QED CORRECTIONS
TO EXCLUSIVE LEPTONIC B DECAYS

Cluster of Excellence PRISMA+

Mainz Institute for Theoretical Physics

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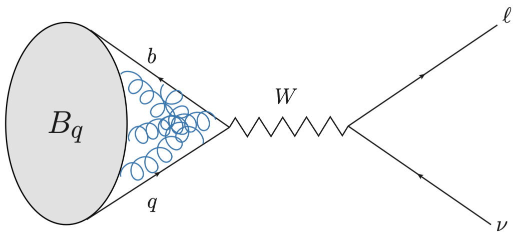
YOUNGST@RS - EFTs and Beyond

Based on arXiv:2212.14430 [hep-ph] and work in progress
In collab. w/ C. Cornella, M. König and M. Neubert (JGU Mainz)

Motivations

- Why leptonic B decays ? $$\mathscr{B}(B^- \to \ell^- \bar{\nu}_\ell) = \tau_B G_F^2 f_B^2 |V_{ub}|^2 \frac{m_B m_\ell^2}{8\pi}\left(1 - \frac{m_\ell^2}{m_B^2}\right)^2$$
  - Direct determination of the CKM element $|V_{ub}|$
  - Chirality-suppressed in the SM → powerful probe of (pseudo) scalar new physics
  - Testing flavor universality in charged current : Belle II will measure the $\ell = \tau, \mu$ channels at $5 - 7\ \%$ [Belle II Physics Book]. FCC-ee prospects are promising.

- Why QED corrections are needed?



  - Pure hadronic effects are simple and well-understood:
$$\langle 0 | \bar{q}\gamma^\mu\gamma_5 b | B_q(p)\rangle = i f_{B_q} p^\mu$$
and f_{B_q} is known with $\mathscr{O}(1\%)$ precision : $f_{B_q} = 189.4 \pm 1.4$ Mev [FNAL/MILC 1712.09262]
  - In the exclusive channel, with strong cuts on additional soft radiations, QED corrections can be sizable and compete with QCD uncertainties → need a precise estimation

A multi-scale process

- Focusing on $\ell = \mu$ case, including QED effects introduce new scales (both static and dynamic) to which $B^- \to \mu^- \bar{\nu}_\mu$ is sensitive :

m_W

m_B

$\mu_{hc} = \sqrt{\Lambda_{QCD} m_B}$

$m_\mu \sim \Lambda_{QCD}$

$\mu_{us} = E_0$

$\mu_{usc} \sim E_0 \frac{m_\mu}{m_B}$

Static scales dictated by the energy cut on the additional final radiation

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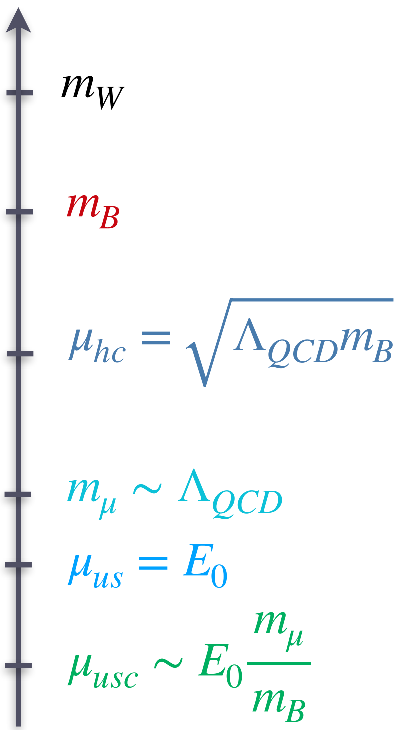
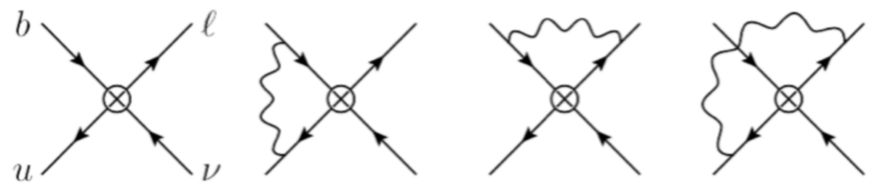
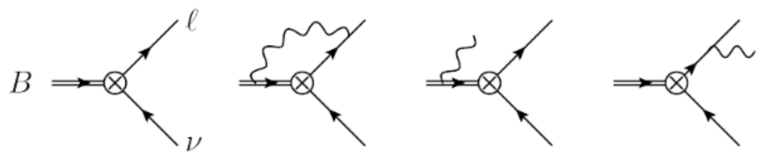




B-meson described as a superposition of Fock-states: $|\bar{u}b\rangle \oplus |\bar{u}bg\rangle \oplus \cdots$

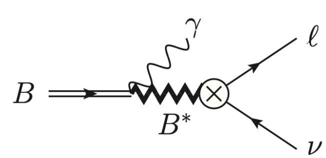
[Beneke et al(2019),JHEP10232]

B-meson described as a point-like pseudo-scalar boson…



[Dai et al (*2022*), Phys.Rev.D 105 3]

…including its first vector excited state as off-shell intermediate propagator !



[Becirevic et al (0907.1845)]

The plan : running down with the scale !

Turning a multi-scale problem to a product of single scale objects by :

- Identifying the appropriate effective description at each scale.
- Performing a step-by-step matching between each EFT.
- Deriving a factorization theorem to break this multi-scale problem into a convolution of single-scale objects.
- Using the renormalisation group to evaluate each object at its natural scale and run it to a common scale to resum logarithms.

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In this talk we are going to embark on an EFT journey :

- Discussing the (re)factorization of the virtual amplitude in the partonic picture.
- Constructing the proper EFT valid at low energy to include the real emissions.
- Deriving the factorization formula for the decay rate.

Fermi theory $\rightarrow$ HQET $\otimes$ SCET$_I$ $\quad \mu \sim m_B$

Power counting: $\lambda = \frac{m_\ell}{m_B} \sim \frac{\Lambda_{QCD}}{m_B}$

Relevant scalings : $p^\mu = (\bar{n}\cdot p, n\cdot p, p_\perp)$

$p \sim \left(1, \lambda^2, \lambda\right)$ **« collinear »,**

$p^2 \sim m_\ell^2 \sim \mathcal{O}(\lambda^2)$

$\rightarrow$ given by the lepton virtuality

$p \sim \left(1, \lambda, \sqrt{\lambda}\right)$ « hard-collinear »,

$p^2 \sim m_\ell m_b \sim \mathcal{O}(\lambda)$

$\rightarrow$ soft and collinear quark X-talk

$p \sim \left(\lambda, \lambda, \lambda\right)$ « soft »,

$p^2 \sim \Lambda_{QCD}^2 \sim \mathcal{O}(\lambda^2)$

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Fermi theory $\rightarrow$ HQET $\otimes$ SCET$_I$ $\quad \mu \sim m_B$

- The b quark is described by a soft HQET field :

$$b(x) \rightarrow e^{-im_b(v\cdot x)}\left(1+\mathcal{O}(\sqrt{\lambda})\right) h_v(x)$$

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- QED corrections $\rightarrow$ hard-collinear momentum exchange between partons & lepton.

$\rightarrow$ need a SCET$_I$ subleading power description for the different modes of the spectator and the lepton :

$$q(x) \rightarrow \left(1+\frac{1}{i\bar{n}\cdot D_s}(i\not{D}_\perp)\frac{\not{\bar{n}}}{2}\right)\xi_C^{(q)}(x)+\left(1+\frac{1}{i\bar{n}\cdot D_s}eQ_q\not{A}_{C\perp}\frac{\not{\bar{n}}}{2}\right)q_s(x)$$

$$\ell(x) \rightarrow \left(1+\frac{1}{i\bar{n}\cdot D_s}(i\not{D}_\perp+m_\ell)\frac{\not{\bar{n}}}{2}\right)\xi_C^{(\ell)}(x)+\left(1+\frac{1}{i\bar{n}\cdot D_s}eQ_q\not{A}_{C\perp}\frac{\not{\bar{n}}}{2}\right)\ell_s(x)$$

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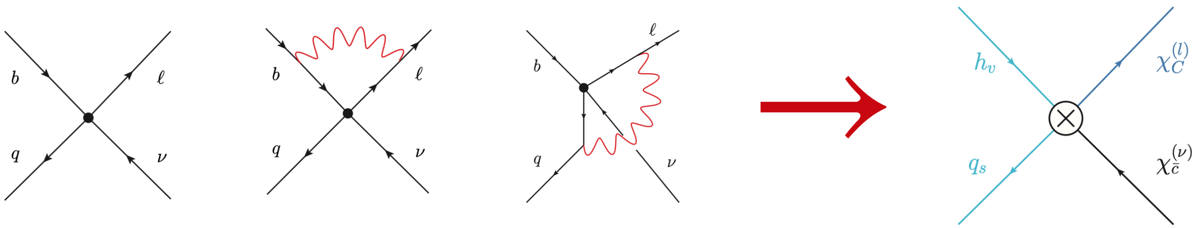
SCET$_I$ operators

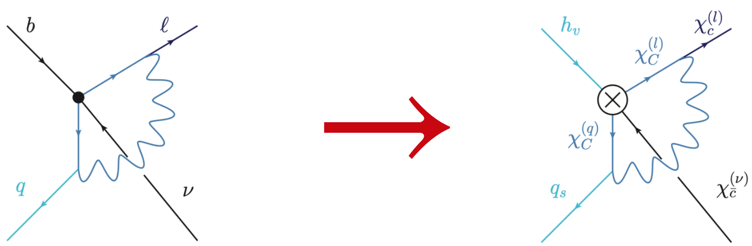
We build our SCET$_I$ basis with the following power counting :

$h_v, q_s \sim \mathcal{O}(\lambda^{3/2})$ $\quad \chi_C \sim \mathcal{O}(\lambda^{1/2})$ $\quad \mathcal{A}_C^{\perp} \sim \mathcal{O}(\lambda^{1/2})$ $\quad \chi_c \sim \mathcal{O}(\lambda)$ $\quad \mathcal{A}_c^{\perp} \sim \mathcal{O}(\lambda)$ $\quad \chi_{\bar{c}} \sim \mathcal{O}(\lambda)$

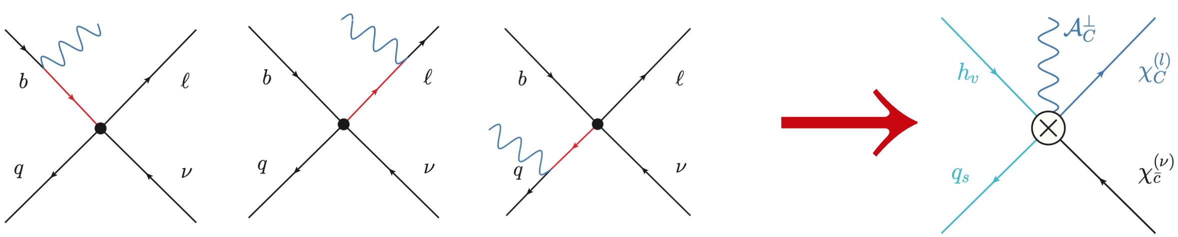
At $\mathcal{O}(\alpha)$, three classes of 4-fermions operators are relevant:



[A] $(\bar{q}_s \ldots h_v)(\bar{\chi}_C \ldots \chi_{\bar{c}})$



[B] $(\bar{\chi}_C{}^{(q)} \ldots h_v)(\bar{\chi}_C{}^{(\ell)} \ldots \chi_{\bar{c}})$



[C] $(\bar{q}_s \ldots h_v)(\bar{\chi}_C{}^{(\ell)} \ldots \chi_{\bar{c}})\mathcal{A}_C^{\perp}$

$\text{SCET}_I \to \text{SCET}_{II}$ $\quad \mu \sim \mu_{hc} \sim \sqrt{\Lambda_{QCD} m_B}$

- At $\mu \sim \mu_{hc}$, we lower the virtuality removing hard-collinear modes $\to$ pure SCET_{II} construction where collinear and soft carry the same virtuality :

$$p_c \sim (1, \lambda^2, \lambda), \qquad p_s \sim (\lambda, \lambda, \lambda), \qquad p_c^2 \sim p_s^2 \sim \mathcal{O}(\lambda^2)$$

- Integrating out hard-collinear propagators introduces non-localities even in soft product :

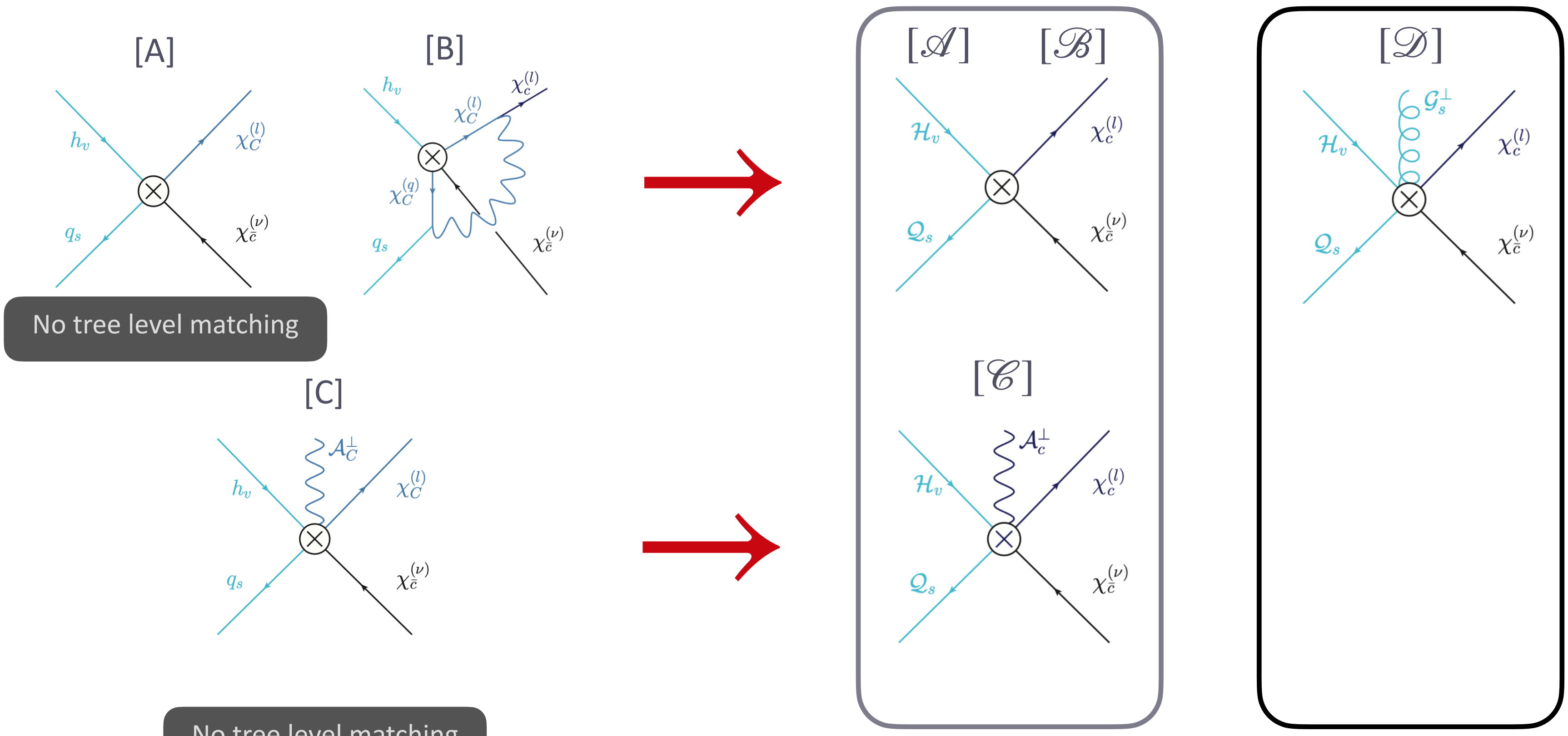
$$\frac{1}{n \cdot \partial} q_s, \left(\frac{1}{n \cdot \partial} \mathcal{G}_s^{\perp}\right)\left(\frac{1}{n \cdot \partial} q_s\right), \cdots$$

$\to$ Contains more fields but are of the same order !

- Contrary to the $B_s \to \mu^+ \mu^-$ case, those structure-dependent contributions to $B^- \to \mu^- \bar{\nu}_\mu$ carry the same suppression as the tree level result and do not overcome the chiral suppression.
  [Beneke, Bobeth, Szafron 2017, 2019]

SCET$_{II}$ basis

- Starting from our SCET$_{I}$ operators, we can construct our SCET$_{II}$ basis :



$$|B\rangle = |\bar{u}b\rangle \oplus |\bar{u}bg\rangle \oplus \cdots$$

Refactorization

- The matrix element of B-type SCET II operators involve an endpoint-divergent integral→ the theory fails to properly separate hard-collinear modes with low energy (hard-collinear fraction $v \to 0$) from soft modes.
- This region can be identified with the energetic limit of the soft region → we rearrange terms between the two (RBS scheme) [Liu, Neubert 2019; Liu, Mecaj, Neubert, Wang 2020; Beneke et al. 2022]

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Jet function

$$[\![\mathcal{J}_{B1}]\!](v,\omega,\Lambda) = \mathcal{J}_{B1}(v,\omega) - \theta(r_\Lambda - v)\delta\mathcal{J}_{B1}(v,\omega)$$

$$[\![\mathcal{J}_{B1}^{3part.}(v,\omega,\omega_g,\Lambda)]\!] = \mathcal{J}_{B1}^{3part.}(v,\omega,\omega_g) - \theta(r_\Lambda - v)\delta\mathcal{J}_{B1}(v,\omega,\omega_g)$$

Soft function

$$\langle O_{\mathscr{A}}^{(1)}\rangle = \left\langle \frac{m_\ell}{(\bar{n}\cdot\mathscr{P}_c)}\left(\bar{q}_s\, \not{\bar{n}} P_L S_n^{(l)\dagger} h_v\right)\left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)}\right)\right\rangle$$

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Subtraction term, singular for $v \to 0$

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Add it back

Soft function

$$\langle O_{\mathscr{A}}^{\Lambda(1)} \rangle = \left\langle \frac{m_\ell}{(\bar{n} \cdot \mathscr{P}_c)} \left(\bar{q}_s \theta(\Lambda + i\bar{n} \cdot \overleftarrow{D}_s) \not{n} P_L S_n^{(l)\dagger} h_v \right) \left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right) \right\rangle$$

[Cornella, König, Neubert 2023]

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$$\langle O_{\mathcal{A}}^{\Lambda(1)}\rangle = \left\langle \frac{m_\ell}{(\bar{n}\cdot\mathcal{P}_c)}\left(\bar{q}_s\theta(\Lambda + i\bar{n}\cdot\overleftarrow{D}_s)\not{n}P_L S_n^{(l)\dagger}h_v\right)\left(\bar{\chi}_c^{(\ell)}P_L\chi_{\bar{c}}^{(\nu)}\right)\right\rangle \propto F_\Lambda(\mu)$$

New hadronic parameter !

[Cornella, König, Neubert 2023]

Factorization formula (virtual corrections)

▸ Taking the matrix elements of our SCET_{II} operators, the virtual corrections can be expressed as :

$$
\begin{aligned}
i\mathscr{A}_{\text{virt}} = \; & H_{A_0}^{(1)}(\mu)\,\langle O_{\mathscr{A}}^{\Lambda(1)}\rangle + H_{A_0}^{(2)}(\mu)\,\langle O_{\mathscr{A}}^{(2)}\rangle \\
& + \int_0^\infty d\omega \int_0^1 dv \left[\sum_{i=1,2} H_{B_0}^{(i)}(v,\mu)\, \mathscr{J}_{B_0}^{(i)}(v,\omega) + H_{B_1}(v,\mu)\, [\![\mathscr{J}_{B_1}(v,\omega,\Lambda)]\!] \right] \langle O_{\mathscr{B}}^{(1)}(\omega)\rangle_s \\
& + \sum_{i=1,2} \int_0^1 du\; H_{C_0}^{(i)}(u,\mu)\,\langle O_{\mathscr{C}}^{(i)}\rangle \\
& + \int_0^\infty d\omega \int_0^\infty d\omega_g \int_0^1 dv\; H_{B_1}(v,\mu)\, [\![\mathscr{J}_{B_1}^{3part.}(v,\omega,\omega_g)]\!]\, \langle O_{\mathscr{D}}(\omega,\omega_g)\rangle
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$$+\int_0^\infty d\omega \int_0^1 dv \left[\sum_{i=1,2} H^{(i)}_{B_0}(v,\mu)\,\mathscr{J}^{(i)}_{B_0}(v,\omega) + H_{B_1}(v,\mu)\,[\![\mathscr{J}_{B_1}(v,\omega,\Lambda)]\!]\right]\langle O^{(1)}_{\mathscr{B}}(\omega)\rangle_s$$

$$+\sum_{i=1,2}\int_0^1 du\; H^{(i)}_{C_0}(u,\mu)\,\langle O^{(i)}_{\mathscr{C}}\rangle$$

$$+\int_0^\infty d\omega \int_0^\infty d\omega_g \int_0^1 dv\; H_{B_1}(v,\mu)\,[\![\mathscr{J}^{3part.}_{B_1}(v,\omega,\omega_g)]\!]\,\langle O_{\mathscr{D}}(\omega,\omega_g)\rangle$$

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$\propto F_\Lambda(\mu)$

$$+ \int_0^\infty d\omega \int_0^1 dv \left[\sum_{i=1,2} H^{(i)}_{B_0}(v,\mu)\, \mathscr{J}^{(i)}_{B_0}(v,\omega) + H_{B_1}(v,\mu)\, [\![\mathscr{J}_{B_1}(v,\omega,\Lambda)]\!] \right] K^{(1)}_{\mathscr{B}}(\mu)\, \langle O^{(1)}_{\mathscr{B}}(\omega)\rangle_s$$

$$+ \sum_{i=1,2} \int_0^1 du\, H^{(i)}_{C_0}(u,\mu)\, K^{(i)}_{\mathscr{C}}(u,\mu)\, \langle O^{(i)}_{\mathscr{A}}\rangle_s$$

$$+ \int_0^\infty d\omega \int_0^\infty d\omega_g \int_0^1 dv\, H_{B_1}(v,\mu)\, [\![\mathscr{J}^{3part.}_{B_1}(v,\omega,\omega_g)]\!]\, K_{\mathscr{D}}(\mu)\, \langle O_{\mathscr{D}}(\omega,\omega_g)\rangle_s$$

Generalized decay constant

- The QED corrected $O_A^{\Lambda(1)}$ operator defines a generalized decay constant :

$$\langle O_A^{\Lambda(1)} \rangle_s \equiv \left\langle 0 \middle| \bar{q}_s \theta(\Lambda + i\bar{n} \cdot \overleftarrow{D}_s) \not{n} P_L S_n^{(l)\dagger} h_v \middle| B^- \right\rangle$$

$$= -\frac{i\sqrt{m_B}}{2} F_\Lambda(\mu) \, \langle 0 | \, S_{v_B}^{(B)} S_n^{(l)\dagger} \, | 0 \rangle$$

$$S_r^{(i)}(x) = \mathscr{P} \exp\left\{ ieQ_i \int_{-\infty}^{0} ds \, r \cdot A_s(x + sr) \right\}$$

- For $\alpha \to 0$, F_Λ reduces to the standard HQET decay constant :

$$\sqrt{m_B} f_B \left[1 + C_F \frac{\alpha_s(m_b)}{2\pi} + \mathcal{O}(\alpha_s^2) \right] = F_\Lambda(m_b) \Big|_{\alpha \to 0} = F_{\rm QCD}(m_b) \, ,$$

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$$S_r^{(i)}(x) = \mathscr{P} \exp\left\{ ieQ_i \int_{-\infty}^0 ds\, r \cdot A_s(x + sr) \right\}$$

- F_Λ is an unknown non-perturbative parameter following evolution equations for Λ and μ :

$$F_\Lambda(\mu) = F_{m_B}(\mu) \left[1 - \frac{\alpha}{4\pi} Q_\ell Q_u \log \frac{\Lambda^2}{m_B^2} \int d\omega \left(\phi_-(\omega) \log \frac{\mu^2}{\omega\sqrt{m_B \Lambda}} + 2 \int d\omega_g \frac{\phi_3(\omega, \omega_g)}{\omega_g^2} \log \frac{\omega}{\omega + \omega_g} \right) + \cdots \right]$$

two-particle LCDA — three-particle LCDA

$$F_{m_B}(\mu) = F_{m_B}(m_b) \left[1 + \frac{\alpha}{8\pi} Q_\ell Q_u \log^2 \frac{\mu^2}{m_b^2} + \ldots \right]$$

- Need to be computed at a single value of Λ and μ. Lattice determination ? QCD SR estimate ?

Low-energy theory : bHLET ⊗ HSET $\mu < \Lambda_{QCD} \sim m_\mu$

- Below $\mu \sim \Lambda_{QCD}$, quarks hadronize and the meson can be described by a charged scalar of mass m_B. We can describe it with a heavy scalar (HS) :

$$\Phi_B(x) \to \frac{e^{-im_B(v\cdot x)}}{\sqrt{2m_B}}\varphi_v(x)$$

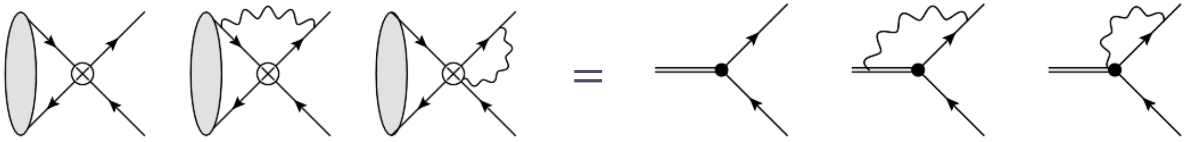
- Since $\Lambda_{\text{QCD}} \sim m_\mu$, also the muon becomes infinitely heavy. We describe it as a boosted heavy lepton field (bHL) :

$$\ell(x) \to \sqrt{\frac{\bar{n}\cdot v_\ell}{2}}e^{-im_\ell(v_\ell\cdot x)}h_{v_\ell}(x)$$

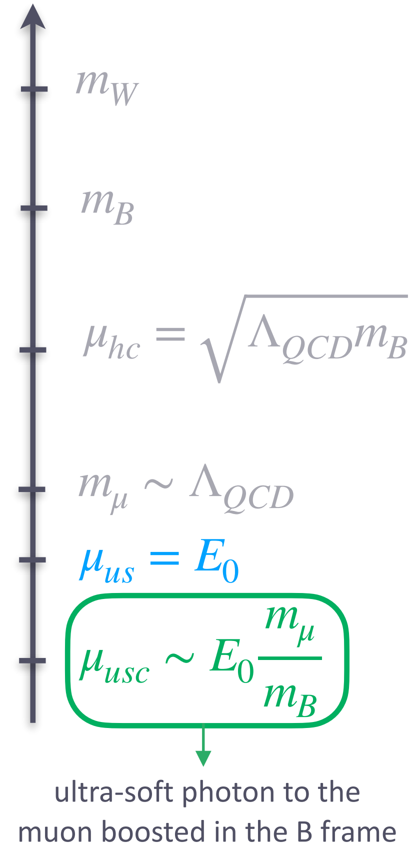
[Fleming et al (hep-ph/0703207); Beneke et al (2305.06401)]

- We match the resulting low energy EFT by taking the hadronic matrix element :

$$\langle \ell\nu | \mathscr{L}_{\text{SCET}_{II}\otimes\text{HQET}} | B\rangle = \langle \ell\nu | \mathscr{L}_{\text{bHLET}\otimes\text{HSET}} | B\rangle$$



A theory of Wilson lines ?



- We can decouple the interactions of the B and the muon with ultrasoft and ultrasoft-collinear photons :

$$\varphi_v(x) \to Y^{(B)}_{v_B}(x) C^{(B)}_{\bar{n}}(x_+)\, \varphi^0_v(x) \qquad h_{v_\ell}(x) \to Y^{(\ell)}_{n}(x_-) C^{(\ell)}_{v_\ell}(x) \frac{\not{n}\not{\bar{n}}}{4} h^0_{v_\ell}(x)$$

$$Y^{(i)}_r(x) = \mathscr{P} \exp\left\{ ieQ_i \int_{-\infty}^{0} ds\, r \cdot A_{us}(x + sr) \right\} \quad C^{(i)}_r(x) = \mathscr{P} \exp\left\{ ieQ_i \int_{-\infty}^{0} ds\, r \cdot A_{usc}(x + sr) \right\}$$

- Real corrections can be expressed as matrix elements of these Wilson lines, convoluted with the measurement function implementing the experimental radiation veto

$$S(E_0, \mu) = \int_0^\infty d\omega_s \int_0^\infty d\omega_{sc}\, \theta\left(\frac{E_0}{2} - \omega_{us} - \omega_{usc} \right) W_{us}(\omega_{us}, \mu) W_{usc}(\omega_{usc}, \mu)$$

Factorization formula…

- Using this framework, we can write the factorization formula for $B^- \to \mu^- \bar{\nu}_\mu$ at the level of the decay rate.

- Real emissions are factorized at the level of the decay rate . Ultrasoft/ultrasoft-collinear logarithms can be resummed in Laplace space ($\sim \mathcal{O}(10\%)$ corrections).

- The virtual corrections already factorize at the amplitude level and appear as an effective Yukawa coupling containing the hard/hard-collinear logarithms (which could be resummed but lead to a $\sim \mathcal{O}(1\%)$ correction).

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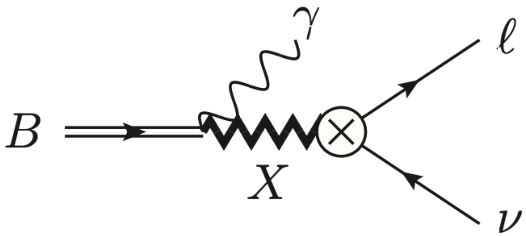
$$S(E_0, \mu) = W_{us}(E_0, \mu) \otimes W_{usc}(E_0, \mu)$$

...incomplete !

- Below Λ_{QCD} , the initial B meson can emit a photon and transition into an excited state X :

[Becirevic et al (0907.1845)]



$$\frac{1}{m_B E_0 - \delta_X^2}$$

- For $E_0 \lesssim \Lambda_{QCD}$, only the B^* needs to be included (heavy-spin symmetry), contributions from higher states being power-suppressed → Pole-dominance approximation

$$\delta_X^2 = \frac{m_X^2 - m_B^2}{2m_B} \lesssim E_0 \qquad \delta_{B^*}^2 \sim \mathcal{O}(\lambda^2) \qquad \delta_{B_1'}^2, \delta_{B_2^*}^2 \sim \mathcal{O}(\lambda)$$

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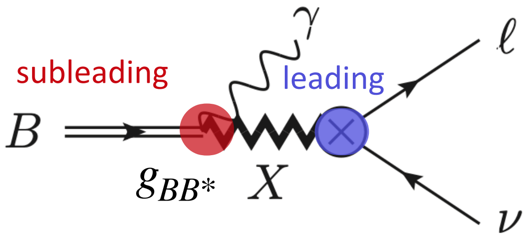

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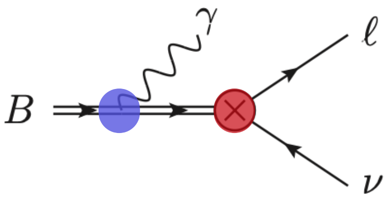
- The $B \rightarrow B^*\gamma$ interaction carries the same power suppression as the effective Yukawa.



- g_{BB^*} is unobserved, has to be estimated by a mixture of QCD sum rules, quark models and lattice QCD.

Factorization formula…. now complete !

$$\Gamma = \Gamma_0\Big(\underbrace{|\,y_B(\mu)\,|^2}_{\text{Non-radiative}} \underbrace{S(E_0,\mu)}_{\text{Radiative}} + \underbrace{|\,y_B^*\,|^2 S_{B*}(E_0)}_{\text{Structure-dependent radiative corrections}}\Big)$$

Non-radiative Radiative Structure-dependent radiative corrections

Factorization formula.... now complete !

$$\Gamma = \Gamma_0\Big(\,|\,y_B(\mu)\,|^2\,S(E_0,\mu) + |\,y_B^*\,|^2\,S_{B*}(E_0)\Big)$$

Non-radiative   Radiative   Structure-dependent radiative corrections

- Virtual QED correction breaks the heavy-spin symmetry for the effective Yukawa couplings :

$$y_B(m_\ell) = 6.30 \cdot 10^{-8}\,(1 \pm 0.05_{V_{ub}} \pm 0.02_{F_{m_B}} \pm 0.01_{f_B})\,,$$

$$y_{B^*}(m_\ell) = 6.69 \cdot 10^{-8}\,(1 \pm 0.05_{V_{ub}} \pm 0.06_{f_{B^*}})\,.$$

We choose $\Lambda = m_B$

$$F_{m_B}(m_b) = (1.00 \pm 0.02) \cdot F_{\mathrm{QCD}}(m_b)\,.$$

Conservative assumption !

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- We get an estimation of the branching ratio for various values of $E_\gamma^{max} = E_0/2$:

$$\mathscr{B}(B^- \to \mu^-\bar\nu_\mu)\Big|_{E_\gamma^{\rm max}=25\,\rm MeV} = 4.00 \cdot 10^{-7}\Big(1 \pm 0.09_{V_{ub}} \pm 0.04_{F_{m_B}} \pm 0.01_{f_B}\Big)$$
$$\mathscr{B}(B^- \to \mu^-\bar\nu_\mu)\Big|_{E_\gamma^{\rm max}=100\,\rm MeV} = 4.16 \cdot 10^{-7}\Big(1 \pm 0.09_{V_{ub}} \pm 0.04_{F_{m_B}} \pm 0.01_{f_B} \pm 0.01_{g_{BB^*}}\Big)$$

Factorization formula…. now complete !

$$\Gamma = \Gamma_0\Big(\,|\,y_B(\mu)\,|^2\, S(E_0,\mu) + |\,y_B^*\,|^2\, S_{B*}(E_0)\Big)$$

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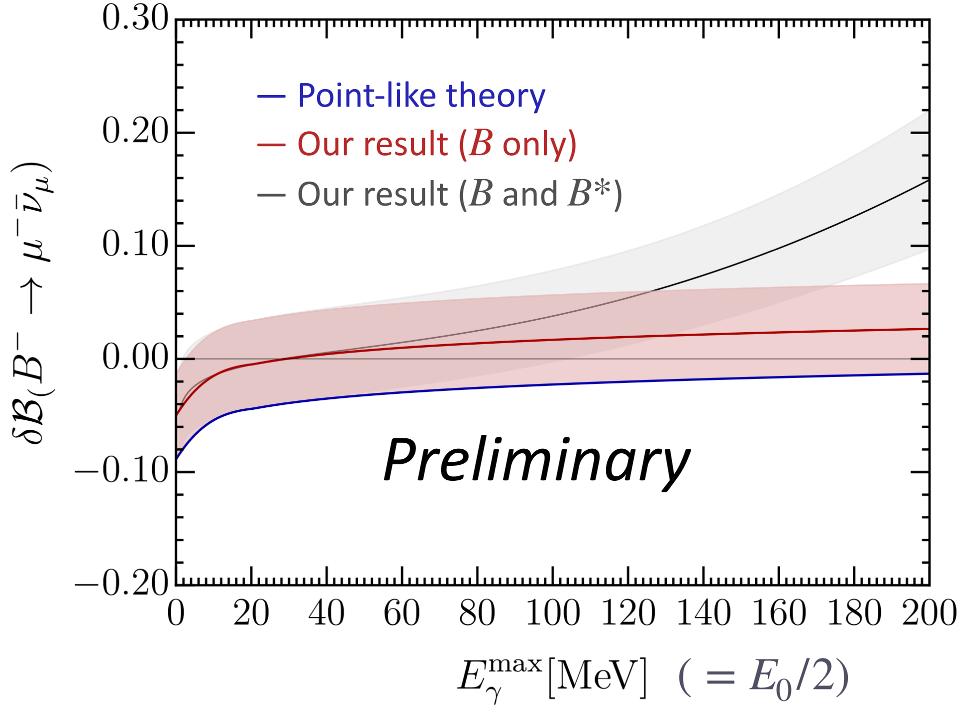
Precision for the extraction of V_{ub}

$$\mathscr{B} = \mathscr{B}_0\Big(|y_B(\mu)|^2 S(E_s,\mu) + |y_B^*|^2 S_{B*}(E_S)\Big)$$

$$\delta\mathscr{B}(B^- \to \mu^-\bar{\nu}_\mu) \equiv \frac{\mathscr{B}(B^- \to \mu^-\bar{\nu}_\mu)}{\mathscr{B}_0(B^- \to \mu^-\bar{\nu}_\mu)} - 1$$

$$F_{m_B}(m_b) = (1.00 \pm 0.02) \cdot F_{\mathrm{QCD}}(m_b)\,.$$

Conservative assumption !



$$S(E_0,\mu) \sim \log\frac{E_0}{m_B}\log\frac{m_\ell}{m_B}$$

$$S_{B*}(E_0) \sim E_0^2$$

Conclusions

- The exclusive leptonic decay $B^- \to \mu^- \bar{\nu}_\mu$, is an important channel for test of physics in SM and beyond. Since future experimental accuracy calls for percent precision in the theory prediction, QED corrections are needed.

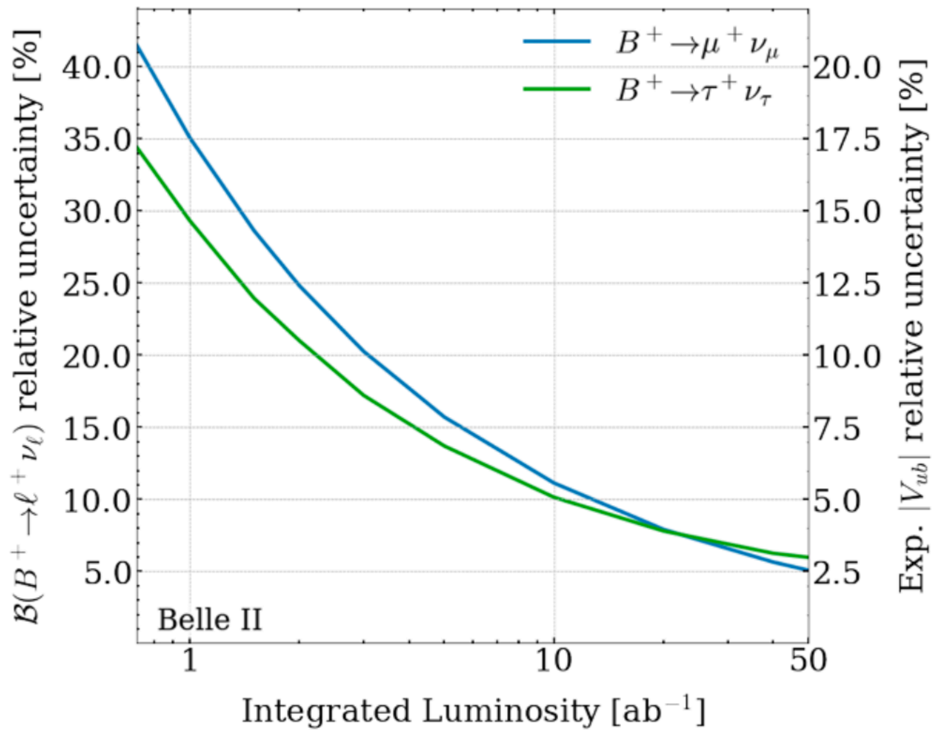
- On top of large logarithms of lepton mass and photon cuts, we find (single logarithmic) structure-dependent effects in the virtual & real corrections:
  - virtual: hard-collinear photons between the lepton and light spectator quark
  - real: B* contribution; gets more important for a looser cut.

- These corrections introduce new uncertainties: they probe the inner structure of the B meson & introduce new hadronic parameters. Could F_Λ be computed on the lattice ?

Thanks for listening !

Backup-slides

Belle II projections



[Belle II Physics Book]

Figure 1: Projection of uncertainties on the branching fractions $\mathcal{B}(B^+ \to \mu^+ + \nu_\mu)$ and $\mathcal{B}(B^+ \to \tau^+ + \nu_\tau)$. The corresponding uncertainty on the experimental value of $|V_{ub}|$ is shown on the right-hand vertical axis.

SCET$_{II}$ basis

$$\mathscr{H}_v = Y_n^{(q)\dagger} h_v\,,\quad \mathscr{Q}_s = Y_n^{(q)\dagger} q_s\,,\quad \mathscr{G}_s^{\mu} = Y_n^{\dagger}(iD_s^{(G)\mu} Y_n)$$
$$\omega = n\cdot p_q\,,\ \omega_g = n\cdot p_g$$

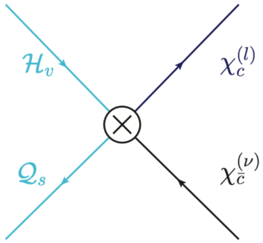
- Starting from our SCET$_I$ operators, we can construct our SCET$_{II}$ basis :

[$\mathscr{A}$]

$$O_{\mathscr{A}}^{(1)} = \frac{m_\ell}{(\bar n\cdot \mathscr{P}_c)}\Big(\bar{\mathscr{Q}}_s\ \not{n} P_L \mathscr{H}_v\Big)\Big(S_n^{\dagger(\ell)}\bar\chi_c^{(\ell)} P_L \chi_{\bar c}^{(\nu)}\Big)$$
$$O_{\mathscr{A}}^{(2)} = m_\ell\Big(\bar{\mathscr{Q}}_s P_R \mathscr{H}_v\Big)\Big(S_n^{\dagger(\ell)}\bar\chi_c^{(\ell)} P_L \chi_{\bar c}^{(\nu)}\Big)$$

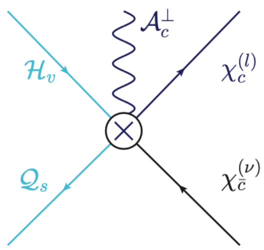
[$\mathscr{B}$]

$$O_{\mathscr{B}}^{(1)}(\omega) = \frac{m_\ell}{(\bar n\cdot \mathscr{P}_c)}\Big(\bar{\mathscr{Q}}_s^{[\omega]}\ \not{n} P_L \mathscr{H}_v\Big)\Big(S_n^{\dagger(\ell)}\bar\chi_c^{(\ell)} P_L \chi_{\bar c}^{(\nu)}\Big)$$
$$O_{\mathscr{B}}^{(2)}(\omega) = m_\ell\Big(\bar{\mathscr{Q}}_s^{[\omega]} P_R \mathscr{H}_v\Big)\Big(S_n^{\dagger(\ell)}\bar\chi_c^{(\ell)} P_L \chi_{\bar c}^{(\nu)}\Big)$$



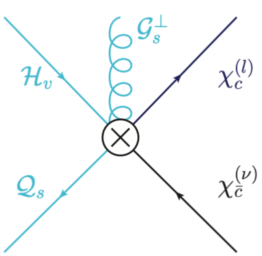
[$\mathscr{C}$]

$$O_{\mathscr{C}}^{(1)}(u,\omega) = \frac{1}{(\bar n\cdot \mathscr{P}_c)}\Big(\bar{\mathscr{Q}}_s^{[\omega]}\ \not{n} P_L \mathscr{H}_v\Big)\Big(S_n^{\dagger(\ell)}\bar\chi_c^{(\ell)} \mathscr{A}_{c\perp}^{[u]} P_L \chi_{\bar c}^{(\nu)}\Big)$$
$$O_{\mathscr{C}}^{(2)}(u,\omega) = \Big(\bar{\mathscr{Q}}_s^{[\omega]} P_R \mathscr{H}_v\Big)\Big(S_n^{\dagger(\ell)}\bar\chi_c^{(\ell)} \mathscr{A}_{c\perp}^{[u]} P_L \chi_{\bar c}^{(\nu)}\Big)$$



[$\mathscr{D}$]

$$O_{\mathscr{D}}^{(1)}(\omega,\omega_g) = m_\ell\Big(\bar{\mathscr{Q}}_s^{[\omega]}\Big(\frac{\not{n}}{in\cdot\partial}\Big(\mathscr{G}_{s\perp}^{[\omega_g]}\Big)^{\mu}\Big) P_R \mathscr{H}_v\Big)\Big(S_n^{\dagger(\ell)}\bar\chi_c^{(\ell)} P_L \chi_{\bar c}^{(\nu)}\Big)$$



Virtual amplitude at NLO QED

$$\mathcal{A}_{\text{virtual}} = i\sqrt{2}\, G_F\, K_{\text{EW}}(\mu) V_{ub} \frac{m_\ell}{m_B} \sqrt{m_B}\, F_\Lambda(\mu) \bar{u}(p_\ell) P_L\, v(p_\nu) \sum_j \mathcal{M}_j(\mu)\,,$$

$$\mathcal{M}_{2\text{p}}(\mu) = 1 + \frac{\alpha}{4\pi} \Bigg\{ Q_b^2 \left[-\frac{3}{2} \ln \frac{\mu^2}{m_b^2} - 2 \right]$$
$$- Q_\ell^2 \left[\frac{1}{\epsilon_{\text{IR}}} \left(2 + \log \frac{m_\ell^2}{m_B^2} \right) - \frac{1}{2} \log^2 \frac{\mu^2}{m_\ell^2} + \frac{3}{2} \log \frac{\mu^2}{m_\ell^2} - \frac{\pi^2}{12} + 2 \right]$$
$$+ Q_b Q_\ell \left[\frac{1}{R} \left(\log \frac{\mu^2}{m_B^2} + 1 + \log R \right) - f(R) + \frac{1}{2} R f'(R) - \log \frac{\mu^2}{m_\ell^2} \right]$$
$$+ Q_\ell Q_u \left[-5 - 2 \log \frac{\mu^2}{m_\ell^2} + \int_0^\infty d\omega\, \phi_-(\omega) \left(\log \frac{\mu^2}{\omega \sqrt{m_B \Lambda}} \log \frac{\Lambda^2}{m_B^2} - 2 \log \frac{\mu^2}{\omega \sqrt{m_B \Lambda}} + \log \frac{\Lambda}{m_B} - 6 - \frac{\pi^2}{3} \right) \right] \Bigg\}$$
$$+ \frac{C_F \alpha_s}{4\pi} \left[-\frac{3}{2} \ln \frac{\mu^2}{m_b^2} - 2 \right],$$

$$R = \frac{m_b}{q_-} = \frac{m_b}{m_B}$$

$$f(R) = \frac{1}{2} \log^2 \frac{\mu^2 R^2}{m_b^2} + \frac{\pi^2}{12} - 2\text{Li}_2(1-R) - \log^2 R + 1$$

$$\mathcal{M}_{3\text{p}}(\mu) = -\frac{\alpha}{\pi} Q_\ell Q_u \left(1 + \log \frac{\Lambda}{m_B} \right) \times \int_0^\infty d\omega \int_0^\infty d\omega' \frac{\phi_3(\omega, \omega')}{\omega'} \left[\frac{1}{\omega'} \ln\left(1 + \frac{\omega'}{\omega} \right) - \frac{1}{\omega + \omega'} \right].$$

Coft Wilson-lines cancellation

▸ One can also define F_Λ using time-like Wilson-lines :

$$-\frac{i\sqrt{m_B}}{2}F_\Lambda(\mu) \equiv \frac{\left\langle 0 \left| \bar{q}_s \theta(\Lambda + i\bar{n}\cdot\overleftarrow{D}_s)\not{n} P_L \mathcal{S}^{(l)\dagger}_{v_\ell} h_v \right| B^- \right\rangle}{\langle 0 | \mathcal{S}^{(B)}_{v_B}\mathcal{S}^{(l)\dagger}_{v_\ell} | 0 \rangle}$$

▸ Those time-like WL can be splitted between soft and soft-collinear (coft) contributions :

$$\mathcal{S}^{(l)}_{v_B}(x) = S^{(B)}_{v_B}(x) C^{(B)}_{\bar{n}}(x_+) = S^{(B)}_{v_B}(x) C^{(b)}_{\bar{n}}(x_+) C^{(q)\dagger}_{\bar{n}}(x_+) \qquad \mathcal{S}^{(l)}_{v_\ell}(x) = S^{(\ell)}_{n}(x_-) C^{(\ell)}_{v_\ell}(x)$$

▸ After soft-collinear decoupling, $\quad h_v \to C^{(b)}_{\bar{n}} h_v \quad , \quad q_s \to C^{(q)}_{\bar{n}} q_s$

contributions from the coft Wilson-lines cancel in the ratio :

$$-\frac{i\sqrt{m_B}}{2}F_\Lambda(\mu) = \frac{\left\langle 0 \left| \bar{q}_s \theta(\Lambda + i\bar{n}\cdot\overleftarrow{D}_s)\not{n} P_L S^{(\ell)\dagger}_{n} h_v \right| B^- \right\rangle}{\langle 0 | S^{(B)}_{v_B} S^{(\ell)\dagger}_{n} | 0 \rangle}$$

A theory of Wilson lines ?

$$Y_r^{(i)}(x) = \mathscr{P}\exp\left\{ieQ_i\int_{-\infty}^{0} ds\, r\cdot A_{us}(x+sr)\right\} \quad C_r^{(i)}(x) = \mathscr{P}\exp\left\{ieQ_i\int_{-\infty}^{0} ds\, r\cdot A_{usc}(x+sr)\right\}$$

- We can decouple the interactions of the B and the muon with ultrasoft and ultrasoft-collinear photons :

$$\varphi_v(x) \to Y_{v_B}^{(B)}(x)C_{\bar{n}}^{(B)}(x_+)\,\varphi_v^0(x) \qquad h_{v_\ell}(x) \to Y_n^{(\ell)}(x\_)C_{v_\ell}^{(\ell)}(x)\,\frac{\not{n}\not{\bar{n}}}{4}h_{v_\ell}^0(x)$$

- The real corrections can be expressed as matrix elements of these Wilson lines :

$$W_s(\omega_{us},\mu) = \left[\sum_{n_{us}=0}^{\infty}\prod_{i=1}^{n_{us}}\int d\Pi_i(q_i)\right]\left|\langle n_{us}\gamma_{us}(q_i)\,|\,Y_{v_B}^{(B)}Y_n^{\dagger(\ell)}\,|\,0\rangle\right|^2\delta(w_{us}-q_0^{us}) \quad , \quad q_0^{us} = \sum_i q_{0i}$$

$$W_{usc}(\omega_{usc},\mu) = \left[\sum_{n_{usc}=0}^{\infty}\prod_{j=1}^{n_{usc}}\int d\Pi_j(q_j)\right]\left|\langle n_{usc}\gamma_{usc}(q_j)\,|\,C_{\bar{n}}^{(B)}C_{v_\ell}^{\dagger(\ell)}\,|\,0\rangle\right|^2\delta(w_{usc}-q_0^{usc}) \quad , \quad q_0^{usc} = \sum_j q_{0j}$$

m_W

m_B

$\mu_{hc} = \sqrt{\Lambda_{QCD}m_B}$

$m_\mu \sim \Lambda_{QCD}$

$\mu_{us} = E_0$

$\mu_{usc} \sim E_0\dfrac{m_\mu}{m_B}$

Soft photon to the muon boosted in the B frame

- These radiative functions are convoluted with the measurement function implementing the experimental radiation veto

$$S(E_0,\mu) = \int_0^\infty d\omega_s\int_0^\infty d\omega_{sc}\,\theta\!\left(\frac{E_0}{2}-\omega_{us}-\omega_{usc}\right)W_{us}(\omega_{us},\mu)W_{usc}(\omega_{usc},\mu)$$

First pole dominance

- This structure-dependent correction can be implemented at the level of the radiative hadronic matrix element $\langle \gamma | J^{\mu}_{A/V} | B \rangle$, where in the quark picture:

$$J^{\mu}_{A} = \bar{q}\gamma^{\mu}\gamma_5 b \qquad J^{\mu}_{V} = \bar{q}\gamma^{\mu} b$$

- In the ultrasoft/ultrasoft-collinear region for the photon, it is reasonable to assume first pole dominance for both matrix elements: [Becirevic et al (0907.1845)]

$$\langle \gamma | J^{\mu}_{V} | B \rangle \simeq \frac{\langle 0 | J^{\mu}_{V} | B^{*} \rangle \overbrace{\langle B^{*} | B\gamma \rangle}^{\sim\, g_{BB^*}}}{q^2 - m^2_{B^*}} + \cdots \qquad \langle \gamma | J^{\mu}_{A} | B \rangle \simeq \frac{\langle 0 | J^{\mu}_{A} | B'_1 \rangle \overbrace{\langle B'_1 | B\gamma \rangle}^{\sim\, g_{BB'_1}}}{q^2 - m^2_{B'_1}} + \cdots$$

- Below Λ_{QCD}, quark currents can be replaced by effective hadronic currents involving meson fields:

$$J^{\mu}_{A} \to J^{\mu}_{Had,A} = f_B D^{\mu}\Phi_B + i m_{B'_1} f_{B'_1} V^{\mu}_1 \qquad J^{\mu}_{V} \to J^{\mu}_{Had,V} = m_{B*} f_{B*} V^{\mu}_{*}$$