# Renormalization group equations via on-shell methods

YOUNGST@RS - EFTs and Beyond

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Based on:

[2312.05206] **L. B.**, G. Levati, P. Mastrolia and P. Paradisi;

[2412.xxxxx] **L. B.**, G. Brunello, G. Levati, P. Mastrolia and P. Paradisi.

December 3, 2025

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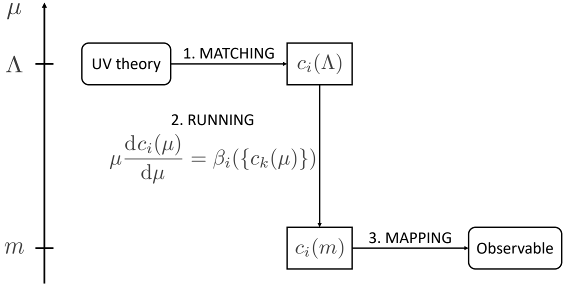
# EFT & Running

- **EFT Approach:** Standard Model as the low-energy description of a more fundamental theory emerging at a large energy scale $\Lambda$

$$\mathcal{L}_{\text{EFT}} = \sum_i \frac{c_i}{\Lambda^{[\mathcal{O}_i]-4}} \mathcal{O}_i \, .$$

- **Running:** The Wilson coefficients $c_i$ need to be evolved from the scale $\Lambda$ down to the experimental scale.

- **EFT Anomalous Dimensions** are crucial for interpreting experimental results.



# On-Shell Methods for Renormalization

**Core Features:**

- **Unitarity Cuts:** Anomalous dimensions are derived from discontinuities of scattering amplitudes.
- **Phase-Space Integration:** Lorentz-invariant phase-space integrals replace full Feynman integrals.
- **Advantages:**
  - Avoid complexities of standard loop calculations by focusing on physical, on-shell states.
  - Gauge invariance is automatic.
  - Explain zeroes in anomalous dimensions $\rightsquigarrow$ Nonrenormalization Theorems based on
    - HELICITY; [Cheung, Shen (15)]
    - LENGTH; [Bern, Parra-Martinez, Sawyer (20)]
    - ANGULAR MOMENTUM. [Jiang, Shu, Xiao, Zheng (21)]

# Limitations and Generalizations

- Originally applied only to massless particles and operators with same dimensions.
- Generalized to include **Leading Mass Effects** via the Higgs low-energy theorem:

$$\mathcal{L}_h^{\text{int}} = -\left(1+\frac{h}{v}\right)\sum_f m_f \bar{f} f \implies \lim_{\{p_h\}\to 0} \mathcal{M}(A \to B + Nh) = \sum_f \left(\frac{m_f}{v}\frac{\partial}{\partial m_f}\right)^N \mathcal{M}(A \to B)\,.$$

- Extended to handle the most **General Operator Mixing**:

$$\mu\frac{\mathrm{d}c_i}{\mathrm{d}\mu} = \sum_{n>0}\frac{1}{n!}\gamma_{i\leftarrow j_1,\ldots,j_n} c_{j_1}\cdots c_{j_n} = \gamma_{i\leftarrow j} c_j + \frac{1}{2}\gamma_{i\leftarrow j,k} c_j c_k + \cdots\,,$$

$$\gamma_{i\leftarrow j_1,\ldots,j_n} = \left.\frac{\partial^n \beta_i}{\partial c_{j_1}\cdots\partial c_{j_n}}\right|_* \,.$$

# $S$-Matrix and Dilatation Operator

- **Form Factor** associated with a local, gauge-invariant operator $\mathcal{O}_i$:

$$F_i(\vec{n};q) = \frac{1}{\Lambda^{[\mathcal{O}_i]-4}} \langle \vec{n}|\mathcal{O}_i(q)|0\rangle \ .$$

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- Exploiting the fundamental relations [Elias-Miró, Ingoldby, Riembau (20)]
  - ◇ **Analyticity:** $F_i^*(\{s_{ij} - i\epsilon\}) = F_i(\{s_{ij} + i\epsilon\})$
  - ◇ **Unitarity:** $\sum_{\vec{n}} \int d\Pi_n \ |\vec{n}\rangle\langle\vec{n}| = \mathbb{1}\,, \quad d\Pi_n = \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3} \frac{1}{2E_i}$
  - ◇ **CPT Theorem:** $\langle\vec{n};\mathrm{out}|\mathcal{O}_i(x)|0\rangle = \langle 0|\mathcal{O}_i^\dagger(-x)|\vec{n};\mathrm{in}\rangle$

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it is possible to show that [Caron-Huot, Wilhelm (16)]

$$e^{-i\pi D} F_i^*(\vec{n}) = (SF_i^*)(\vec{n}) \left( = \sum_{\vec{m}} \int d\Pi_m \ \langle \vec{n}|S|\vec{m}\rangle\, F_i^*(\vec{m}) \right)$$

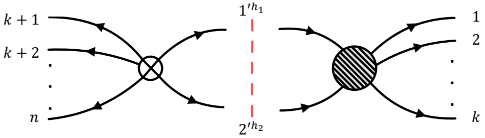
where $S = \mathbb{1} + i\mathcal{M}$ is the **$S$-matrix** and $D = \sum_i p_i \cdot \partial/\partial p_i$ is the **Dilatation Operator**.

# Master Formulae

**Linear operator mixing**

$$\left(\gamma^{(1)}_{i\leftarrow j} - \delta_{ij}\gamma^{(1)}_{i,\mathrm{IR}}\right) F_i\big|^{(0)}_{*} = -\frac{1}{\pi}(\mathcal{M}F_j)\big|^{(1)}_{*}$$

$$(\mathcal{M}F_j)^{(1)}(\vec{n}) = \sum_k \sum_{h_1,h_2}$$




$+$ permutations

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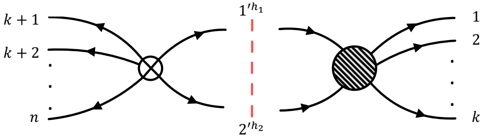



$$+ \ \text{permutations}$$

## Nonlinear operator mixing

$$\gamma^{(1)}_{i\leftarrow j,k} F_i\big|^{(0)}_{*} = -\frac{1}{\pi}\frac{\partial}{\partial c_k}\bigg|_{*}(\mathcal{M}F_j)^{(1)}$$

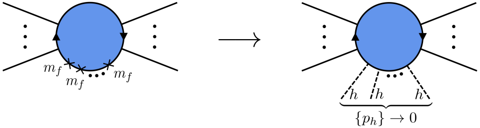
The extension for multiple operator insertions $\gamma_{i\leftarrow j_1,\ldots,j_n}$ is straightforward.

# Leading Mass Effects

The amplitude requires $N$ **fermion mass insertions** not to vanish

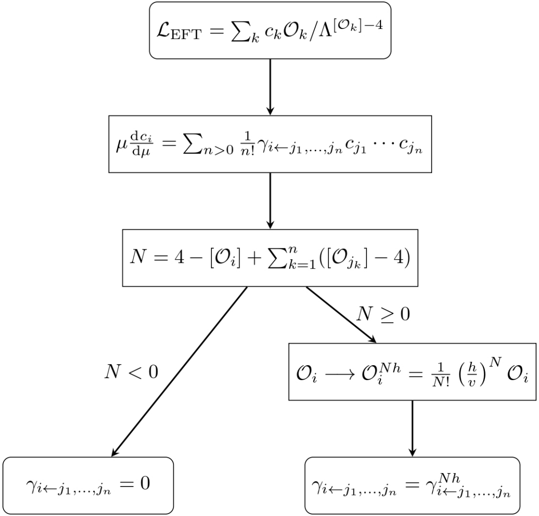
$$\Downarrow$$

Consider an equivalent amplitude entailing $N$ **extra massless Higgs fields**




$$\mathcal{L}_h^{\text{int}} = -\left(1 + \frac{h}{v}\right) \sum_f m_f \bar{f} f$$

$N$: superficial degree of divergence.



# EFT for Axion-Like Particles

Most general CP-violating and $SU(3)_c \times U(1)_{\text{em}}$ invariant dimension-5 Lagrangian:

$$\mathcal{L}_{\text{EFT}} = \frac{\tilde{\mathcal{C}}_\gamma}{\Lambda}\phi F\tilde{F} + \frac{\tilde{\mathcal{C}}_g}{\Lambda}\phi G\tilde{G} + \mathcal{Y}_P^{ij}\phi\bar{f}_i i\gamma_5 f_j + \frac{\mathcal{C}_\gamma}{\Lambda}\phi FF + \frac{\mathcal{C}_g}{\Lambda}\phi GG + \mathcal{Y}_S^{ij}\phi\bar{f}_i f_j \,.$$

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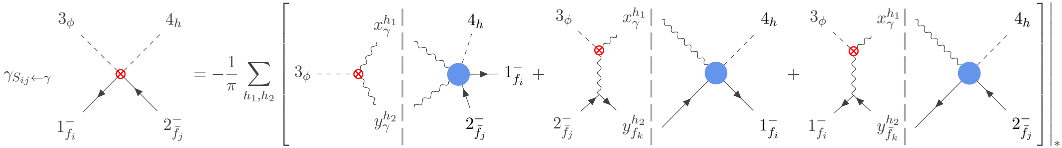
Let's consider $\phi\bar{f}_i f_j \leftarrow \phi FF$: we need $N = (4 - [\phi\bar{f}f]) + ([\phi FF] - 4) = 1$ mass insertion.
$\phi\bar{f}_i f_j$ is substituted with $\frac{h}{v}\phi\bar{f}_i f_j$.



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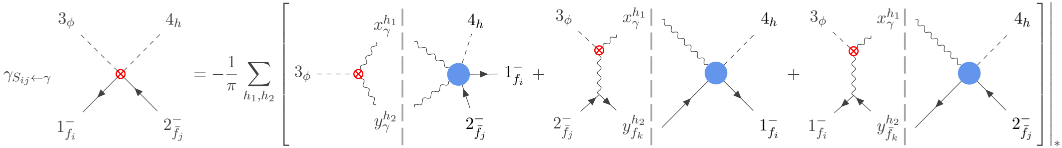


1. The 1st contribution is zero by setting the off-shell momentum $q = 0$: $g_1 = 0$.
2. The integrand of the 2nd contribution is

$$g_2 = \frac{4}{\Lambda}e^2 Q_f^2 y_i \delta^{ij}\,\frac{\langle 1\, y\rangle^2\langle 2\, x\rangle^2}{\langle 1\, x\rangle\langle 2\, y\rangle\langle x\, y\rangle}\,.$$

3. The 3rd is recast from the 2nd: $\int d\Pi_2\, g_3 = -\int d\Pi_2\, g_2|_{1\leftrightarrow 2}$.

# Phase-Space Integral via Stokes Theorem

Efficient way to perform the integral [Mastrolia (09)]:

1. Parameterize the internal spinors as:

$$\begin{pmatrix}\lambda_x \\ \lambda_y\end{pmatrix} = \frac{1}{\sqrt{1+z\bar{z}}}\begin{pmatrix} 1 & \bar{z} \\ -z & 1\end{pmatrix}\begin{pmatrix}\lambda_1 \\ \lambda_4\end{pmatrix} \implies g_2(z,\bar{z}) = \frac{4}{\Lambda} e^2 Q_f^2 y_i \delta^{ij} \frac{(\langle 1\,2\rangle - \bar{z}\langle 2\,4\rangle)^2}{\bar{z}(1+z\bar{z})(z\langle 1\,2\rangle + \langle 2\,4\rangle)}\,.$$

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2. Integrate in $\bar{z}$, keeping only rational contributions:

$$G_{2\,rat}(z,\bar{z}) = \int d\bar{z}\,\frac{g_2(z,\bar{z})}{(1+z\bar{z})^2} = \frac{2}{\Lambda}e^2 Q_f^2 y_i \delta^{ij}\frac{z(3+2z\bar{z})\langle 1\,2\rangle - (1+2z\bar{z})\langle 2\,4\rangle}{z^2(1+z\bar{z})^2}\,.$$

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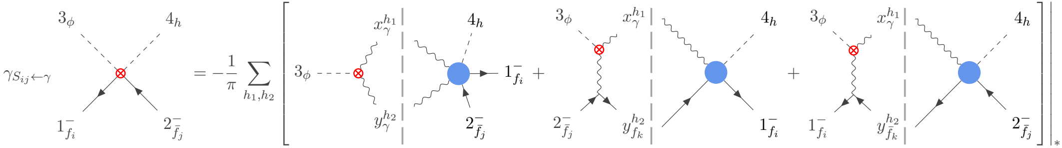
3. Apply Residue Theorem by summing over the $z$-poles $\mathcal{P}_{G_2}$ of $G_{2\,rat}$:

$$\int d\Pi_2\, g_2 = -\frac{1}{8\pi} \sum_{z_0 \in \mathcal{P}_{G_2}} \mathrm{Res}_{(z,\bar{z})=(z_0,z_0^*)} G_{2\,rat}(z,\bar{z}) = -\frac{3e^2 Q_f^2}{4\pi\Lambda} y_i \delta^{ij} \langle 1\,2\rangle \,.$$

# $\phi \bar{f} f \leftarrow \phi F F$ and $\phi \bar{f} i\gamma_5 f \leftarrow \phi F \tilde{F}$




$$\gamma_{S_{ij}\leftarrow\gamma}\frac{\langle 1\, 2\rangle}{v} = -\frac{1}{\pi}\int d\Pi_2\,(g_1+g_2+g_3) = \frac{3e^2Q_f^2}{2\pi^2\Lambda}y_i\delta^{ij}\langle 1\, 2\rangle$$

$$\implies \quad \gamma_{S_{ij}\leftarrow\gamma} = \frac{3e^2Q_f^2}{2\pi^2}\frac{m_i}{\Lambda}\delta^{ij} \quad \implies \quad \mu\frac{\mathrm{d}\mathcal{Y}_S^{ij}}{\mathrm{d}\mu} \supset \gamma_{S_{ij}\leftarrow\gamma}\mathcal{C}_\gamma = \frac{3e^2Q_f^2}{2\pi^2}\frac{m_i}{\Lambda}\delta^{ij}\mathcal{C}_\gamma\,.$$

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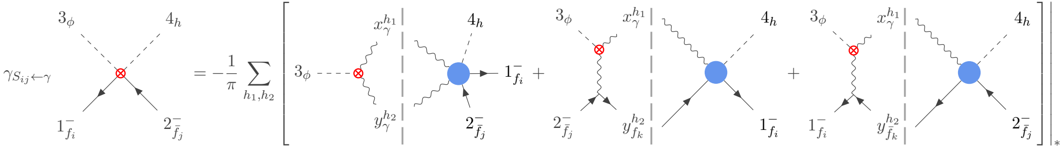


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The CP-counterpart $\phi\bar{f}_i i\gamma_5 f_j \leftarrow \phi F\tilde{F}$ comes for free!

$$\begin{cases} F_{P_{ij}}(1^-_{f_i},2^-_{\bar{f}_j},3_\phi) = -iF_{S_{ij}}(1^-_{f_i},2^-_{\bar{f}_j},3_\phi) \\ F_{\tilde{\gamma}}(1^-_\gamma,2^-_\gamma,3_\phi) = iF_\gamma(1^-_\gamma,2^-_\gamma,3_\phi) \end{cases} \quad \Longrightarrow \quad \gamma_{P_{ij}\leftarrow\tilde{\gamma}} = -\gamma_{S_{ij}\leftarrow\gamma}\,.$$

# Conclusions

**Summary:**

- Derived a master formula for general operator mixings up to 2-loop order.
- Leading mass effects included in massless limit via Higgs low-energy theorem.
- Implemented Stokes integration as an efficient tool for phase-space cut-integrals.
- Established a connection between anomalous dimensions of CP-dual operators.
- Validated findings by reproducing established results in popular EFTs.

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**Future Prospects:**

- Future experimental advances will improve limits on low-energy observables (*e.g.* flavor-violating processes and electric dipole moments) by orders of magnitude.
- Higher-order contributions are crucial for the precise assessment of new physics effects.
- While this is a very challenging task when approached with standard techniques, on-shell and unitarity-based methods offer a simpler, more efficient, and elegant way to reach this goal.

*Thank you for your attention!*

*Q&A*

# Selection Rules: Dimension-6 Operators

| | $F^3$ | $\phi^2 F^2$ | $F\phi\psi^2$ | $D^2\phi^4$ | $D\phi^2\psi^2$ | $\psi^4$ | $\phi^3\psi^2$ | $\phi^6$ |
|---|---|---|---|---|---|---|---|---|
| $F^3$ | | $\times_1$ | (2) | $\times_2$ | $\times_2$ | $\times_2$ | $\times_3$ | $\times_3$ |
| $\phi^2 F^2$ | | | | | | | (2) | $\times_2$ |
| $F\phi\psi^2$ | | | | | | | $\times_1$ | $\times_3$ |
| $D^2\phi^4$ | | | | | | | $\times_1$ | $\times_2$ |
| $D\phi^2\psi^2$ | | | | | | | $\times_1$ | (3) |
| $\psi^4$ | | | | | | | (2) | (4) |
| $\phi^3\psi^2$ | | | | | | | | (2) |
| $\phi^6$ | | | | | | | | |

Table: From [Bern, Parra-Martinez, Sawyer (20)]. Dimension-6 operator mixing pattern. Operators labeling the rows are renormalized by the operators labeling the columns.

- $\times_L$: length selection rules apply at $L$-loop order
- $(L)$: no diagrams before $L$ loops, but renormalization is possible at that order
- Light-gray: zero at one loop due to helicity selection rules

# Spinor-Helicity Formalism

The 4-momentum of an on-shell state is mapped onto a $2 \times 2$ matrix

$$p^\mu = (p^0, \vec{p}) \qquad \longrightarrow \qquad p^{\dot{\alpha}\alpha} = \bar{\sigma}_\mu^{\dot{\alpha}\alpha} p^\mu = \begin{pmatrix} p^0 + p^3 & p^1 - ip^2 \\ p^1 + ip^2 & p^0 - p^3 \end{pmatrix},$$

where $\bar{\sigma}^{\mu\,\dot{\alpha}\alpha} = (\mathbb{1}, -\vec{\sigma})^{\dot{\alpha}\alpha}$. If the particle is massless then

$$p^2 = \det\left(p^{\dot{\alpha}\alpha}\right) = m^2 = 0 \qquad \Longrightarrow \qquad p^{\dot{\alpha}\alpha} = \tilde{\lambda}^{\dot{\alpha}}\lambda^\alpha,$$

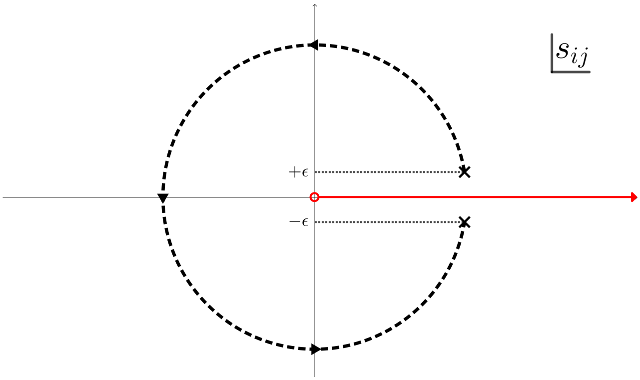
where $\lambda, \tilde{\lambda}$ are commuting Weyl spinors known as helicity spinors.
The angle and square inner products are Lorentz invariant

$$\langle i\, j\rangle \equiv \lambda_i^\alpha \lambda_{j\,\alpha} = \epsilon_{\alpha\beta}\lambda_i^\alpha\lambda_j^\beta = -\langle j\, i\rangle, \qquad [i\, j] \equiv \tilde{\lambda}_{i\,\dot{\alpha}}\tilde{\lambda}_j^{\dot{\alpha}} = -\epsilon_{\dot{\alpha}\dot{\beta}}\tilde{\lambda}_i^{\dot{\alpha}}\tilde{\lambda}_j^{\dot{\beta}} = -[j\, i].$$

The Mandelstam invariants can thus be written as

$$s_{ij} \equiv (p_i + p_j)^2 = 2p_i \cdot p_j = \langle i\, j\rangle [j\, i].$$

# Dilatation Operator & Complex Rotations




The **Dilatation Operator**

$$D = \sum_i p_i \cdot \frac{\partial}{\partial p_i}$$

generates the **Complex Rotations**:

$$p_i \to e^{i\alpha} p_i \implies F_{\mathcal{O}} \to e^{i\alpha D} F_{\mathcal{O}} \,.$$

For $\alpha = \pi$ their infinitesimal imaginary part $\epsilon$ changes sign:

$$F_{\mathcal{O}}(\{s_{ij} - i\epsilon\}) = e^{i\pi D} F_{\mathcal{O}}(\{s_{ij} + i\epsilon\}) \,.$$

# Nonperturbative Relations

## $S$&$D$ Relation

$$(e^{-i\pi D} - 1)F_i^* = i(\mathcal{M}F_i^*)$$

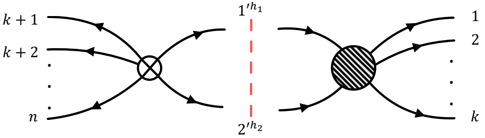
- In dimensional regularization and in absence of masses, $D \simeq -\mu\,\partial/\partial\mu$, which implies

## Callan-Symanzik Equation

$$DF_j = \left(\frac{\partial \beta_i}{\partial c_j} - \delta_{ij}\gamma_{i,\mathrm{IR}} + \delta_{ij}\beta_g \frac{\partial}{\partial g}\right) F_i$$

- Can be combined and expanded, *e.g.* at one-loop

$$\left(\frac{\partial \beta_i^{(1)}}{\partial c_j} - \delta_{ij}\gamma_{i,\mathrm{IR}}^{(1)} + \delta_{ij}\beta_g^{(1)} \frac{\partial}{\partial g}\right) F_i^{(0)} = -\frac{1}{\pi}(\mathcal{M}F_j)^{(1)}$$



# IR Anomalous Dimensions

- In theories with massless fields, IR singularities originate from configurations where loop momenta become soft or collinear.
- The IR anomalous dimension only depends on the external state $\langle \vec{n}|$

$$\gamma_{\mathrm{IR}}^{(1)}(\{s_{ij}\};\mu) = \frac{g^2}{4\pi^2}\sum_{i<j} T_i^a T_j^a \log\frac{\mu^2}{-s_{ij}} + \sum_i \gamma_{i,\mathrm{coll.}}^{(1)} \,.$$

- Since the stress-energy tensor $T_{\mu\nu}$ is UV protected, $\gamma_{\mathrm{IR}}$ can be computed as

$$\gamma_{\mathrm{IR}}^{(1)} F_{T_{\mu\nu}}^{(0)}(\vec{n}) = \frac{1}{\pi}(\mathcal{M}F_{T_{\mu\nu}})^{(1)}(\vec{n}) \,.$$