Two loop effective action at dimension six

YOUNGST@RS - EFTs and Beyond

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Outline

- BSM vs. SMEFT
- Two loop vacuum diagrams $\Rightarrow$ Heat-kernel method
- Example cases - Complex Triplet and 2HDM
- Implementation in CoDex $\Rightarrow$ Futurework

BSM vs. SMEFT

Why do we need SMEFT?

SMEFT captures the effects of the UV theories(BSM) at lower energy scale which can be experimentally measured.

Dimension six corrections at tree level and even the one-loop corrections are not enough to get closer to the full theory.

Thus we look for even more precision calculations.

- Two loop effects at a fixed order.
- Higher order operators at tree and one loop level.

Heat-kernel method

$$\mathcal{L} = \frac{1}{2}\phi D^2\phi + \frac{1}{2}M^2\phi^2 + V(\phi), \quad \text{and} \quad \Delta = \frac{\delta^2\mathcal{L}}{\delta\phi^2}.$$

$$\Delta = D^2 + M^2 + U$$

- The Δ is a second-order elliptic operator.
- It has a positive definite spectrum.

We define the heat kernel as

$$K(t, x, y, \Delta) = \sum_n e^{-\Delta t}\,\tilde{\phi}_n(x)\,\tilde{\phi}_n(y)\,,$$

This satisfies the heat equation

$$(\partial_t + \Delta_x)K(t, x, y, \Delta) = 0$$

Where,

$$K(0, x, y, \Delta) = \delta(x, y)$$

The $\tilde{\phi}_n$ are the eigenstates of the operator Δ. Note that $t > 0$ is a parameter, not to be confused with time, when defining the heat equation.

Heat-kernel method

$$K(t,x,y,\Delta) = \int \frac{d^d p}{(4\pi^2 t)^{d/2}} e^{\frac{(x-y)^2}{4t}} e^{-M^2 t} e^{p^2} \left[1 + \sum_{n=1}^{\infty} (-1)^n f_n(t,\mathcal{A}) \right],$$

where

$$f_n(t,\mathcal{A}) = \int_0^t ds_1 \int_0^{s_1} ds_2 \cdots \int_0^{s_{n-1}} ds_n \, \mathcal{A}(s_1)\mathcal{A}(s_2)\cdots\mathcal{A}(s_n) \,,$$

and

$$\mathcal{A}(t) = e^{M^2 t}(D^2 + 2i\, p \cdot D/\sqrt{t} + U)e^{-M^2 t} \,.$$

After performing the Gaussian momentum integral, over the $f_n(t,\mathcal{A})$, we get a polynomial in t that allows the HK to be written in the following form-

$$K(t,x,y,\Delta) = \frac{1}{(4\pi t)^{d/2}} e^{\frac{(x-y)^2}{4t}} e^{-M^2 t} \sum_{n=0}^{\infty} \frac{(-t)^n}{n!} \tilde{b}_n.$$

The $\tilde{b}_n$'s are the Generalized Heat-Kernel Coefficients (g-HKC).

Ref:Upalaparna Banerjee, Joydeep Chakrabortty,Shakeel Ur Rahaman, Kaanapuli Ramkumar, 2311.12757

Heat-kernel method

We can define an "Interacting propagator" using the HK as

$$G(x,y) = \int_0^\infty dt \; K(t,x,y,\Delta)\,.$$

The Green's function can also be rewritten in the desired form as

$$G(x,y) = \sum_{n=0}^{\infty} g_n(x,y)\tilde{b}_n(x,y),$$

using the previous equations, with $\tilde{b}_n$ being generalized heat kernel coefficient (g-HKC) and component Green's functions are given by -

$$\begin{aligned} g_n(x,y) &= \int_0^\infty dt \frac{1}{(4\pi t)^{\frac{d}{2}}} e^{\frac{z^2}{4t}} e^{-M^2 t} \frac{(-t)^n}{n!} \\ &= \frac{(-1)^n 2^{\frac{d}{2}-n}}{(4\pi)^{\frac{d}{2}} n!} \left(\frac{M}{z}\right)^{\frac{d}{2}-n-1} \mathcal{K}_{\frac{d}{2}-n-1}(Mz). \end{aligned}$$

Here, $z^2 = -|x-y|^2$ and $\mathcal{K}_n(z)$ being modified Bessel function of second kind.

Heat-kernel method

The key features

- The divergences of each diagram can be determined by analyzing the algebraic singularities of Green's functions $g_n(x, y)$.
- The number of diagrams reduces significantly.

The one loop correction of the effective action can easily be calculated as follows.

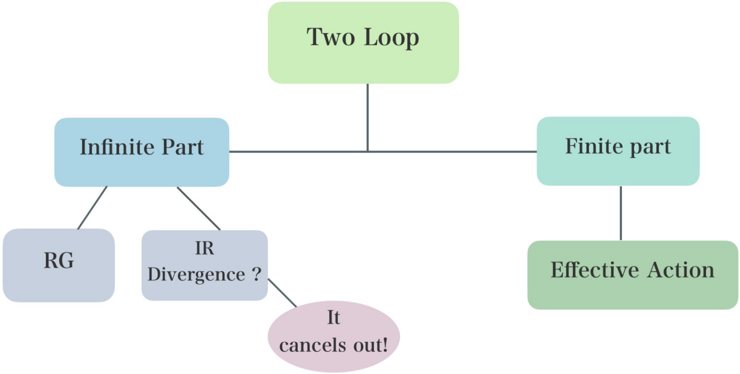
$$\mathcal{L}_{(1)} = c_s \text{tr} \int_0^\infty \frac{dt}{t} K(t, x, x, \Delta).$$

After performing the integral, we get

$$\mathcal{L}_{(1)} = \frac{c_s}{(4\pi)^{d/2}} \sum_{k=0}^{\infty} M^{d-2k} \frac{(-1)^k}{k!} \Gamma[k - d/2] \, \text{tr}[b_k].$$

Ref: Chakrabortty et al, 2306.09103

Computation of the two loop vacuum diagrams



Ref: Chakrabortty et al,2404.02734

Computation of the two loop vacuum diagrams

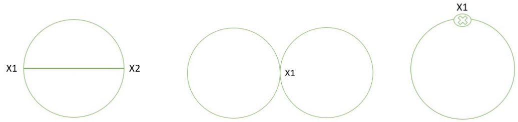


Figure: (a): Sunset Diagram, (b): Infinity Diagram, (c): Counter-term Diagram

Sunset diagram:

$$\begin{aligned}\mathcal{L}^{a}_{(2)} &\subset -\frac{1}{12}\mathrm{tr}\Big[\int d^d x_1 d^d x_2 V_{(3)}(x_1) G(x_1,x_2)^3 V_{(3)}(x_2)\Big] \\ &= -\frac{1}{12}\mathrm{tr}\Big[\int d^d x_1 d^d x_2 V_{(3)}(x_1)\Big(g_0(x_1,x_2)^3 \tilde{b}_0(x_1,x_2)^3 + 3g_0(x_1,x_2)^2 g_1(x_1,x_2)\tilde{b}_0(x_1,x_2)^2\tilde{b}_1(x_1,x_2) \\ &+ 3\alpha g_0(x_1,x_2)^2\tilde{b}_0(x_1,x_2)^2 F(x,y)\Big) V_{(3)}(x_2)\Big].\end{aligned}$$

We've only written the terms that are singular at zero. Here $V_{(3)}(x)$ is the three point vertex function defined as

$$V_{(3)}(x) = \frac{\delta^3 \mathcal{L}}{\delta \phi^3}$$

Computation of the two loop vacuum diagrams

Infinity diagram:

$$\mathcal{L}^{b}_{(2)} \subset \frac{1}{8}\mathrm{tr}\Big[\int d^d x_1 \, V_{(4)}(x_1) G(x_1, x_1)^2\Big],$$

The $V_{(4)}(x)$ is the four point vertex function defined as

$$V_{(4)}(x) = \frac{\delta^4 \mathcal{L}}{\delta \phi^4}$$

counter-term diagram:

$$\mathcal{L}^{ct}_{(2)} \subset \frac{1}{2}\mathrm{tr}\left[\int d^d x_1 \, V^{(ct-1)}_{(2)}(x_1) G(x_1, x_1)\right].$$

The $V^{(ct-1)}_{(2)}(x)$ is the counter-term vertex function defined as

$$V^{(ct-1)}_{(2)}(x) = \frac{\partial^2 \mathcal{L}_{(1)}}{\partial \phi^2}.$$

Computation of the two loop vacuum diagrams

After adding all the contribution, we write the total lagrangian as

$$\mathcal{L} \supset \alpha^2 \,\mathrm{tr}\left[\mathcal{C}_4 M^4 + \mathcal{C}_2 M^2 + \mathcal{C}_0 M^0 + \sum_{n=1,2\ldots} \mathcal{C}_{-2n} M^{-2n}\right],$$

with

$$\mathcal{C}_4 = \mathcal{C}_4^{[\![U_{\phi\phi}]\!]}\, U_{\phi\phi},$$

$$\mathcal{C}_2 = \mathcal{C}_2^{[\![U_\phi^2]\!]}\, U_\phi^2 + \mathcal{C}_2^{[\![UU_{\phi\phi}]\!]}\, UU_{\phi\phi},$$

$$\begin{aligned}\mathcal{C}_0 &= \mathcal{C}_0^{[\![U_\phi D^2 U_\phi]\!]}\, U_\phi D^2 U_\phi + \mathcal{C}_0^{[\![UU_\phi^2]\!]}\, UU_\phi^2 \\ &\quad + \mathcal{C}_0^{[\![U^2 U_{\phi\phi}]\!]}\, U^2 U_{\phi\phi} + \mathcal{C}_0^{[\![U_{\mu\mu} U_{\phi\phi}]\!]}\, U_{\mu\mu} U_{\phi\phi} + \mathcal{C}_0^{[\![G_{\mu\nu}^2 U_{\phi\phi}]\!]}\, G_{\mu\nu}^2 U_{\phi\phi}, \\ &\quad \cdot \quad \cdot \quad \cdot\end{aligned}$$

The $\mathcal{C}_i^{[\![\mathcal{O}]\!]}$s are the Wilson coefficients.

Example cases - Complex Triplet and 2HDM

- We consider two models to compute the two-loop effective action, the electroweak complex triplet model($Y_\Delta = 1$) and the two Higgs doublet model (2HDM).

- We use the method of CDE(covariant derivative expansion) to get the effective operators after integrating out the heavy field.

- We present the result in the given way

$$\mathcal{L} \supset \alpha^2\, \mathcal{O}_a \mathcal{C}_a,$$

Example cases - Complex Triplet and 2HDM

Models	$\mathcal{O}_6$	$\mathcal{O}_T$	$\mathcal{O}_R$	$\mathcal{O}_H$	$\mathcal{O}_{WW}$	$\mathcal{O}_{BB}$	$\mathcal{O}_{WB}$	$\mathcal{O}_W$	$\mathcal{O}_B$
Complex Triplet	✓	✓	✓	✓	✓	✓	✓	✓	✓
2HDM	✓	✓	✓	✓	✓	✓	✓	✓	✓

Table: Dimension-six CP-conserving bosonic operators in case of two models

The operators purely involving the gauge fields are considered here.

Models	$\mathcal{Q}_{quqd}^{(1)}$	$\mathcal{Q}_{ll}$	$\mathcal{Q}_{e\phi}$	$\mathcal{Q}_{u\phi}$	$\mathcal{Q}_{d\phi}$
Complex Triplet	✓	✗	✗	✗	✗
2HDM	✗	✓	✓	✓	✓

Table: Dimension-six CP-conserving fermionic operators in case of two models

Example cases - Complex Triplet and 2HDM

Dim-six Ops.($\mathcal{O}_a$)	Wilson coefficients ($\mathcal{C}_a$)
$\mathcal{O}_6$	$\frac{\mu_\Delta^2}{16m_\Delta^4}\left(2\mathcal{C}_0^{[\![UU_\phi^2]\!]} - \mathcal{C}_2^{[\![UU_{\phi\phi}]\!]} - 2\mathcal{C}_2^{[\![U_\phi^2]\!]}\right)(4\lambda_2+3\lambda_3)^2(2\lambda_1+\lambda_4)$ $+\frac{\mu_\Delta^2}{4m_\Delta^4}\mathcal{C}_0^{[\![U^2U_{\phi\phi}]\!]}(4\lambda_2+3\lambda_3)\left(2\lambda_1\lambda_2+\frac{3\lambda_1\lambda_3}{2}+\lambda_2\lambda_4+\frac{3\lambda_3\lambda_4}{4}\right)$ $+\frac{3}{16m_\Delta^2}\mathcal{C}_{-2}^{[\![U^3U_{\phi\phi}]\!]}(4\lambda_2+3\lambda_3)\left(\lambda_1^3+\frac{3\lambda_4^3}{8}+\frac{3\lambda_1^2\lambda_4}{2}+\frac{5\lambda_1\lambda_4^2}{4}\right)$
$\mathcal{O}_H$	$\frac{\mu_\Delta^2}{2m_\Delta^4}\left(2\mathcal{C}_2^{[\![U_\phi^2]\!]} + \mathcal{C}_2^{[\![UU_{\phi\phi}]\!]} - \mathcal{C}_0^{[\![U_\phi D^2U_\phi]\!]}\right)(4\lambda_2+3\lambda_3)^2$ $-\frac{1}{m_\Delta^2}\mathcal{C}_{-2}^{[\![U_\mu^2U_{\phi\phi}]\!]}(4\lambda_2+3\lambda_3)\left(\frac{3\lambda_1^2}{4}+\frac{3\lambda_4^2}{16}+\frac{3\lambda_1\lambda_4}{4}\right)$
$\mathcal{O}_T$	$\frac{\lambda_4^2}{8m_\Delta^2}\mathcal{C}_{-2}^{[\![U_\mu^2U_{\phi\phi}]\!]}(4\lambda_2+3\lambda_3)$
$\mathcal{O}_R$	$\frac{\lambda_4^2}{4m_\Delta^2}\mathcal{C}_{-2}^{[\![U_\mu^2U_{\phi\phi}]\!]}(4\lambda_2+3\lambda_3)$

Table: Dimension-six CP-conserving bosonic operators and their corresponding two-loop Wilson coefficients for the extension of the SM by an electroweak complex triplet scalar with hypercharge $Y_\Delta = 1$.

Dim-six Ops. ($\mathcal{O}_a$)	Wilson coefficients ($\mathcal{C}_a$)
$(\overline{l}_L^{T\,p}C\,l_L^s)(\overline{l}_L^{T\,r}C\,l_L^q)$	$-\frac{Y_{pq}^*Y_{rs}}{8m_\Delta^4}\left(\mathcal{C}_2^{[\![UU_{\phi\phi}]\!]} + 2\mathcal{C}_2^{[\![U_\phi^2]\!]}\right)(4\lambda_2+3\lambda_3)^2$

Table: Dimension-six CP-conserving leptonic operators and their corresponding two-loop Wilson coefficients for complex triplet model with hypercharge $Y_\Delta = 1$.

Futurework

- We'll implement these results into mathcing tool CoDeX.
- For our follow up work, we aim to get the two loop effective action after integrating out fermions.

Thank you for listening!

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