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Institute for Theoretical Physics

Two-loop running effects in Higgs physics in the SMEFT

Based on arXiv:2408.03252 with R. Gröber, M. K. Mandal

YOUNGST@RS - EFTs and Beyond

Stefano Di Noi | 4 December 2024

Outline

Introduction

- The **Standard Model** (**SM**) must be extended.
- **Effective Field Theories** (**EFTs**): search for NP with minimal UV assumptions.

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- **Effective Field Theories** (**EFTs**): search for NP with minimal UV assumptions.

- This talk focuses on Standard Model Effective Field Theory (**SMEFT**) at dim 6 (**Warsaw basis** [Grzadkowski,Iskrzynski,Misiak,Rosiek,'10]),

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{\mathfrak{D}_i=6} \frac{\mathcal{C}_i}{\Lambda^2}\mathcal{O}_i.$$

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- The scale dependence of the coefficients is encoded in the **Renormalization Group Equations** (**RGEs**):

$$\mu \frac{d\mathcal{C}_i(\mu)}{d\mu} = \frac{1}{16\pi^2} \Gamma_{ij}(\mu) \mathcal{C}_j(\mu).$$

- At dim=6, the system is linear: $\Gamma_{ij}(\mu)$, **Anomalous Dimension Matrix**.(**ADM**)

Structure of the ADM in the SMEFT

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- $\Gamma_{ij}(\mu)$: known at 1-loop [(Alonso),Jenkins,Manohar,Trott,'13] and automatized [Fuentes-Martín,Ruiz-Femenia,Vicente,Virto,'20], [Aebischer,Kumar,Straub,'18], [SDN,Silvestrini,'22].
- Some preliminary two-loop results available in [Bern,Parra-Martinez,Sawyer,'19], [Bern,Parra-Martinez,Sawyer,'20], [SDN,Gröber,Heinrich,Lang,Vitti,'23], [Jenkins,Manohar,Naterop,Pagès,'23], [Born,Fuentes-Martín,Kvedaraitė,Thomsen,'24],[Talk by U. Haisch @ EFT WG].

Running effects in the SMEFT

- Several SMEFT studies show the importance of $\mathcal{O}(\alpha_s)$ running effects: [Grazzini,Ilnicka,Spira,'18],[Battaglia,Grazzini,Spira,Wiesemann,'21],[Aoude et al.,'22],[Maltoni,Ventura,Vryonidou,'24],[Heinrich,Lang,'24].

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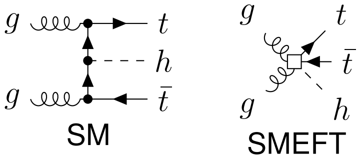
- Top-Yukawa induced running can be important ($pp \to \bar{t}th$ in [SDN,Gröber,'23]).
- SM Tree-level, SMEFT tree-level.



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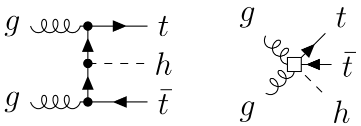
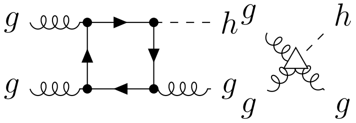


SM SMEFT

- Higgs+jet/di-Higgs: **SM One-loop**, SMEFT tree-level.



SM SMEFT

Power counting in Higgs production

- Assumption on the UV completion: renormalizable and weakly coupled
  [Arzt,Einhorn,Wudka,'95],[Craig,Jiang,Li,Sutherland,'20],[Buchalla,Heinrich,Müller-Salditt,Pandler,'22].

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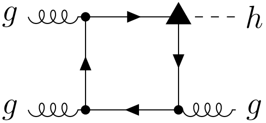
- $\blacktriangle = \mathcal{O}_{tH} \equiv (\bar{Q}_L \tilde{H} t_R)\left(H^\dagger H\right)$: Tree-level generated but enters at one-loop level.

$$\frac{\mathcal{C}_{tH}}{\Lambda^2} \sim \frac{1}{\Lambda^2} \longrightarrow$$



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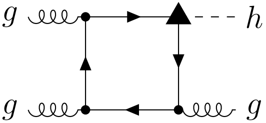

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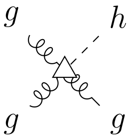

$$\sim 1 \times \frac{\mathcal{C}_{HG}}{\Lambda^2} \sim \frac{1}{16\pi^2} \times \frac{1}{\Lambda^2}$$

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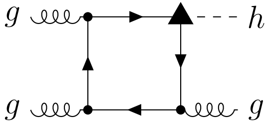

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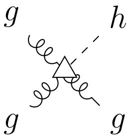
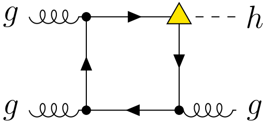


$$\sim 1 \times \frac{\mathcal{C}_{HG}}{\Lambda^2} \sim \frac{1}{16\pi^2} \times \frac{1}{\Lambda^2}$$

- $\mathcal{O}_{tH}$, $\mathcal{O}_{HG}$ **contribute at the same order to Higgs+jet production!**

Running in Higgs production

- 1L running in $\mathcal{C}_{tH}$:  $\sim \frac{1}{16\pi^2}\frac{1}{16\pi^2}\log\left(\frac{\mu_R^2}{\Lambda^2}\right)\frac{1}{\Lambda^2}$: **2L effect.**

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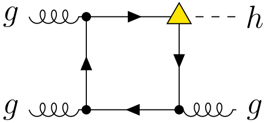
- 1L running in $\mathcal{C}_{tH}$: 

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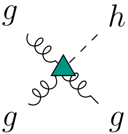
- We need 2L running operators in $\mathcal{C}_{HG}$: 

$\sim \left(\frac{1}{16\pi^2}\right)^2 \log\left(\frac{\mu_R^2}{\Lambda^2}\right)\frac{1}{\Lambda^2}$ from potentially tree-level generated operators.

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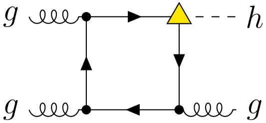

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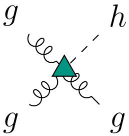

$$\sim \left(\frac{1}{16\pi^2}\right)^2 \log\left(\frac{\mu_R^2}{\Lambda^2}\right)\frac{1}{\Lambda^2}$$ from potentially tree-level generated operators.

- $\mathcal{C}_{4t} \xrightarrow{2L} \mathcal{C}_{HG}$ computed in [SDN,Gröber,Heinrich,Lang,Vitti,'23], **this work** : $\mathcal{C}_{tH} \xrightarrow{2L} \mathcal{C}_{HG}$.

Continuation to D dimensions schemes for γ_5

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  - **Algebraically consistent.**
  - **Breaks Ward ids/chiral symmetries** ([Larin,'93], [Olgoso,Vecchi,'24]).
  - **Computationally demanding.**

Two-loop running of Higgs-gluon coupling: method

- Determining the ADM $\equiv$ computing divergent parts: $\delta\mathcal{C}_{HG} \sim \frac{1}{\epsilon}\gamma_{HG,tH}\mathcal{C}_{tH}$.

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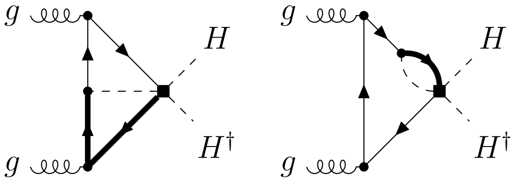
- Computation in the unbroken phase: easier, massless fermions.
- (thin arrow) = RH field t_R, b_R, (thick arrow) = LH field Q_L.
- Two families, 4 Master Integrals (MIs) each.
- The result does not depend on the γ_5 scheme.



(a) Family 1 (b) Family 2

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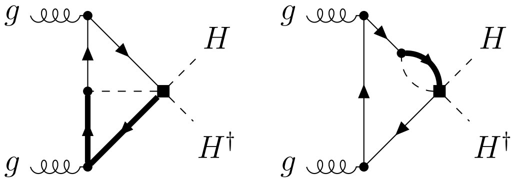

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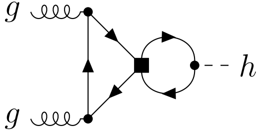
- Two families, 4 Master Integrals (MIs) each.
- The result does not depend on the γ_5 scheme.
- Differential eqs for the MIs via the Magnus method.
- Solving the system, the MIs can be expressed in terms of Harmonic PolyLogarithms.
- Result cross-checked numerically with `AMFlow` [Liu, Ma, '22].

Two-loop running of Higgs-gluon coupling: result



- RGE@2L for $\mathcal{C}_{HG}$ combining the result in this work with the result in [SDN,Gröber,Heinrich,Lang,Vitti,'23] we have:

$$\mu\frac{d\mathcal{C}_{HG}}{d\mu} \supset 3\left(\frac{1}{16\pi^2}\right)^2 g_s^2\left[\mathcal{C}_{tH}Y_t + \mathcal{C}_{tH}^*Y_t^* + \mathcal{C}_{bH}Y_b + \mathcal{C}_{bH}^*Y_b^*\right] -4\left(\frac{1}{16\pi^2}\right)^2 g_s^2 Y_t Y_t^* \delta_{\mathrm{X}}\left(\mathcal{C}_{Qt}^{(1)} - \frac{1}{6}\mathcal{C}_{Qt}^{(8)}\right),$$

$$\delta_{\mathrm{X}} = \begin{cases} 1 & (\mathrm{NDR}), \\ 0 & (\mathrm{BMHV}). \end{cases}$$

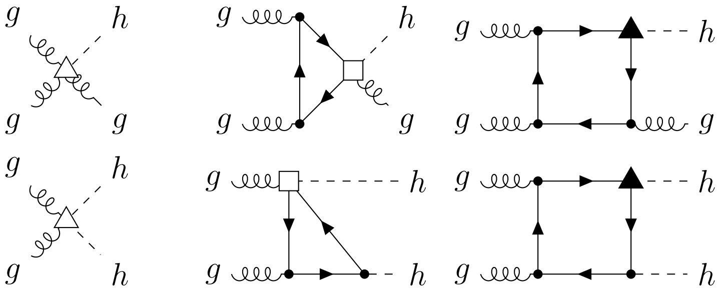
- $\mathcal{O}_{tH}$, $\mathcal{O}_{4t}$ potentially **tree-level generated** if the UV completion is renormalizable and weakly coupled [Arzt,Einhorn,Wudka,'95],[Craig,Jiang,Li,Sutherland,'20],[Buchalla,Heinrich,Müller-Salditt,Pandler,'22].

Phenomenological impact: set-up

- We study the two-loop running effects for:

$\frac{d\sigma}{dp_T}(pp \to hj)$ using the results in [Grazzini,Ilnicka,Spira,'18].

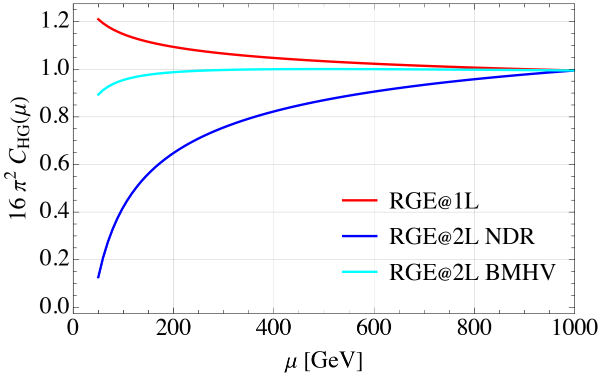
$\frac{d\sigma}{dm_{hh}}(pp \to hh)$ using a private version of hpair [Dawson,Dittmaier,Spira,'98].

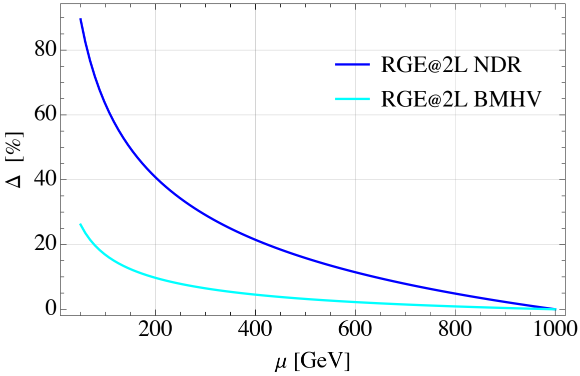


$$\Lambda = 1\,\mathrm{TeV}, \quad \mu_R = \mu_F = \begin{cases} \sqrt{m_h^2 + p_T^2}/2 \text{ for } pp \to hj, \\ m_{hh}/2 \text{ for } pp \to hh. \end{cases}$$

Running of $\mathcal{C}_{HG}$: 1L vs 2L

$$\text{S1}: \quad \mathcal{C}_{tH}(\Lambda) = 1,\ \mathcal{C}_{HG}(\Lambda) = 1/16\pi^2,\ \mathcal{C}_{tG}(\Lambda) = -1/16\pi^2,\ \mathcal{C}_{Qt(1,8)}(\Lambda) = -10.$$

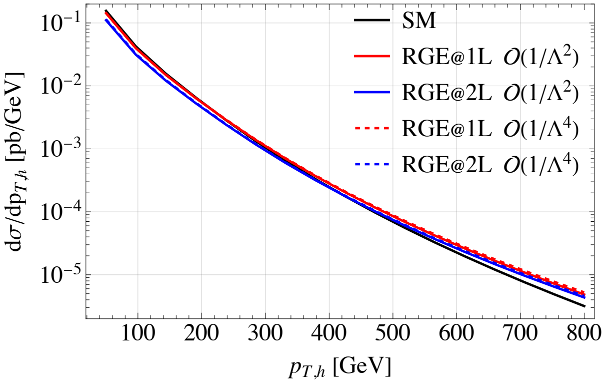


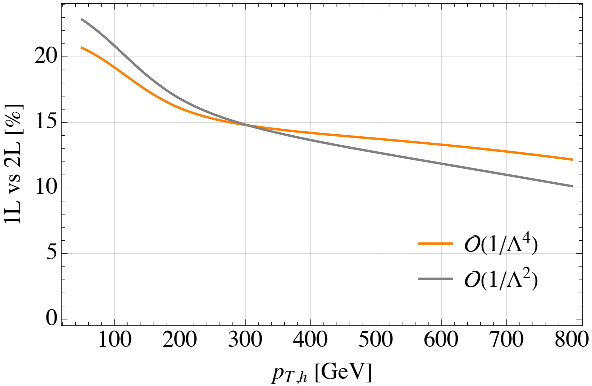


- Running computed with a private version of `RGESolver` [SDN,Silvestrini,'22].

Higgs+jet: 1L vs 2L (NDR)

$$\mu \frac{d\mathcal{C}_{HG}}{d\mu} \supset 3\left(\frac{1}{16\pi^2}\right)^2 g_s^2 \times 2\mathcal{C}_{tH} Y_t - 4\left(\frac{1}{16\pi^2}\right)^2 g_s^2 Y_t^2 \left(\mathcal{C}_{Qt}^{(1)} - \frac{1}{6}\mathcal{C}_{Qt}^{(8)}\right)$$





Higgs+jet: linear vs quadratic order in $1/\Lambda^2$

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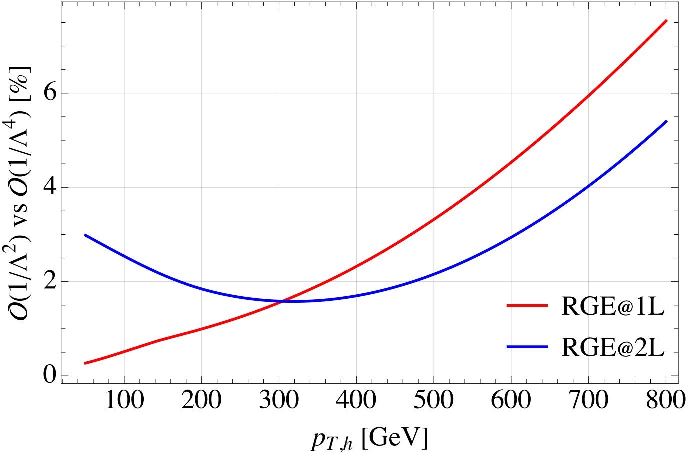

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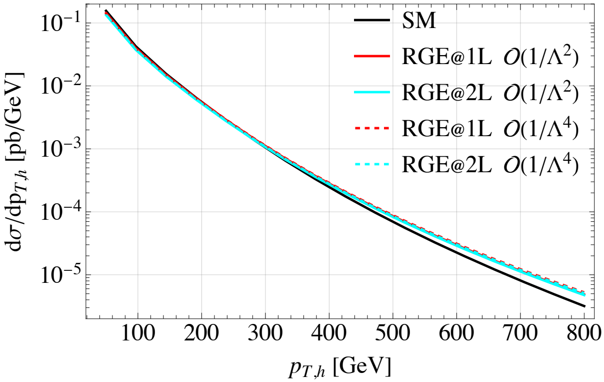

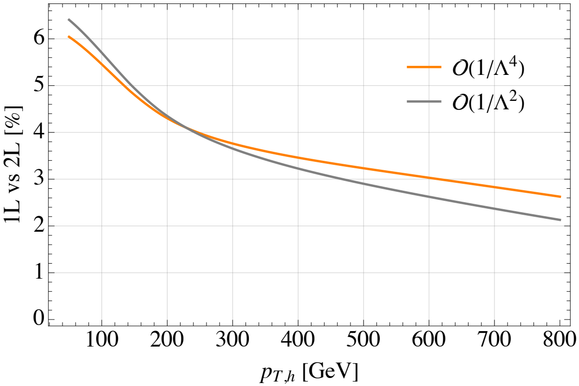

Higgs+jet: fixed vs dynamical renormalization scale

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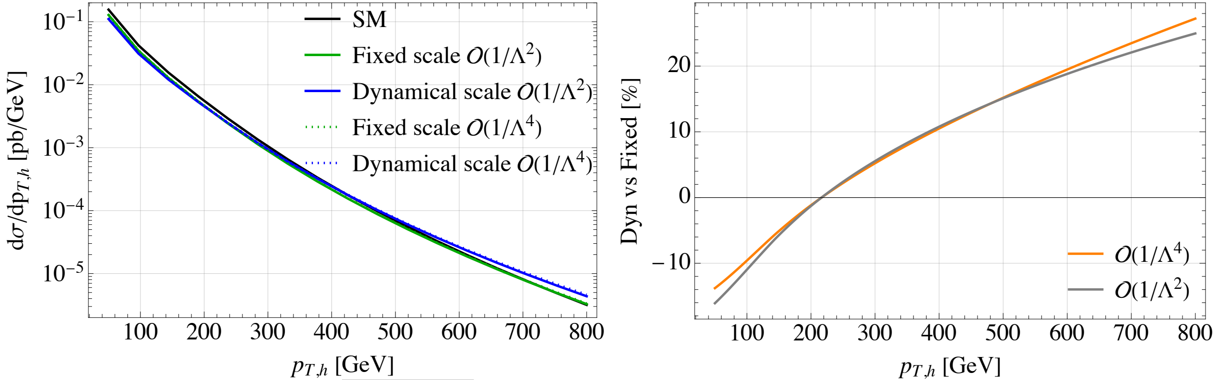
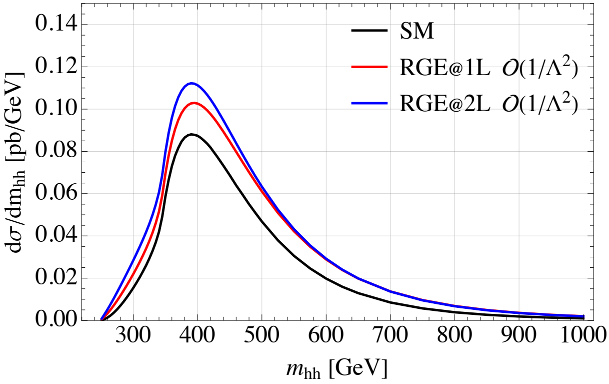
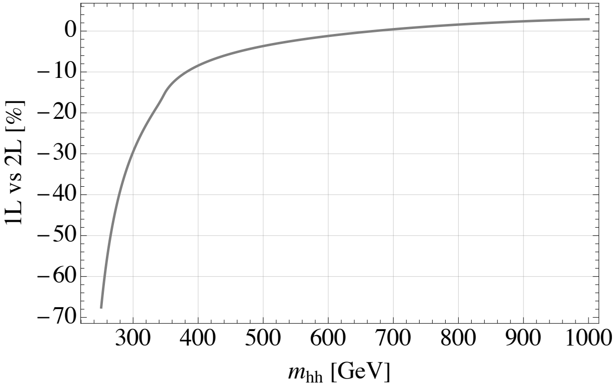
$$\mu_R = \sqrt{m_h^2 + p_T^2/2} \quad \text{(dynamical)} \quad \text{vs} \quad \mu_R = m_h \quad \text{(fixed)}$$



- Same result when $\sqrt{p_{T,h}^2 + m_h^2}/2 = m_h \to p_{T,h} \simeq 216.5\,\text{GeV}.$

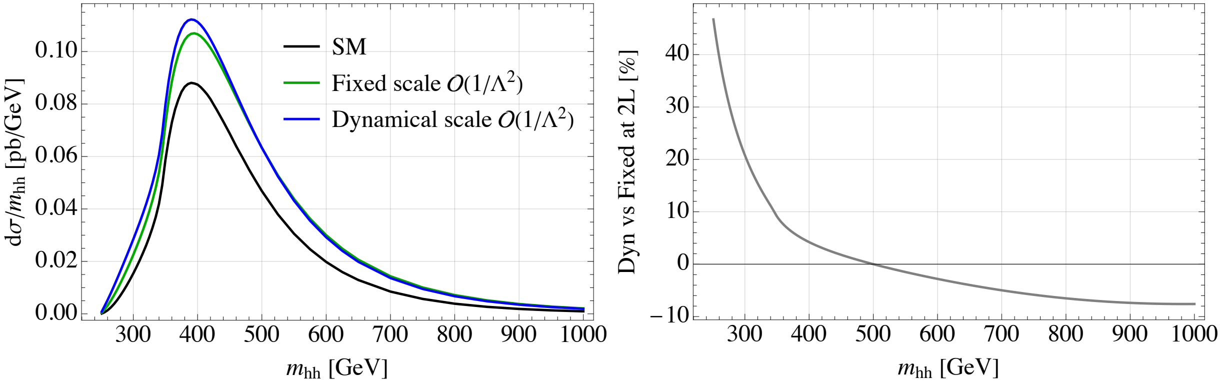
Di-Higgs: 1L vs 2L (NDR)





Di-Higgs: fixed vs dynamical renormalization scale

$$\mu_R = m_{hh}/2 \quad \text{(dynamical)} \quad \text{vs} \quad \mu_R = 2m_h \quad \text{(fixed)}$$



- Same result when $m_{hh}/2 = 2m_h \rightarrow m_{hh} = 500\,\text{GeV}$.

Summary

- We present the two-loop contribution to the running of the Higgs-gluon coupling $\mathcal{C}_{HG}$ proportional to the (potentially tree-level generated) Yukawa-like coefficient $\mathcal{C}_{tH}$.
- This work is a direct extension of [SDN,Gröber,Heinrich,Lang,Vitti,'23].

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- This work is a direct extension of [SDN,Gröber,Heinrich,Lang,Vitti,'23].

- We study the phenomenological impact in differential obsevables in $pp \to hj/hh$.
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Summary

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- Outlook: full two-loop RGE for the Higgs-gluon coupling.

Thanks for your attention!

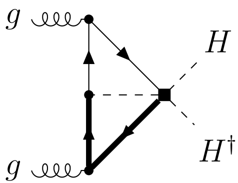
Back-up slides

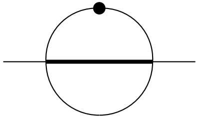
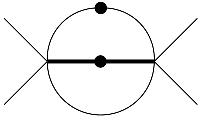
Full set of SMEFT operators

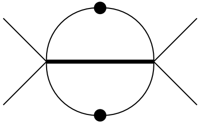
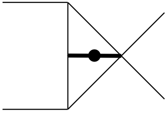
$$\mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H), \quad \mathcal{O}_{HD} = (H^\dagger D_\mu H)^*(H^\dagger D^\mu H), \quad \mathcal{O}_H = (H^\dagger H)^3,$$
$$\mathcal{O}_{tH} = (\bar{Q}_L \tilde{H} t_R)\left(H^\dagger H\right), \quad \mathcal{O}_{bH} = (\bar{Q}_L H b_R)\left(H^\dagger H\right),$$
$$\mathcal{O}_{HG} = \left(H^\dagger H\right) G^A_{\mu\nu} G^{\mu\nu,A}, \quad \mathcal{O}_{tG} = \bar{Q}_L \tilde{H} \sigma_{\mu\nu} T^A t_R G^{\mu\nu,A},$$
$$\mathcal{O}_{Qt(1)} = \left(\bar{Q}_L \gamma^\mu Q_L\right)\left(\bar{t}_R \gamma_\mu t_R\right), \quad \mathcal{O}_{Qt(8)} = \left(\bar{Q}_L \gamma^\mu T^A Q_L\right)\left(\bar{t}_R \gamma_\mu T^A Q_R\right) .$$

MIs Family 1

- Full expression in [SDN,Gröber,Mandal,'24].
- **Right:** thin line=RH field t_R, b_R, thick line=LH field Q_L.
- **Below:** thin line=massless propagator, thick line=Higgs propagator.

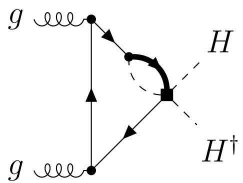


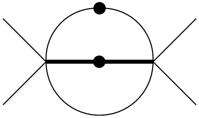
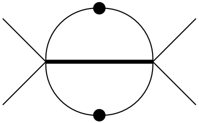
$\tau_{0,2,1,0,1,0,0} =$  , $\tau_{1,0,2,0,2,0,0} =$  ,

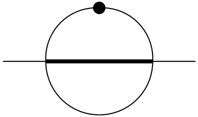
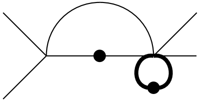
$\tau_{2,0,1,0,2,0,0} =$  , $\tau_{1,1,2,1,1,0,0} =$  .

MIs Family 2

- Full expression in [SDN,Gröber,Mandal,'24].
- **Right:** thin line=RH field t_R, b_R, thick line=LH field Q_L.
- **Below:** thin line=massless propagator, thick line=Higgs propagator.



$\tau_{0,0,2,1,2,0,0} =$  , $\tau_{0,0,2,2,1,0,0} =$  ,

$\tau_{0,2,0,1,1,0,0} =$  , $\tau_{2,0,1,0,2,0,0} =$  .

RGESolver [S.D.N.,Silvestrini,'22]

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- A C++ library that performs RG evolution of SMEFT coefficients (dim6, 1 loop).
- General flavour structure (assuming L, B conservation).
- Numerical and approximate solutions of the RGEs with unprecedented efficiency.
- Back-rotation effects easily implemented.