Precision Lattice Calculations

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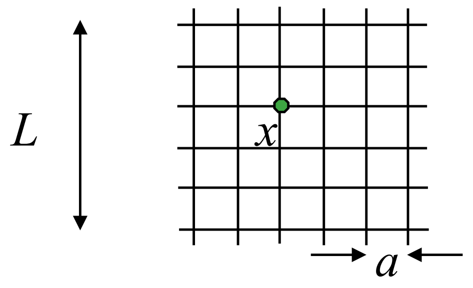
Lecture 2: Symanzik EFT — an example

Outline

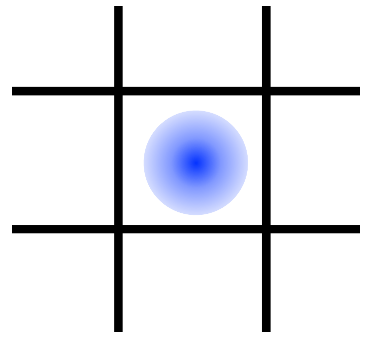
- Motivation: Open Questions
- The role of (lattice) QCD in flavor physics
  - History
  - Challenges
- Examples
  - Leptonic kaon, pion decay
  - Semileptonic kaon decay
  - First row CKM unitarity
  - Semileptonic D-meson decay
  - Leptonic B-meson decay

  - muon g-2 ➠ Lecture 3



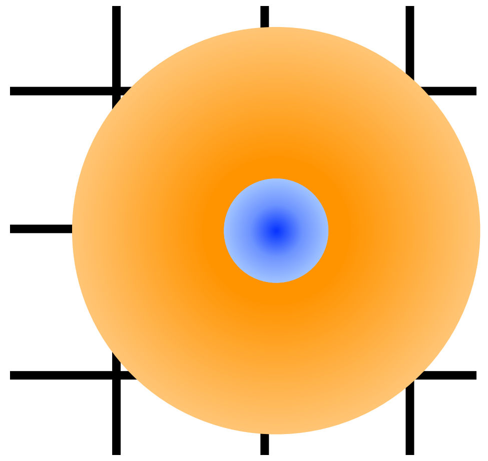
Lattice QCD Introduction: heavy quarks

b quark



$m_b \gtrsim a^{-1} \gg \Lambda$ ➠ leading discretization errors $\sim (am_b)^2$
(using same action as for light quarks)

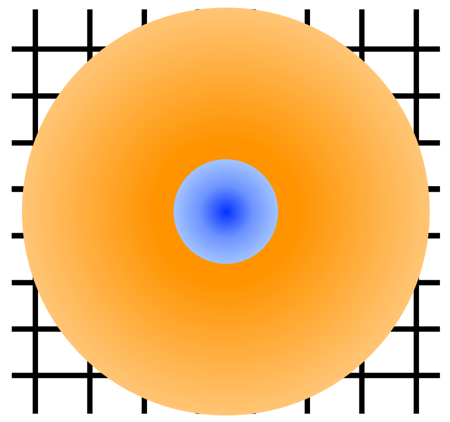
B meson



use EFT (HQET, NRQCD) ➠ Λ/m_b expansion

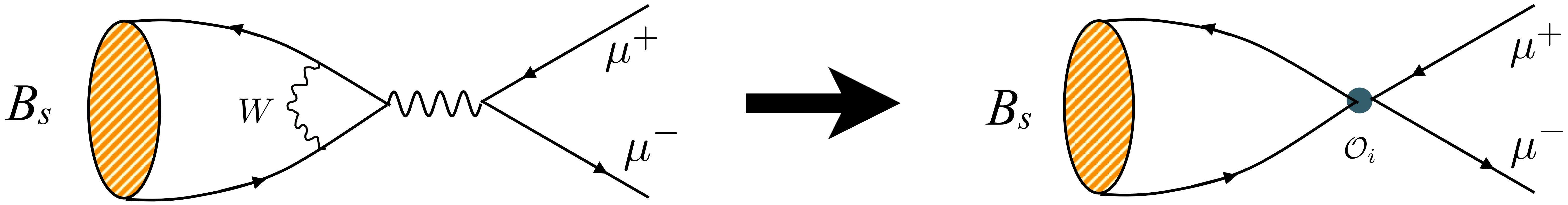
EFTs co-developed continuum/lattice

- lattice HQET, NRQCD: use EFT to construct lattice action
  complicated continuum limit
  nontrivial matching and renormalization ➠ (few-5)% errors
- relativistic heavy quarks: Fermilab (1996), also Tsukuba (2003), RHQ (2006)
  matching relativistic lattice action via HQET to continuum
  nontrivial matching and renormalization ➠ (1-3)% errors



$a^{-1} > m_b \gg \Lambda$ + highly improved light quark action
➠ same action for all quarks
➠ simple renormalization (Ward identities) ➠ < 1% errors

Rare leptonic decay $B_s \to \mu^+\mu^-$

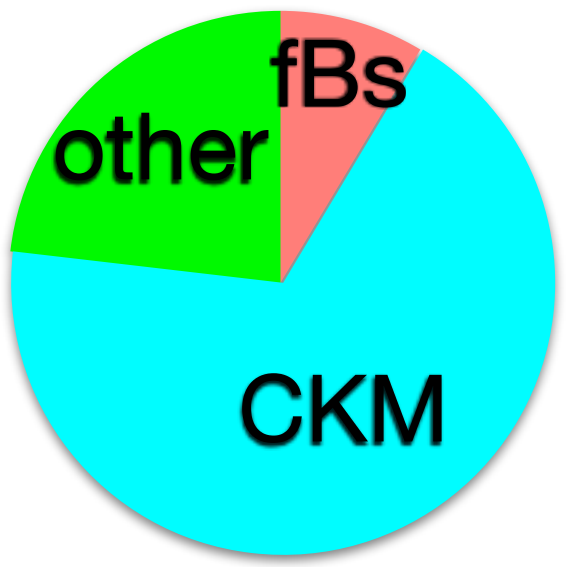


Standard Model predictions: Buras, et al [arXiv:1303.3820, JHEP 2013], Bobeth, et al [arXiv:1311.0903, PRL 2014; arXiv:2104.09521], Beneke et al [arXiv:1908.07011, JHEP 2019].

$$\overline{\mathcal{B}}(B_q \to \mu\bar{\mu}) = \frac{|\mathcal{N}_q|^2\, M_{B_q}^3\, f_{B_q}^2}{8\pi\, \Gamma_q^H} \beta_{q\mu} \left(\frac{m_\mu}{M_{B_q}}\right)^2 \left|C_{10}^{\rm eff}\right|^2, \qquad \beta_{q\mu} \equiv \sqrt{1 - \frac{4\, m_\mu^2}{M_{B_q}^2}},$$

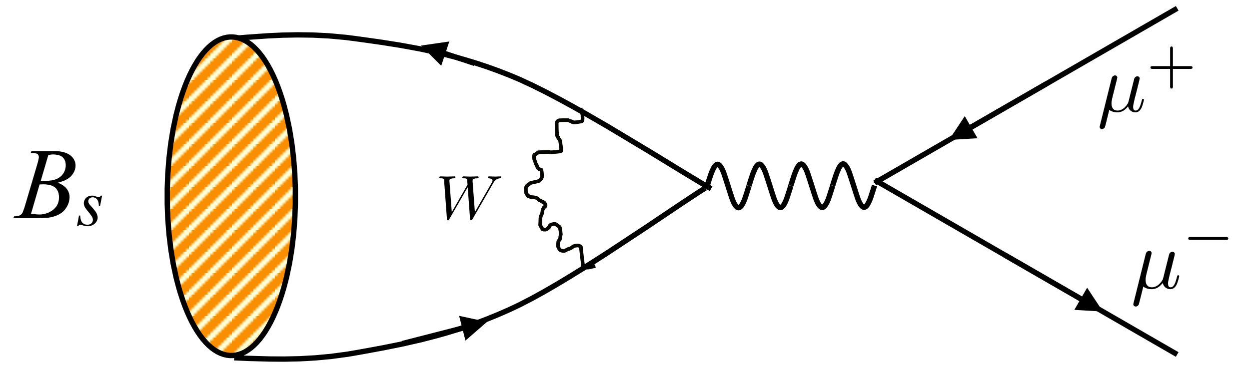
$$\overline{\mathcal{B}}(B_s \to \mu\bar{\mu})_{\rm SM} = (3.66 \pm 0.14)\cdot 10^{-9}$$

error^2



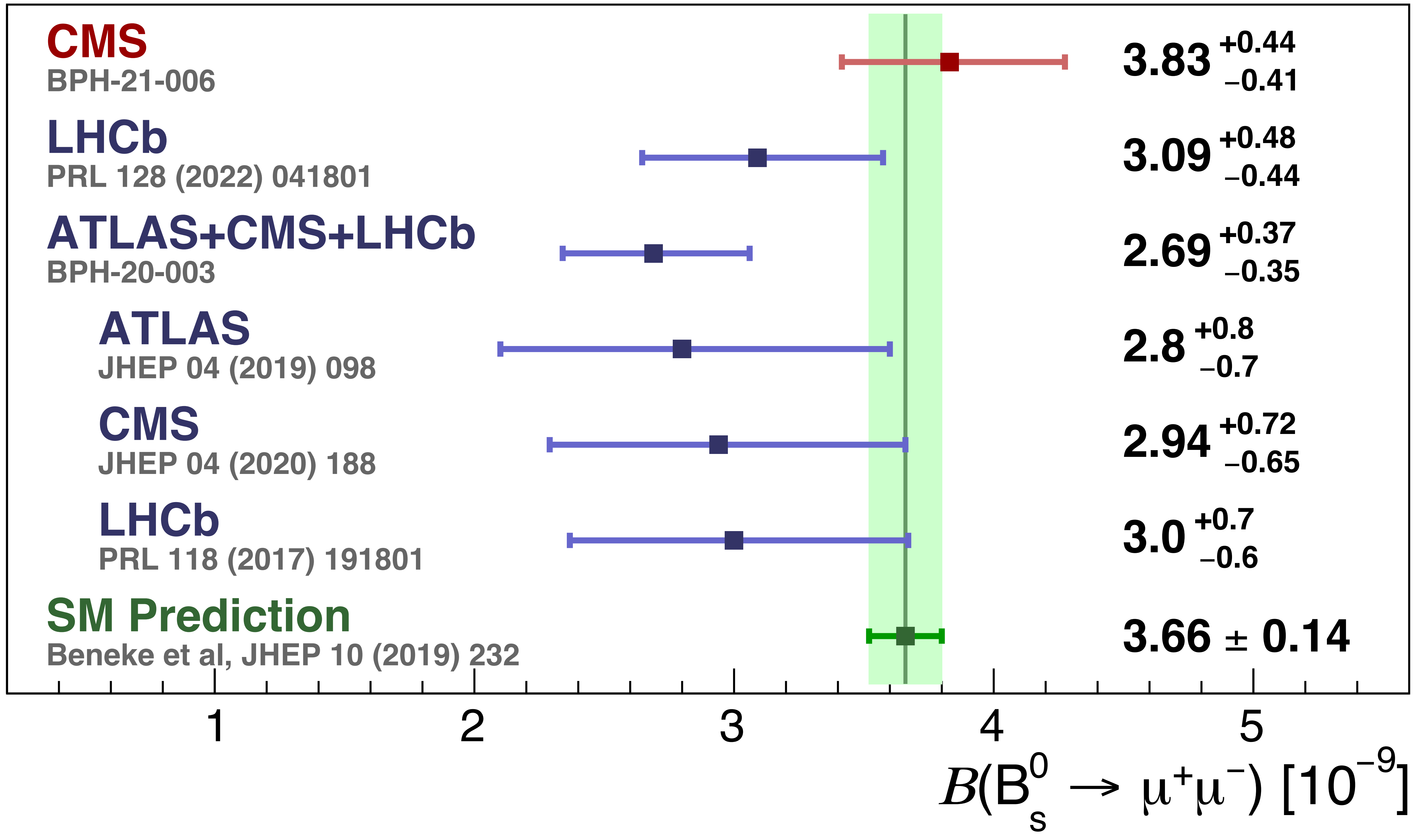
- includes structure-dependent QED corrections
- dominant uncertainty due to $|V_{cb}|$
- LQCD decay constant sub dominant source of uncertainty

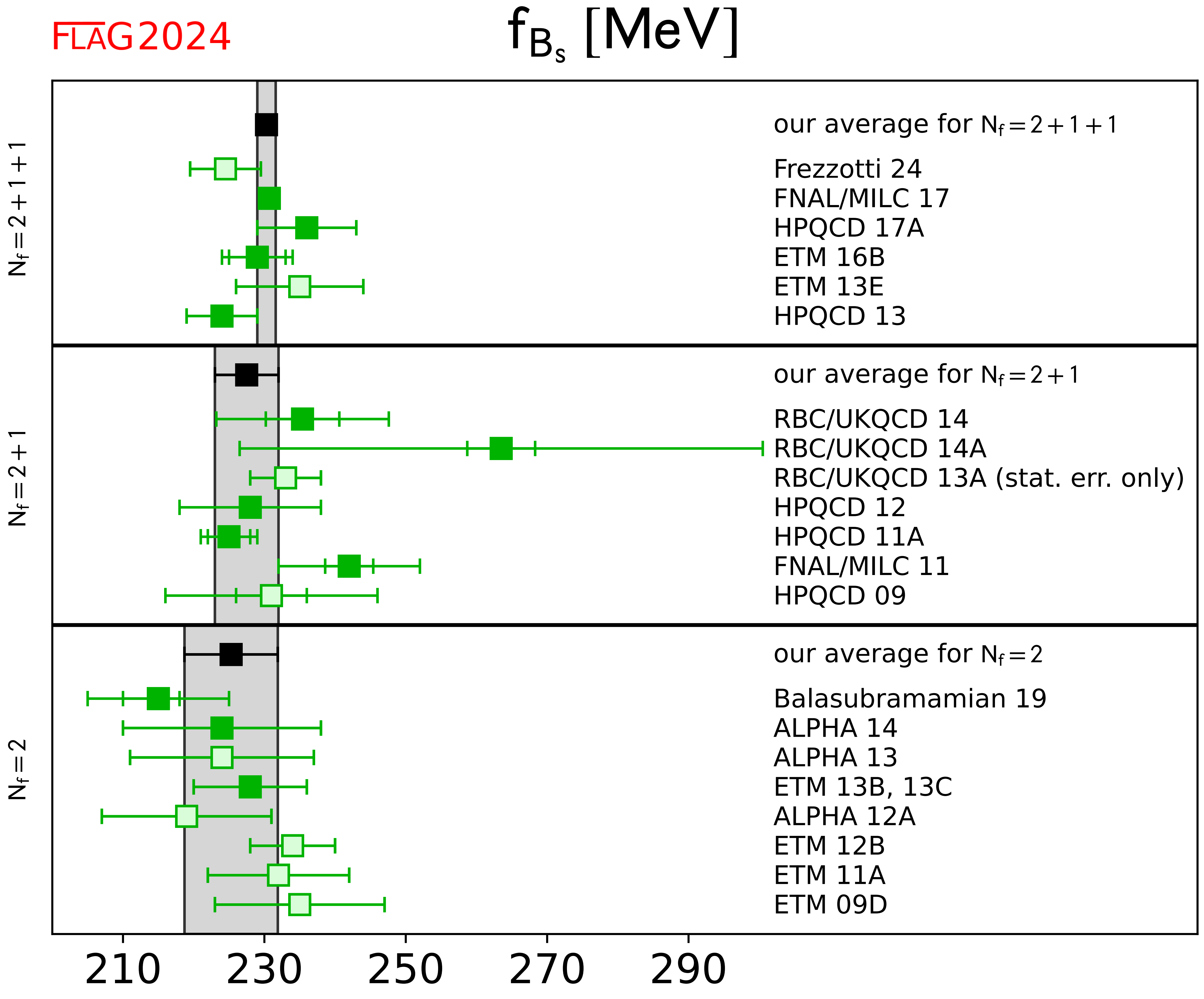
Rare leptonic decay $B_s \to \mu\mu$



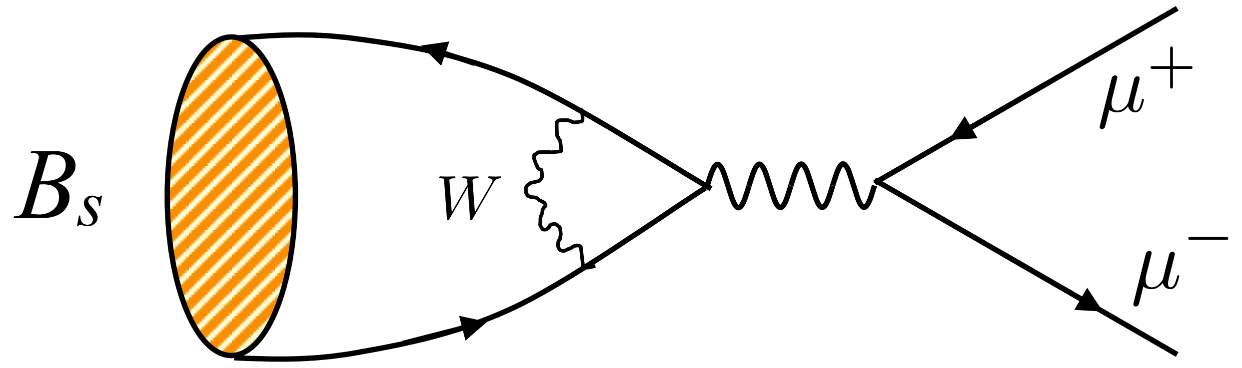
S. Aoki et al [FLAG 2024 review, arXiv:2411.04268]

[CMS, Phys Lett B 2023]





Rare leptonic decay $B_s \to \mu\mu$



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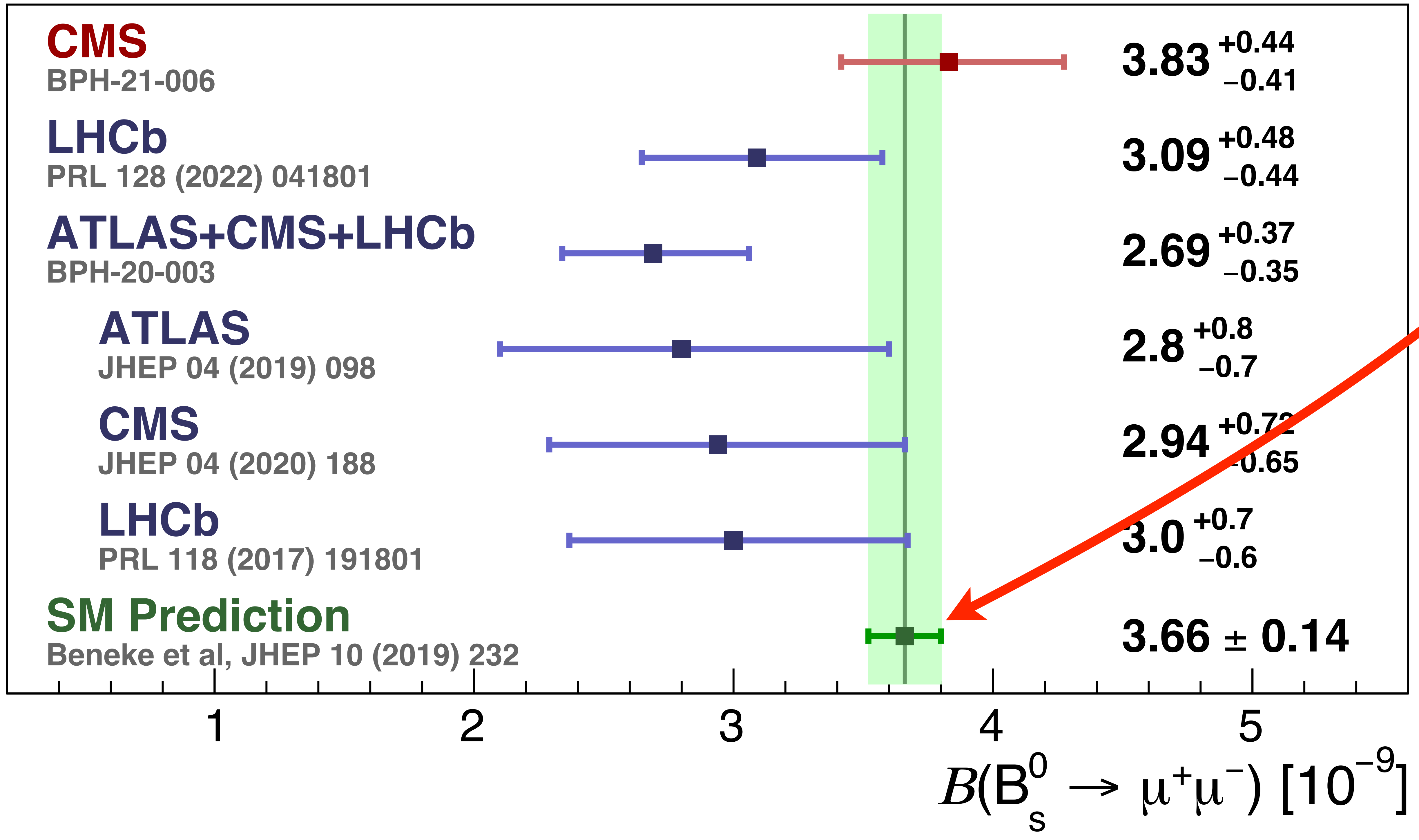

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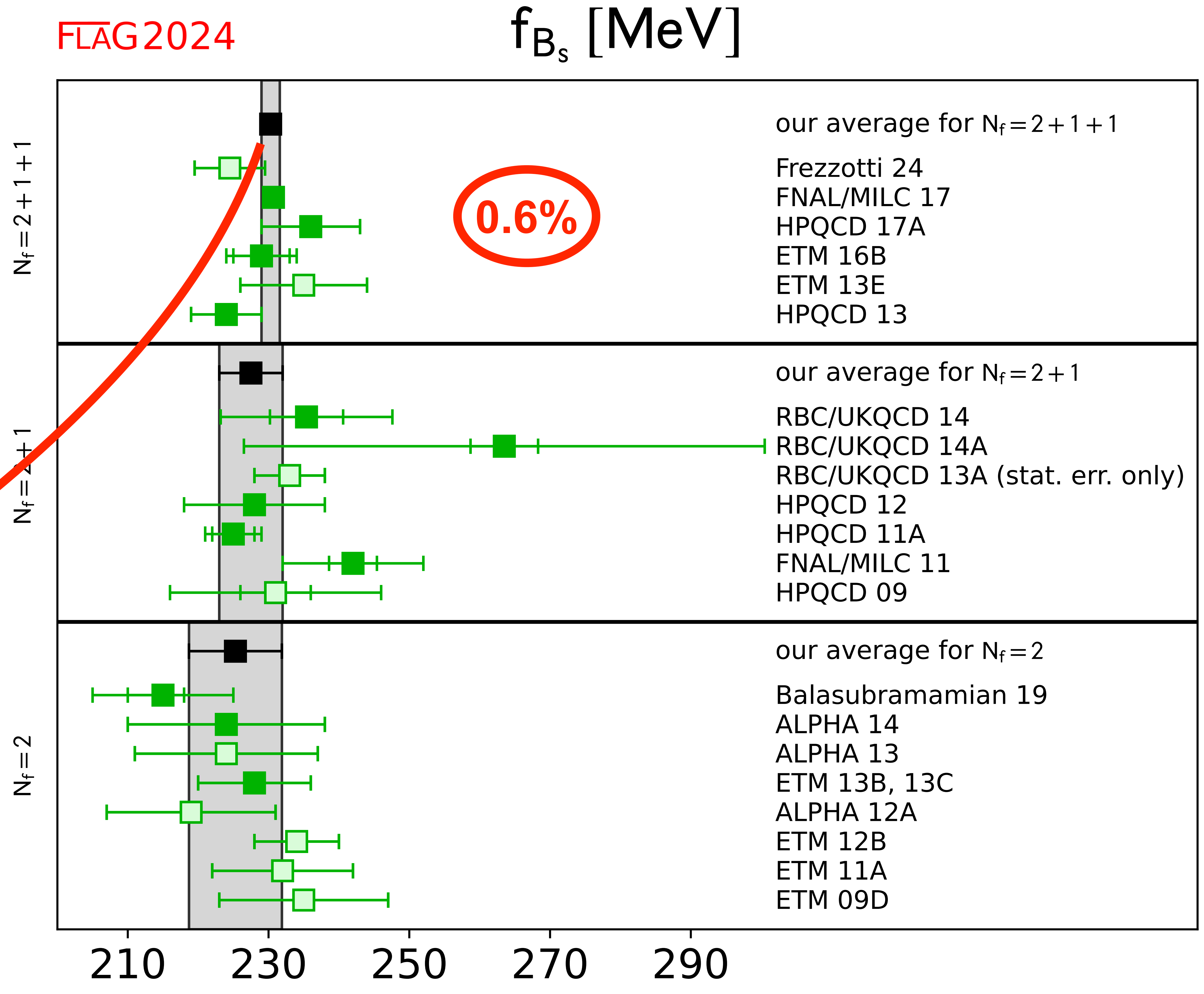


Heavy Quark discretization errors for HISQ fermions

Follana et al [HPQCD, hep-lat/0610092, 2007 PRD]
Monahan et al [arXiv:1211.6966, 2013 PRD]
Bazavov et al [FNAL/MILC, arXiv:1712.09262, 2018 PRD]

Tree-level HISQ action:

$$S = \sum_x \overline{\psi}(x) \left\{ \sum_\mu \gamma_\mu \left[a\Delta_\mu - \frac{\mathbb{N}}{6} a^3 \Delta_\mu^3 \right] + am_0 \right\} \psi(x)$$

$$a\Delta_\mu \psi(x) = \tfrac{1}{2}[\psi(x+\hat{\mu}a) - \psi(x-\hat{\mu}a)],$$

Naik term:

$$m_1 \equiv E(0) \qquad \frac{m_1}{m_2} = \lim_{\boldsymbol{p}\to \boldsymbol{0}} \frac{E^2(\boldsymbol{p}) - E(0)^2}{\boldsymbol{p}^2} = 1 \quad \Rightarrow \quad am_0 = \sinh(am_1)\,\frac{1+\sqrt{1+3X}}{3}$$

$$\Rightarrow \quad \mathbb{N}(am_1) = \frac{4 - 2\sqrt{1+3X(am_1)}}{\sinh^2(am_1)} \qquad X(am_1) = \frac{2am_1}{\sinh(2am_1)}$$

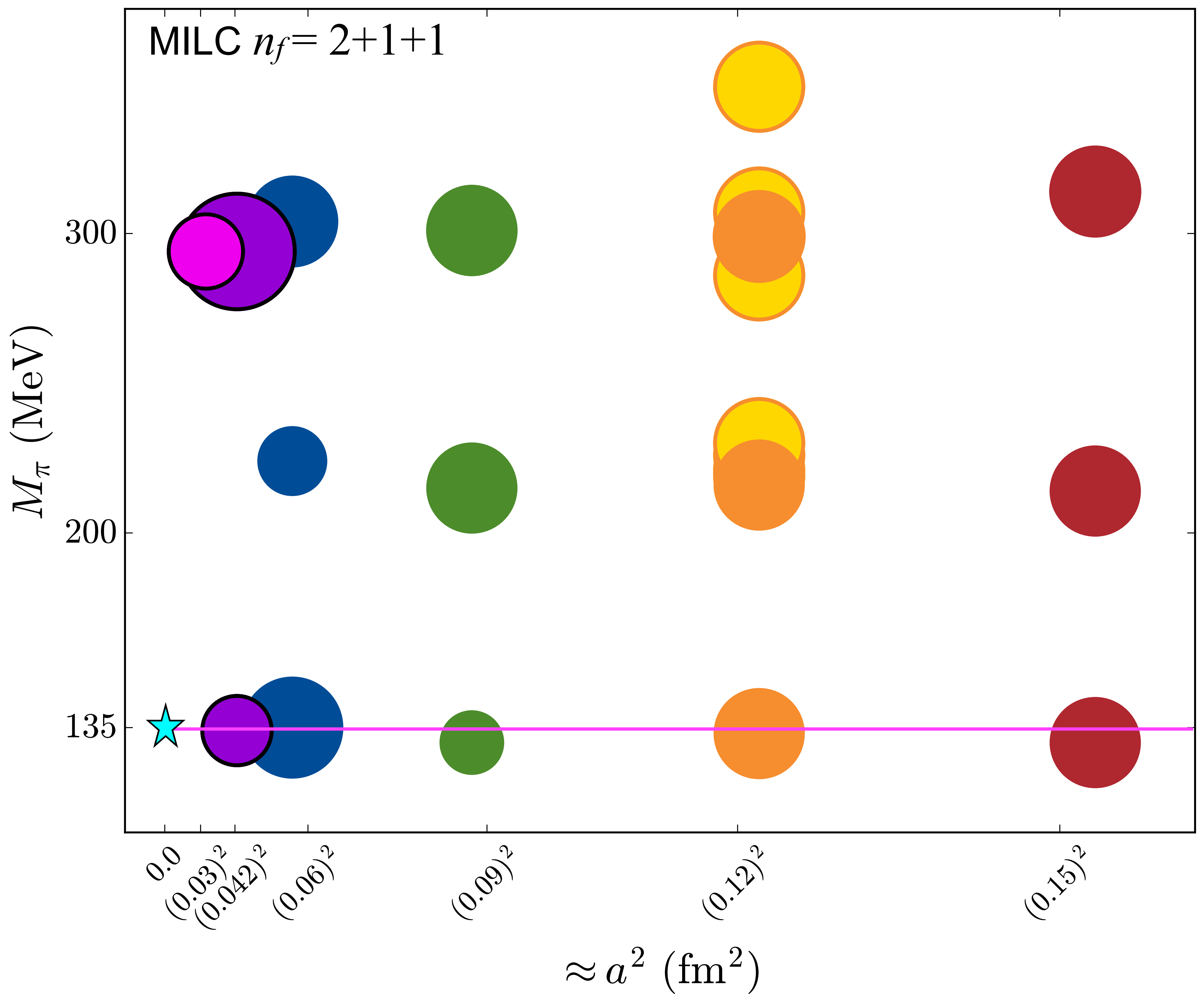
Normalization of heavy-light bilinears:

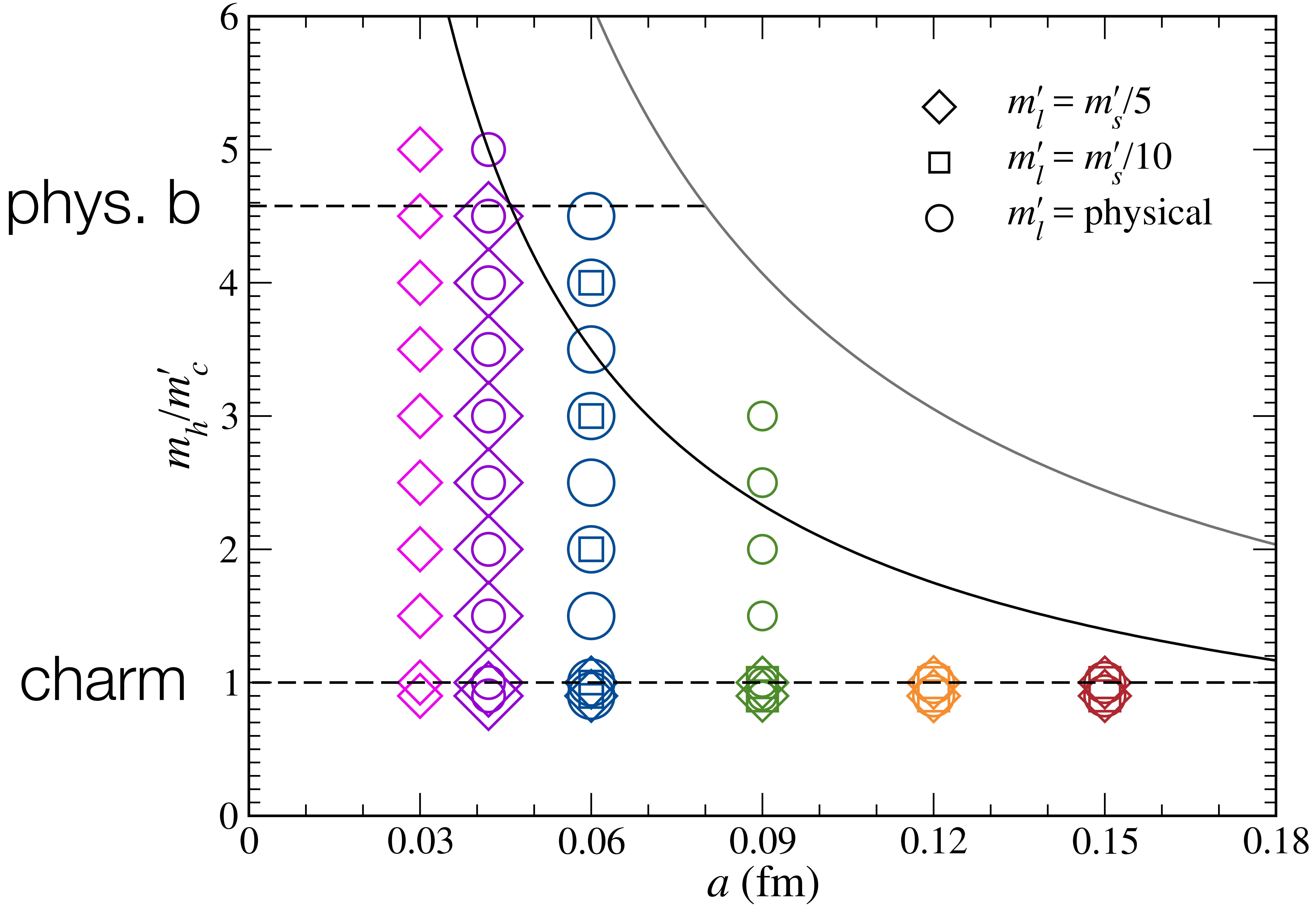
$$Z_{J_{hx}} = \widetilde{\mathcal{C}h}_h^{1/2} \qquad \widetilde{\mathcal{C}h} = \cosh am_1 \left(1 - \tfrac{1}{2}\mathbb{N}\sinh^2 am_1\right)$$

remaining discretization errors:
$\sim (am_h)^4, \alpha_s(am_h)^2$

B, D meson decay constant results

A. Bazavov et al [FNAL/MILC, arXiv:1712.09262, 2018 PRD]





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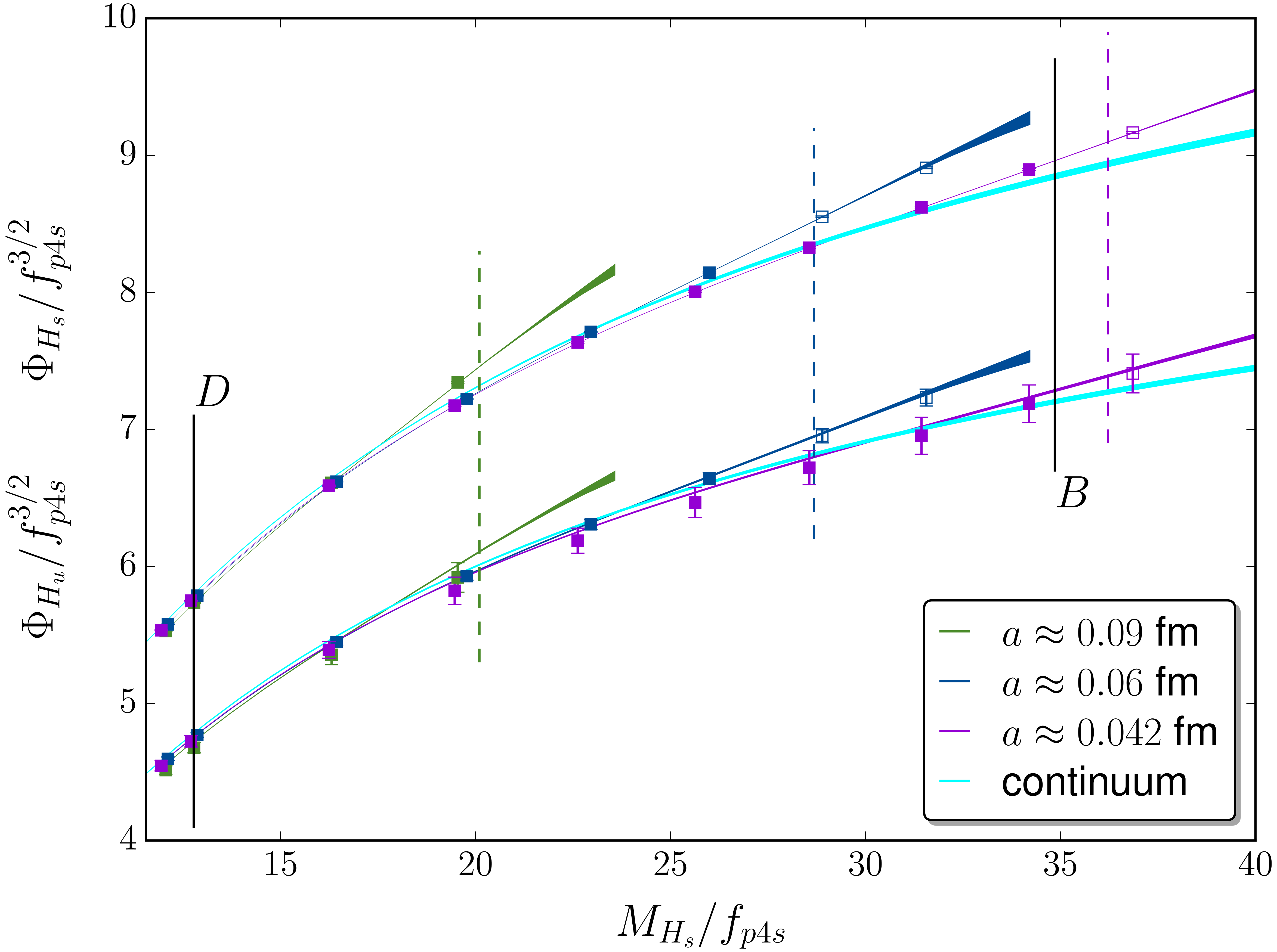

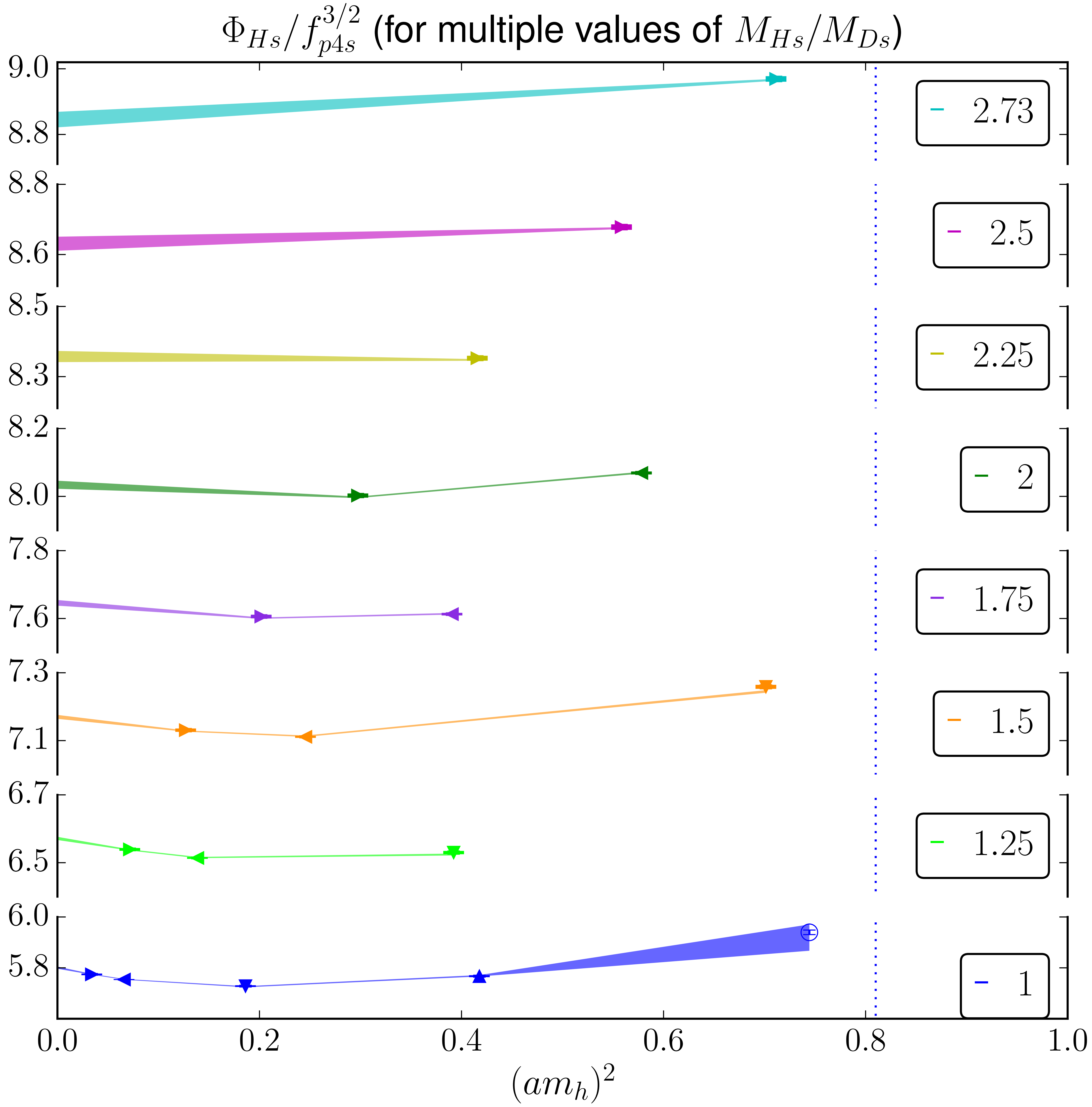

- small errors due to **physical light quark masses**
- improved quark action with small discretization errors
- no renormalization (Ward identity)