Universidad de Granada

High Energy Theory

Efficient on-shell matching

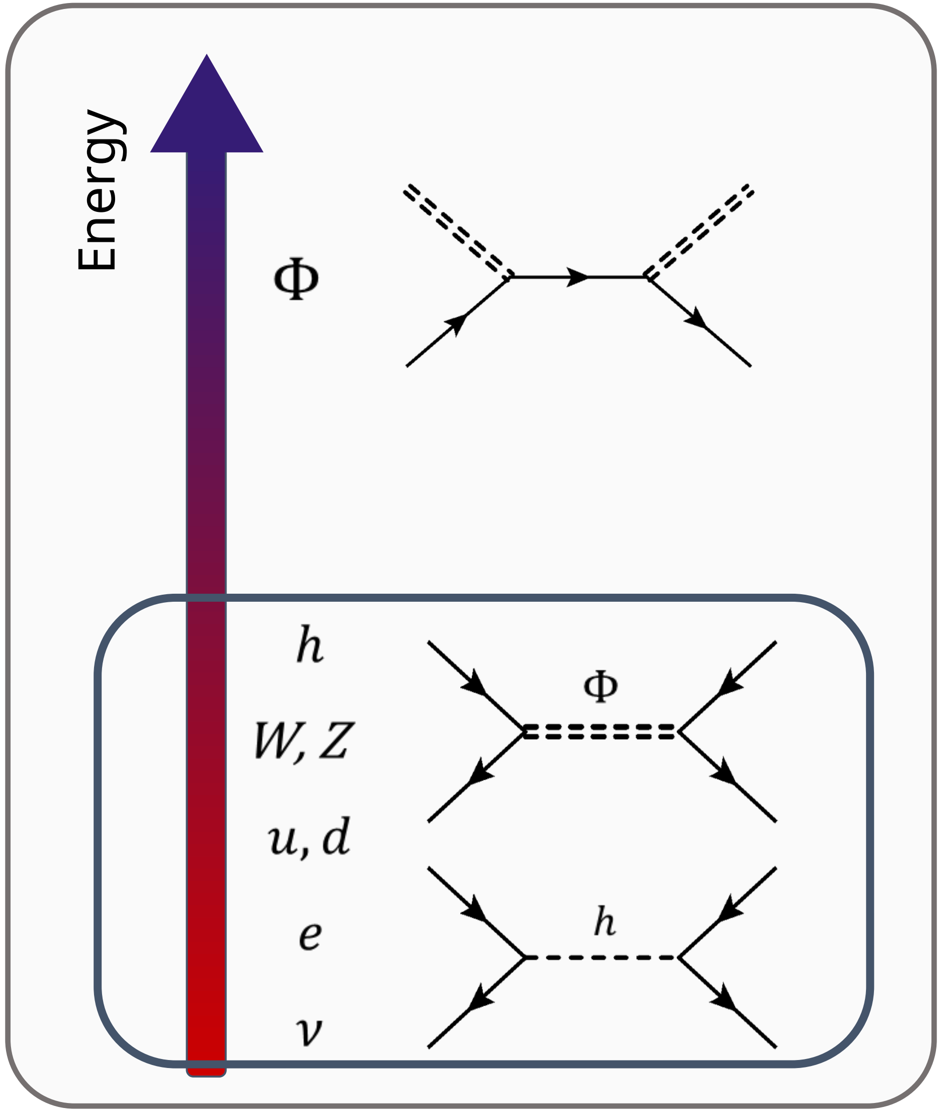
Fuensanta Vilches Bravo

with M. Chala, J. López-Miras and J. Santiago [2411.12798]

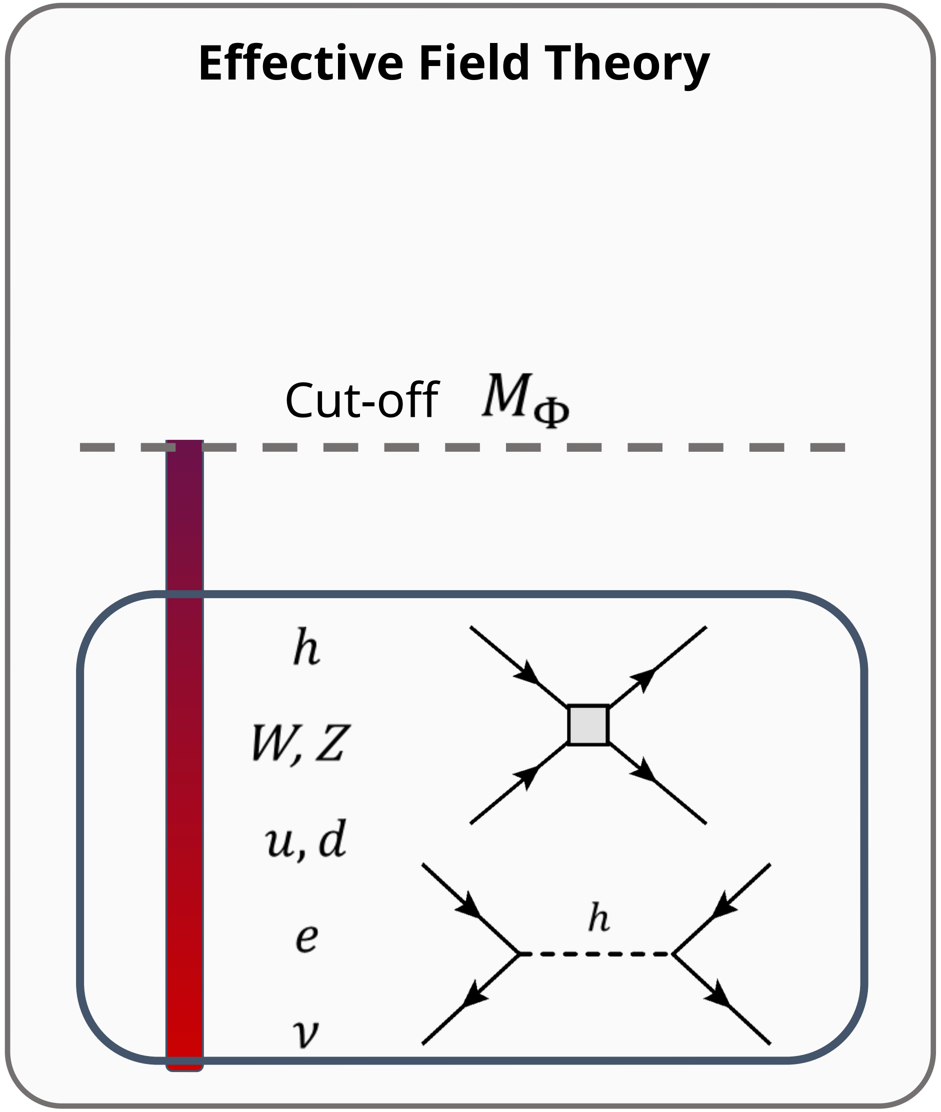
YOUNGST@RS - EFTs and Beyond

Junta de Andalucía

EFTs and matching



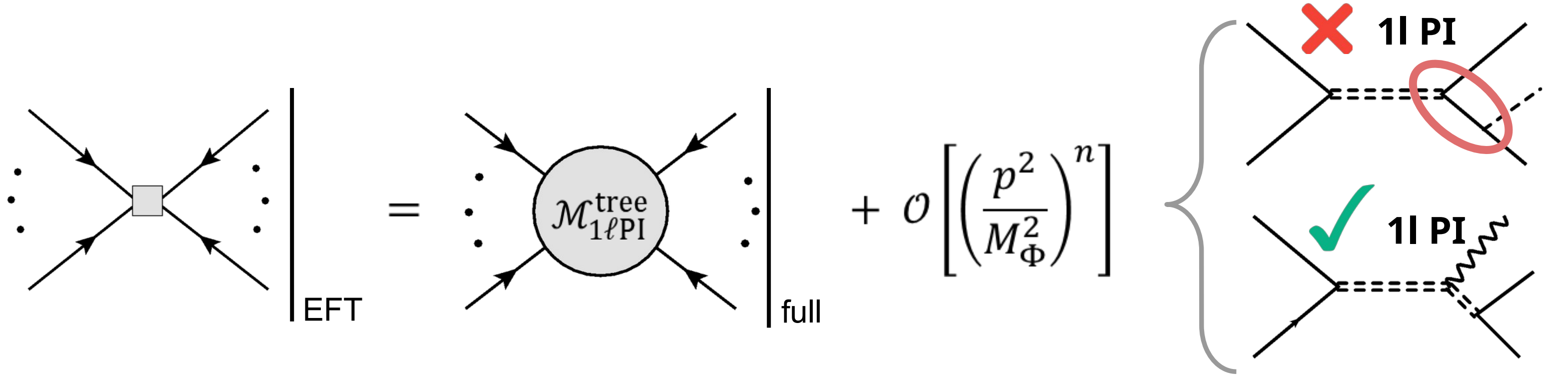
Matching



Off-shell matching

$$\frac{p^2}{M_\Phi^2} \ll 1$$

Tree-level matching:



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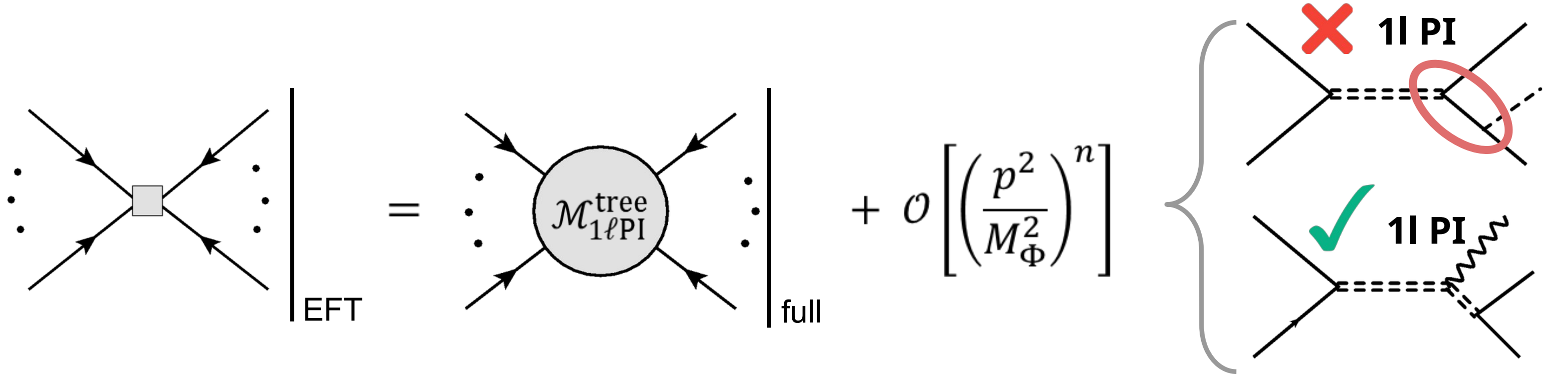
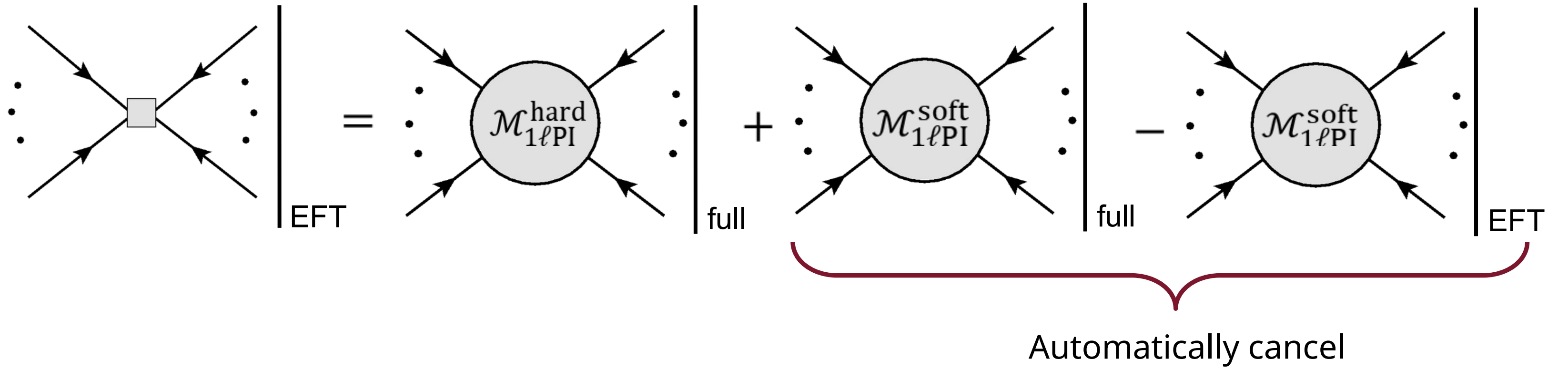


- **One-loop matching:**
  (method of regions)

  hard: $k \sim M_\Phi \gg p, m$

  soft: $M_\Phi \gg k \sim p, m$



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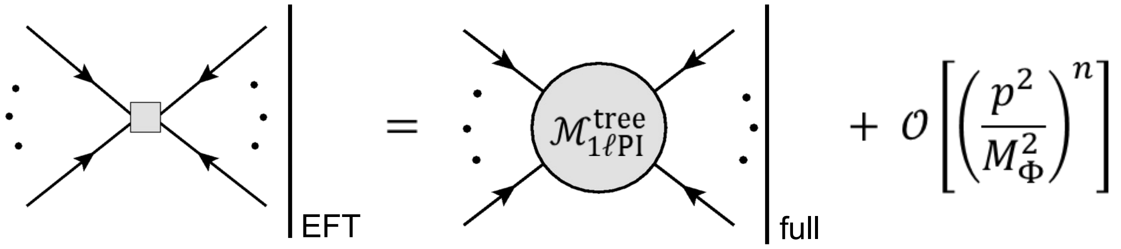

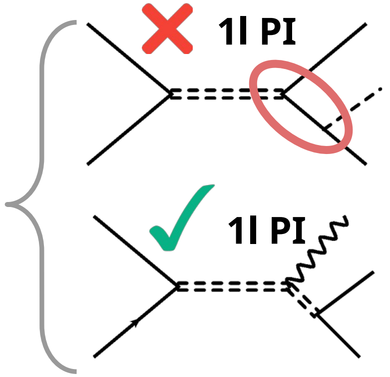

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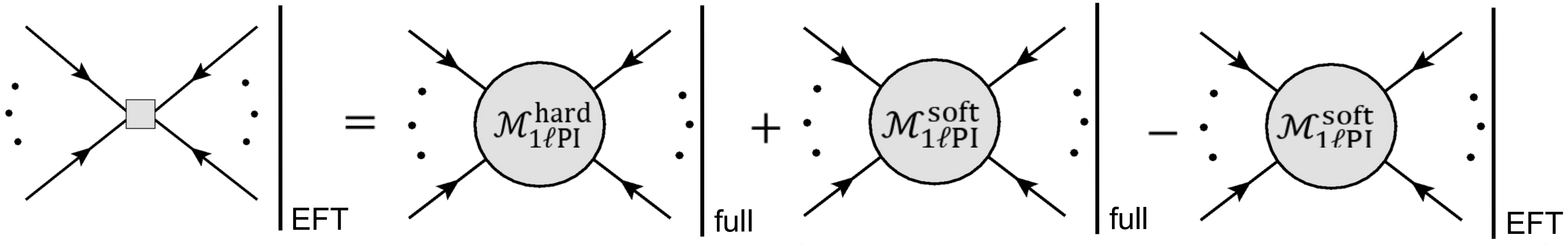

Automatically cancel

The matching results in values for couplings in the EFT → $C_1, C_2, \ldots, C_i, R_1, R_2, \ldots, R_j$ → $C_1, C_2, \ldots, C_i$

Some correspond to **redundant** operators

On-shell matching

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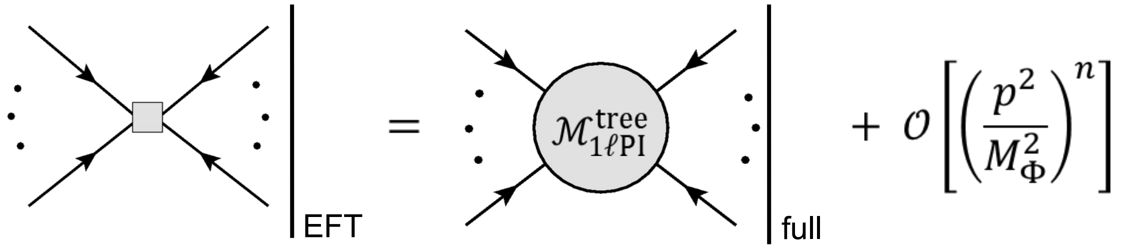

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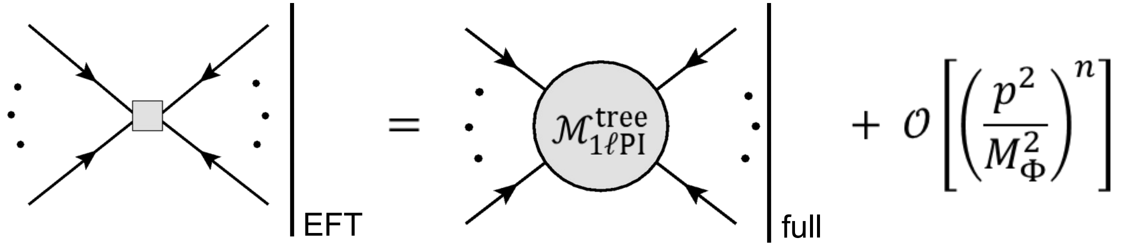

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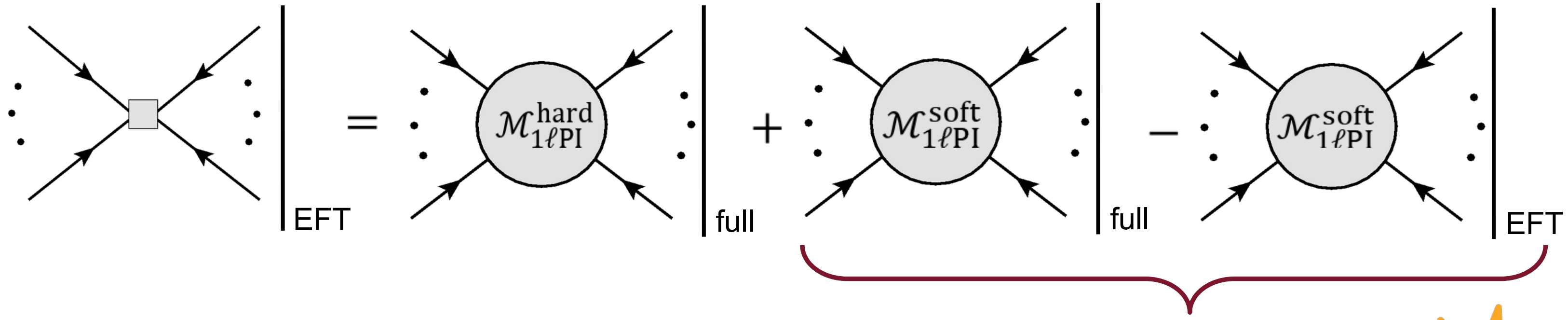

Almost cancel
EVANESCENT CONTRIBUTIONS

On-shell matching

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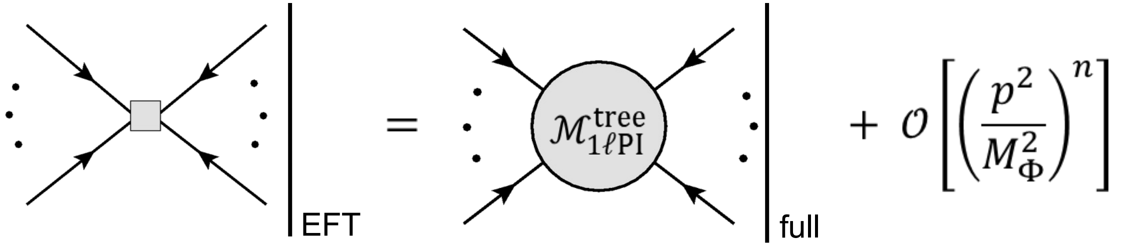

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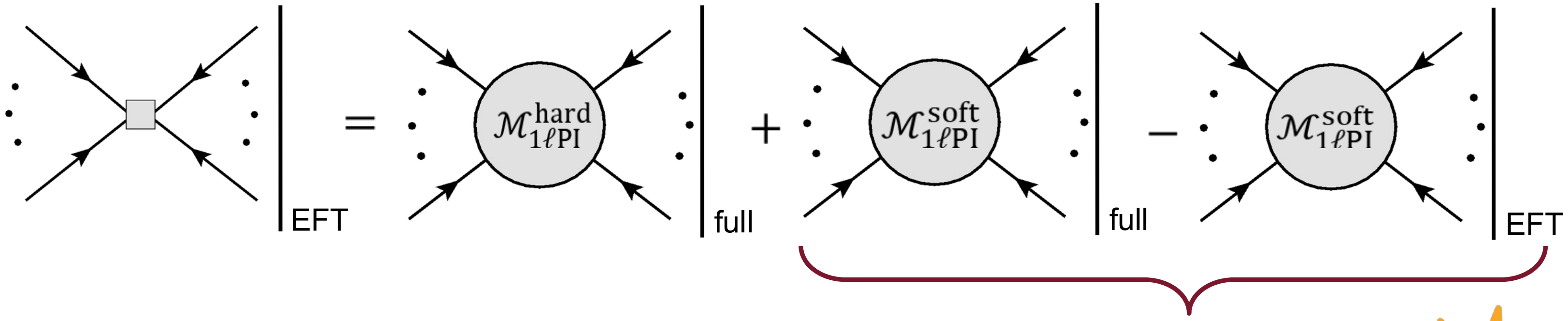


Almost cancel
EVANESCENT CONTRIBUTIONS

On-shell condition: $p^2 = m^2_{\text{phys}}$

Wavefunction factors: $\mathcal{M} = \left(\sqrt{Z}\right)^n \mathcal{M}_{\text{amp}}$

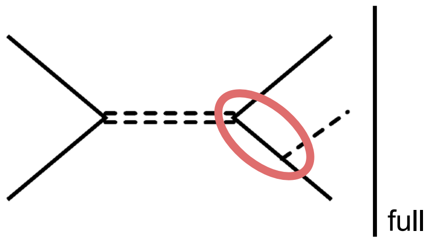
$\rightarrow$ $C_1, C_2, \ldots, C_i$

(no redundancies!)

Slide 6

On-shell matching: non-localities

Light bridges ➔



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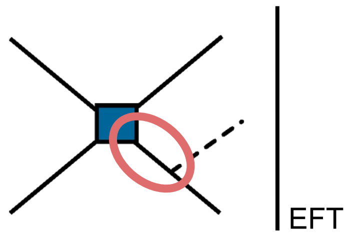
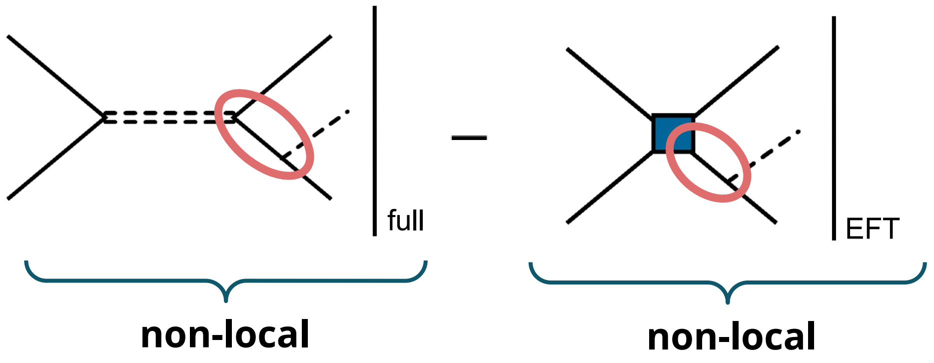


= **local** **(polynomial in external momenta)**

non-local **non-local**

On-shell matching: non-localities

Light bridges →



= **local** **(polynomial in external momenta)**

- **Difficult** to follow this cancelation **analitycally** → **Substitution of random-generated kinematics**
- The procedure is to be **numerical but exact** → **Rational kinematics**

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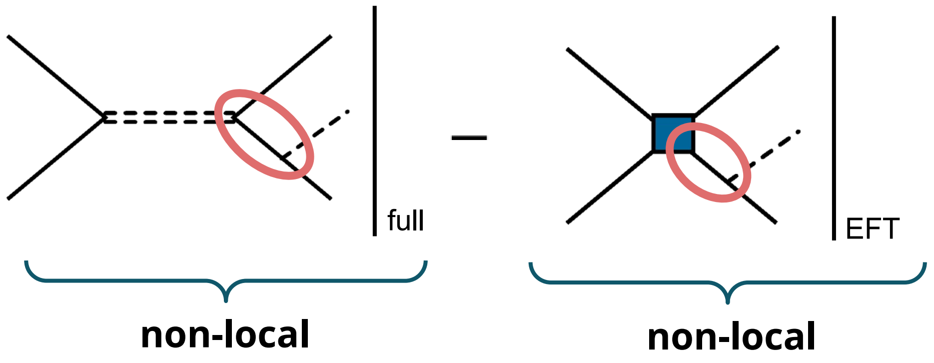

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Spinor Helicity Formalism
[*arXiv:2304.01589, arXiv:2202.02681*]

Rational values for { **momenta**, **polarizations**, **spinors** } with symbolic masses m_i

Satisfying...

Momentum conservation
On-shell condition
Dirac equation
Transversality

$$\mathcal{M} = \alpha p_2 \cdot p_3 + \frac{\beta^2}{(p_1+p_4)^2 - m^2} = \frac{9044503}{1681920} m^2 \alpha - \frac{840960}{8203543} \frac{\beta^2}{m^2}$$

Evanescent shifts

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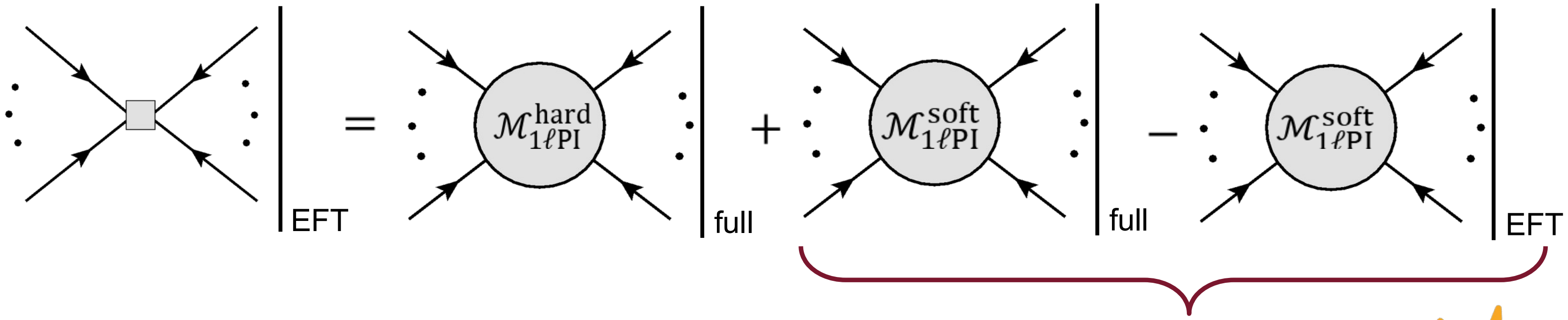
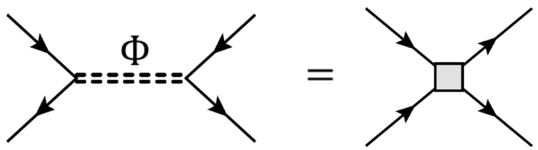


Almost cancel
EVANESCENT CONTRIBUTIONS

Copy of SM Higgs: $\mathcal{L} = \mathcal{L}_{\text{SM}} + D_\mu \Phi^\dagger D^\mu \Phi - M^2 \Phi^\dagger \Phi - \left(\mathcal{Y}^{pr} \bar{\ell}^p \Phi e^r + \text{h.c.} \right)$



$$(\bar{\ell}^p e^r)(\bar{e}^s \ell^t) = -\frac{1}{2} (\bar{\ell}^p \gamma^\mu \ell^r)(\bar{e}^s \gamma_\mu e^t)$$ **(d=4)**

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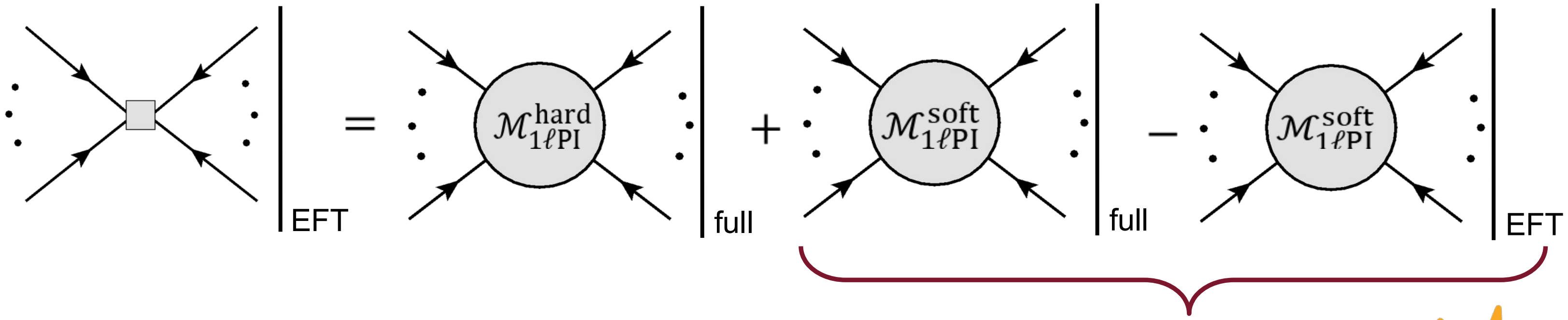

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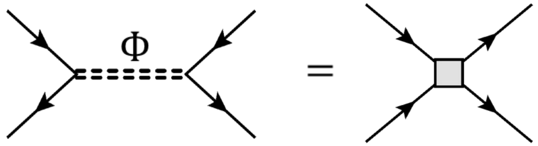

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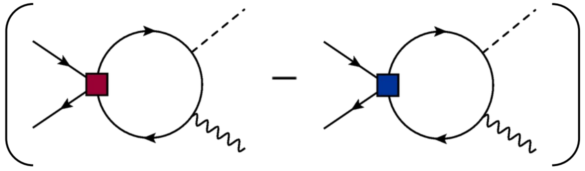

$\neq 0$ → ΔC_{eB}

Only **finite parts** coming from $\epsilon \times \dfrac{1}{\epsilon_{\text{UV}}}$

An example: one-loop matching to SMEFT

Copy of SM Higgs: $\mathcal{L} = \mathcal{L}_{\rm SM} + D_\mu \Phi^\dagger D^\mu \Phi - M^2 \Phi^\dagger \Phi - \left(\mathcal{Y}^{pr} \bar{\ell}^p \Phi e^r + \text{h.c.}\right)$

Operators to match...

$$\mathcal{O}_\lambda = -(H^\dagger H)^2, \qquad \mathcal{O}_{He} = (\bar{e}\gamma^\mu e)(H^\dagger \mathrm{i}\overleftrightarrow{D}_\mu H),$$

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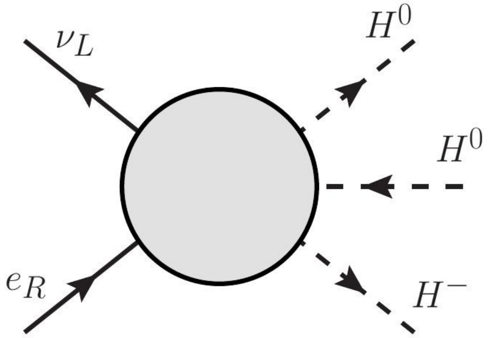
$$\mathcal{O}_{eB} = (\bar{\ell}\sigma^{\mu\nu} e) H B_{\mu\nu}, \qquad \mathcal{O}_{H\ell}^{(3)} = (\bar{\ell}\gamma^\mu \sigma^I \ell)(H^\dagger \mathrm{i}\overleftrightarrow{D}^I_\mu H),$$

$$\mathcal{O}_{eW} = (\bar{\ell}\sigma^{\mu\nu} e)\sigma^I H W^I_{\mu\nu}, \qquad \mathcal{O}_{eH} = (H^\dagger H)\bar{\ell} e H,$$

...with the **amplitude**



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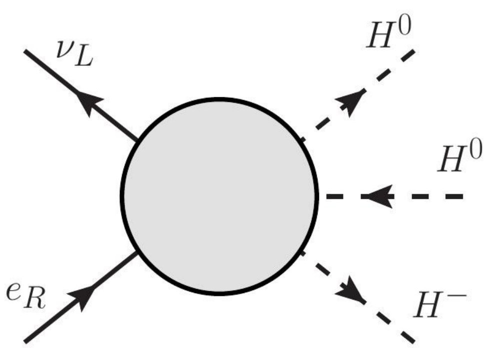

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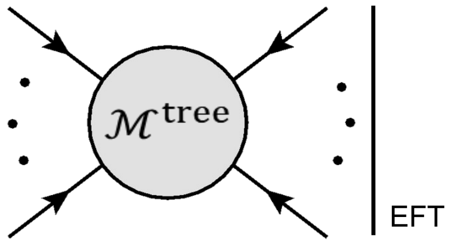

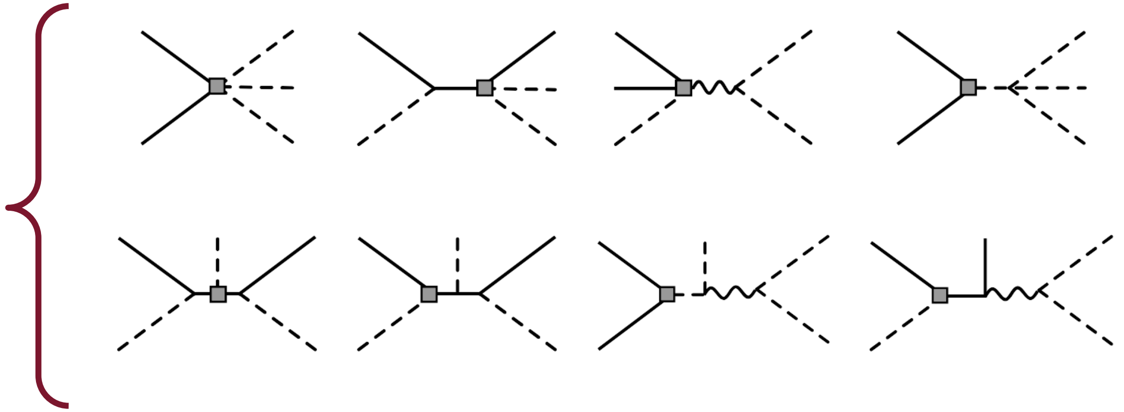

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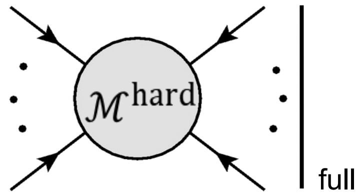
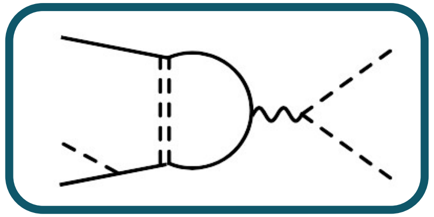
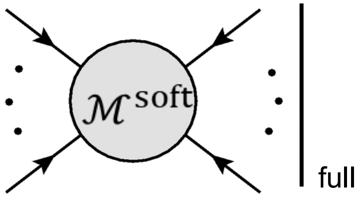


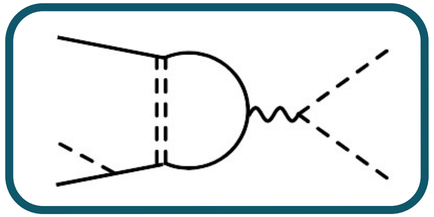


$$\sim \int_k \frac{1}{k^2 - M_\Phi^2} \frac{\not{k} + \not{p}_1}{(k+p_1)^2} \frac{\not{k} + \not{p}_2}{(k+p_2)^2}$$

$$\sim \int_k \frac{(\not{k} + \not{p}_1)(\not{k} + \not{p}_2)}{k^4 (k^2 - M_\Phi^2)} \left(1 - \frac{p_1^2 + 2k \cdot p_1}{k^2} + \ldots\right) \left(1 - \frac{p_2^2 + 2k \cdot p_2}{k^2} + \ldots\right)$$

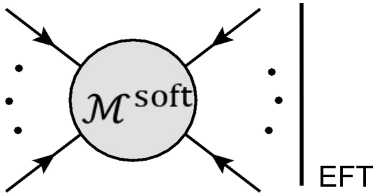
In the

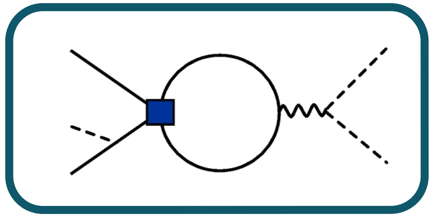




$$\sim \frac{-1}{M_\Phi^2} \int_k \frac{\not{k} + \not{p}_1}{(k+p_1)^2} \frac{\not{k} + \not{p}_2}{(k+p_2)^2} \Bigg|_{\text{UV pole}}$$

In the





$$\sim C_{\ell e} \int_k \frac{\not{k}}{k^2} \frac{\not{k} + \not{p}_2}{(k+p_2)^2} \Bigg|_{\text{UV pole}}$$

An example: one-loop matching to SMEFT

We solve for effective couplings **perturbatively** in the EFT order:

$$\lambda \to \lambda + \frac{g_2^4 m_H^2}{960\pi^2 M^2}\,,$$
$$y_e^{pr} \to y_e^{pr} - \frac{1}{128\pi^2}\left(\mathcal{Y}\mathcal{Y}^\dagger y_e + 2y_e\mathcal{Y}^\dagger\mathcal{Y}\right)^{pr} + \frac{m_H^2}{32\pi^2 M^2}\mathcal{Y}^{pr}\left(\mathcal{Y}^\dagger\right)^{st} y_e^{ts}\,,$$
$$c_{HD} \to -\frac{g_1^4}{1920\pi^2 M^2}\,,$$
$$c_{H\Box} \to -\frac{1}{7680\pi^2 M^2}(g_1^4 + 3g_2^4)\,,$$
$$c_{He}^{pr} \to \frac{g_1^4}{1920\pi^2 M^2}\delta^{pr} + \frac{7g_1^2}{576\pi^2 M^2}\left(\mathcal{Y}^\dagger\mathcal{Y}\right)^{pr} + \frac{1}{192\pi^2 M^2}\left(6\mathcal{Y}^\dagger y_e y_e^\dagger\mathcal{Y} + y^\dagger\mathcal{Y}\mathcal{Y}^\dagger y_e\right)^{pr}\,,$$
$$c_{Hl}^{(1)pr} \to \frac{g_1^4}{3840\pi^2 M^2}\delta^{pr} + \frac{17g_1^2}{1152\pi^2 M^2}\left(\mathcal{Y}\mathcal{Y}^\dagger\right)^{pr} - \frac{1}{192\pi^2 M^2}\left(6\mathcal{Y}y_e^\dagger y_e\mathcal{Y}^\dagger + y_e\mathcal{Y}^\dagger\mathcal{Y}y_e^\dagger\right)^{pr}\,,$$
$$c_{Hl}^{(3)pr} \to -\frac{g_2^4}{3840\pi^2 M^2}\delta^{pr} + \frac{g_2^2}{1152\pi^2 M^2}\left(\mathcal{Y}\mathcal{Y}^\dagger\right)^{pr} - \frac{1}{192\pi^2 M^2}\left(y_e\mathcal{Y}^\dagger\mathcal{Y}y_e^\dagger\right)^{pr}\,,$$
$$c_{eB}^{pr} \to -\frac{g_1}{768\pi^2 M^2}\left(5\mathcal{Y}\mathcal{Y}^\dagger y_e + 2y_e\mathcal{Y}^\dagger\mathcal{Y}\right)^{pr} + \frac{3g_1}{128\pi^2 M^2}\mathcal{Y}^{pr}\left(\mathcal{Y}^\dagger\right)^{st} y_e^{ts}\,,$$
$$c_{eW}^{pr} \to -\frac{5g_2}{768\pi^2 M^2}\left(\mathcal{Y}\mathcal{Y}^\dagger y_e\right)^{pr} - \frac{g_2}{128\pi^2 M^2}\mathcal{Y}^{pr}\left(\mathcal{Y}^\dagger\right)^{st} y_e^{ts}\,,$$
$$c_{eH}^{pr} \to -\frac{g_2^4}{3840\pi^2 M^2}y_e^{pr} + \frac{1}{192\pi^2 M^2}\left(y_e y_e^\dagger\mathcal{Y}\mathcal{Y}^\dagger y_e + 2y_e\mathcal{Y}^\dagger\mathcal{Y}y^\dagger y - 3\mathcal{Y}y_e^\dagger y_e\mathcal{Y}^\dagger y_e - 3y_e\mathcal{Y}^\dagger y_e y_e^\dagger\mathcal{Y}\right)^{pr} + \frac{1}{16\pi^2 M^2}\mathcal{Y}^{pr} y_e^{st}\left(y_e^\dagger\right)^{tu} y_e^{uv}\left(\mathcal{Y}^\dagger\right)^{vs} - \frac{\lambda}{32\pi^2 M^2}\mathcal{Y}^{pr}\left(\mathcal{Y}^\dagger\right)^{st} y_e^{ts}\,.$$

Cross-check with
Matchete [arXiv:2212.04510]
and
MatchMakerEFT [arXiv:2112.10787]

A second example: reduction of Green's basis

Green's basis minimal set of operators to match amplitudes **off-shell.**

Physical basis minimal set of operators to match amplitudes **on-shell.**

Field redefinitions

Tedious
Non systematic
Hard to automate

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$\mathcal{L}_{\text{full}} \supset$ operators in Green's basis

$\mathcal{L}_{\text{phys}} \supset$ operators in physical basis

Match both theories **on-shell** at **tree level**

We find the couplings of $\mathcal{L}_{\text{phys}}$ in terms of those in $\mathcal{L}_{\text{full}}$ → ***REDUCTION OF GREEN'S BASIS***

- **Example I:** Real scalar singlet with Z_2 symmetry up to dimension 8
- **Example II:** Bosonic sector in the SMEFT up to dimension 8

A second example: reduction of Green's basis

BOSONIC SECTOR

Cross-check with [2003.12525v5]

X^3		X^2H^2		H^2D^4	
$\mathcal{O}_{3G}$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{HG}$	$G_{\mu\nu}^A G^{A\mu\nu}(H^\dagger H)$	$\mathcal{O}_{DH}$	$(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$
$\mathcal{O}_{3\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}_{\mu\nu}^A G^{A\mu\nu}(H^\dagger H)$	H^4D^2	
$\mathcal{O}_{3W}$	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{HW}$	$W_{\mu\nu}^I W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{3\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}_{\mu\nu}^I W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger(H^\dagger D_\mu H)$
X^2D^2		$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$
$\mathcal{O}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G_{\rho\nu}^A)$	$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}''_{HD}$	$(H^\dagger H)D_\mu(H^\dagger i\overleftrightarrow{D}^\mu H)$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W_{\rho\nu}^I)$	$\mathcal{O}_{HWB}$	$W_{\mu\nu}^I B^{\mu\nu}(H^\dagger\sigma^I H)$	H^6	
$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$	$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}_{\mu\nu}^I B^{\mu\nu}(H^\dagger\sigma^I H)$	$\mathcal{O}_H$	$(H^\dagger H)^3$
		H^2XD^2			
		$\mathcal{O}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu^I H)$		
		$\mathcal{O}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$		

V. Gherardi, D. Marzocca y E. Venturini (2021) [2003.12525v5]

X^2H^2

$$c_{HB} \to c_{HB} - 2m_0^2 c_{HB} r_{DH}$$
$$c_{H\widetilde{B}} \to c_{H\widetilde{B}} - 2m_0^2 c_{H\widetilde{B}} r_{DH}$$

H^4D^2

$$c_{H\Box} \to c_{H\Box} - \frac{1}{8}g'^2 r_{2B} + \frac{1}{2}g' r_{BDH} - m_0^2(4c_{H\Box}r_{DH} + g' r_{BDH} r_{DH} + 2r_{DH}r'_{HD})$$
$$c_{HD} \to c_{HD} - \frac{1}{2}g'^2 r_{2B} + 2g' r_{BDH} - m_0^2(4c_{HD}r_{DH} + 4g' r_{BDH} r_{DH})$$

Reduction of dim. 6 operators up to dim. 8!!

H^6

$$c_H \to c_H + \lambda^2 r_{DH} + \lambda r'_{HD} + m_0^2\Big(\frac{1}{4}g'^2 c_{HD} r_{2B} - \frac{1}{16}g'^4 r_{2B}^2 - \frac{1}{2}g' c_{HD} r_{BDH} + \frac{1}{2}g'^3 r_{2B} r_{BDH}$$
$$- \frac{3}{4}g'^2 r_{BDH}^2 - 6c_H r_{DH} - \lambda c_{HD} r_{DH} + 8\lambda c_{H\Box} r_{DH} + g'\lambda r_{BDH} r_{DH} - 11\lambda^2 r_{DH}^2$$
$$- \frac{1}{2}c_{HD} r'_{HD} + 4c_{H\Box} r'_{HD} + \frac{1}{2}g' r_{BDH} r'_{HD} - 9\lambda r_{DH} r'_{HD} - \frac{1}{4}r'^2_{HD} - r''^2_{HD}\Big)$$

Conclusions

- On-shell matching is a **diagrammatic alternative** that allows to compute the **coefficients directly into the physical basis**.
- **More diagrams** to be computed in a single amplitude, but **several operators can be matched** at the **same time**.
- **No** need of defining **evanescent operators** or using **background field method** for finite matching.
- Useful for **renormalization** and computing **beta functions** (see [arXiv:2409.15408]).
- Can be used to compute the **reduction to any physical basis** from any redundant basis.

  ↳ Code in Mathematica in progress (***RGB***) based on Feynrules+FeynArts+FeynCalc (with J. López-Miras)

Universidad de Granada

High Energy Theory

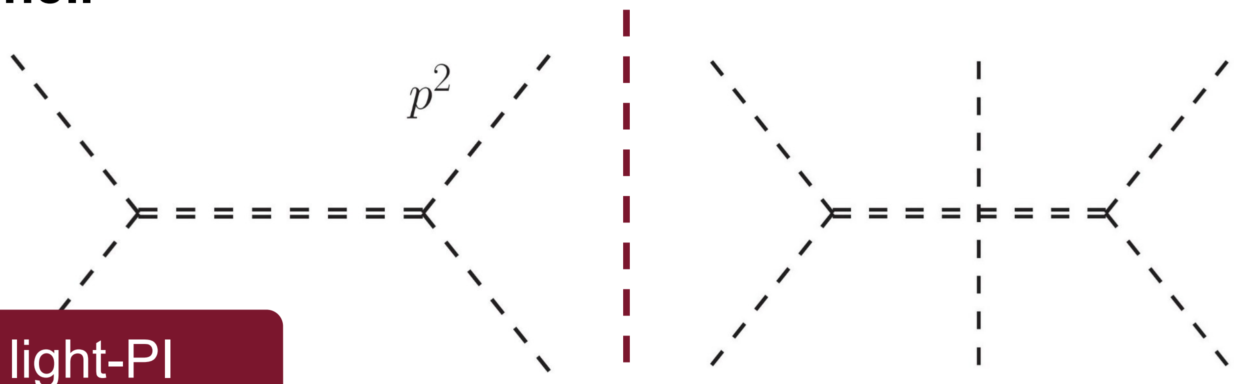
THANKS FOR YOUR ATTENTION!

Matching theories: Off-Shell vs On-shell

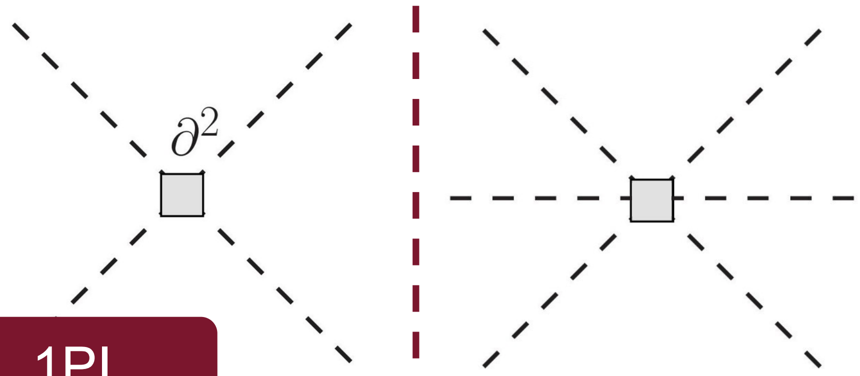
UV

IR

Off-shell

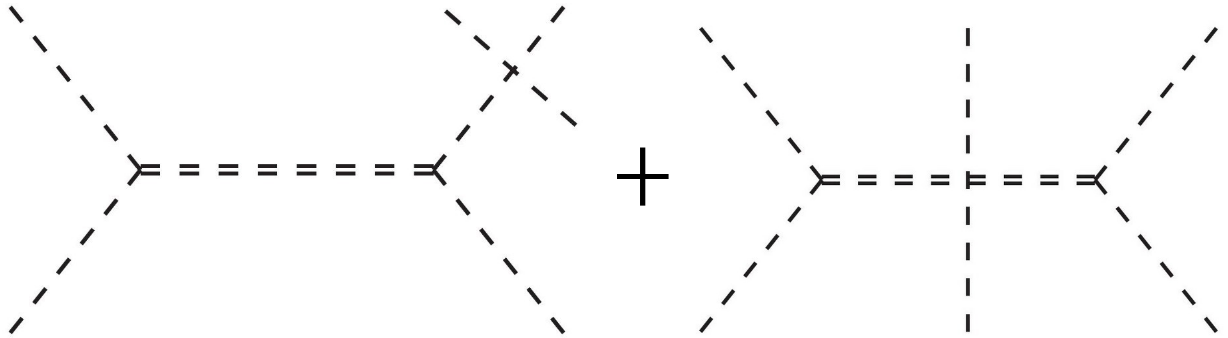


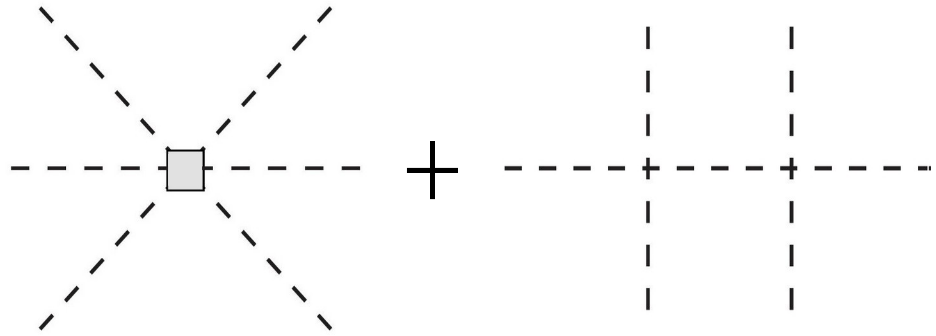
1 light-PI



1PI

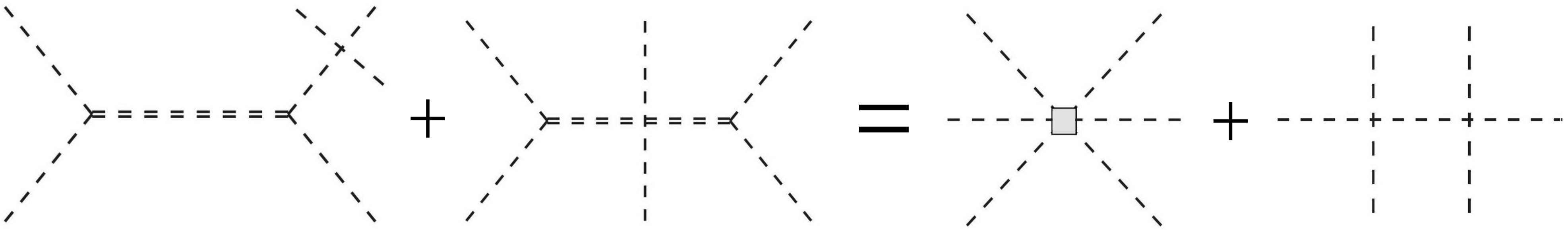
On-shell





Matching theories: Off-Shell vs On-shell

On-Shell:



Off-Shell	On-Shell
Large number of operators (Green’s basis)	Smaller set of operators (physical basis)
Small number of diagrams (1IPI in UV, 1PI in IR)	All diagrams (light bridges too)
Contribution directly local in momenta	Delicate cancelation of non-local contributions
Need of Background Field Method	
Need of evanescent operators	

Some results in the SMEFT

Dimension 4 up to dimension 8 (H and B)

$$m_0^2 \to m_0^2 - m_0^4 r_{DH} + 2m_0^6 r_{DH}^2$$

$$\lambda \to \lambda - m_0^2(4\lambda r_{DH} + 2r'_{HD}) + m_0^4(16\lambda r_{DH}^2 + 10 r_{DH} r'_{HD})$$

$$y_E \to y_E(1 - m_o^2 r_{DH} + \frac{5}{2} m_0^4 r_{DH}^2)$$

Some results in the SMEFT

Dimension 6 up to dimension 8 (H and B)

X^2H^2

$$c_{HB} \to c_{HB} - 2m_0^2 c_{HB} r_{DH}$$
$$c_{H\widetilde{B}} \to c_{H\widetilde{B}} - 2m_0^2 c_{H\widetilde{B}} r_{DH}$$

H^4D^2

$$c_{H\Box} \to c_{H\Box} - \frac{1}{8}g'^2 r_{2B} + \frac{1}{2}g' r_{BDH} - m_0^2(4c_{H\Box} r_{DH} + g' r_{BDH} r_{DH} + 2r_{DH} r'_{HD})$$
$$c_{HD} \to c_{HD} - \frac{1}{2}g'^2 r_{2B} + 2g' r_{BDH} - m_0^2(4c_{HD} r_{DH} + 4g' r_{BDH} r_{DH})$$

H^6

$$\begin{aligned} c_H \to c_H + \lambda^2 r_{DH} + \lambda r'_{HD} + m_0^2 \Big(& \frac{1}{4}g'^2 c_{HD} r_{2B} - \frac{1}{16}g'^4 r_{2B}^2 - \frac{1}{2}g' c_{HD} r_{BDH} + \frac{1}{2}g'^3 r_{2B} r_{BDH} \\ & - \frac{3}{4}g'^2 r_{BDH}^2 - 6c_H r_{DH} - \lambda c_{HD} r_{DH} + 8\lambda c_{H\Box} r_{DH} + g'\lambda r_{BDH} r_{DH} - 11\lambda^2 r_{DH}^2 \\ & - \frac{1}{2}c_{HD} r'_{HD} + 4c_{H\Box} r'_{HD} + \frac{1}{2}g' r_{BDH} r'_{HD} - 9\lambda r_{DH} r'_{HD} - \frac{1}{4}r'^2_{HD} - r''^2_{HD} \Big) \end{aligned}$$

Some results in the SMEFT

Dimension 8 (H and B)

XH^4D^2

$$c^{(1)}_{BH^4D^2} \to c^{(1)}_{BH^4D^2} - 4g' c_{HB} r_{2B} + \frac{1}{2} g'^3 r^2_{2B} + 8 c_{HB} r_{BDH} - 2 g'^2 r_{2B} r_{BDH} + 2 g' r^2_{BDH}$$

$$c^{(2)}_{BH^4D^2} \to -4g' c_{H\widetilde{B}} r_{2B} + 8 c_{H\widetilde{B}} r_{BDH}$$

$X^2H^2D^2$

$$c^{(1)}_{B^2H^2D^2} \to 0$$

$$c^{(2)}_{B^2H^2D^2} \to 0$$

$$c^{(3)}_{B^2H^2D^2} \to 0$$

X^4

$$c^{(1)}_{B^4} \to 0$$

$$c^{(2)}_{B^4} \to 0$$

$$c^{(3)}_{B^4} \to 0$$

X^2H^4

$$c^{(1)}_{B^2H^4} \to -c_{HB} g'^2 r_{2B} + \frac{1}{16} g'^4 r^2_{2B} + 2 c_{HB} g' r_{BDH} - \frac{1}{4} g'^3 r_{2B} r_{BDH} + \frac{1}{4} g'^2 r^2_{BDH} - 2 c_{HB} \lambda r_{DH} - c_{HB} r'_{HD}$$

$$c^{(2)}_{B^2H^4} \to -g'^2 c_{H\widetilde{B}} r_{2B} + 2 g' c_{H\widetilde{B}} r_{BDH} - 2\lambda c_{H\widetilde{B}} r_{BDH} - c_{H\widetilde{B}} r'_{HD}$$

Some results in the SMEFT

Dimension 8 (H and B)

$H^4 D^4$

$$c_{H^4}^{(1)} \to \frac{1}{2} g'^2 r_{2B}^2 - 2g' r_{2B} r_{BDH} + 2 r_{BDH}^2$$

$$c_{H^4}^{(2)} \to -\frac{1}{2} g'^2 r_{2B}^2 + 2g' r_{2B} r_{BDH} - 2 r_{BDH}^2$$

$$c_{H^4}^{(3)} \to 0$$

$H^6 D^2$

$$c_{H^6}^{(1)} \to -\frac{3}{4} g'^2 c_{HD} r_{2B} + \frac{3}{16} g'^4 r_{2B}^2 + \frac{3}{2} g' c_{HD} r_{BDH} - \frac{3}{2} g'^3 r_{2B} r_{BDH} + \frac{9}{4} g'^2 r_{BDH}^2 - \lambda c_{HD} r_{DH}$$
$$- 8\lambda c_{H\Box} r_{DH} - 3g' \lambda r_{BDH} r_{DH} + \lambda^2 r_{DH}^2 - \frac{1}{2} c_{HD} r'_{HD} - 4 c_{H\Box} r'_{HD} - \frac{3}{2} g' r_{BDH} r'_{HD}$$
$$- 3\lambda r_{DH} r'_{HD} - \frac{7}{4} r'^2_{HD} + r''^2_{HD}$$

$$c_{H^6}^{(2)} \to -\frac{1}{2} g'^2 c_{HD} r_{2B} + \frac{1}{8} g'^4 r_{2B}^2 + g' c_{HD} r_{BDH} - g'^3 r_{2B} r_{BDH} + \frac{3}{2} g'^2 r_{BDH}^2 - 2\lambda c_{HD} r_{DH}$$
$$- 2g' \lambda r_{BDH} r_{DH} - c_{HD} r'_{HD} - g' r_{BDH} r'_{HD}$$

Generation of random momenta

$$SL(2,\mathbb{C}) \cong SU(2)_L \times SU(2)_R \quad \left\{ \begin{array}{c} \lambda \in SU(2)_L \\ \tilde{\lambda} \in SU(2)_R \end{array} \right\} \quad \begin{array}{l} \lambda^\alpha = \varepsilon^{\alpha\beta}\lambda_\beta \\ \tilde{\lambda}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\tilde{\lambda}^{\dot{\beta}} \end{array}$$

Massless momenta :

$$P_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} \quad \Rightarrow \quad P = p_\mu \sigma^\mu = \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix}$$

Massive momenta :

$$P^\mu := q^\mu + \frac{m^2}{2q \cdot k} k^\mu \qquad \begin{aligned} q^2, k^2 &= 0 \\ q_{\alpha\dot{\alpha}} &= \lambda_\alpha \lambda_{\dot{\alpha}} \\ k_{\alpha\dot{\alpha}} &= \mu_\alpha \tilde{\mu}_{\dot{\alpha}} \end{aligned}$$

Evanescent operators

$$\mathcal{R} = \alpha\, \mathcal{O} \xrightarrow{d = 4 - 2\epsilon} \mathcal{R} = \alpha \mathcal{O} + \mathcal{E}$$

$\mathcal{O}(\epsilon)$

Additional finite local contributions in loop amplitudes

$$IR^{(0)} + IR^{(1)} = UV^{(0)} + UV^{(1)}$$

$$IR^{(0)} + IR^{(1)}_{soft} = UV^{(0)} + UV^{(1)}_{hard} + UV^{(1)}_{soft}$$

We take the hard region

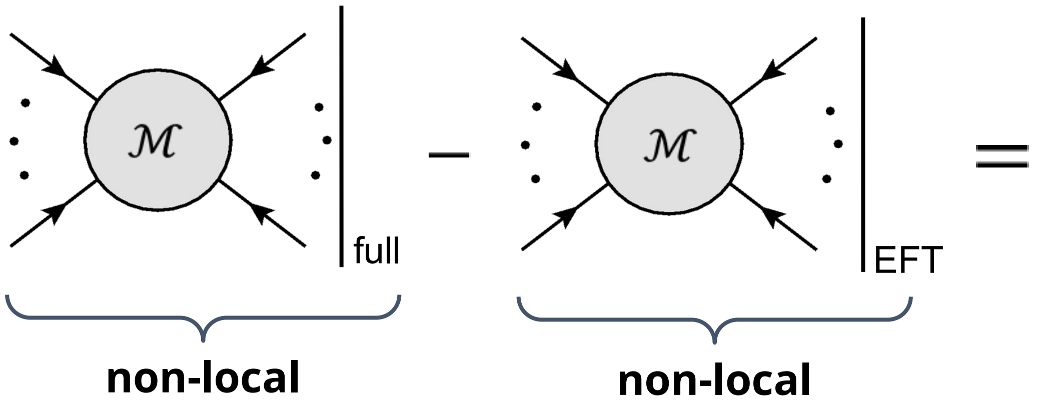
$$\int \mathcal{O} = \frac{1}{\epsilon}(a + b_{\mathcal{O}}\epsilon)$$

$$\int \mathcal{R} = \frac{1}{\epsilon}(a + b_{\mathcal{R}}\epsilon)$$

$$\int \mathcal{R} - \mathcal{O} = \frac{1}{\epsilon}(b_{\mathcal{R}}\epsilon - b_{\mathcal{O}}\epsilon) = b$$

On-shell matching: non-localities

Light bridges ➡



local **(polynomial in external momenta)**

- **Difficult to follow this cancelation analitycally** ➡ **Substitution of random-generated kinematics**
- **The procedure is to be numerical but exact** ➡ **Rational kinematics**

Spinor Helicity Formalism [*arXiv:2304.01589, arXiv:2202.02681*]

$$p^\mu = \frac{1}{2}\langle p\, \sigma^\mu\, p]$$

$$\varepsilon^\mu = \frac{1}{\sqrt{2}}\frac{\langle r\, \sigma^\mu\, p]}{\langle r\, p\rangle}$$

$$|p\rangle = P_L u(p)$$
$$|p] = P_R u(p)$$