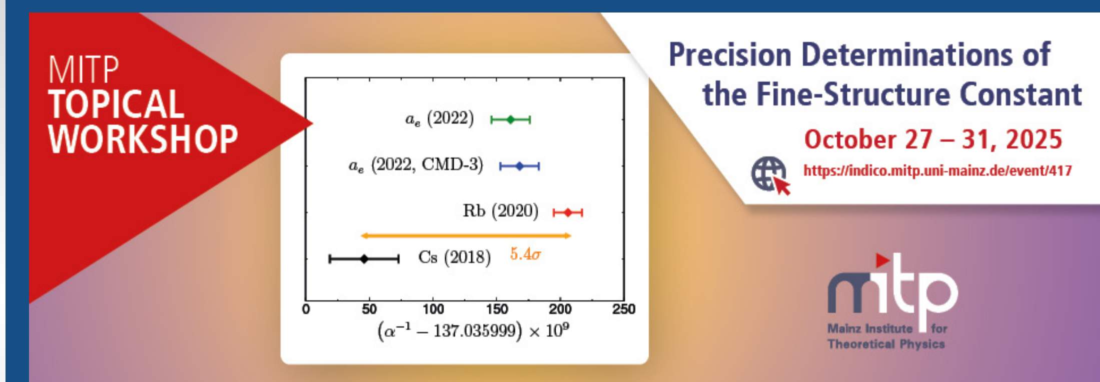
The HVP contribution to the lepton anomalous magnetic moments from first principles

Davide Giusti

JÜLICH Forschungszentrum

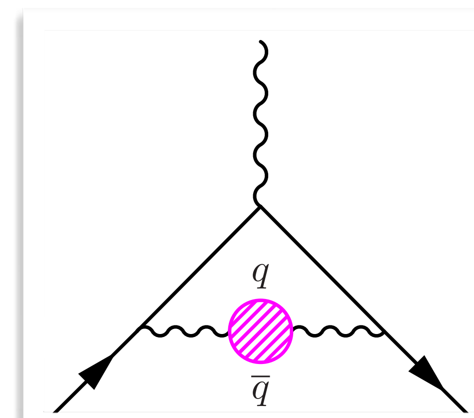


Precision Determination of the Fine-Structure Constant

Mainz

30th October 2025

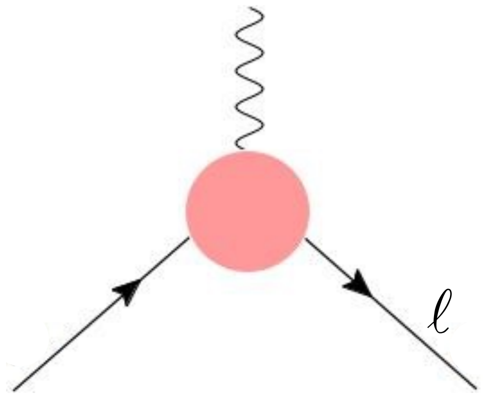
OUTLINE



- Introduction
- HVP from the lattice & window obs.
- The BMW/DMZ-24 calculation

Introduction

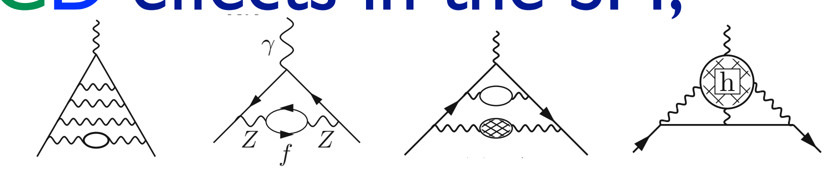
Lepton magnetic anomaly

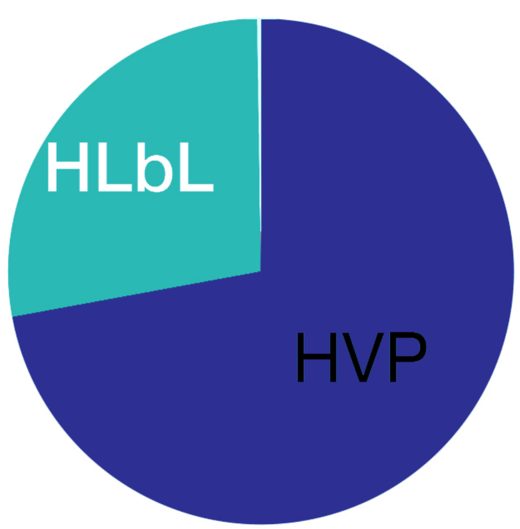


$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

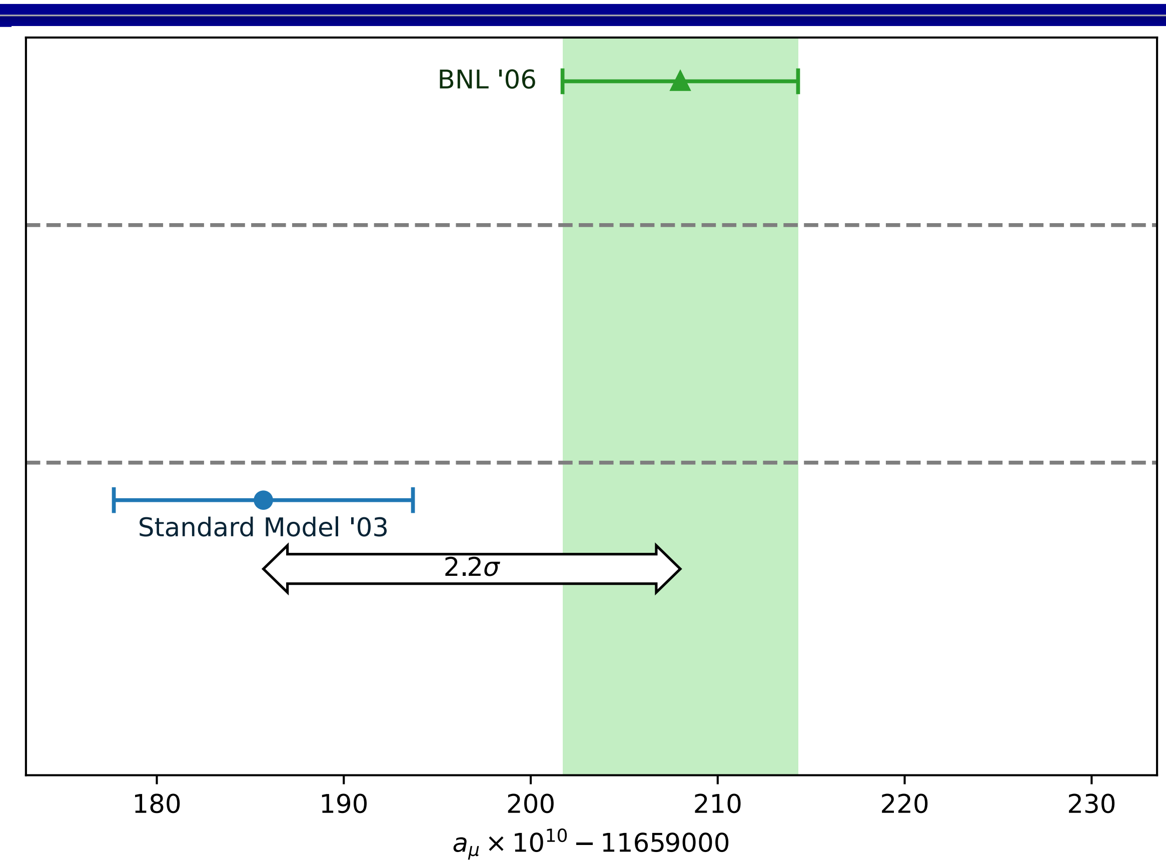
<u>lepton anomalous magnetic moment:</u> $a_\ell \equiv \frac{g_\ell - 2}{2} = F_2(0)$

- is generated by quantum loops;
- receives contribution from QED, EW and QCD effects in the SM;
- is a sensitive probe of new physics





Theory error dominated by hadronic physics



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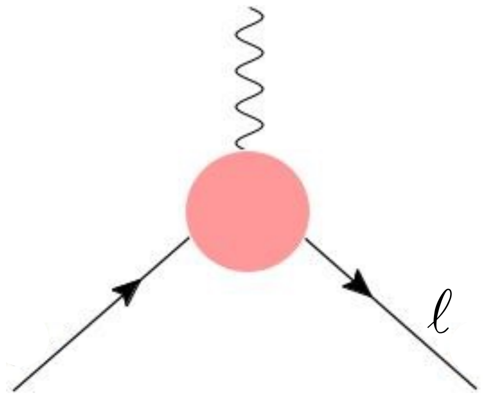

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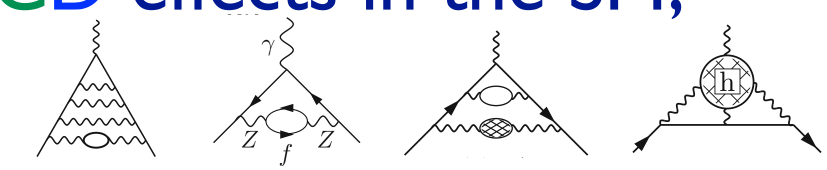

SM contributions to $a_\mu[\times 10^{10}]$

5-loop QED	11 658 471.8931(104)
2-loop EW	15.36(10)
HVP LO	693.1(4.0)
HVP NLO	-9.83(7)
HVP NNLO	1.24(1)
HLbL	9.2(1.8)

Aoyama *et al.* [WP] 2020

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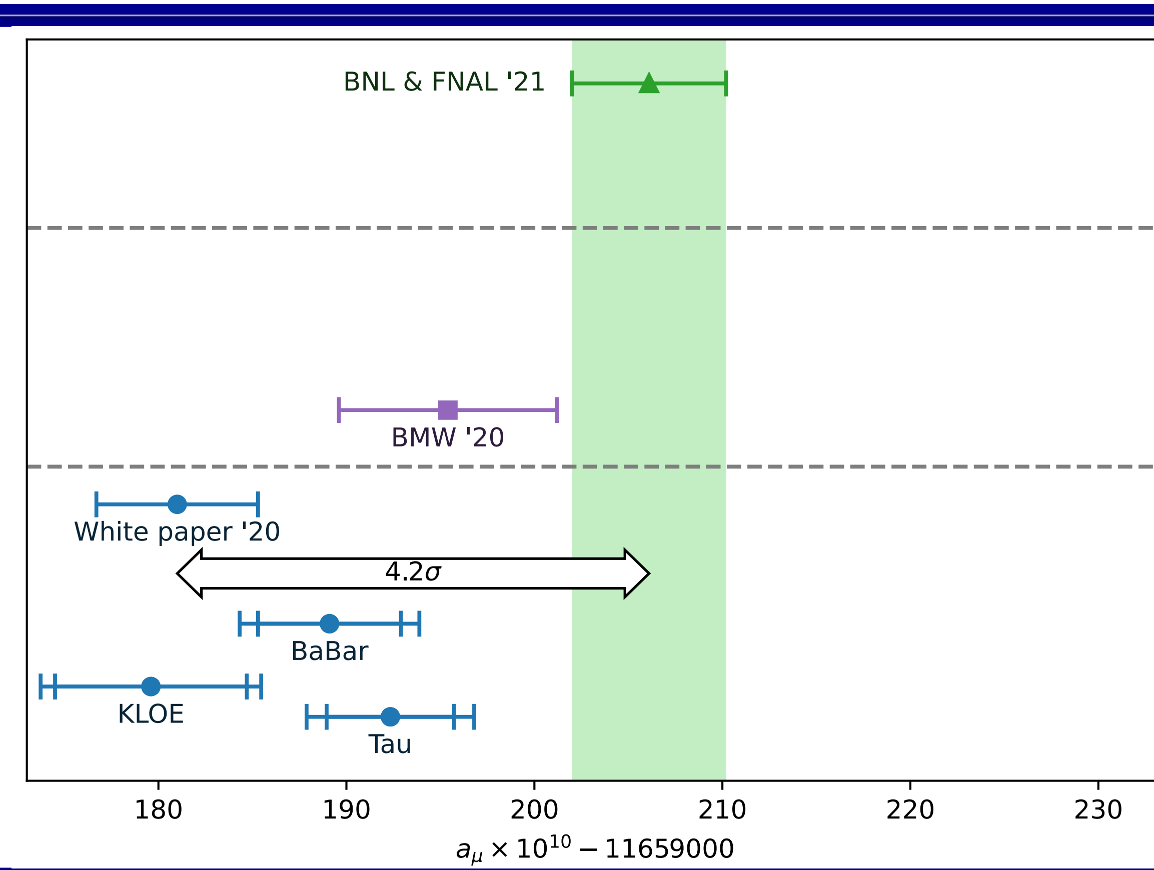

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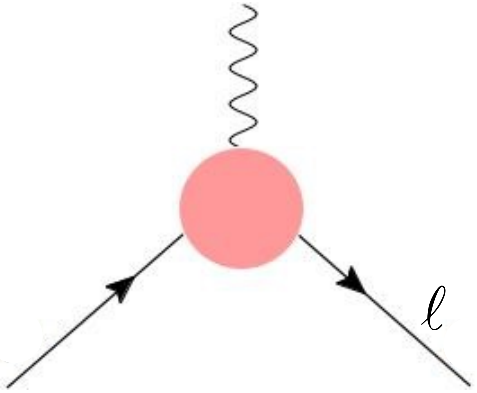

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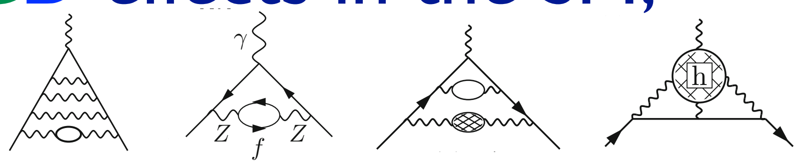

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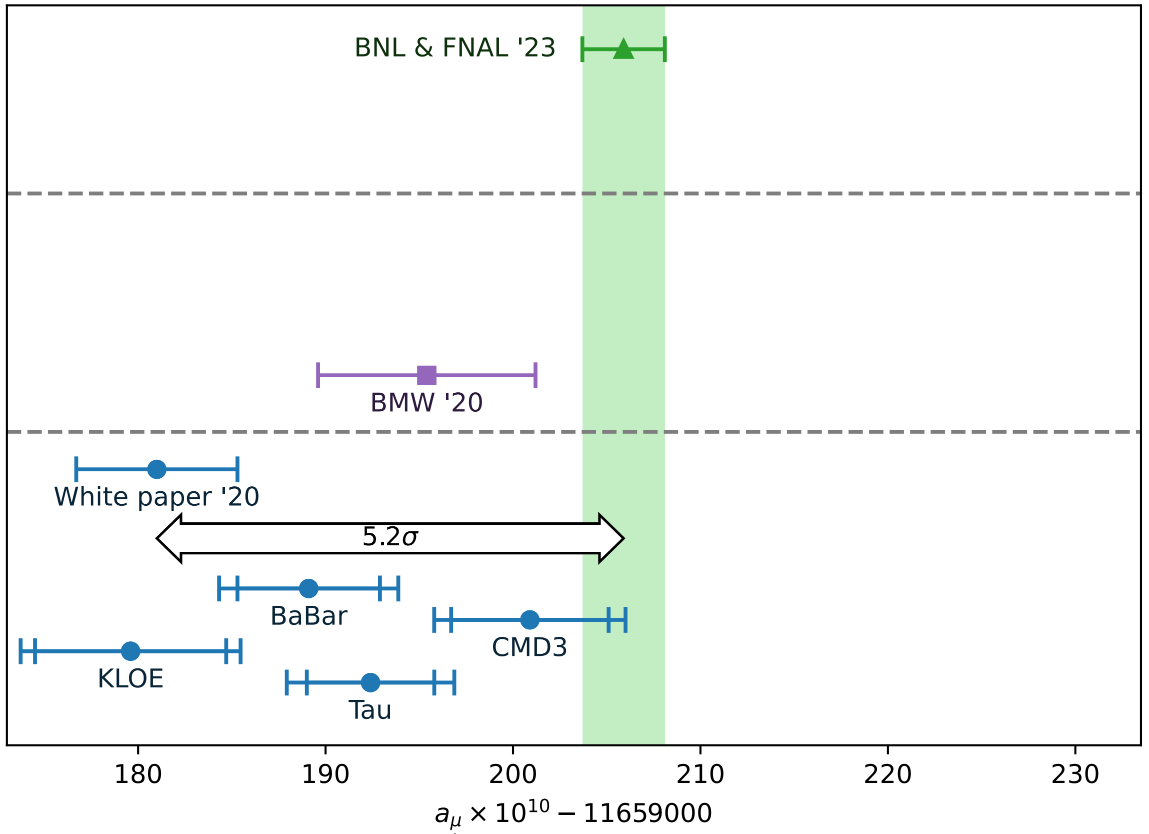

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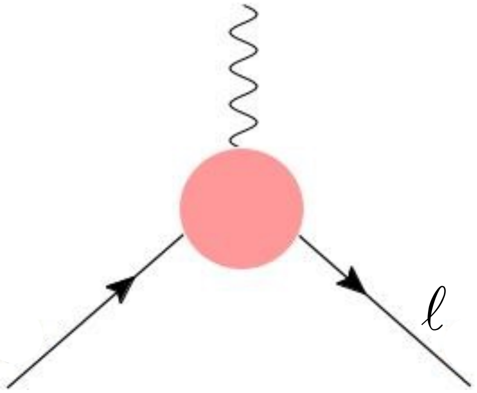

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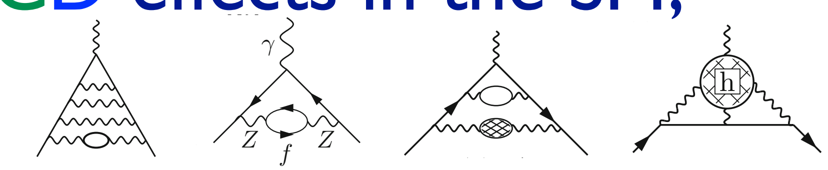

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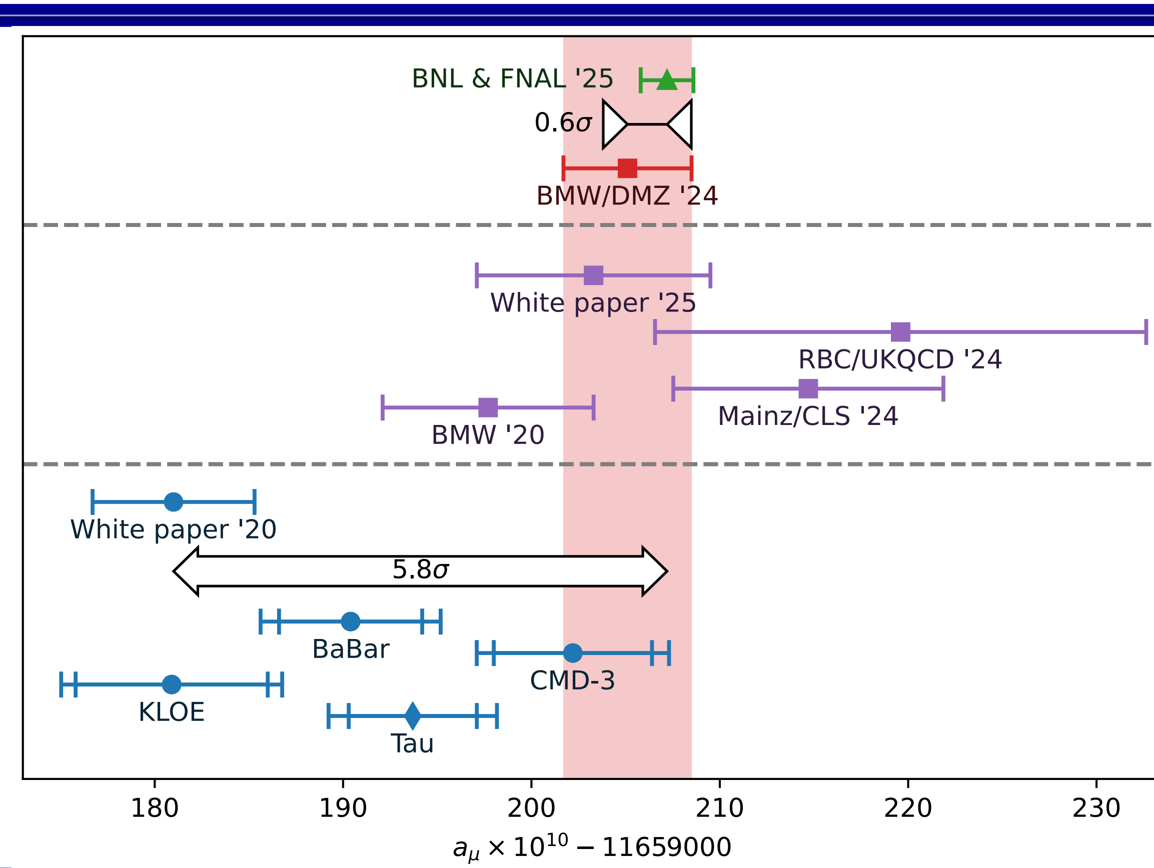
5-loop QED	11 658 471.88(2)
2-loop EW	15.44(4)
HVP LO	713.2(6.1)
HVP NLO	-9.96(13)
HVP NNLO	1.24(1)
HLbL	11.55(99)

Aliberti *et al.* [WP] 2025

Theory error dominated by hadronic physics



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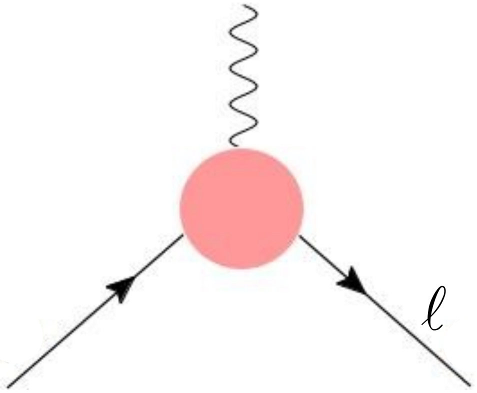

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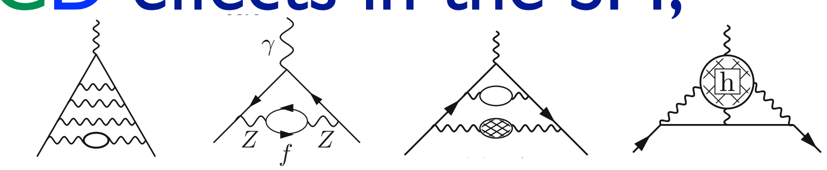

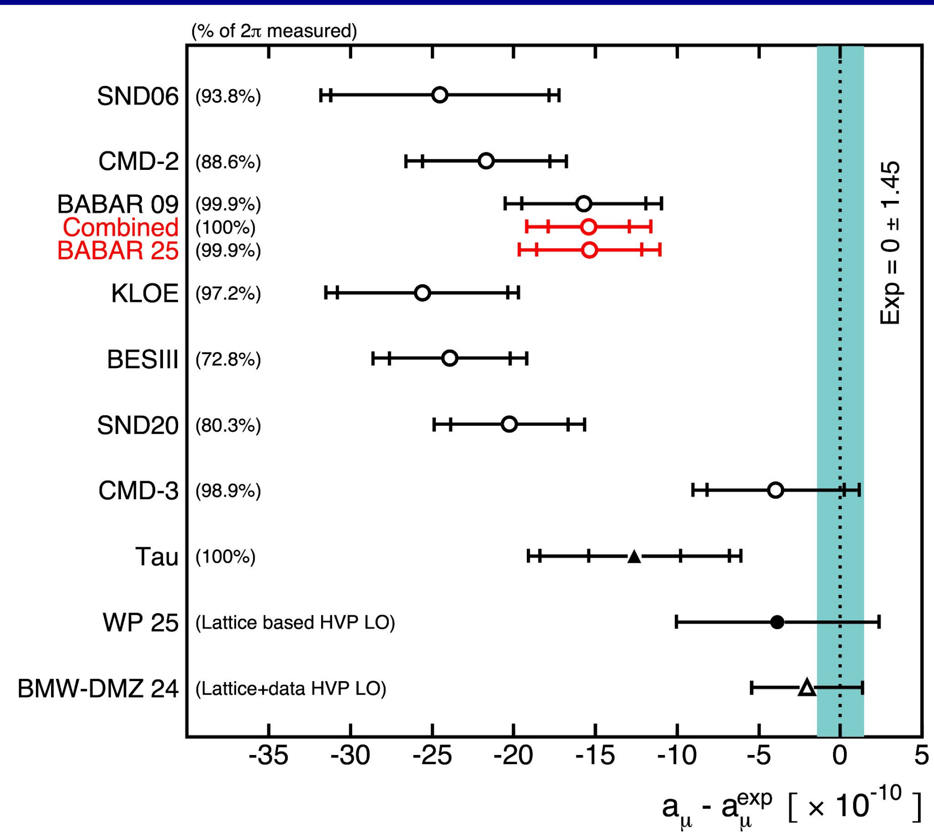

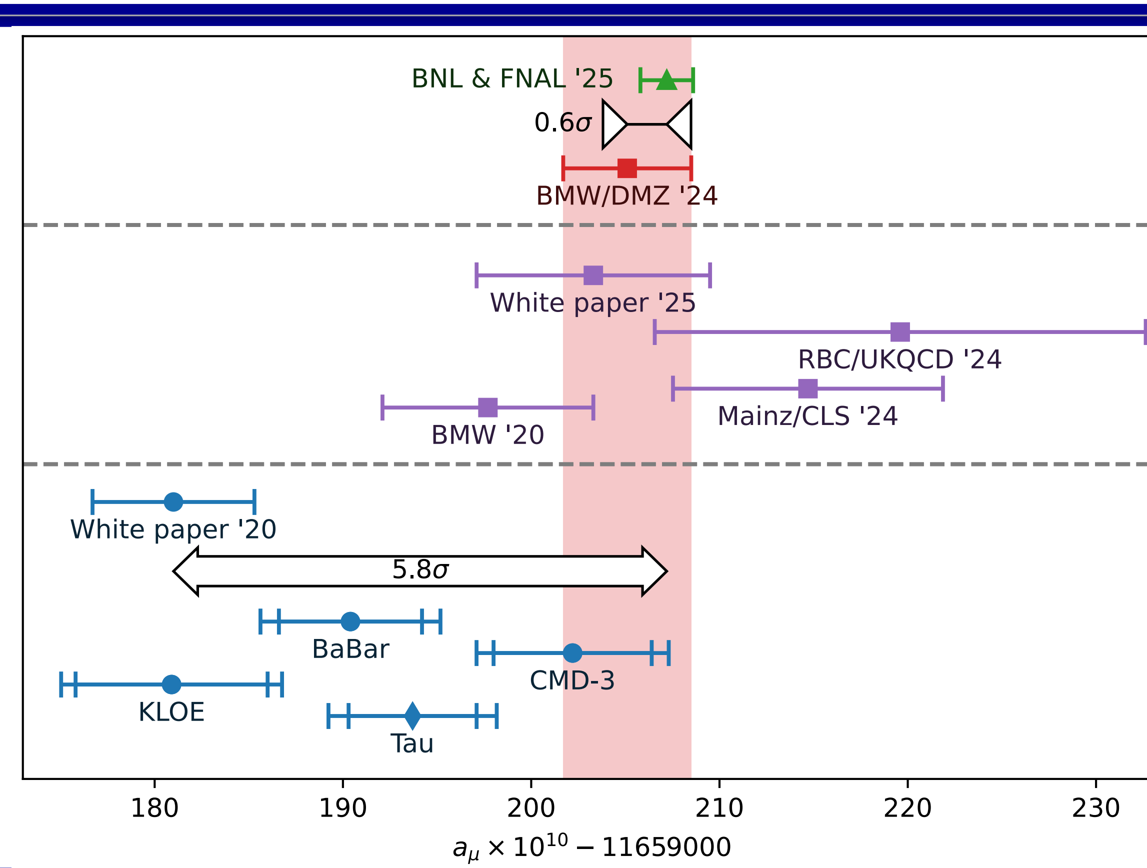

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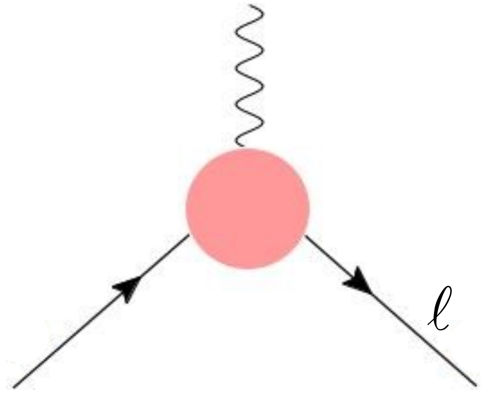

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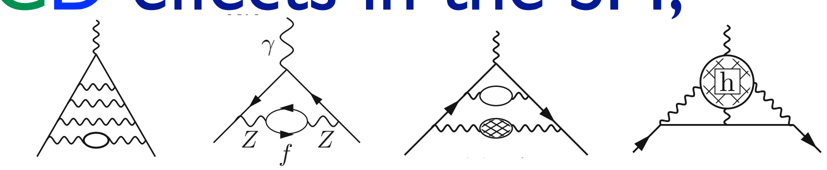
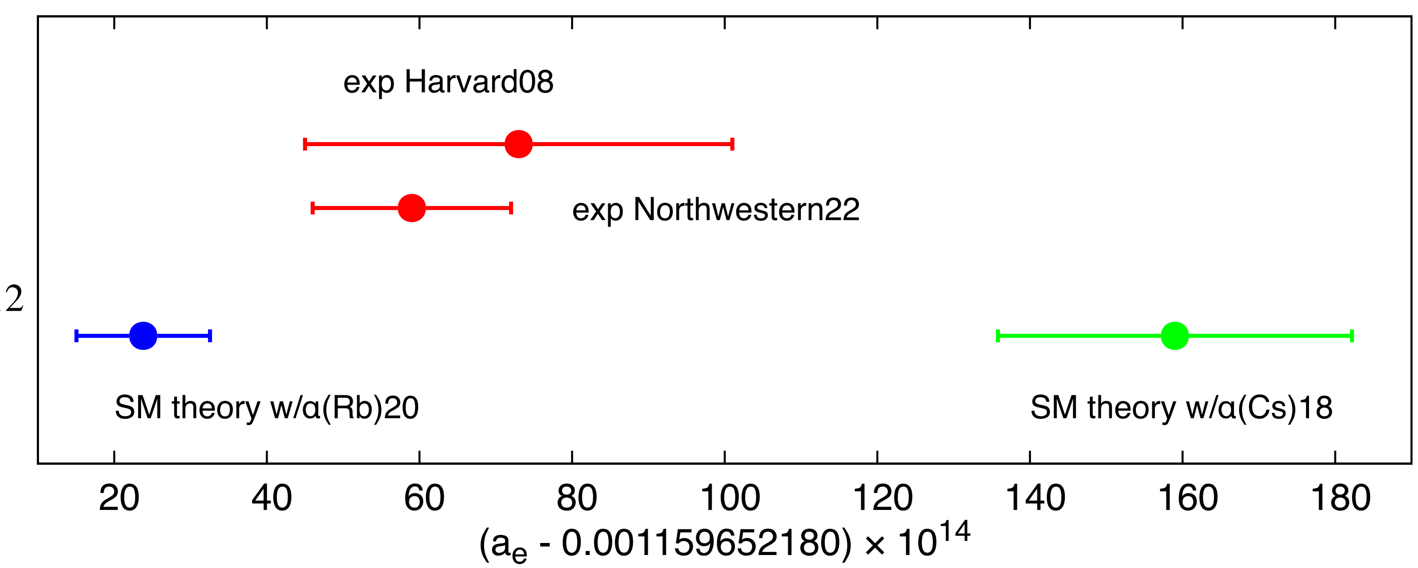


$$a_e^{\rm SM}[\alpha({\rm Cs})] = 1\,159\,652\,181.59(23)(0)(3)\times 10^{-12}$$

$$a_e^{\rm SM}[\alpha({\rm Rb})] = 1\,159\,652\,180.238(82)(4)(30)\times 10^{-12}$$



Aliberti *et al.* [WP] 2025

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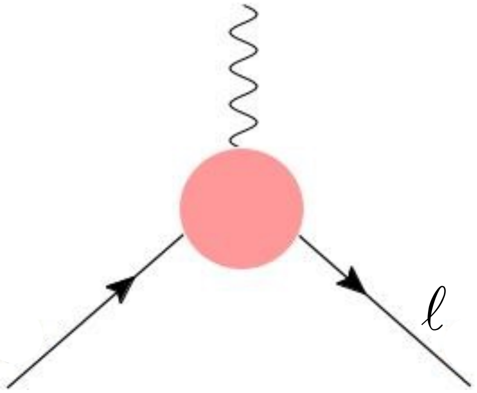

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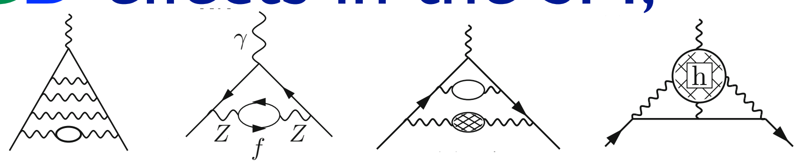

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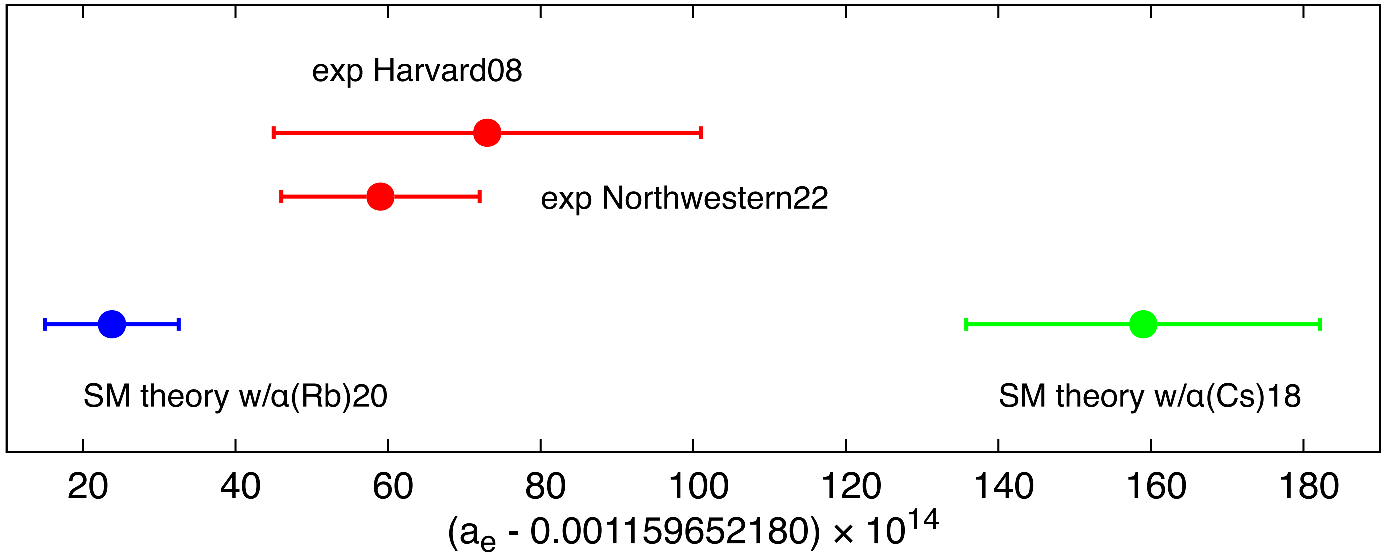
Hadronic error dominated by HVP contribution



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Hadronic contributions

$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{EW}} = 719.8(1.5) \times 10^{-10} \stackrel{?}{=} a_\mu^{\text{had}}$$

Clearly right order of magnitude:

$$a_\mu^{\text{had}} = O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) = O\left(10^{-7}\right)$$

(already Gourdin & de Rafael '69 found $a_\mu^{\text{had}} = 650(50) \times 10^{-10}$)

Huge challenge: theory of strong interaction between quarks and gluons, QCD, hugely nonlinear at energies relevant for a_μ

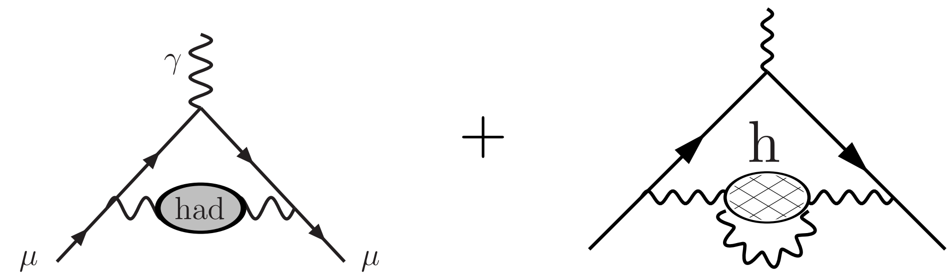
$\rightarrow$ perturbative methods used for electromagnetic and weak interactions do not work

$\rightarrow$ need nonperturbative approaches

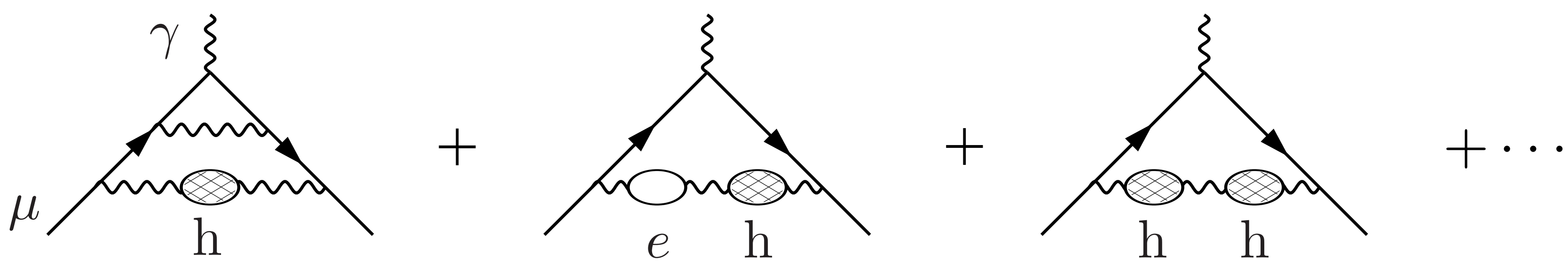
Write

$$a_\mu^{\text{had}} = a_\mu^{\text{LO-HVP}} + a_\mu^{\text{HO-HVP}} + a_\mu^{\text{HLbyL}} + O\left(\left(\frac{\alpha}{\pi}\right)^4\right)$$

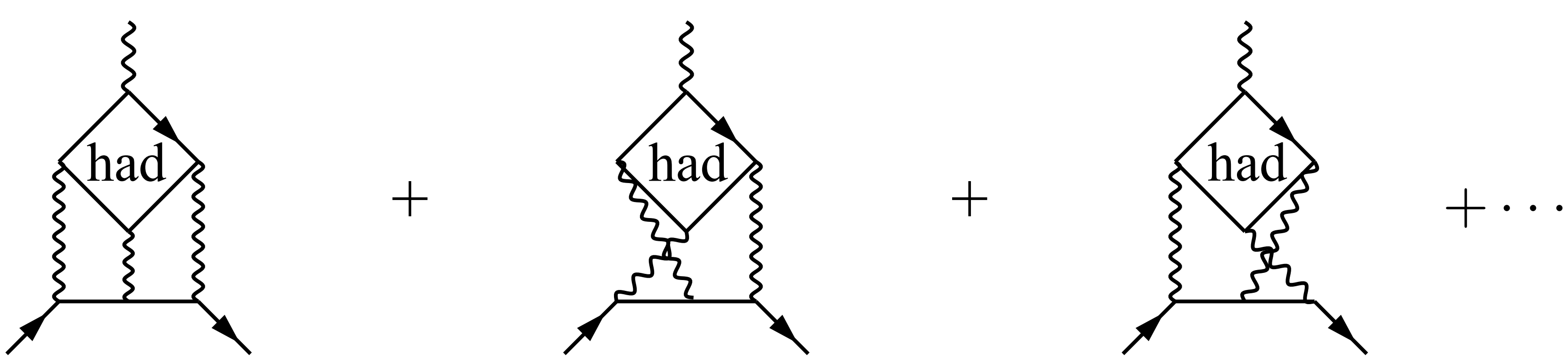
Hadronic contributions: diagrams



$\rightarrow a_\mu^{\text{LO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^2\right)$

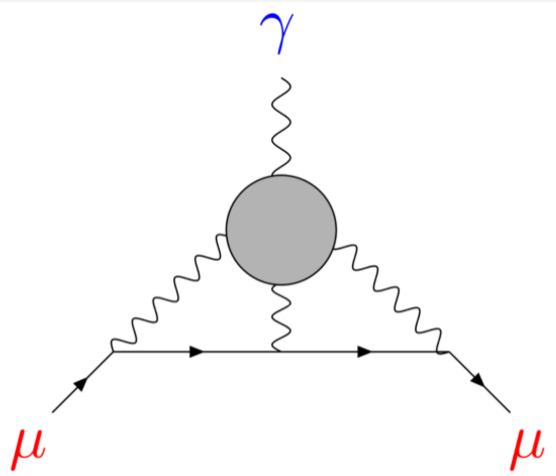


$\rightarrow a_\mu^{\text{NLO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

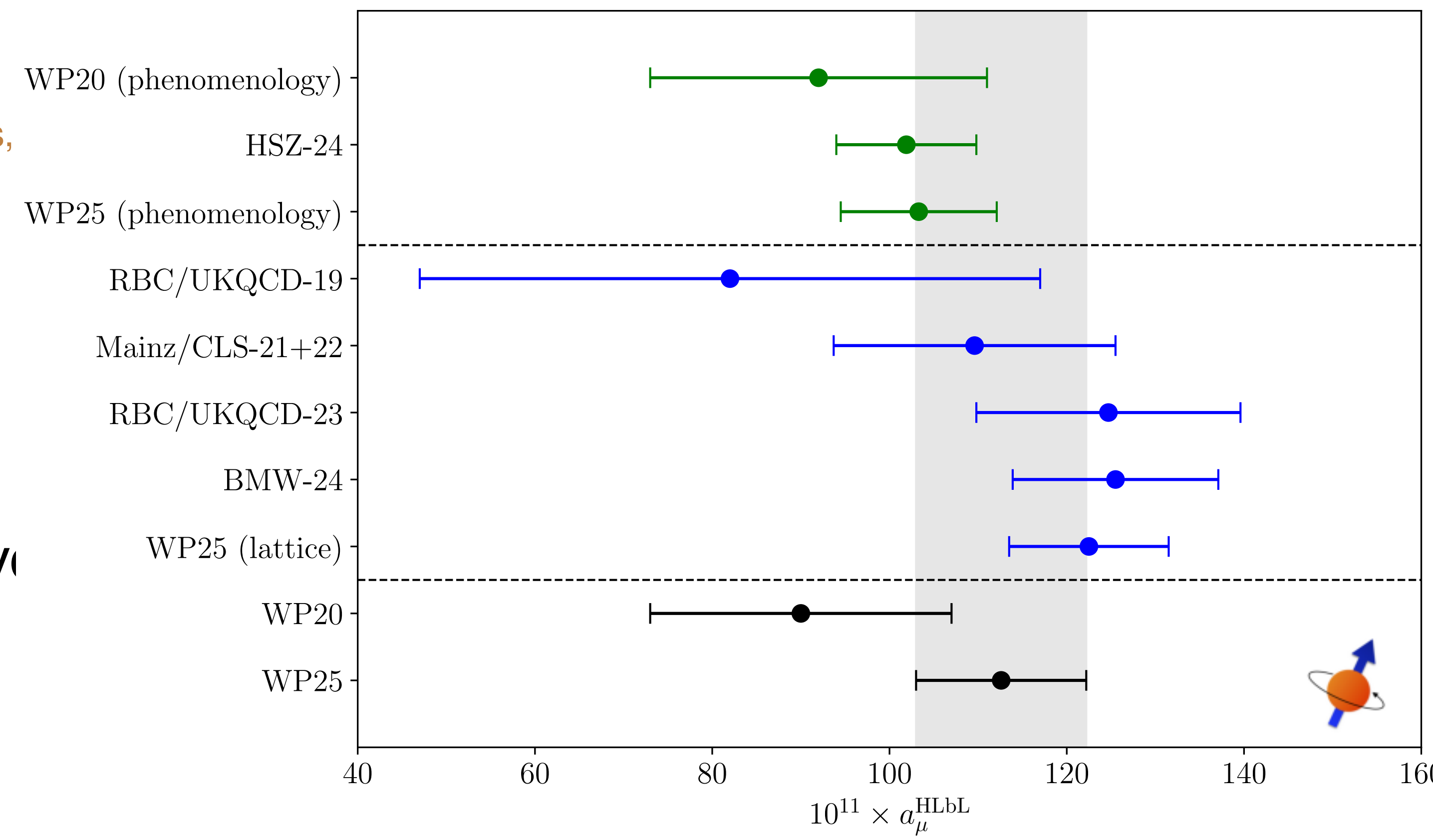


$\rightarrow a_\mu^{\text{HLbL}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

Hadronic light-by-light



- HLbL much more complicated than HVP, but ultimate precision needed is $\simeq$ 10% instead of $\simeq$ 0.2%
- For many years, only accessible to models of QCD w/ difficult to estimate systematics (Prades et al '09):
  $a_\mu^{\text{HLbL}} = 10.5(2.6) \times 10^{-10}$
- Also, lattice QCD calculations were exploratory and incomplete
- Tremendous progress in past 5 years:
  - $\rightarrow$ Phenomenology: rigorous data driven approach [Colangelo, Hoferichter, Kubis, Procura, Stoffer,... '15-'20]
  - $\rightarrow$ Lattice: three solid lattice calculations
- All agree w/ older model results but error estimate much more solid and will improve
- Agreed upon average w/ NLO HLbL and conservative error estimates
- $a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{EW}} - a_\mu^{\text{HLbL}} = 708.3(1.8) \times 10^{-10} \stackrel{?}{=} a_\mu^{\text{HVP}}$



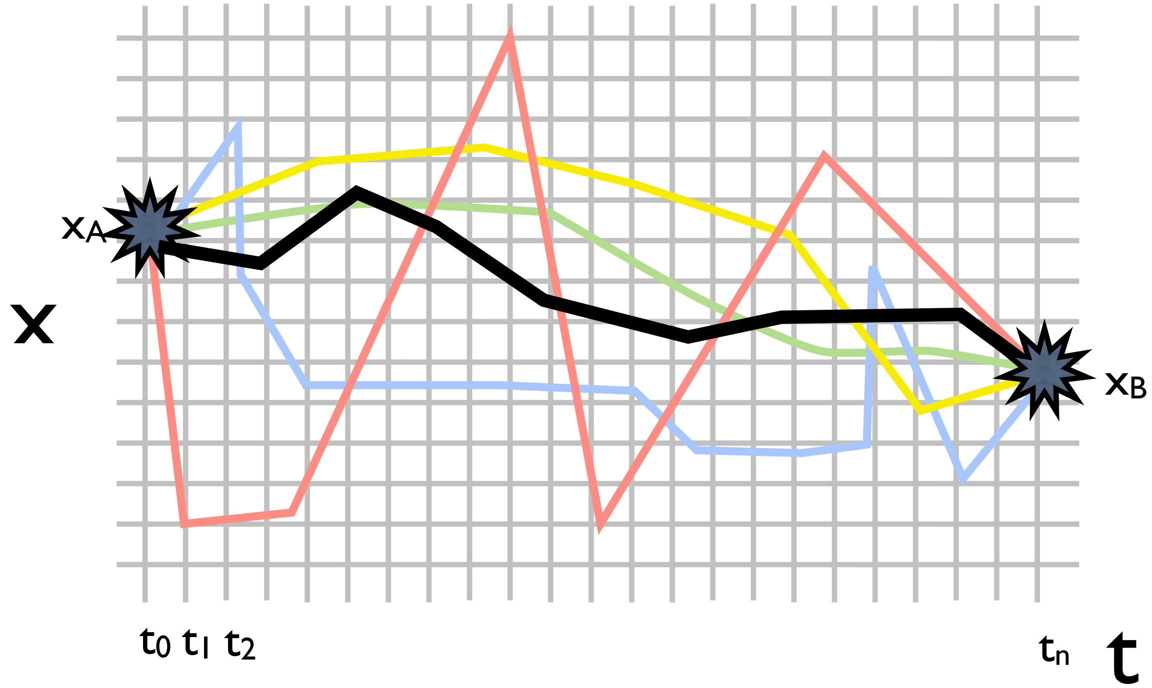
Aliberti *et al.* [WP] 2025

Small interlude:
Lattice QCD

Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- Discretise QCD onto 4D space-time lattice
- QCD equations ⟷ integrals over the values of quark and gluon fields on each site/link (QCD path integral)
- ~10^{12} variables (for state-of-the-art)

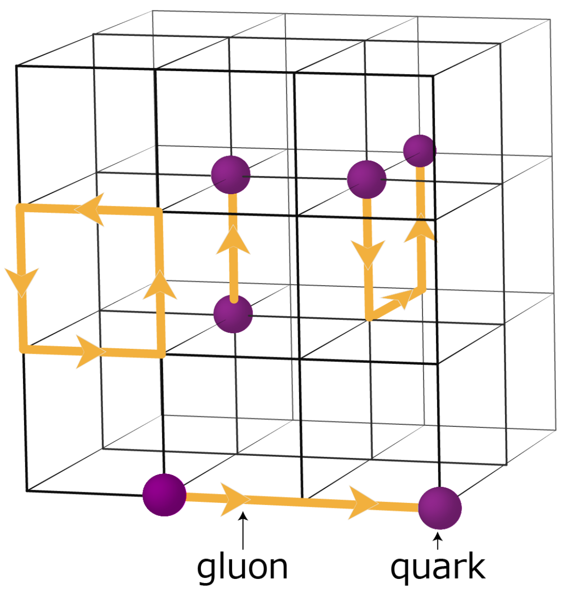


- Evaluate by importance sampling
- Paths near classical action dominate
- Calculate physics on a set (ensemble) of samples of the quark and gluon fields

Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- Euclidean space-time $t \to i\tau$
- Finite lattice spacing a
- Volume $L^3 \times T = 64^3 \times 128$
- Boundary conditions



Approximate the QCD path integral by **Monte Carlo**

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\overline{\psi} \mathcal{D}\psi \mathcal{O}[A, \overline{\psi}\psi] e^{-S[A,\overline{\psi}\psi]} \quad \rightarrow \quad \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\mathrm{conf}}} \sum_{i}^{N_{\mathrm{conf}}} \mathcal{O}([U^i])$$

with field configurations U^i distributed according to $e^{-S[U]}$

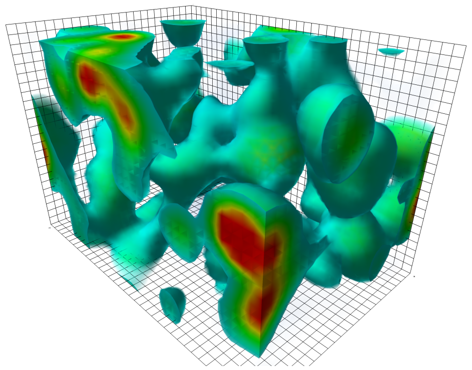
Lattice QCD

Workflow of a lattice QCD calculation

1. Generate field configurations via Hybrid Monte Carlo
   - Leadership-class computing
   - ~100K cores or 1000GPUs, 10's of TF-years
   - O(100-1000) configurations, each ~10-100GB

2. Compute propagators
   - Large sparse matrix inversion
   - ~few 100s GPUs
   - 10x field config in size, many per config

3. Contract into correlation functions
   - ~few GPUs
   - O(100k-1M) copies



Lattice QCD action density, Leinweber, CSSM, Adelaide, 2003

Hadrons are emergent phenomena of statistical average over background gluon configurations

- 1 year on supercomputer ~ 100k years on laptop

JUPITER Booster: Rank 4 on TOP500 2025

Challenges of a full lattice calculation

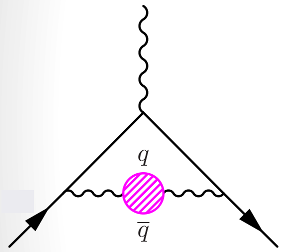
To make contact with experiment need:

- **A valid approximation to the SM**
  - $\rightarrow$ at least u, d, s in the sea w/ $m_u = m_d \ll m_s$ (N_f=2+1) $\Rightarrow \sigma \sim 1\%$
  - $\rightarrow$ better also include c (N_f=2+1+1) & $m_u \leq m_d$ & EM $\Rightarrow \sigma \sim 0.1\%$
- **u & d w/ masses well w/in $SU(2)$ chiral regime** : $\sigma_\chi \sim (M_\pi/4\pi F_\pi)^2$
  - $\rightarrow$ $M_\pi \sim 135\,\mathrm{MeV}$ or many $M_\pi \leq 400\,\mathrm{MeV}$ w/ $M_\pi^{\mathrm{min}} < 200\,\mathrm{MeV}$ for $M_\pi \to 135\,\mathrm{MeV}$
- **a $\rightarrow$ 0** : $\sigma_a \sim (a\Lambda_{\mathrm{QCD}})^n,\ (am_q)^n,\ (a|\vec{p}|)^n$ w/ $a^{-1} \sim 2 \div 4\,\mathrm{fm}$
  - $\rightarrow$ at least 3 a's $\leq 0.1\,\mathrm{fm}$ for $a \to 0$
- **L $\rightarrow \infty$** : $\sigma_L \sim (M_\pi/4\pi F_\pi)^2 \times e^{-LM_\pi}$ for stable hadrons, $\sim 1/L^n$ for resonances, QED, ...
  - $\rightarrow$ many L w/ $(LM_\pi)^{\mathrm{max}} \gtrsim 4$ for stable hadrons & better otherwise to allow for $L \to \infty$
- These requirements $\Rightarrow O(10^{12})$ **dofs** that have to be integrated over
- **Renormalization** : best done nonperturbatively
- **A signal** : $\sigma_{\mathrm{stat}} \sim 1/\sqrt{N_{\mathrm{meas}}}$, reduce w/ $N_{\mathrm{meas}} \to \infty$

HVP from the lattice

&

Window observables

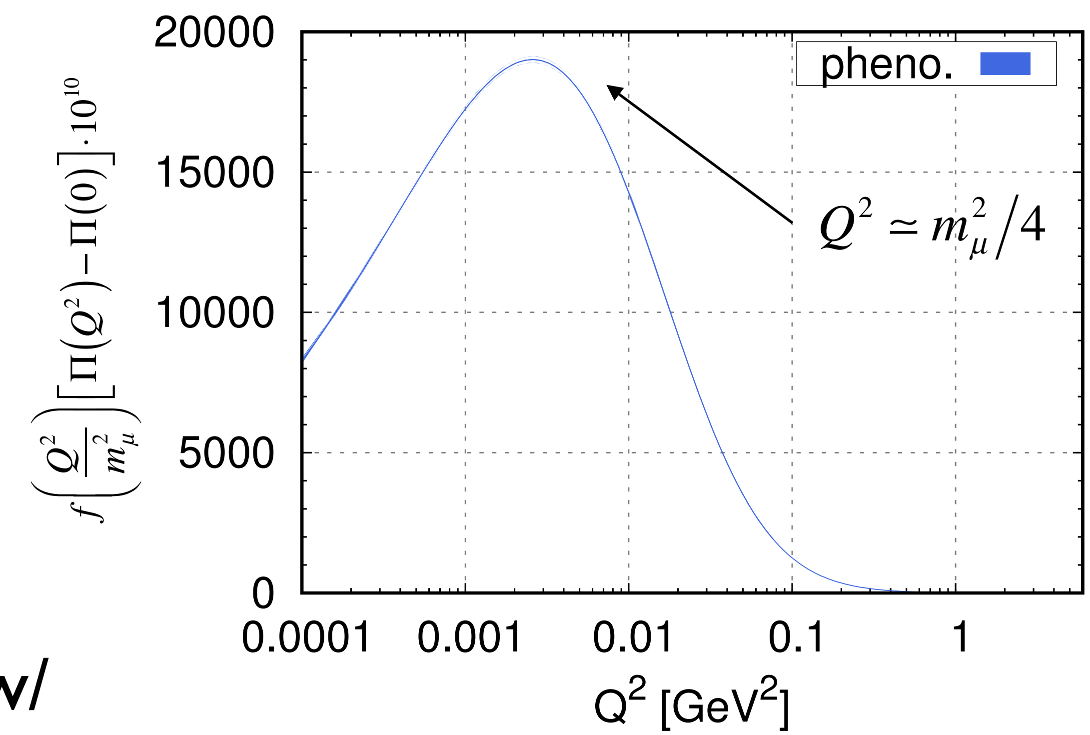


HVP from LQCD

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x} \langle J_\mu(x) J_\nu(0)\rangle = \left[\delta_{\mu\nu}Q^2 - Q_\mu Q_\nu\right]\Pi(Q^2)$$

$$a_\ell^{\mathrm{HVP,LO}} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \frac{1}{m_\ell^2} f\left(\frac{Q^2}{m_\ell^2}\right)\left[\Pi(Q^2) - \Pi(0)\right]$$

B. E. Lautrup *et al.*, 1972



F. Jegerlehner, "alphaQEDc17"

FV & $a \neq 0$: A. discrete momenta ($Q_{\min} = 2\pi/T > m_\mu/2$); B. $\Pi_{\mu\nu}(0) \neq 0$ in FV contaminates $\Pi(Q^2) \sim \Pi_{\mu\nu}(Q)/Q^2$ for $Q^2 \to 0$ w/ very large FV effects; C. $\Pi(0) \sim \ln(a)$

Time-Momentum Representation

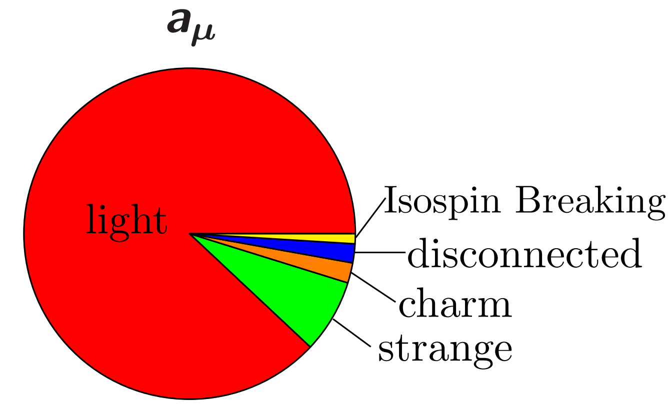
$$a_\ell^{\mathrm{HVP,LO}} = 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t)\, V(t)$$

$$V(t) \equiv \frac{1}{3}\sum_{i=1,2,3} \int d\vec{x}\, \langle J_i(\vec{x},t) J_i(0)\rangle$$

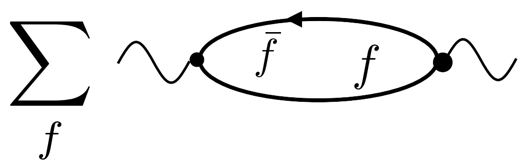
D. Bernecker and H. B. Meyer, 2011

Time-Momentum Representation

- No reliance on exp. data, except for hadronic quantities used to calibrate the simulation ($M_\pi, M_K, M_{nucl}, \ldots$)

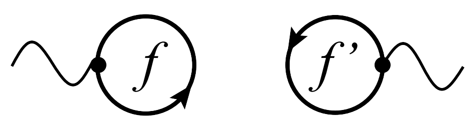


- Can perform an explicit quark flavor separation of $a_\mu^{\text{HVP,LO}}$

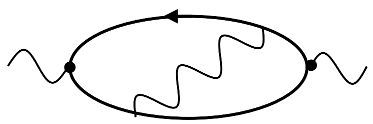


light-quark connected $a_\mu^{\text{HVP,LO}}(\text{ud}) \sim 90\,\%$ of total

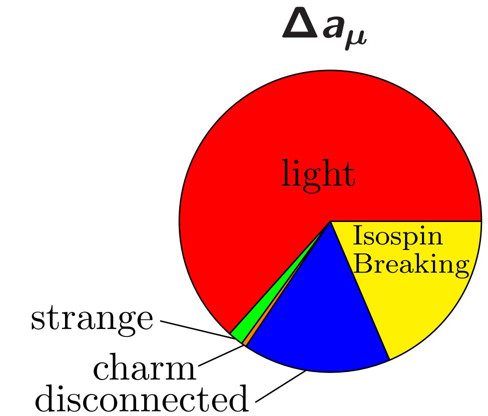
s,c-quark connected $a_\mu^{\text{HVP,LO}}(\text{s, c}) \sim 8\,\%\,, 2\,\%$ of total



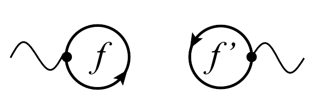
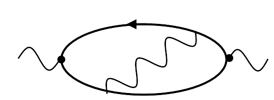
disconnected $a_{\mu,disc}^{\text{HVP,LO}} \sim 2\,\%$ of total

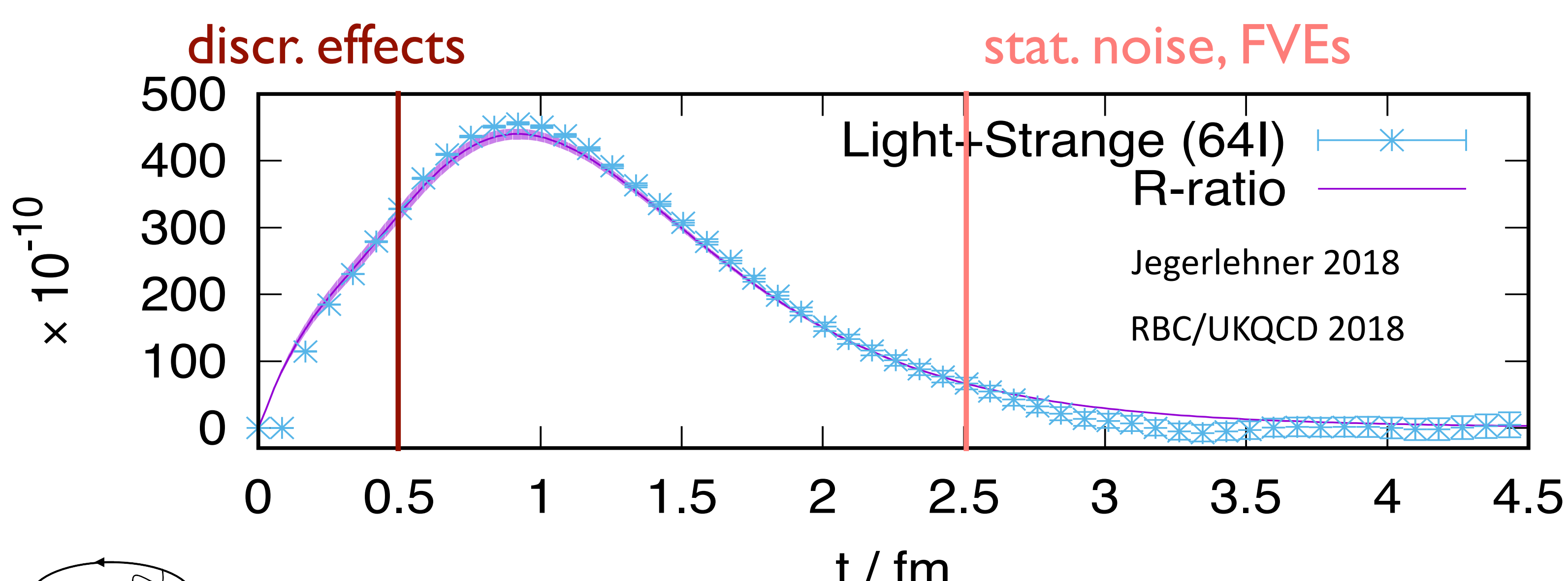


IB ($m_u \neq m_d + \text{QED}$) $\delta a_\mu^{\text{HVP,LO}} \sim 1\,\%$ of total



Challenges:

- sub-percent stat. precision
  exp. growing StN ratio in $V(t)$ as $t \to \infty$
- correct for FVEs, control discr. effects (scale setting and continuum extrap.)
- quark-disconn. diagrams 
  control stat. & stochastic noise
- isospin-breaking: $m_u \neq m_d,\ \alpha_{em} \neq 0$ 



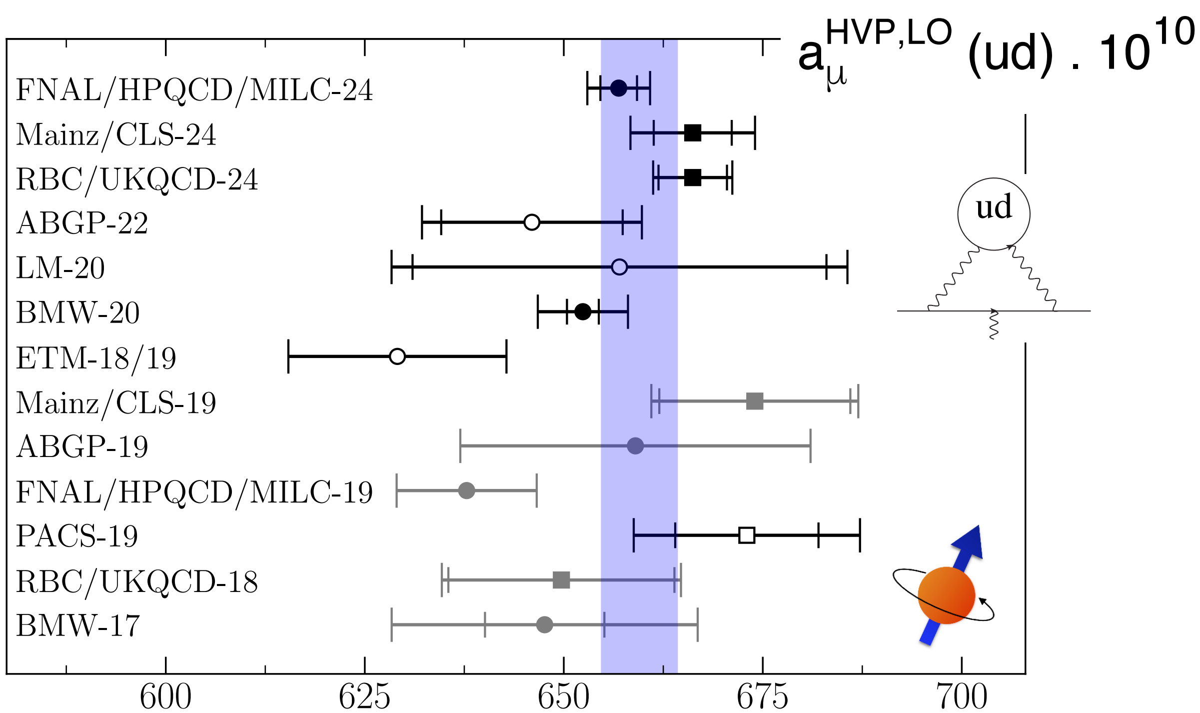
Results for each contribution

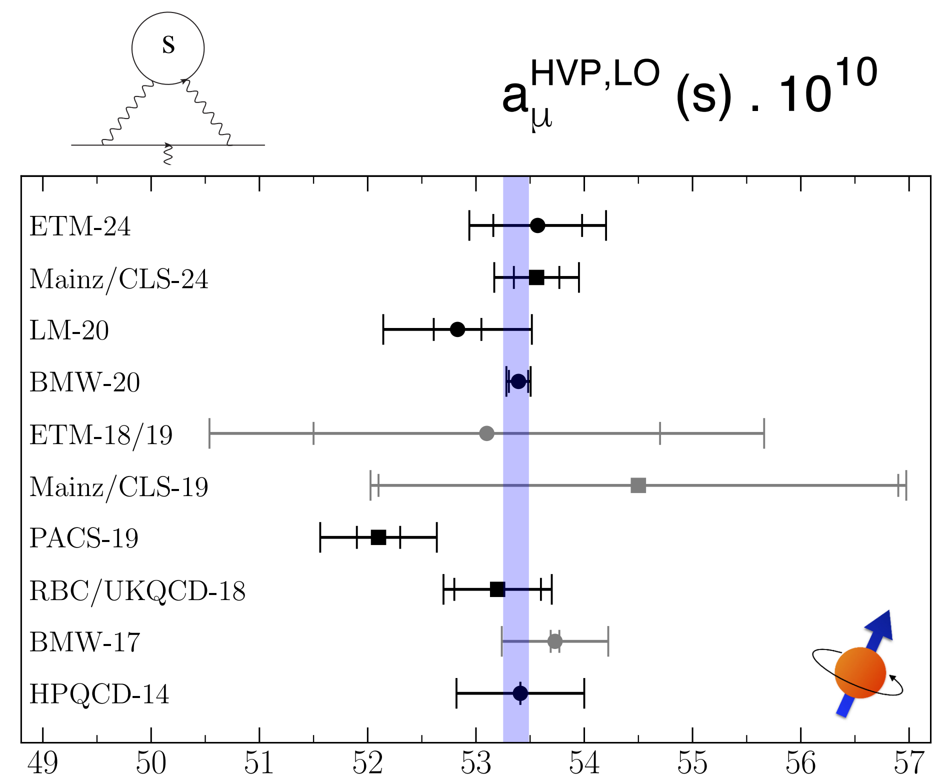
prescription. We define *isospin-symmetric QCD (isoQCD)* to be the $\alpha = 0$ theory where quark masses are tuned to reproduce the following complete set of inputs

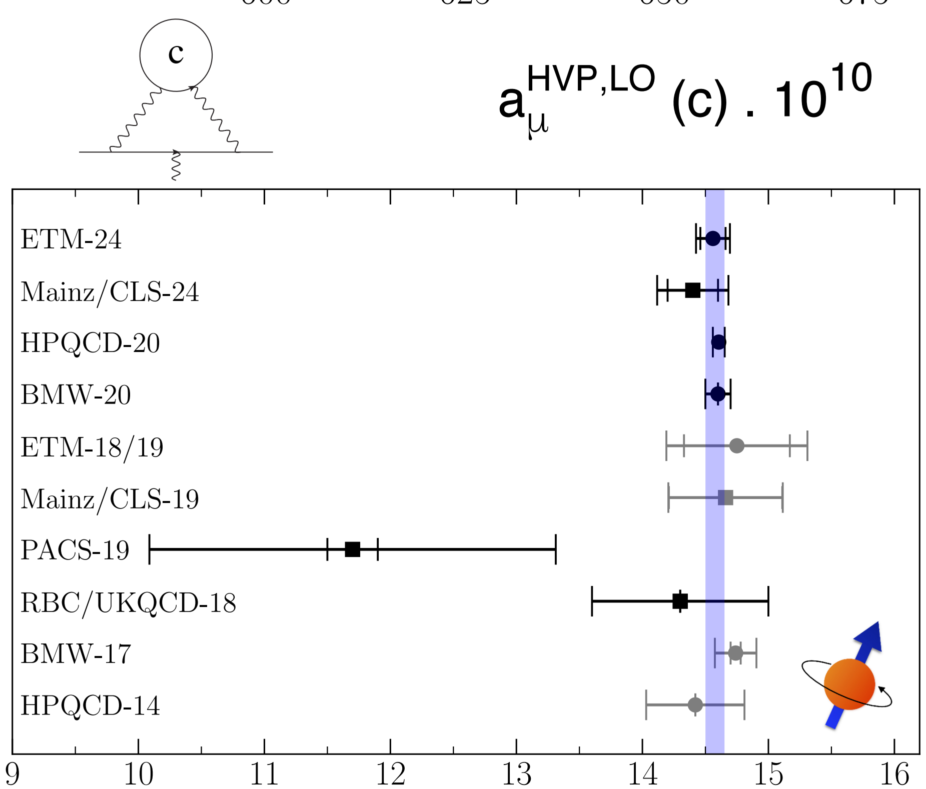
$$M_{\pi^+} = 135.0\ \text{MeV}\,, \quad M_{K^0} = M_{K^+} = 494.6\ \text{MeV}\,, \quad M_{D_s^+} = 1967\ \text{MeV}\,, \quad \text{and} \quad w_0 = 0.17236\ \text{fm}\,, \qquad (3.9)$$

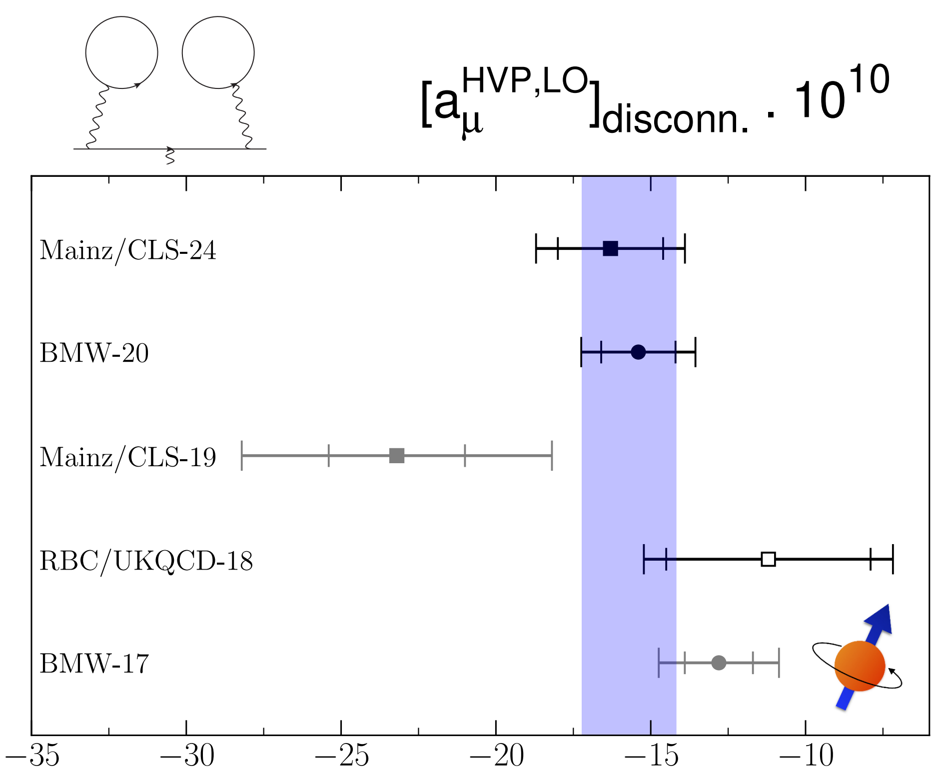
where w_0 is the Wilson flow scale introduced in Ref. [424]. We call this prescription the *WP25 scheme*. The only

WP '25









Windows "on the g-2 mystery"

Restrict integration over Euclidean time to sub-intervals

→ reduce/enhance sensitivity to systematic effects

$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{SD} + a_\mu^{W} + a_\mu^{LD}$$

$$a_\mu^{SD}(f; t_0, \Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t) \left[1 - \Theta\left(t, t_0, \Delta\right)\right]$$

$$a_\mu^{W}(f; t_0, t_1, \Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t) \left[\Theta\left(t, t_0, \Delta\right) - \Theta\left(t, t_1, \Delta\right)\right]$$

$$a_\mu^{LD}(f; t_1, \Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t)\, \Theta\left(t, t_1, \Delta\right)$$

$$\Theta\left(t, t', \Delta\right) = \frac{1}{1 + e^{-2(t-t')/\Delta}}$$

"Standard" choice:

$$t_0 = 0.4 \text{ fm} \quad t_1 = 1.0 \text{ fm}$$

$$\Delta = 0.15 \text{ fm}$$

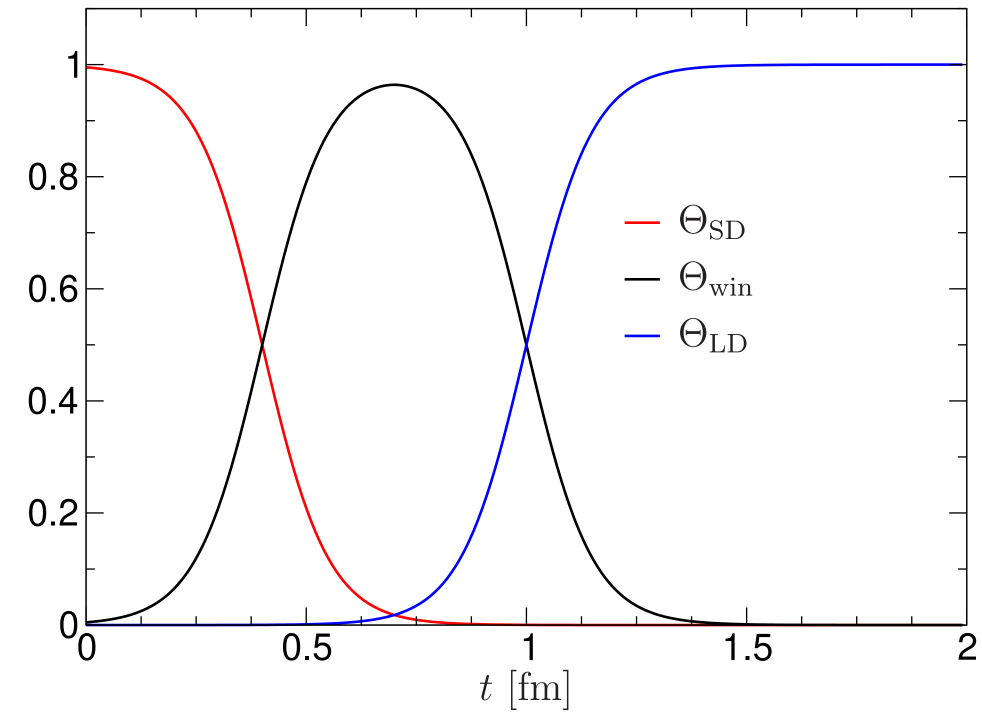
RBC/UKQCD 2018

Intermediate window

- Reduced FVEs
- Much better StN ratio

→ Precision test of different lattice calculations

→ Commensurate uncertainties compared to dispersive evaluations

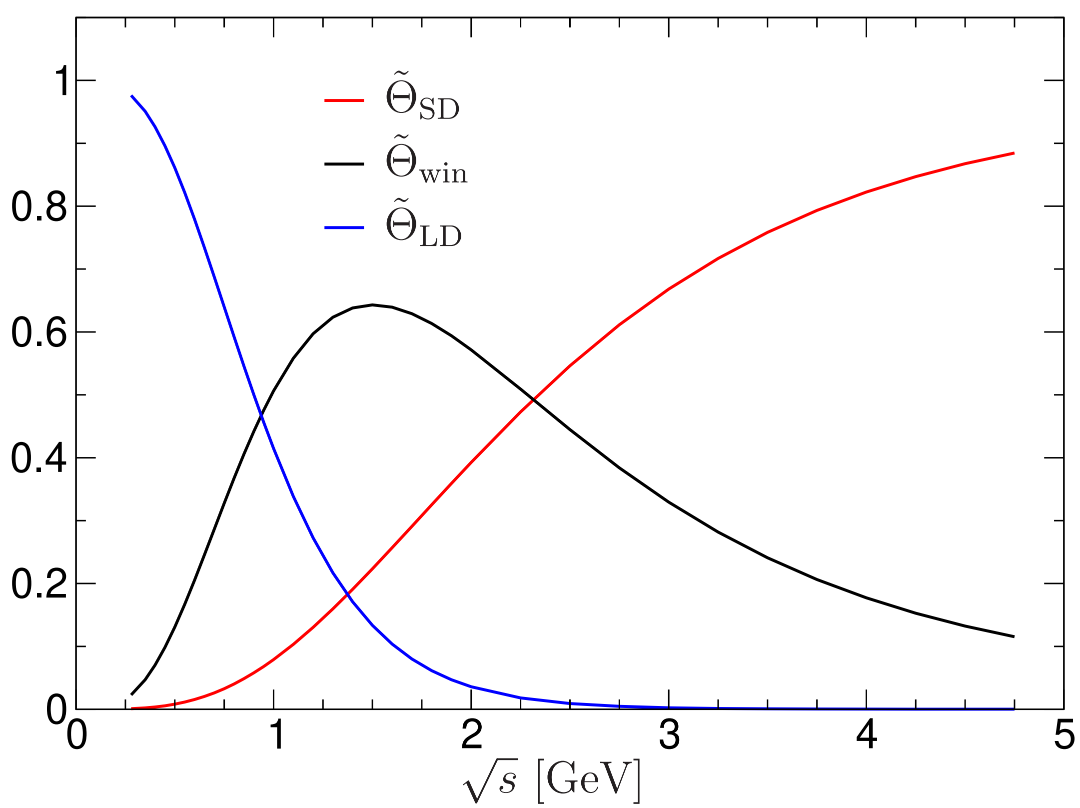


Comparison with R-ratio

$$V(t) = \frac{1}{12\pi^2}\int_{M_{\pi^0}}^{\infty} d(\sqrt{s})\, R(s)\, s\, e^{-\sqrt{s}t} \qquad R(s) = \frac{3s}{4\pi\alpha_{em}^2}\sigma(s, e^+e^- \to hadrons)$$

Insert $V(t)$ into the expression for TMR

$$a_{\mu,win}^{\mathrm{HVP,LO}} = 4\alpha_{em}^2 \int_{M_{\pi^0}}^{\infty} d(\sqrt{s}) R(s) \frac{1}{12\pi^2}\, s \int_0^{\infty} dt\, \tilde{f}(t)\, \Theta_{win}(t)\, e^{-\sqrt{s}t}$$

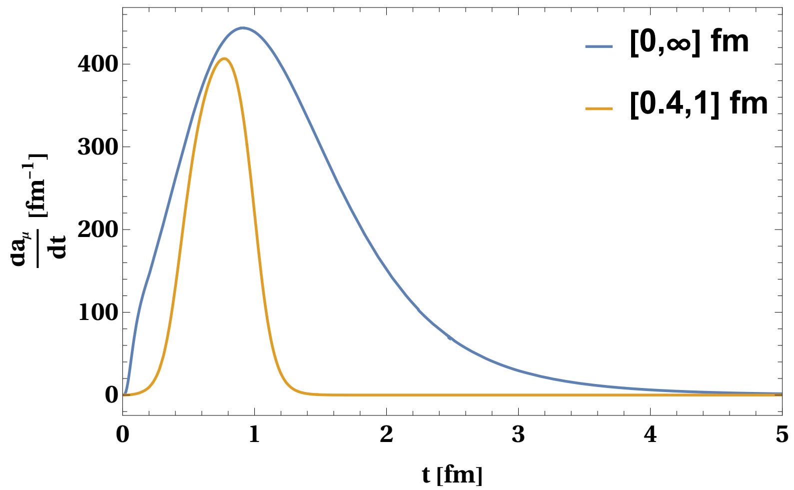


Colangelo *et al.* 2022

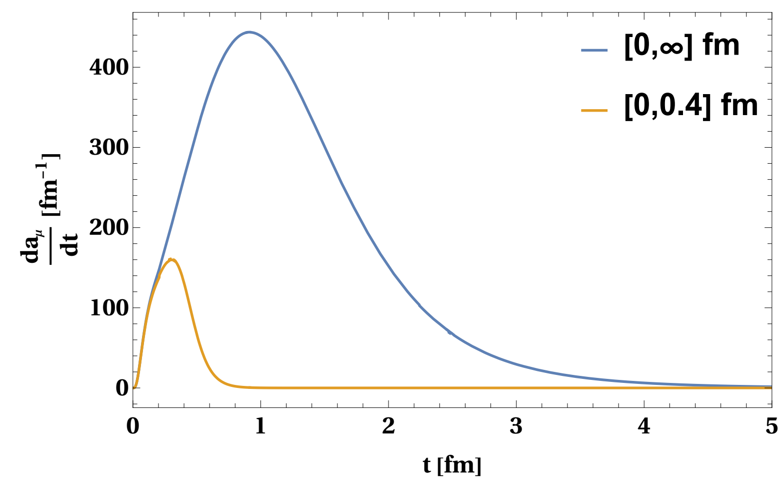
	$a^{\mathrm{HVP}}_{\mathrm{SD}}$	$a^{\mathrm{HVP}}_{\mathrm{int}}$	$a^{\mathrm{HVP}}_{\mathrm{LD}}$	$a^{\mathrm{HVP}}_{\mathrm{total}}$
All channels	68.4(5) [9.9%]	229.4(1.4) [33.1%]	395.1(2.4) [57.0%]	693.0(3.9) [100%]
2π below 1.0 GeV	13.7(1) [2.8%]	138.3(1.2) [28.0%]	342.3(2.3) [69.2%]	494.3(3.6) [100%]
3π below 1.8 GeV	2.5(1) [5.5%]	18.5(4) [39.9%]	25.3(6) [54.6%]	46.4(1.0) [100%]
White Paper [1]	–	–	–	693.1(4.0)
RBC/UKQCD [24]	–	231.9(1.5)	–	715.4(18.7)
BMWc [36]	–	236.7(1.4)	–	707.5(5.5)
BMWc/KNT [7, 36]	–	229.7(1.3)	–	–
Mainz/CLS [99]	–	237.30(1.46)	–	–
ETMC [100]	69.33(29)	235.0(1.1)	–	–

Benchmarking of lattice calculations: windows

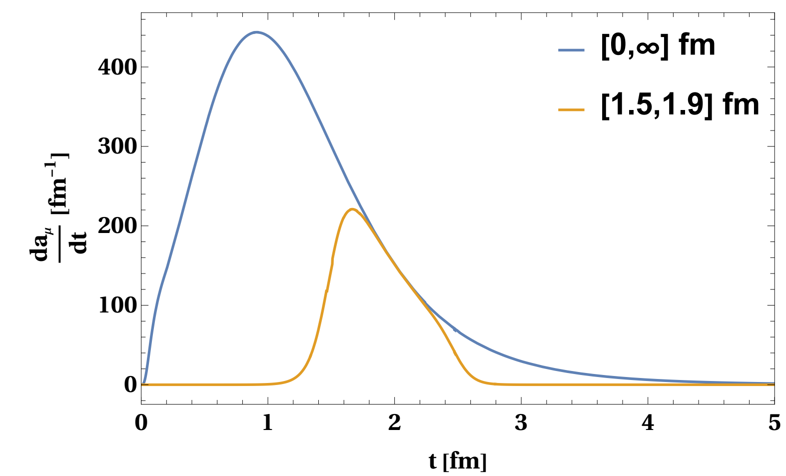
$0 \rightarrow 0.4$ fm



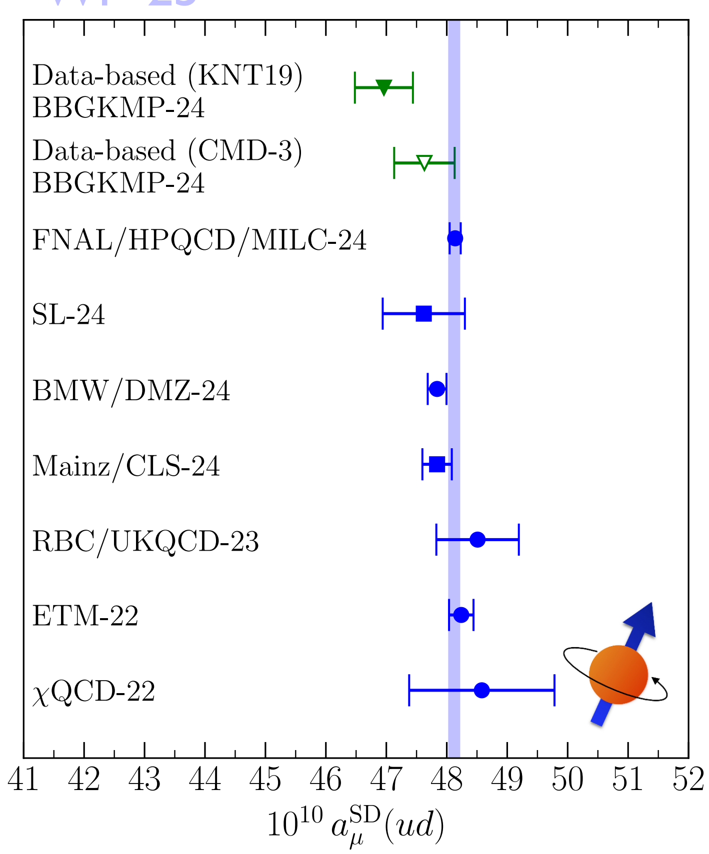
$0.4 \rightarrow 1$ fm

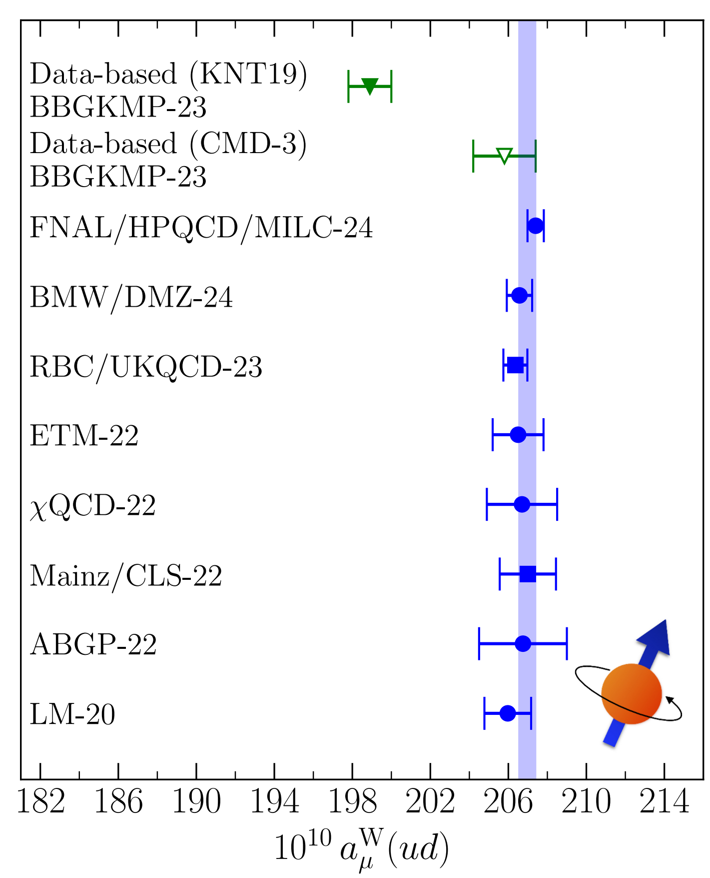


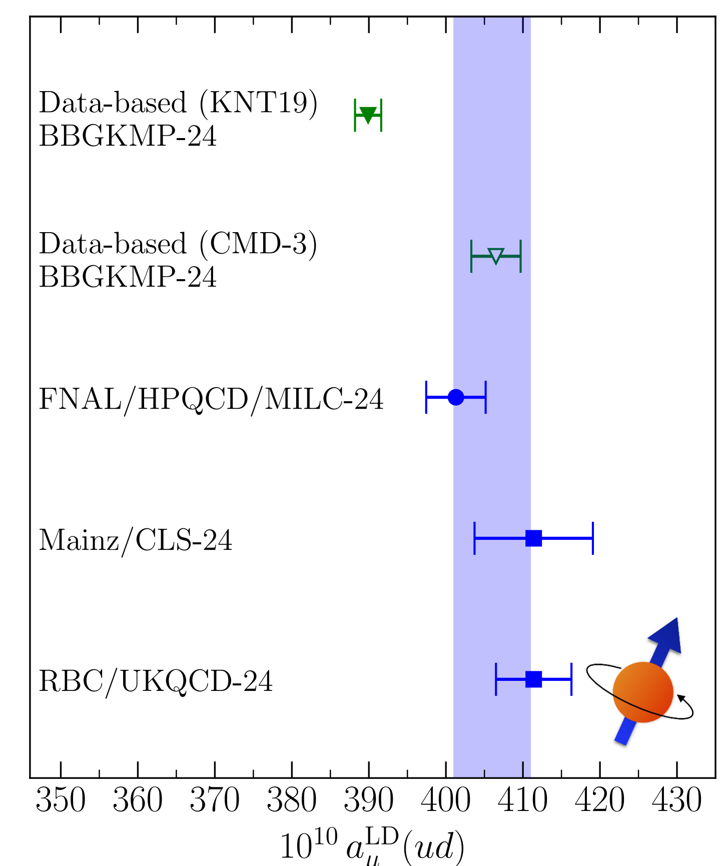
$1 \rightarrow \infty$ fm



WP '25





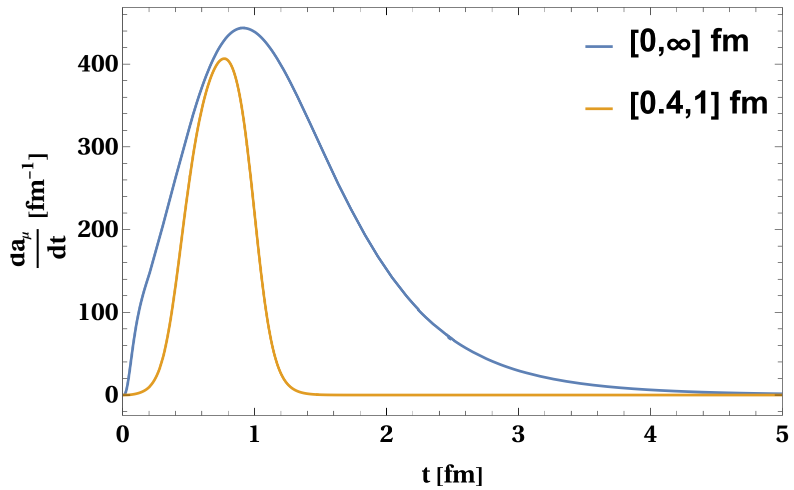


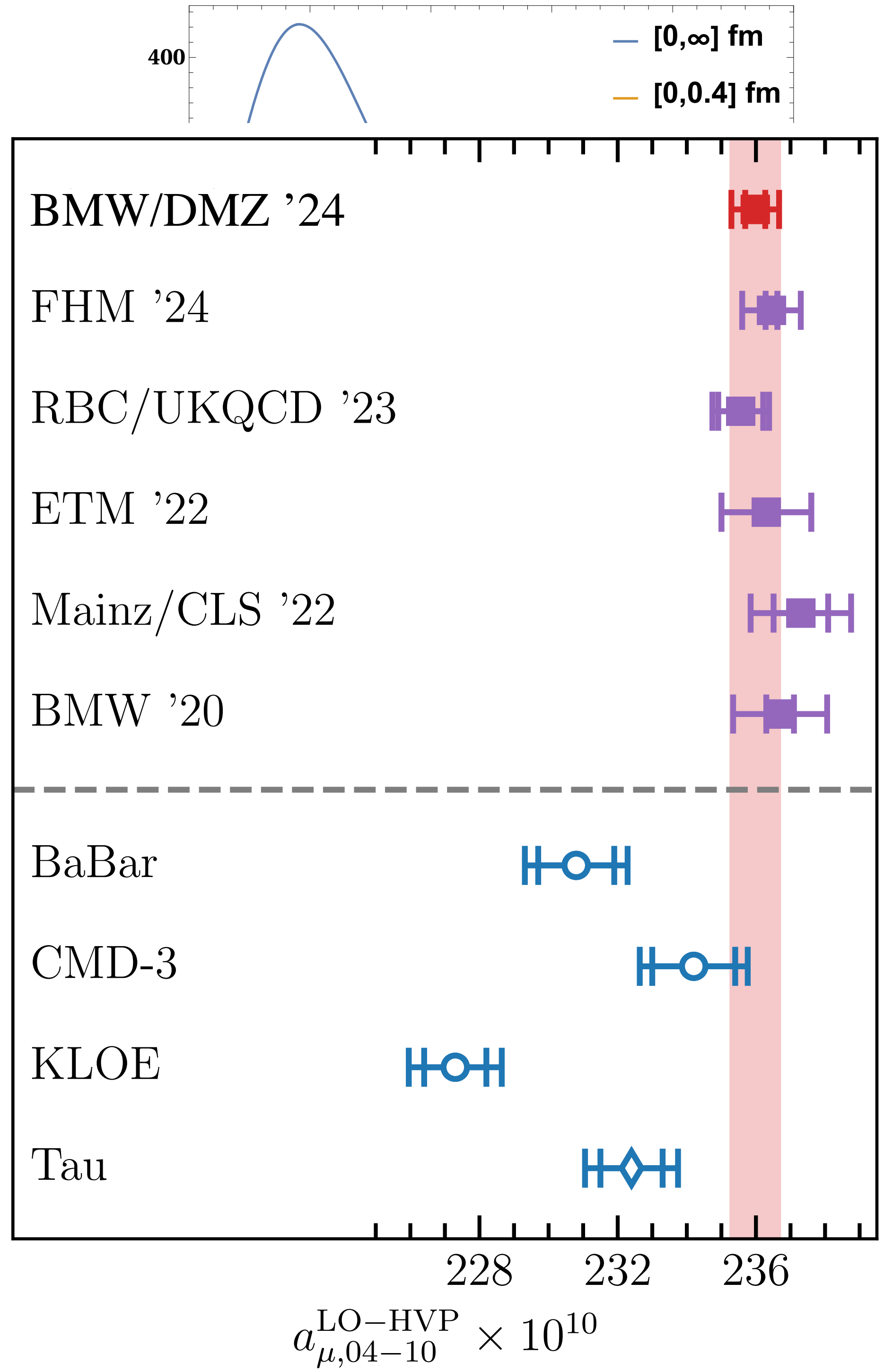
Benchmarking of lattice calculations: windows

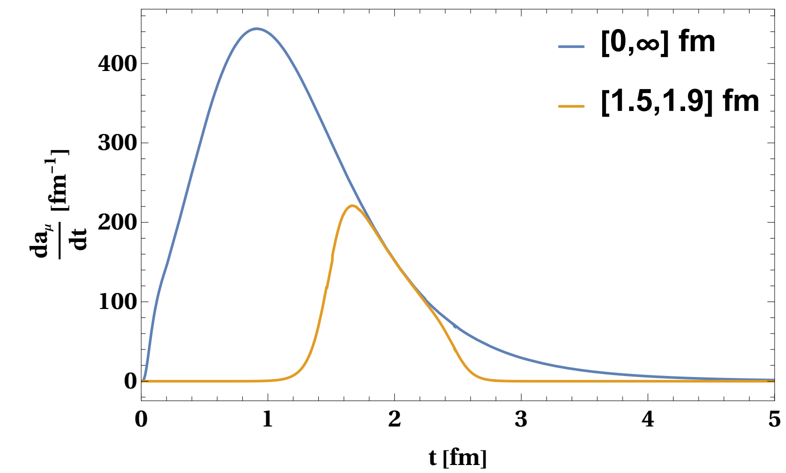
$0 \rightarrow 0.4$ fm

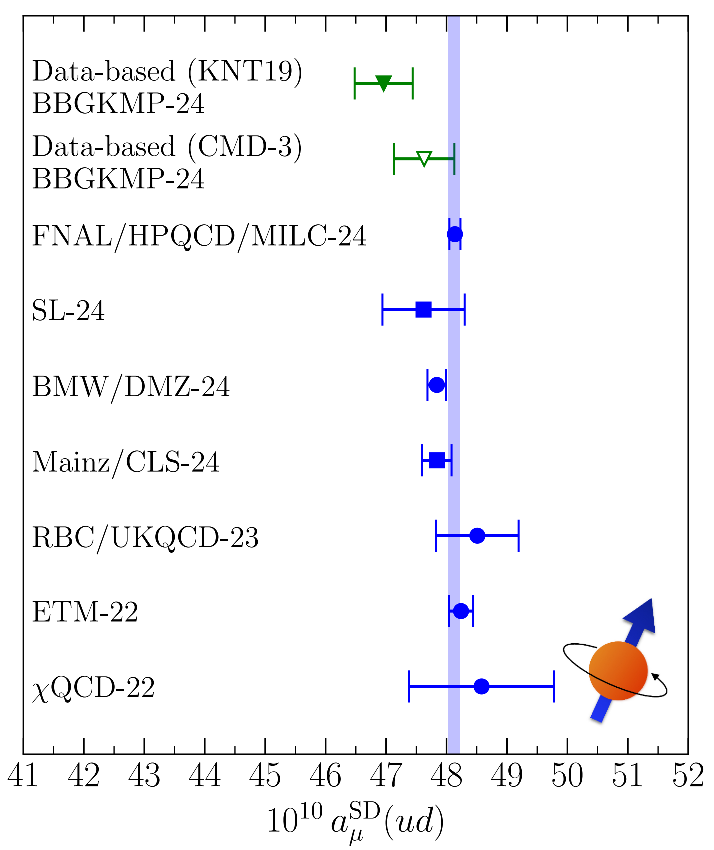
$0.4 \rightarrow 1$ fm

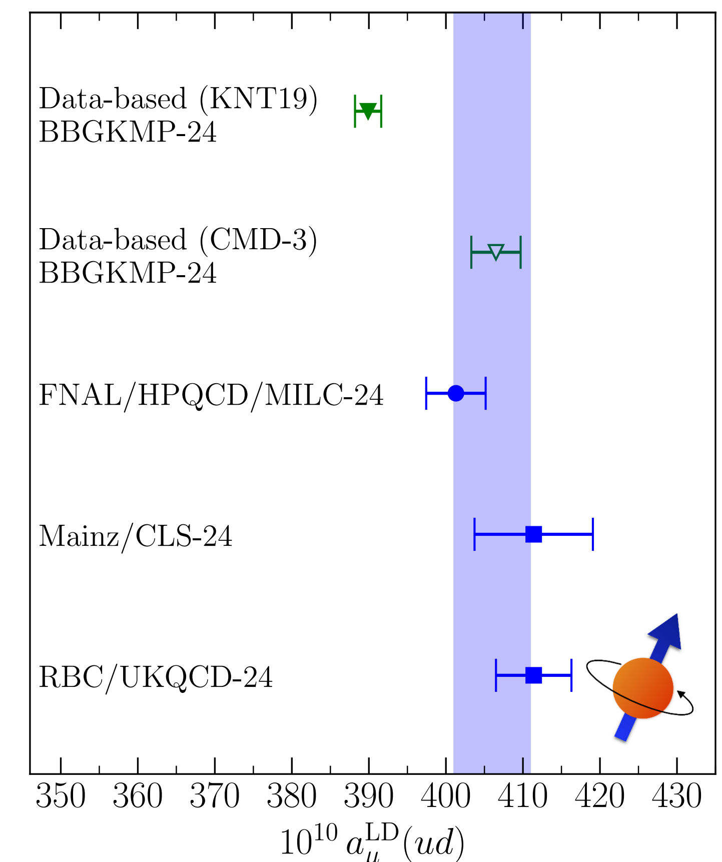
$1 \rightarrow \infty$ fm





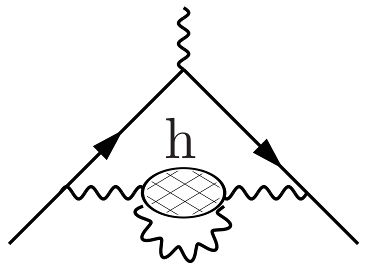


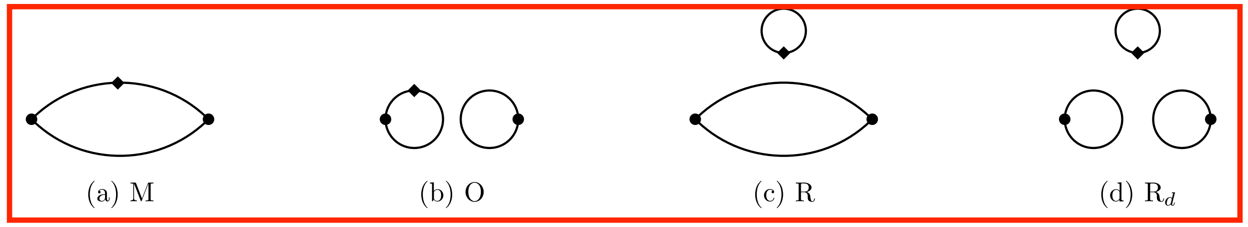




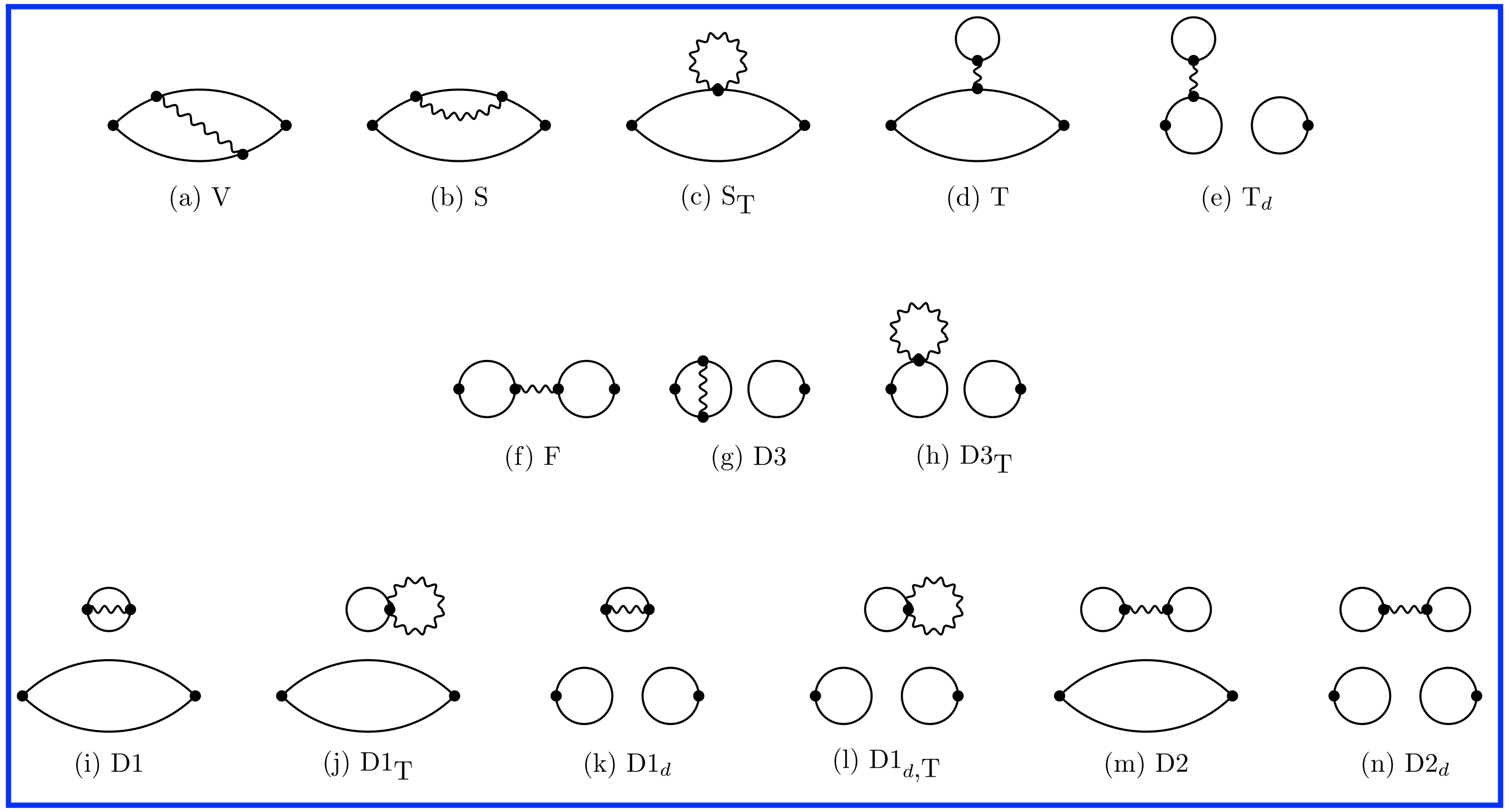
A. Boccaletti *et al.* 2024

Isospin-breaking contributions

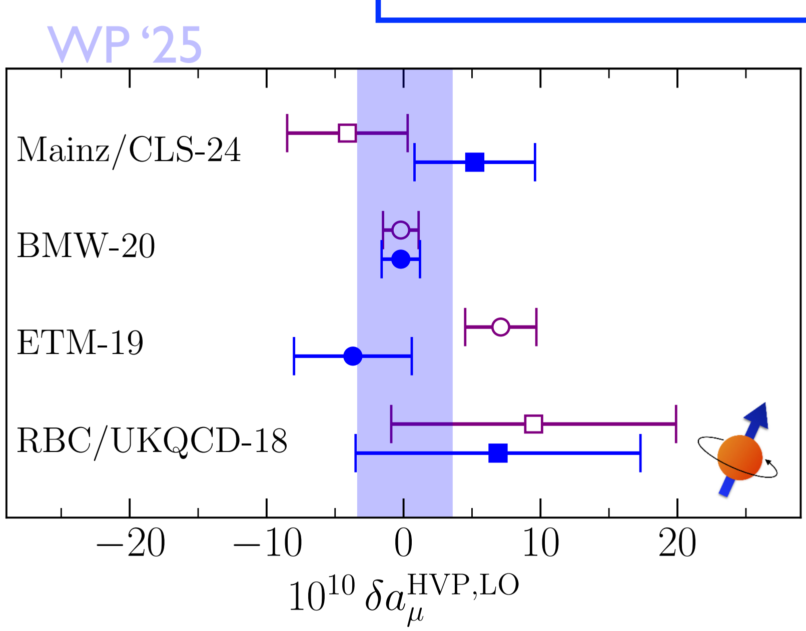




$m_d - m_u$



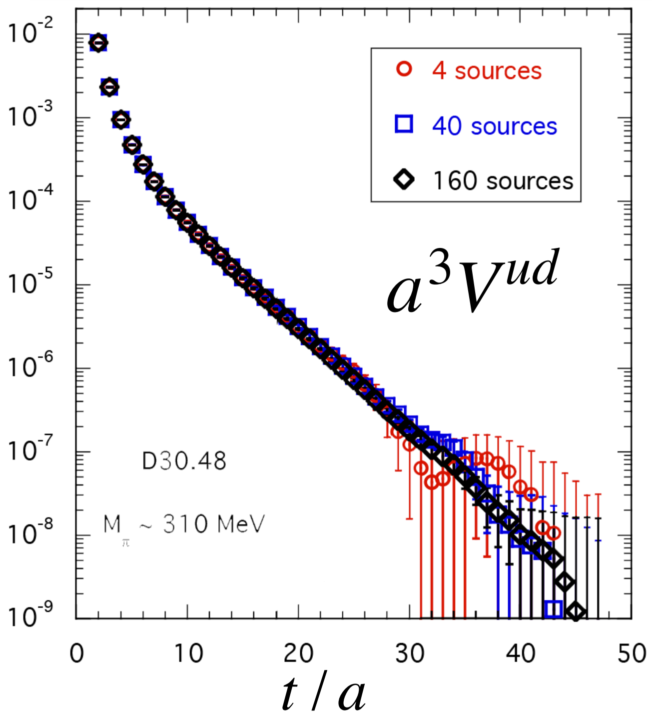
α_{em}



- Small overall value due to large cancellations
- Large statistical uncertainties
- More calculations are in progress

Beyond the muon g-2

Correlator representation



$$a_\ell^{\mathrm{HVP}} = \sum_{f=u,d,s,c} 4\alpha_{em}^2 \left\{ \sum_{t=0}^{T_{data}} \tilde{f}(t) V^f(t) + \sum_{t=T_{data}+a}^{\infty} \tilde{f}(t) V^f(t) \right\}$$

DG *et al.* 2018

lattice data   analytic representation

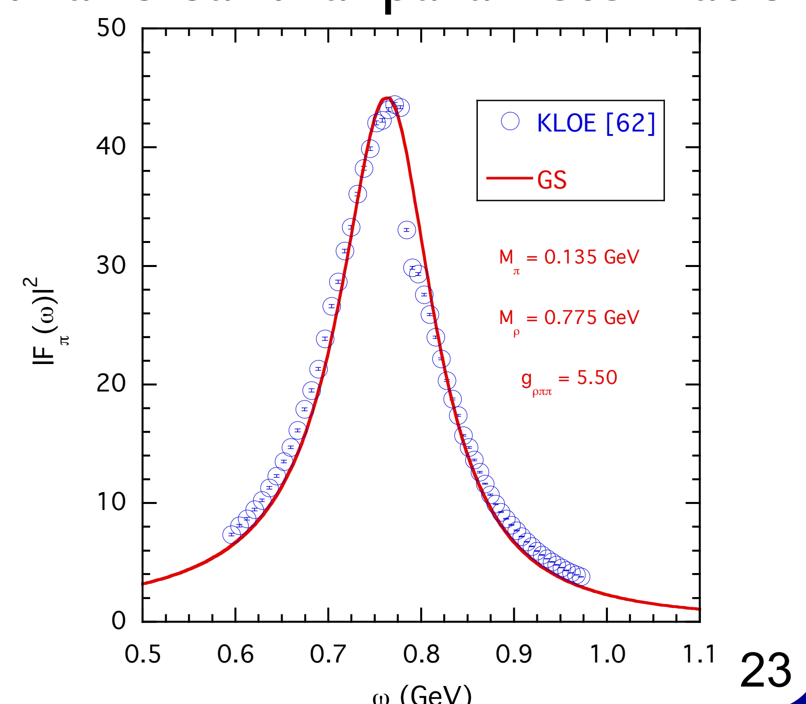
$$V^{ud}(t) = V_{dual}(t) + V_{\pi\pi}(t)$$

long time distances

$$V_{\pi\pi}(t) = \sum_n \nu_n |A_n|^2 e^{-\omega_n t}$$

L. Lellouch and M. Lüscher, 2001

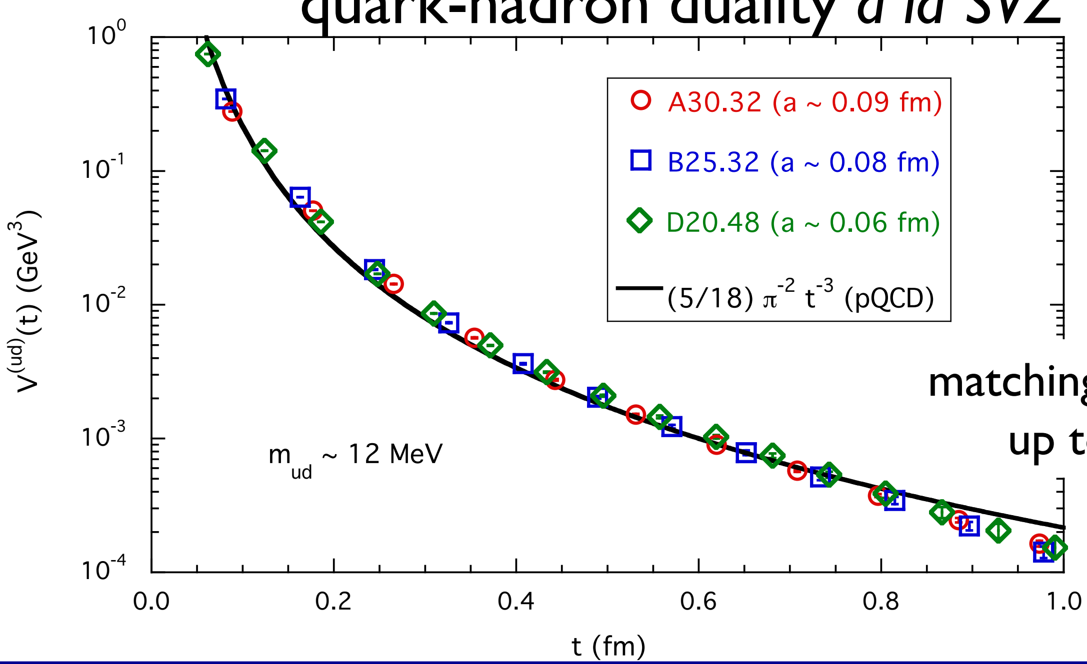
Gounaris-Sakurai parameterization



low and intermediate time distances

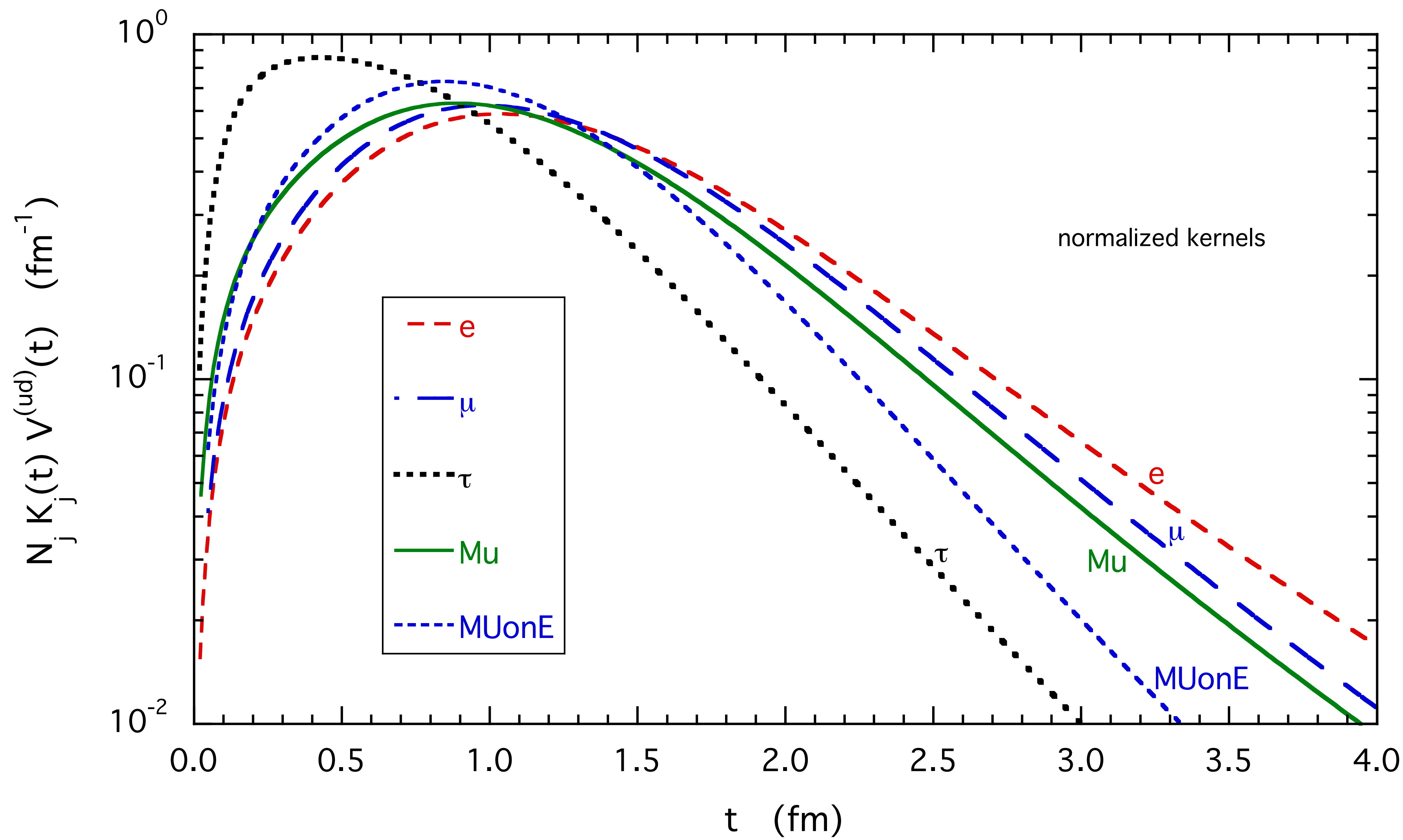
$$V_{dual}(t) \equiv \frac{1}{24\pi^2} \int_{s_{dual}}^{\infty} ds \sqrt{s}\; e^{-\sqrt{s}t} R^{pQCD}(s)$$

quark-hadron duality *à la SVZ*   SVZ, 1979



matching with pQCD up to t ~1 fm

Comparison of lepton HVP contributions



- Electron g-2: huge sensitivity to low-energy two-pion states
- Tau g-2: more severe discretization effects (sensitivity to larger momenta)

a_e^{HVP} and a_τ^{HVP} : Lattice results

ArXiv:1910.03874

LO

f	$a_e^{\text{HVP}}(f)\cdot 10^{14}$	$a_\tau^{\text{HVP}}(f)\cdot 10^{8}$
ud	170.7 (3.9)	273.3 (6.6)
s	13.5 (0.8)	36.2 (1.1)
c	3.5 (0.2)	25.8 (0.8)
disc	−3.8 (0.4)	−2.4 (0.3)

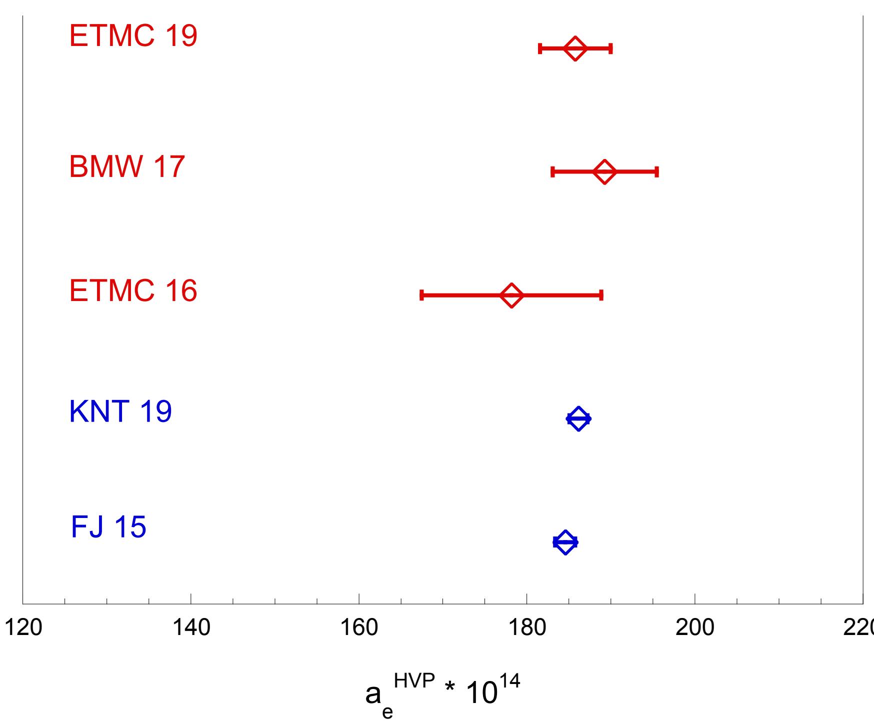
IB

f	$\delta a_e^{\text{HVP}}(f)\cdot 10^{14}$	$\delta a_\tau^{\text{HVP}}(f)\cdot 10^{8}$
ud	1.9 (0.8)	3.0 (1.1)
s	−0.002 (0.001)	0.001 (0.002)
c	0.004 (0.001)	0.032 (0.006)
total	1.9 (1.0)	3.0 (1.3)

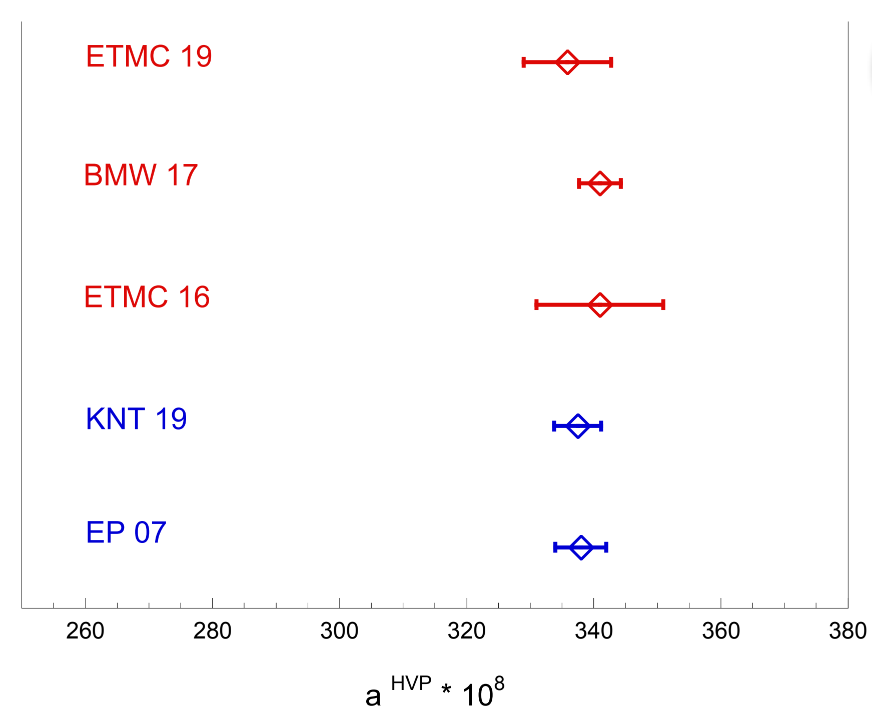
$$a_e^{\text{HVP}} = 185.8(4.2)\cdot 10^{-14}$$

$$a_\tau^{\text{HVP}} = 335.9(6.9)\cdot 10^{-8}$$

e



τ



$a_e^{\rm HVP}$ and $a_\tau^{\rm HVP}$: Lattice results

ArXiv:1910.03874

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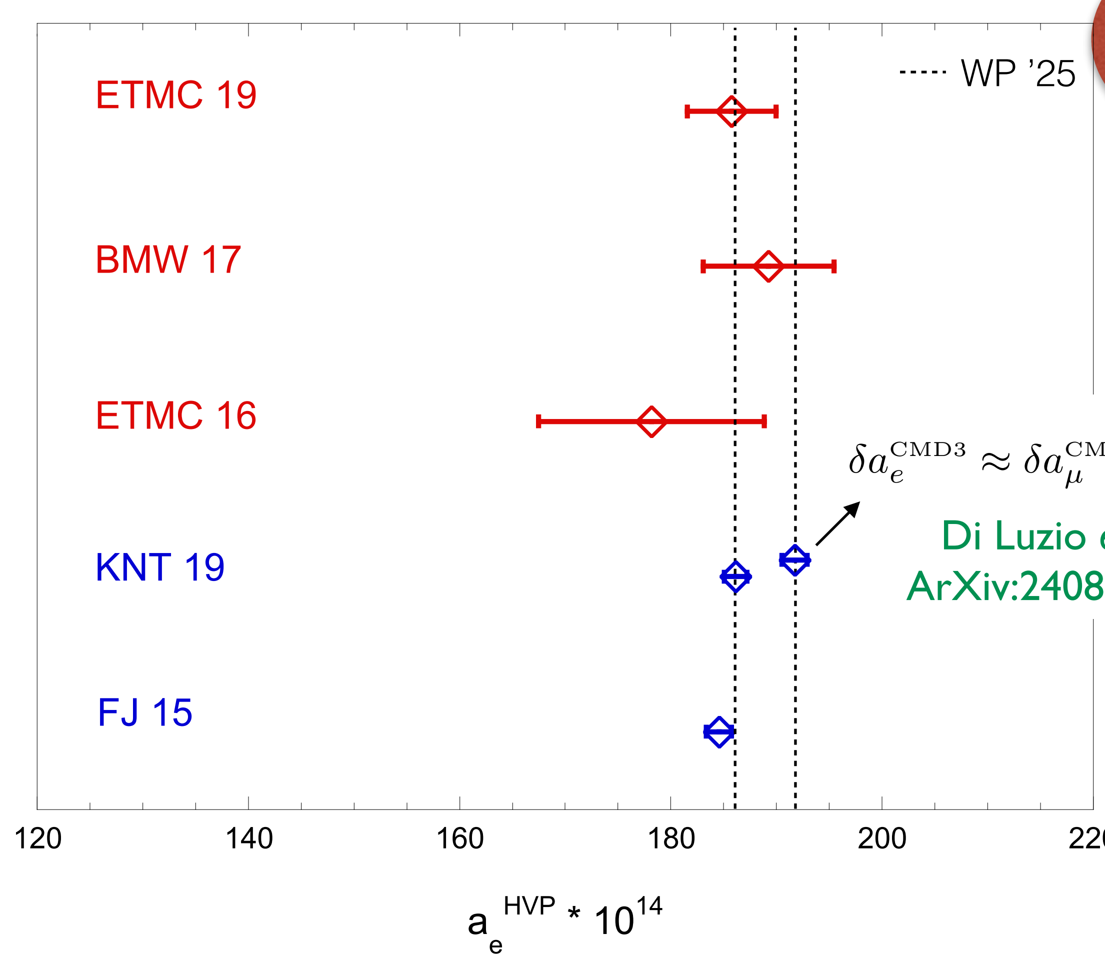
e

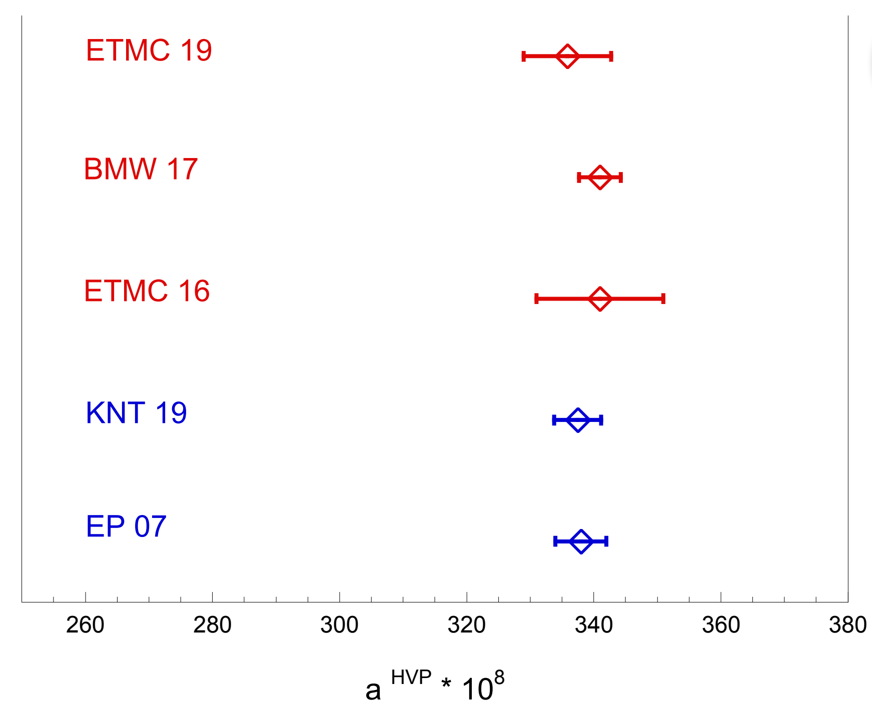


$$\delta a_e^{\rm CMD3} \approx \delta a_\mu^{\rm CMD3}\left(\frac{m_e}{m_\mu}\right)^2$$

Di Luzio *et al.*, ArXiv:2408.01123

$$a_\tau^{\rm HVP} = 335.9(6.9)\cdot 10^{-8}$$

τ



Muonium hyperfine splitting

$$a_{Mu} \equiv \frac{1}{2}\left[\frac{\Delta\nu_{Mu}}{\nu_F} - 1\right] \qquad \nu_F = \frac{16}{3} c R_\infty \, \alpha_{em}^2 \frac{m_e}{m_\mu}\left(1 + \frac{m_e}{m_\mu}\right)^{-3}$$

$$\Delta\nu_{Mu}^{exp} = 4\ 463\ 302\ 776\ [51]\ \text{Hz} \qquad a_{Mu}^{exp} = 10\ 624\ 150\ (56)_{\Delta\nu_{Mu}^{exp}}\ (568)_{\nu_F}\ [571] \cdot 10^{-10}$$

$$\Delta\nu_{Mu}^{SM} = 4\ 463\ 302\ 872\ [515]\ \text{Hz} \qquad a_{Mu}^{SM} = 10\ 624\ 305\ (95)_{QED}\ (3)_{had}\ [95] \cdot 10^{-10}$$

$$a_{Mu}^{HVP,LO} = 4\alpha_{em}^2 \int_0^\infty dt K_{Mu}(t) V(t)$$

$$K_{Mu}(t) = \frac{m_e}{m_\mu} t^2 \int_0^1 dx\, \frac{16 - 6x - x^2}{2x^2}\left[1 - j_0^2\left(\frac{m_\mu t}{2}\frac{x}{\sqrt{1-x}}\right)\right]$$

Lattice QCD+QED

$$\Delta\nu_{Mu}^{\text{HVP,LO}} = 231.7\ (4.5)\ \text{Hz}$$

DG and S. Simula

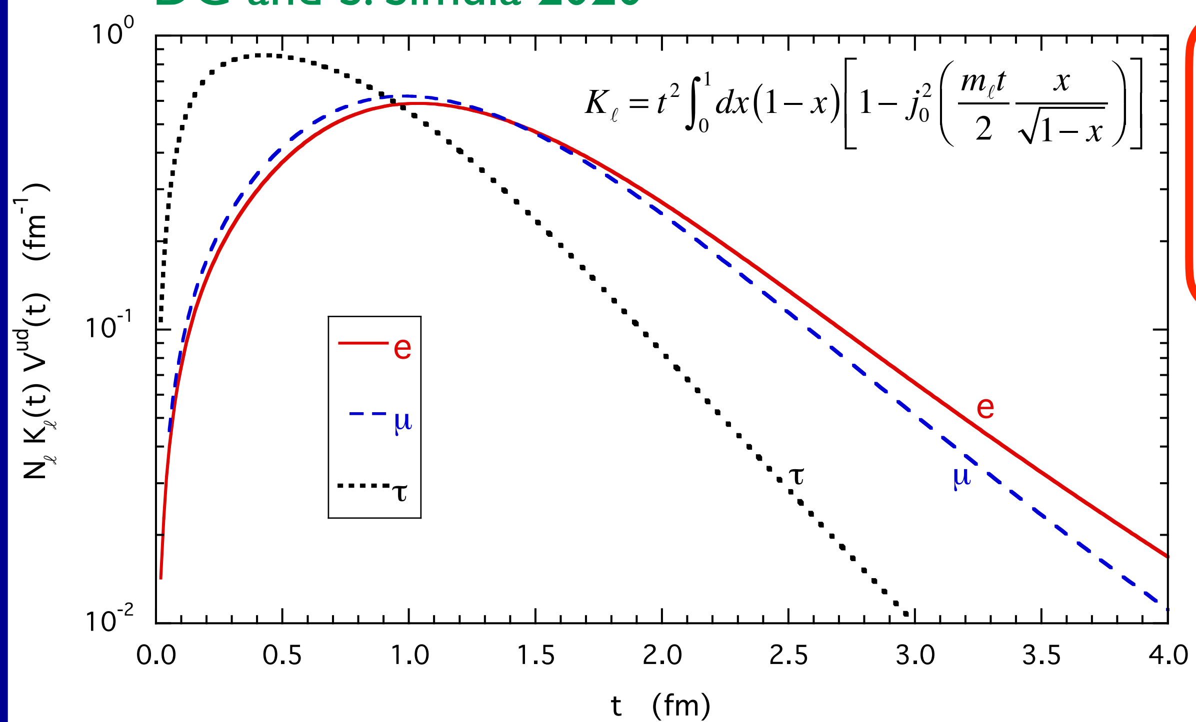
NEW!

$$\Delta\nu_{Mu}^{\text{HVP,LO}} = 232.04\ (0.82)\ \text{Hz}$$

KNT '19

Ratio electron/muon

DG and S. Simula 2020



$$R_{e/\mu} \equiv \left(\frac{m_\mu}{m_e}\right)^2 \frac{a_e^{\rm HVP}}{a_\mu^{\rm HVP}}$$

- numerator and denominator share the same hadronic input
- hadronic uncertainties strongly correlated (~ 98%) and largely cancel out

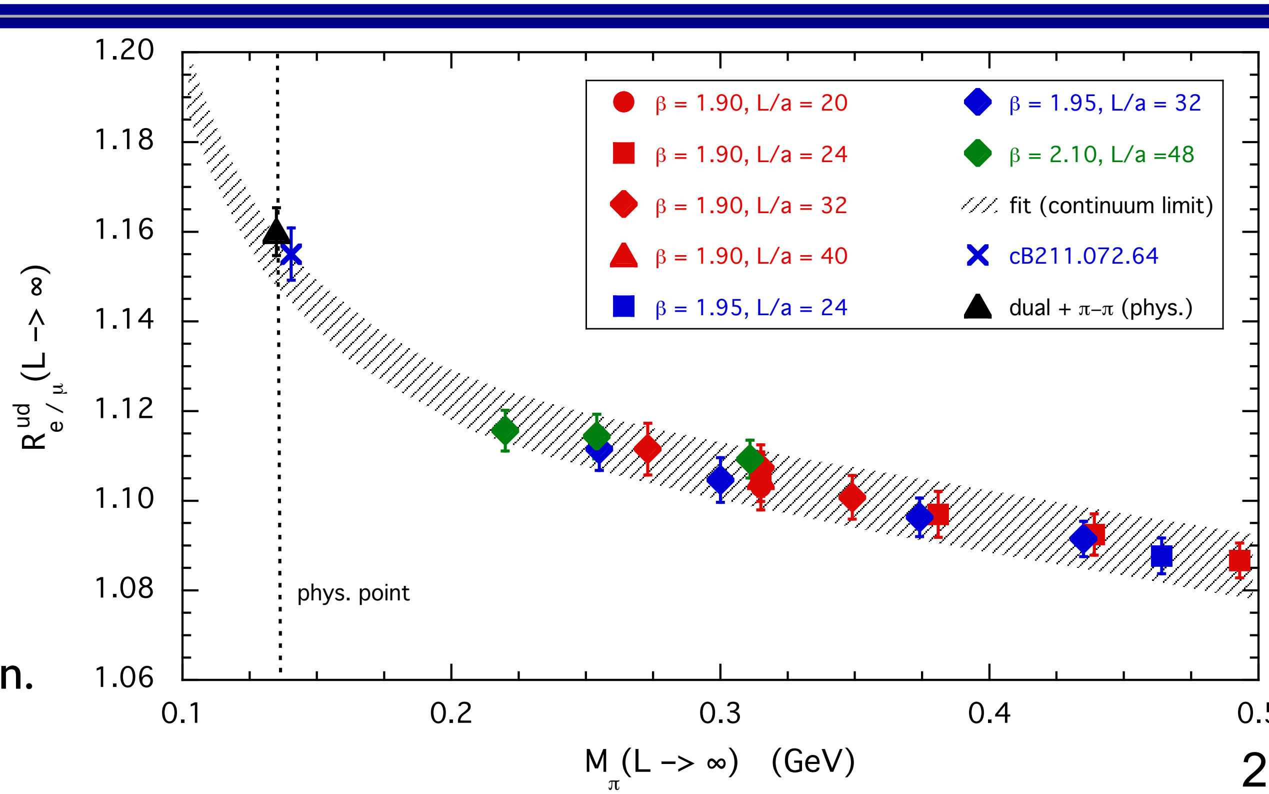
$$R_{e/\mu} \equiv R_{e/\mu}^{ud} \cdot \widetilde{R}_{e/\mu}$$

$$R_{e/\mu}^{ud} \equiv \left(\frac{m_\mu}{m_e}\right)^2 \frac{a_e^{\rm HVP}(ud)}{a_\mu^{\rm HVP}(ud)}$$

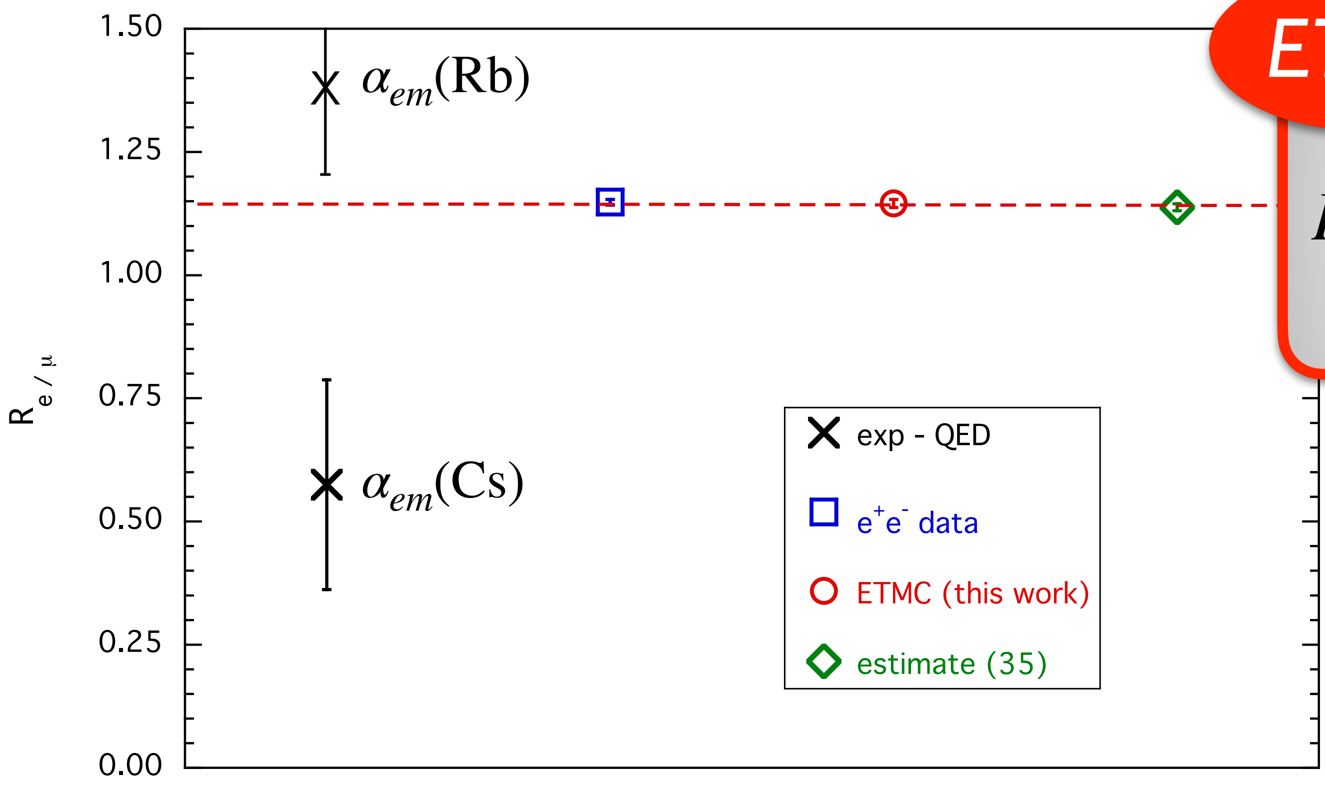
$$\widetilde{R}_{e/\mu} \equiv \frac{1 + \sum_{j=s,c,IB,disc} \frac{a_e^{\rm HVP}(j)}{a_e^{\rm HVP}(ud)}}{1 + \sum_{j=s,c,IB,disc} \frac{a_\mu^{\rm HVP}(j)}{a_\mu^{\rm HVP}(ud)}}$$

$R_{e/\mu}^{ud}$

- Precision of the data ≈ 4 times better than the individual HVP terms
- Discretization and scale setting errors play a minor role
- Non-trivial pion mass dependence
- Visible FVEs, removed using the analytic representation. The correction does not exceed ~ 1.3%



Results



ETMC

$$R_{e/\mu} = \left(\frac{m_\mu}{m_e}\right)^2 \frac{a_e^{\mathrm{HVP}}}{a_\mu^{\mathrm{HVP}}} = 1.1456(63)_{stat}(54)_{syst}\,[83]$$

$$R_{e/\mu} = 1.1381(72)$$

based on BMW 17
(assuming 100% correlation)

$$R_{e/\mu}^{e^+e^-} = 1.148343(62)$$

KNT 19

- The ratio $R_{e/\mu}$ is less sensitive to possible tensions between lattice and dispersive results occurring for the individual HVP terms
- Resolution of α_{em}(Cs) - α_{em}(Rb) discrepancy and twofold improvement in a_e^{exp} are necessary to make meaningful comparison

$R_{e/\mu}$	$R_{e/\tau}$	$R_{\mu/\tau}$
1.1456 (83)	6.69 (20)	5.83 (17)

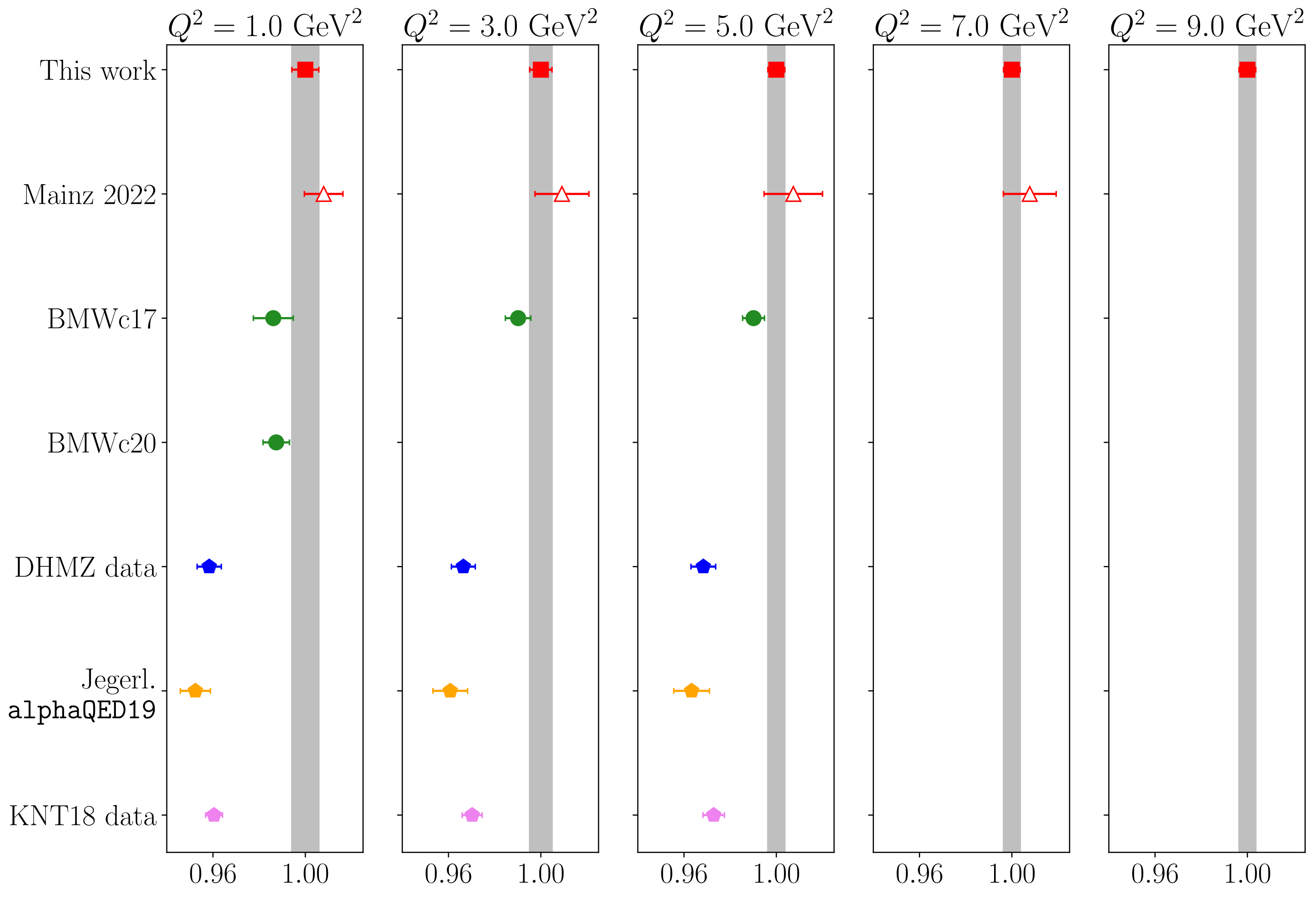
→ numerator and denominator almost uncorrelated

Hadronic running of α_{em} from the lattice

Comparison with other determinations at space-like momenta

Ratio of $\Delta\alpha_{\mathrm{had}}$ divided by our results for different Q^2

- Agreement with Mainz 2022
  [M. Cè et al., 2021]
- 1-2 σ disagreement between this work and [Borsanyi et al., 2018; Borsanyi et al., 2021]
- 3-6 σ tension between this work and [Keshavarzi, Nomura, and Teubner 2020; Davier et al. 2020; Jegerlehner 2020]



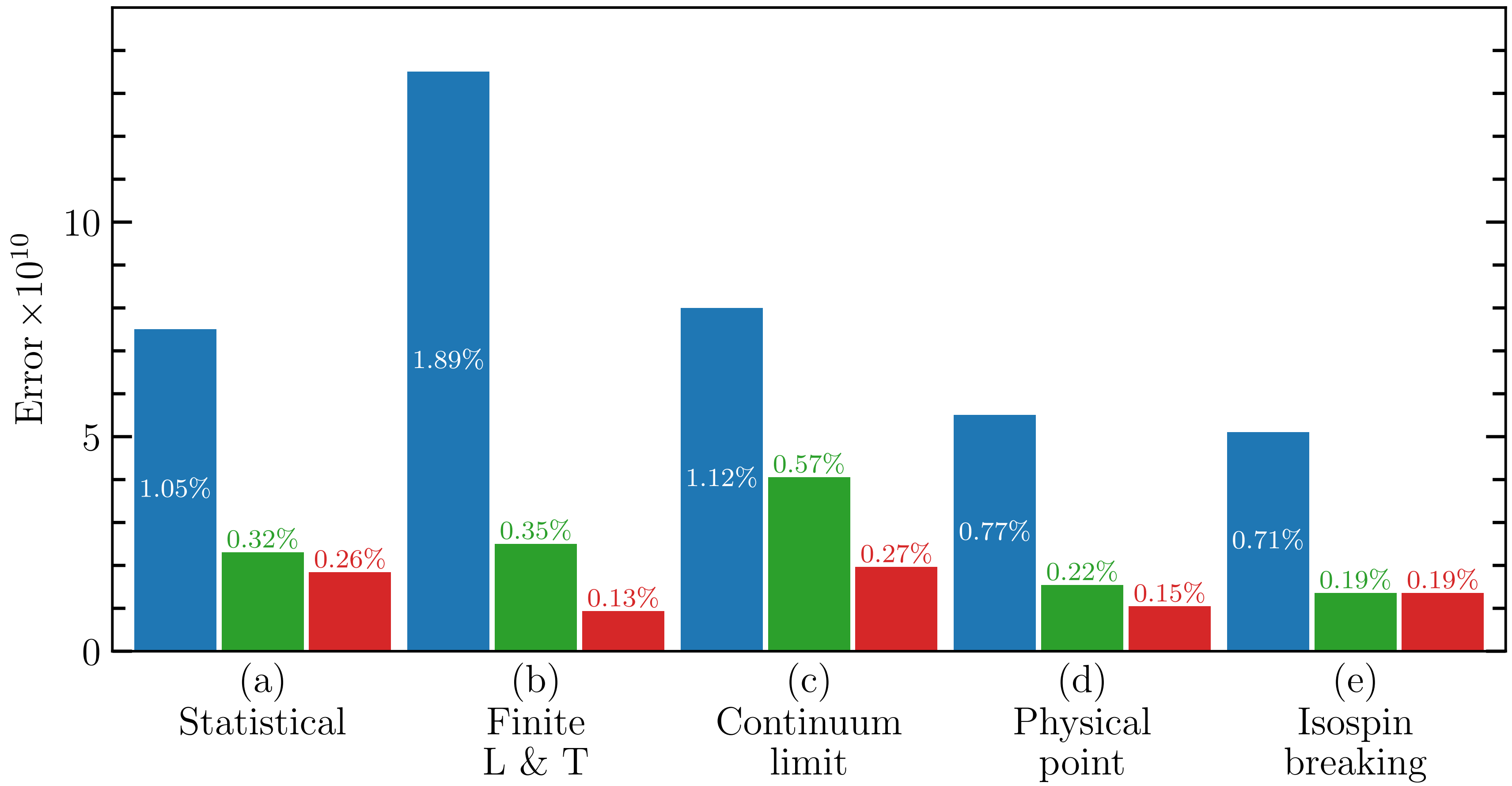
Talk @Muon g-2 TI in Orsay

The BMW/DMZ-24 calculation

BMW/DMZ-24 calculation

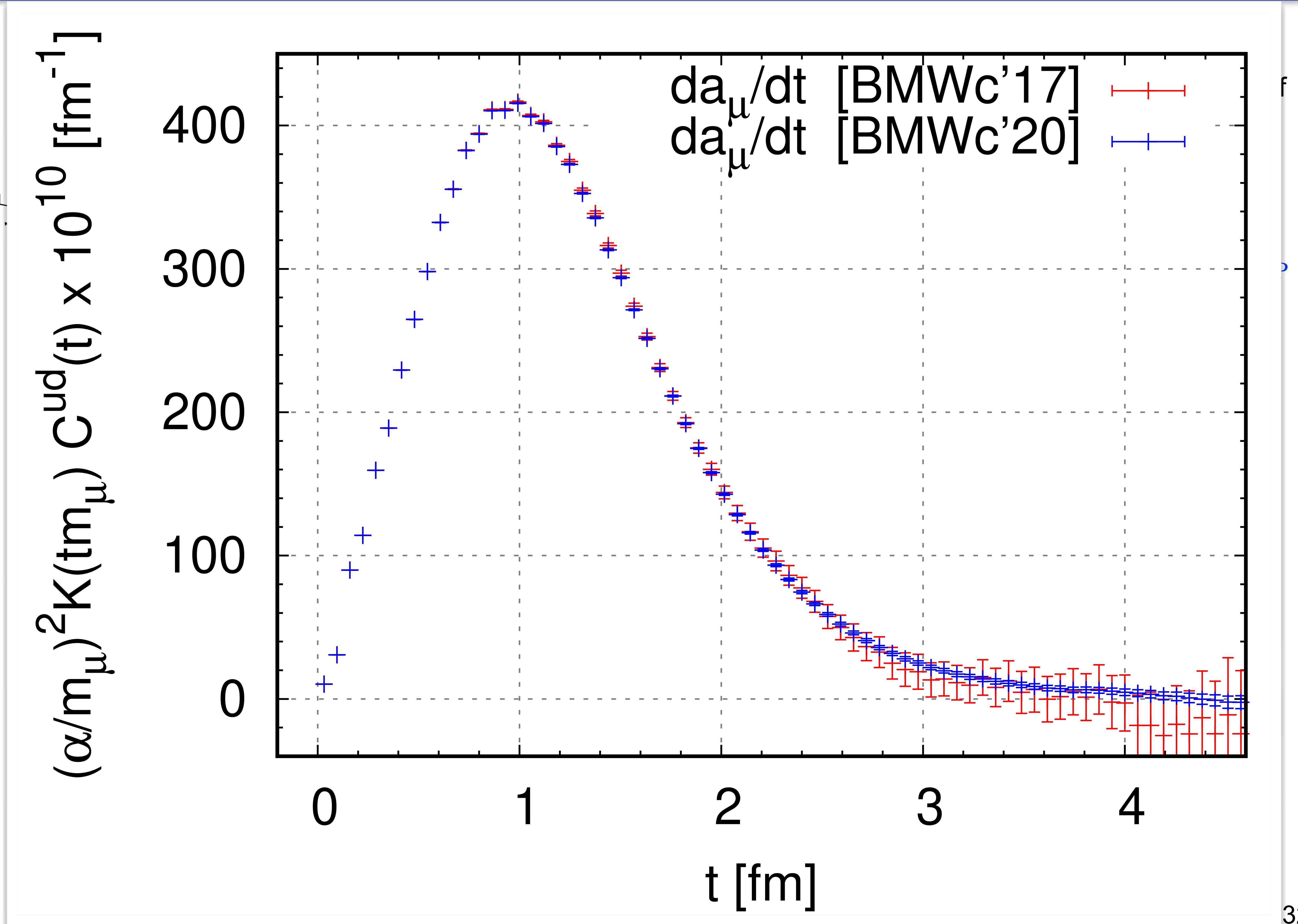
High precision calculation of the hadronic vacuum polarisation contribution to the muon anomaly

A. Boccaletti[1,2], Sz. Borsanyi[1], M. Davier[3], Z. Fodor[1,4,5,2,6,7,*], F. Frech[1], A. Gérardin[8], D. Giusti[2,9], A.Yu. Kotov[2], L. Lellouch[8], Th. Lippert[2], A. Lupo[8], B. Malaescu[10], S. Mutzel[8,11], A. Portelli[12,13], A. Risch[1], M. Sjö[8], F. Stokes[2,14], K.K. Szabo[1,2], B.C. Toth[1], G. Wang[8], Z. Zhang[3]

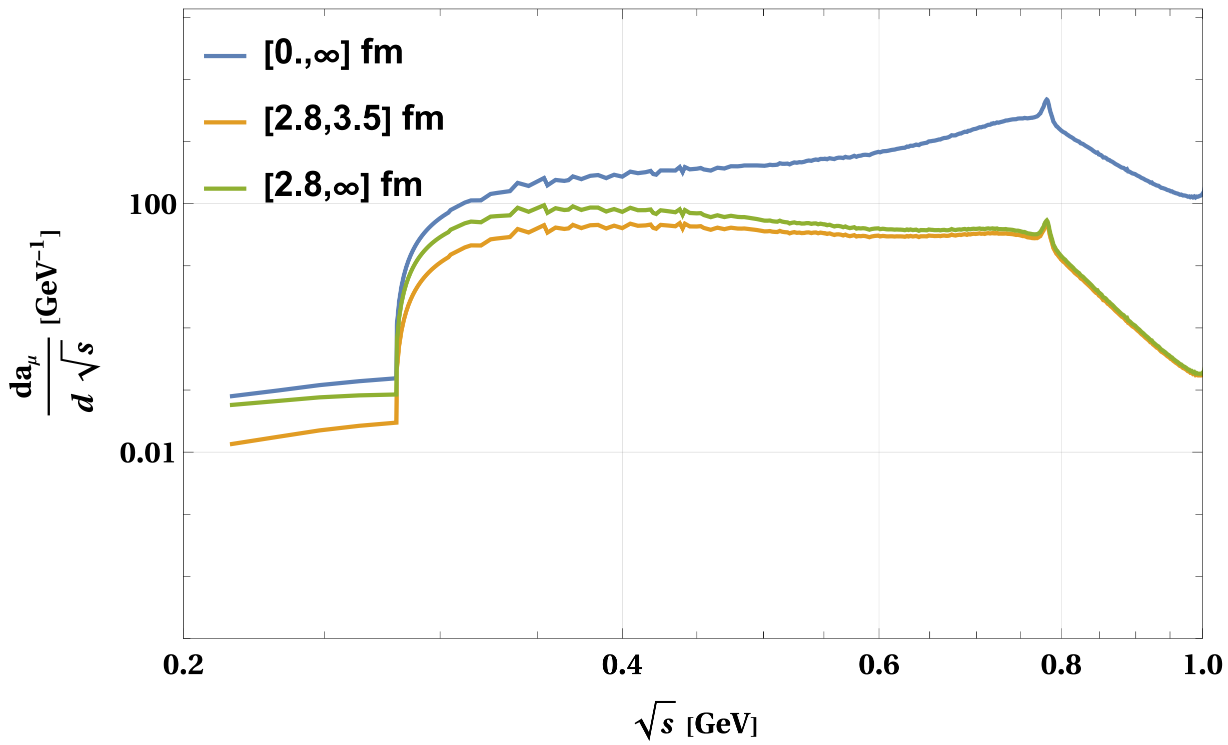


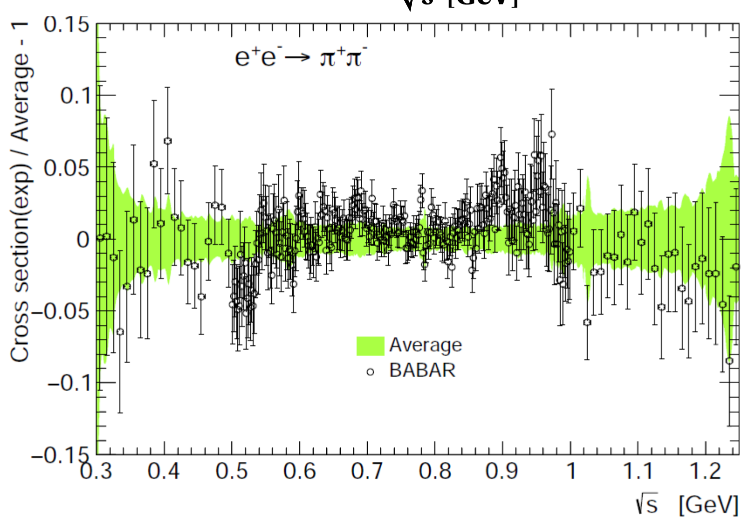
- New lattice spacing $a = 0.048$ fm (same cost as all of BMWc '20) $\longrightarrow$ divides a^2 effects by 2
- Over 30,000 gauge configurations, 10's of millions of measurements

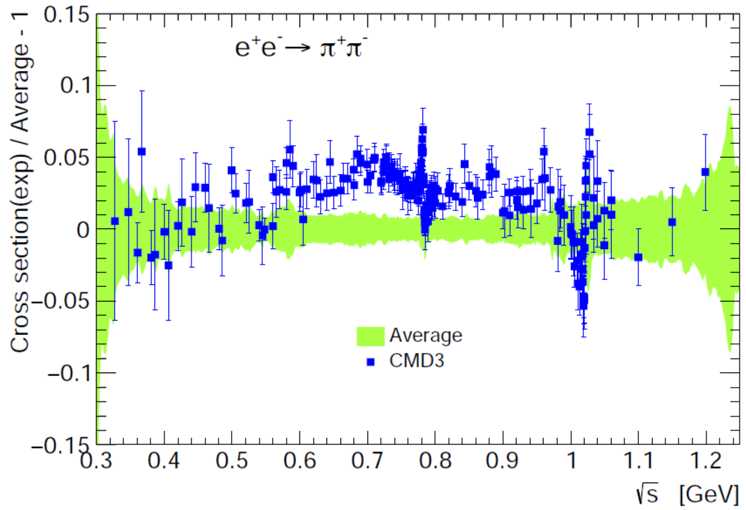
Tail contribution



Tail contribution

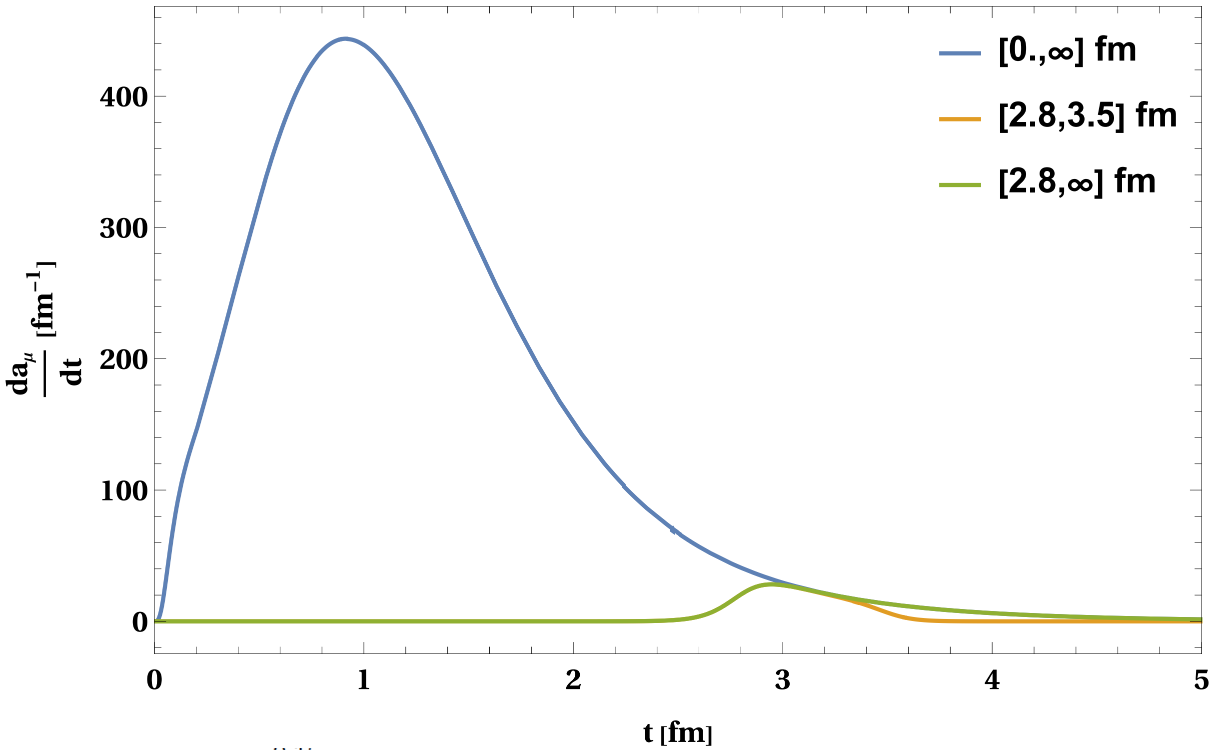


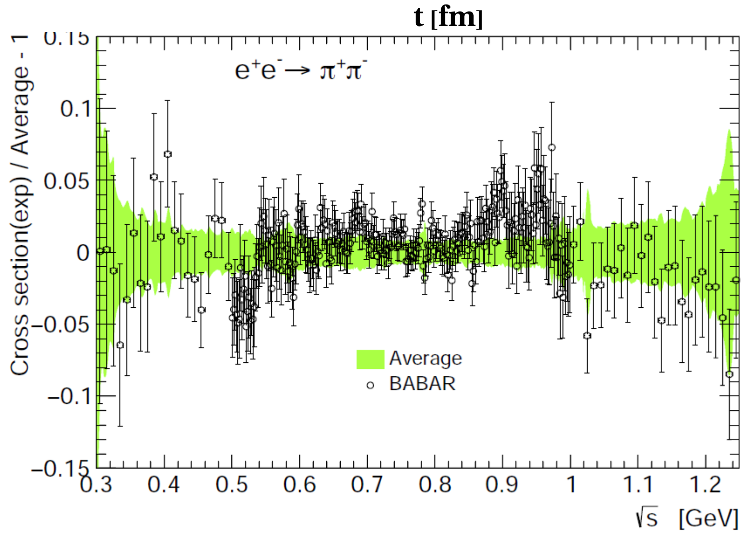


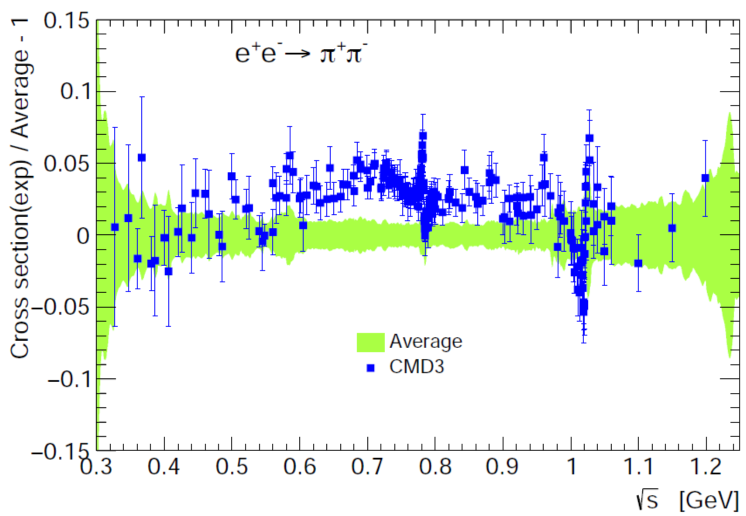


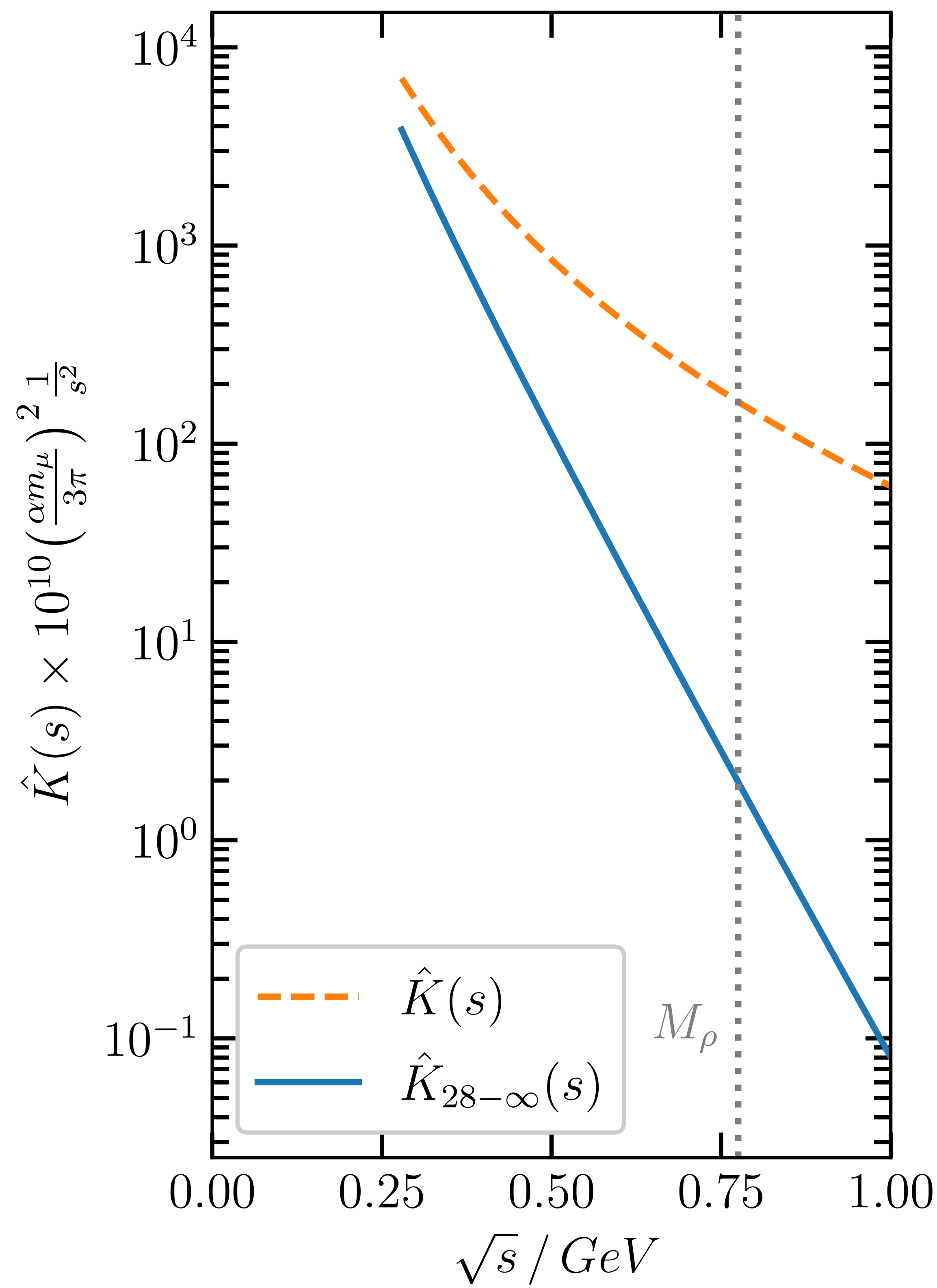
- Lattice computation up to $t = 2.8\ \mathrm{fm} : > 95\,\%$ of final result for $a_\mu^{\mathrm{LO-HVP}}$
- Tail $a_{\mu,28-\infty}^{\mathrm{LO-HVP}}$ computed using $e^+e^- \to \mathrm{hadrons}$ for $t > 2.8\ \mathrm{fm} : \lesssim 5\,\%$ of final result for $a_\mu^{\mathrm{LO-HVP}}$
- Tail dominated by cross section below ρ peak: $\sim 75\,\%$ for $\sqrt{s} \leq 0.63\ \mathrm{GeV}$
- All measurements agree to within 1.4σ for $\sqrt{s} \lesssim 0.55\ \mathrm{GeV}$. Tensions that plague $a_\mu^{\mathrm{LO-HVP}}$ & $a_{\mu,\mathrm{win}}^{\mathrm{LO-HVP}}$ not present here
- Partial tail $a_{\mu,28-35}^{\mathrm{LO-HVP}}$ for comparison with lattice; dominated by cross section below ρ peak: $\sim 70\,\%$ for $\sqrt{s} \leq 0.63\ \mathrm{GeV}$

Tail contribution

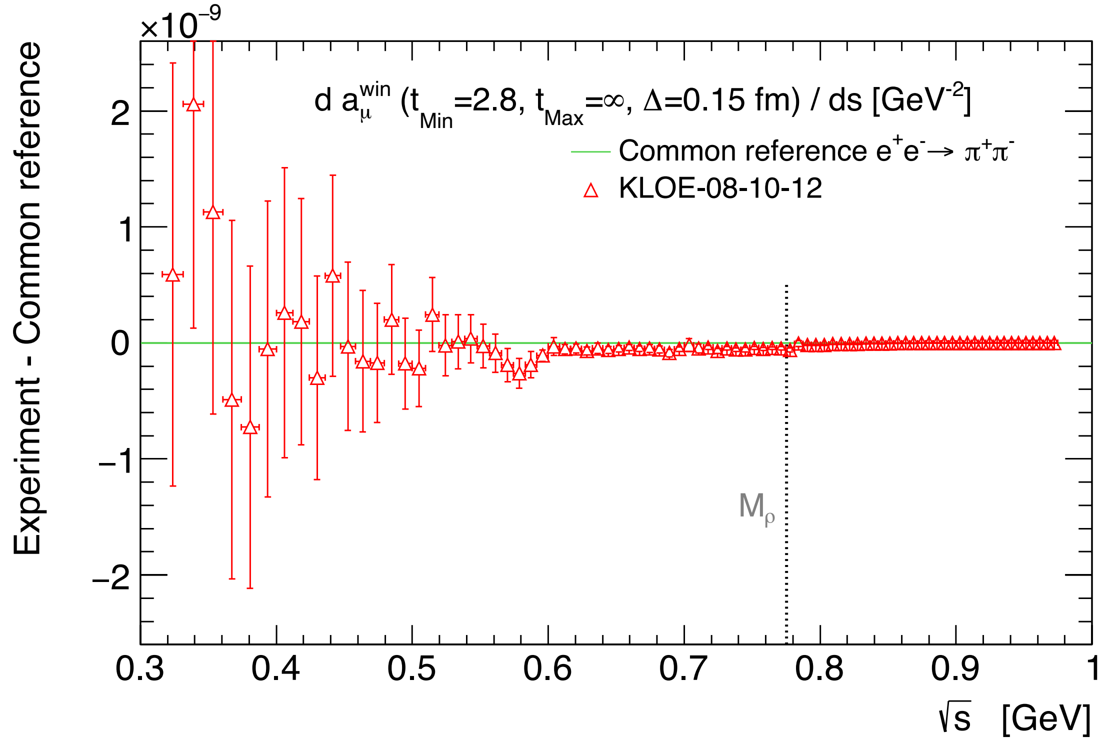


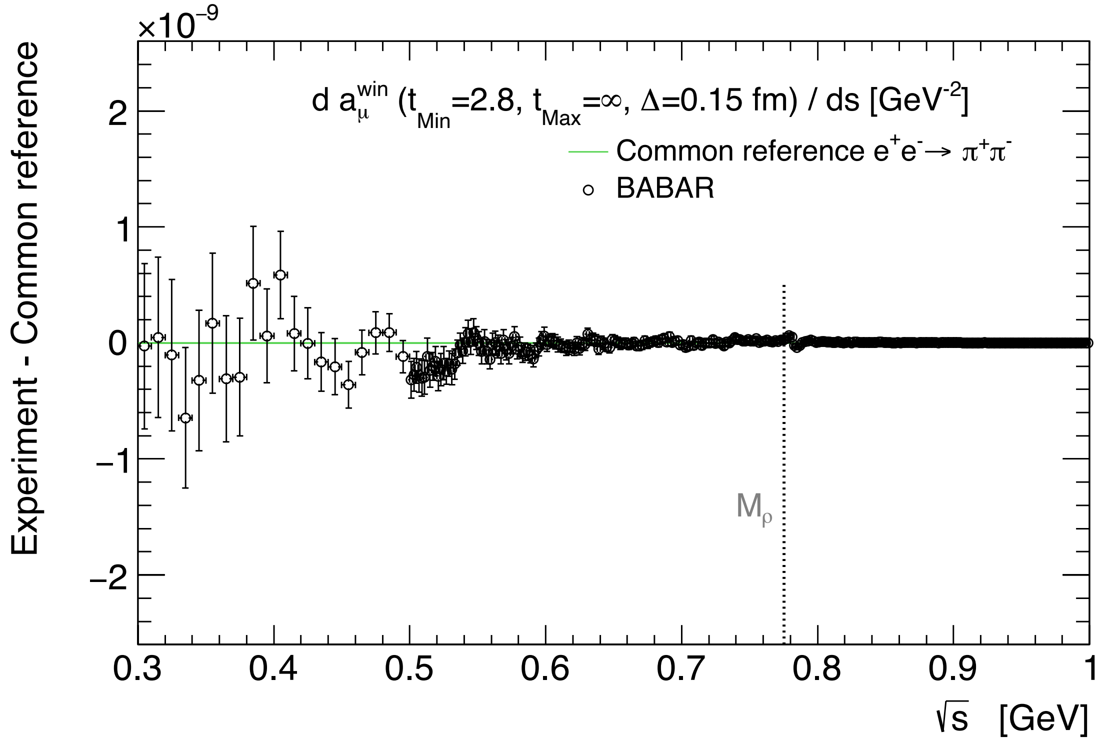


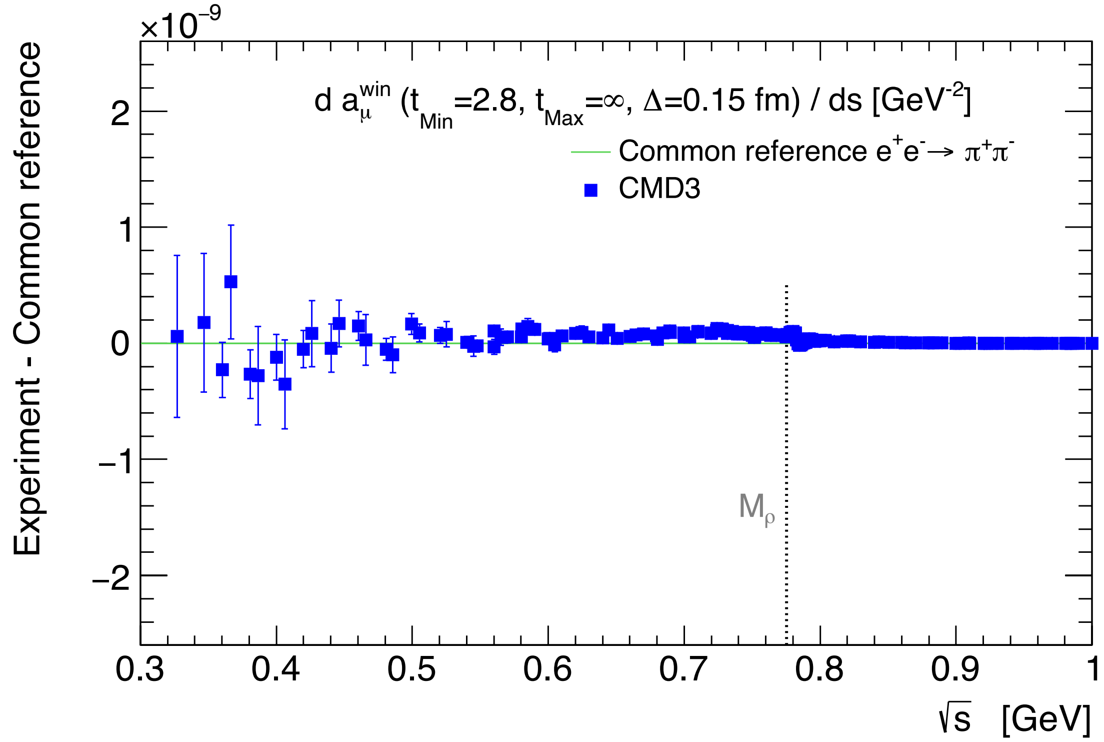


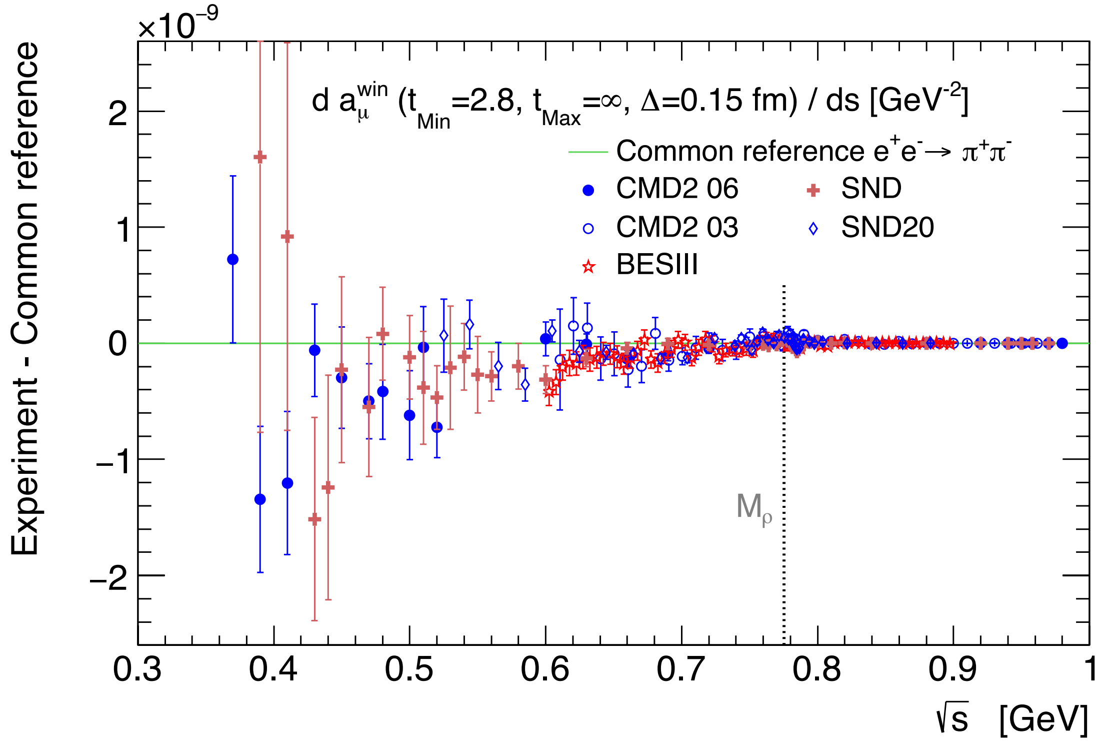


Cross section and the tail

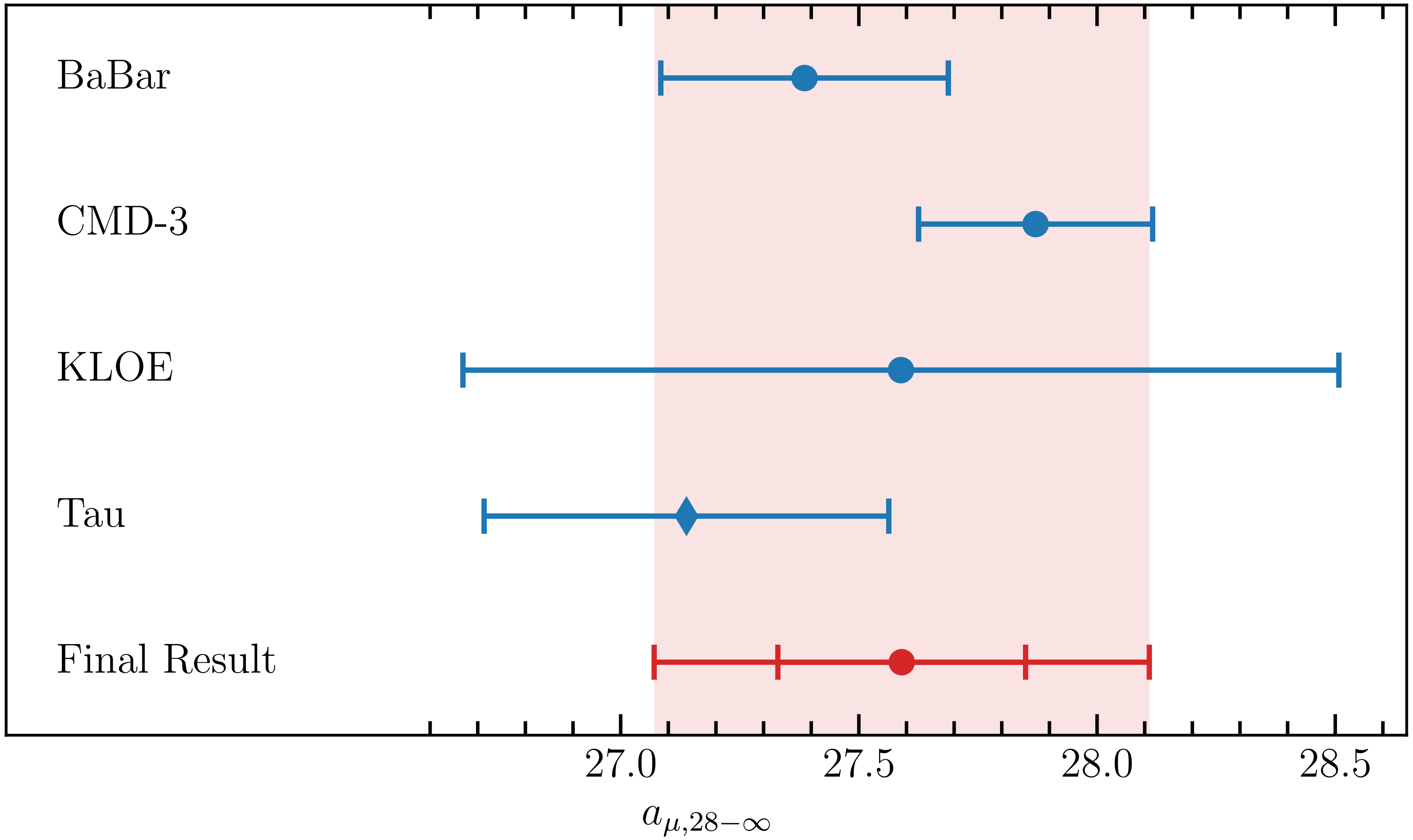




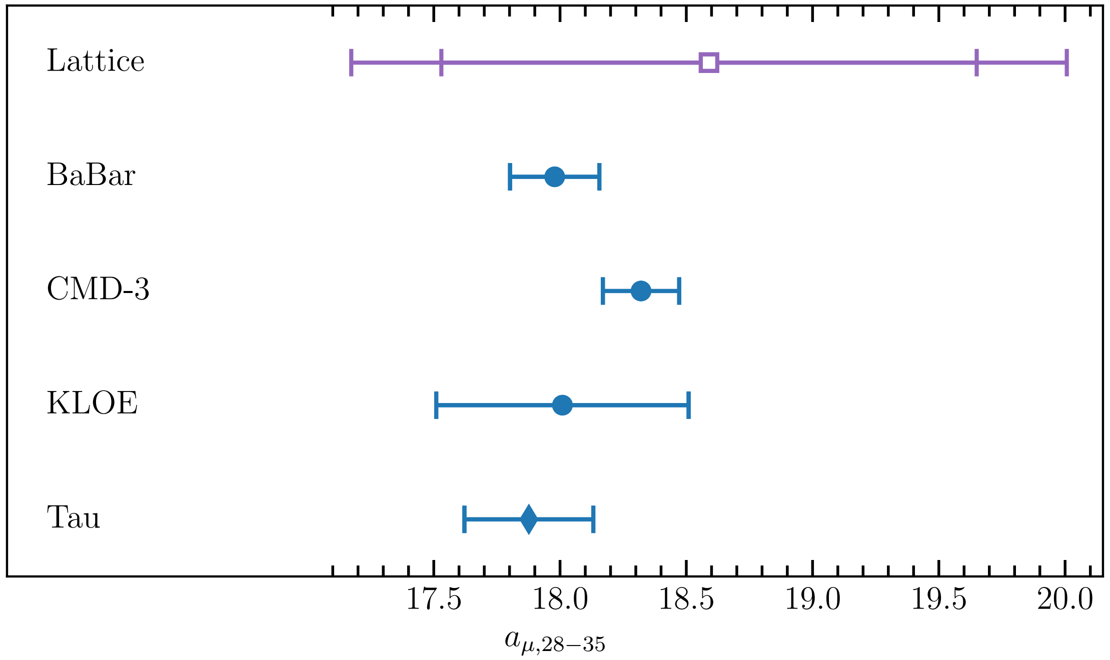




Data-driven tail

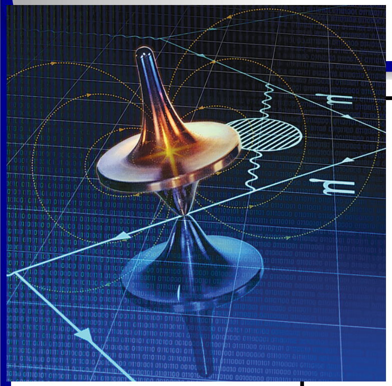


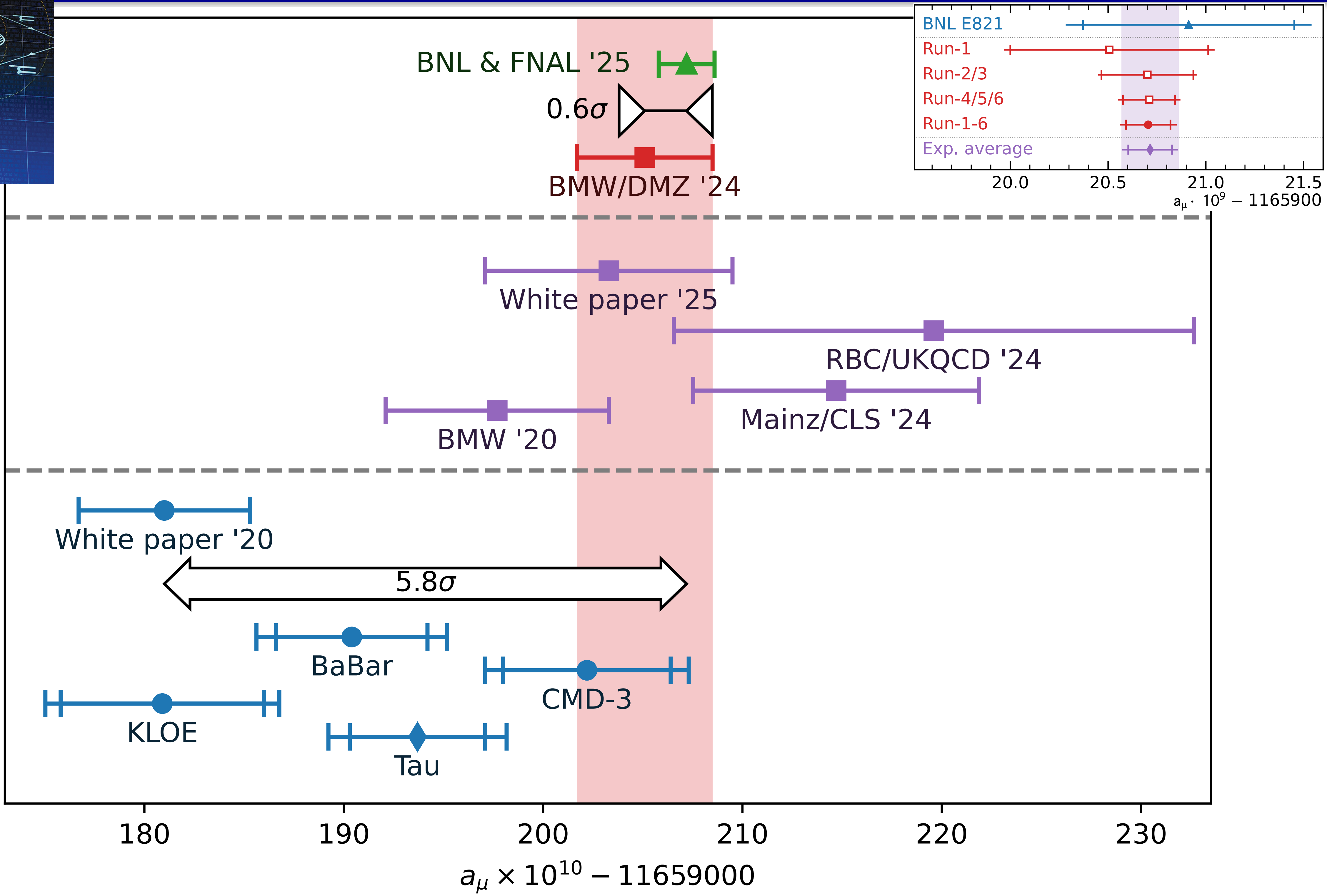
- Only $\lesssim$ 5% of final result for a_μ
- Contributes $\sim$ 65% to total squared uncertainty improvement: 5.5 $\rightarrow$ 3.2
- Even if the error was arbitrarily doubled, the effect on total uncertainty would be insignificant



July 12, 2024: unblinding

BMW-DMZ '24 vs g-2 experiment

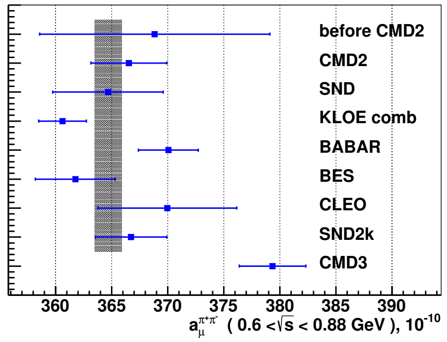




Indicates Standard Model confirmed to 0.29 ppm!

Summary and Outlook

- Tremendous progress in lattice calculations of HVP (and HLbL!) contributions
- New BMW-DMZ calculation to 0.44% w/ fully blinded analysis, confirming the SM to 0.29 ppm
- Good agreement between lattice calculations for various windows
- Compared to WP '20, in WP '25 the SM prediction is dominated by lattice calculations, w/ consolidated averages from many independent groups
- Hybrid lattice-dispersive approaches could soon provide competitive estimates for other "leptonic" observables
- Awaiting new KLOE, BESIII, Belle II, CMD3, SND2 data/analysis to clarify tensions in $\pi^+\pi^-$



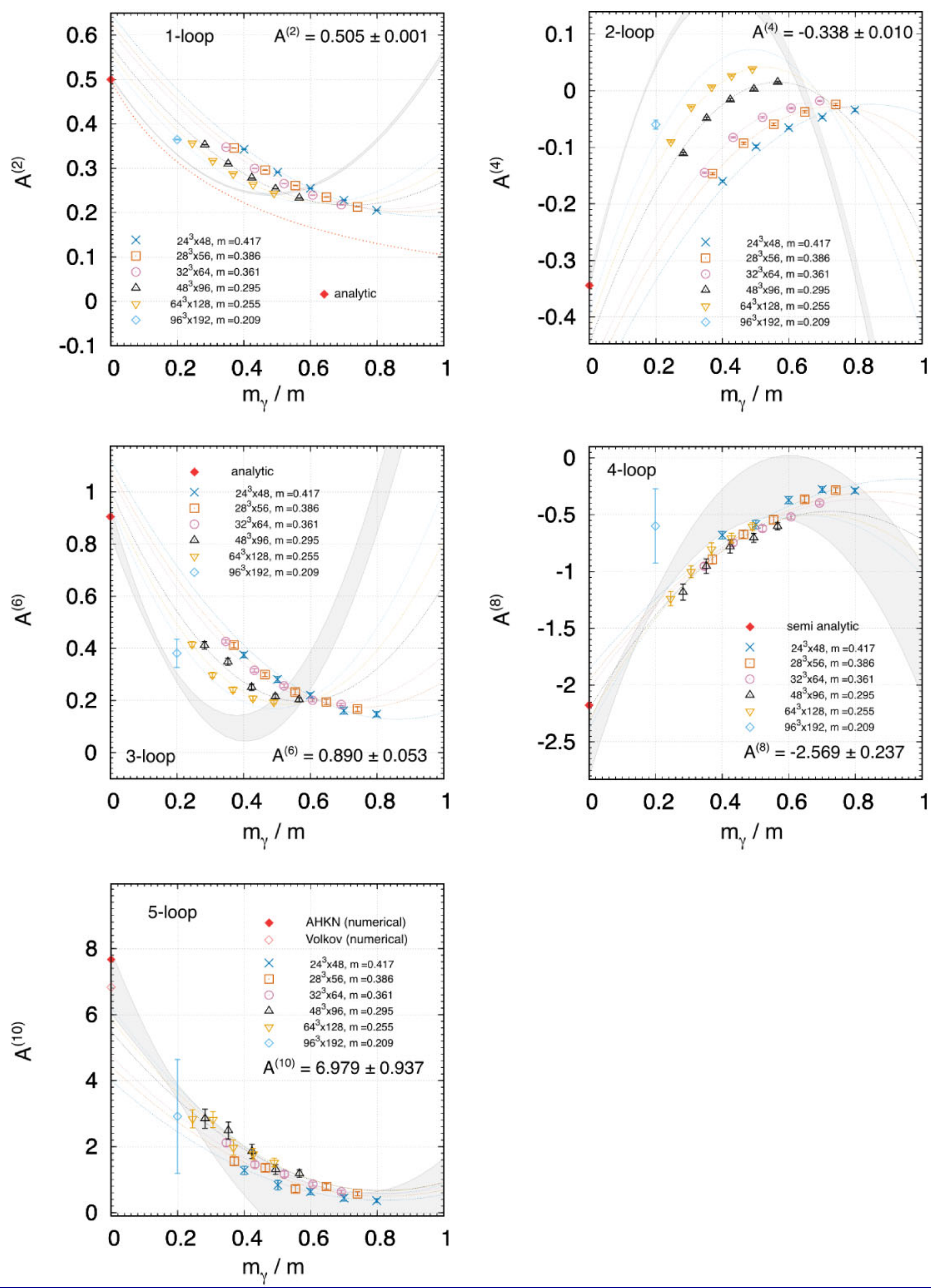
- $\mu e \to \mu e$ experiment MUonE very important for experimental cross-check and complementarity w/ LQCD

Backup slides

QED multiloop from the lattice

PTEP **2025**, 013B02 R. Kitano

Diagrams w/o lepton loops only



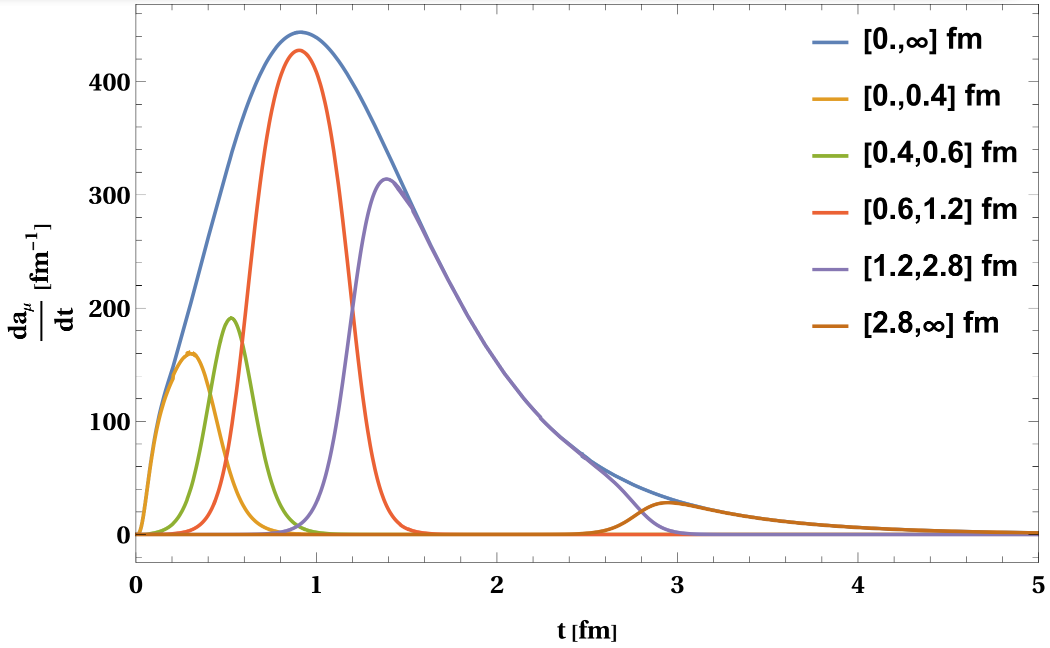
Strategy for improvement

- New simulations on finer lattice spacing: $128^3 \times 192$ w/ $a = 0.048\,\mathrm{fm}$
- Completely revamped analysis vs BMWc '20
- Break up analysis into optimized set of windows: $0-0.4$, $0.4-0.6$, $0.6-1.2$, $1.2-2.8\,\mathrm{fm}$
- Combined fit to $a_{\mu,\text{win},04\text{-}06}^{\text{LO-HVP}}$, $a_{\mu,\text{win},06\text{-}12}^{\text{LO-HVP}}$, $a_{\mu,\text{win},12\text{-}28}^{\text{LO-HVP}}$
- Continuum extrapolate $I = 0$ instead of disconnected

$\rightarrow$ reduces statistical uncertainty
$\rightarrow$ reduces $a \rightarrow 0$ error

- Data-driven evaluation of tail: $a_{\mu,28\text{-}\infty}^{\text{LO-HVP}}$ (proposed and used w/ $1\,\mathrm{fm} \rightarrow \infty$ [RBC/UKQCD '18])

$\rightarrow$ reduces FV effect $18.5(2.5) \rightarrow 9.3(9)$, i.e. cv $\div 2$ & err $\div 3$
$\rightarrow$ reduces LD noise
$\rightarrow$ reduces LD taste breaking and $a \rightarrow 0$ error



[plot made w/ KNT '18 data set]

Fully blinded analysis:

- Independent blinding by factor $\pm 3\%$ on correlator for each window and component, including data-driven tail
- $\gtrsim 2$ independent analyses of all blinded $a_\mu^{\text{LO-HVP}}$ contributions (and of other aspects)
- Once agreement reached, partial unblinding to allow sum of contributions
- Full unblinding on July 12, 2024, w/ automatic script that made appropriate changes in all figures and text
- Paper submitted to arXiv on July 15, 2024

Summary of all contributions [BMW/DMZ-24]

light and disconnected $00-28$	$618.6(1.9)(2.3)[3.0]$	this work, Equation (34)
strange $00-28$	$53.19(13)(16)[21]$	this work, Equation (37)
charm $00-28$	$14.64(24)(28)[37]$	this work, Equation (40)
light qed	$-1.57(42)(35)$	[5], Table 15 corrected in Equation (45)
light sib	$6.60(63)(53)$	[5], Table 15
disconnected qed	$-0.58(14)(10)$	[5], Table 15
disconnected sib	$-4.67(54)(69)$	[5], Table 15
disconnected charm	$0.0(1)$	[31], Section 4 in Supp. Mat.
strange qed	$-0.0136(86)(76)$	[5], Table 15
charm qed	$0.0182(36)$	[43]
bottom	$0.271(37)$	[44]
tail from data-driven $28-\infty$	$27.59(17)(9)[26]$	this work, Equation (50)
total	$714.1(2.2)(2.5)[3.3]$	

ArXiv:2407.10913

$$a_\mu^{\text{LO-HVP}} \times 10^{10} = 714.1(2.2)(2.5)[3.3] \quad [0.46\%]$$