Updates to the KNTW Dispersive HVP Analysis and Implications for the Evaluation of a_e^{HVP} and α

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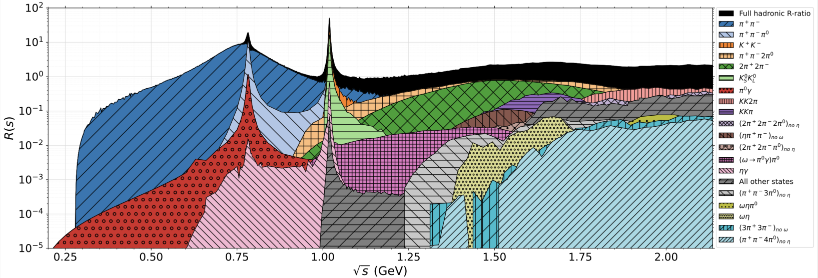
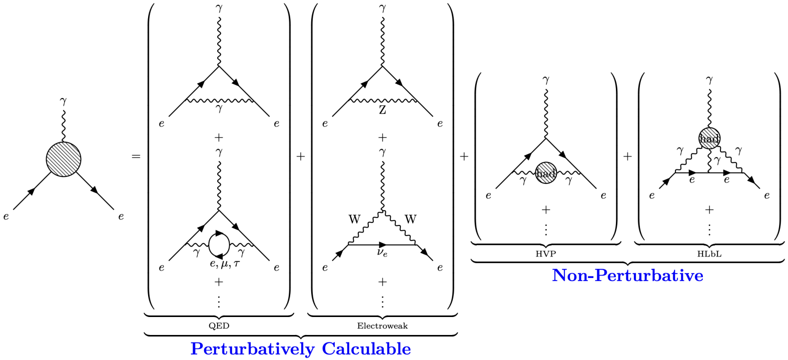
Science and Technology Facilities Council

LEVERHULME TRUST



Hadronic Vacuum Polarisation

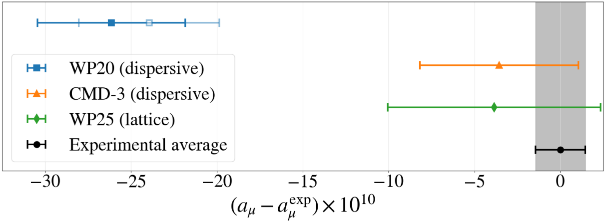


$$a_e \Longleftrightarrow \alpha$$

Overview of Methods

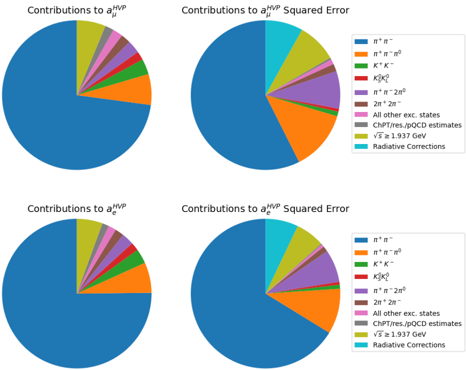
Problem: QCD is non-perturbative at low $\sqrt{s}$.
Implication: HVP of photon cannot be calculated in loop integrals etc.

- Solution: discretise spacetime and perform lattice QCD calculation.
  - → See Davide's talk after coffee...
- Solution: Measure t-channel $e - \mu$ scattering.
  - → Upcoming MUonE experiment.
- Solution: dispersion integral over the $e^+e^- \to$ *hadrons* cross section.
  - → Dispersive/data-driven methods...
  - → The main focus of this presentation – including KNT**W** data compilation.



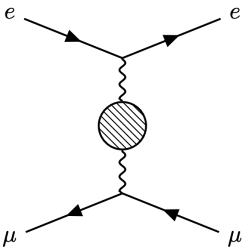
Disclaimer:

Plots may show a_μ rather than a_e or α.
However, $K_e(s) \sim ({}^{m_e}/{}_{m_\mu})^2 K_\mu(s)$.
The same general trends/arguments hold.



MUonE – Background

Problem: QCD is non-perturbative at low $\sqrt{s}$.
Implication: HVP of photon cannot be calculated in loop integrals etc.



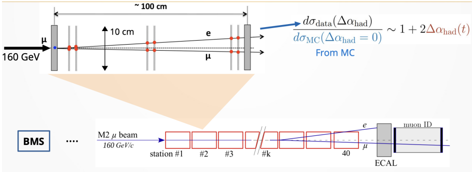
$$\Longrightarrow \Delta\alpha(t) = \Delta\alpha_{\text{lep}}(t) + \Delta\alpha_{\text{had}}(t) + \Delta\alpha_{\text{top}}(t)$$

Calculate (timelike) HVP using measured running of α (spacelike).

$$a_e^{\text{HVP}} = \frac{\alpha(0)}{\pi} \int_0^1 dx \left\{ (1-x)\Delta\alpha_{\text{had}}^{(5)}\left(t(x)\right) \right\} \quad \text{with} \quad t(x) = \frac{x^2 m_e^2}{x-1} < 0$$

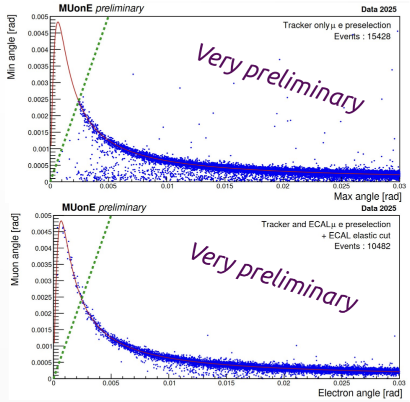
MUonE – Setup and Expected Results

Problem: QCD is non-perturbative at low $\sqrt{s}$.
Implication: HVP of photon cannot be calculated in loop integrals etc.



MUonE, R. Pilato - 2025 Muon $g-2$ TI Workshop

- 3yr post LS3 $\implies$ a_μ^{HVP} at 0.5%.
- Measure scattering angles and compare differential distributions to MC.
- Successful test run complete - test analysis for $\alpha_{\text{lep}}(t) \sim 10\alpha_{\text{had.}}(t)$.
- Presently tags angle *size*, μID feasibility being studied.



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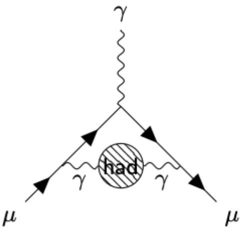
Dispersive Method

Problem: QCD is non-perturbative at low $\sqrt{s}$.
Implication: HVP of photon cannot be calculated in loop integrals etc.

- Solution: dispersion integral over the $e^+e^- \to$ *hadrons* cross section.

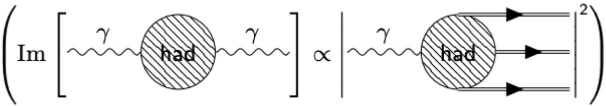


$$\propto \Pi^{had.}_{\mu\nu}(q^2) \quad \textit{by definition}$$

$$\propto \Pi^{had.}(q^2) \quad \textit{due to gauge invariance}$$

$$\propto \int \frac{\mathrm{Im}\left\{\Pi^{had.}(s)\right\}}{s\,(s-q^2-i\varepsilon)}ds \quad \textit{by analyticity and Cauchy's theorem}$$

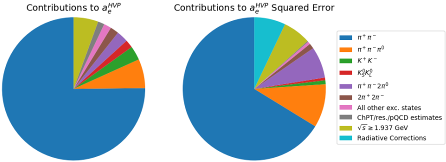
$$\propto \int \frac{\sigma^0_{had.}(s)}{s-q^2-i\varepsilon}ds \quad \textit{by unitarity} \implies \textit{the Optical Theorem}$$

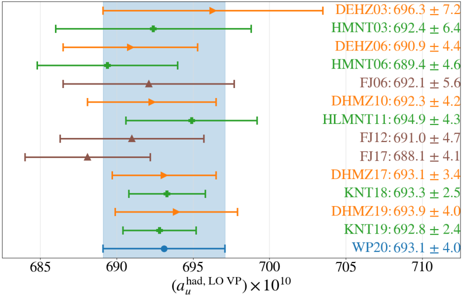


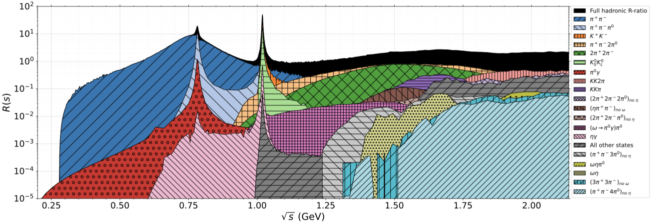
- For > 50 years, low energy $e^+e^- \to$ *hadrons* data have been collected...

Hadronic Data

- $\sim$ 11 300 data points in $>$ 50 hadronic channels.
- Dominated ($\sim$ 75%) by $e^+e^- \to \pi^+\pi^-$.

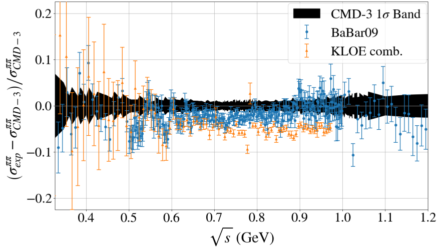


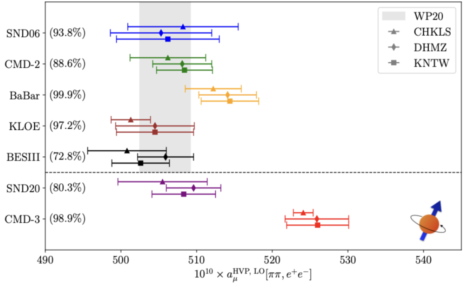




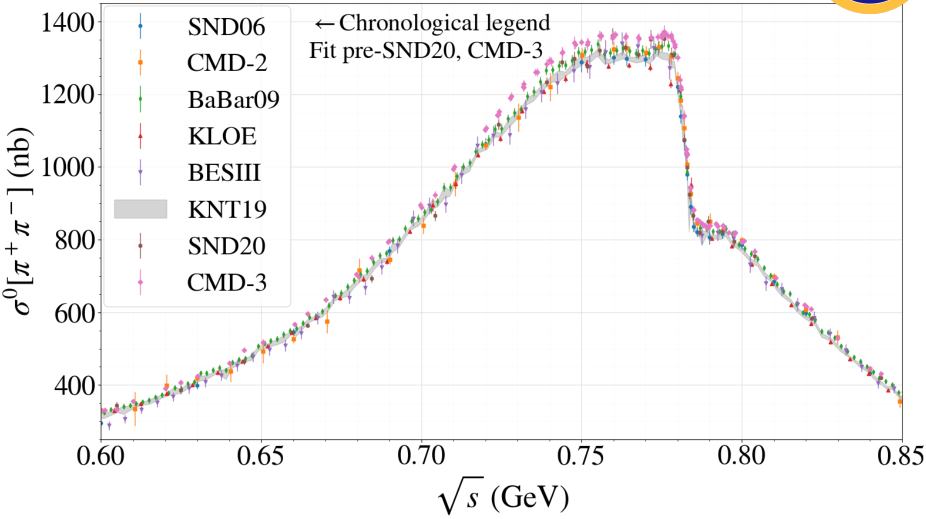
Tensions in $\pi^+\pi^-$

- Historic Problem: $\sim 2.5\sigma$ KLOE/BaBar tension.
- Historic Solution: local error inflation; additional ‘ad-hoc’ systematic.
- **Current Problem:** $> 5\sigma$ KLOE/CMD-3 tension, $\sim 2.5\sigma$ BaBar/CMD-3 tension!
  - CMD-3 ‘corroborated’ by new SND preliminary.
  - BaBar confirm their earlier result.
- **Current Solution:** None as yet...
  - Nothing suggests earlier data is defective.
  - Dispersive method is robust.
- WP25 recommends lattice value – **not** the end of the story.
- Need to understand dispersive or incomplete understanding of $g-2$s.
- Tensions must be resolved/managed to make dispersive predictions...

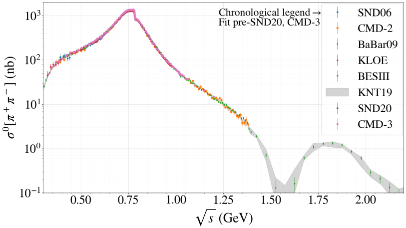


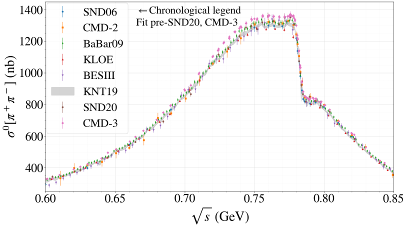


WP25, R. Aliberti et al - Phys. Rept. 1143 (2025)

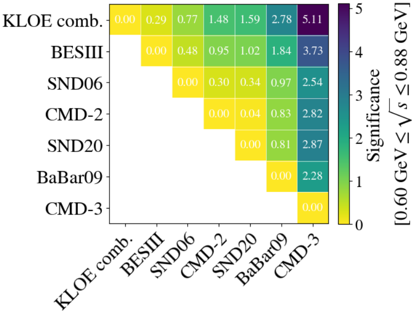

SND06
CMD-2
BaBar09
KLOE
BESIII
KNT19
SND20
CMD-3
←Chronological legend
Fit pre-SND20, CMD-3
1400
1200
1000
800
600
400
0.60
0.65
0.70
0.75
0.80
0.85
$\sigma^0[\pi^+\pi^-]$ (nb)
$\sqrt{s}$ (GeV)

The $\pi^+\pi^-$ Channel



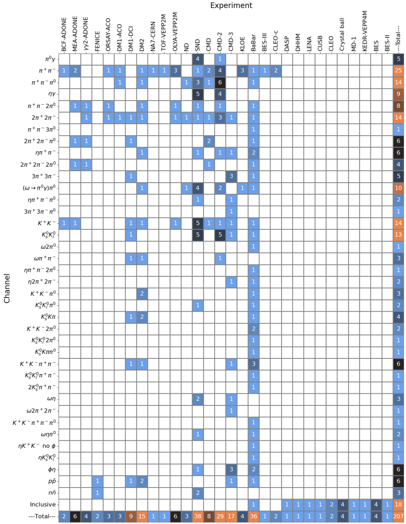


- Shown: most precise datasets and KNT**19** combination $\implies$ two-pion spectrum.
- Zoom in on important ρ-resonance region.
- Demonstrates universal CMD-3 excess.
- **Cannot** average such datasets without improved understanding of tensions.



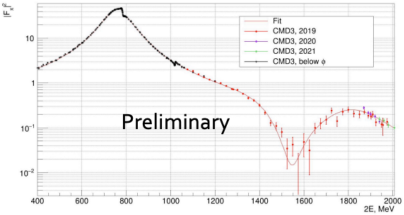
Low Energy Collider Experiments

- Interested in e^+e^- at $\sqrt{s} \lesssim 11.2$ GeV.
- Particularly, exclusive measurements $\lesssim 2$ GeV.
- Rough procedure:
  - Collect dataset (high luminosity to maximise available statistics).
  - Identify final state kinematics – Monte Carlo, prior measurement, theoretical calculation.
  - Apply selection cuts to discard background and keep signal.
  - Apply corrections: radiative events, discarded signal, missed data...
  - Correct event counts at each $\sqrt{s}$, calculate cross section using luminosity.
  - Calculate stat., syst. uncertainties (and correlations).
- Present experiments return/aim for $\sim 0.5\%$ in leading $\pi^+\pi^-$ channel.

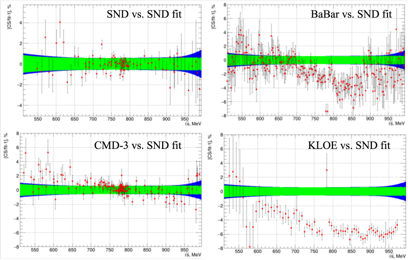


Direct Scan Experiments ($\pi^+\pi^-$)

- All experiments pre-2007 are DS (CMD(-2), OLYA, Crystal ball, DM1/2, MEA, $\gamma\gamma2$, ...).
- Modern DS experiments: SND (post-upgrade) and CMD-3.
- Principle: beam energies changed to achieve each measured $\sqrt{s}$ value.
- Procedure very much as previously outlined.
- Statistics typically uncorrelated point-to-point, systematics less clear.
- Energy limited compared to radiative return...
- Recent: CMD-3, SND preliminary.
- Upcoming ($\sim$ 2 years): new CMD-3 measurements (high energy and w/ more statistics).

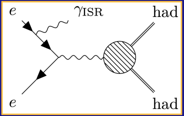


CMD-3, E. Solodov - 2025 Muon $g-2$ TI Workshop

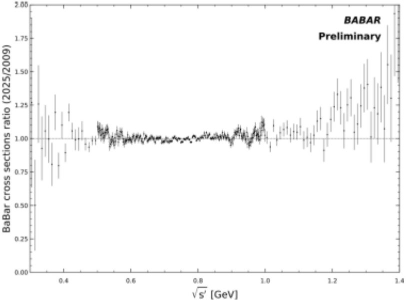


SND, A. Kupich - 2025 Muon $g-2$ TI Workshop

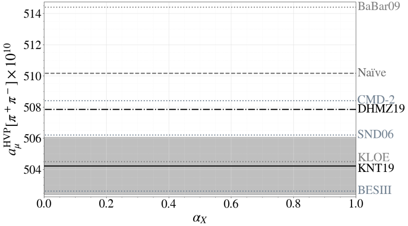
Radiative Return Experiments ($\pi^+\pi^-$)



- KLOE, BaBar, BESIII and Belle are RR (or 'ISR method') experiments.
- All ran/run on resonances (ϕ, Υ, ψ) $\implies$ high luminosity but 'wrong' energy.
- ISR method:
  - High energy $e^\pm$ beam emits a (tagged) photon from the initial state.
  - Cross section of $e^+e^- \to X\gamma_{\text{ISR}}$ measured at $M_{\phi,\psi,\Upsilon,\ldots}$.
  - Radiator Function relates M measurement to cross section at energy of interest.
- Requires good understanding of radiative corrections $\implies$ RMCL2 initiative.
- Unfolding $\implies$ correlated statistics, single dataset $\implies$ correlated systematics.
- Recent: BaBar measurement preliminary.
- Upcoming ($\sim$ 2 years): new KLOE, BESIII and Belle measurements.



BaBar, A. Pinto - 2025 Muon $g-2$ TI Workshop



CHKLS Recent Work

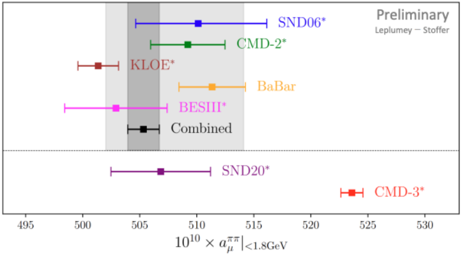
Unitarity & analyticity $\Longrightarrow$ constraints on spectral lineshapes $\Longrightarrow$ dispersive fit.

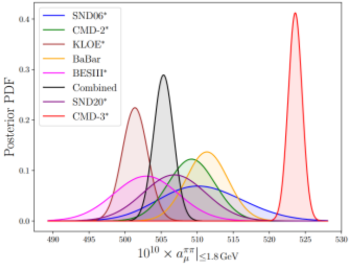
$$F_\pi^V(s) = \underbrace{\Omega_1^1(s)}_{\text{2 param.s}} \times \underbrace{G_{\omega(\phi)}(s)}_{\text{3(6) param.s}} \times G_{\text{in}}(s)$$

- Improved inelastic function:

$$G_{\text{in}}(z) = \frac{1}{\phi(z)} \frac{P_N(z)}{\prod_j (z - z_j)(z - z_j^*)}$$

  for OF $\phi(s)$, polynomial P_N and poles s_j.
- Bayesian parameter interference for improved fitting.
- Fit **exacerbates** CMD-3 tension.
- Strong correlation of lattice windows $\Longrightarrow$ potential issues with lattice-dispersive hybrid approach.



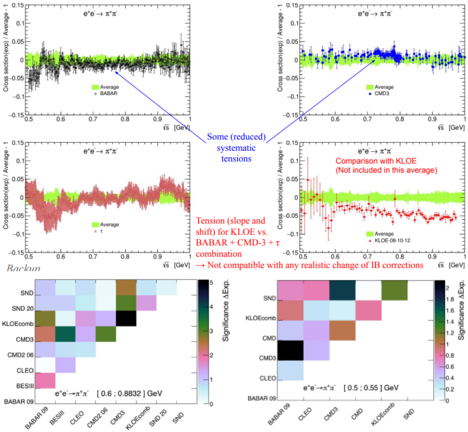


T. Leplumey - 8th Plenary Workshop of the Muon g − 2 TI

DHMZ Recent Work

Dataset interpolating splines averaged, weighting only by local correlations.

- Significant effort to better understand tensions.
- Proposed combination using only τ, BaBar and CMD-3:
  - Agreement was quite good.
  - However, reasons for KLOE/BESIII exclusion disfavoured.
  - Recent τ work $\implies$ potential large shift and change of landscape.
- Local tensions assessment:
  - Measurements exhibit different levels of tension in different regions.
  - Most significant tensions are on and above ρ peak.
  - Datasets are compatible at low energies.



B. Malaescu - 8th Plenary Workshop of the Muon $g-2$ TI

KNTW Data Compilation

- KNTW philosophy is to be data-driven $\implies$ minimal modelling assumptions.
- Combination procedure:
  - All non-defective data used.
  - Radiative corrections from robust routines.
  - Clustering – dynamic data-driven combination.
  - Fitting – incorporate full correlation information whilst avoiding bias.
  - Integrate, e.g.

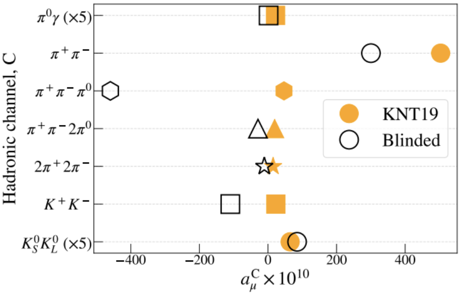
$$a_e^{\mathrm{HVP,LO}} = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty} ds\left\{\sigma^0(s)K_e(s)\right\}$$

- Analysis choices:
  - Re-binning procedure;
  - Fitting procedure - correlations;
  - Use of additional constraints;
  - Error inflation;
  - Interpolation/integration...

$\implies$ changes in central value.

- Need to make sure we make optimal choices $\implies$ exhaustive reanalysis.
- Avoid bias $\implies$ blinded analysis.
- Channel dependent blinding kernel B_i:

$$a_e^{\mathrm{blind}}[i] = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty} ds\left\{\sigma_i^0(s)K_e(s)B_i(s)\right\}$$

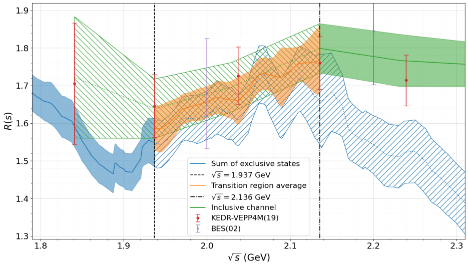


KNTW Phys. Rev. D 111 L011901 (2025)

"Re-Baselining"

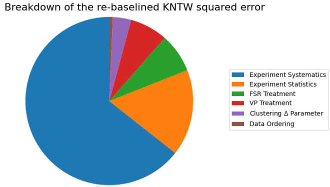
Correction/Completion

- Full code rewrite FORTRAN→Python.
- (*Minor*) Corrections of KNT19 analysis:
  - Checks of database against literature.
  - More detailed systematic covariance matrix construction.
- Completions of KNT19 analysis features:
  - Lagrange polynomial interpolation of all resonances.
  - Exclusive/inclusive transition region.

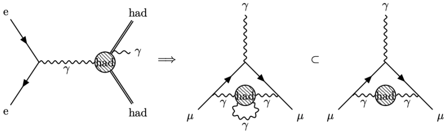


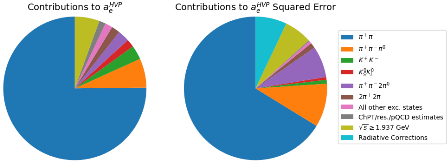
Method Systematics

- Quantify quality of current method.
- KNTW clustering procedure (simplified)
  - Optimised cluster with Δ for each channel.
  - Work through ordered points – may join clusters if with Δ, else form new.
  - Continues until fully rebinned.
- Two 'unfixed' aspects: Δ and ordering.
- Straightforward to estimate systematics.
- $<$ 5% of KNT19 squared error.



FSR for Dispersive HVP



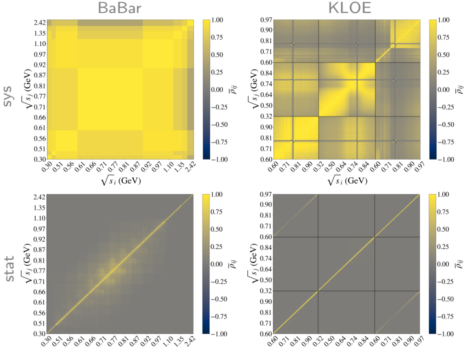


- Need cross sections to include FSR.
- Need to add it in where it has been excluded/removed.
- KNT19: $F \times$ sQED in $\pi^+\pi^-$.
- Hard ($F \times$ sQED) FSR negligible in $K\bar{K}$.
- KNT19 treating $\pi^+\pi^-$ & $K\bar{K}$ corrects nearly 80% of a_e contributions.
- Adding $\pi^+\pi^-\pi^0$ and Inclusive channel treatments $\Longrightarrow$ 93%.

- Inclusive Channel (> 2 GeV):
  - Approx. described by QCD $\Longrightarrow$ fermionic FSR correction.
  - Most datasets include FSR, four need hard adding in.
  - $\Longrightarrow$ 20% drop in $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ error!
- $\pi^+\pi^-\pi^0$:
  - MH et al. derive 3π FSR correction.
  - Again only hard correction needed – can be derived.
  - Necessary corrections $< 0.1\%$.

Correlation Study: Motivation and Procedure

- Systematics estimated by experiments $\implies$ correlations inherently uncertain.
- Syst. corr. use impacts DHMZ/KNTW.
- Historic understanding: full syst. correlations $\implies$ lower value.
- WP20 featured KLOE-BaBar systematic partially as correlations proxy.



Analysis choice impacts final value

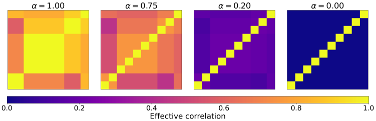
↓

Quantify systematic for new KNTW analysis

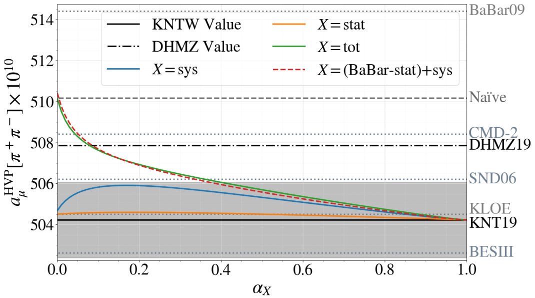
- Develop procedure now (with WP20 data) for use in KNTW202X.
- Will *then* be CMD-3 inclusive.
- Define simple procedure for conservative estimate.

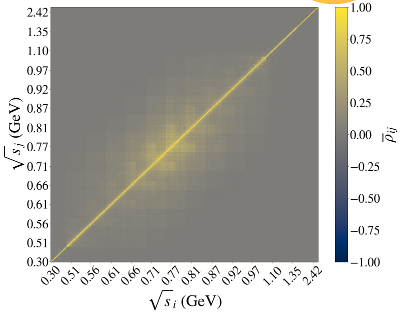
$$\tilde{C}_{ij} = \alpha\, C_{ij} + (1-\alpha)\,\mathrm{diag}\left[C_{ij}\right]$$

$$0 \leq \alpha \leq 1$$



Correlation Study: Results and Implications





- Maximum deviation $\iff$ conservative uncertainty.

$$\underbrace{d^{\rho} a_\mu^{\pi\pi}}_{1.7\times 10^{-10}} < \underbrace{d^{\text{K-B}} a_\mu^{\pi\pi}}_{2.8\times 10^{-10}}$$

- Prevents future error overestimates; does not explain naïve average difference...
- Variation of just α_{stat} – minimal effect.
- Emergent effect when varying stat and sys together (tot).
- Arises from mid-range **BaBar** statistics.
- Statistical correlations confidently known – should not be varied.

KLOE-BaBar over-conservative. Global shifts from BaBar permitted $\because$ stat. corrs.

Implications of the New Studies

KNTW

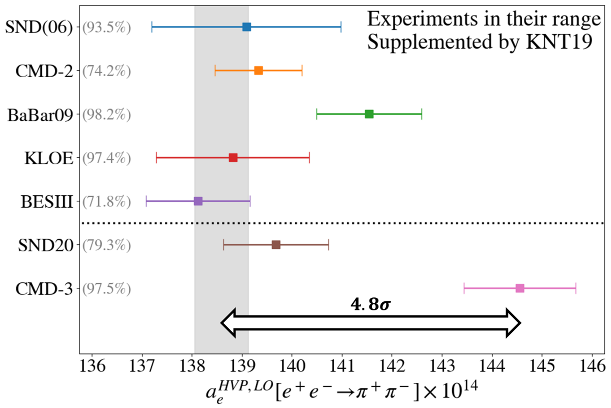
- Exact numbers cannot yet be given for changes in central values:
  - Changes due to the re-baselining are small.
  - New methods may be brought in for KNTW202x – method uncertainties should still be at $\sim$ 5% level.
  - FSR and VP – radiative correction uncertainties potentially reduced $\times 3$.
  - Correlation strength uncertainty – first of potentially many to quantify effects of 'analysis choices'.
- A new result will be presented when we are confident all procedures are optimised and have unblinded.

General

- Work proceeds for all dispersive analysis groups.
- Developments with τ isospin breaking corrections may permit use of these additional datasets (timescale $\mathcal{O}($ few years$)$).
- Once dispersive tensions are understood, precision may optimised by a lattice-dispersive hybrid.

What impact will this have on a_e and α?

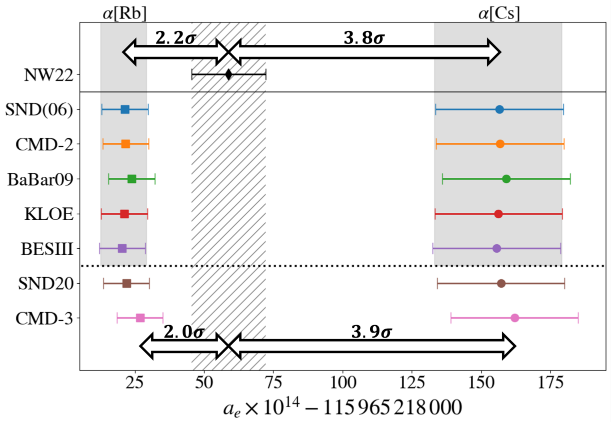
Dispersive Status of of a_e^{HVP}



Values evaluated in the KNTW framework. Each dataset is used from its lowest energy up to the lower of its maximum and 1 GeV. Bracketed percentages indicate the fraction of $a_e^{\text{HVP,LO}}$[KNT19] this energy range corresponds to. The shaded grey band indicates $a_e^{\text{HVP,LO}}$[KNT19] and its error.

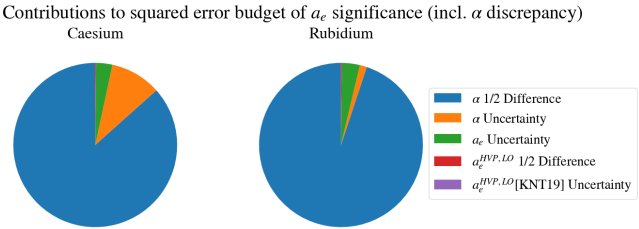
- $K_e(s)$ weights small $\sqrt{s}$ more heavily then $K_\mu(s)$.
- Larger errors and smaller differences here $\Longrightarrow$ smaller 'headline' tension.
- Considerable cross section level tensions still stymie combination.
- Tension is still uncomfortably large: more work still needed to produce a_e^{HVP} value.
- When does this tension become relevant?

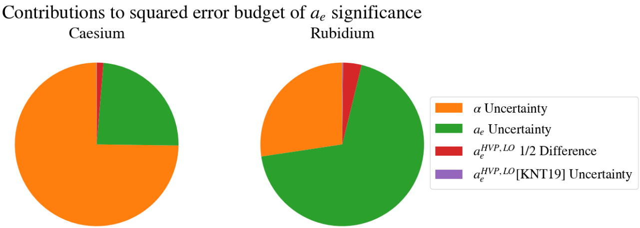
Dispersive Status of of a_e



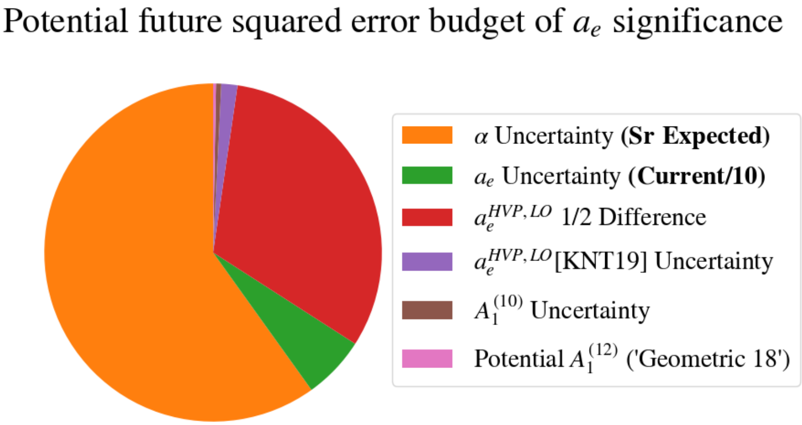
- Dispersive discrepancies not presently relevant at the level of a_e uncertainty.
- Hierarchy of significance affecting effects:
  1) α[Rb]-α[Cs] difference;
  2) α[Rb] and α[Cs] uncertainties;
  3) Experimental precision of a_e;
  4) Dispersive discrepancies;
  5) Precision of data inputs.
- Situation will be much the same for α.

Hierarchy of Effects

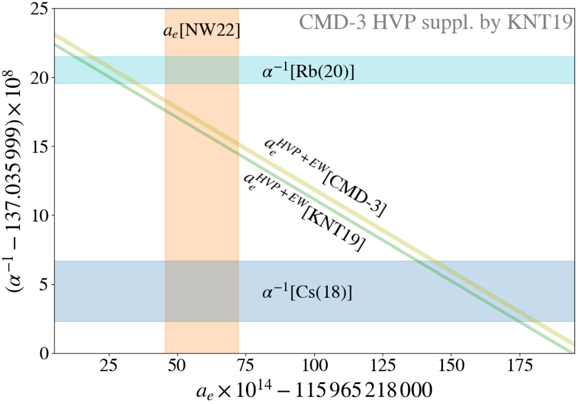




Future Hierarchy...



Dispersive Status of α



- Axes show a_e, α^{-1} measurements & errors.
- Diagonal HVP values: LO KNT19 & supplemented CMD-3.
- Diagonals errors are HVP LO→NNLO, EW & $A_1^{(10)}$.
- Potential for future three-way tension illustrated.
- CMD-3 tension w/ a_e would imply larger 'BSM' than a_μ.

Conclusions

Dispersive is not dead!

- Dispersive $g-2$ faces challenges due to underlying non-trivial dataset discrepancies.
- Analysis groups are working to handle tensions.
- KNTW are addressing these with a full detailed re-analysis under blinding.
- Significant at the level of a_e^{HVP} but not a_e.
- Potential for future three-way tension between measured a_e and α, and theory a_e^{had}.