

# Precision nucleon and nuclear structure from light (muonic) atoms



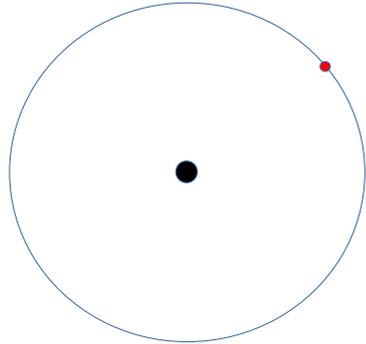
Randolf Pohl

Johannes Gutenberg  
Universität Mainz

MITP  
Workshop on alpha  
30.10.25



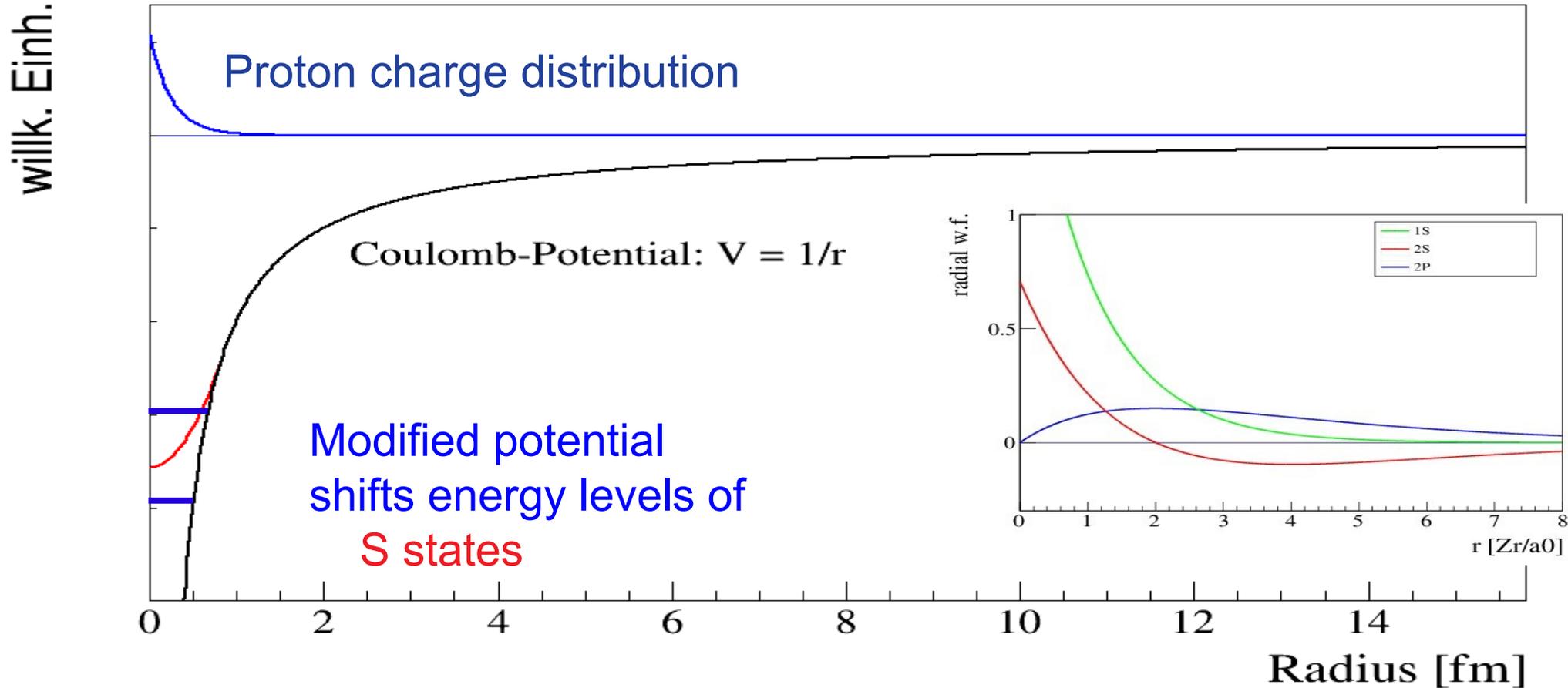
# Hydrogen-like atoms



“There's a reason physicists are so successful with what they do, and that is they study the hydrogen atom and the helium ion and then they stop.”

- Richard Feynman

# Proton Radius and Hydrogen



# Muonic Atoms

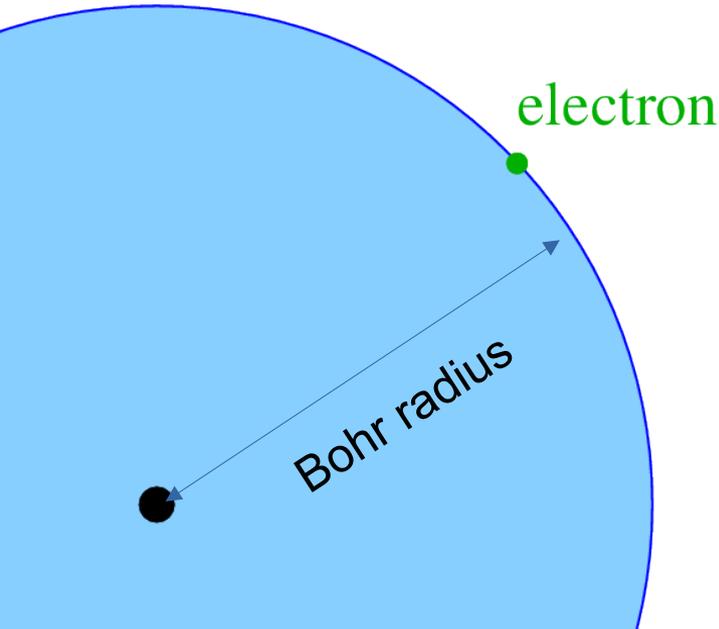
A **bare** nucleus, orbited by **one negative muon**.

Hydrogen theory!

# Muonic atoms in a nutshell

Regular hydrogen:

Bohr radius  $\sim 50'000$  x nuclear radius



Muonic hydrogen:

Muon **mass** = **200** \* electron mass

Bohr **radius** = **1/200** of H

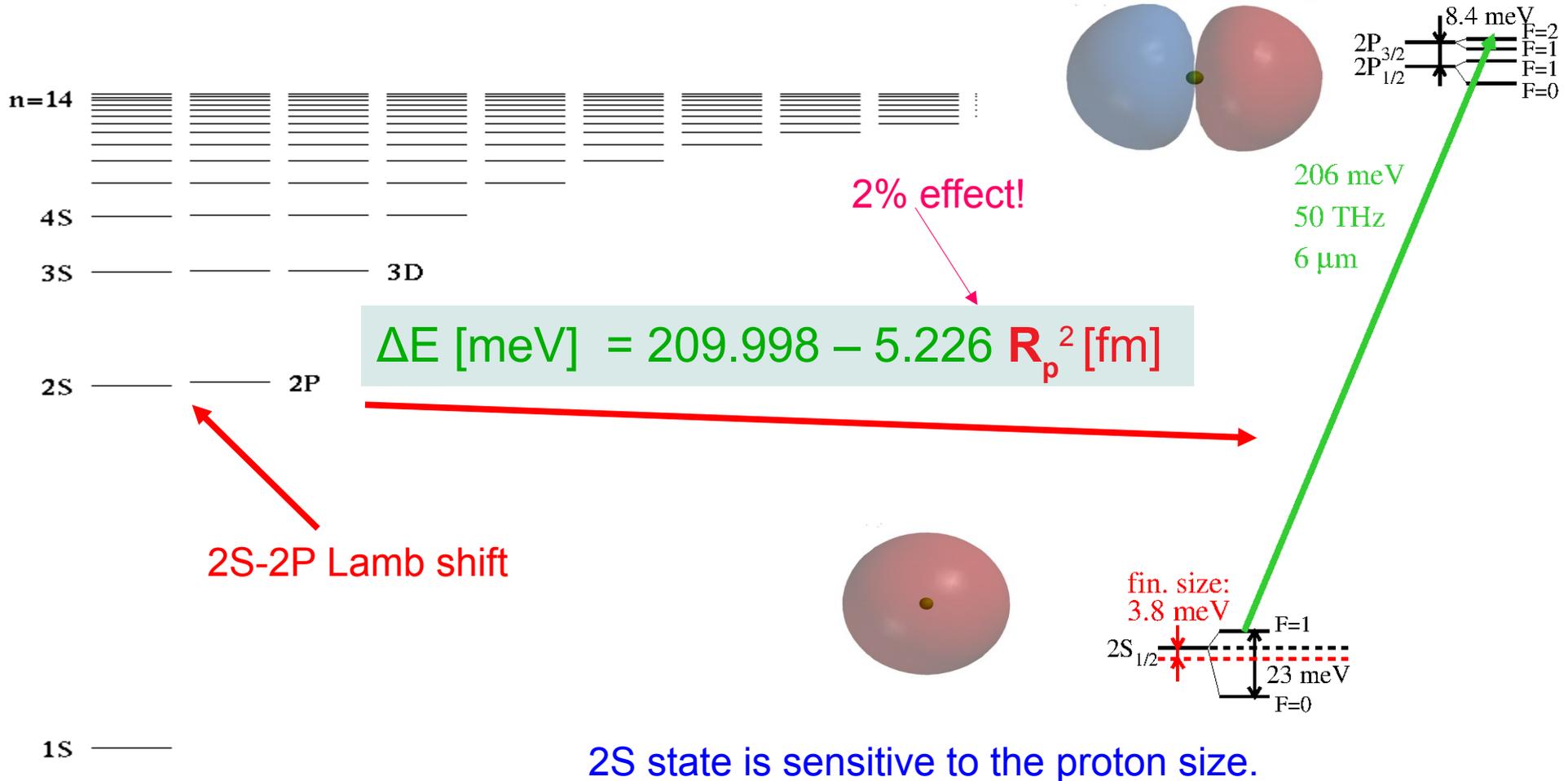
**200<sup>3</sup>** = a **ten million times** more sensitive to nuclear size & structure

==> Our (laser) spectroscopy at **10<sup>-5</sup>** level can compete with **10<sup>-12</sup>** from normal atoms

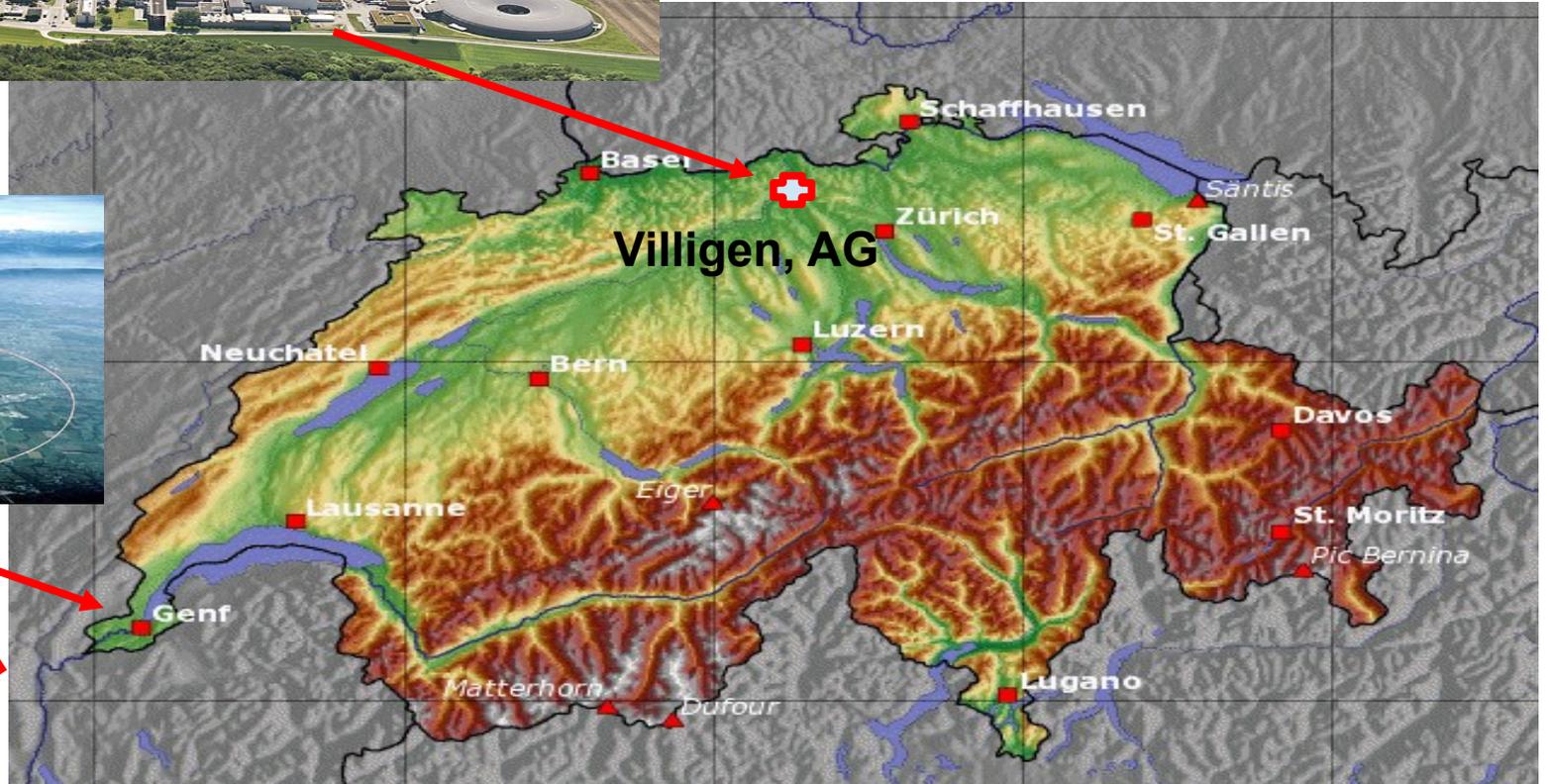


Vastly not to scale!!

# Lamb shift in Muonic Hydrogen

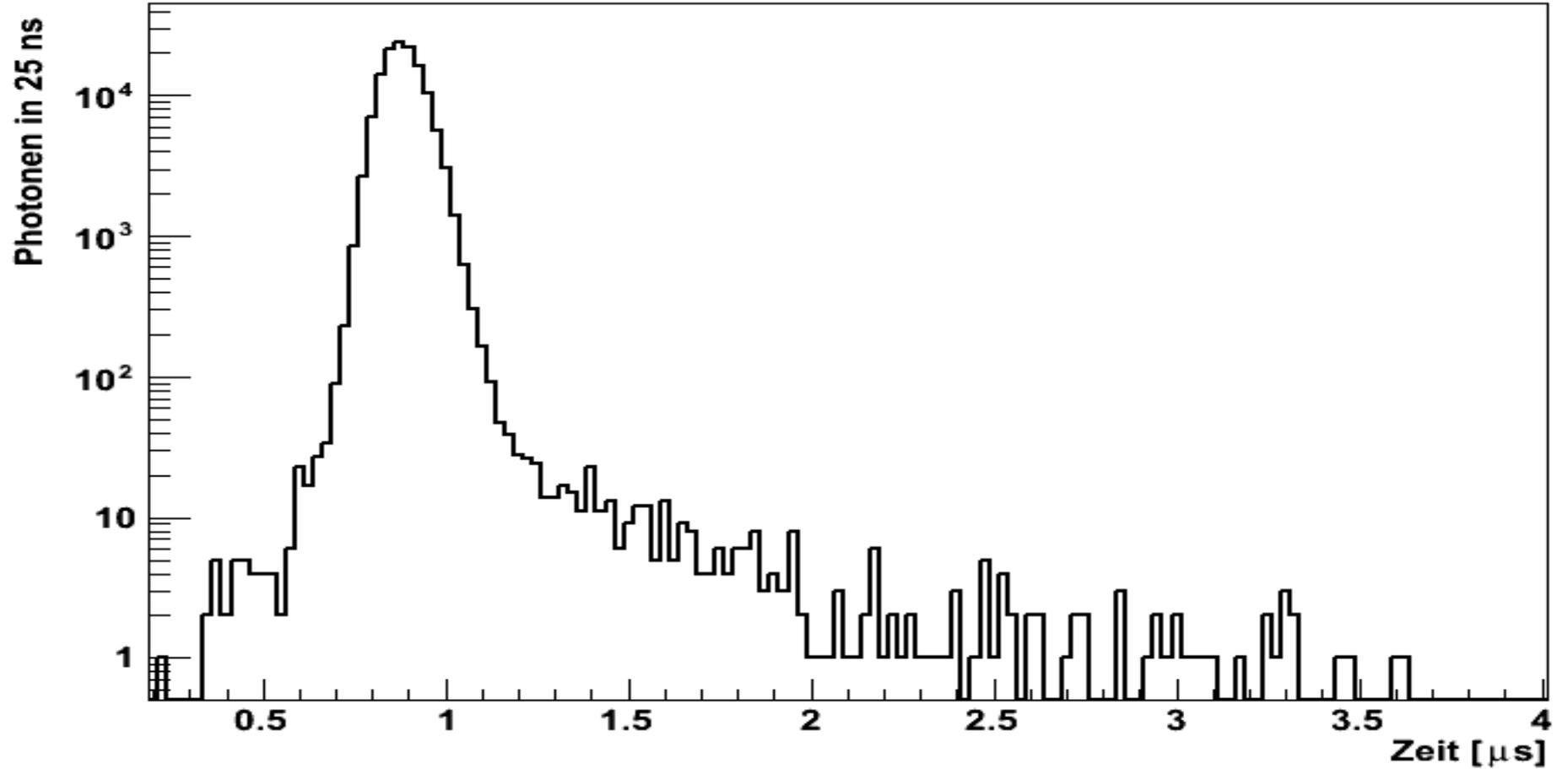


# The accelerator at PSI



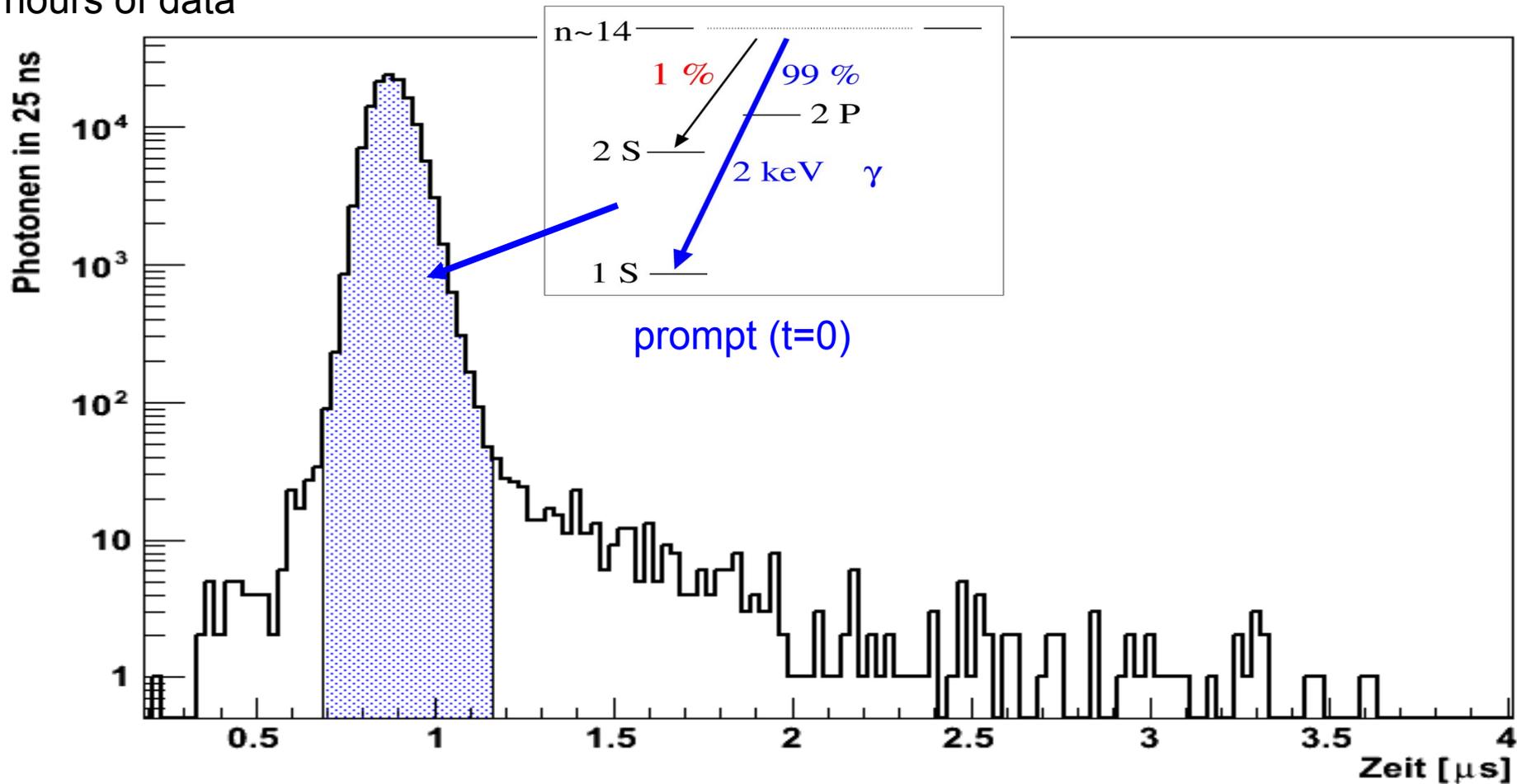
# Time Spectra

13 hours of data

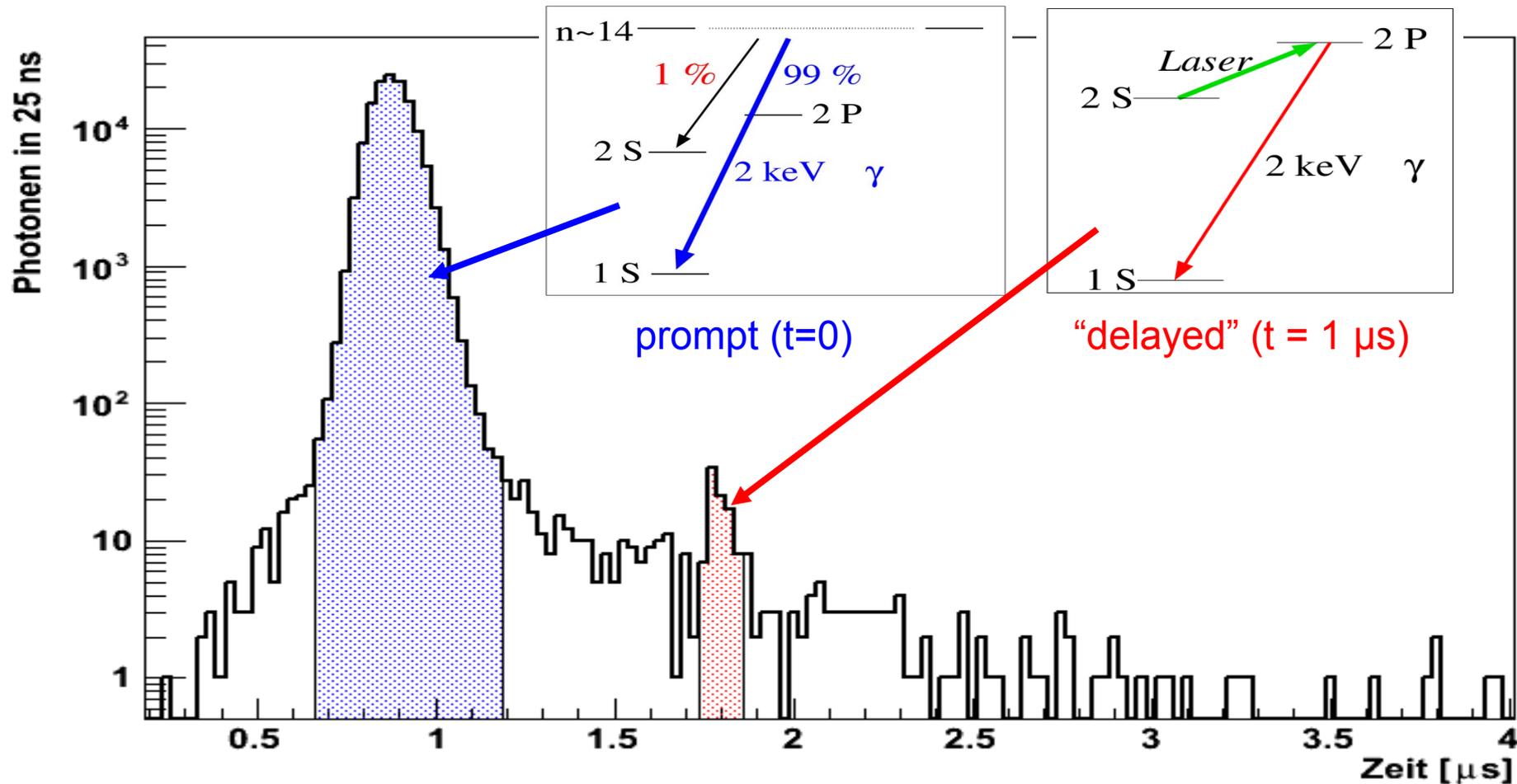


# Time Spectra

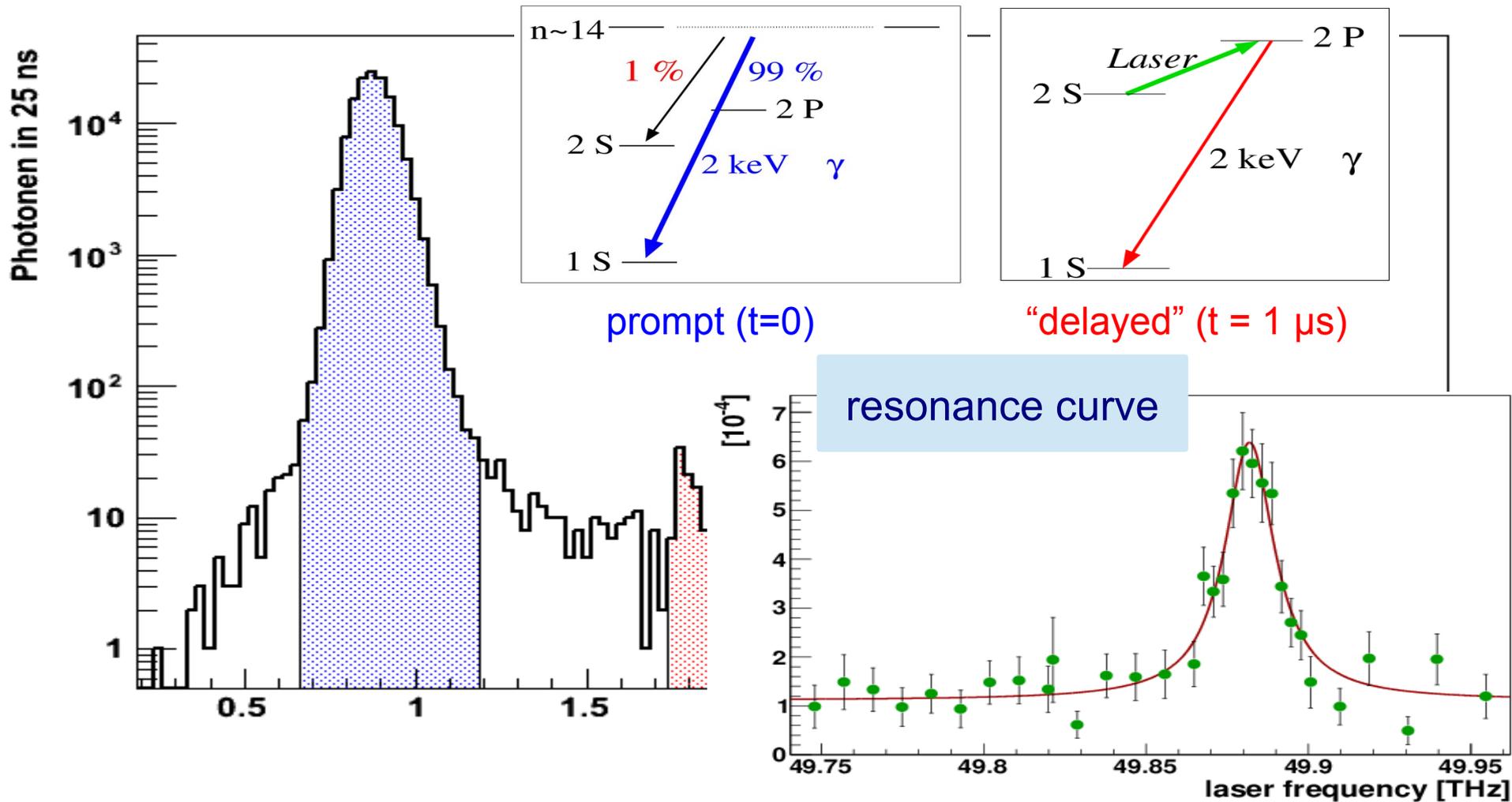
13 hours of data



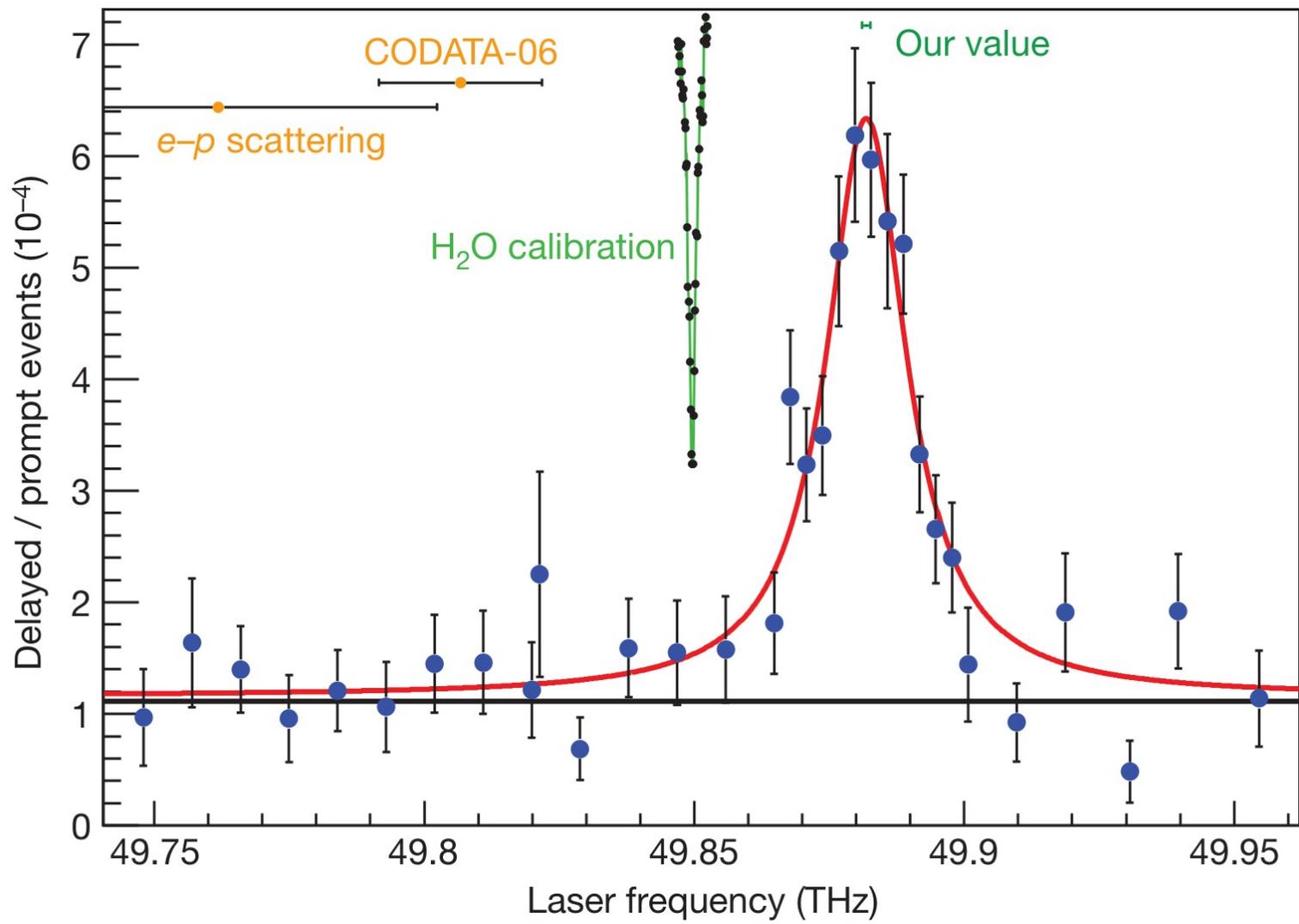
# Time Spectra



# Time Spectra



# Resonance

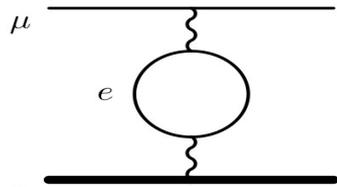


# Theory in muonic H

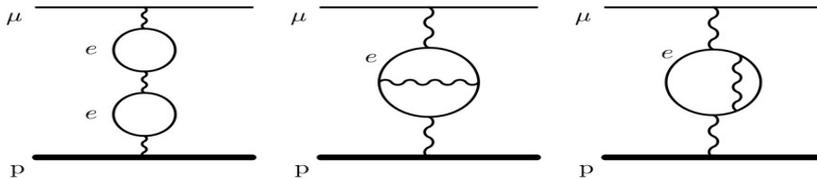
(D,  $^3,^4\text{He}^+$  similar)

$$\Delta E_{\text{Lamb}} = 206.0344 (3) \text{ meV}_{\text{QED}} + 0.0289 (25) \text{ meV}_{\text{TPE}} - 5.2259 (1) \text{ meV/fm}^2 * R_p^2$$

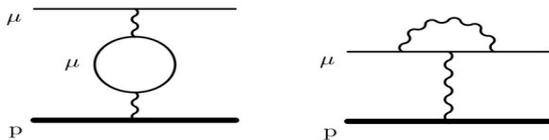
Uehling



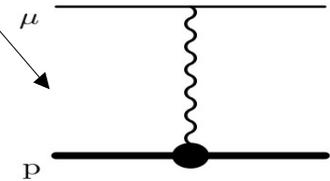
Källen-Sabry



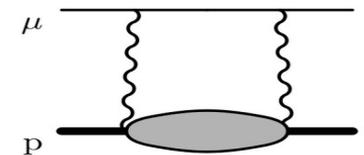
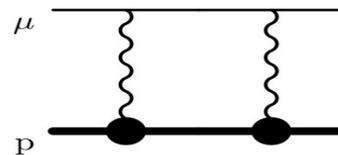
Muon SE+VP



and 20+ more....



Proton form factor



elastic and inelastic two-photon exchange  
(Friar moment and polarizability)

# Theory

TABLE I. Contributions to the  $2P_{1/2} - 2S_{1/2}$  energy difference  $E_L$  in meV, with the charge radii  $r_C$  given in fm. All corrections larger than 3% of the overall uncertainty are included. Theoretical predictions for  $E_L$  are  $E_L(\text{theo}) = E_{\text{QED}} + Cr_C^2 + E_{\text{NS}}$ . The last two rows show the values of  $r_C$  determined from a comparison of  $E_L(\text{theo})$  to  $E_L(\text{exp})$ .

Section	Order	Correction	$\mu\text{H}$	$\mu\text{D}$	$\mu^3\text{He}^+$	$\mu^4\text{He}^+$
III.A	$\alpha(Z\alpha)^2$	eVP <sup>(1)</sup>	205.007 38	227.634 70	1641.886 2	1665.773 1
III.A	$\alpha^2(Z\alpha)^2$	eVP <sup>(2)</sup>	1.658 85	1.838 04	13.084 3	13.276 9
III.A	$\alpha^3(Z\alpha)^2$	eVP <sup>(3)</sup>	0.007 52	0.008 42(7)	0.073 0(30)	0.074 0(30)
III.B	$(Z, Z^2, Z^3)\alpha^5$	Light-by-light eVP	-0.000 89(2)	-0.000 96(2)	-0.013 4(6)	-0.013 6(6)
III.C	$(Z\alpha)^4$	Recoil	0.057 47	0.067 22	0.126 5	0.295 2
III.D	$\alpha(Z\alpha)^4$	Relativistic with eVP <sup>(1)</sup>	0.018 76	0.021 78	0.509 3	0.521 1
III.E	$\alpha^2(Z\alpha)^4$	Relativistic with eVP <sup>(2)</sup>	0.000 17	0.000 20	0.005 6	0.005 7
III.F	$\alpha(Z\alpha)^4$	$\mu\text{SE}^{(1)} + \mu\text{VP}^{(1)}$ , LO	-0.663 45	-0.769 43	-10.652 5	-10.926 0
III.G	$\alpha(Z\alpha)^5$	$\mu\text{SE}^{(1)} + \mu\text{VP}^{(1)}$ , NLO	-0.004 43	-0.005 18	-0.174 9	-0.179 7
III.H	$\alpha^2(Z\alpha)^4$	$\mu\text{VP}^{(1)}$ with eVP <sup>(1)</sup>	0.000 13	0.000 15	0.003 8	0.003 9
III.I	$\alpha^2(Z\alpha)^4$	$\mu\text{SE}^{(1)}$ with eVP <sup>(1)</sup>	-0.002 54	-0.003 06	-0.062 7	-0.064 6
III.J	$(Z\alpha)^5$	Recoil	-0.044 97	-0.026 60	-0.558 1	-0.433 0
III.K	$\alpha(Z\alpha)^5$	Recoil with eVP <sup>(1)</sup>	0.000 14(14)	0.000 09(9)	0.004 9(49)	0.003 9(39)
III.L	$Z^2\alpha(Z\alpha)^4$	nSE <sup>(1)</sup>	-0.009 92	-0.003 10	-0.084 0	-0.050 5
III.M	$\alpha^2(Z\alpha)^4$	$\mu F_1^{(2)}, \mu F_2^{(2)}, \mu\text{VP}^{(2)}$	-0.001 58	-0.001 84	-0.031 1	-0.031 9
III.N	$(Z\alpha)^6$	Pure recoil	0.000 09	0.000 04	0.001 9	0.001 4
III.O	$\alpha(Z\alpha)^5$	Radiative recoil	0.000 22	0.000 13	0.002 9	0.002 3
III.P	$\alpha(Z\alpha)^4$	hVP	0.011 36(27)	0.013 28(32)	0.224 1(53)	0.230 3(54)
III.Q	$\alpha^2(Z\alpha)^4$	hVP with eVP <sup>(1)</sup>	0.000 09	0.000 10	0.002 6(1)	0.002 7(1)
IV.A	$(Z\alpha)^4$	$r_C^2$	-5.197 5 $r_p^2$	-6.073 2 $r_d^2$	-102.523 $r_h^2$	-105.322 $r_a^2$
IV.B	$\alpha(Z\alpha)^4$	eVP <sup>(1)</sup> with $r_C^2$	-0.028 2 $r_p^2$	-0.034 0 $r_d^2$	-0.851 $r_h^2$	-0.878 $r_a^2$
IV.C	$\alpha^2(Z\alpha)^4$	eVP <sup>(2)</sup> with $r_C^2$	-0.000 2 $r_p^2$	-0.000 2 $r_d^2$	-0.009(1) $r_h^2$	-0.009(1) $r_a^2$
V.A	$(Z\alpha)^5$	TPE	0.029 2(25)	1.979(20)	16.38(31)	9.76(40)
V.B	$\alpha^2(Z\alpha)^4$	Coulomb distortion	0.0	-0.261	-1.010	-0.536
V.C	$(Z\alpha)^6$	3PE	-0.001 3(3)	-0.002 2(9)	-0.214(214)	-0.165(165)
V.D	$\alpha(Z\alpha)^5$	eVP <sup>(1)</sup> with TPE	0.000 6(1)	0.027 5(4)	0.266(24)	0.158(12)
V.E	$\alpha(Z\alpha)^5$	$\mu\text{SE}^{(1)} + \mu\text{VP}^{(1)}$ with TPE	0.000 4	0.002 6(3)	0.077(8)	0.059(6)
III	$E_{\text{QED}}$	Point nucleus	206.034 4(3)	228.774 0(3)	1644.348(8)	1668.491(7)
IV	$Cr_C^2$	Finite size	-5.225 9 $r_p^2$	-6.107 4 $r_d^2$	-103.383 $r_h^2$	-106.209 $r_a^2$
V	$E_{\text{NS}}$	Nuclear structure	0.028 9(25)	1.750 3(200)	15.499(378)	9.276(433)
	$E_L$ (exp)	Experiment <sup>a</sup>	202.370 6(23)	202.878 5(34)	1258.598(48)	1378.521(48)
	$r_C$	This review	0.840 60(39)	2.127 58(78)	1.970 07(94)	1.678 6(12)
	$r_C$	Previous work <sup>a</sup>	0.840 87(39)	2.125 62(78)	1.970 07(94)	1.678 24(83)

Pachucki, Lensky, Hagelstein, LiMuli, Bacca, RP, RMP 96 (2024)

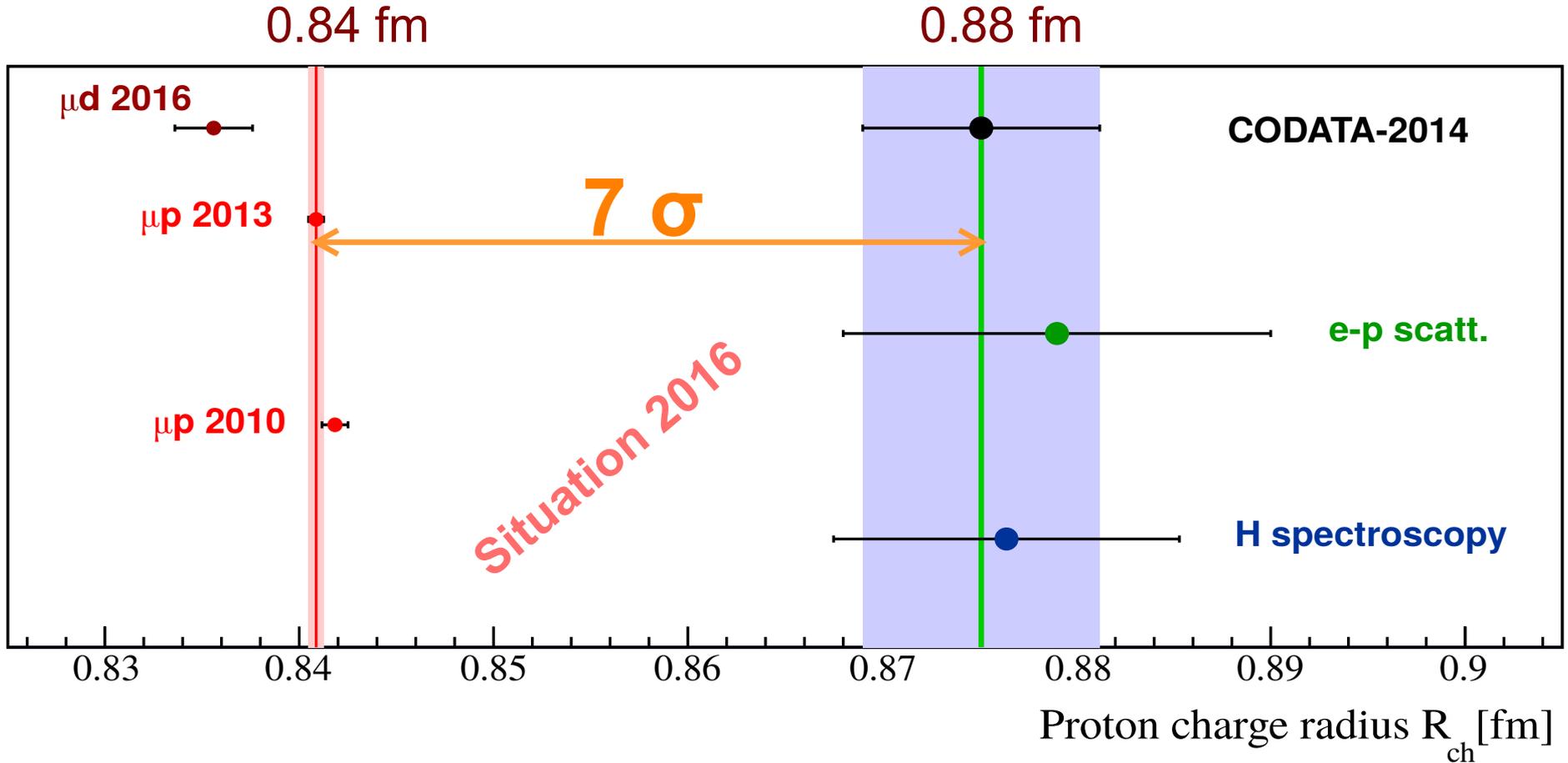
# Theory

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Pachucki, Lensky, Hagelstein, LiMuli, Bacca, RP, RMP 96 (2024)

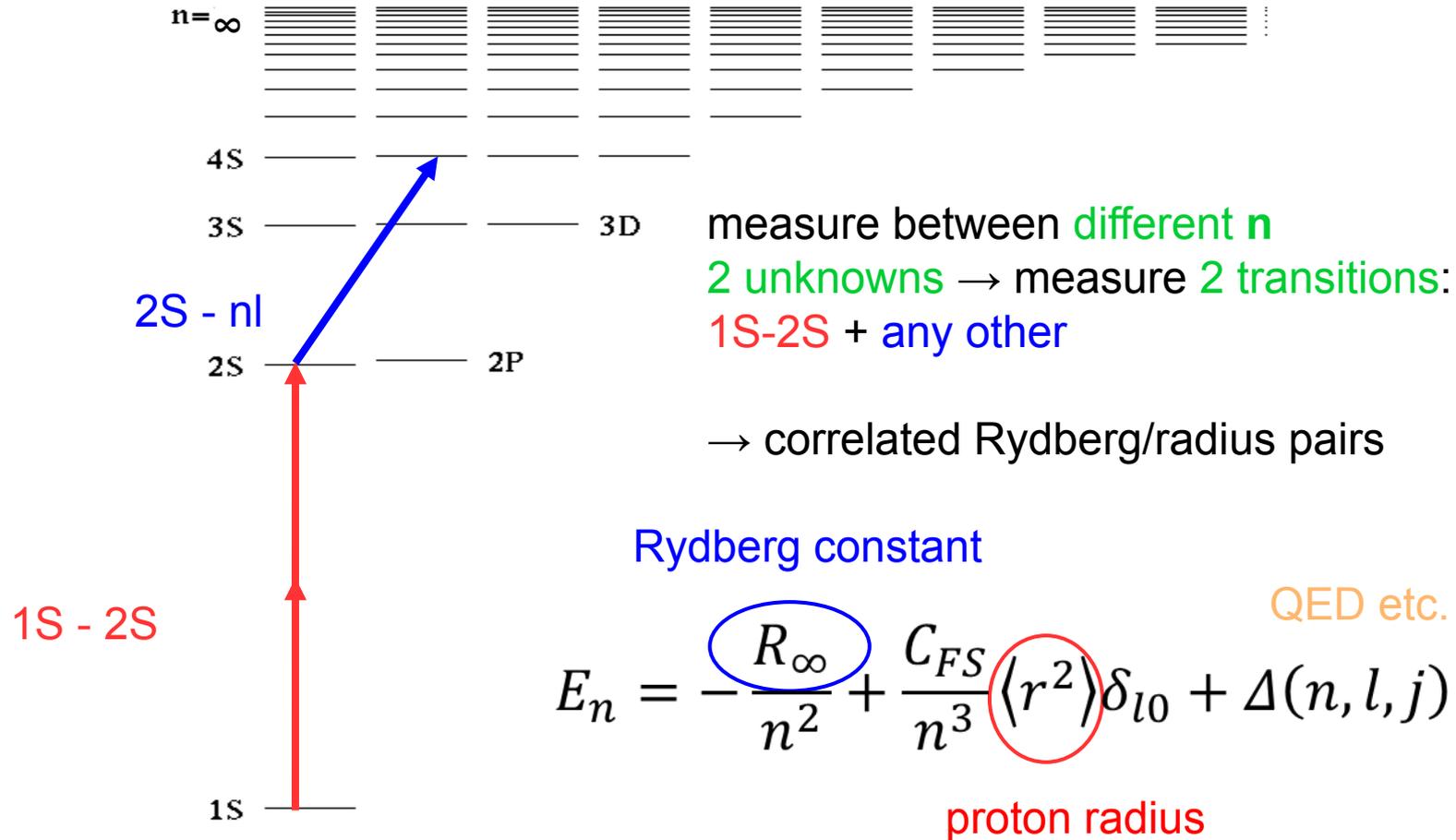
# The “Proton Radius Puzzle”



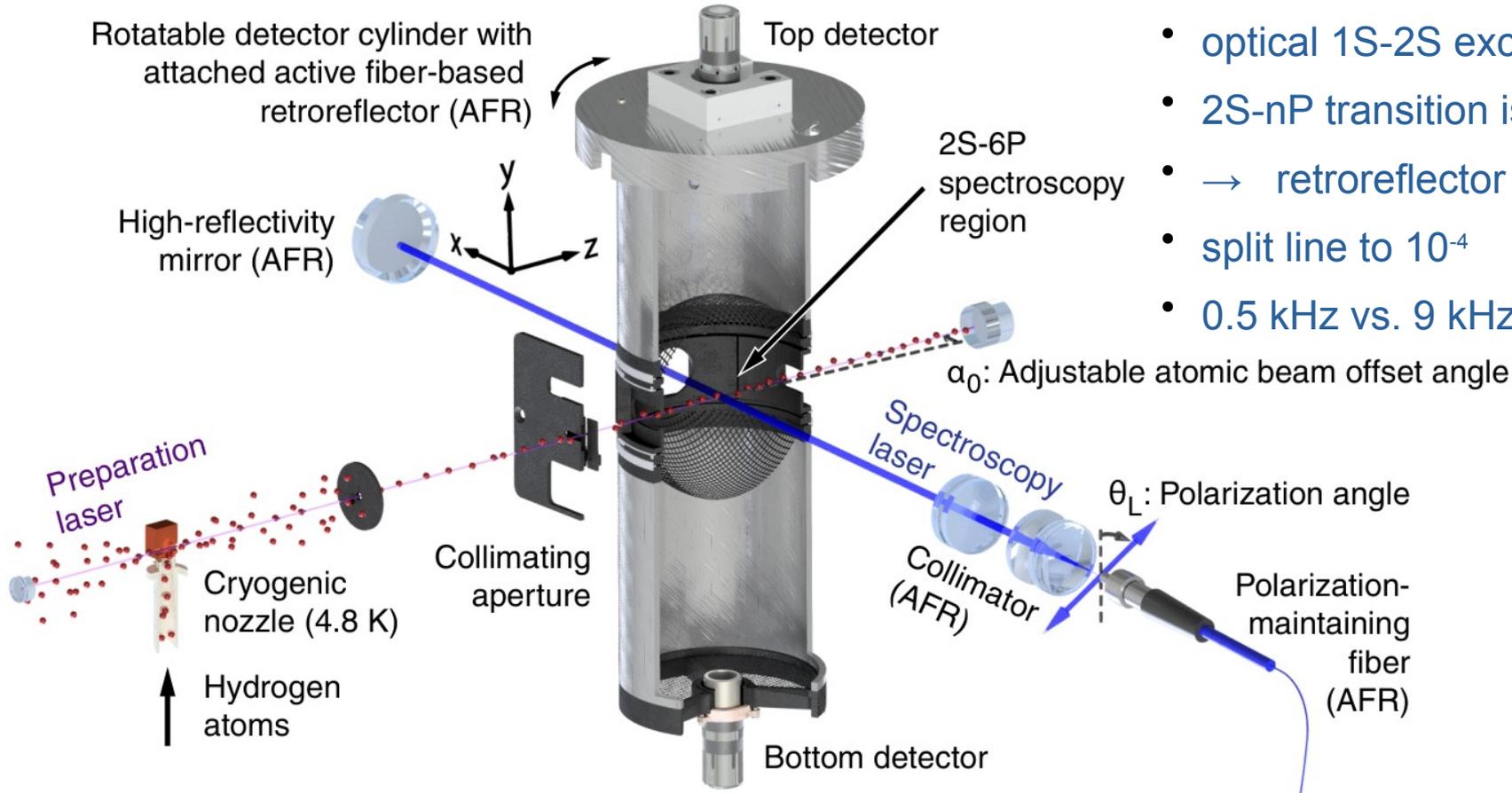
$\mu d$  2016: RP et al (CREMA Coll.) Science 353, 669 (2016)

$\mu p$  2013: A. Antognini, RP et al (CREMA Coll.) Science 339, 417 (2013)

# Proton radius from **ordinary** hydrogen

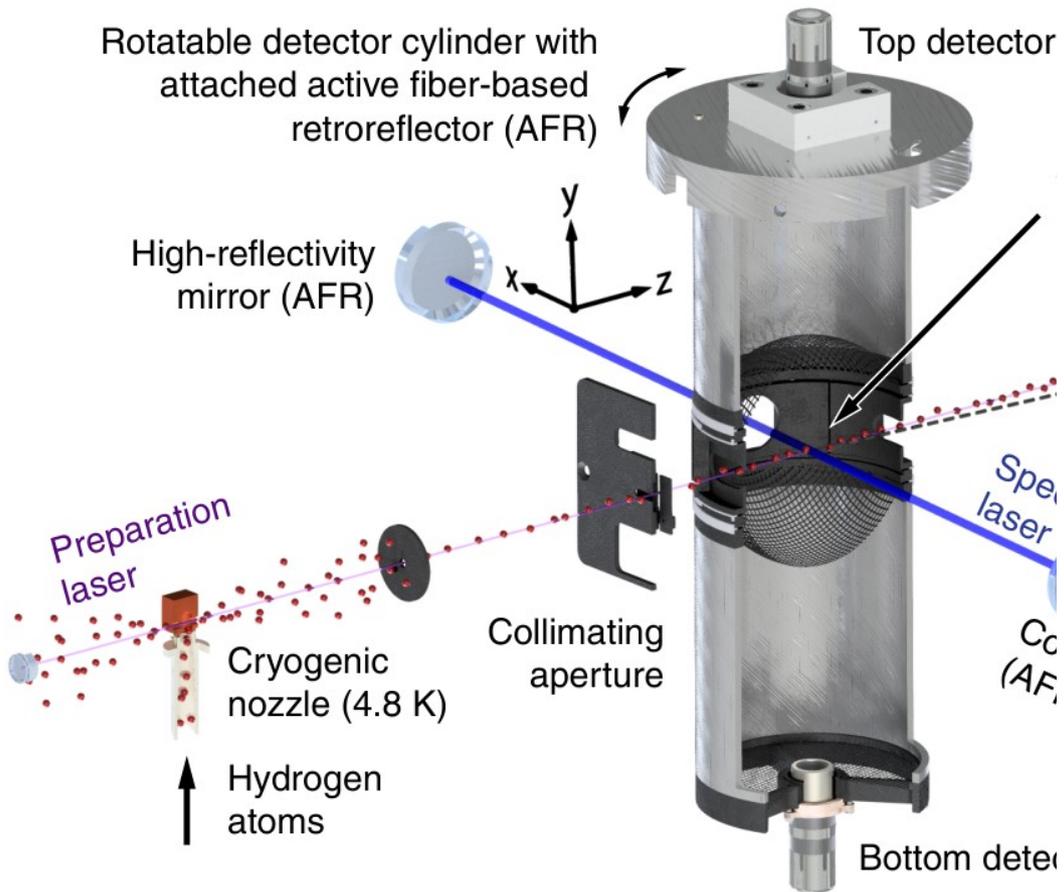


# (Normal) Hydrogen 2S-nP in Garching

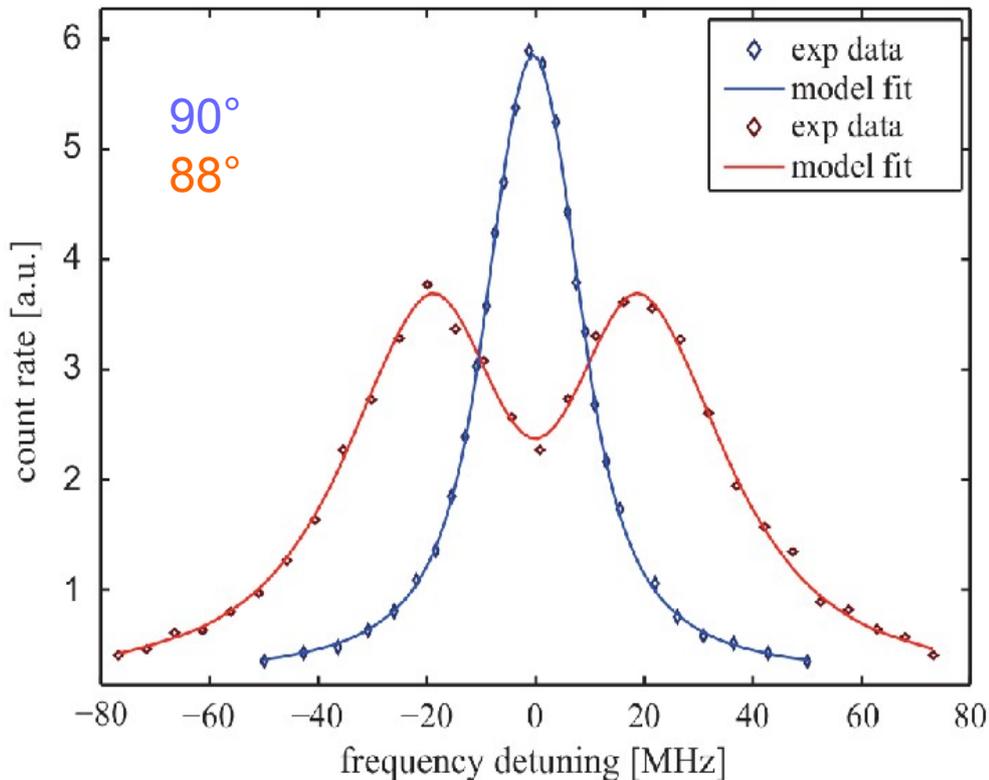


- cryogenic H beam (6 K)
- optical 1S-2S excitation (2S, F=0)
- 2S-nP transition is 1-photon
- → retroreflector (“AFR”)
- split line to  $10^{-4}$
- 0.5 kHz vs. 9 kHz PRP

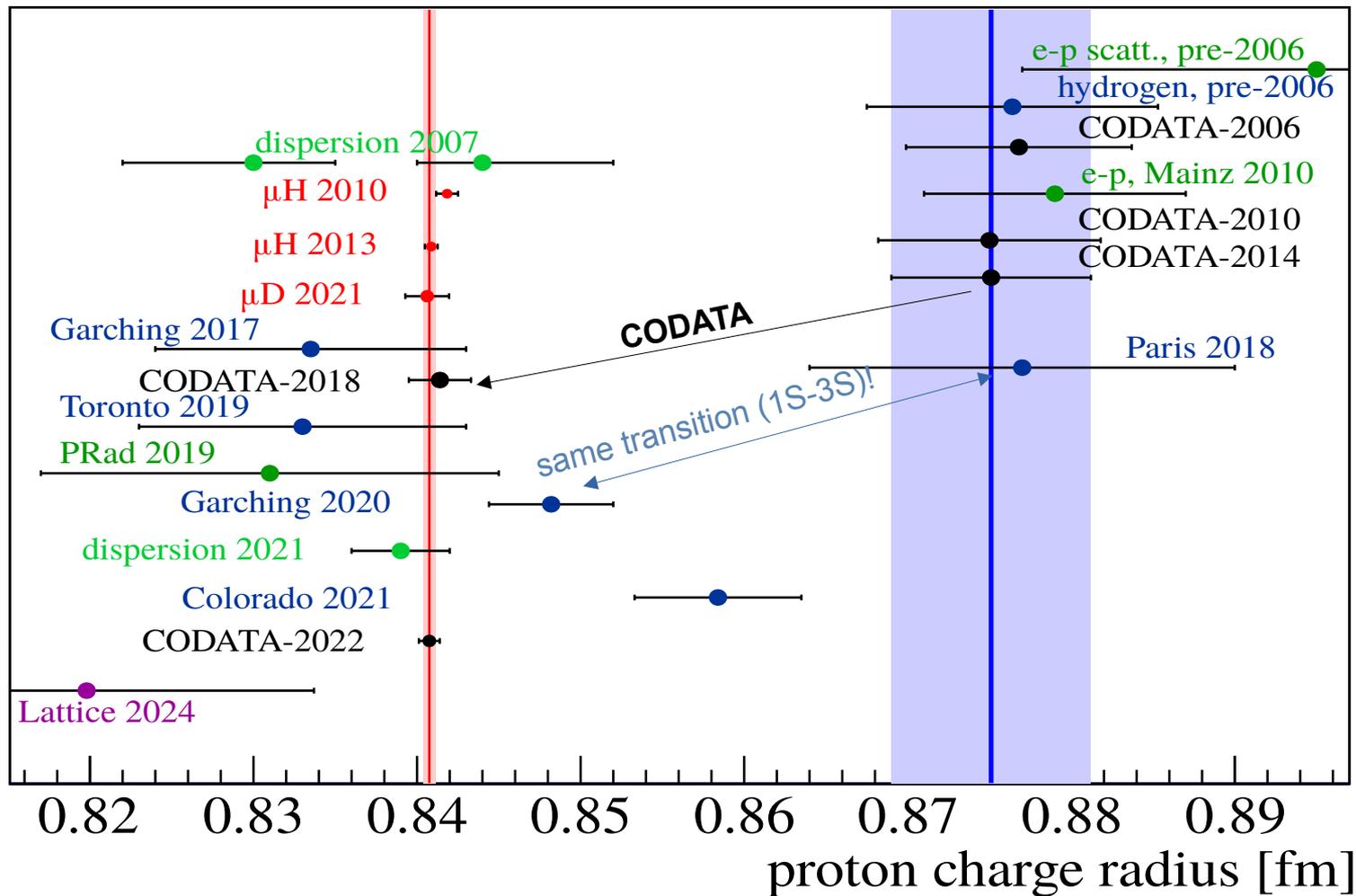
# Hydrogen 2S-nP in Garching



## 1<sup>st</sup> order Doppler cancellation (2S-4P)



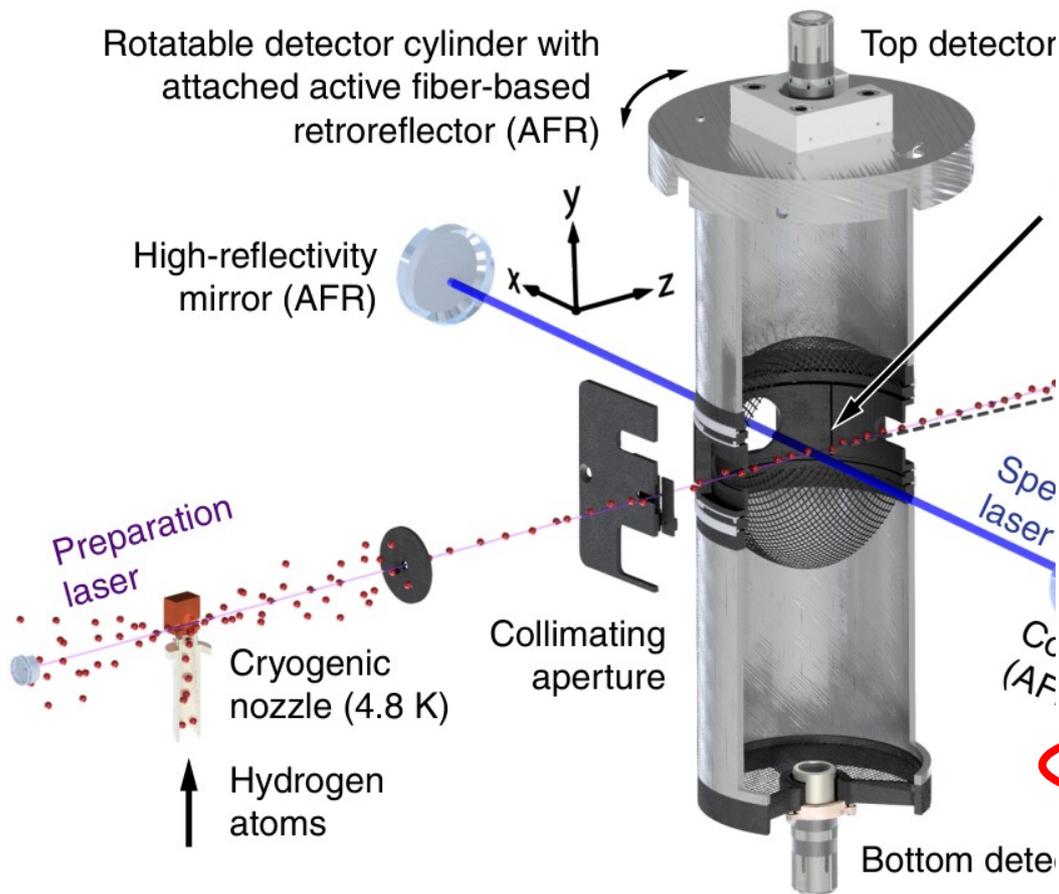
# The proton radius situation “yesterday”



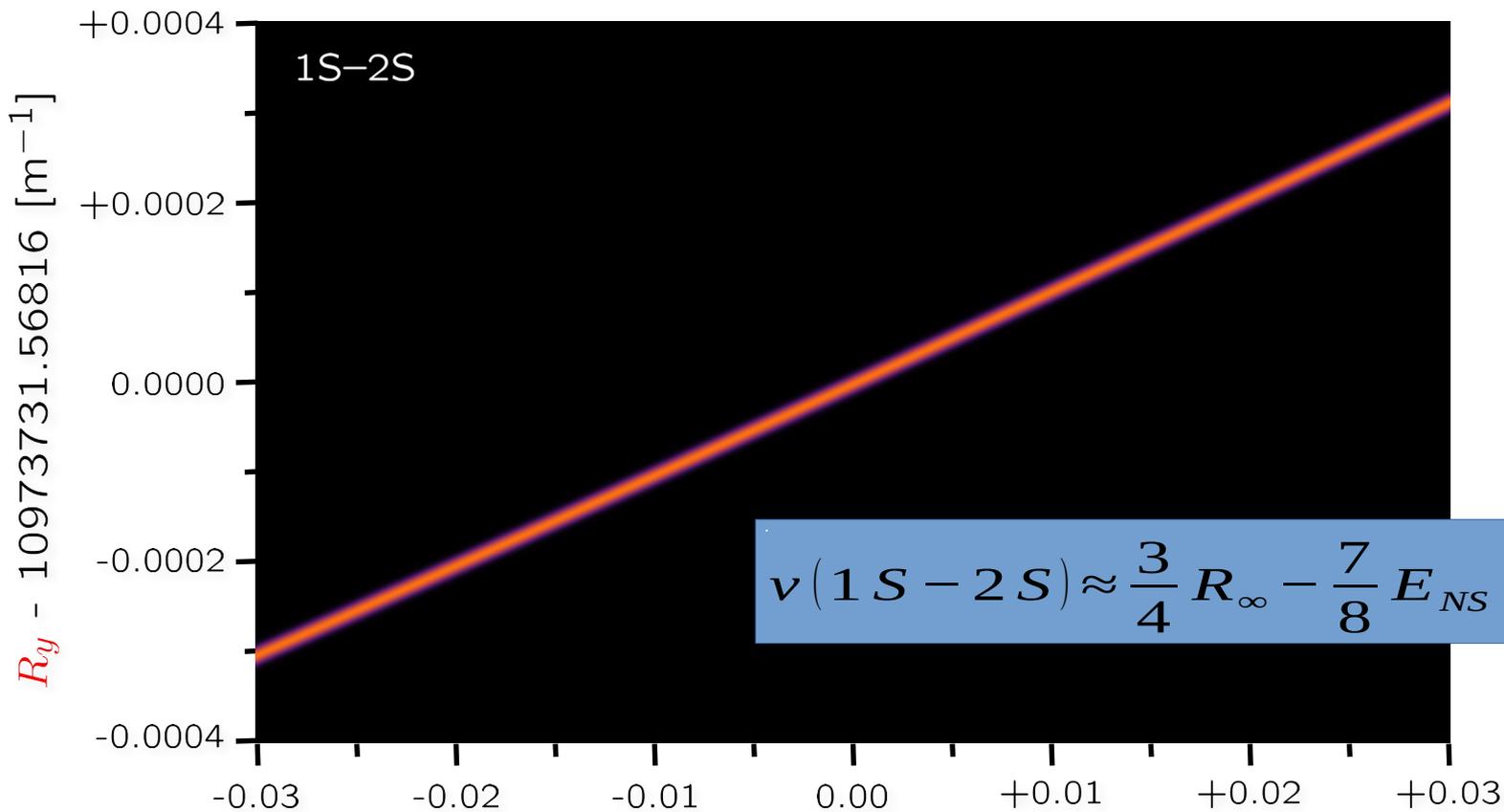
Not really “solved”

# NEW Hydrogen 2S-6P in Garching

## Systematics (2S-6P)

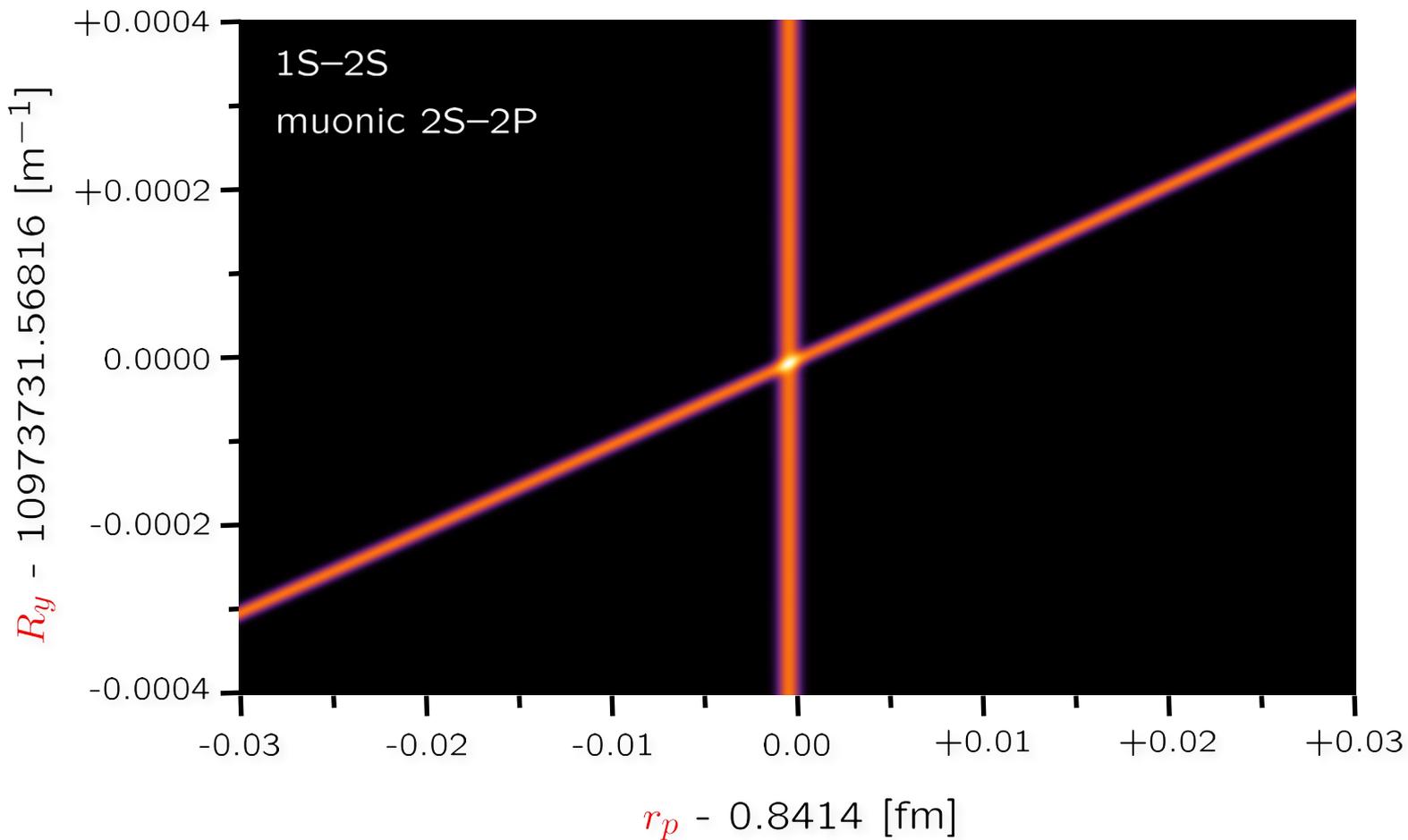


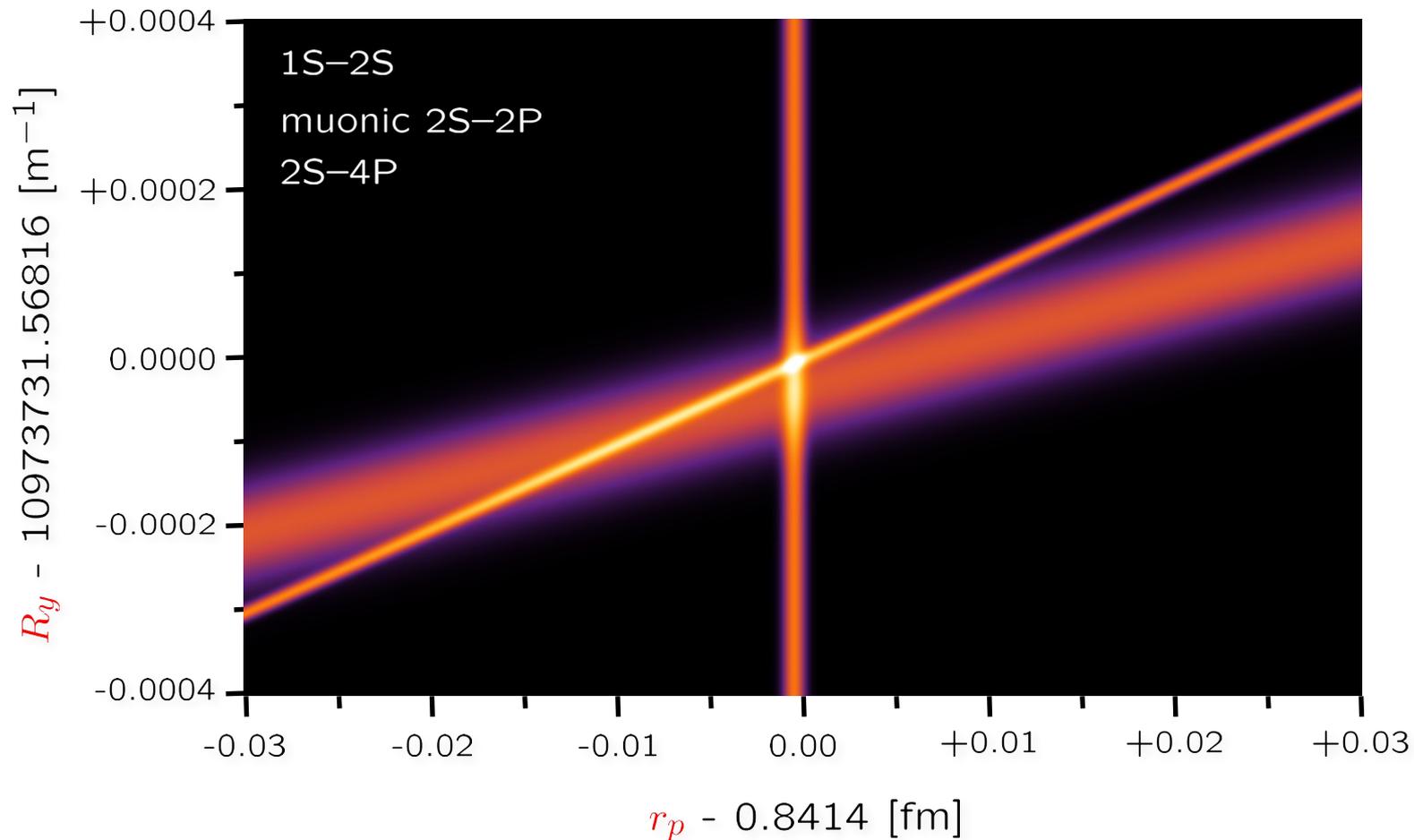
Contribution	$\Delta\nu$ (kHz)	$\sigma$ (kHz)
First-order Doppler shift	0.34	0.43
Extrapolation (statistical)	0.34	0.43
Simulation of atom speeds	—	0.01
Simulation corrections	1.05	0.17
Light force shift	1.15	0.17
Quantum interference shift	0.05	0.02
Second-order Doppler shift	-0.14	0.01
dc-Stark shift	0.05	0.07
BBR-induced shift	0.28	0.01
Zeeman shift	0.00	0.08
Pressure shift	0.00	0.02
Sampling bias	0.00	0.06
Signal background	0.00	0.03
Laser spectrum	0.00	0.07
Frequency standard	0.02	0.01
<b>Subtotal (experiment-specific contributions)</b>	<b>1.74</b>	<b>0.48</b>
Recoil shift	-1 176.03	0.00
HFS correction $\Delta\nu_{\text{HFS}}(\nu_{2\text{S-6P}})$	-132 985.25	0.01
<b>Total (all contributions)</b>	<b>-134 159.54</b>	<b>0.48</b>

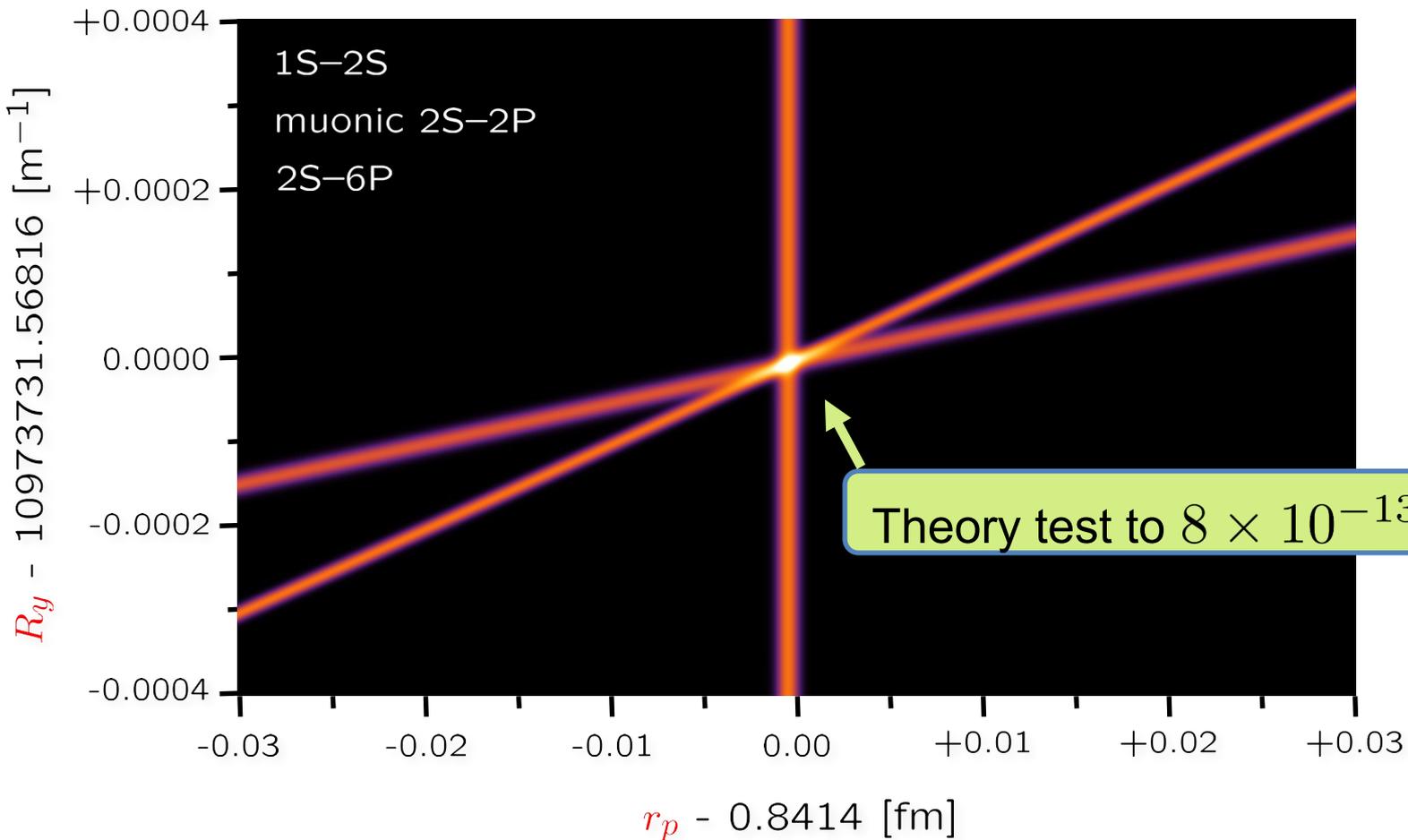


$$E_n = -\frac{R_{\infty}}{n^2} + \frac{C_{FS}}{n^3} \langle r^2 \rangle \delta_{l0} + \Delta(n, l, j)$$

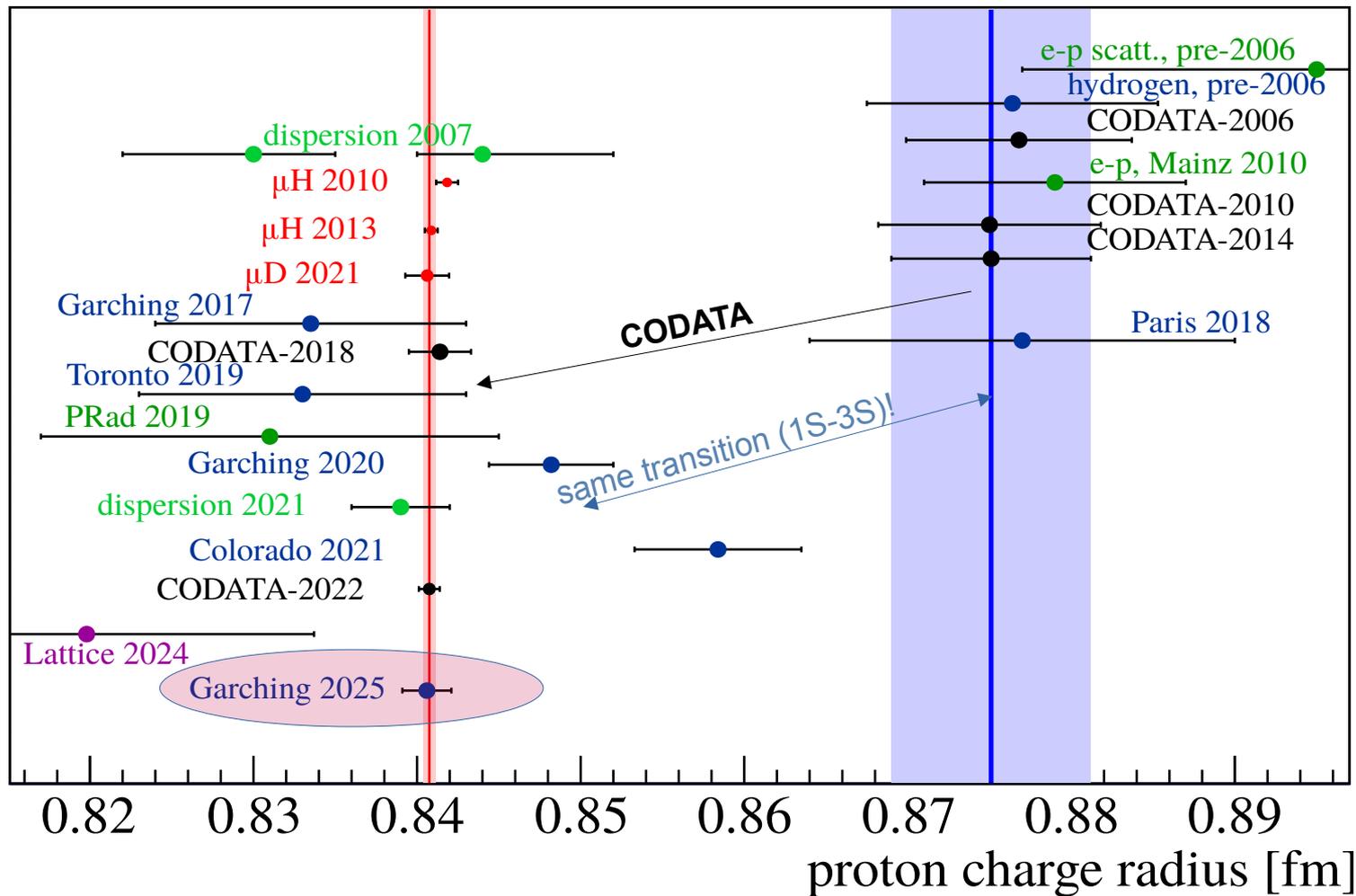
$$r_p - 0.8414 \text{ [fm]}$$







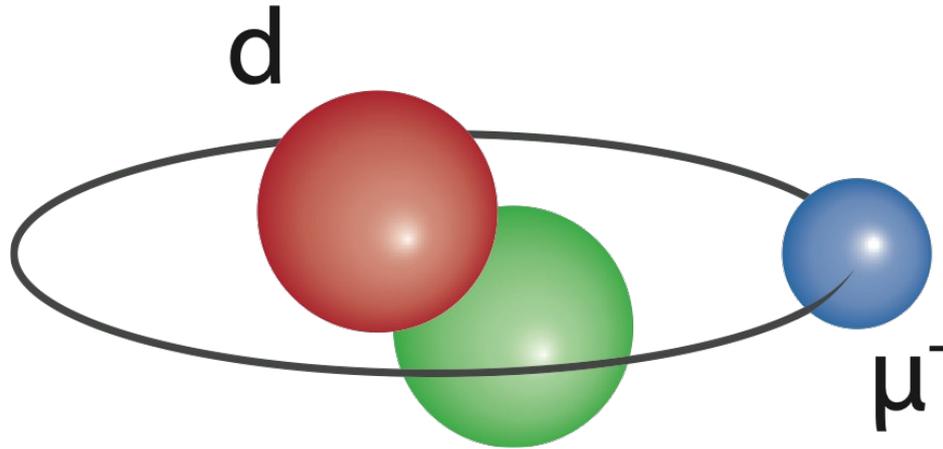
# The proton radius situation today



Not really “solved”

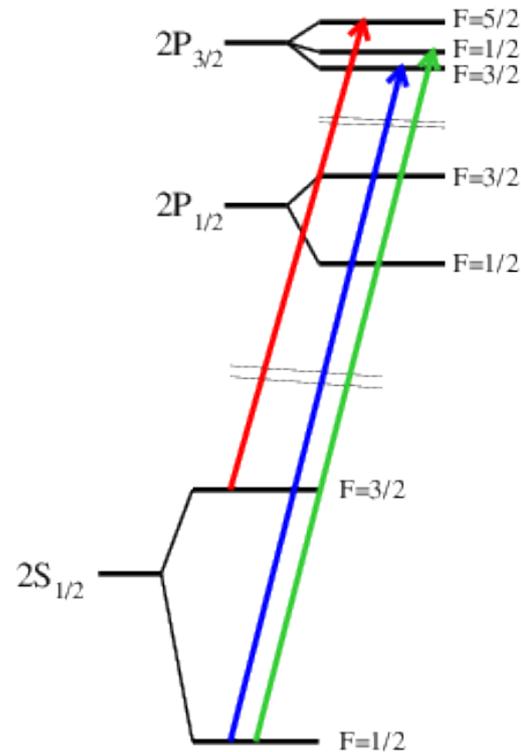
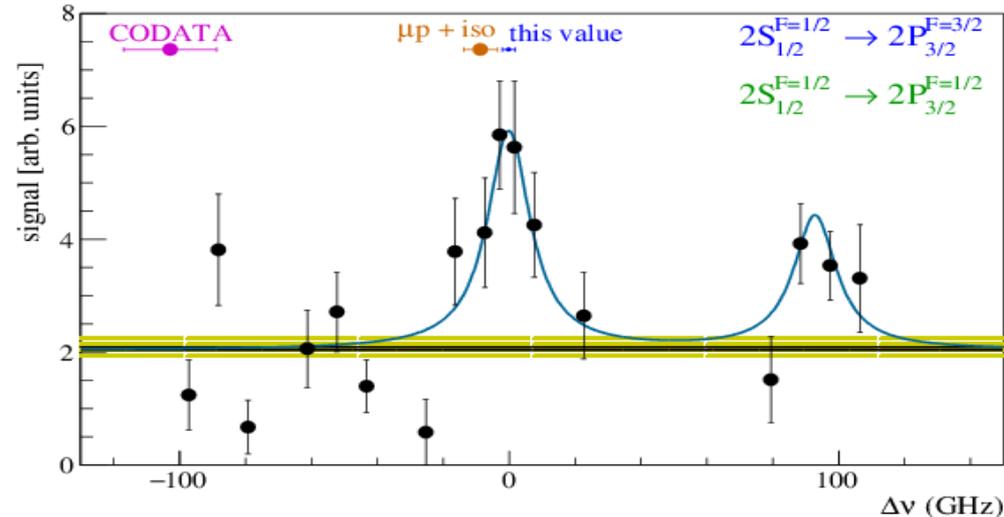
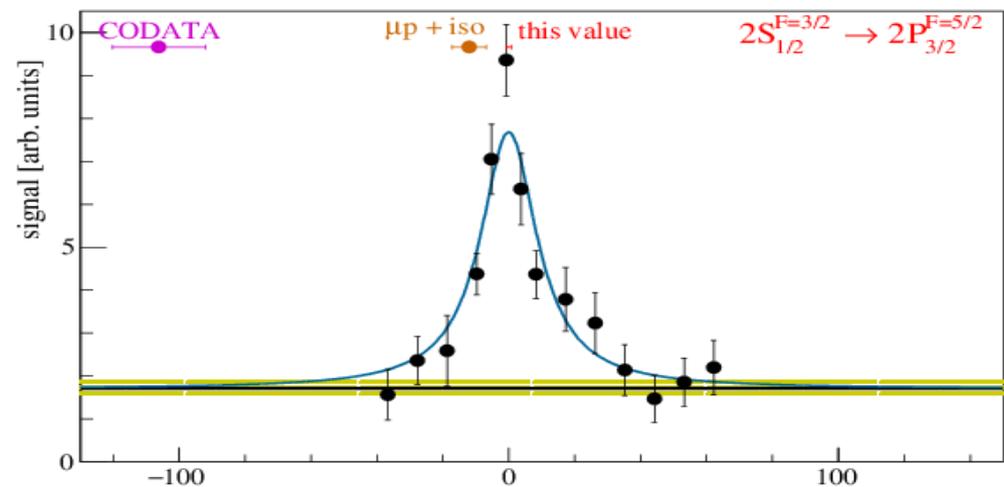
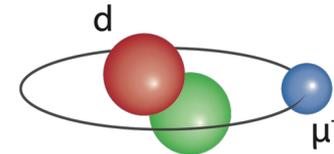
H (2S-6P) Nature  
(accepted)

# Muonic Deuterium



Pohl et al. (CREMA), Science (2016)

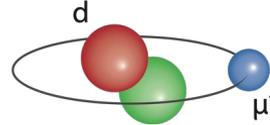
# 2.5 transitions in muonic D



$$\Delta E_{LS}^{\text{exp}} = 202.8785(31)_{\text{stat}}(14)_{\text{syst}} \text{ meV}$$

$$\Delta E_{\text{HFS}}^{\text{exp}} = 6.2747(70)_{\text{stat}}(20)_{\text{syst}} \text{ meV}$$

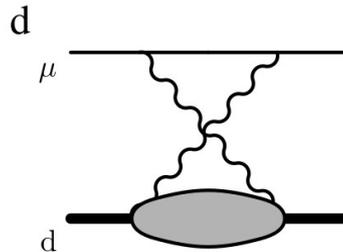
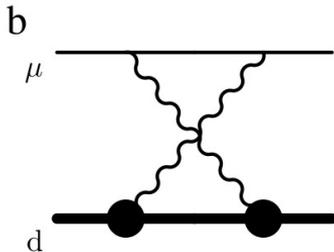
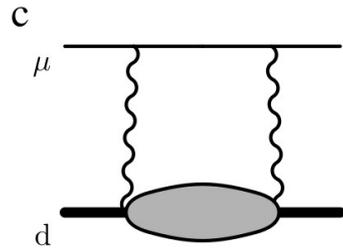
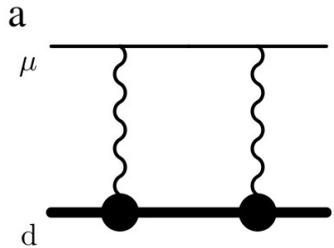
# Theory: Lamb shift in muonic D



$$\Delta E_{\text{Lamb}}^{\mu\text{D}} = 228.7740 (3) \text{ meV}_{\text{QED}} + 1.7503 (200) \text{ meV}_{\text{TPE}} - 6.1074 \text{ meV/fm}^2 * R_d^2$$

$$\Delta E_{\text{LS}}^{\text{exp}} = 202.8785(31)_{\text{stat}} (14)_{\text{syst}} \text{ meV}$$

Nuclear structure **two (and three!)-photon contributions** to the Lamb shift in muonic deuterium.



Pachucki, RP et al, arXiv 2212.13782

see also Krauth, RP et al. (2016) using calculations from Pachucki (2011), Friar (2013), Carlson, Gorchtein, Vanderhaeghen (2014), Hernandez et al. (2014), Pachucki + Wienczek (2015)

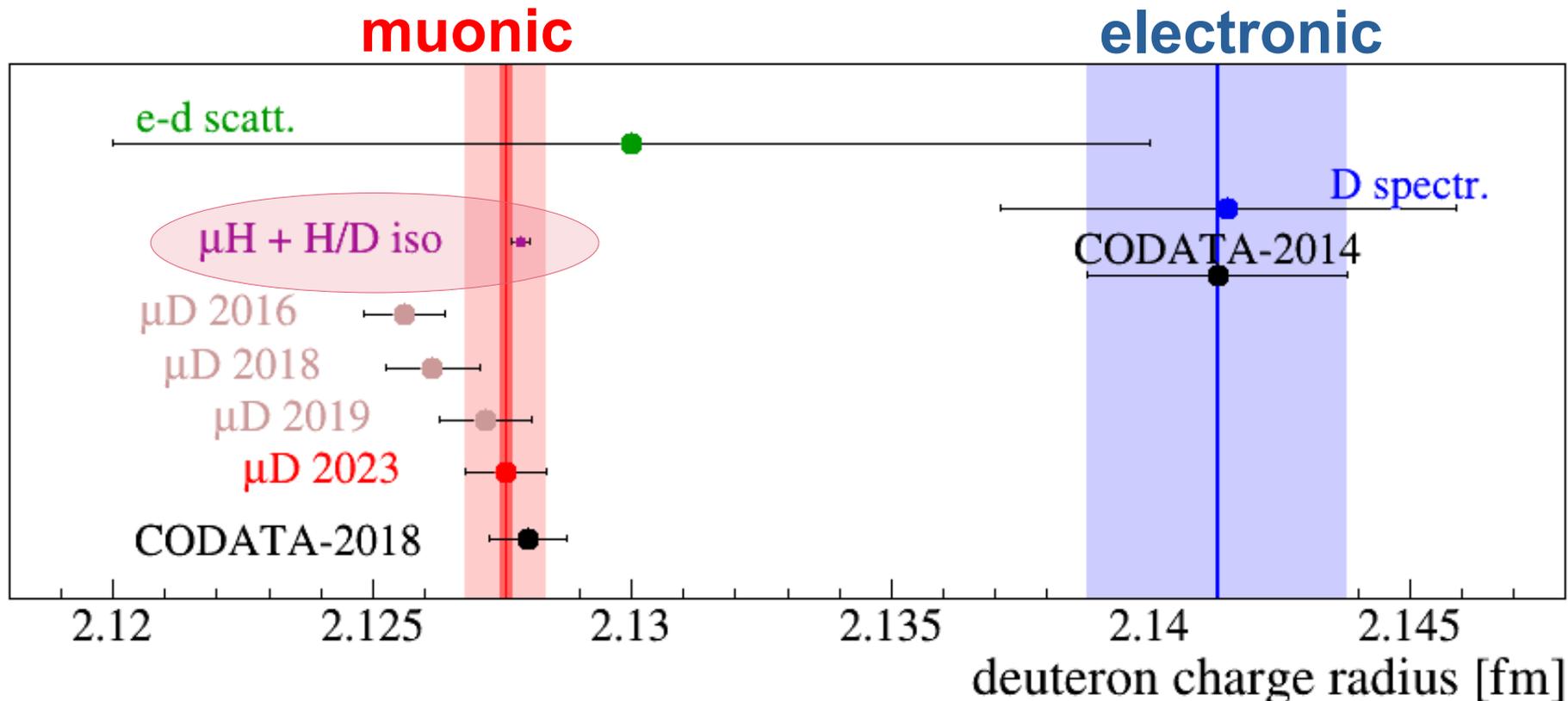
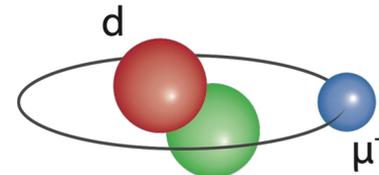
+ Pachucki et al., PRA 97, 062511 (2018): Sizeable three-photon !!

+ Hernandez et al., PLB 778, 377 (2018):  $\chi$ EFT

+ Kalinowski (2019): eVP to nucl. struct.

+ Acharya et al., PRC 103, 024001 (2021)  
 $\chi$ EFT + Dispersion relations

# Muonic Deuterium

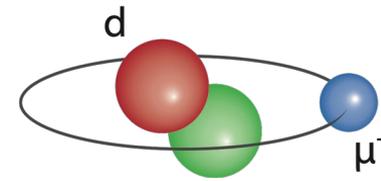


$\mu\text{D}$ : 2.12758 (13)<sub>exp</sub> (78)<sub>theo</sub> fm

$\mu\text{H} + \text{H}/\text{D}(1\text{S}-2\text{S})$ : 2.12785 (17) fm

$$r_d^2 - r_p^2 = 3.8207(3)_{\text{theo}} \text{ fm}^2$$

# Theory in muonic D



$$\Delta E_{\text{Lamb}}^{\mu\text{D}} = 228.7740 (3) \text{ meV}_{\text{QED}} + 1.7503 (200) \text{ meV}_{\text{TPE}} - 6.1074 \text{ meV/fm}^2 * R_d^2$$

$\Delta E_{\text{TPE}} (\text{theo}) = 1.7503 \pm 0.0200 \text{ meV}$  **Bacca group**  
**vs.**  $\pm 0.0034 \text{ meV}$  **experimental uncertainty**

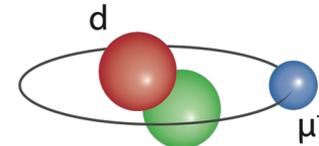
(1) **charge radius**, using **calculated TPE**

$$r_d (\mu\text{D}) = 2.12758 (13)_{\text{exp}} (78)_{\text{theo}} \text{ fm}$$

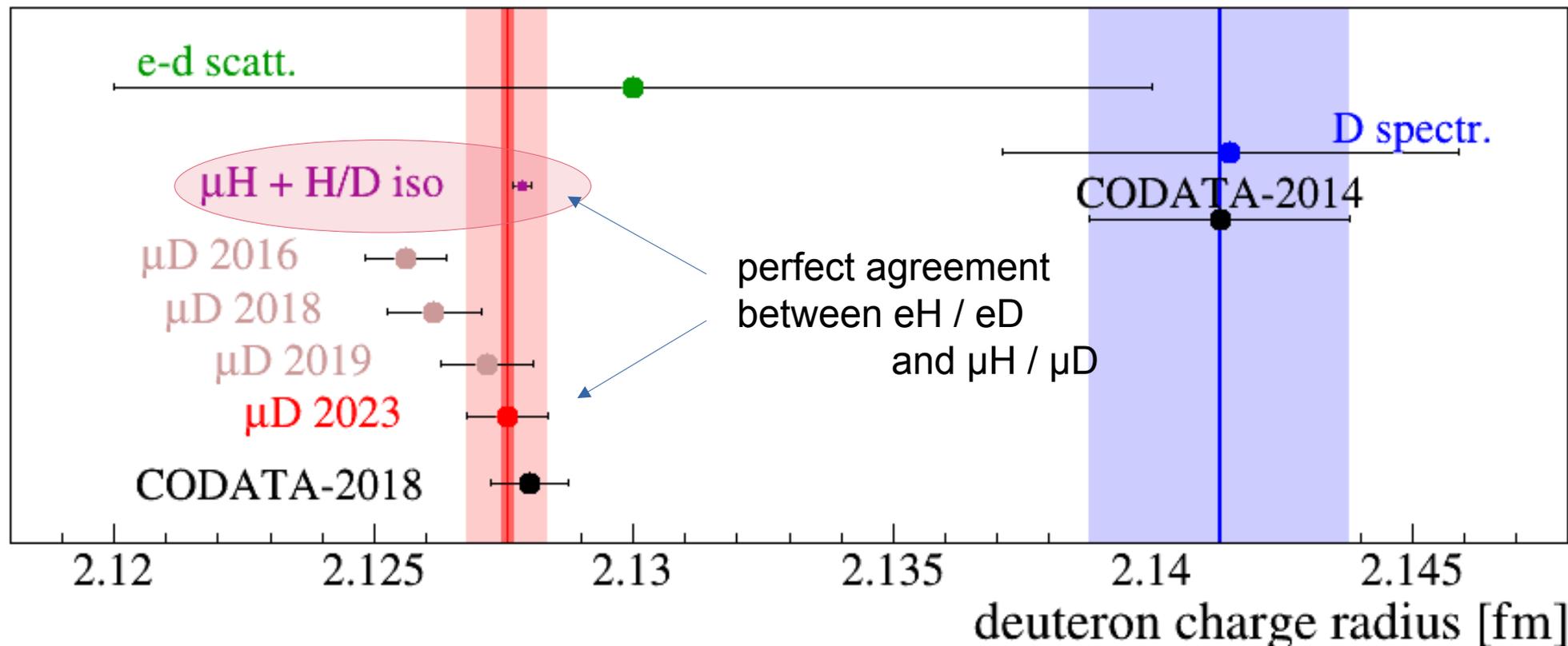
(2) **polarizability**, using **charge radius from isotope shift**

$$\begin{aligned} \Delta E_{\text{TPE}} (\text{theo}) &= 1.7503 (200) \text{ meV vs.} \\ \Delta E_{\text{TPE}} (\text{exp}) &= 1.7591 (59) \text{ meV} \quad \text{3x more accurate} \end{aligned}$$

# Muonic Deuterium



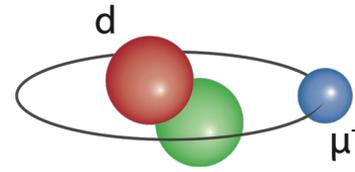
**muonic**



$$r_d^2 - r_p^2 = 3.8207(3) \text{ fm}^2 \text{ H / D } \quad 1\text{S-2S isotope shift}$$

$$3.8200(31) \text{ fm}^2 \quad \mu\text{H} / \mu\text{D } 2\text{S-2P isotope shift } (0.2 \sigma)$$

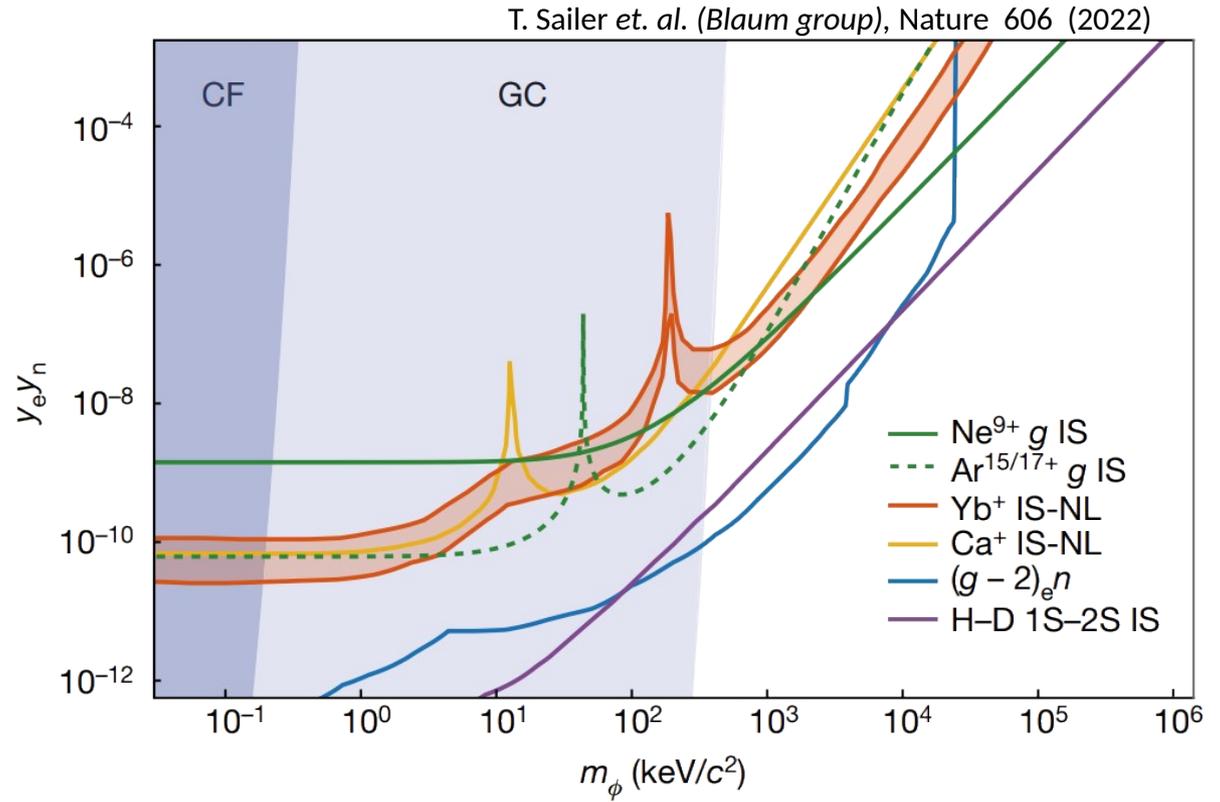
# H/D isotope shift



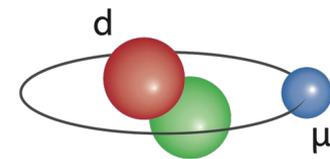
electronic H/D (1S-2S):  $r_d^2 - r_p^2 = 3.8207(3)_{\text{theo}} \text{ fm}^2$

muonic H/D (2S-2P):  $r_d^2 - r_p^2 = 3.8200(7)_{\text{exp}}(30)_{\text{theo}} \text{ fm}^2$

→ Best bound on 5<sup>th</sup> force



# HFS in muonic D



PHYSICAL REVIEW A **98**, 062513 (2018)

## Nuclear-structure corrections to the hyperfine splitting in muonic deuterium

Marcin Kalinowski\* and Krzysztof Pachucki†

Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland

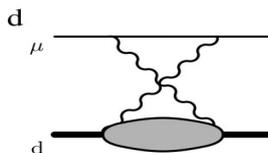
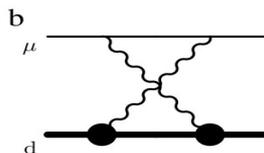
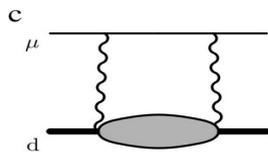
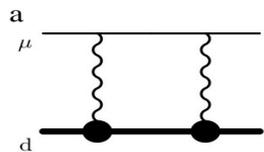
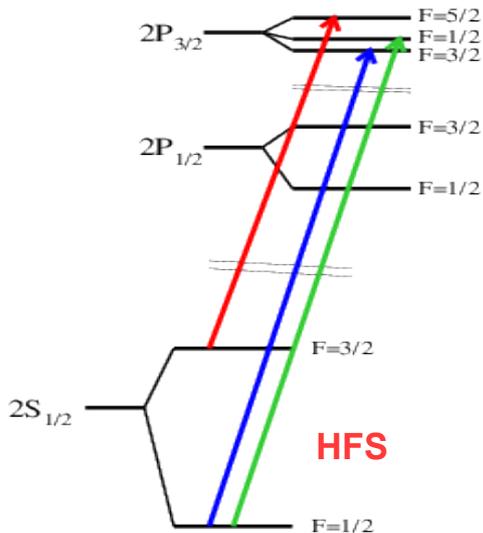
Vladimir A. Yerokhin

Center for Advanced Studies, Peter the Great St. Petersburg Polytechnic University, 195251 St. Petersburg, Russia

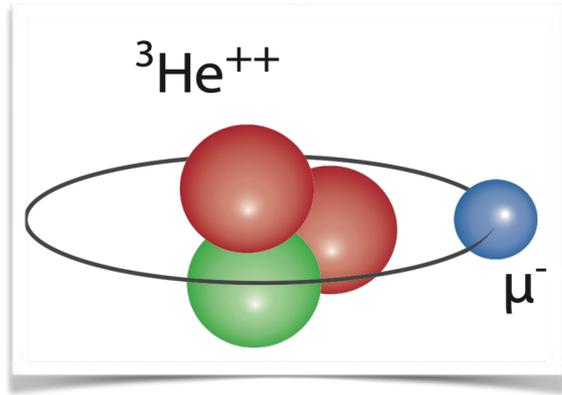
(Received 15 October 2018; revised manuscript received 7 November 2018; published 17 December 2018)

Nuclear structure corrections of orders  $Z\alpha E_F$  and  $(Z\alpha)^2 E_F$  are calculated for the hyperfine splitting of the muonic deuterium. The obtained results disagree with previous calculations and lead to a  $5\sigma$  disagreement with the current experimental value of the  $2S$  hyperfine splitting in muonic deuterium.

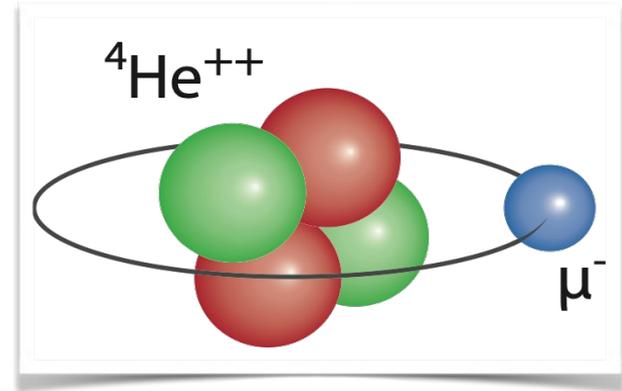
**5 $\sigma$  disagreement between theory and experiment !!!**



# Muonic Helium-3 and -4

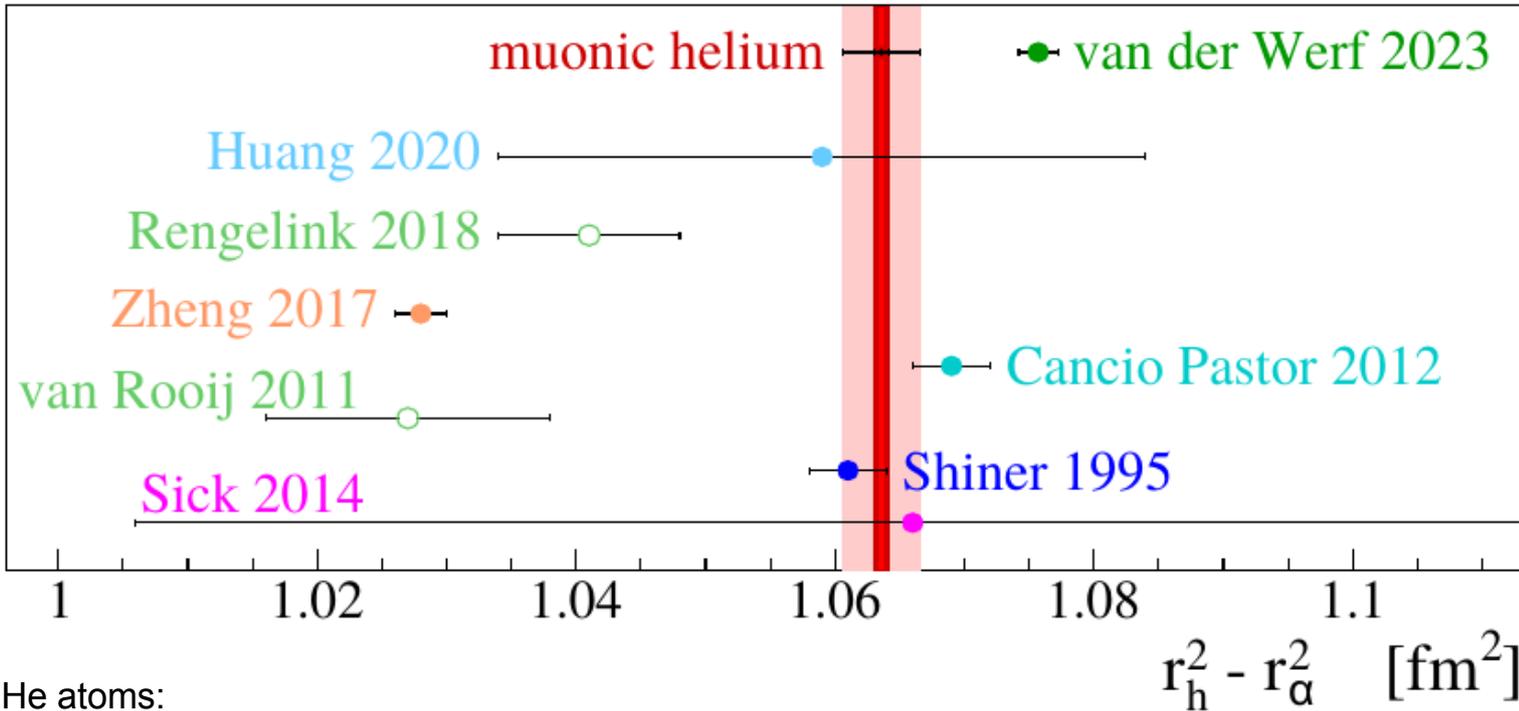


**Schuhmann et al. (CREMA)**  
**Science 388, 854 (2025)**  
**arXiv 2305.11679**



**Krauth et al. (CREMA)**  
**Nature 589, 527 (2021)**

# Helium-3 – Helium-4 Isotope Shift



normal He atoms:

van der Werf et al.: Science (2025 / arXiv 2306.02333)

Huang: PRA 101, 062507 (2020)

Rengelink: Nature Physics 14, 1132 (2018)

Zheng: PRL 119, 263002 (2017)

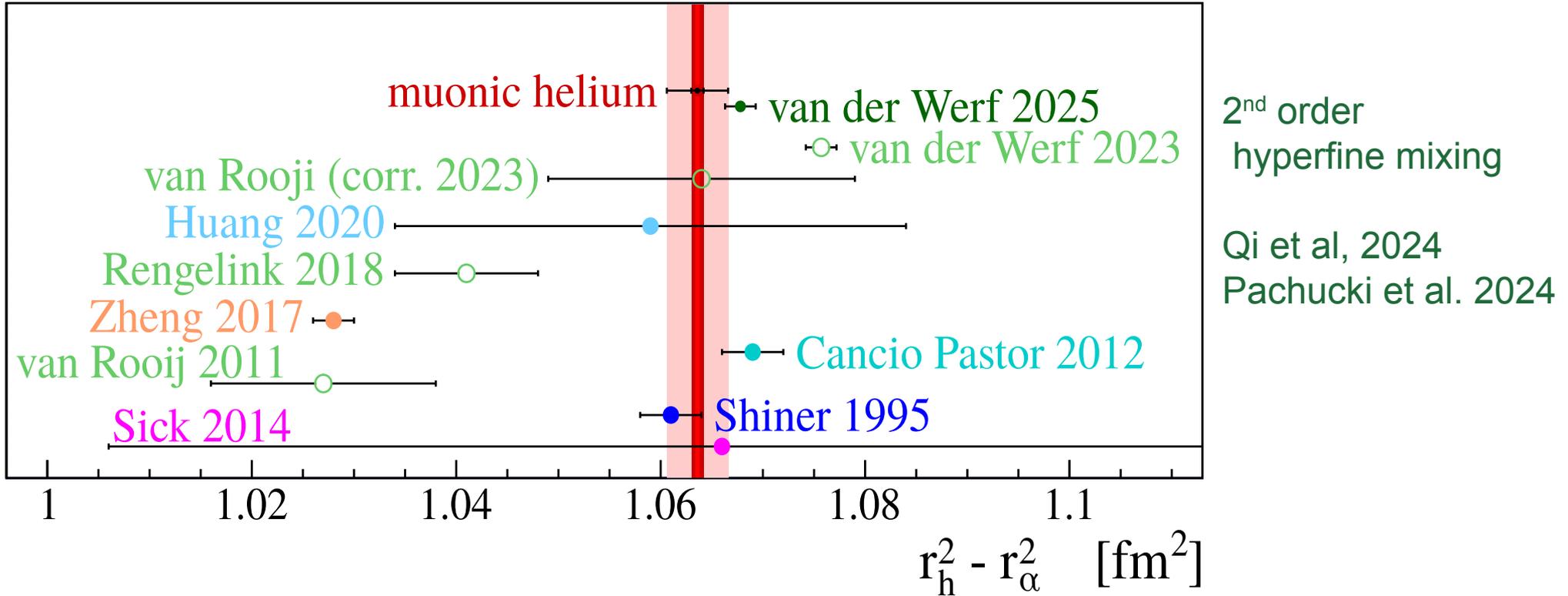
van Rooij: Science 333, 196 (2011)

Cancio Pastor: PRL 108, 143001 (2012)

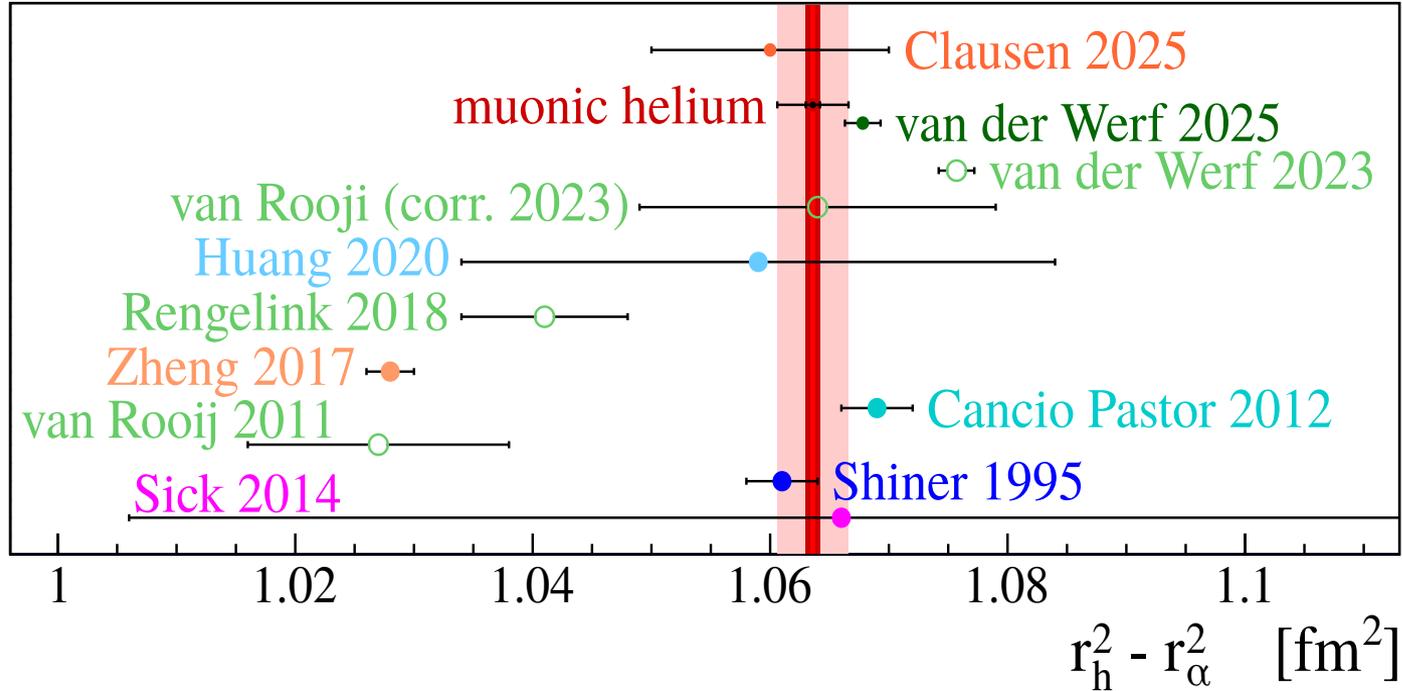
Shiner: PRL 74, 3553 (1995)

CREMA Coll., Science 388, 852 (2025)

# Helium-3 – Helium-4 Isotope Shift



# Helium-3 – Helium-4 Isotope Shift

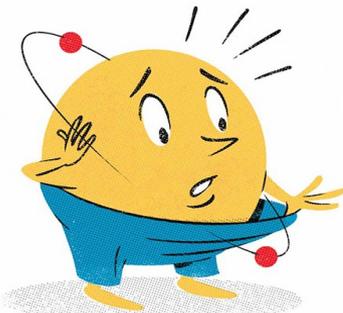


# Intermediate conclusions

Muonic atoms / ions provide:

- **~10x more accurate charge radii**, when combined with **calculated polarizability**

	${}^3\text{He}$ 1.9701* ( 10) <del>1.9730 (160)</del>	${}^4\text{He}$ 1.6786 ( 12) <del>1.6810 ( 40)</del>
${}^1\text{H}$ 0.8406 ( 4) <del>0.8751 (61)</del>	${}^2\text{D}$ 2.1279 ( 2) <del>2.1413 (25)</del>	${}^3\text{T}$ 1.7550 (860)



# Intermediate conclusions

Muonic atoms / ions provide:

- **~10x more accurate charge radii**, when combined with **calculated polarizability**
- **few times more accurate nuclear polarizability (TPE)**,
- when combined with **charge radius from regular atoms**

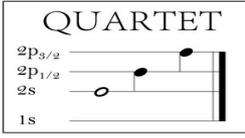
- $$\Delta E(\text{meas}) = \text{QED} + \text{TPE} - C * R_{\alpha}^2 / \text{fm}^2$$

$$\text{TPE} = 2\text{PE} + 3\text{PE}$$

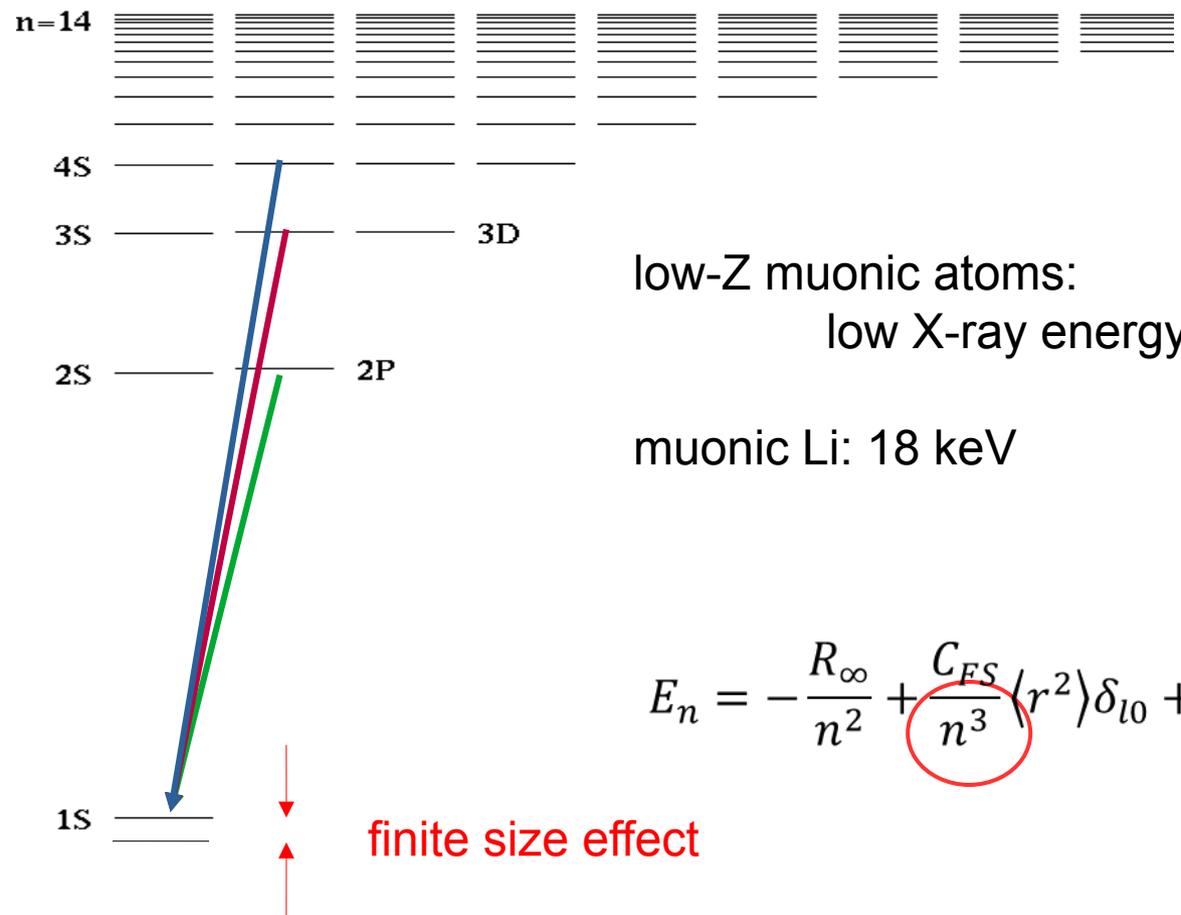
**Muonic atoms are a novel tool for proton and new-nucleon properties!**

From **Laser** to

**X-ray spectroscopy**



# X-ray spectroscopy



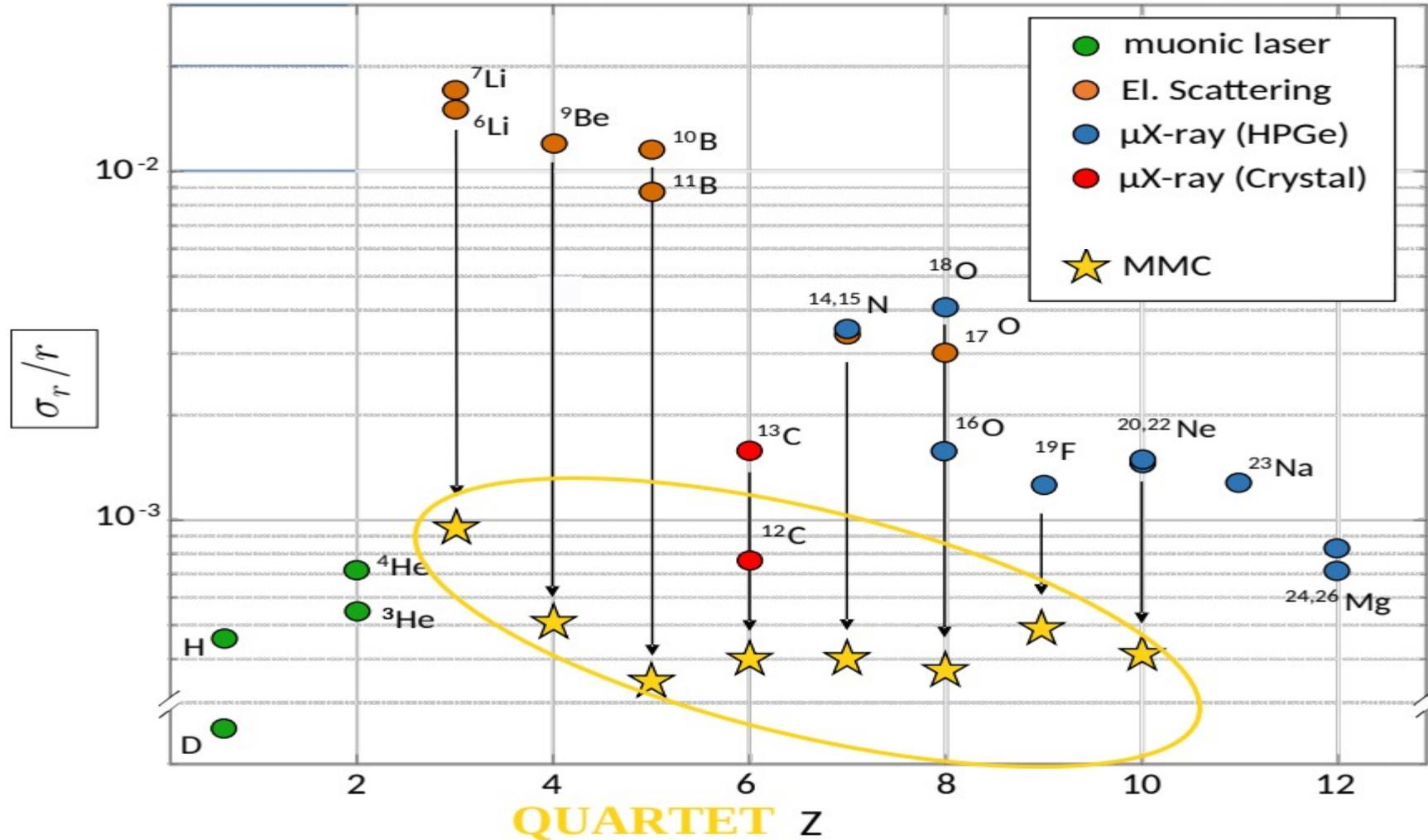
low-Z muonic atoms:  
low X-ray energy → difficult!

muonic Li: 18 keV

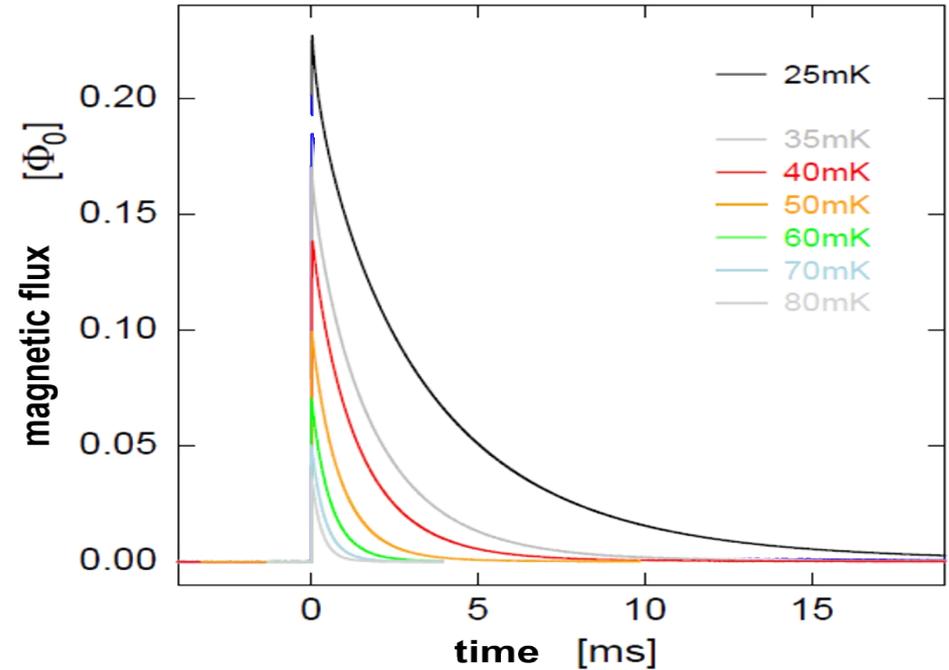
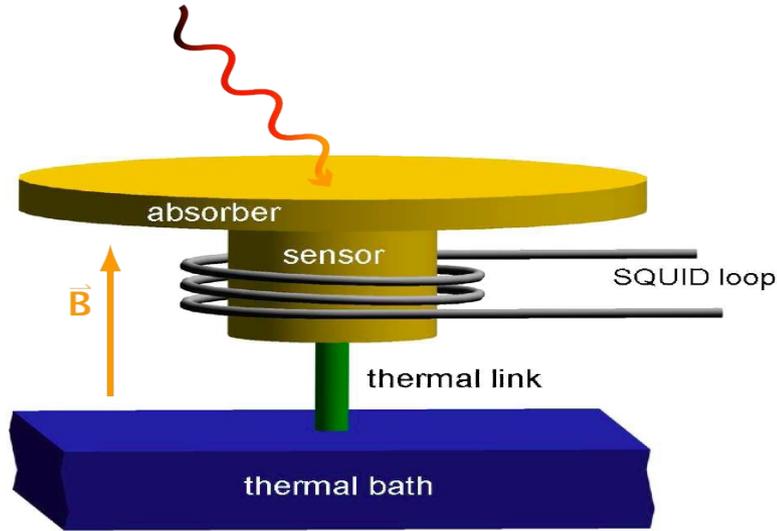
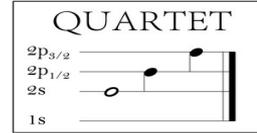
$$E_n = -\frac{R_\infty}{n^2} + \frac{C_{FS}}{n^3} \langle r^2 \rangle \delta_{l0} + \Delta(n, l, j)$$

finite size effect

# QUARTET: X-ray spectroscopy



# Metallic Magnetic Calorimeters (MMCs)

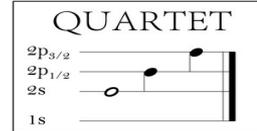


Magnetization of paramagnetic material:

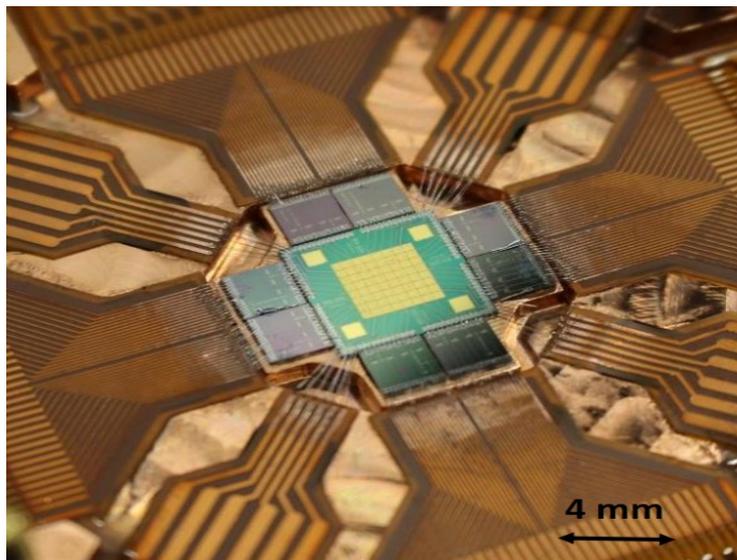


Decay time 3ms@30mK  
Keep rates < 10 Hz  
per pixel to avoid pileup

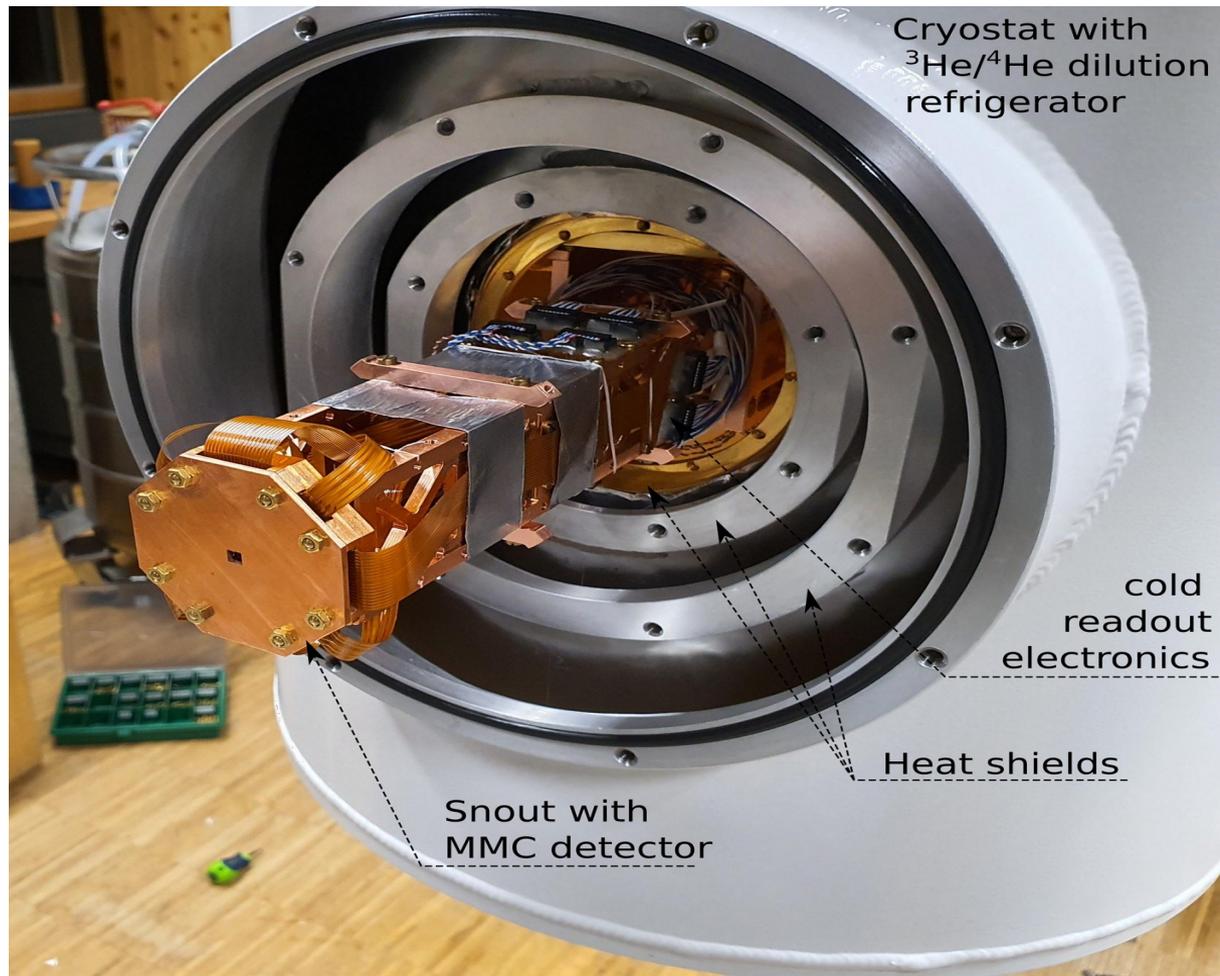
# Metallic Magnetic Calorimeters (MMCs)



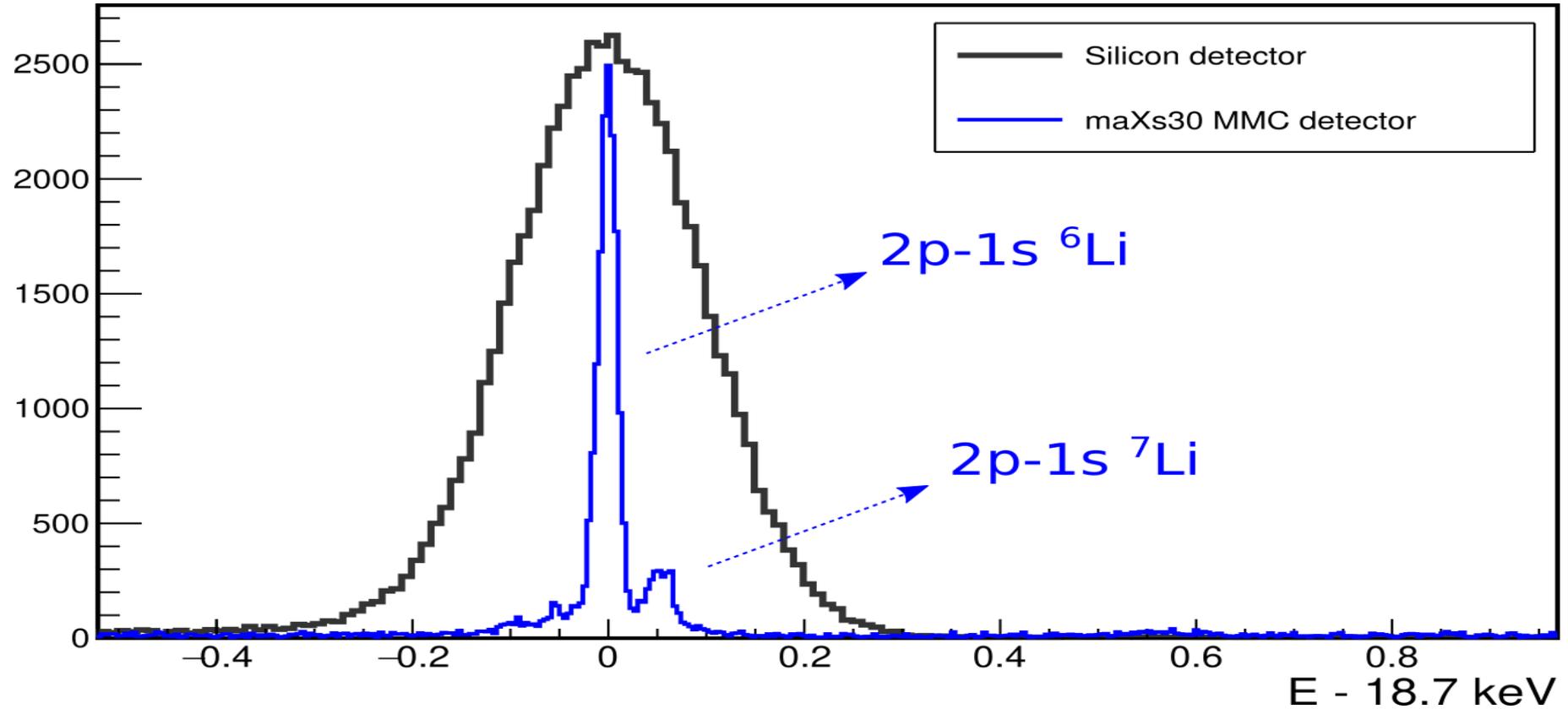
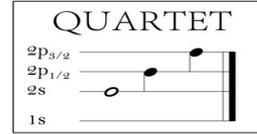
pixel array, area  $16\text{mm}^2$



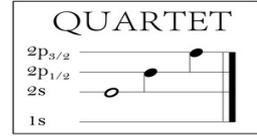
High efficiency ( $>90\%$ ) for photons  $10\text{-}60\text{ keV}$   
(2025: up to  $>170\text{ keV}$ )



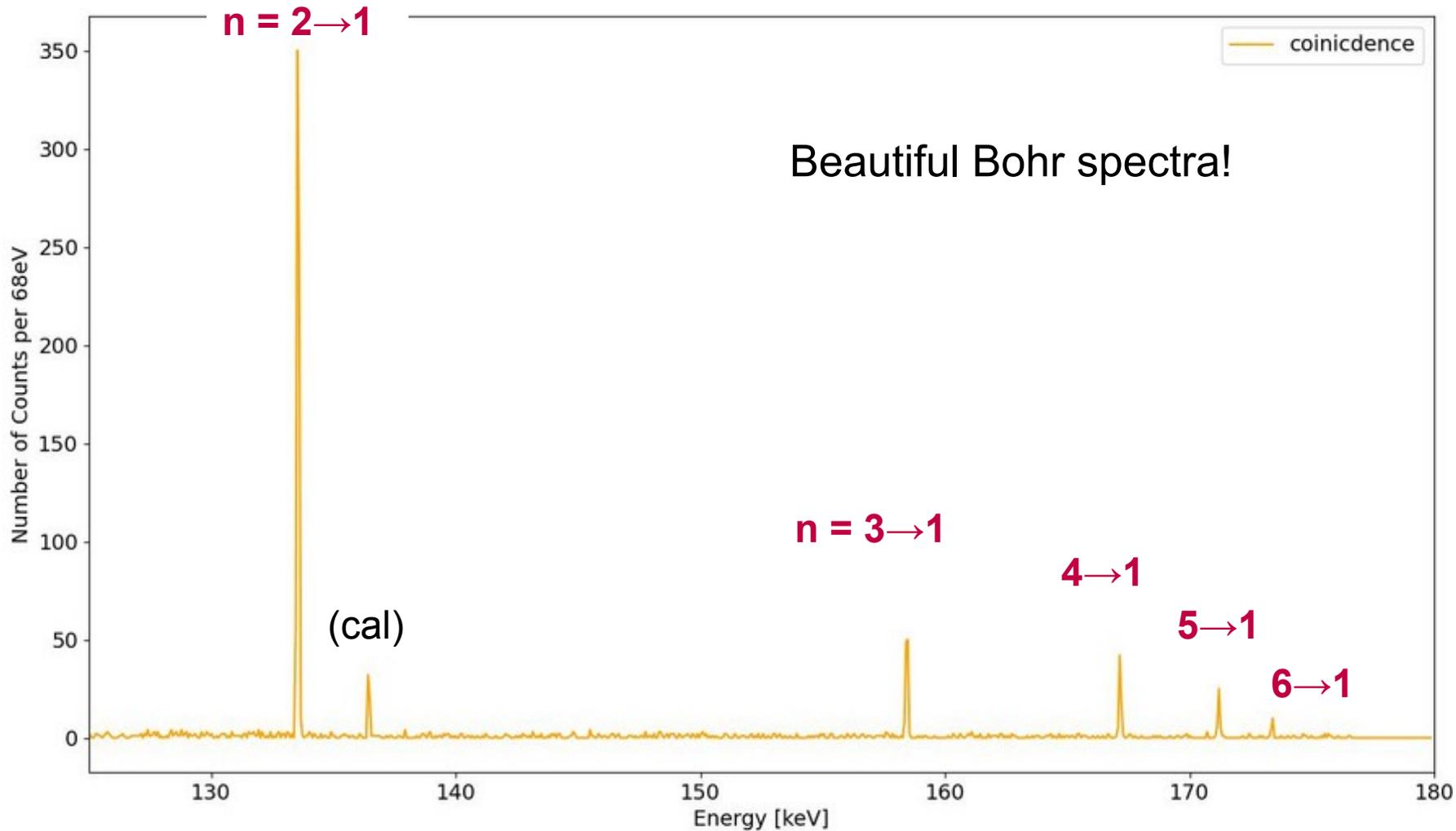
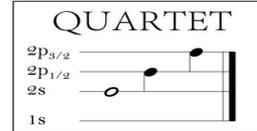
# MMC have *excellent* energy resolution



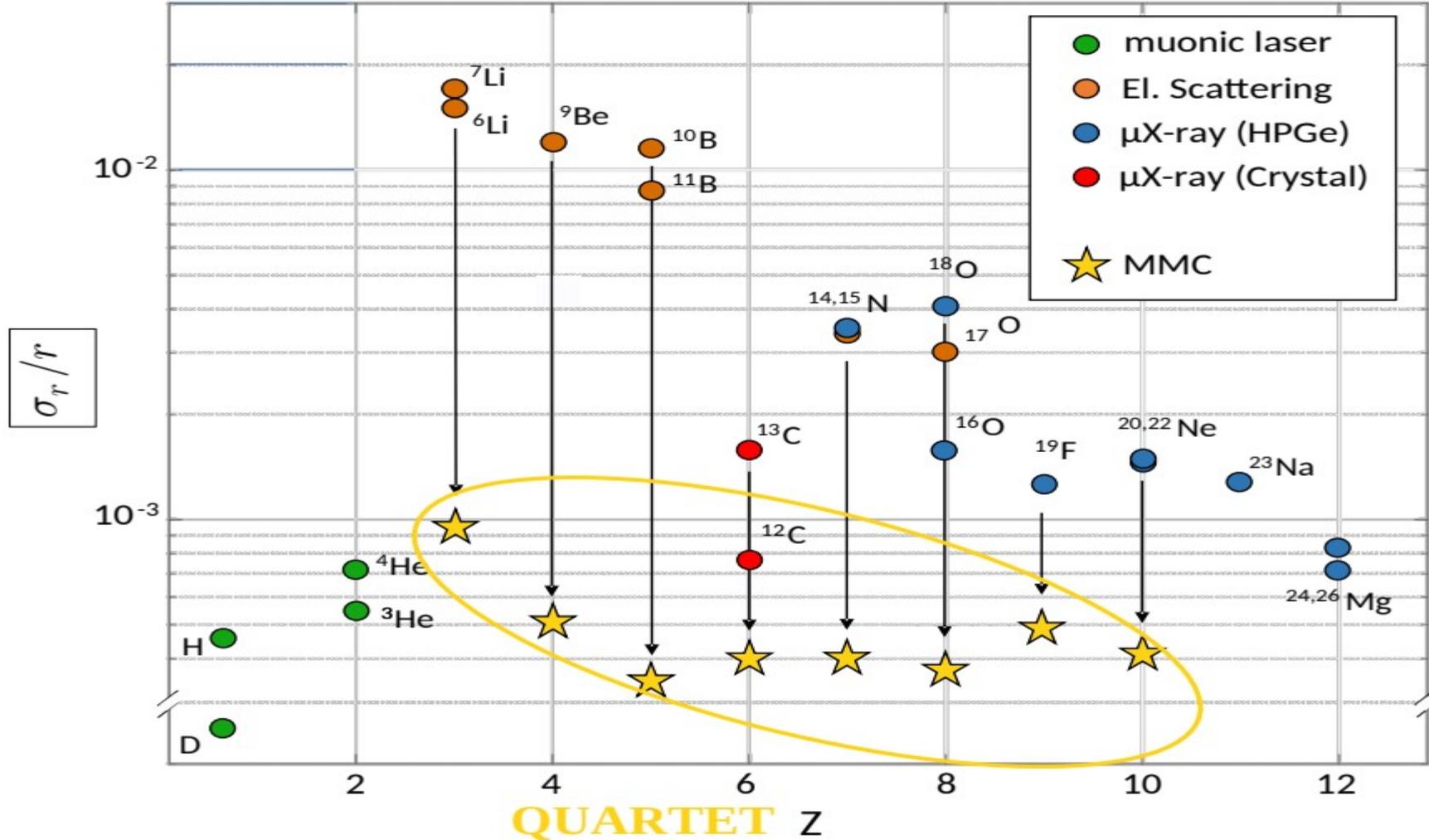
# muonic oxygen (2025)



# muonic $^{17}\text{O}$ , online spectrum (taken 3 weeks ago)



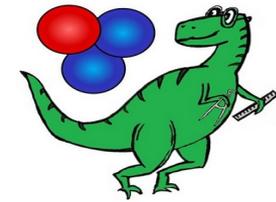
# QUARTET: X-ray spectroscopy



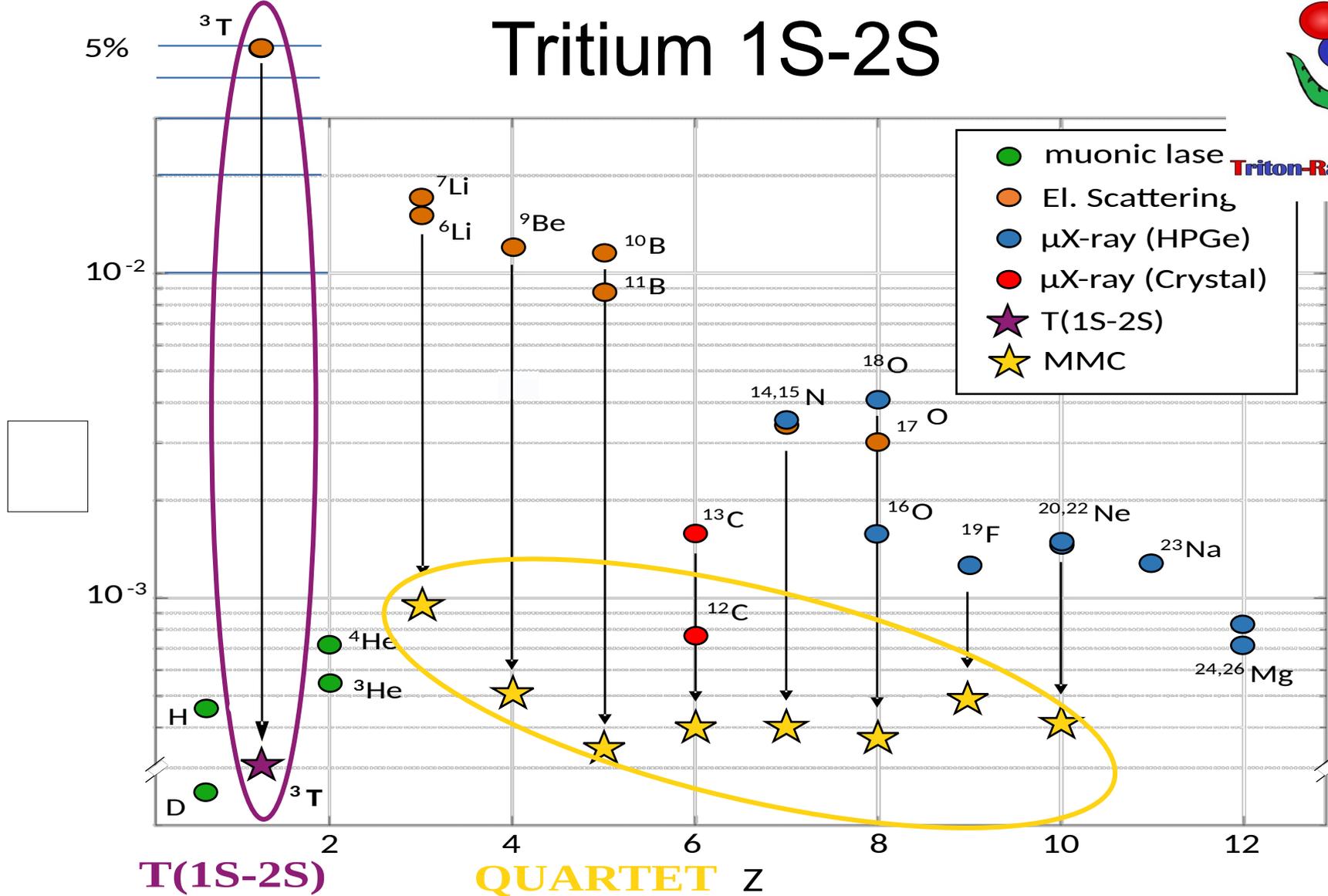
# The “missing link”

	${}^3\text{He}$ 1.9701* ( 10) <del>1.9730 (160)</del>	${}^4\text{He}$ 1.6786 ( 12) <del>1.6810 ( 40)</del>
${}^1\text{H}$ 0.8406 ( 4) <del>0.8751 (61)</del>	${}^2\text{D}$ 2.1279 ( 2) <del>2.1413 (25)</del>	${}^3\text{T}$ 1.7550 (860)

# Tritium 1S-2S



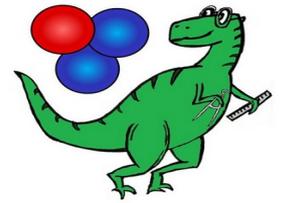
**Triton-Radius Experiment  
Mainz**



**T(1S-2S)**

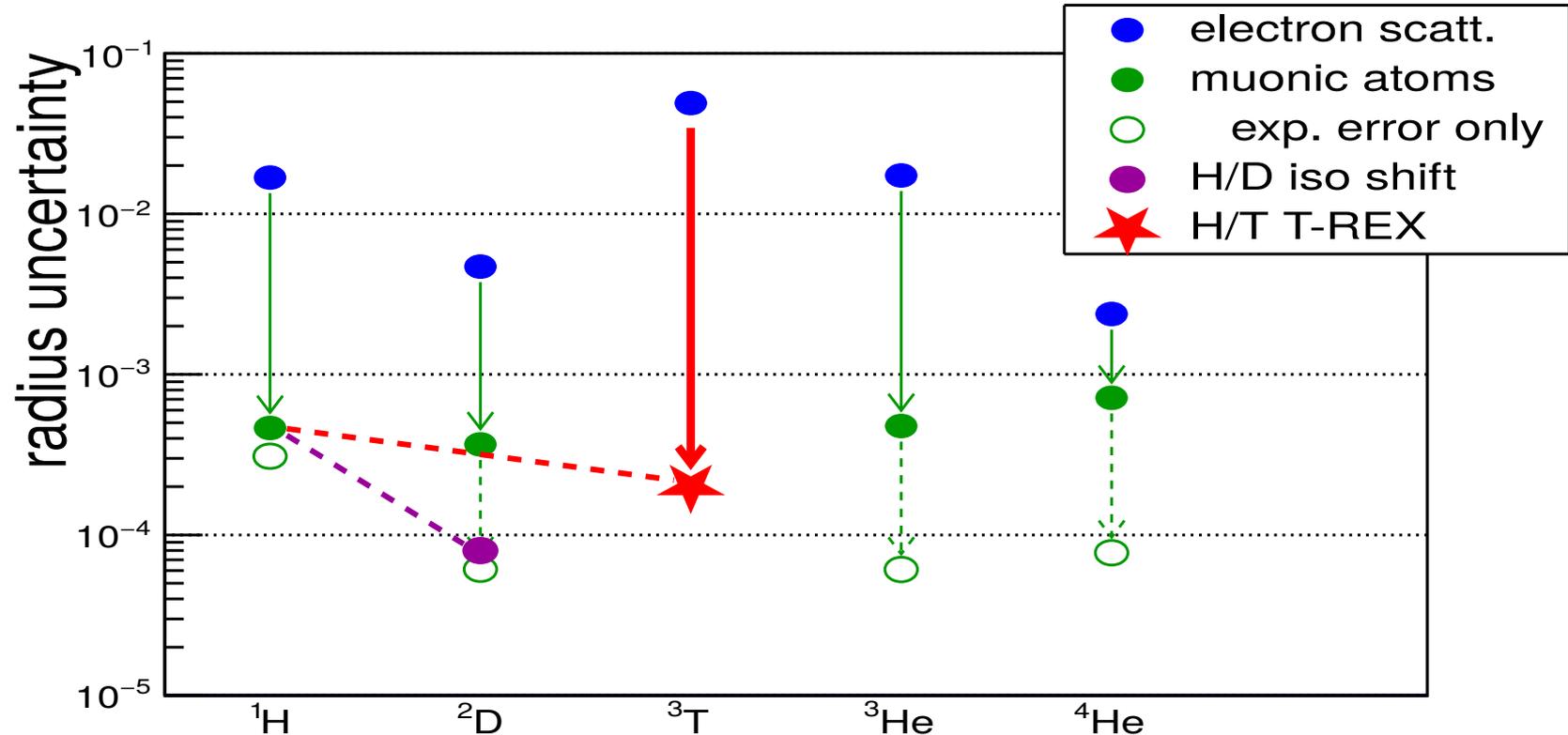
**QUARTET Z**

# Tritium 1S-2S

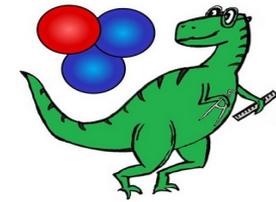


Triton-Radius Experiment  
Mainz

measure 1S-2S in **ordinary** tritium to  
determine the **isotope H/T shift**



# Tritium 1S-2S

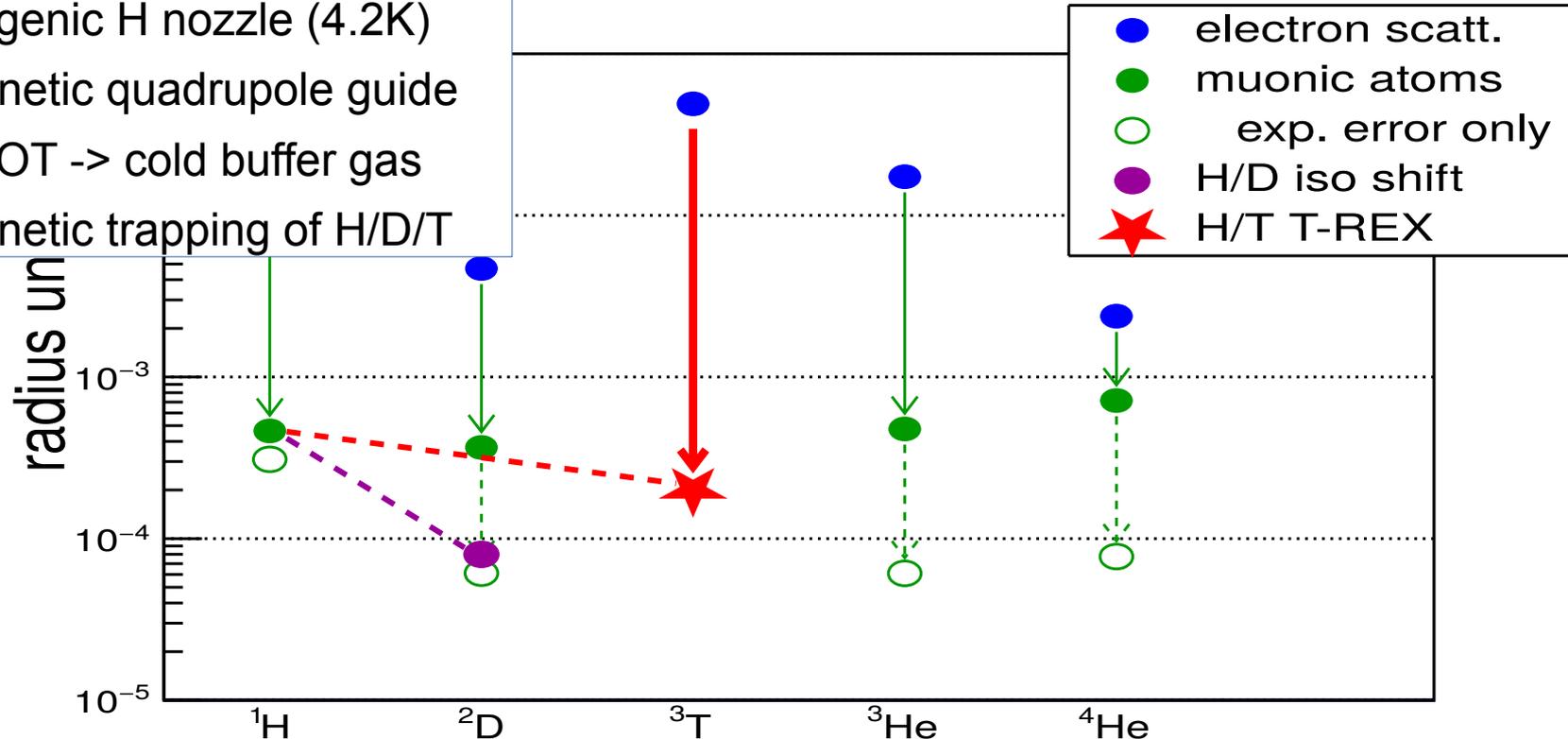


Triton-Radius Experiment  
Mainz

**400x better triton radius  
with 1 kHz measurement**

(vs. 0.01 kHz for H, D)

- cryogenic H nozzle (4.2K)
- magnetic quadrupole guide
- Li MOT -> cold buffer gas
- magnetic trapping of H/D/T

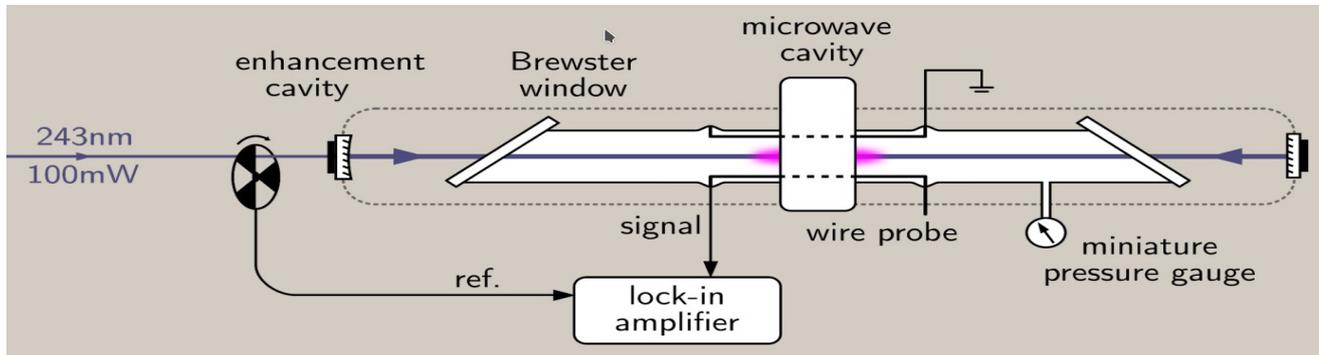


# 1st step: Somewhat better triton radius

	$^3\text{He}$ 1.9679* ( 14) <del>1.9730 (160)</del>	$^4\text{He}$ 1.6782 ( 8) <del>1.6810 ( 40)</del>
$^1\text{H}$ 0.8409 ( 4) <del>0.8751 (61)</del>	$^2\text{D}$ 2.1279 ( 2) <del>2.1413 (25)</del>	$^3\text{T}$ <b>1.7xxx (200)</b> <del>1.7550 (860)</del>

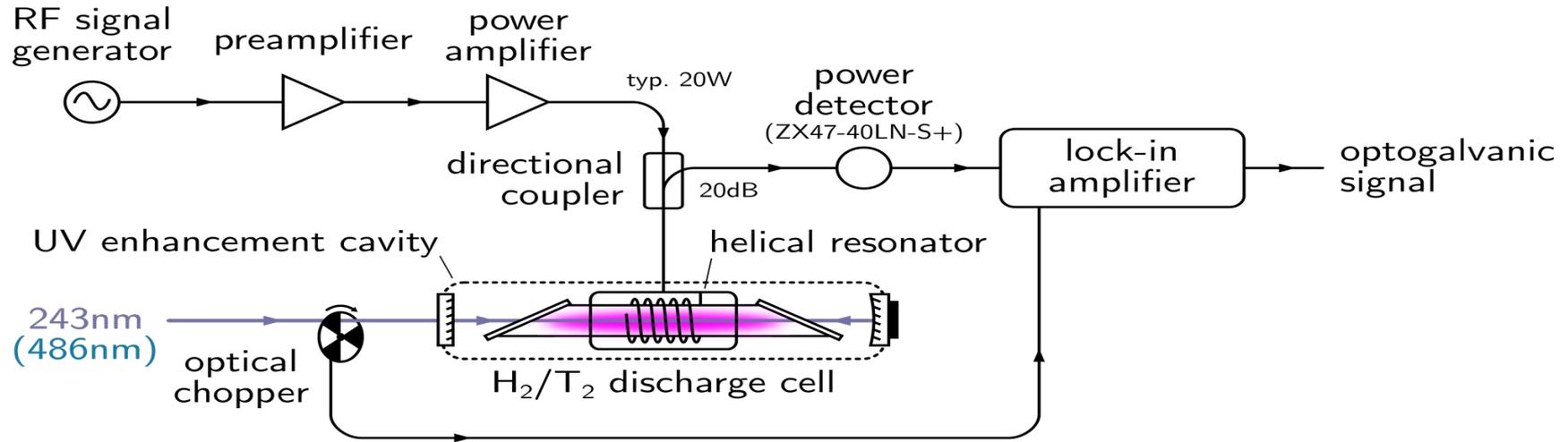
**4x better radius  
with 100 kHz measurement**

(vs. 0.01 kHz for H, D)



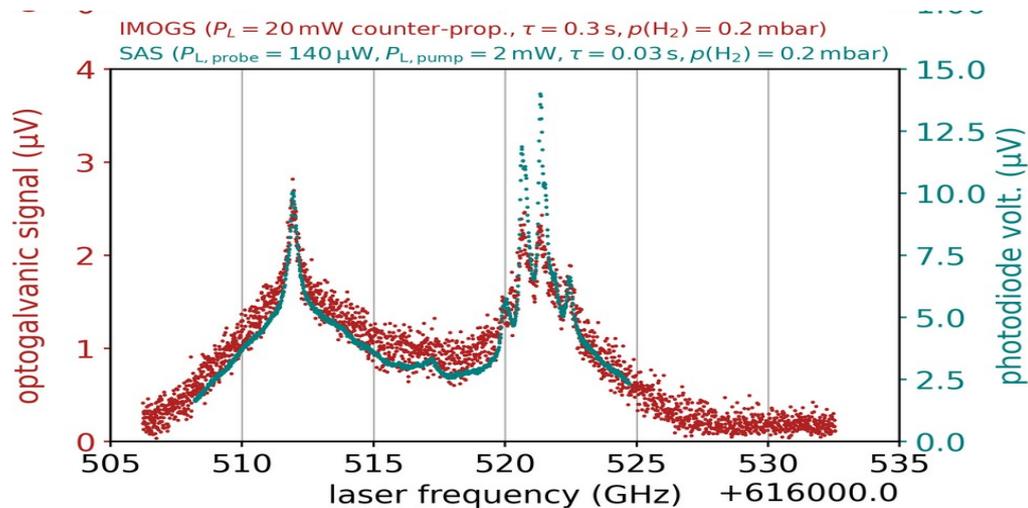
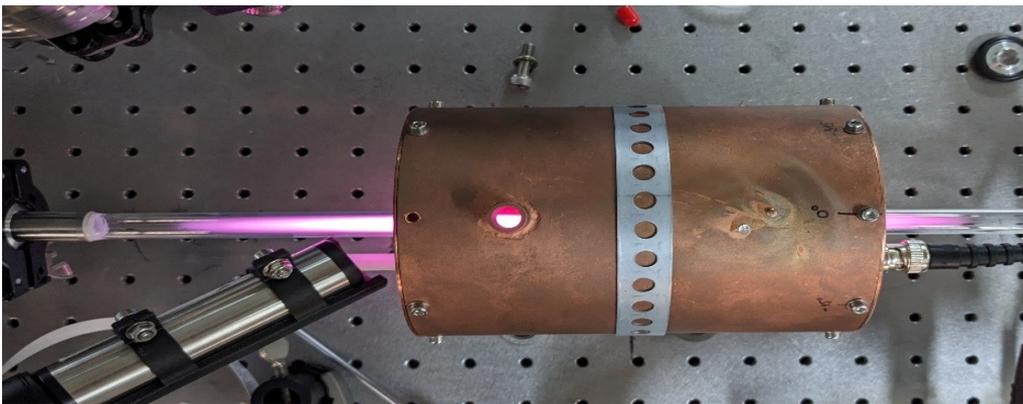
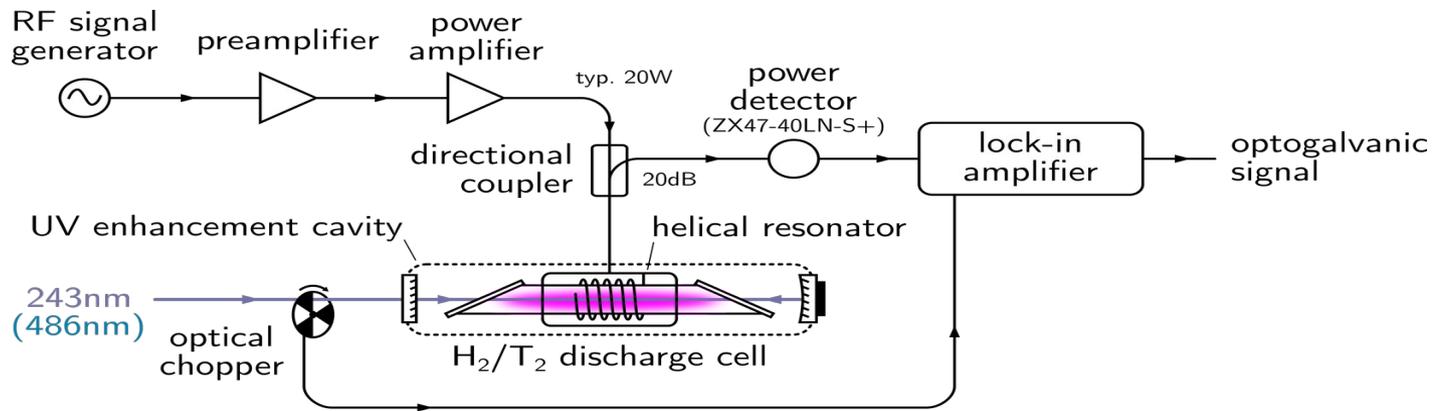
- Optogalvanic spectroscopy in a cell
- Syst. extrapolation w/ H,D
- Tritium confined.

# T(1S-2S) in a cell: Optogalvanic spectroscopy



- Optogalvanic spectroscopy in a cell
- → Don't need to observe fluorescence 😊
- → Tritium confined 😊
- → BUT: large systematics! ↓
- HOWEVER: We KNOW the results for H and D → can test our syst. effects 😊

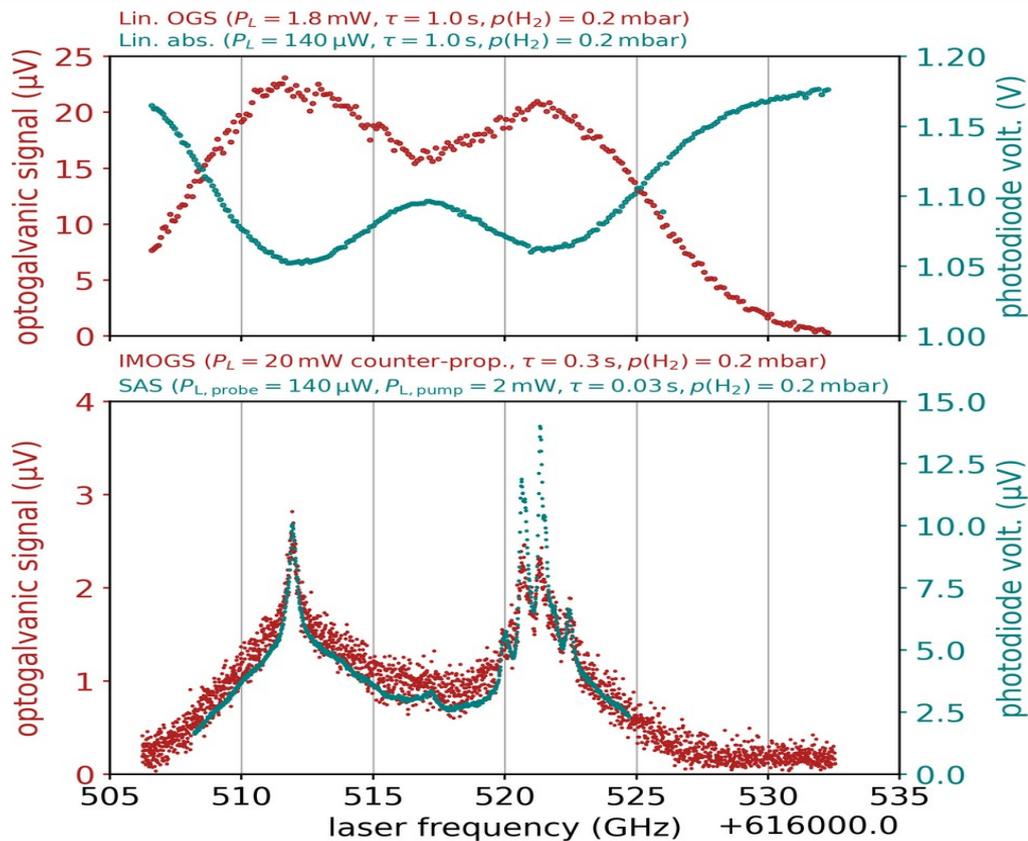
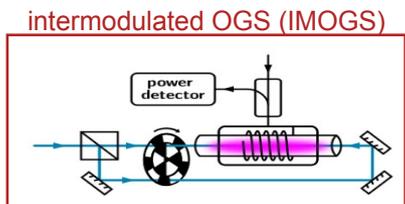
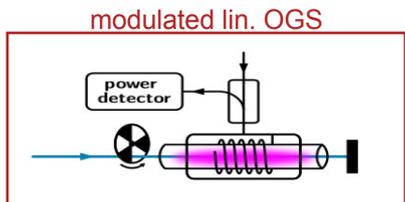
# T(1S-2S) in a cell: Optogalvanic spectroscopy



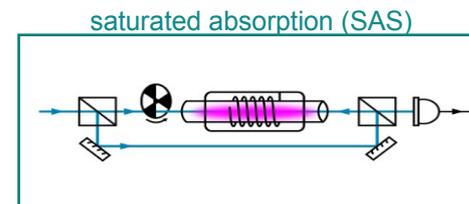
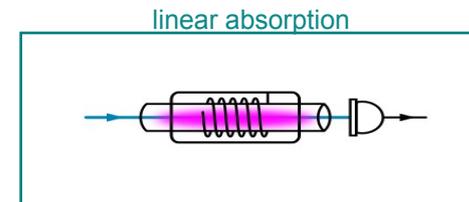
# Hydrogen/Tritium Laser Spectroscopy in RF Discharge Cell

## Balmer- $\beta$ Transition / Overview

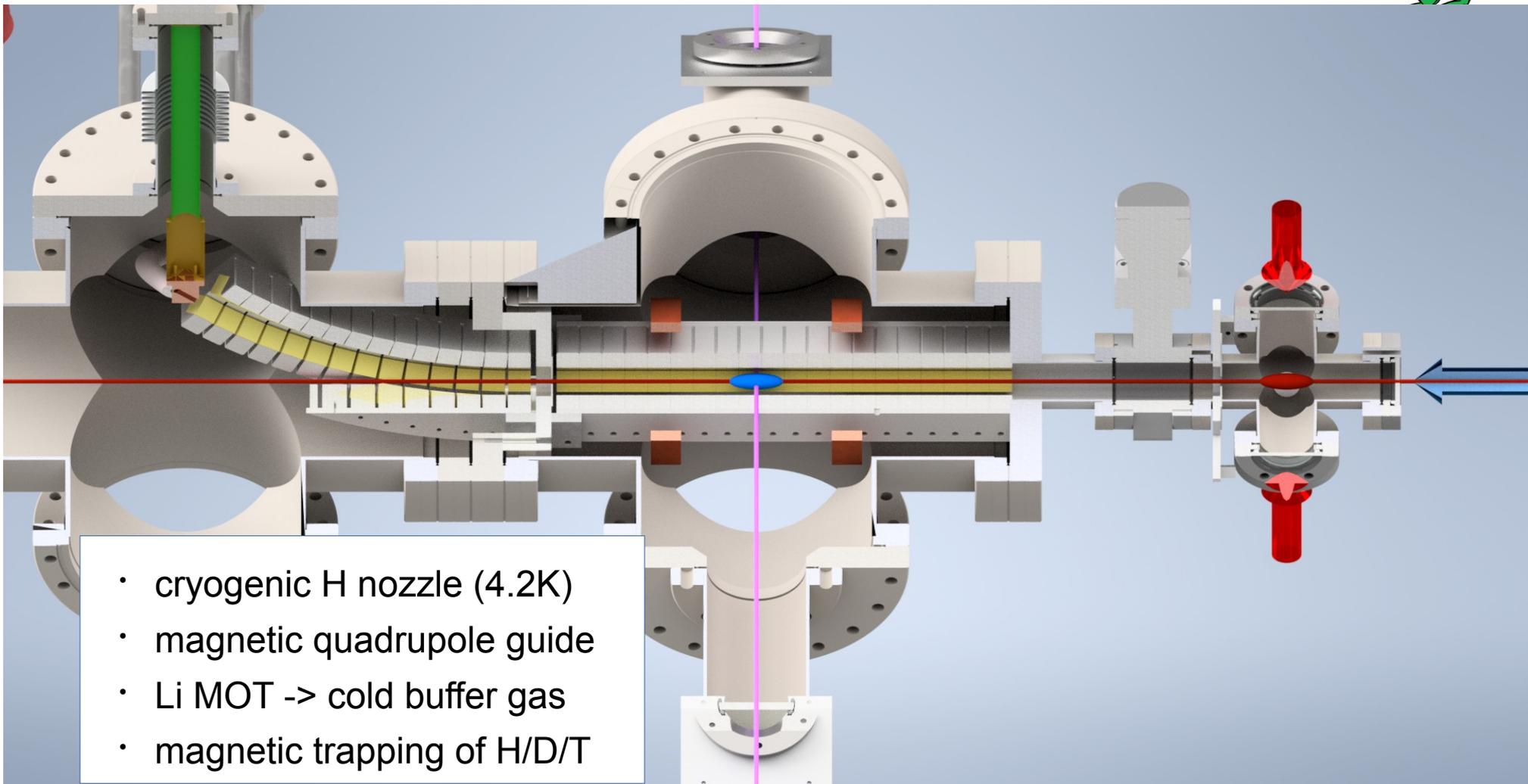
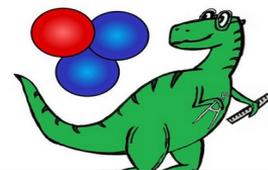
### optogalvanic spectroscopy (OGS)



### absorption spectroscopy

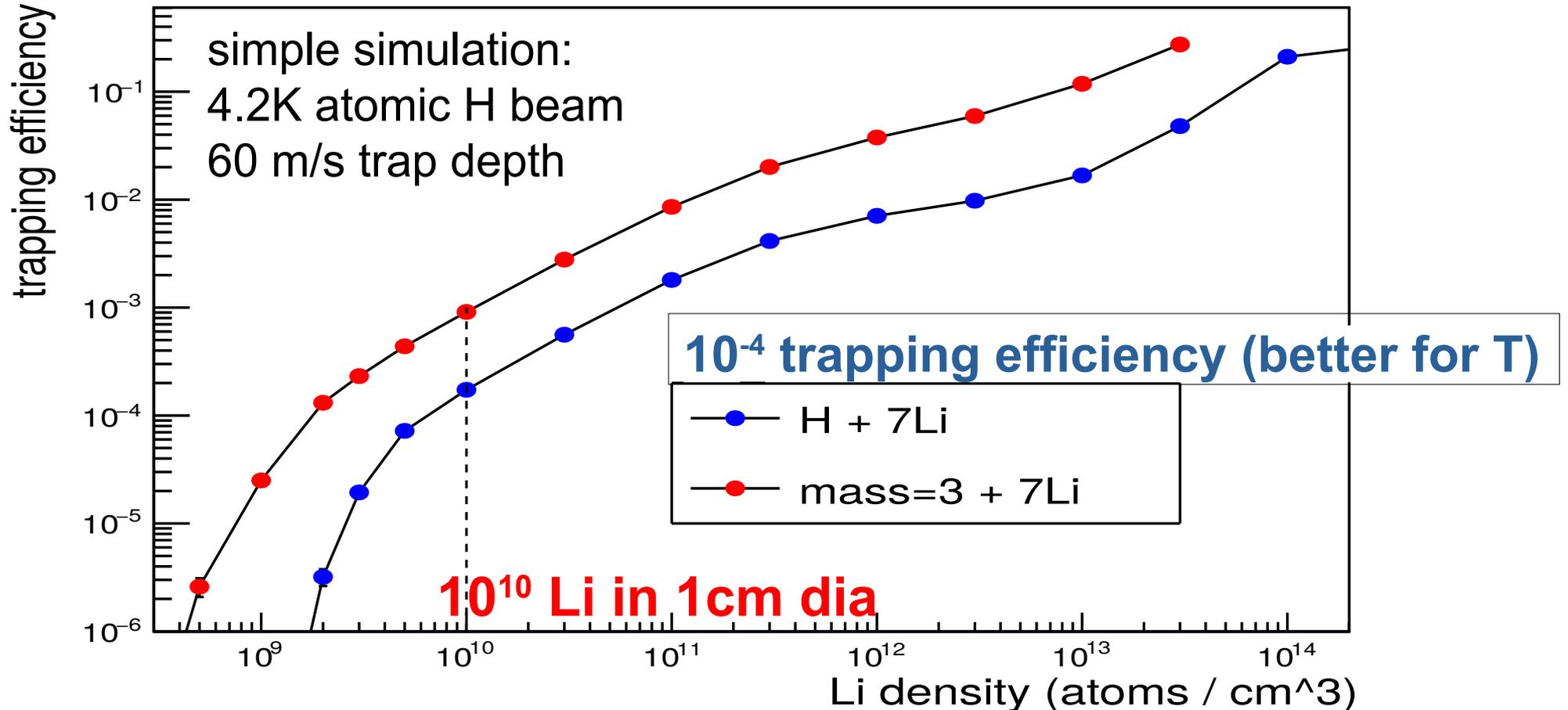


# Trapping and spectroscopy

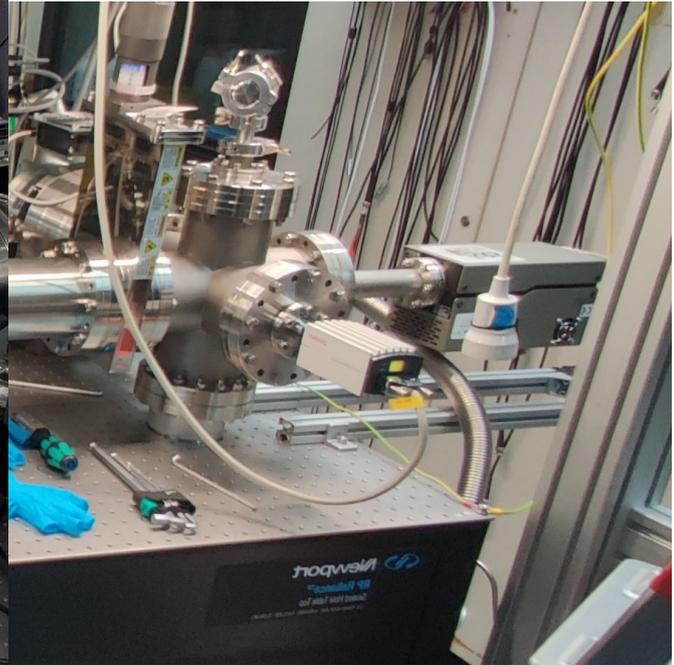
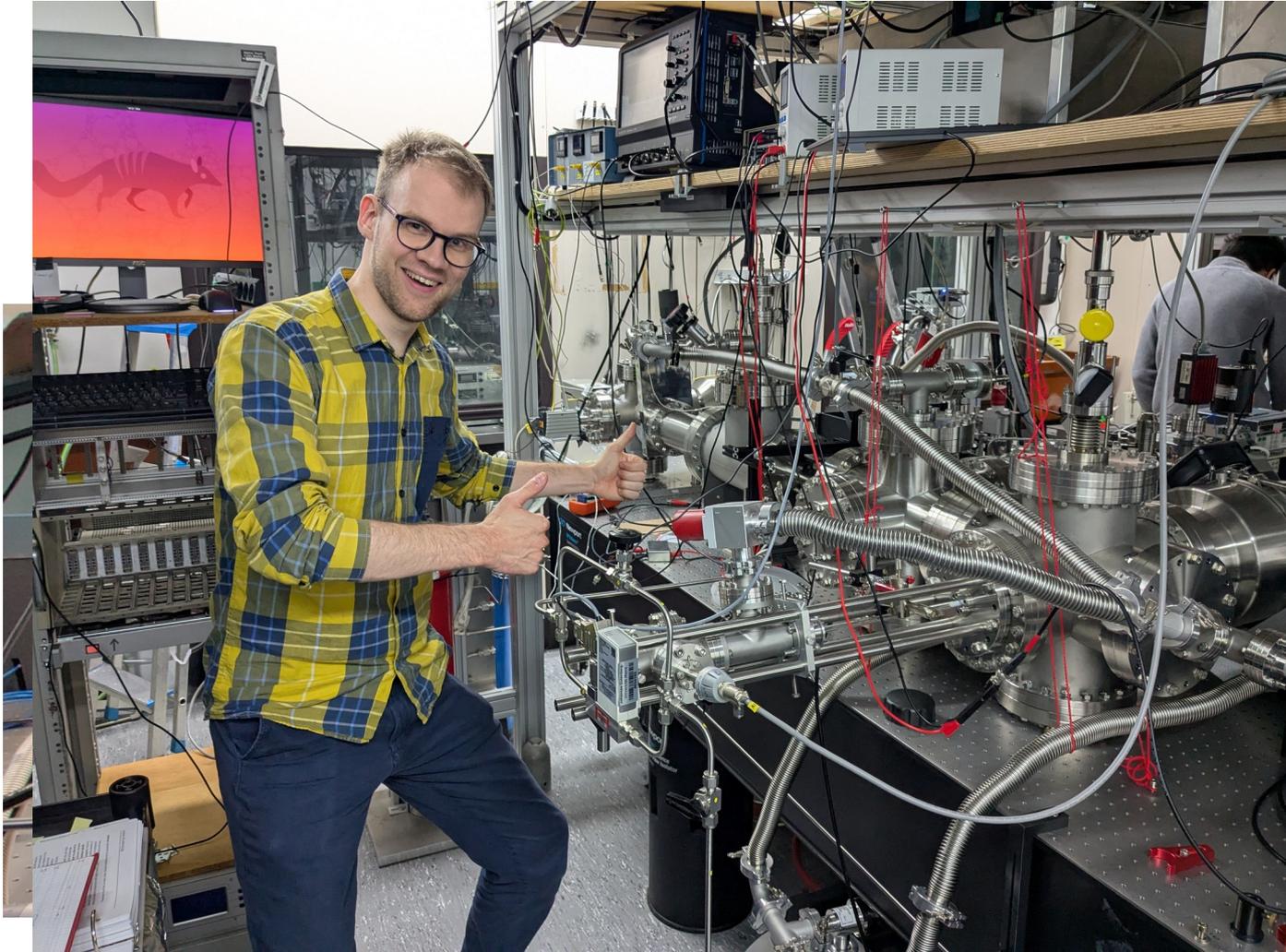


- cryogenic H nozzle (4.2K)
- magnetic quadrupole guide
- Li MOT -> cold buffer gas
- magnetic trapping of H/D/T

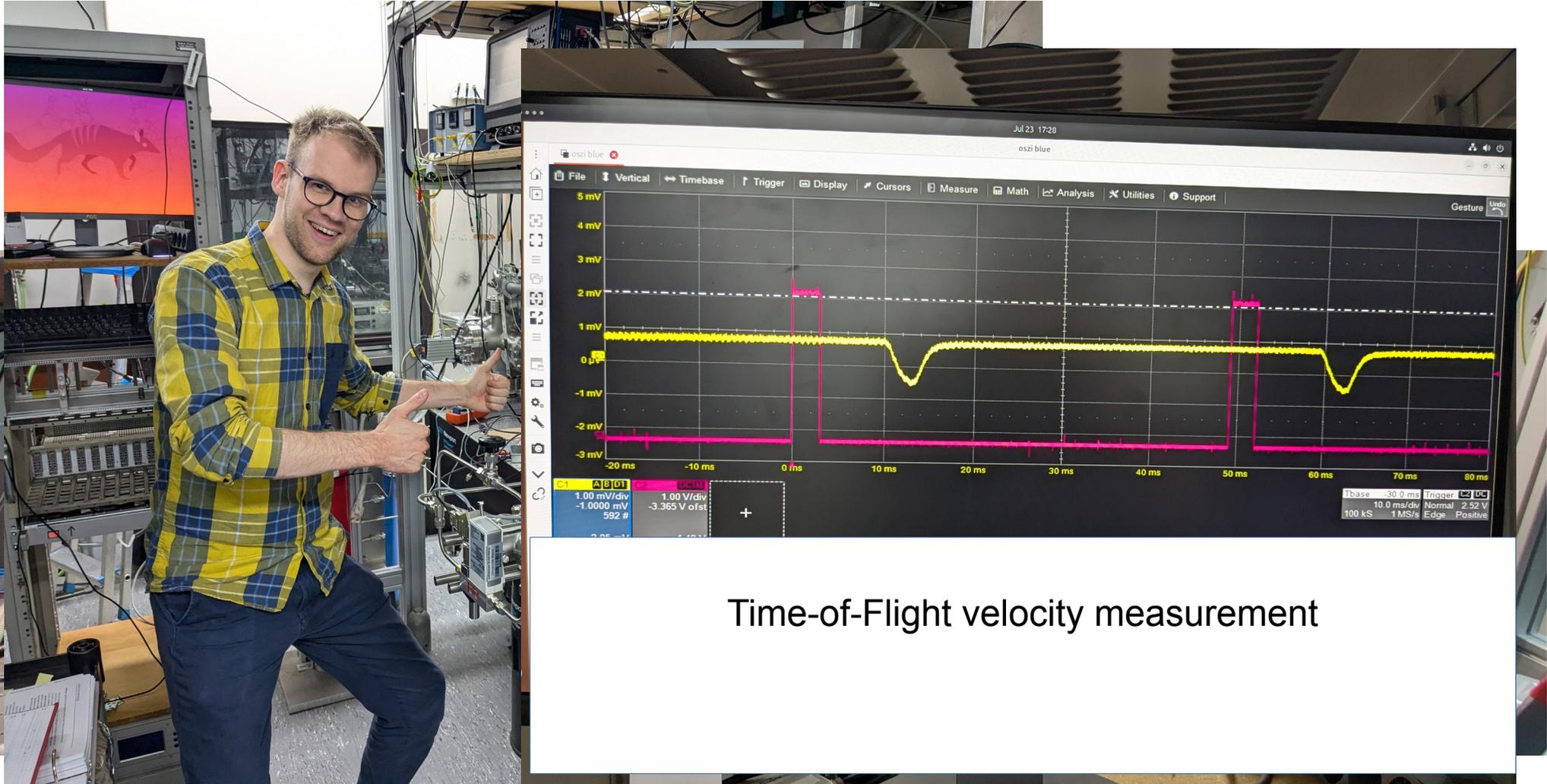
# Simulated trapping efficiency



# Towards H Beam Characterization

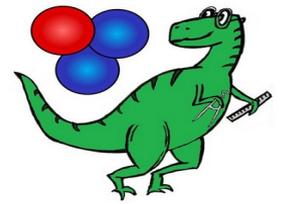


# Towards H Beam Characterization

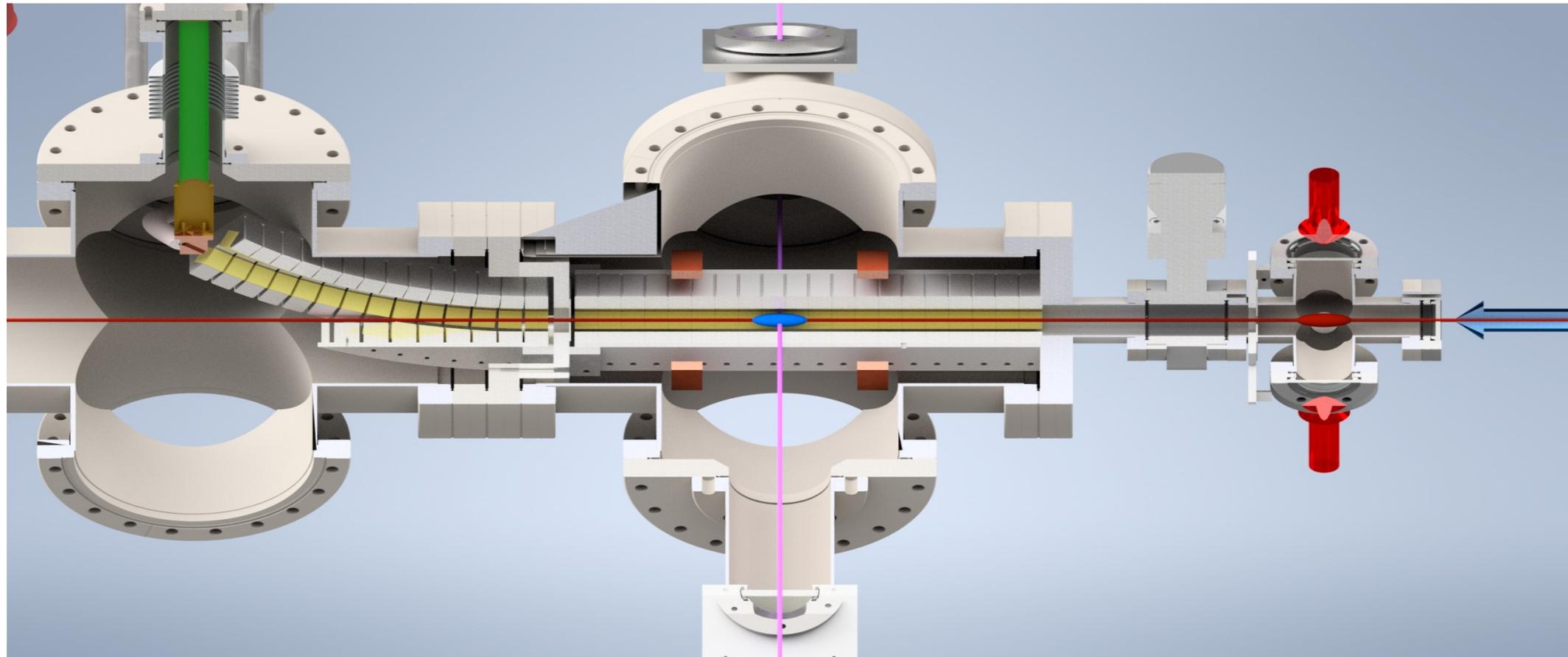


Time-of-Flight velocity measurement

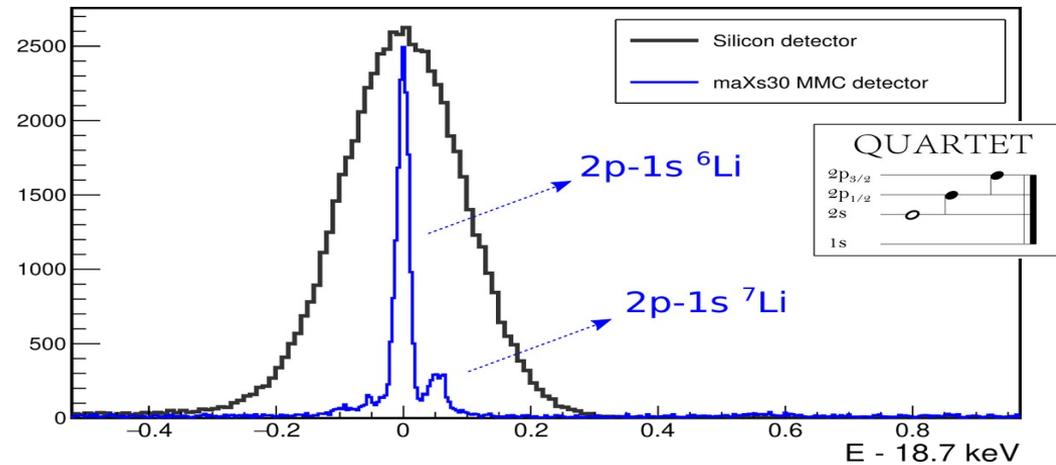
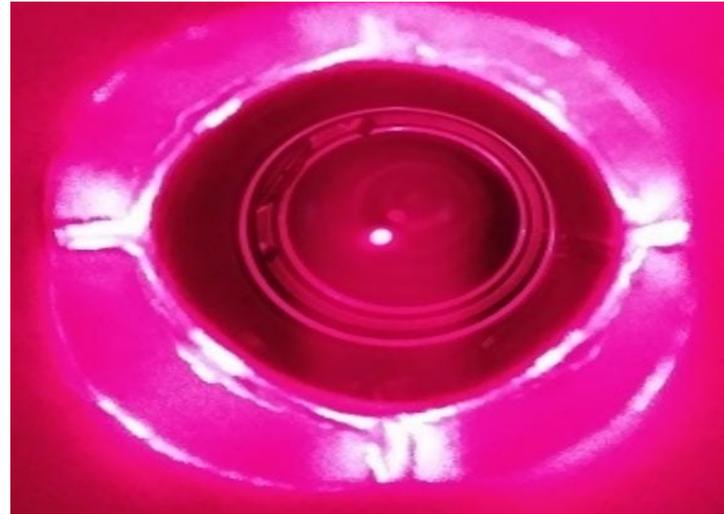
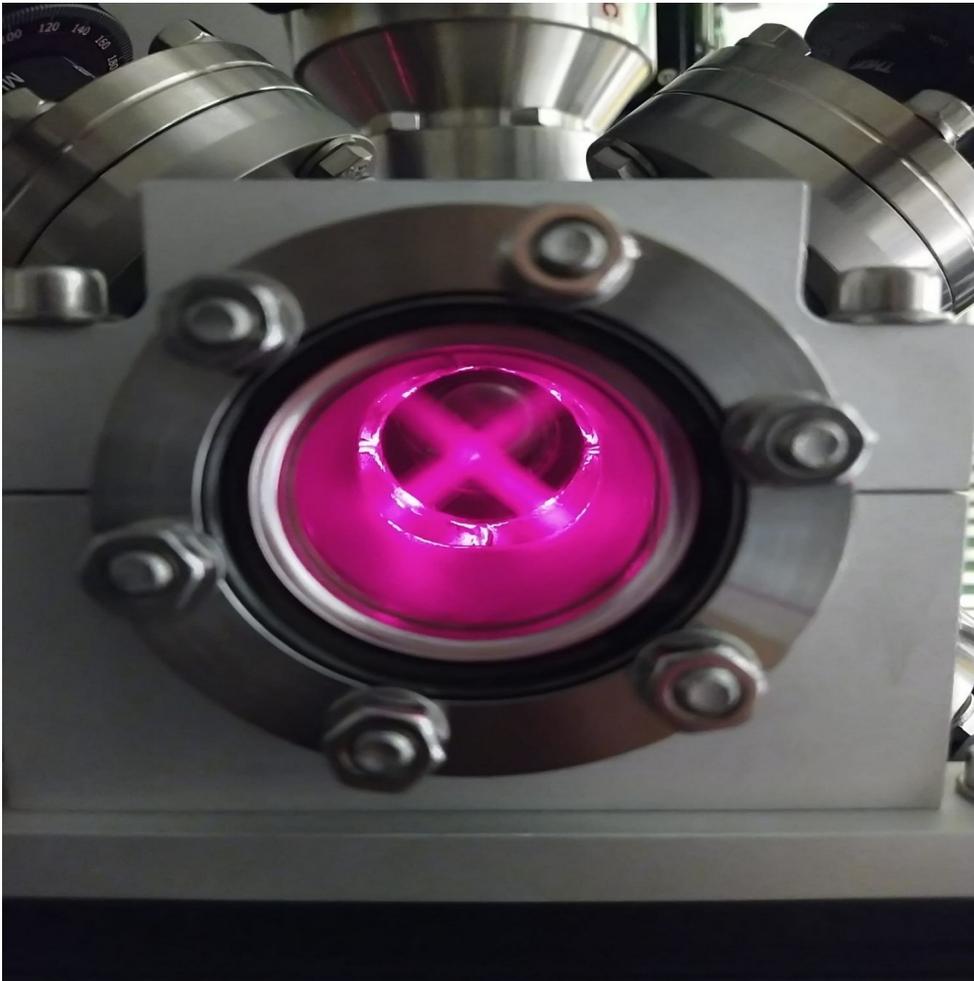
# Towards trapping and spectroscopy



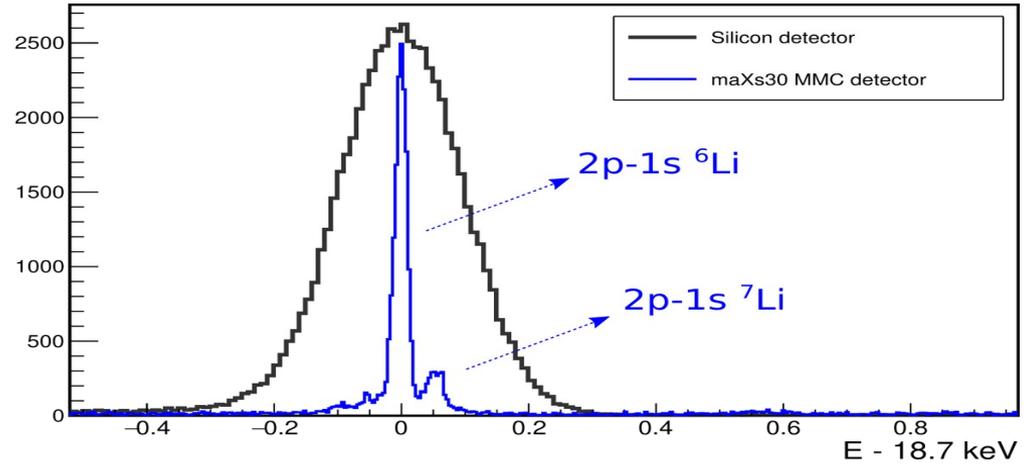
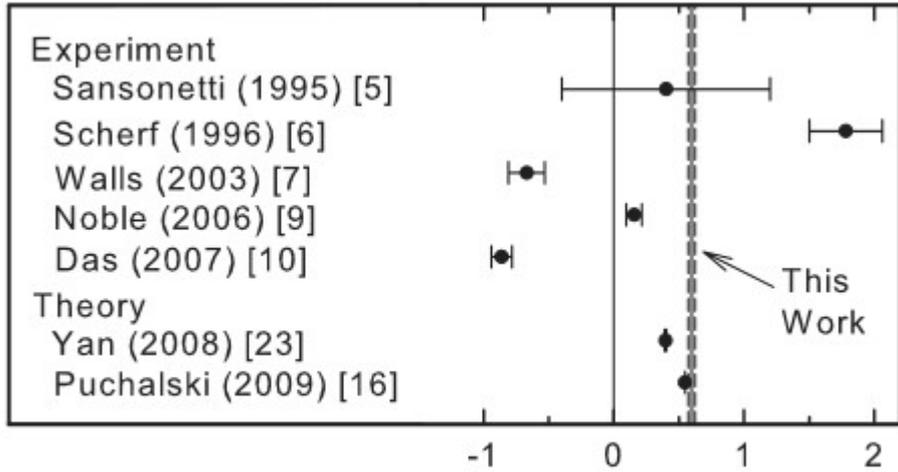
**Triton-Radius EXperiment**  
**Mainz**



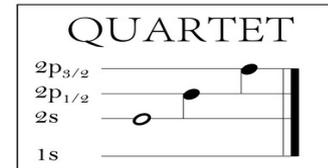
# Atomic Lithium



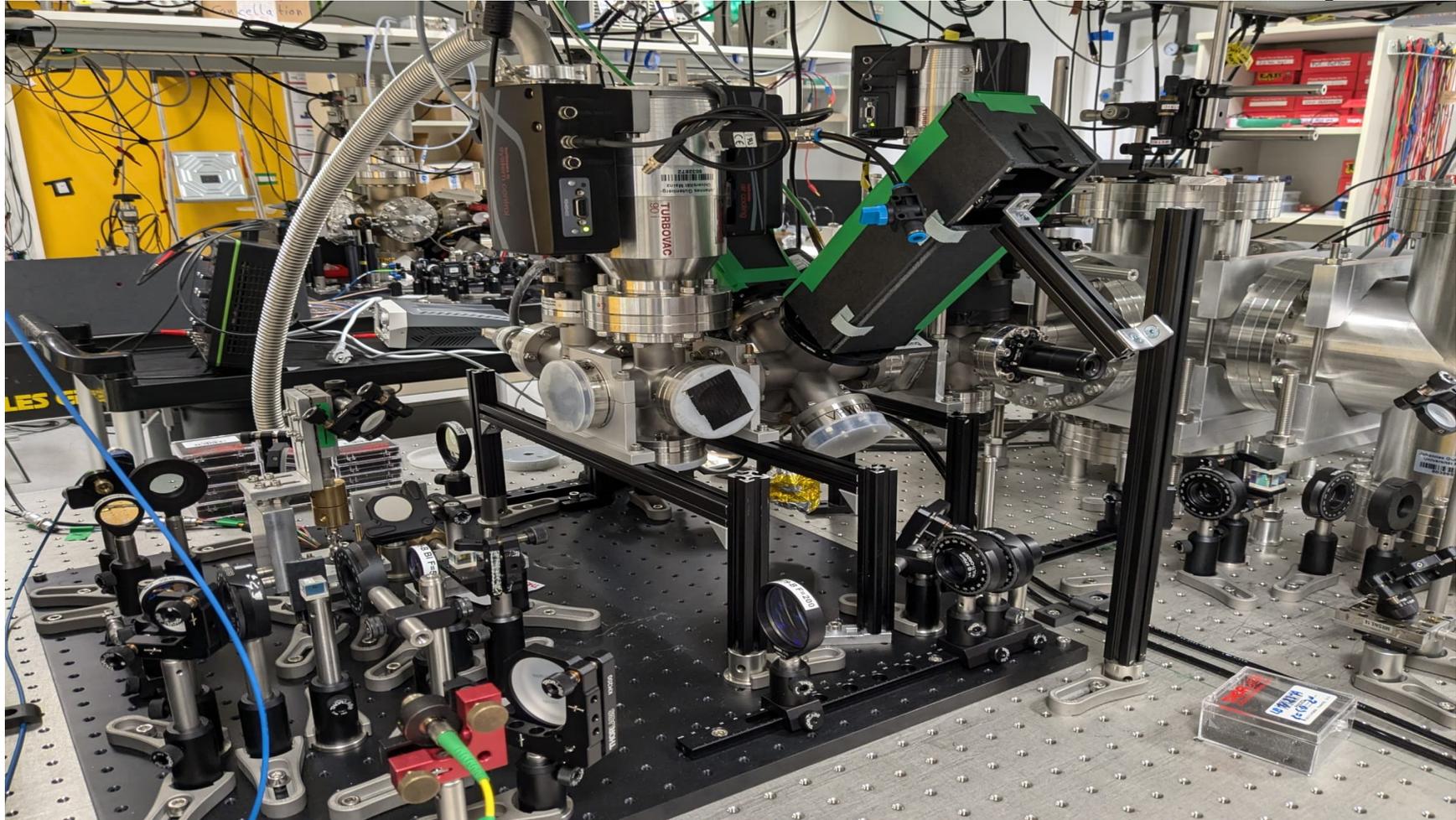
# Li-6 Li-7 isotope shift



Sansonetti et al., PRL (2011)

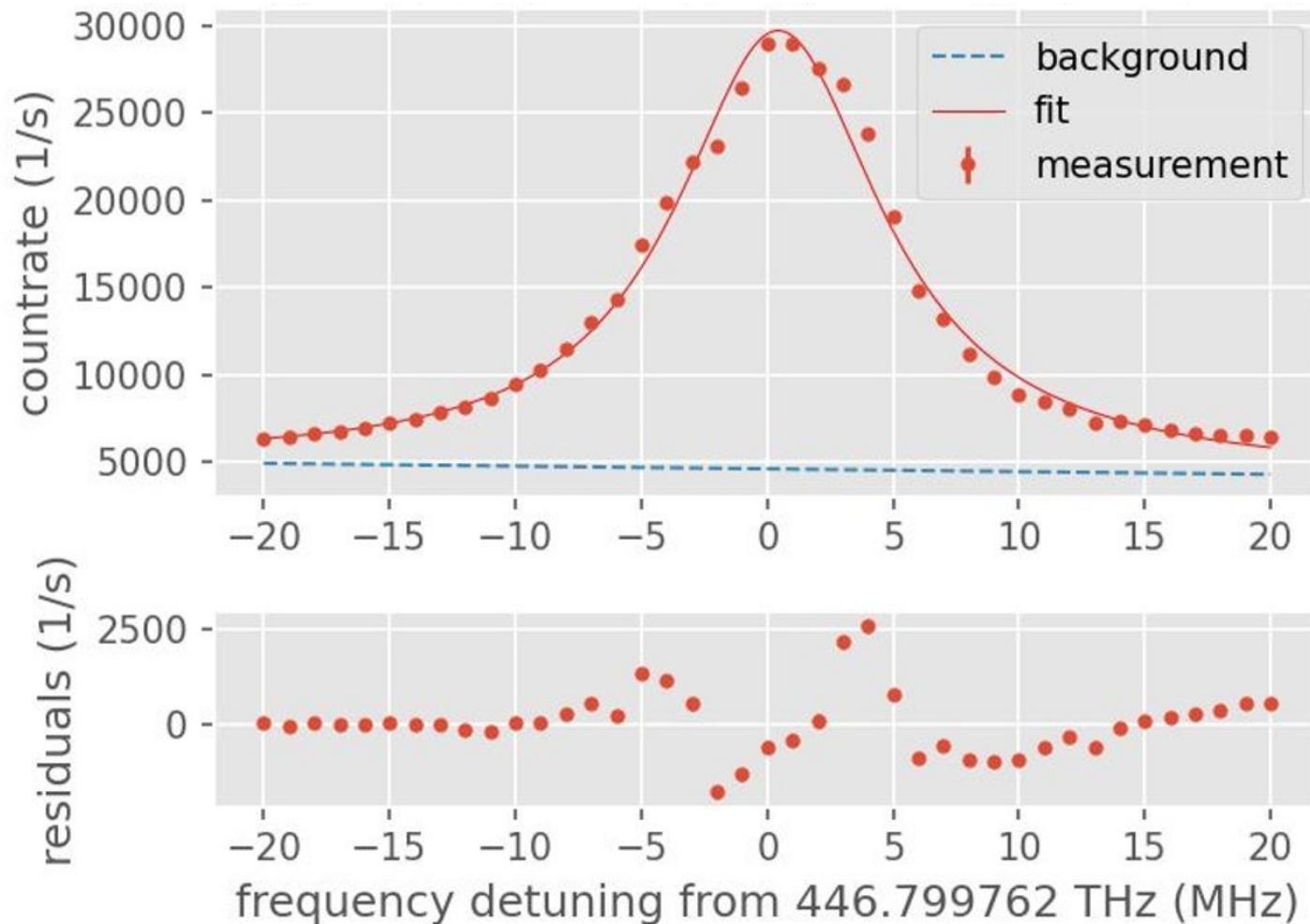


# 2D MOT + spectroscopy (AFR)



2D-MOT design: Tiecke, Walraven et al, PRA 80, 013409 (2009)

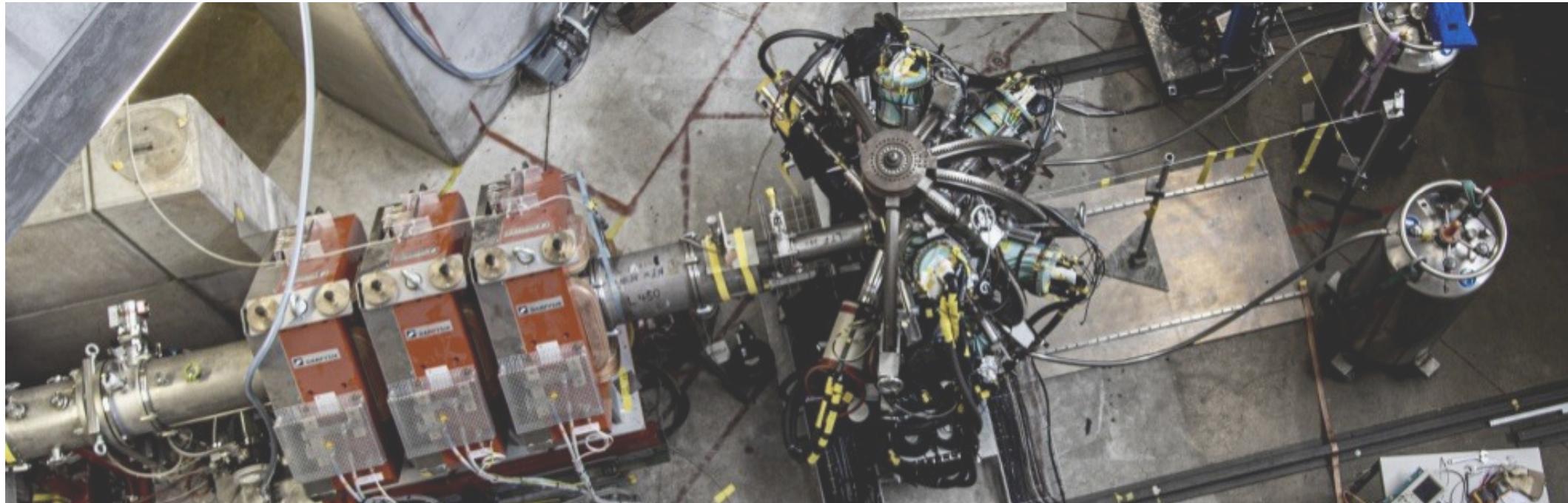
# Our 1<sup>st</sup> spectroscopy on cold ${}^6\text{Li}$



# muX

X-rays from **O(10s of  $\mu\text{g}$ ) target material** using Ge detectors  
(Miniball)

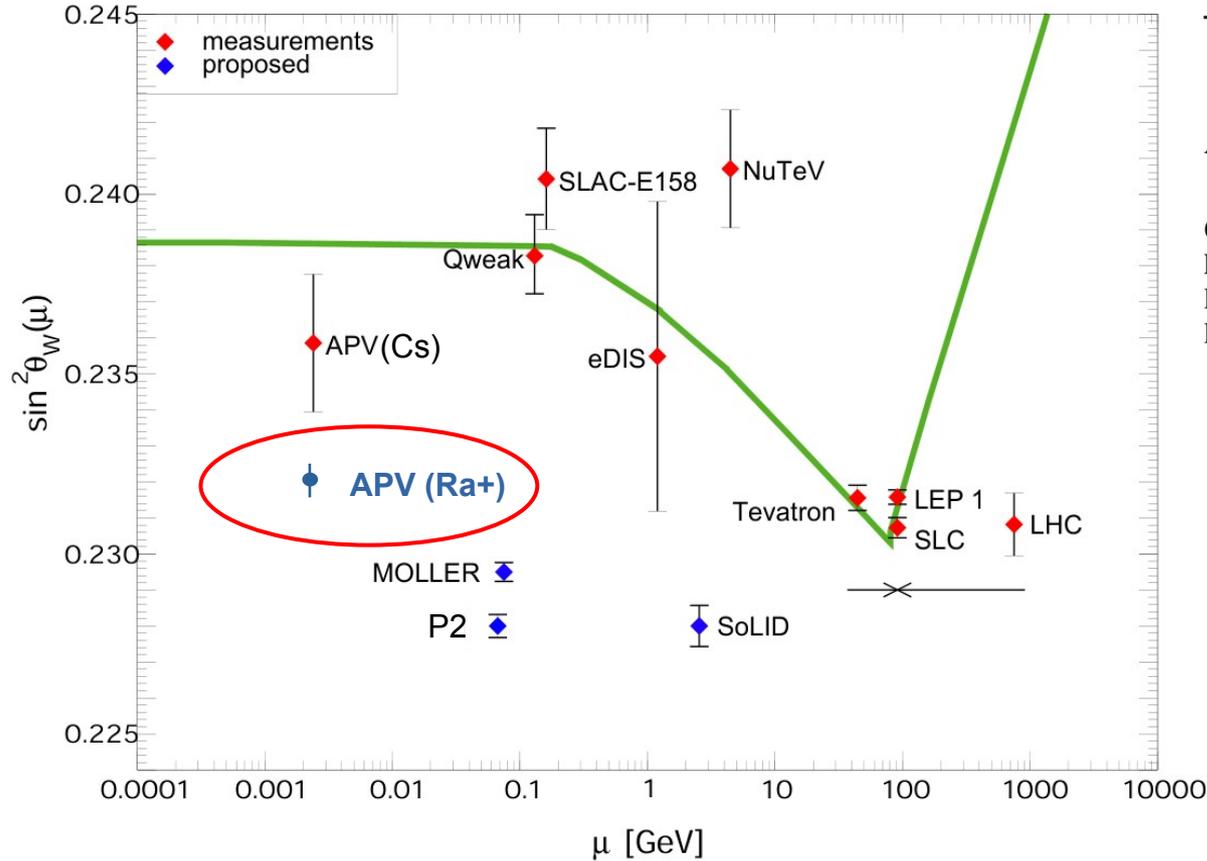
→ rare, or radioactive



# muX: Radii of O(10μg) material

rare stuff, or radioactive isotopes

Hyperfine Interact (2011) 199:9–19  
DOI 10.1007/s10751-011-0296-6



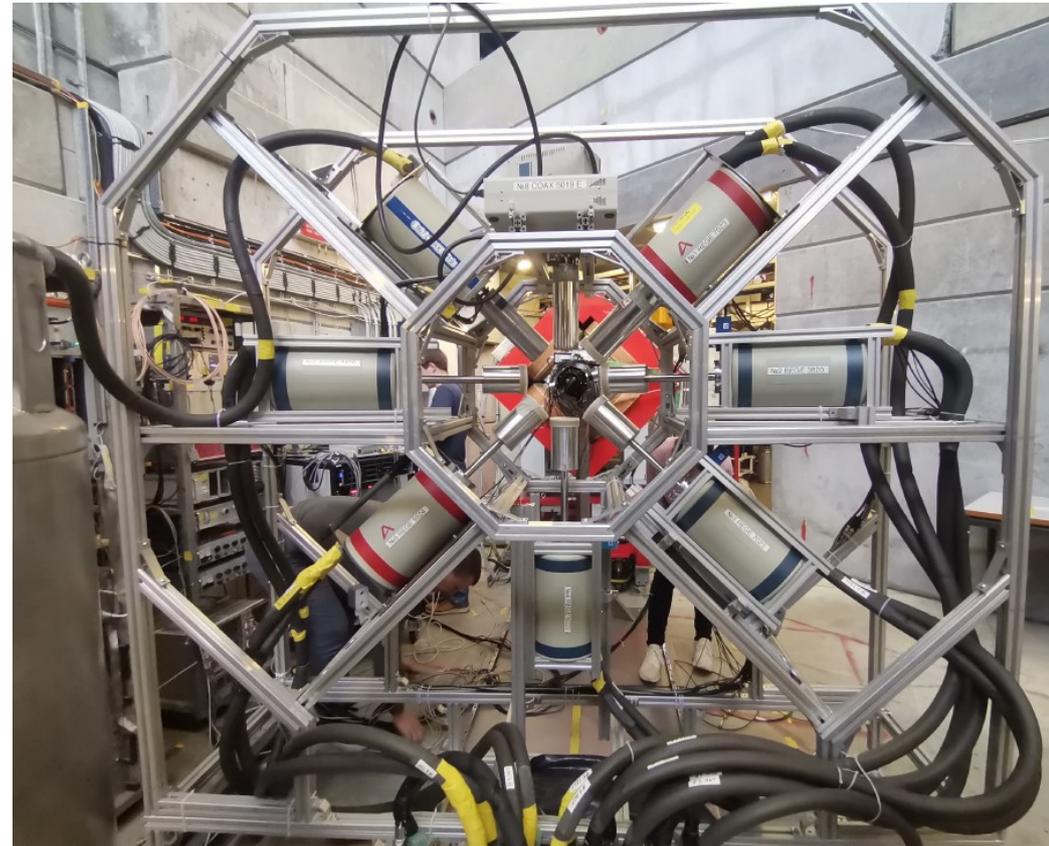
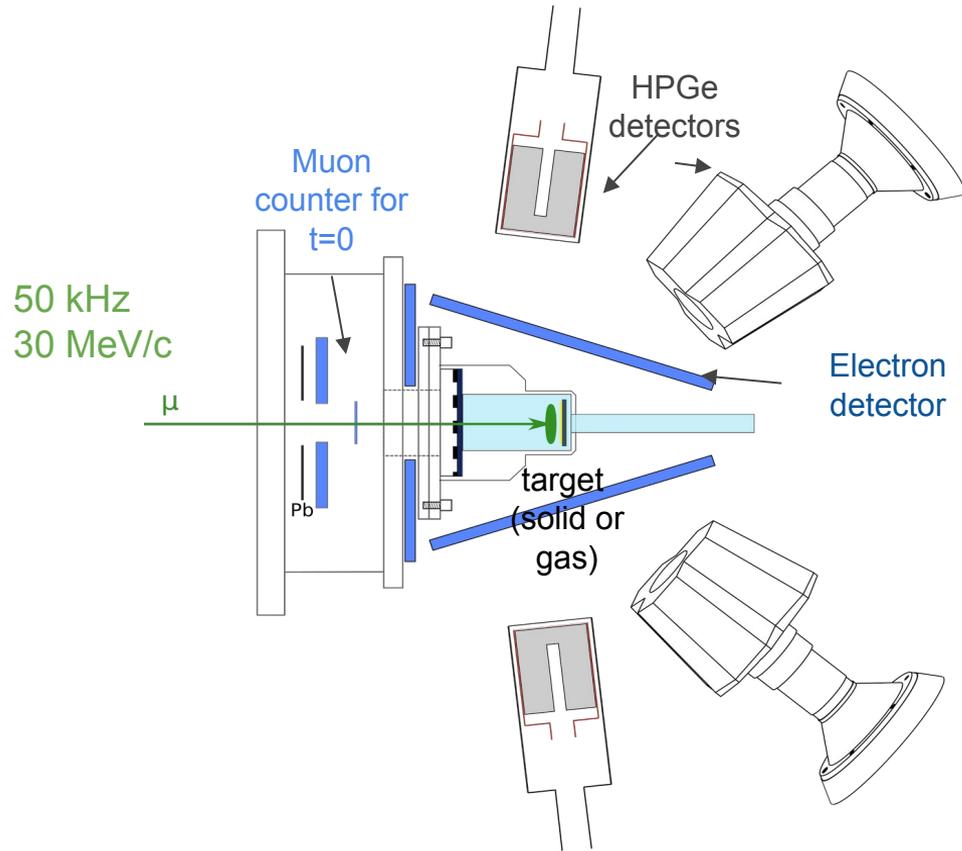
## Atomic parity violation in a single trapped radium ion

O. O. Versolato · L. W. Wansbeck · G. S. Giri · J. E. van den Berg ·  
 D. J. van der Hoek · K. Jungmann · W. L. Kruithof · C. J. G. Onderwater ·  
 B. K. Sahoo · B. Santra · P. D. Shilling · R. G. E. Timmermans ·  
 L. Willmann · H. W. Wilschut

INPUT: need  $\delta R$  to 0.2%

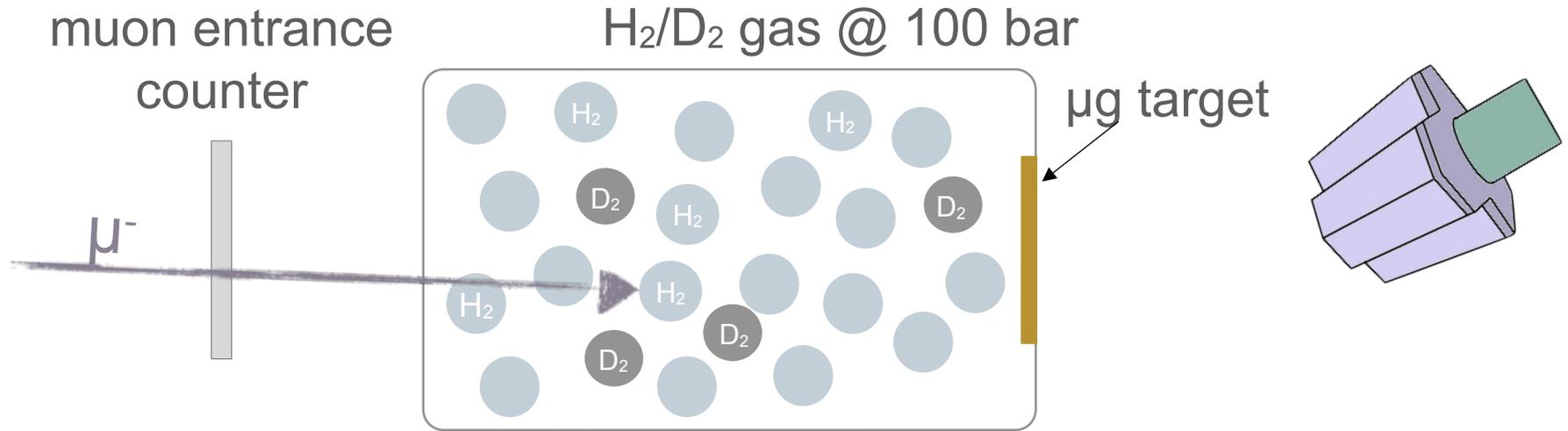
$$E1_{\text{PNC}} = K_r Z^3 Q_w$$

# muX Setup



Miniball detector array

# Muon transfer to microgram targets



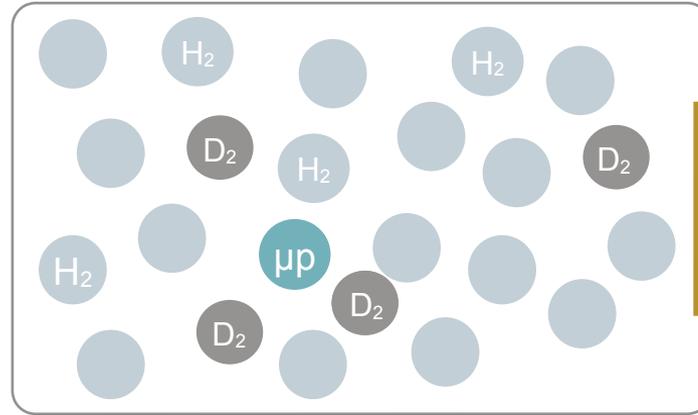
1.  $\mu^-$  stops in 100 bar of H<sub>2</sub> + 0.25% D<sub>2</sub> & forms muonic hydrogen  $\mu p$
2. transfer to deuterium  $\mu p \rightarrow \mu d$
3.  $\mu d$  moves almost freely in the H<sub>2</sub> gas
4. transfer to high-Z element  $\mu d \rightarrow \mu Z$  when hitting target & emission of x rays during the atomic cascade

# Muon transfer to microgram targets

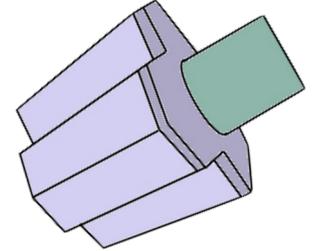
muon entrance  
counter



H<sub>2</sub>/D<sub>2</sub> gas @ 100 bar



μg target



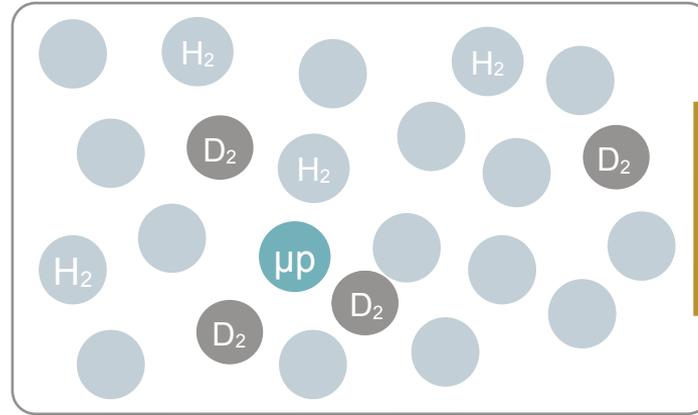
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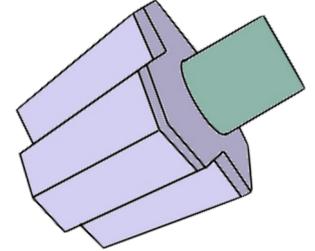
muon entrance  
counter



H<sub>2</sub>/D<sub>2</sub> gas @ 100 bar



μg target



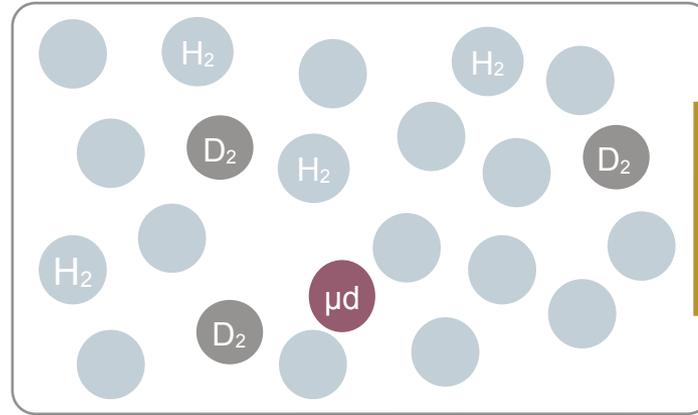
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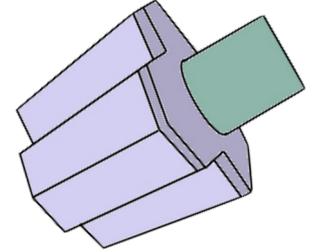
muon entrance  
counter



H<sub>2</sub>/D<sub>2</sub> gas @ 100 bar



μg target

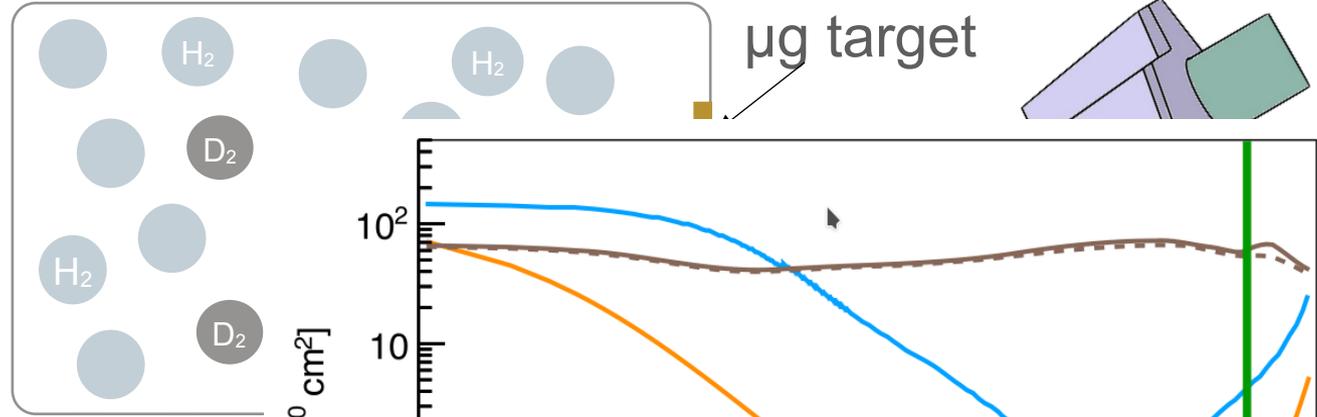


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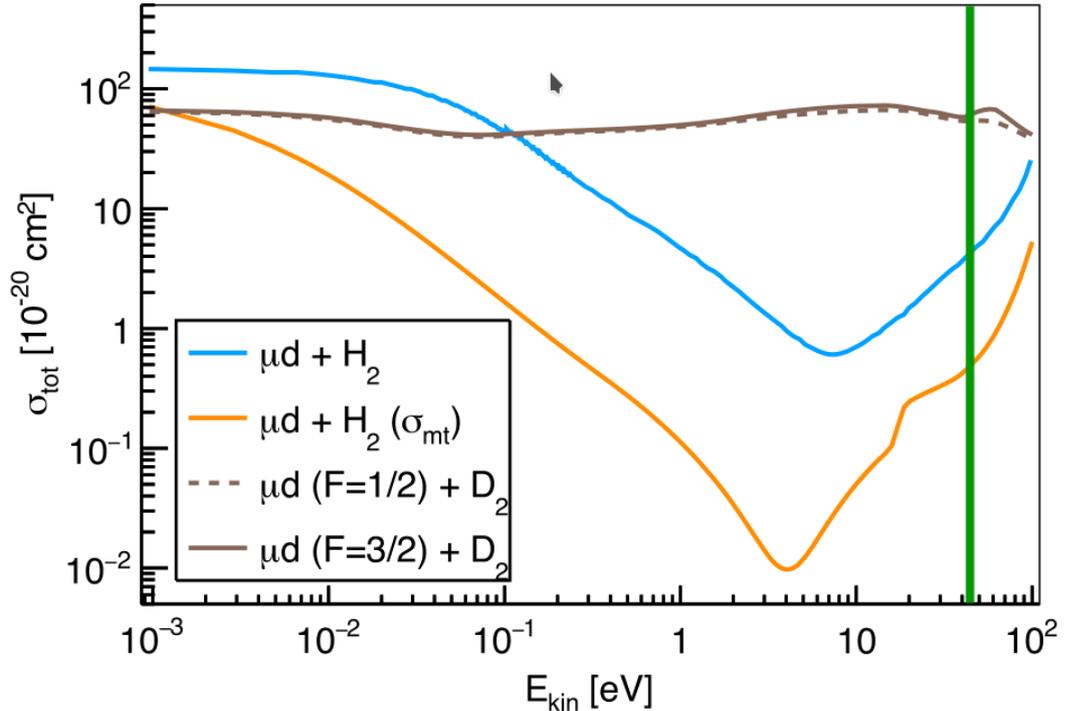
# Muon transfer to microgram targets

muon entrance  
counter

H<sub>2</sub>/D<sub>2</sub> gas @ 100 bar



1.  $\mu^-$  stops in 100 bar of H<sub>2</sub> + 0.25%
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emission of x rays during the atoi

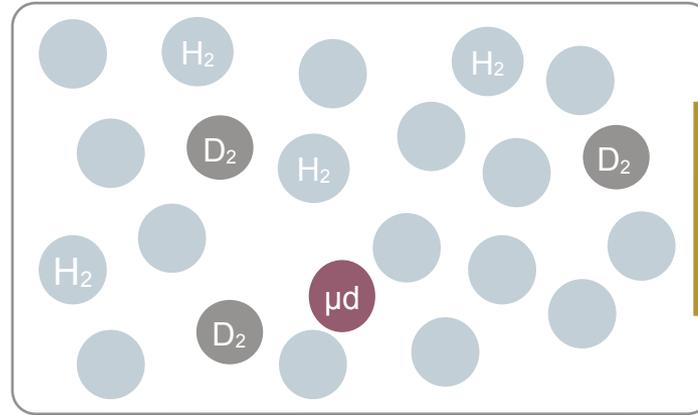


# Muon transfer to microgram targets

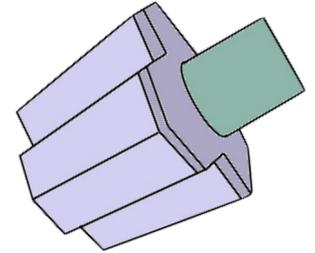
muon entrance  
counter



H<sub>2</sub>/D<sub>2</sub> gas @ 100 bar



μg target



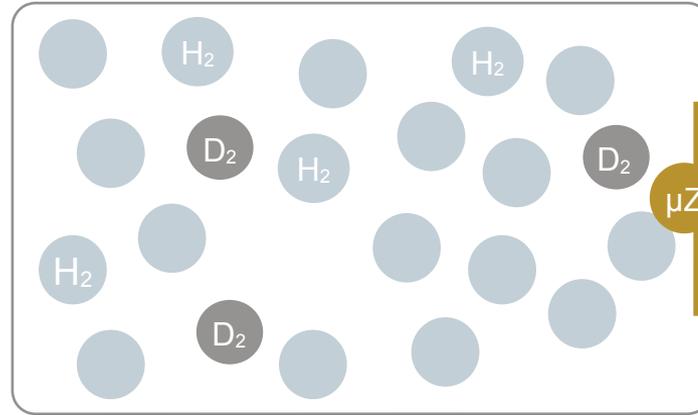
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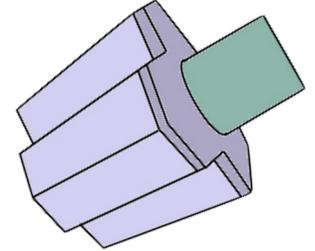
muon entrance  
counter



H<sub>2</sub>/D<sub>2</sub> gas @ 100 bar

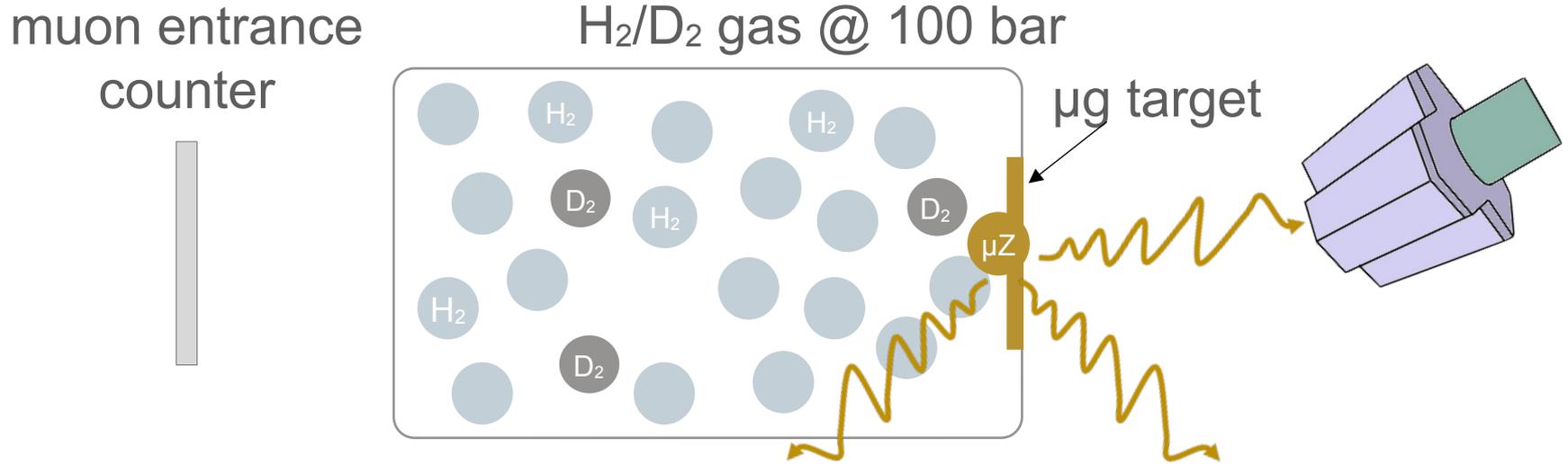


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# Proof-of-principle

So far problems with the uniformity of Ra target.

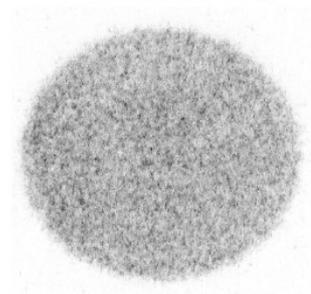
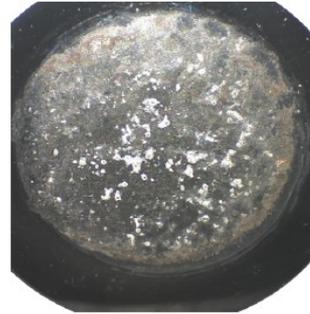
Measured Re and Cm

$$Q(^{185}\text{Re}) = 2.07(5) \text{ b}$$

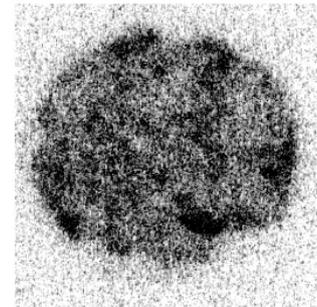
$$Q(^{187}\text{Re}) = 1.94(5) \text{ b}$$

**First physics result:**

PRC 101, 054313 (2020)



15  $\mu\text{g}$  of  $^{248}\text{Cm}$



4  $\mu\text{g}$  of  $^{226}\text{Ra}$

# Nuclear charge radius in $^{185,187}\text{Re}$

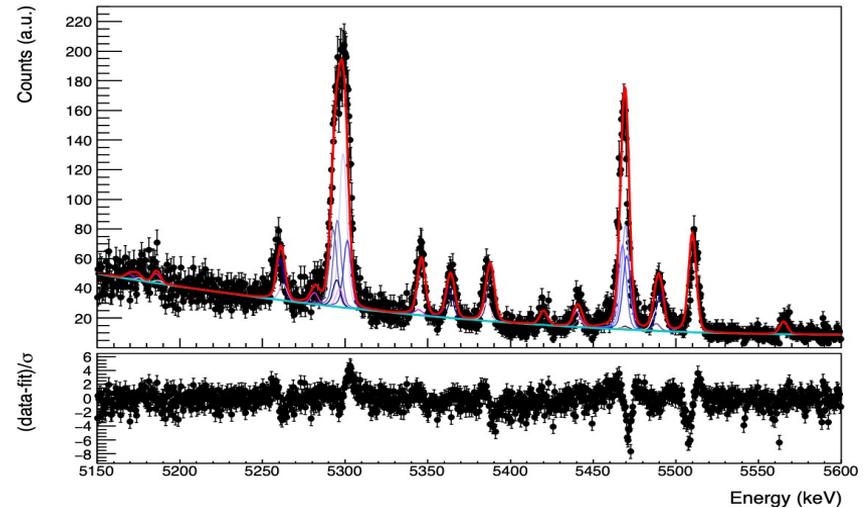
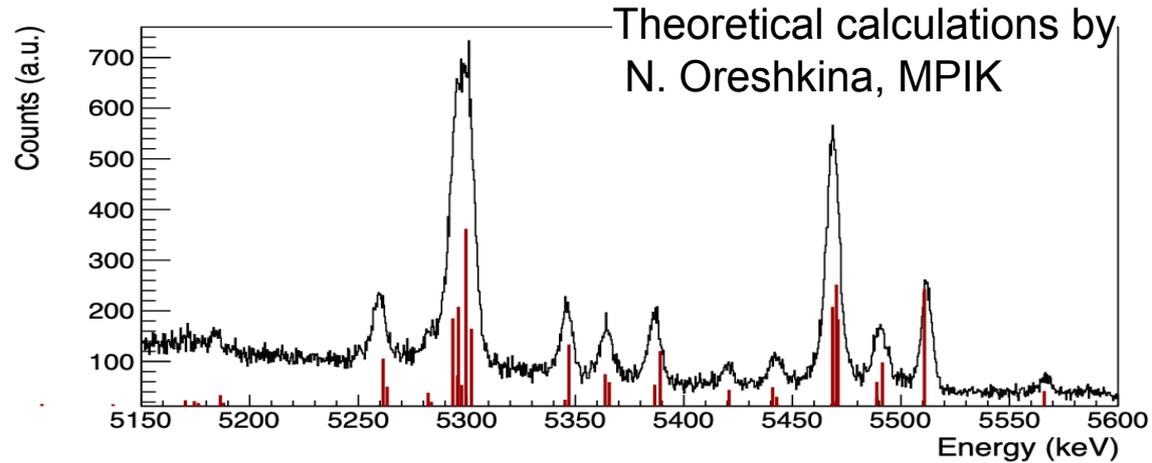
The extraction of the nuclear charge radius from the analysis of the  $2p1s$  hyperfine transitions

Preliminary results:

$$R(^{185}\text{Re}) = 5.297(2)_{\text{stat}}(6)_{\text{sys}} \text{ fm}$$

$$R(^{187}\text{Re}) = 5.288(2)_{\text{stat}}(4)_{\text{sys}} \text{ fm}$$

Not all the systematics are taken into account.



# Conclusions

muonic H, D,  $^3\text{He}$ ,  $^4\text{He}$  by laser done  $\rightarrow$  improved charge radii  
nuclear polarizability

muonic Li, Be, ....  $\rightarrow$  better charge radii by QUARTET X-rays and MMCs

muonic high-Z , also for rare or radioactive nuclei!  $\rightarrow$  muX

$^3\text{T}$  charge radius: missing link  $Z=1 \rightarrow 2$

$\text{T}_2^+$  molecular ions (F. Schmid) or  $\text{T}(1\text{S}-2\text{S})$  in Mainz

$^{6,7}\text{Li}$  isotope shift: new effort in Mainz using cold Li beams + AFR

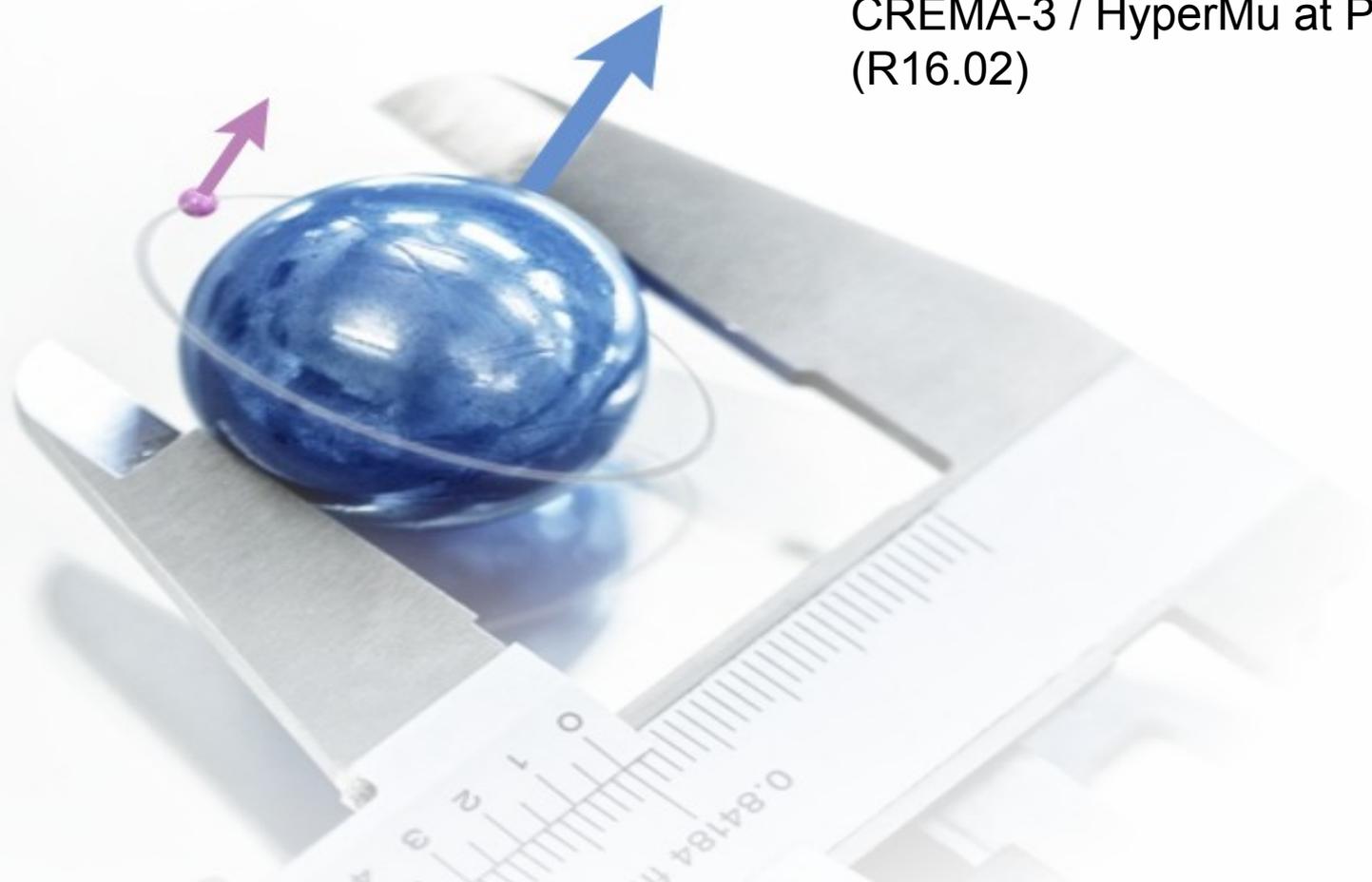
Thanks a lot  
for your attention



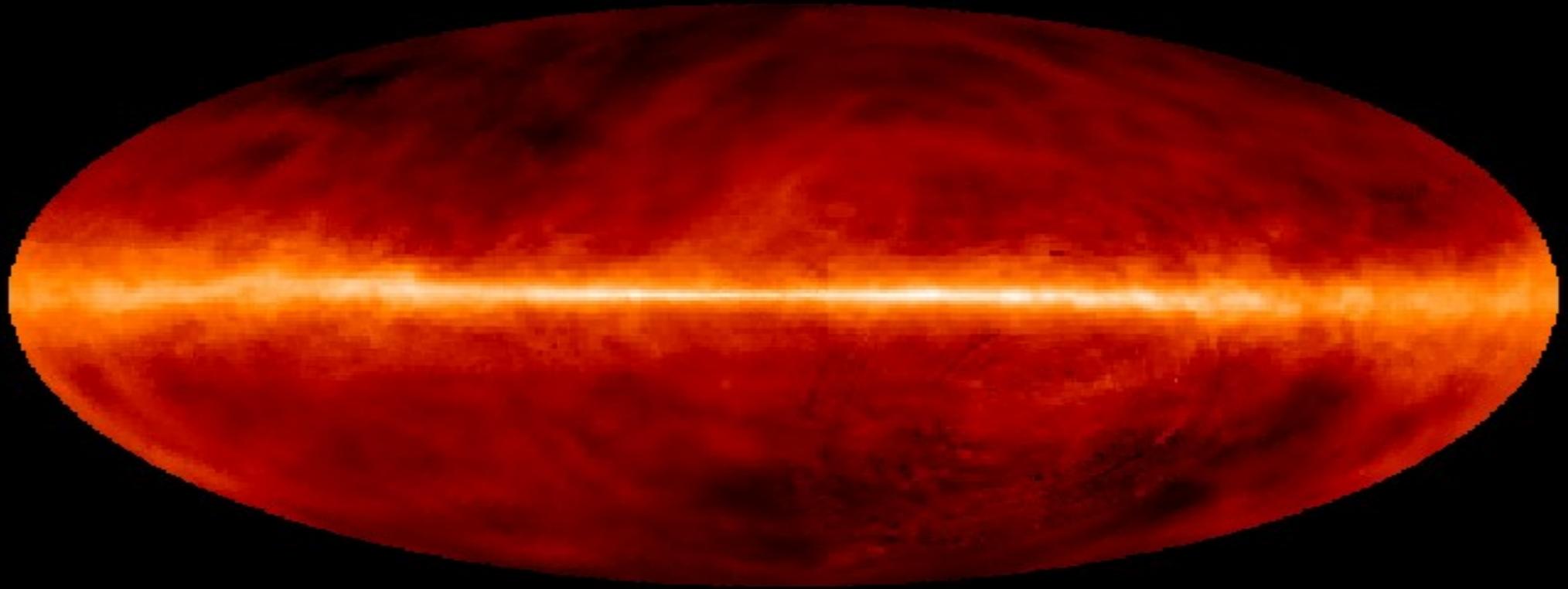
Backup

# Hyperfine structure in muonic H

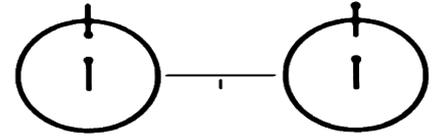
CREMA-3 / HyperMu at PSI  
(R16.02)



# The sky in hydrogen



# Hyperfine structure in H / $\mu\text{p}$



The **21 cm line** in hydrogen (1S hyperfine splitting) has been **measured** to **12 digits** (0.001 Hz) in **1971**:

$$\nu_{\text{exp}} = 1\,420\,405.751\,766\,7 \pm 0.000\,001 \text{ kHz}$$

Essen et al., Nature 229, 110 (1971)

**QED test** is limited to **6 digits** (800 Hz) because of **proton structure** effects:

$$\nu_{\text{theo}} = 1\,420\,403.1 \pm 0.6_{\text{proton size}} \pm 0.4_{\text{polarizability}} \text{ kHz}$$

Eides et al., Springer Tracts 222, 217 (2007)

# Proton Zemach radius

HFS depends on “Zemach” radius:

$$\Delta E = -2(Z\alpha)m\langle r \rangle_{(2)} E_F$$

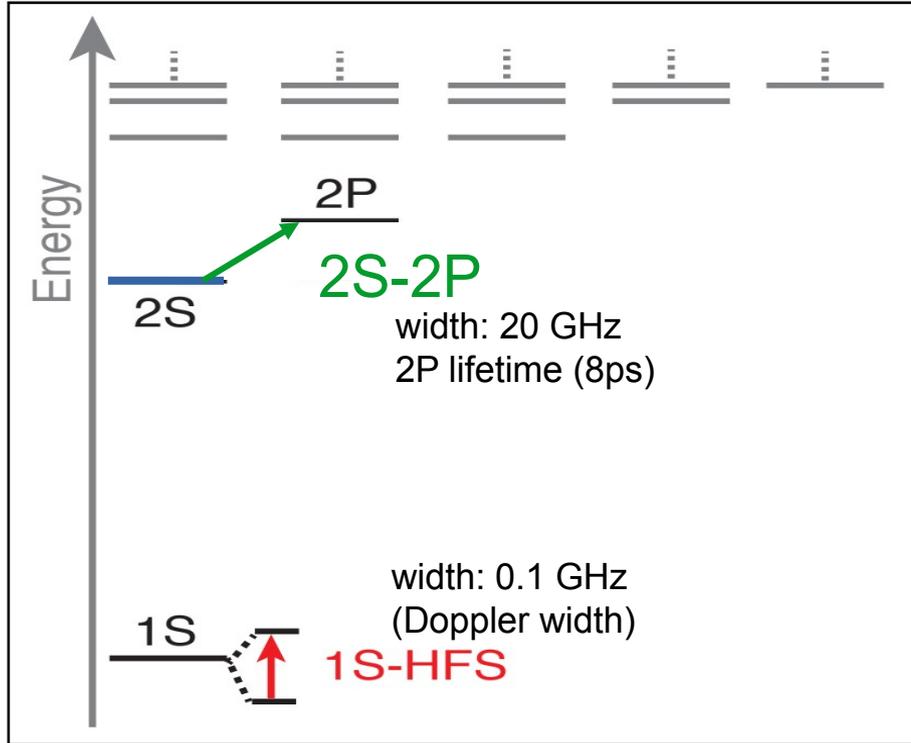
$$\langle r \rangle_{(2)} = \int d^3r d^3r' \rho_E(r) \rho_M(r') |r - r'|$$

Zemach, Phys. Rev. 104, 1771 (1956)

$$\Delta E = \frac{8(Z\alpha)m}{\pi n^3} E_F \int_0^\infty \frac{dk}{k^2} \left[ \frac{G_E(-k^2) G_M(-k^2)}{1+\kappa} \right]$$

Form factors and momentum space

# From charge to magnetic properties



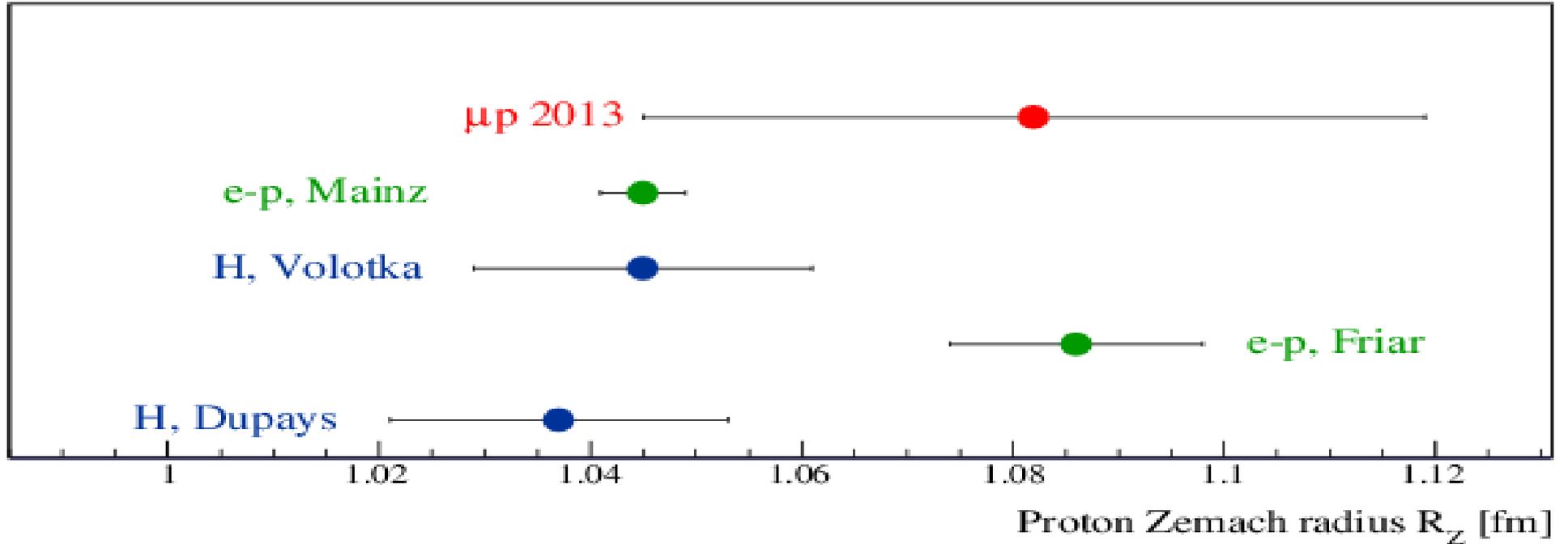
2S-2P = Lamb shift

is sensitive to CHARGE radius

1S-HFS = Hyperfine splitting

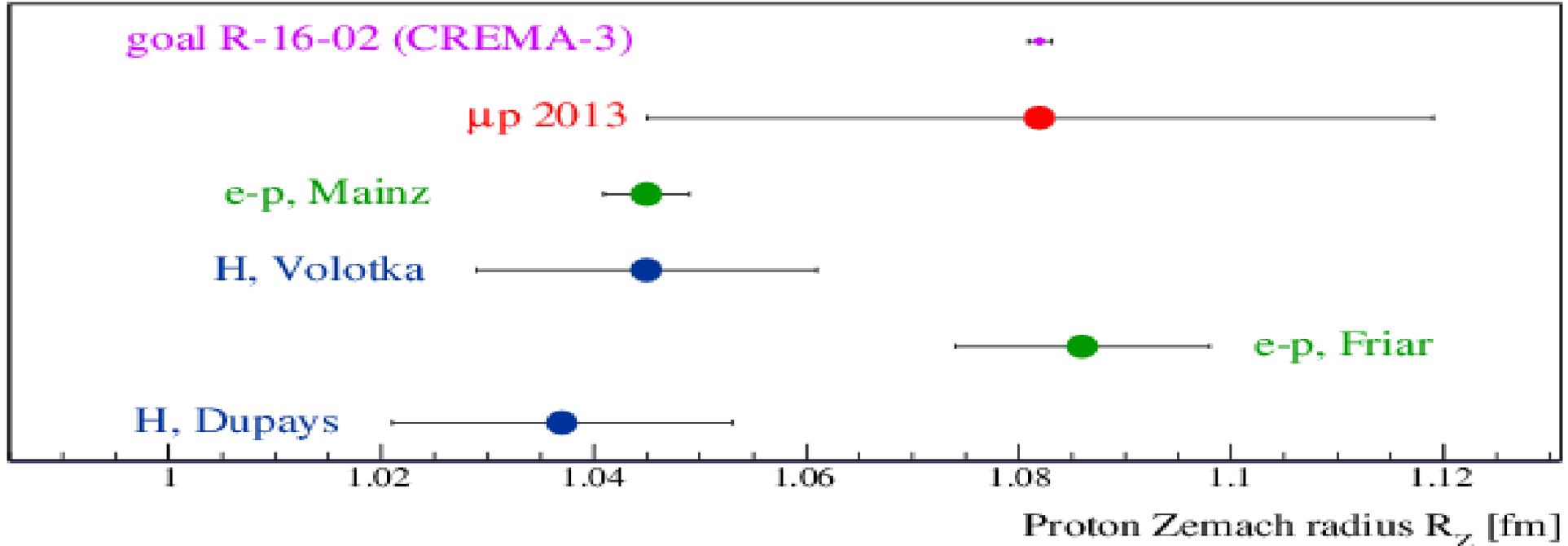
is sensitive to ZEMACH radius

# Proton Zemach radius from $\mu p$



$\mu p$  2013: Antognini et al. (CREMA Coll.), Science 339, 417 (2013)

# Proton Zemach radius from $\mu p$



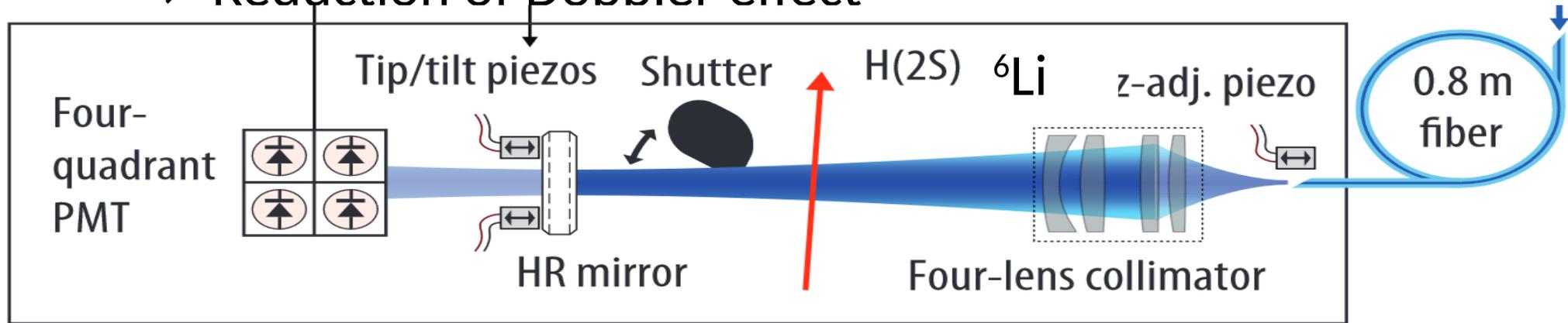
PSI Exp. R-16-02: Antognini, RP et al. (CREMA-3 / HyperMu)

see e.g. Schmidt, RP et al., J. Phys. Conf. Ser 1138, 012010 (2018); arXiv 1808.07240

also: FAMU @ RIKEN/RAL, and a Collaboration at J-PARC

# AFR: Active Fiber-based Retroreflector

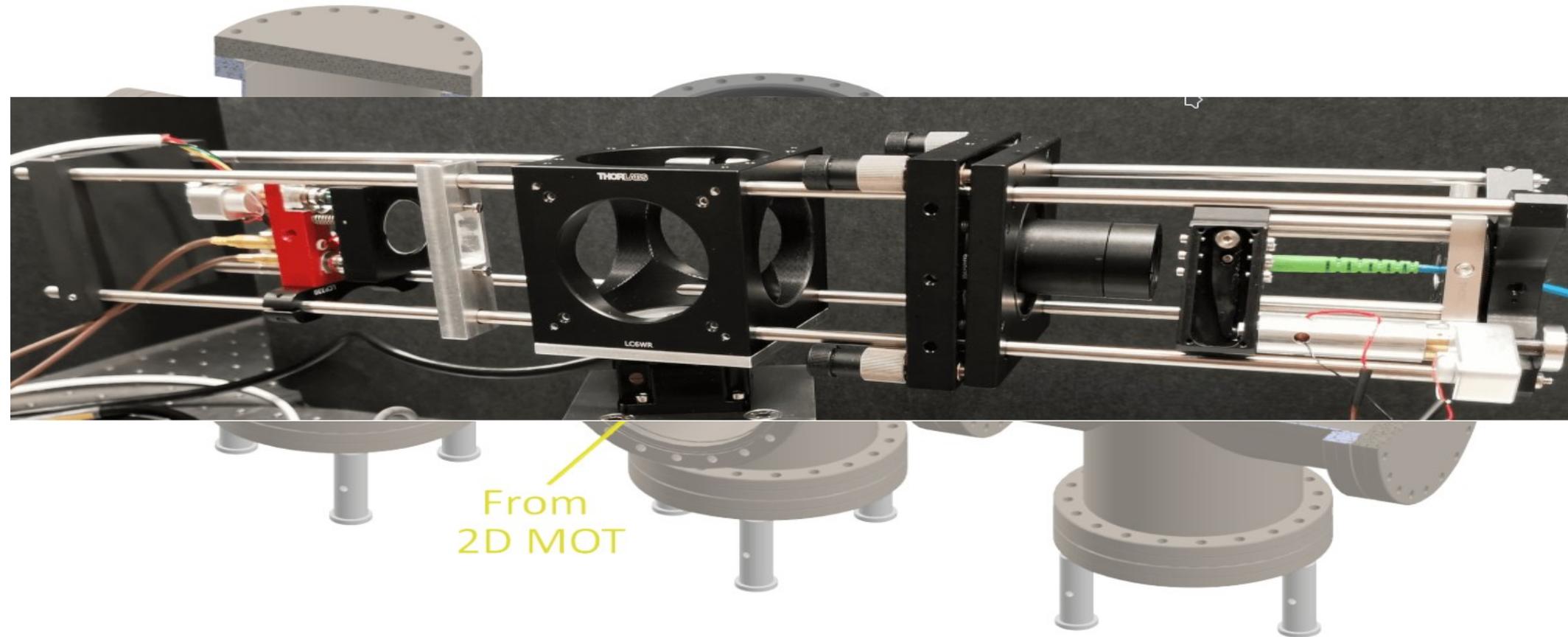
- Use of Active fiber-based Retroreflector:  
→ Reduction of Doppler effect



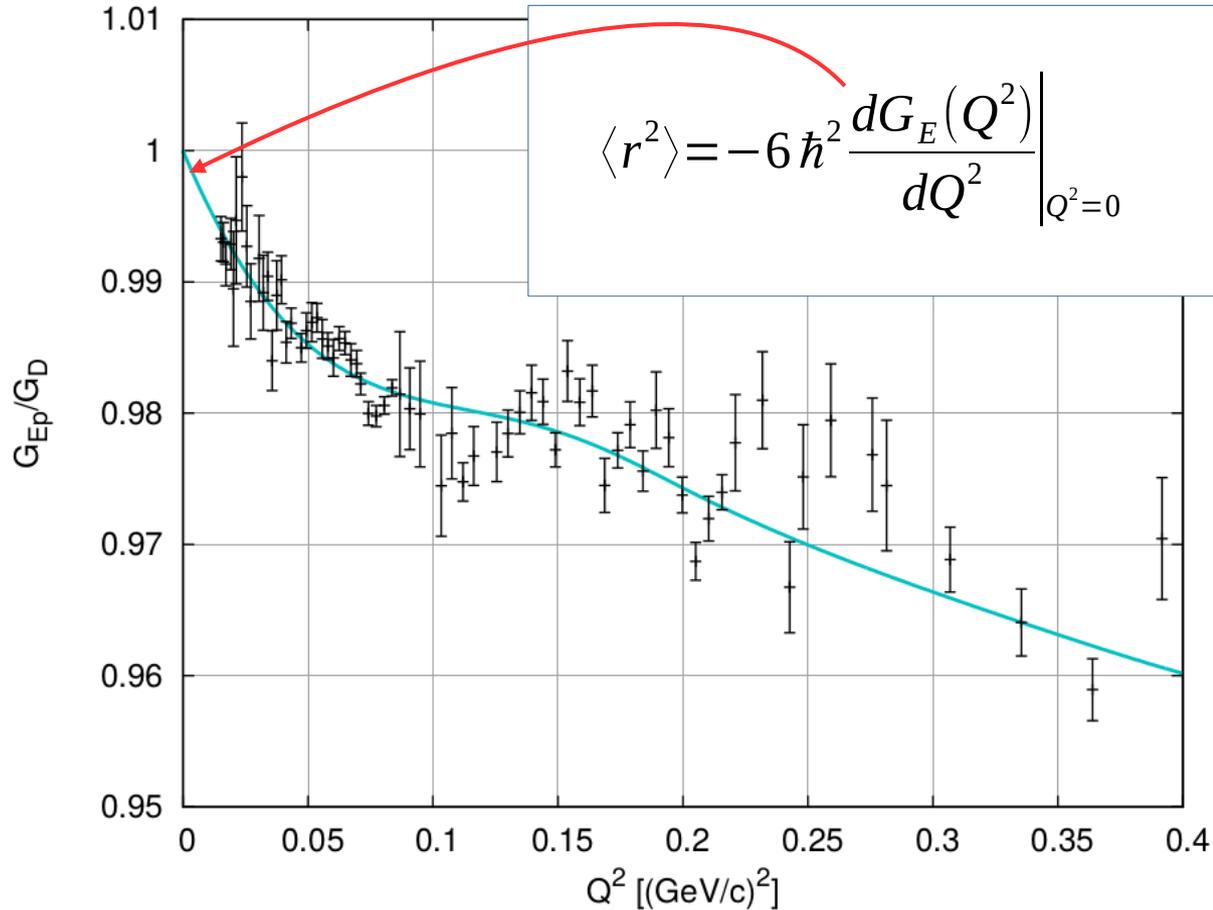
[A. Beyer, "Active fiber-based retroreflector providing phase-retracing anti-parallel laser beams for precision spectroscopy", 2016]

[V. Wirthl, "Improved active fiber-based retroreflector with intensity stabilization and a polarization monitor for the near UV", 2022]

# AFR: Actively stabilized Fiber-based Retro-Reflector



# Electron scattering

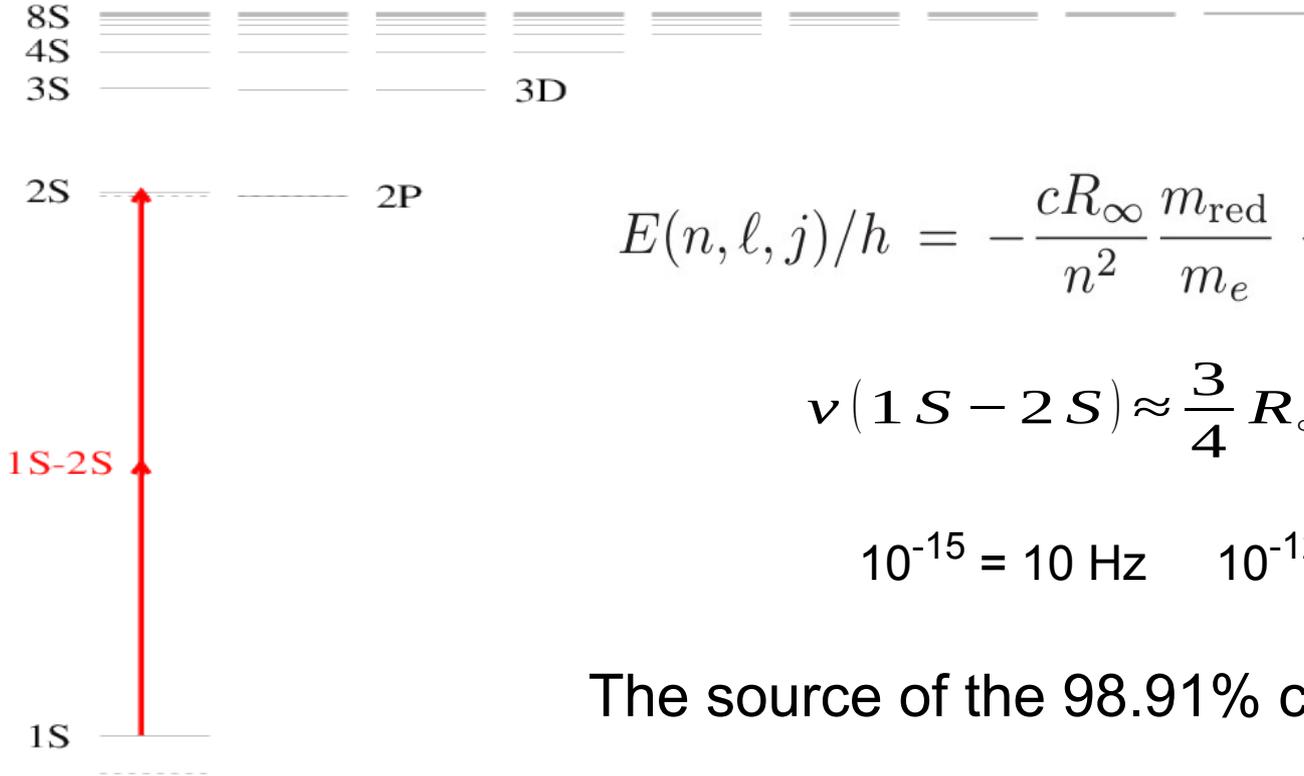


extrapolation to  $Q^2 = 0$  required

Vanderhaeghen, Walcher: 1008.4225

Mainz MAMI data 2010

# Correlation between $R_\infty$ and $R_p / R_d$



$$E(n, \ell, j)/h = -\frac{cR_\infty}{n^2} \frac{m_{\text{red}}}{m_e} + \frac{E_{NS}}{n^3} \delta_{\ell 0} + \Delta(n, \ell, j). \quad (7)$$

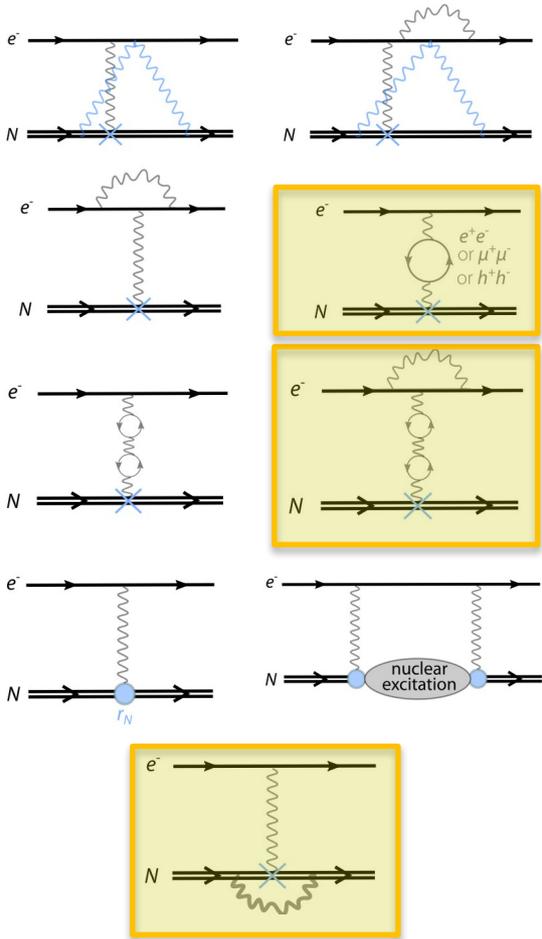
$$\nu(1S - 2S) \approx \frac{3}{4} R_\infty - \frac{7}{8} E_{NS}$$

$$10^{-15} = 10 \text{ Hz} \quad 10^{-12} = 20 \text{ kHz}$$

The source of the 98.91% correlation of  $R_\infty$  and  $R_p$

[Pohl et al., Metrologia 54, L1 (2017)]

# Hydrogen 2S-6P: which contributions are being tested



	Hydrogen $2S_{1/2}-6P_{1/2}$ (Hz)
Dirac (with $m_e \rightarrow m_{\text{red}}$ )	730 691 021 696 054
Rel. nuclear recoil	1 129 173
Radiative recoil	1540
1-loop QED	
self-energy	-1 071 679 859
vacuum-polarization	26 853 088
$\mu^+ \mu^-$ vacuum-pol.	634
hadronic vacuum-pol.	425
2-loop QED	-90 477
3-loop QED	-236
Finite nuclear size	
$\propto \alpha^4$	-138 394
$\propto \alpha^5$	5
$\propto \alpha^6$	-74
Nuclear polarizability	
$\propto \alpha^5$	8
$\propto \alpha^6$	-49
Nuclear self-energy	-584
Total	730 689 977 771 255
Theory uncertainty	199

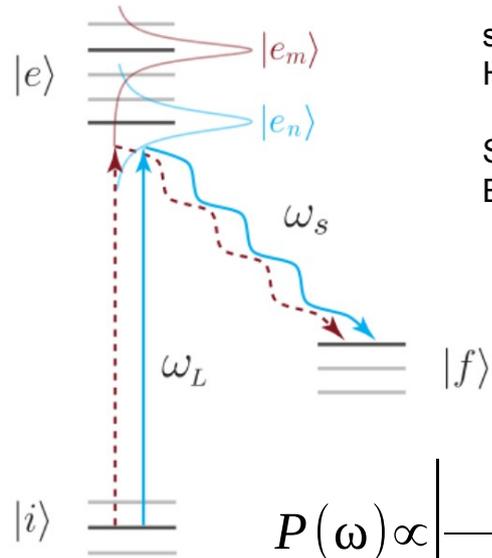
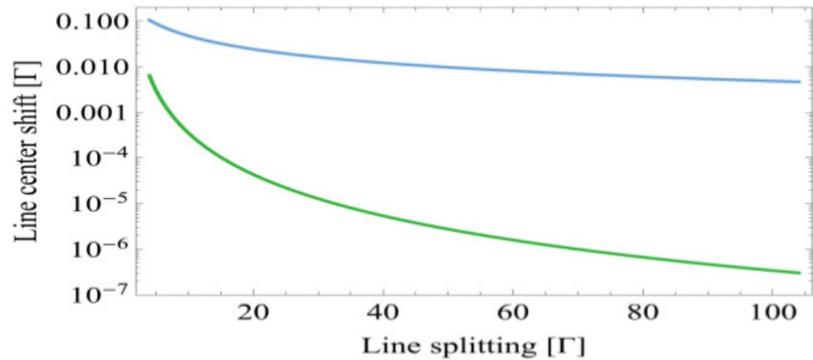
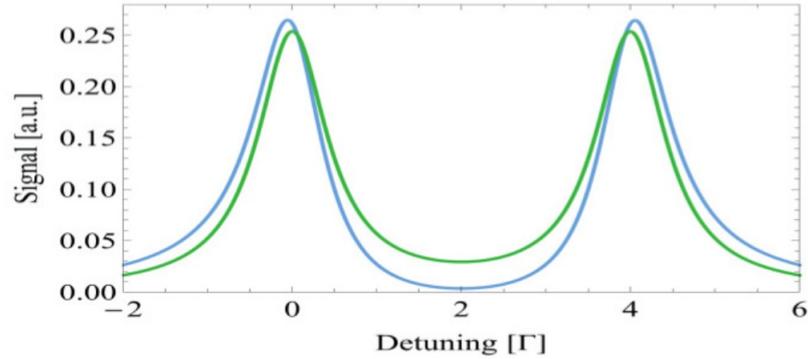
Our 2S-6P meas. uncert.:  
**490 Hz**

Start seeing muons and hadrons in vacuum

Start seeing 3-loop bound-state vacuum effects

Start seeing Nuclear self-energy

# Quantum Interference in 2S-nP



see

Horbatsch, Hessels, PRA 82, 052519 (2010);

PRA 84, 032508 ('11); PRA 86 040501 ('12)

Sansonetti et al., PRL 107, 021001 (2011)

Brown et al., PRA 87, 032504 (2013)

$$P(\omega) \propto \left| \frac{(\vec{d}_1 \vec{E}_0) \vec{d}_1}{\omega_1 - \omega_L + i\gamma_1/2} + \frac{(\vec{d}_2 \vec{E}_0) \vec{d}_2 e^{i\Delta\Phi}}{\omega_2 - \omega_L + i\gamma_2/2} \right|^2$$

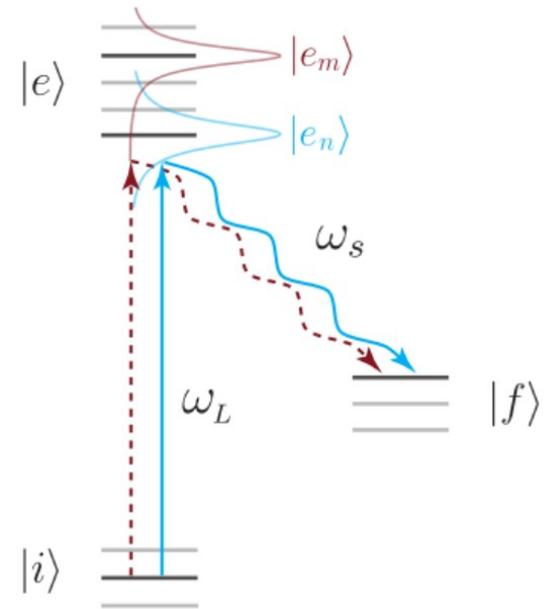
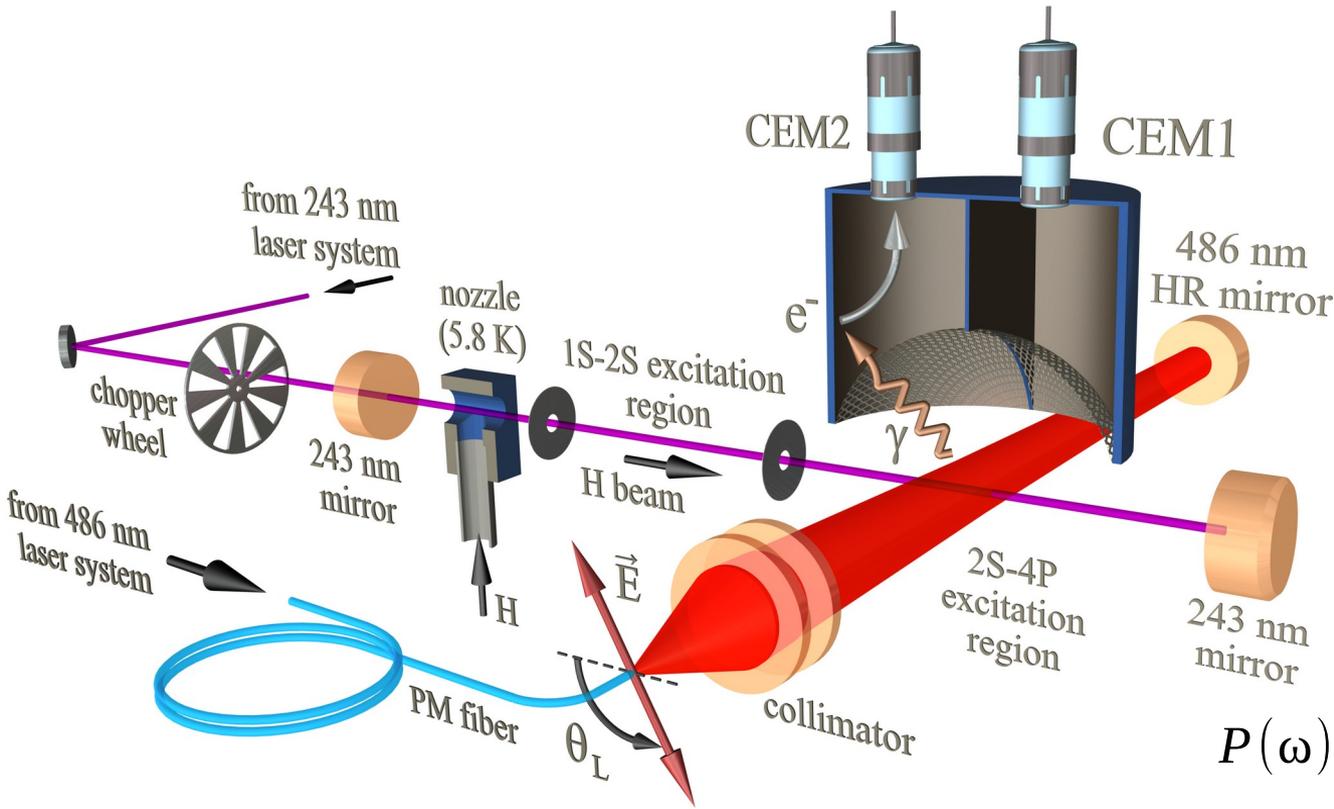
= Lorentzian(1) + Lorentzian(2)

+ cross-term (QI)

Fitting this with 2 Lorentzians creates

**line shifts**

# Studying Quantum Interference in 2S-4P



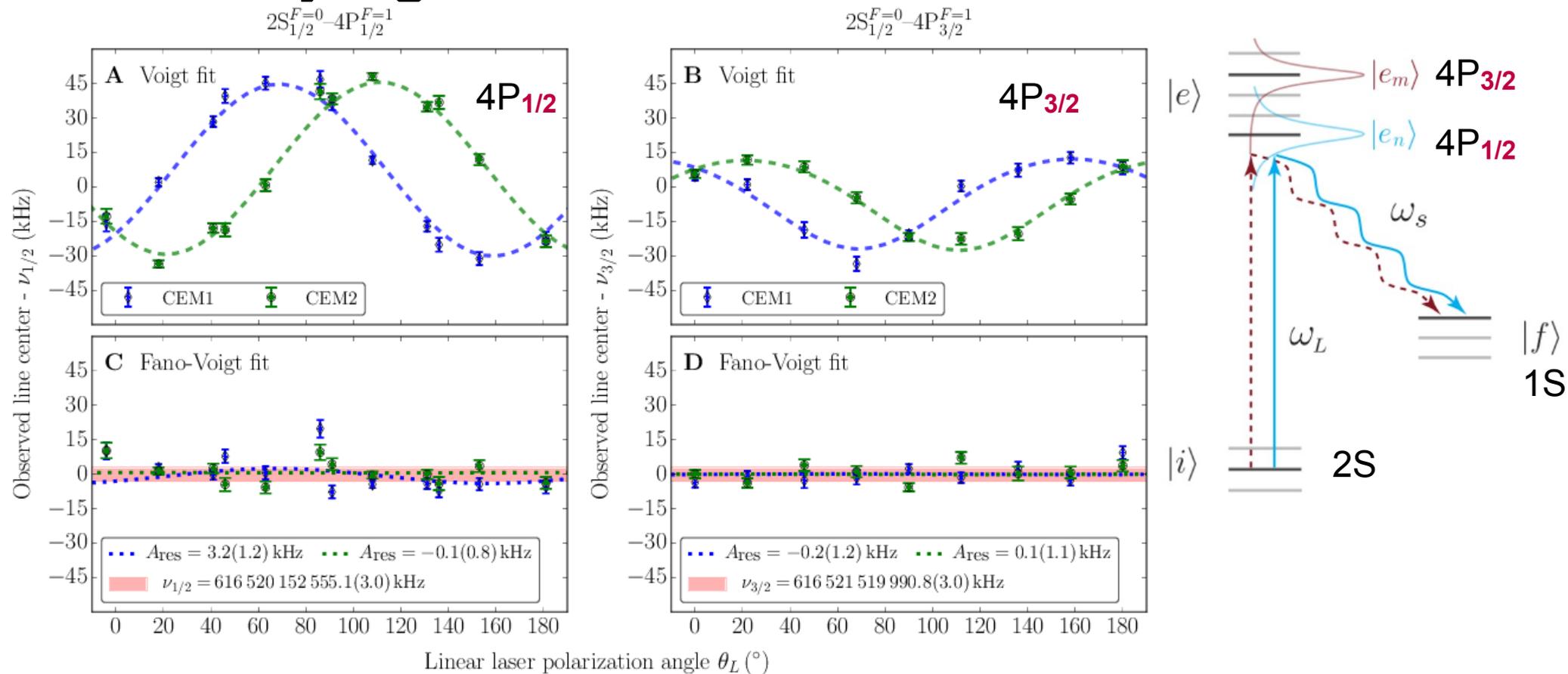
$$P(\omega) \propto \left| \frac{(\vec{d}_1 \vec{E}_0) \vec{d}_1}{\omega_1 - \omega_L + i\gamma_1/2} + \frac{(\vec{d}_2 \vec{E}_0) \vec{d}_2 e^{i\Delta\Phi}}{\omega_2 - \omega_L + i\gamma_2/2} \right|^2$$

= Lorentzian(1) + Lorentzian(2)

+ cross-term (QI)

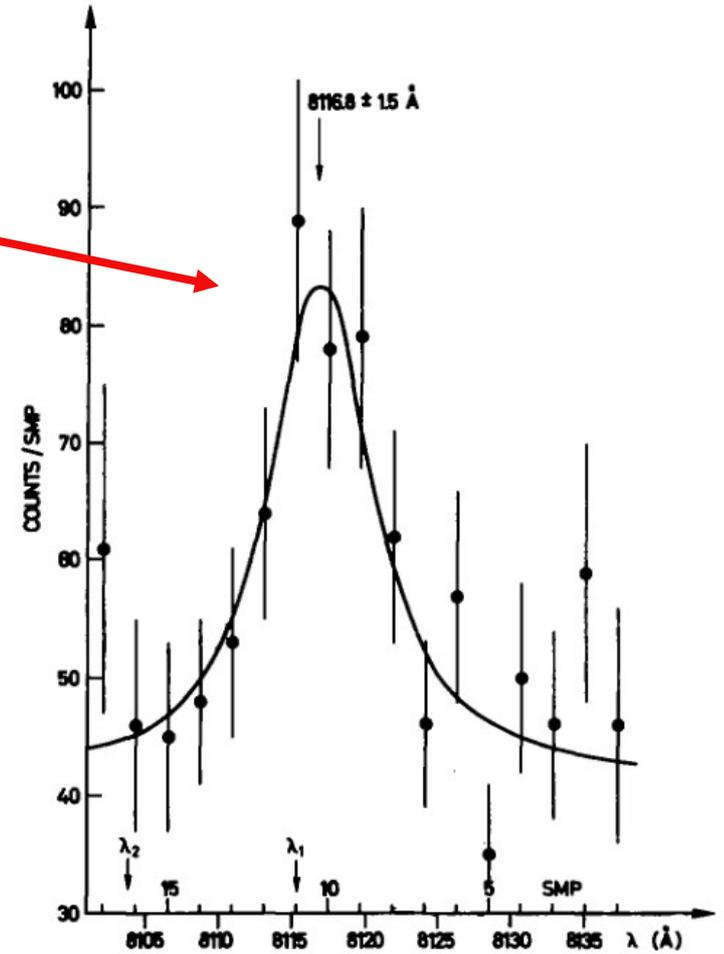
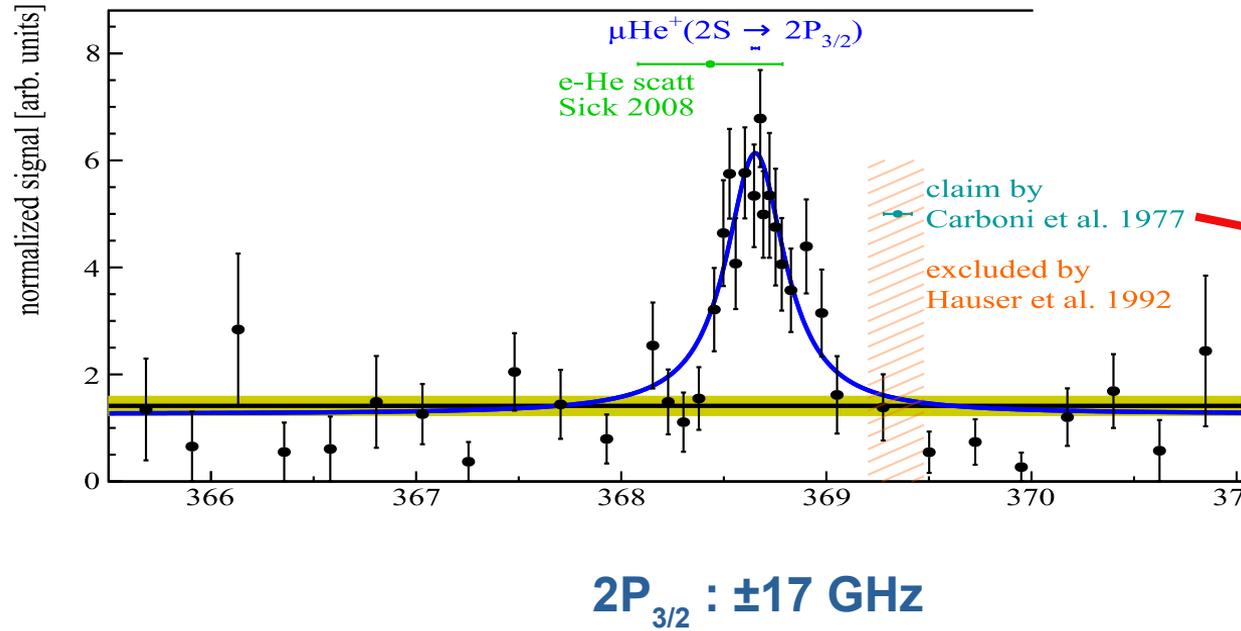
Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

# Studying Quantum Interference in 2S-4P

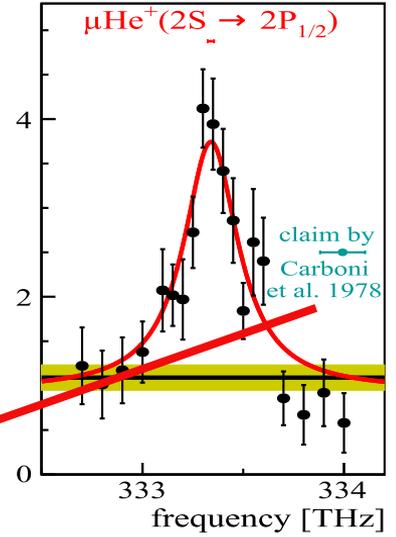
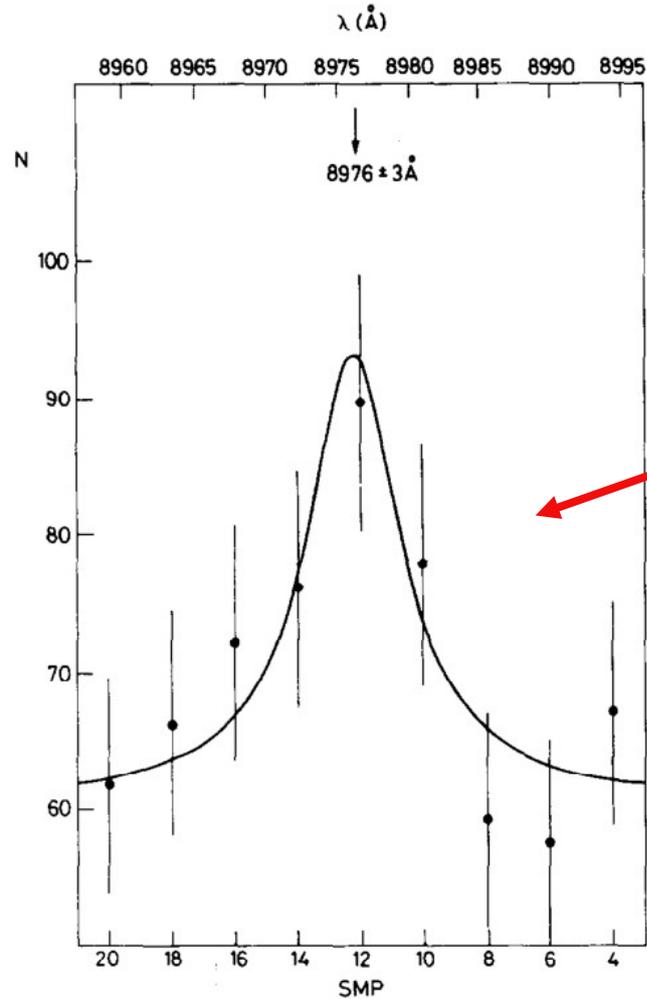
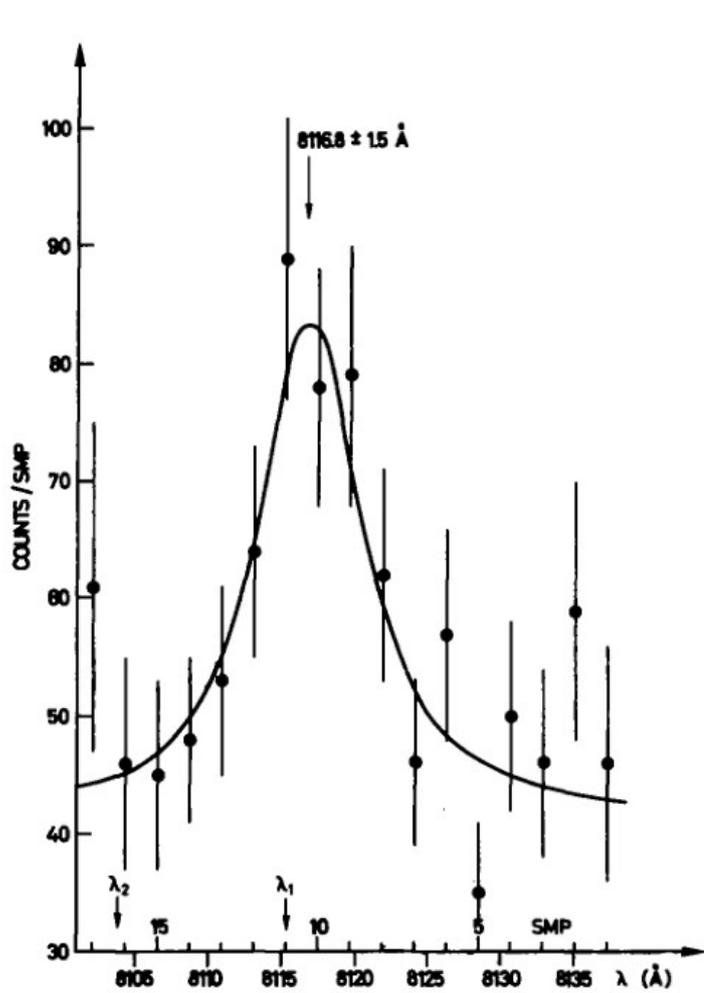


Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

# muonic $^4\text{He}$ ions



# muonic $^4\text{He}$ ions

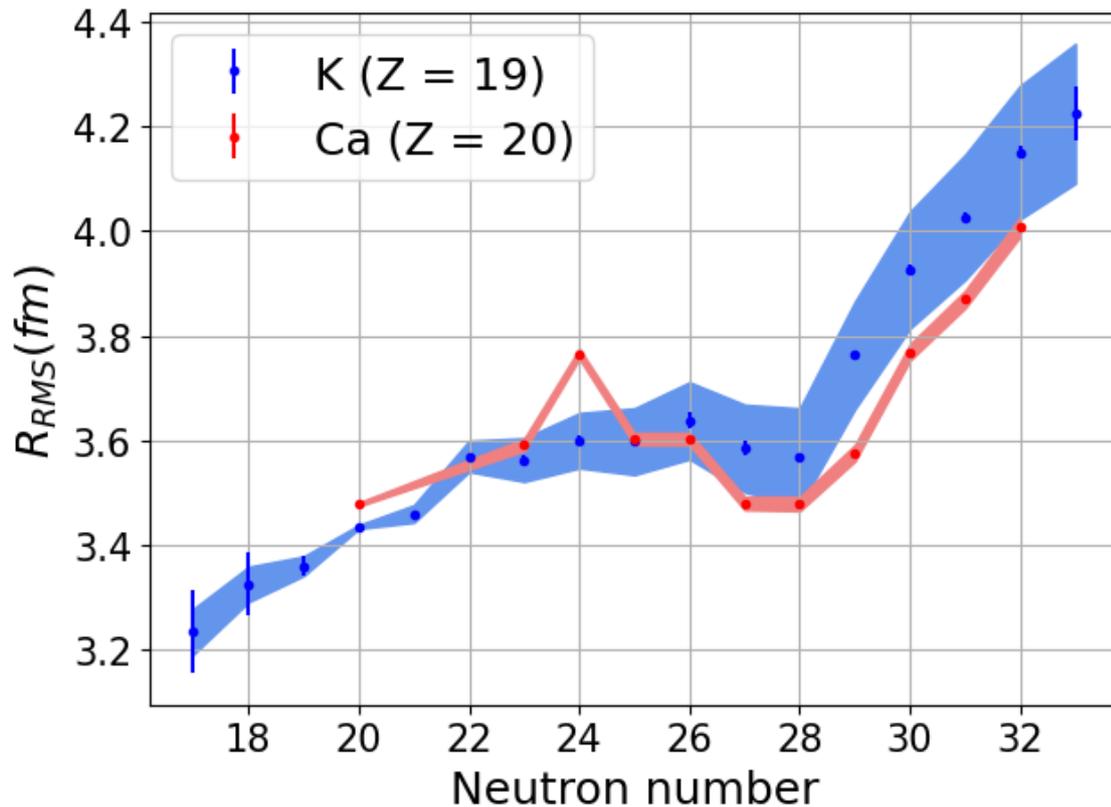


**$2P_{1/2} : \pm 15 \text{ GHz}$**

# ReferenceRadii

muX with lighter nuclei

absolute radii and Kir



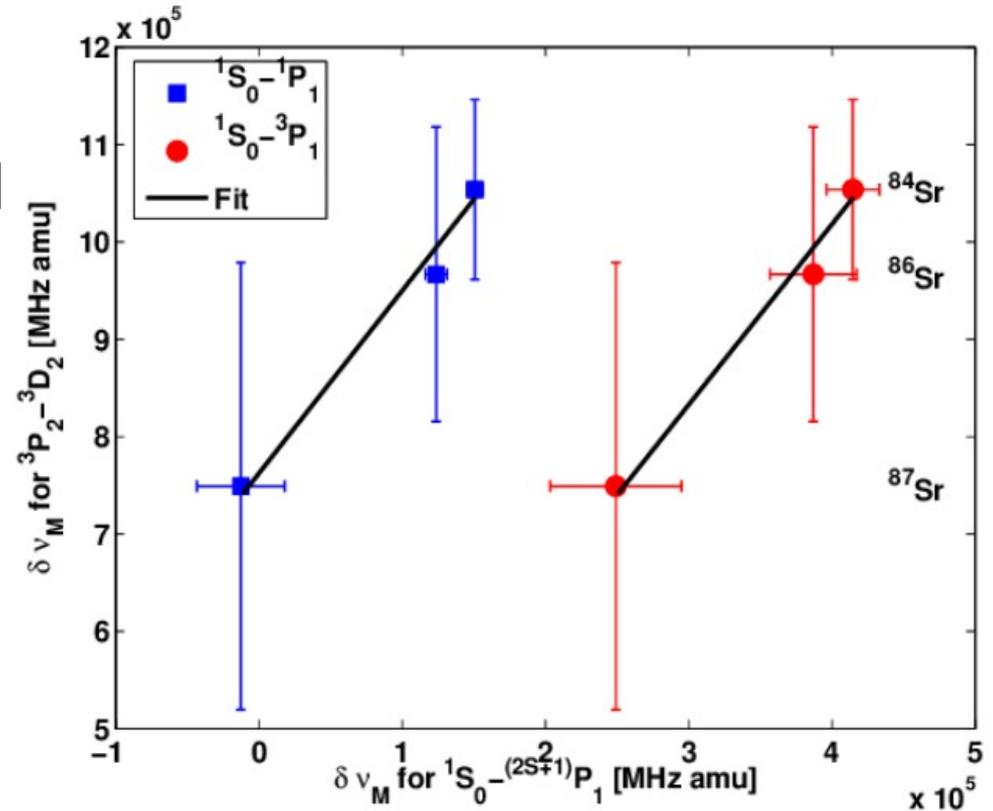
# Reference Radii

muX with lighter nuclei

$$\delta \langle r^2 \rangle^{A,A'} = \frac{1}{F_i} \left( \delta v_{A,A'} - \frac{A - A'}{A A'} M_i \right) \quad \text{g p l}$$

$M_i$  Mass shift

$F_i$  Field shift



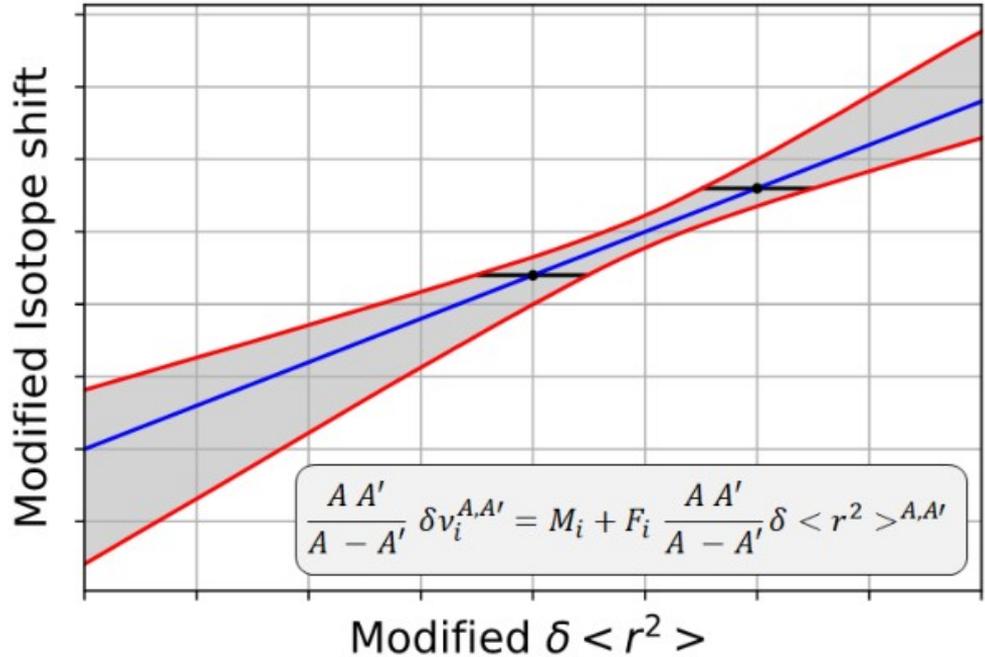
different for each element and transition

# ReferenceRadii

## Modified King plot

$$\frac{A-A'}{AA'} \delta v_i^{A,A'} = M_i + F_i \frac{A-A'}{AA'} \delta \langle r^2 \rangle^{A,A'}$$

- Mass shift: intercept
- Field shift: slope
- Absolute charge radii
  - One  $\rightarrow$  Absolute values
  - Two  $\rightarrow \frac{M_i}{F_i}$
  - Three  $\rightarrow M_i$  and  $F_i$



muonic  $^{16}\text{O}$ ,  $^{17}\text{O}$ ,  $^{18}\text{O}$

