Spectroscopy and scattering: applications (day 3)

Jeremy R. Green

Deutsches Elektronen-Synchrotron DESY

Frontiers and Challenges in Lattice Gauge Theory
MITP Summer School 2025
July 21 to August 1, 2025

Chalkboard notes

Two-point correlation functions

- spectral decomposition
- variance
- variational method

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- single-hadron interpolators
- smearing
- two-hadron interpolators

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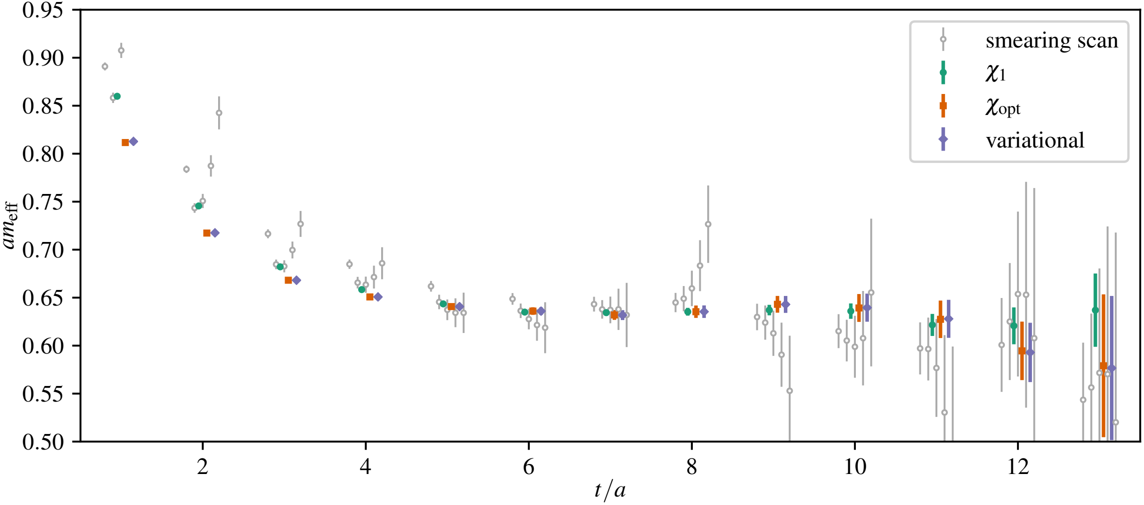
Computing correlation functions

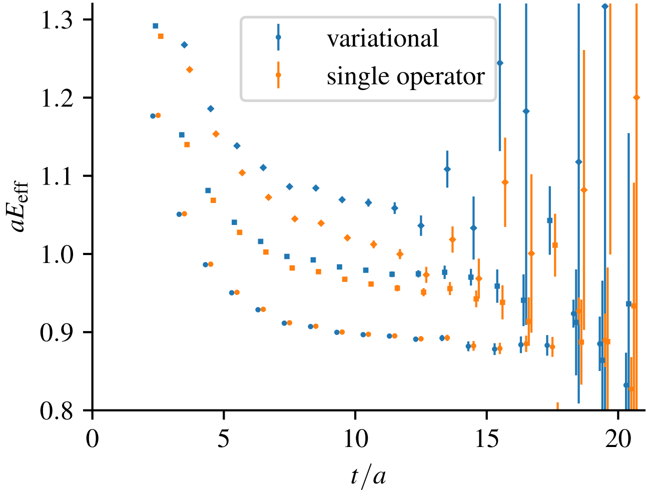
- example Wick contraction
- distillation

Dependence on smearing width



JRG *et al.*, Phys. Rev. D **100**, 074510 (2019) [1907.11950]

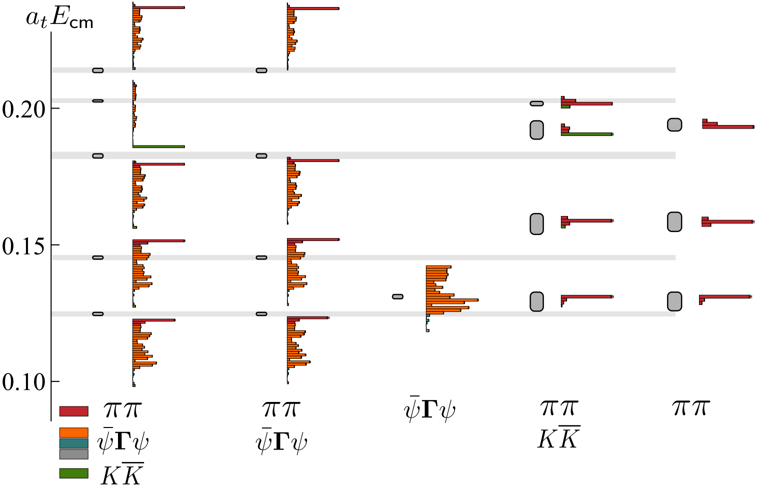
Importance of variational method



Variational approach essential for excited states.

Single operators can also fail to obtain ground state.

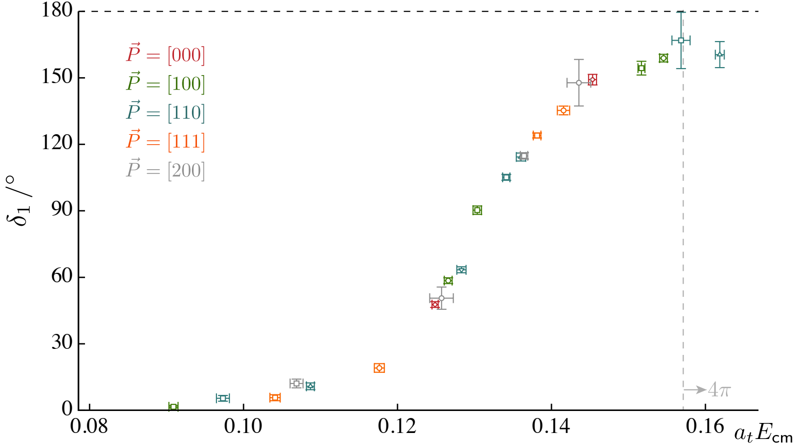
Effect of operator choice



quantum numbers of $\rho(770)$
$\boldsymbol{P} = (0, 0, 0)\ T_1^-$

D. J. Wilson *et al.* (Hadron Spectrum Collaboration),
Phys. Rev. D **92**, 094502 (2015) [1507.02599]

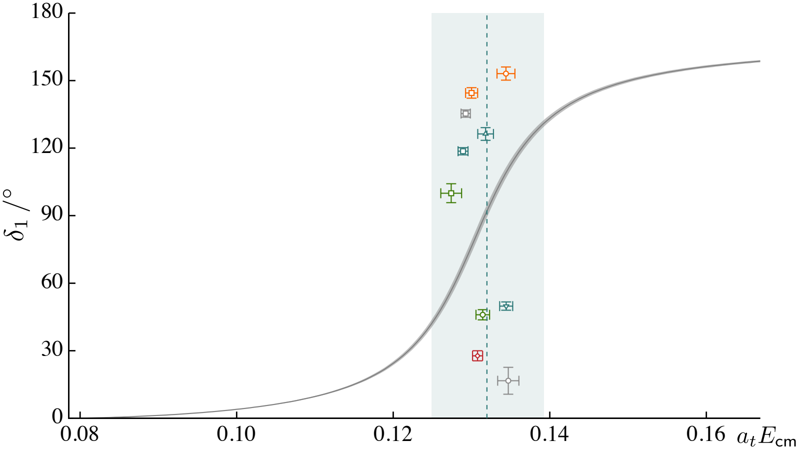
Effect of operator choice



$\pi\pi$ p-wave phase shift
Full set of operators.

D. J. Wilson *et al.* (Hadron Spectrum Collaboration), Phys. Rev. D **92**, 094502 (2015) [1507.02599]

Effect of operator choice

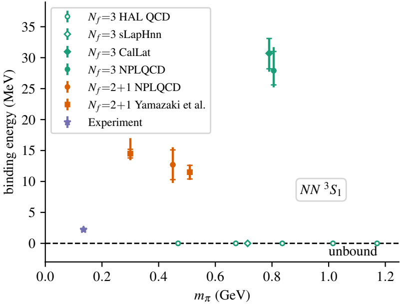


$\pi\pi$ p-wave phase shift
Only local operators.

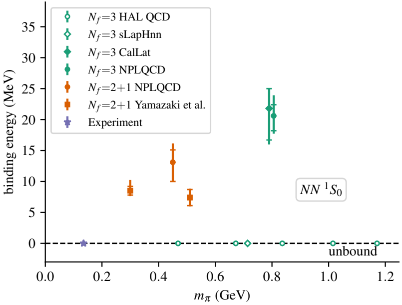
D. J. Wilson *et al.* (Hadron Spectrum Collaboration),
Phys. Rev. D **92**, 094502 (2015) [1507.02599]

Nucleon-nucleon scattering from LQCD: past calculations

Decade-long controversy over presence of NN bound states at heavy quark masses.



deuteron



dineutron

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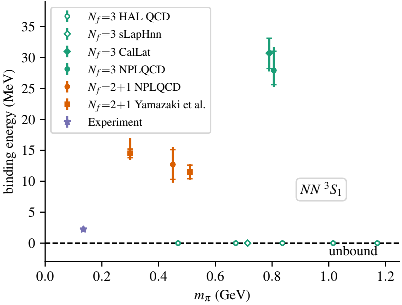

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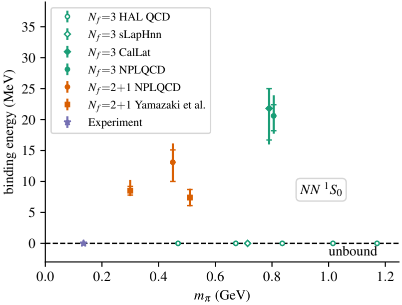

dineutron

Disagreement about simplest warm-up problem for nuclear physics on the lattice.

Experiment: $B_d \approx 2.2$ MeV known for 90 years. J. Chadwick and M. Goldhaber, Nature **134**, 237–238 (1934)

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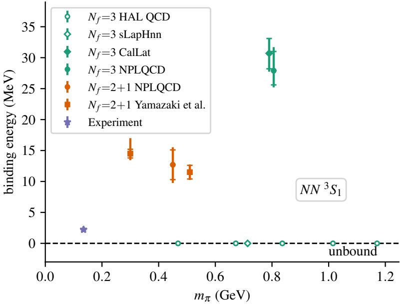

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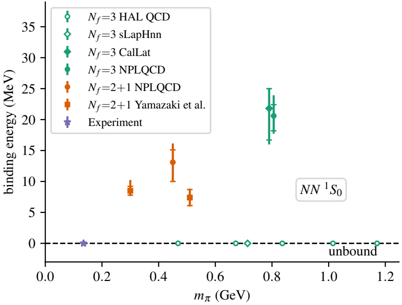

dineutron

No calculation performed using more than one lattice spacing.

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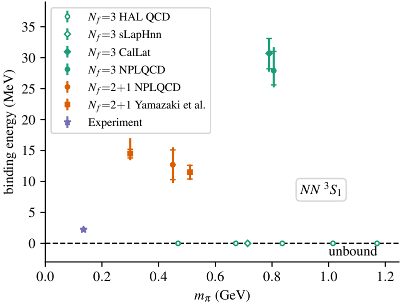

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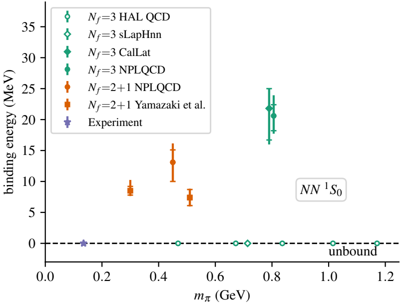


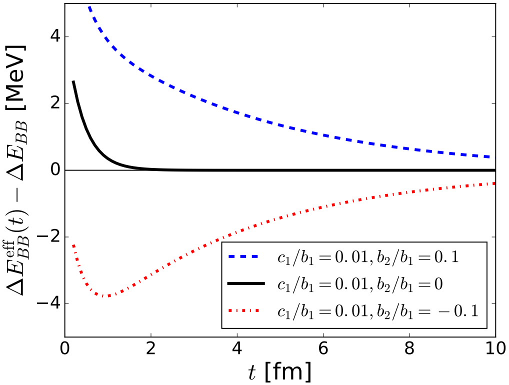
dineutron

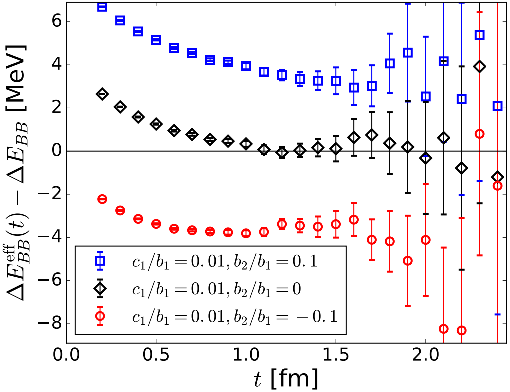
No calculation performed using more than one lattice spacing.

All calculations that obtain bound states use $\langle O_{BB}(t) O_H^\dagger(0)\rangle$ asymmetric correlation functions.

What can go wrong?

T. Iritani *et al.* (HAL QCD), Mirage in temporal correlation functions for baryon-baryon interactions in lattice QCD, JHEP **2016**, 101 (2016) [1607.06371] (CC BY 4.0)

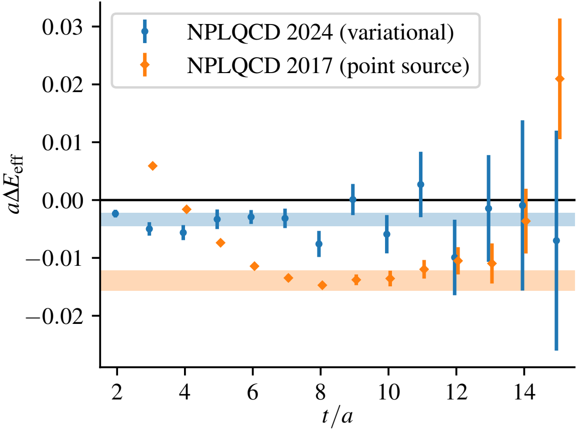




Mock data: effective mass for correlator $C(t) = b_1 + b_2 e^{-\delta E_{\rm el} t} + c_1 e^{-\delta E_{\rm inel} t}$.
"elastic" excitation $\delta E_{\rm el} = 50$ MeV
"inelastic" excitation $\delta E_{\rm inel} = 500$ MeV

Point sources versus variational method with bilocal interpolators

Old and new methods used on same ensemble.



Data extracted from

W. Detmold *et al.* (NPLQCD), PRD 111, 114501 (2025) [2404.12039]
M. L. Wagman *et al.* (NPLQCD), PRD 96, 114510 (2017) [1706.06550]

$m_\pi \approx 800$ MeV.

Old calculation:

$^1S_0$ bound state with $B_{nn} \approx 21$ MeV.

New calculation consistent with unbound NN.

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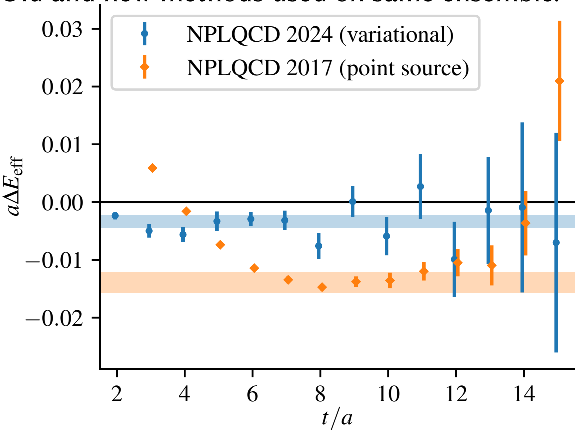

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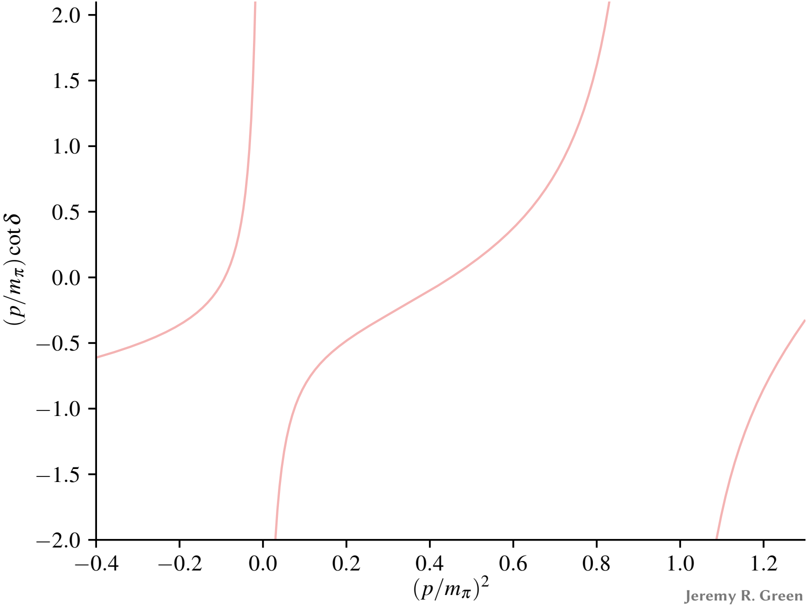
Several variational baryon-baryon calculations done:

A. Francis, JRG *et al.*, PRD 99, 074505 (2019) [1805.03966]
B. Hörz *et al.* (sLapHnn), PRC 103, 014003 (2021) [2009.11825]
JRG *et al.*, PRL 127, 242003 (2021) [2103.01054]
S. Amarasinghe *et al.* (NPLQCD), PRD 107, 094508 (2023) [2108.10835]
W. Detmold *et al.* (NPLQCD), 2404.12039
Z.-Y. Wang @ Lattice 2024 Y. Geng (CLQCD) @ Lattice 2024

Largely consistent picture:
no NN bound state at heavy m_π.

Finite-volume quantization (spin zero s-wave)



Truncated to s wave:

$$p\cot\delta_0(p) = \frac{2}{\sqrt{\pi}L\gamma} Z_{00}^{PL/(2\pi)}\left(1, \left(\frac{pL}{2\pi}\right)^2\right),$$

where $p^2 = (E_{\mathrm{cm}}/2)^2 - m^2$.

baryon-baryon
$\boldsymbol{P} = (0,0,0)\ A_1^+$

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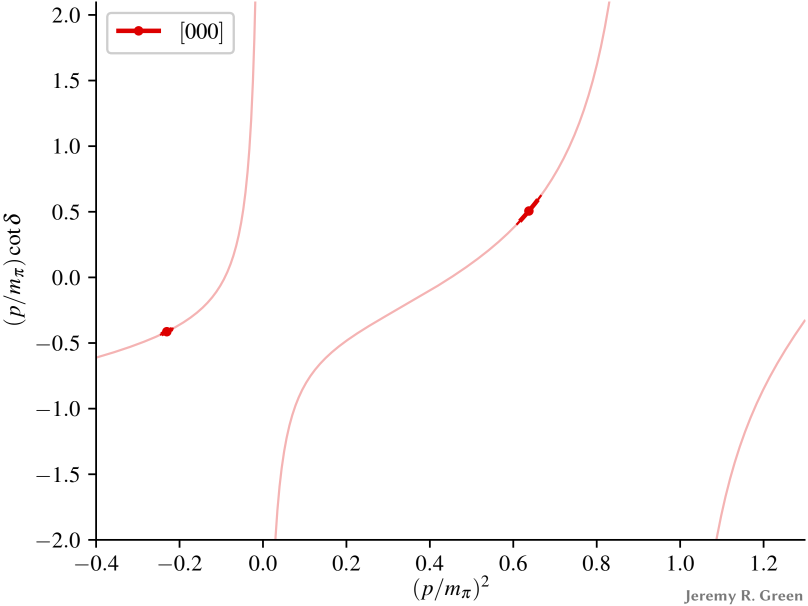

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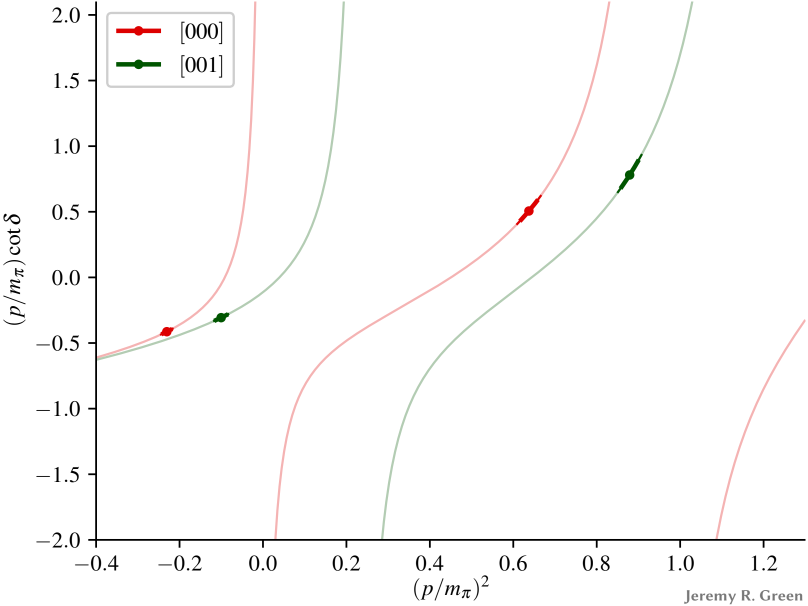

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$\boldsymbol{P} = (0,0,0)\ A_1^+$,

$\boldsymbol{P} = \frac{2\pi}{L}(0,0,1)\ A_1$,

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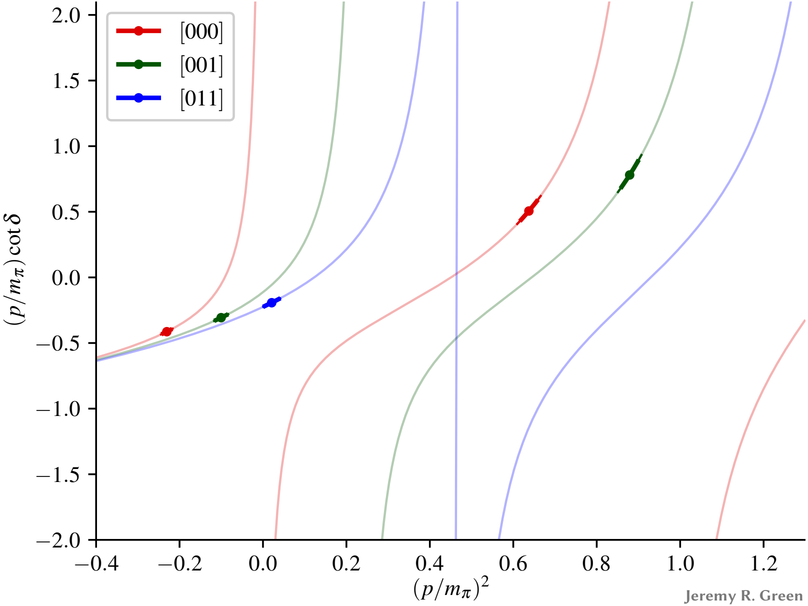

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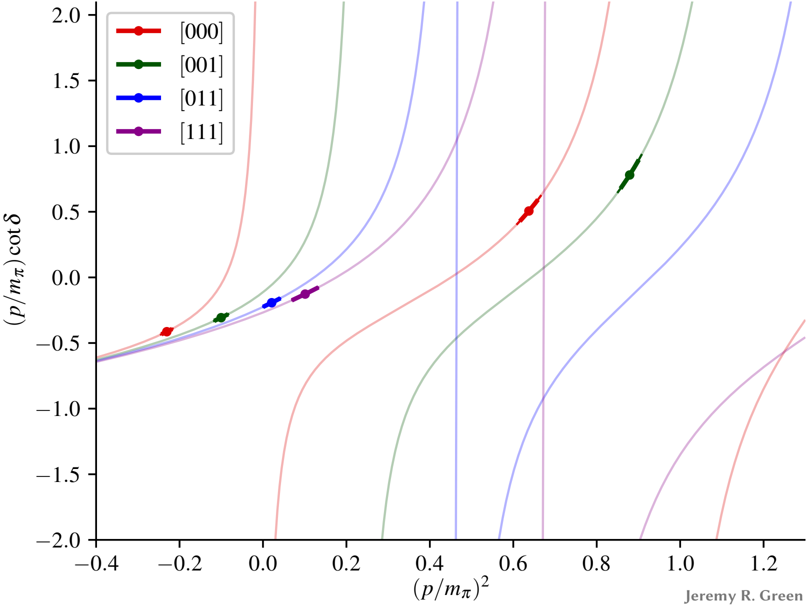

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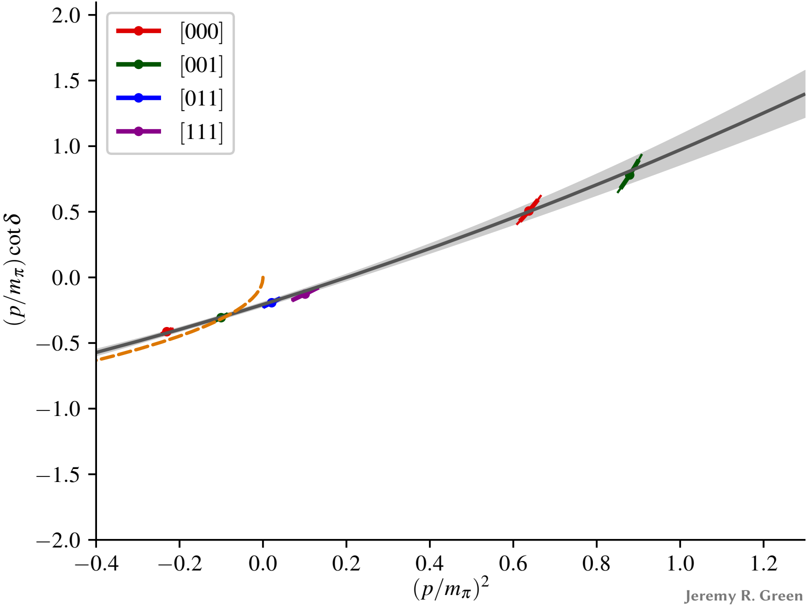
Phase shift and bound state (s-wave)

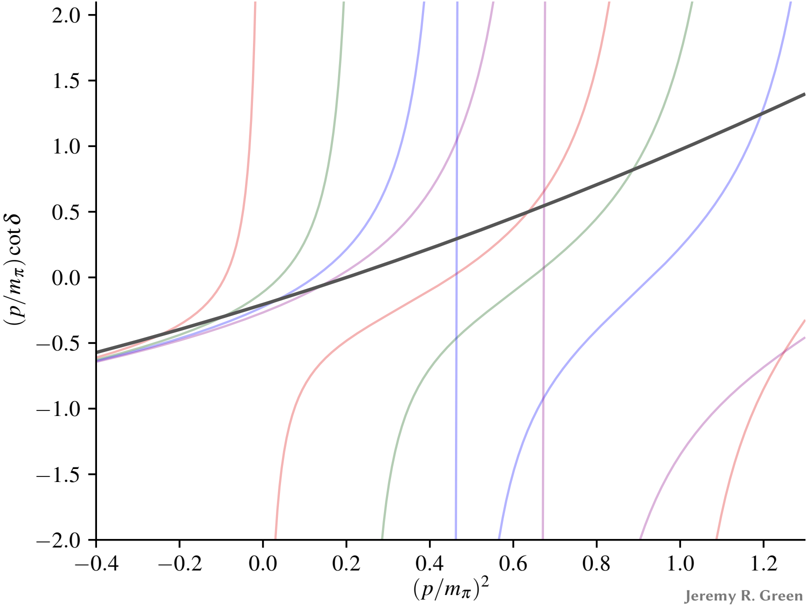


Tricky defining χ^2 to measure fit quality: both p^2 and $\delta_0(p^2)$ have errors and they are strongly correlated.

bound state:

$$p \cot \delta_0(p) = -\sqrt{-p^2}$$

Spectrum method



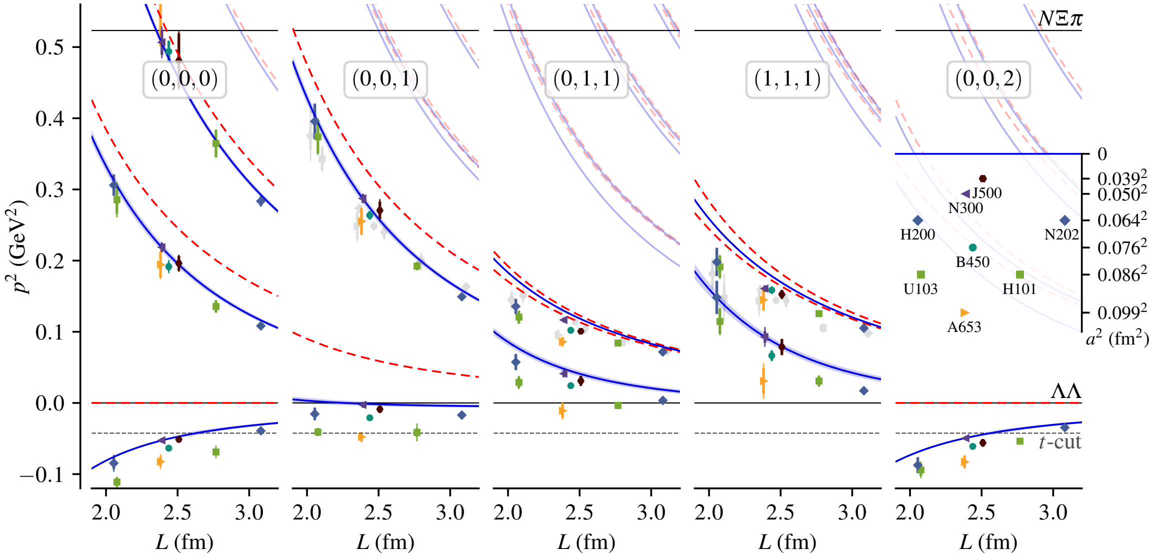
Robust approach: find solutions to quantization condition.

$$p\cot\delta_0(p^2) = f(p^2,\{c_i\}) \longrightarrow E_n(\{c_i\})$$

Then fit the lattice energies.

Also works for multiple partial waves and coupled channels.

Spectrum: *H* dibaryon quantum numbers



JRG *et al.*, Phys. Rev. Lett. **127**, 242003 (2021) [2103.01054]

Summary

How to study two-hadron system:

1. Choose the desired infinite-volume quantum numbers.
2. Work out the relevant irreps in rest frame and moving frames.
3. Construct local interpolators in those irreps.
4. Construct towers of bilocal interpolators in those irreps.
5. Compute the correlator matrix $C_{ij}(t) \equiv \langle \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)\rangle$ using distillation or some other method.
6. Use the GEVP to find the lowest few energy levels in each irrep.
7. Build parameterized models for the relevant scattering amplitudes.
8. Solve quantization conditions to find the finite-volume spectra corresponding to the models.
9. Tune the model parameters to fit the lattice energies.

References

- “Angular momentum on the lattice: The case of nonzero linear momentum”
  D. C. Moore and G. T. Fleming, Phys. Rev. D **73**, 014504 (2006) [Erratum: Phys. Rev. D **74**, 079905 (2006)], arXiv:hep-lat/0507018.
- “Extended hadron and two-hadron operators of definite momentum for spectrum calculations in lattice QCD”
  C. Morningstar *et al.*, Phys. Rev. D **88**, 014511 (2013), arXiv:1303.6816.
- “Novel quark-field creation operator construction for hadronic physics in lattice QCD”
  M. Peardon *et al.* (Hadron Spectrum Collaboration), Phys. Rev. D **80**, 054506 (2009), arXiv:0905.2160.