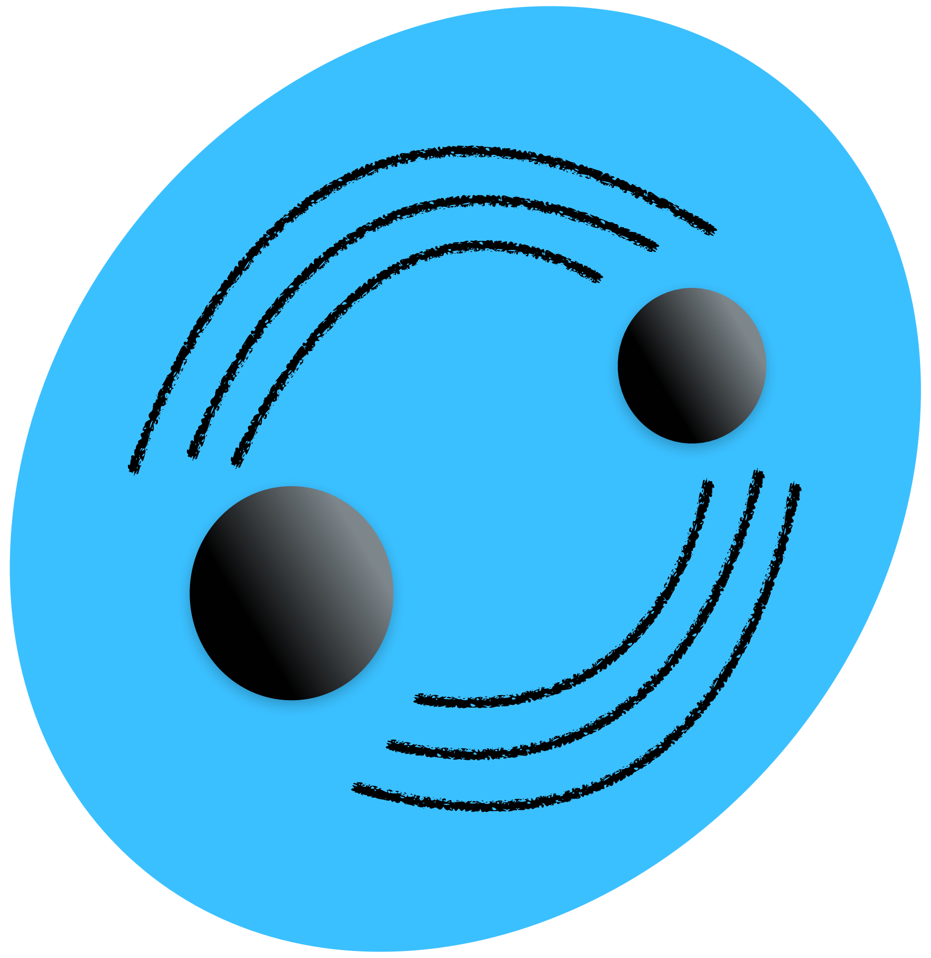
Gravitational Waves from Amplitudes and EFT

Dimitrios Kosmopoulos

Département de Physique Théorique, Université de Genève

EFTs and Beyond - December 5, 2024

Gravitational Waves & Particle Physics

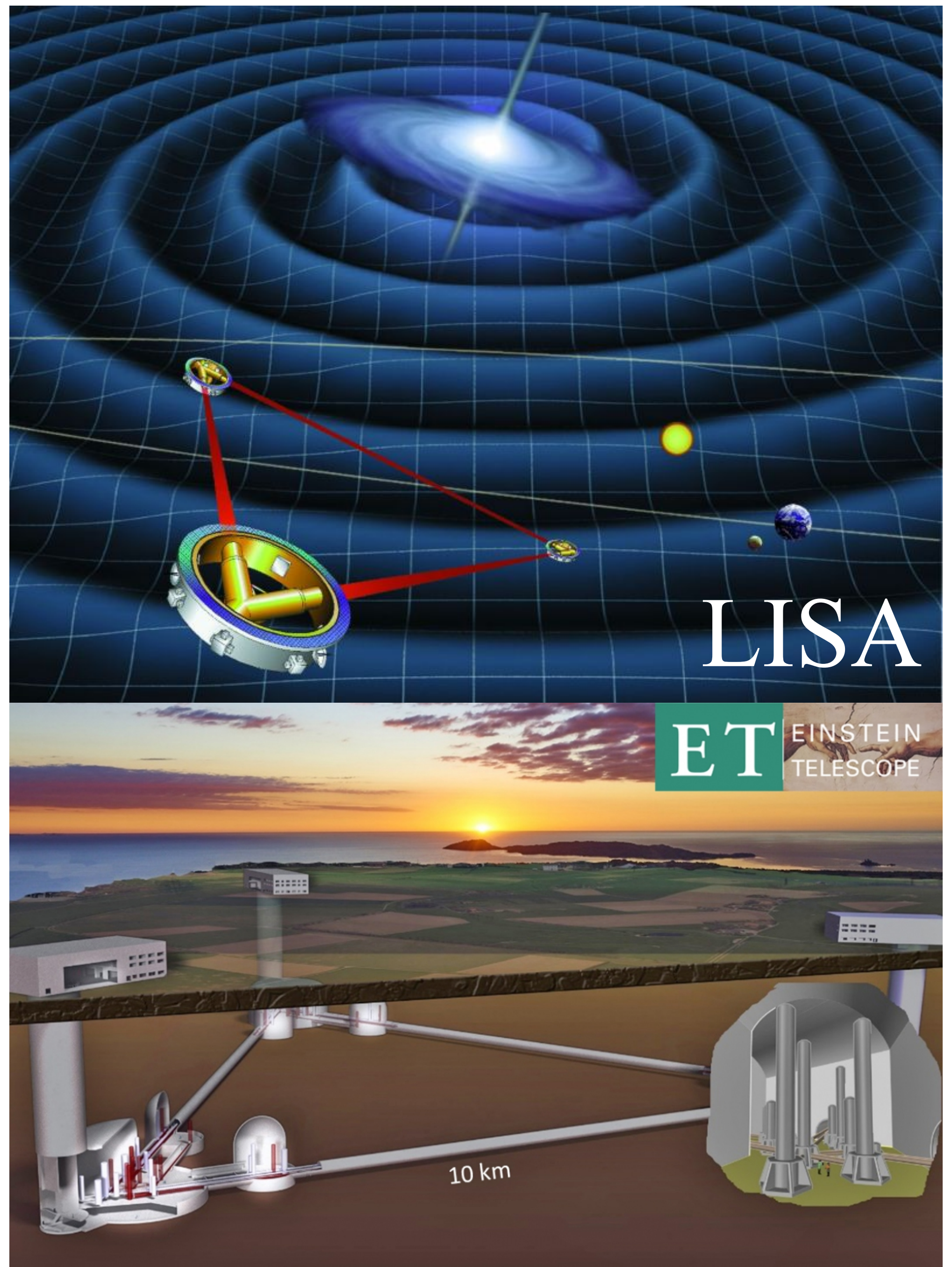


Era of gravitational-wave astronomy

New insights into fundamental open questions

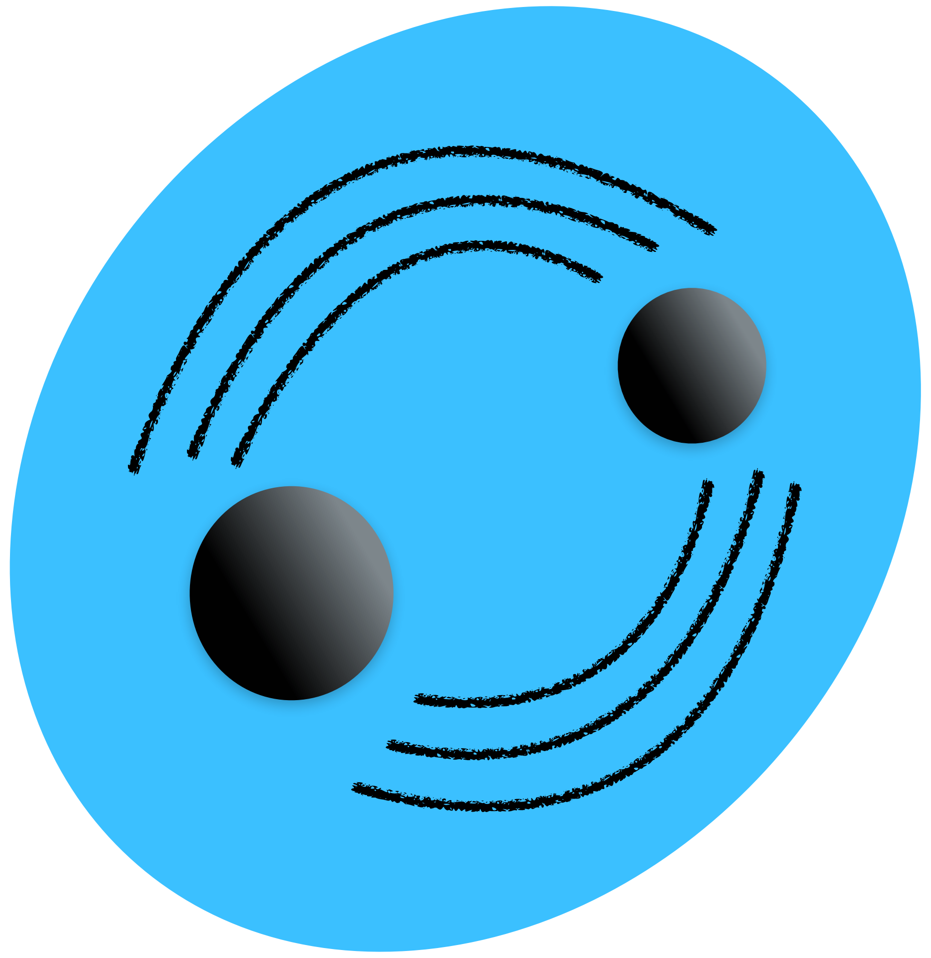
Present experiments

Future experiments



… and many more!

Gravitational Waves & Particle Physics



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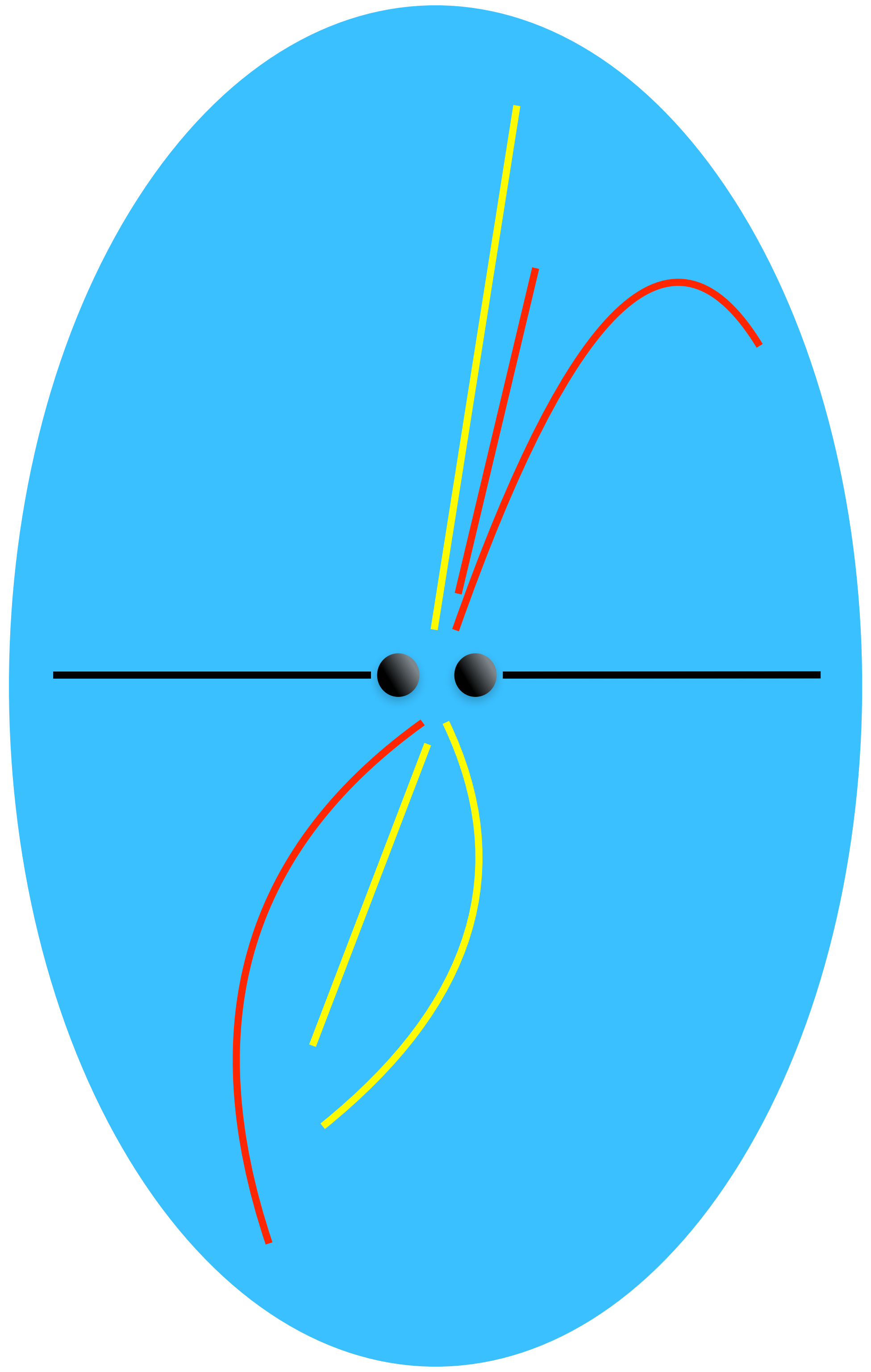
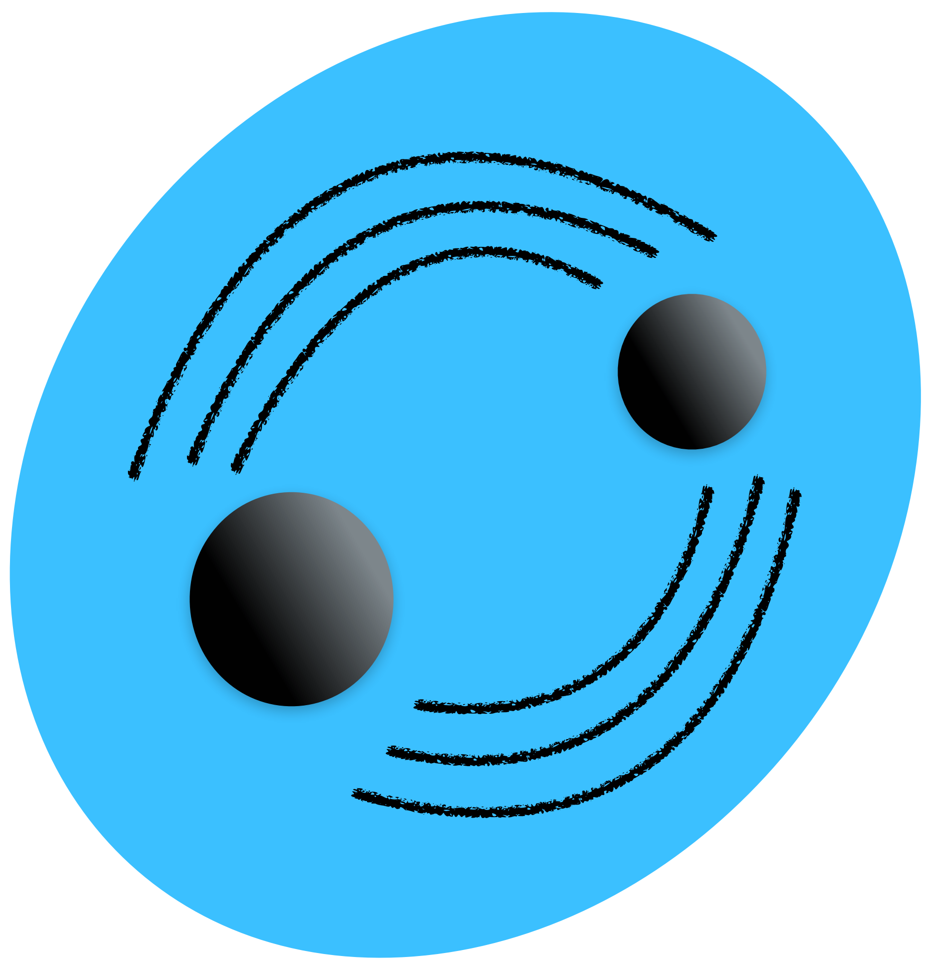


Collider physics: From proton-proton scattering to the Standard Model and beyond

Gravitational-wave physics: From black hole-black hole merging to General Relativity and beyond

Gravitational Waves & Particle Physics



Era of gravitational-wave astronomy

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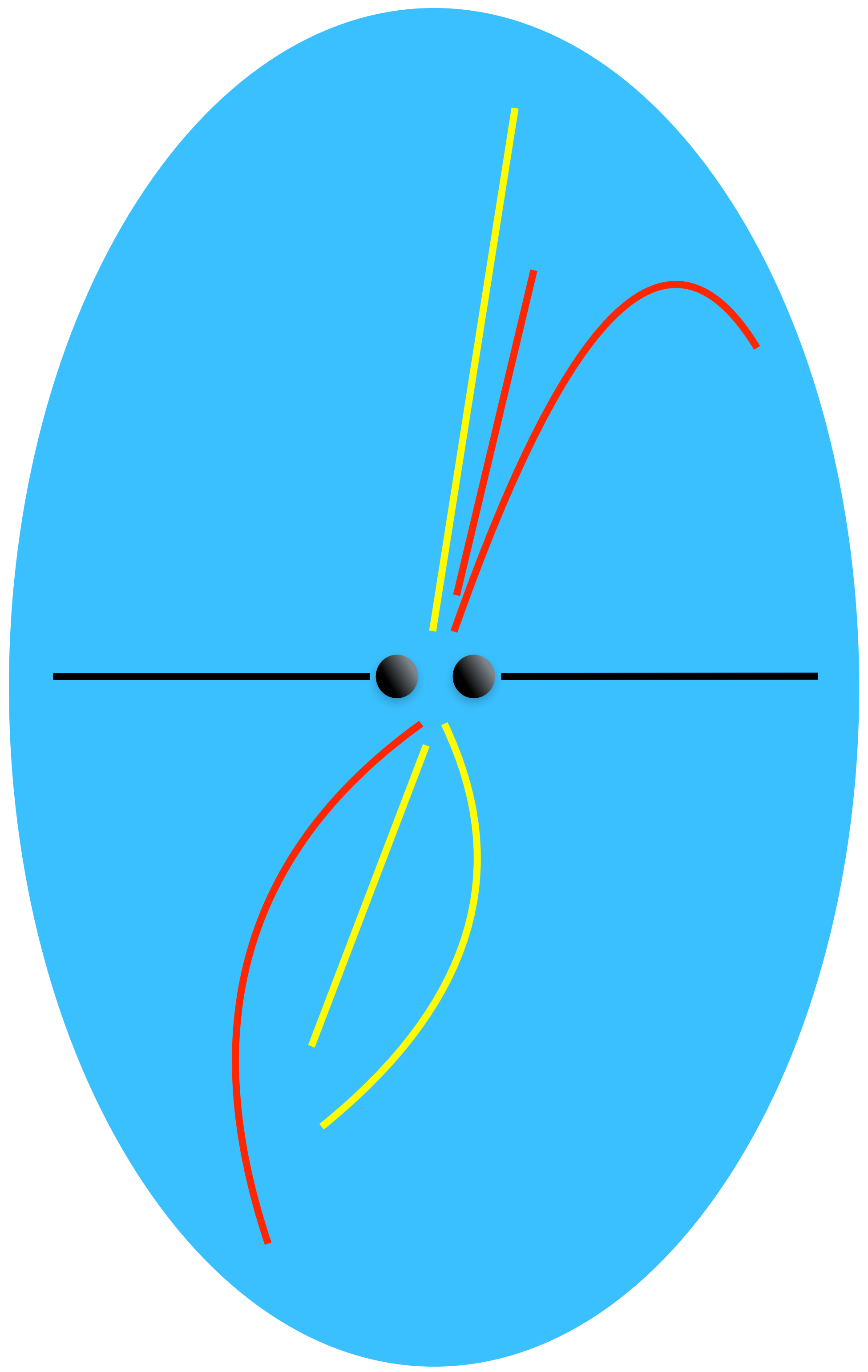
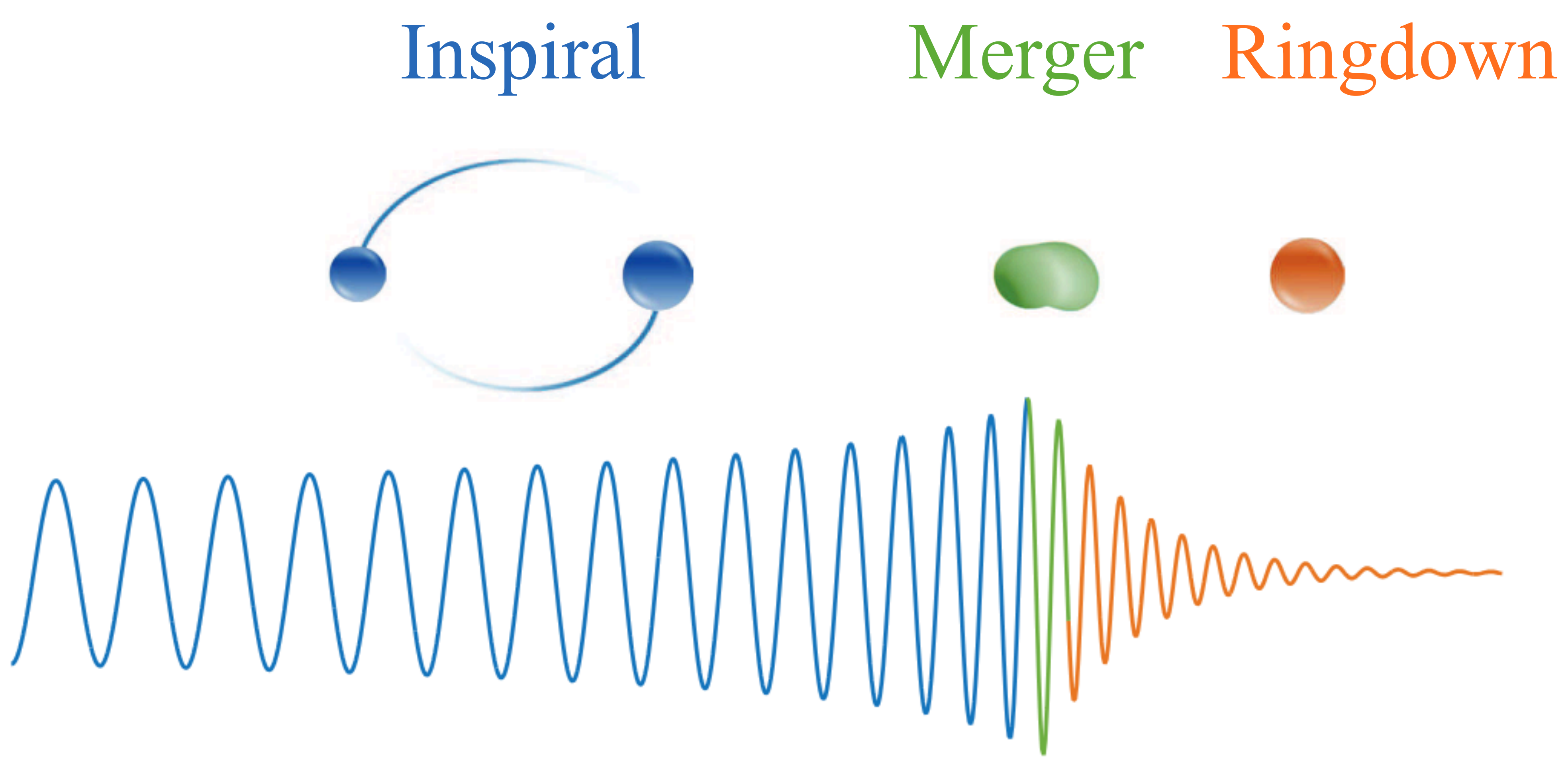


Collider physics: From proton-proton scattering to the Standard Model and beyond

Gravitational-wave physics: From black hole-black hole merging to General Relativity and beyond

Quantum Field Theory for Gravitational-Wave Science

Gravitational-Wave Analysis



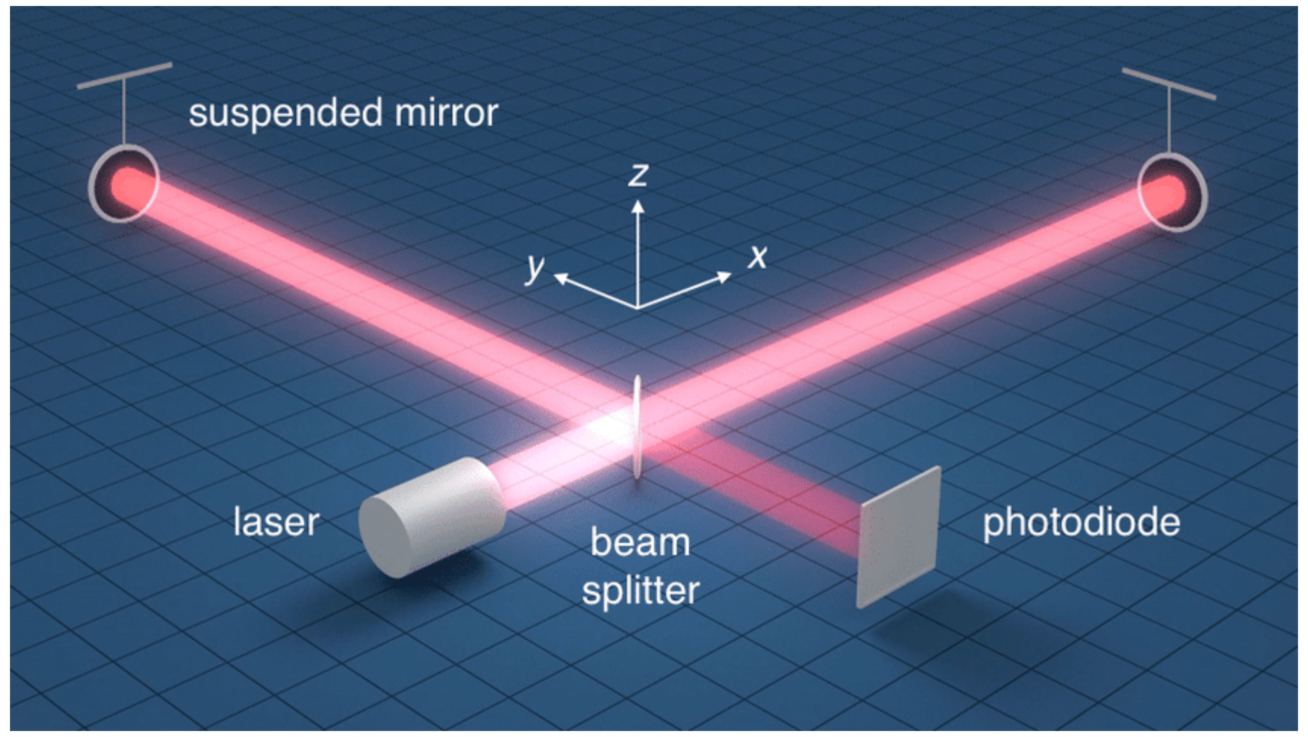
Credit: ANTELIS, MORENO

From the Binary's Evolution to the Observed Waveform

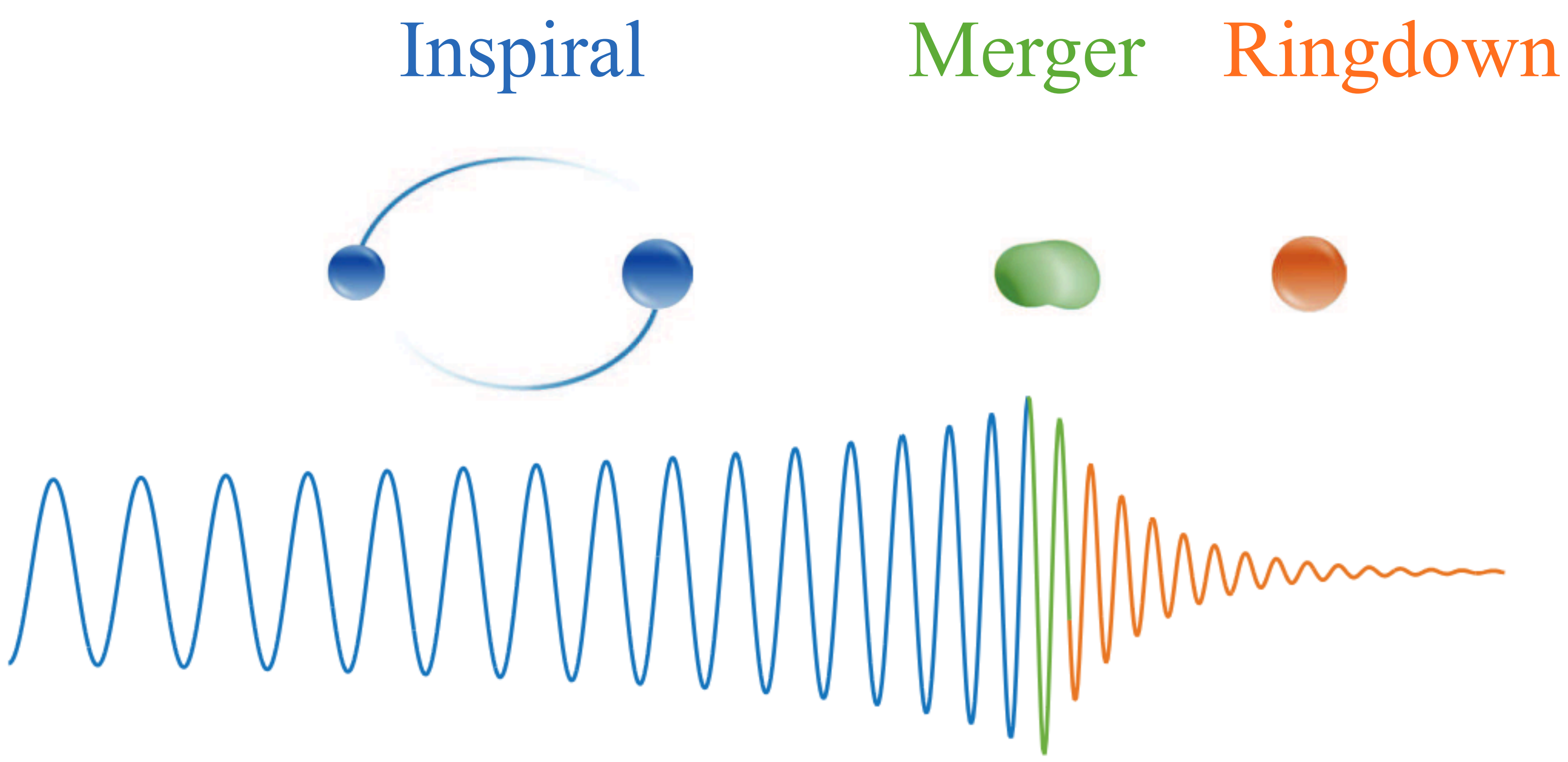
Gravitational-Wave Analysis

Quadrupole radiation formula

$$h \sim \frac{G}{r}\ddot{Q}$$



Credit: https://www.ligo.caltech.edu/



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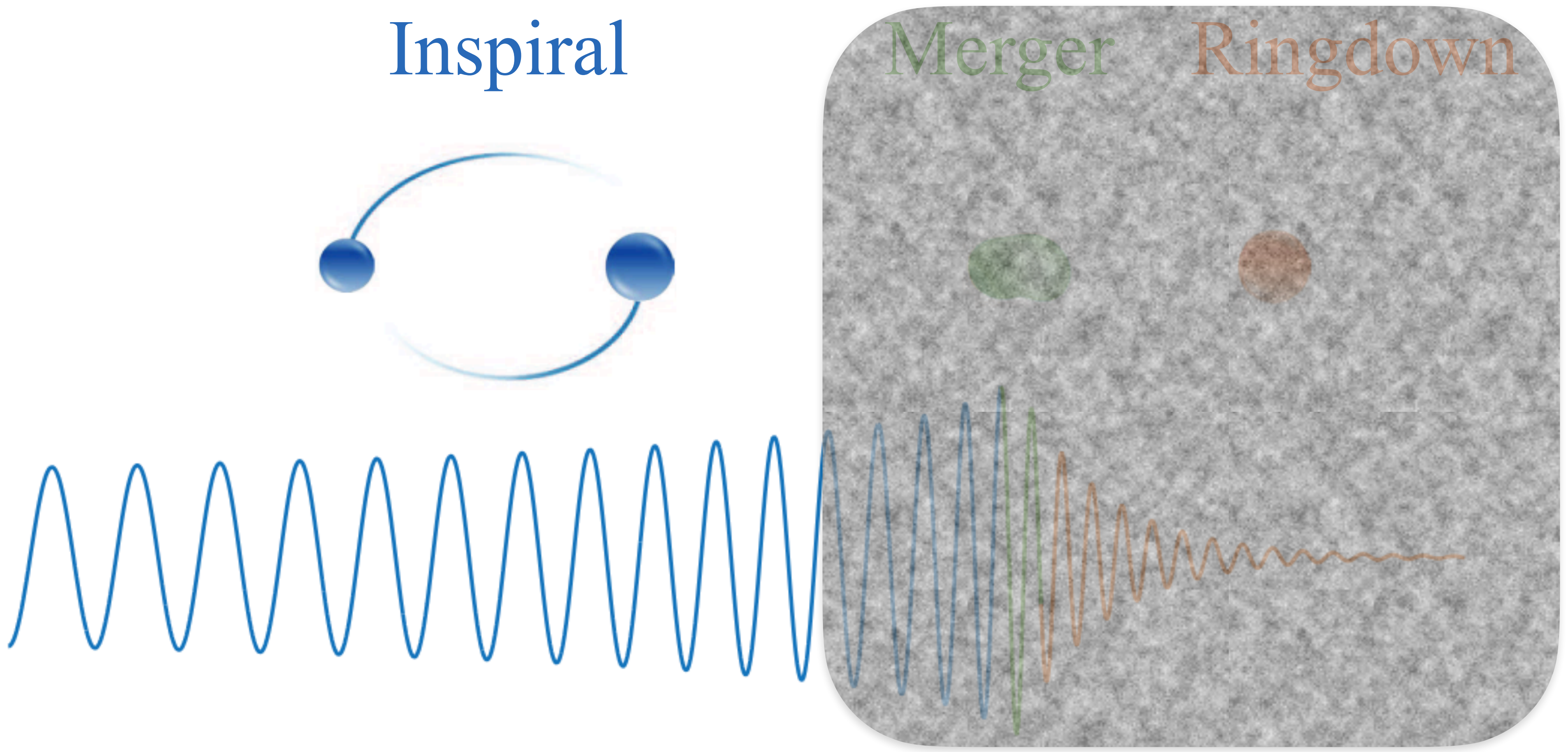

Credit: ANTELIS, MORENO

Adiabatic approximation → Conservative motion via a Hamiltonian H

Adiabatic approximation → Energy loss due to radiation

From the Binary's Evolution to the Observed Waveform

Gravitational-Wave Analysis

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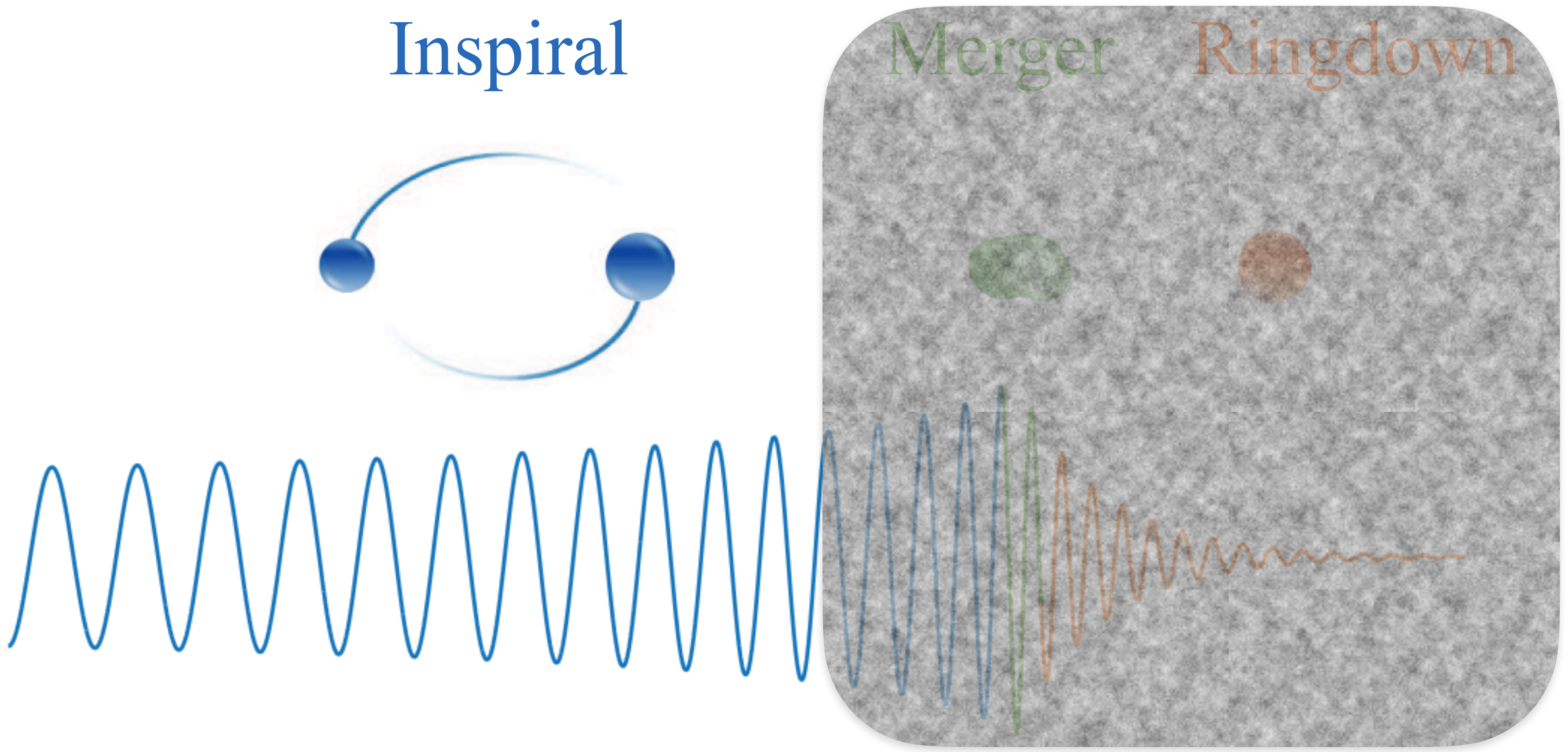

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Adiabatic approximation → Conservative motion via a Hamiltonian H ← This talk
→ Energy loss due to radiation

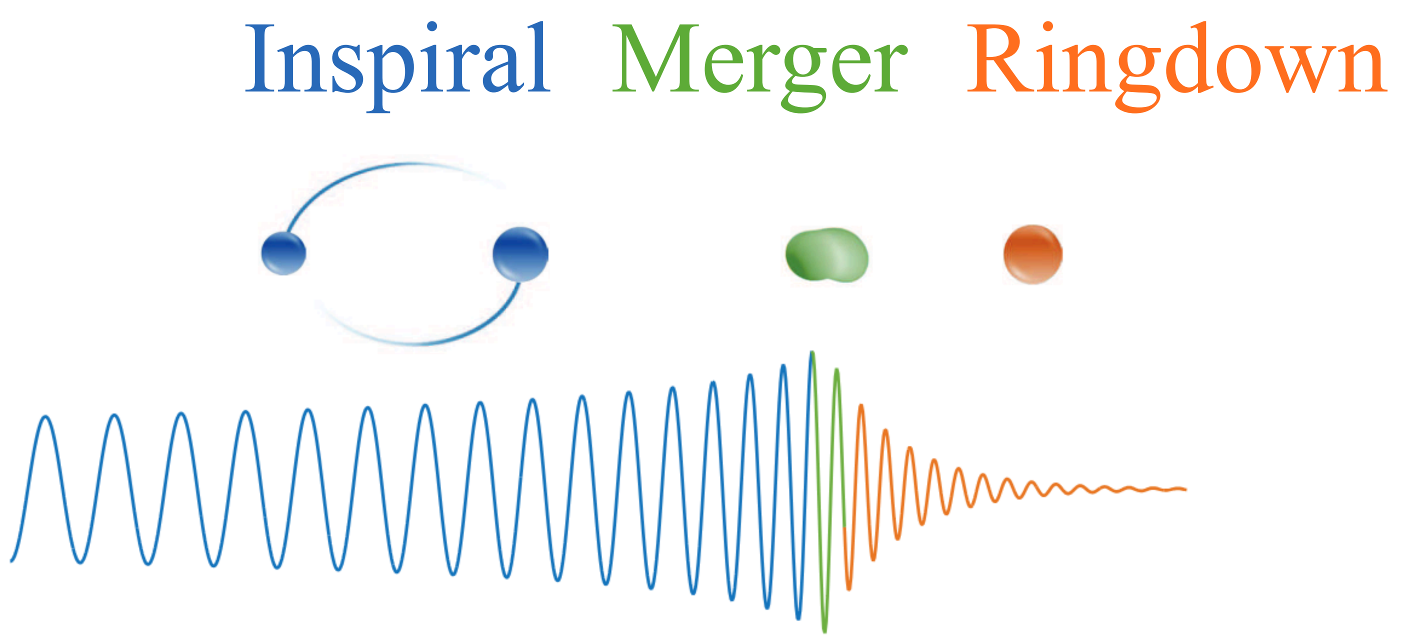
From the Binary's Evolution to the Observed Waveform

Theoretical Gravitational-Wave Analysis

Theoretical template

+

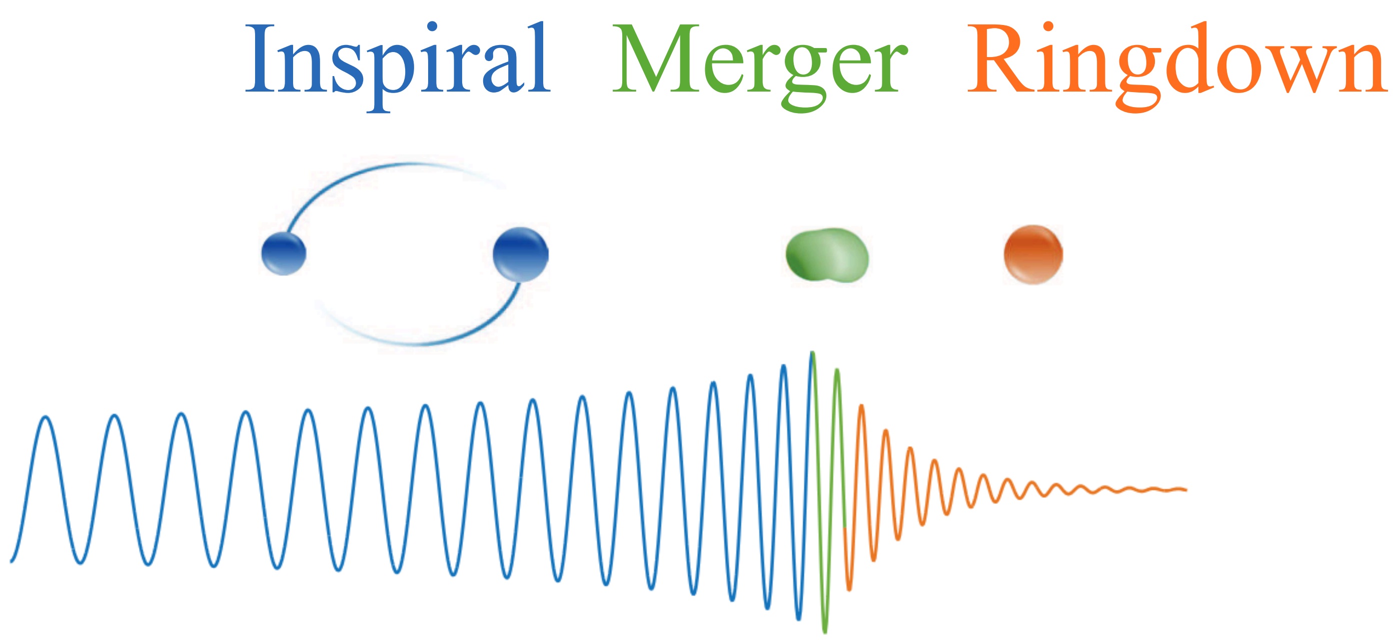
Measurement



Credit: ANTELIS, MORENO

Theoretical Gravitational-Wave Analysis

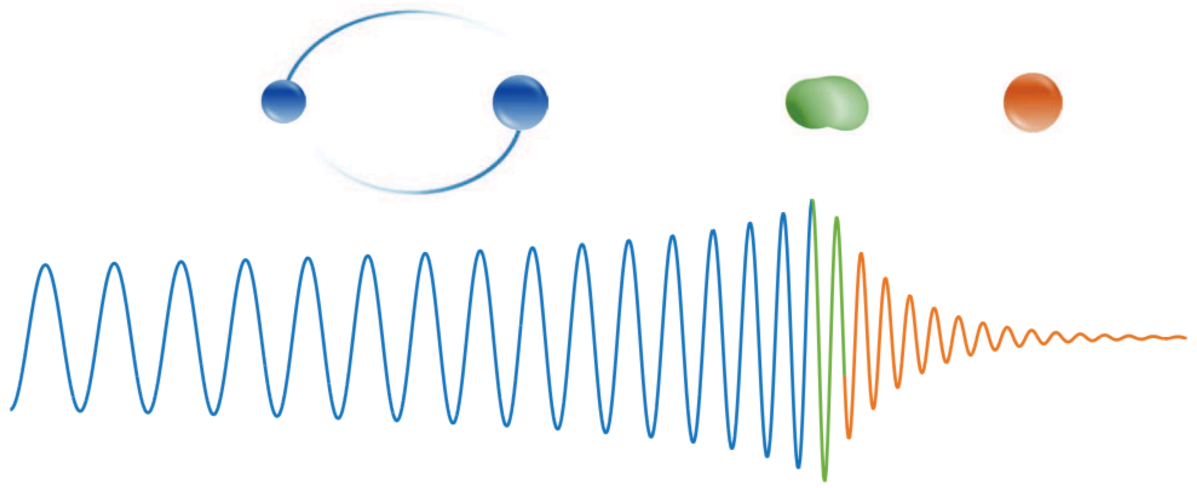
This talk → Theoretical template + Measurement →



Credit: ANTELIS, MORENO

Theoretical Gravitational-Wave Analysis

This talk → Theoretical template →



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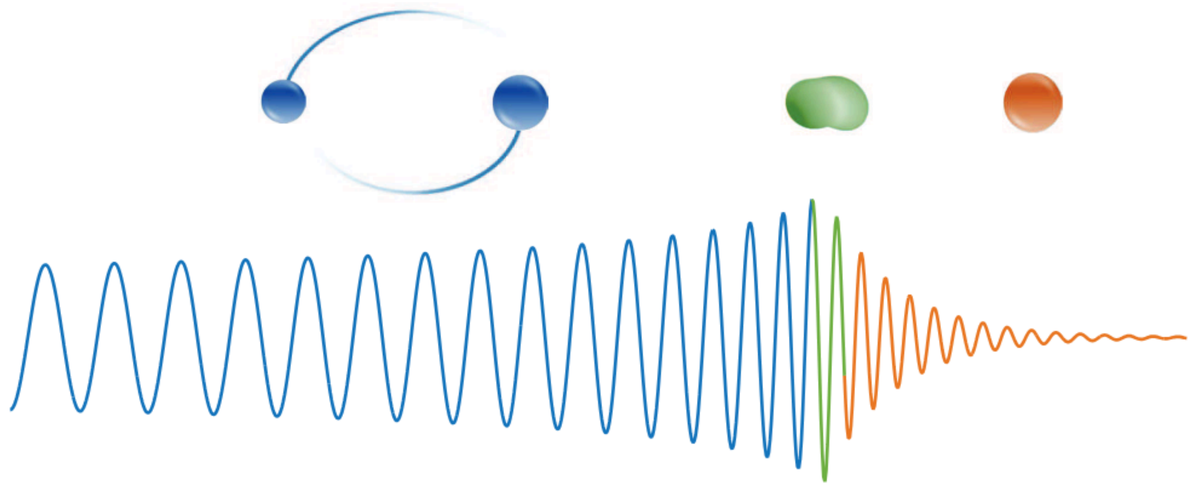
Numerical Relativity (NR)

[Pretorius (2025)]

The 'truth'
Resource intensive

Theoretical Gravitational-Wave Analysis

This talk → Theoretical template →



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<u>Numerical Relativity (NR)</u>

[Pretorius (2025)]

The ‘truth’
Resource intensive

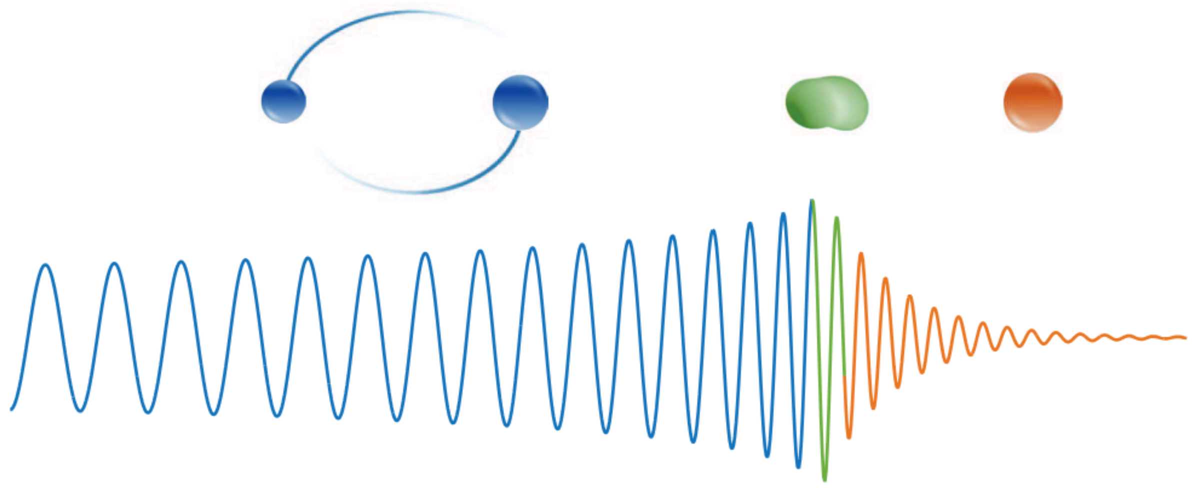
<u>Effective One Body (EOB)</u>

[Buonanno, Damour (1998)]

Model built on theoretical calculations
Verified against NR
Resource efficient

Theoretical Gravitational-Wave Analysis

Theoretical template →



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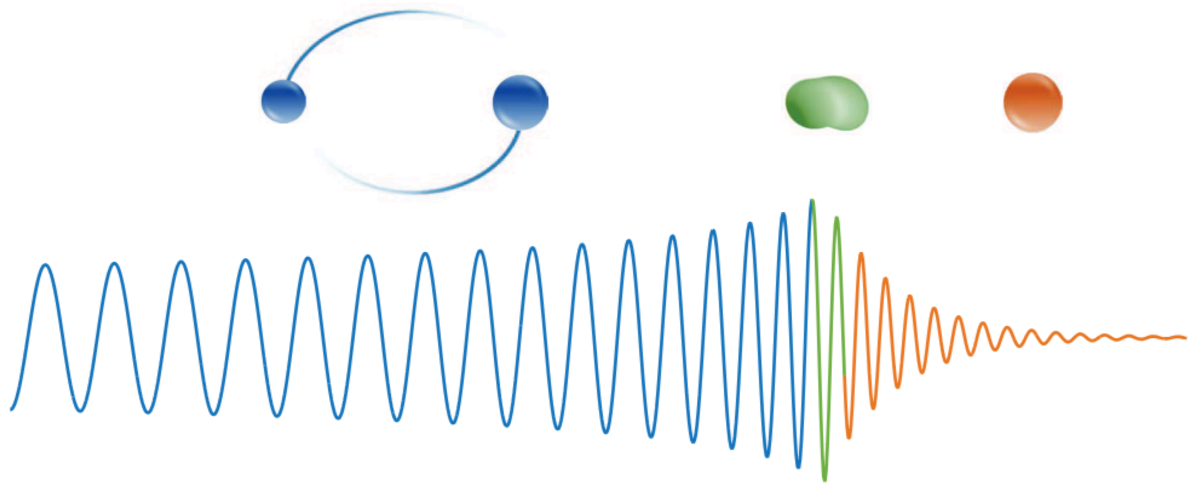
↑ This talk

Theoretical Gravitational-Wave Analysis

Theoretical template →



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Numerical Relativity (NR)

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Effective One Body (EOB)

[Buonanno, Damour (1998)]

This talk → EOB → Observed waveform

LIGO-Virgo-KAGRA
theoretical analysis

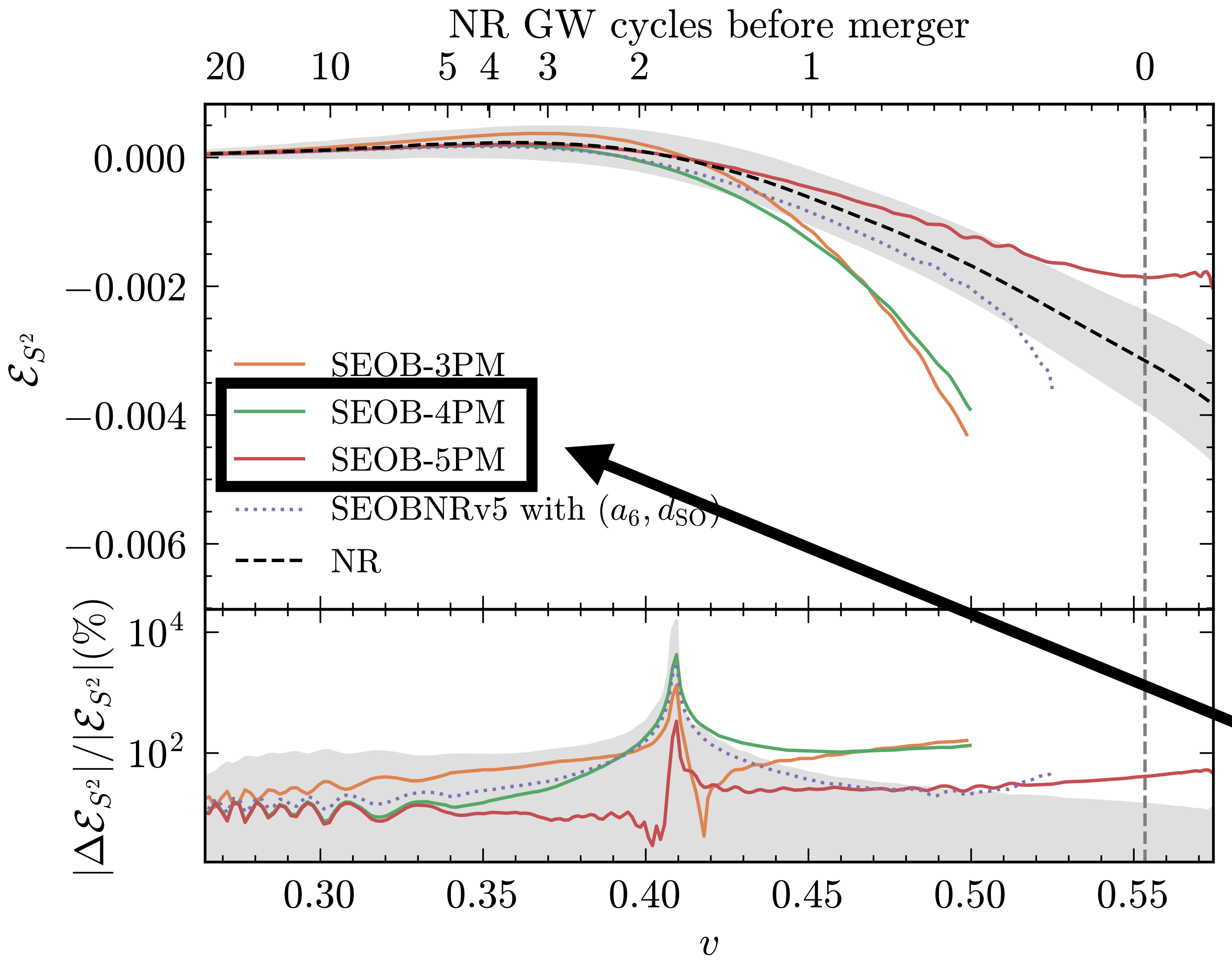
LIGO-Virgo-KAGRA
experimental analysis

Theoretical Gravitational-Wave Analysis

EOB & NR comparison

[Buonanno, Mogull, Patil, Pompili (2024)]

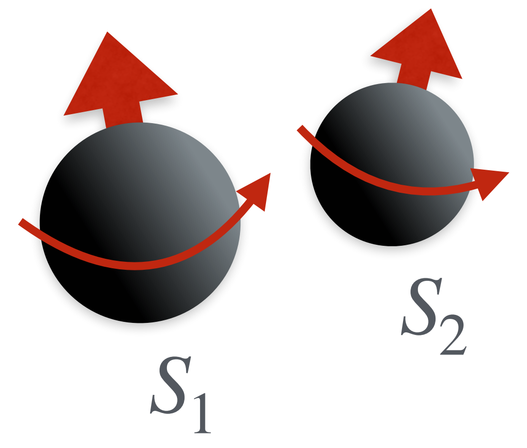
Binding energy



Relative velocity

$n\text{PM} = \mathcal{O}\left(G^a S^b\right)$, with $a + b = n$

Physical Post-Minkowiskian counting



Imports [**DK**, Luna (2021)]

EOB NR

EOB: Effective One Body
NR: Numerical Relativity

The Precision Frontier

Spin ($|\mathbf{S}|/|\mathbf{L}|$) →

Gravitational coupling $\left(\frac{GM}{r}\right)$ ↓

	S^0	S^1	S^2	S^3	S^4	S^5
G^1	✓	✓	✓	✓	✓	✓
G^2	✓	✓	✓	✓	✓	Partial
G^3	✓	✓	✓			
G^4	✓	✓				
G^5	Partial					

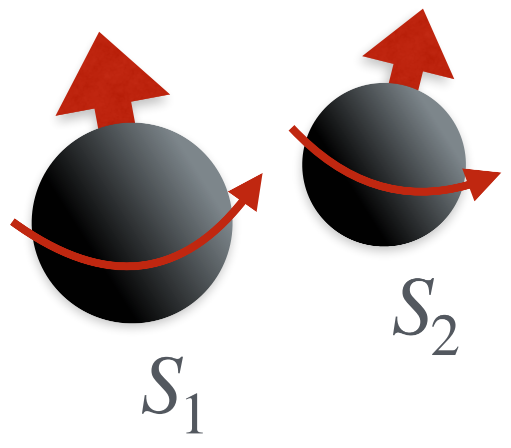
[**DK**, Luna (2021)] [Bern, **DK**, Luna, Roiban, Teng (2022)]

Precision goal for future gravitational-wave detectors (e.g., Cosmic Explorer, Einstein Telescope, Lisa)

$n\text{PM} = \mathcal{O}\left(G^a S^b\right)$, with $a + b = n$

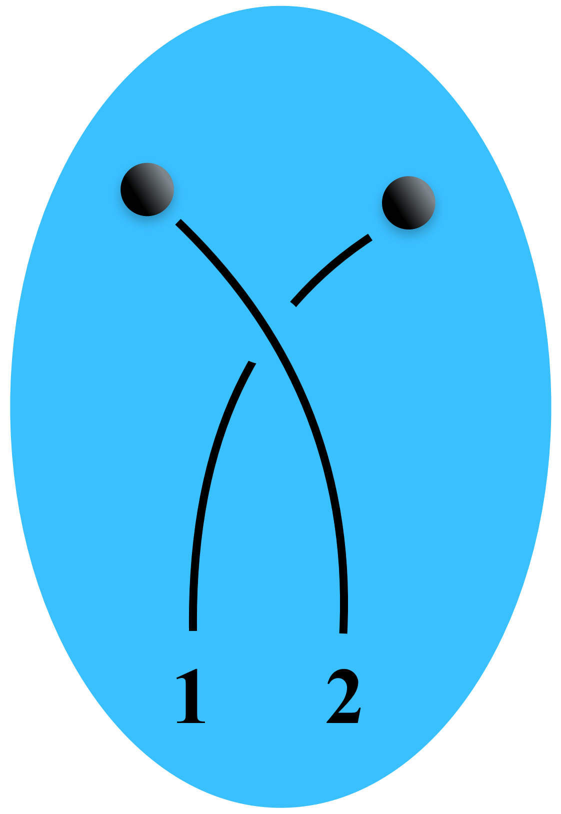
Physical Post-Minkowiskian counting

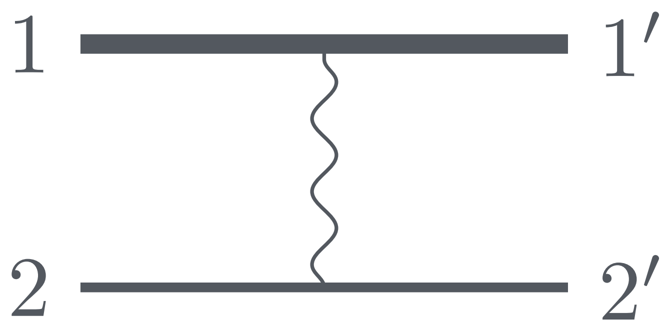
Same color = Same physical PM order



From Quantum Amplitudes to Classical Hamiltonians

Amplitude to Potential: Tree-Level Matching



$\mathscr{A}_4 \sim$ 

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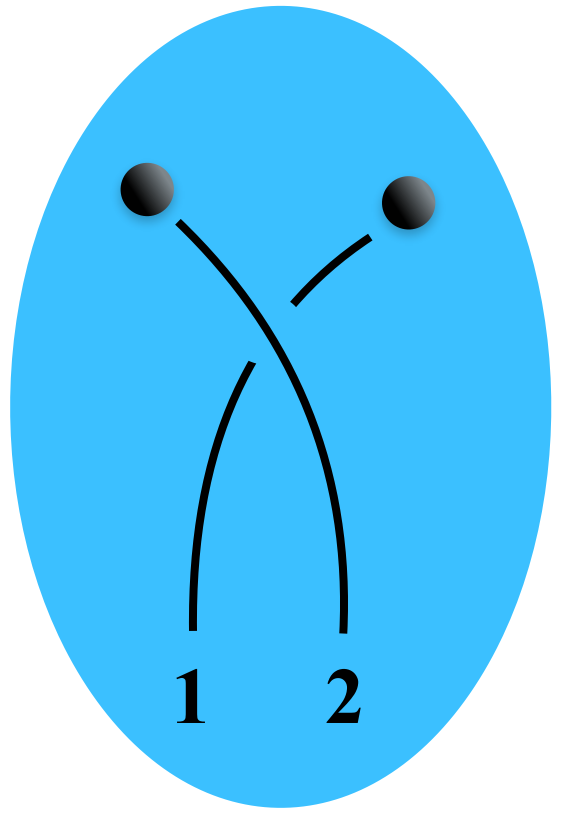

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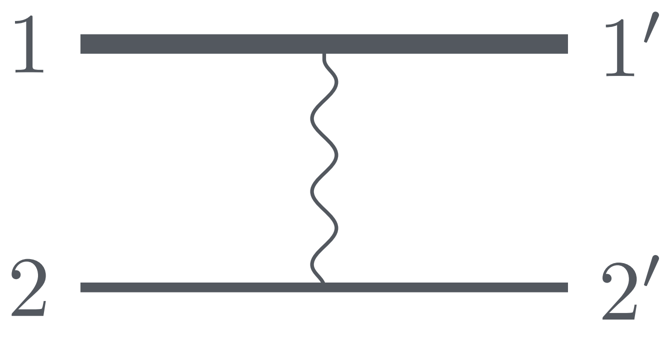
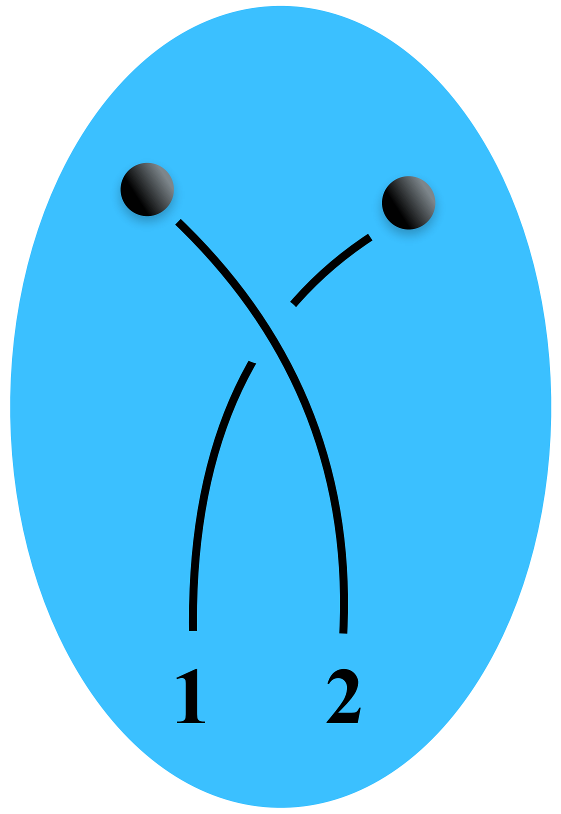
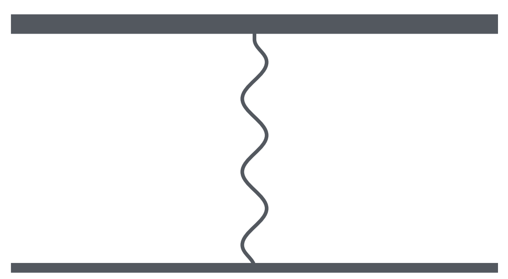


$$\sim G \frac{p_1^\mu p_1^\nu \mathscr{P}_{\mu\nu\alpha\beta} p_2^\alpha p_2^\beta}{q^2}$$

Amplitude to Potential: Tree-Level Matching



$\mathcal{A}_4 \sim$



$$\sim G \frac{p_1^\mu p_1^\nu \mathscr{P}_{\mu\nu\alpha\beta} p_2^\alpha p_2^\beta}{q^2} \sim E_1 E_2 \left(\frac{G\, n(\mathbf{p})}{\mathbf{q}^2} \right)$$

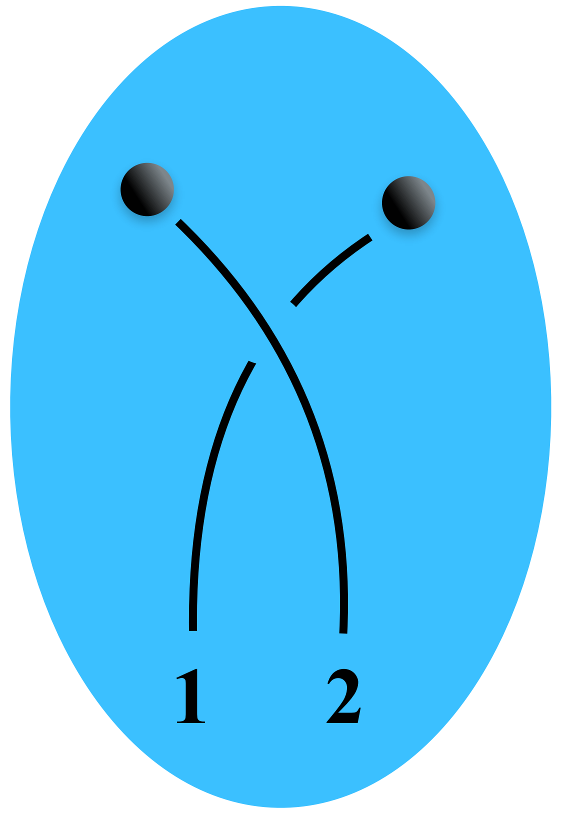
(CoM)

(CoM)

$p_1 = \left(E_1, \mathbf{p}\right)$

$p_2 = \left(E_2, -\mathbf{p}\right)$

Amplitude to Potential: Tree-Level Matching



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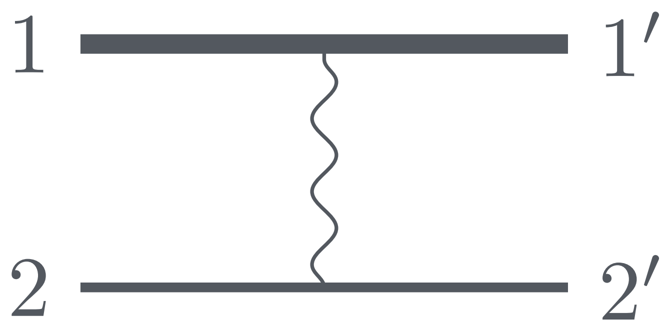

$$\sim G\frac{p_1^\mu p_1^\nu \mathcal{P}_{\mu\nu\alpha\beta} p_2^\alpha p_2^\beta}{q^2} \sim E_1E_2\left(\frac{G\,n(\mathbf{p})}{\mathbf{q}^2}\right) \sim E_1E_2V(\mathbf{p},\mathbf{q})$$

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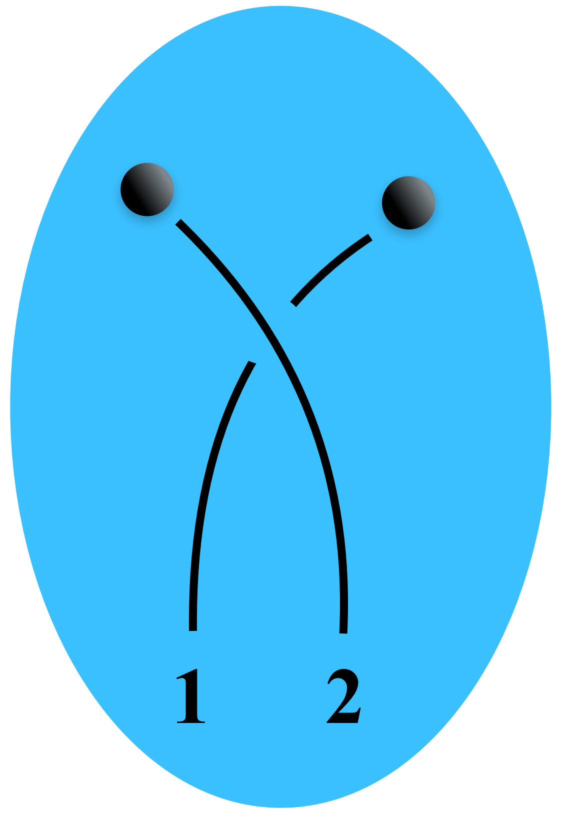
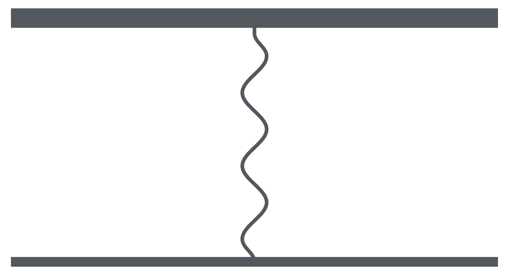


$\mathscr{A}_4 \sim$  $\sim G \dfrac{p_1^\mu p_1^\nu \mathscr{P}_{\mu\nu\alpha\beta} p_2^\alpha p_2^\beta}{q^2} \sim E_1 E_2 \left(\dfrac{G\, n(\mathbf{p})}{\mathbf{q}^2} \right) \sim E_1 E_2 V(\mathbf{p}, \mathbf{q})$

(CoM)

1PM potential $\quad V(\mathbf{p}, \mathbf{r}) = -\dfrac{G\, n(\mathbf{p})}{r}, \quad n(\mathbf{p}) = m_1 m_2 + \mathcal{O}\left(\mathbf{p}^2\right)$

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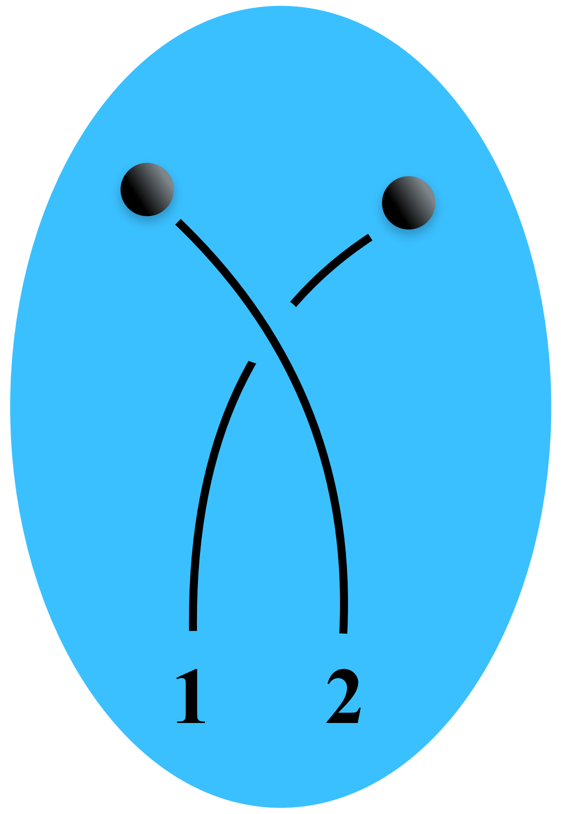

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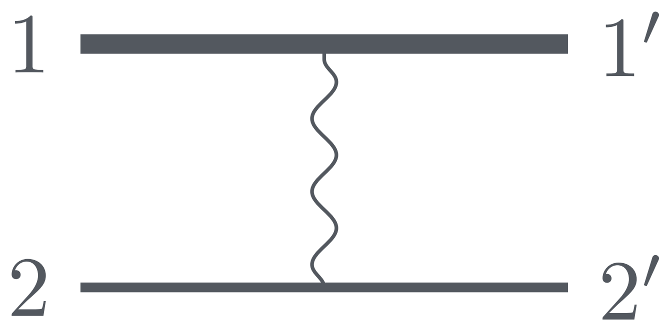

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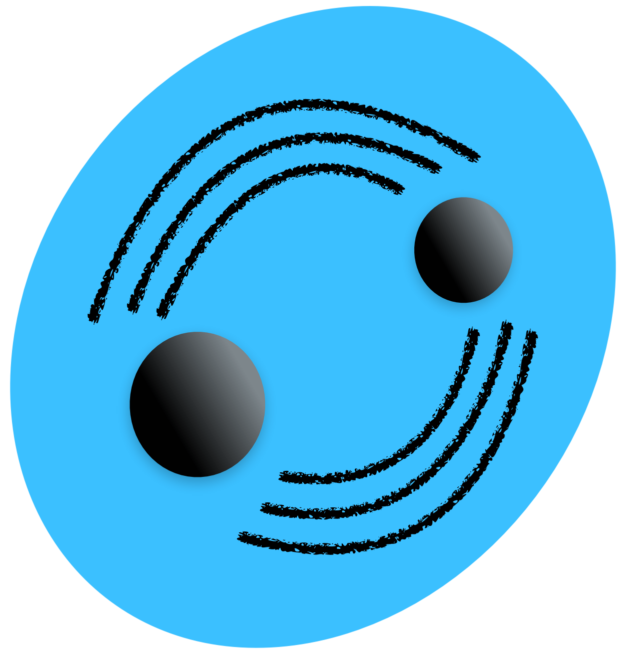

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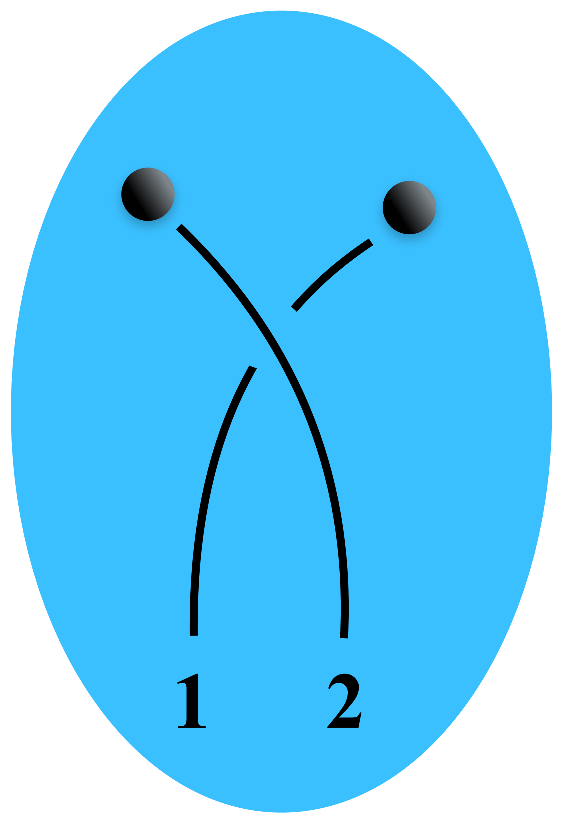
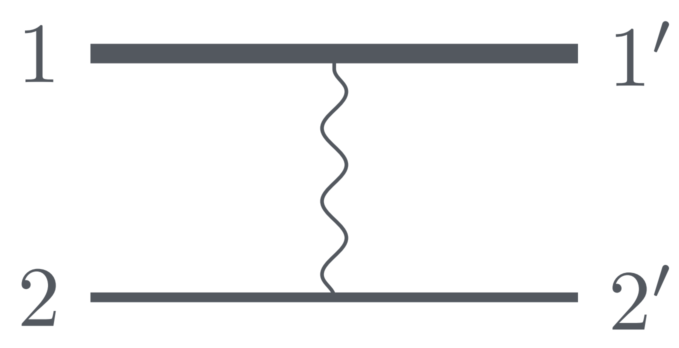


$\mathcal{A}_4 \sim$ 

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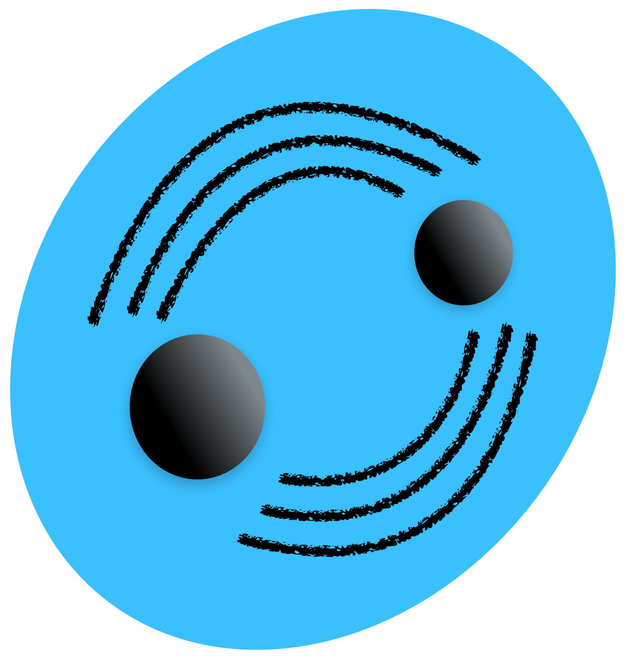
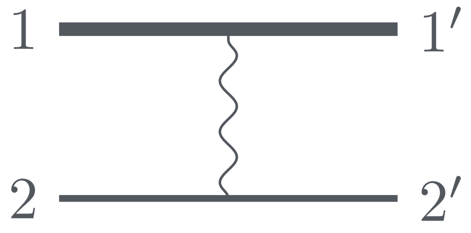
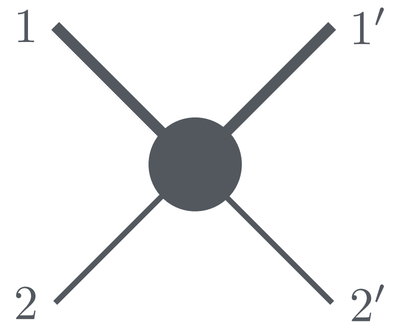


Full theory

$\mathcal{A}_4^{\text{FT}} \sim$ 

Effective theory

$\mathcal{A}_4^{\text{EFT}} \sim$ 

$\sim V(\mathbf{p},\mathbf{q})$

(CoM)

$p_1 = \left(E_1, \mathbf{p}\right)$

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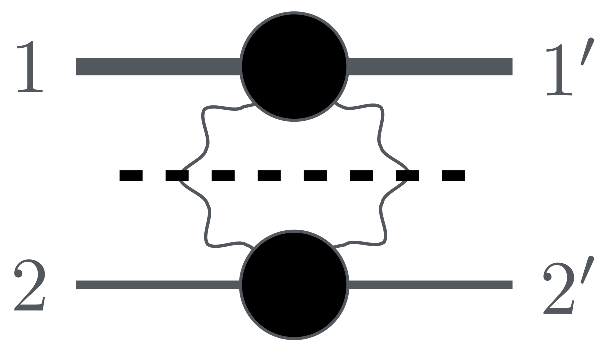
Amplitude to Potential: Loop-Level Matching

Full theory

$\mathscr{A}_4^{\mathrm{FT}} \sim$ 

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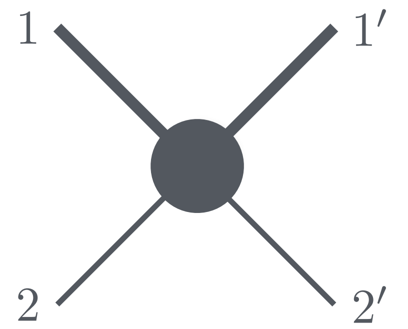
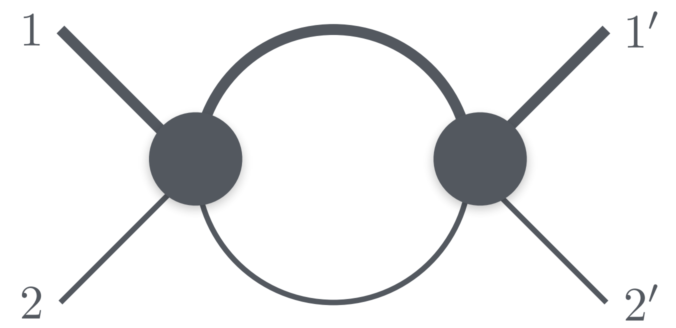
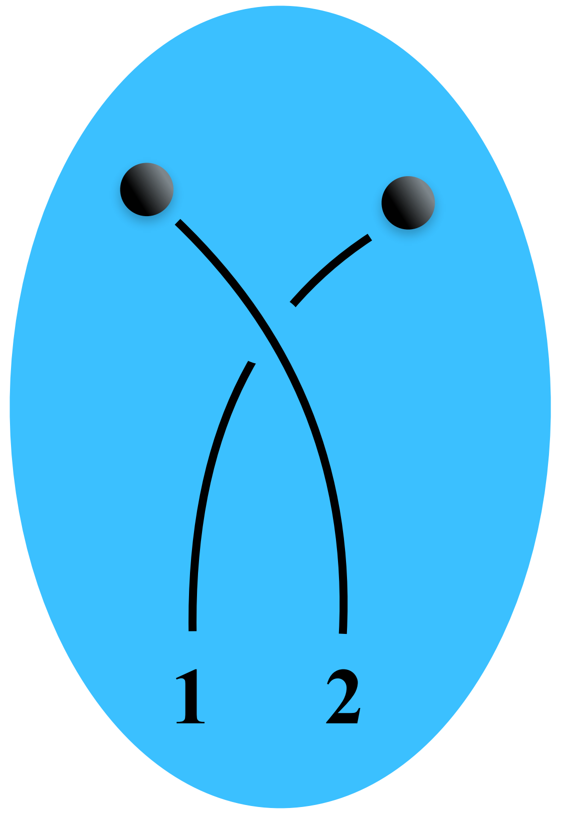
$\mathscr{A}_4^{\mathrm{EFT}} \sim$ 

$+$ 

$+ \quad \dots$



The $\mathcal{O}\left(G^2\right)$ potential

$$\delta V(\mathbf{p}, \mathbf{r}) = -\frac{G^2\, \tilde{n}(\mathbf{p})}{r^2}$$

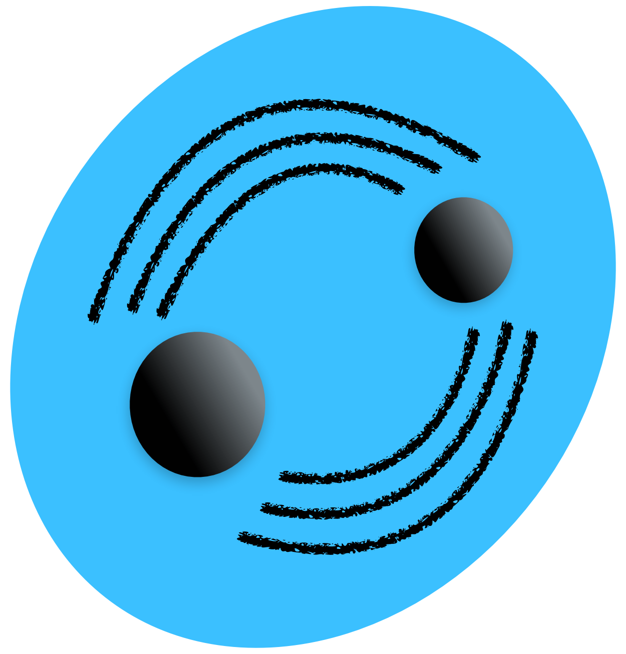
[Cheung, Rothstein, Solon (2018)]

(Non-spinning case)

Obtaining the $\mathcal{O}\left(G^2\right)$ Potential

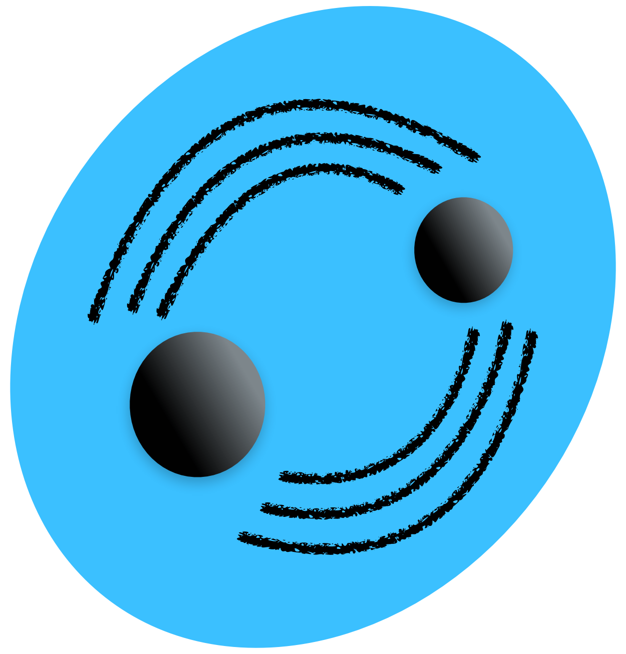
Modeling Non-Spinning Compact Objects in General Relativity

Modeling Compact Objects



Matter (traditionally): $S_M = -m \int d\tau$ Worldline picture

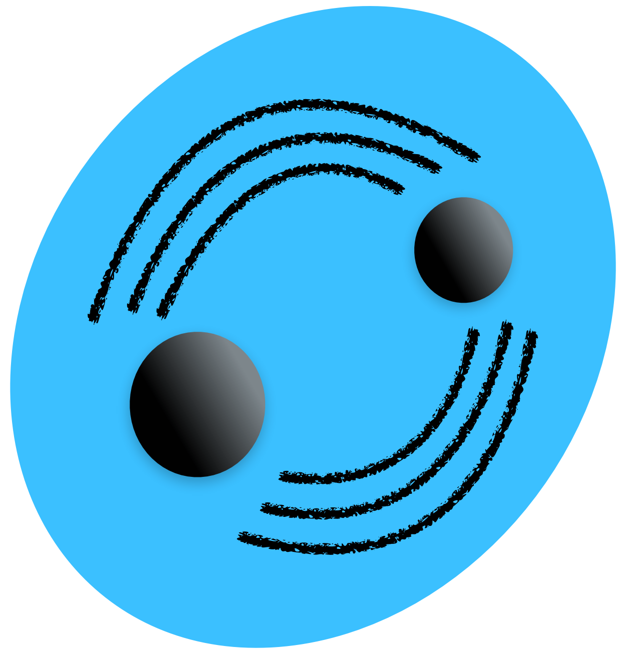
Modeling Compact Objects



Matter (traditionally): $S_M = -m \int d\tau$ Worldline picture

Effective description:
Replace black hole/neutron star with point particle.

Modeling Compact Objects



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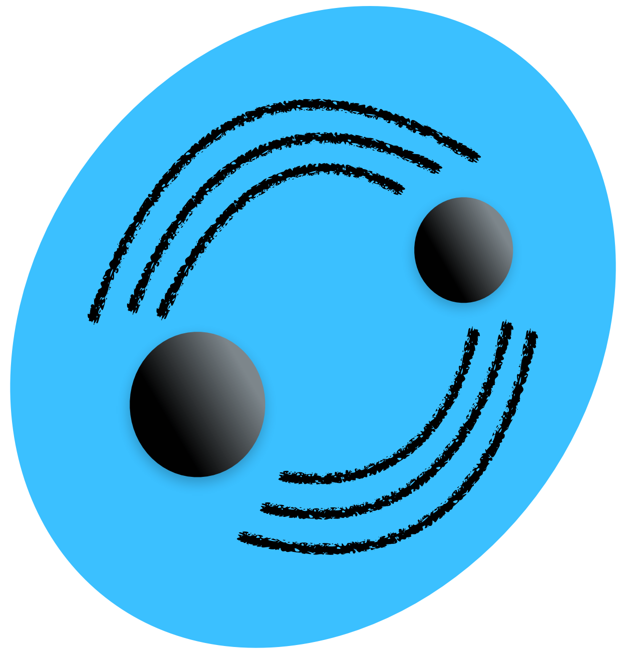
Matter: $S_M = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right)$ QFT picture

Modeling Compact Objects



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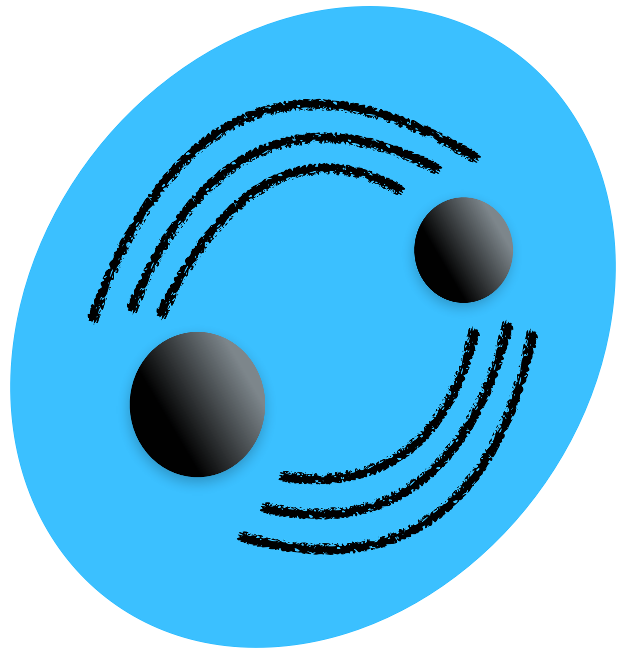
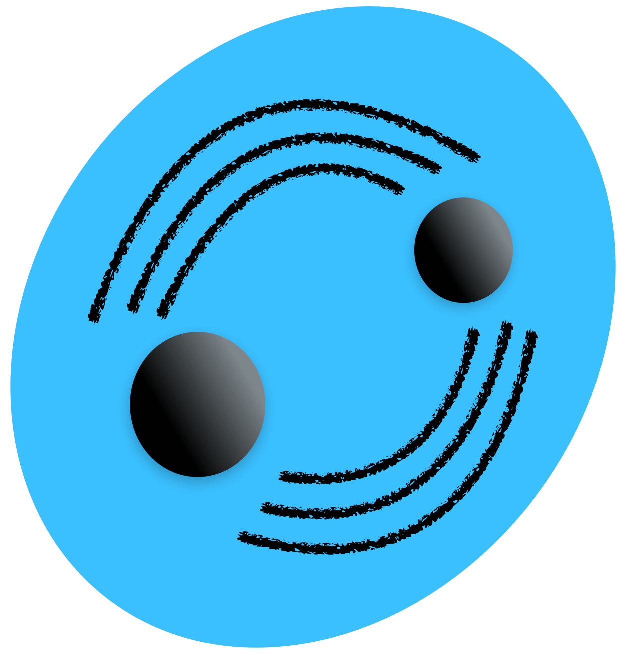
QFT/Amplitudes tools: Unitarity, Double copy, On-shell recursion, … + Integration technology from collider physics ⟶ New results

Computing the Amplitude



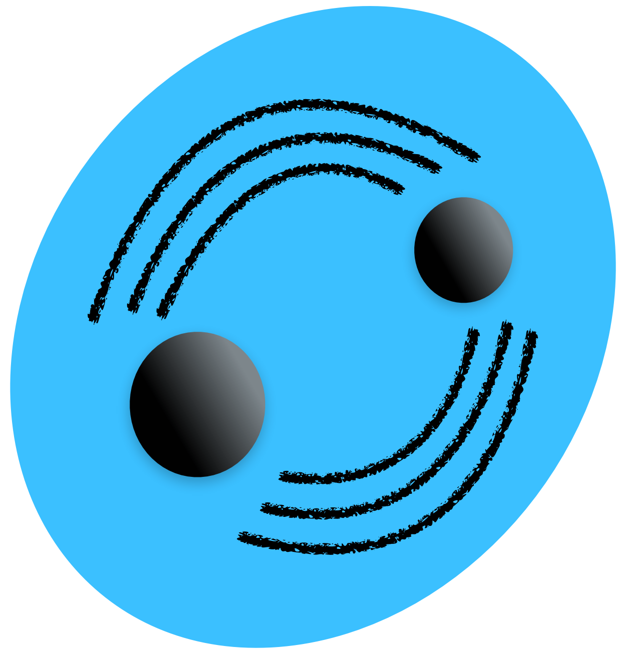
$$S_M = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right)$$

Computing the Amplitude



$$S_M = \int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2\right)$$

Computing the Amplitude



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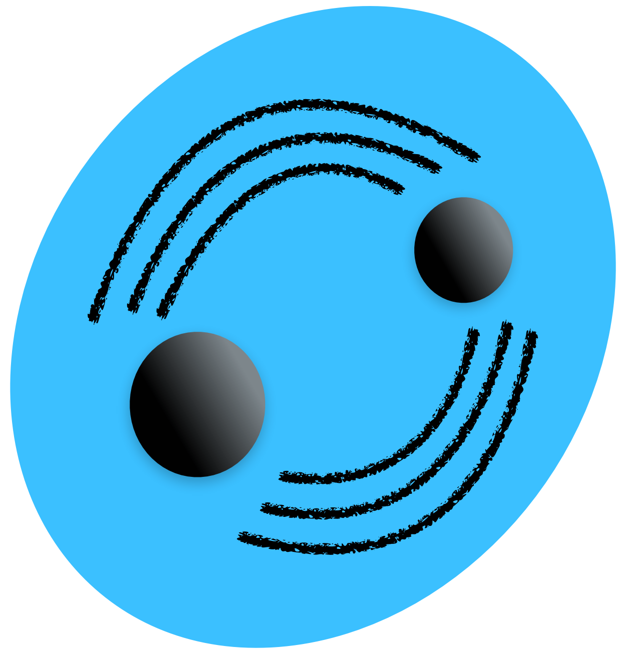

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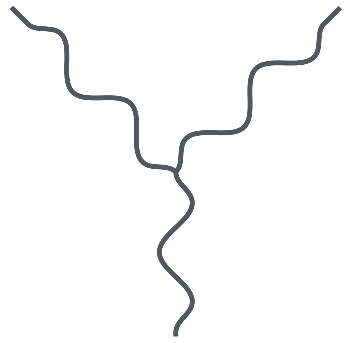
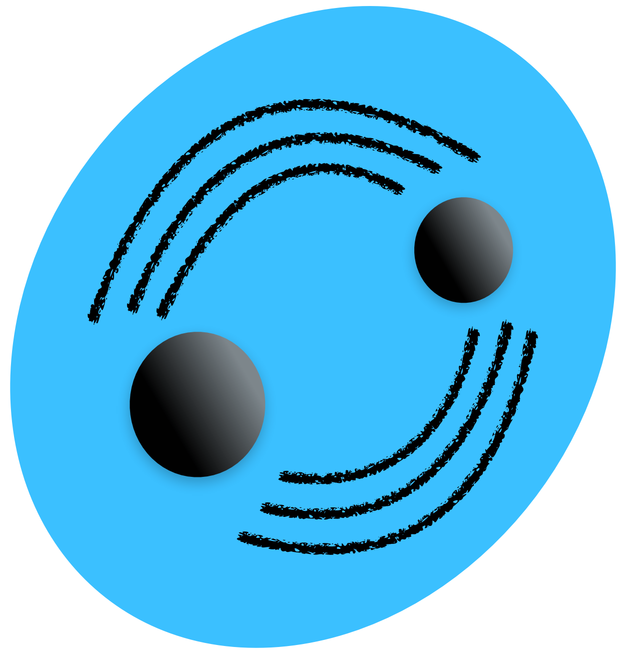


The Amplitude at $\mathcal{O}\left(G^2\right)$ Required for Extracting the Hamiltonian

Computing the Amplitude



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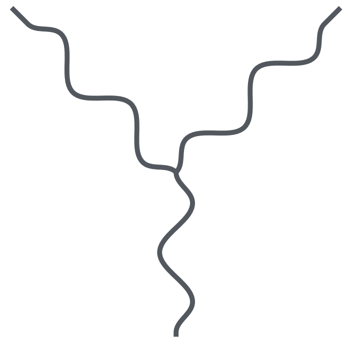
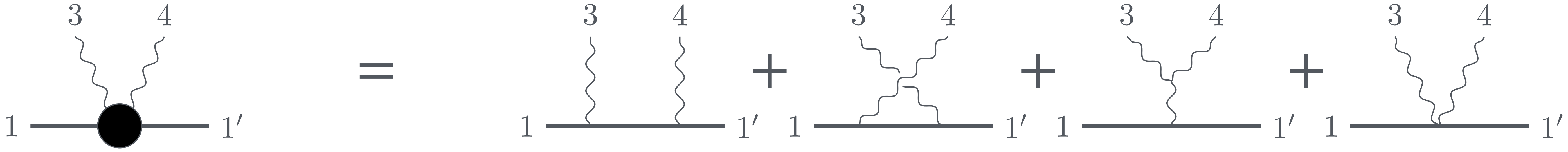


$\mathscr{A}^{\mathrm{C}}$ =



$\mathscr{A}^{\mathrm{C}} \leftrightarrow$ Compton
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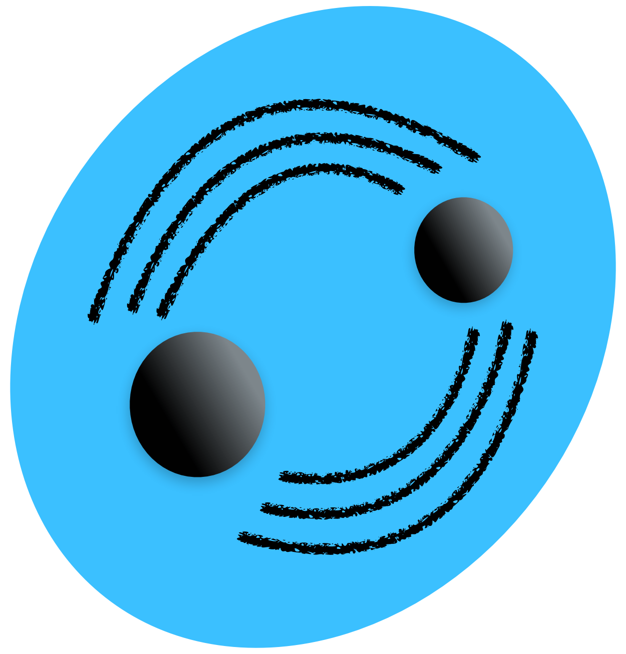
The Amplitude at $\mathcal{O}\left(G^2\right)$ Required for Extracting the Hamiltonian

Computing the Amplitude



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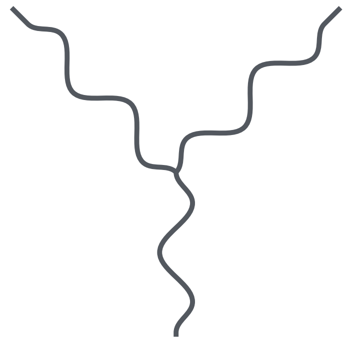

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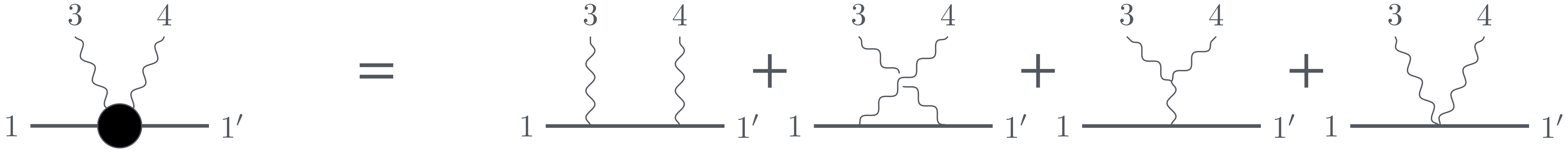
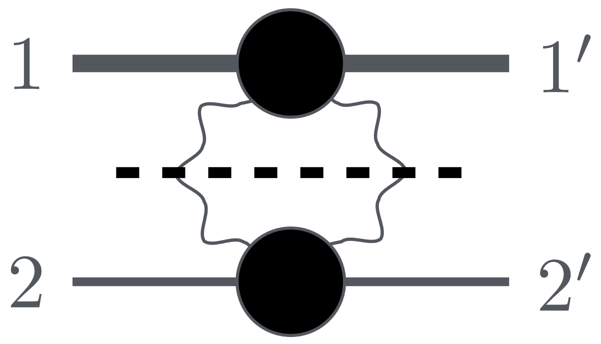


$\mathscr{A}_4^{\mathrm{FT}} \sim$



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The Amplitude at $\mathcal{O}\left(G^2\right)$ Required for Extracting the Hamiltonian

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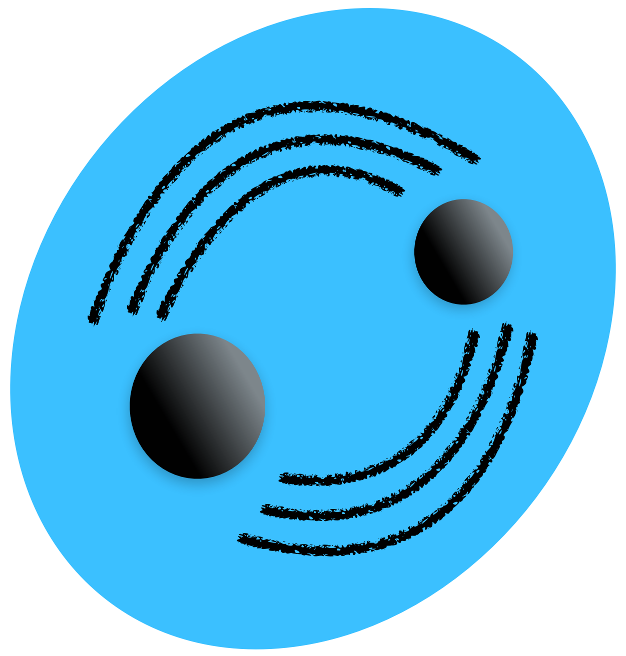

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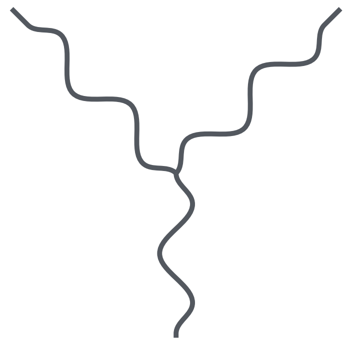

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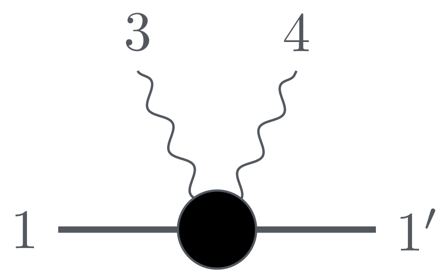

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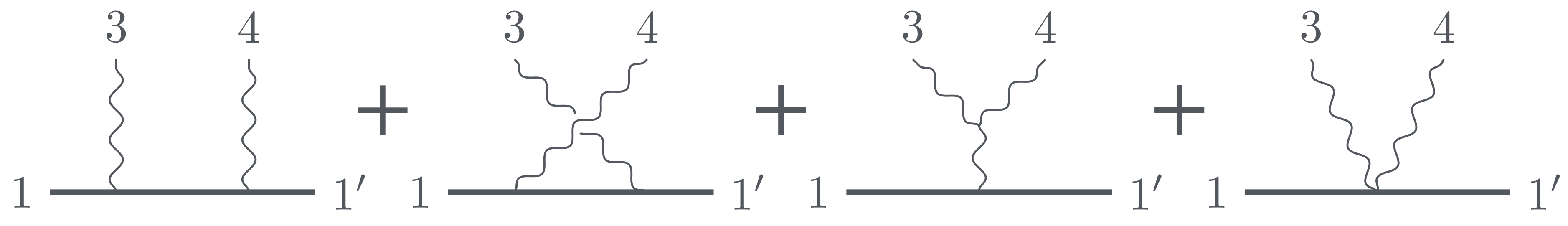

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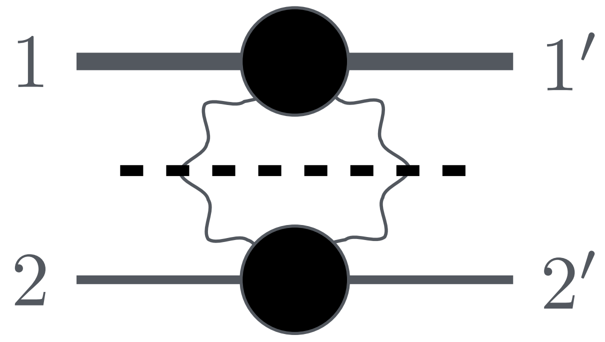


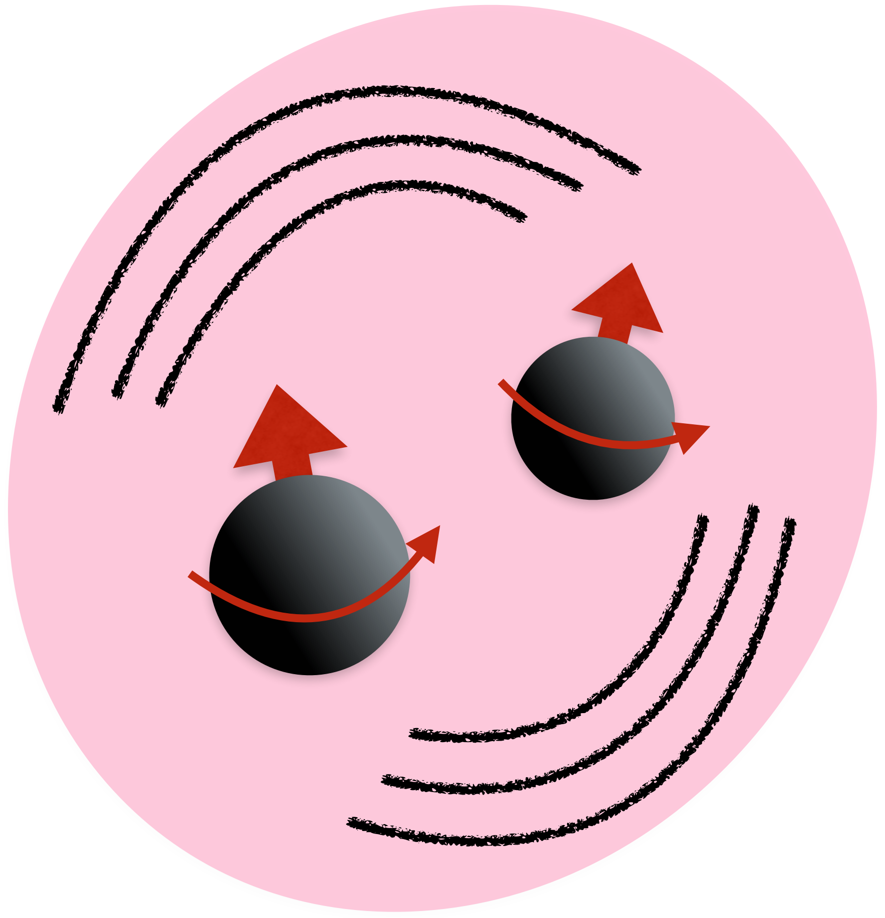
$\mathscr{A}_4^{\mathrm{EFT}} = \mathscr{A}_4^{\mathrm{FT}} \Rightarrow H$

$\mathscr{A}^{\mathrm{C}} \leftrightarrow$ Compton
$\mathscr{A}_4^{\mathrm{FT}} \leftrightarrow$ Full theory

The Amplitude at $\mathcal{O}\left(G^2\right)$ Required for Extracting the Hamiltonian

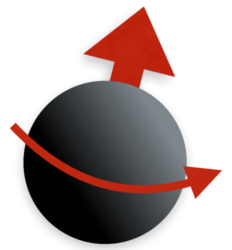
Modeling Spinning Compact Objects in General Relativity

Modeling Spinning Compact Objects

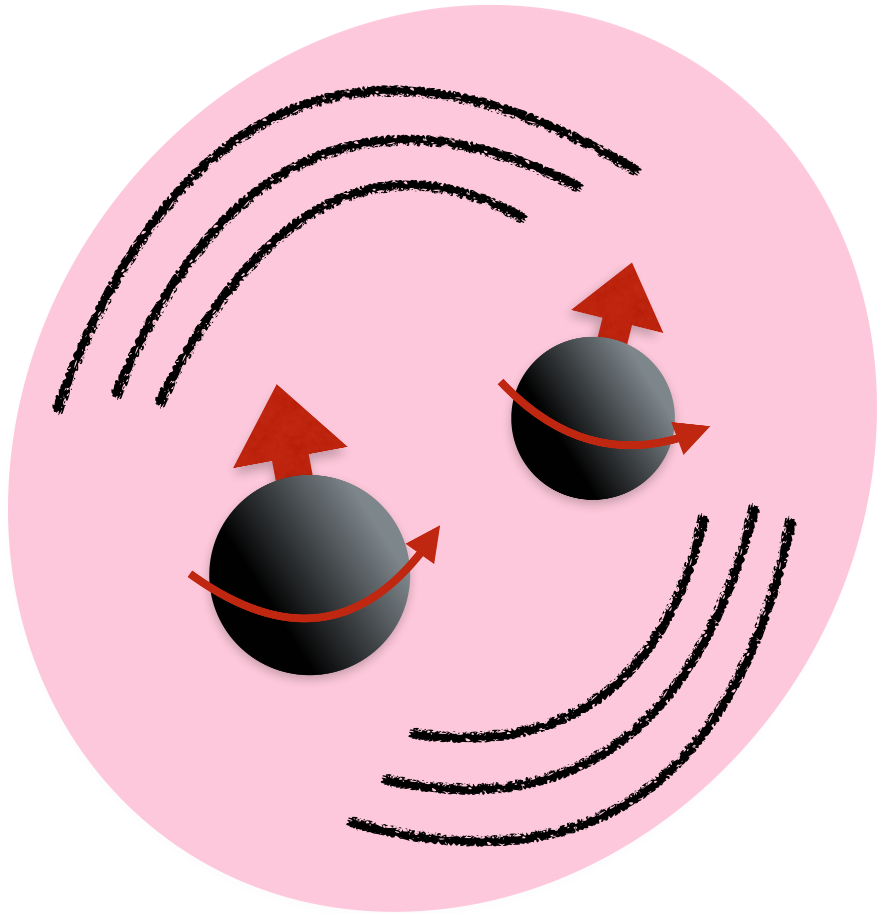


Worldline picture: $$S_M = \int d\tau \left(-m + \frac{1}{2}\Omega_{\mu\nu}S^{\mu\nu} + \ldots \right)$$

QFT picture: $$S_M = \int d^4x\sqrt{-g} \left(\frac{1}{2}g^{\mu\nu}(\nabla_\mu\phi_s)\cdot(\nabla_\nu\phi_s) - \frac{1}{2}m^2\phi_s\cdot\phi_s + \ldots \right)$$

$\phi_s^{\mu_1\ldots\mu_s} \quad \leftrightarrow$ 

Modeling Spinning Compact Objects



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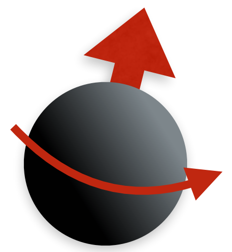
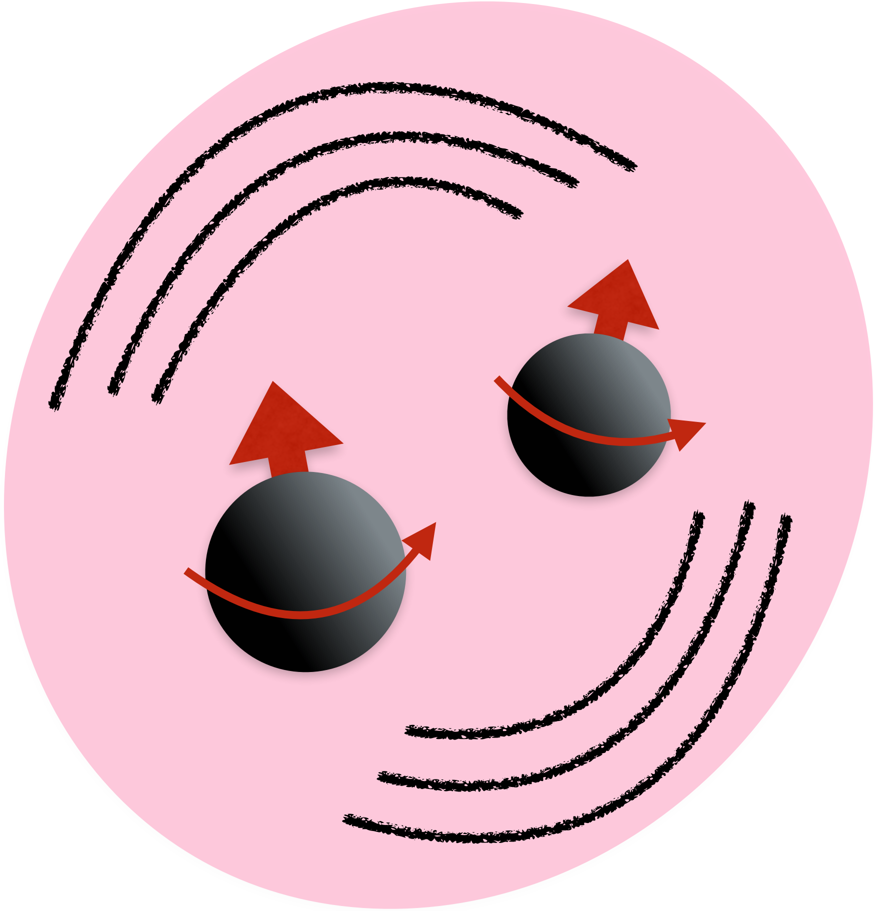
$\phi_s^{\mu_1\ldots\mu_s} \quad \leftrightarrow$ 

States in rest frame:

$\{\,|s,s\rangle, |s,s-1\rangle, \ldots |s,-s\rangle\}$

Modeling Spinning Compact Objects



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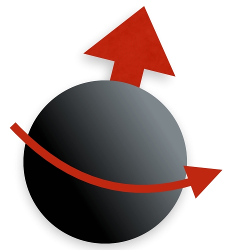
$\phi_s^{\mu_1 \dots \mu_s} \quad \leftrightarrow$ 

<u>States in rest frame:</u>

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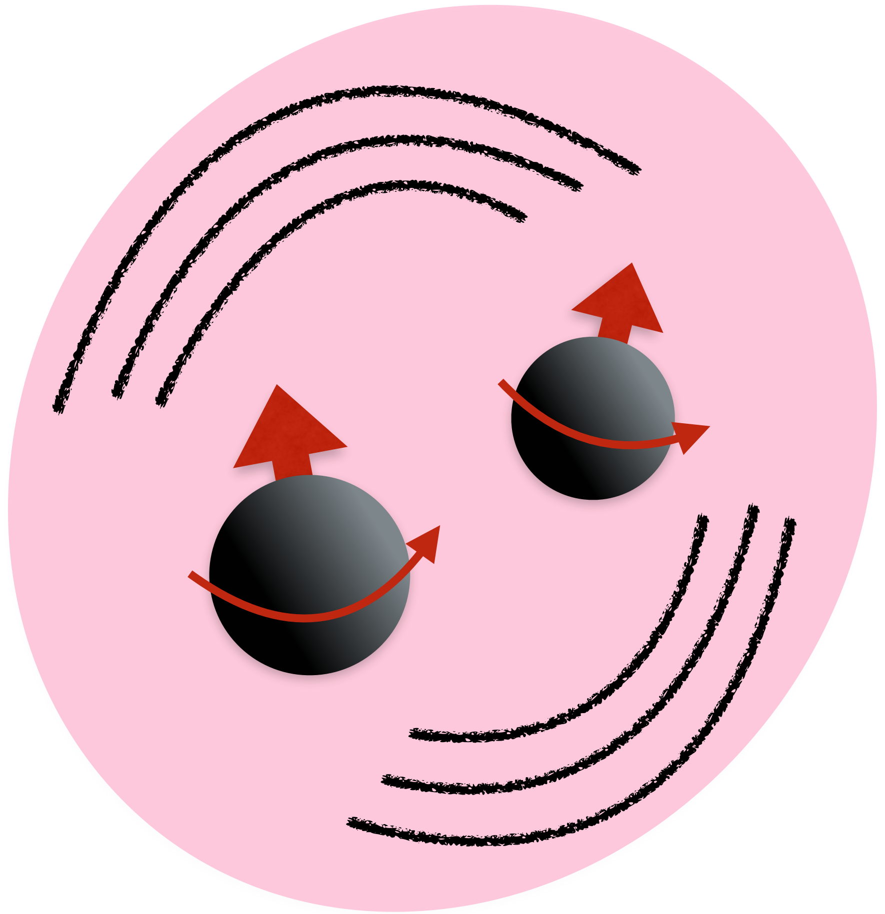
$\leftrightarrow$ spin magnitude fixed,
spin orientation variable

Modeling Spinning Compact Objects



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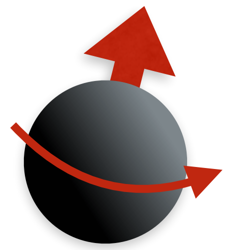
$\phi_s^{\mu_1\ldots\mu_s} \leftrightarrow$ 

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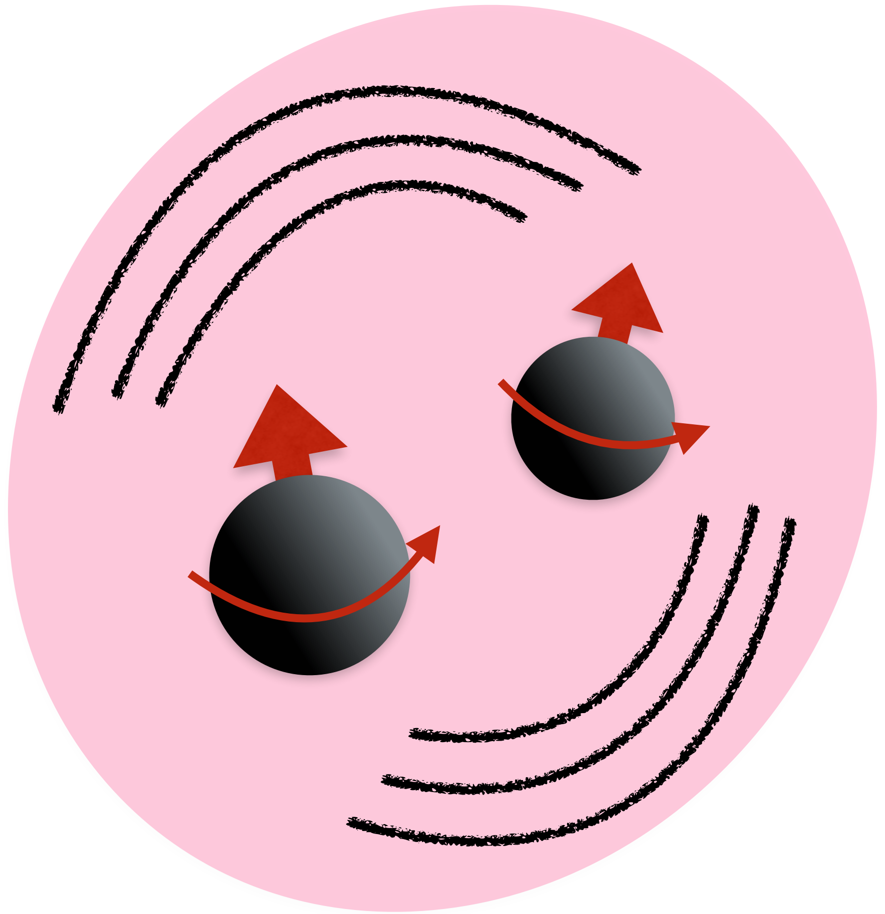
GW190412: $m_1 \sim 30M_\odot, \chi_1 \sim 0.4$

$s_1 = 2\chi_1 G m_1^2 \sim 10^{79}$

$\hbar = c = 1$

Modeling Spinning Compact Objects



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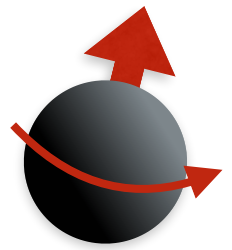
$\phi_s^{\mu_1\ldots\mu_s}$ $\leftrightarrow$ 

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Number of indices

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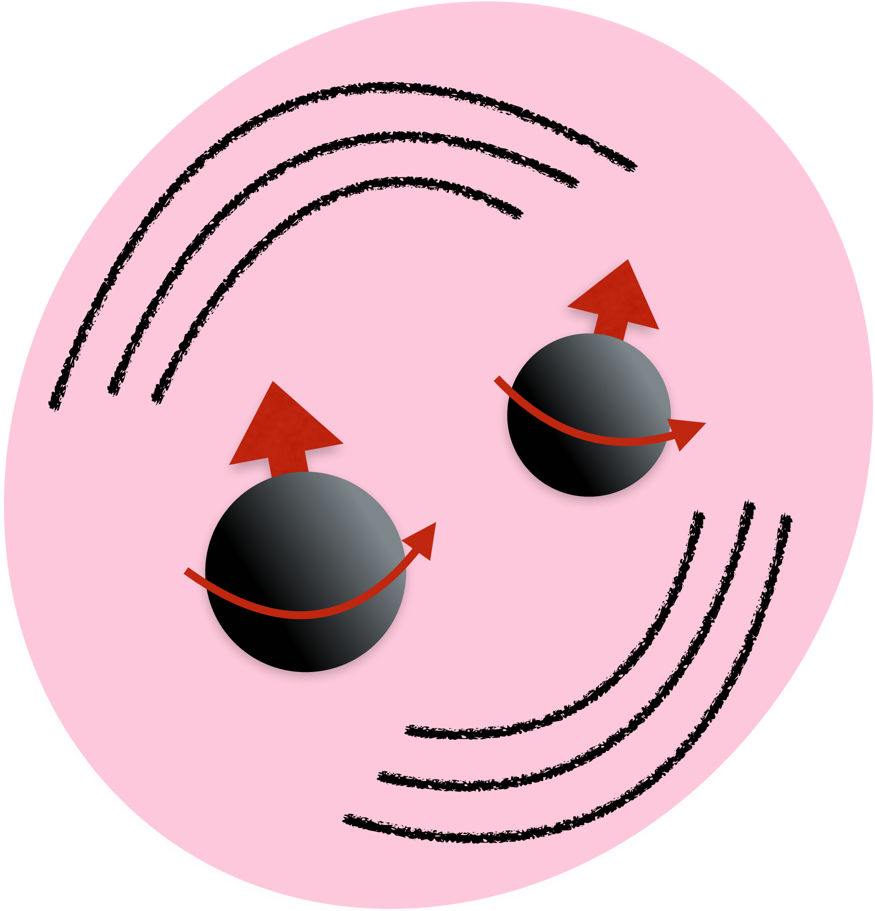
Example Calculation at $\mathcal{O}\left(S^2\right)$



$$\mathbb{S}^a \equiv -\frac{i}{2m}\epsilon^{abcd}M_{cd}\nabla_b$$

$$S_M = \int d^4x\sqrt{-g}\left(\ldots - \frac{C_{ES^2}}{2m^2}R_{f_1af_2b}\nabla^a\phi_s\,\mathbb{S}^{(f_1}\mathbb{S}^{f_2)}\nabla^b\phi_s + \ldots\right)$$

[**DK**, Luna (2021)]

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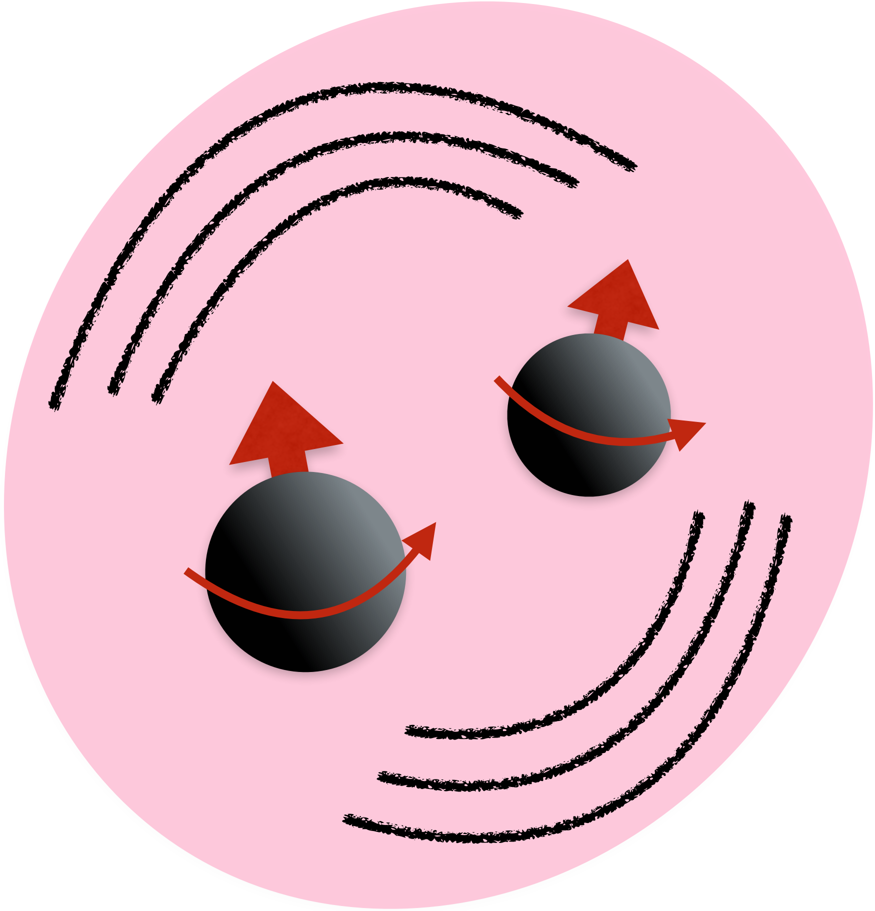

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$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}$$

$T^{\mu\nu}$: Stress tensor ↔ Details of the body ↔ Equation of state

[**DK**, Luna (2021)]

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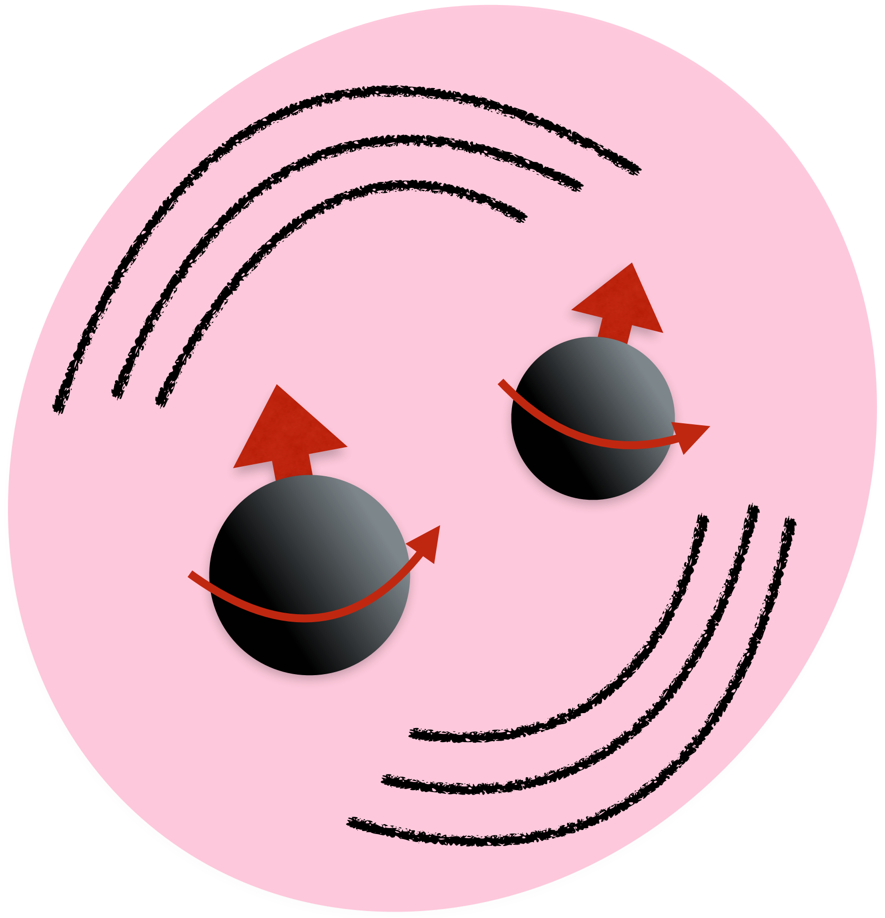

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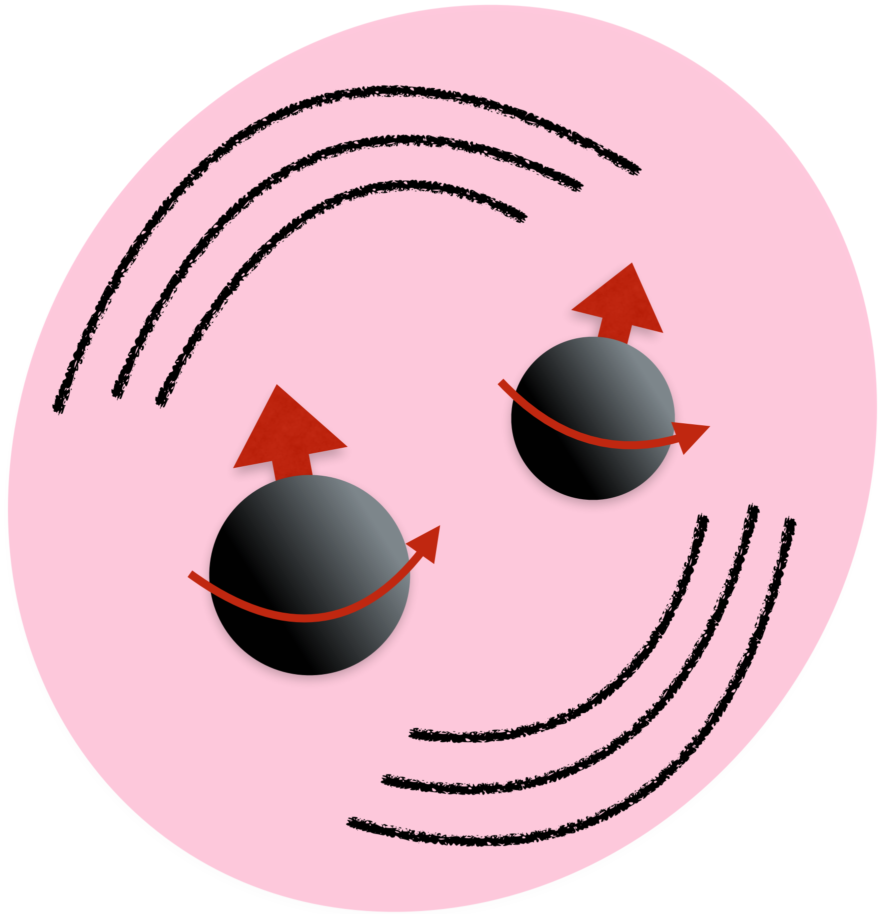

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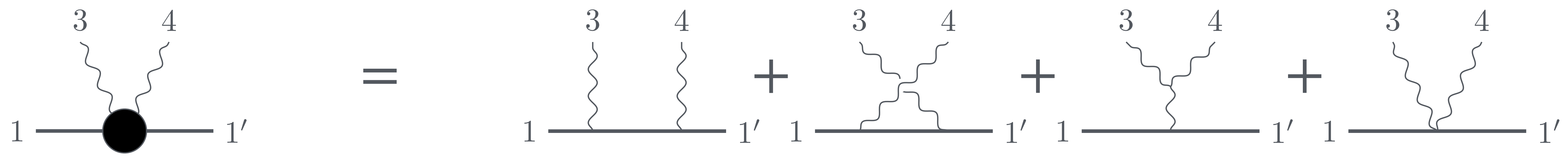

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[**DK**, Luna (2021)]

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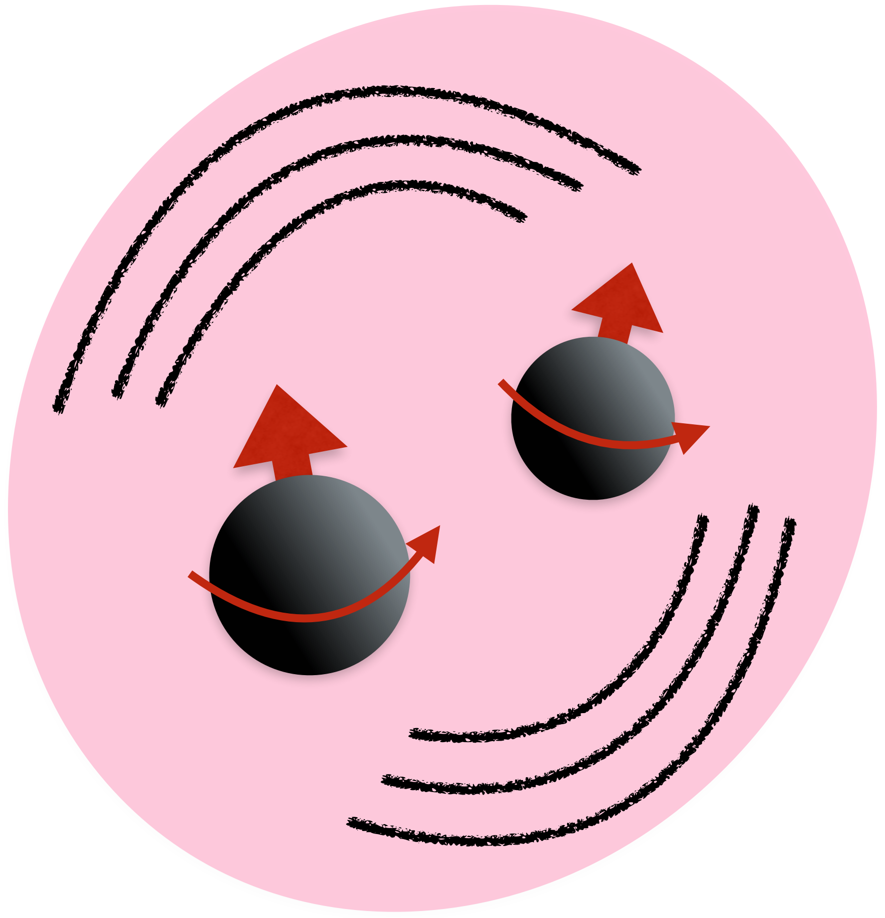

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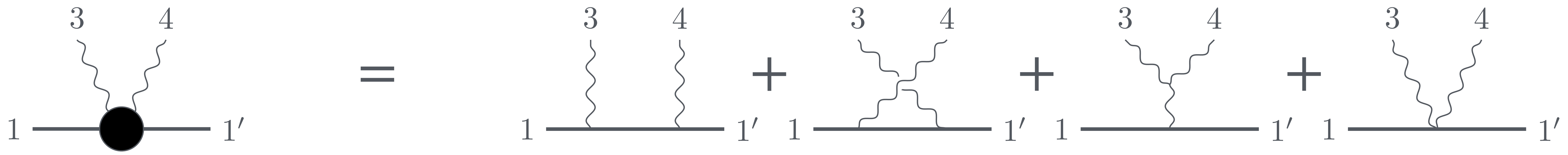

$$\bar{\mathscr{E}}_{1'}\cdot M^{\mu\nu}\cdot\mathscr{E}_1 \sim S_1^{\mu\nu}$$

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[**DK**, Luna (2021)]

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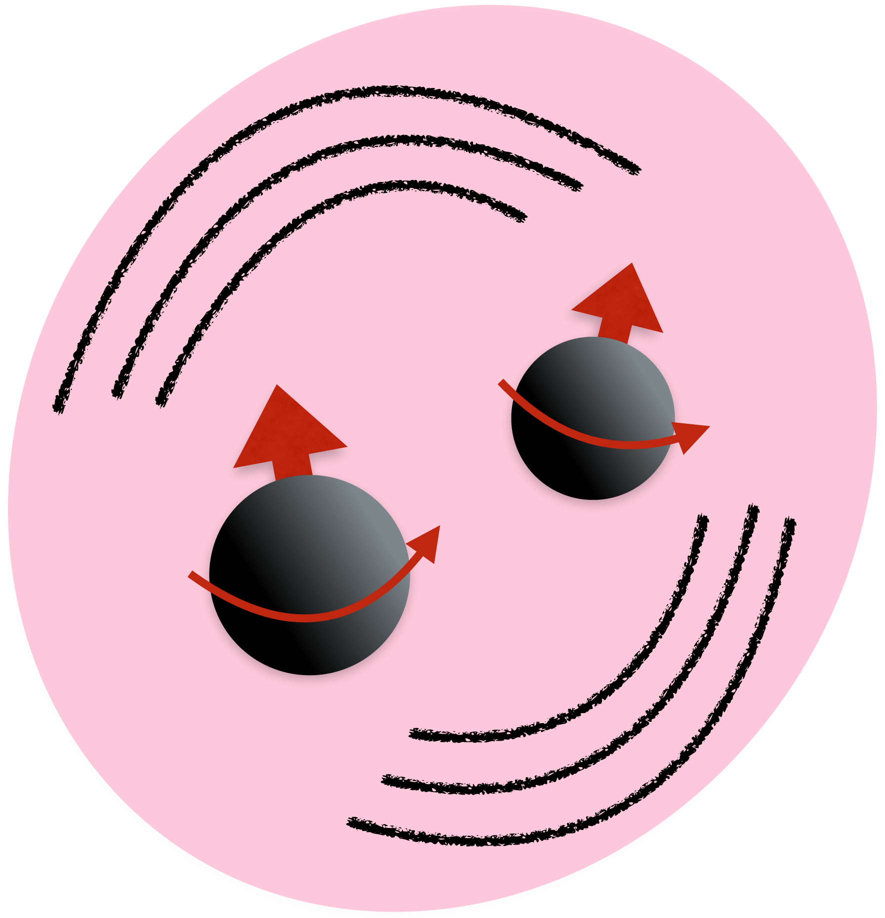

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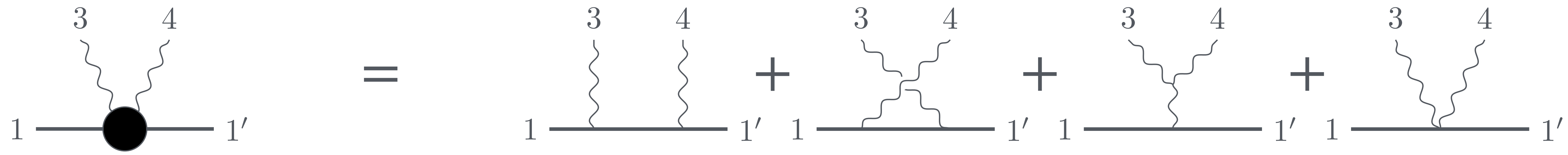

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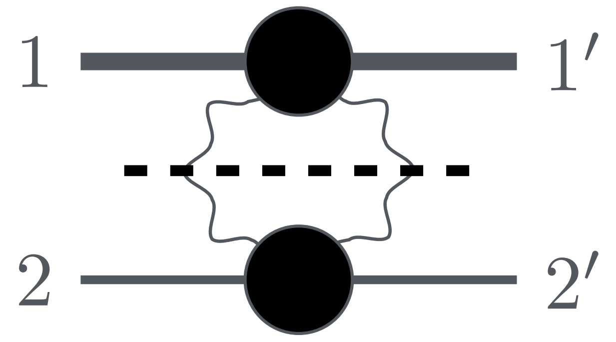
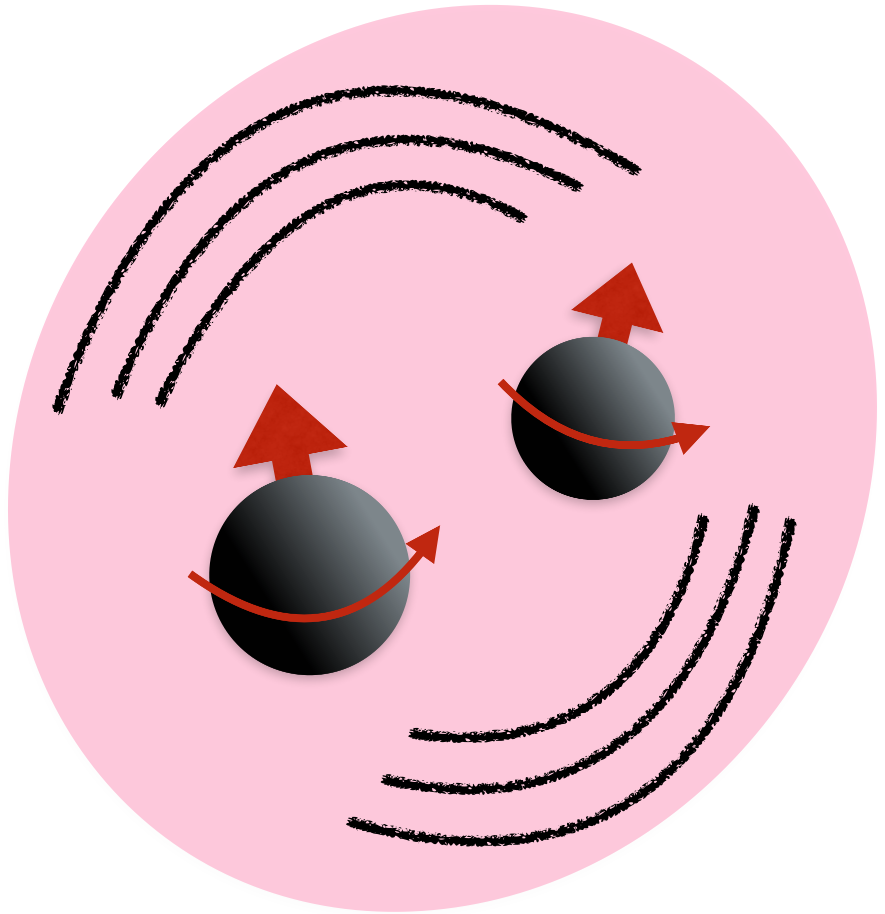


$T^{\mu\nu}$: Stress tensor ↔ Details of the body ↔ Equation of state

[**DK**, Luna (2021)]

Example Calculation at $\mathcal{O}\left(S^2\right)$



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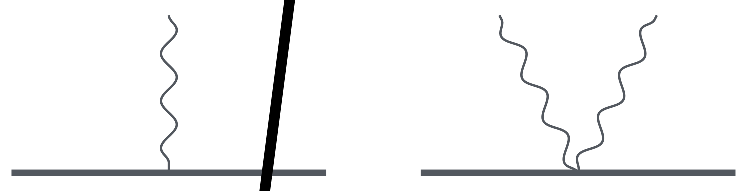

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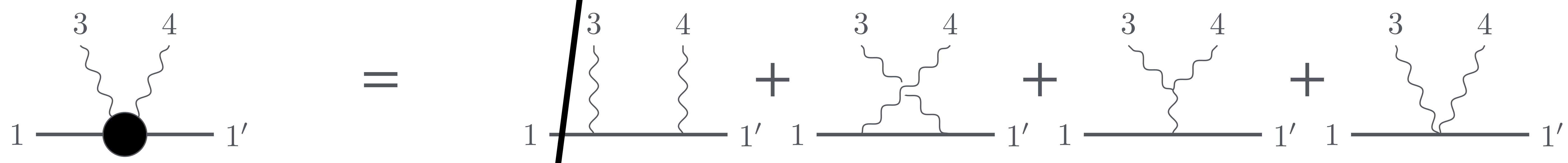

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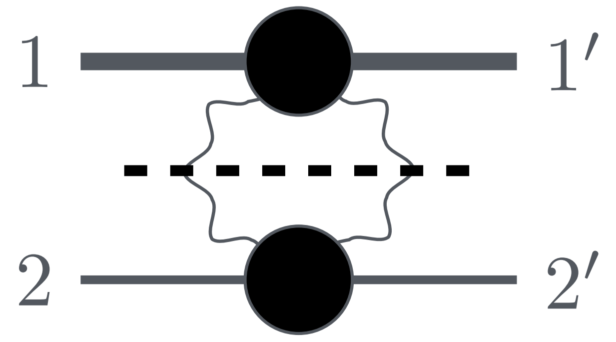
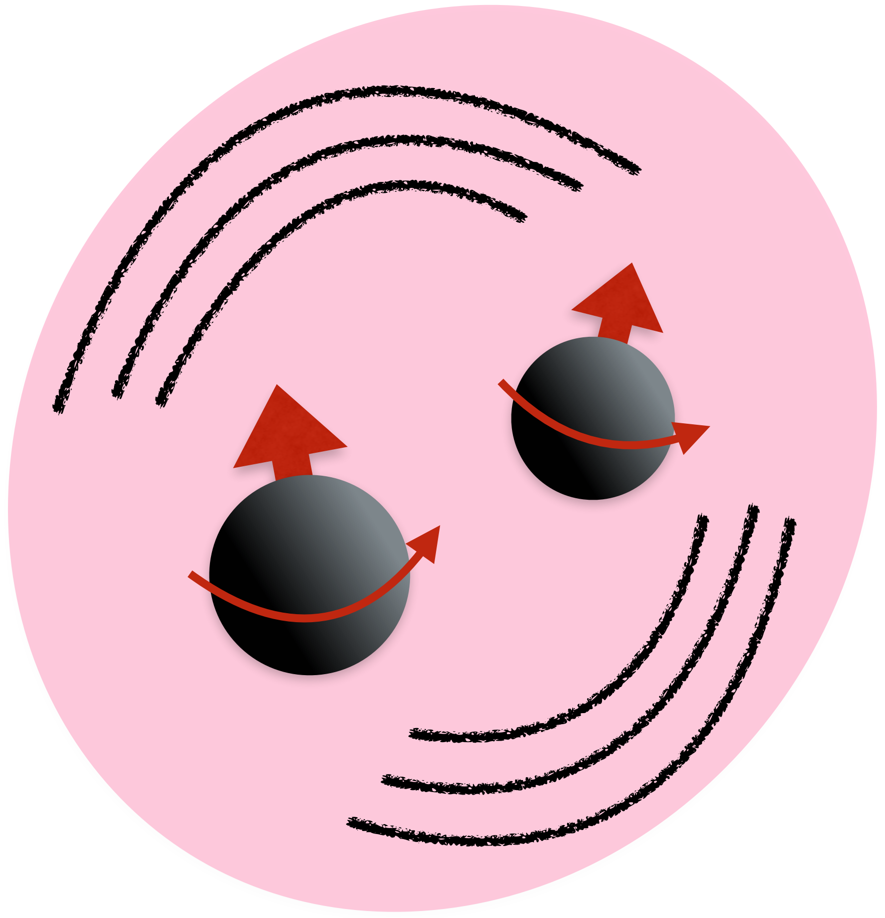


$T^{\mu\nu}$: Stress tensor ↔ Details of the body ↔ Equation of state

[**DK**, Luna (2021)]

Hamiltonian for Spinning Objects



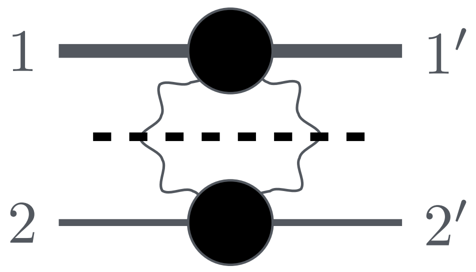
$$H = \sqrt{\mathbf{p}^2 + m_1^2} + \sqrt{\mathbf{p}^2 + m_2^2} + V^{(0)}(\mathbf{r}^2, \mathbf{p}^2) + V^{(1,1)}(\mathbf{r}^2, \mathbf{p}^2)\frac{\mathbf{L}\cdot\mathbf{S}_1}{\mathbf{r}^2} + \ldots$$

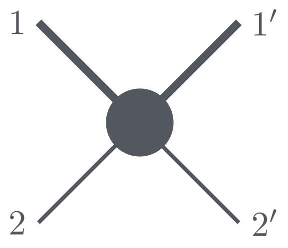
$$+V^{(2,4)}(\mathbf{r}^2, \mathbf{p}^2)\frac{(\mathbf{r}\cdot\mathbf{S}_1)^2}{\mathbf{r}^4} + \ldots$$

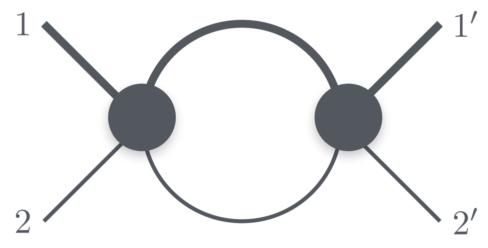
[**DK**, Luna (2021)]
$\mathcal{O}\left(S^2\right)$

$$+V^{(5,22)}(\mathbf{r}^2, \mathbf{p}^2)\frac{(\mathbf{L}\cdot\mathbf{S}_1)(\mathbf{p}\cdot\mathbf{S}_1)^2(\mathbf{r}\cdot\mathbf{S}_2)^2}{\mathbf{r}^6} + \ldots$$

[Bern, **DK**, Luna, Roiban, Teng (2022)]
$\mathcal{O}\left(S^5\right)$

$\mathscr{A}_4^{\mathrm{FT}} \sim$ 

$\mathscr{A}_4^{\mathrm{EFT}} \sim$ 

$+$ 

$+ \quad \ldots$

$\mathscr{A}_4^{\mathrm{EFT}} = \mathscr{A}_4^{\mathrm{FT}} \Rightarrow H$

Sample Results for the Hamiltonian

S^n	1	2	3	4	5
Number of spin structures	2	9	18	43	86

$$H = \ldots + V^{(2,4)}(\mathbf{r}^2, \mathbf{p}^2)\frac{(\mathbf{r}\cdot\mathbf{S}_1)^2}{\mathbf{r}^4} + \ldots$$

$$V^A(\mathbf{r}^2, \mathbf{p}^2) = \frac{G}{|\mathbf{r}|}c_1^A(\mathbf{p}^2) + \left(\frac{G}{|\mathbf{r}|}\right)^2 c_2^A(\mathbf{p}^2) + \mathcal{O}(G^3)$$

$$E = \sqrt{\mathbf{p}^2 + m_1^2} + \sqrt{\mathbf{p}^2 + m_2^2} \qquad \sigma = \frac{\mathbf{p}^2 + \sqrt{\mathbf{p}^2 + m_1^2}\sqrt{\mathbf{p}^2 + m_2^2}}{m_1 m_2}$$

$$\begin{aligned}
c_2{}^{(2,4)} ={}& \\
-\Big(\Big(\Big(&(-1+\sigma)\, m_2\, (m_1^2+2\sigma m_1 m_2+m_2^2)^5\, (4\,(1+\sigma)\,(3\sigma^2(-7+15\sigma^2)+(-12-29\sigma^2+53\sigma^4)\,C_{ES^2})\, m_1^8+4\, m_1^7 \\
&(\sigma(1+\sigma)\,(3E\sigma(-7+15\sigma^2)+(-42+4\sigma+65\sigma^2+161\sigma^4)\,m_2)+C_{ES^2}\,(E(-12-12\sigma-29\sigma^2- \\
&29\sigma^3+53\sigma^4+53\sigma^5)+\sigma(-142-130\sigma-251\sigma^2-367\sigma^3+529\sigma^4+649\sigma^5)\,m_2))+ \\
&m_1^6 m_2\,((1+\sigma)\,(8E\sigma(-21+2\sigma+43\sigma^2+58\sigma^4)+(-132+32\sigma-291\sigma^2-208\sigma^3+3766\sigma^4+ \\
&480\sigma^5-1079\sigma^6)\,m_2)+C_{ES^2}\,(8E\sigma(-65-59\sigma-111\sigma^2-169\sigma^3+238\sigma^4+298\sigma^5)+ \\
&(-32+64\sigma-3823\sigma^2-4511\sigma^3-346\sigma^4-1706\sigma^5+6569\sigma^6+8969\sigma^7)\,m_2))+ \\
&4 m_2^7\,(-(1+\sigma)\,(E\sigma(-12+5\sigma-122\sigma^2+33\sigma^3+108\sigma^4-90\sigma^5+60\sigma^6)- \\
&(4+40\sigma^2-39\sigma^3+98\sigma^4+61\sigma^5-176\sigma^6+30\sigma^7)\,m_2)+ \\
&C_{ES^2}\,(E\sigma(-9-6\sigma-56\sigma^2-76\sigma^3+123\sigma^4+66\sigma^5-30\sigma^6+60\sigma^7)- \\
&(3+3\sigma+27\sigma^2+18\sigma^3-3\sigma^4+81\sigma^5-85\sigma^6-146\sigma^7+30\sigma^8)\,m_2))- \\
&m_1 m_2^6\,((1+\sigma)\,(E(16+40\sigma-1115\sigma^2+560\sigma^3-1710\sigma^4-936\sigma^5+2705\sigma^6-720\sigma^7+624\sigma^8)- \\
&\sigma(240-324\sigma+2121\sigma^2-220\sigma^3+66\sigma^4+1568\sigma^5-3083\sigma^6+240\sigma^7)\,m_2)+ \\
&C_{ES^2}\,(E(36+12\sigma+813\sigma^2+853\sigma^3-90\sigma^4+1454\sigma^5-1671\sigma^6-2895\sigma^7+96\sigma^8-624\sigma^9)+ \\
&\sigma(300+192\sigma+1023\sigma^2+1863\sigma^3-1446\sigma^4-578\sigma^5-1045\sigma^6-2853\sigma^7+240\sigma^8)\,m_2))+ \\
&m_1^5 m_2^2\,(-(1+\sigma)\,(E(132-32\sigma+81\sigma^2+224\sigma^3-3290\sigma^4-480\sigma^5+1453\sigma^6)+ \\
&(-16+978\sigma+428\sigma^2-5371\sigma^3-100\sigma^4-4732\sigma^5-1800\sigma^6+5621\sigma^7)\,m_2)+ \\
&C_{ES^2}\,(E(-8+88\sigma-3269\sigma^2-4005\sigma^3+378\sigma^4-518\sigma^5+4771\sigma^6+6691\sigma^7)+(48-1182\sigma- \\
&1130\sigma^2-10133\sigma^3-14197\sigma^4+8352\sigma^5+9220\sigma^6+8435\sigma^7+12875\sigma^8)\,m_2))+ \\
&m_1^4 m_2^3\,(-(1+\sigma)\,(2E(-8+381\sigma+234\sigma^2-2577\sigma^3-166\sigma^4-757\sigma^5-660\sigma^6+2013\sigma^7)+ \\
&(207+232\sigma-788\sigma^2+1420\sigma^3-14648\sigma^4-2812\sigma^5+6780\sigma^6-2280\sigma^7+5849\sigma^8)\,m_2)+ \\
&C_{ES^2}\,(2E(24-469\sigma-503\sigma^2-3357\sigma^3-4893\sigma^4+3371\sigma^5+4017\sigma^6+2255\sigma^7+3635\sigma^8)+ \\
&(-111+201\sigma-5760\sigma^2-7780\sigma^3-7440\sigma^4-13576\sigma^5+ \\
&16480\sigma^6+22844\sigma^7+4351\sigma^8+8071\sigma^9)\,m_2))+ \\
&m_1^2 m_2^5\,(-(1+\sigma)\,(2E(10-47\sigma+354\sigma^2-2533\sigma^3-130\sigma^4+75\sigma^5-1134\sigma^6+2569\sigma^7- \\
&180\sigma^8+168\sigma^9)+(32+180\sigma-2043\sigma^2+1436\sigma^3-7232\sigma^4-2308\sigma^5+5885\sigma^6- \\
&2404\sigma^7+4342\sigma^8-120\sigma^9)\,m_2)+C_{ES^2}\,(2E(6-351\sigma-247\sigma^2-1451\sigma^3-2783\sigma^4+ \\
&1529\sigma^5+1485\sigma^6+1509\sigma^7+3075\sigma^8-12\sigma^9+168\sigma^{10})+(-96+12\sigma-2469\sigma^2- \\
&2925\sigma^3-1608\sigma^4-5912\sigma^5+6199\sigma^6+9191\sigma^7+1358\sigma^8+4242\sigma^9-120\sigma^{10})\,m_2))+ \\
&m_1^3 m_2^4\,(-(1+\sigma)\,(E(149+264\sigma-1398\sigma^2+816\sigma^3-8406\sigma^4-2120\sigma^5+6438\sigma^6- \\
&1200\sigma^7+2497\sigma^8)+(12+229\sigma+1220\sigma^2-8640\sigma^3+100\sigma^4- \\
&5856\sigma^5-4652\sigma^6+12192\sigma^7-1080\sigma^8+1819\sigma^9)\,m_2)+ \\
&C_{ES^2}\,(E(-113+103\sigma-3430\sigma^2-4942\sigma^3-2650\sigma^4-5890\sigma^5+8706\sigma^6+12842\sigma^7+ \\
&1407\sigma^8+3087\sigma^9)+(36-1233\sigma-921\sigma^2-7852\sigma^3-13020\sigma^4+ \\
&4916\sigma^5+3412\sigma^6+10028\sigma^7+17272\sigma^8+509\sigma^9+1829\sigma^{10})\,m_2))))\Big)\Big/ \\
&\Big(8E^5(-1+\sigma^2)\, m_1\, (\sigma m_1+m_2)^3\, (m_1+\sigma m_2)^3\, (m_1^2+m_2(E\sigma+m_2)+m_1(E+2\sigma m_2))^3\Big)\Big)
\end{aligned}$$

Hamiltonian Term at $\mathcal{O}\left(G^2 S^2\right)$

[**DK**, Luna (2021)]

Sample Results for the Hamiltonian

S^n	1	2	3	4	5
Number of spin structures	2	9	18	43	86

$$H = \ldots + V^{(2,4)}(\mathbf{r}^2, \mathbf{p}^2)\frac{(\mathbf{r}\cdot\mathbf{S}_1)^2}{\mathbf{r}^4} + \ldots$$

$$V^A(\mathbf{r}^2, \mathbf{p}^2) = \frac{G}{|\mathbf{r}|}c_1^A(\mathbf{p}^2) + \left(\frac{G}{|\mathbf{r}|}\right)^2 c_2^A(\mathbf{p}^2) + \mathcal{O}(G^3)$$

$$E = \sqrt{\mathbf{p}^2 + m_1^2} + \sqrt{\mathbf{p}^2 + m_2^2} \qquad \sigma = \frac{\mathbf{p}^2 + \sqrt{\mathbf{p}^2 + m_1^2}\sqrt{\mathbf{p}^2 + m_2^2}}{m_1 m_2}$$

$$\begin{aligned}
&c_2{}^{(2,4)} = \\
&-\Big(\Big((-1+\sigma)\, m_2\, (m_1^2 + 2\sigma m_1 m_2 + m_2^2)^5 \Big(4(1+\sigma)\big(3\sigma^2(-7+15\sigma^2) + (-12-29\sigma^2+53\sigma^4)C_{ES^2}\big) m_1^8 + 4 m_1^7 \\
&\quad \big(\sigma(1+\sigma)\big(3\mathrm{E}\sigma(-7+15\sigma^2) + (-42+4\sigma+65\sigma^2+161\sigma^4)m_2\big) + C_{ES^2}\big(\mathrm{E}(-12-12\sigma-29\sigma^2 - \\
&\qquad 29\sigma^3+53\sigma^4+53\sigma^5) + \sigma(-142-130\sigma-251\sigma^2-367\sigma^3+529\sigma^4+649\sigma^5)m_2\big)\big) + \\
&\quad m_1^6 m_2 \big((1+\sigma)\big(8\mathrm{E}\sigma(-21+2\sigma+43\sigma^2+58\sigma^4) + (-132+32\sigma-291\sigma^2-208\sigma^3+3766\sigma^4 + \\
&\qquad 480\sigma^5-1079\sigma^6)m_2\big) + C_{ES^2}\big(8\mathrm{E}\sigma(-65-59\sigma-111\sigma^2-169\sigma^3+238\sigma^4+298\sigma^5) + \\
&\qquad (-32+64\sigma-3823\sigma^2-4511\sigma^3-346\sigma^4-1706\sigma^5+6569\sigma^6+8969\sigma^7)m_2\big)\big) + \\
&\quad 4 m_2^7 \big(-(1+\sigma)\big(\mathrm{E}\sigma(-12+5\sigma-122\sigma^2+33\sigma^3+108\sigma^4-90\sigma^5+60\sigma^6) - \\
&\qquad (4+40\sigma^2-39\sigma^3+98\sigma^4+61\sigma^5-176\sigma^6+30\sigma^7)m_2\big) + \\
&\quad C_{ES^2}\big(\mathrm{E}\sigma(-9-6\sigma-56\sigma^2-76\sigma^3+123\sigma^4+66\sigma^5-30\sigma^6+60\sigma^7) - \\
&\qquad (3+3\sigma+27\sigma^2+18\sigma^3-3\sigma^4+81\sigma^5-85\sigma^6-146\sigma^7+30\sigma^8)m_2\big)\big) - \\
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&\qquad \sigma(240-324\sigma+2121\sigma^2-220\sigma^3+66\sigma^4+1568\sigma^5-3083\sigma^6+240\sigma^7)m_2\big) + \\
&\quad C_{ES^2}\big(\mathrm{E}(36+12\sigma+813\sigma^2+853\sigma^3-90\sigma^4+1454\sigma^5-1671\sigma^6-2895\sigma^7+96\sigma^8-624\sigma^9) + \\
&\qquad \sigma(300+192\sigma+1023\sigma^2+1863\sigma^3-1446\sigma^4-578\sigma^5-1045\sigma^6-2853\sigma^7+240\sigma^8)m_2\big)\big) + \\
&\quad m_1^5 m_2^2 \big(-(1+\sigma)\big(\mathrm{E}(132-32\sigma+81\sigma^2+224\sigma^3-3290\sigma^4-480\sigma^5+1453\sigma^6) + \\
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&\qquad 1130\sigma^2-10\,133\sigma^3-14\,197\sigma^4+8352\sigma^5+9220\sigma^6+8435\sigma^7+12\,875\sigma^8)m_2\big)\big) + \\
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&\quad C_{ES^2}\big(\mathrm{E}(-113+103\sigma-3430\sigma^2-4942\sigma^3-2650\sigma^4-5890\sigma^5+8706\sigma^6+12\,842\sigma^7 + \\
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&\qquad 4916\sigma^5+3412\sigma^6+10\,028\sigma^7+17\,272\sigma^8+509\sigma^9+1829\sigma^{10})m_2\big)\big)\Big)\Big)\Big) \Big/ \\
&\Big(8\mathrm{E}^5(-1+\sigma^2)\,m_1(\sigma m_1+m_2)^3(m_1+\sigma m_2)^3\big(m_1^2+m_2(\mathrm{E}\sigma+m_2)+m_1(\mathrm{E}+2\sigma m_2)\big)^3\Big)\Big)
\end{aligned}$$

C_{ES^2}

Hamiltonian Term at $\mathcal{O}\left(G^2S^2\right)$

[**DK**, Luna (2021)]

Sample Results for the Hamiltonian

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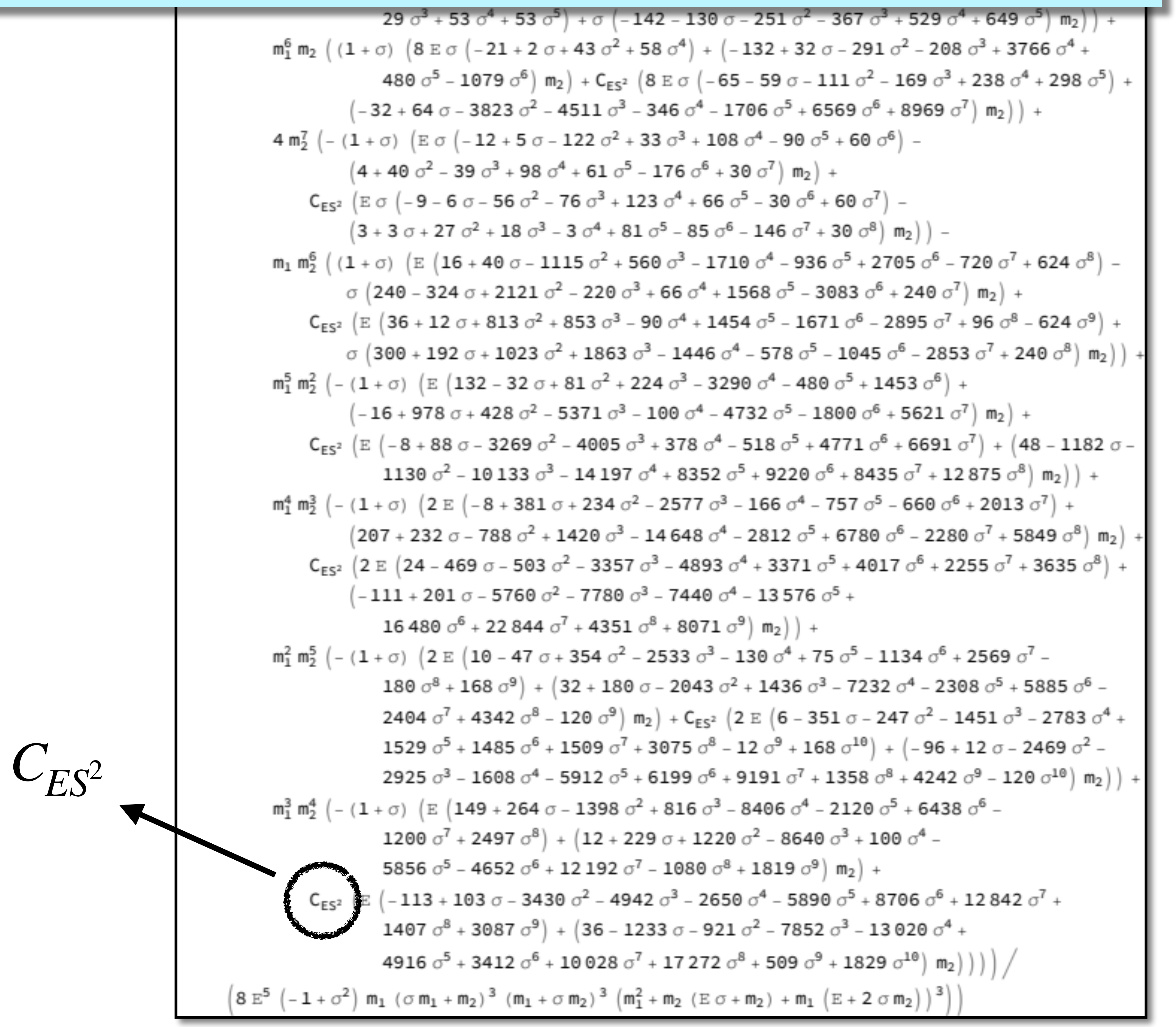
Pushing the state of the art in modeling binary systems

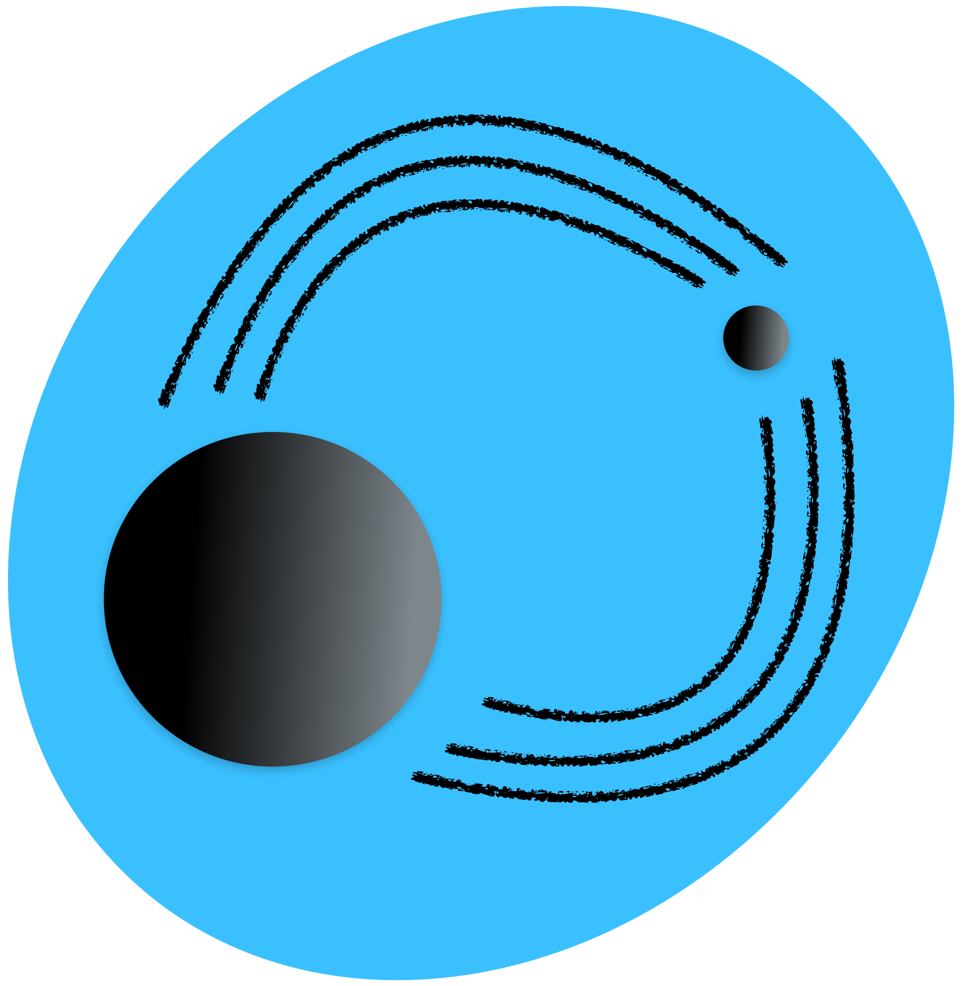


Hamiltonian Term at $\mathcal{O}\left(G^2S^2\right)$

[**DK**, Luna (2021)]

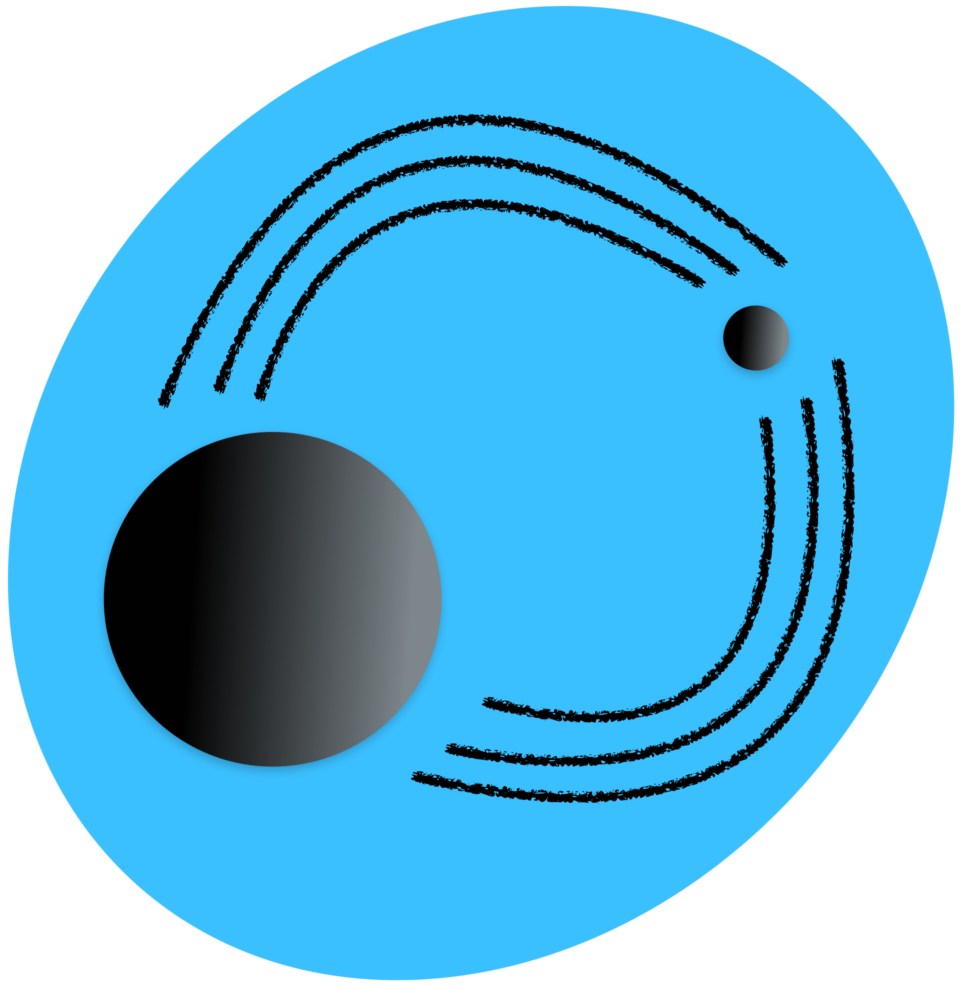
Quantum Field Theory around a Black Hole

EFT for Extreme-Mass-Ratio Inspirals



EFT in $m/M \ll 1$ — Gravitational Self-Force (SF) expansion

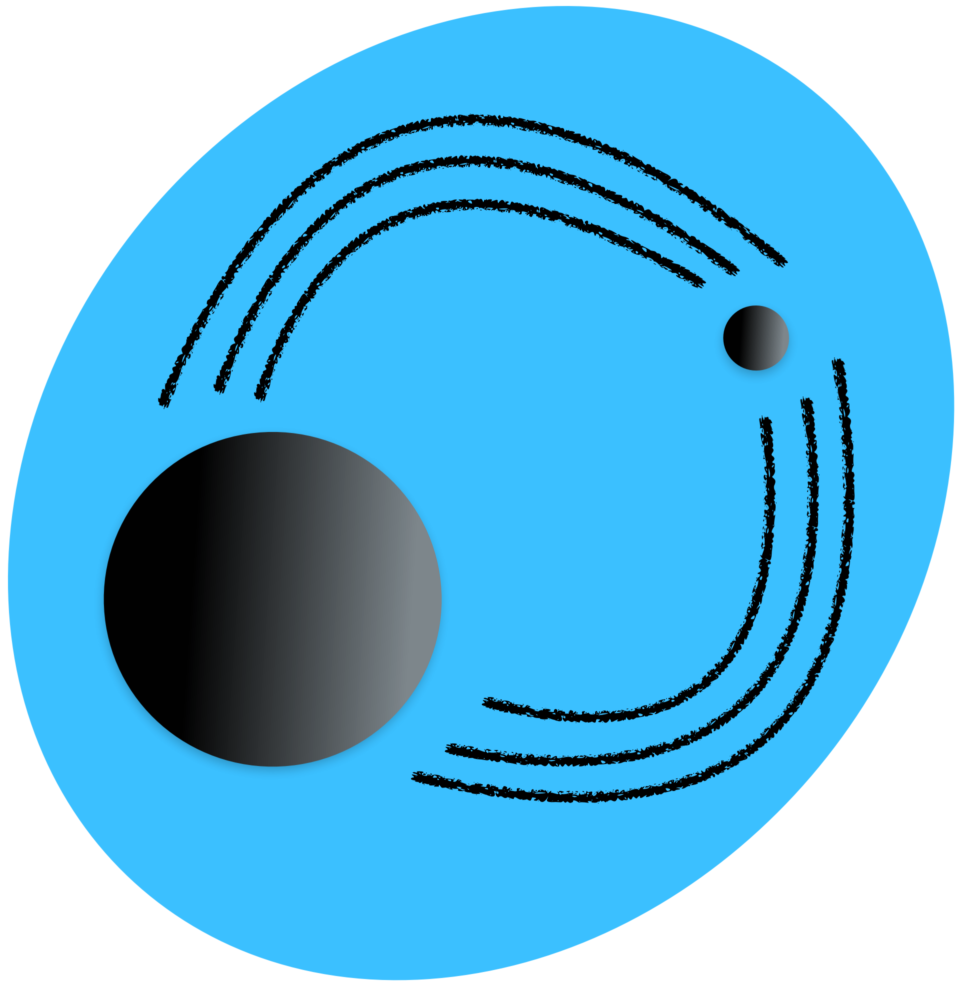
EFT for Extreme-Mass-Ratio Inspirals



EFT in $m/M \ll 1$ — Gravitational Self-Force (SF) expansion

Motion = Motion in black-hole background + $\mathcal{O}\,(m/M)$

EFT for Extreme-Mass-Ratio Inspirals

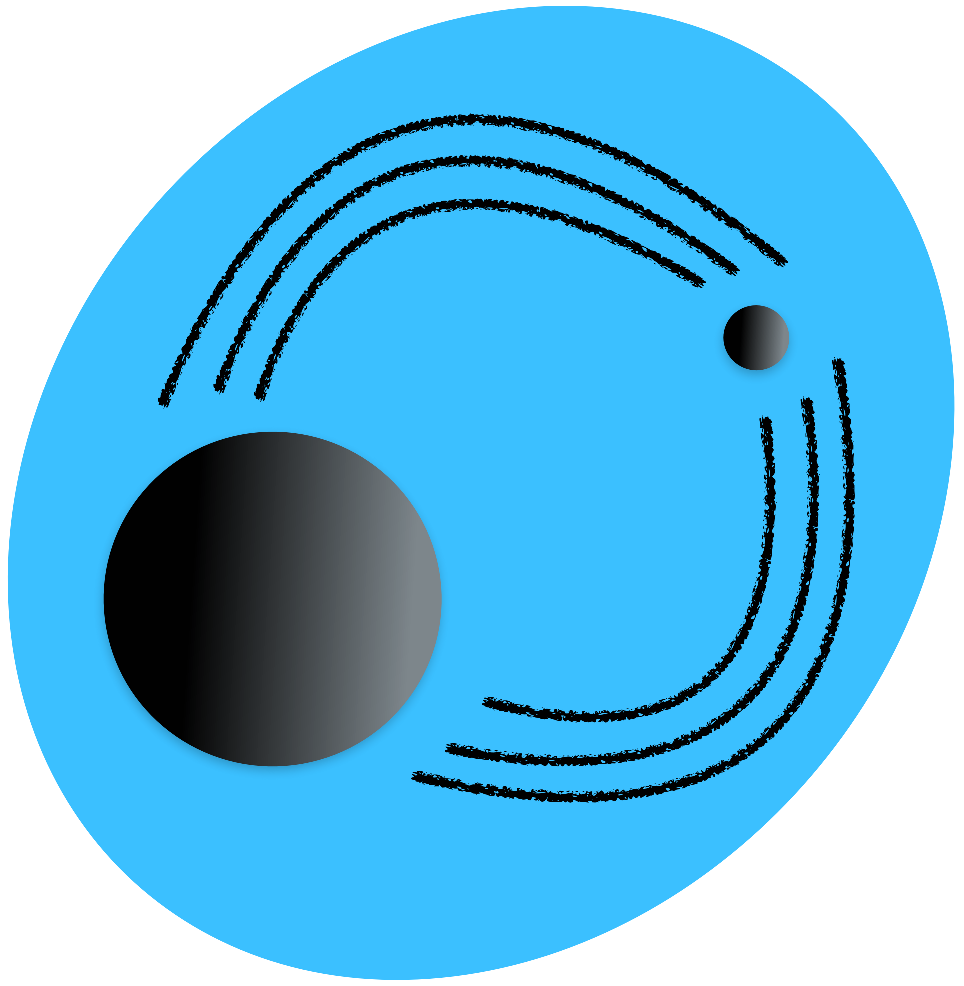


EFT in $m/M \ll 1$ — Gravitational Self-Force (SF) expansion

Motion = Motion in black-hole background + $\mathcal{O}(m/M)$

Black hole in General Relativity: $\bar{g}_{\mu\nu}dx^{\mu}dx^{\nu} = (1 - \frac{2GM}{r})dt^2 - (1 - \frac{2GM}{r})^{-1}dr^2 - r^2 d\Omega^2$

EFT for Extreme-Mass-Ratio Inspirals



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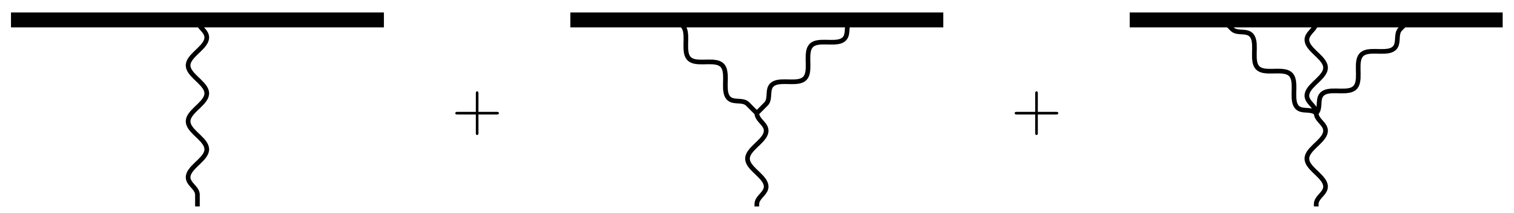
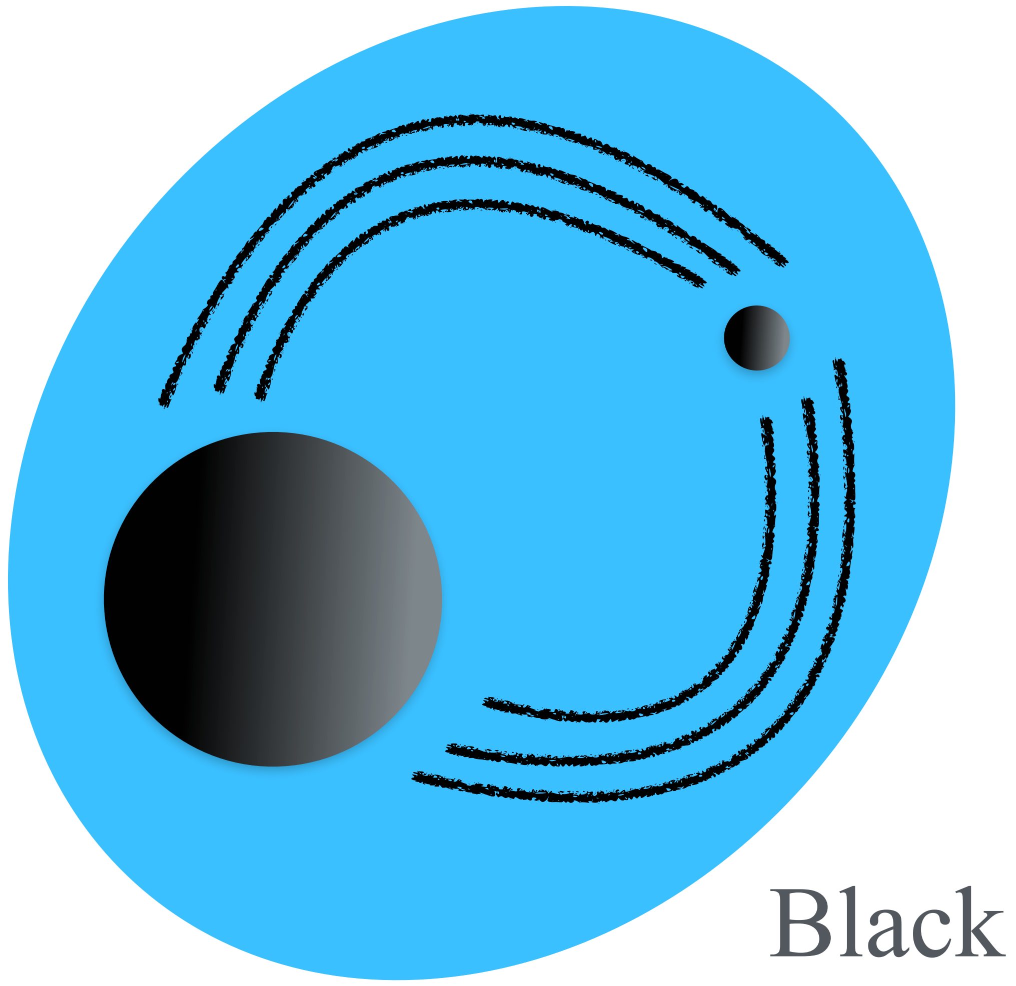
Black hole in Quantum Field Theory:  $+ \cdots \approx \bar{g}_{\mu\nu} - \eta_{\mu\nu}$ [Duff (1973)]

EFT for Extreme-Mass-Ratio Inspirals



EFT in $m/M \ll 1$ — Gravitational Self-Force (SF) expansion

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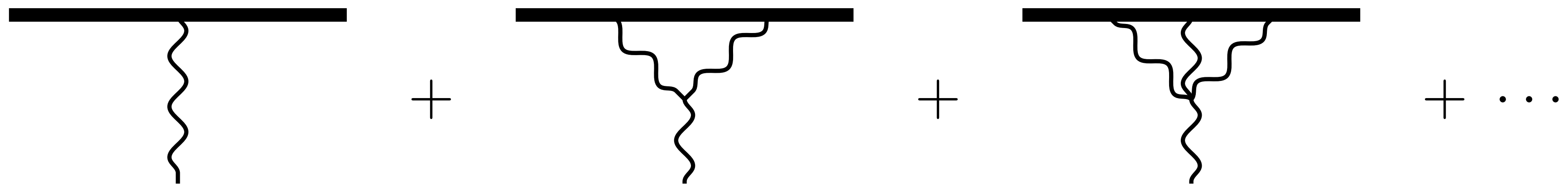
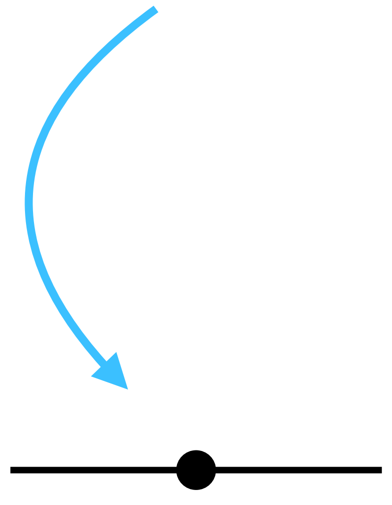
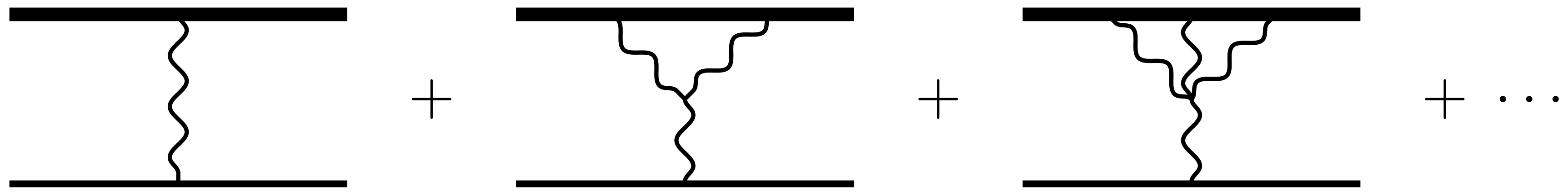
Black hole in General Relativity: $\bar{g}_{\mu\nu}dx^\mu dx^\nu = (1 - \frac{2GM}{r})dt^2 - (1 - \frac{2GM}{r})^{-1}dr^2 - r^2 d\Omega^2$

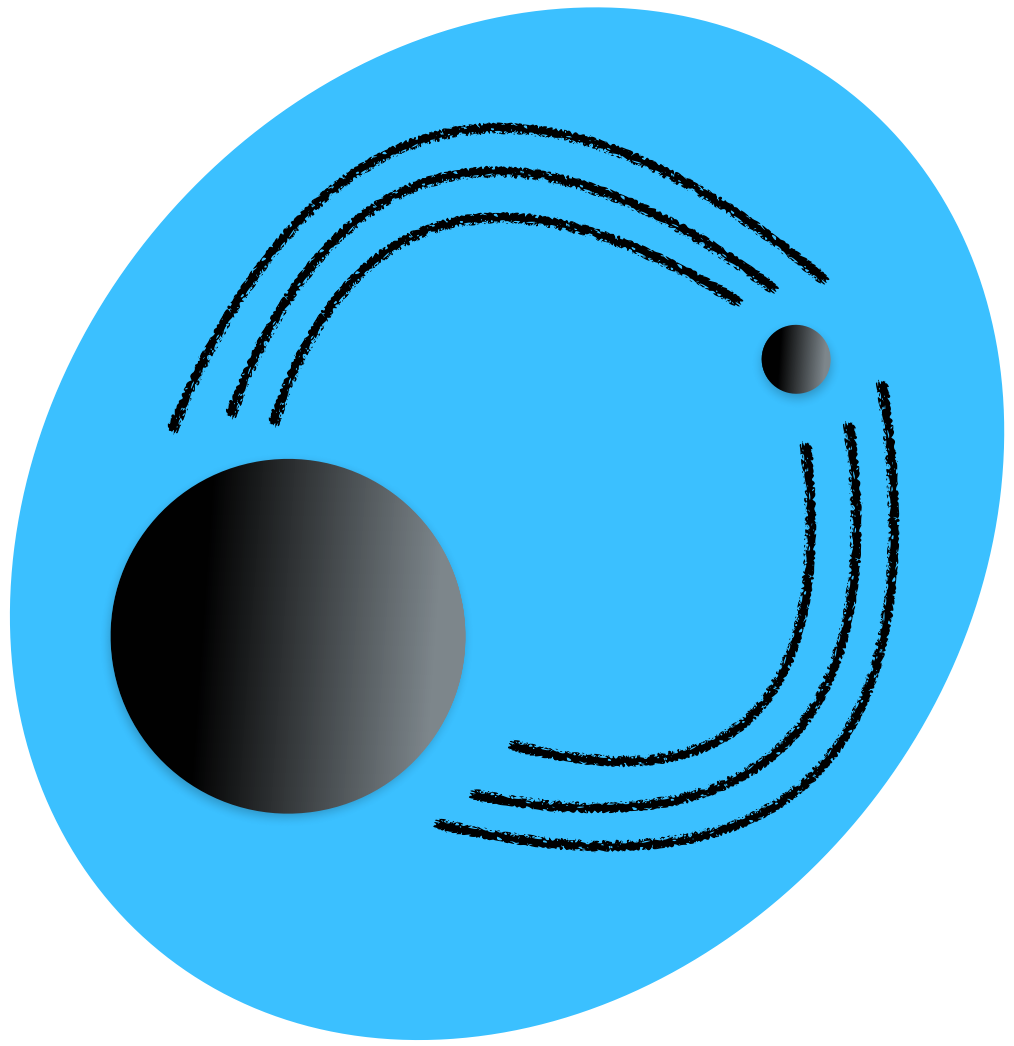
Black hole in Quantum Field Theory:  $\approx$ $\bar{g}_{\mu\nu} - \eta_{\mu\nu}$ [Duff (1973)]



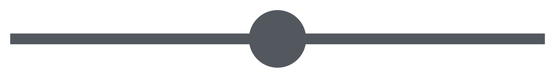
New QFT framework:  $=$ [**DK**, Solon (2023)]

Combining Efficient Flat-Space Methods & Exact Black-Hole Solutions

Manifest Power Counting



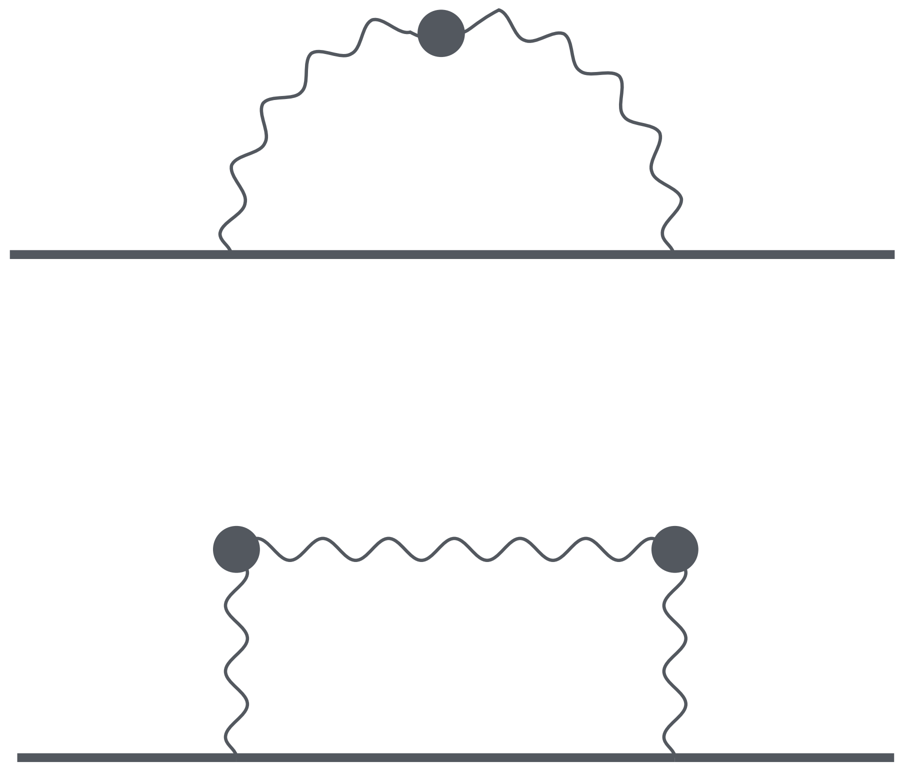
<u>Tree level</u>



Geodesic

0SF - $\mathcal{O}\left((m/M)^0\right)$

<u>1-loop level</u>



⋮

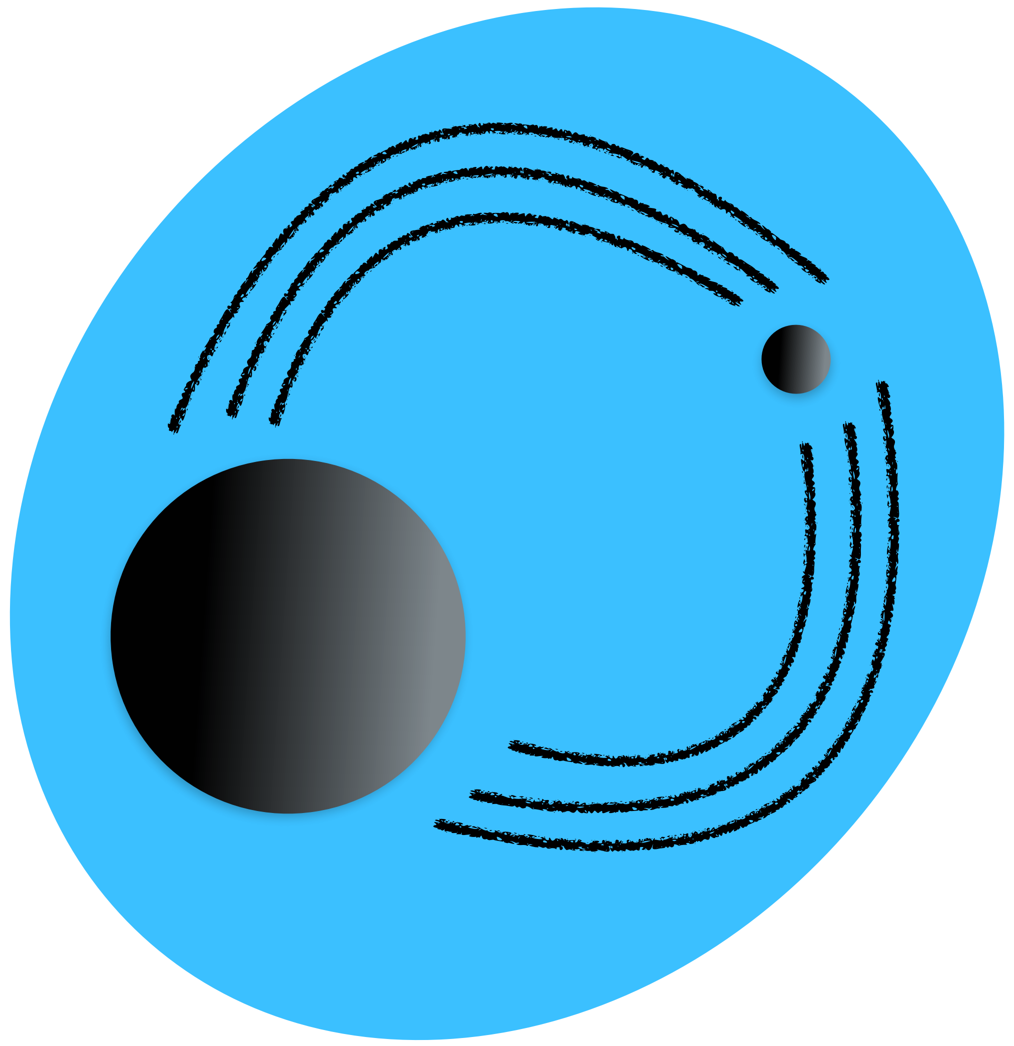
1SF - $\mathcal{O}\left((m/M)^1\right)$

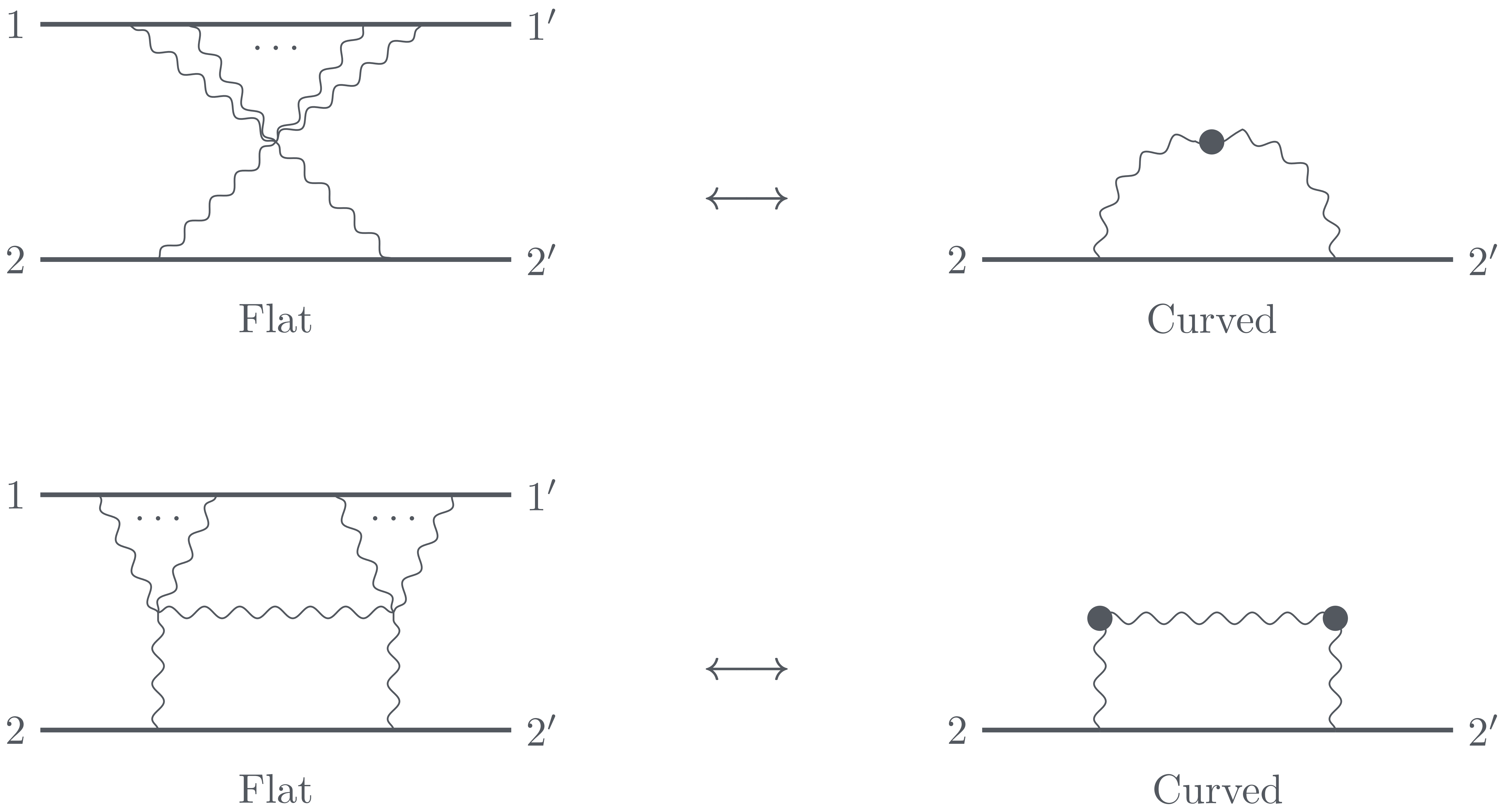
…

[**DK**, Solon (2023)]

Diagram Loops Count SF order

Systematic Resummation

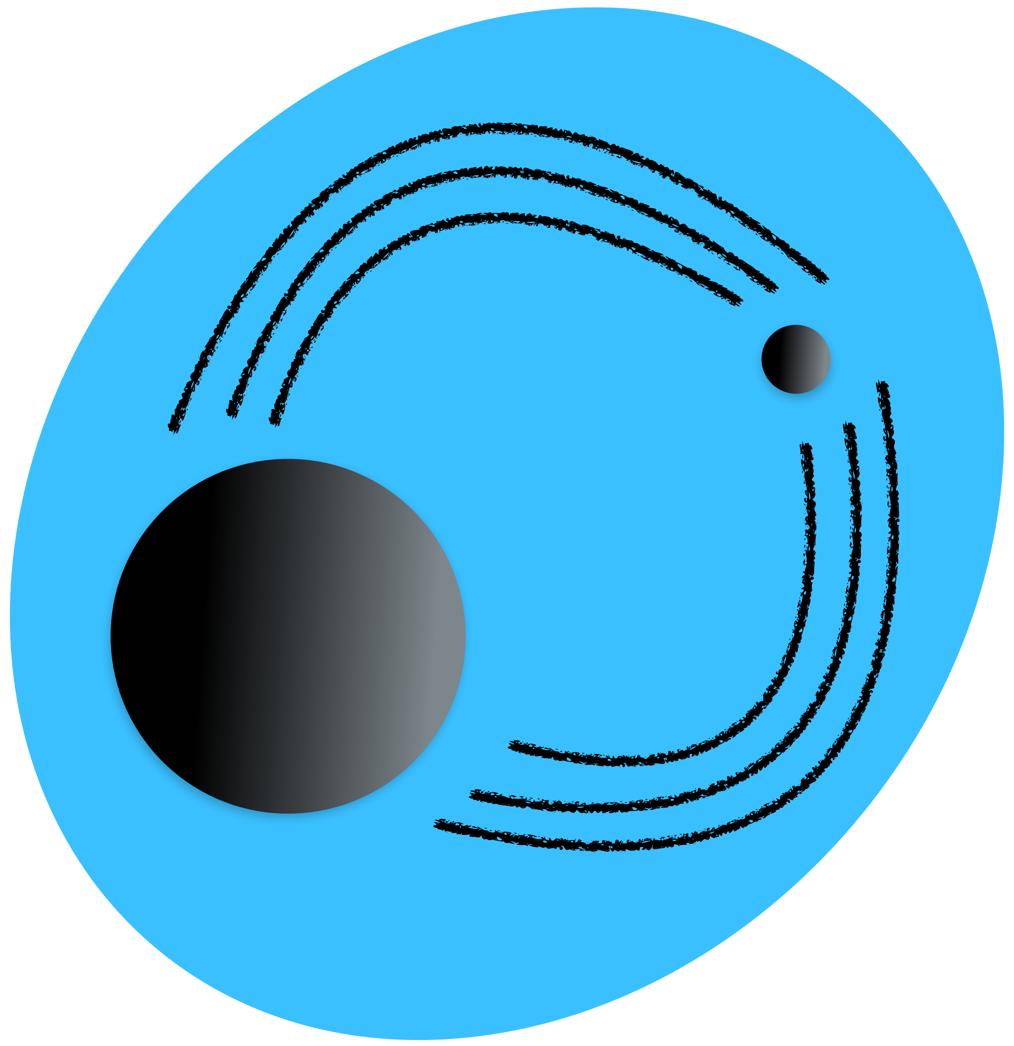




[**DK**, Solon (2023)]

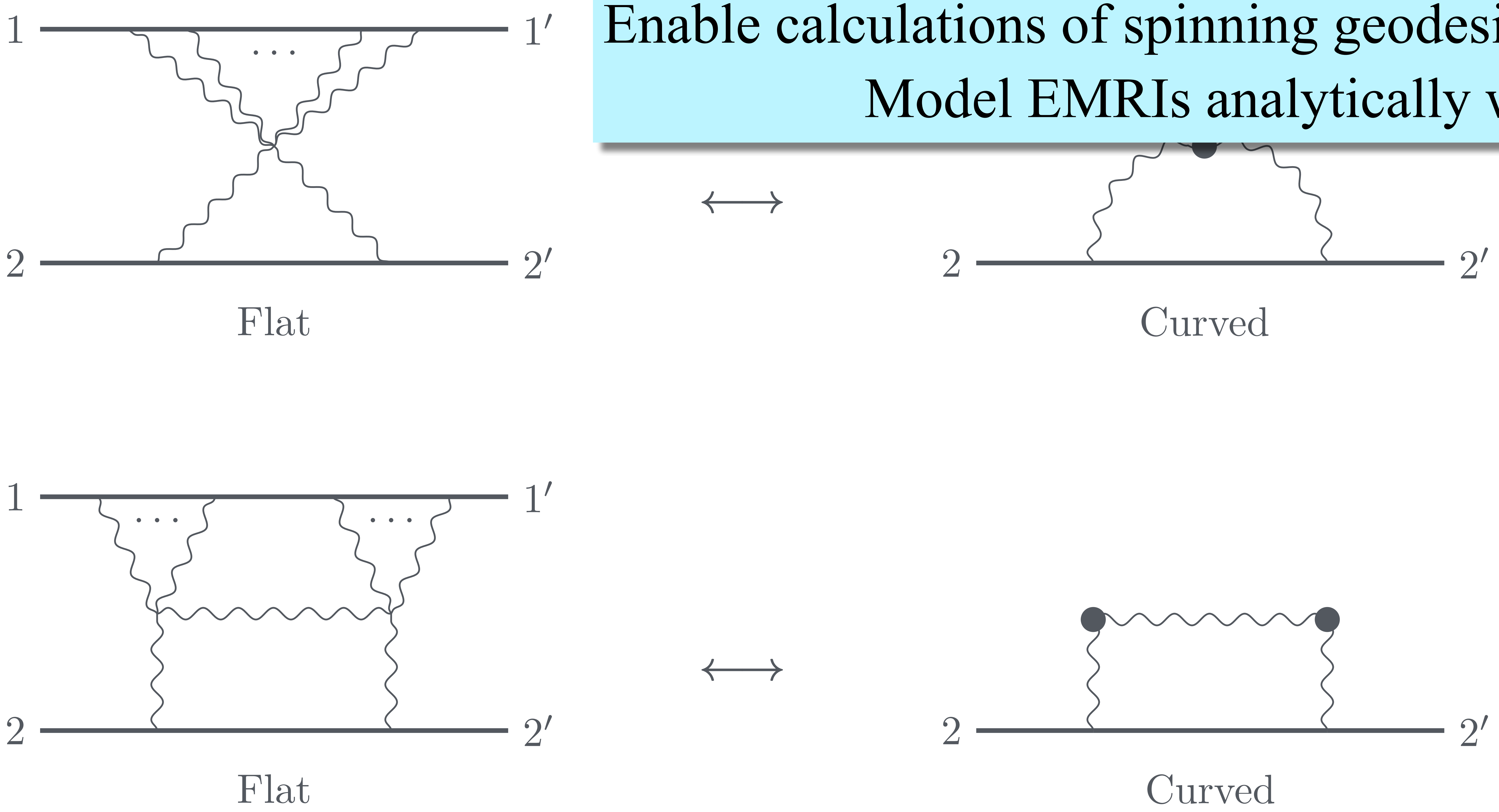
Benefits: Classes of Diagrams Combined & Generically Fewer Integrals

Systematic Resummation



Extreme-mass-ratio inspirals (EMRIs)

Streamline calculations at high orders
Enable calculations of spinning geodesics in Kerr background
Model EMRIs analytically within QFT

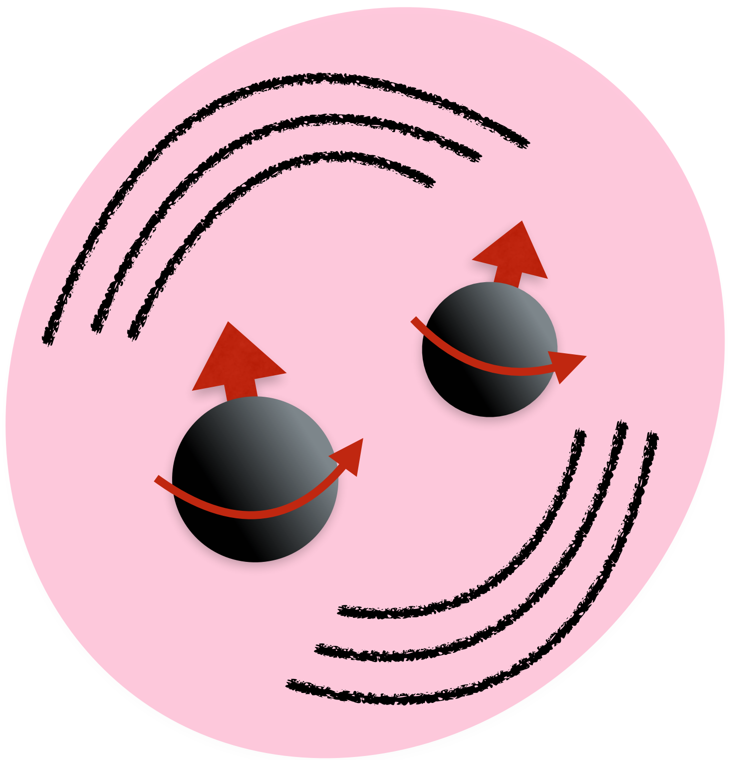


[**DK**, Solon (2023)]

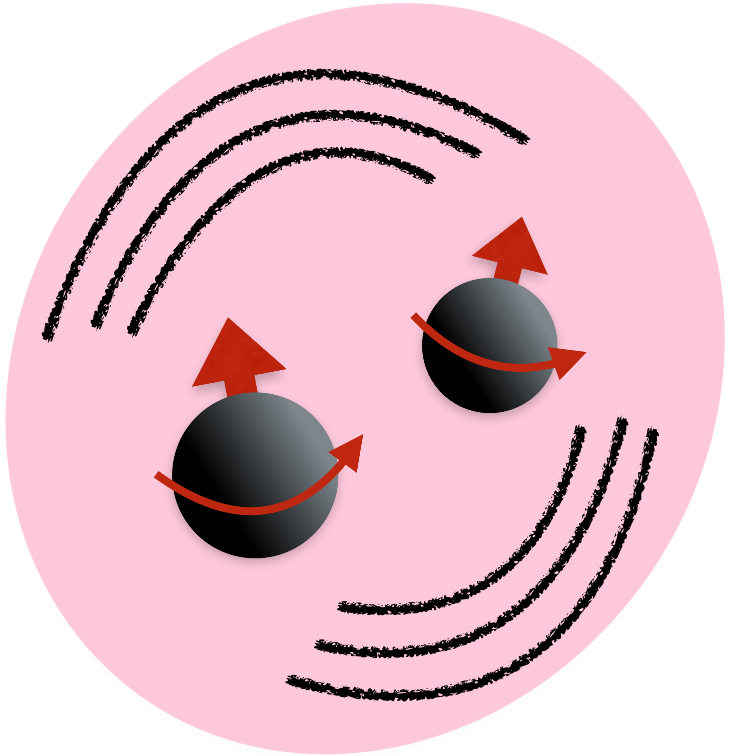
Benefits: Classes of Diagrams Combined & Generically Fewer Integrals

Outlook

Conclusions



EFT for Gravitational-Wave Science

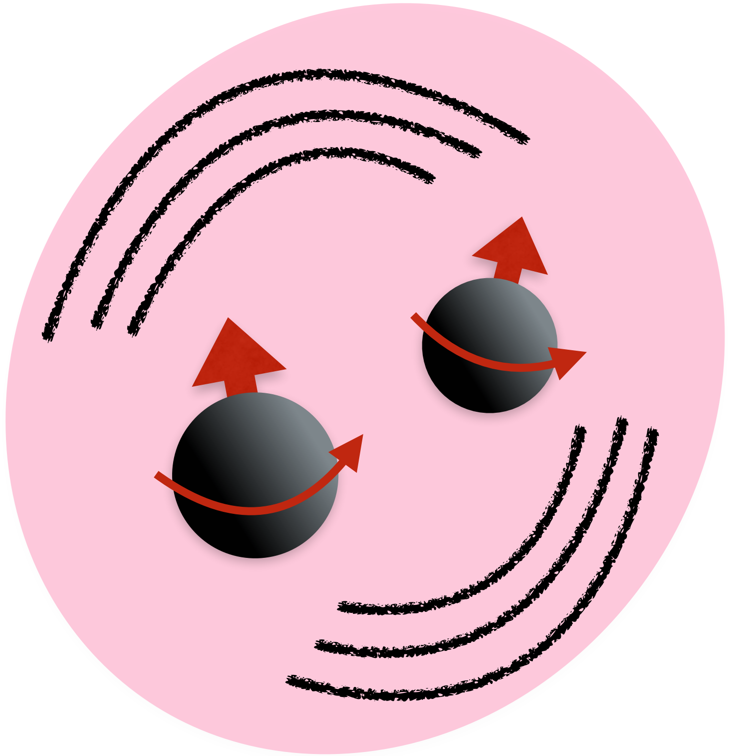


Conclusions

EFT for Gravitational-Wave Science

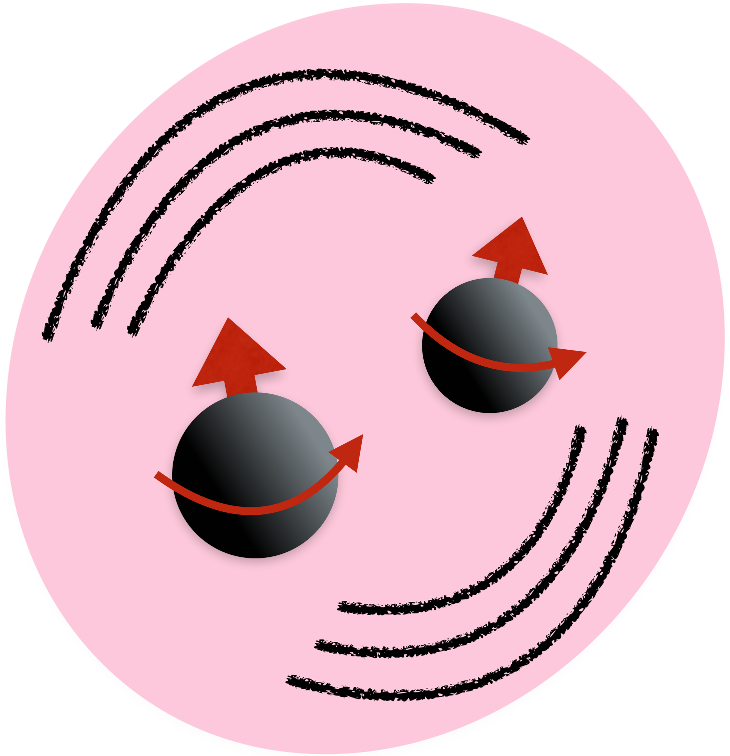
- Pushed the state-of-the-art in modeling binary systems

Conclusions



EFT for Gravitational-Wave Science

- Pushed the state-of-the-art in modeling binary systems
- Designed EFT for expanding in m/M and resumming classes of contributions to all orders in G



Conclusions

EFT for Gravitational-Wave Science

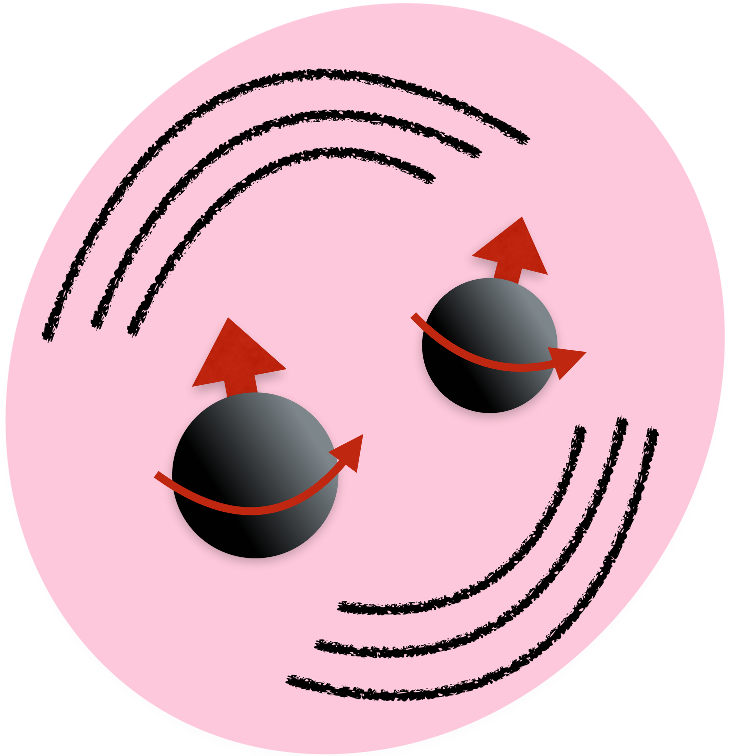
- Pushed the state-of-the-art in modeling binary systems
- Designed EFT for expanding in m/M and resumming classes of contributions to all orders in G
- Discovered new phenomena that may manifest in the waveform ⇒ Spin-magnitude change

[Bern, **DK**, Luna, Roiban, Scheopner, Teng, Vines (2023)]

[Alaverdian, Bern, **DK**, Luna, Roiban, Scheopner, Teng (2024)]

[Alaverdian, Bern, **DK**, Luna, Roiban, Scheopner, Teng (to appear)]

Conclusions



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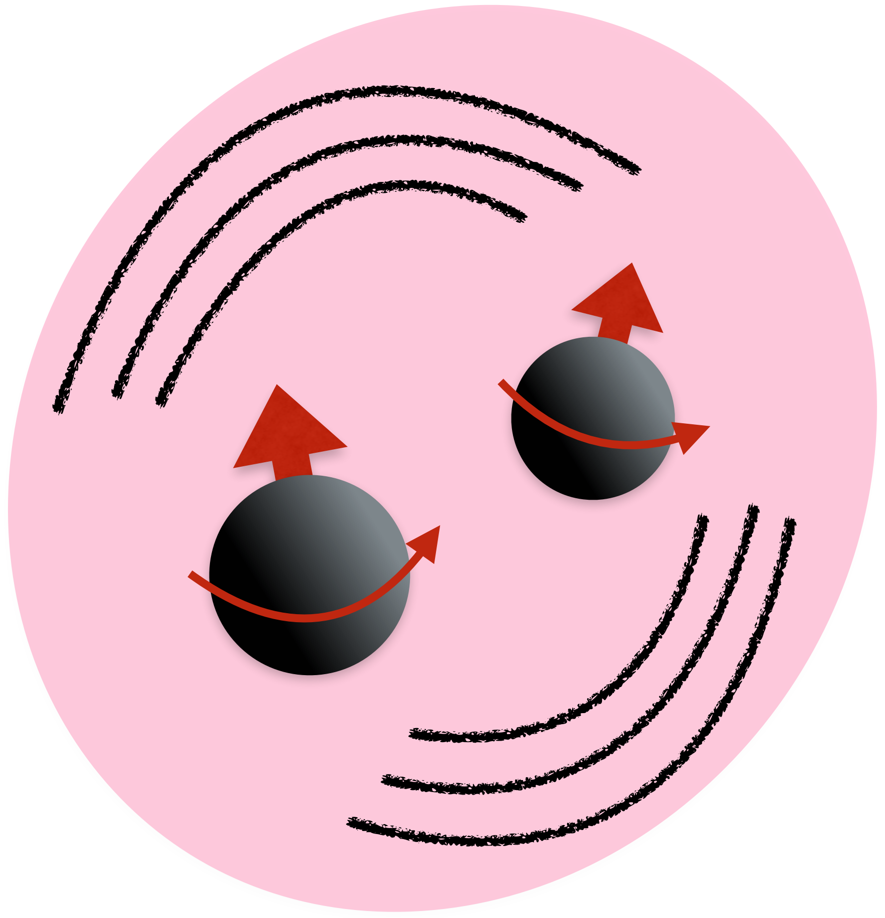
[Alaverdian, Bern, **DK**, Luna, Roiban, Scheopner, Teng (to appear)]

Thank you!

Backup Slides

Modeling Spin-Magnitude Change in Orbital Evolution

Modeling Spin-Magnitude Change



$$S_M = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} (\nabla_\mu \Phi) \cdot (\nabla_\nu \Phi) - \frac{1}{2} m^2 \Phi \cdot \Phi + \ldots \right)$$

$$\Phi \sim \begin{pmatrix} \vdots \\ \phi_s \\ \phi_{s-1} \\ \vdots \end{pmatrix}$$

[Bern, **DK**, Luna, Roiban, Scheopner, Teng, Vines (2023)]

[Alaverdian, Bern, **DK**, Luna, Roiban, Scheopner, Teng (2024)]

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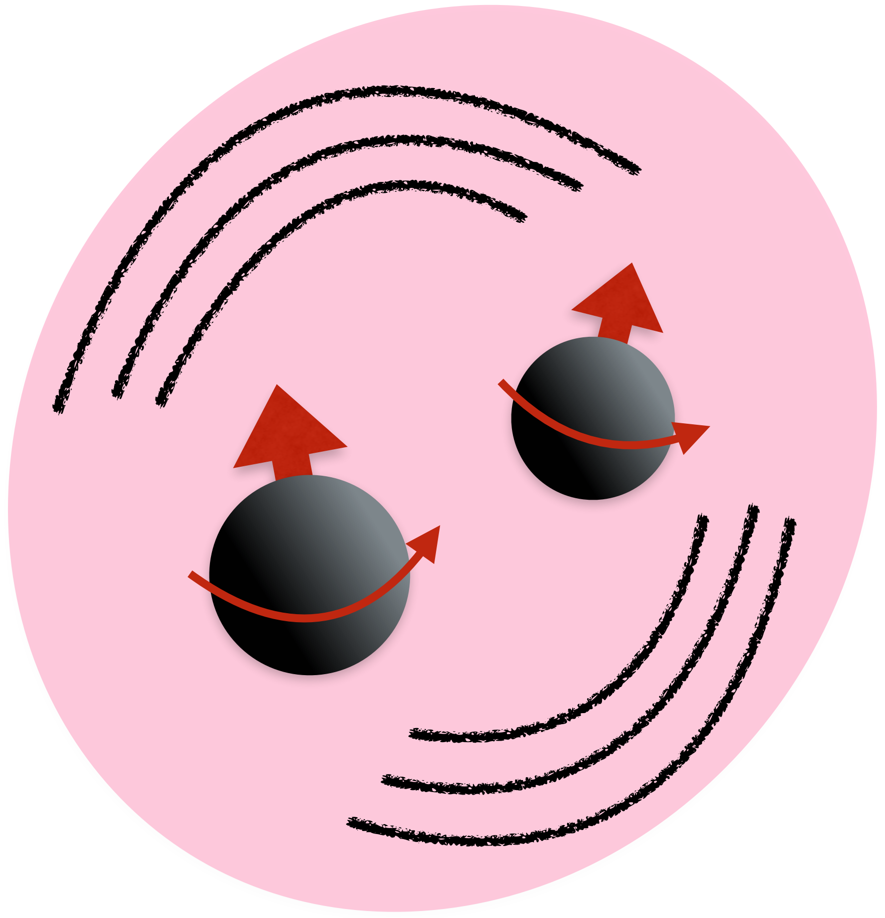
Modeling Spin-Magnitude Change



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<u>States in rest frame:</u>

$$\{\ldots, |s, s_z \in \{-s, \ldots, s\}\rangle,$$

$$|s-1, s_z \in \{-s+1, \ldots, s-1\}\rangle, \ldots\}$$

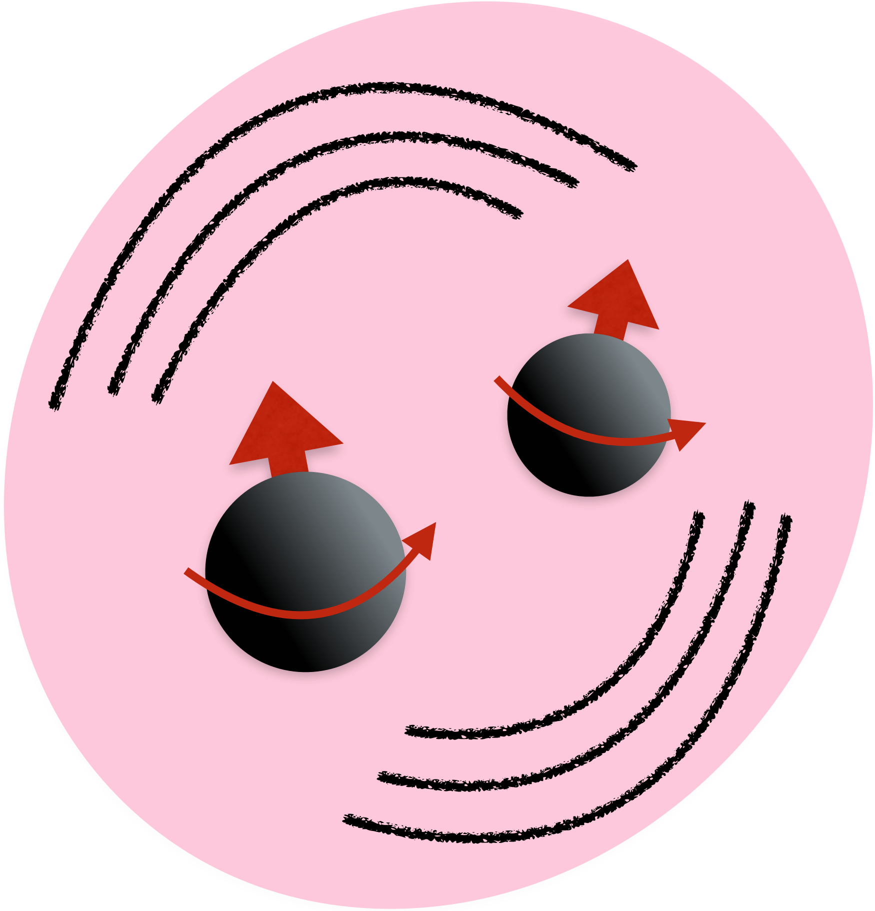
$\leftrightarrow$ spin magnitude variable

[Bern, **DK**, Luna, Roiban, Scheopner, Teng, Vines (2023)]

[Alaverdian, Bern, **DK**, Luna, Roiban, Scheopner, Teng (2024)]

[Alaverdian, Bern, **DK**, Luna, Roiban, Scheopner, Teng (to appear)]

Modeling Spin-Magnitude Change



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ϕ_s - Irreducible representation

of the Little Group

Φ - Reducible representation

$\leftrightarrow$ spin magnitude variable

[Bern, **DK**, Luna, Roiban, Scheopner, Teng, Vines (2023)]

[Alaverdian, Bern, **DK**, Luna, Roiban, Scheopner, Teng (2024)]

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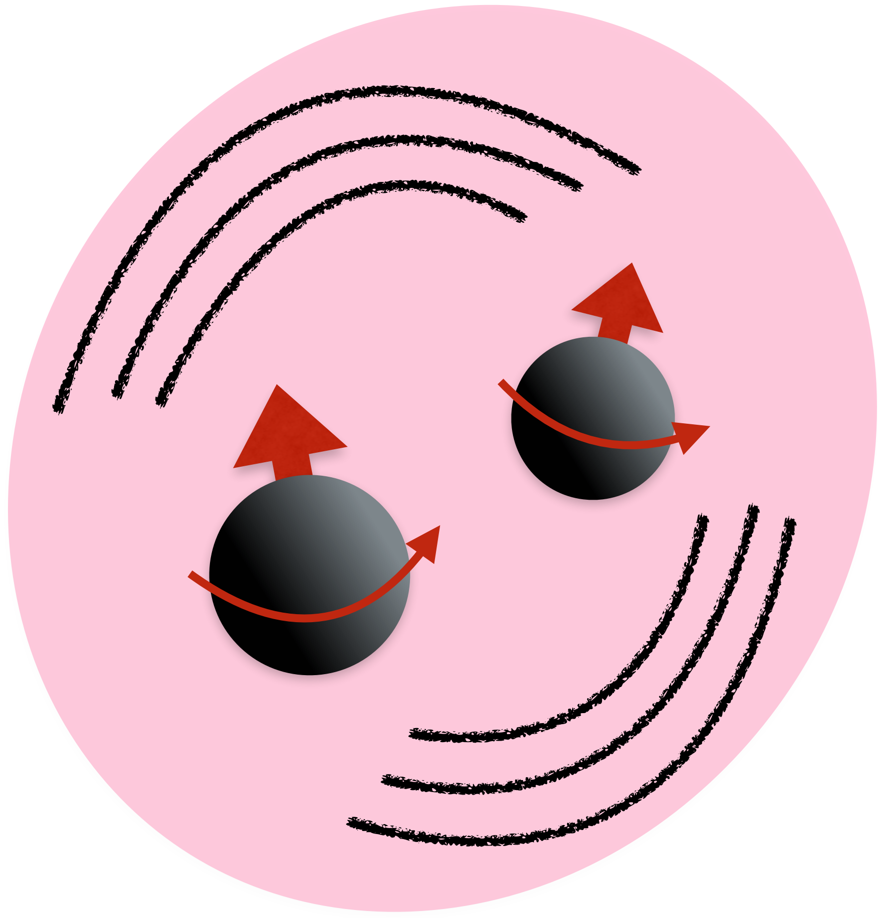
Modeling Spin-Magnitude Change



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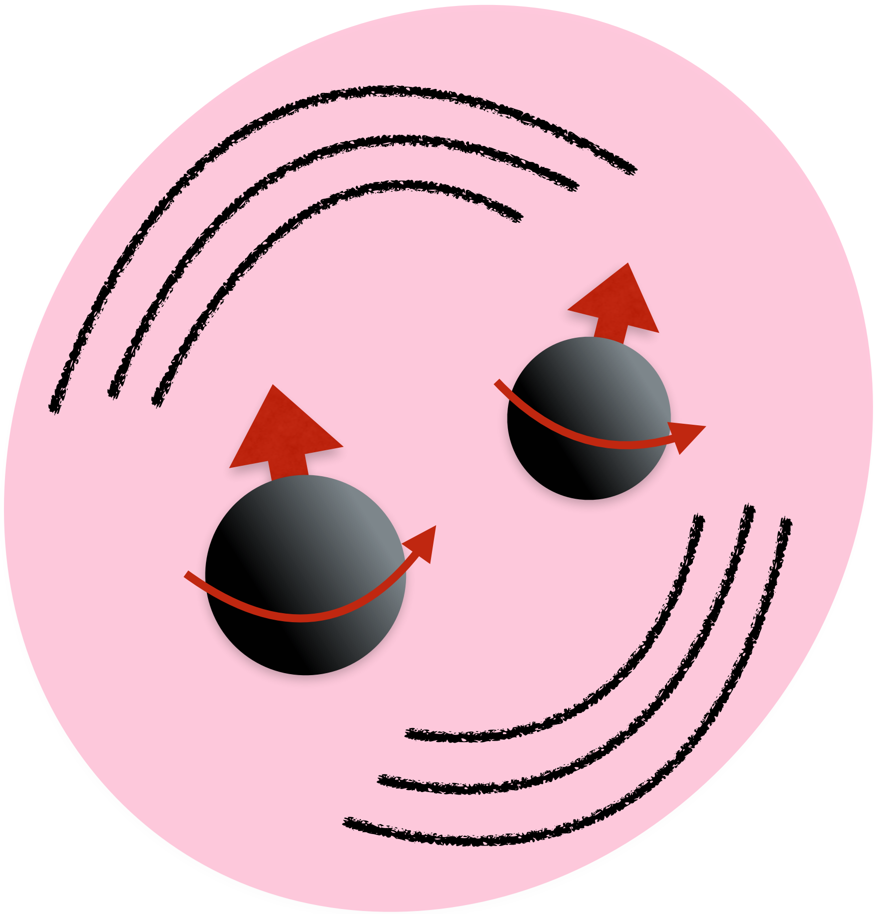
Prediction: Spin-magnitude change w/o energy absorption/dissipation

[Bern, **DK**, Luna, Roiban, Scheopner, Teng, Vines (2023)]

[Alaverdian, Bern, **DK**, Luna, Roiban, Scheopner, Teng (2024)]

[Alaverdian, Bern, **DK**, Luna, Roiban, Scheopner, Teng (to appear)]

New Structure for Compact Objects



$$H = H\left(\mathbf{r}, \mathbf{p}, \mathbf{S}_1, \mathbf{S}_2\right) + V^{(1,3)}(\mathbf{r}^2, \mathbf{p}^2)\frac{\mathbf{r}\cdot\mathbf{K}_1}{\mathbf{r}^2} + \ldots$$

$$V^A(\mathbf{r}^2, \mathbf{p}^2) = \frac{G}{|\mathbf{r}|}c_1^A(\mathbf{p}^2) + \left(\frac{G}{|\mathbf{r}|}\right)^2 c_2^A(\mathbf{p}^2) + \mathcal{O}(G^3)$$

[Bern, **DK**, Luna, Roiban, Scheopner, Teng, Vines (2023)]

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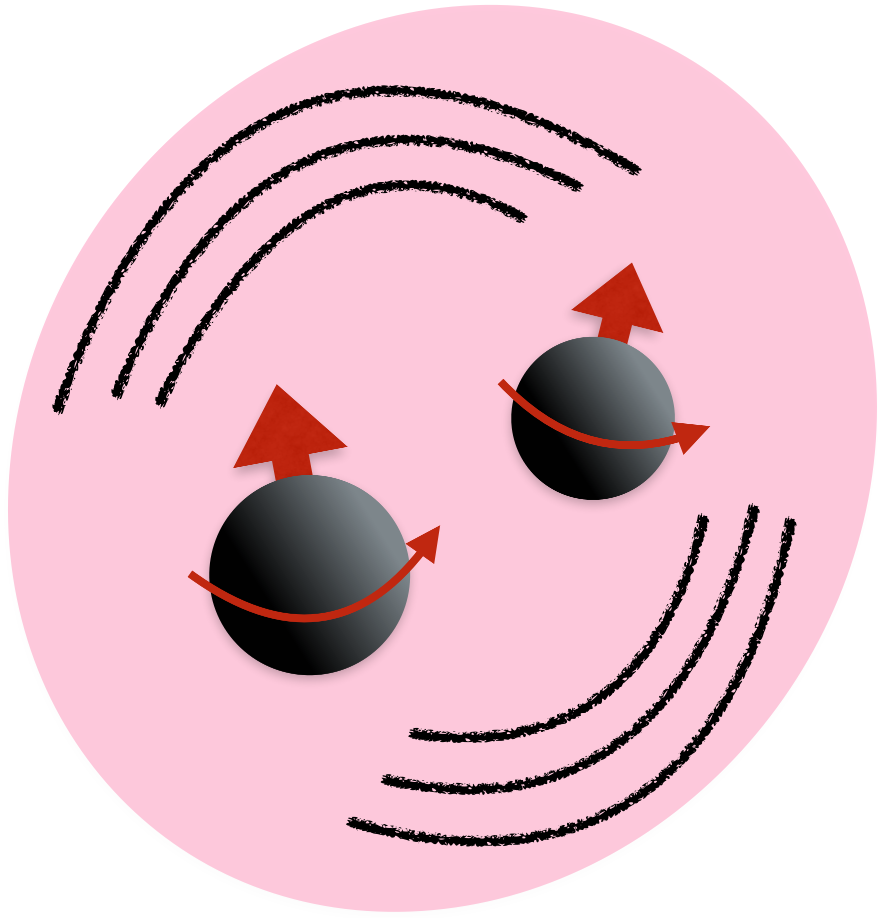

Additional multipolar structure

$$H = H\left(\mathbf{r}, \mathbf{p}, \mathbf{S}_1, \mathbf{S}_2\right) + V^{(1,3)}(\mathbf{r}^2, \mathbf{p}^2)\frac{\mathbf{r}\cdot\mathbf{K}_1}{\mathbf{r}^2} + \dots$$

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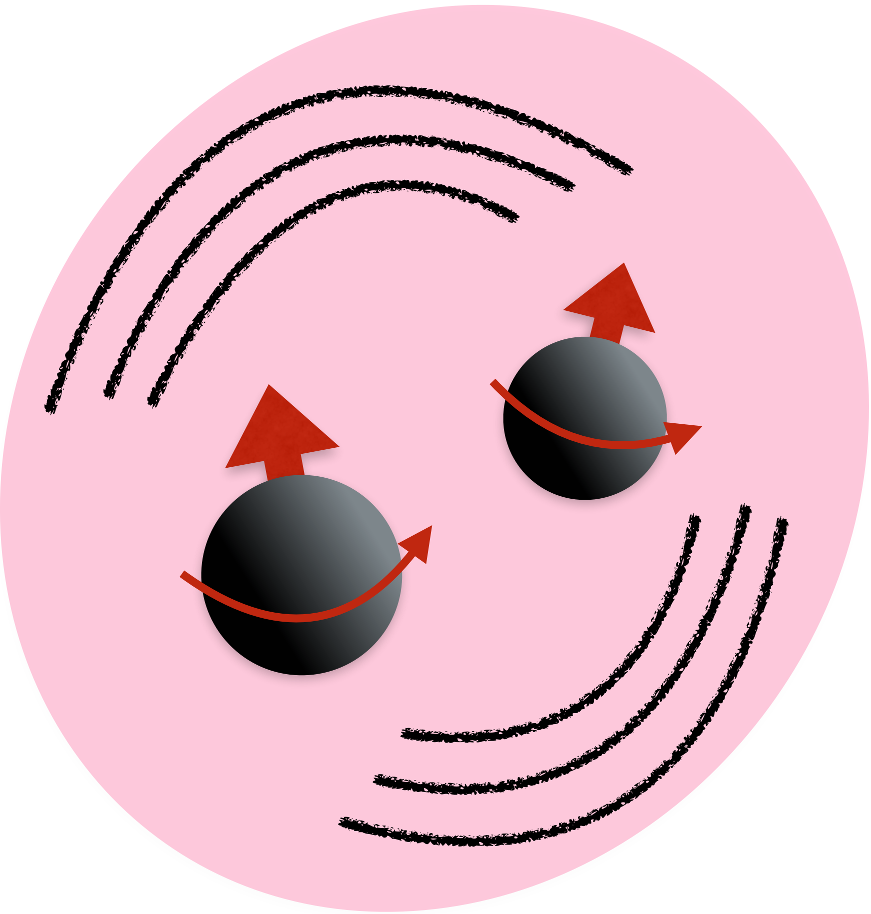

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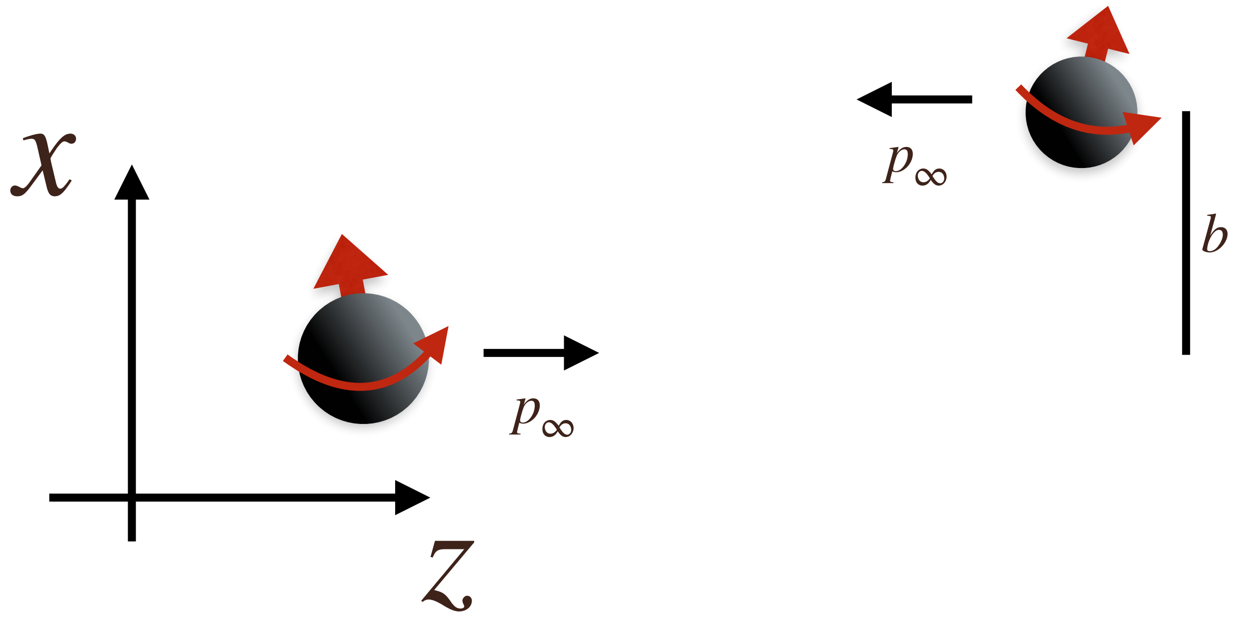


$$\Delta\mathbf{S}_1^2 = \frac{4GE_1E_2\left(K_{1z}^{(0)}S_{1y}^{(0)} - K_{1y}^{(0)}S_{1z}^{(0)}\right)c_1^{(1,3)}(p_\infty^2)}{b\,p_\infty(E_1+E_2)} + \ldots$$

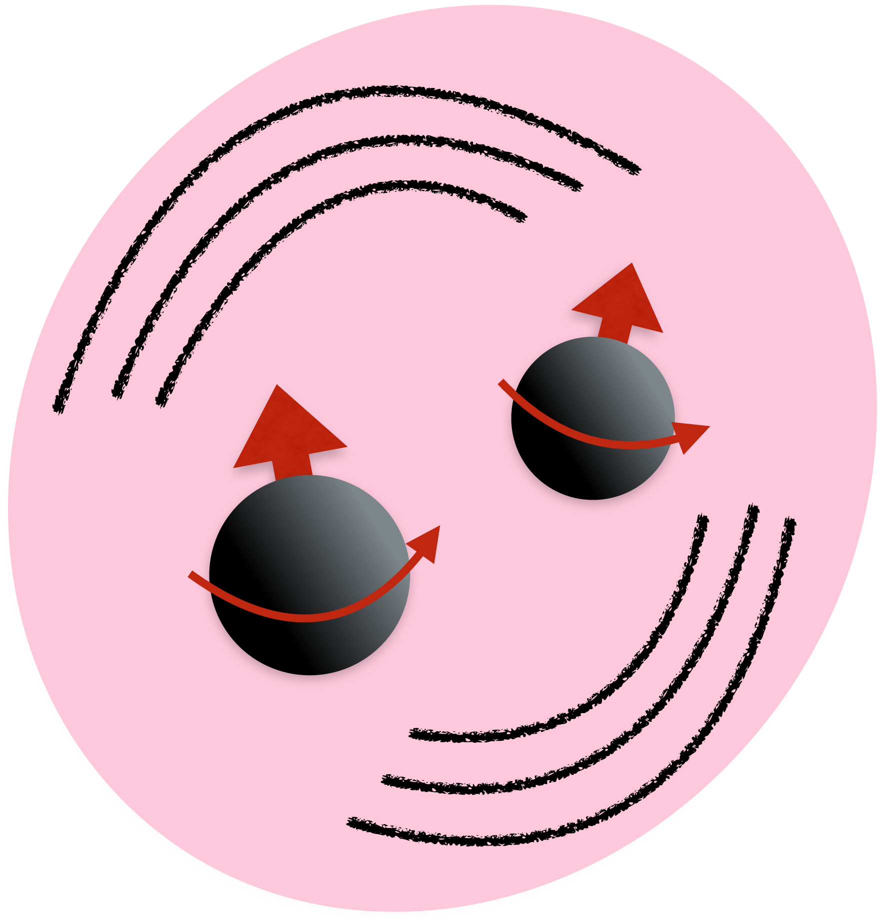
[Bern, **DK**, Luna, Roiban, Scheopner, Teng, Vines (2023)]

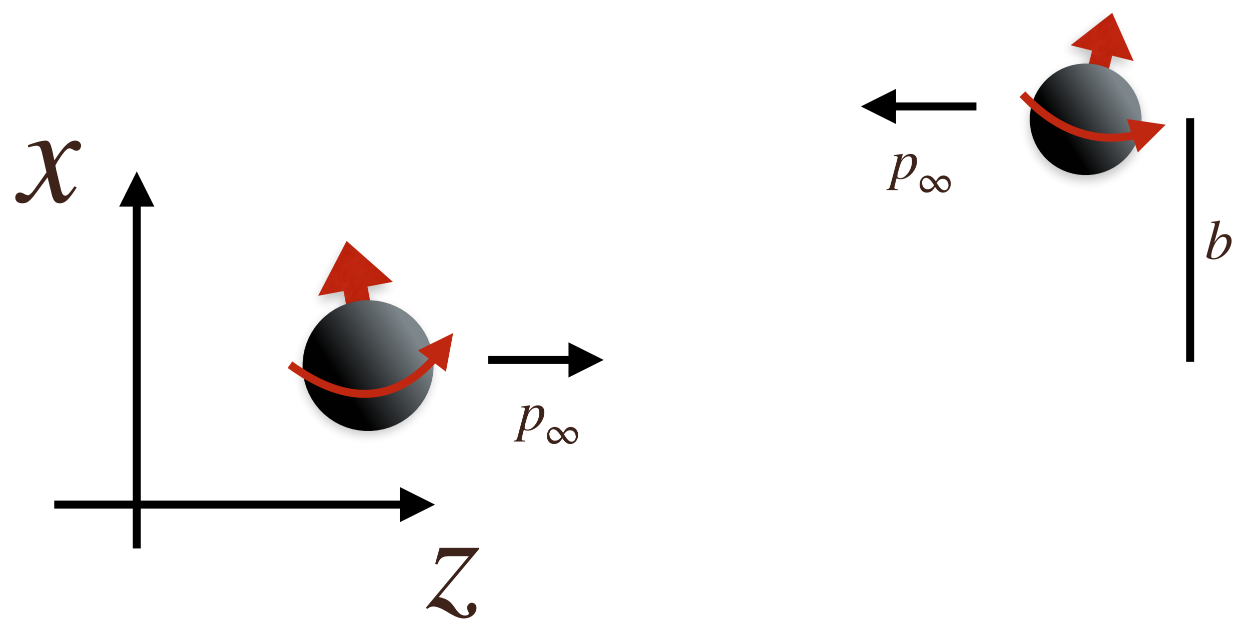
[Alaverdian, Bern, **DK**, Luna, Roiban, Scheopner, Teng (2024)]

[Alaverdian, Bern, **DK**, Luna, Roiban, Scheopner, Teng (to appear)]

Novel Effect in Conservative Dynamics

New Structure Manifests in the Waveform



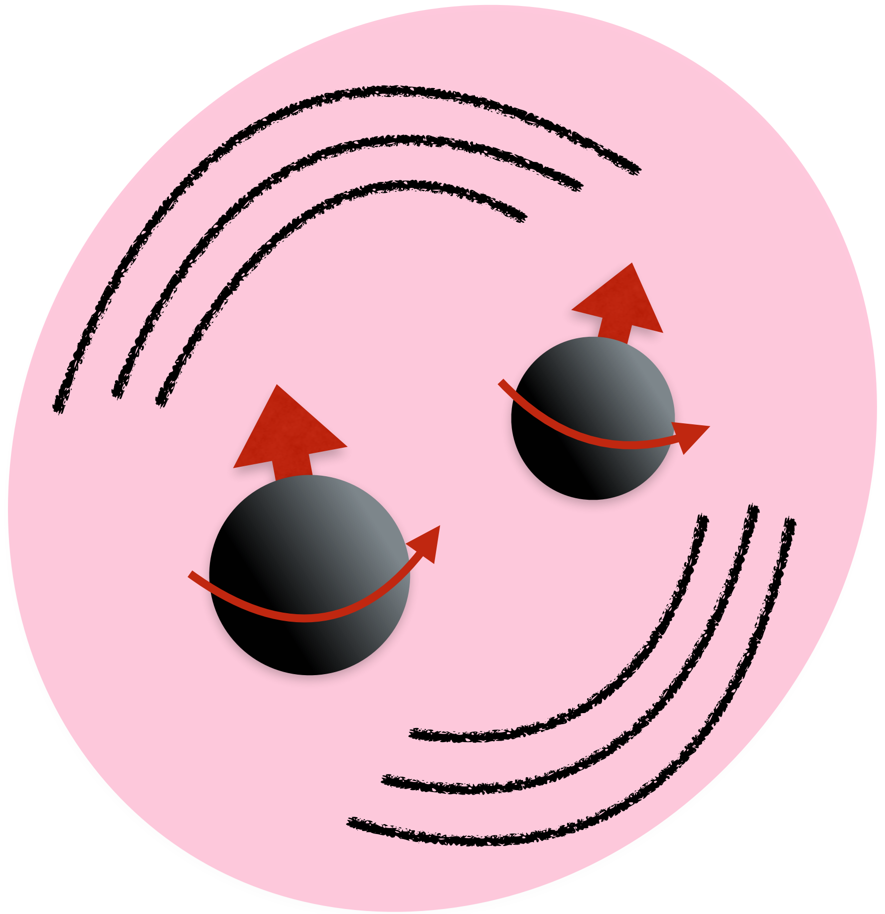


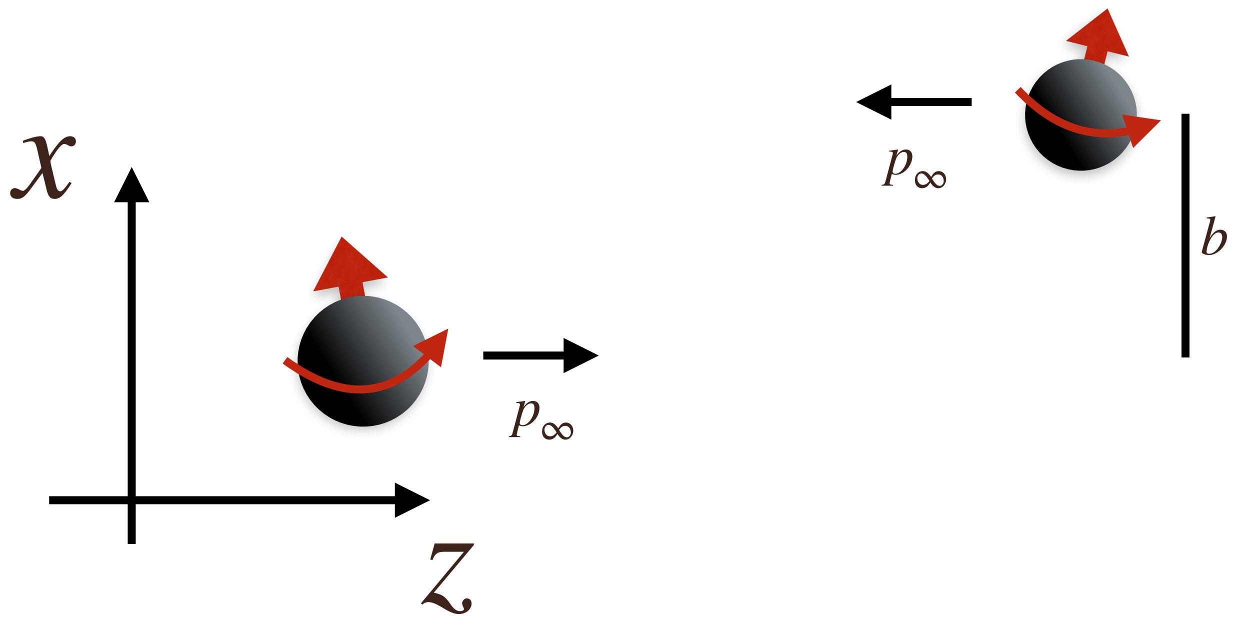
[Alaverdian, Bern, **DK**, Luna, Roiban, Scheopner, Teng (2024)]

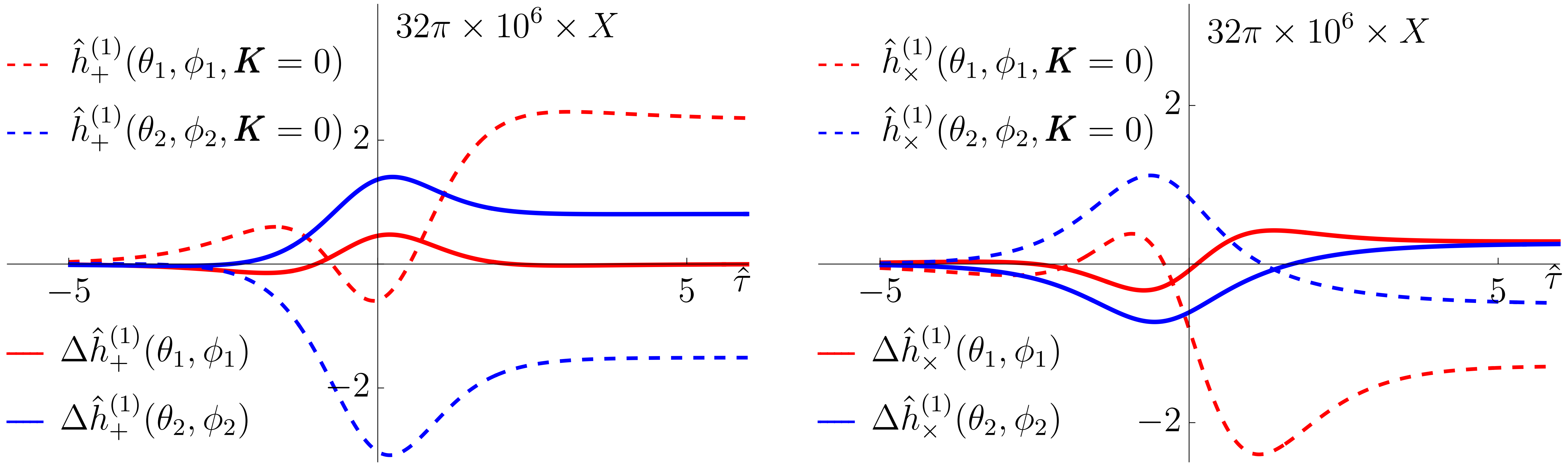
[Alaverdian, Bern, **DK**, Luna, Roiban, Scheopner, Teng (to appear)]

Large Modification to Observables

New Structure Manifests in the Waveform







$$\Delta h^{(1)}_{+/\times} = h^{(1)}_{+/\times}(\ldots, \mathbf{K} = \mathbf{S}) - h^{(1)}_{+/\times}(\ldots, \mathbf{K} = 0)$$

[Alaverdian, Bern, **DK**, Luna, Roiban, Scheopner, Teng (2024)]

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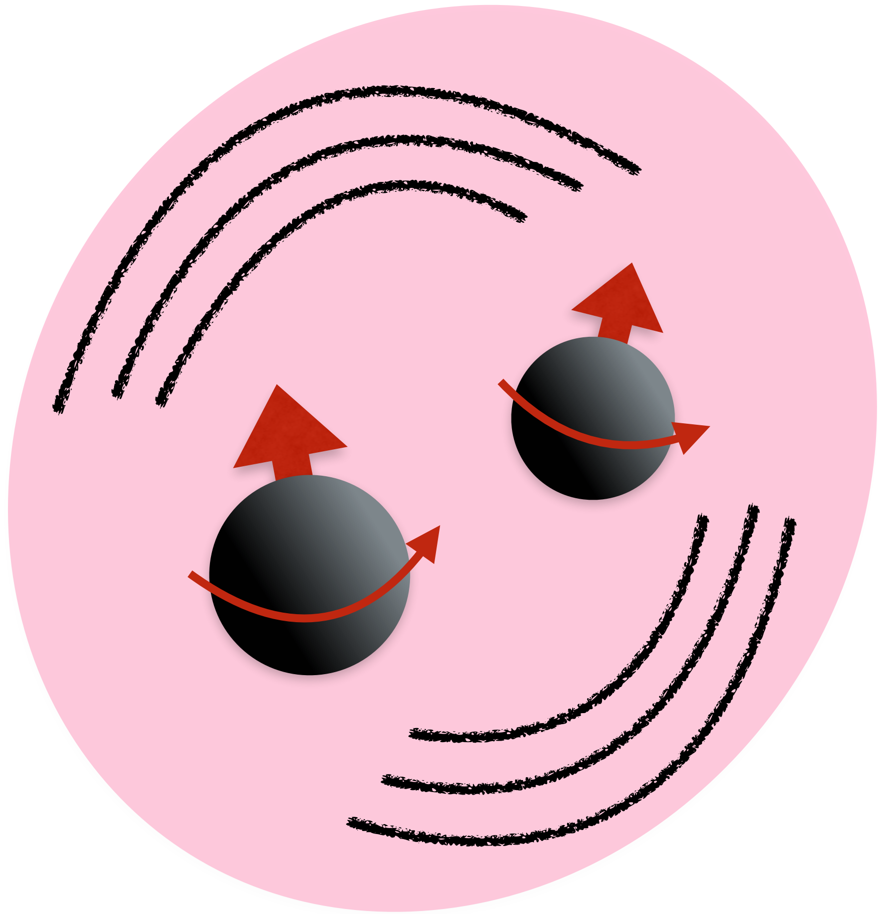
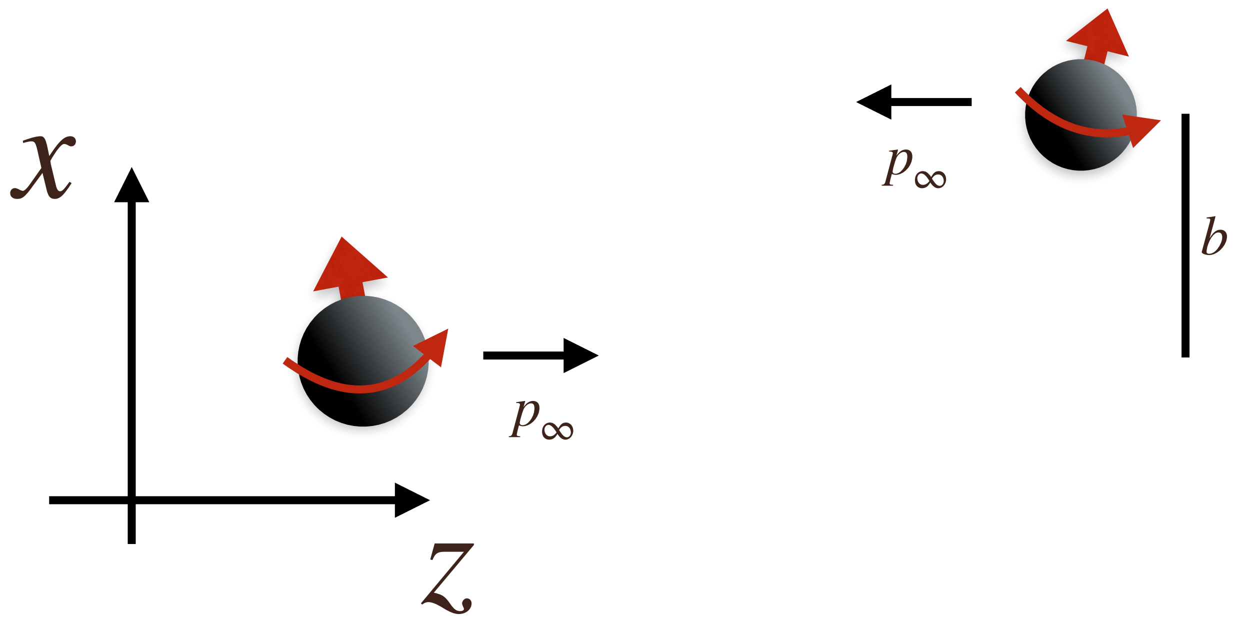
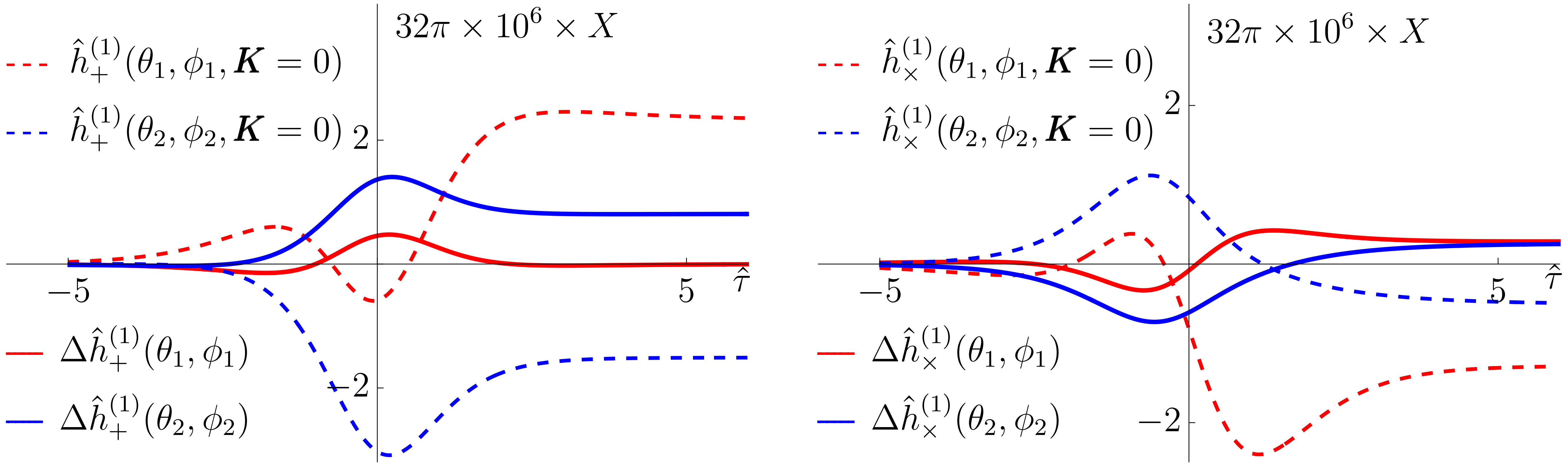




Spin dynamics are much richer than previously thought



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