Probing QED and nuclear physics with heavy muonic atoms

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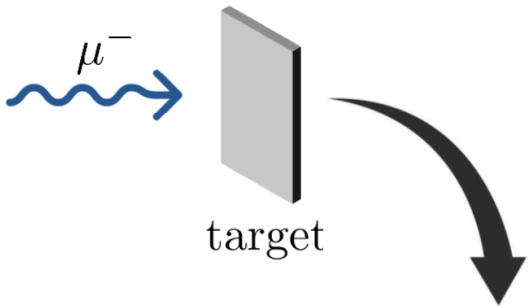
Precision Determination of the Fine-structure Constant
Mainz, October 29, 2025

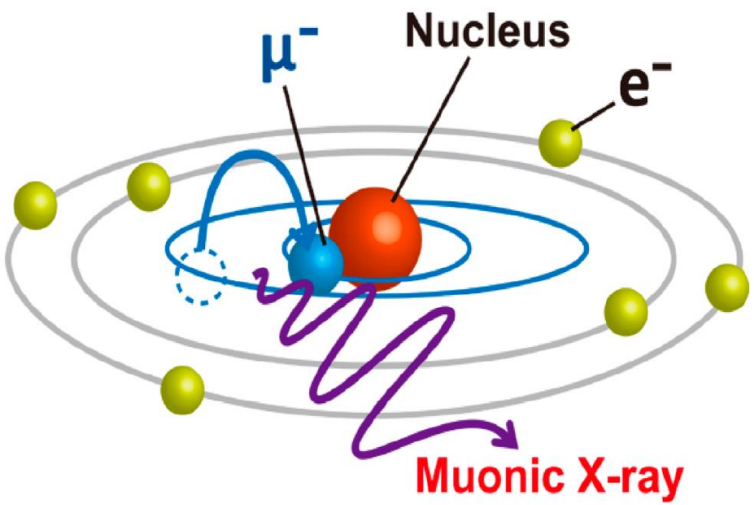
Outline

"Live fast, die young!"

https://www.particlezoo.net

Access to muonic atoms





http://www.mdpi.com/2412-382-X/1/1/11/htm

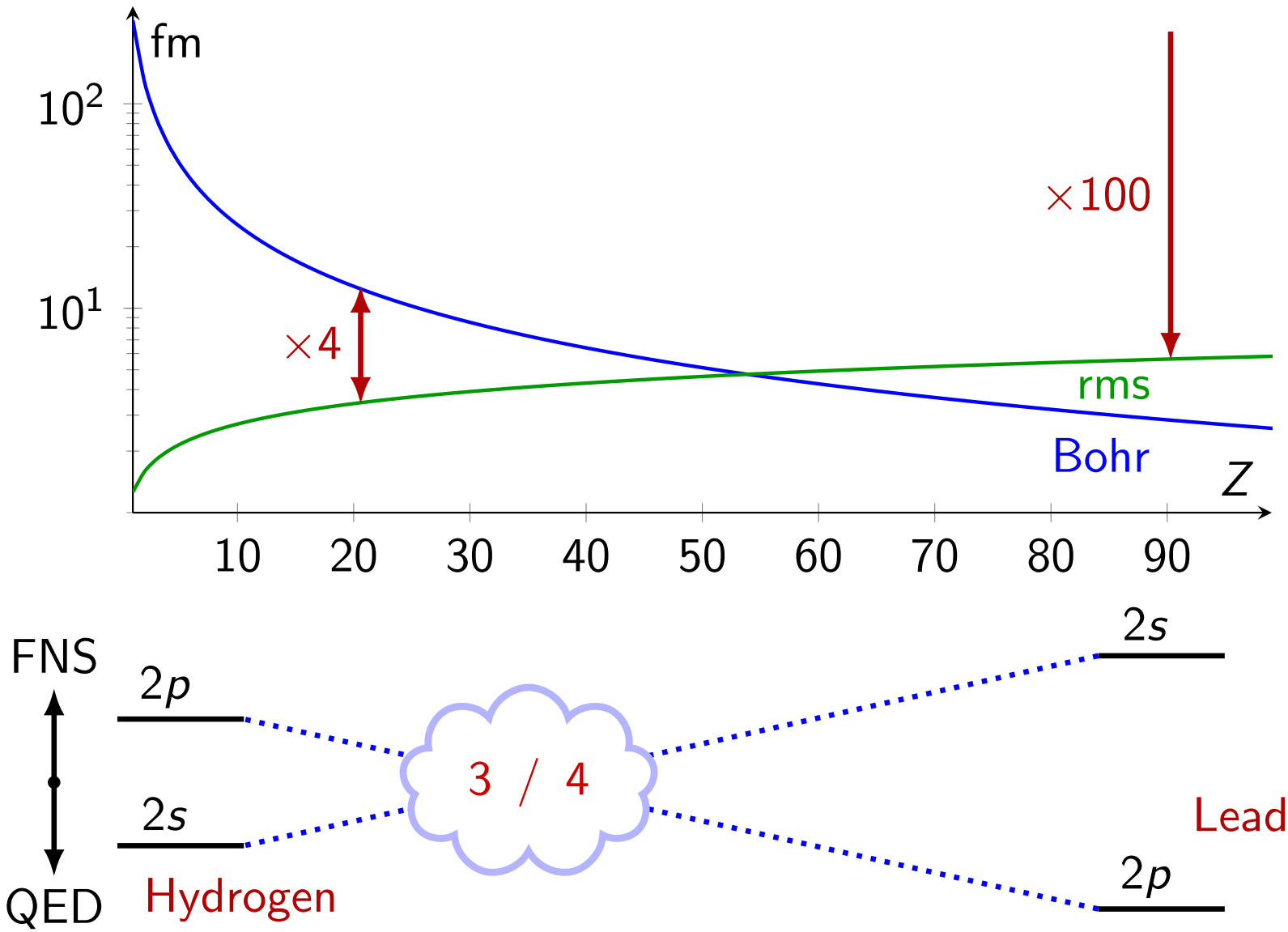
- "heavy electron"
- capture and cascade: $10^{-12} - 10^{-9}$ s
- lifetime: $0.1 - 2.2$ μs
- decay channels

$$\mu^- \to e^- + \overline{\nu}_e + \nu_\mu$$

$$\mu^- + p \to n + \nu_\mu$$

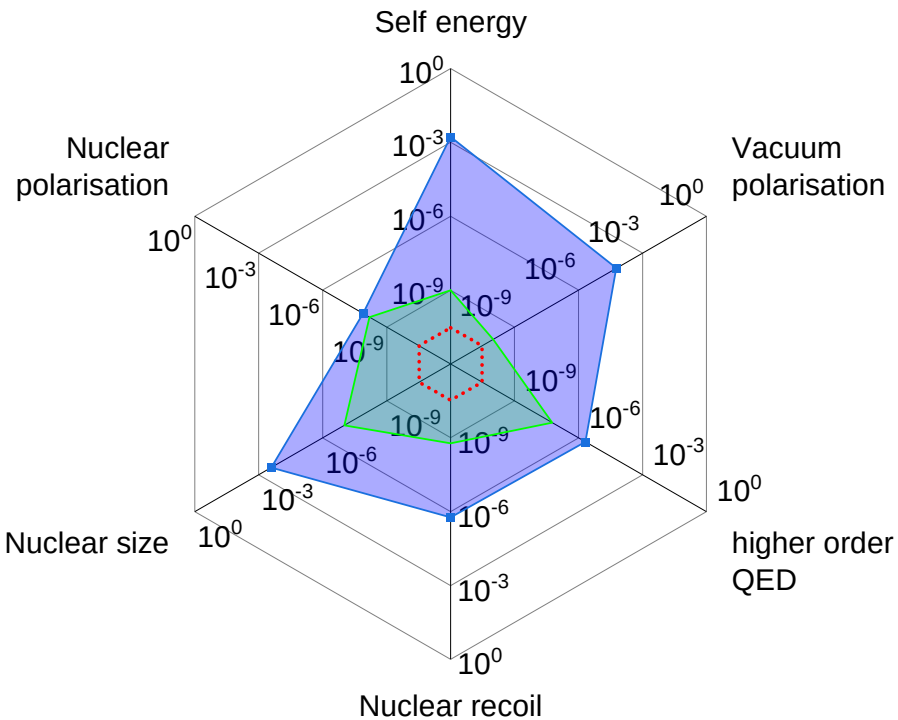
- some electrons far away
- always H-like
- passive spectroscopy

How different are muonic atoms compared to electronic?

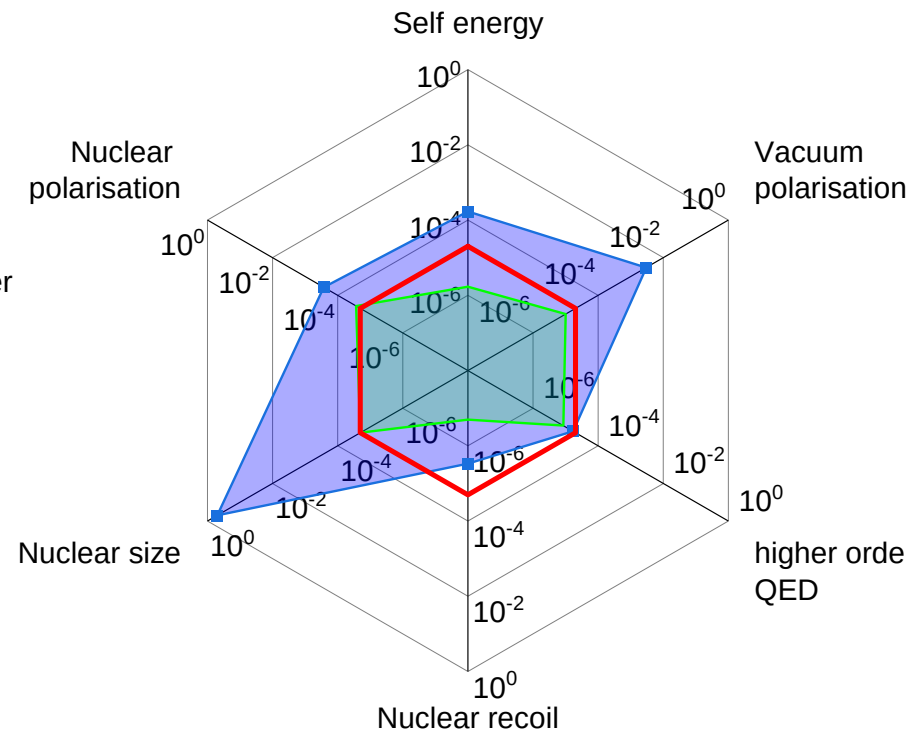


QED and nuclear effects' scaling

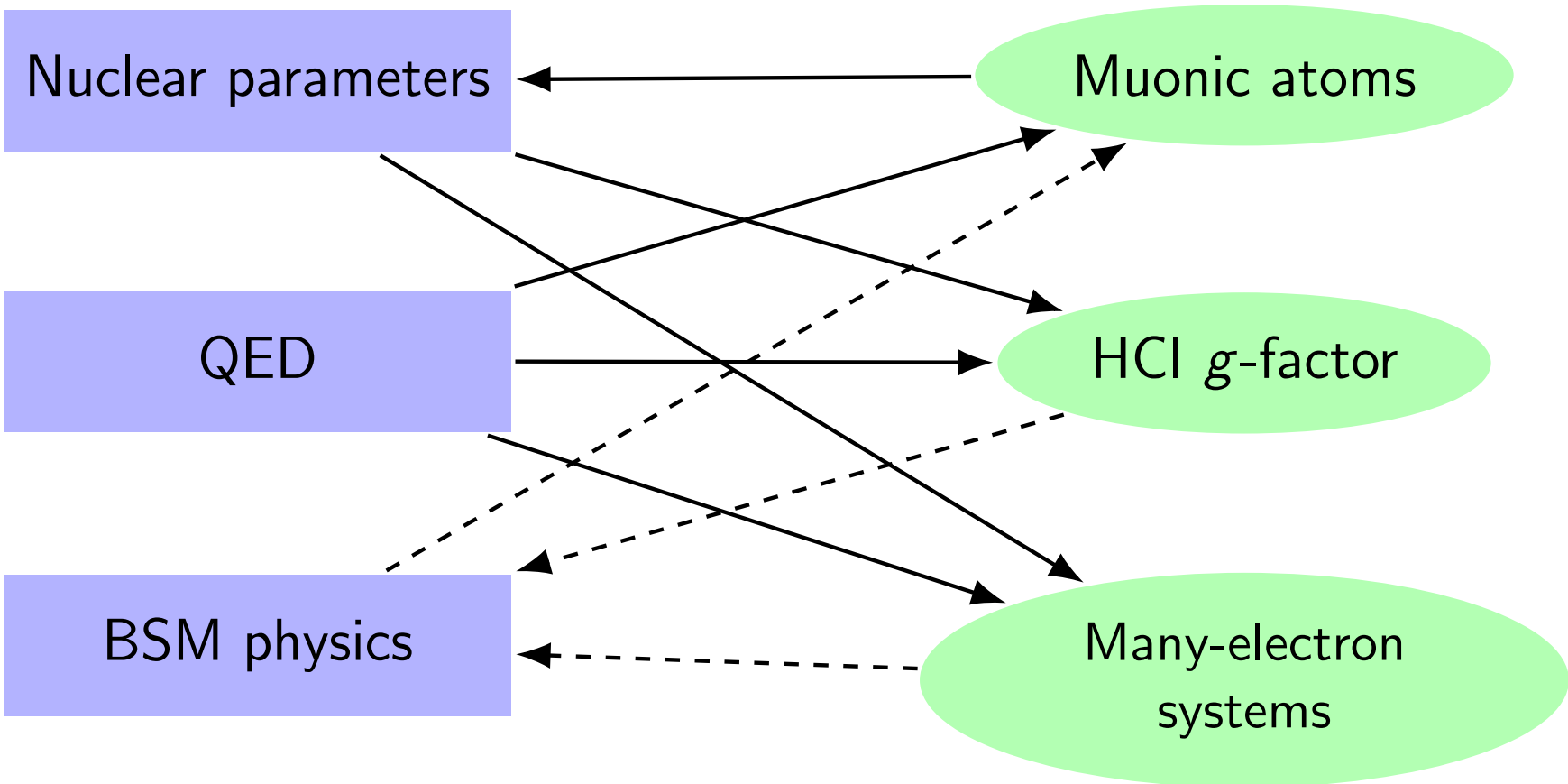
(A) *g*-factor hydrogenlike $^{208}Pb^{81+}$



(B) $2p_{3/2} \rightarrow 1s_{1/2}$ muonic ^{208}Pb

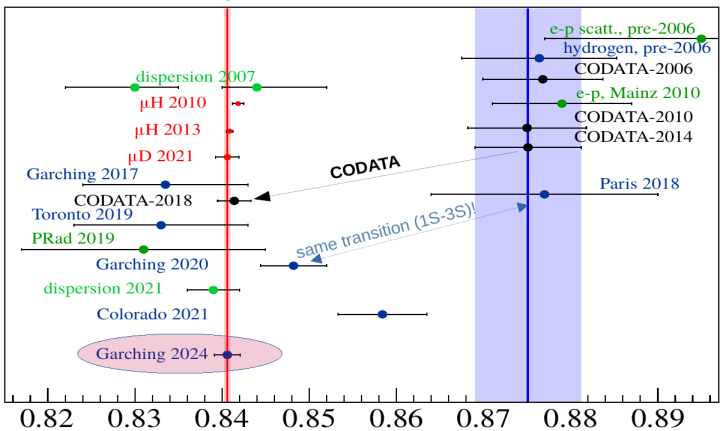


Connections



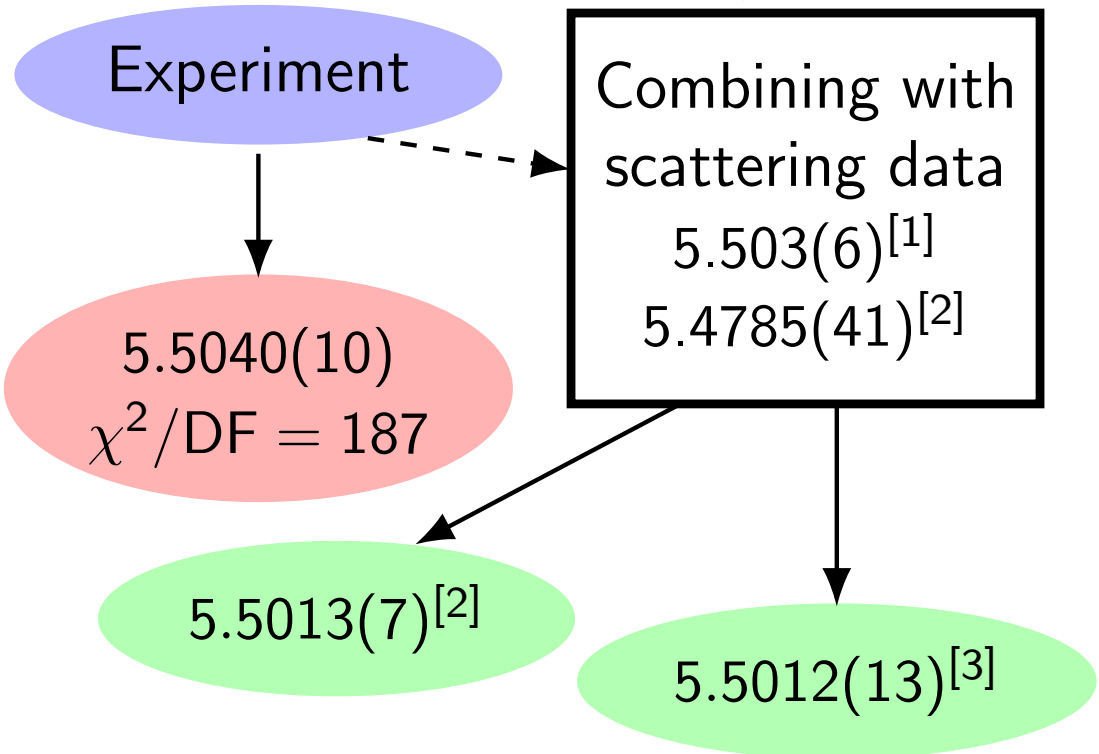
Muonic anomalies

Proton radius puzzle



From: Randolf Pohl

Muonic fine-structure anomaly, Pb



Observed in ^{208}Pb, ^{120}Sn, and ^{90}Zr
Blamed it on theory (NP)

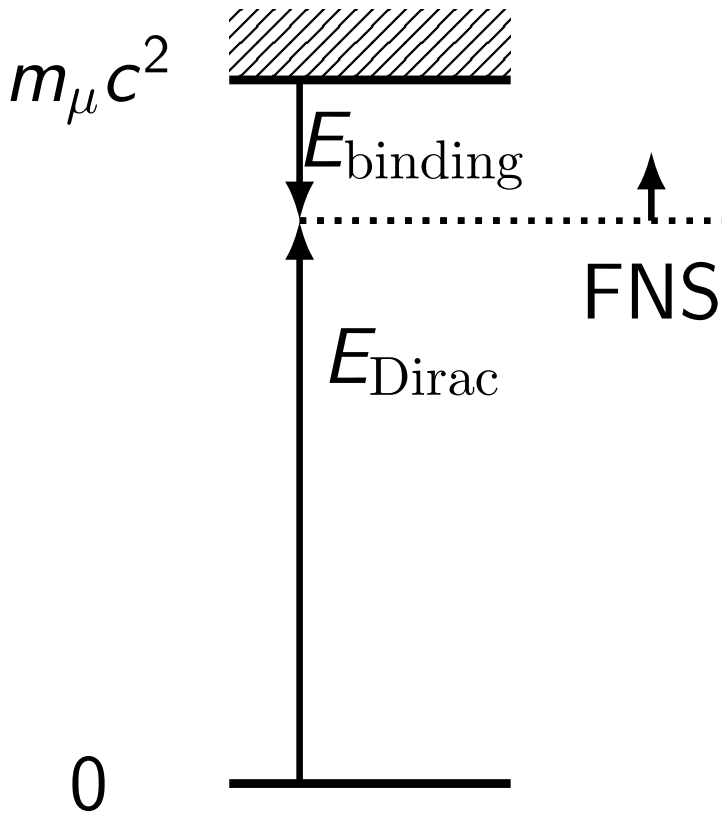
[1] Bergem *et al.*, PRC **37**, 2821 (1988)
[2] Fricke and Bernhardt, At. Data Nucl. Data Tables **60**, 177 (1995)
[3] Angeli and Marinova, At. Data Nucl. Data Tables **99**, 69 (2013)

Historical data

Source	Value	Comment
Belicard 1967	5.38(3), 5.39(3), 5.42(5), 5.54	e scattering, obtained from different electron incident energies and 3pF nuclear model
Anderson 1969	5.4978(30)	muonic atoms
Heisenberg 1969	5.535, 5.539, 5.546	e scattering, 3pF and 4pF models
	5.501, 5.502	e scattering + muonic data
Dreher 1974	5.498(15), 5.520(23), 5.498(10), 5.514(28)	e scattering, different energies and fitting models
Kessler 1975	5.5097(6)	muonic atoms, $\chi^2/N = 0.11$[1]
Euteneuer 1976	5.500(24)	e scattering 3pF, $\chi^2_{\min} \approx 24$
	5.494(24)	"model independent"
Euteneuer 1978	5.4927(75)	e scattering, $\overline{\chi^2} \approx 20$
	5.5032(16)	e scattering + muonic, $\overline{\chi^2} \approx 22$
Bergem 1988	5.5040(11)	muonic atoms, $\chi^2/\mathrm{DoF} \approx 200$

[1] all lines with $3d$ were excluded to improve the quality of the fit

Dirac value and nuclear size



- Muons are close to the nucleus, relativistic $\rightarrow$ Dirac equation
- $m_\mu c^2 \approx 100$ MeV
  $E_{\text{Dirac}} \approx 80$ MeV
  $E_{\text{binding}} \approx 20$ MeV
- Extended nucleus: sphere, Fermi, deformed Fermi

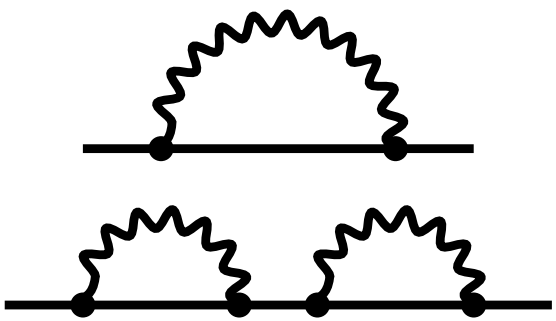
$$\rho^F_{a,c}(r_\mu) = \frac{N}{1 + \mathrm{e}^{(r-c)/a}}$$

- $\Delta E_{\text{FNS}} \approx 10$ MeV

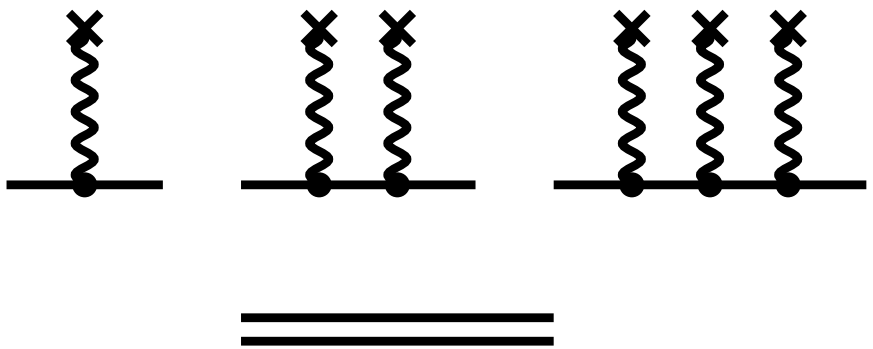
How can we do calculations?

Perturbation theory:

in α



in αZ



"Furry picture"

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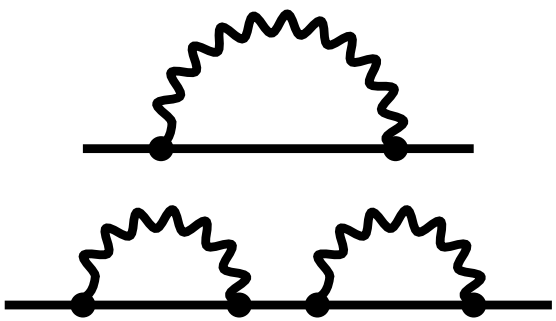

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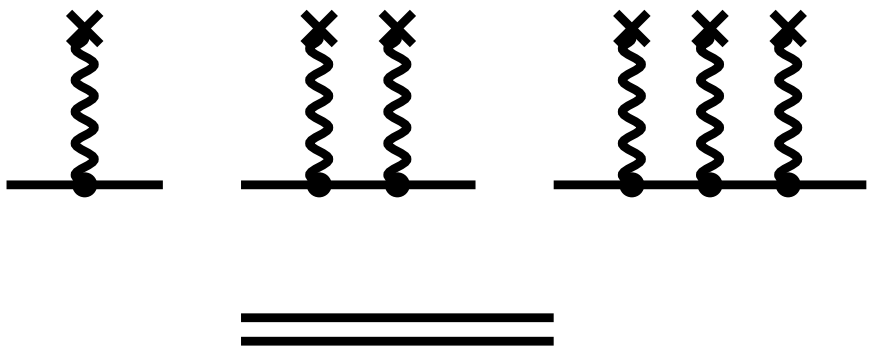

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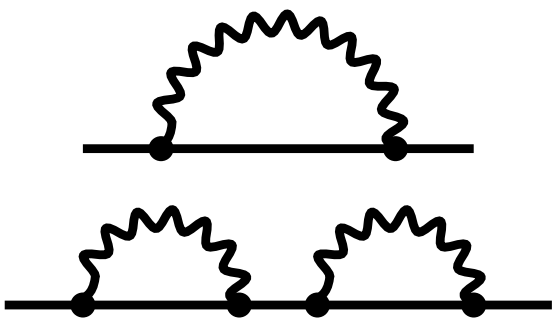

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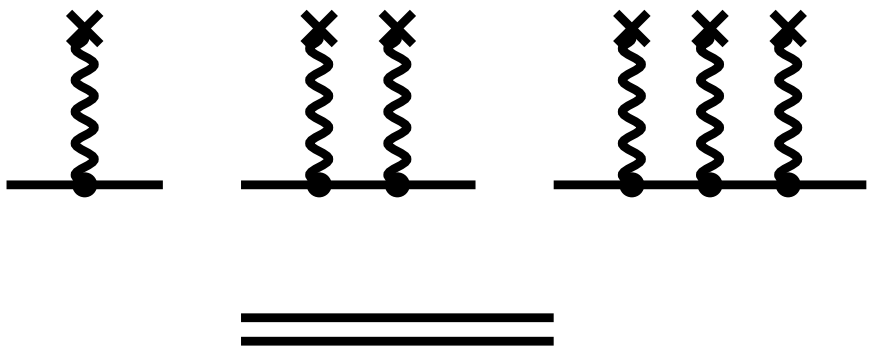

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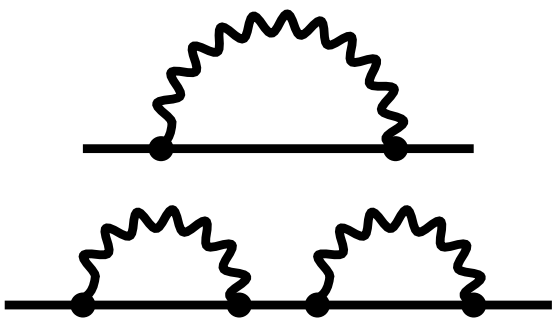

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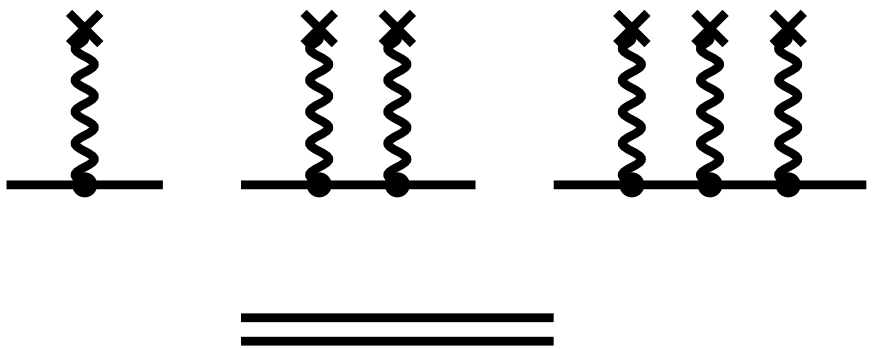

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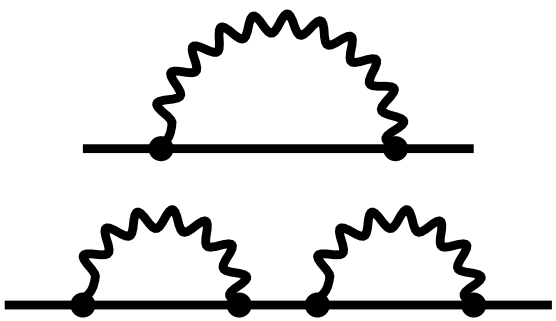

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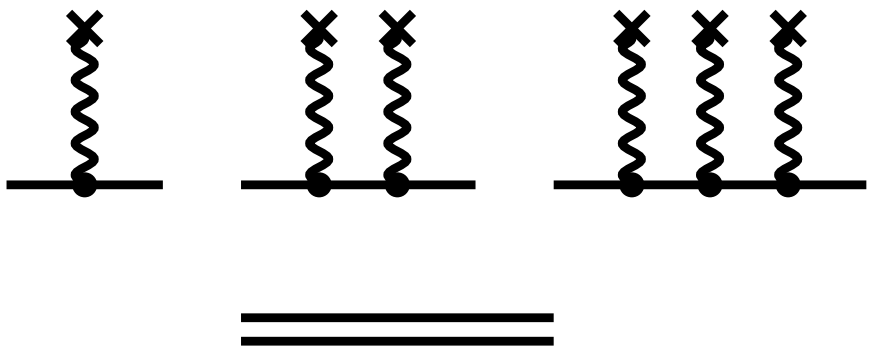

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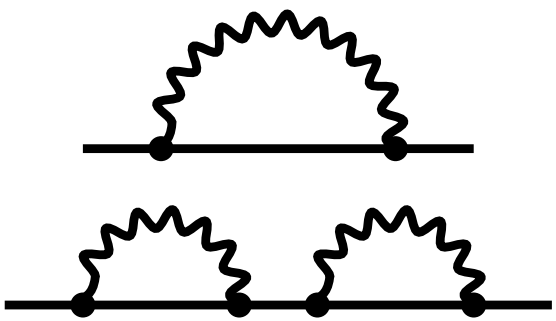

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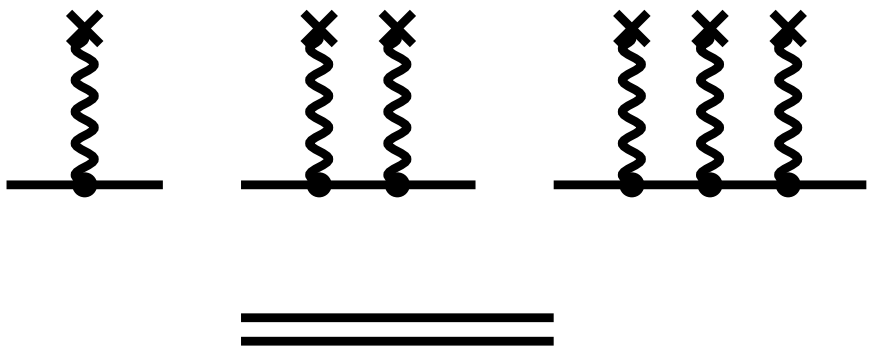

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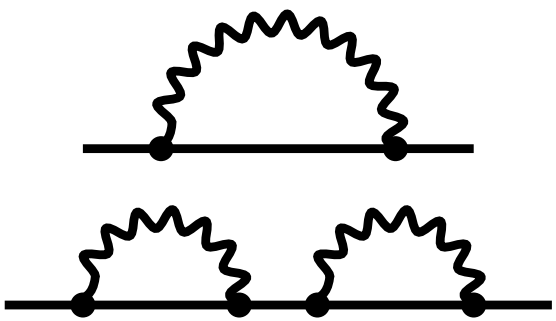

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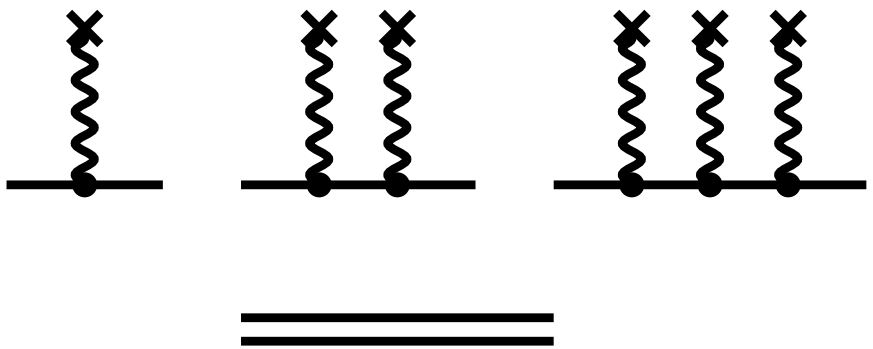

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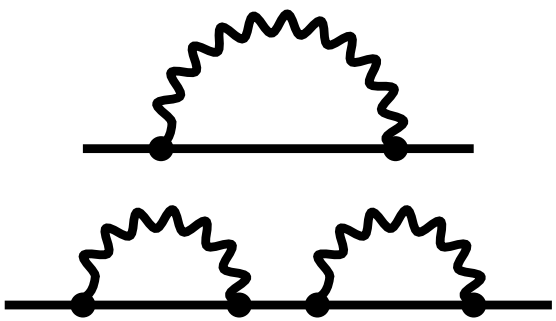

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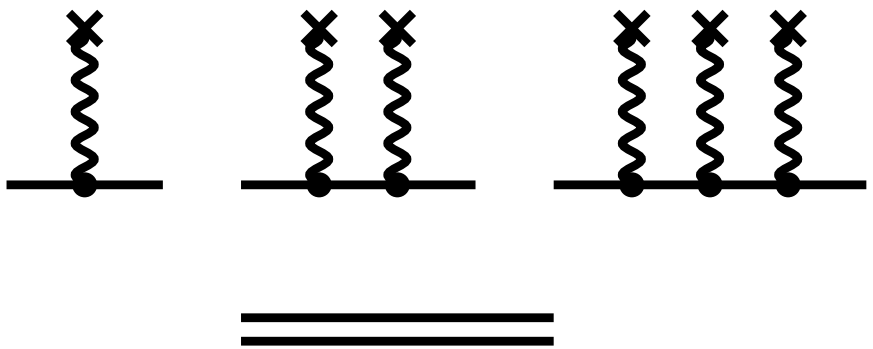

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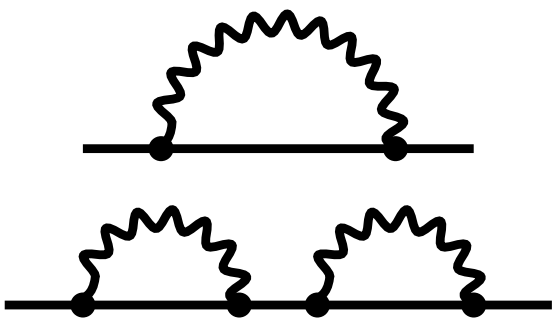

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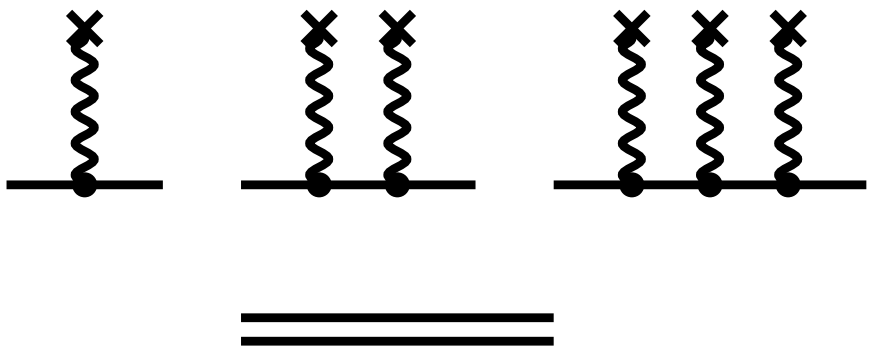

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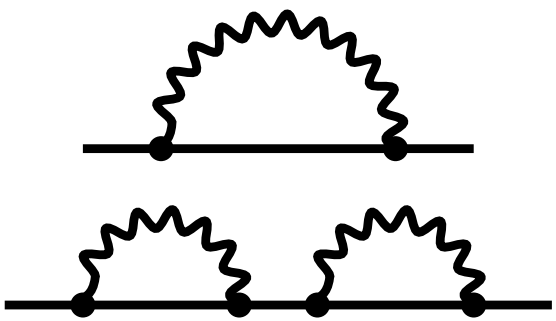

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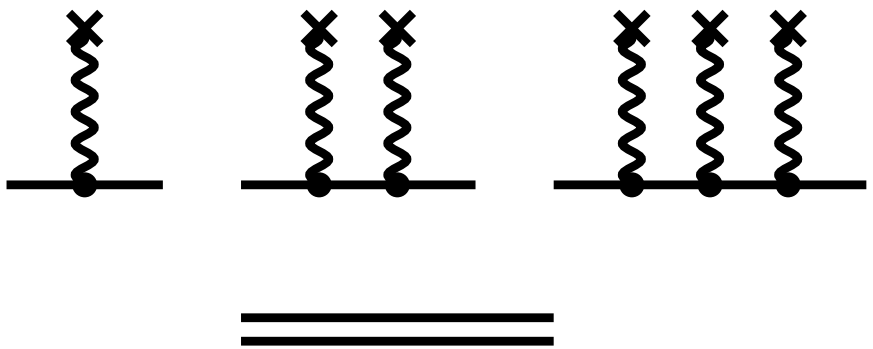

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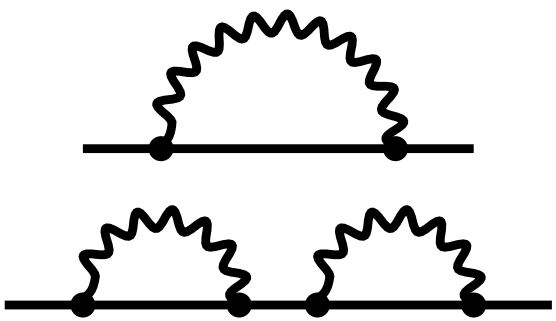

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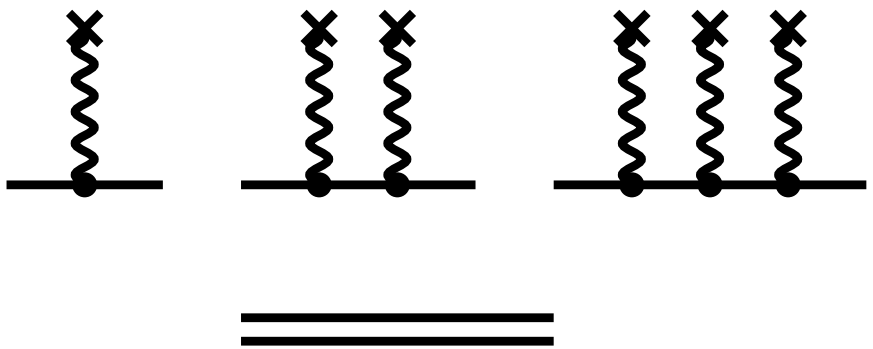
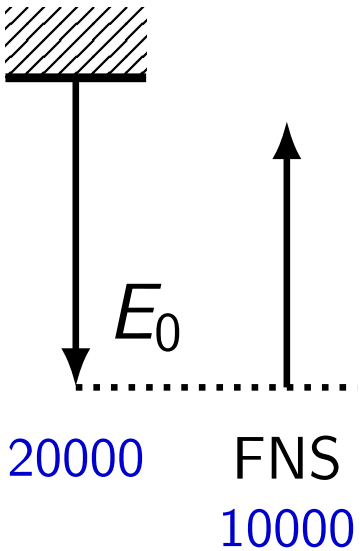


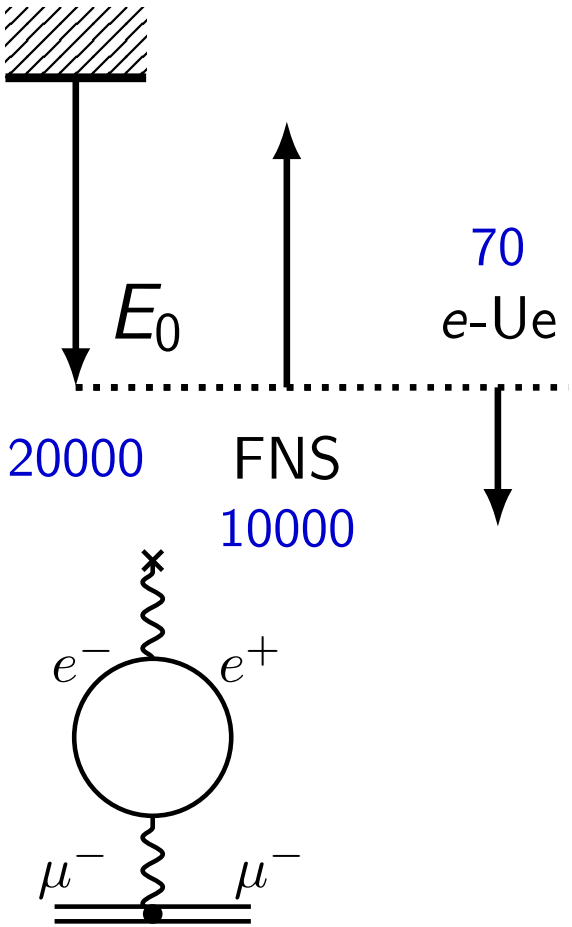
"Furry picture"

The two approaches can be independent or complimentary!

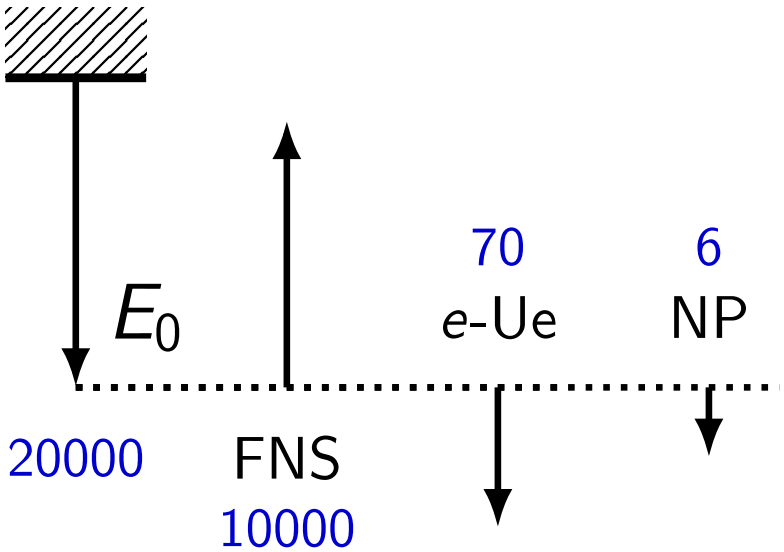
QED effects:



QED effects: Uehling



QED effects: Nuclear polarization

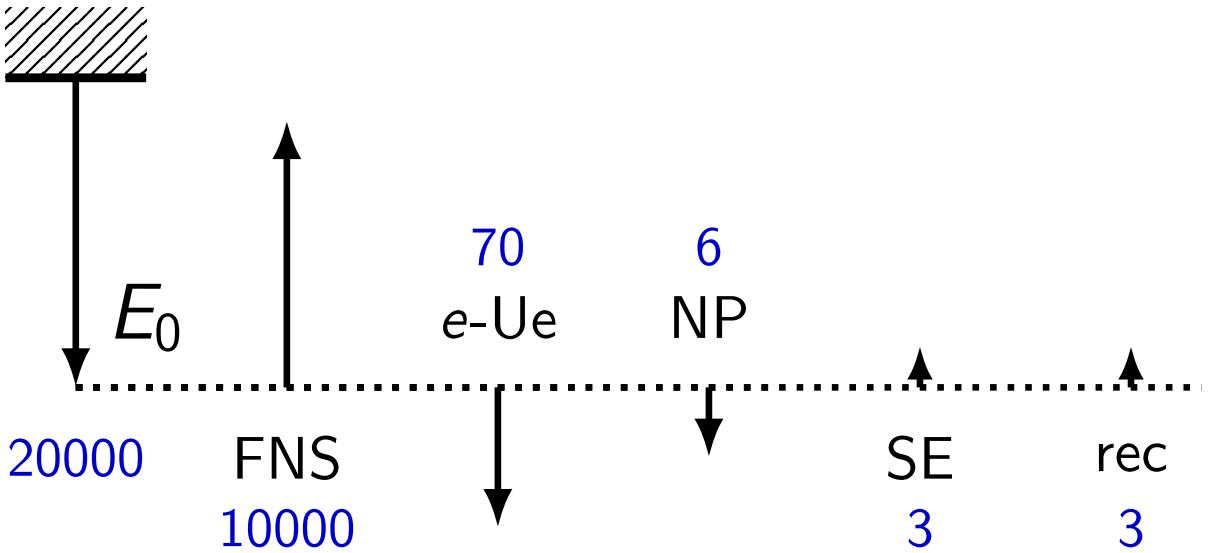


Improvements:

- field-theory approach, including transverse part
- state-of-art muonic and nuclear input, model dependence
- $0^+, 1^-, 2^+, 3^-, 4^+, 5^-$ and 1^+ excitation modes
- 4252 eV $\rightarrow$ 5712 eV

Valuev *et al.*, PRL **128** 203001 (2022)

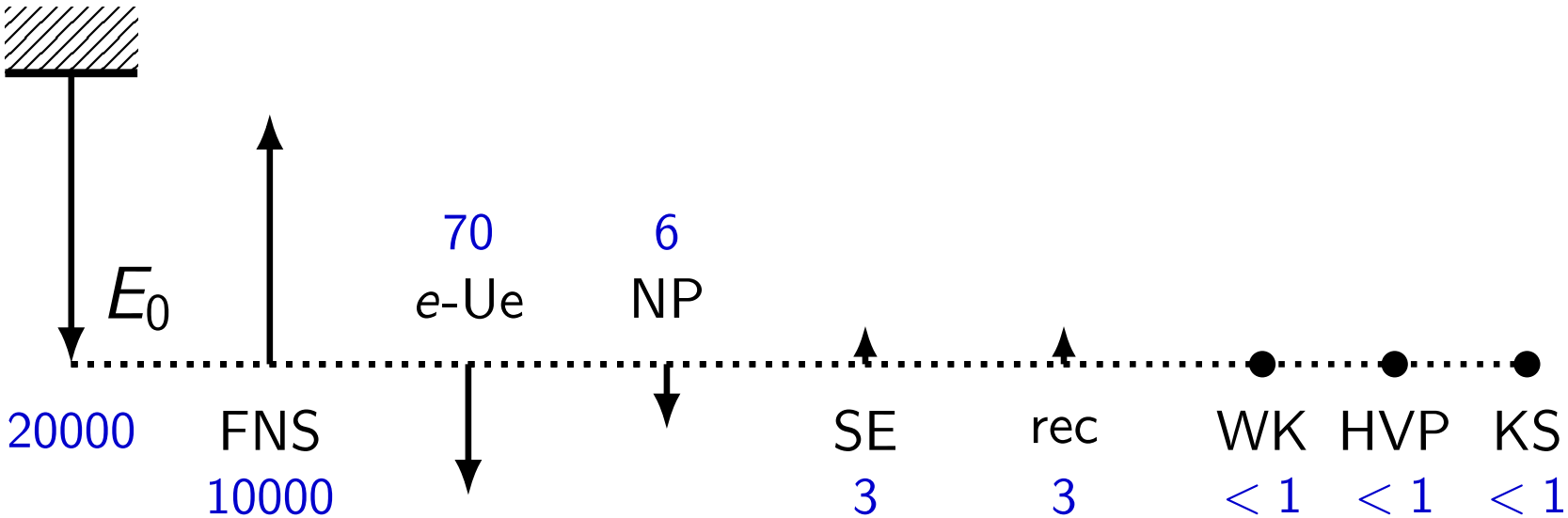
QED effects: Self energy and recoil

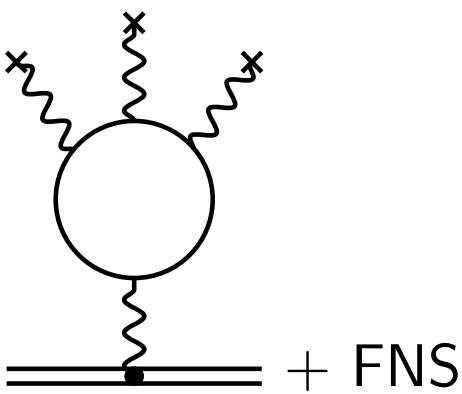


- rigorous QED calculations
- $\Delta E_{\rm SE}$ =3270(160)[1], 3373[2] eV $\rightarrow$ 3225(15)[3] eV
- $\Delta E_{\rm rec}$ =385[4]* eV $\rightarrow$ 3902[5] eV

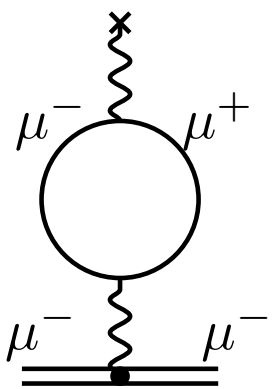
[1] Cheng *et al.*, PRA **17**, 489 (1978)
[2] Haga *et al.*, PRC **75**, 044315 (2007)
[3] Oreshkina, PRR **4**, L042040 (2022)
[4] Bergem *et al.*, PRC **37**, 2821 (1988)
[5] Yerokhin and Oreshkina, PRA **108**, 052824 (2023)

QED effects: sub-leading

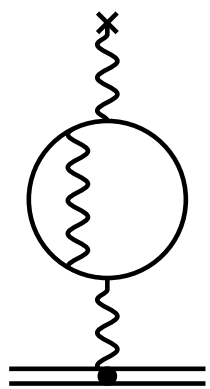




+ FNS



+ hadronic



Recoil with reduced mass: all orders in m/M

Schödinger equation:

$$H\psi = E\psi$$

$$\left(-\frac{\hbar^2}{2m_e}\nabla_e - \frac{\hbar^2}{2m_N}\nabla_N + V(|\mathbf{r}_e - \mathbf{r}_N|)\right)\psi(\mathbf{r}_e, \mathbf{r}_N) = E\psi(\mathbf{r}_e, \mathbf{r}_N)$$

$$\{\mathbf{r}_e, \mathbf{r}_N\} \to \left\{\mathbf{r} = \mathbf{r}_e - \mathbf{r}_N, \mathbf{R} = \frac{m_e\mathbf{r}_e + m_N\mathbf{r}_N}{m_e + m_N}\right\}$$

$$\left(-\frac{\hbar^2}{2\mu}\nabla + V(r)\right)\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

Dirac equation:

$$H_D = \alpha\mathbf{p} + \beta m + V$$

Recoil with reduced mass: all orders in m/M

Schödinger equation:

$$H\psi(\mathbf{r}_1, \mathbf{r}_2) = E\psi(\mathbf{r}_1, \mathbf{r}_2)$$

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$$\left(-\frac{\hbar^2}{2\mu}\nabla + V(r)\right)\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

Dirac equation:

$$H_D = \alpha\mathbf{p} + \beta m + V$$

Relativistic recoil: the first order in m/M

PHYSICAL REVIEW LETTERS **130,** 053002 (2023)

QED Theory of the Nuclear Recoil with Finite Size

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Vladimir A. Yerokhin
Peter the Great St. Petersburg Polytechnic University, Polytekhnicheskaya 29, 195251 St. Petersburg, Russia

$$\Delta E_{\mathrm{rec,LO}} = \langle a|\frac{\mathbf{p}^2}{2m}|a\rangle$$

$$\Delta E_{\mathrm{rec,HO}} = \text{some other integral}$$

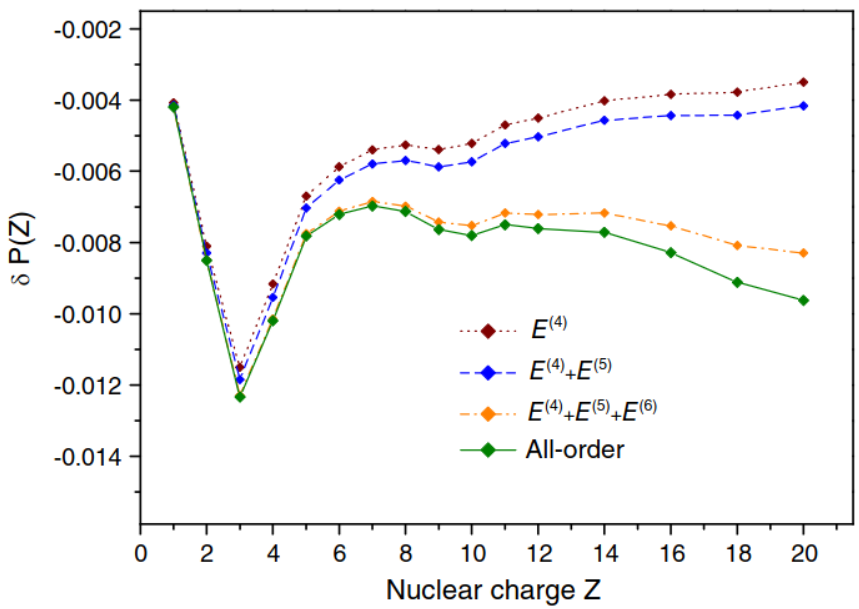


FIG. 1. Finite-size nuclear recoil correction for the $1s$ state of H-like ions, in terms of function $\delta P = E_{\mathrm{recfns}}/[(m^2/M)(Z\alpha)^5/\pi]$.

Polarization or polarizability?

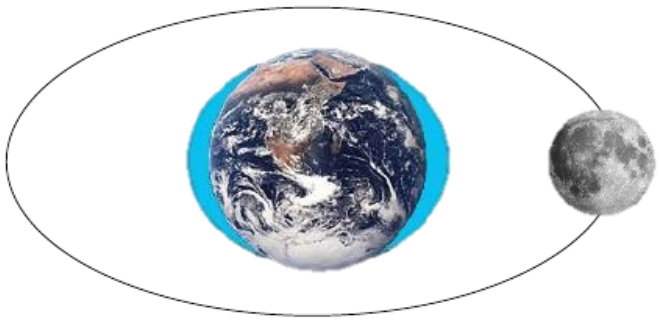


Image source: www.universetoday.com

- nucleus plus muon together
- "measure of entanglement"
- sometimes can be separated (approximately)

Nuclear polarization $\approx$ Nuclear polarizability $\times$ muonic factor

Nuclear polarization on a nutshell

$$V_{\rm Coul}(r) = -\frac{\alpha Z}{r}$$

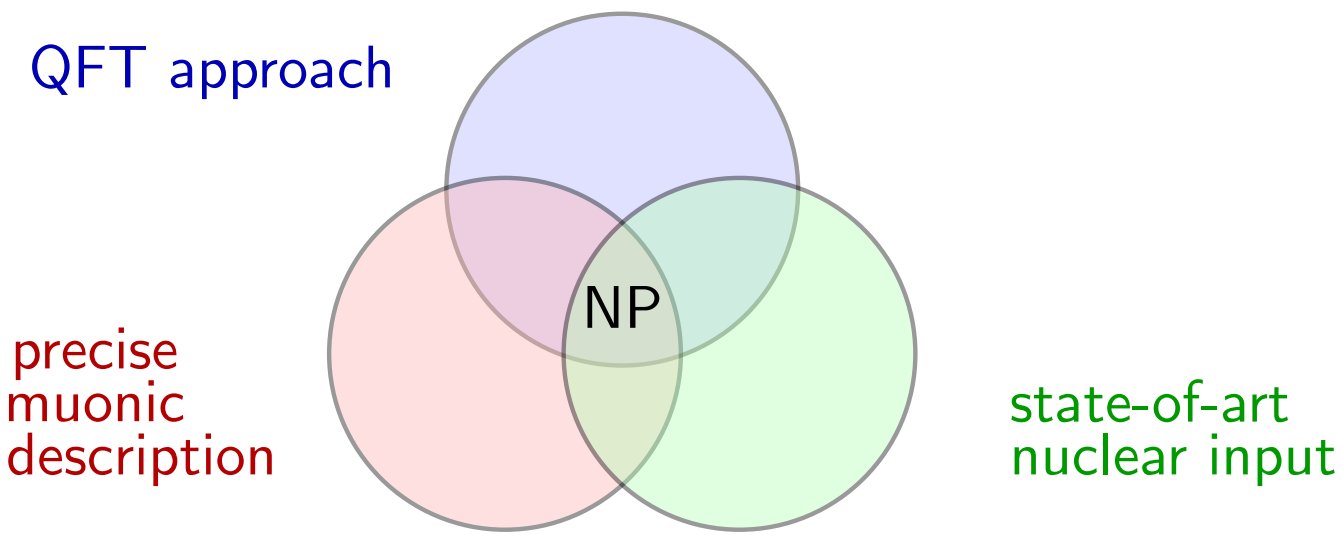
$$V_{\rm ext}(r) = -\alpha \int \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}'$$

Longitudinal (Coulomb) part

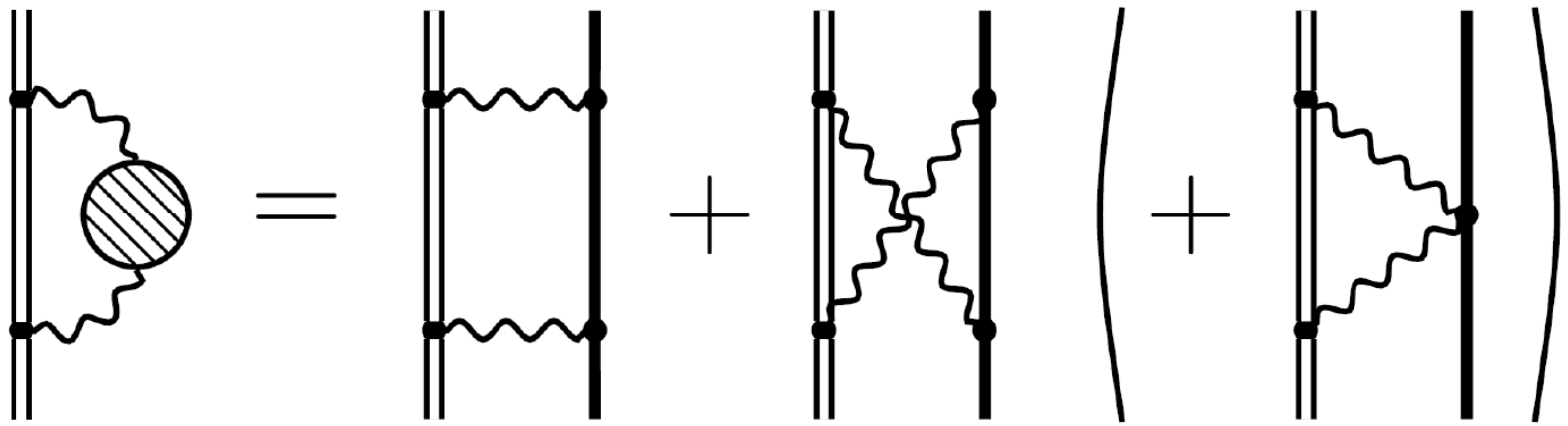
$$\Delta E_{\rm NP} = \sum_{nN} \frac{|\langle aA|\delta V|nN\rangle|^2}{E_{aA} - E_{nN}}$$

$$V_{\rm NP}(r) = -\alpha \sum_{Z} \frac{1}{|\mathbf{r} - \mathbf{r}_{\mathbf{N_i}}|}$$

Transverse part: $H = H_N + \boldsymbol{\alpha}(\mathbf{p} - e\mathbf{A}(\mathbf{r}, \mathbf{r}_{N_i})) + \beta m_\mu + V(\mathbf{r}, \mathbf{r}_{N_i})$



Nuclear polarization: in scary formulas



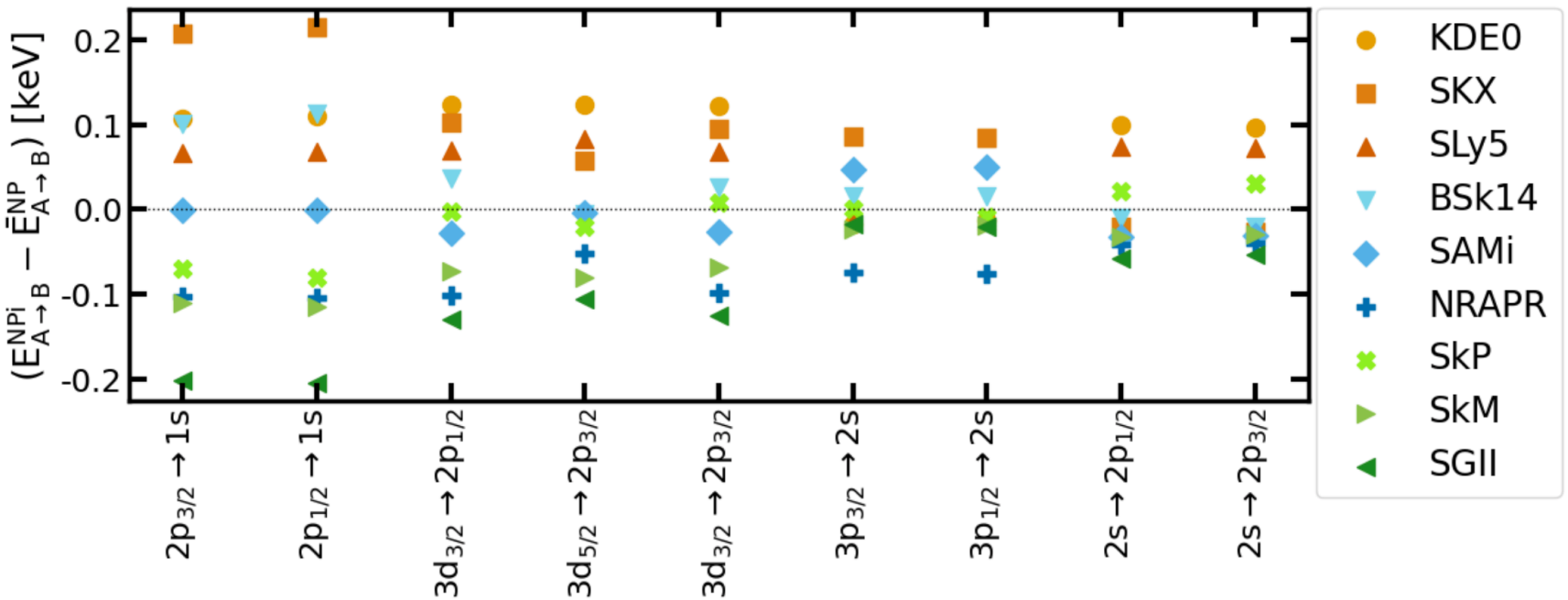
$$\Delta E^{\mathrm{L}}_{\mathrm{NP}} = -i(4\pi\alpha)^2 \sum_{i'I'} \iint \frac{d\boldsymbol{q}\, d\boldsymbol{q}'}{(2\pi)^6} \int \frac{d\omega}{2\pi} \frac{D_{\mu\xi}(\omega,\boldsymbol{q}) D_{\zeta\nu}(\omega,\boldsymbol{q}') \langle iI|j^\mu_m(-\boldsymbol{q}) J^\xi_N(\boldsymbol{q})|i'I'\rangle \langle i'I'|J^\zeta_N(-\boldsymbol{q}') j^\nu_m(\boldsymbol{q}')|iI\rangle}{(\omega+\omega_m - iE_{i'}\epsilon)(\omega-\omega_N+i\epsilon)},$$

$$\Delta E^{\mathrm{X}}_{\mathrm{NP}} = +i(4\pi\alpha)^2 \sum_{i'I'} \iint \frac{d\boldsymbol{q}\, d\boldsymbol{q}'}{(2\pi)^6} \int \frac{d\omega}{2\pi} \frac{D_{\mu\xi}(\omega,\boldsymbol{q}) D_{\zeta\nu}(\omega,\boldsymbol{q}') \langle iI'|j^\mu_m(-\boldsymbol{q}) J^\xi_N(\boldsymbol{q})|i'I\rangle \langle i'I|J^\zeta_N(-\boldsymbol{q}') j^\nu_m(\boldsymbol{q}')|iI'\rangle}{(\omega+\omega_m - iE_{i'}\epsilon)(\omega+\omega_N-i\epsilon)},$$

$$\Delta E^{\mathrm{SG}}_{\mathrm{NP}} = -i(4\pi\alpha)^2 \sum_{i'} \iint \frac{d\boldsymbol{q}\, d\boldsymbol{q}'}{(2\pi)^6} \int \frac{d\omega}{2\pi} \frac{D_{\mu\xi}(\omega,\boldsymbol{q}) \delta^{\xi\zeta} D_{\zeta\nu}(\omega,\boldsymbol{q}') \langle i|j^\mu_m(-\boldsymbol{q})|i'\rangle \langle i'|j^\nu_m(\boldsymbol{q}')|i\rangle}{(\omega+\omega_m - iE_{i'}\epsilon)} \frac{\langle I|\rho_N(\boldsymbol{q}-\boldsymbol{q}')|I\rangle}{m_p}.$$

summations over entire muonic (i') and nuclear (I') spectra

Nuclear polarization: results

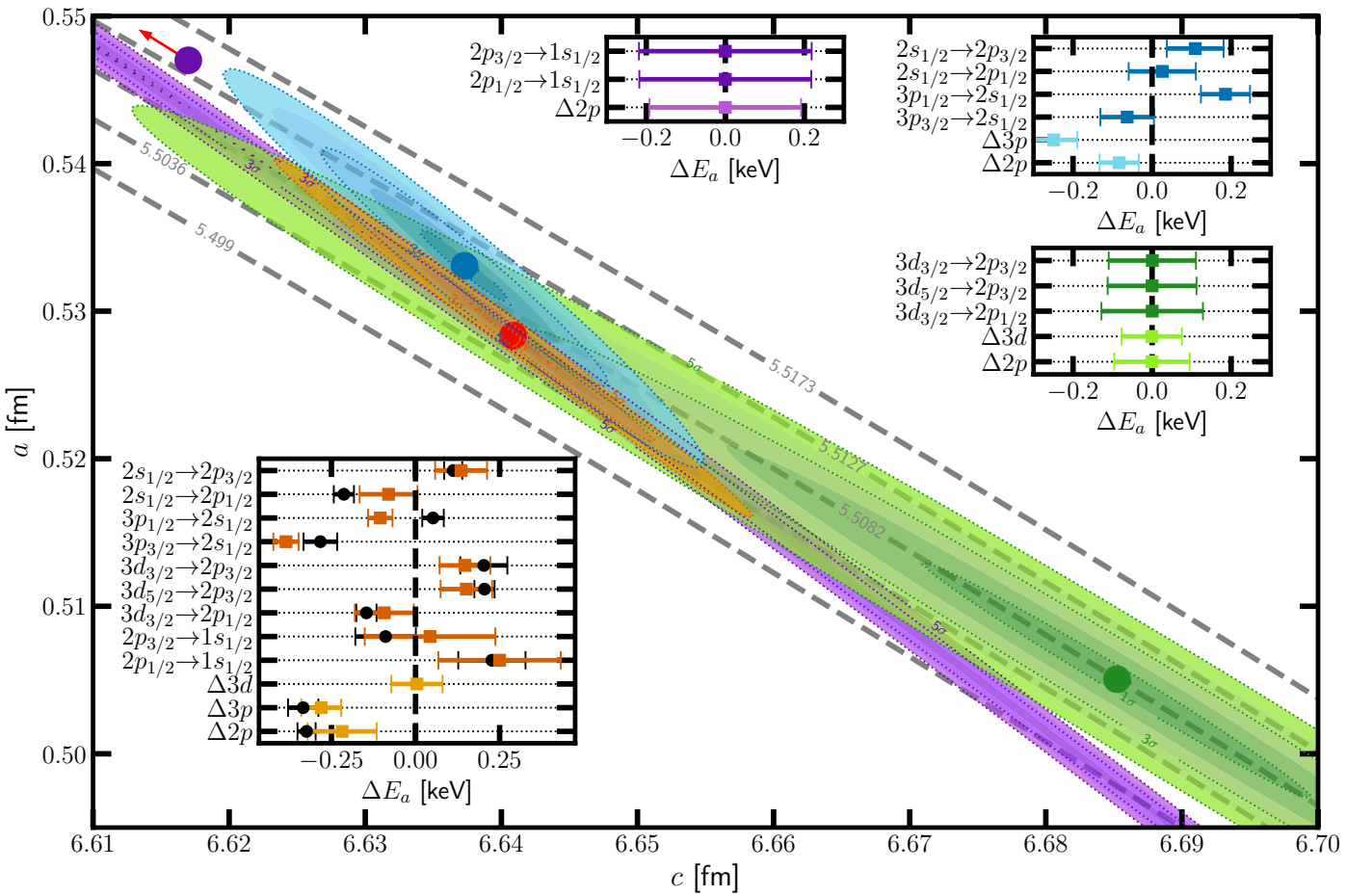


Experimental data

Set	Transition	Energy (keV)
Run I	$2p_{3/2} - 1s_{1/2}$	5962.854(90)
	$2p_{1/2} - 1s_{1/2}$	5778.058(100)
Run II	$3d_{3/2} - 2p_{1/2}$	2642.332(30)
	$3d_{5/2} - 2p_{3/2}$	2500.590(30)
	$3d_{3/2} - 2p_{3/2}$	2457.569(70)
Run III	$3p_{3/2} - 2s_{1/2}$	1507.754(50)
	$3p_{1/2} - 2s_{1/2}$	1460.558(32)
	$2s_{1/2} - 2p_{1/2}$	1215.330(30)
	$2s_{1/2} - 2p_{3/2}$	1030.543(27)

Bergem *et al.*, PRC **37**, 2821 (1988)

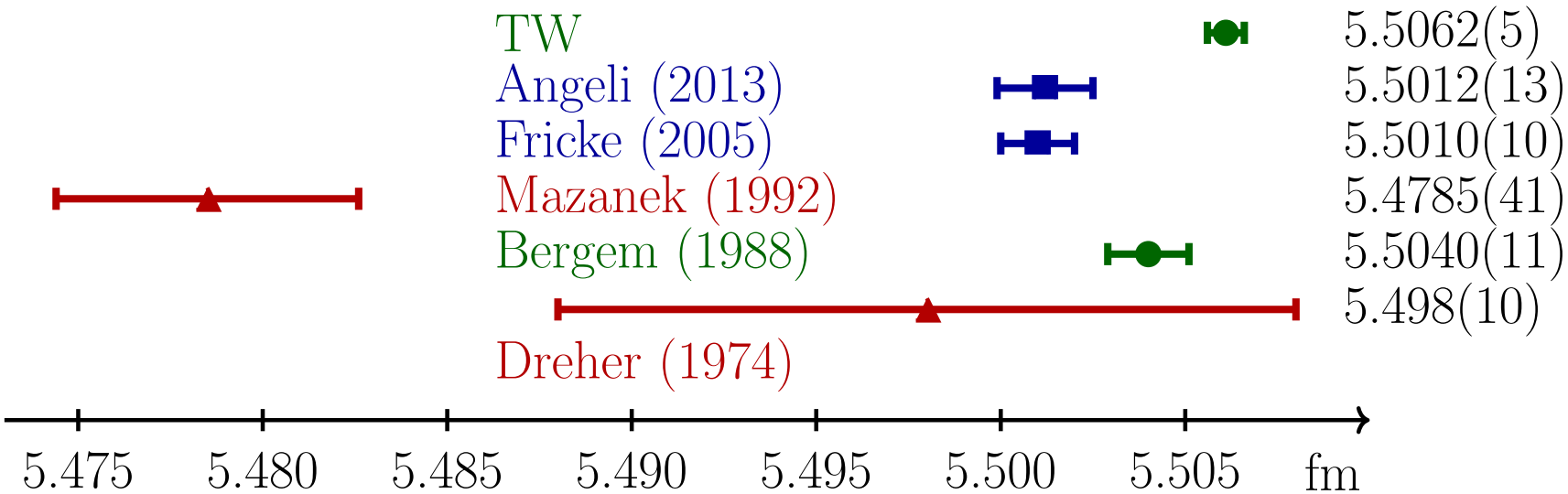
Results for ^{208}Pb



Purple: $2p_{1,3/2} - 1s_{1/2}$, green: $3d_{3,5/2} - 2p_{1,3/2}$,
blue: $3p_{1,3/2} - 2s_{1/2}$ and $2s_{1/2} - 2p_{3,1/2}$, red: total

Sun *et al.*, *PRL* **135, 163002 (2025)**

Compared with old data



- Mild tension with Bergem
- 3-4σ with Angeli, Fricke
- $\chi^2/\mathrm{DoF} = 9.5$: factor of 20 improvement
- Evidence against the existence of the anomaly

Normalized fit quality $\chi^2/\mathrm{DoF} = 1$: value 5.5062(17) fm
reduced model-dependence

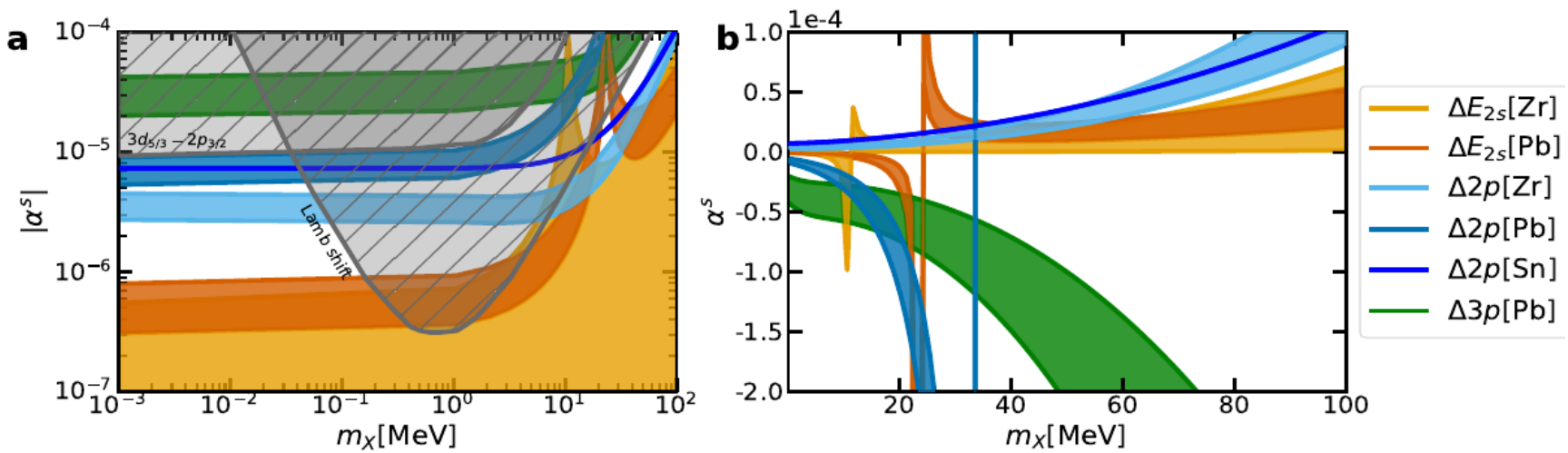
Sun *et al.*, *PRL* **135, 163002 (2025)**

Muonic atoms anomalies: H vs Pb

BSM as (not) a solution of the anomaly

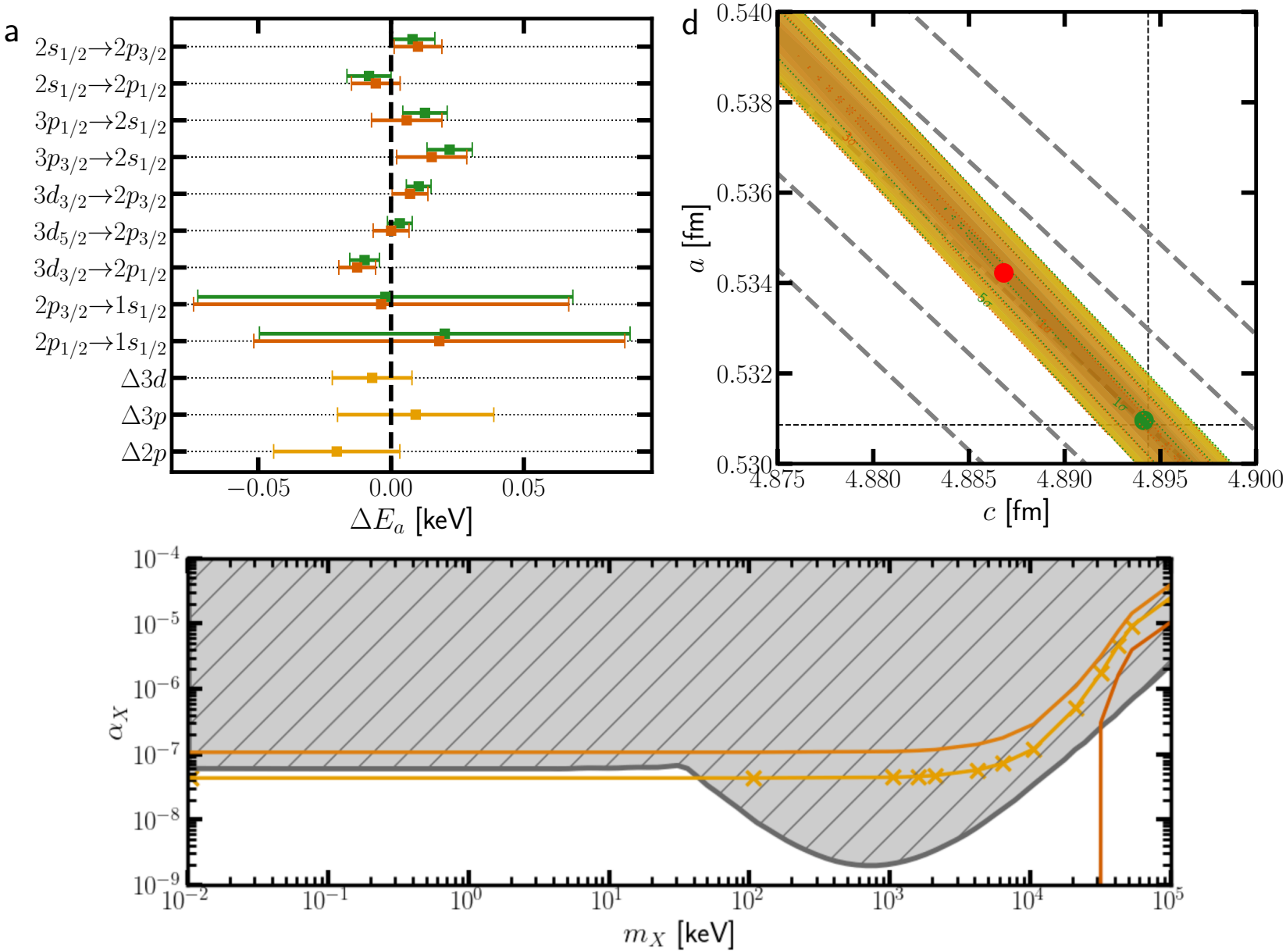
Scalar/vector new boson

$$V_{\mathrm{BSM}} = \alpha_{\mathrm{BSM}} \hbar c \frac{exp[-m_{\mathrm{BSM}} cr/\hbar]}{r} \gamma_0$$



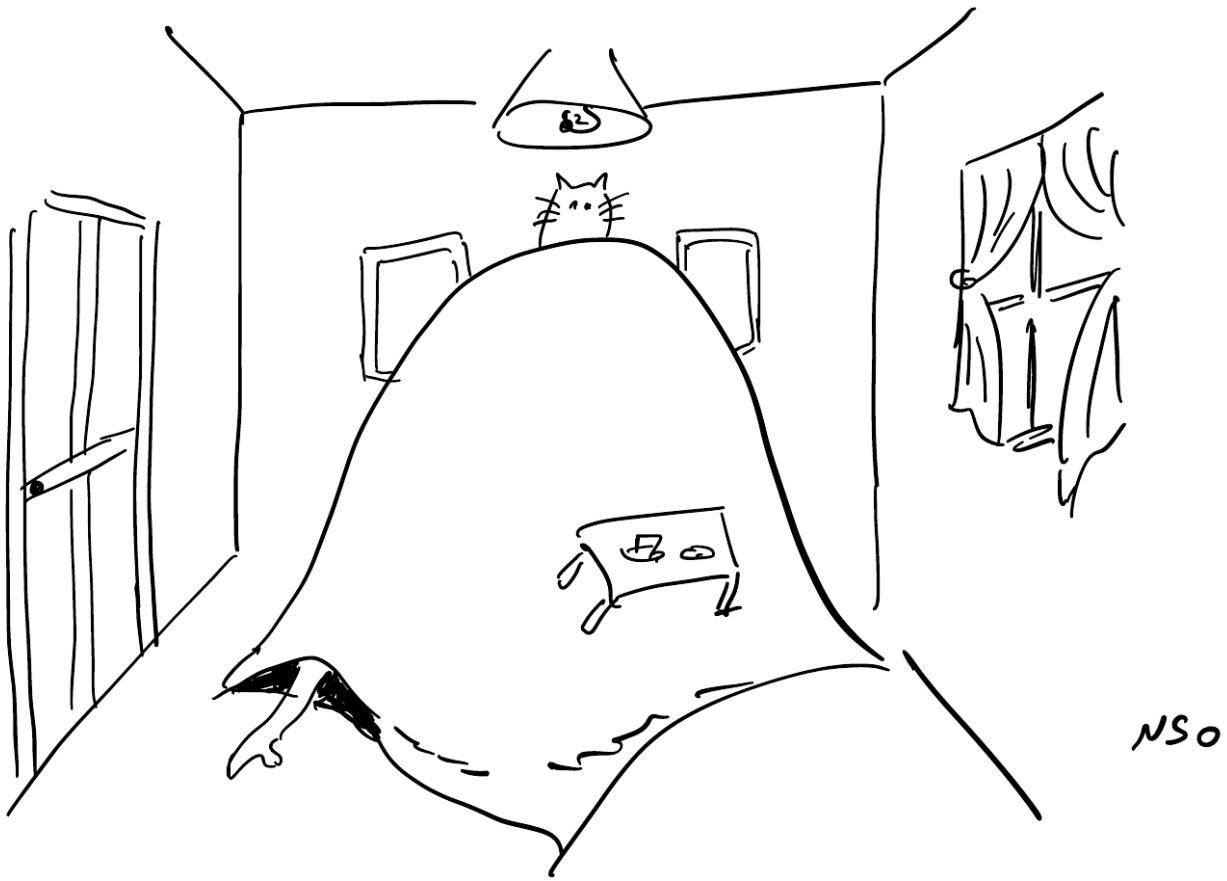
Beyer *et al.*, *Phys. Lett. B* **854, 138746 (2024)**

Self-consistent probe of BSM PRELIMINARY



Outlook

- "heavy electron" QED
- Anomaly is almost resolved
- New recommended rms for ^{208}Pb: 5.5062(17) fm
- A step forward systematic reevaluation of all rms radii



Acknowledgments

Experiments and experimentalists

My hard-working team

and you for your attention