Inclusive Measurement of R

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Precision Determinations of the Fine-structure Constant, Mainz

Oct. 30, 2025

Outline

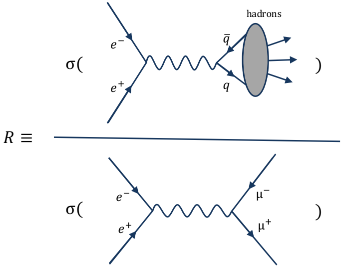
- **Introduction**
- **Inclusive R with energy-scan method**
- **Inclusive R via ISR technique**
- **Summary**

Definition of R

The R value is defined as the leading-order production cross section ratio of inclusive hadrons and muon pairs in the annihilation of electron-positron:

$$R \equiv \frac{\sigma^0(e^+e^- \to \text{hadrons})}{\sigma^0(e^+e^- \to \mu^+\mu^-)} \equiv \frac{\sigma^0_{\text{had}}}{\sigma^0_{\mu\mu}}$$

That is,



A direct result from the QED:

$$\sigma^0_{\mu\mu}(s) = \frac{4\pi\alpha^2}{3s}\frac{\beta_\mu(3-\beta_\mu^2)}{2}, \text{ with } \beta_\mu = \sqrt{1-4m_\mu^2/s}$$

Running of QED coupling constant: $\Delta\alpha(s)$

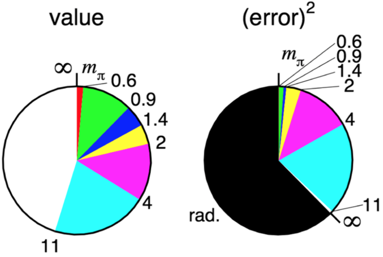
The contributions to $\Delta\alpha(s)$ is distinguished to three pieces:

$$\Delta\alpha(s) = 1 - \alpha(0)/\alpha(s) = \Delta\alpha_{\text{lepton}}(s) + \Delta\alpha_{\text{had}}^{(5)}(s) + \Delta\alpha_{\text{top}}(s)$$

- $\Delta\alpha_{\text{lepton}}(s)$ is calculated with perturbative approach and $\Delta\alpha_{\text{top}}(s)$ is usually small
- $\Delta\alpha_{\text{had}}^{(5)}(s)$ should be calculated by using R value at low energy:

$$\Delta\alpha_{\text{had}}^{(5)}(s) = -\frac{\alpha s}{3\pi}\,\text{Re}\int_{E_{\text{th}}^2}^{\infty} ds' \frac{R(s')}{s'(s' - s - i\varepsilon)}$$

Fractional contribution to $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$:



Eur. Phys. J. C 80, 241 (2020)

Source	Contribution($\times 10^4$)
$\Delta\alpha_{\text{lepton}}(M_Z^2)$	314.979 ± 0.002
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$	276.0 ± 1.0
$\Delta\alpha_{\text{top}}(M_Z^2)$	-0.7180 ± 0.0054

The $\Delta\alpha_{\text{had}}^{(5)}(s)$ is sensitive with R value over all energy region!

Moun anomaly: $a_\mu^{\text{LO-HVP}}$

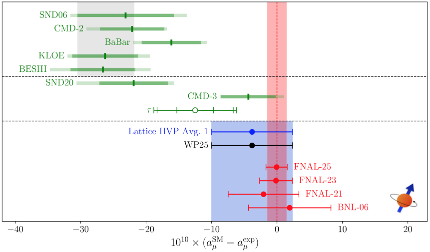
Based on the optical theorem, the leading-order hadronic VP contribution to muon anomaly, i.e., $a_\mu^{\text{LO-HVP}}$, is evaluated with R:

$$a_\mu^{\text{LO-HVP}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{E_{\text{th}}^2}^{\infty} \mathrm{d}s \frac{R(s)K(s)}{s^2}$$

- $R(s)$ in low energy region ($\sqrt{s} < 1$ GeV) contributes to $a_\mu^{\text{LO-HVP}}$ significantly.
- After sufficient high energy, e.g., 12 GeV, $a_\mu^{\text{LO-HVP}}$ is calculated according to pQCD.

Phys. Rep. 1143, 1 (2025)

Source	Value ($\times 10^{11}$)
QED	116584718.8(2)
EW	154.4(4)
HVP LO (e^+e^-)	6931(40)
HVP LO (lattice)	7132(61)
HVP NLO (e^+e^-)	−99.6(1.3)
HVP NNLO (e^+e^-)	12.4(1)
HLbL	115.5(9.9)
a_μ^{SM}	116592033(62)
a_μ^{exp}	116592071.5(14.5)
Δa_μ	38(63)



QCD coupling constant: $\alpha_s(s)$

According to pQCD, R is predicted with the coupling constant $\alpha_s(s)$:

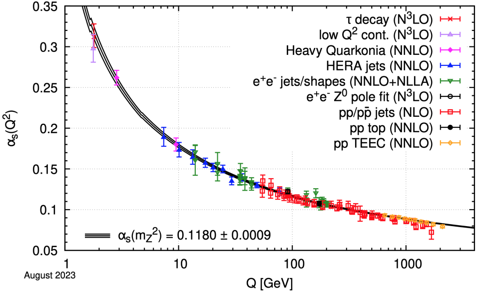
$$R_{\text{QCD}}(s) = N_c \sum_f Q_f^2 \left[1 + \left(\frac{\alpha_s(s)}{\pi}\right) + r_1\left(\frac{\alpha_s(s)}{\pi}\right)^2 + r_2\left(\frac{\alpha_s(s)}{\pi}\right)^3 + r_3\left(\frac{\alpha_s(s)}{\pi}\right)^4\right] + \mathcal{O}(\alpha_s^5(s)),$$

where $N_c = 3$ and Q_f are number of colors and charge carried by each of N_f activated quarks.

$$r_1 = 1.9857 - 0.1152 N_f,$$

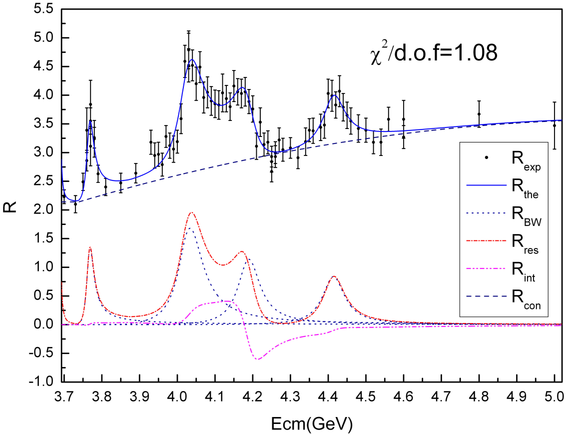
$$r_2 = -6.63694 - 1.20013 N_f - 0.00518 N_f^2 - 1.240\left(\sum Q_f\right)^2 / \left(3\sum Q_f^2\right),$$

$$r_3 = -156.61 + 18.775 N_f - 0.7974 N_f^2 + 0.0215 N_f^3 - (17.828 - 0.575 N_f) \times \left(\sum Q_f\right)^2 / \left(3\sum Q_f^2\right)$$

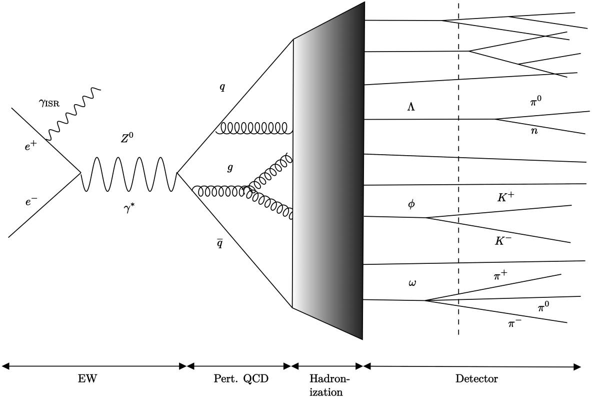


More physics potential of R

- Search for higher excitation states or exotic states with $J^{PC} = 1^{--}$
- Constraint charm quark mass with precise R value in open-charm region



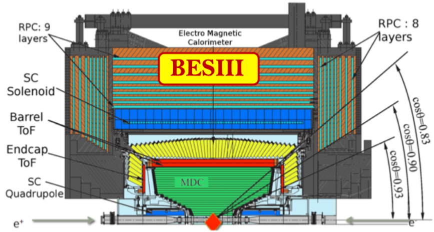
Production of a hadronic event

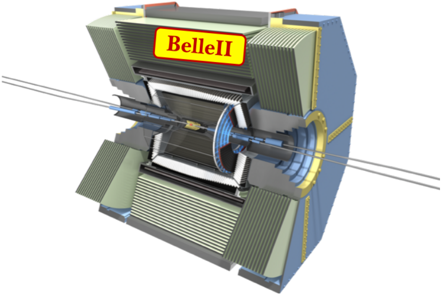


- R value measurement is based-on experimental data taken at e^+e^- collider
- Initial state radiation reduces the c.m. energy thereby requires a correction
- Hadronization of partons are simulated by phenomenological models

Detection of a hadronic event

In annihilation of e^+e^-, a hadronic event is detected by a composited detector:





- Unstable initial state hadrons are produced at interaction point and decay in detector
- Not all the relatively stable hadrons are detected due to limited acceptance
- Various background processes are also stored in data sample, e.g., QED events
- Not all the collision events are recorded by detector due to imperfect trigger system

Determination of R value

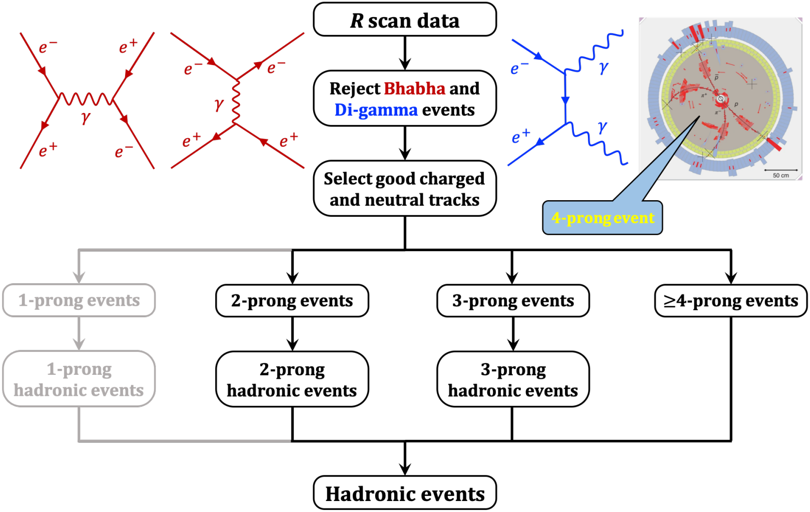
Experimentally, R value is determined by

$$R = \frac{N_{\text{had}}^{\text{obs}} - N_{\text{bkg}}}{\sigma_{\mu\mu}^{0}\mathcal{L}_{\text{int.}}\varepsilon_{\text{trig}}\varepsilon_{\text{had}}(1+\delta)}$$

- $N_{\text{had}}^{\text{obs}}$: Numbers of observed hadronic events.
- N_{bkg}: Number of the residual background events.
- $\sigma_{\mu\mu}^{0}(s) = 86.85\ \text{nb}/s$: Leading order QED cross section for $e^+e^- \to \mu^+\mu^-$.
- $\mathcal{L}_{\text{int.}}$: Integrated luminosity is measured by analyzing Bhabha events.
- $\varepsilon_{\text{trig}}$: Trigger efficiency $\sim 100\%$.
- ε_{had}: Detection efficiency of the hadronic events.
- $(1+\delta)$: ISR correction factor.

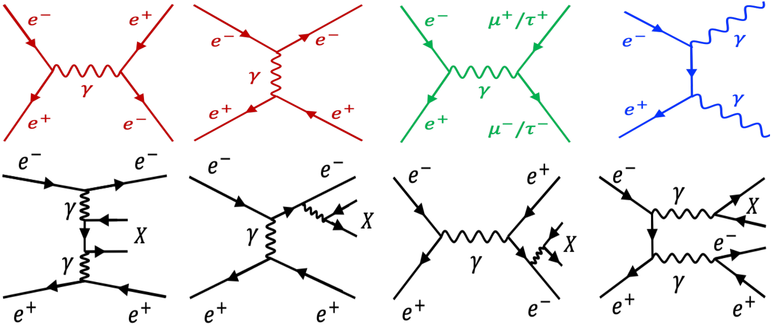
Event selection strategy

For an inclusive measurement, the signal event is not selected specifically:



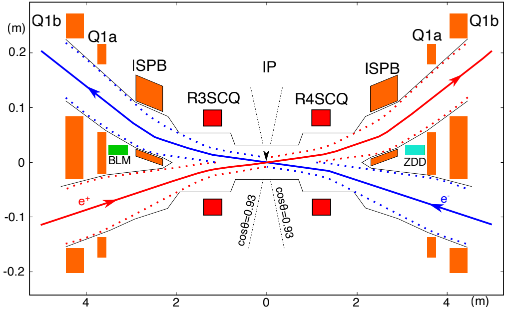
QED-related background processes

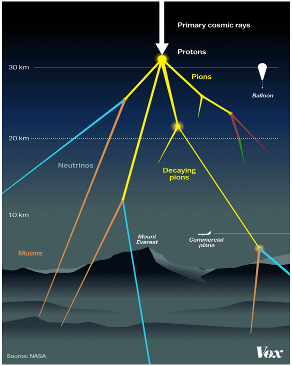
In data, there are various QED-related background processes could contribute to signal:



- QED processes $e^+e^- \to e^+e^-$, $\mu^+\mu^-$, $\tau^+\tau^-$, and $\gamma\gamma$ have specific features in detector.
- **Two-photon** processes, with $X = e^+e^-$, $\mu^+\mu^-$, and hadrons, have relatively low cross sections and detection efficiencies.
- Although dedicated detection criteria are applied, there are still residual background.
- These backgrounds are well known and estimated by dedicated MC simulations.

Beam background and cosmic ray





- Interaction between beam and materials in the interaction region produces hadrons
- Muon beams from cosmic rays will penetrate the detector and be recorded
- Cosmic events are easily identified and excluded from data sample
- A sophisticated method is used to estimate residual beam-associated background

Evaluate luminosity via QED process

Integrated luminosity of a data sample:

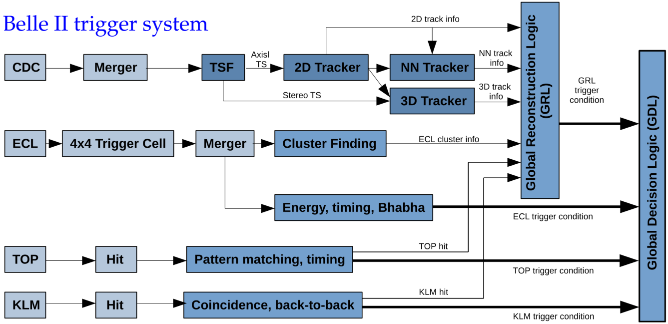
- is proportional to the number of collisions of data sample
- is essential for various cross-section related analysis
- limits the uncertainty of high-precision cross section measurements, such as $e^+e^- \to \pi^+\pi^-$ which contributes to $a_\mu^{\text{LO-HVP}}$ significantly.
- is usually measured by employing $e^+e^- \to e^+e^-$, $\mu^+\mu^-$, and $\gamma\gamma$ processes, where different sub-detectors are used to identify the signals and suppress the backgrounds
- is evaluated as

$$\mathcal{L}_{\text{int.}} = \frac{N_{\text{sig}}^{\text{obs}}}{\varepsilon_{\text{sig}}\,\sigma_{\text{sig}}^{\text{obs}}}$$

where the observed cross section of signal, i.e., $\sigma_{\text{sig}}^{\text{obs}}$, is well known

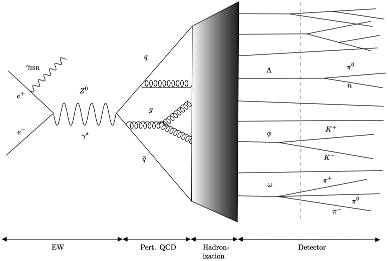
Trigger system of a composite detector

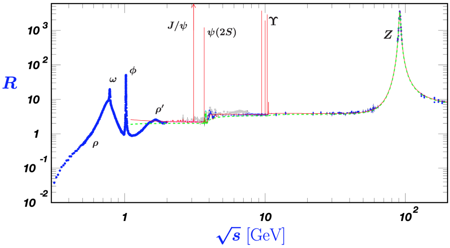
Level 1 trigger decision is made to readout raw data recorded in sub-detectors:



- Preliminary info provided by sub-detectors are used to form trigger conditions
- Different trigger conditions are combined to form various trigger channels
- Info of event stored in pipeline will be readout once a L1 signal is issued
- Typical hadronic event usually triggers various trigger channels at the same time

Signal simulation: the most challenging task



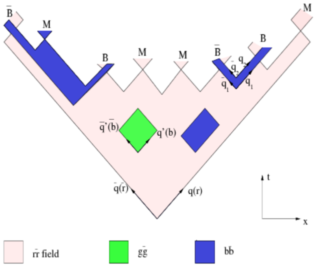


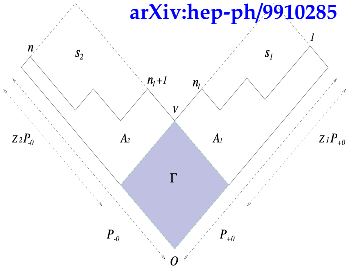
Reliable simulation of inclusive hadronic event at c.m. energy $\sqrt{s}$ requires:

- precise probability of emitting an ISR photon with specific energy and angle.
- accurate knowledge of all the allowed 1^{--} resonance states below $\sqrt{s}$.
- an effective phenomenological model to realize hadronization of partons.
- reliable production fractions and kinematic variables of some few-body channels, e.g., $e^+e^- \to \pi^+\pi^-$, 3π, 4π, K^+K^-, and almost all the open-charm channels.
- comparable multiplicities and momenta distributions of $p(\bar{p})$, $K^\pm$, and $\pi^\pm$ to data

Inclusive simulation model: LUARLW

An inclusive simulation model of hadronic events at low c.m. energy ($\sqrt{s} < 5$ GeV):





Main features of the LUARLW model:

- ✓ A self-consistent inclusive generator developed based on JETSET.
- ✓ Initial-state radiation (ISR) process is implemented from $2m_\pi$ to given $\sqrt{s}$.
- ✓ Kinematic quantities of initial hadrons are sampled by the Lund area law.
- ✓ Phenomenological parameters are tuned based on comparisons with data.

Functions of LUARLW

- After the ISR, resonance or continuum process is generated at the effective c.m. energy: $s' = s(1-x)$, where x is the energy ratio over E_{beam} carried by the ISR photon

$$e^+e^- \Rightarrow \gamma^* \Rightarrow \begin{cases} q\bar{q} \Rightarrow \text{string} \Rightarrow \text{hadrons} \\ gq\bar{q} \Rightarrow \text{string} + \text{string} \Rightarrow \text{hadrons} \end{cases}$$

$$e^+e^- \to \gamma^* \to \rho(770), \omega(782), \phi(1020), \cdots, \rho(1700), J/\psi, \psi', \psi'', \cdots$$

- Production probabilities of continuum and resonance processes are proportional to $\sigma^0_{\text{con}}(s')$ and $\sigma^0_{\text{res}}(s')$, respectively. The decay of above low mass resonances are modelled by branching fractions recorded in PDG.
- Decays of J/ψ, ψ' are simulated by Lund area law with specific branching fractions.

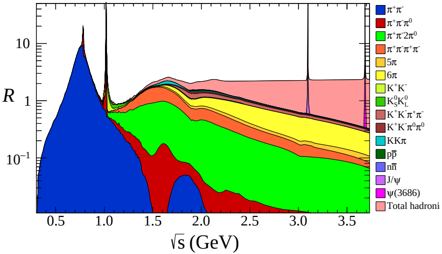
$$e^+e^- \Rightarrow J/\psi, \psi' \Rightarrow \begin{cases} \gamma^* \Rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^- \\ \gamma^* \Rightarrow q\bar{q} \Rightarrow \text{string} \Rightarrow \text{hadrons} \\ ggg \Rightarrow \text{string} + \text{string} + \text{string} \Rightarrow \text{hadrons} \\ \gamma gg \Rightarrow \text{string} + \text{string} \Rightarrow \text{hadrons} \\ \gamma\eta_c \Rightarrow gg \Rightarrow \text{string} + \text{string} \Rightarrow \text{hadrons} \\ \gamma + \text{exclusive radiative decay channels} \end{cases}$$

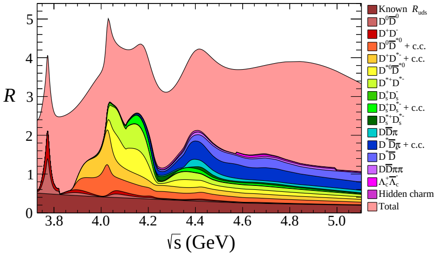
- ψ'' and heavier charmonium states decay to a pair of charmed mesons.

$\sigma^0_{\text{res}}(s)$ is modeled by BW function with parameters cited from PDG

Exclusive point of view: the HYBRID model

An alternative model is proposed by replacing the phenomenological hadronization scheme with published experimental data of exclusive measurements:





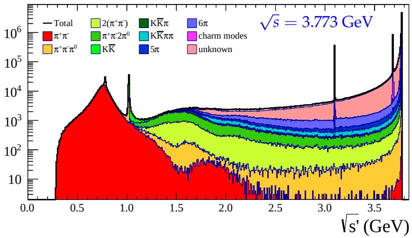
The main features of the HYBRID model:

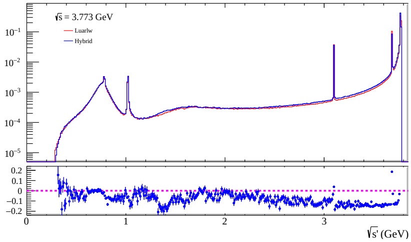
- ✓ Combination of THREE different well-established simulation models.
- ✓ As much as currently known experimental knowledges are implemented, especially the almost complete open-charm processes have been measured.
- ✓ Different ISR and VP schemes are used for a conservative cross check.

Construction of the HYBRID model

- HYBRID is consisted of CONEXC, PHOKHARA and LUARLW components.
- PHOKHARA is used to simulate 10 exclusive processes with known cross sections and intermediate states, $e^+e^- \to 2\pi, 3\pi, 4\pi$ etc..
- CONEXC simulates more than 100 exclusive processes with known cross sections, such as $e^+e^- \to K^+K^-\pi^0$, $K^0_S K^\pm \pi^\mp$, 5π, 6π below open-charm threshold, $D\bar{D}$, $D\bar{D}^*$, $\bar{D}D^*$, $D^*\bar{D}^*$, $D_s^+ D_s^-$, $D_s^\pm D_s^{*\mp}$, $D_s^{*+} D_s^{*-}$, $D\bar{D}\pi$, $D\bar{D}\pi\pi$, $D\bar{D}^*\pi$, $D^*\bar{D}\pi$, $\Lambda_c^+ \bar{\Lambda}_c^-$, and some hidden charm channels $\pi\pi J/\psi$, $\pi\pi\psi(3686)$, KKJ/ψ, $\pi\pi h_c$.
- As much as exclusive channels containing intermediate states are implemented in CONEXC with their contributions to the related inclusive channels are excluded.
- LUARLW model is partially used to simulate remain unknown processes, in which a set of chosen parameters are tuned after comparing HYBRID simulations with data.
- Processes simulated by PHOKHARA or CONEXC are prohibited in LUARLW to avoid excessive generation of some specific processes.
- Residual double-generatings among the three components are negligible.

The effective energy spectrum after ISR





- A plenty of exclusive channels are precisely simulated by HYBRID.
- Two simulation models result in consistent effective energy spectrum.
- Slight difference in $\sqrt{s'}$ spectrum is caused by different ISR schemes.

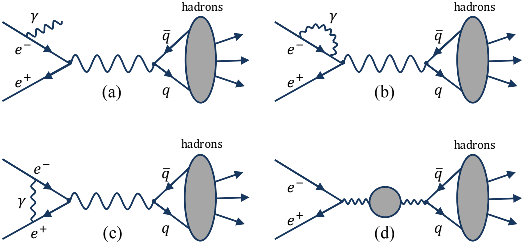
Consistent $\sqrt{s'}$ spectrum implies reliable simulation of hadronic events!

ISR effects in LUARLW: Feynman Diagram

Definition of initial-state radiation (ISR) factors:

$$(1+\delta)(s) \equiv \sigma^{\text{tot}}_{\text{had}}(s)/\sigma^{0}_{\text{had}}(s)$$

In LUARLW, the Feynman Diagram (FD) scheme is used to simulate ISR correction and calculate $(1+\delta)$. Following ISR procedures are considered:



The total hadronic cross section measured by experiment is the total effect of all these diagrams:

$$\sigma^{\text{tot}}_{\text{had}}(s) = \frac{\delta_{\text{vert}}\,\sigma^{0}_{\text{had}}(s)}{|1-\Pi(s)|^2} + \int_0^{x_m} \frac{F_{\text{FD}}(x,s)\,\sigma^{0}_{\text{had}}(s')}{|1-\Pi(s')|^2}\text{d}x, \text{ and } F_{\text{FD}}(x,s) \equiv \beta\frac{x^{\beta}}{x}\left(1-x+\frac{x^2}{2}\right)$$

ISR effects in HYBRID: Structure Function

The Structure Function (SF) scheme of ISR correction is implemented in HYBRID model:

$$\sigma^{\rm tot}_{\rm had}(s) = \int_0^{x_m} F_{\rm SF}(x,s)\,\frac{\sigma^0_{\rm had}(s')}{|1-\Pi(s')|^2}\,\mathrm{d}x.$$

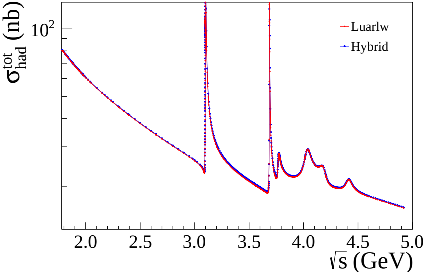
Taking the parameterization scheme given in **Nucl. Phys. B318, 1 (1989)** as an example:

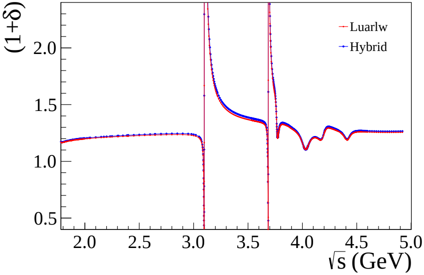
$$F_{\rm SF}(x,s) = \beta x^{\beta-1}\Delta - \beta\left(1-\frac{1}{2}x\right) - \frac{1}{8}\beta^2\left[4(2-x)\ln x + \frac{1+3(1-x)^2}{x}\ln(1-x) + 6 - x\right]$$

where

$$\Delta = 1 + \frac{\alpha}{\pi}\left(\frac{3}{2}L + \frac{\pi^2}{3} - 2\right) + \left(\frac{\alpha}{\pi}\right)^2\left\{\left[\frac{9}{8} - 2\zeta(2)\right]L^2 + \left[-\frac{45}{16} + \frac{11}{2}\zeta(2) + 3\zeta(3)\right]L - \frac{6}{5}\left[\zeta(2)\right]^2 - \frac{9}{2}\zeta(3) - 6\zeta(2)\ln 2 + \frac{3}{8}\zeta(2) + \frac{57}{12}\right\}.$$

ISR correction factors:





- Same input hadronic cross section $\sigma^0_{had}(s)$ but different VP operators $\Pi(s)$.
- Consistent total cross sections and ISR factors are obtained with FD and SF schemes.

According to the experimental definition of R, its uncertainty is roughly expressed as

$$\left(\frac{\Delta R}{R}\right)^2_{\text{sys}} = \left(\frac{\Delta \tilde{N}}{\tilde{N}}\right)^2 + \left(\frac{\Delta \sigma^0_{\mu\mu}}{\sigma^0_{\mu\mu}}\right)^2 + \left(\frac{\Delta \mathcal{L}_{\text{int.}}}{\mathcal{L}_{\text{int.}}}\right)^2 + \left(\frac{\Delta \varepsilon_{\text{trig}}}{\varepsilon_{\text{trig}}}\right)^2 + \left(\frac{\Delta \varepsilon_{\text{had}}}{\varepsilon_{\text{had}}}\right)^2 + \left[\frac{\Delta(1+\delta)}{(1+\delta)}\right]^2,$$

where

$$\tilde{N} = \frac{N^{\text{net}}_{\text{had}}}{\varepsilon_{\text{had}}} = \frac{N^{\text{obs}}_{\text{had}} - N_{\text{bkg}}}{\varepsilon_{\text{had}}}$$

In practice, the uncertainties are addressed in following different aspects:

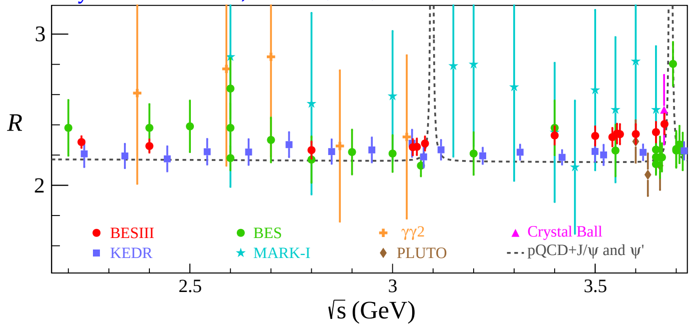
- Event selection: all implemented selection criteria are slightly varied, $0.40 \sim 0.80\%$.
- Background estimation: use different methods and simulation models, $0.30 \sim 0.40\%$.
- $\sigma^0_{\mu\mu}$: uncertainty is negligible due to the high precision of QED.
- $\mathcal{L}_{\text{int.}}$: uncertainty is directly cited from published results, $0.80 \to 0.50\%$.
- $\varepsilon_{\text{trig}}$: approaches to 100% with an uncertainty less than 0.10%.
- Signal simulation: differences of R resulted by LUARLW and HYBRID is taken, $1.00 \sim 2.50\%$.
- ISR factor: considered in calculation precision and uncertainty in $\sigma^0_{\text{had}}(s)$, $0.50 \sim 1.00\%$.

A total uncertainty no larger than 3.0% could be achieved.

Measured R values between 2.2 ~ 3.7 GeV

After successfully constructing the LUARLW and HYBRID models, the first bunch of R has been published in 2022:

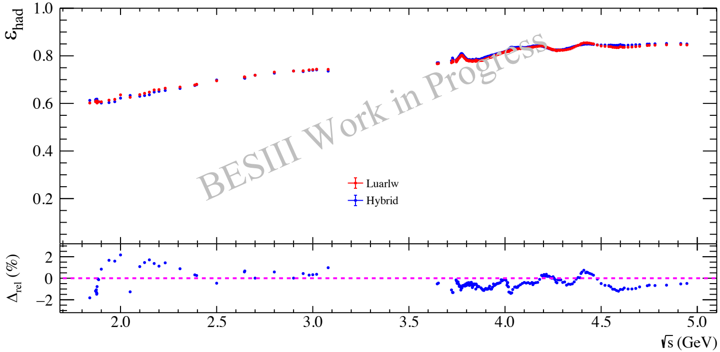
Phys. Rev. Lett. 128, 062004 (2022)



- The accuracy is better than 2.6% below 3.1 GeV and 3.0% above.
- Larger than the pQCD prediction by 2.7σ between 3.4 ~ 3.6 GeV.
- A plenty of checks have been carried out before and after the publication for R above 3.4 GeV, the results are found to be solid.

More results are on the way

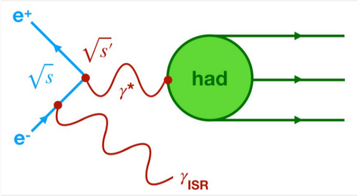
There are more scan data sets (~200 points) are taken at BESIII, where both the LUARLW and HYBRID models are ready:

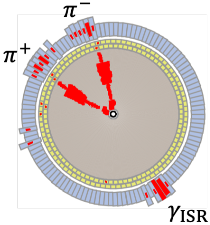


- Reliable and consistent $\varepsilon_{\rm had}$ is obtained from $\sqrt{s} = 1.8$ to 5.0 GeV.
- The R value could be precisely determined at these data points, where those below 2.0 GeV and above 3.7 GeV are of great interests.

A new idea: measure R via ISR technique

The ISR approach could access the R value below $\sqrt{s} = 2.0$ GeV:





In practice, the signal selection strategy is:

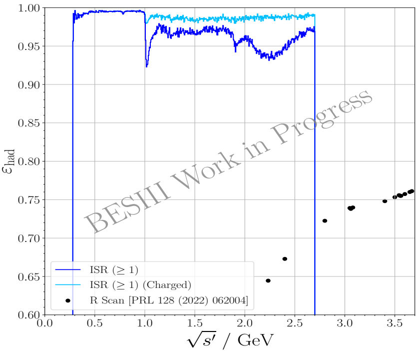
- require an energetic photon in barrel of EMC: $E_\gamma > 1.2$ GeV and $|\cos\theta| < 0.8$.
- there should be at least one charged track in barrel region of the detector.
- suppress Bhabha and Di-gamma background veto **meson decays** into photons.
- reconstruct mass of hadronic final states from recoil of the ISR photon:

$$s' = m_{\text{had}}^2 = s - 2E_\gamma\sqrt{s}$$

R measurement via ISR technique

Advantages:

- ✓ Very high detection efficiency due to the sufficient boost of ISR photon.
- ✓ Less reliant on the simulation of the hadronic events in data sample
- ✓ Single measurement accesses m_{had} down to threshold of $\pi^+\pi^-$
- ✓ Fully inclusive for final state radiation and higher order ISR effects
- ✓ Independent of previous R analysis based on energy scan method

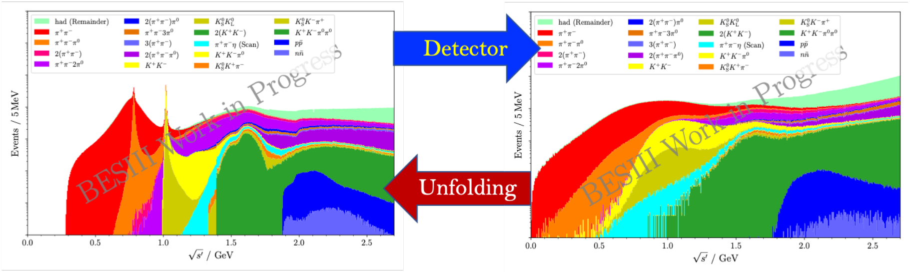


Challenges:

- Significant QED backgrounds due to their higher cross sections: dedicated PID needed.
- Background from non-ISR hadronic events containing π^0/η: dedicated vetoes
- Limited resolution in m_{had} due to high energy of ISR photon: unfolding of m_{had}

R measurement via ISR technique

Unfolding is a powerful approach to recover the truth m_{had} spectrum from detected one:

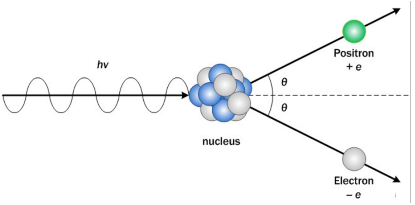


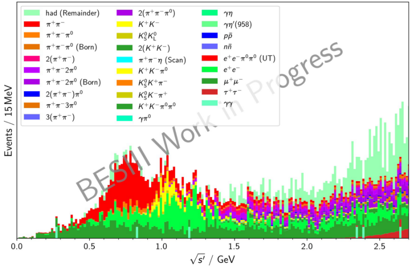
- Large smearing is caused by detector: track lost, photon energy leakage, and so on.
- An un-biased unfolding is crucial to recover the true hadronic mass spectrum.
- Fractions and shapes of $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$ channels in the signal MC samples producing response matrix are modified to test the unfolding method.
- Unfolded spectra keep unchanged within the corresponding standard deviation.

Improve m_{had} by photon conversion event

An alternative method to tag the ISR photon:

- Energetic ISR photon could convert to a pair of e^+e^- via interaction with detector material, especially beam-pipe.
- Tracks of produced e^+e^- pair could be well reconstructed by the tracking sub-detectors
- Improve the m_{had} resolution by large factors thanks to precisely measured e^+ and e^- momenta
- As a result, the statistics is significantly reduced due to the low probability of photon conversion
- High potential for the new high-statistics data sets at BESIII and Belle II





Summary

- Precise R values are highly desired by $\alpha_{\text{QED}}(s)$ and a_μ^{HVP} evaluations.
- JGU-Mainz group plays an important role in R measurement.
- New R value results from $\sqrt{s} = 1.8$ to 5.0 GeV are around the corner.
- Numerous efforts are made in R measurement via ISR technique:
  - QED and non-ISR hadronic background are understood
  - Unfolding approach is effective and robust in extracting truth m_{had}
  - Few percent accuracy is targeted to shed light on current discrepancy in obtained a_μ^{HVP} between data-driven and Lattice QCD
  - Higher potential at Belle-II by tagging conversion events of ISR photon

Thanks for your attention!