Searching for New Physics with Nuclear Lineshape Data

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based on work with

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E. Fuchs, Ekkehard Peik, Johannes Tiedau (PTB, LUH)

November 18, 2024

Outline

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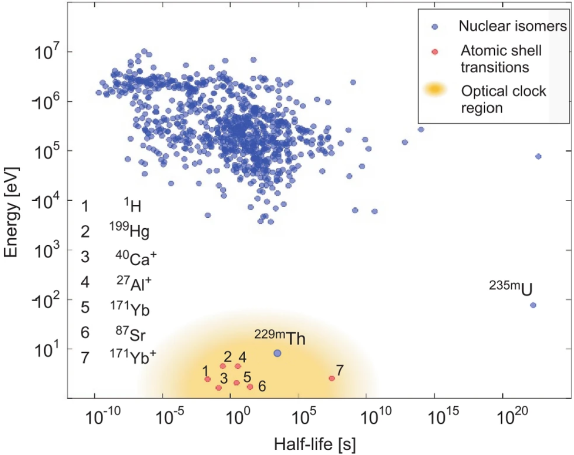
Fine-Tuning in Thorium-229?



Nuclear transition energy in ^{229}Th:

$$\Delta E = \Delta E_{\text{EM}} + \Delta E_{\text{nuc}}$$

$$\Delta E \ll |\Delta E_{\text{EM}}| \sim |\Delta E_{\text{nuc}}|$$

$$8 \text{ eV} \ll 0.1 \text{ MeV}$$

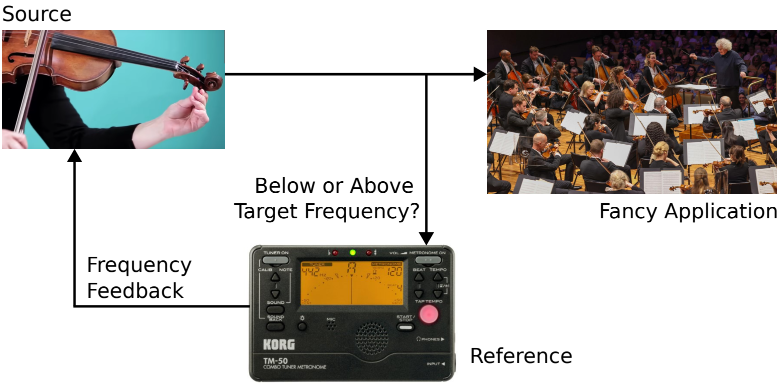
$\Rightarrow$ **Fine tuning?**

$\Rightarrow$ **Exceptionally sensitive probe of QCD at low energy?**

Progression of precision $\delta\nu/\nu$:

10^{-1} (2020), 10^{-3} (2022, ISOLDE), 10^{-6} (March 2024, PTB), 10^{-11} (June 2024, JILA)

What Is A Clock?



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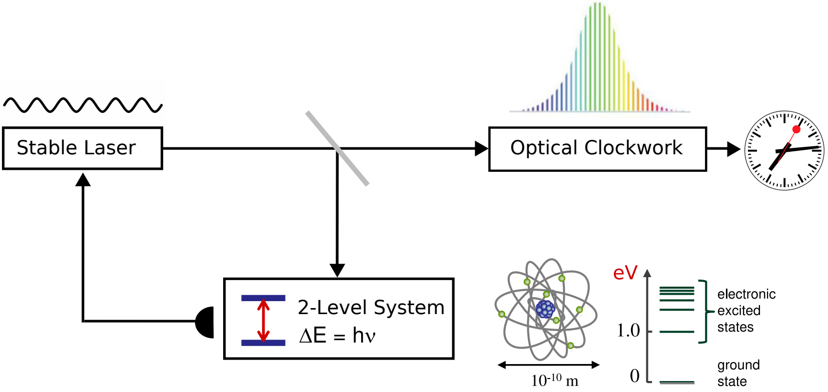
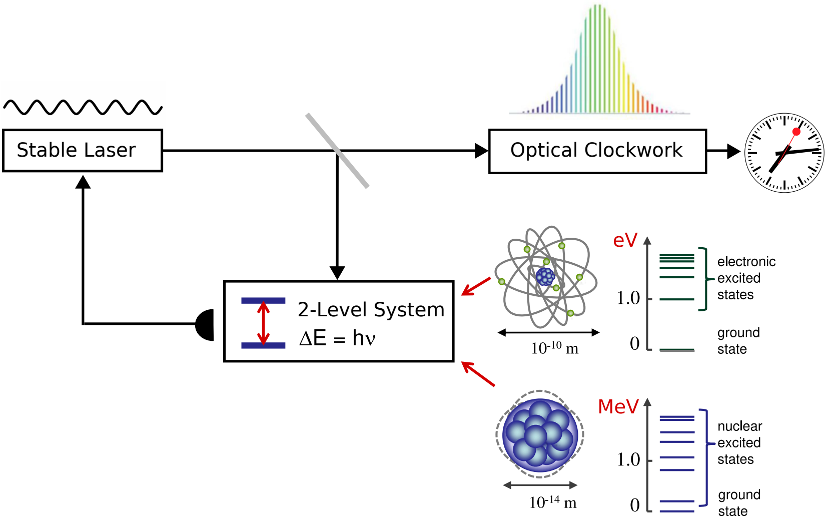


Towards A Nuclear Clock



Outline

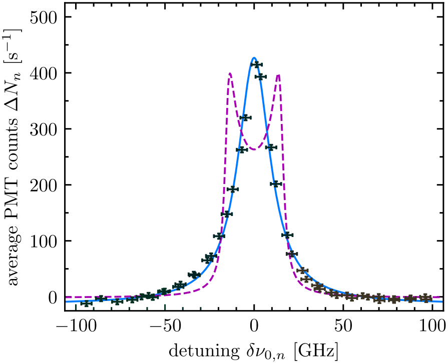
Which Lineshape?

Lineshape Search for New Physics

Lineshape Bounds

Nuclear Lineshape Analysis in the Limit $\delta\nu_{\mathrm{DM}} \gg \nu_{\mathrm{DM}}$

$$\nu(t) \simeq \nu_0 + \delta\nu_{\mathrm{DM}} \cos\left(2\pi\nu_{\mathrm{DM}} t + \varphi_{\mathrm{DM}}\right)$$



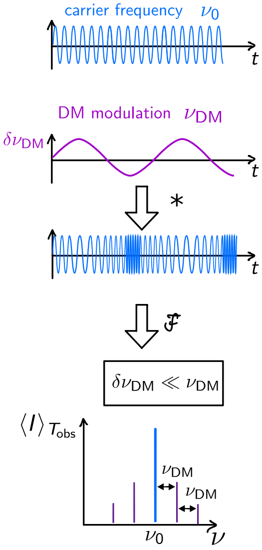
- In absence of DM,
  $I(\nu) = \delta(\nu - \nu_0)$
- In presence of DM,
  average over $T_{\mathrm{DM}} = 1/\nu_{\mathrm{DM}}$:

$$\langle I(\nu)\rangle_{T_{\mathrm{DM}}} = \int_0^{T_{\mathrm{DM}}} \frac{\mathrm{d}t}{T_{\mathrm{DM}}} \delta(\nu - \nu(t))$$

$$= \frac{\theta\left(1 - \left|\frac{\nu - \nu_0}{\delta\nu_{\mathrm{DM}}}\right|\right)/\pi}{\sqrt{\delta\nu_{\mathrm{DM}}^2 - (\nu - \nu_0)^2}}$$

$\Rightarrow$ Convolve with resonance lineshape

Sidebands in the Limit $\delta\nu_{\mathrm{DM}} \ll \nu_{\mathrm{DM}}$



Transition frequency:

$$\nu(t) = \exp\{-i2\pi\nu_0 t - i\alpha \sin(2\pi\nu \mathrm{DM} t)\}$$

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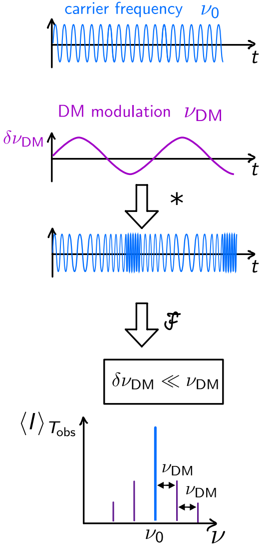

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$$= \sum_{n=-\infty}^{\infty} J_n(\alpha) \exp\{-2\pi i(\nu_0 + n\nu \mathrm{DM})t\},$$

J_n: n^{th} Bessel function, $\alpha = \frac{\delta\nu_{\mathrm{DM}}}{\nu_{\mathrm{DM}}}$ modulation index

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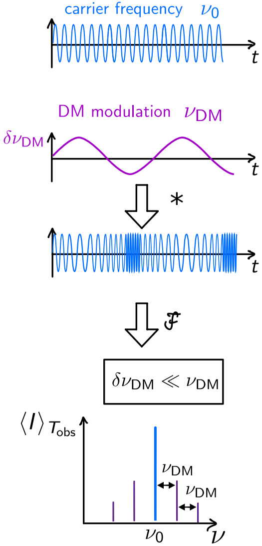

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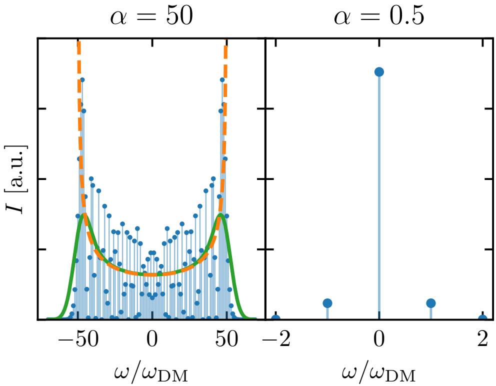
$\Rightarrow$ The lineshape is convoluted with

$$I(\nu) = \sum_{n=-\infty}^{\infty} |J_n(\alpha)|^2 \delta\left[\nu - (\nu_0 + n\nu_{\mathrm{DM}})\right]$$

Nuclear Lineshape Analysis Regimes

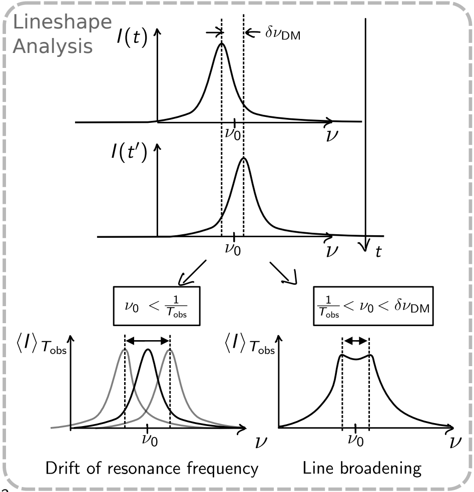
$$\alpha = \frac{\delta\nu_{\text{DM}}}{\nu_{\text{DM}}} \text{ modulation index}$$

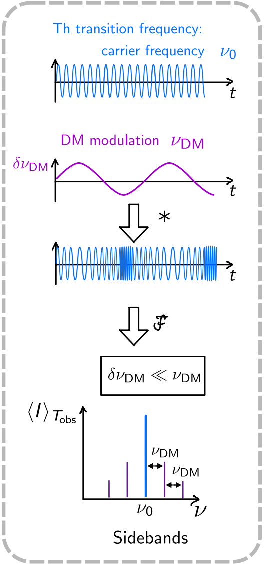
DM amplitude $\gg$ DM frequency | DM amplitude $\ll$ DM frequency



Nuclear Lineshape Analysis Regimes: Future

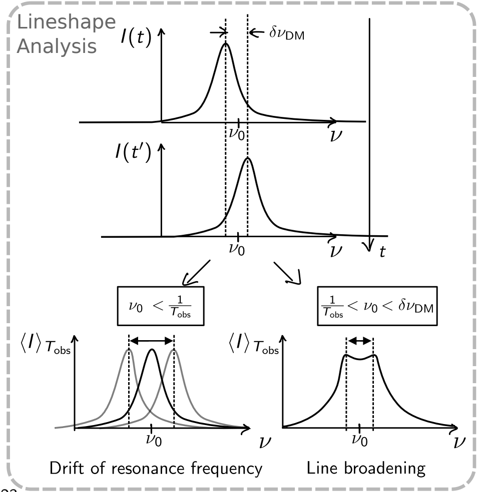
$$\nu(t) = \nu_0 + \delta\nu_{DM} \cos\left(2\pi\nu_{DM} t + \varphi_{DM}\right)$$



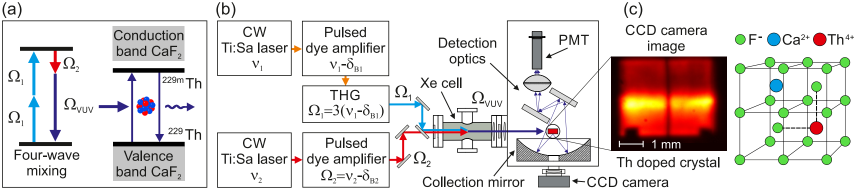


Nuclear Lineshape Analysis Regimes: Current

$$\nu(t) = \nu_0 + \delta\nu_{DM} \cos(2\pi\nu_{DM} t + \varphi_{DM})$$



Experimental Setup



(a,b) VUV laser spectroscopy of the isomeric state in Th-doped cristals. (c) False color image of the cristal during VUV laser excitation, cristal structure.

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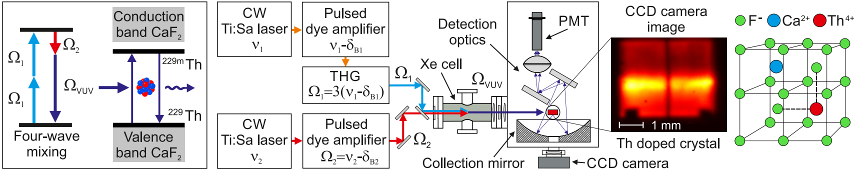

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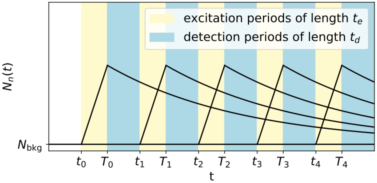
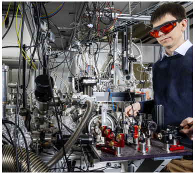
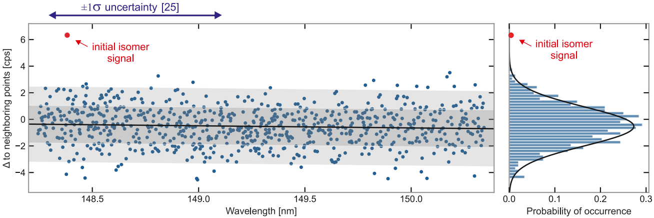


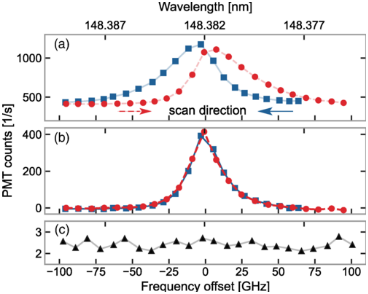
- ^{229}Th-doped cristal irradiated for t_e
- Laser turned off
- Fluorescence photons from isomer decays detected during t_d
- Change laser frequency, repeat

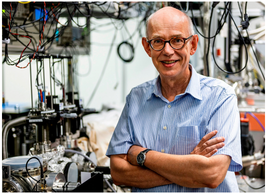
$\Rightarrow$ Record excitation spectrum of ^{229}Th nuclear resonance

Finding the Resonance



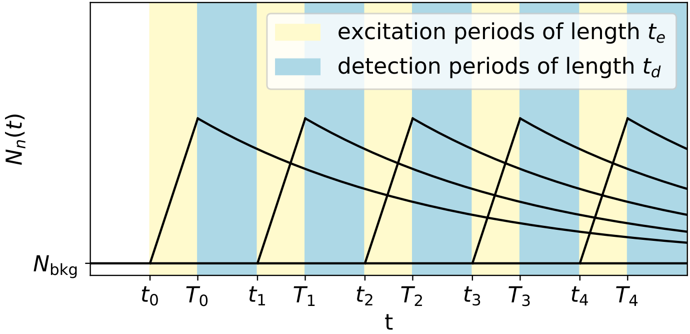






Nuclear Lineshape Analysis of PTB Data

⇒ Take into account experimental procedure



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$$\Delta N_n \equiv (N_n - N_{\text{bkg}}) - e^{-\frac{t_e + t_d}{\tau}} (N_{n-1} - N_{\text{bkg}})$$

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- Approximate laser profile as a Lorentzian with peak frequency ν_L, linewidth Δ_L, detuning from the ^{229}Th resonance: $\delta\nu = \nu_L - \nu_{\mathrm{Th}}$

$$\Gamma(t) \propto \frac{1}{1 + 4\left(\frac{\delta\nu(t)}{\Delta_L}\right)^2}$$

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- Substituting $x = (t_n - t)/t_e$, absorbing const. prefactors in $\mathcal{N}$,

$$\Delta N_n = \Delta N_{\mathrm{offset}} + \mathcal{N} \int_0^1 \mathrm{d}x \frac{e^{-x t_e/\tau}}{1 + 4\left(\frac{\delta\nu(x)}{\Delta_L}\right)^2}$$

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- Detuning of the laser from the ^{229}Th resonance during n^{th} measurement:

$$\delta\nu_n = \delta\nu_{0,n} + \delta\nu_{\text{offset}} + \delta\nu_{\text{DM}} \cos(2\pi\nu_{\text{DM}} t_e x - \varphi_n)$$

φ_n: DM phase at the beginning of the n^{th} detection period.
$\Rightarrow$ If DM oscillations are coherent, $\varphi_n = \varphi_{\text{DM}} + 2\pi n \nu_{\text{DM}}(t_e + t_d)$

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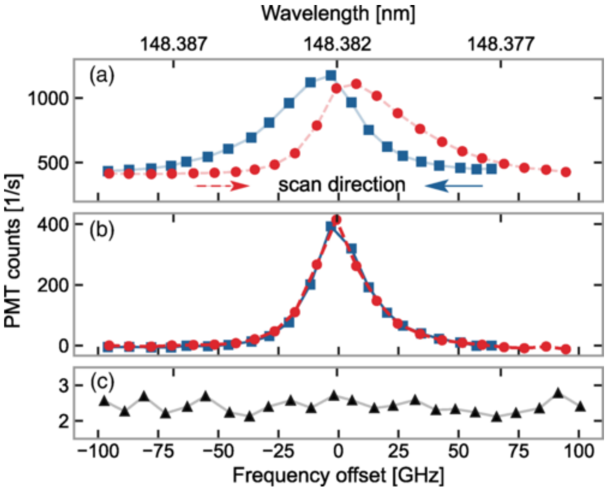
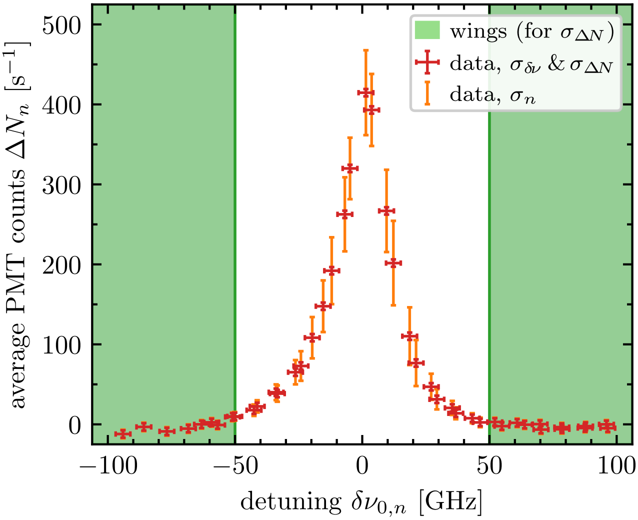
$\Rightarrow$ Fit (MCMC/ODR)

Nuclear Lineshape Analysis

First laser-excitation of a nuclear transition: PTB 2024 [PRL 132, 182501]



Nuclear Lineshape Analysis



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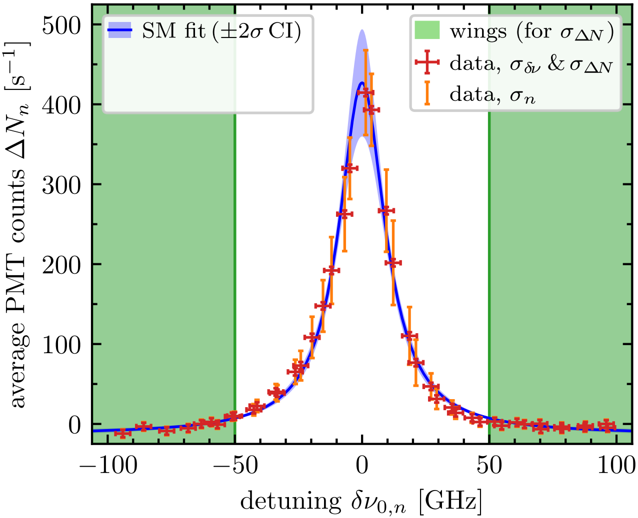

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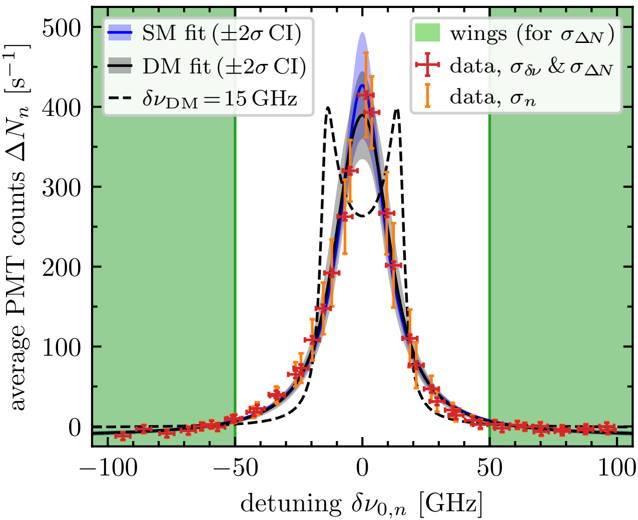
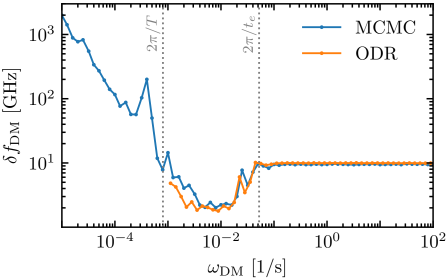


Outline

MCMC vs. ODR

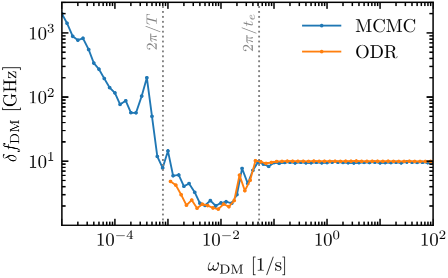


t_e: excitation time

T: time delay betw.

forward/backward scans

MCMC vs. ODR



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- νDM $< T^{-1}$, **"drift regime":** DM oscillation longer than duration of experiment

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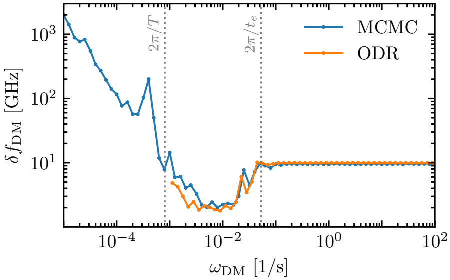

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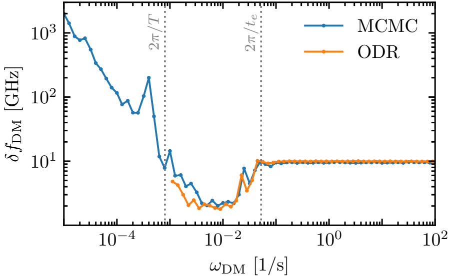

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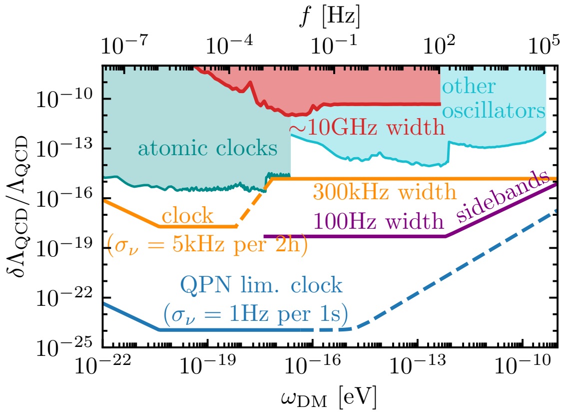
- $t_e^{-1} < \nu\mathrm{DM}$, **"broadening regime":** Experiment cannot track DM oscillation in time, but DM oscillation leads to broadening of resonance

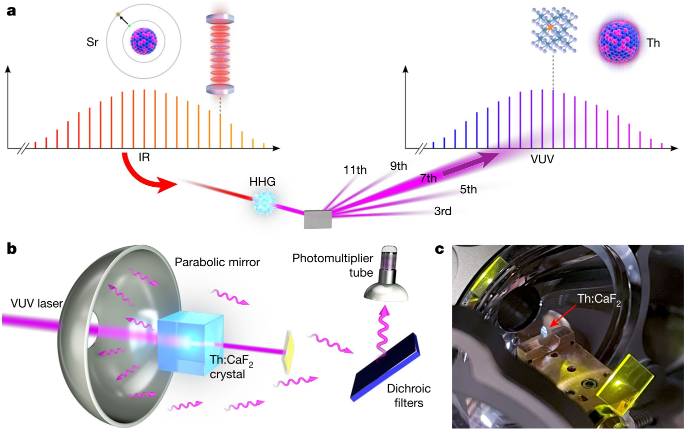
Bounds on Ultralight Scalar Coupling to QCD



$$\frac{\delta\nu_{\rm DM}}{\nu_0} = K_{\Lambda_{\rm QCD}} \frac{\delta\Lambda_{\rm QCD}}{\Lambda_{\rm QCD}} \approx 10^5 \frac{\delta\Lambda_{\rm QCD}}{\Lambda_{\rm QCD}}$$

^{229}Th Nuclear Transition vs. ^{87}Sr Atomic Clock

[arXiv:2406.18719]



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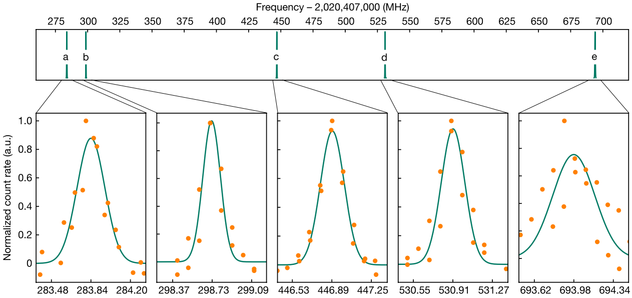
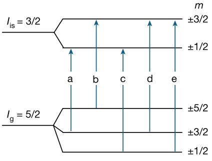
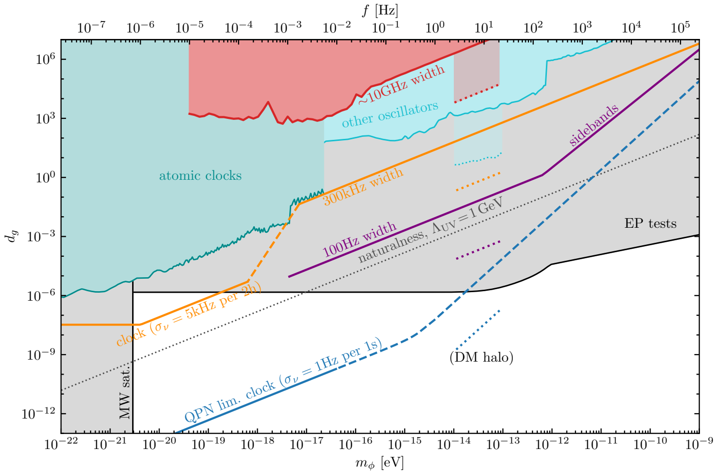




m_g	m_{is}	ν_{Th} (MHz)
3/2	1/2	2,020,407,283.847(4)
5/2	3/2	2,020,407,298.727(4)
1/2	1/2	2,020,407,446.895(4)
3/2	3/2	2,020,407,530.918(4)
1/2	3/2	2,020,407,693.98(2)

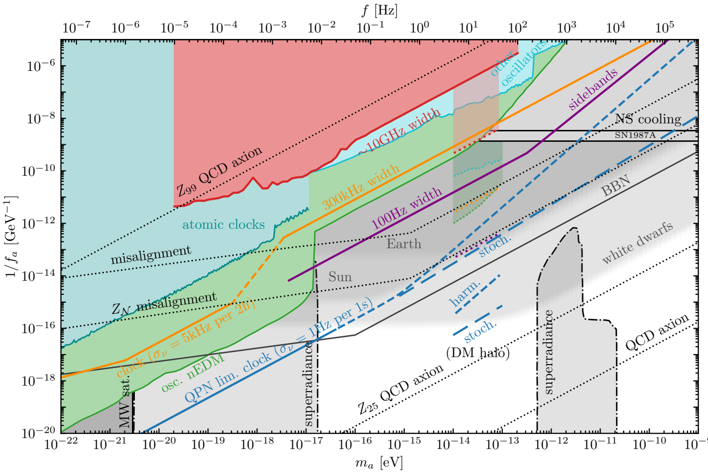
$$\frac{\nu_{\mathrm{Th}}}{\nu_{\mathrm{Sr}}} = 4.707072615078(5)$$

Bounds on Ultralight Scalar Coupling to QCD



$$\mathcal{L}_\phi \supset -d_g \frac{\phi}{M_{Pl}} \frac{\sqrt{\pi}\beta_s}{g_s} G^A_{\mu\nu} G^{A\mu\nu} \quad \Rightarrow \quad \frac{\delta\nu_{DM}}{\nu_0} \approx 10^5 d_g \frac{\sqrt{4\pi}}{M_{Pl}}$$

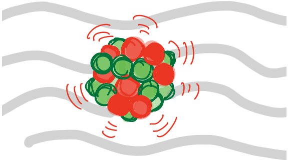
Bounds on Axions



$$\mathcal{L}_a \supset \frac{g_s^2}{32\pi^2}\frac{a}{f_a}G^A_{\mu\nu}\tilde{G}^{A\mu\nu} \qquad \Rightarrow \qquad \frac{\delta\nu_{\rm DM}}{\nu_0} \approx 2\times 10^5\,\frac{\delta m_\pi^2}{m_\pi^2} \approx -2.2\times 10^4\,\theta^2$$

Conclusions

Nuclear clocks are sensitive probes of light new scalars coupling to **QCD**:



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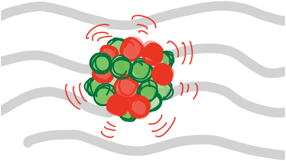

Can be searched for using a nuclear line-shape analysis in different regimes:

- light ULDM: drift regime
- “heavy” ULDM with large amplitude: line broadening
- “heavy” ULDM with small amplitude: sidebands

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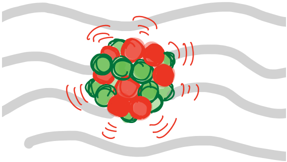

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...And on the way we can learn about

Fine-tuning in thorium, interaction between light & matter, cristals

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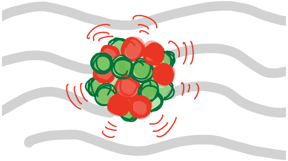

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...And on the way we can learn about

Fine-tuning in thorium, interaction between light & matter, cristals
And find applications for: gravitational physics, geodesy, navigation,...

Check out our paper on the arXiv:

Th-229 & ULDM: `https://arxiv.org/abs/2407.15924`

Thank you for your attention.

Backup slides

Ultralight Scalars

- Motivation: dilaton, relaxion, axion
- Assume ϕ makes up dark matter & is light ($m_\phi \lesssim 1$ eV)
  - $\Rightarrow$ Average number of DM particles in a de Broglie volume $\lambda_{\rm dB}^3$ in a Milky-Way-like environment:

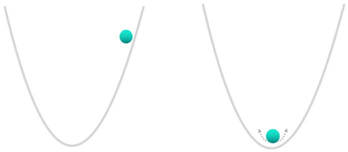
$$N_{\rm dB} \sim \left(\frac{34\ \mathrm{eV}}{m}\right)^4 \left(\frac{250\ \mathrm{km/s}}{v}\right)^3$$

  - $\Rightarrow$ For sub-eV dark matter, huge $N_{\rm dB} \Rightarrow$ DM $\simeq$ classical field

(see e.g. [arXiv:2101.11735])

- Simplest possible Lagrangian:

$$\mathcal{L}_\phi \supset \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}m_\phi^2\phi^2$$



ϕ oscillates around potential minimum:

$$\phi(t,x) \sim \phi_0 \cos(m_\phi t)$$

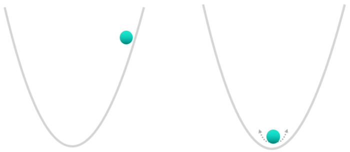
Ultralight Scalars

- ϕ interacts with the Standard Model:

$$\mathcal{L}_\phi \supset \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}m_\phi^2\phi^2 + \frac{\phi}{M_{\rm Pl}}\left[\frac{d_e}{4e^2}F_{\mu\nu}F^{\mu\nu} - \frac{d_g\beta_s}{2g_s}G^a_{\mu\nu}G^{a\mu\nu} - d_{m_e}m_e\bar{e}e - \sum_{q=u,d}\left(d_{m_q}+\gamma_{m_q}d_g\right)m_q\bar{q}q\right]$$

- ϕ oscillates around potential minimum:

$$\phi(t,x) \sim \phi_0 \cos(m_\phi t)$$



$\Rightarrow$ Oscillating fundamental constants

$\Rightarrow$ Oscillating transition frequencies

$$\nu(t) \sim \nu_0 \left(1 + \left(K_g d_g + K_e d_e + \ldots\right)\phi(t)/M_{\rm Pl}\right)$$

$$\Rightarrow \nu(t) \simeq \nu_0 + \delta\nu_{\rm DM}\cos\left(2\pi\nu_{\rm DM}t + \varphi_{\rm DM}\right)$$

Axion

$$\mathcal{L}_a \supset \frac{g_s^2}{32\pi^2}\frac{a}{f_a} G^A_{\mu\nu}\tilde{G}^{A,\mu\nu}\,, \qquad \tilde{G} \text{ : dual of the gluon field strength}$$

Confinement of QCD:

⇒ Interactions between pions & axion

⇒ Axion-dependent pion mass [Nucl.Phys.B 171 (1980) 253-272,arXiv:0811.1599]

$$m_\pi^2(\theta) = B\sqrt{m_u^2 + m_d^2 + 2m_u m_d\cos(\theta)}\,, \qquad \theta = a/f_a\,, \quad B = \langle\bar{q}q\rangle/f_\pi^2$$

Expanding around $\theta = 0$, axion potential $V(\theta) = -m_\pi^2(\theta) f_\pi^2$ at LO.

⇒ Axion oscillation induces oscillation of pion mass ⇒ $\frac{\delta m_\pi^2}{m_\pi^2} = -\frac{m_u m_d}{2(m_u+m_d)^2}\theta^2$

⇒ Oscillating transition frequency: $\frac{\delta\nu}{\nu} \simeq K_\pi \frac{\delta m_\pi^2}{m_\pi^2}$

[Nucl.Phys.B 171 (1980) 253-272, arXiv:0709.0077, 0807.4943, 2211.05174]

Sensitivity of Nuclear Clocks to New Physics

[arXiv:2012.09304,2407.17526]

$$\Delta E = \Delta E_{\mathrm{EM}} + \Delta E_{\mathrm{nuc}}$$

$$\Delta E \ll |\Delta E_{\mathrm{EM}}| \sim |\Delta E_{\mathrm{nuc}}|$$

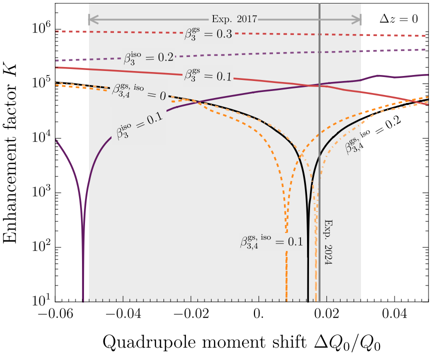
$$8\ \mathrm{eV} \ll 0.1\ \mathrm{MeV}$$

$$K_{\mathrm{EM}} \equiv \frac{1}{\Delta E}\frac{\partial \Delta E_{\mathrm{EM}}}{\partial \log \alpha_{\mathrm{EM}}} \simeq \frac{\Delta E_{\mathrm{EM}}}{\Delta E} \sim 10^5$$

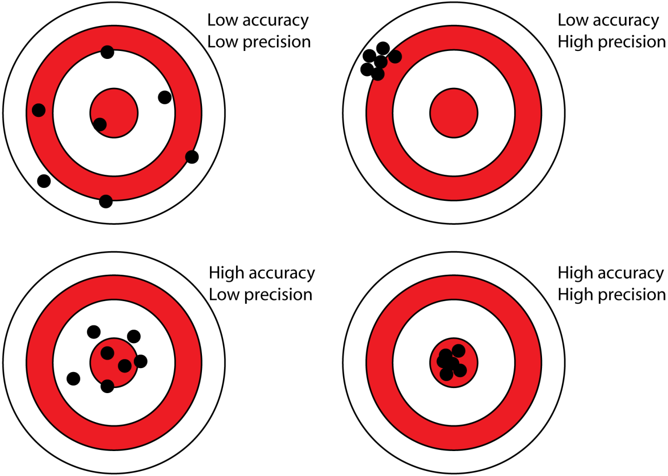
$$K_s^{\mathrm{EM}} \equiv \frac{1}{\Delta E}\frac{\partial \Delta E_{\mathrm{EM}}}{\partial \log \alpha_s} \sim \beta \frac{\Delta E_{\mathrm{EM}}}{\Delta E} \sim \beta K_{\mathrm{EM}}\,,$$

$$\beta \sim \mathcal{O}(1)?$$

$$K_s^{\mathrm{nuc}} \equiv \frac{1}{\Delta E}\frac{\partial \Delta E_{\mathrm{nuc}}}{\partial \log \alpha_s} \sim K_s^{\mathrm{EM}}?$$



Precision vs. Accuracy

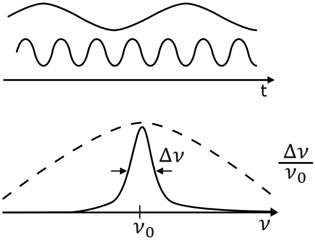


Optical Clock Stability

Achieve higher **precision** through

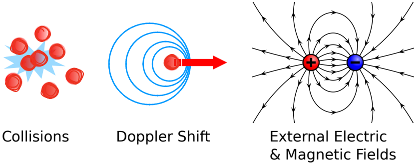
- Higher frequency ($\nu \to \infty$)
- Narrower linewidth ($\Delta\nu \to 0$)

$\Rightarrow$ Relevant quantity: fractional uncertainty $\frac{\Delta\nu}{\nu_0}$



Achieve higher **accuracy** by

- Cooling
- Trapping
- Shielding from / accounting for external fields



Advantages of Nuclear Clocks wrt. Atomic Clocks

+ Higher frequency $\Rightarrow$ **Higher stability**
+ Use solids? $\Rightarrow$ Higher statistics $\Rightarrow$ **Higher stability**
+ Strong vs. electromagnetic force $\Rightarrow$ Nucleus less polarisable than atom $\Rightarrow$ **Higher accuracy**
+ Nucleus $\ll$ Atom $\Rightarrow$ Shielded from external fields $\Rightarrow$ **Higher accuracy**
+ Low transition frequency due to accidental cancellation (?)

$$\Delta E = \Delta E_{\text{EM}} + \Delta E_{\text{nuc}} \qquad \Delta E \ll |\Delta E_{\text{EM}}| \sim |\Delta E_{\text{nuc}}|$$

$$8\text{ eV} \ll 0.1\text{ MeV}$$

$\Rightarrow$ **Extraordinary sensitivity to new physics?**

[arXiv:2012.09304,2407.17526]

+ Probes QCD $\Rightarrow$ **Sensitive to NP coupling to QCD**

Sidebands in the Limit $\delta\nu_{\mathrm{DM}} \ll \nu_{\mathrm{DM}}$

Transition frequency:

$$\nu(t) = \exp\left\{-i2\pi\nu_0 t - i\alpha\sin(2\pi\nu\mathrm{DM}t)\right\}$$
$$= \sum_{n=-\infty}^{\infty} J_n(\alpha)\exp\left\{-2\pi i(\nu_0 + n\nu\mathrm{DM})t\right\},$$

J_n: n^{th} Bessel function, $\alpha = \frac{\delta\nu_{\mathrm{DM}}}{\nu_{\mathrm{DM}}}$ modulation index

⇒ The lineshape is convoluted with

$$I(\nu) = \sum_{n=-\infty}^{\infty} |J_n(\alpha)|^2 \delta\left[\nu - (\nu_0 + n\nu_{\mathrm{DM}})\right]$$

⇒ The transition can be resonantly driven at frequencies $\nu = \nu_0 + n\nu_{\mathrm{DM}}$, but at a rate suppressed by $|J_n(\alpha)|^2$

⇒ If sideband with relative intensity $\Delta I/I$ to main peak detected, constrain $\delta\nu_{\mathrm{DM}} \lesssim \frac{\nu_{\mathrm{DM}}}{2}\sqrt{\frac{\Delta I}{I}}$