Functional Multi-Loops Matching

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with J. Fuentes-Martín,
A.Palavrić and
A.E. Thomsen
[2412.XXXXX]

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FTAE High Energy Theory

UNIVERSITAS GRANATENSIS 1531 PLUS ULTRA

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Effective Field Theory

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They represent our best connection between theory and experiment

$$\mathcal{L}_{\rm EFT} = \mathcal{L}_{d\leq 4} + \sum_{\ell=0}^{\infty}\sum_{n=5}^{\infty}\frac{C^{(\ell,n)}}{\Lambda^{n-4}}\mathcal{O}_n$$

Bottom up: Parameterize our lack of knowledge

Top Down: Separate scales for precision measurements

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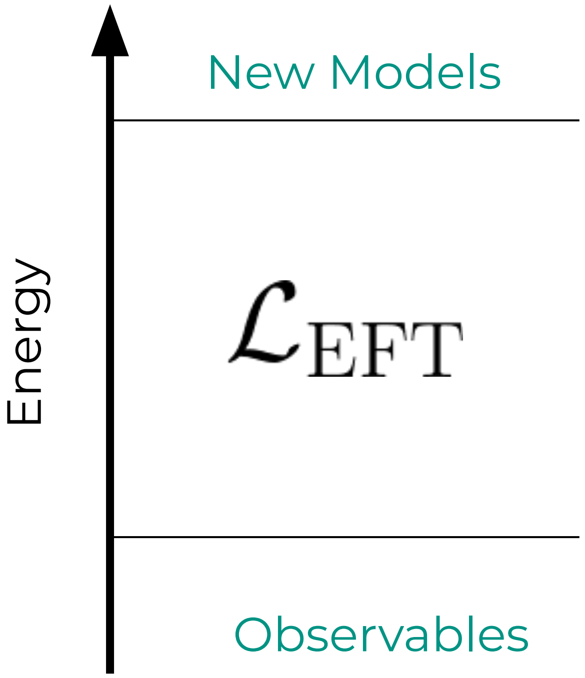
Matching



- Connection of UV theories to low energy observables
- Automated up to one-loop

[Carmona et al-2112.10787]

[Fuentes-Martín et al-2212.04510]

- Running of the theory via RG evolution

Matching

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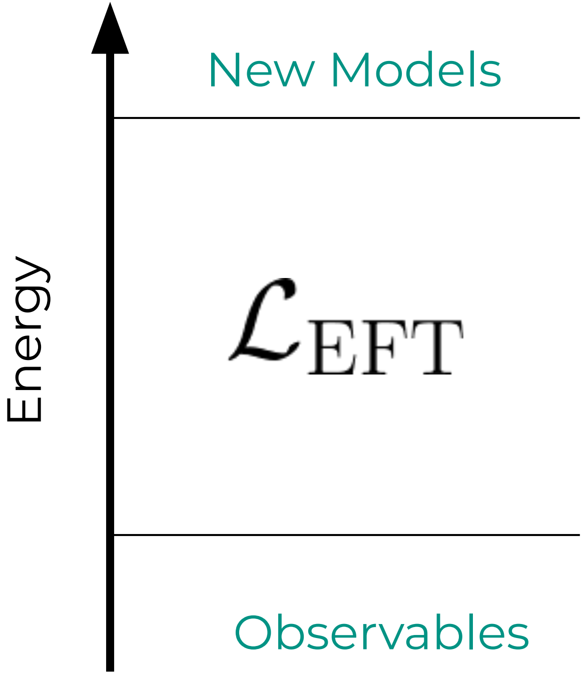

Why **more loops**?

- There is physics we are losing at the one loop level
- Scheme independence one loop matching demands two loop running.
- Precision test of UV models are also required for future high-luminosity experiments.

Amplitude Matching

$$\mathcal{L}_{\rm UV}(z_h, z_l) \xrightarrow{q_i << \Lambda} \{\mathcal{A}_{\rm UV}(q_i)\}$$

Matching: Determining Wilson Coefficients

$$\mathcal{L}_{\rm EFT}(z_l) \longrightarrow \{\mathcal{A}_{\rm EFT}(q_i)\}$$

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Feynman Diagrams

- Well-established and automated.
- Ansatz: Redundancies, redefinitions…
- Breaking of gauge symmetry in intermediate steps: additional computations

Functional Matching

Quantum Effective Action

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We are going to **"Integrate Out"** the heavy fields

$$e^{i\Gamma_{\mathrm{UV}}} = \int [D\Phi][D\phi] \exp\left(\int \mathrm{d}^d x \mathcal{L}_{\mathrm{UV}}[\Phi, \phi] \right)$$

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Method of Regions: Multiscale (m<<M) integrals can be separated in regions

$$\Gamma_{\rm UV} = \Gamma_{\rm UV}\Big|_{\rm hard} + \Gamma_{\rm UV}\Big|_{\rm soft}$$

Quantum Effective Action

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Matching Condition [Fuentes-Martín, Palavrić , Eller Thomsen-2311.13630]

$$S_{\mathrm{EFT}} = \Gamma_{\mathrm{UV}}[\bar{\Phi}[\phi], \phi]\Big|_{\mathrm{hard}}$$

Quantum Effective Action

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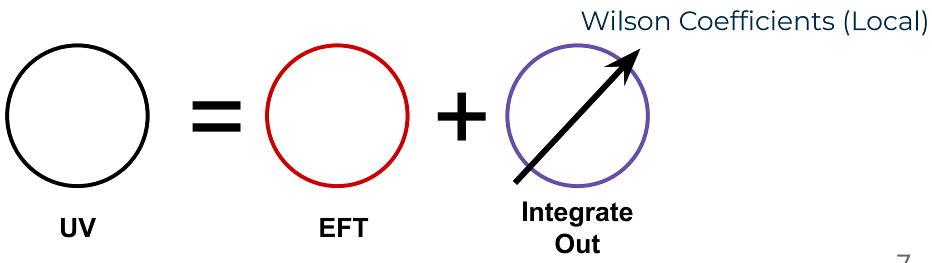

Background Field Method

$$\phi = \bar{\phi} + \hat{\phi}$$

$\bar{\phi}$ **Classical Configuration:** Tree Level

$\hat{\phi}$ **Quantum Fluctuation:** Loops

Expanding the Lagrangian

$$\mathcal{L}_{\mathrm{UV}}(\bar{\phi} + \hat{\phi}) = \mathcal{L}_{\mathrm{UV}}(\bar{\phi}) + \frac{1}{2}\phi_i Q_{ij} \phi_j + \ldots$$

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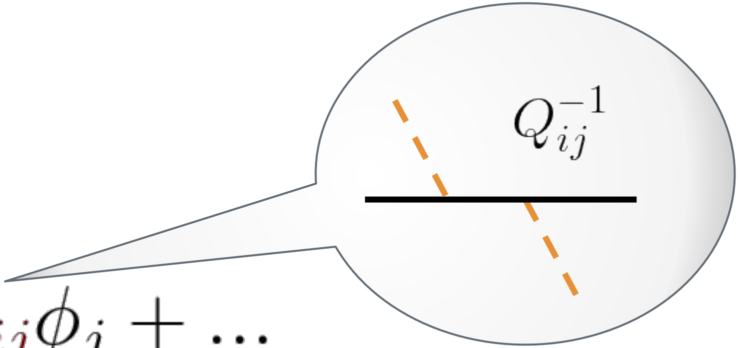

- **One-loop**

$$\exp(i\Gamma^{(1)}_{\mathrm{UV}}[\hat{\phi}]) = \int [D\phi] \exp \frac{1}{2} \left(\int \mathrm{d}^d x \phi_i Q_{ij} \phi_j \right)$$

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↓ Gaussian Integration

$$\Gamma^{(1)}_{\mathrm{UV}} = \frac{i}{2}\mathrm{STr}\ln Q$$

Evaluation of Traces

[Fuentes-Martín, AMS, Palavrić , Eller Thomsen]

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Differential operators under a Gauge Symmetry

$$Q_{ij}^{ab}(x,y) = \mathcal{Q}_{ij}^{ac}(x,P_x)\delta_c{}^b(x,y)$$

Evaluation of Traces

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b

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Locality and gauge invariance of the action: Functional Traces are dressed loop integrals

$$\mathrm{Tr}\ln Q\big|_{\rm hard} = \int_{x,y} \delta_b{}^a(x,y)\ \ln \mathcal{Q}_{ii}^{bc}(x,P_x)\delta_c{}^a(x,y)$$

$$= \int_{x,k} \ln \mathcal{Q}_{ii}^{ab}(x,P_x+k)U_b{}^a(x,y)\Big|_{x=y}$$

Evaluation of Traces

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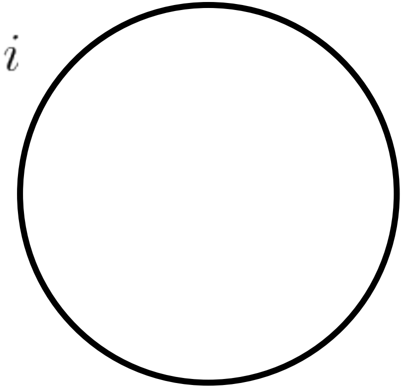
OPE around k~Λ
=
Explicit Gauge Invariance

Diagrammatically,

$\ln Q_{ii}$



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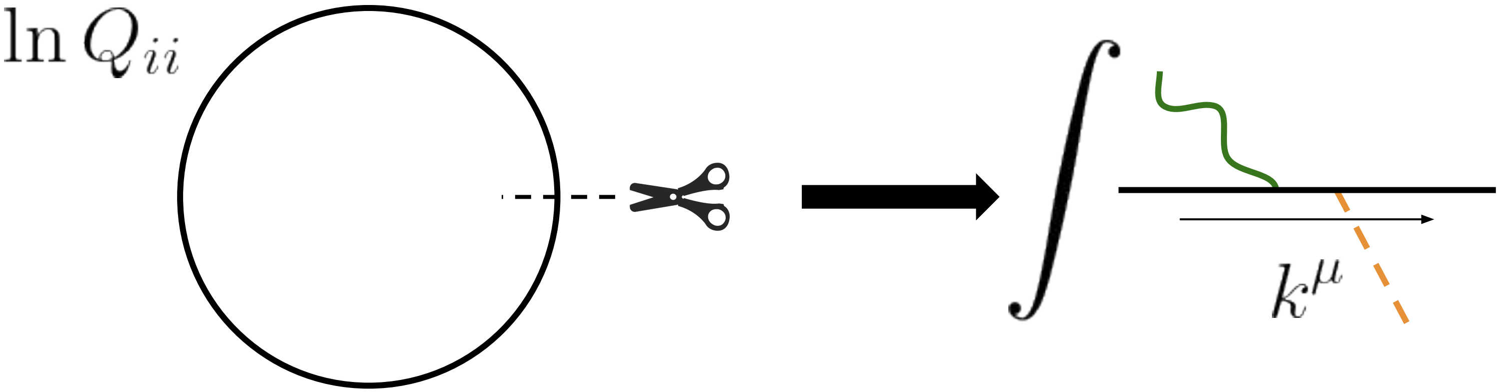

Operators traced in different points of spacetime remain local by a momentum shift operation

- Two Loops: More Topologies involved

$$\Gamma^{(2)}_{UV}[\bar{\phi}] = \frac{i}{2} Q^{-1}_{ij} B_{ij} - \frac{1}{8} Q^{-1}_{ij} D_{ijkl} Q^{-1}_{kl} + \frac{1}{12} C_{ijk} Q^{-1}_{il} Q^{-1}_{jm} Q^{-1}_{kn} C_{lmn}$$

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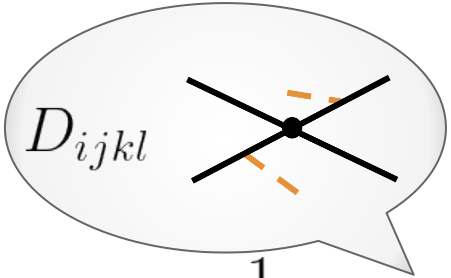

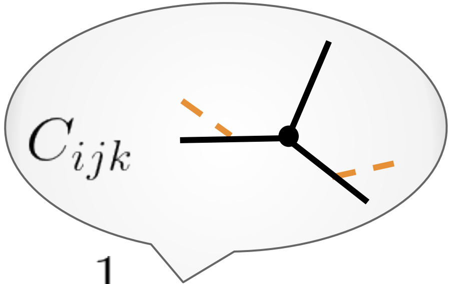

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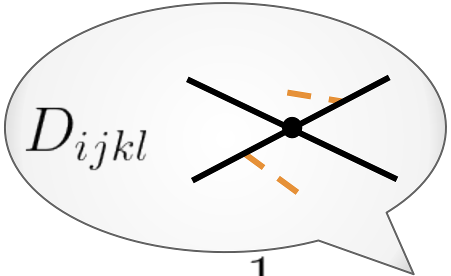

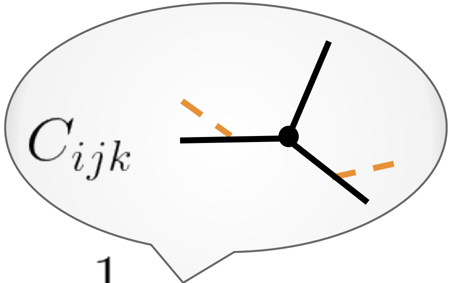

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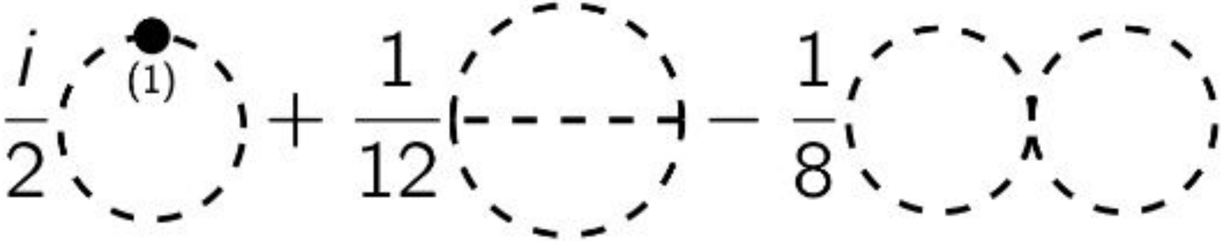

$$G_{\mathrm{ss}}\big|_{\mathrm{hard}} = \sum_{n,m,n'm'} (-1)^{n+m} \int_x \int_{k,\ell} C^{(n,m)}_{abc} \mathcal{Q}^{-1}_{aa'}(y, P_y - k - \ell) C^{(n',m')}_{a'b'c'}(y)$$

$$\times \big[(P_x + k)^m \mathcal{Q}^{-1}_{be}(x, P_x + k)(P_x + k)^{m'} U_{b'}{}^{e}(x, y)\big]$$

$$\times \big[(P_x + \ell)^n \mathcal{Q}^{-1}_{cf}(x, P_x + \ell)(P_x + \ell)^{n'} U_{c'}{}^{f}(x, y)\big]\big|_{x=y}$$

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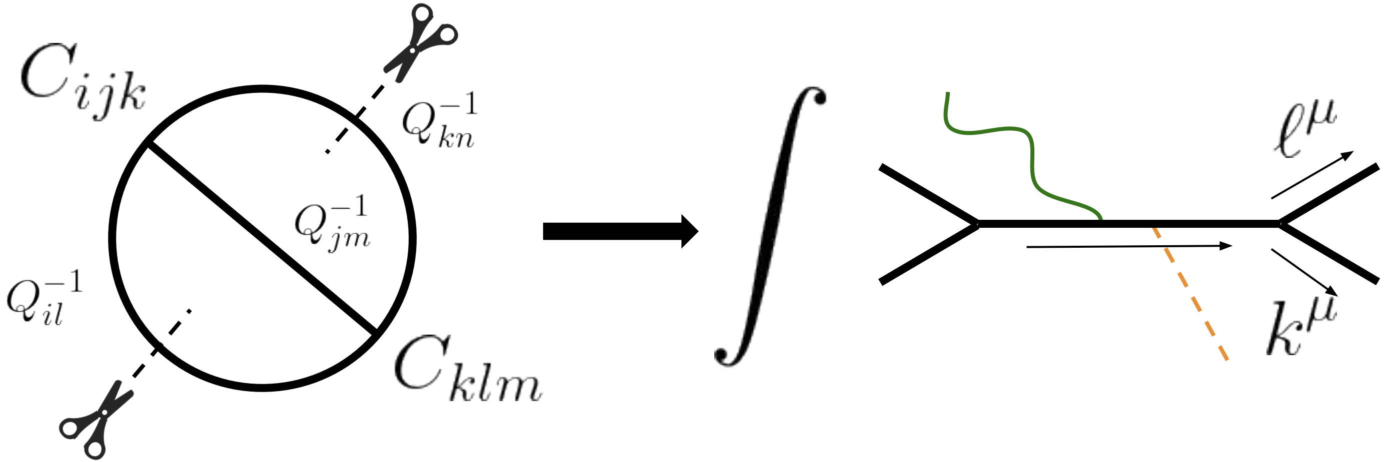

To take home

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- Functional matching is a powerful alternative for multi-loop computations.
- No need of ansatz but still we need basis reduction.
- Well suited for automation: Matchete.
- Gauge theories are fully understood: future work in other theories.

Thank you!