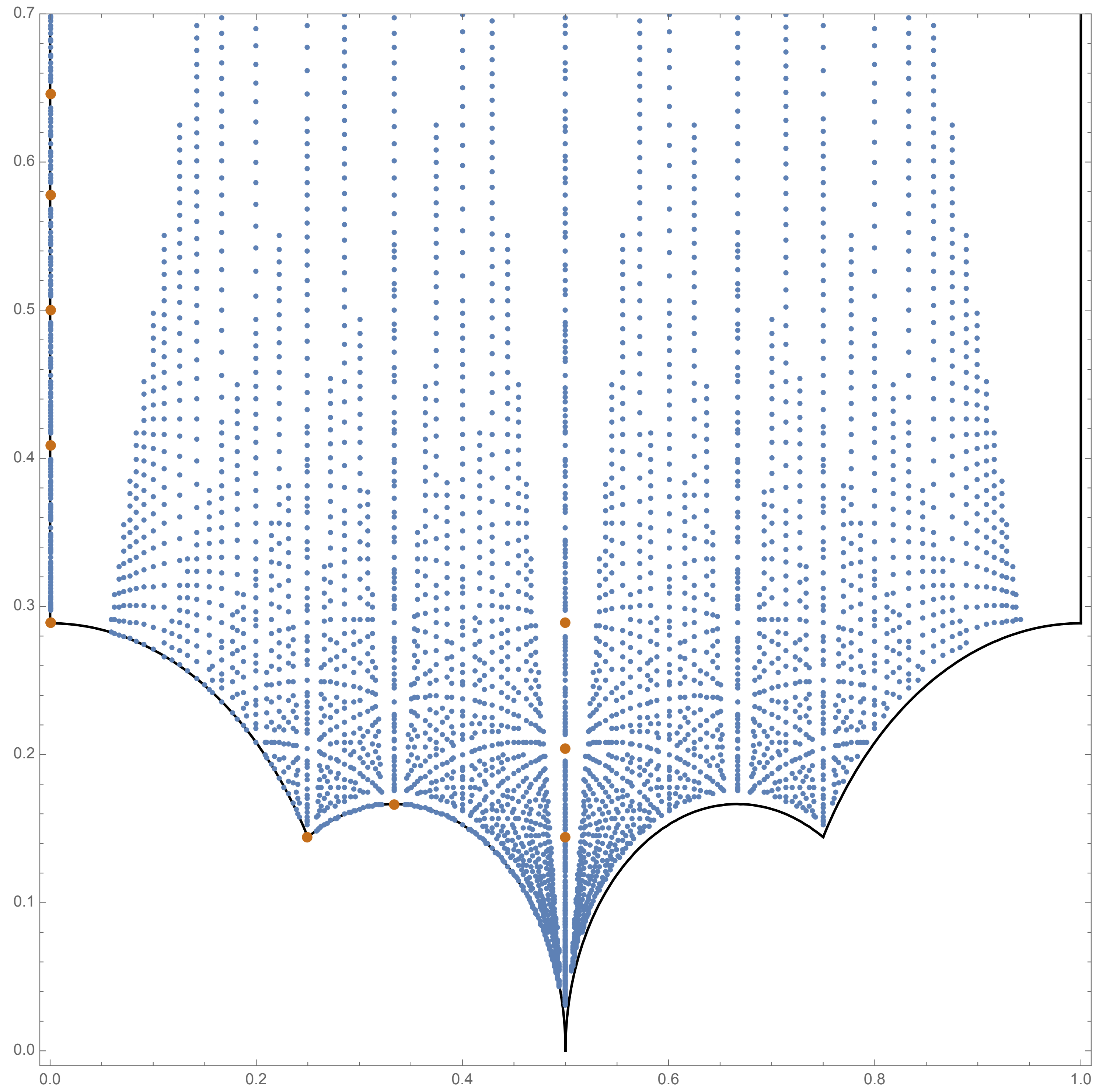
Charting F-theory landscapes



Damian van de Heisteeg

Based on:

2404.03456

2404.12422 with **Thomas Grimm**

MITP YOUNGST@RS - Physics and Number Theory

Effective Field Theories from String Theory

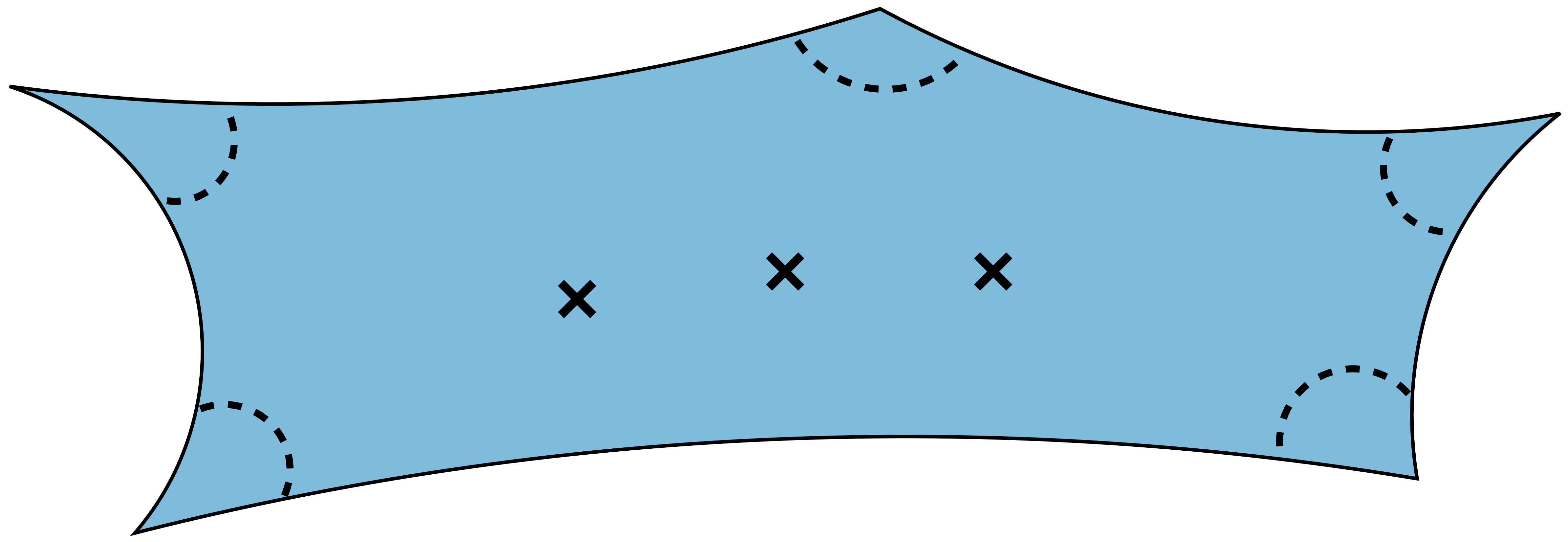
EFT from reducing string theory on a Calabi—Yau manifold Y_D:

$$S_{4\mathrm{d}} = \int d^4x\sqrt{g}\left(R - K(\phi)\,\partial_\mu\phi\,\partial^\nu\phi - \frac{1}{g^2(\phi)}F_{\mu\nu}F^{\mu\nu} - \theta(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu} - V(\phi)\right)$$

Physical couplings $K(\phi), g(\phi), \theta(\phi), V(\phi)$ $\Longrightarrow$ vary with Y_D-deformations ϕ

Much recent progress:

- Breakdown of EFT near boundaries
- special points/loci in the interior



F-theory on Calabi-Yau fourfolds

Kähler potential and flux superpotential:

$$e^{-K_{\rm cs}} = \int_{Y_4} \bar{\Omega}(\bar{z}) \wedge \Omega(z) = \bar{\mathbf{\Pi}}^T(\bar{z})\Sigma\mathbf{\Pi}(z)$$

$$\Omega(z) \in H^{4,0}$$

$$W = \int_{Y_4} G_4 \wedge \Omega(z) = \mathbf{G}_4^T \Sigma \mathbf{\Pi}(z)$$

Dependence on **complex structure moduli** encoded in period vector:

$$\Pi^I(z) = \int_{\Gamma_I} \Omega(z)$$

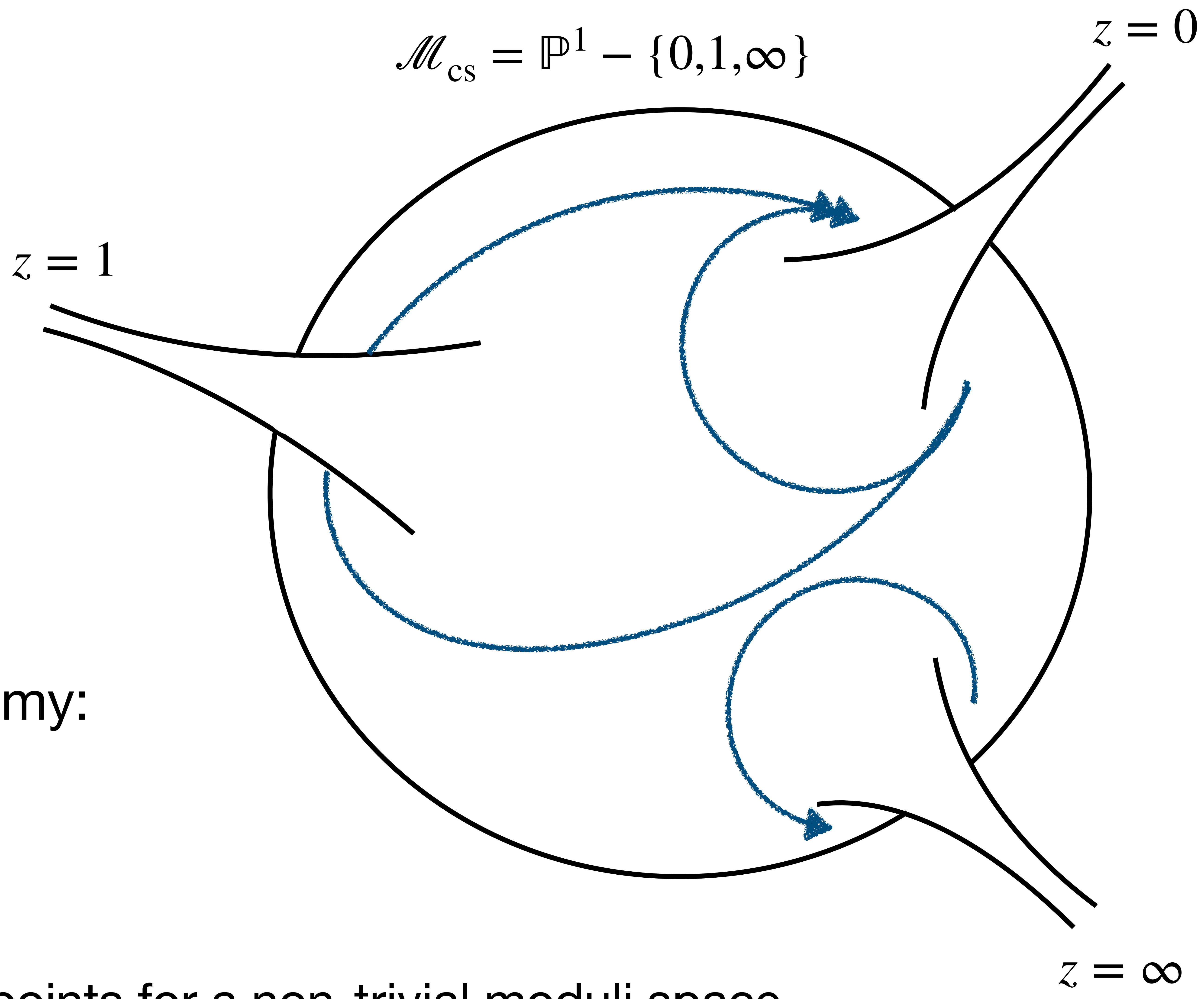
$$\Gamma_I \in H_4(Y_4, \mathbb{Z})$$

Monodromies

Circling a boundary point induces a monodromy:

$$\mathbf{\Pi}(z) \mapsto \mathbf{\Pi}(e^{2\pi i} z) = M \cdot \mathbf{\Pi}(z)$$

$(M \in SL(2,\mathbb{Z}), Sp(2m,\mathbb{Z}), SO(m,n;\mathbb{Z}))$



Equivalent loops have same monodromy:

$$M_0 M_1 = (M_\infty)^{-1}$$

Side-remark: need at least **three** singular points for a non-trivial moduli space

(monodromy group must be infinite order and completely reducible [Griffiths, '70])

Large complex structure periods

Periods in LCS regime:

[Gerhardus, Jockers '16;
Cota, Klemm, Schimannek '18;
Marchesano, Prieto, Wiesner '21]

$$\Pi_{\rm LCS} = \begin{pmatrix} 1 \\ -t \\ -\frac{1}{2}t^2 - \frac{1}{2}t + \frac{c_2}{24\kappa} \\ \frac{\kappa}{6}t^3 + \frac{\kappa}{4}t^2 + \frac{\kappa}{8}t + \frac{ic_3\zeta(3)}{8\pi^3} - \frac{c_2}{48} \\ \frac{\kappa}{24}t^4 + \frac{c_2}{48}t^2 + \frac{ic_3 t\zeta(3)}{8\pi^3} - \frac{c_4}{3456} - \frac{5}{12} \end{pmatrix}$$

(covering coordinate: $z = e^{2\pi i t}$)

Monodromy under $t \mapsto t+1$:

$$M_{\rm LCS}(\kappa, c_2) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ \frac{1}{24}\left(c_2 + 13\kappa\right) & -\frac{\kappa}{2} & -\kappa & 1 & 0 \\ \frac{1}{24}\left(c_2 + \kappa\right) & -\frac{1}{24}\left(c_2+\kappa\right) & 0 & 1 & 1 \end{pmatrix}$$

Encode **topological data** of mirror Calabi-Yau

Plan for the talk

1. **Landscape of moduli spaces:**

   Calabi-Yau fourfolds with $\mathscr{M}_{\text{cs}} = \mathbb{P}^1 \backslash \{0, 1, \infty\}$

2. **Moduli space as a landscape:**

   Flux vacua with vanishing superpotentials in F-theory

1. Landscape of moduli spaces

Calabi-Yau fourfolds with $\mathscr{M}_{\rm cs} = \mathbb{P}^1 \backslash \{0,1,\infty\}$

[DvdH '24]

Finiteness of monodromy groups

- (Non-effective) Finiteness theorem by [Deligne '81]

For a given moduli space with fixed singularity structure, there are only finitely many monodromy groups possible.

- Effective method for enumerating Calabi-Yau threefolds with $\mathscr{M}_{\rm cs} = \mathbb{P}^1 \backslash \{0,1,\infty\}$ [Doran, Morgan '05]

  - *Mirror symmetry constrains LCS and conifold monodromy*
  - ***Quasi-unipotence*** *of monodromy around infinity*

  } *14 Calabi-Yau threefolds*

→ apply to Calabi-Yau fourfold moduli spaces [DvdH '24]

Quasi-unipotence of monodromies

Driving principle behind classification: quasi-unipotence

$$(M^l - \mathbb{I})^d \neq 0\,, \qquad (M^l - \mathbb{I})^{d+1} = 0\,,$$

geometric proof by [Landman, '73]
group-theoretic proof by [Schmid, '73]

- Nilpotence degree $d = 0,1,\ldots,4$ (complex dimension of Calabi-Yau manifold)
- Finite order $l = 1,2,3,4,5,6,8,10,12$ (possible orders for a $GL(5,\mathbb{Q})$ matrix)

Warm-up: T2 monodromies

- Monodromies in $SL(2,\mathbb{Z})$:

$$M_0 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \qquad M_1 = \begin{pmatrix} 1 & -\kappa \\ 0 & 1 \end{pmatrix} \qquad M_\infty = (M_0 M_1)^{-1} = \begin{pmatrix} 1-\kappa & \kappa \\ -1 & 1 \end{pmatrix}$$

- Check quasi-unipotence condition for degree $d = 0,1$, finite order $l = 1,2,3,4,6$,

An example, $d = 0, l = 3$: $M_\infty^3 - 1 = (\kappa - 3) \begin{pmatrix} 2\kappa - \kappa^2 & \kappa^2 - \kappa \\ 1-\kappa & \kappa \end{pmatrix} = 0\,,$

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- Check quasi-unipotence condition for degree $d = 0,1$, finite order $l = 1,2,3,4,6$,

$$M_\infty^3 - 1 = (\kappa - 3) \begin{pmatrix} 2\kappa - \kappa^2 & \kappa^2 - \kappa \\ 1 - \kappa & \kappa \end{pmatrix} = 0\,,$$

$$M_\infty^4 - 1 = (\kappa - 2) \begin{pmatrix} \kappa^3 - 5\kappa^2 + 5\kappa & -\kappa^3 + 4\kappa^2 - 2\kappa \\ \kappa^2 - 4\kappa + 2 & 3\kappa - \kappa^2 \end{pmatrix} = 0\,,$$

$$M_\infty^6 - 1 = (\kappa - 1)(\kappa - 3) \begin{pmatrix} \kappa^4 - 7\kappa^3 + 14\kappa^2 - 7\kappa & -\kappa^4 + 6\kappa^3 - 9\kappa^2 + 2\kappa \\ \kappa^3 - 6\kappa^2 + 9\kappa - 2 & -\kappa^3 + 5\kappa^2 - 5\kappa \end{pmatrix} = 0\,,$$

$$(M_\infty^2 - 1)^2 = (\kappa - 4) \begin{pmatrix} \kappa^3 - 3\kappa^2 + \kappa & 2\kappa^2 - \kappa^3 \\ \kappa^2 - 2\kappa & \kappa - \kappa^2 \end{pmatrix} = 0\,,$$

$\Longrightarrow$ solutions $\kappa = 3,2,1,4$

Warm-up: T2 periods

Periods are solutions to the hypergeometric differential operator

$$L = \theta^2 - \mu z(\theta + a_1)(\theta + a_2) \qquad \theta = z\frac{d}{dz}$$

$\Longrightarrow$ L fixed by eigenvalues of M_∞: $e^{2\pi i a_1}, e^{2\pi i a_2}$

Periods are given by **hypergeometric functions**:

$$\varpi_0 = {}_2F_1\left(a_1, a_2; 1; \mu z\right)\ , \qquad \varpi_1 = \frac{i}{\sqrt{\kappa}} \cdot {}_2F_1\left(a_1, a_2; 1; 1 - \mu z\right)$$

Reverse-engineer geometries

[Hosono, Klemm, Theisen, Yau ’93]

Expand fundamental period in large complex structure regime:

degree of hypersurface

(example: $\kappa = 1$)

$$\varpi_0 = \sum_{n=0}^{\infty} \frac{(6n)!}{n!(2n)!(3n)!} z^n = 1 + 60z + 13860z^2 + 4084080z^3 + \mathcal{O}(z^4)$$

weights of projective space

$\Longrightarrow$ complete intersection Calabi-Yau $X_6(1,2,3)$: sextic in $\mathbb{P}^2[1,2,3]$

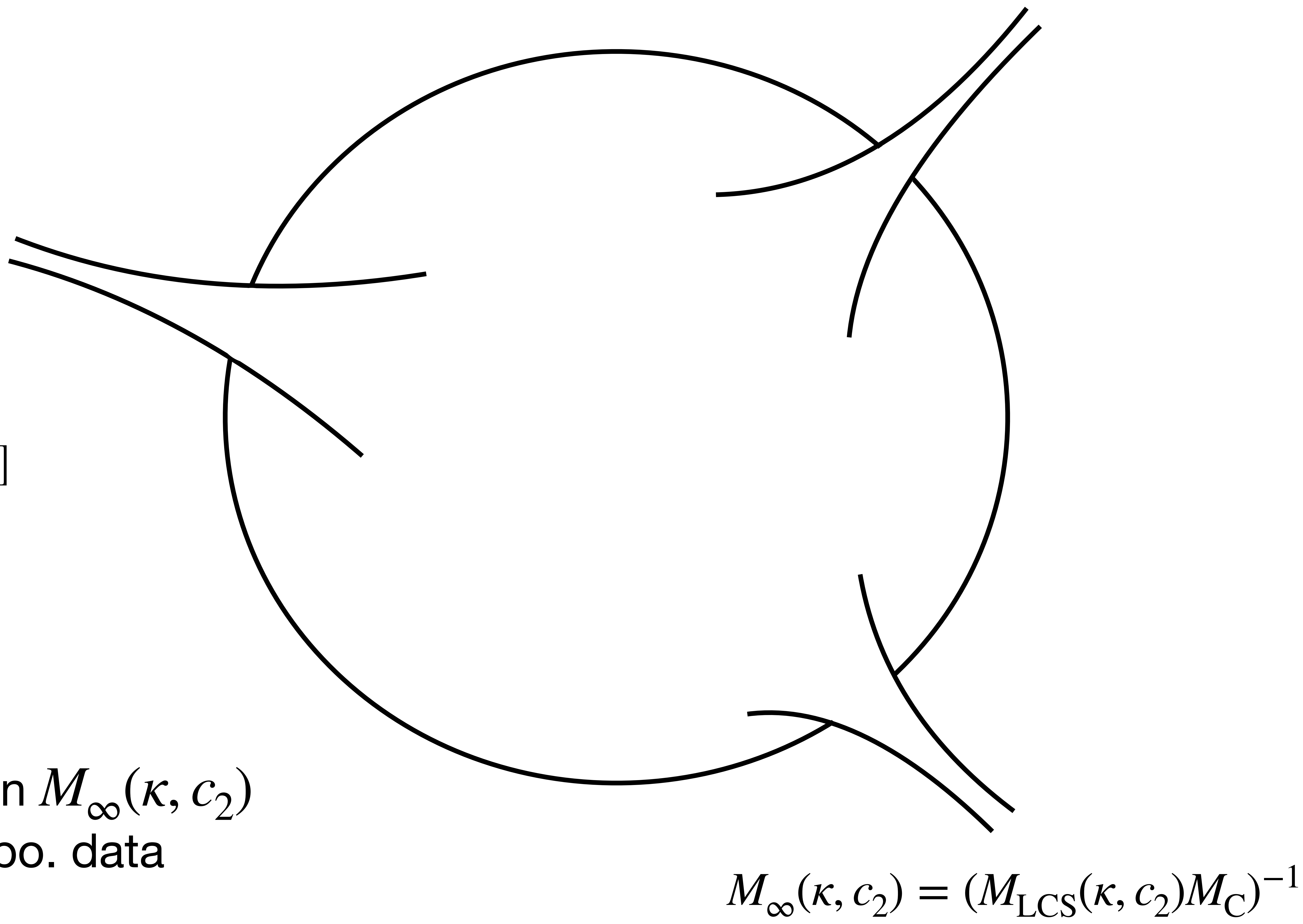
Warm-up: T2 landscape

(a_1, a_2)	$(\frac{1}{6}, \frac{5}{6})$	$(\frac{1}{4}, \frac{3}{4})$	$(\frac{1}{3}, \frac{2}{3})$	$(\frac{1}{2}, \frac{1}{2})$
κ	1	2	3	4
μ	432	64	27	16
(d, l)	$(0, 6)$	$(0, 4)$	$(0, 3)$	$(1, 2)$
Modular group	$\Gamma_1(1)$	$\Gamma_1(2)$	$\Gamma_1(3)$	$\Gamma_1(4)$
Elliptic curve	$X_6(1, 2, 3)$	$X_4(1^2, 2)$	$X_3(1^3)$	$X_{2,2}(1^4)$

Back to Calabi-Yau fourfolds

$$M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

[Grimm, Ha, Klemm, Klevers '09]



$\Longrightarrow$ impose quasi-unipotence on $M_\infty(\kappa, c_2)$ and solve for topo. data

Example

Impose a finite order monodromy of order $l = 6$:

$$(M_\infty(\kappa, c_2))^6 - \mathbb{I} = 0$$

$\Longrightarrow$ polynomial set of equations for κ and c_2

Only 1 solution: $\kappa = 6,\ c_2 = 90$

$\Longrightarrow$ data of the sextic in $\mathbb{P}^5$, (without doing a geometrical computation)

Landscape of monodromy groups

[DvdH, '24]

(κ, a)	(6,4)	(4,4)	(2,3)	(10,5)	(2,4)	(4,3)	(12,5)
degree d	0						
order l	6	8	10		12		

(a) *Finite order monodromies.*

(κ, a)	(8,4)	(2,2)	(18,6)	(16,6)	(8,5)	(24,7)	(32,8)
degree d	1			2			4
order l	4	6		4	6		2

(b) *Infinite order monodromies.*

$a = (\kappa + c_2)/24$

Computing the periods

- Periods solve the hypergeometric equation:

$$L = \theta^5 - \mu z(\theta + a_1)(\theta + a_2)(\theta + a_3)(\theta + a_4)(\theta + a_5) \qquad \theta = z\frac{d}{dz}$$

Fundamental period solution:

$$\Pi^0(z) = {}_5F_4(a_1, \ldots, a_5; 1^4; \mu z)$$

- Can determine the CICY from series expansion of this period
- Other 4 periods have similar expressions in hypergeometric functions

Calabi-Yau fourfold landscape

a_1, a_2, a_3, a_4, a_5	Type	Mirror	μ	(κ, a)	c_2	c_3	c_4
$\frac{1}{5}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{4}{5}$	F	$X_{2,5}(1^7)$	$2^2 5^5$	$(10, 5)$	110	-420	2190
$\frac{1}{10}, \frac{3}{10}, \frac{1}{2}, \frac{7}{10}, \frac{9}{10}$	F	$X_{10}(1^5, 5)$	$2^{10} 5^5$	$(2, 3)$	70	-580	5910
$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	LCS	$X_{2^5}(1^{10})$	2^{10}	$(32, 8)$	160	-320	960
$\frac{1}{3}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{2}{3}$	CY3	$X_{2,3,3}(1^8)$	$2^2 3^6$	$(18, 6)$	126	-324	1206
$\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{2}{3}$	C	$X_{2,2,2,3}(1^9)$	$2^6 3^3$	$(24, 7)$	144	-336	1152
$\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{4}$	C	$X_{2,2,4}(1^8)$	2^{12}	$(16, 6)$	128	-384	1632
$\frac{1}{8}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{7}{8}$	F	$X_{2,8}(1^6, 4)^*$	2^{18}	$(4, 4)$	92	-600	4908
$\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}$	F	$X_6(1^6)$	6^6	$(6, 4)$	90	-420	2610
$\frac{1}{12}, \frac{5}{12}, \frac{1}{2}, \frac{7}{12}, \frac{11}{12}$	F	$X_{2,2,12}(1^6, 4, 6)^{**}$	$2^{14} 3^6$	$(2, 4)$	94	-972	11814
$\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{3}{4}$	CY3	$X_{4,4}(1^6, 2)$	2^{14}	$(8, 4)$	88	-304	1464
$\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}$	F	$X_{3,4}(1^7)$	$2^8 3^3$	$(12, 5)$	108	-336	1476
$\frac{1}{6}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{5}{6}$	F	$X_{4,6}(1^5, 2, 3)^*$	$2^{12} 3^3$	$(4, 3)$	68	-320	2028
$\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{5}{6}$	CY3	$X_{6,6}(1^4, 2, 3^2)^*$	$2^{10} 3^3$	$(2, 2)$	46	-244	1734
$\frac{1}{6}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{5}{6}$	C	$X_{2,2,6}(1^7, 3)^*$	$2^{10} 3^6$	$(8, 5)$	112	-528	3264

- 9 CY4 already known

  [Cabo-Bizet, Klemm, Lopes ’14]
- 5 CY4 are new

  [DvdH ’24]

Phases at infinity

- **LCS point:** another maximally unipotent point, $d = 4$

- **CY3-point:** weak string-coupling limit of a **rigid** Calabi-Yau orientifold, $d = 1$

- **Conifold-point:** finite distance point, but infinite order monodromy, $d = 2$

- **Landau-Ginzburg point:** finite order monodromy, $d = 0$

$\Longrightarrow$ for each phase an example worked out in [DvdH, '24]

CY3-point of $X_{6,6}(1^4,2,3^2)$

Period expansion around the CY3-point:

$$\Pi(\tau) = \begin{pmatrix} 1 \\ \frac{1}{2}+\frac{i\sqrt{3}}{2} \\ 0 \\ \tau \\ (\frac{1}{2}+\frac{i\sqrt{3}}{2})\tau \end{pmatrix} + \frac{i}{\sqrt{3}} \begin{pmatrix} 0 \\ 0 \\ -1 \\ -\frac{2}{3} \\ \frac{1}{3} \end{pmatrix} + \mathcal{O}(e^{2\pi i \tau}) \qquad \tau = \log[z]/2\pi i$$

- **Rigid Calabi-Yau threefold** with period vector $(1, \frac{1}{2}+\frac{i\sqrt{3}}{2})$
- Complex structure coordinate parametrizes the **string coupling**

D7-brane superpotential

Fourfold periods are known to encode **open-string** physics

[Grimm-Ha-Klemm-Klevers ’09; Alim-Hecht-Jockers-Mayr-Mertens-Soroush ’09; Jockers-Mayr-Walcher ’09; Clinghler-Donagi-Wijnholt ’12]

Remaining period: superpotential induced by **worldvolume flux** of D7-branes

$$W_{\mathrm{D7}} = q_{\mathrm{D7}} \frac{\sqrt{z}}{\pi^2} \, {}_5F_4\left(\tfrac{1}{2}^5; \tfrac{2}{3}^2, \tfrac{4}{3}^2; -2^{10}3^3 z\right) = \frac{q_{\mathrm{D7}}}{\pi^2}\sqrt{z} \sum_{k=0}^{\infty} \frac{\Gamma\left(k+\frac{1}{2}\right)^5}{\sqrt{\pi}\,\Gamma(k+1)\Gamma\left(k+\frac{2}{3}\right)^2 \Gamma\left(k+\frac{4}{3}\right)^2} (-2^{10}3^3 z)^k$$

$$= \frac{q_{\mathrm{D7}}}{\pi^2}\sqrt{z}\left(1 - \frac{2187}{2}z + \frac{9298091736}{1225}z^2 - \frac{4236443047215}{49}z^3 + \mathcal{O}(z^4)\right) \qquad z = e^{2\pi i \tau}$$

2. Moduli Space as a Landscape

Flux vacua with vanishing superpotentials in F-theory

[Grimm, DvdH '24]

Special loci in Calabi-Yau moduli spaces

- Moduli stabilization $\Longrightarrow$ flux vacua: $D_{z^I} W = \int_{Y_4} G_4 \wedge D_{z^I} \Omega = 0$

$$G_4 \in H^4(Y_4, \mathbb{Z}) \cap (H^{4,0} \oplus H^{2,2} \oplus H^{0,4})$$

- Black hole physics $\Longrightarrow$ attractor points: $\partial_{z^I} |Z(Q)| = \int_{Y_3} Q \wedge D_{z^I} \Omega = 0$

$$Q \in H^3(Y_3, \mathbb{Z}) \cap (H^{3,0} \oplus H^{0,3})$$

Rank-two attractors: [Moore '98]

- Rational CFTs & Complex Multiplication points

[Gukov, Vafa '02]

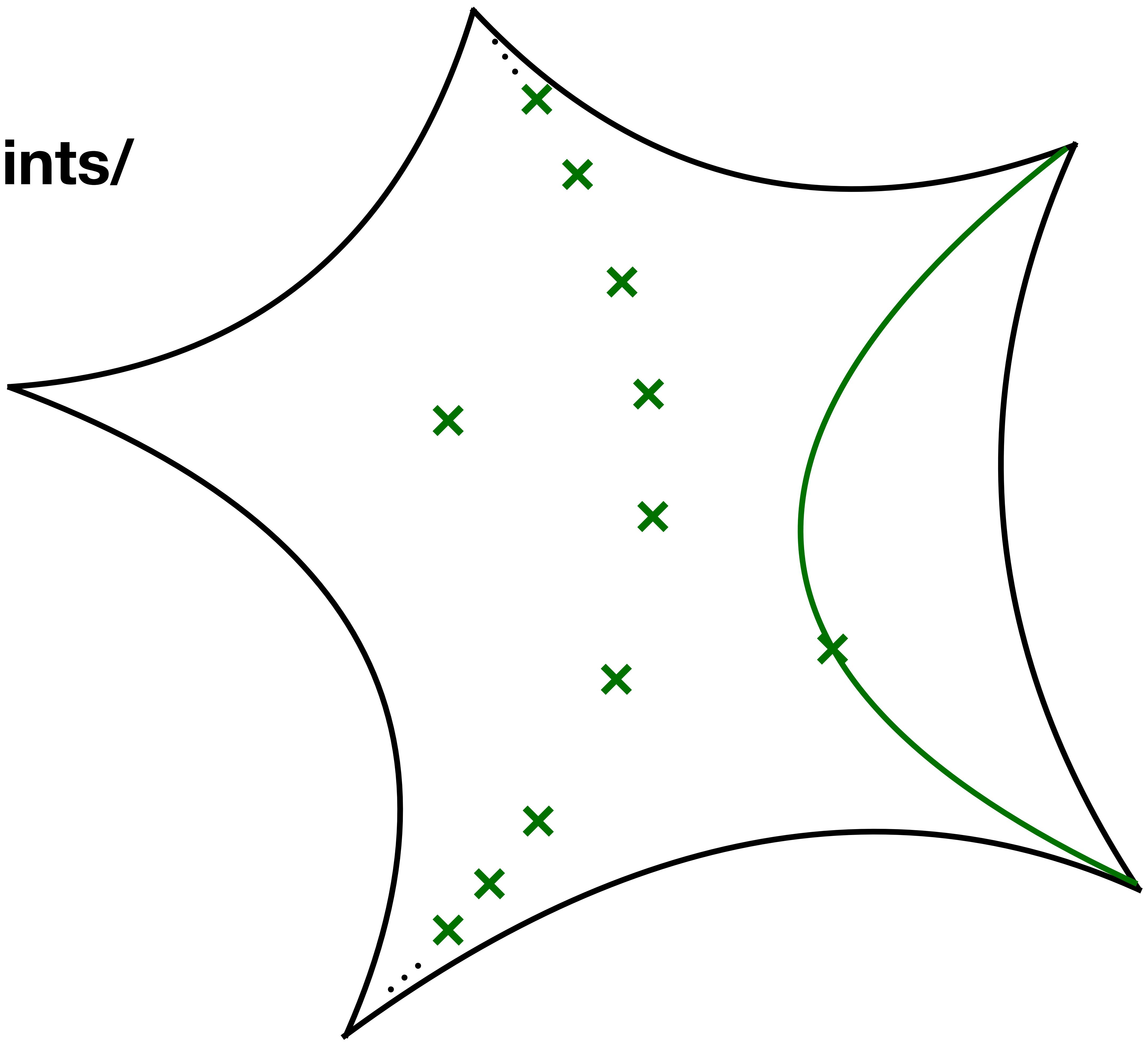
(Rough) distribution of special points

- Moduli spaces are littered with **special points/ submanifolds** of physical relevance

  e.g. flux vacua and attractor points

- "Special enough" loci are, generically, **scarce** in the moduli space

  e.g. flux vacua with $W = 0$ and rank-two attractors



Flux potential

Scalar potential for moduli:

$$V = e^K \; K^{a\bar{b}} D_a W D_{\bar{b}} W = \int_{Y_4} G_4 \wedge \star G_4 - \int_{Y_4} G_4 \wedge G_4$$

- Global minima:
  - vanishing F-terms $D_a W = \partial_a W + \partial_a K \, W = 0$
  - self-dual fluxes $G_4 \in H^4(Y_4, \mathbb{Z}) \cap (H^{4,0} \oplus H^{2,2} \oplus H^{0,4})$
- Supersymmetric vacua
  - Vanishing superpotential $W = \partial_a W = 0$
  - Hodge class $G_4 \in H^4(Y_4, \mathbb{Z}) \cap H^{2,2}$

Current state of the flux landscape

- Type IIB Flux vacua with $W = 0$ on orbifold loci

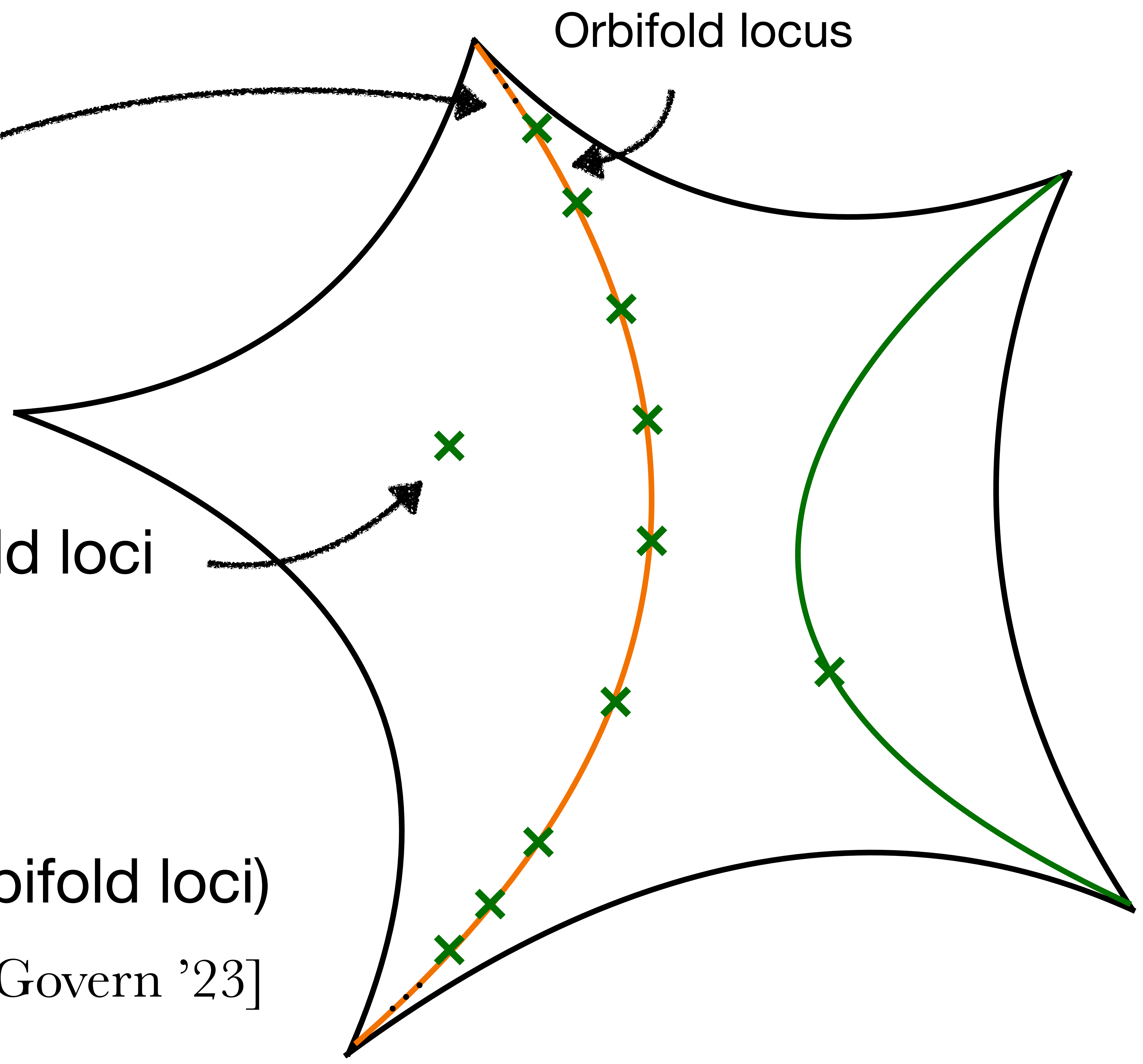
  [DeWolfe, Giryavets, Kachru, Taylor ’04; DeWolfe ’05; Palti ’06; …; Rajaguru, Sengupta, Wrase ’24; Becker, Brady, Graña, Morros, Sengupta ’24]

- Type IIB flux vacua with $W = 0$ away from orbifold loci

  [Candelas, de la Ossa, Elmi, Van Straten ’19; Bönisch, Elmi, Kashani-Poor, Klemm ‘22]

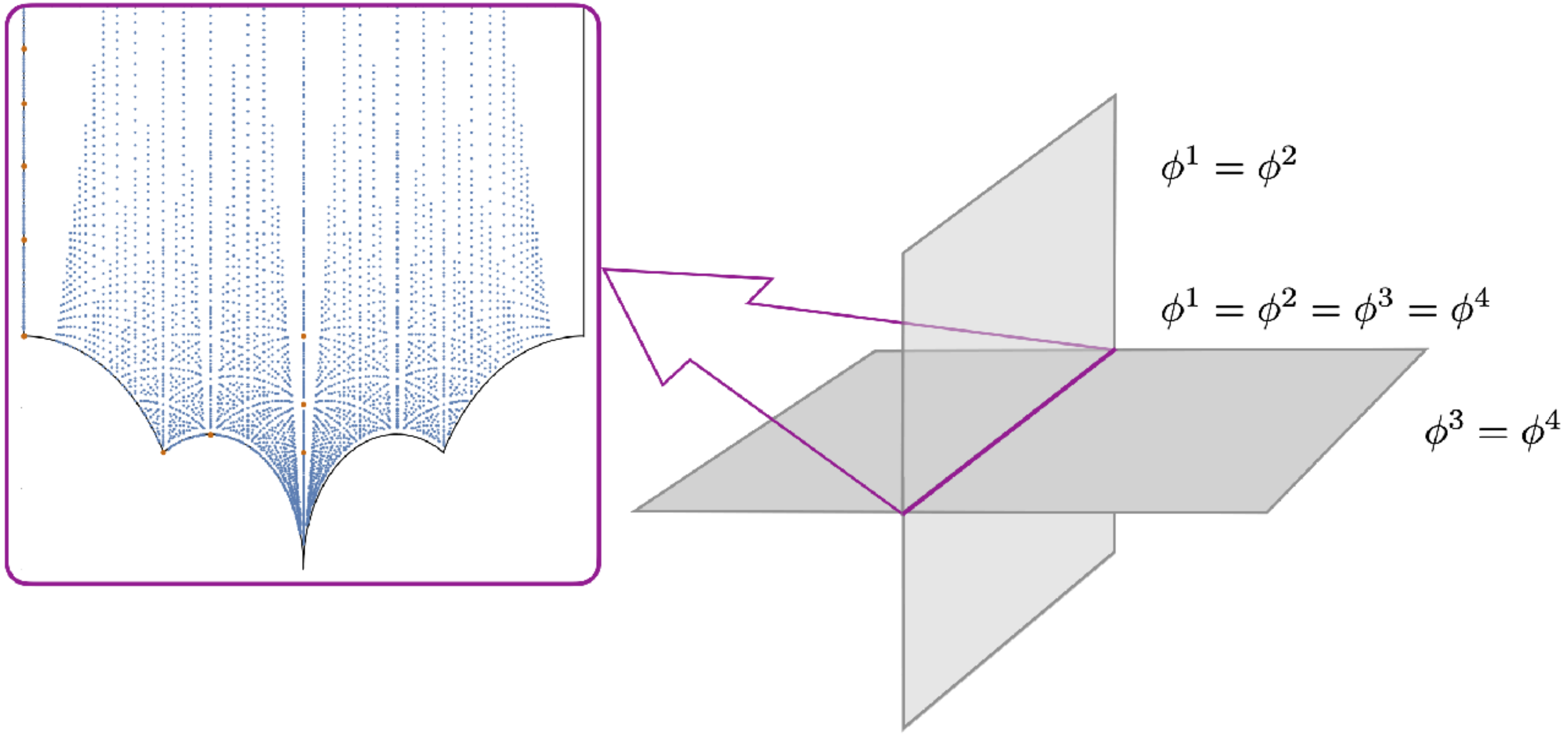
- Extended Type IIB flux vacua with $W = 0$ (on orbifold loci)

  [Kachru, Nally, Yang ’20; Candelas, de la Ossa, Kuusela, McGovern ’23]



Our goal: an F-theory flux landscape [Grimm, DvdH ’24]

How? Search along the diagonal locus in moduli space:



Calabi-Yau fourfold of Hulek-Verrill

- **Hulek-Verrill fourfold:** $(X^1, \ldots, X^6) \in \mathbb{T}^5 = \mathbb{P}^5 \backslash \{X_1 \cdots X_6 = 0\}$

$$(X^1 + X^2 + X^3 + X^4 + X^5 + X^6)\left(\frac{\phi^1}{X^1} + \frac{\phi^2}{X^2} + \frac{\phi^3}{X^3} + \frac{\phi^4}{X^4} + \frac{\phi^5}{X^5} + \frac{\phi^6}{X^6}\right) = 1$$

$\Longrightarrow$ S_6 permutation symmetry under exchanging moduli and coordinates

- **Periods:** expanded in large complex structure regime [Jockers, Kotlewski, Kuusela '23]

$$\mathbf{\Pi} = \begin{pmatrix} \Pi^0 \\ \Pi^I \\ \Pi^{IJ} \\ \Pi_I \\ \Pi_0 \end{pmatrix}$$

$$\Pi^0 = \sum_{n_1,\ldots,n_6=0}^{\infty} \left(\frac{(n_1 + \ldots + n_6)!}{n_1! \cdots n_6!}\right)^2 (\phi^1)^{n_1} \cdots (\phi^6)^{n_6}$$

$$\Pi^I = \Pi^0 \frac{\log \phi^I}{2\pi i} + 2 \sum_{n_1,\ldots,n_6=0}^{\infty} \left(\frac{(n_1 + \ldots + n_6)!}{n_1! \cdots n_6!}\right)^2 (H_{n_1+\ldots+n_6} - H_{n_I})(\phi^1)^{n_1} \cdots (\phi^6)^{n_6}$$

Symmetry in the periods

Monodromy symmetry:

$$M_{\text{swap}} \cdot \mathbf{\Pi}(\phi^1, \phi^2, \phi^i) = \mathbf{\Pi}(\phi^2, \phi^1, \phi^i)$$

$\Longrightarrow$ decompose based on orbifold charge:

$$\mathbf{\Pi}(\phi) = \mathbf{\Pi}_+(\phi) + \mathbf{\Pi}_-(\phi)\,, \qquad \mathbf{\Pi}_\pm(\phi) = \frac{1}{2}(1 \pm M_{\text{swap}})\mathbf{\Pi}(\phi)$$

On symmetry locus:

- Vanishing conditions: $\mathbf{\Pi}_-(\phi)\big|_{\phi^1=\phi^2} = \partial_+\mathbf{\Pi}_-(\phi)\big|_{\phi^1=\phi^2} = \partial_i\mathbf{\Pi}_-(\phi)\big|_{\phi^1=\phi^2} = \partial_-\mathbf{\Pi}_+(\phi)\big|_{\phi^1=\phi^2} = 0$
- K3 period vector: $\partial_-\mathbf{\Pi}_-\big|_{\phi^1=\phi^2} = \varpi^0\mathbf{v}^- + \varpi^i\mathbf{v}_i - \varpi_0\mathbf{v}_-$

K3 surface inside Calabi-Yau fourfold

- K3 fundamental period from Calabi-Yau fourfold period

$$\varpi^0 = \partial_-(\Pi^1 - \Pi^2)\Big|_{\phi^1=\phi^2} = \sum_{n_3,n_4,n_5,n_6} \left(\frac{(n_3+\ldots+n_6)!}{n_3!\cdots n_6!}\right)^2 (\phi^3)^{n_3}\cdots(\phi^6)^{n_6}$$

$\Longrightarrow$ other periods follow similarly

- K3 submanifold along $X_1 = -X_2$ and $\phi^1 = \phi^2$:

$$(X^1+X^2+X^3+X^4+X^5+X^6)\left(\frac{\phi^1}{X^1}+\frac{\phi^2}{X^2}+\frac{\phi^3}{X^3}+\frac{\phi^4}{X^4}+\frac{\phi^5}{X^5}+\frac{\phi^6}{X^6}\right)=1$$

$$\Downarrow$$

$$(X_3+X_4+X_5+X_6)\left(\frac{\phi_3}{X_3}+\frac{\phi_4}{X_4}+\frac{\phi_5}{X_5}+\frac{\phi_6}{X_6}\right)=1$$

Finding flux vacua

Turn on $\mathbb{Z}_2$-odd flux: $\mathbf{G}_4 = q^0\mathbf{v}^- + q^i\mathbf{v}_i - (q_0 + q^0)\mathbf{v}_-$

- Most F-terms and superpotential vanish automatically:

$$W|_{\phi^1=\phi^2} = \partial_+ W|_{\phi^1=\phi^2} = \partial_i W|_{\phi^1=\phi^2} = 0$$

- Remaining F-term reduces to K3 superpotential:

$$\partial_- W|_{\phi^1=\phi^2} = W_{\mathrm{K3}} = \varpi^0\left(q_0 + 2q_i\mathfrak{t}^i + 2q^0\sum_{i<j}\mathfrak{t}^i\mathfrak{t}^j\right)$$

exact by K3 mirror map

Scalar potential: $V\big|_{\phi^1=\phi^2} = \mathcal{V}_{\mathrm{b}}^{-2}\ e^{K_{\mathrm{K3}}}|W_{\mathrm{K3}}|^2 = \dfrac{\mathcal{V}_{\mathrm{b}}^{-2}}{\sum_{i<j}\mathrm{Im}\,\mathfrak{t}^i\,\mathrm{Im}\,\mathfrak{t}^j}\left|q_0 + 2q_i\mathfrak{t}^i + 2q^0\sum_{i<j}\mathfrak{t}^i\mathfrak{t}^j\right|^2$

Stabilizing all moduli — flux choice

Turn on most general flux compatible with S_6-symmetry:

$$\mathbf{G}_4^{\mathrm{vac}} = (0, a_I, b_I + b_J, a_I + c_I, 0)$$

- S_6 symmetry condition: $\sum_{I=1}^{6} a_I = \sum_{I=1}^{6} b_I = \sum_{I=1}^{6} c_I = 0$
- Same solution for all six moduli: $(a_I, b_I, c_I) = n_I(a, b, c)$
- Minimal tadpole contribution: $n_I = (1, -1, 1, -1, 1, -1)$

Flux vacua

VEV for diagonal modulus: $\mathfrak{t} = -\frac{b}{a} \pm i\frac{\sqrt{ac-12b^2}}{2\sqrt{3}a}$

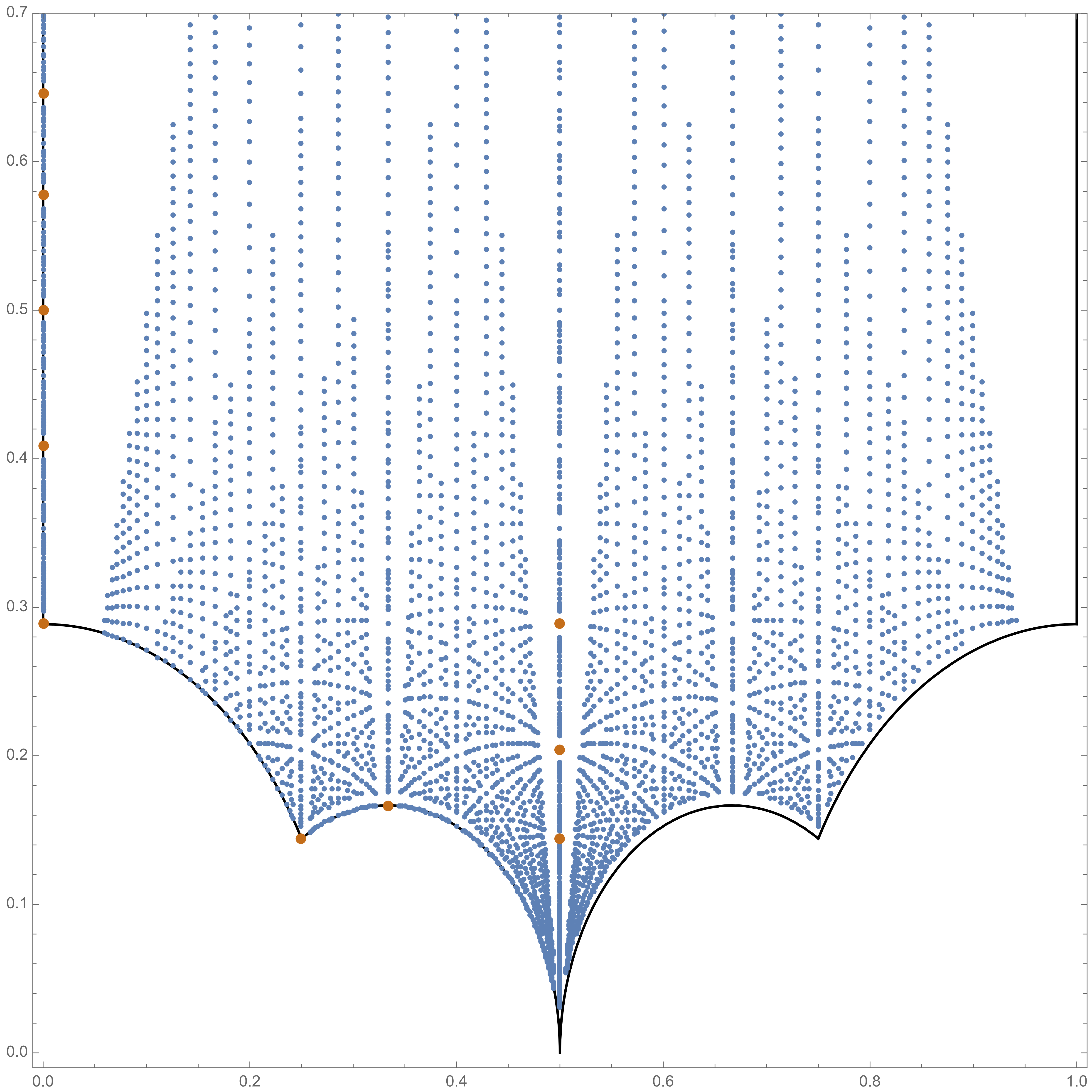
Tadpole contribution of fluxes: $L^{\text{vac}} = 12(ac - 12b^2)$

- Physical solution requires positive tadpole contribution $\left(L = \frac{1}{2}\int_{Y_4} G_4 \wedge G_4 \leq \frac{\chi(Y_4)}{24}\right)$
- Finiteness of vacua follows from:
  - Imposing the tadpole bound $L^{\text{vac}} \leq 60$
  - Restricting to fundamental domain

Generally proven by:
[Cattani, Deligne, Kaplan '95] for $W = 0$
[Bakker, Grimm, Schnell, Tsimerman '21] for $W \neq 0$

determined in [Verrill '96]

Flux vacuum landscape of HV4



$$\hat{L}^{\mathrm{vac}} = L^{\mathrm{vac}}/12$$

- Red dots: $\hat{L}^{\mathrm{vac}} \leq 5$
- Blue dots: $\hat{L}^{\mathrm{vac}} \leq 300$

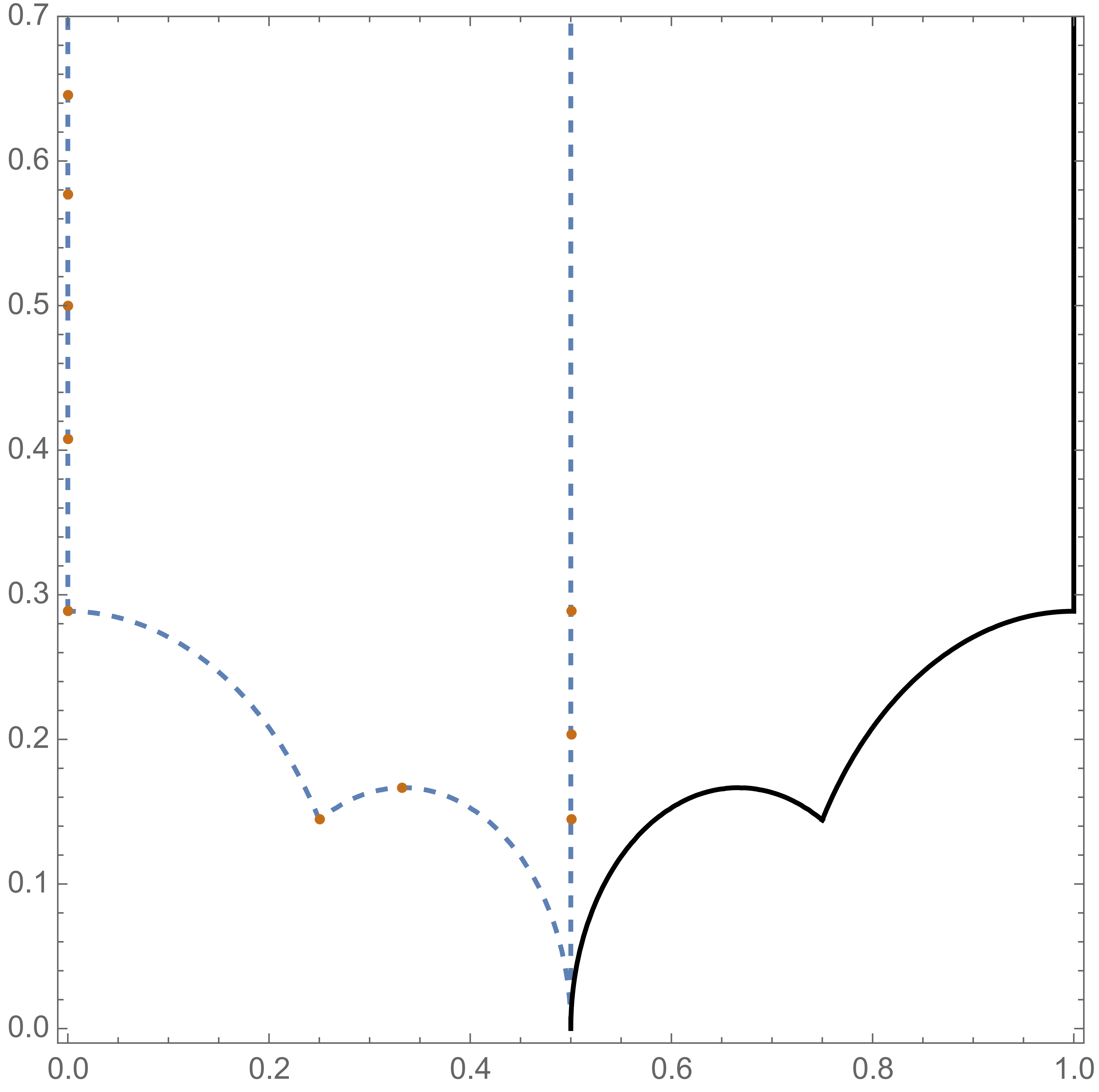
Landscape below tadpole bound

$\mathfrak{t}$	$\frac{i}{2\sqrt{3}}$ *	$\frac{i}{\sqrt{6}}$	$\frac{1}{2}+\frac{i}{2\sqrt{6}}$	$\frac{i}{2}$	$\frac{1}{3}+\frac{i}{6}$	$\frac{i}{\sqrt{3}}$	$\frac{1}{4}+\frac{i}{4\sqrt{3}}$ *	$\frac{1}{2}+\frac{i}{4\sqrt{3}}$	$\frac{1}{2}+\frac{i}{2\sqrt{3}}$	$\frac{i\sqrt{15}}{6}$
ϕ	$\frac{1}{16}$	$\frac{3\sqrt{3}-5}{4}$	$-\frac{5+3\sqrt{3}}{4}$	$\frac{1}{4}-\frac{\sqrt{3}}{8}$	$\frac{1}{4}+\frac{\sqrt{3}}{8}$	$\frac{1}{22+9\sqrt{6}}$	$\frac{1}{4}$	$-\frac{22+9\sqrt{6}}{2}$	$-\frac{1}{2}$	$\frac{1}{64}$
a	1	2	7	3	5	4	4	13	8	5
b	0	0	1	0	1	0	1	2	1	0
c	1	1	2	1	3	1	4	4	2	1
$\hat{L}^{\text{vac}}$	1	2	2	3	3	4	4	4	4	5

*located at a conifold point

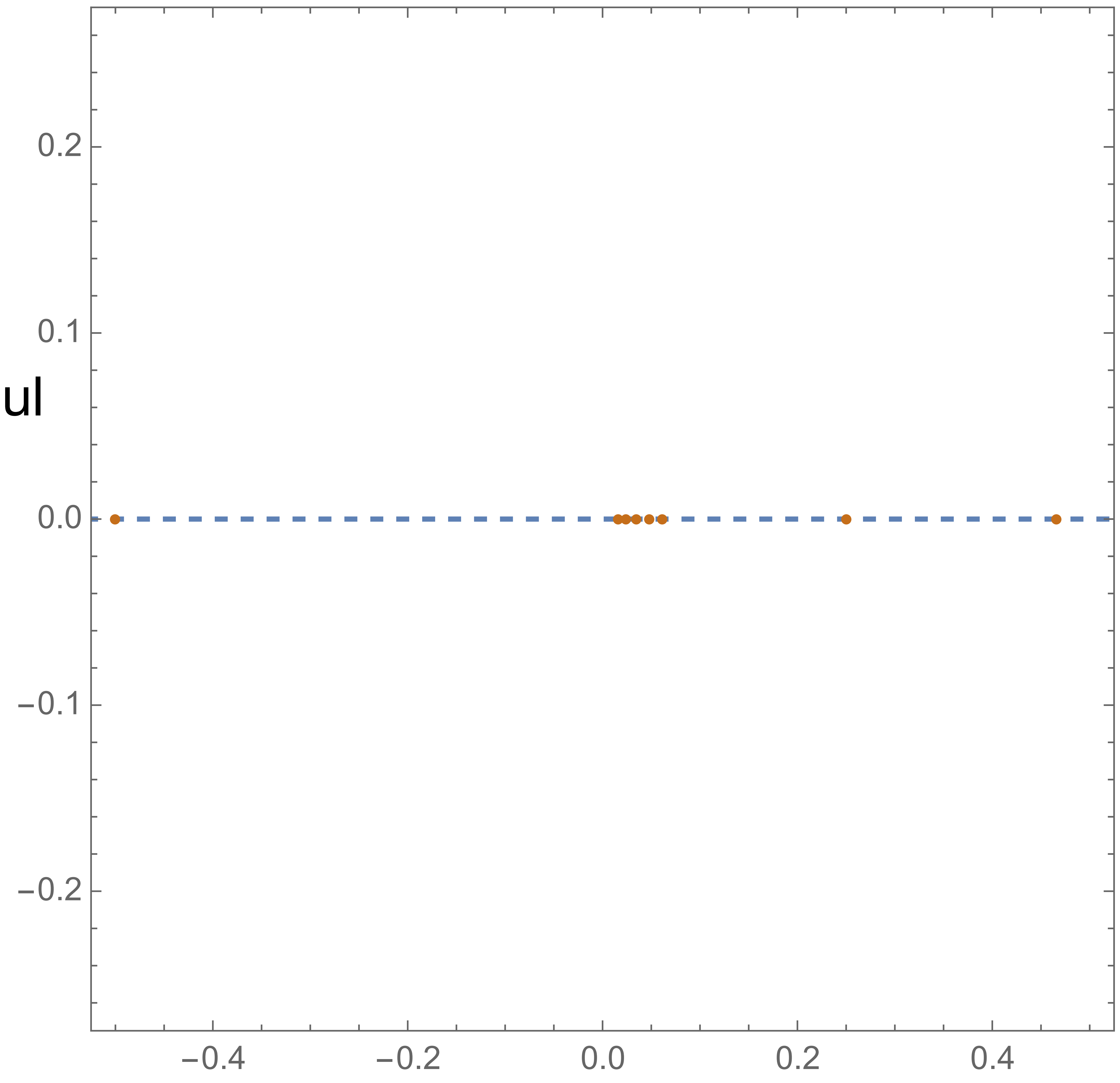
Vacua on the boundary

$$\phi(t) = -\frac{\eta(2t+1)^6\eta(6t+3)^6}{\eta\left(t+\frac{1}{2}\right)^6\eta\left(3t+\frac{3}{2}\right)^6}$$



Hauptmodul

$\Longrightarrow$



CP invariance of the CY landscape

Theta angle in effective action: $S_4 \supset \int \sqrt{g} d^4x \, \text{Re}(\mathfrak{t}) F_{\mu\nu} \tilde{F}^{\mu\nu}$

CP invariance $\implies$ only certain rational values allowed for Re$(\mathfrak{t})$

- N=2 rigid supergravity theories have theta-angle Re$(\mathfrak{t}) = 0, \frac{1}{2}$ [Cecotti, Vafa '18]
- Vacua of Hulek-Verrill fourfold below tadpole bound: Re$(\mathfrak{t}) = 0, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$

See also: [Bönisch, Elmi, Kashani-Poor, Klemm '22]

Underlying structure: inverse mirror map takes real values $\phi(\mathfrak{t}) \in \mathbb{R}$

Exponential corrections and symmetries

Supersymmetric Genericity Conjecture [Palti, Vafa, Weigand '20]

Whenever certain corrections are allowed by supersymmetry considerations in a given theory, the vanishing of these terms is due to some relation to a **higher-supersymmetric** theory

How does this fit with our flux landscape?

- Presence of discrete symmetries $\Longleftrightarrow$ polynomial K3 periods
- A third avatar: **density** of the flux vacua in moduli space [Grimm, DvdH '24]

Mathematical underpinnings

Algebraicity of Hodge loci: [Cattani, Deligne, Kaplan '95]

Locus of vacua with $W = 0$ must be algebraic in moduli space

- This algebraicity appears in the algebraic coordinates ϕ^I on the moduli space
- We observe something stronger: algebraicity in mirror coordinates $\mathfrak{t}^I(\phi)$

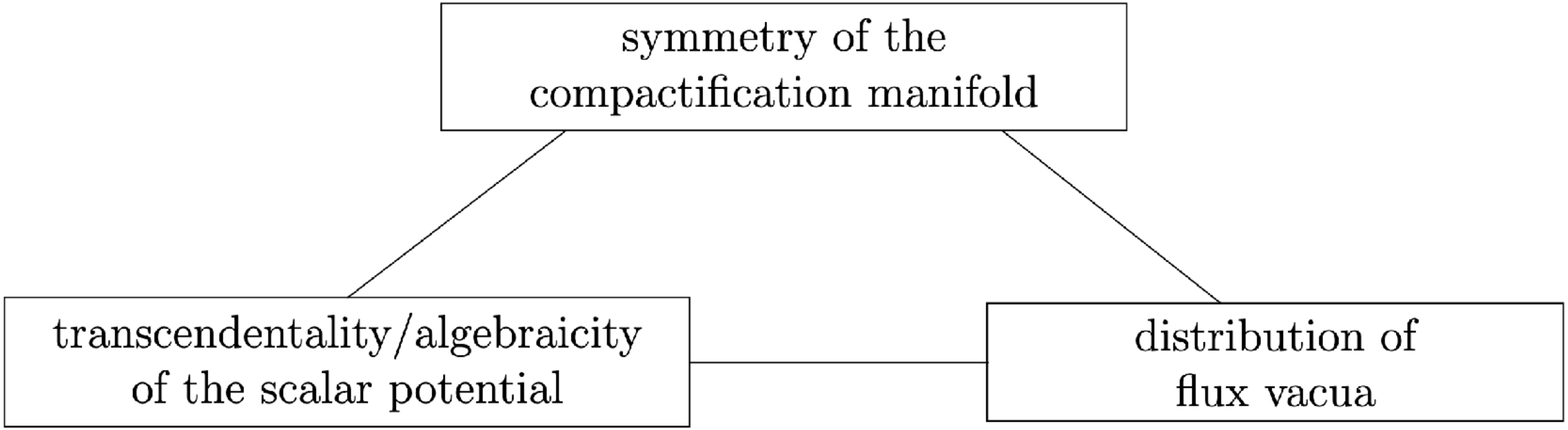
Distribution of Hodge loci: [Baldi, Klingler, Ullmo '21]

Dense vacua must lie on a higher-symmetry locus in moduli space with $\ell = 1,2$

- Crucial measure of transcendentality: level ℓ of the Hodge structure

  $\ell = 1$: elliptic curves, K3; $\ell \geq 3$: Calabi-Yau threefolds and higher

Conclusions



Thank you for your attention!