Black hole scattering at fifth PM and first self-force order

Work with Mathias Driesse, Gustav Jakobsen, Albrecht Klemm, Gustav Mogull, Christoph Nega, Jan Plefka, Johann Usovitsch

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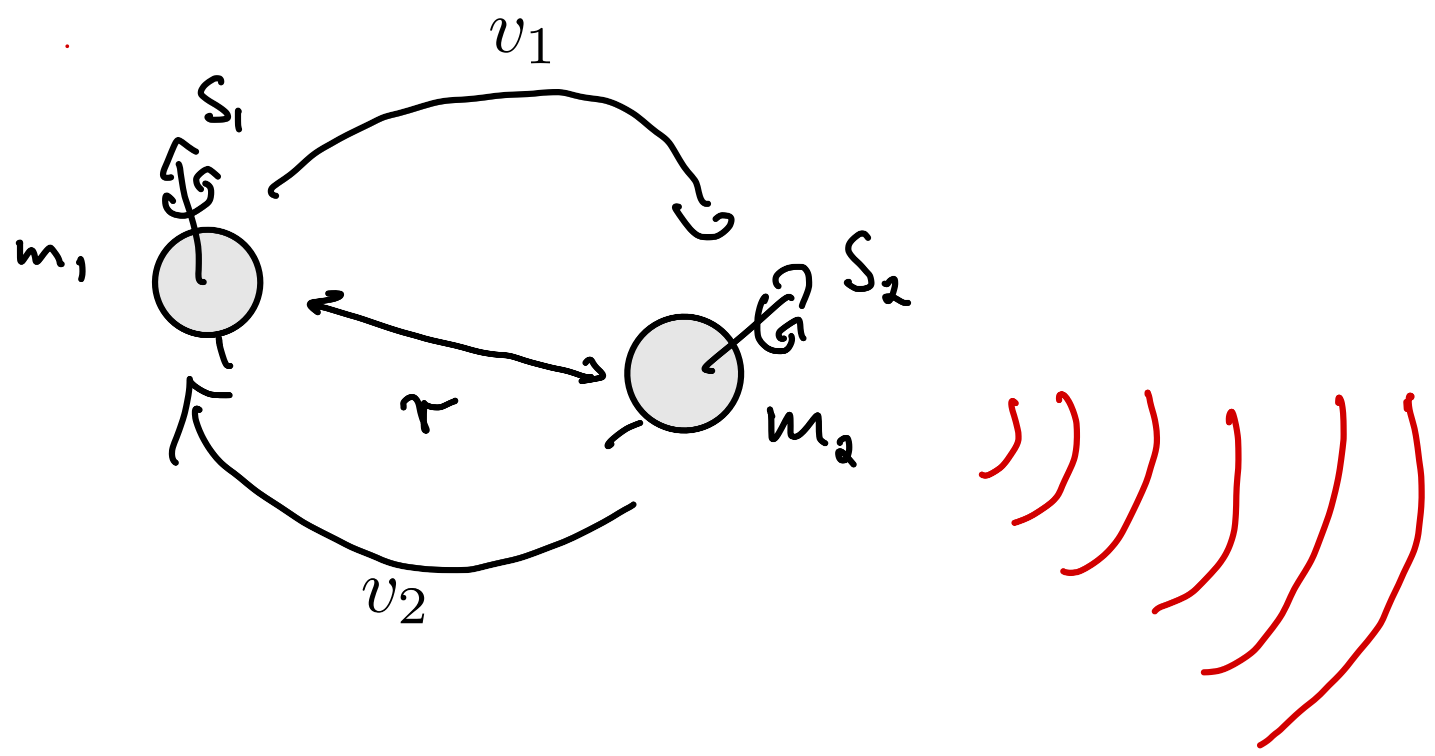
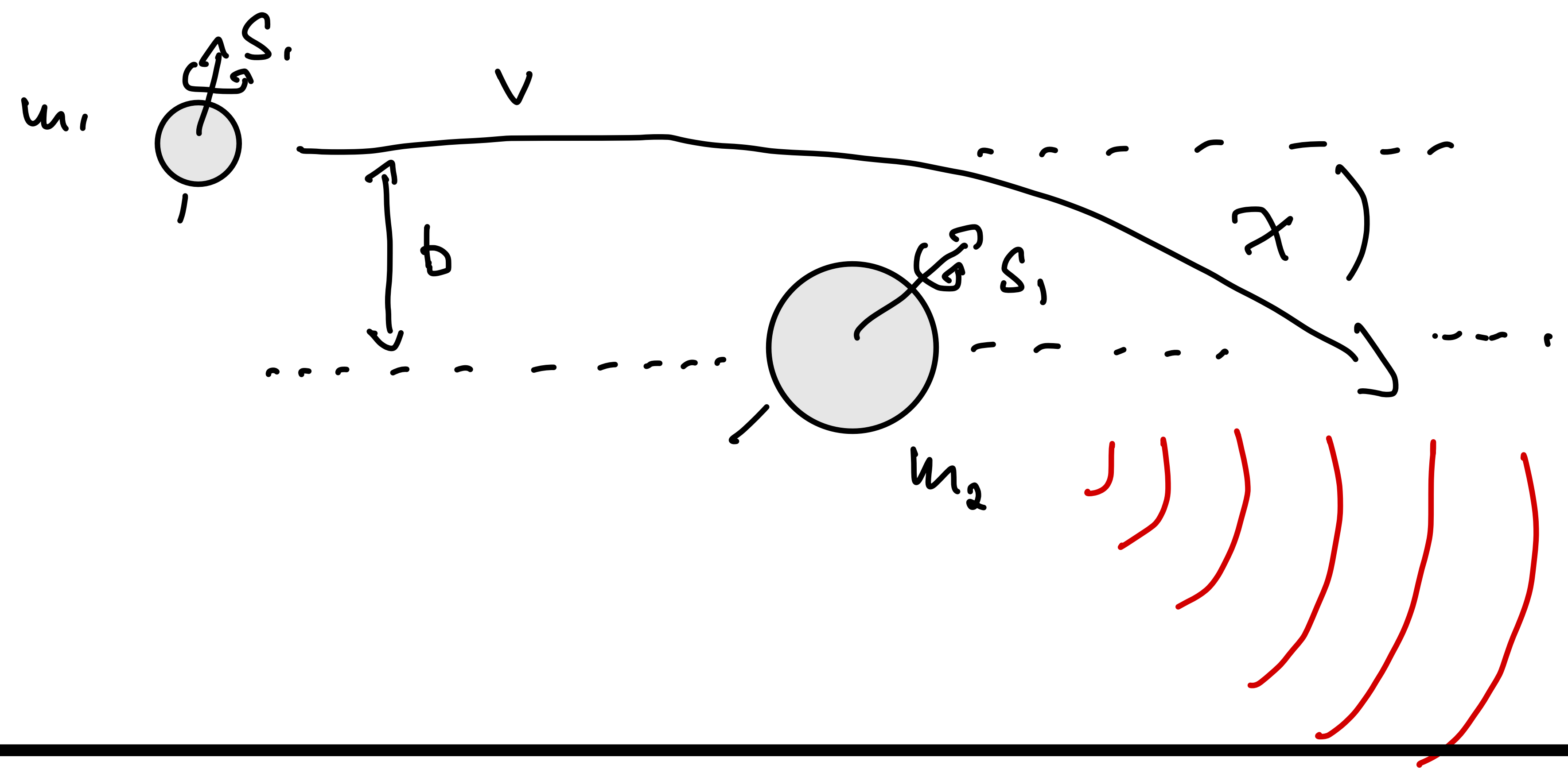
2403.07781 PRL 132 (2024) 24, Driesse, Jakobsen, Mogull, Plefka, **BS**, Usovitsch

2401.07899 PRD 109 (2024) 12, Klemm, Nega, **BS**, Plefka

The general relativistic 2-body problem

Bound Case: Post-Newtonian expansion in v and G





Unbound Case: Post-Minkowskian weak field expansion in G

➡Mapping back to bound case possible

➡Generation of waveforms via e.g. EOB approach

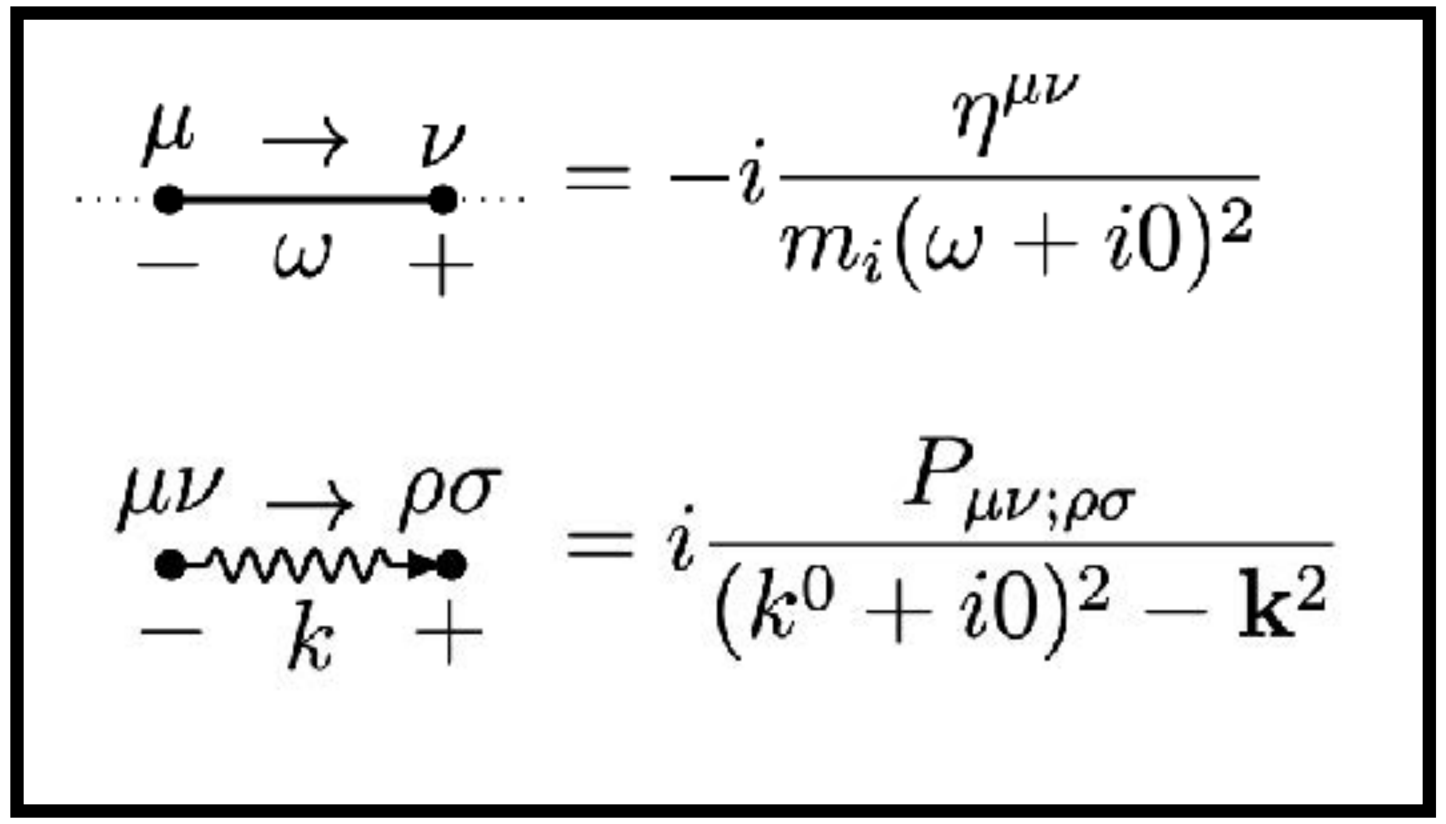
Worldline Quantum Field Theory

Mogull,Plefka,Steinhoff [2010.02865]

$$S = -\sum_{i=1}^{2} \int d\tau_i \sqrt{g_{\mu\nu}\dot{x}_i^{\mu}(\tau_i)\dot{x}_i^{\nu}(\tau_i)} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + S_{\text{g.f.}}$$

Point particle approximation

Bulk gravity & gauge fixing



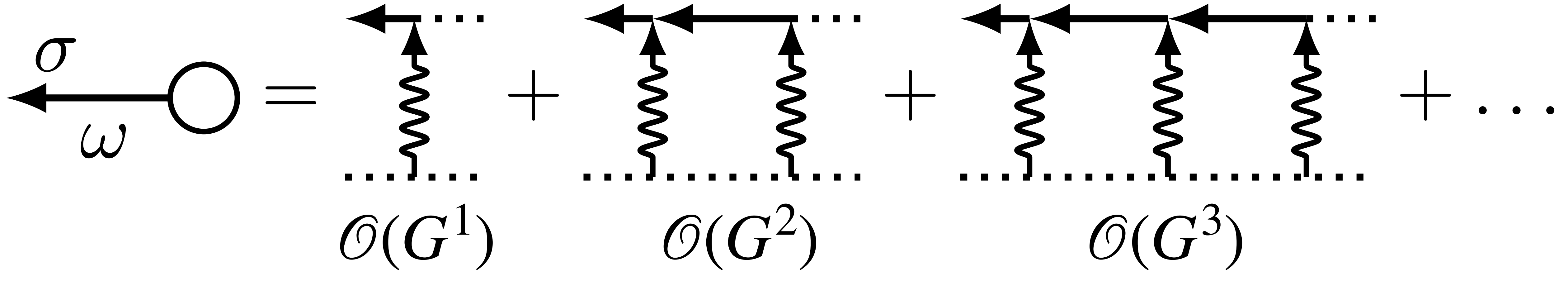
WQFT: Quantization of deflection and perturbation of flat space

$$x_i^{\mu}(\tau) = b_i^{\mu} + v_i^{\mu}\tau + z_i^{\mu}(\tau) \qquad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\,h_{\mu\nu}$$

Tree-level one-point functions solve **classical** equations of motion.
Very easy classical limit!

$$\Delta p_1^{\mu} = \lim_{\omega\to 0}\omega^2 < z_1^{\mu}(\omega) >$$

$\langle z^{\sigma}(\omega)\rangle =$



Retarded propagators + Causality flow!

Scattering at 5PM

$$\Delta p_i^{(5)\mu} = m_1 m_2 \left(m_1^4 \Delta p_{0SF}^{(5)\mu} + m_1^3 m_2 \Delta p_{1SF}^{(5)\mu} + m_1^2 m_2^2 \Delta p_{2SF}^{(5)\mu} + m_1 m_2^3 \Delta \bar{p}_{1SF}^{(5)\mu} + m_2^4 \Delta \bar{p}_{0SF}^{(5)\mu} \right)$$

Scattering at 5PM

$$\Delta p_i^{(5)\mu} = m_1 m_2 \left(m_1^4 \Delta p_{0SF}^{(5)\mu} + m_1^3 m_2 \Delta p_{1SF}^{(5)\mu} + m_1^2 m_2^2 \Delta p_{2SF}^{(5)\mu} + m_1 m_2^3 \Delta \bar{p}_{1SF}^{(5)\mu} + m_2^4 \Delta \bar{p}_{0SF}^{(5)\mu} \right)$$

<u>Geodesic motion</u>

Scattering at 5PM

$$\nu = m_1 m_2/(m_1 + m_2)$$

$\mathcal{O}(\nu^0)$ $\mathcal{O}(\nu)$ $\mathcal{O}(\nu^2)$ $\mathcal{O}(\nu)$ $\mathcal{O}(\nu^0)$

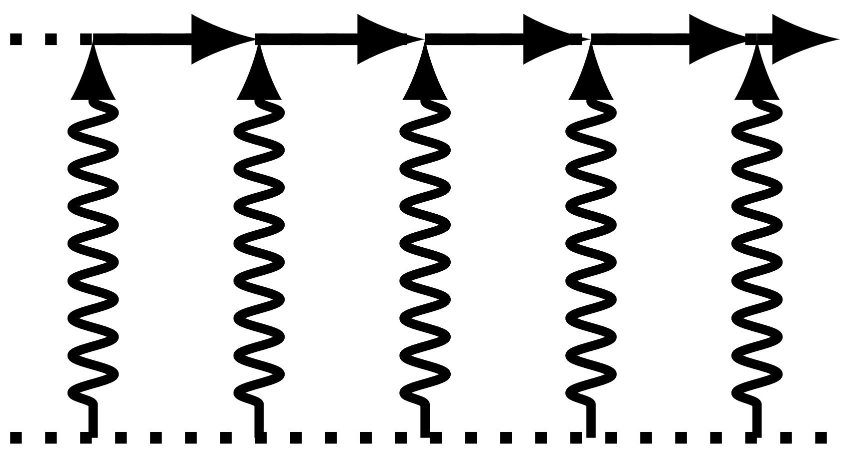
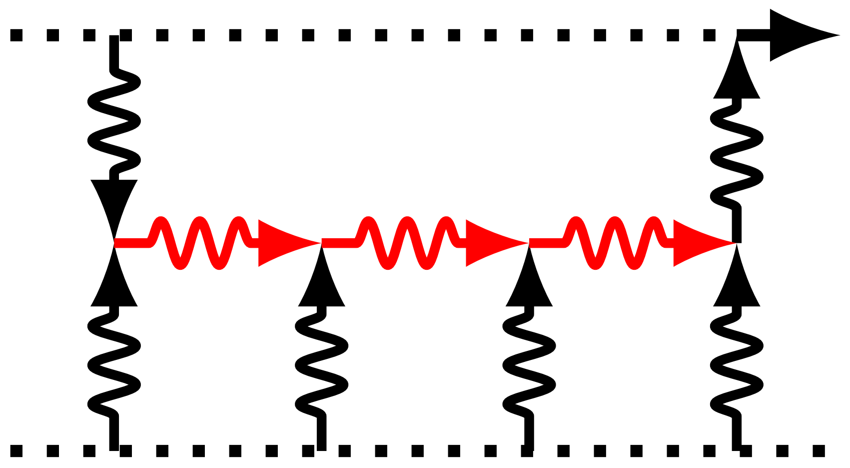
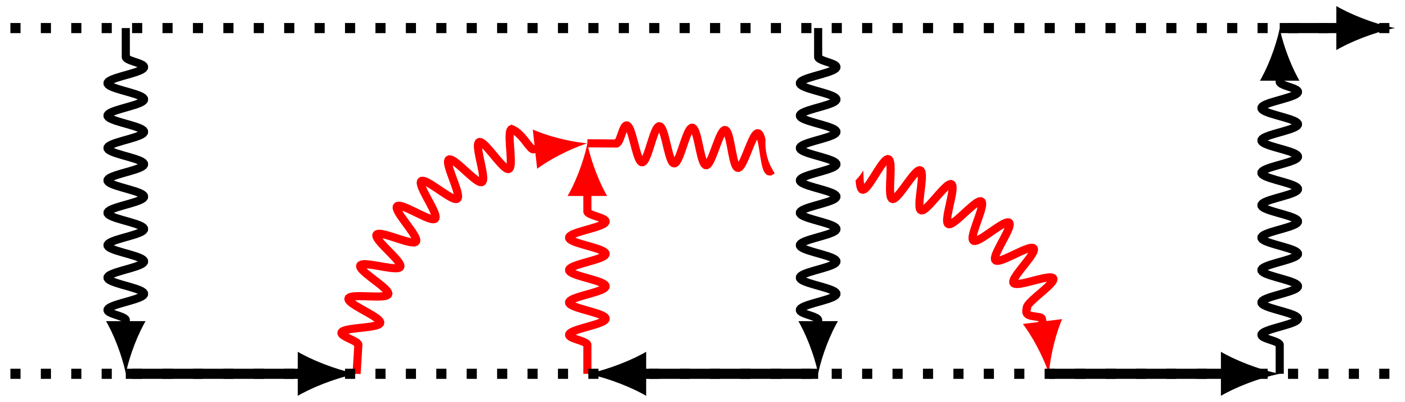
$$\Delta p_i^{(5)\mu} = m_1 m_2 \left(m_1^4 \Delta p_{0SF}^{(5)\mu} + m_1^3 m_2 \Delta p_{1SF}^{(5)\mu} + m_1^2 m_2^2 \Delta p_{2SF}^{(5)\mu} + m_1 m_2^3 \Delta \bar{p}_{1SF}^{(5)\mu} + m_2^4 \Delta \bar{p}_{0SF}^{(5)\mu} \right)$$

Scattering at 5PM

$$\nu = m_1 m_2/(m_1 + m_2)$$

$$\mathcal{O}(\nu^0) \quad \mathcal{O}(\nu) \quad \mathcal{O}(\nu^2) \quad \mathcal{O}(\nu) \quad \mathcal{O}(\nu^0)$$

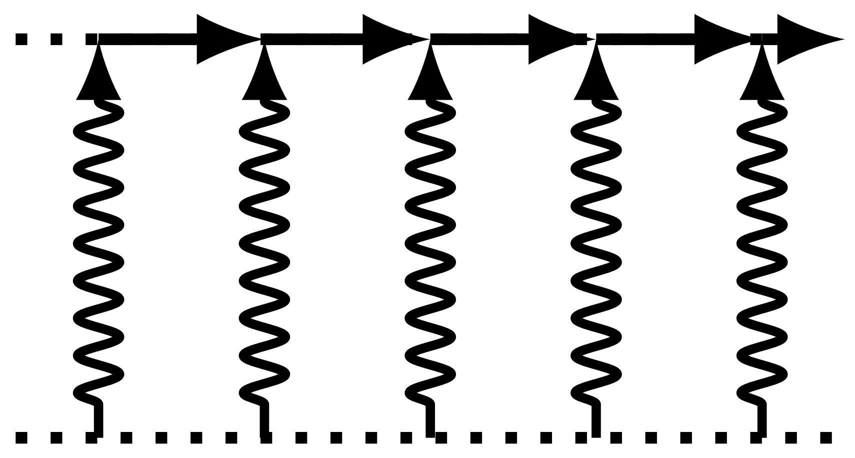
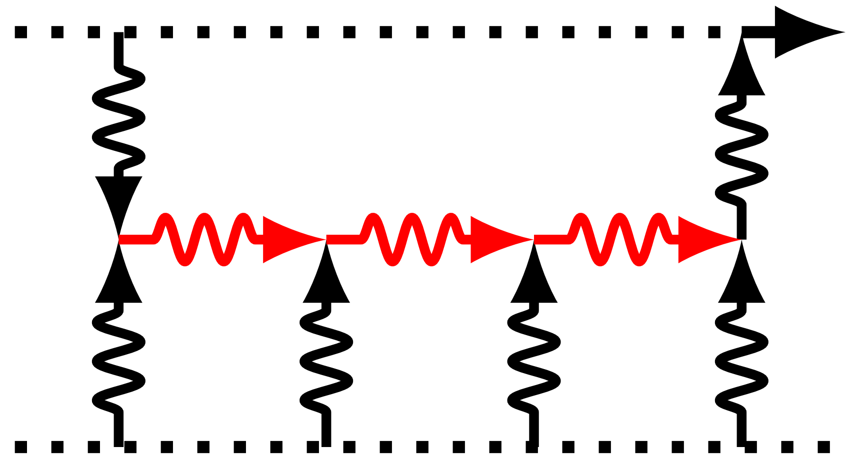
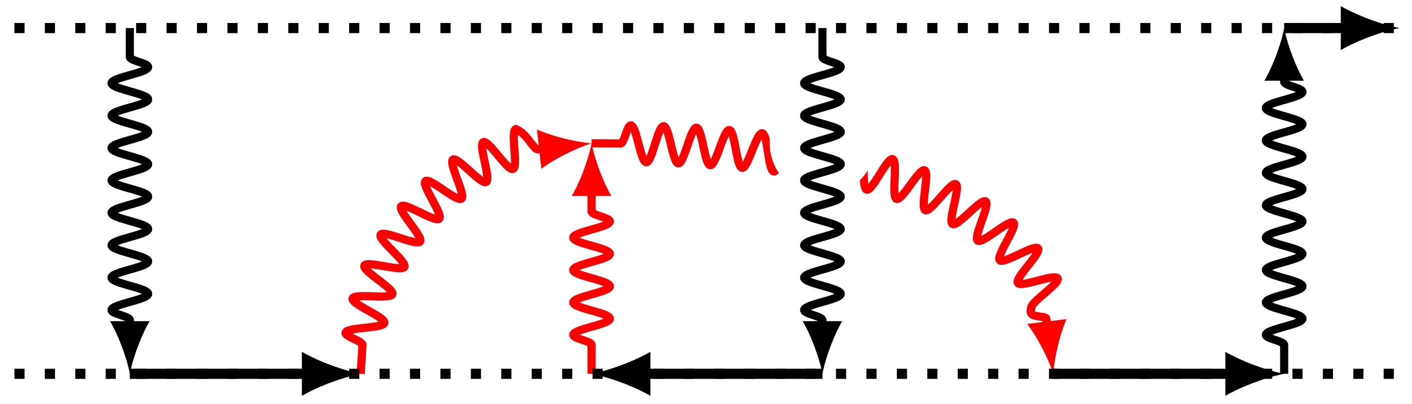
$$\Delta p_i^{(5)\mu} = m_1 m_2 \left(m_1^4 \Delta p_{0SF}^{(5)\mu} + m_1^3 m_2 \Delta p_{1SF}^{(5)\mu} + m_1^2 m_2^2 \Delta p_{2SF}^{(5)\mu} + m_1 m_2^3 \Delta \bar{p}_{1SF}^{(5)\mu} + m_2^4 \Delta \bar{p}_{0SF}^{(5)\mu} \right)$$

0SF - geodesic motion	1SF - $\mathcal{O}(\nu)$	2SF - $\mathcal{O}(\nu^2)$
		
• Trivial scale dependence • 1 planar family • 60 MIs	• 1 planar family • ~500 MIs	• planar + non-planar • >1000 MIs

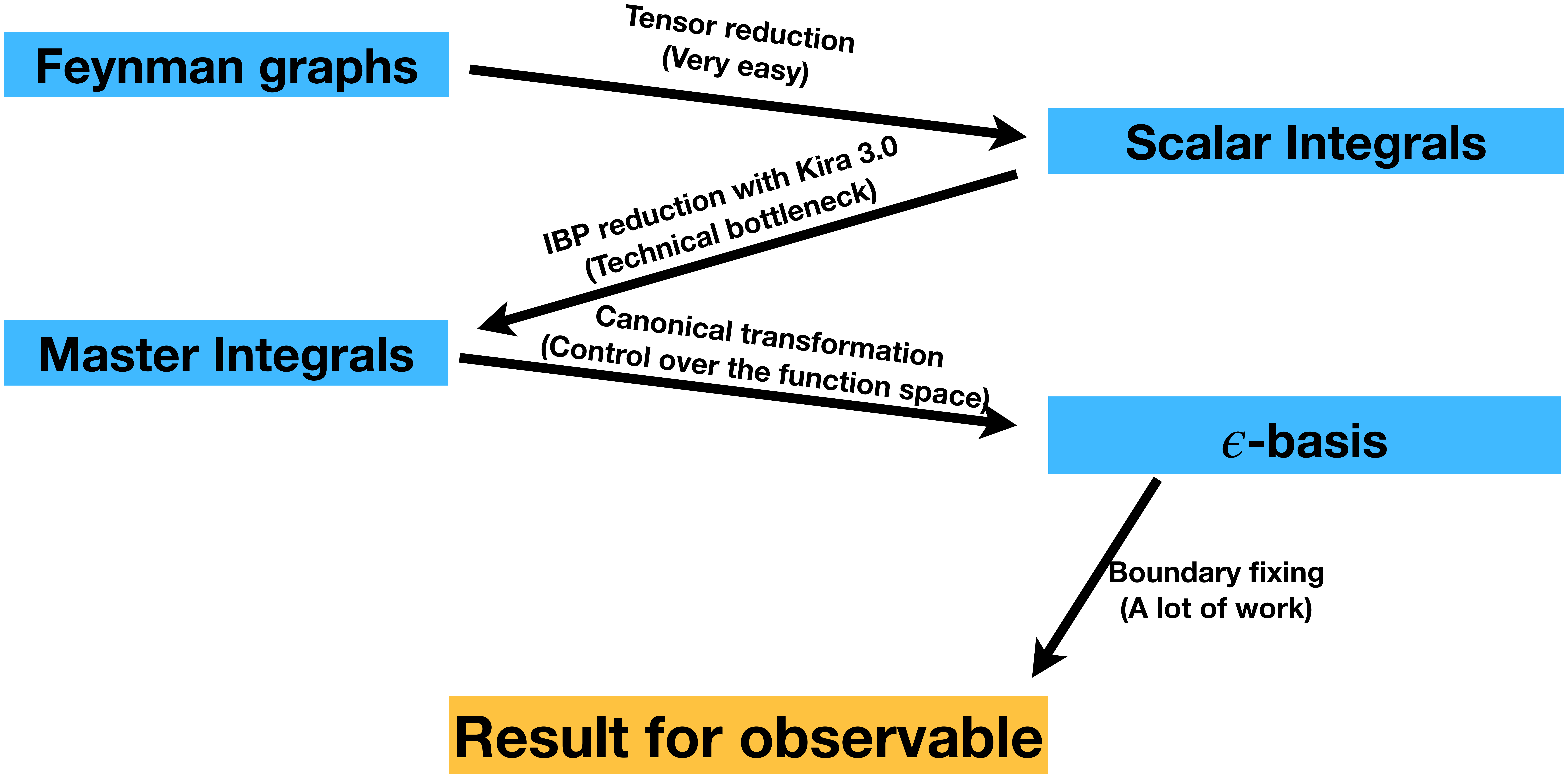
Scattering at 5PM

$$\nu = m_1 m_2/(m_1 + m_2)$$

$$\Delta p_i^{(5)\mu} = m_1 m_2 \Big(\underbrace{m_1^4 \Delta p_{0SF}^{(5)\mu}}_{\mathcal{O}(\nu^0)} + \underbrace{\boxed{m_1^3 m_2 \Delta p_{1SF}^{(5)\mu}}}_{\mathcal{O}(\nu)} + \underbrace{m_1^2 m_2^2 \Delta p_{2SF}^{(5)\mu}}_{\mathcal{O}(\nu^2)} + \underbrace{\boxed{m_1 m_2^3 \Delta \bar{p}_{1SF}^{(5)\mu}}}_{\mathcal{O}(\nu)} + \underbrace{m_2^4 \Delta \bar{p}_{0SF}^{(5)\mu}}_{\mathcal{O}(\nu^0)} \Big)$$

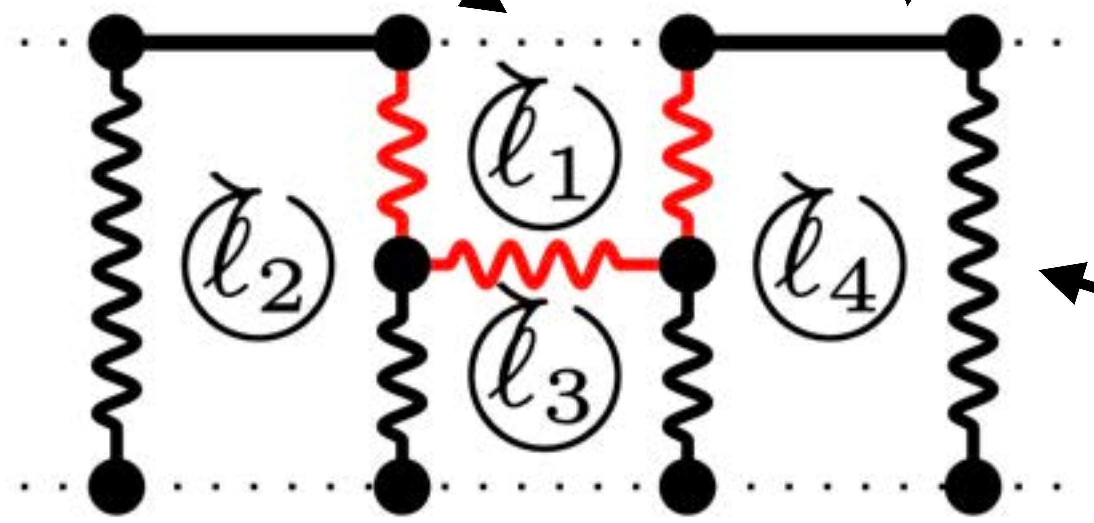
0SF - geodesic motion	1SF - $\mathcal{O}(\nu)$	2SF - $\mathcal{O}(\nu^2)$
		
• Trivial scale dependence • 1 planar family • 60 MIs	• 1 planar family • ~500 MIs	• planar + non-planar • >1000 MIs

Workflow


Feynman graphs
Tensor reduction
(Very easy)
Scalar Integrals
IBP reduction with Kira 3.0
(Technical bottleneck)
Master Integrals
Canonical transformation
(Control over the function space)
ϵ-basis
Boundary fixing
(A lot of work)
Result for observable

Integral family at first self force order

$$\mathcal{I}^{\{\sigma\}}_{\{n\}} = \int_{l_1 \cdots \ell_4} \frac{\delta^{(\bar{n}_1-1)}(\ell_1 \cdot v_1)\delta^{(\bar{n}_1-1)}(\ell_2 \cdot v_2)\delta^{(\bar{n}_1-1)}(\ell_3 \cdot v_2)\delta^{(\bar{n}_1-1)}(\ell_4 \cdot v_2)}{\prod_{i=1}^{4} D_i^{n_i}(\sigma_i) \prod_{I<J} D_{IJ}^{n_{IJ}}}$$



Linear propagators:

$$D_i = \ell_i \cdot v_{\bar{j}_i} + \sigma_i i0^+$$

Graviton propagators:

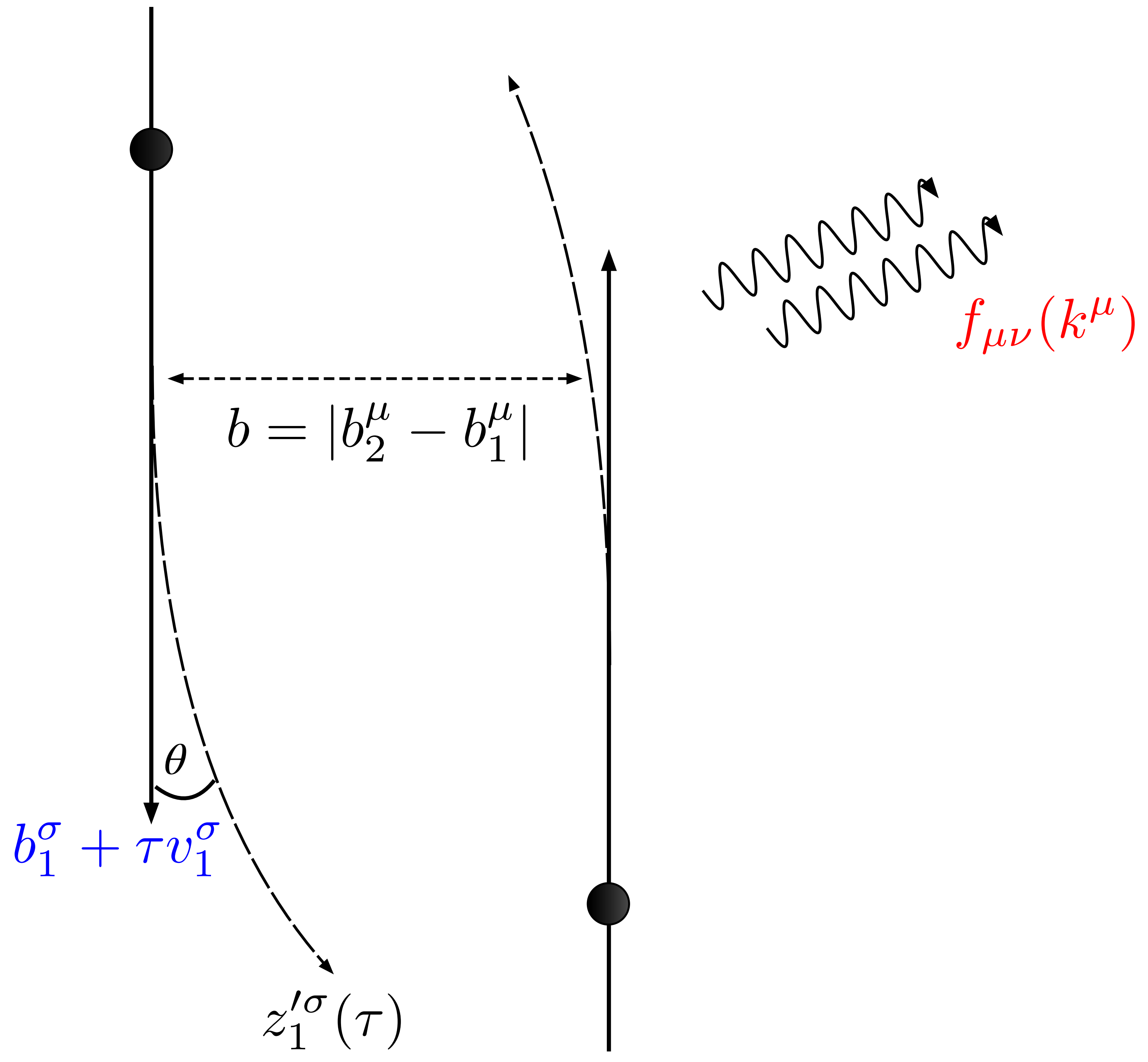
$$D_{IJ} = (\ell_I - \ell_J)^2$$

$$D_{qi} = (\ell_i + q)^2$$

$$D_{0i} = \ell_i^2$$

- **Single scale:** $v_1 \cdot v_2 = \gamma = 1/2(x + x^{-1})$
- Retarded propagator make integrals (pseudo-)real

"Parity" of integrals



Observables: θ E_{rad}

$$\Delta p^\mu = c_b b^\mu + c_{v_1} v_1^\mu + c_{v_2} v_2^\mu$$

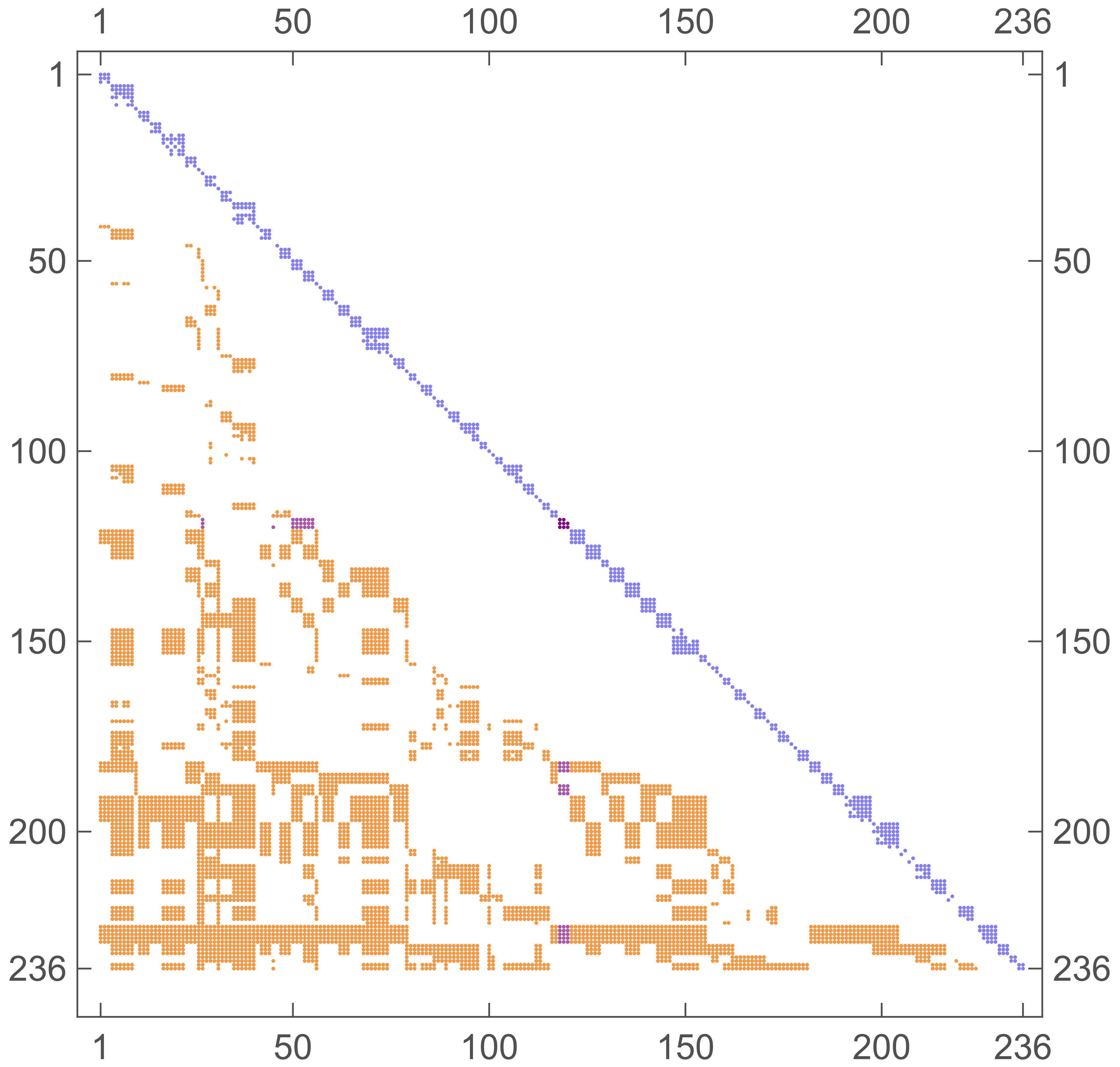
- Scattering angle θ
- Integrals purely **real**, even number of lin. prop.
- Conservative + dissipative effects

- Radiated energy E_{rad}
- Integrals purely **imaginary**, odd number of lin. prop.
- Dissipative effects + lower order results

Two distinct integral families of 232+234 masters with same propagators

Differential Equations

$d = 4 - 2\epsilon$



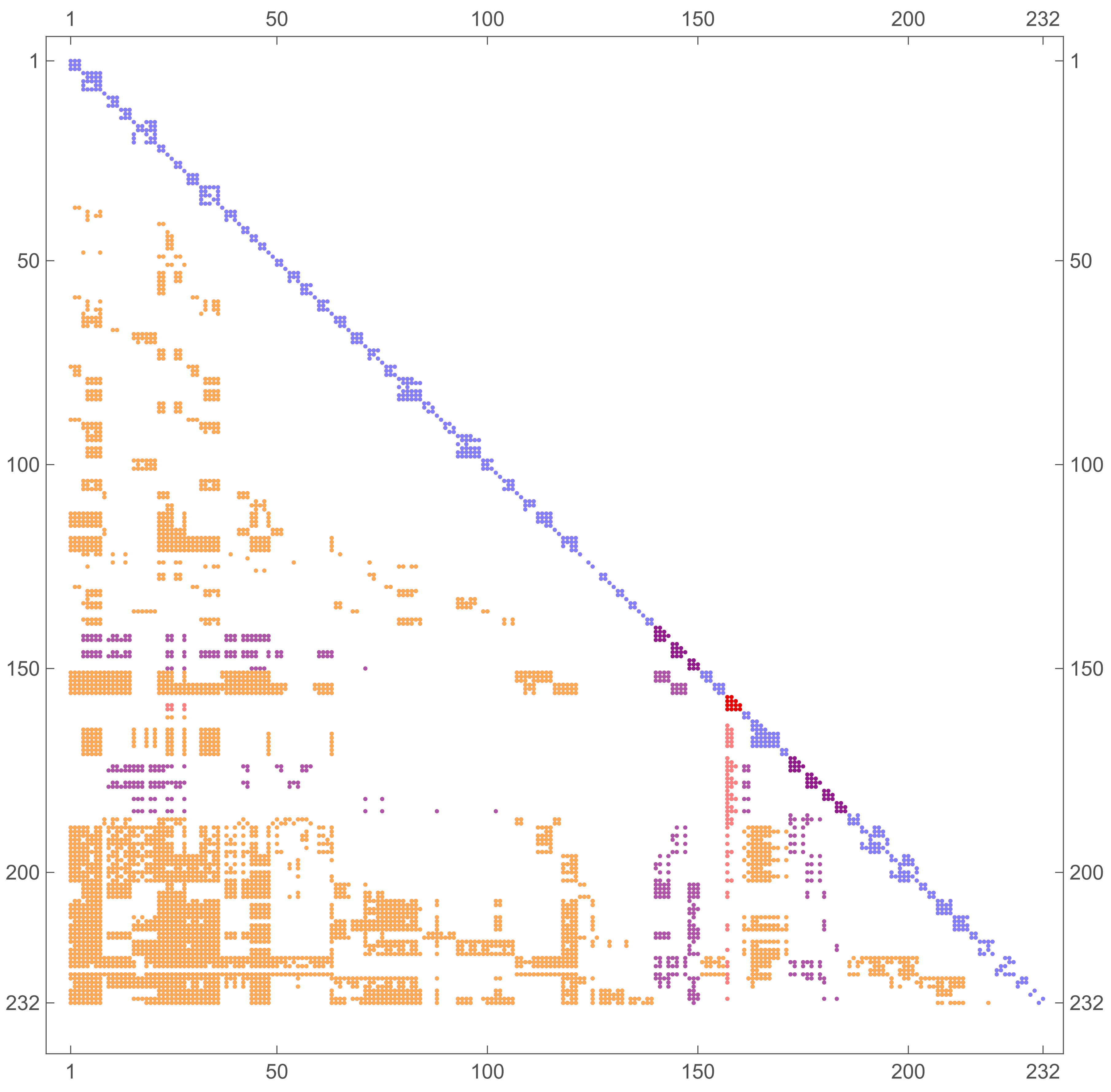
Even parity

$$\frac{d\vec{I}}{dx} = M(x,\epsilon)\vec{I}$$

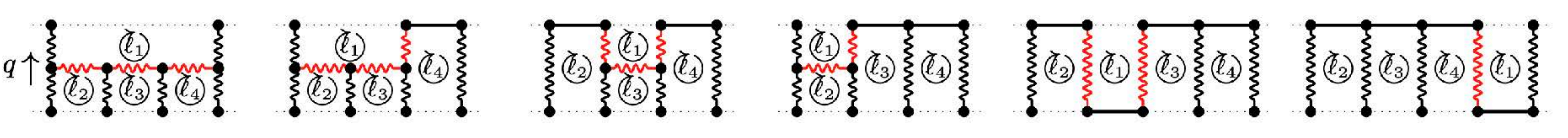
[Henn],[Gehrmann, Remiddi]

T

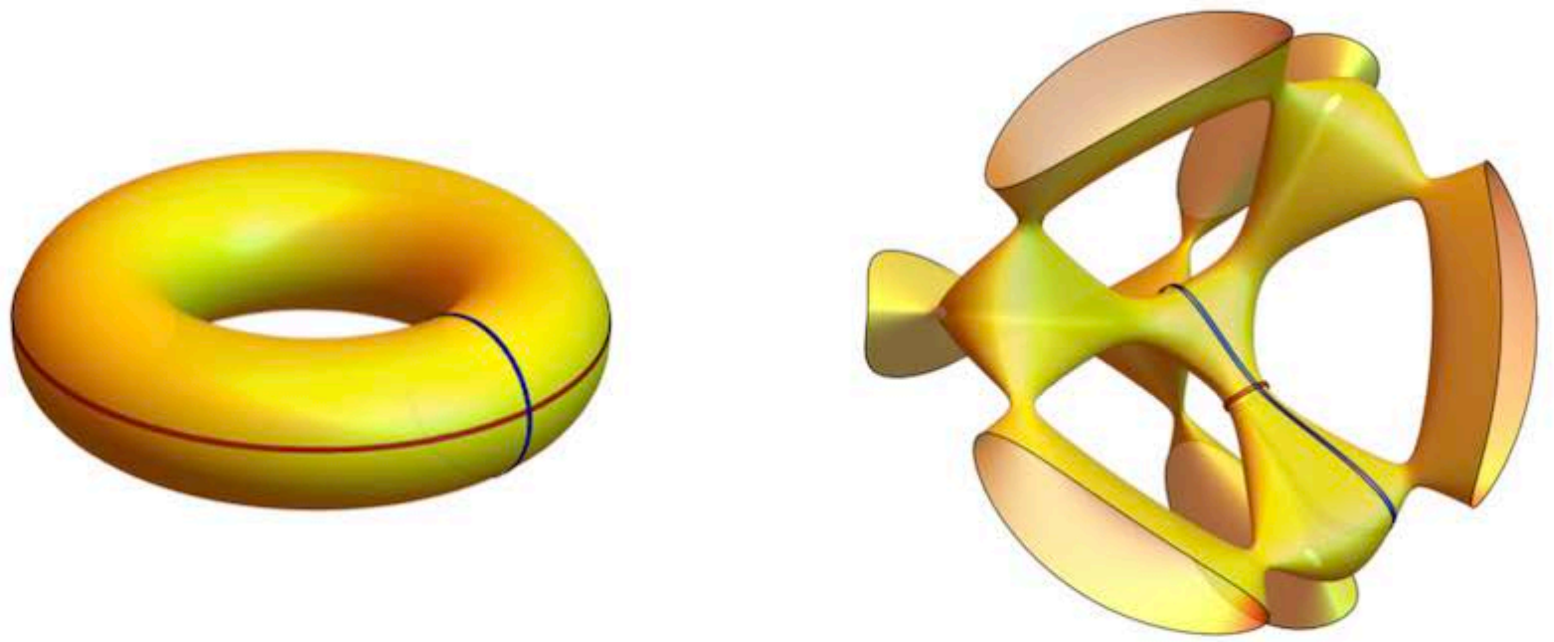
$$\frac{d\vec{J}}{dx} = \epsilon\tilde{M}(x)\vec{J}$$



Odd parity

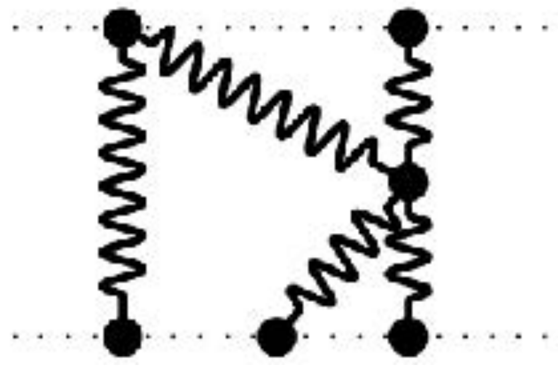


CY in the Sky



4PM K3 sector:

(Dlapa,Kaelin,Liu, Porto) [2106.082776]
(Bern,Parra-Martinez, Robin, Ruf, Shen, Solon, Zeng) [2101.07254]



$$\theta = x\frac{\mathrm{d}}{\mathrm{d}x}$$

$$\mathscr{L}^{(3)} = \mathrm{Sym}^2(\mathscr{L}^{(2)})$$

$$\mathscr{L}^{(2)} = \theta^2 - z(\theta + \frac{1}{2})^2$$

$$[\theta^3 - 2x^2(2 + 4\theta + 3\theta^2 + \theta^3) + x^4(2 + \theta)^3]I_1|_{\epsilon=0} = 0$$

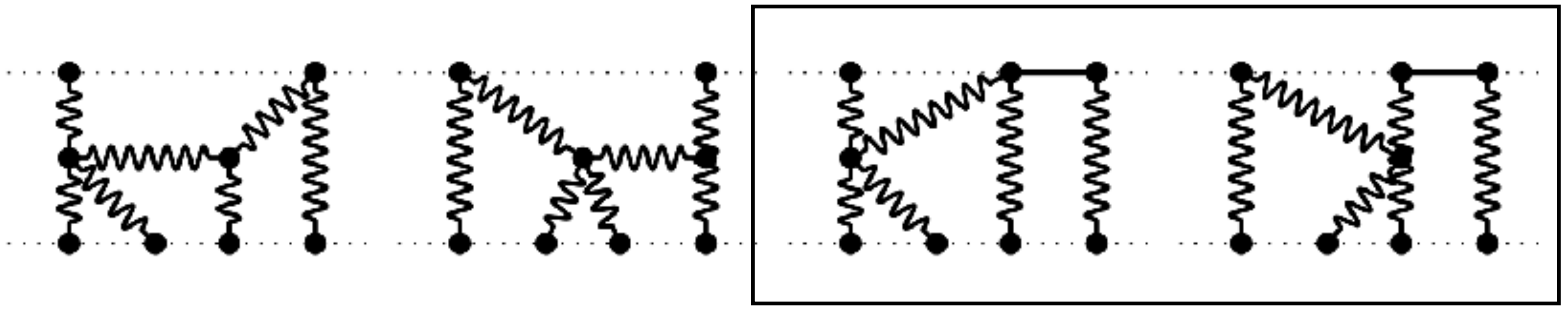
$$= \pi^4 \epsilon x K^2(1 - x^2) + \mathscr{O}(\epsilon^2)$$

$$I_1|_{\epsilon=0} = const. \cdot \varpi_{K3}(x)$$

➡ **Solution factorizes into elliptic functions of first and second type**

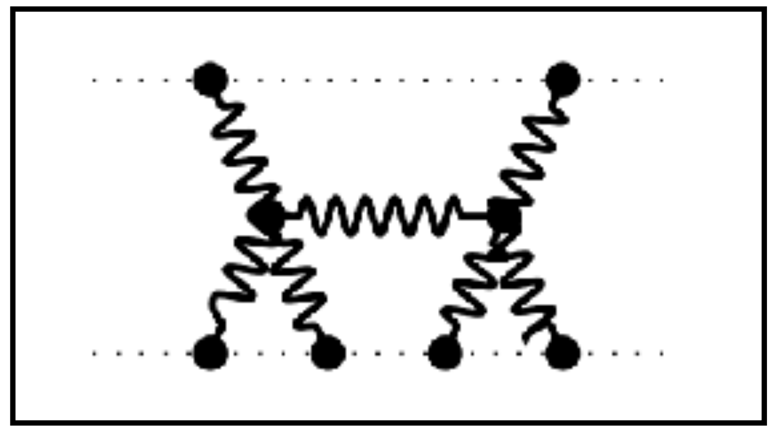
5PM-1SF K3 sectors:

Odd integrals:



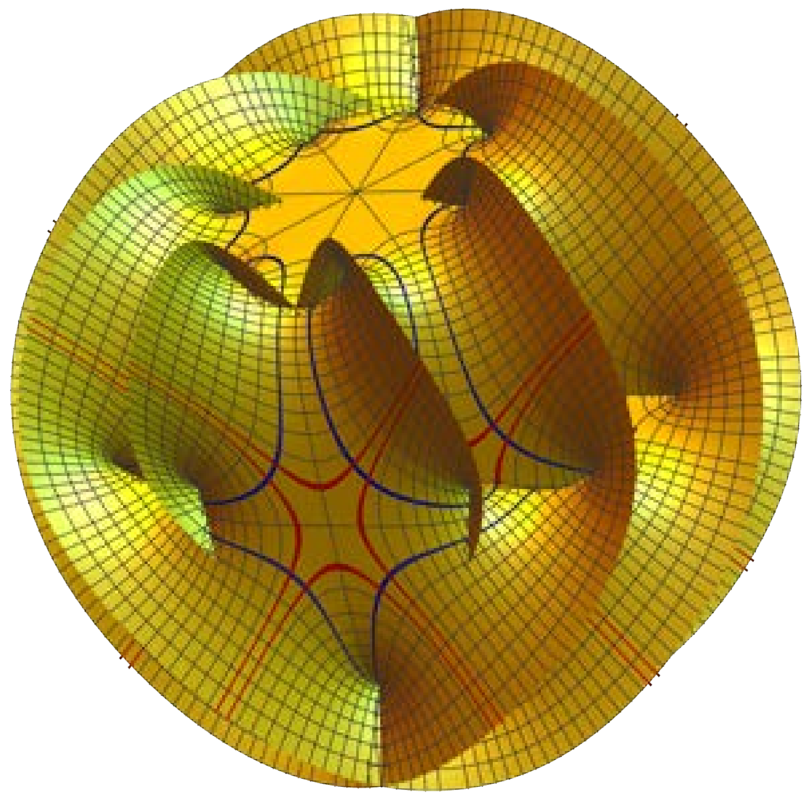
Conservative contribution

Even integrals:



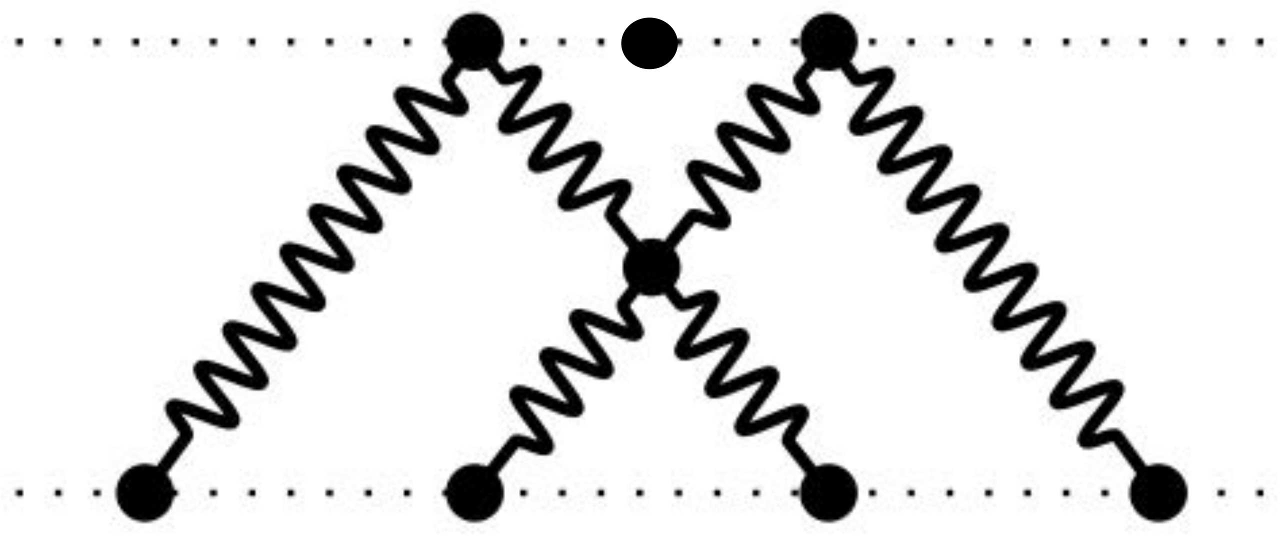
$$= -1/(1024\pi^2)xK^2(1 - x^2) + \mathscr{O}(\epsilon)$$

CY in the Sky



CY3 geometry:

(Klemm, Nega, **BS**, Plefka) [2401.07899]



$$\mathscr{L}^{(4)} = \mathscr{L}^{(2)} * \mathscr{L}^{(2)}$$

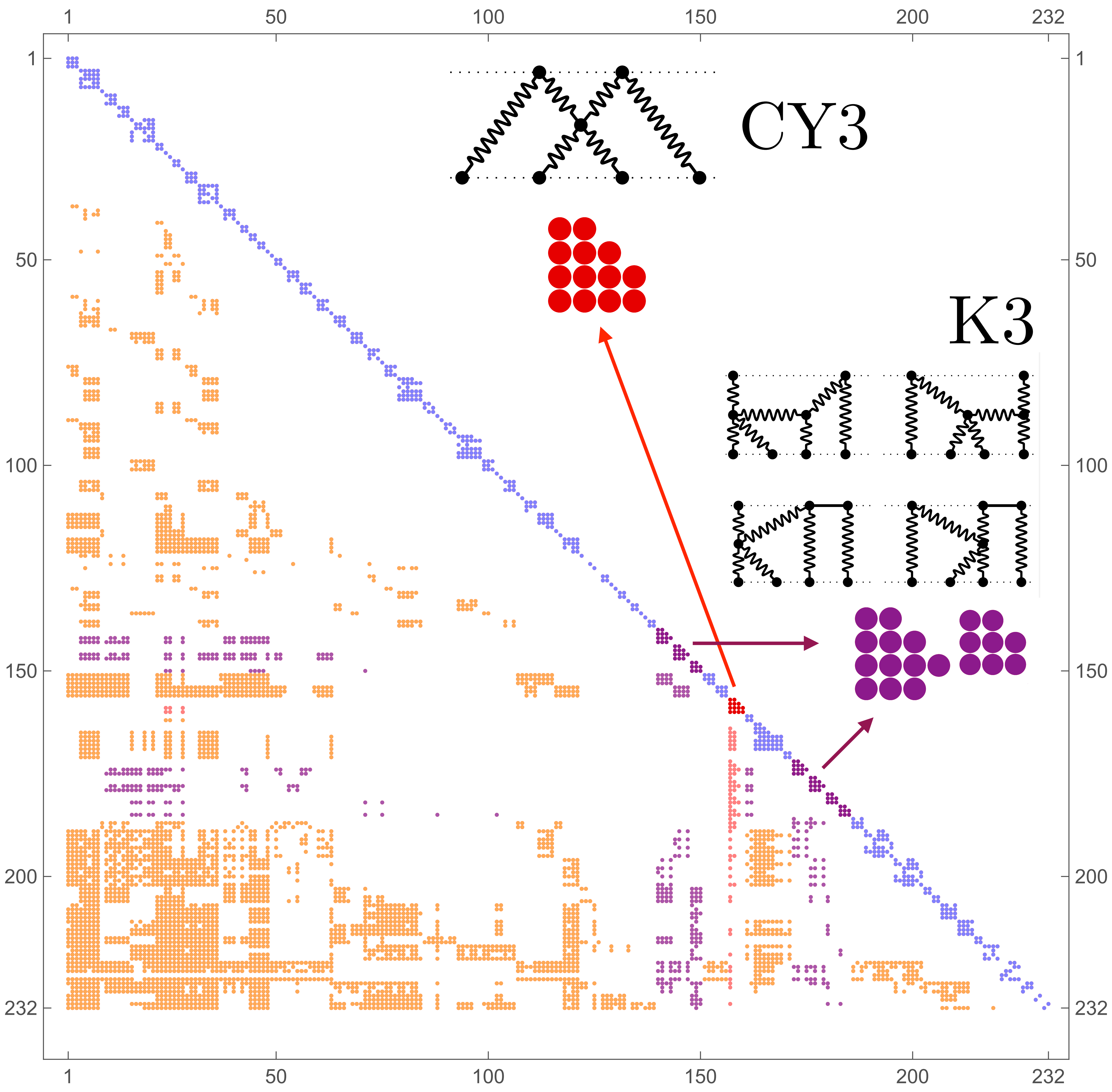
$$[(\theta - 1)^4 - x^4(\theta + 1)^4]I_1\,|_{\epsilon=0} = 0$$

Canonical form via unipotent/semi-simple split

(Goerges, Nega, Tancredi, Wagner) [2305.14090]

Extra functions needed for canonical form:

$$\alpha_1 = \frac{\varpi_0^2}{x(\varpi_0\varpi_1' - \varpi_0'\varpi_1)}, \ldots$$



Results

Driesse, Jakobsen, Mogull, Plefka, BS, Usovitsch [2403.07781]:
Driesse, Jakobsen, Klemm, Mogull, Nega, Plefka, BS, Usovitsch [2411.*****]:

Scattering angle:

$$\theta^{(5)} = \frac{M^5\Gamma}{b^5}\left(\theta^{(5,0)} + \nu\theta^{(5,1)}\left(+ \nu^2\theta^{(5,2)}\right) + \nu^3\Gamma^{-2}\theta^{(5,3)}\right)$$

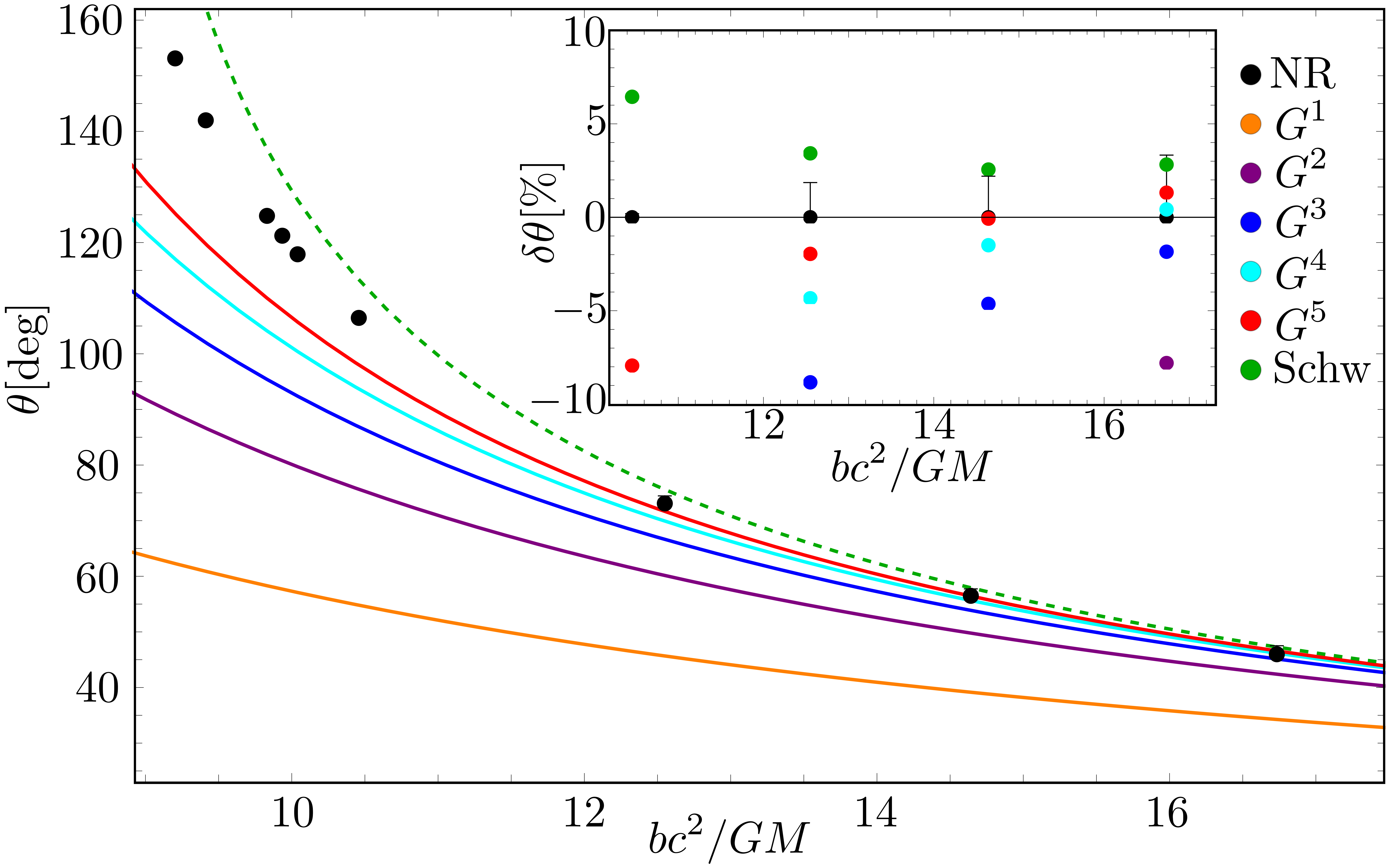
0SF 1SF 2SF 1SF-recoil

2SF → Only missing piece!

- **Elliptics drop out** (they were present at 4PM)
- also seen in $\mathcal{N}=8$ SUGRA (Bern, Herrmann, Roiban, Ruf, Smirnov, Zeng) [2406.01554]

Function space:

	3PM-1SF	4PM-1SF	5PM-1SF
Kernels of iterated integrals	$\frac{1}{x}, \frac{1}{1-x}, \frac{1}{1+x}$	$\frac{1}{x}, \frac{1}{1-x}, \frac{1}{1+x}, \frac{x}{x^2+1}$	$\frac{1}{x}, \frac{1}{1-x}, \frac{1}{1+x}, \frac{x}{x^2+1}$
Max. Weight	1	2	3
Functions from rotation		$K^2, E^2, E\cdot K$	



160
140
120
100
80
60
40
θ[deg]
10
12
14
16
bc²/GM
10
5
0
−5
−10
δθ[%]
12
14
16
bc²/GM
NR
G¹
G²
G³
G⁴
G⁵
Schw

Results

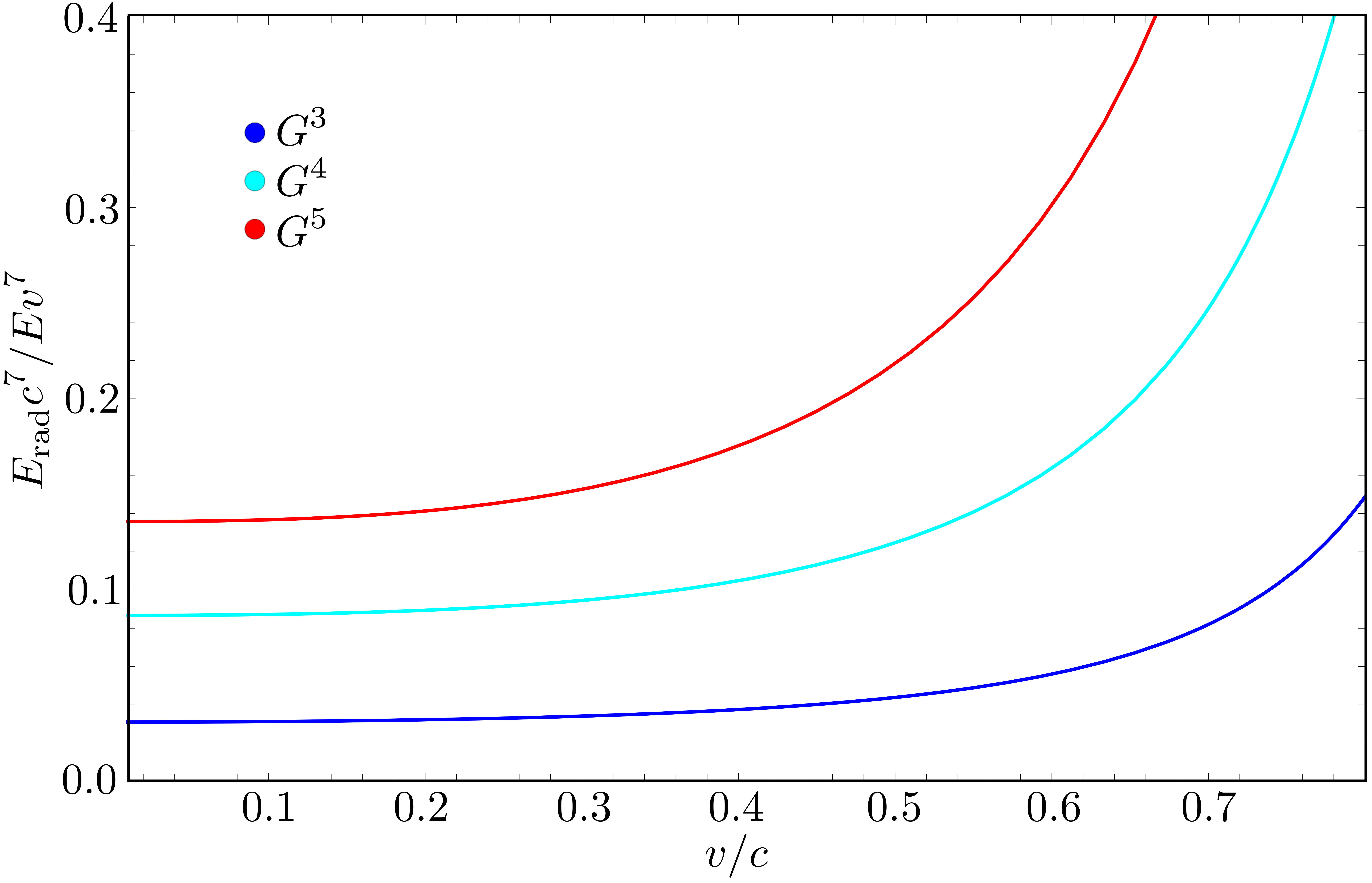
Dissipative results:

Driesse, Jakobsen, Klemm, Mogull, Nega, Plefka, BS, Usovitsch [2411.*****]:

$$P_{rad} = \nu \sum c_k(x) f(x)$$

Function space:

	3PM-1SF	4PM-1SF	5PM-1SF
Kernels of iterated integrals	$\frac{1}{x}, \frac{1}{1-x}, \frac{1}{1+x}$	$\frac{1}{x}, \frac{1}{1-x}, \frac{1}{1+x}, \frac{x}{x^2+1}$	$\frac{1}{x}, \frac{1}{1-x}, \frac{1}{1+x}, \frac{x}{x^2+1}$ $+g(K^2, \omega_{0,CY3}, \alpha, G_i \ldots)$
Max. Weight	1	2	4
Functions from rotation			$K^2, E^2, E \cdot K, \omega_{0,CY3}, \alpha, G_i$ $\omega'_{0,CY3}, \alpha', G'_i, \ldots$



G^3
G^4
G^5
$E_{\rm rad}c^7/Ev^7$
v/c
0.0
0.1
0.2
0.3
0.4
0.1
0.2
0.3
0.4
0.5
0.6
0.7

Summary

- complete 5PM-1SF including dissipative effects
- Control of the function space plays an important role in classical gravitational scattering state of the art calculations
- First instance in classical gravitational scattering in which CY3 makes appearance in physical observable
- Elliptics surprisingly drop out of scattering angle
- Extended special function space in dissipative sector!