

Evanescent operators at NLO

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Outline

- 1 Introduction
- 2 Origin of Evanescent operators
- 3 Application
- 4 Evanescent-free schemes
- 5 Summary

based on: [2208.10513](#), [2211.01379](#), [2401.16904](#)

in collaboration with Marko Pesut and Zach Polonsky

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Motivation

Matching

One-loop level

RGE running

At two-loop level

Basis changes

At one-loop

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Example: Fierz transformations

Four-fermion (4F) operators

$$\mathcal{O}_{4F} = \mathcal{F}\mathcal{O}_{4F}$$

Example

$$(\bar{q}_1^\alpha \gamma_\mu P_L q_2^\beta)(\bar{q}_3^\beta \gamma^\mu P_R q_4^\alpha) = -2(\bar{q}_1 P_R q_4)(\bar{q}_3 P_L q_2)$$

$$d = 4 - 2\epsilon$$

$$\mathcal{O}_{4F} = \mathcal{F}\mathcal{O}_{4F} + E_{\mathcal{O}}$$

Evanescent operators

Definition

$$E_{\mathcal{O}} = \mathcal{O}_{4F} - \mathcal{F}\mathcal{O}_{4F}$$

Evanescent

$$E_{\mathcal{O}} \xrightarrow{d \rightarrow 4} 0$$

Basis

$$\{\mathcal{O}_j, E_i\}$$

Evanescent operators: Complication

E_i

Finite contributions from one-loop insertions

Scheme dependence

ADMs and matching

Solution

Interpret finite contributions as one-loop shifts in \mathcal{F}

Traditional way

Basis

$$\{\mathcal{O}_j, E_i\}$$

Fierz identities

$$\mathcal{O}_i = \mathcal{F}\mathcal{O}_i + E_i$$

Finite contributions

Resulting from E_i

Novel way: One-loop Fierz identities

JA/Pesut: 2208.10513

Basis

$$\{\mathcal{O}_j\}$$

Fierz identities

$$\mathcal{O}_i = \mathcal{F}\mathcal{O}_i + \frac{\alpha_s}{4\pi} \sum_j a_j \mathcal{O}_j$$

One-loop shifts

Expressed in physical basis

Computation

One-loop corrections

$$LO, LFO$$

Taking difference

$$LO - LFO = LE$$

Finite shifts

$$LE = \frac{\alpha_s}{4\pi} \sum_i a_i \mathcal{O}_i$$

Operators

Four-fermion operators

4q, SL, 4l

Dirac structures

vector, scalar, tensor

Colour

singlet and crossed

Basis

$$\gamma_\mu P_A \otimes \gamma^\mu P_B, \quad P_A \otimes P_B, \quad \sigma_{\mu\nu} P_A \otimes \sigma^{\mu\nu} P_A \quad A, B = L, R$$

Contributions

QCD, QED

General scheme

$$\gamma_\mu \gamma_\nu \gamma_\rho P_L \otimes \gamma^\mu \gamma^\nu \gamma^\rho P_L = 4(4 - a_1 \epsilon) \gamma_\mu P_L \otimes \gamma^\mu P_L$$

Dipole operators

JA/Pesut/Polonsky: 2211.01379

Basis

$$\gamma_\mu P_A \otimes \gamma^\mu P_B, \quad P_A \otimes P_B, \quad \sigma_{\mu\nu} P_A \otimes \sigma^{\mu\nu} P_A \quad A, B = L, R$$

Contributions

QCD, QED

Dipoles

$$D_{q_1 q_2 G}^B = \frac{1}{g_s} m_q (\bar{q}_1 \sigma^{\mu\nu} P_B T^A q_2) G_{\mu\nu}^A$$

$$D_{f_1 f_2 \gamma}^B = \frac{1}{e} m_f (\bar{f}_1 \sigma^{\mu\nu} P_B f_2) F_{\mu\nu}$$

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LQ matching

Scalar leptoquark

$$\mathcal{L}_{q\ell}^{LQ} = \bar{q} (\Gamma_L^S P_L + \Gamma_R^S P_R) \ell \Phi^* + \text{h.c.}$$

Matching

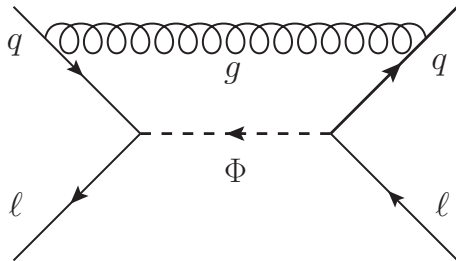
SL operators

QCD corrections

$\sim 10\%$ corrections

JA/Crivellin/Greub: 1811.08907

Matching: Example



Issue

Matching: LQ basis

$$\tilde{O}_S^{AB} = (\bar{q} P_A l)(\bar{l} P_B q)$$

$$\tilde{O}_T^A = (\bar{q} \sigma_{\mu\nu} P_A l)(\bar{l} \sigma^{\mu\nu} P_A q)$$

Running: SM basis

$$O_S^{AB} = (\bar{q} P_A q)(\bar{l} P_B l)$$

$$O_T^A = (\bar{q} \sigma_{\mu\nu} P_A q)(\bar{l} \sigma^{\mu\nu} P_A l)$$

Combine results

One-loop Fierz

Basis change

Tree-level Fierz

$$R_0 = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{8} \\ -6 & \frac{1}{2} \end{pmatrix}$$

One-loop Fierz

$$R_1 = \begin{pmatrix} 0 & \frac{N_c^2 - 1}{16N_c} \\ \frac{7 - 7N_c^2}{N_c} & 0 \end{pmatrix}$$

$$\begin{pmatrix} O_S^{AA} \\ O_T^A \end{pmatrix} = R_0 \begin{pmatrix} \tilde{O}_S^{AA} \\ \tilde{O}_T^A \end{pmatrix} + \frac{\alpha_s}{4\pi} R_1 \begin{pmatrix} \tilde{O}_S^{AA} \\ \tilde{O}_T^A \end{pmatrix}$$

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Issue: EV-to-physical mixing

Evanescient operators

Needed for mapping to physical basis

EV insertions

generate physical operators

Renormalization

Subtract physical contributions

Prescription

$$(\Gamma_{S_i}) \otimes (\Gamma_{S_j}) = C_{ij}^{k\ell} (\Gamma_{O_k}) \otimes (\Gamma_{O_\ell})$$

Coefficients

$$C_{ij}^{k\ell} = a_{ij}^{k\ell} + \varepsilon b_{ij}^{k\ell}$$

$$a_{ij}^{k\ell} = \text{four-dim. part}$$

$$b_{ij}^{k\ell} = \text{arbitrary}$$

Usage

Iterative application of relations \leftrightarrow no EV-to-physical mixing

Physical Operator Insertion

$$Q = (\Gamma_1) \otimes (\Gamma_2)$$

$$\Rightarrow \langle Q \rangle^{(1)} \sim (\Gamma' \Gamma_1 \Gamma') \otimes (\Gamma' \Gamma_2 \Gamma') = C_i(\Gamma_i) \otimes (\Gamma_i)$$

EV definition

$$E := (\Gamma' \Gamma_1 \Gamma') \otimes (\Gamma' \Gamma_2 \Gamma') - C_i(\Gamma_i) \otimes (\Gamma_i)$$

$$\Rightarrow \langle E \rangle^{(1)} \sim (\Gamma'' \Gamma' \Gamma_1 \Gamma' \Gamma'') \otimes (\Gamma'' \Gamma' \Gamma_2 \Gamma' \Gamma'') - C_i(\Gamma'' \Gamma_i \Gamma'') \otimes (\Gamma'' \Gamma_i \Gamma'') = (\Gamma'' \Gamma' \Gamma_1 \Gamma' \Gamma'') \otimes (\Gamma'' \Gamma' \Gamma_2 \Gamma' \Gamma'') - C_i C'_i(\Gamma_i) \otimes (\Gamma_i)$$

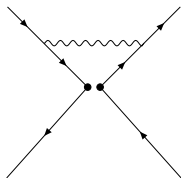
Vanishing mixing

$$E' := (\Gamma'' \Gamma' \Gamma_1 \Gamma' \Gamma'') \otimes (\Gamma'' \Gamma' \Gamma_2 \Gamma' \Gamma'') - K_i(\Gamma_i) \otimes (\Gamma_i)$$

$$\Rightarrow \langle E \rangle^{(1)} \sim \langle E' \rangle^{(1)} + (K_i - C_i C'_i)(\Gamma_i) \otimes (\Gamma_i)$$

→ no mixing if $K_i = C_i C'_i$

Example: Scalar and Tensor 1L insertion

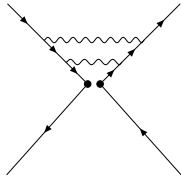


Prescription

$$(P_L \gamma^\mu \gamma^\nu) \otimes (\gamma_\nu \gamma_\mu P_L) = (4 - 2\epsilon)(P_L) \otimes (P_L) + (1 + \epsilon b_{t1})(\sigma^{\mu\nu} P_L) \otimes (\sigma_{\mu\nu} P_L)$$

$$\begin{aligned} & (\sigma^{\mu\nu} P_L \gamma^\alpha \gamma^\beta) \otimes (\gamma_\beta \gamma_\alpha \sigma_{\mu\nu} P_L) = \\ & (48 + \epsilon b_{s2})(P_L) \otimes (P_L) + (12 + \epsilon b_{t2})(\sigma^{\mu\nu} P_L) \otimes (\sigma_{\mu\nu} P_L) \end{aligned}$$

Scalar 2L insertion



More complicated structure

$$\begin{aligned} & (P_L \gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta) \otimes (\gamma_\beta \gamma_\alpha \gamma_\nu \gamma_\mu P_L) \\ &= (4 - 2\epsilon) (P_L \gamma^\alpha \gamma^\beta) \otimes (\gamma_\beta \gamma_\alpha P_L) \\ &\quad + (1 + \epsilon b_{t1}) (\sigma^{\mu\nu} P_L \gamma^\alpha \gamma^\beta) \otimes (\sigma_{\mu\nu} \gamma_\beta \gamma_\alpha P_L) \\ &= (64 - \{16 + b_{s2} + 48b_{t1}\} \epsilon) (P_L) \otimes (P_L) \\ &\quad + (16 - \{2 + 16b_{t1} + b_{t2}\} \epsilon) (\sigma^{\mu\nu} P_L) \otimes (\sigma_{\mu\nu} P_L) \end{aligned}$$

Advantages

Conceptually simpler

No insertions/renormalization of EVs

Independent of treatment of γ_5

Can be used in combination with HV

Algorithmic procedure

Automation

Results from literature

Two-Loop QCD ADM for $\Delta F = 1$

four-quark operators

Herrlich/Nierste: [hep-ph/9604330](#)

Two-Loop QCD ADM for $\Delta F = 2$

Double-insertions from $\Delta F = 1$ operators

Herrlich/Nierste: [hep-ph/9604330](#)

Various matching calculations

Fix scheme constants

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Summary

Evanescent operators

Shifts in Fierz transformations

EV-free schemes

Prescription method

Simplifications

No EV renormalization, automation