The Ferroaxionic Force

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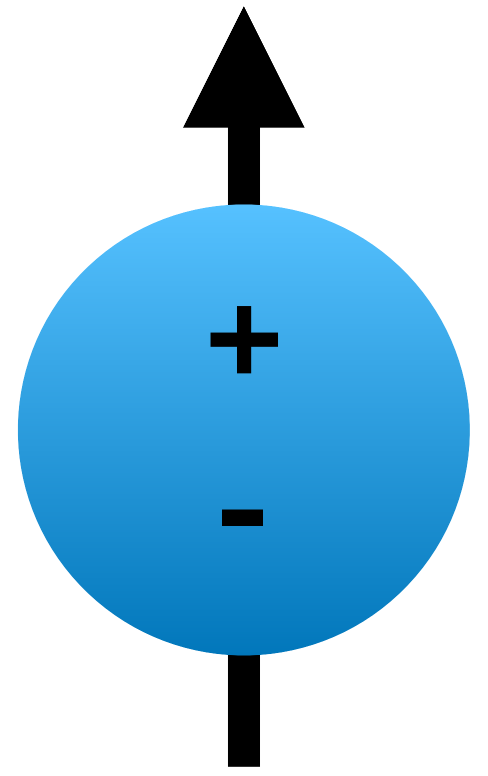
Based on work w/ A. Arvanitaki, J. Engel, A. Geraci, A. Hepburn and K. Van Tilburg

The Strong CP Problem



- $\mathscr{L} \supset \frac{\theta_{QCD}}{32\pi^2} G\tilde{G}$
- Generates a neutron electric dipole moment (EDM) $d_n \simeq e\,\theta_{QCD} \times 10^{-18}\,\mathrm{m}$
- Experimental bound: $\theta_{QCD} < 10^{-10}$
- $d_n = 0$ would mean that both parity (P) and time reversal (T) symmetries are preserved by QCD
- The electroweak sector violates P and T, so why not QCD?

The QCD Axion

- $$\mathscr{L} \supset \frac{a(x)}{32\pi^2 f_a}\tilde{G}G$$

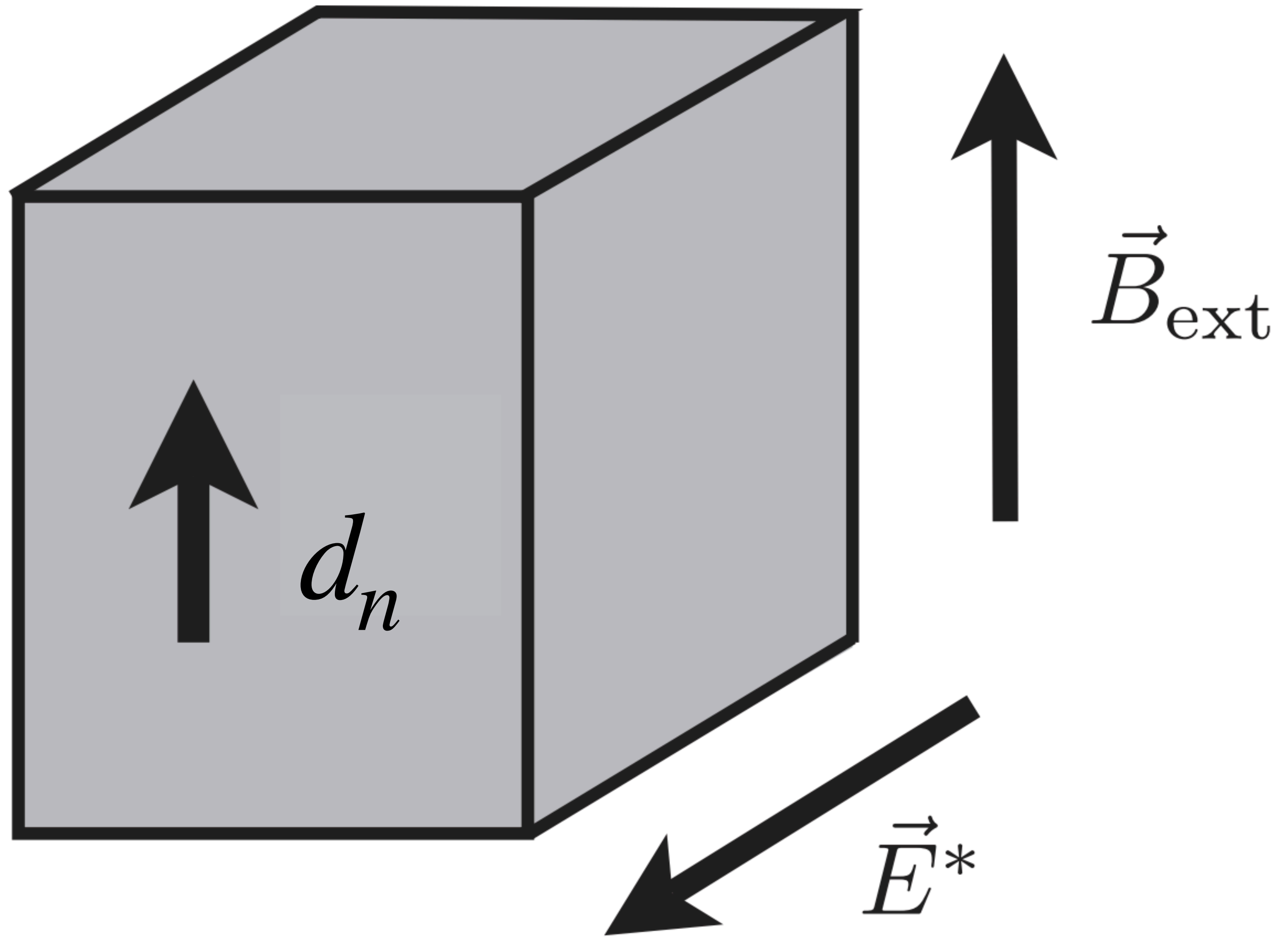
  (f_a = axion decay constant)

- The axion field *dynamically* solves the strong CP problem.

- QCD predicts a mass for the axion of $m_a \sim 6 \times 10^{-11} eV \left(\frac{10^{17} GeV}{f_a}\right)$

- Can be the dark matter and can mediate new forces

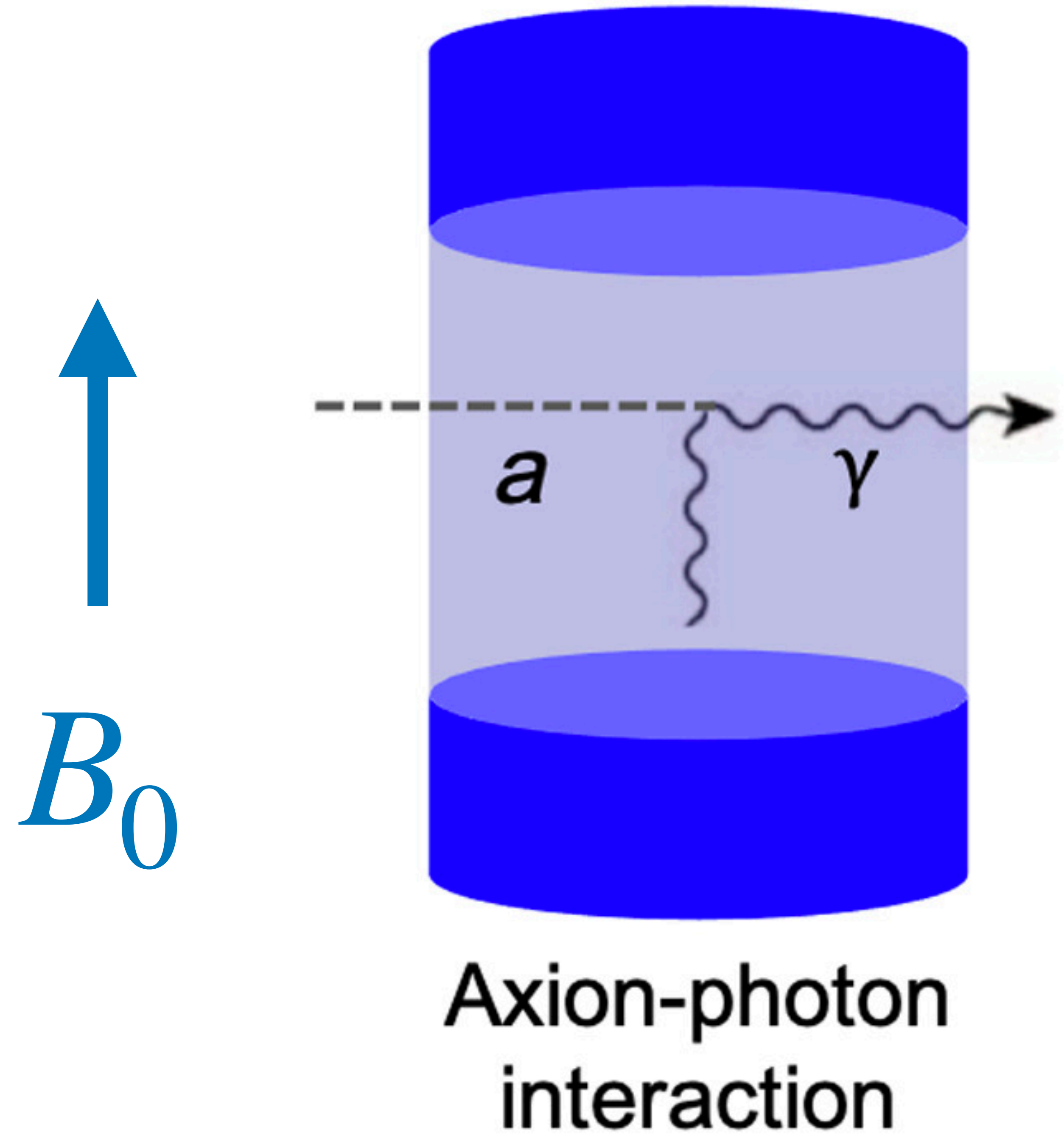
Gluon Couplings: $\mathscr{L} \supset \frac{\alpha_s^2}{32\pi^2}\frac{a}{f_a}G\tilde{G}$



$$\tau = d_n(t) \times E^*$$

CASPER-electric

Photon Couplings: $\mathscr{L} \supset -\frac{g_{a\gamma\gamma}}{4} aF\tilde{F}$



<u>ADMX</u>, DM Radio, CAST, IAXO (solar), ALPS…

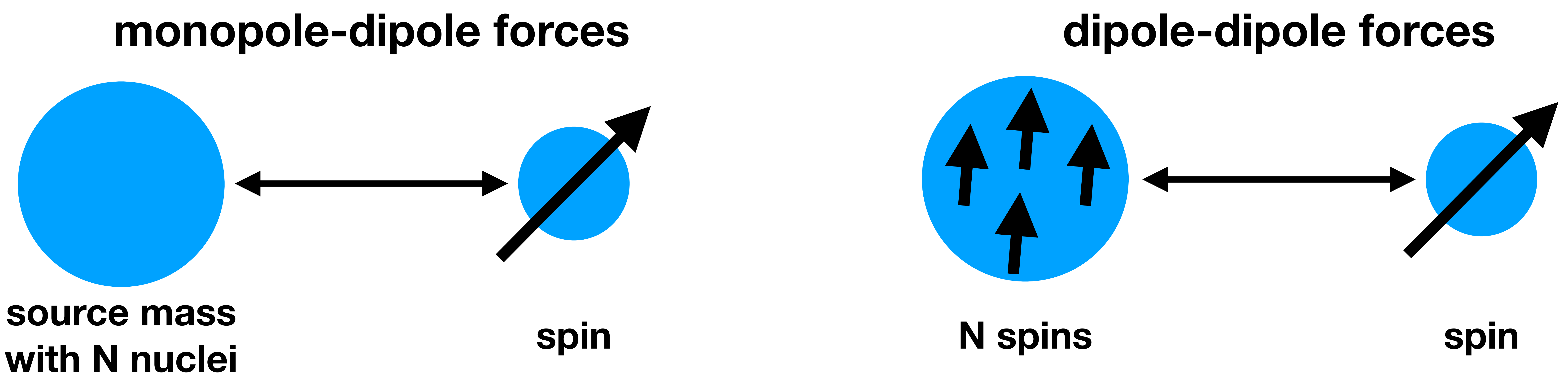
Image: Semertzidis et al. 2022

Fermion Couplings:

$$-g_s a \bar{\psi} \psi + \frac{g_p}{2m_\psi} \partial_\mu a \bar{\psi} \gamma^\mu \gamma_5 \psi$$

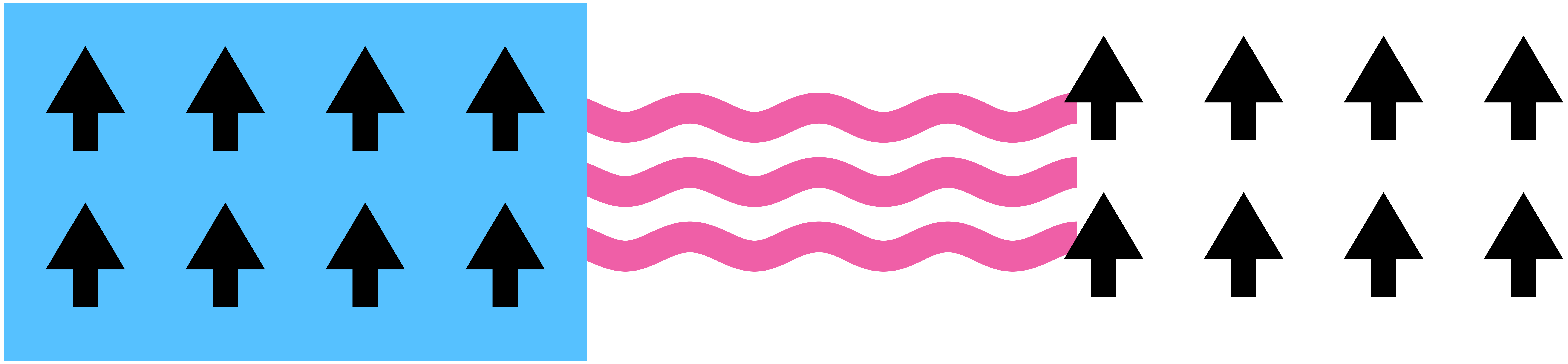
NR limit

$$\frac{g_p}{2m_\psi} \sigma_\psi \cdot \left[\nabla a + \dot{a} \frac{\mathbf{p}_\psi}{m_\psi} \right]$$



<u>ARIADNE,</u> CASPEr-wind, QUAX

The Ferroaxionic Force

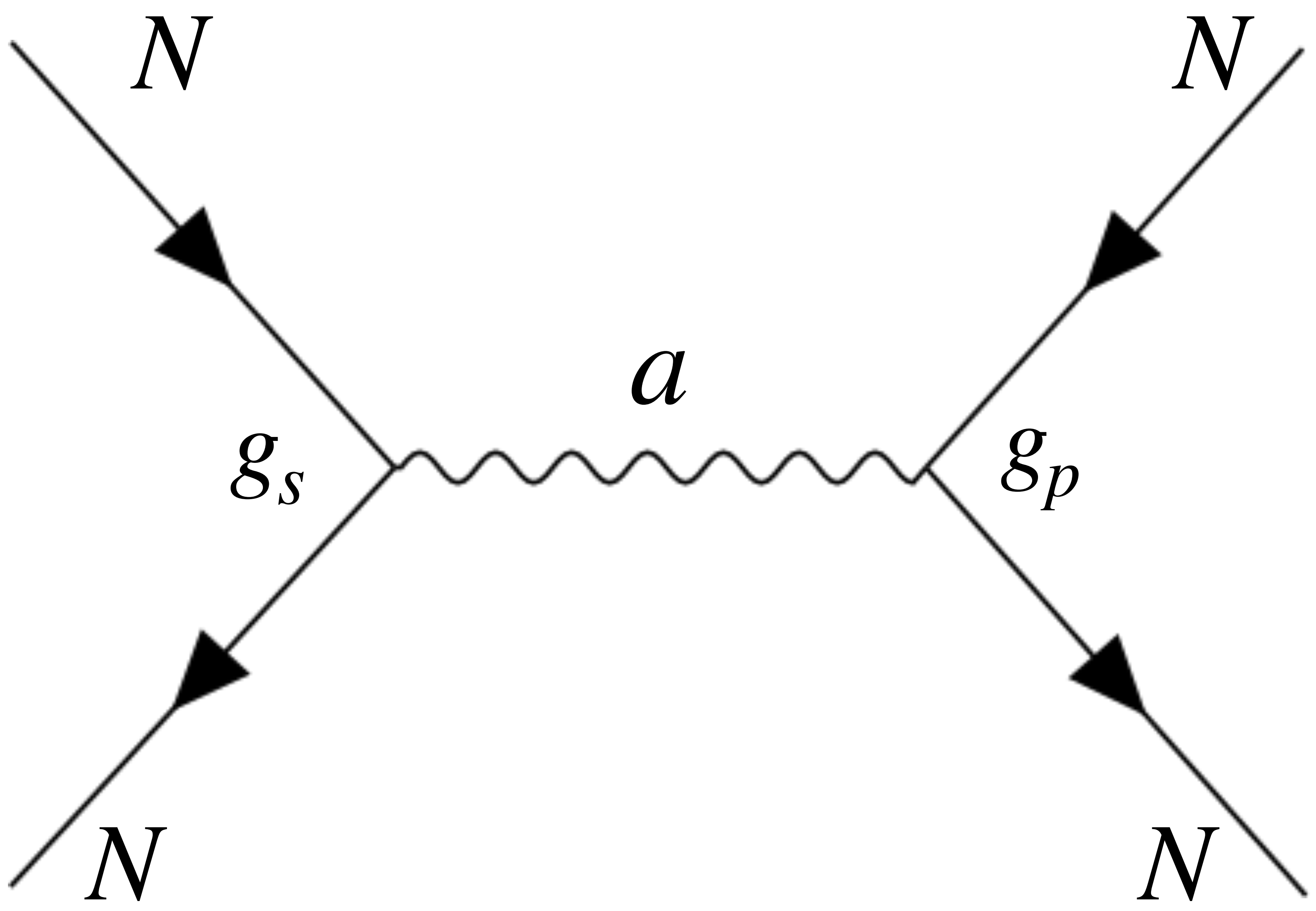


$$\mathscr{L} \supset \frac{a}{f_a} G\tilde{G}$$

$$\mathscr{L} \supset \frac{g_p}{2m_N} \partial_\mu a \bar{N} \gamma^\mu \gamma_5 N$$

Axion-mediated forces

$$\mathscr{L} \supset \frac{(\partial a)^2}{2} - \frac{m_a^2 a^2}{2} - g_s a \bar{N} N + \frac{g_p}{2m_N} \partial_\mu a \bar{N} \gamma^\mu \gamma_5 N$$



$$U_{sp} = \frac{g_s g_p}{8\pi m_N} \left(\frac{m_a}{r} + \frac{1}{r^2} \right) e^{-r\,m_a} (\hat{\sigma} \cdot \hat{r})$$

Axion-mediated forces

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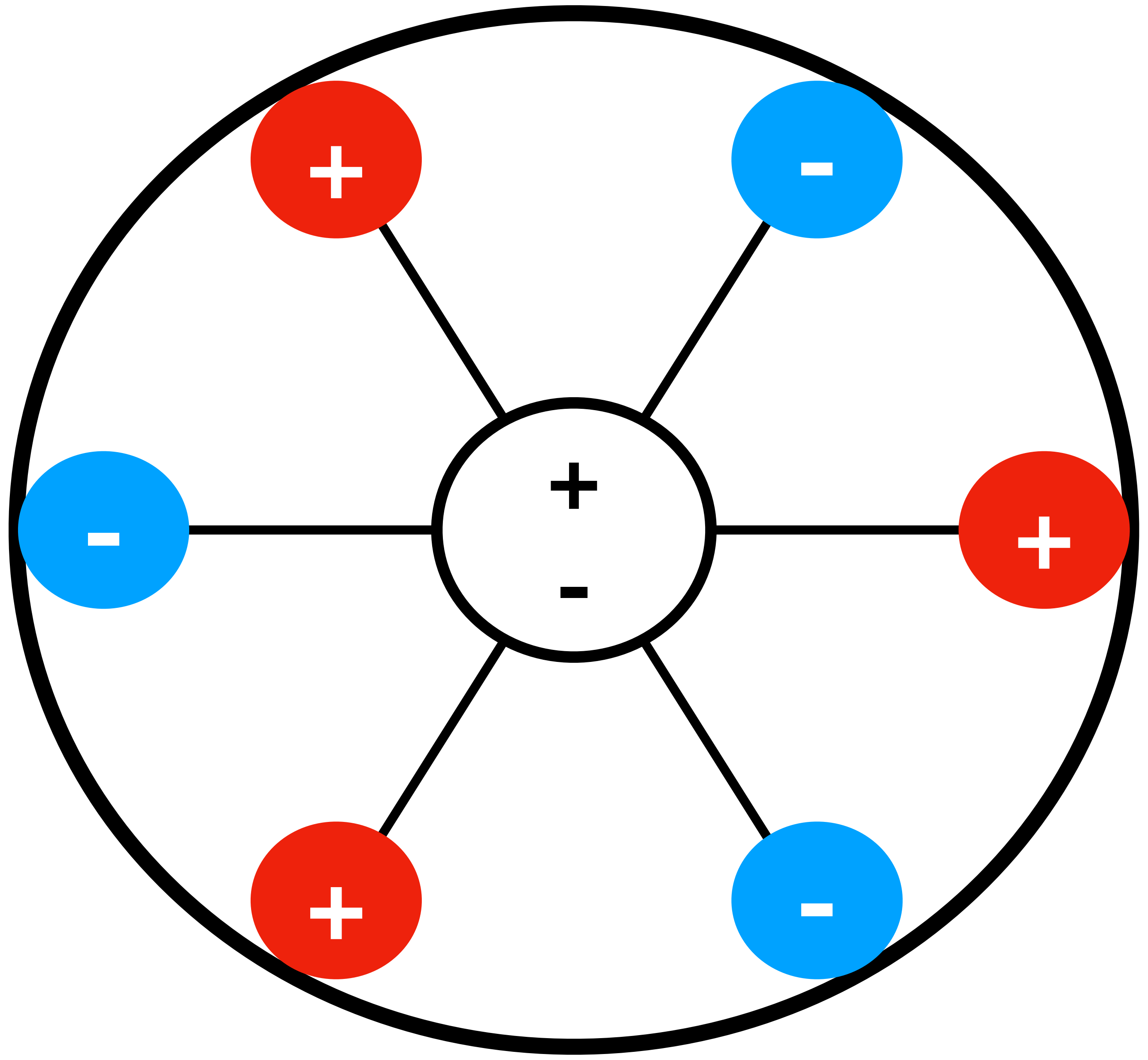
P & T odd　　P & T even

$$g_s \sim 10^{-30} \frac{10^9\, GeV}{f_a}$$ (from CKM)

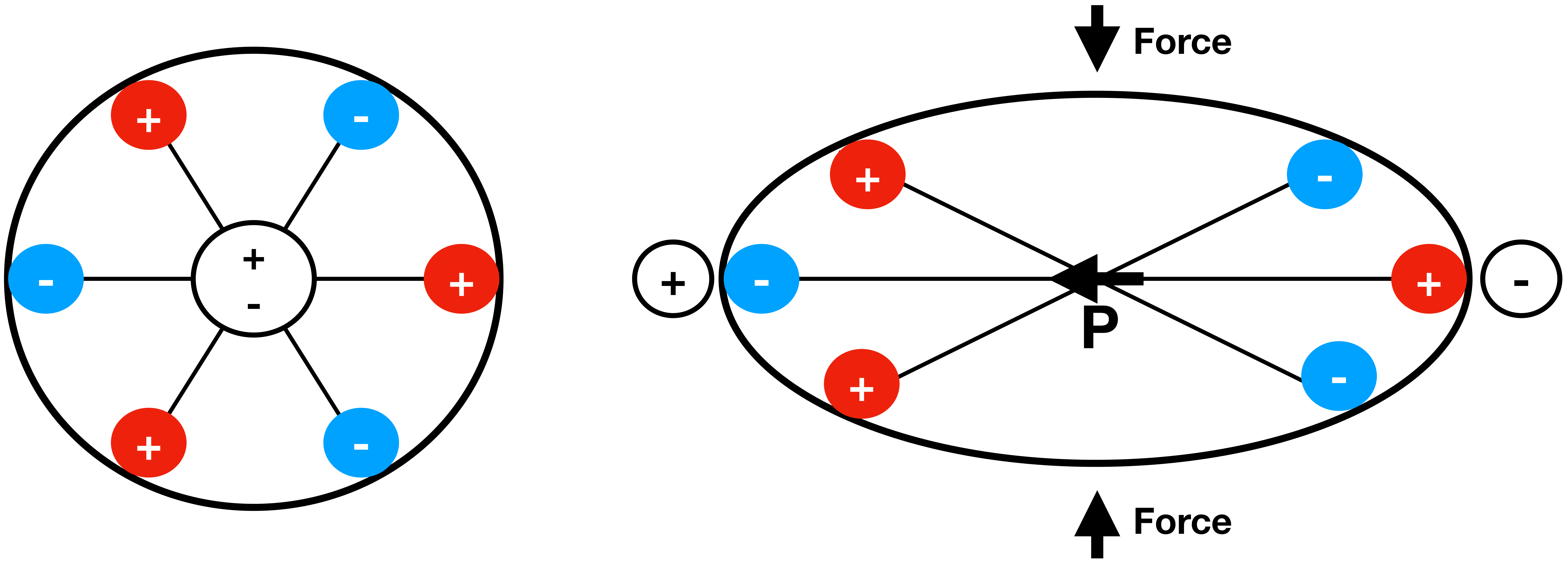
Idea: what if P and T violation comes from a piezoelectric crystal with polarised nuclear spins?

Piezoelectric Crystals



- Crystal structure breaks parity symmetry $(x, y, z) \neq (-x, -y, -z)$

Piezoelectric Crystals



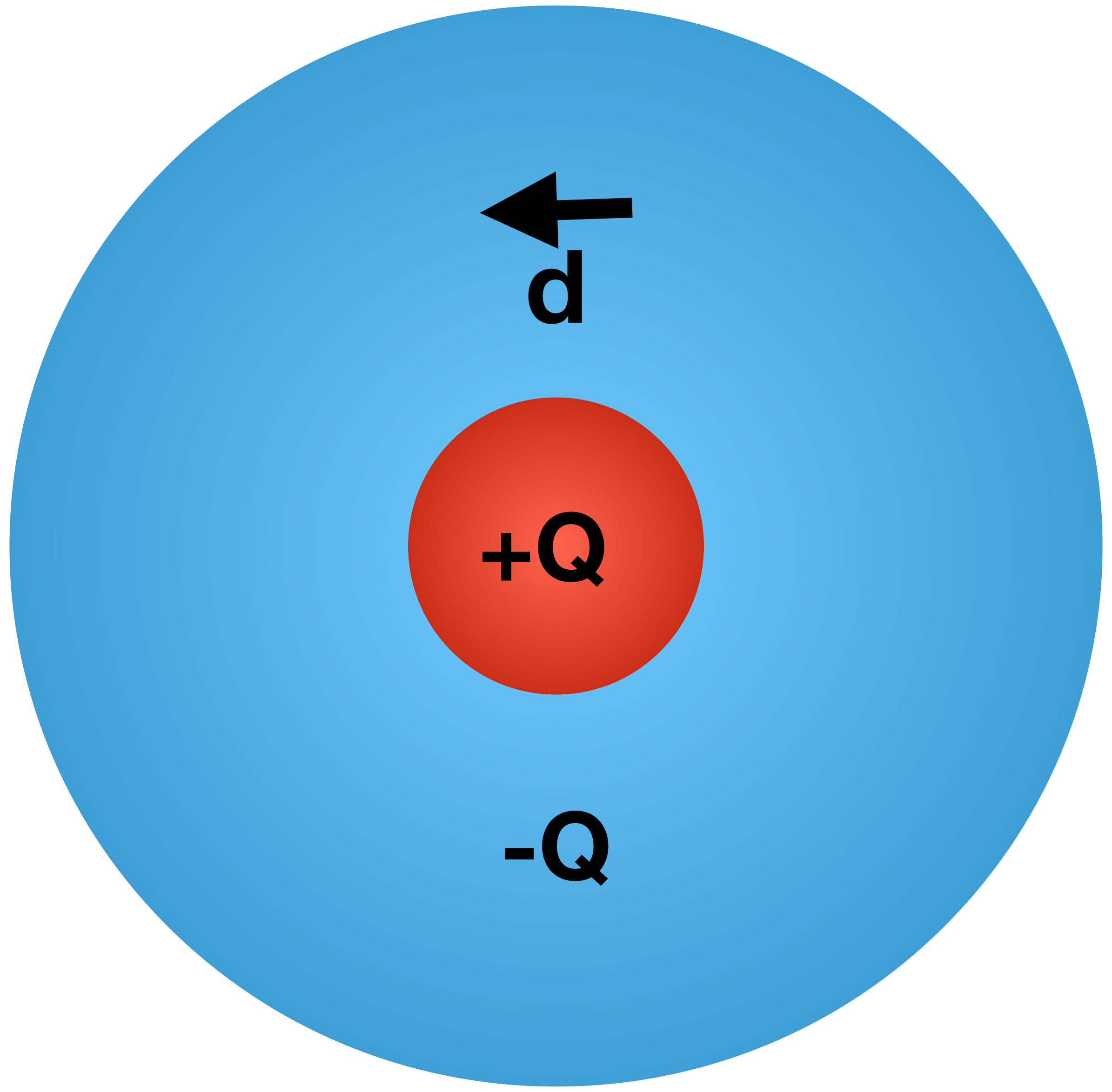
- Crystal structure breaks parity symmetry $(x, y, z) \neq (-x, -y, -z)$

- Deformation causes electric dipole moment across unit cell (and vice versa).

How does the axion couple to a piezoelectric material?

- Expectation: a coupling that looks like a nuclear EDM interacting with crystal electrons…

Schiff's theorem: If we treat an atom as a system of **static, point-like** particles, nuclear EDM is shielded by electron cloud [*Schiff* 1963].



Resolution: finite size effects

Electrostatic (scalar) potential:
Nuclear Schiff Moment

Magnetic (vector) potential:
Nuclear Magnetic Quadrupole Moment

Schiff Moment

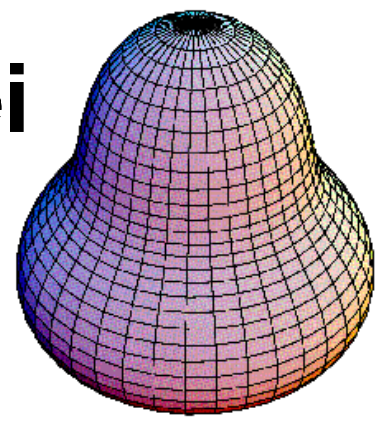
$$V_e = 4\pi e\, \mathsf{S} \cdot \nabla(\delta_e(\mathbf{r}))$$

$$\mathsf{S} \sim e\frac{\bar{\theta}_a}{m_N}R_0^2 \propto A^{2/3}$$

non-deformed nuclei

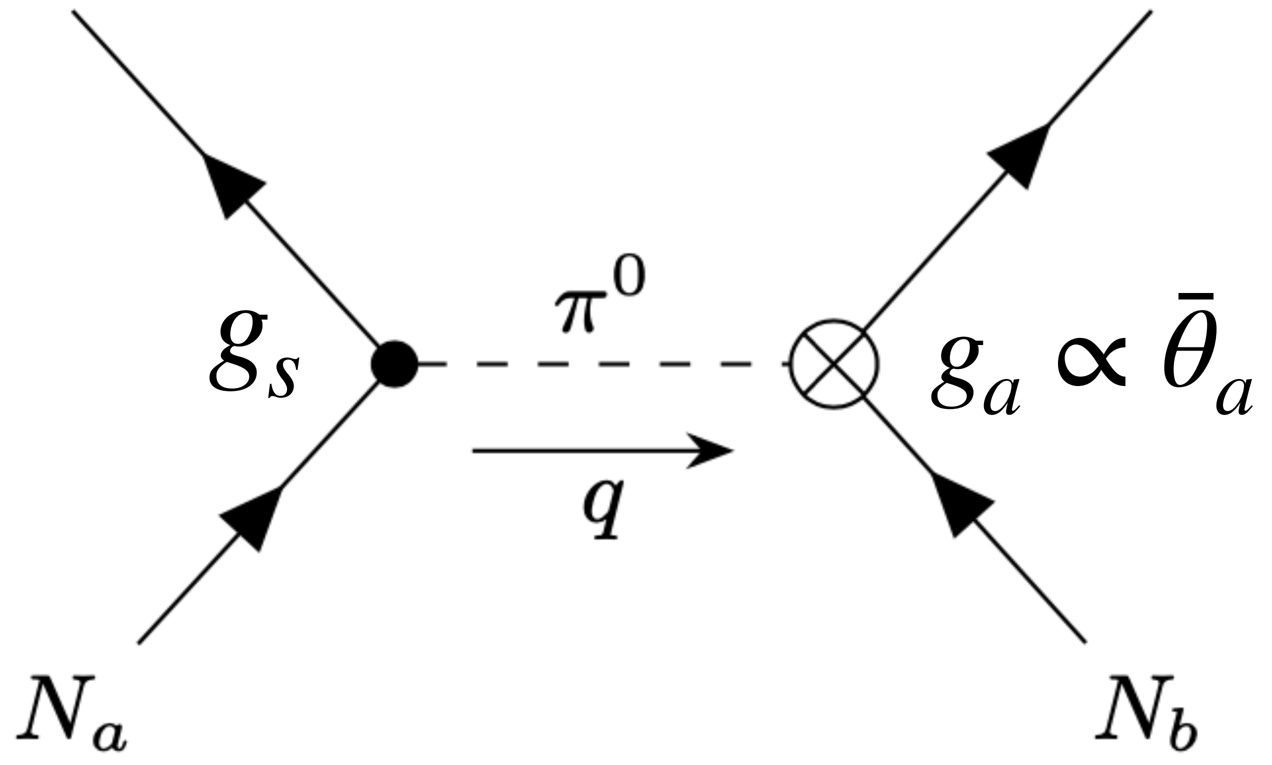
$$\mathsf{S} \sim eZ\frac{\bar{\theta}_a}{m_N}R_0^2 \propto Z\, A^{2/3}$$

pear shaped nuclei



$$\sim (0.01 - 1) \times \bar{\theta}_a\, e\, fm^3$$

Pion-nucleon forces:



$$\left(\bar{\theta}_a \equiv \frac{a}{f_a}\right)$$

Schiff Moment

- In a piezoelectric crystal, the ground state electron wave function is a mixture of opposite parity orbitals ϵ_s and ϵ_p:

$$|\psi\rangle_e = \epsilon_s |s\rangle + \epsilon_p |p\rangle$$

$$\rho_S = n_S \frac{4\pi e}{f_a} \frac{\partial S}{\partial \theta_a} \epsilon_s \epsilon_p^* \mathcal{M}_S \cdot \hat{\mathbf{I}} + c.c$$

- **Effective in-medium energy density** (ρ_S)
- **Number density of Schiff moments** (n_S)
- **Schiff moment** ($\partial S / \partial \theta_a$)
- $\epsilon_s \epsilon_p^*$: **~O(1)**
- **Electronic matrix element** ($\mathcal{M}_S$): $\neq 0$ **in a ferroelectric crystal**
- **Nuclear spin polarization** ($\hat{\mathbf{I}}$)

Magnetic Quadrupole Moment

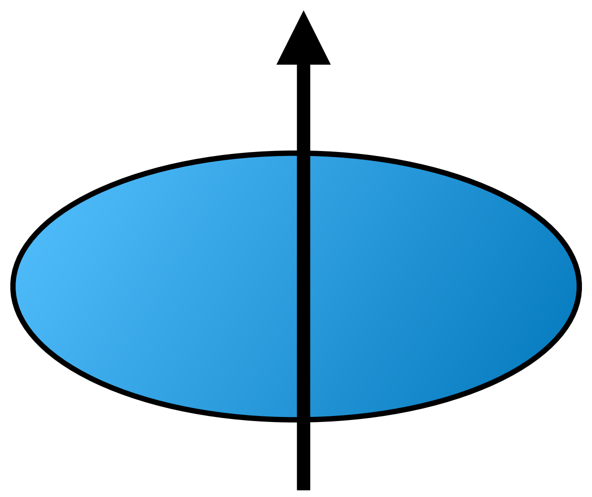
$$V_{\mathsf{M}} = \frac{e\,\mathsf{M}}{4I(2I-1)}\left[I_m I_n + I_n I_m - \frac{2}{3}\delta_{mn} I(I+1)\right] \times t_{mn}(\sigma_e, \hat{r}_e)$$

$$\mathsf{M} \sim 10 \frac{\bar{\theta}_a}{m_N} \mu_N$$

non-deformed nuclei

$$\mathsf{M} \sim Z^{2/3}\, 10 \frac{\bar{\theta}_a}{m_N} \mu_N$$

rugby-ball shaped nuclei



$$\sim (0.1 - 1) \times \bar{\theta}_a\, e\, fm^2$$

$$\mu_n = \frac{e}{2m_p}$$ **= nuclear magnetic moment**

MQM: Atomic Mixing

$$|\psi\rangle_e = \epsilon_s |s\rangle + \epsilon_p |p\rangle$$

MQM

Density of MQM nuclei

Electronic matrix element

$$\rho_\mathrm{M} = e\, n_\mathrm{M} \frac{\partial \mathrm{M}}{\partial \theta_a} t_{mn}(I_N) \times \epsilon_s \epsilon_p^* A_{mn}(\sigma_e, \hat{r}_e) \quad + c.c.$$

$\neq 0$ in a magnetic piezoelectric crystal

Effective in-medium energy density

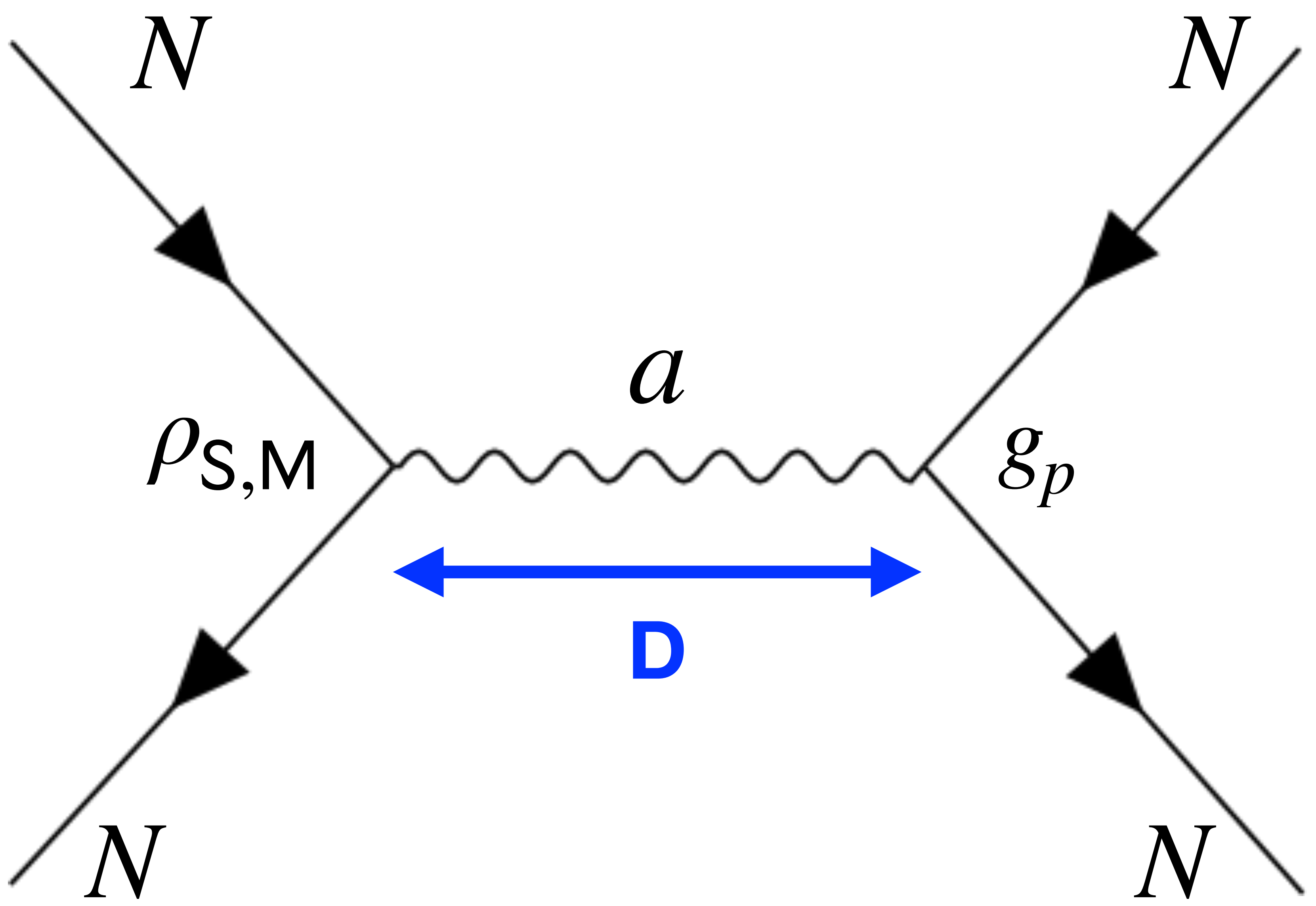
Nuclear quadrupole tensor

Take a uniform slab of material with a big transverse area:

$$\left(\Box + m_a^2\right) a\,(t, \mathbf{x}) = -\frac{\rho_{\mathsf{S}} + \rho_{\mathsf{M}}}{f_a} \equiv j(t, \mathbf{x})$$

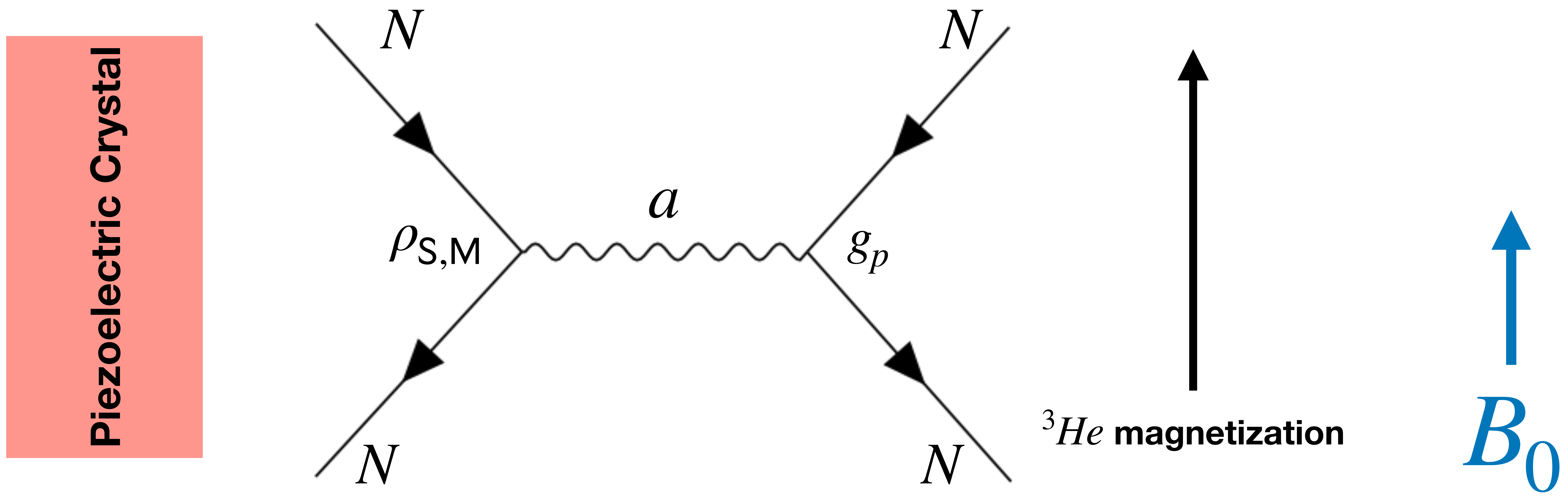
$$\nabla \bar{\theta}_a \simeq -\hat{\mathbf{D}} \frac{j}{2 m_a f_a} e^{-m_a D}$$

$$H \supset -\frac{g_p}{m_N} \sigma_N \cdot \nabla \bar{\theta}_a, \qquad g_p \equiv \frac{c_N m_N}{f_a}$$



Like a B-field, but unaffected by magnetic shielding!

Nuclear Magnetic Resonance

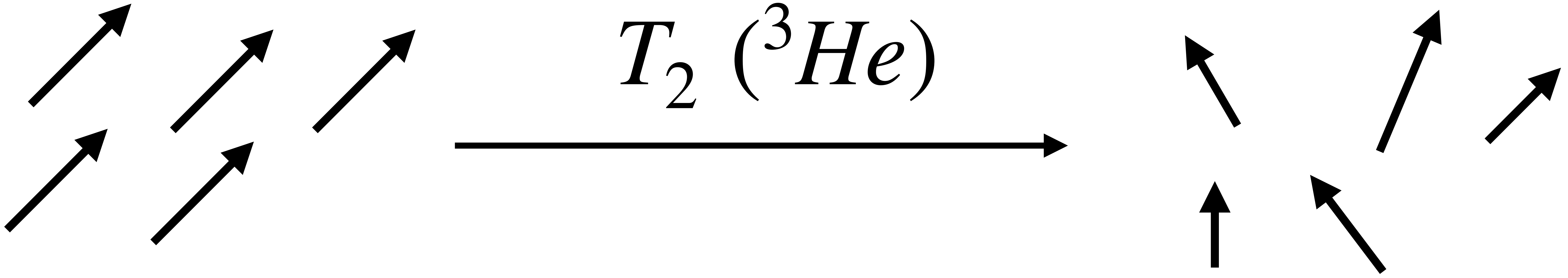


By moving the crystal at the Larmor frequency $\omega = -\gamma_N B_0$, we pick up a resonantly enhanced, off-axis magnetization

The separation between the crystal and NMR sample sets the range of axion masses that we are sensitive to.

Noise

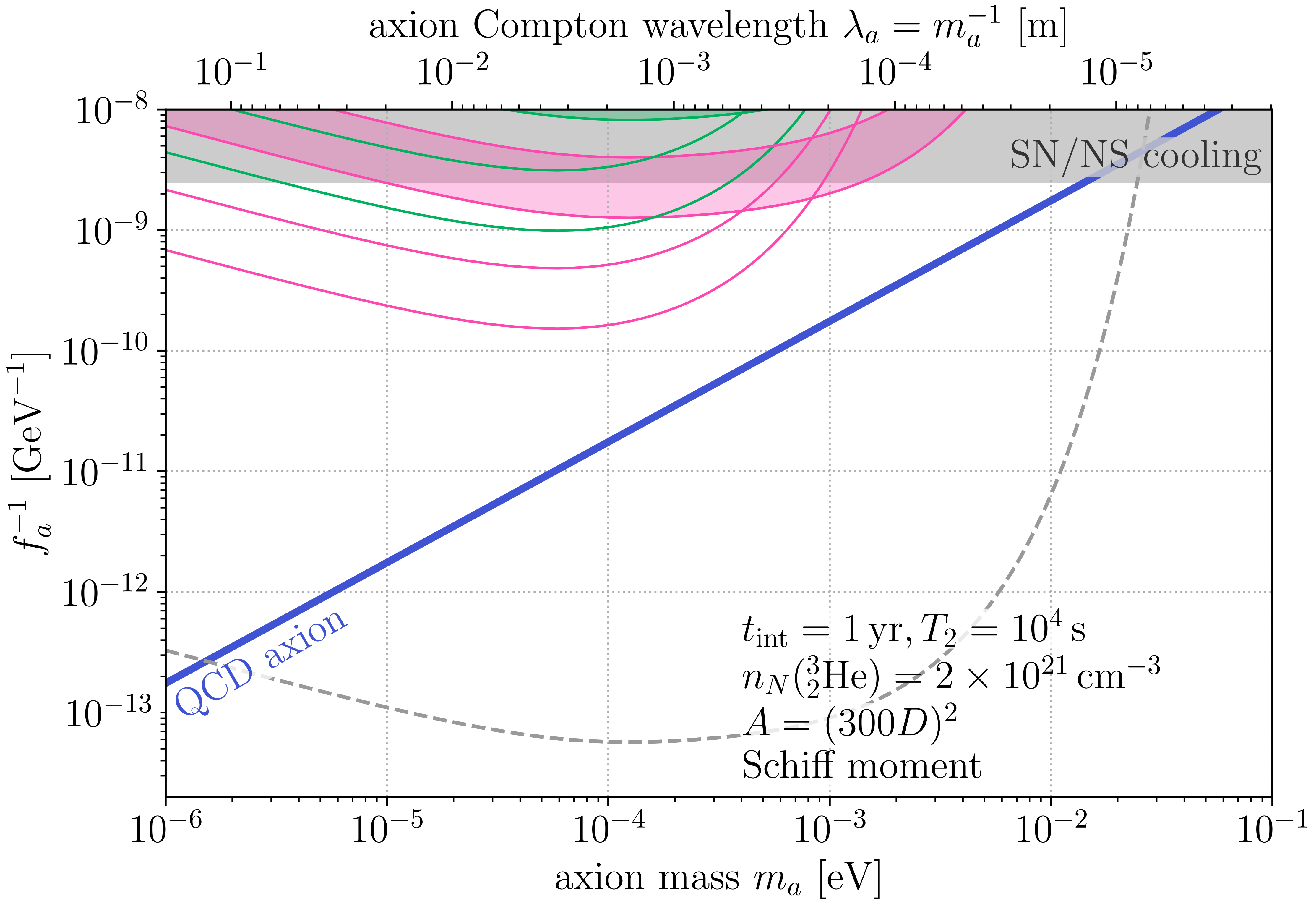
The main noise source is transverse spin projection noise from the sample itself



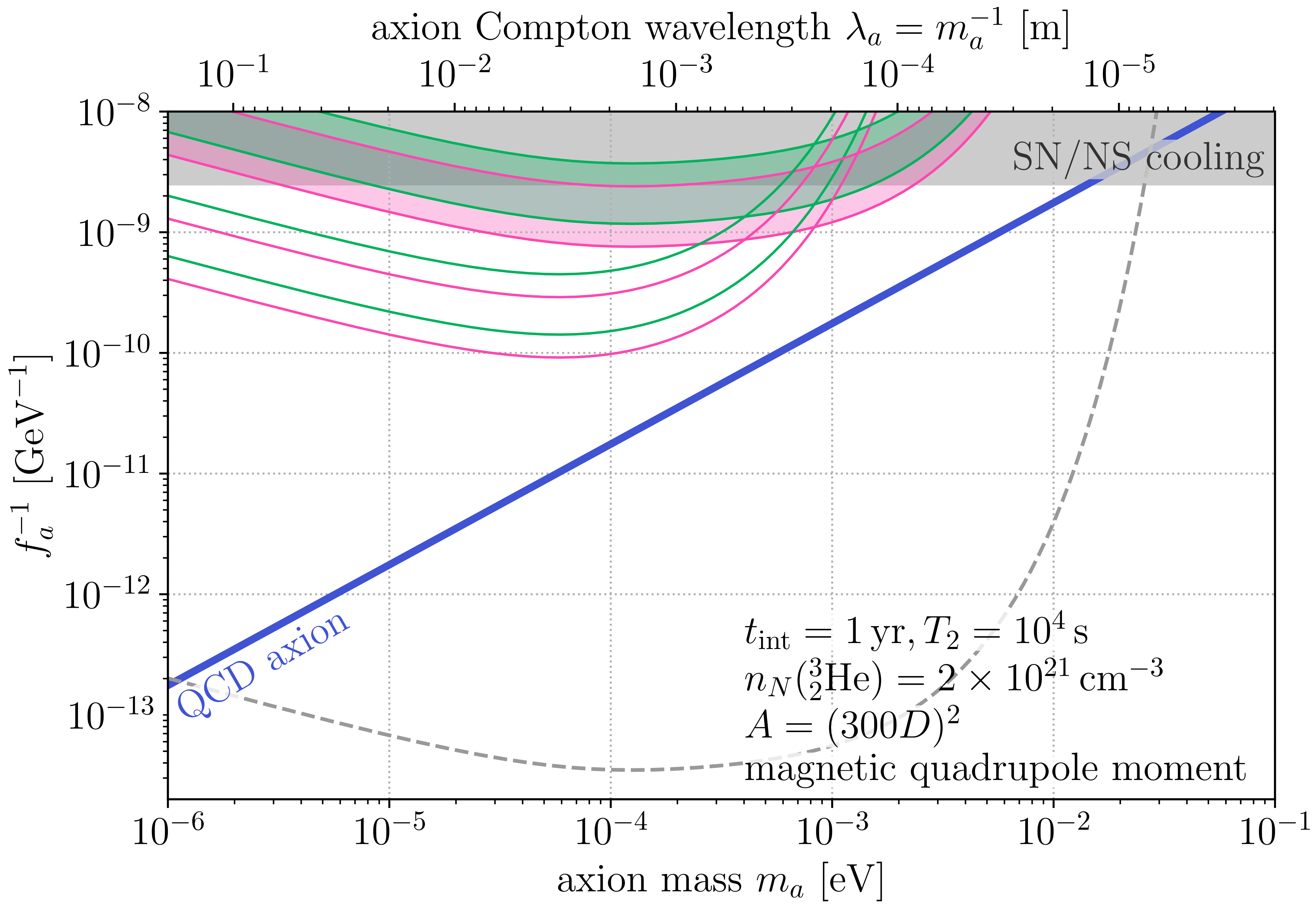
$$B_{\min} \approx 10^{-20}\ \mathrm{T} \times \sqrt{\left(\frac{b}{1\ \mathrm{Hz}}\right)\left(\frac{1\ mm^3}{V_{^3He}}\right)\left(\frac{10^{22}\ cm^{-3}}{n_{^3He}}\right)\left(\frac{1000\ s}{T_{2_{^3He}}}\right)}$$

$NpIO_5$ $Eu_{0.5}Ba_{0.5}TiO_3$

Schiff moment:

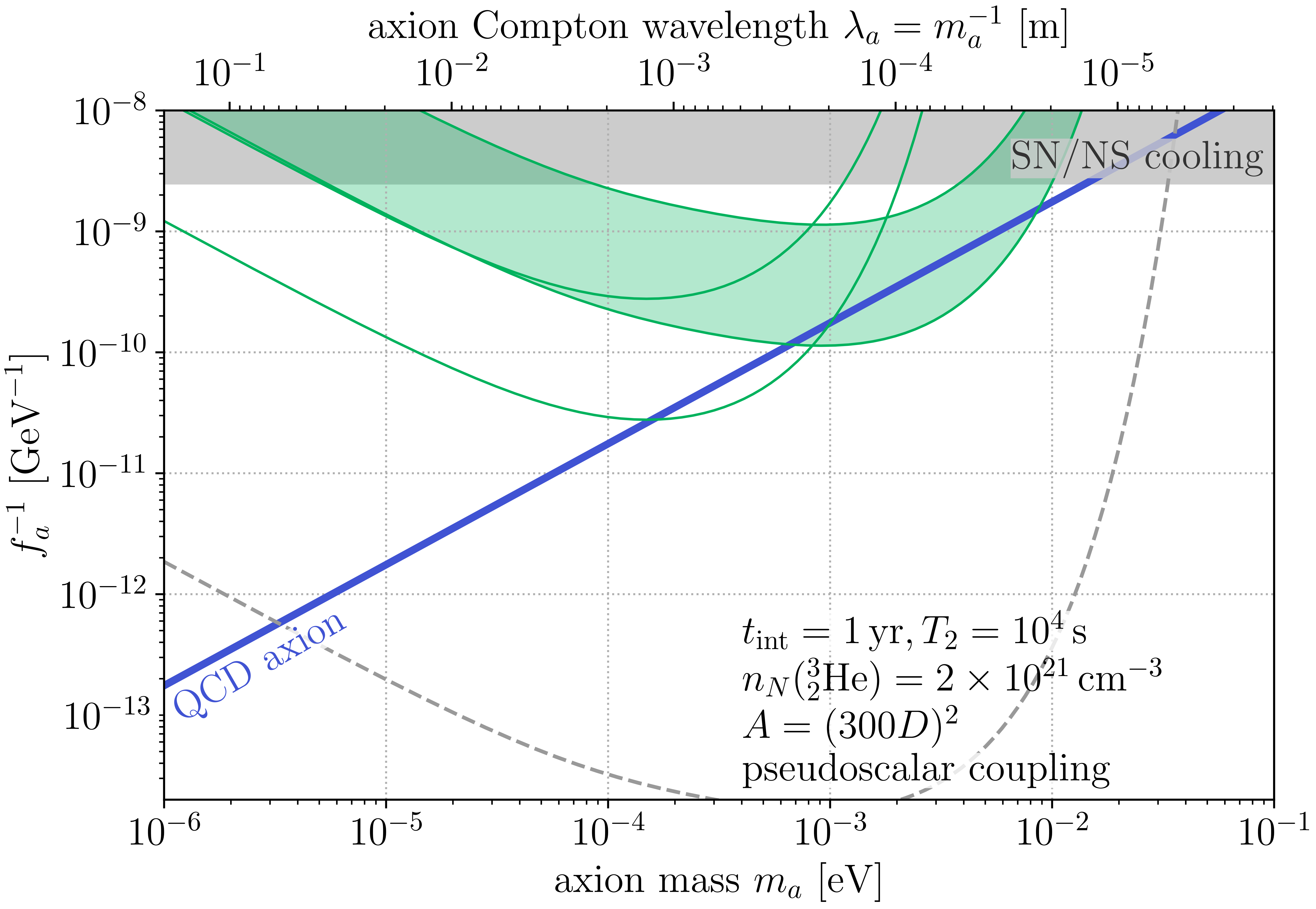


MQM:



$$“g_s” = \frac{\rho_S + \rho_M}{f_a n_N} \lesssim 10^7 \times g_s^{CKM}$$

Dipole-Dipole ($g_p \times g_p$)

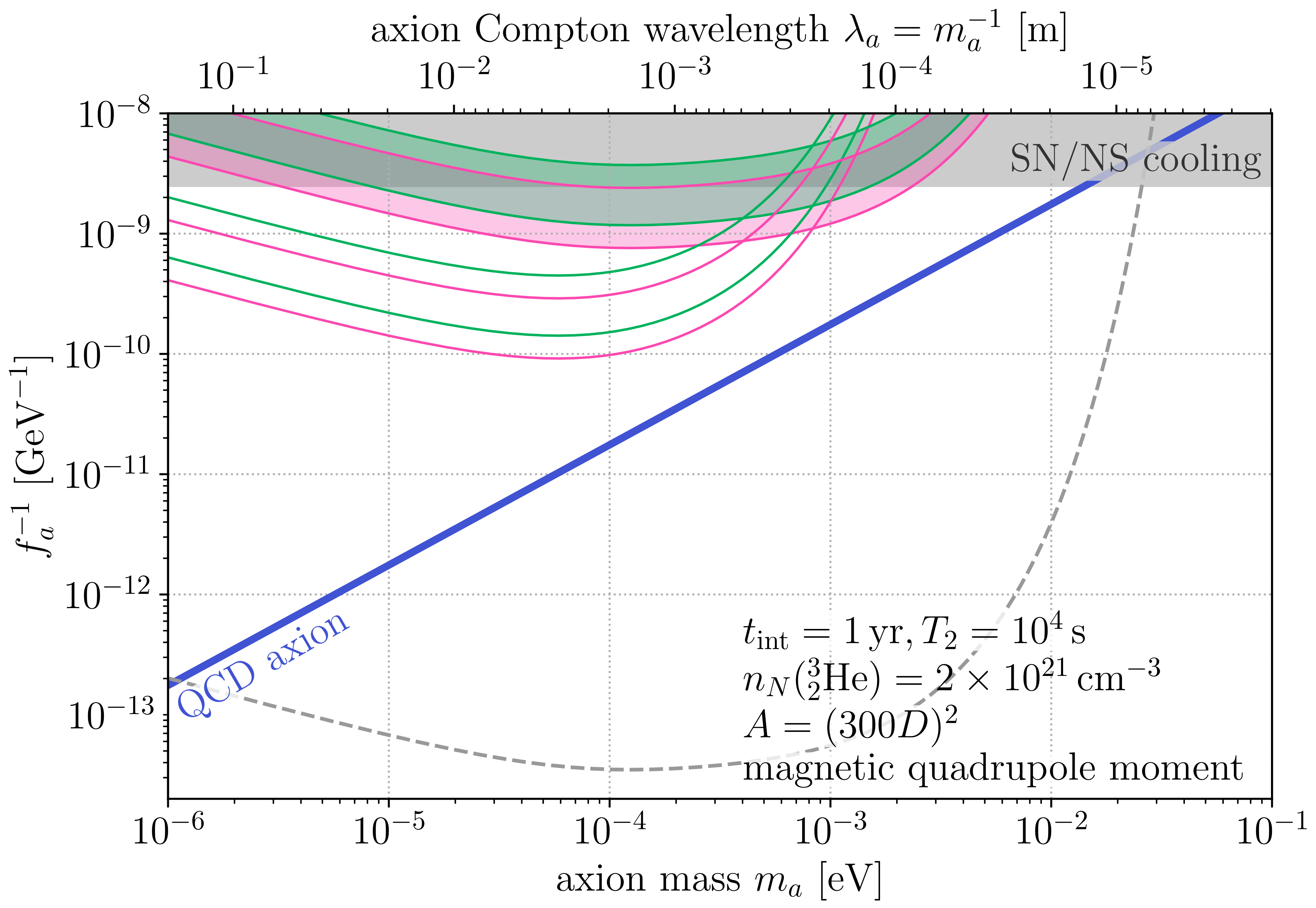


What's next?

- In progress: precise Schiff/MQM calculations for stable, deformed nuclei
- Density functional theory (DFT) calculations of electron wave functions near nuclei
- Further experimental investigation of candidate materials
- Squeezing protocol to reach SQUID limited noise sensitivity

Summary

- Piezoelectric crystals can generate an in-medium scalar coupling of the QCD axion, via its model-independent gluon coupling, that is much larger than the scalar coupling in vacuum.



- By sourcing virtual axions, we generate a new force that can be searched for using nuclear magnetic resonance techniques.

- While this effect does not require the QCD axion to be the dark matter, it probes a mass range where it could be.

- Complimentary to cavity experiments and astrophysical probes