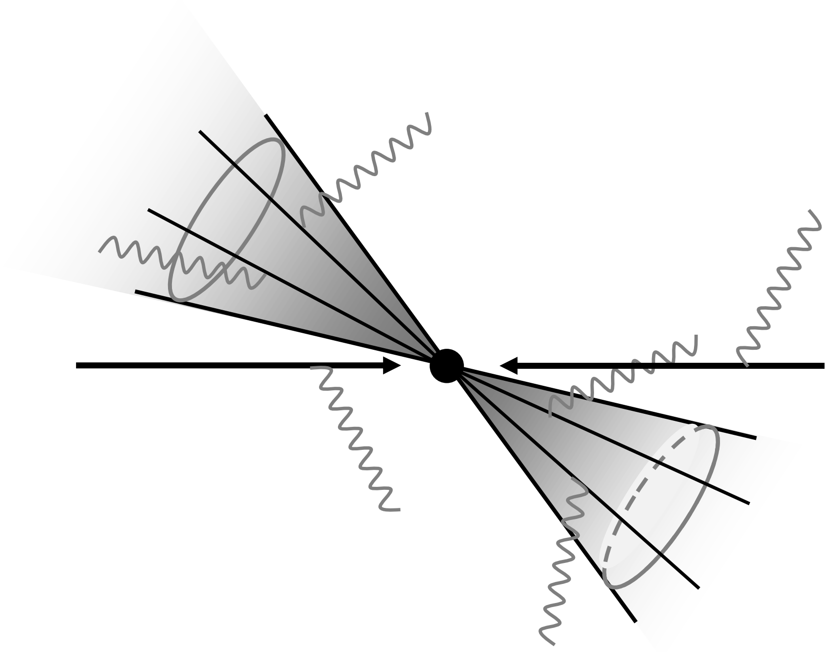
Romy Grünhofer | Chiemsee 2025

Soft-Collinear Effective Theory

At the LHC: collide highly energetic protons, $pp \to \text{jets}$

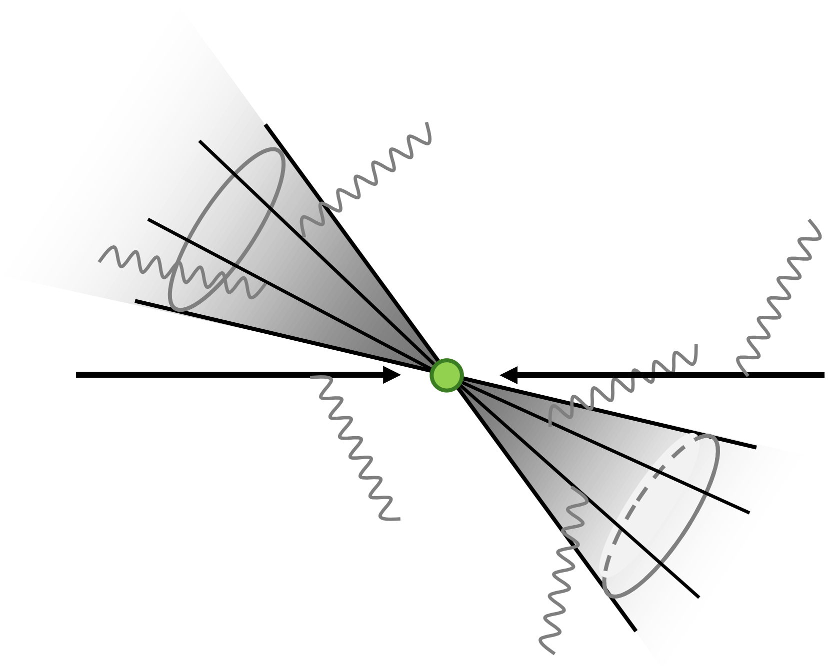
$\Longrightarrow$ Different scales



Soft-Collinear Effective Theory

At the LHC: collide highly energetic protons, $pp \to \text{jets}$

$\Longrightarrow$ Different scales: ∘ Hard scattering



Hard:

$$p^2 = Q^2$$

Invariant mass

Q: high energy scale

Soft-Collinear Effective Theory

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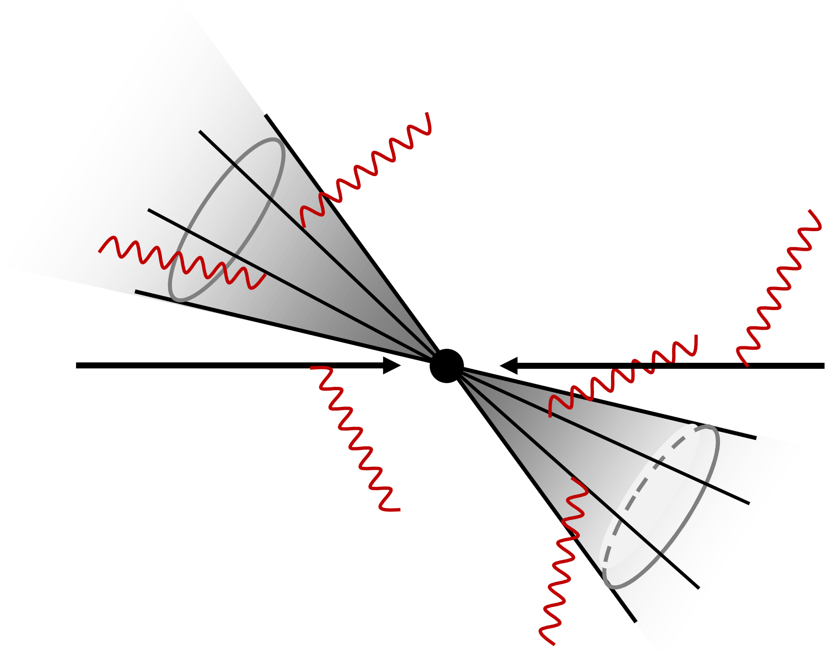
- Hard scattering
- Soft gluons



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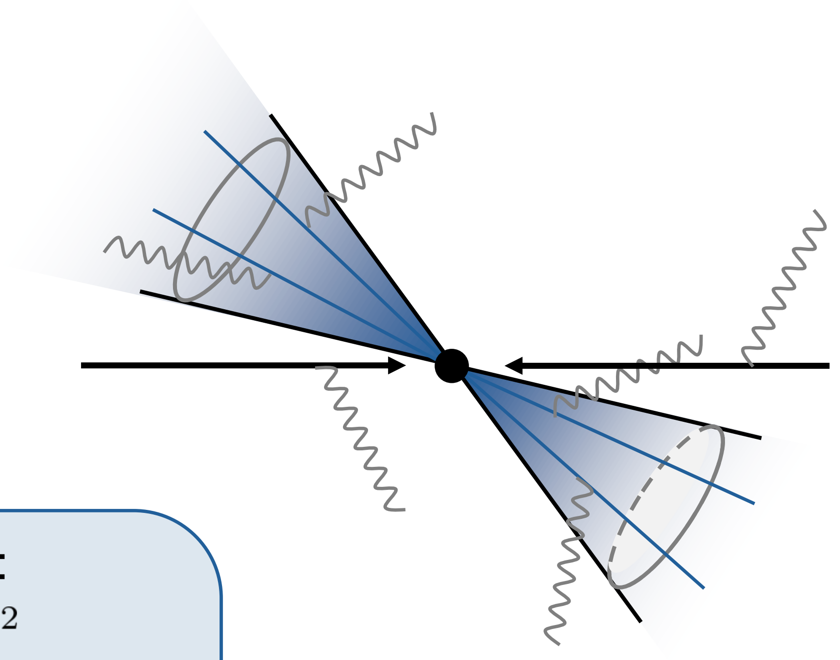

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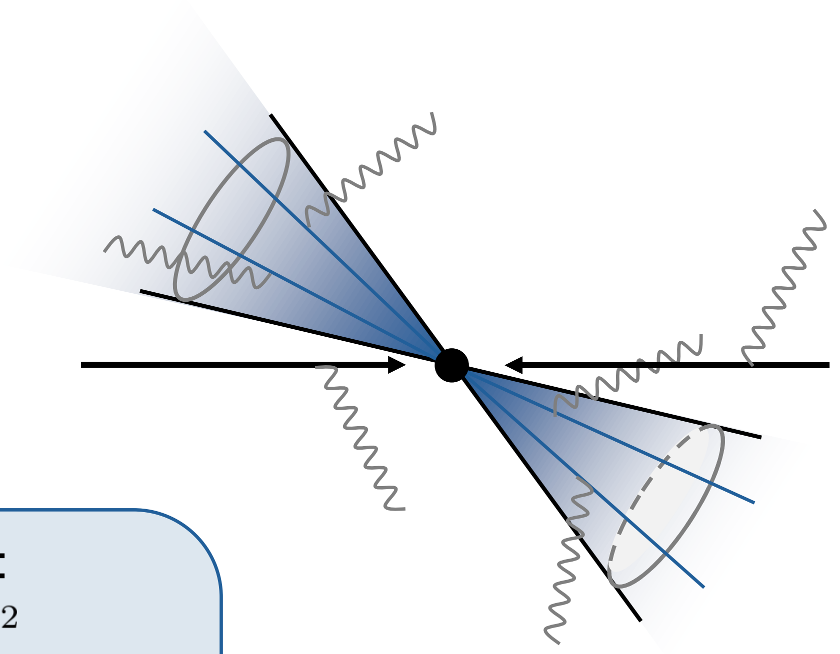

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But: high energy into a light-like direction

$$n_\mu = (1,0,0,1) \Longrightarrow n^2 = 0$$

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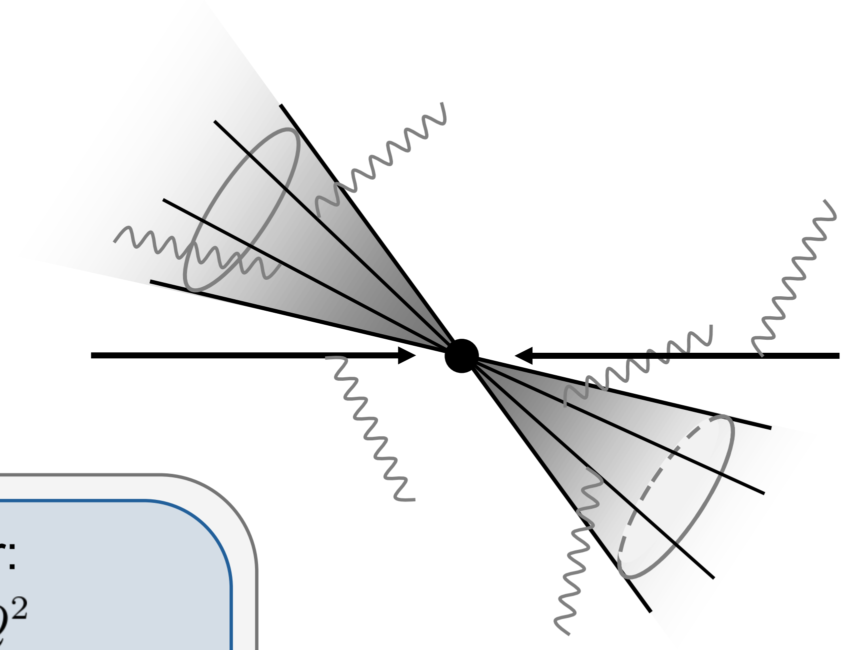
Soft-Collinear Effective Theory



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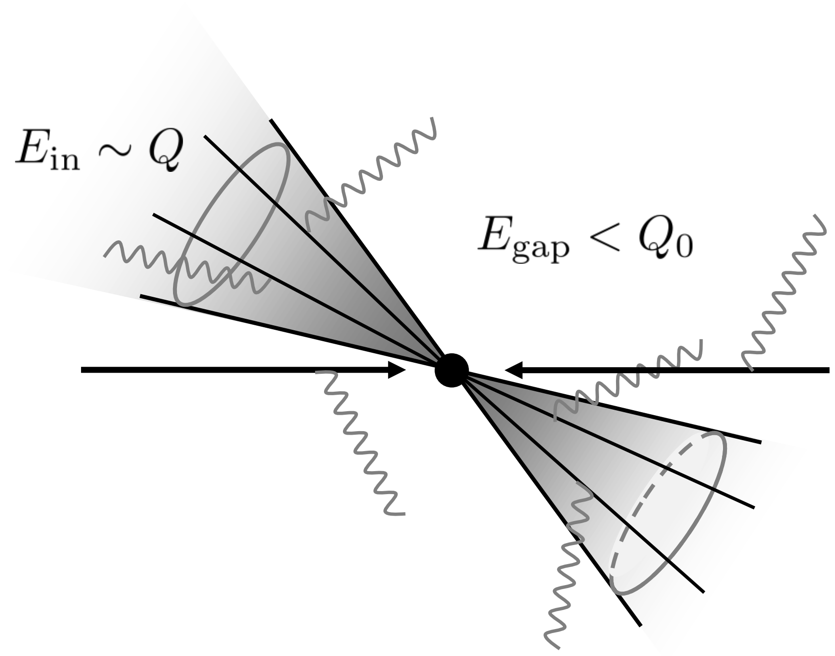
$\Longrightarrow$ Soft-collinear effective theory (SCET)

Super-Leading Logarithms in $t\bar{t}$ -Production

Origin of SLLs

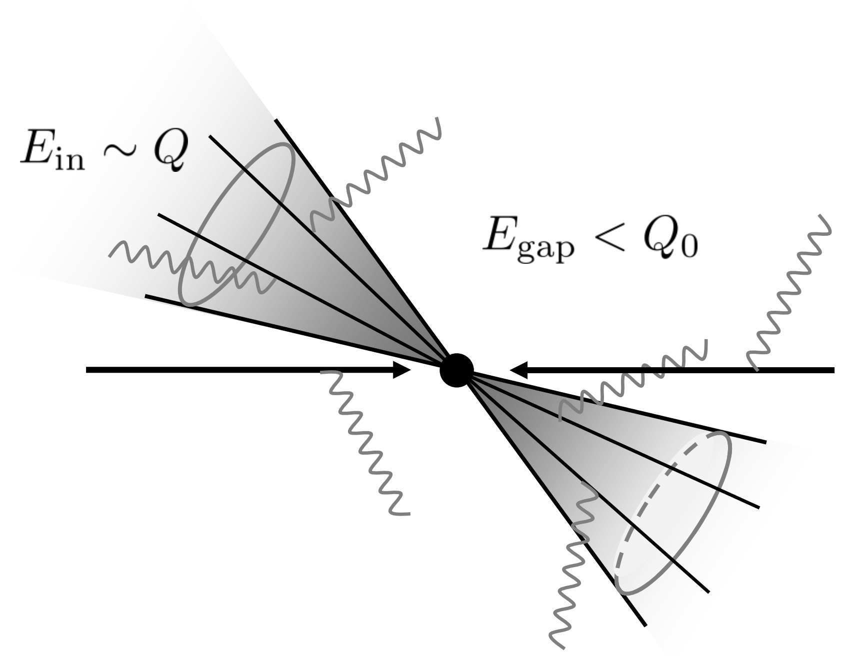


Super-Leading Logarithms in $t\bar{t}$ -Production

Origin of SLLs

Large Logarithms in $pp \to \text{jets}$ processes: $L = \ln\left(\frac{Q}{Q_0}\right) \gg 1$

$$\sigma = \sigma_{\text{Born}} \times \left\{1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \alpha_s^4 L^5 + \alpha_s^5 L^7 + \dots\right\}$$



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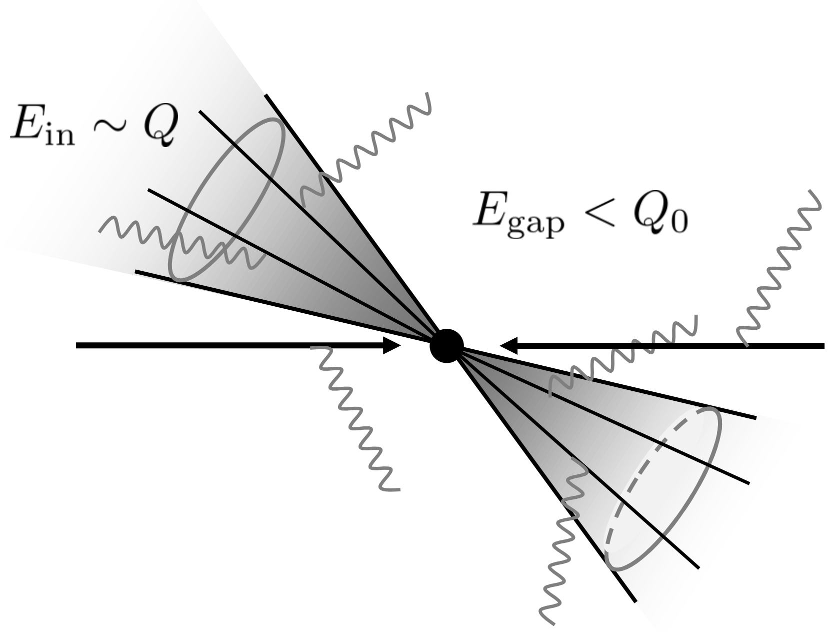
Formally larger than $\mathcal{O}(1)$



⟹ Resum these logarithms!

Super-Leading Logarithms in $t\bar{t}$ -Production

Origin of SLLs

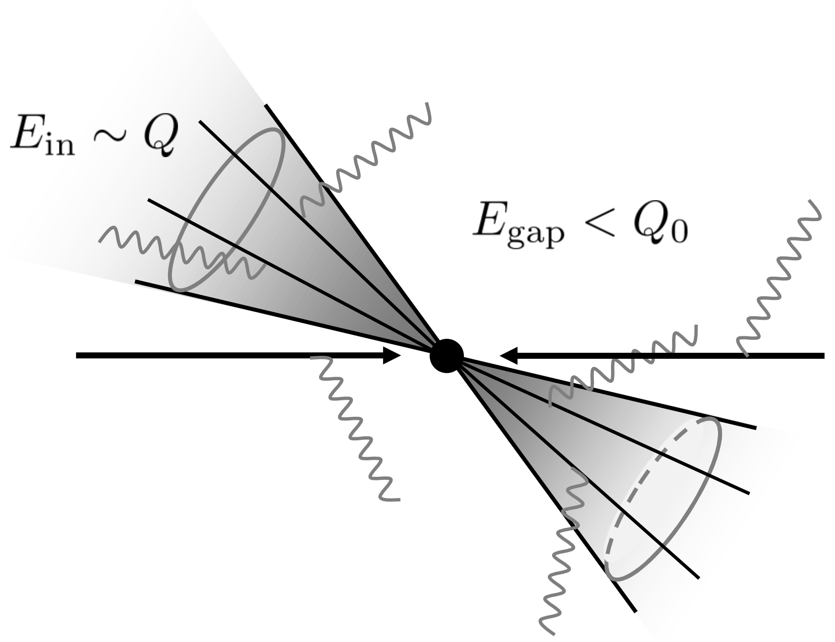
Large Logarithms in $pp \to \text{jets}$ processes: $\ln\left(\frac{Q}{Q_0}\right) \gg 1$

Partonic cross section evaluated at $\mu = \mu_s \sim Q_0$:

Hard function

Trivial low energy matrix element

$$\hat{\sigma}^{\text{SLL}}_{2\to M} = \langle \boldsymbol{\mathcal{H}}_{2\to M}(Q, \mu = \mu_s) \otimes \mathbf{1} \rangle$$



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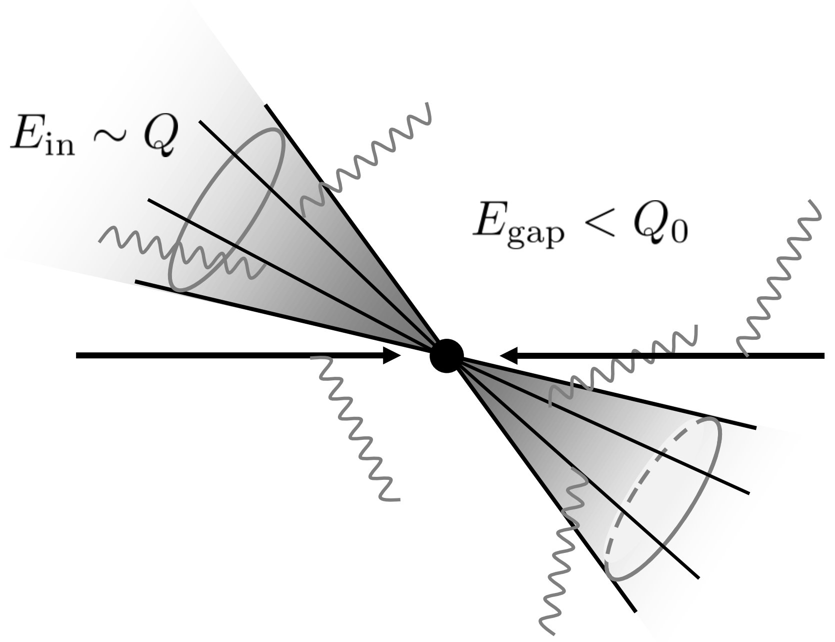

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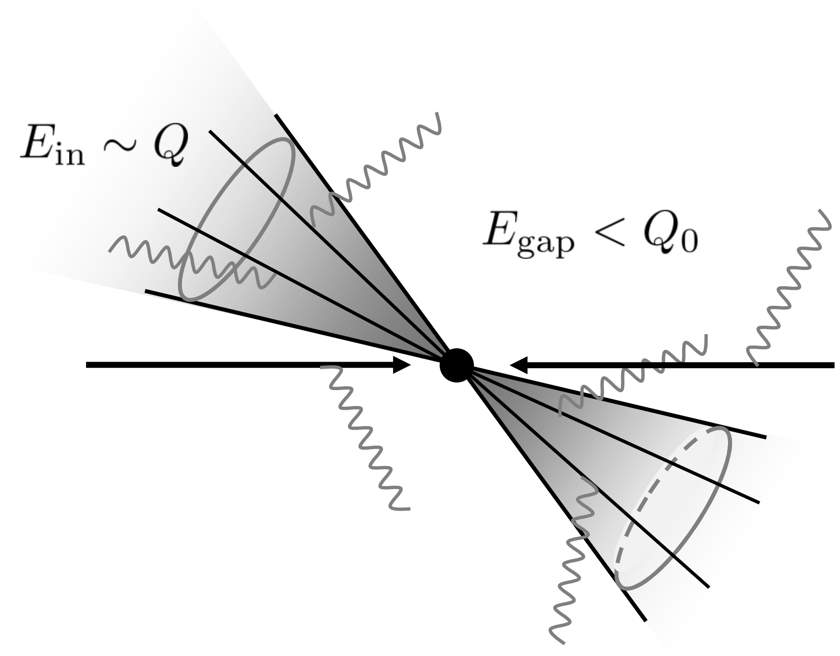
Natural scale: $\mu_h \sim Q$



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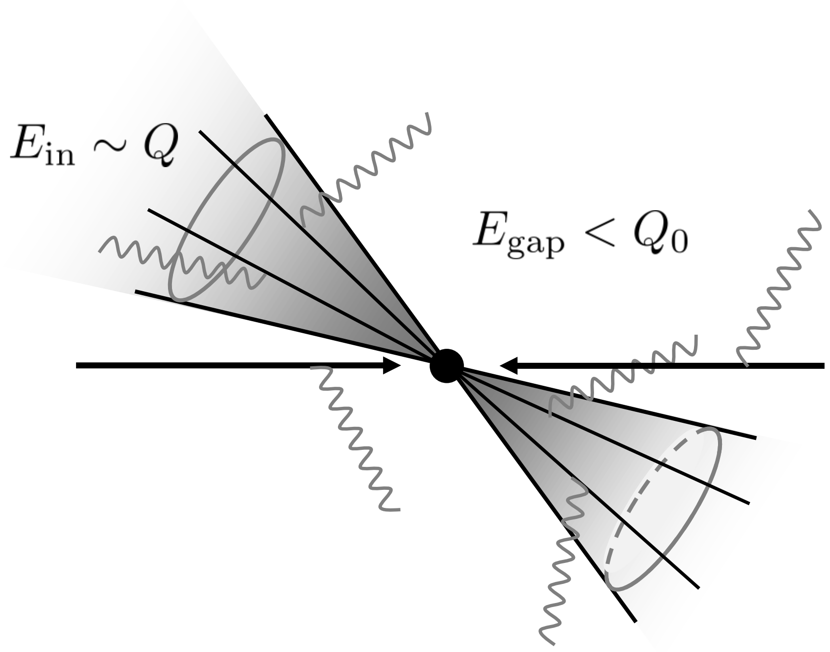

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Large logs!

$\Longrightarrow$ use $\boldsymbol{\mathcal{H}}_{2\to M}(Q, \mu_s) = \boldsymbol{\mathcal{H}}_{2\to M}(Q, \mu_h) \times \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \boldsymbol{\Gamma}^{\mathcal{H}}(\mu)\right]$

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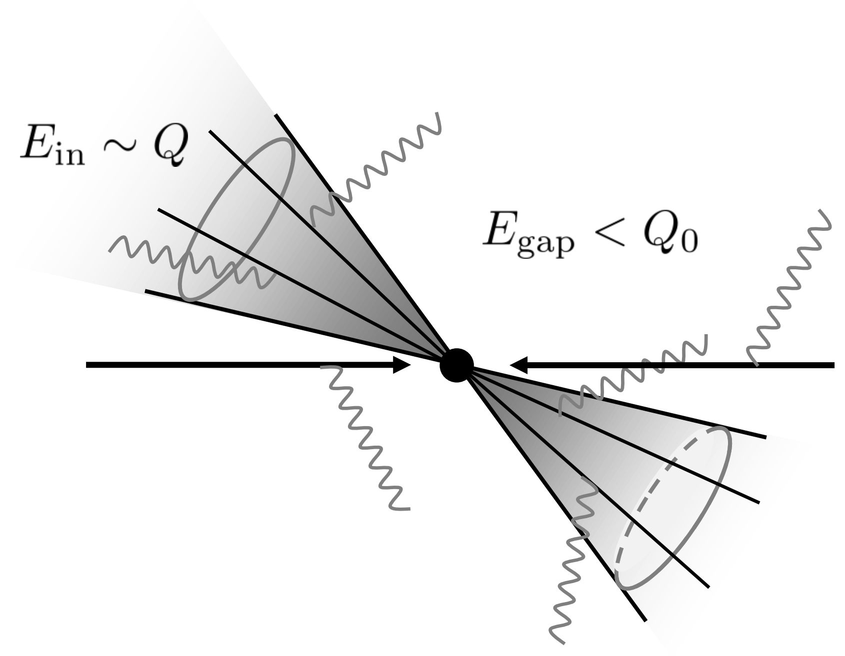

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Anomalous dimension matrix

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Super-Leading Logarithms in $t\bar{t}$ -Production

Anomalous Dimension

$$\boldsymbol{\mathcal{H}}(Q,\mu_s) = \boldsymbol{\mathcal{H}}(Q,\mu_h) \times \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h} \frac{\mathrm{d}\mu}{\mu} \boldsymbol{\Gamma}^{\mathcal{H}}(\mu)\right]$$

$$\boldsymbol{\Gamma}^{\mathcal{H}}(\xi_1,\xi_2) = \delta(1-\xi_1)\delta(1-\xi_2)\boldsymbol{\Gamma}^S + \delta(1-\xi_2)\boldsymbol{\Gamma}_1^C + \delta(1-\xi_1)\boldsymbol{\Gamma}_2^C$$

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Collinear parts $\Longrightarrow$ subleading effects

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Coulomb phase $\Longrightarrow$ only for massive partons!

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Our work

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Cusp contribution

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Colour Traces

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- Use soft emission (introduces Q_0) $\overset{?}{\Longrightarrow} \langle \boldsymbol{\mathcal{H}}(\mu_h)\,(\mathbf{\Gamma}^c)^n \overline{\mathbf{\Gamma}} \otimes \mathbf{1}\rangle$
- Use Glauber phase $\overset{?}{\Longrightarrow} \langle \boldsymbol{\mathcal{H}}(\mu_h)\,(\mathbf{\Gamma}^c)^n \mathbf{V}^G \overline{\mathbf{\Gamma}} \otimes \mathbf{1}\rangle$

$$\langle \ldots \mathbf{\Gamma}^c \otimes \mathbf{1}\rangle = 0$$
$$\langle \ldots \mathbf{V}^G \otimes \mathbf{1}\rangle = 0$$
$$\langle \ldots \mathbf{V}^{\text{Coul}} \otimes \mathbf{1}\rangle = 0$$

cross section should be real

$$[\mathbf{\Gamma}^c, \overline{\mathbf{\Gamma}}] = 0$$
$$[\mathbf{\Gamma}^c, \mathbf{V}^{\text{Coul}}] = 0$$
$$[\mathbf{V}^G, \mathbf{V}^{\text{Coul}}] = 0$$

Super-Leading Logarithms in $t\bar{t}$ -Production

Colour Traces

$$\boldsymbol{\Gamma}^S(\mu) = \frac{\alpha_s(\mu)}{4\pi}\left[\overline{\boldsymbol{\Gamma}} + \gamma_0 \boldsymbol{V}^G + \gamma_0 \boldsymbol{V}^{\text{Coul}} + \gamma_0 \boldsymbol{\Gamma}^c \ln\left(\frac{\mu^2}{\mu_h^2}\right)\right] + \mathcal{O}(\alpha_s^2)$$

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- Use second phase $\Longrightarrow$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\mathbf{V}^G\,(\boldsymbol{\Gamma}^c)^n\,\mathbf{V}^G\overline{\boldsymbol{\Gamma}} \otimes \mathbf{1}\rangle$ + different orderings

  $\Longrightarrow$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\mathbf{V}^{\text{Coul}}(\boldsymbol{\Gamma}^c)^n\,\mathbf{V}^G\overline{\boldsymbol{\Gamma}} \otimes \mathbf{1}\rangle$ + different orderings

Super-Leading Logarithms in $t\bar{t}$ -Production

Colour Traces

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  - $\Longrightarrow \langle \boldsymbol{\mathcal{H}}(\mu_h)\mathbf{V}^{\text{Coul}}\mathbf{V}^{\text{Coul}}\overline{\mathbf{\Gamma}} \otimes \mathbf{1}\rangle$

Super-Leading Logarithms in $t\bar{t}$ -Production

Colour Traces

$$\mathbf{\Gamma}^S(\mu) = \frac{\alpha_s(\mu)}{4\pi}\left[\overline{\mathbf{\Gamma}} + \gamma_0 \boldsymbol{V}^G + \gamma_0 \boldsymbol{V}^{\text{Coul}} + \gamma_0 \mathbf{\Gamma}^c \ln\left(\frac{\mu^2}{\mu_h^2}\right)\right] + \mathcal{O}(\alpha_s^2)$$

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$\Longrightarrow \langle \mathcal{H}(\mu_h)\mathbf{V}^{\text{Coul}}\,(\mathbf{\Gamma}^c)^n\,\mathbf{V}^G \overline{\mathbf{\Gamma}} \otimes \mathbf{1}\rangle + \text{different orderings}$

$\Longrightarrow \langle \mathcal{H}(\mu_h)\mathbf{V}^{\text{Coul}}\mathbf{V}^{\text{Coul}} \overline{\mathbf{\Gamma}} \otimes \mathbf{1}\rangle$

Only for massive partons!

Super-Leading Logarithms in $t\bar{t}$ -Production

$2 \to t\bar{t}$ Processes

For $t\bar{t}$-production:

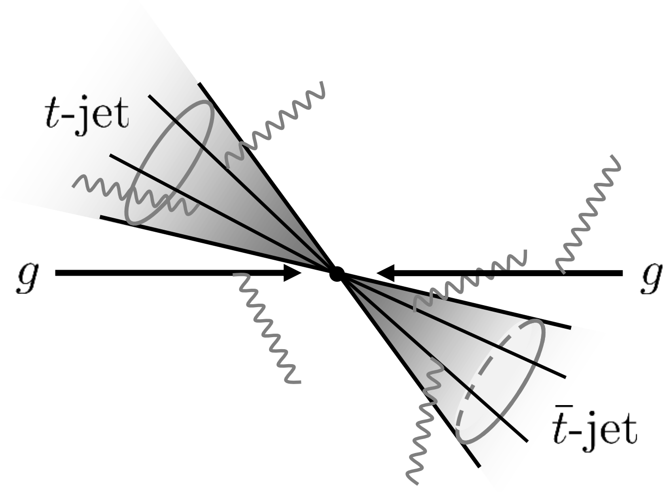
- $q\bar{q} \to t\bar{t}$
- $gg \to t\bar{t}$

Super-Leading Logarithms in $t\bar{t}$ -Production

$2 \to t\bar{t}$ Processes

For $t\bar{t}$-production:

- $q\bar{q} \to t\bar{t}$ vanishes: colour trace $= 0$
- $gg \to t\bar{t}$



Super-Leading Logarithms in $t\bar{t}$ -Production

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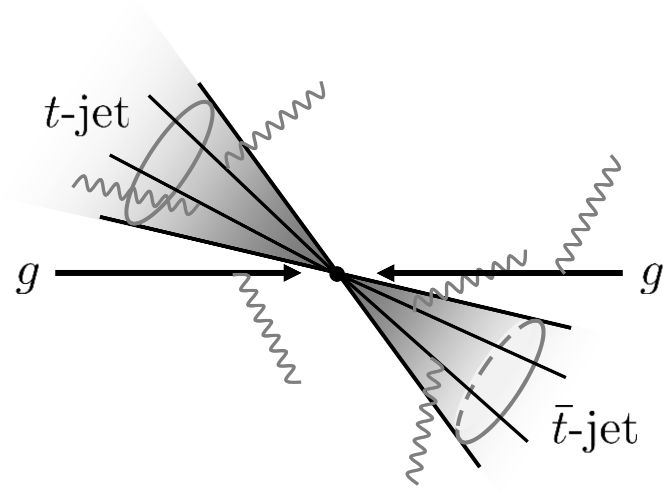
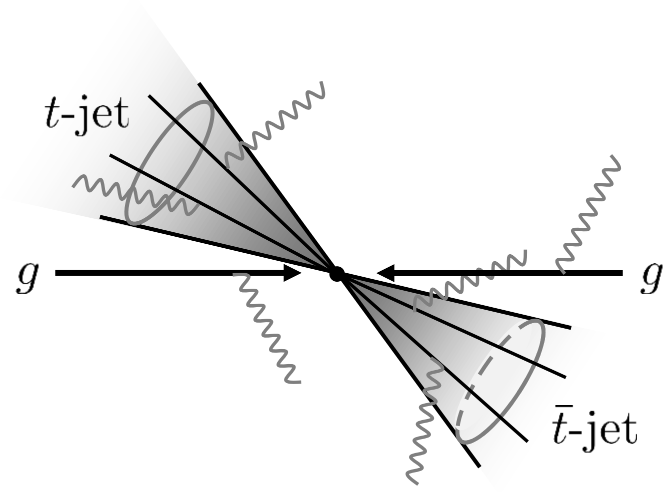


Center-of-mass frame: kinematics encoded in

- $\beta \equiv \beta_t = \beta_{\bar{t}}$ where $\beta_I = \sqrt{1 - \frac{m_I^2}{E_I^2}}$
- $\eta \equiv \eta_t = -\eta_{\bar{t}}$ where $\eta_I = \text{artanh}\left(\cos(\theta_I)\right)$

Super-Leading Logarithms in $t\bar{t}$ -Production

$2 \to t\bar{t}$ Processes



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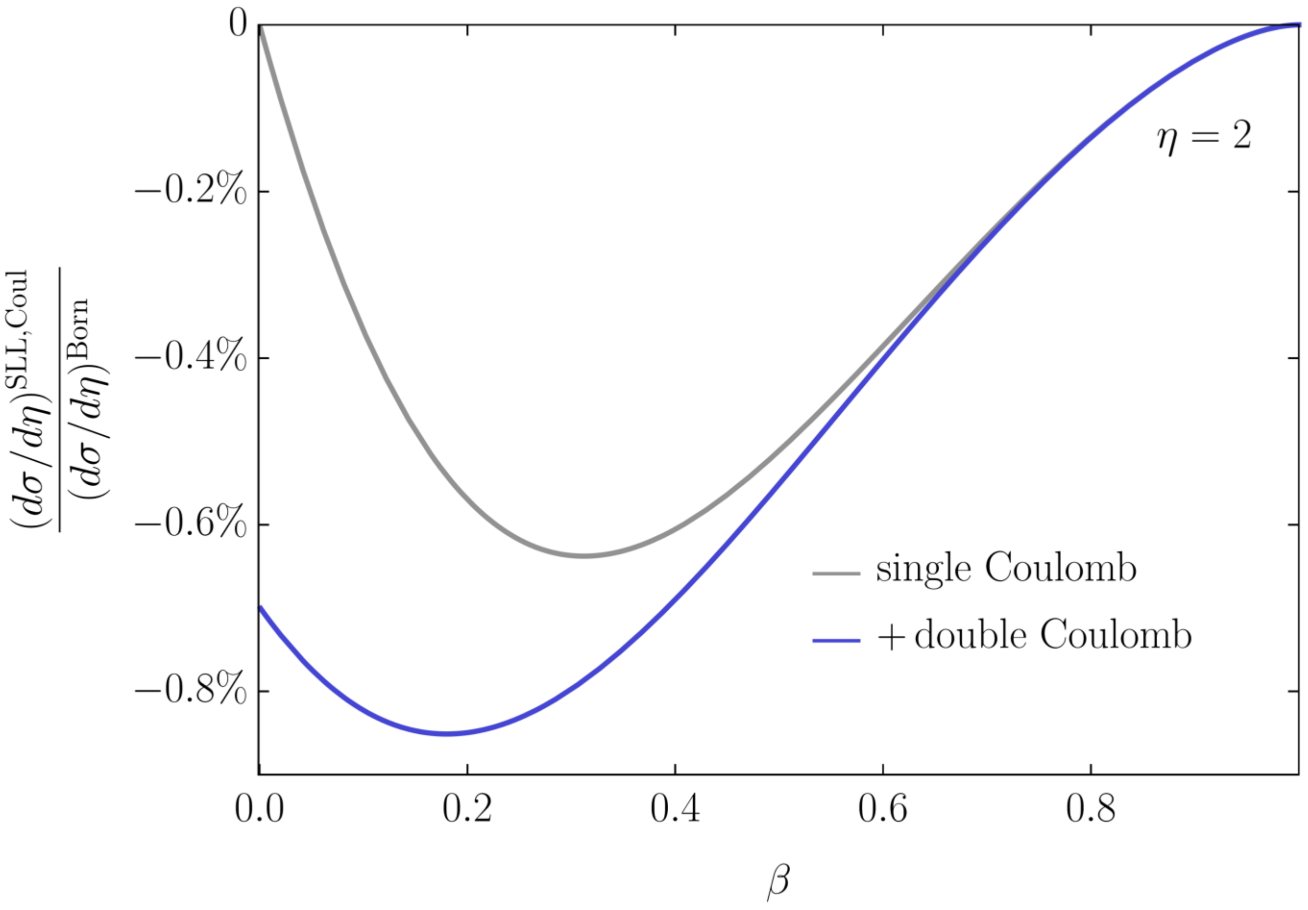
Veto region between the jets: $\eta \in [-1, 1]$

Soft scale: $\mu_s = 20\ \mathrm{GeV}$

Hard scale: $\mu_h = \frac{2m_t}{\sqrt{1-\beta^2}}$

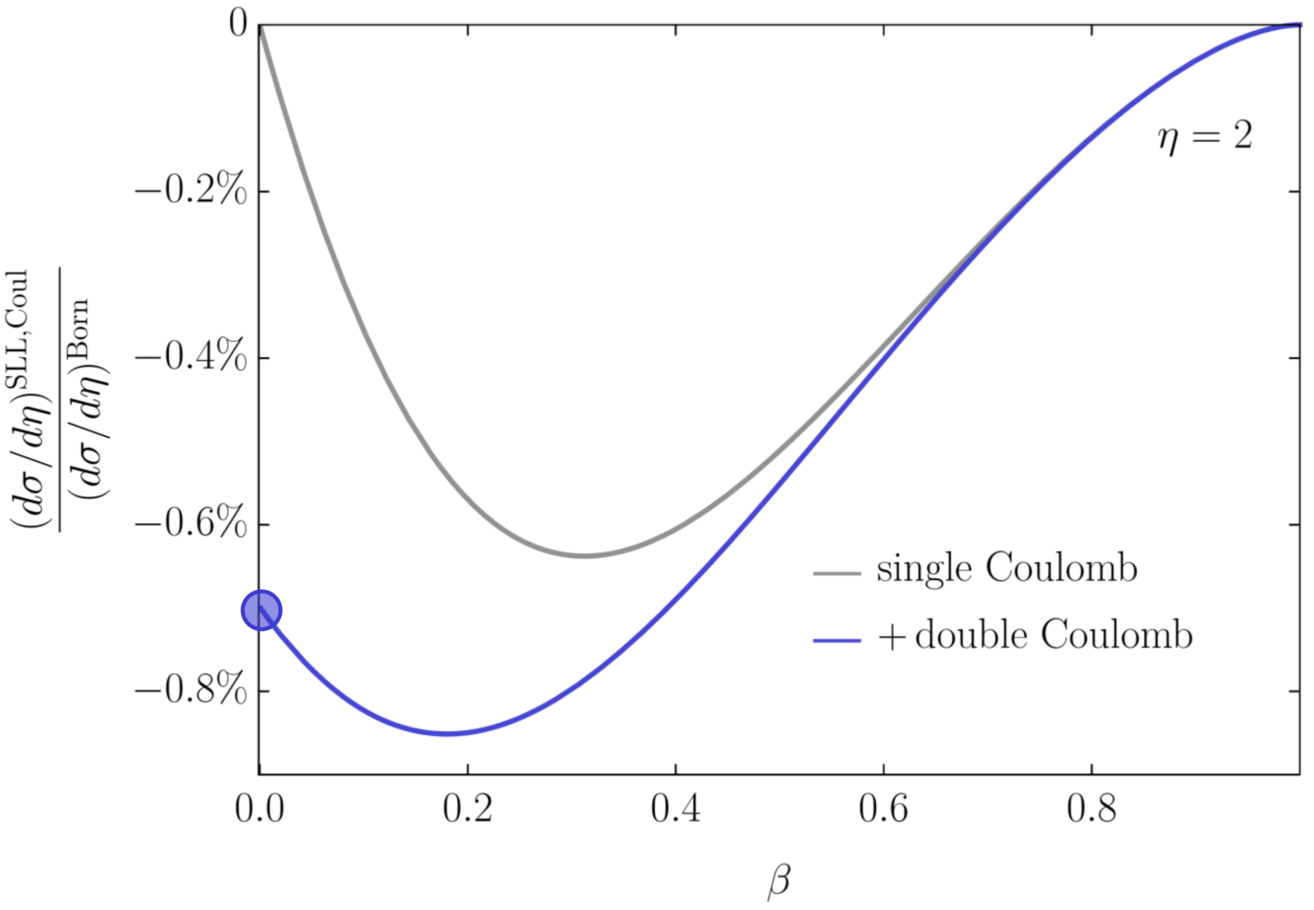
Numerical Effects

$gg \to t\bar{t}$



Numerical Effects

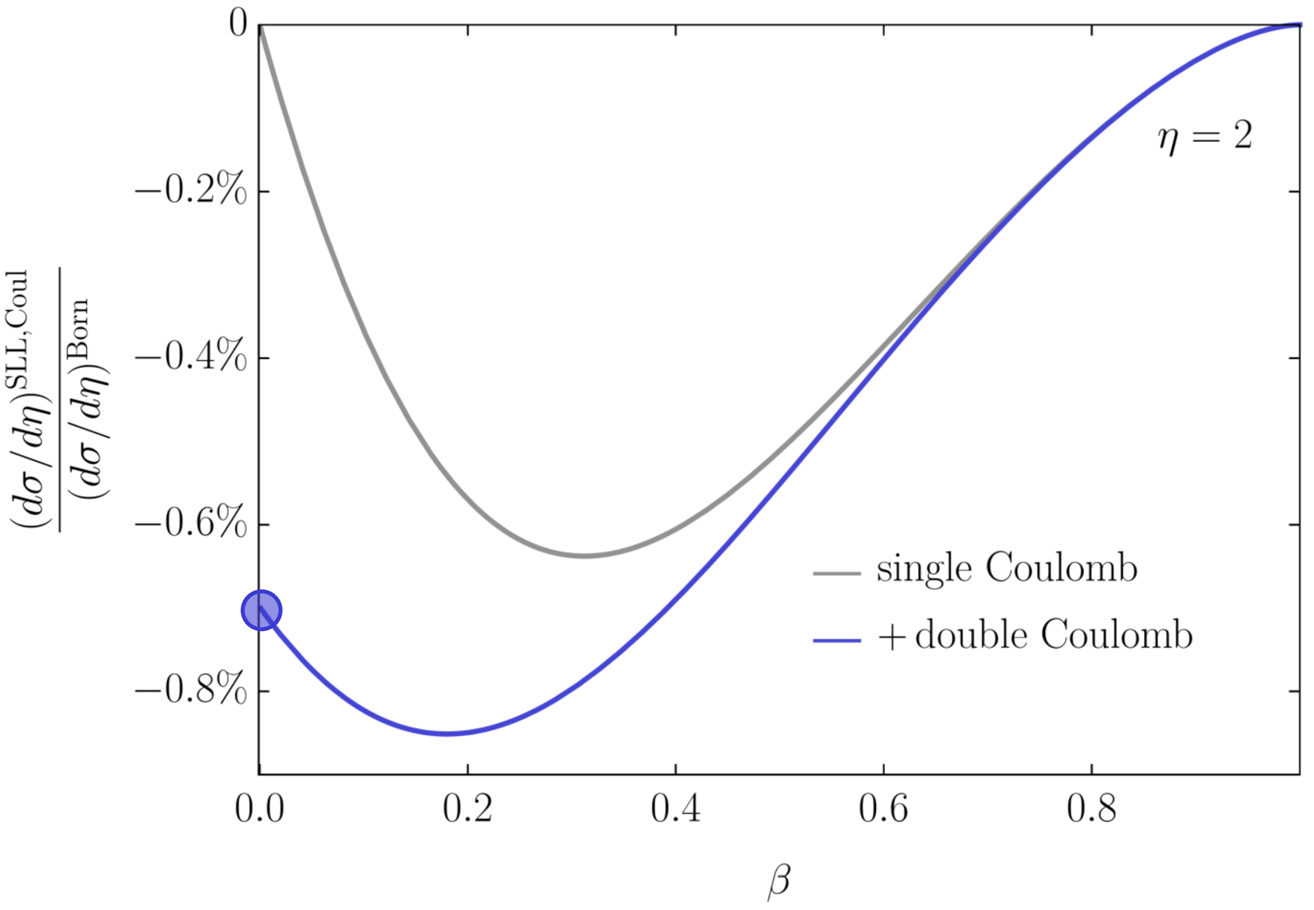
$gg \to t\bar{t}$



$$\mathbf{V}^{\text{Coul}} = -i\pi \left(\boldsymbol{T}_{t,L} \cdot \boldsymbol{T}_{\bar{t},L} - \boldsymbol{T}_{t,R} \cdot \boldsymbol{T}_{\bar{t},R} \right) v_{t\bar{t}}$$

Numerical Effects

$gg \to t\bar{t}$



Colour generators of t and $\bar{t}$

$$\mathbf{V}^{\text{Coul}} = -i\pi \left(\boldsymbol{T}_{t,L} \cdot \boldsymbol{T}_{\bar{t},L} - \boldsymbol{T}_{t,R} \cdot \boldsymbol{T}_{\bar{t},R} \right) v_{t\bar{t}}$$

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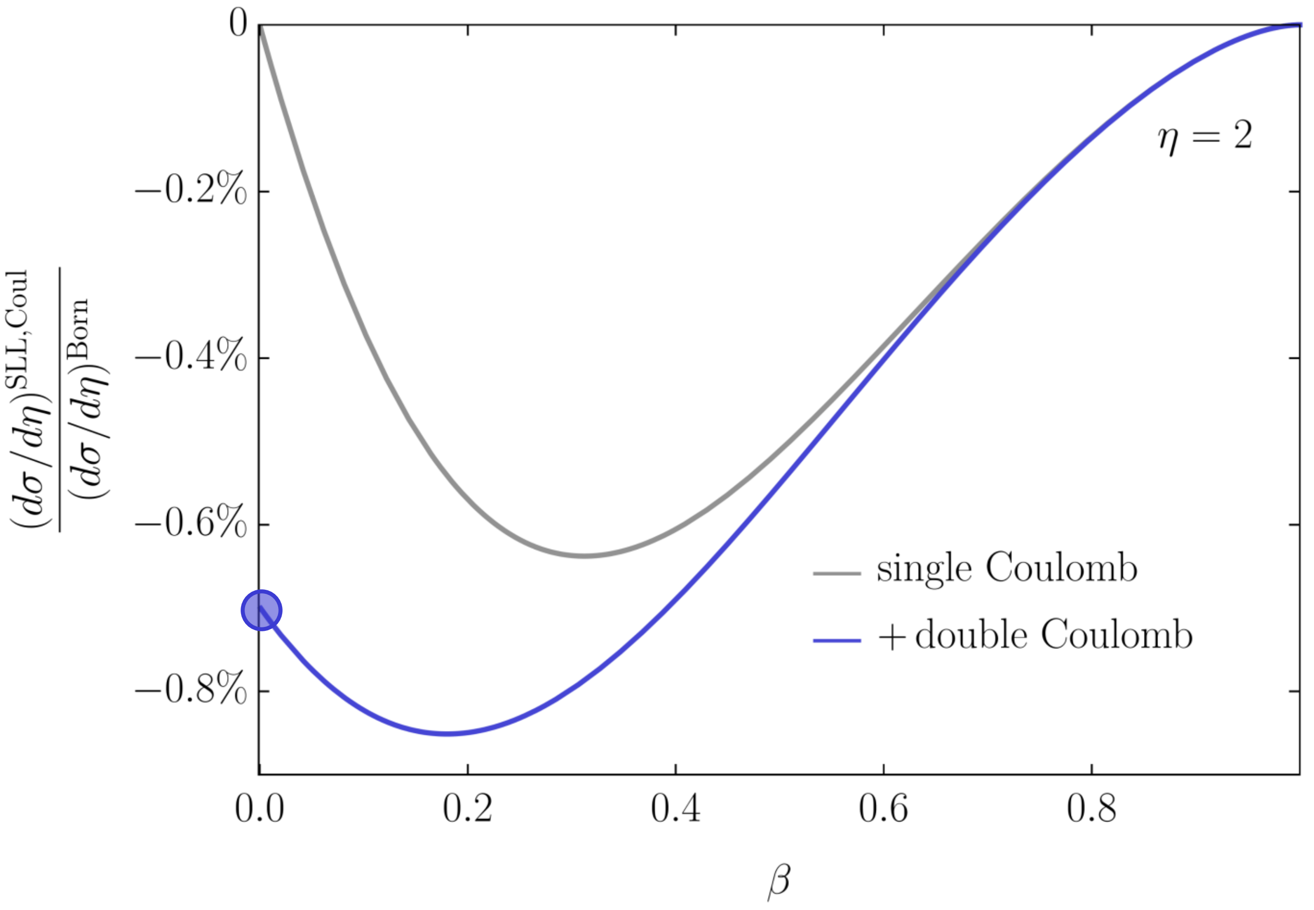

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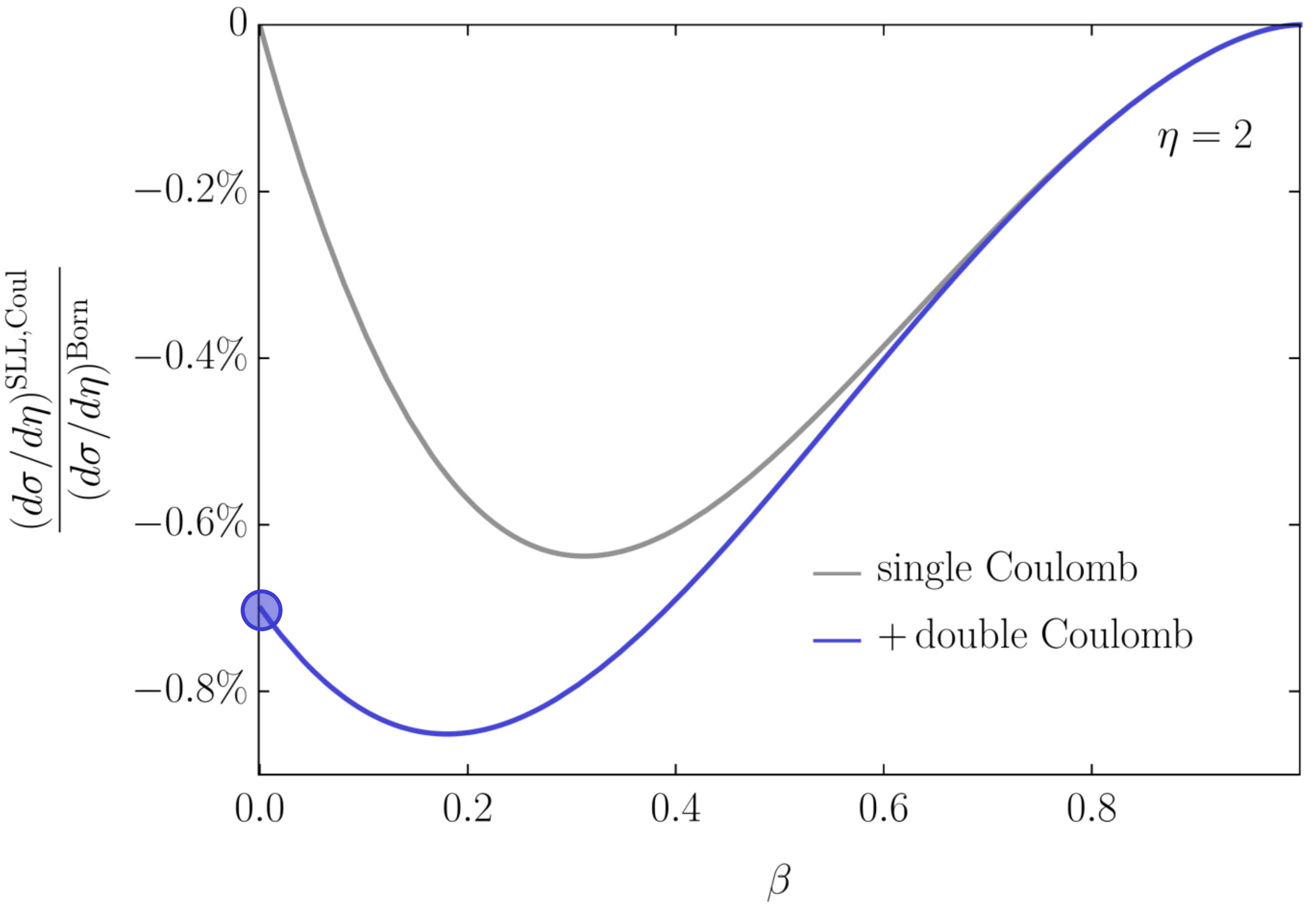
Kinematical factor $v_{t\bar{t}} = \frac{(1-\beta)^2}{2\beta}$

$\Longrightarrow$ diverges for $\beta \to 0$

Numerical Effects

$gg \to t\bar{t}$



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$$\mathbf{V}^{\text{Coul}} = -i\pi \left(\boldsymbol{T}_{t,L} \cdot \boldsymbol{T}_{\bar{t},L} - \boldsymbol{T}_{t,R} \cdot \boldsymbol{T}_{\bar{t},R} \right) v_{t\bar{t}}$$

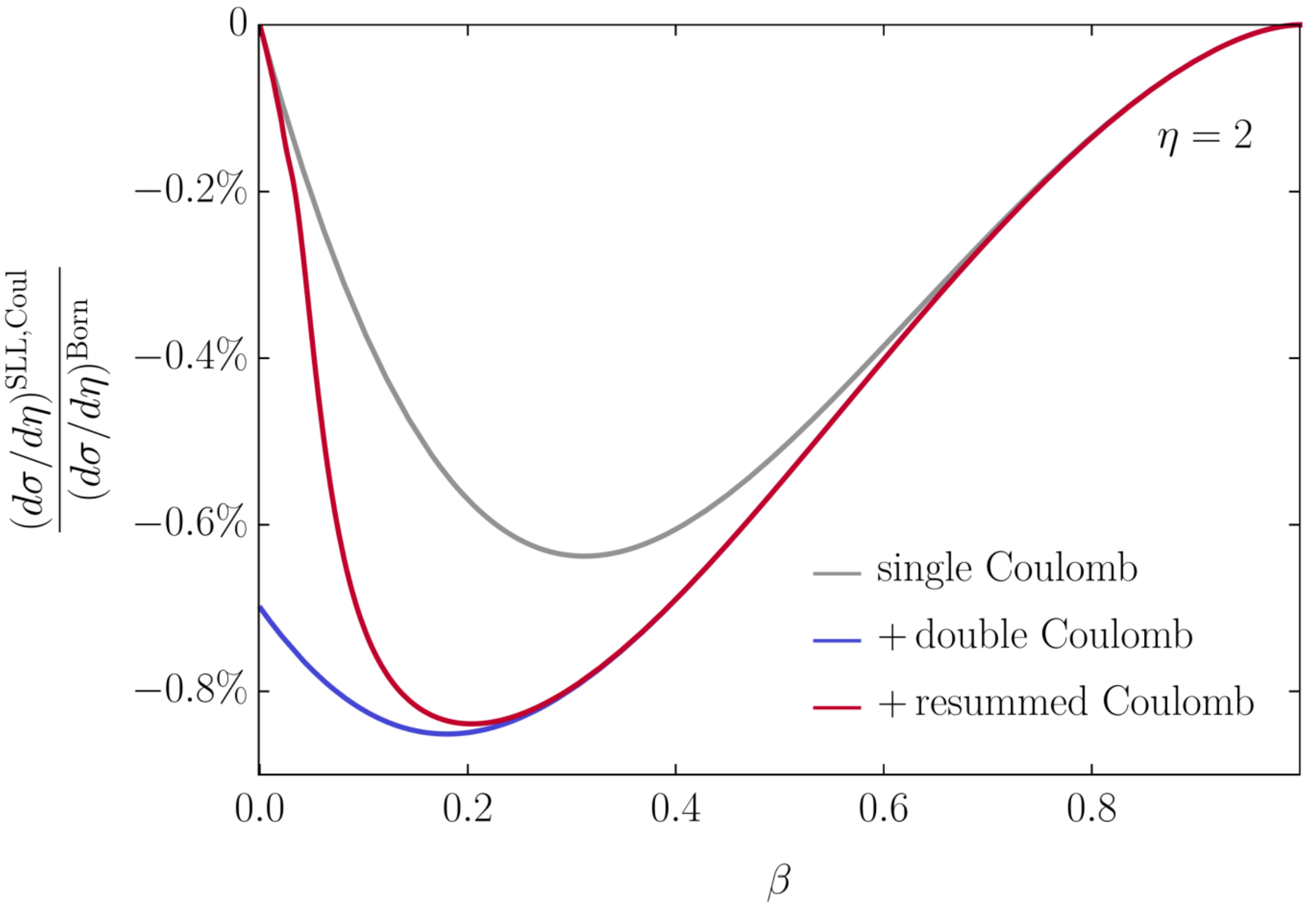
Kinematical factor $v_{t\bar{t}} = \frac{(1-\beta)^2}{2\beta}$

$\Longrightarrow$ diverges for $\beta \to 0$

- $\left(\mathbf{V}^{\text{Coul}}\right)^1 : \ \mathcal{O}(\beta^1) \Longrightarrow$ 0 for $\beta \to 0$
- $\left(\mathbf{V}^{\text{Coul}}\right)^2 : \ \mathcal{O}(\beta^0) \Longrightarrow$ constant for $\beta \to 0$
- $\left(\mathbf{V}^{\text{Coul}}\right)^3 : \ \mathcal{O}(\beta^{-1}) \Longrightarrow$ diverges for $\beta \to 0$
- $\ldots$

Numerical Effects

$gg \to t\bar{t}$



Colour generators of t and $\bar{t}$

$$\mathbf{V}^{\text{Coul}} = -i\pi \left(\boldsymbol{T}_{t,L} \cdot \boldsymbol{T}_{\bar{t},L} - \boldsymbol{T}_{t,R} \cdot \boldsymbol{T}_{\bar{t},R} \right) v_{t\bar{t}}$$

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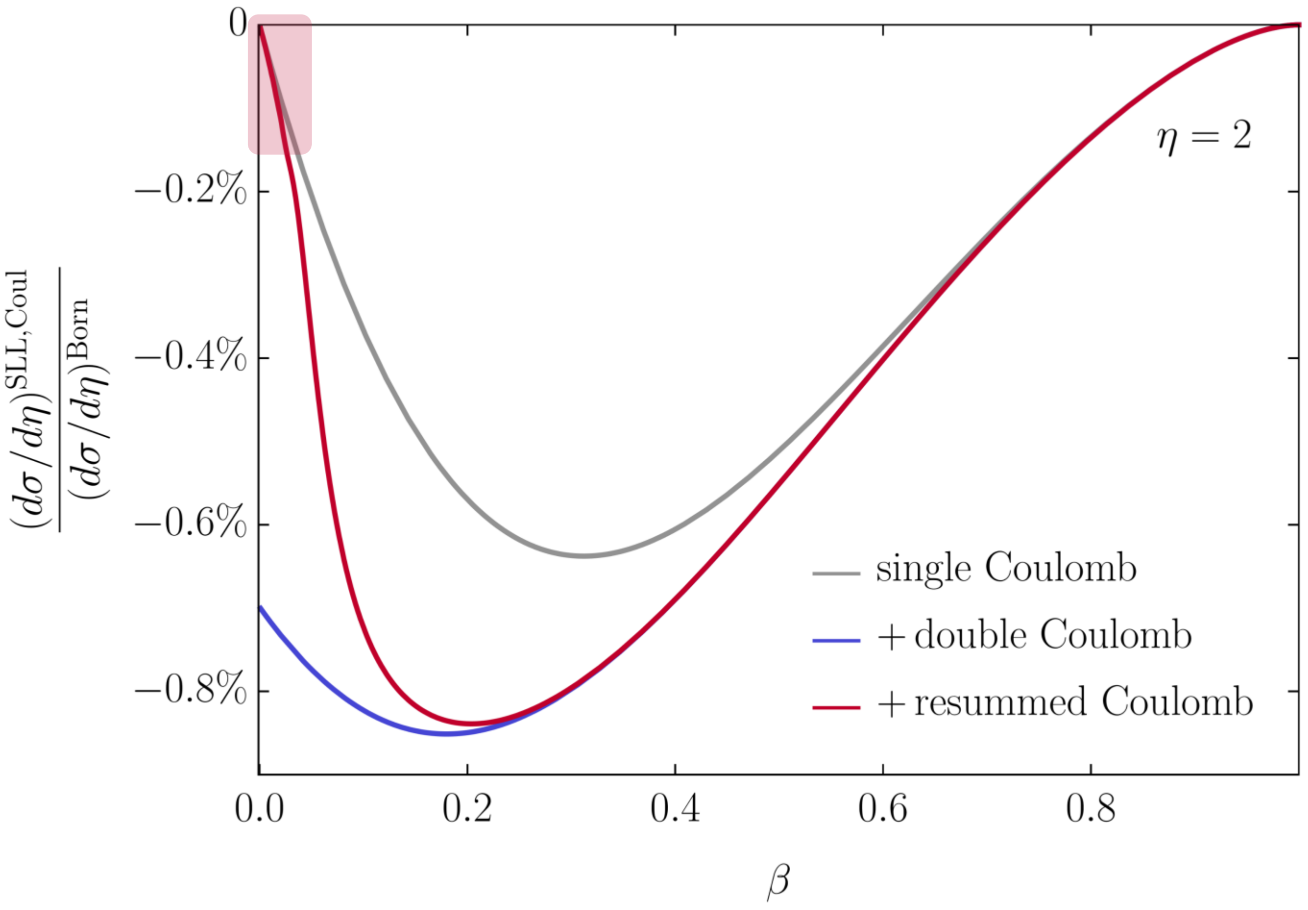
- $\Longrightarrow$ Sommerfeld effect
- $\Longrightarrow$ Resummation of arbitrary (even) number of Coulomb insertions!

$$\langle \boldsymbol{\mathcal{H}}(\mu_h) \left(\mathbf{V}^{\text{Coul}} \right)^{2n} \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$$

Numerical Effects

$gg \to t\bar{t}$



Colour generators of t and $\bar{t}$

$$\mathbf{V}^{\text{Coul}} = -i\pi \left(\boldsymbol{T}_{t,L} \cdot \boldsymbol{T}_{\bar{t},L} - \boldsymbol{T}_{t,R} \cdot \boldsymbol{T}_{\bar{t},R} \right) v_{t\bar{t}}$$

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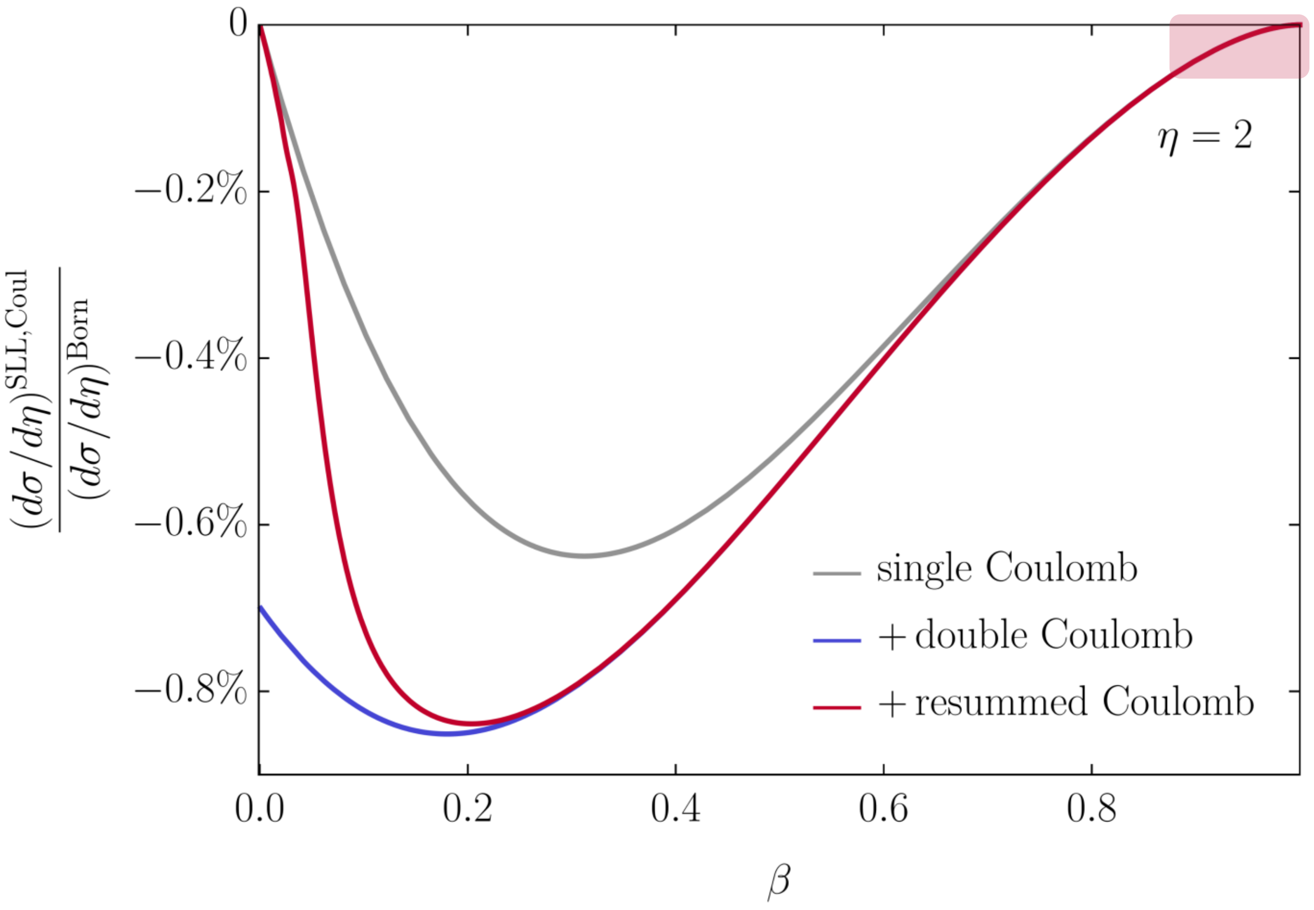


Colour generators of t and $\bar{t}$

$$\mathbf{V}^{\text{Coul}} = -i\pi \left(\boldsymbol{T}_{t,L} \cdot \boldsymbol{T}_{\bar{t},L} - \boldsymbol{T}_{t,R} \cdot \boldsymbol{T}_{\bar{t},R} \right) v_{t\bar{t}}$$

Kinematical factor $v_{t\bar{t}} = \frac{(1-\beta)^2}{2\beta}$

⟹ diverges for $\beta \to 0$

⟹ Sommerfeld effect

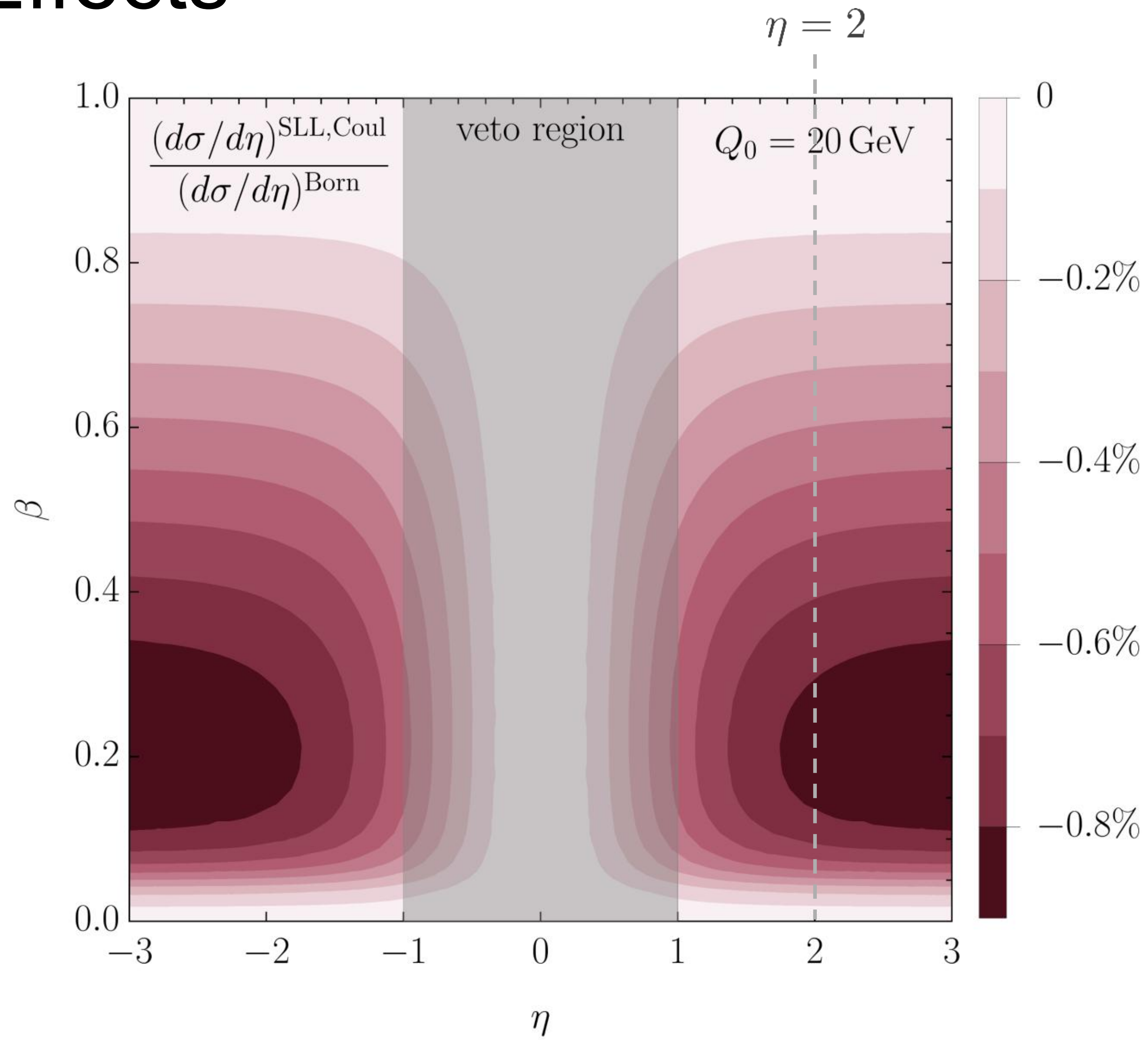
⟹ Resummation of arbitrary (even) number of Coulomb insertions!

$$\langle \boldsymbol{\mathcal{H}}(\mu_h) \left(\mathbf{V}^{\text{Coul}} \right)^{2n} \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$$

Numerical Effects

$gg \to t\bar{t}$

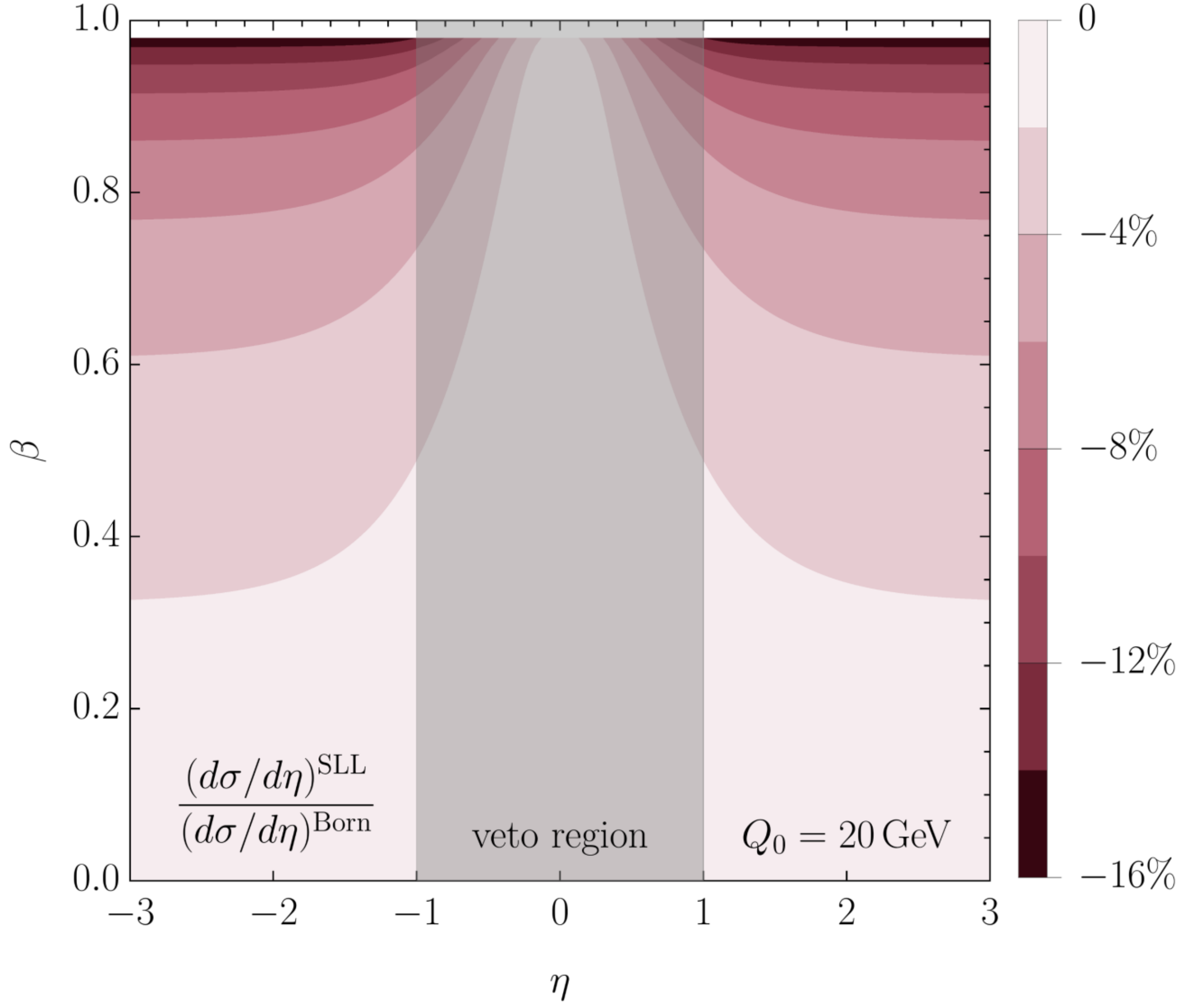
Coulomb SLLs



Numerical Effects

$gg \to t\bar{t}$

Coulomb and Glauber SLLs



Conclusion

Super-leading logarithms at hadron colliders:

$$\sigma = \sigma_{\text{Born}} \times \left\{1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \boxed{\alpha_s^4 L^5 + \alpha_s^5 L^7 + \dots}\right\}$$

New source of super-leading logarithms for massive final states

$\Longrightarrow$ Require resummation close to threshold (Sommerfeld effect)

Numerical impact:

- $q\bar{q} \to t\bar{t}$: no contribution
- $gg \to t\bar{t}$: up to $\sim 1\%$ effects in the differential cross section

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Super-leading logarithms at hadron colliders:

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Thank you for your attention!

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Backup-Slides

Anomalous Dimension

$$\overline{\boldsymbol{\Gamma}} = \frac{1}{2}\gamma_0 \sum_{\alpha,\beta} (\boldsymbol{T}_{\alpha,L} \cdot \boldsymbol{T}_{\beta,L} + \boldsymbol{T}_{\alpha,R} \cdot \boldsymbol{T}_{\beta,R}) \int \frac{d^2\Omega_k}{4\pi} \overline{W}^k_{\alpha\beta} - 4 \sum_{\alpha,\beta} \theta_{\text{hard}}(n_k) \overline{W}^k_{\alpha\beta} \boldsymbol{T}_{\alpha,L} \circ \boldsymbol{T}_{\beta,R}$$

$$\boldsymbol{\Gamma}^c = \sum_{i=1,2} \left[C_i \, \mathbf{1} - \delta(n_i - n_k) \, \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R} \right]$$

$$\boldsymbol{V}^G = -2\pi i \, (\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R})$$

$$\boldsymbol{V}^{\text{Coul}} = -\frac{1}{2}\pi i \sum_{(IJ)} (\boldsymbol{T}_{I,L} \cdot \boldsymbol{T}_{J,L} - \boldsymbol{T}_{I,R} \cdot \boldsymbol{T}_{J,R}) \, v_{IJ}$$

Cross section

$$U^c(1;\mu_i,\mu_j) = \exp\left[N_c \int_{\mu_j}^{\mu_i} \frac{d\mu}{\mu} \gamma_{\text{cusp}}(\alpha_s(\mu)) \ln\left(\frac{\mu^2}{\mu_h^2}\right)\right]$$

$$\left(\frac{d\sigma}{d\eta}\right)^{\text{SLL,Coul}} = -\frac{1}{\cosh^2(\eta)} \frac{\beta}{32\pi M^2} \frac{1}{\mathcal{N}_1\mathcal{N}_2} \times \Big\{16\pi^2 \,\text{Tr}\big(\boldsymbol{\mathcal{H}}_{2\to 2}(\mu_h)\boldsymbol{X}^{\text{Coul}}\big) \int_1^{x_s} \frac{dx}{x} \frac{1}{\beta_0^3} U^c(1;\mu_h,\mu)\left(\ln^2(x_s) - \ln^2(x)\right) + \frac{3}{2}\pi^2 \,\text{Tr}\big(\boldsymbol{\mathcal{H}}_{2\to 2}(\mu_h)\boldsymbol{X}^{\text{2Coul}}\big) \frac{1}{\beta_0^3} \ln^3(x_s)\Big\}$$

where $\boldsymbol{X}_{2\to t\bar{t}}^{\text{Coul}} = J_{43} v_{t\bar{t}} f^{abe} f^{cde} \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_3^c \boldsymbol{T}_4^d$

$$\boldsymbol{X}_{2\to t\bar{t}}^{\text{2Coul}} = v_{t\bar{t}}^2 f^{abe} f^{cde} \left(\boldsymbol{T}_3^c\{\boldsymbol{T}_4^b,\boldsymbol{T}_4^d\} - \boldsymbol{T}_4^c\{\boldsymbol{T}_3^b,\boldsymbol{T}_3^d\}\right)\left(\tilde{J}_1^{34}\boldsymbol{T}_1^a + \tilde{J}_2^{34}\boldsymbol{T}_2^a\right)$$