Systematic effects in the measurement of the Cesium recoil frequency

Jack Roth, UC Berkeley

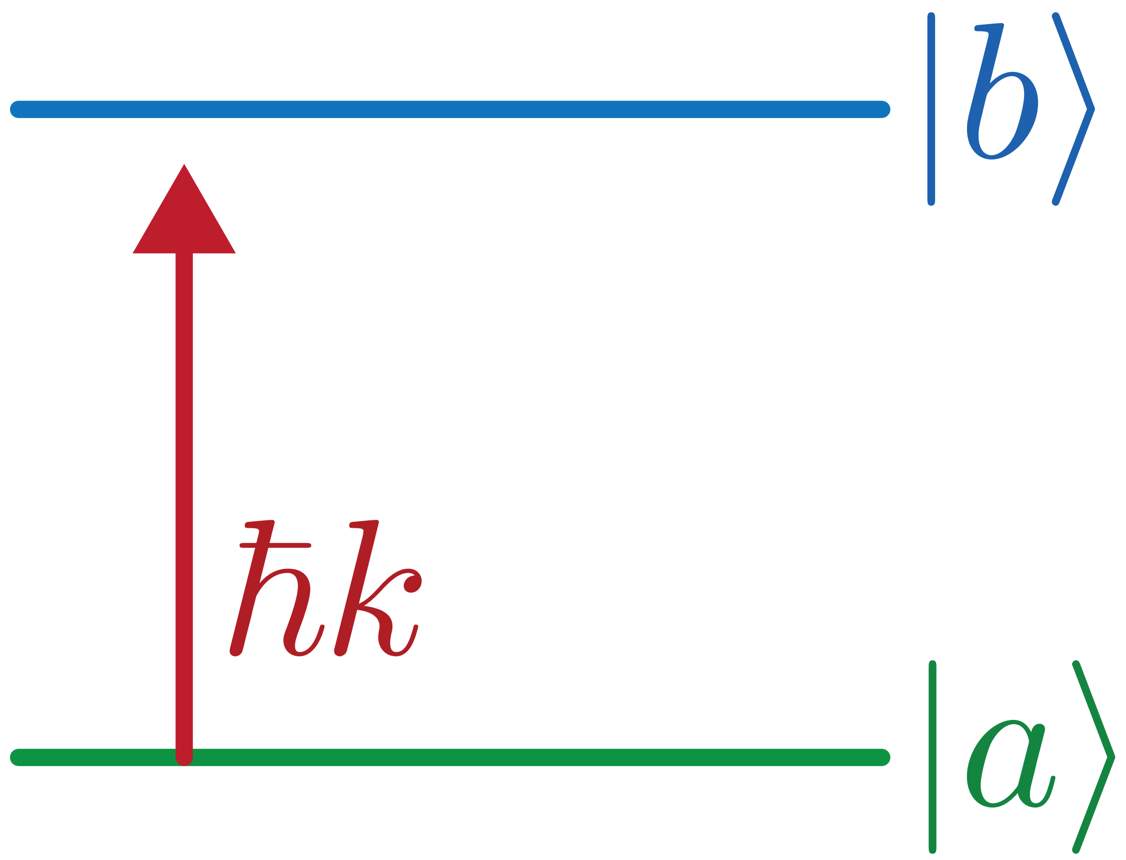
Precision Determinations of the Fine-structure Constant
Mainz Institute for Theoretical Physics, 28 October 2025

Berkeley
UNIVERSITY OF CALIFORNIA

Outline

- Review of the measurement technique
- Review of systematic effects
- Approach to understanding difficult systematics
- Outlook

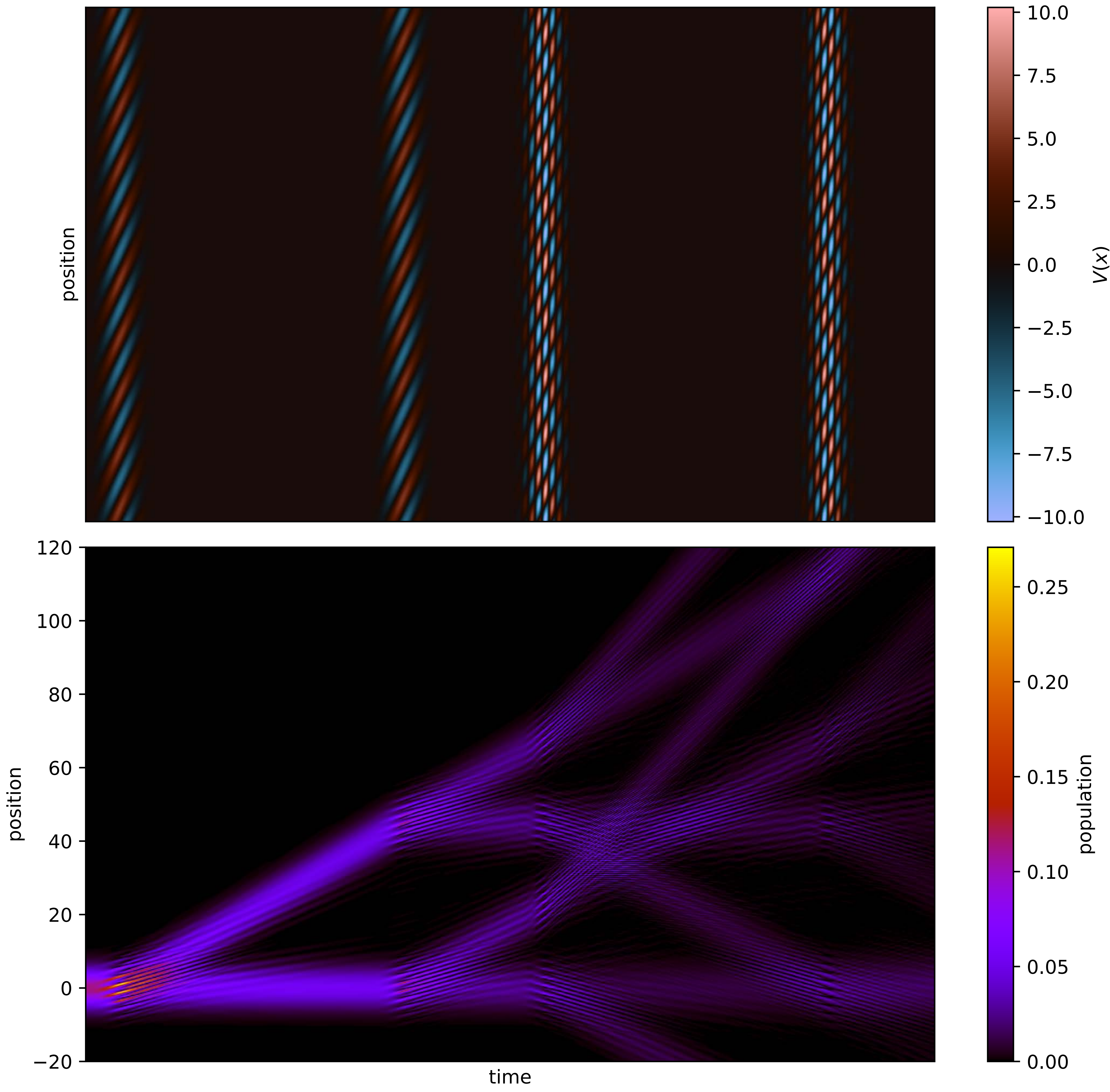
Bragg beamsplitters



Use AC Stark effect to create potential from laser

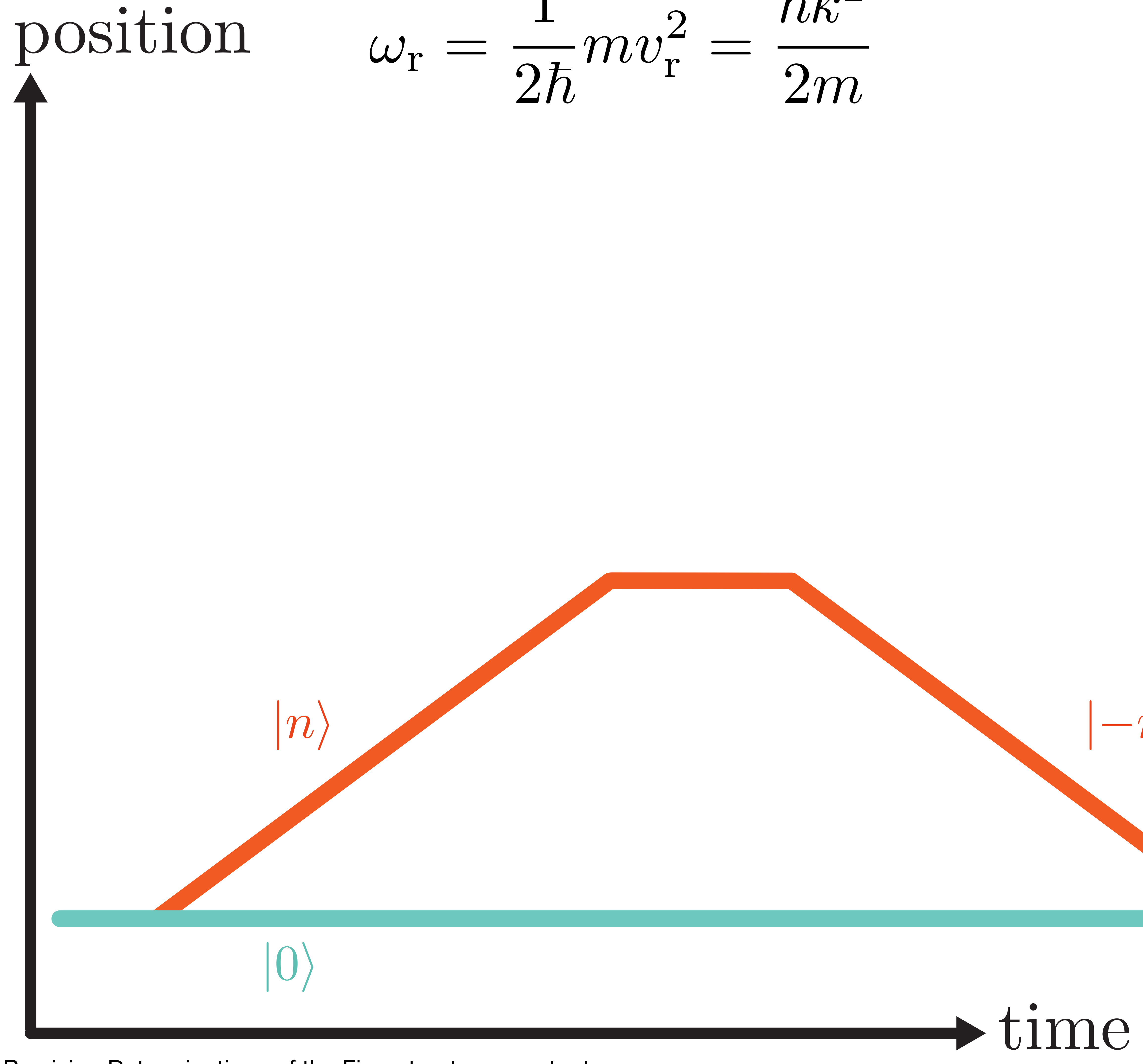
$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \left[\frac{\hat{p}^2}{2m} + V(x,t)\right]\Psi(x,t)$$

$$V(x,t) = 2\hbar\Omega_{\text{eff}}(t)\cos^2(kx - \delta t/2)$$



Simultaneous conjugate interferometer geometry

$$\omega_{\mathrm{r}} = \frac{1}{2\hbar} m v_{\mathrm{r}}^2 = \frac{\hbar k^2}{2m}$$



$$\phi_{\mathrm{free}} = \frac{1}{\hbar} \int_0^T \mathcal{L}\, \mathrm{d}t$$

$$\phi_{\mathrm{laser}} = n(k_1 - k_2)z - n(\omega_1 - \omega_2)t$$

$$\Phi_{\mathrm{RBI}} = 8n^2\omega_{\mathrm{r}}T - n\omega_{\mathrm{m}}T + \phi_{\mathrm{gravity}}$$

Simultaneous conjugate interferometer geometry

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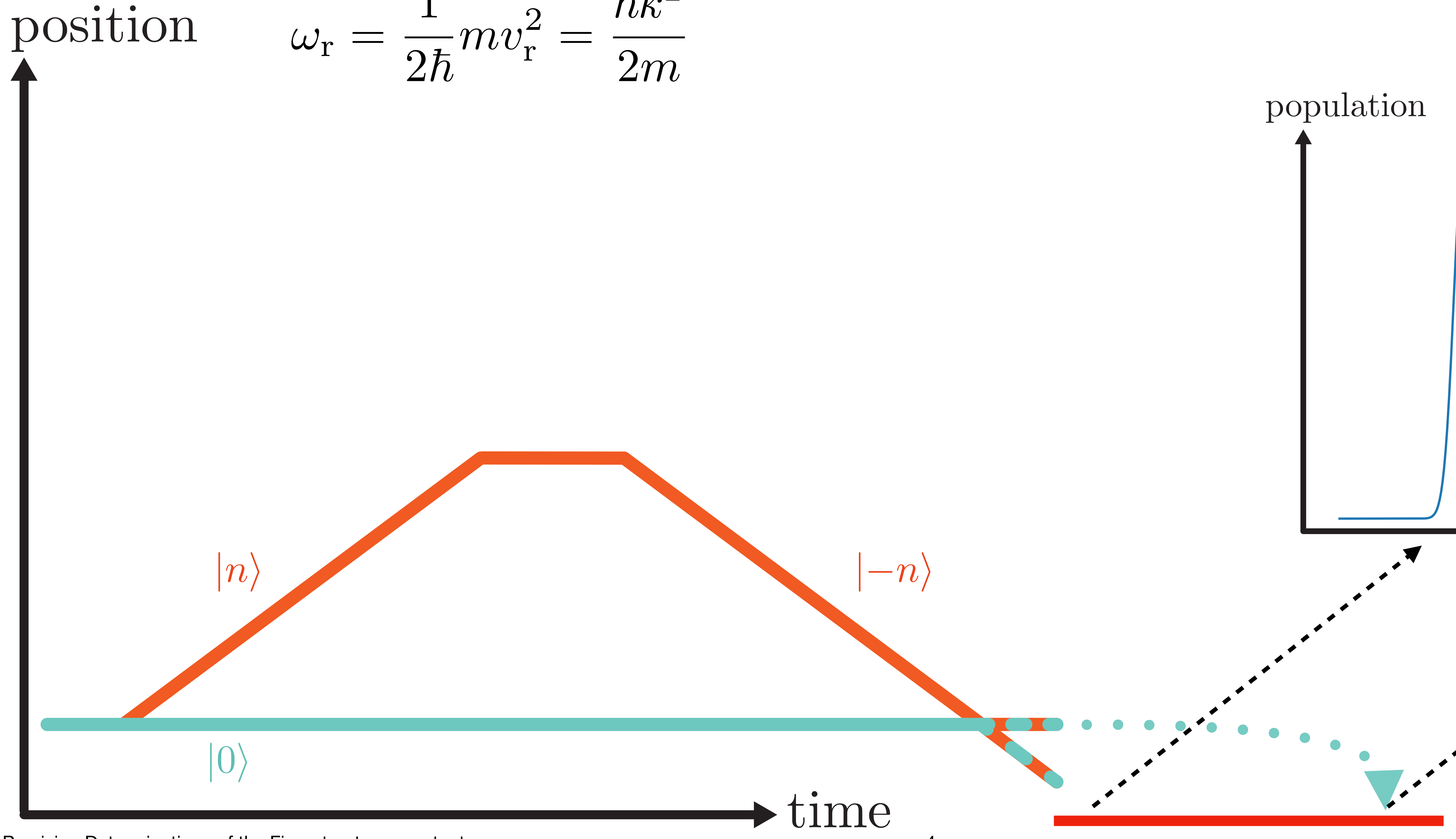

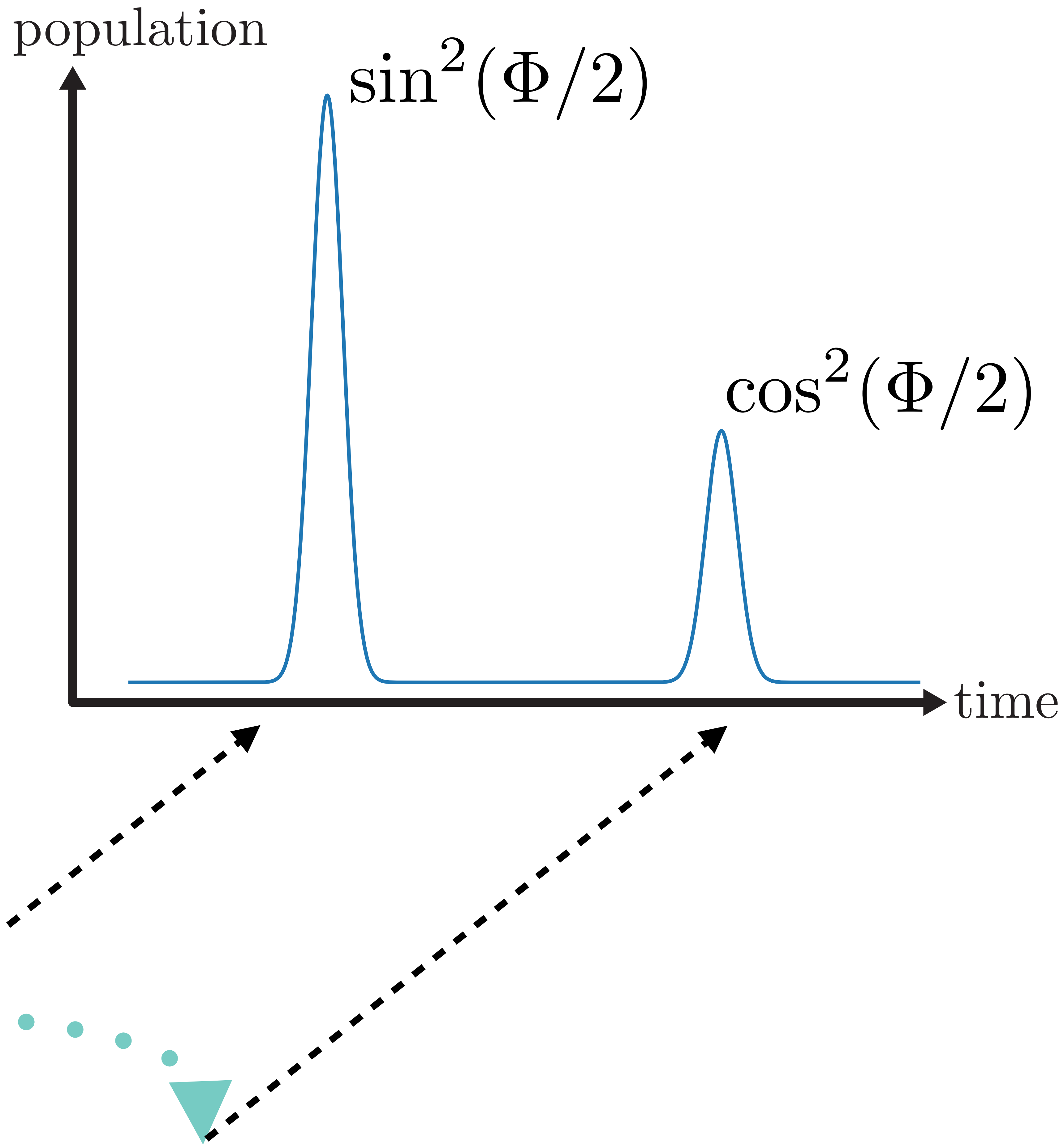

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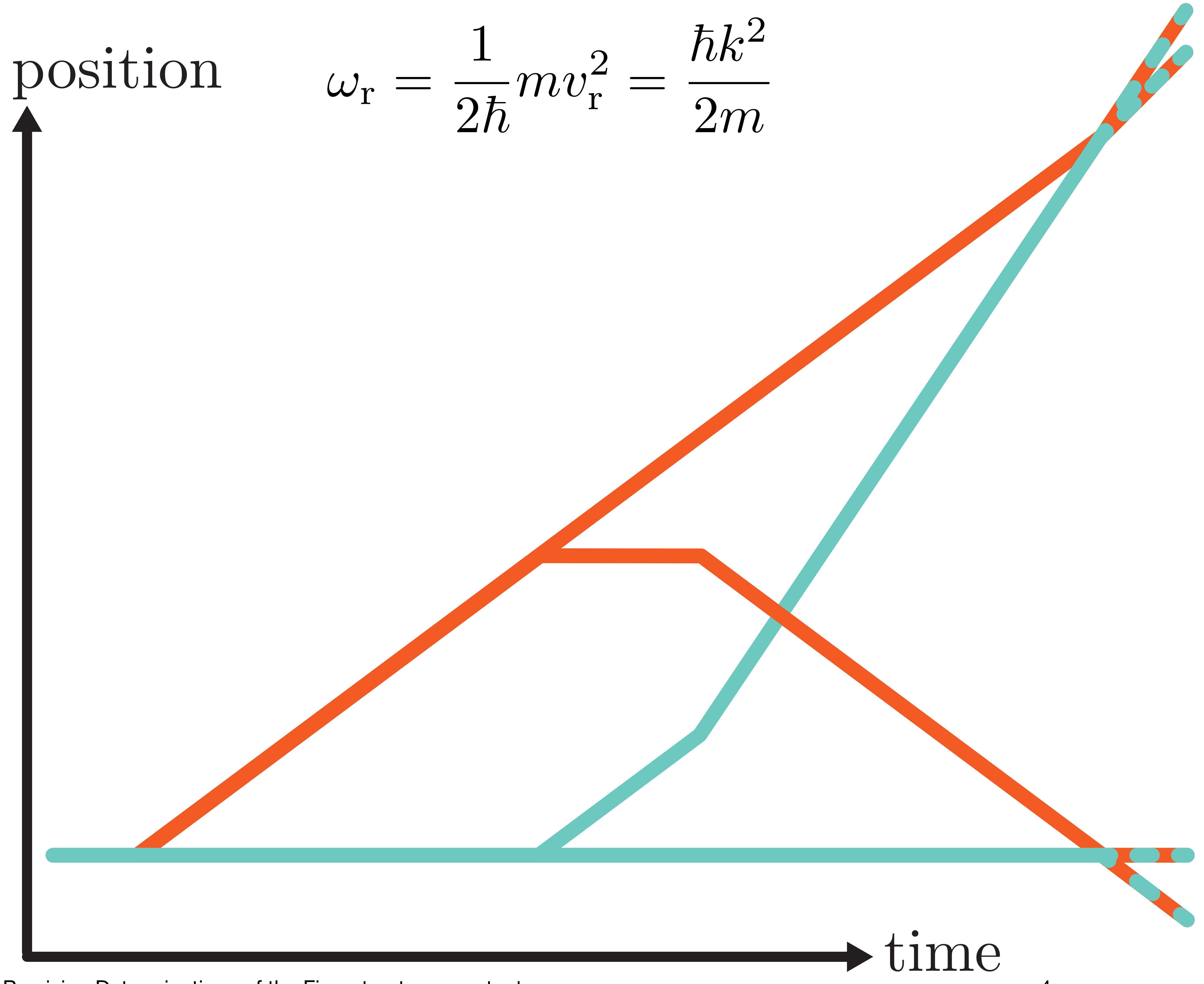
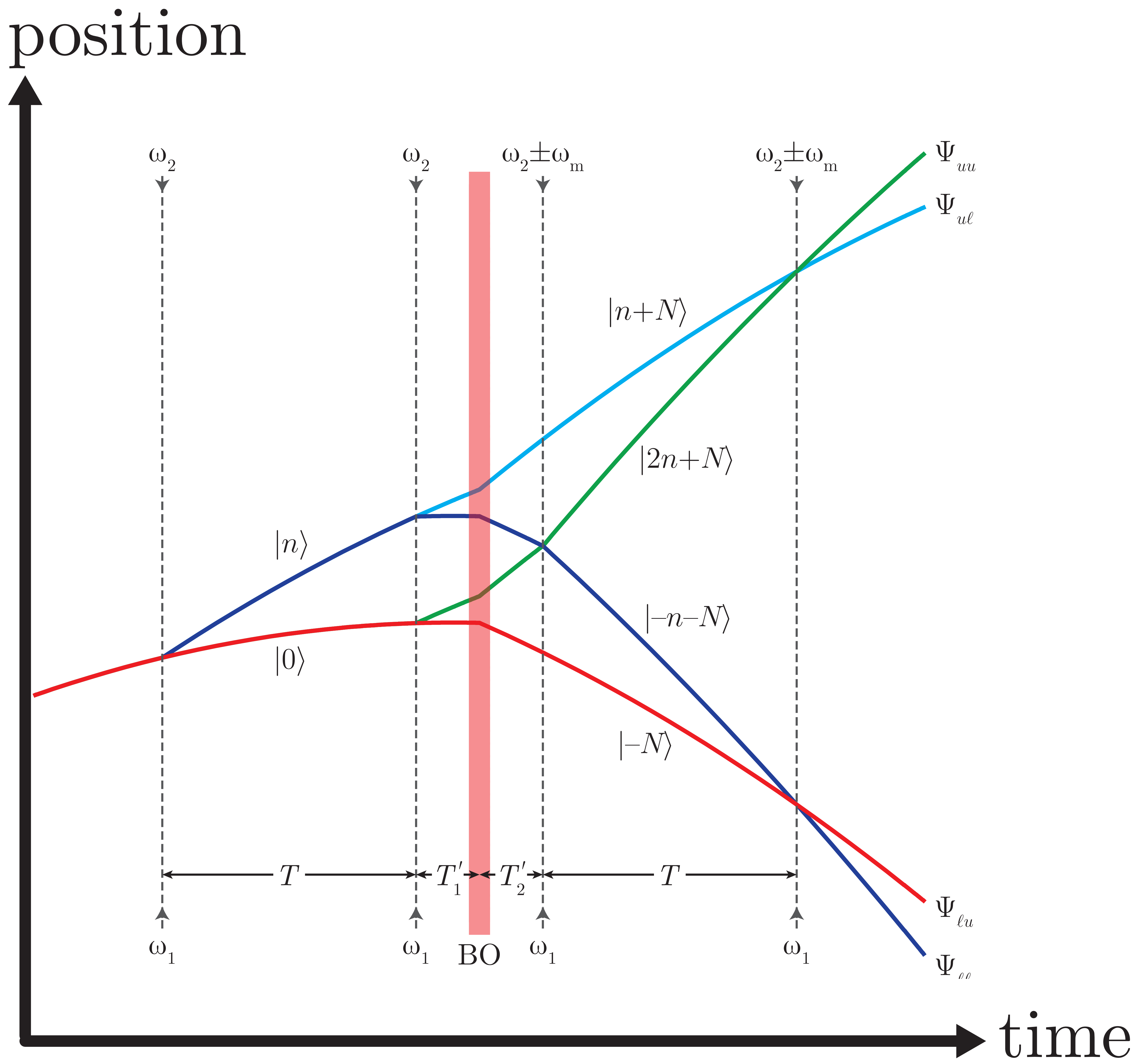


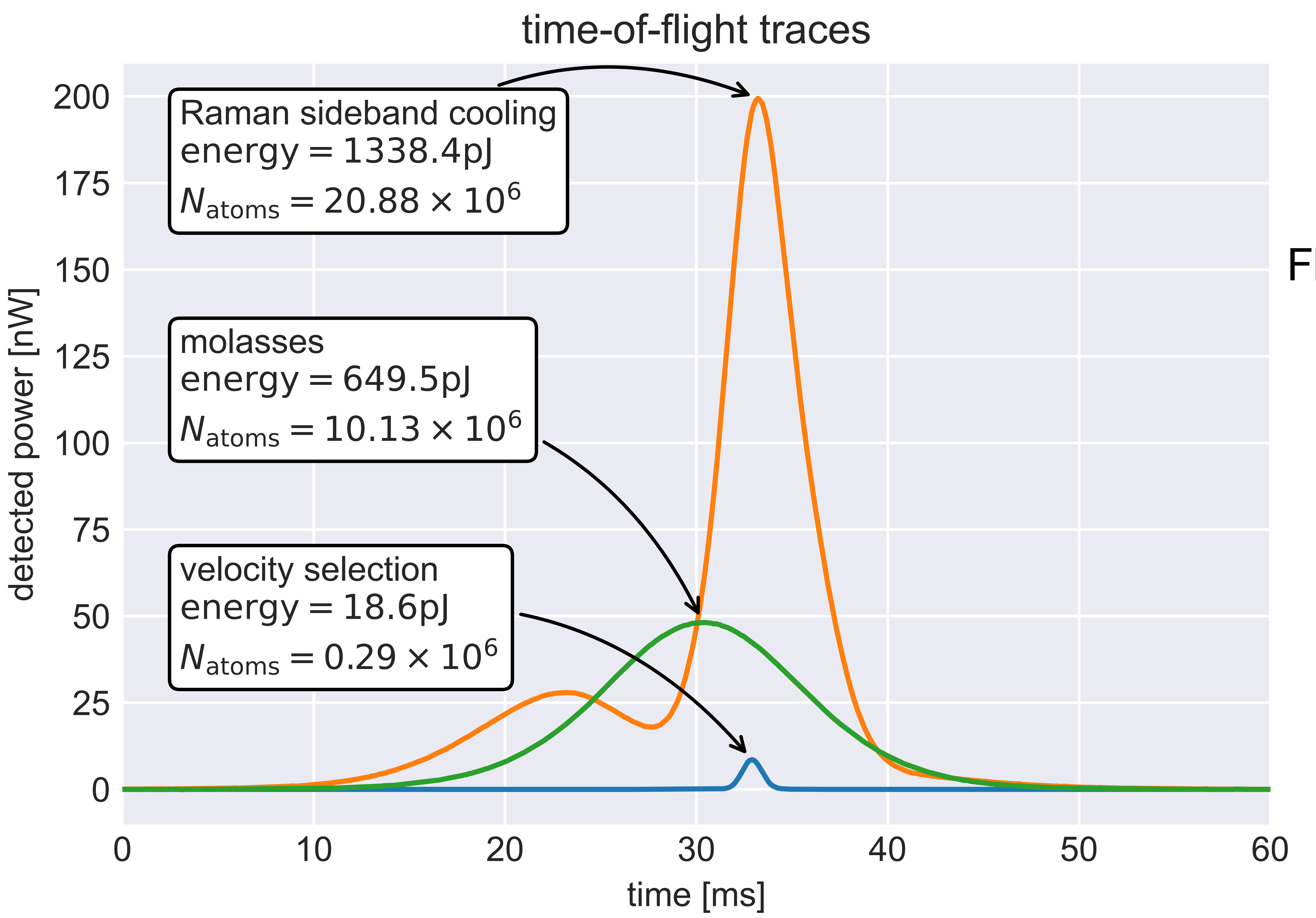
$$\begin{aligned}\Phi_{\rm SCRBI} &= \Phi_{\text{lower RBI}} - \Phi_{\text{upper RBI}} \\ &= 16 n^2 \omega_{\rm r} T - 2 n \omega_{\rm m} T\end{aligned}$$

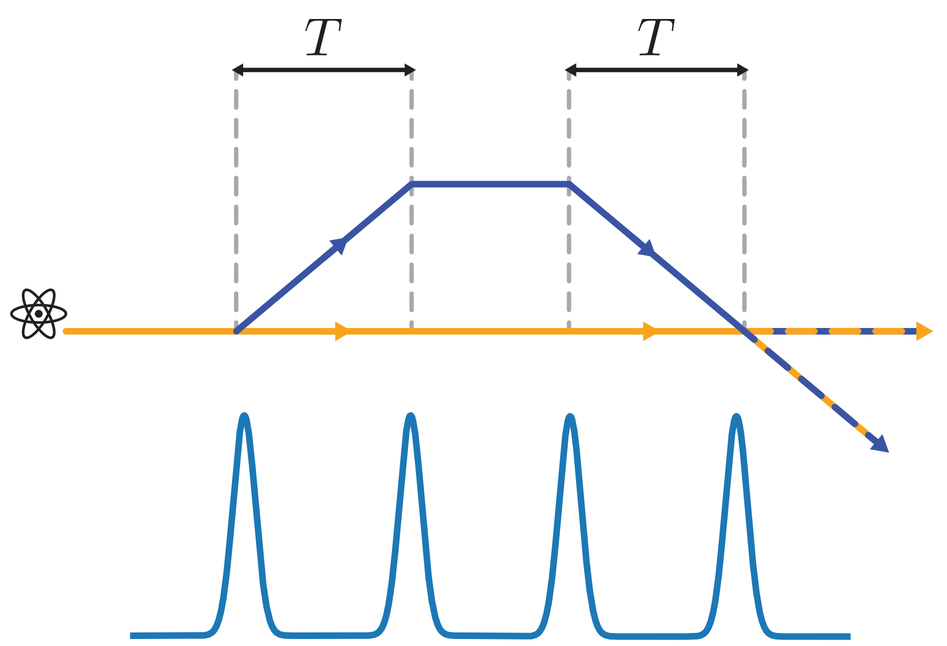
Simultaneous conjugate interferometer geometry

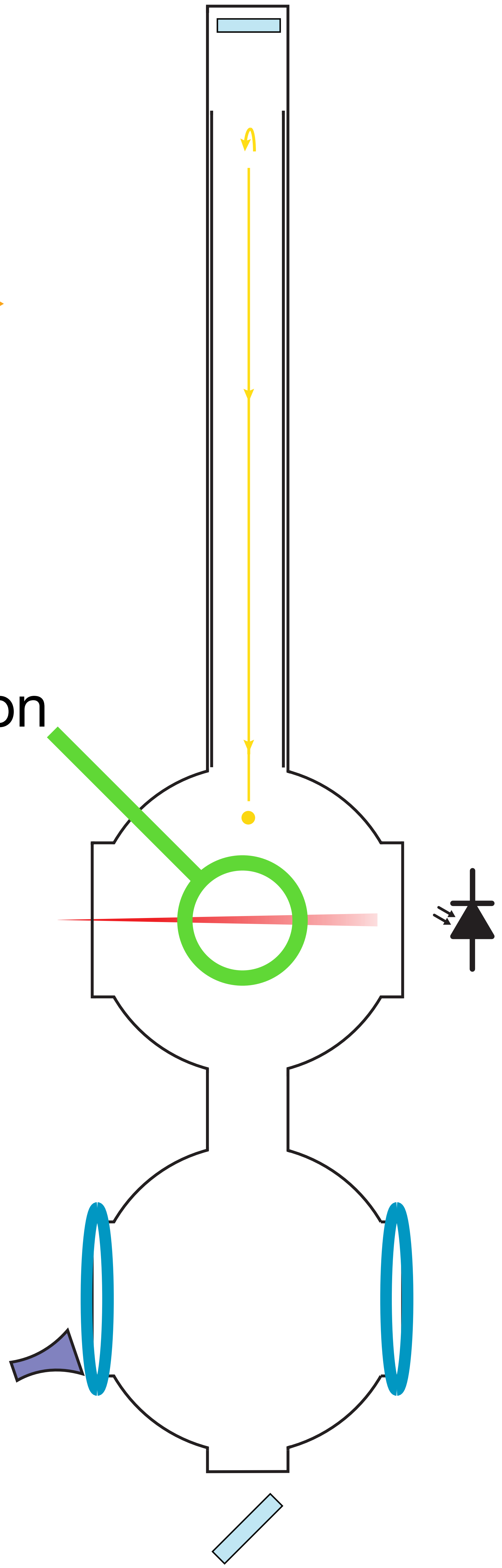


$$\Phi_{\mathrm{SCRBI}} = 16n(n+N)\omega_{\mathrm{r}}T - 2n\omega_{\mathrm{m}}T$$

Atomic fountain







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- Approach to negating/understanding difficult systematics
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Overview of systematic effects

"Straightforward"
- Laser frequency
- Acceleration gradient
- Beam alignment
- Bloch oscillation light shift
- Density shift
- Index of refraction
- Sagnac effect
- Modulation frequency wavenumber

"Difficult"
- Gouy phase
- Speckle phase
- Thermal motion of atoms
- Non-Gaussian waveform
- Parasitic interferometers

Effect	Section	$\delta\alpha/\alpha$ (ppb)
This study		
Laser frequency	1	−0.24 ± 0.03
Acceleration gradient	4A	−1.79 ± 0.02
Gouy phase	3	−2.60 ± 0.03
Beam alignment	5	0.05 ± 0.03
Bloch oscillation light shift	6	0 ± 0.002
Density shift	7	0 ± 0.003
Index of refraction	8	0 ± 0.03
Speckle phase shift	4B	0 ± 0.04
Sagnac effect	9	0 ± 0.001
Modulation frequency wave number	10	0 ± 0.001
Thermal motion of atoms	11	0 ± 0.08
Non-Gaussian waveform	13	0 ± 0.03
Parasitic interferometers	14	0 ± 0.03
Total systematic error	All previous	−4.58 ± 0.12
Statistical error	N/A	±0.16
Other studies		
Electron mass (*16*)	N/A	±0.02
Cesium mass (*6*, *15*)	N/A	±0.03
Rydberg constant (*6*)	N/A	±0.003
Combined result		
Total uncertainty in α	N/A	±0.20

“Straightforward” systematics

- Laser frequency $\boxed{-0.24 \pm 0.03\,\mathrm{ppb}}$
- Acceleration gradient
- Beam alignment
- Index of refraction

$$\frac{\hbar}{m_{\mathrm{Cs}}} = \frac{2\omega_{\mathrm{r}}}{k^2}$$

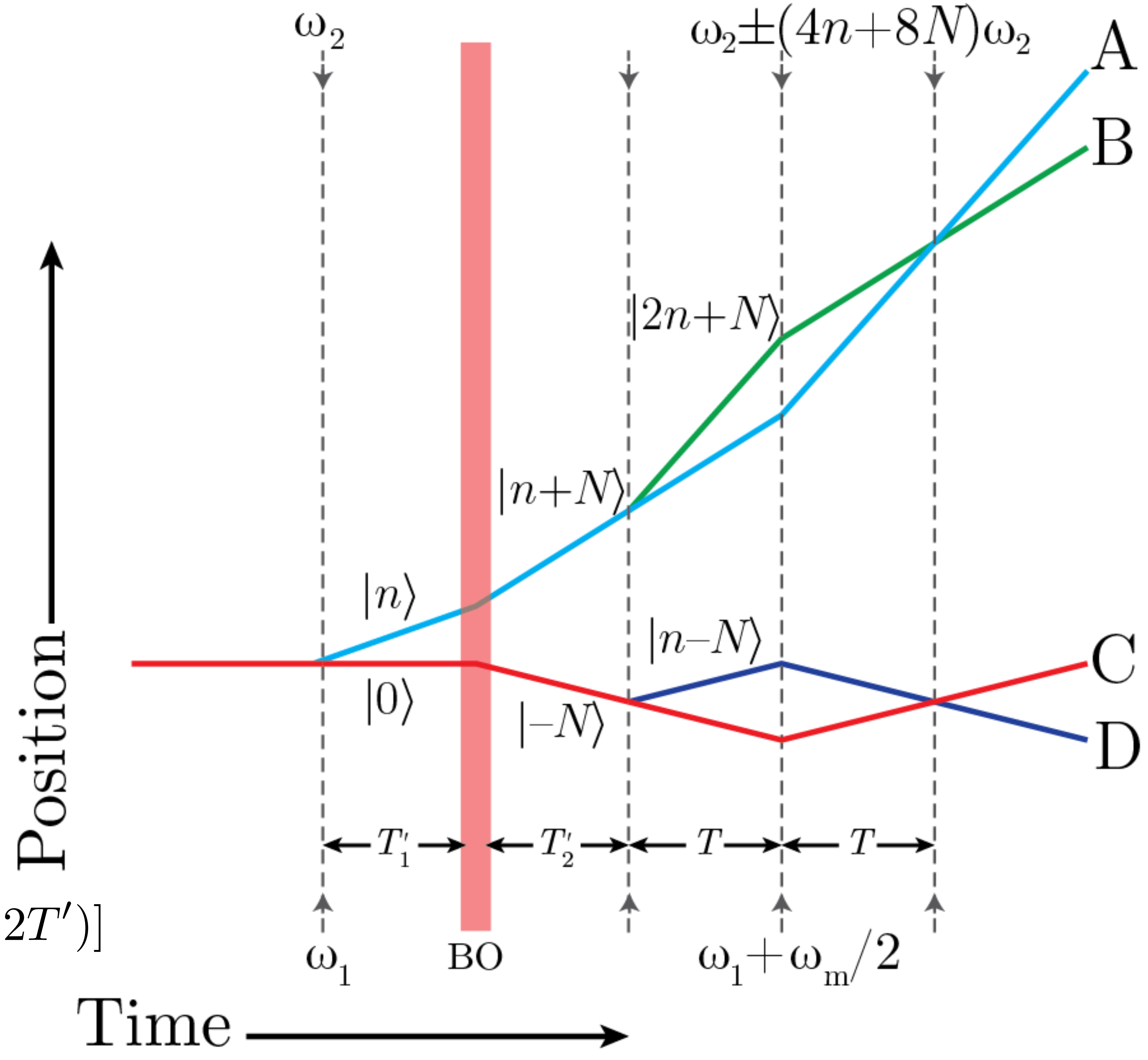
$$\frac{\delta(\hbar/m_{\mathrm{Cs}})}{\hbar/m_{\mathrm{Cs}}} = 4\frac{\delta\omega_{\mathrm{laser}}}{\omega_{\mathrm{laser}}}$$

New comb will allow constant frequency monitoring

Aside: blind measurement of k

“Straightforward” systematics

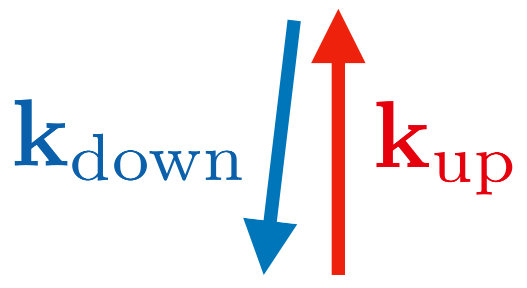
- ~~Laser frequency~~ $\boxed{-0.24 \pm 0.03\,\text{ppb}}$
- Acceleration gradient $\boxed{-1.79 \pm 0.02\,\text{ppb}}$
- Beam alignment
- Index of refraction



$$\Phi_{\text{dual MZ}} = 8n\omega_r\gamma T^2[N(2T - NT_{\rm B} + 2T') + n(T + 2T')] + \frac{8gn(n+2N)T^2\omega_{\rm r}}{c}$$

“Straightforward” systematics

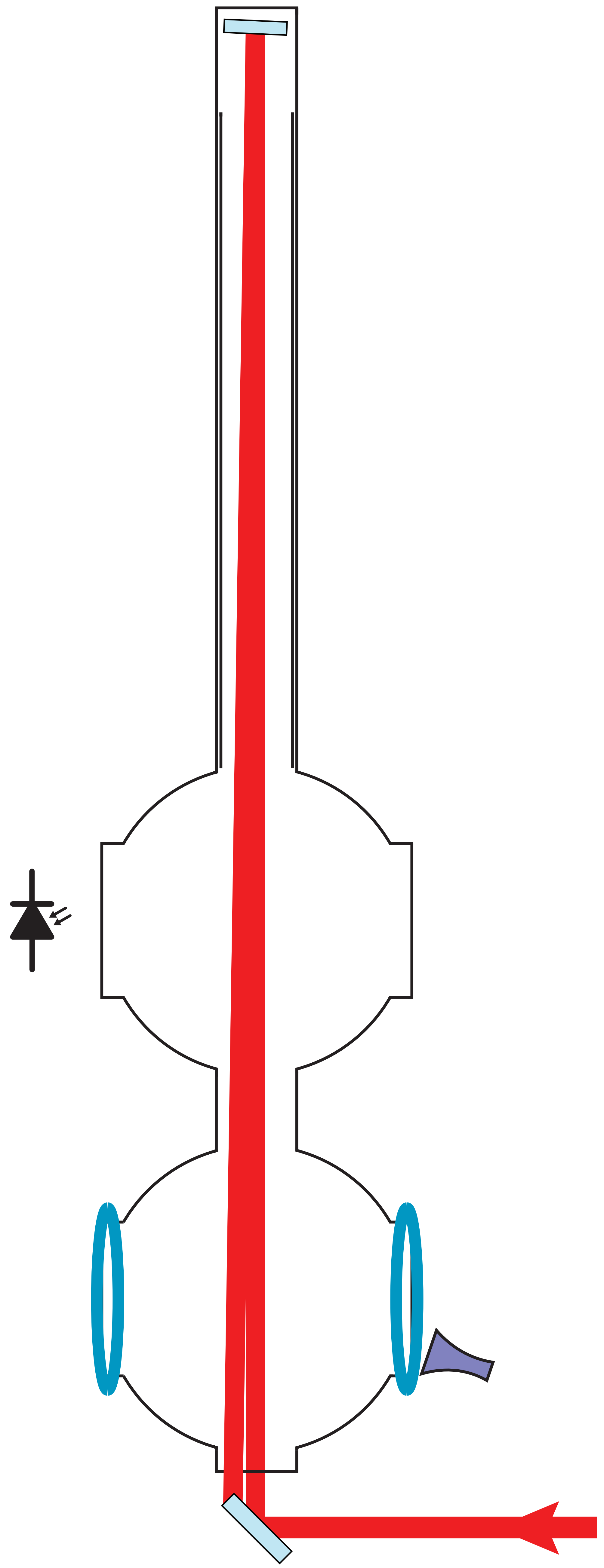
- ~~Laser frequency~~ $\boxed{-0.24 \pm 0.03\,\text{ppb}}$
- ~~Acceleration gradient~~ $\boxed{-1.79 \pm 0.02\,\text{ppb}}$
- Beam alignment
- Index of refraction



$$\mathbf{k}_{\text{eff}} = \mathbf{k}_{\text{up}} - \mathbf{k}_{\text{down}}$$

$$k_{\text{eff}}^2 = 4k^2 - k^2\theta^2$$

Backfiber coupling constrains θ to 12 μrad



“Straightforward” systematics

- ~~Laser frequency~~ $\boxed{-0.24 \pm 0.03\,\mathrm{ppb}}$
- ~~Acceleration gradient~~ $\boxed{-1.79 \pm 0.02\,\mathrm{ppb}}$
- ~~Beam alignment~~ $\boxed{0.05 \pm 0.03\,\mathrm{ppb}}$
- Index of refraction

Dispersion of laser due to background Cesium atoms changes the refractive index

$$n - 1 = \frac{\rho\sigma_0}{4k}\frac{\Gamma}{\Delta}$$

Bounded by assuming all vacuum pressure is due to cesium.

Overview of systematic effects

“Straightforward”

Laser frequency
Acceleration gradient
Beam alignment
Bloch llation ligh
Density
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“Difficult”

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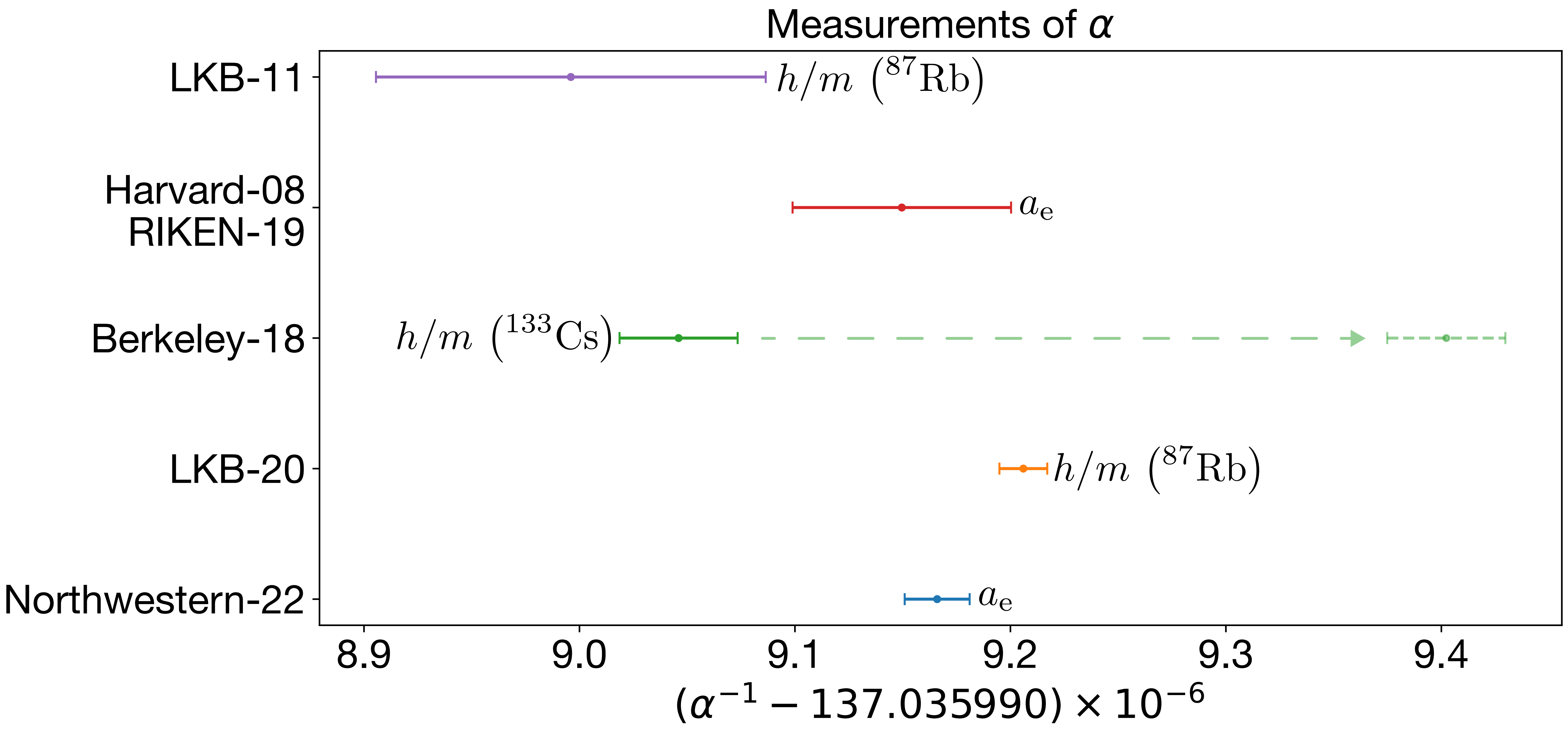
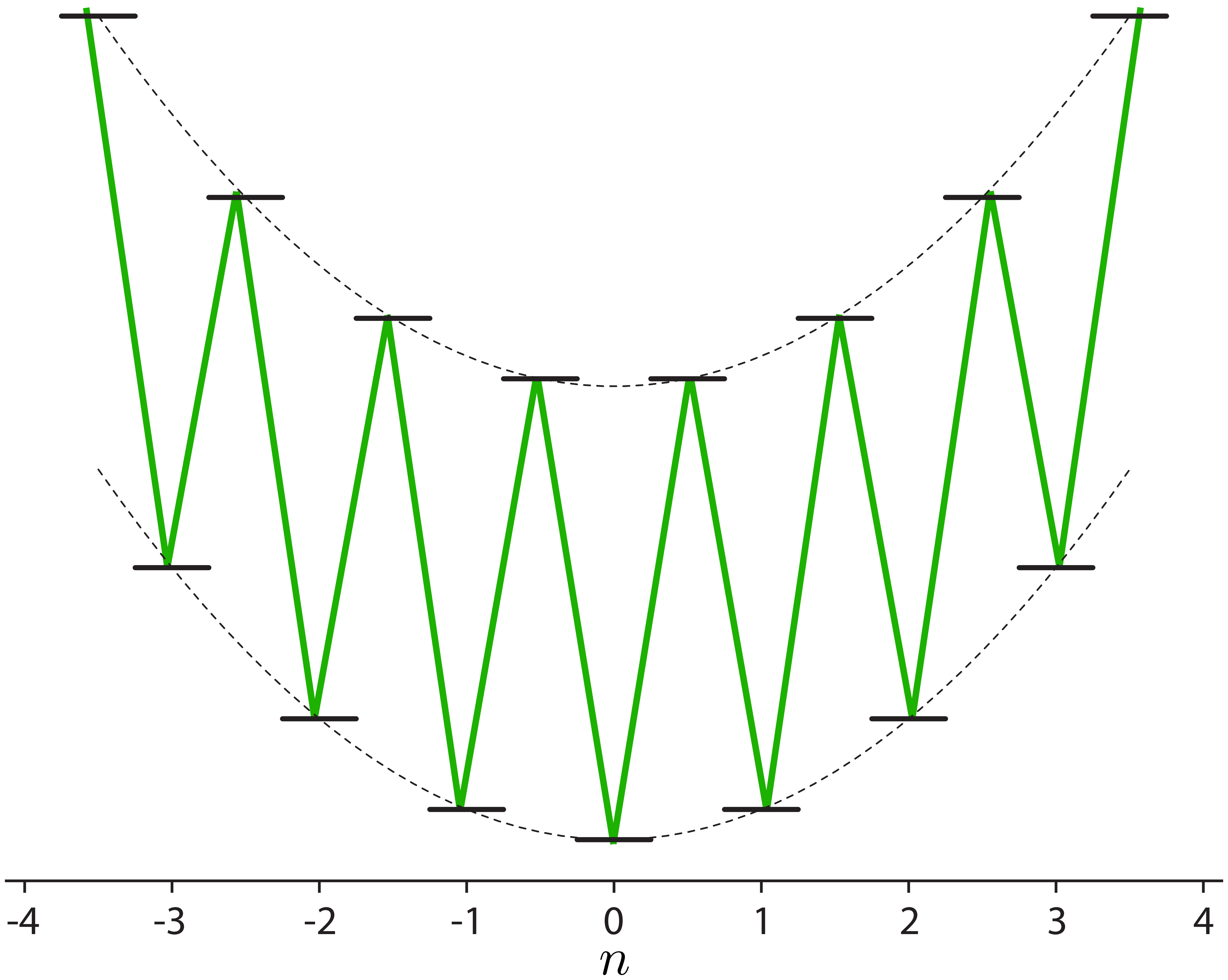
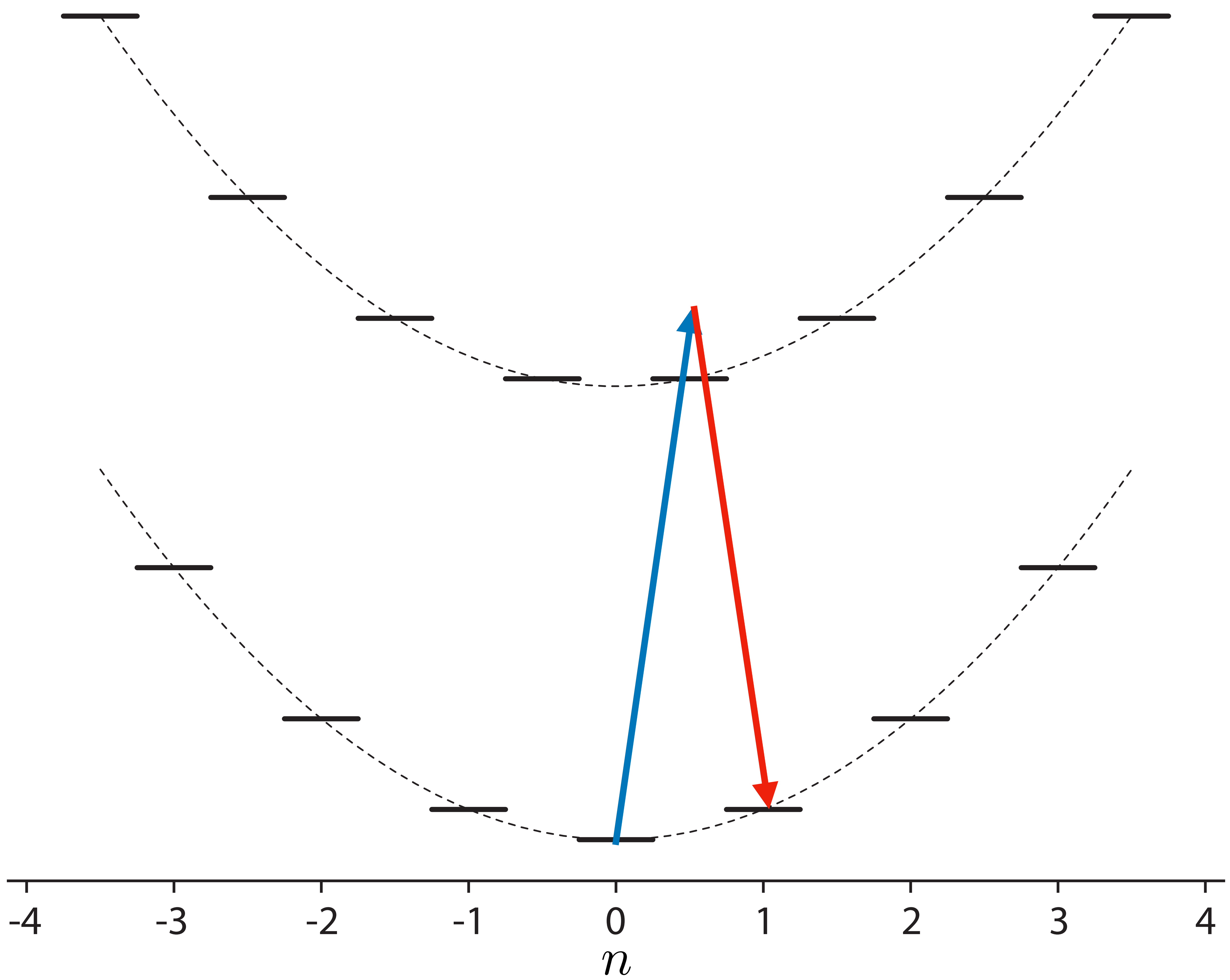


Bragg beamsplitter problems

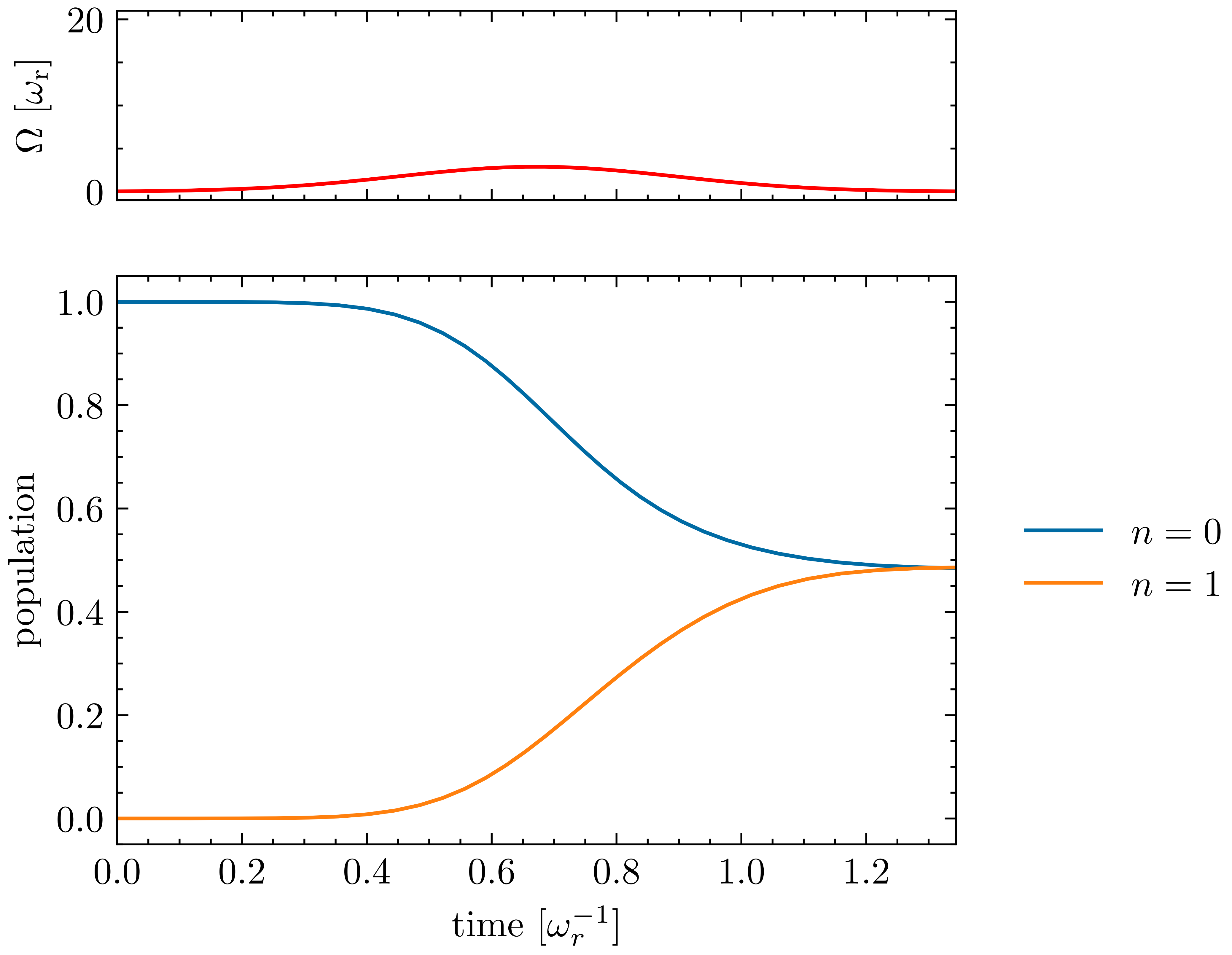
$$H = \frac{(2n\hbar k)^2}{2m} + \hbar\omega_e \left|e\right\rangle\left\langle e\right| + \mathbf{E}(t)\cdot\mathbf{d}\left|e\right\rangle\left\langle g\right| + \mathrm{h.c.}$$



Bragg beamsplitter problems



$$H = \frac{(2n\hbar k)^2}{2m} + \hbar\omega_e \left|e\right\rangle\!\left\langle e\right| + \mathbf{E}(t)\cdot\mathbf{d}\left|e\right\rangle\!\left\langle g\right| + \text{h.c.}$$



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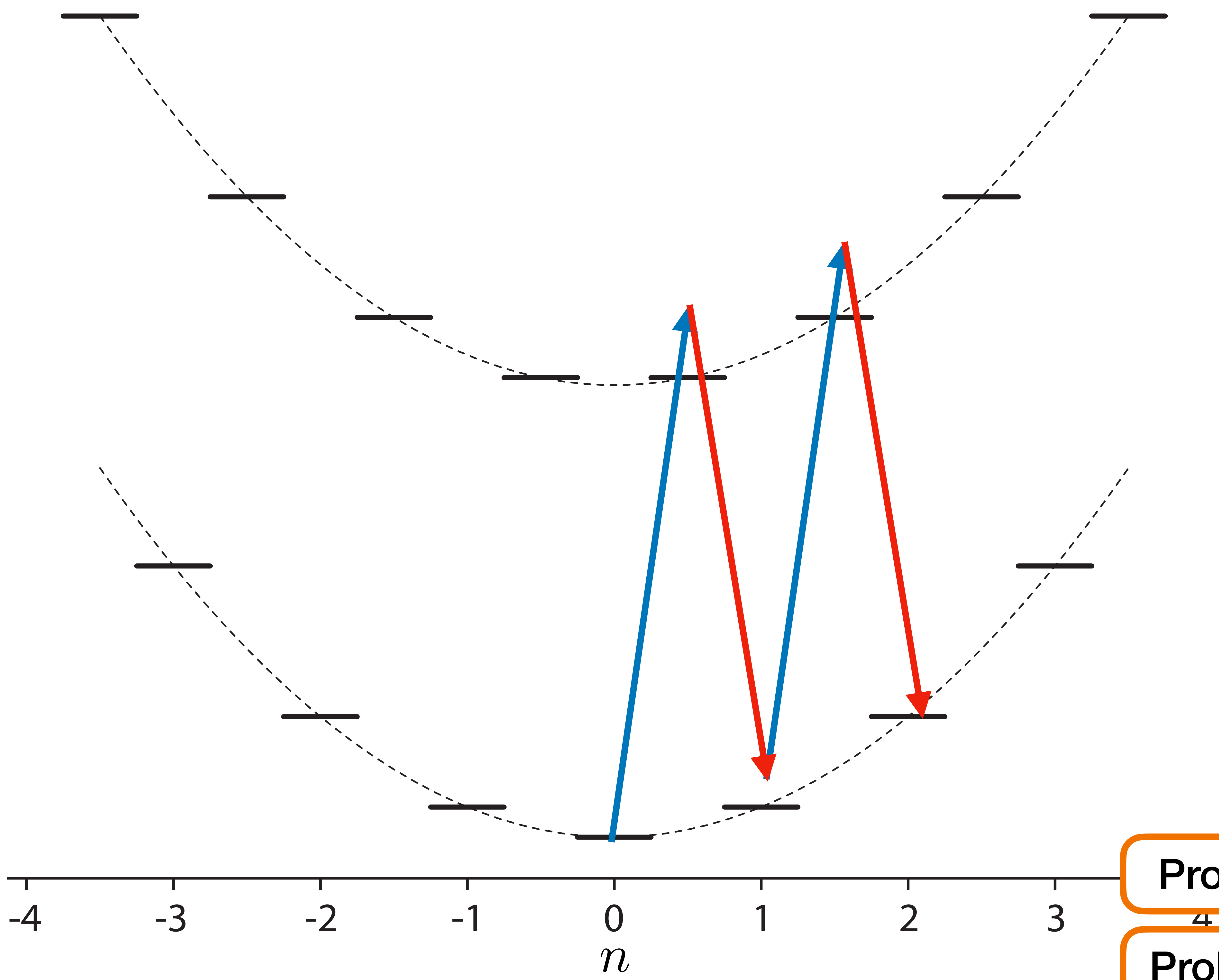
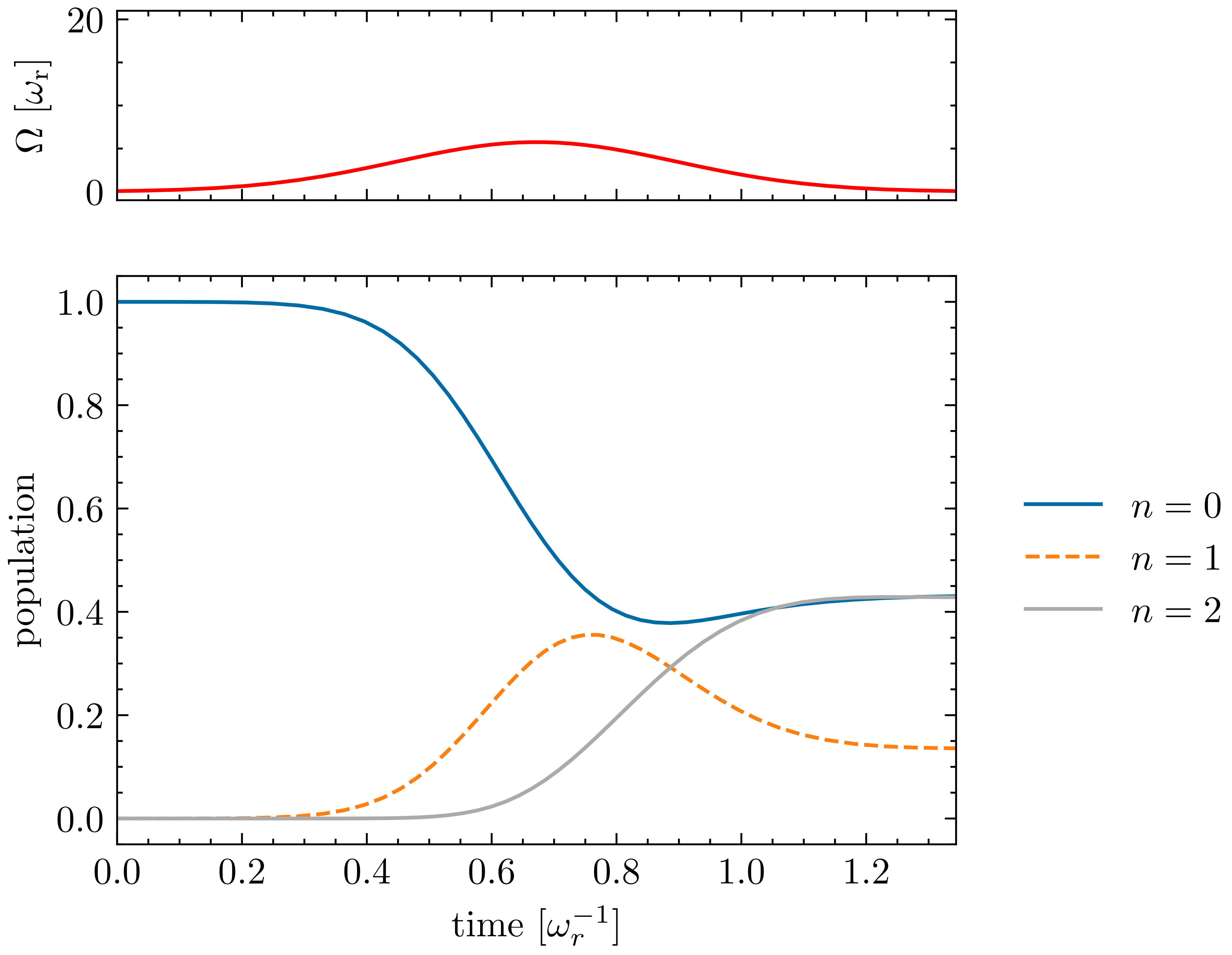


$$H = \frac{(2n\hbar k)^2}{2m} + \hbar\omega_e \, |e\rangle\langle e| + \mathbf{E}(t)\cdot\mathbf{d}\, |e\rangle\langle g| + \text{h.c.}$$



Problem #1: dynamics during pulse will effect imparted phase

Problem #2: beamsplitter outputs unwanted momentum states

Anomalous phase

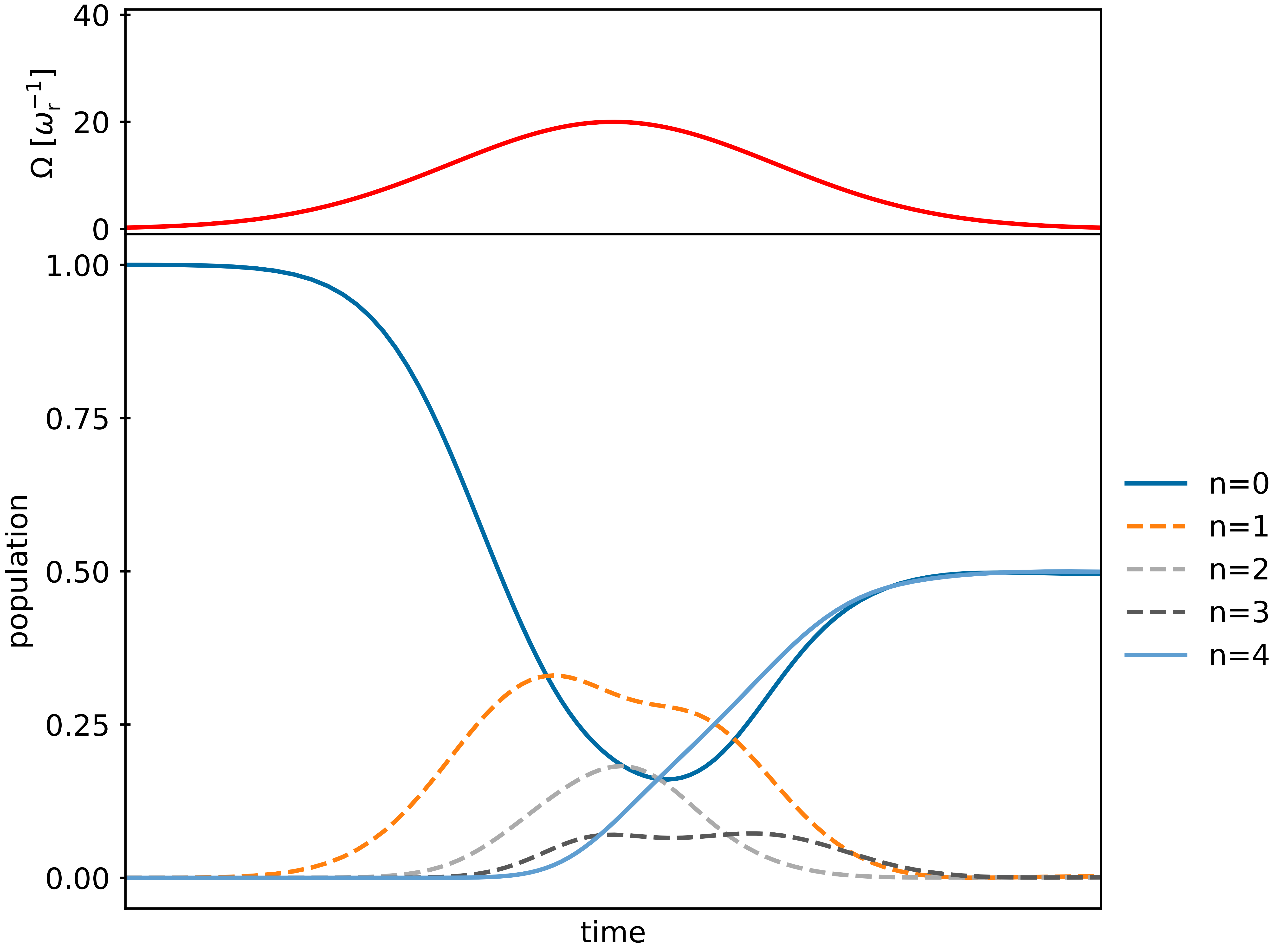
$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \left[\frac{\hat{p}^2}{2m} + V(x,t)\right]\Psi(x,t)$$

$$V(x,t) = 2\hbar\Omega_{\text{eff}}(t)\cos^2(kx - \delta t/2)$$

$$|\psi\rangle = \sum_{n=-\infty}^{\infty} c_n(t)e^{i2nkx}\,|n\rangle$$

$$\begin{aligned}
&\vdots\\
\dot{c}_0 &= i\frac{\Omega_{\text{eff}}}{2}\left[c_{-1}e^{i(-\delta-4)t} + c_1e^{i(\delta-4)t}\right]\\
\dot{c}_1 &= i\frac{\Omega_{\text{eff}}}{2}\left[c_0e^{i(-\delta+4)t} + c_2e^{i(\delta-12)t}\right]\\
&\vdots\\
\dot{c}_j &= i\frac{\Omega_{\text{eff}}}{2}\left[c_{j-1}e^{i(-\delta+8j-4)t} + c_{j+1}e^{i(\delta-8j-4)t}\right]\\
&\vdots\\
\dot{c}_{n-1} &= i\frac{\Omega_{\text{eff}}}{2}\left[c_{n-2}e^{i(-\delta+8n-12)t} + c_ne^{i(\delta-8n+4)t}\right]\\
\dot{c}_n &= i\frac{\Omega_{\text{eff}}}{2}\left[c_{n-1}e^{i(-\delta+8n-4)t} + c_{n+1}e^{i(\delta-8n-4)t}\right]\\
&\vdots
\end{aligned}$$

Anomalous phase



$$\vdots$$

$$\dot{c}_0 = i\frac{\Omega_{\text{eff}}}{2}\left[c_{-1}e^{i(-\delta-4)t} + c_1 e^{i(\delta-4)t}\right]$$

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$$\vdots$$

$$\dot{c}_j = i\frac{\Omega_{\text{eff}}}{2}\left[c_{j-1}e^{i(-\delta+8j-4)t} + c_{j+1} e^{i(\delta-8j-4)t}\right]$$

$$\vdots$$

$$\dot{c}_{n-1} = i\frac{\Omega_{\text{eff}}}{2}\left[c_{n-2}e^{i(-\delta+8n-12)t} + c_n e^{i(\delta-8n+4)t}\right]$$

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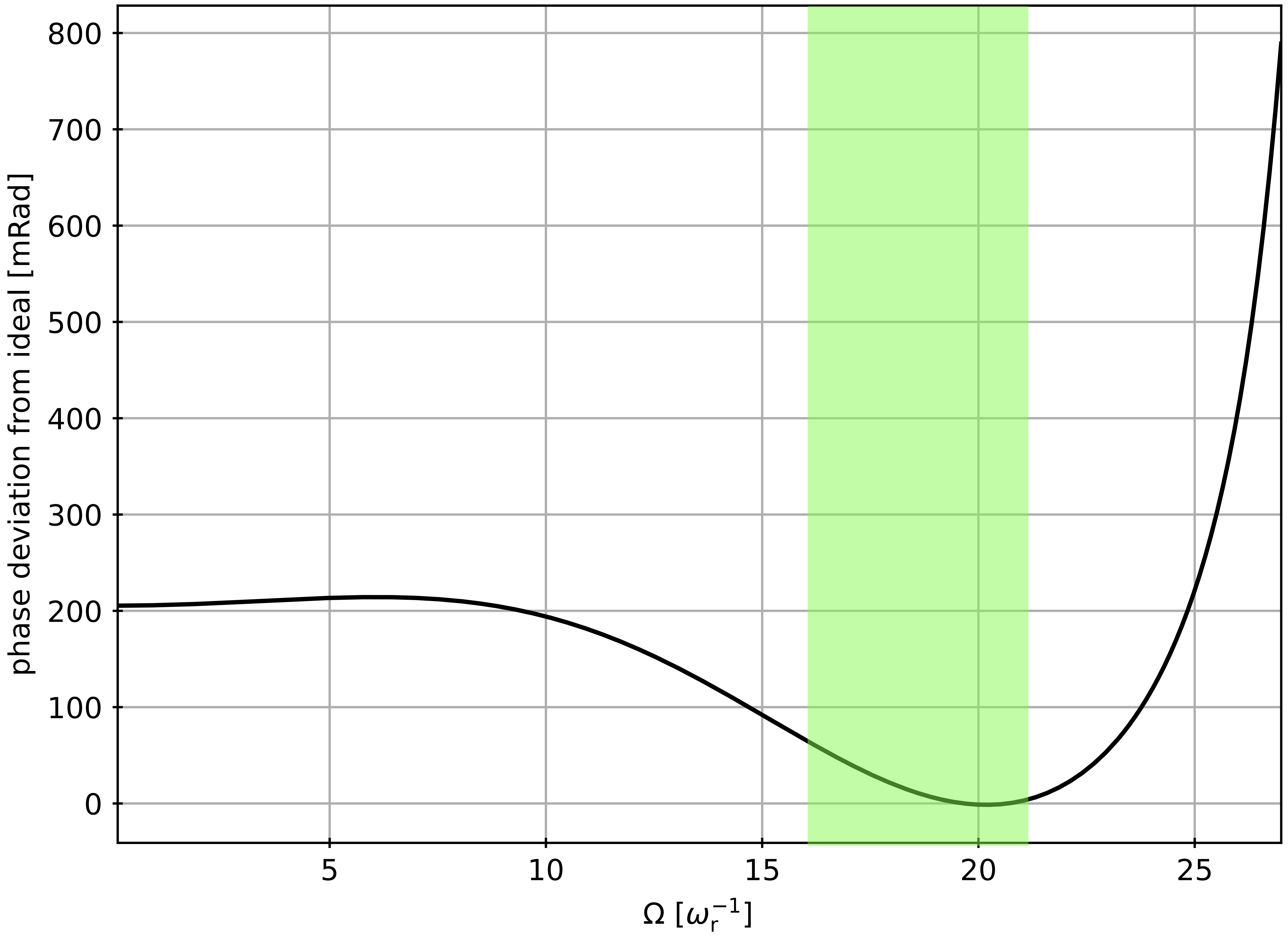
$$\vdots$$

Anomalous phase

2018 measurement systematic uncertainty of 120ppt at Φ_{total} corresponds to ellipse angle to 3mRad

$|\phi_{n=4}|^2 > 0.2$



$$\vdots$$

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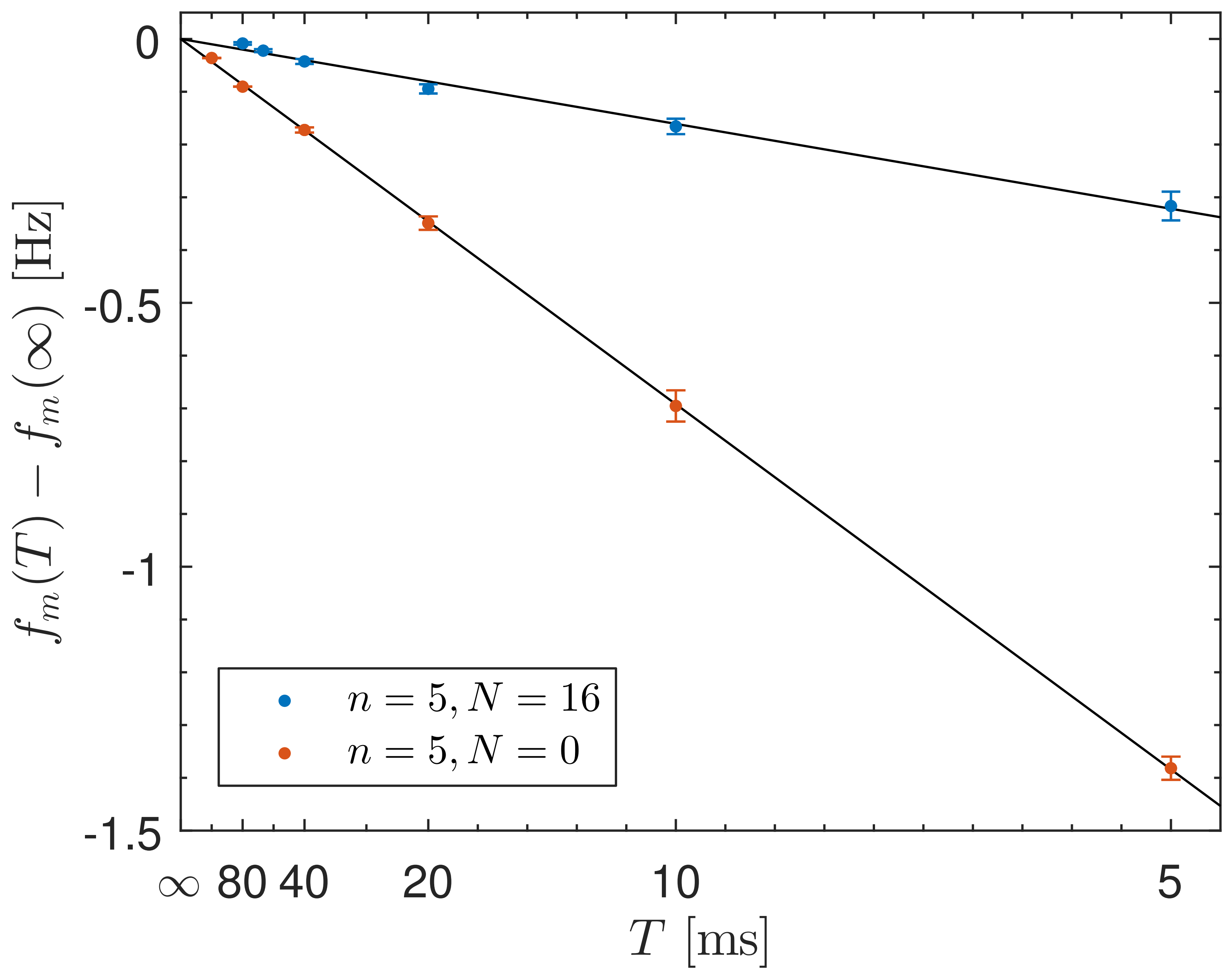
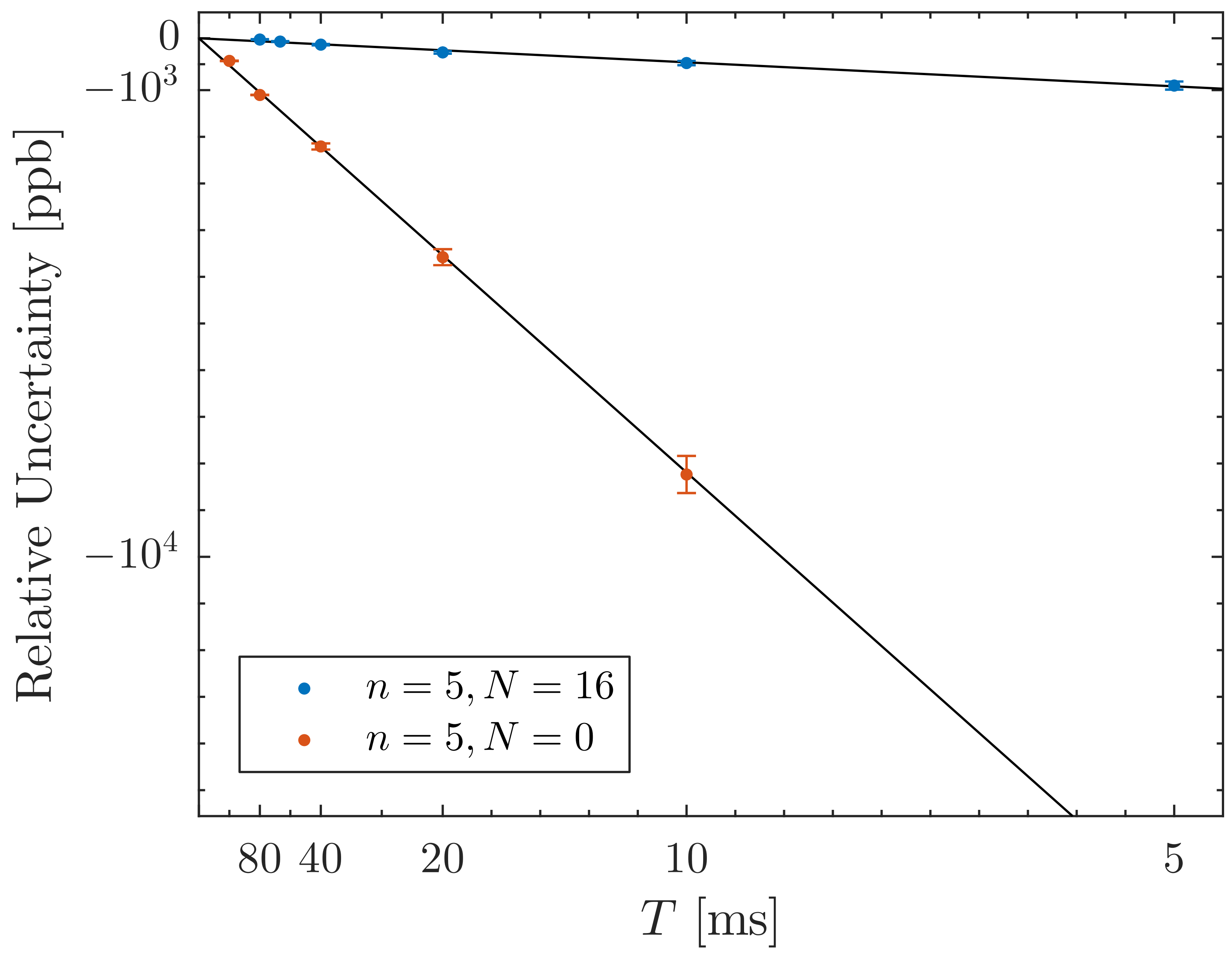
&\vdots
\end{aligned}$$

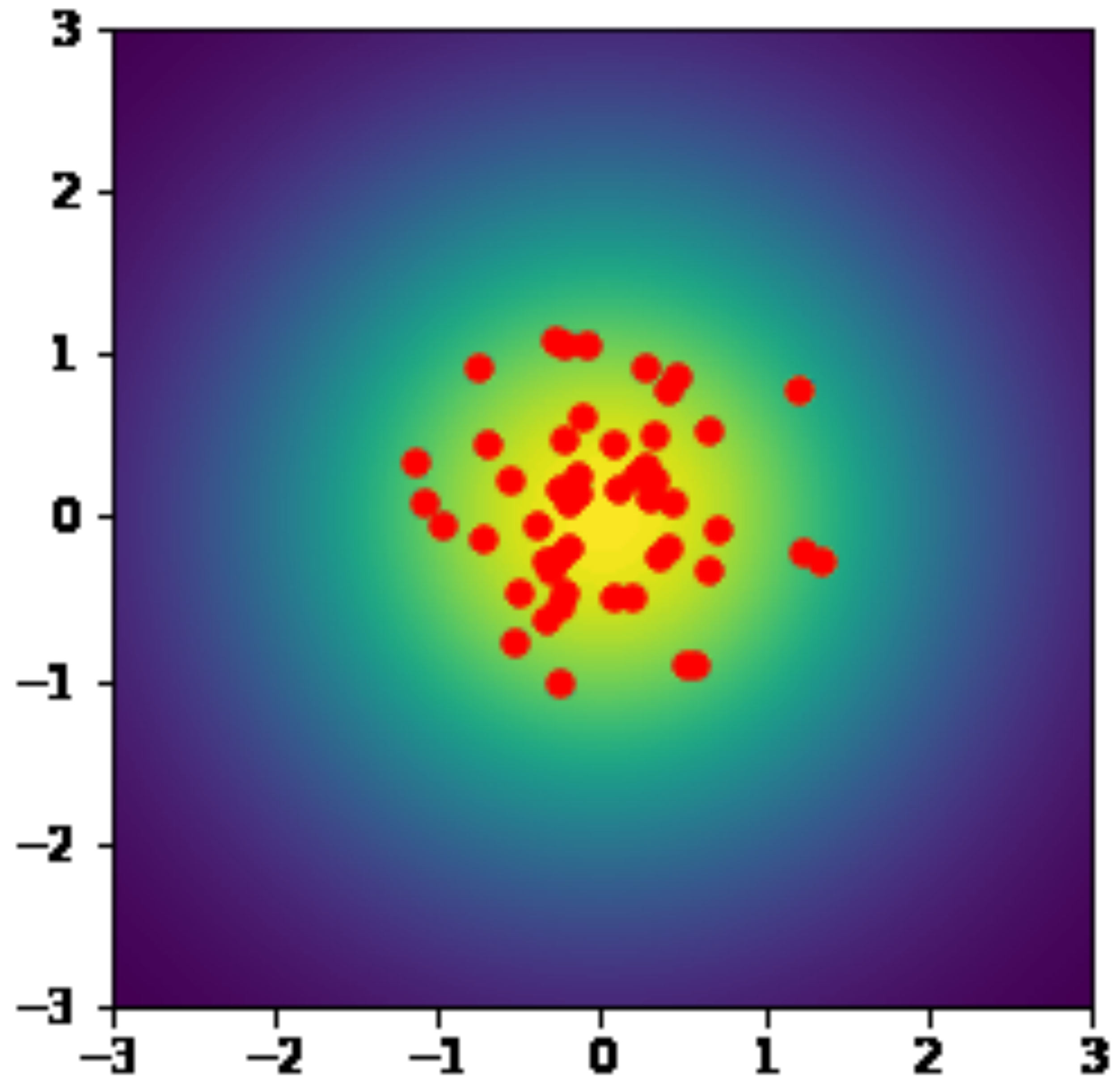
Anomalous phase previous analysis

$\boxed{0 \pm 0.08\,\mathrm{ppb}}$





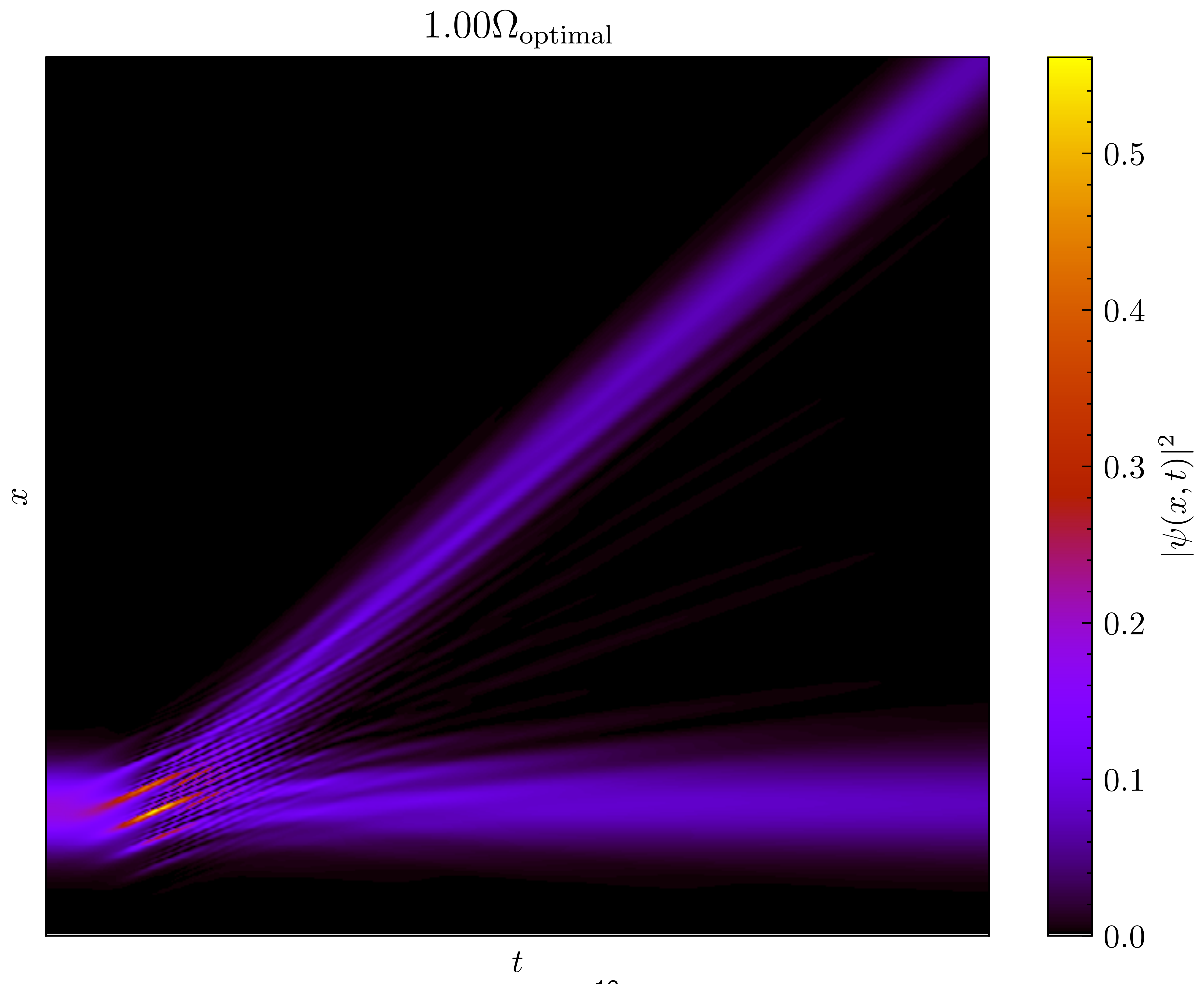
Source: Brian Estey thesis work



Source: Zack Pagel

$$\Phi_{\mathrm{SCI}} = 16n(n+N)\omega_r T - 2n\omega_m T + \phi_{\mathrm{diffraction}} \xrightarrow{\text{Zero } \Phi_{\mathrm{SCI}}} \omega_m = 8(n+N)\omega_r + \frac{\phi_{\mathrm{diffraction}}}{2nT}$$

Parasitic ports



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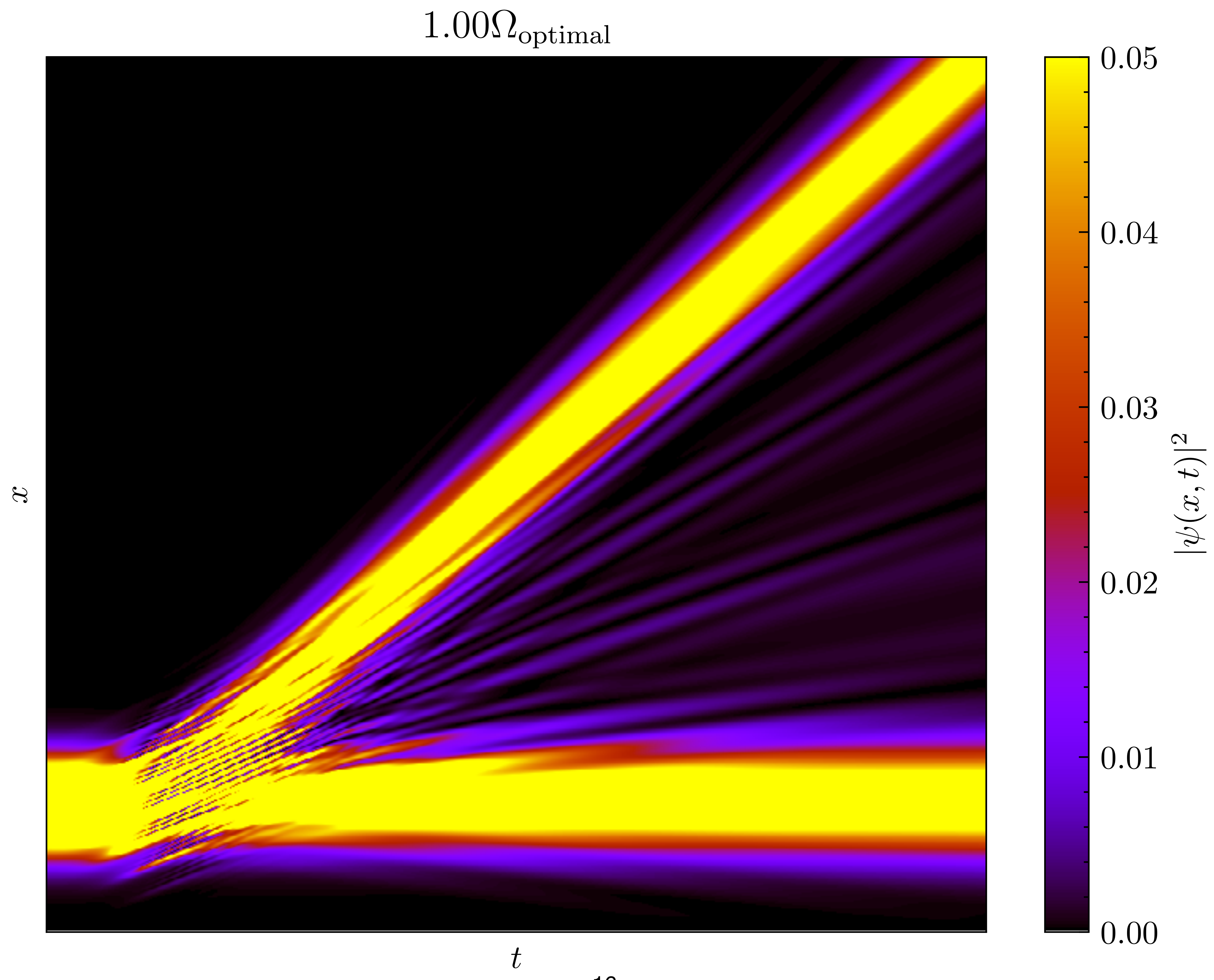

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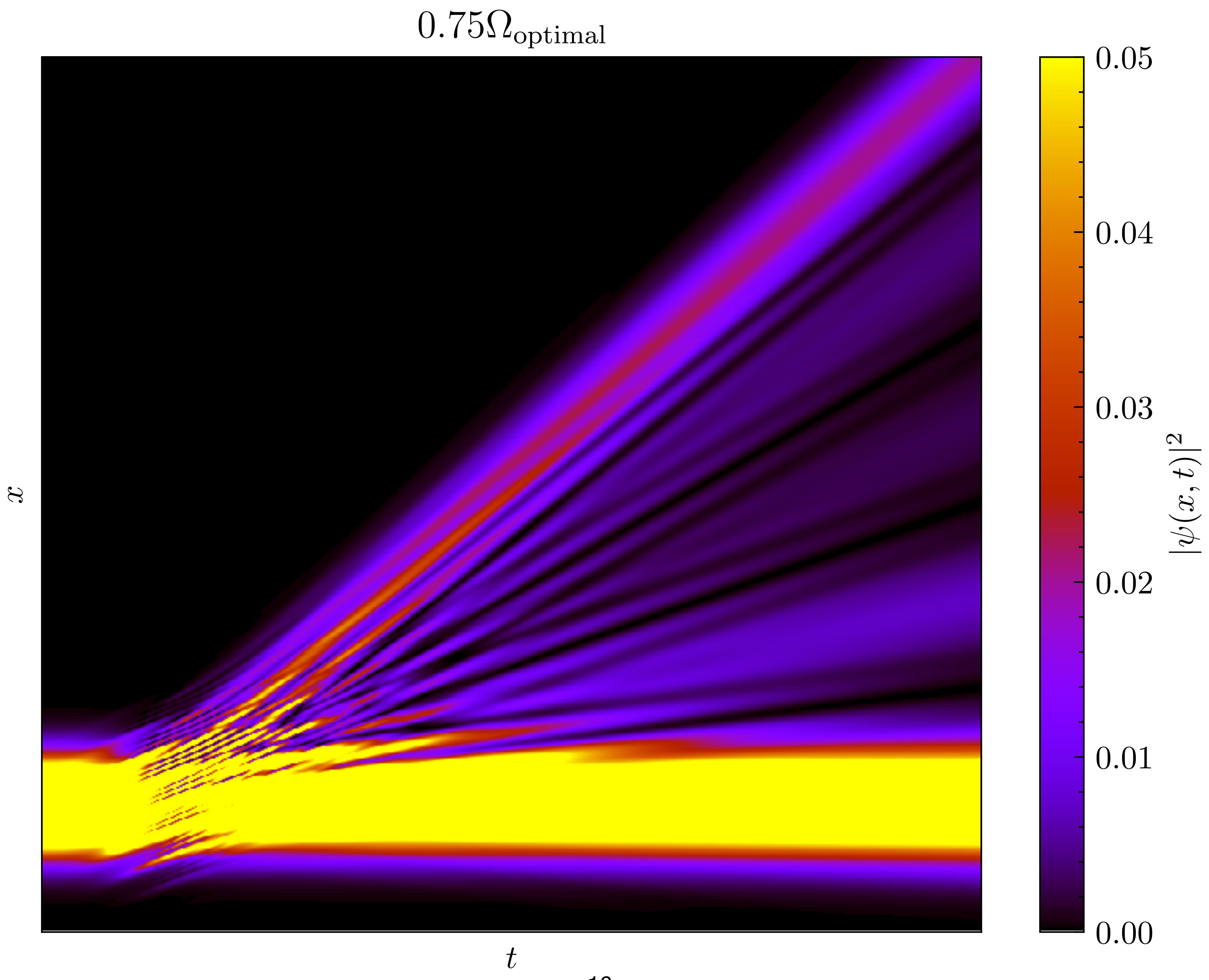

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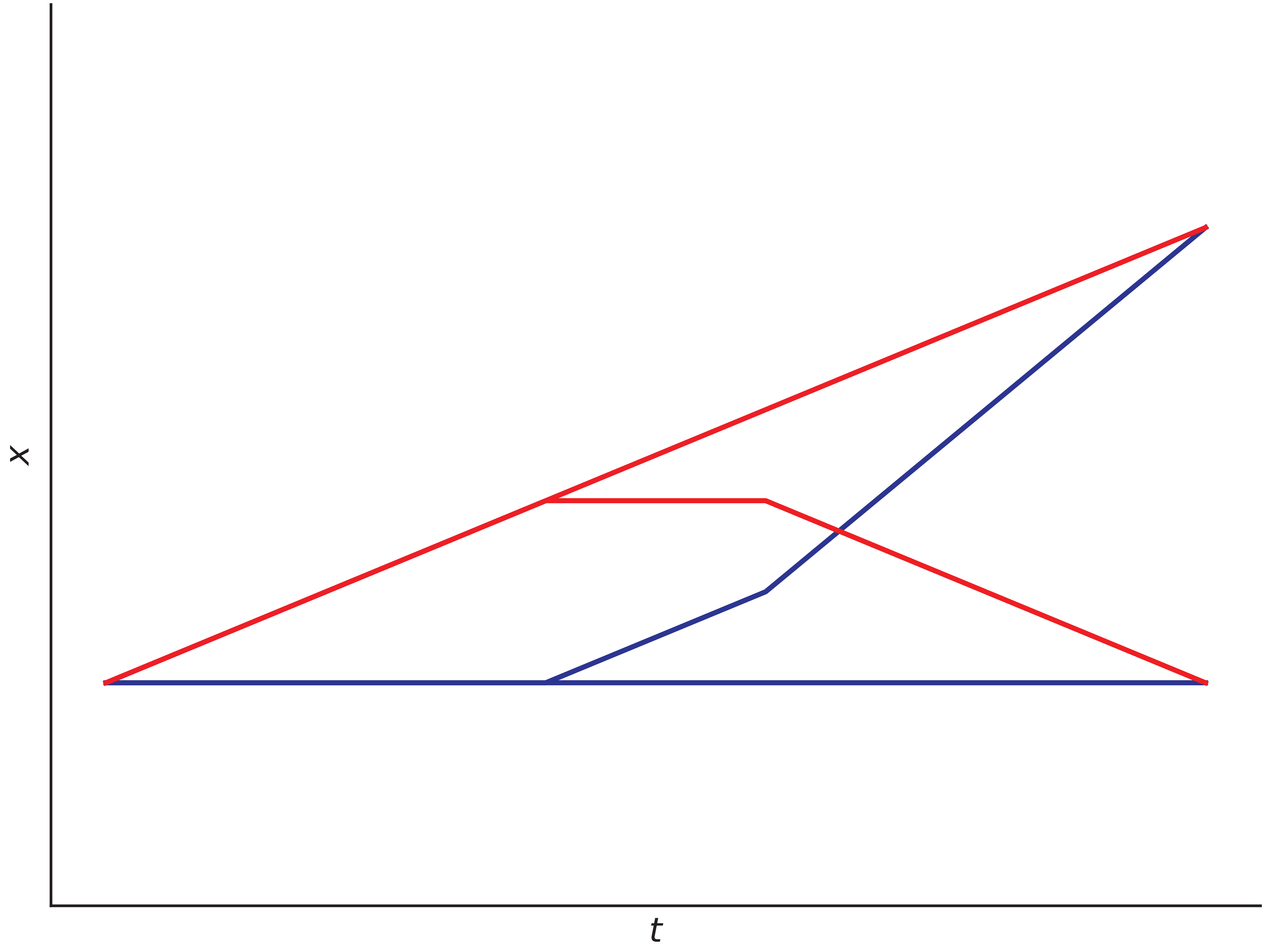

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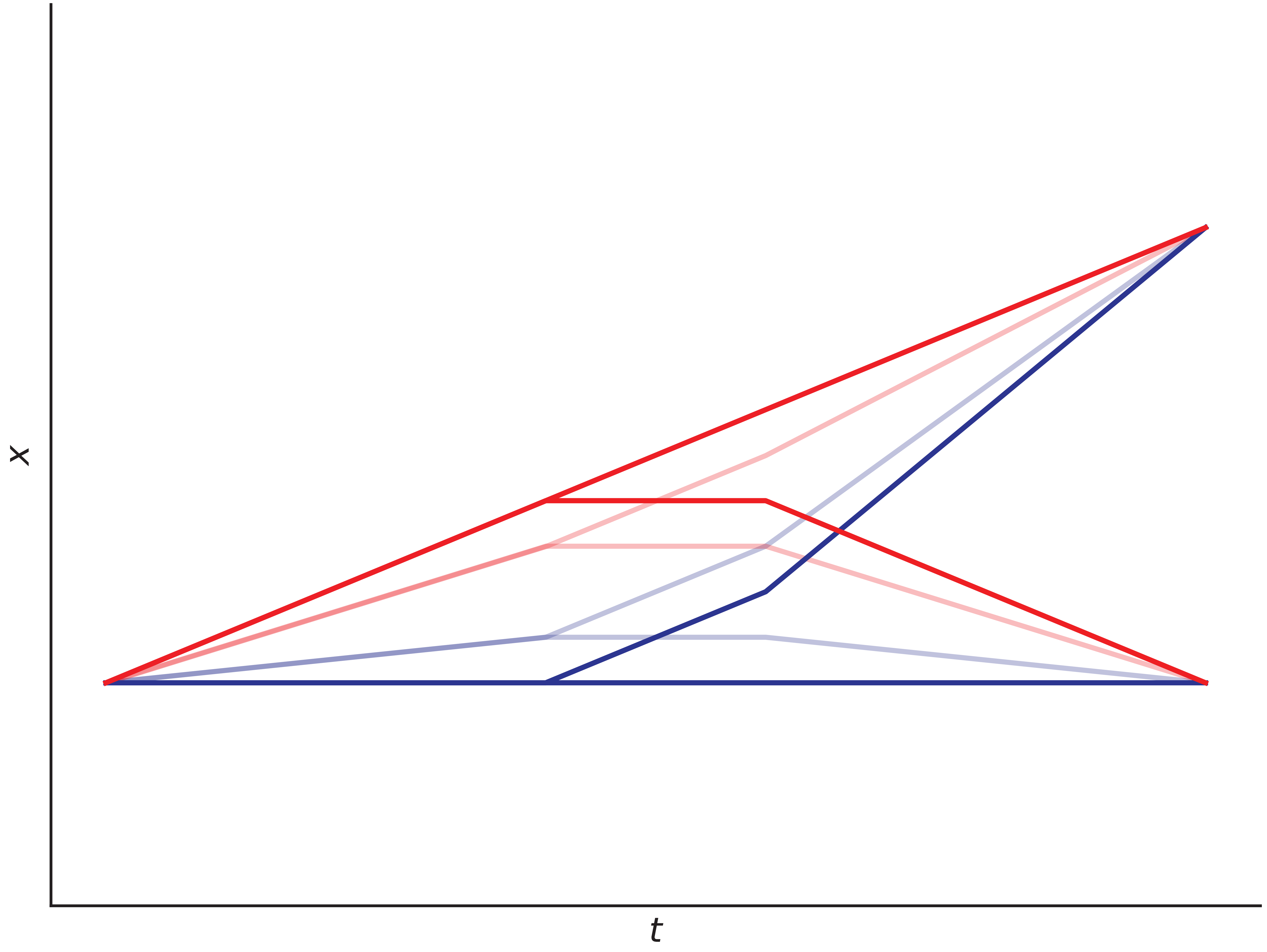

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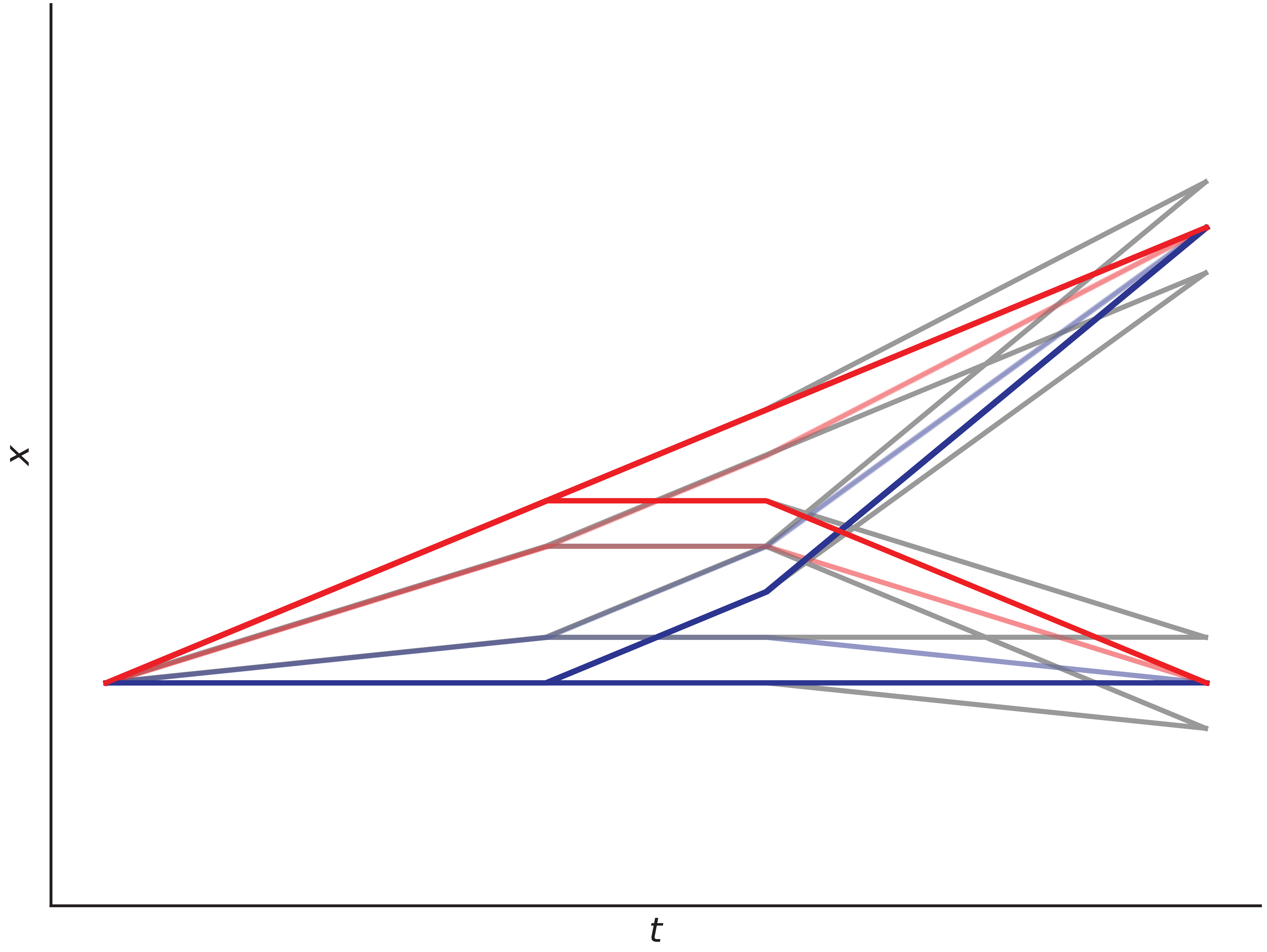
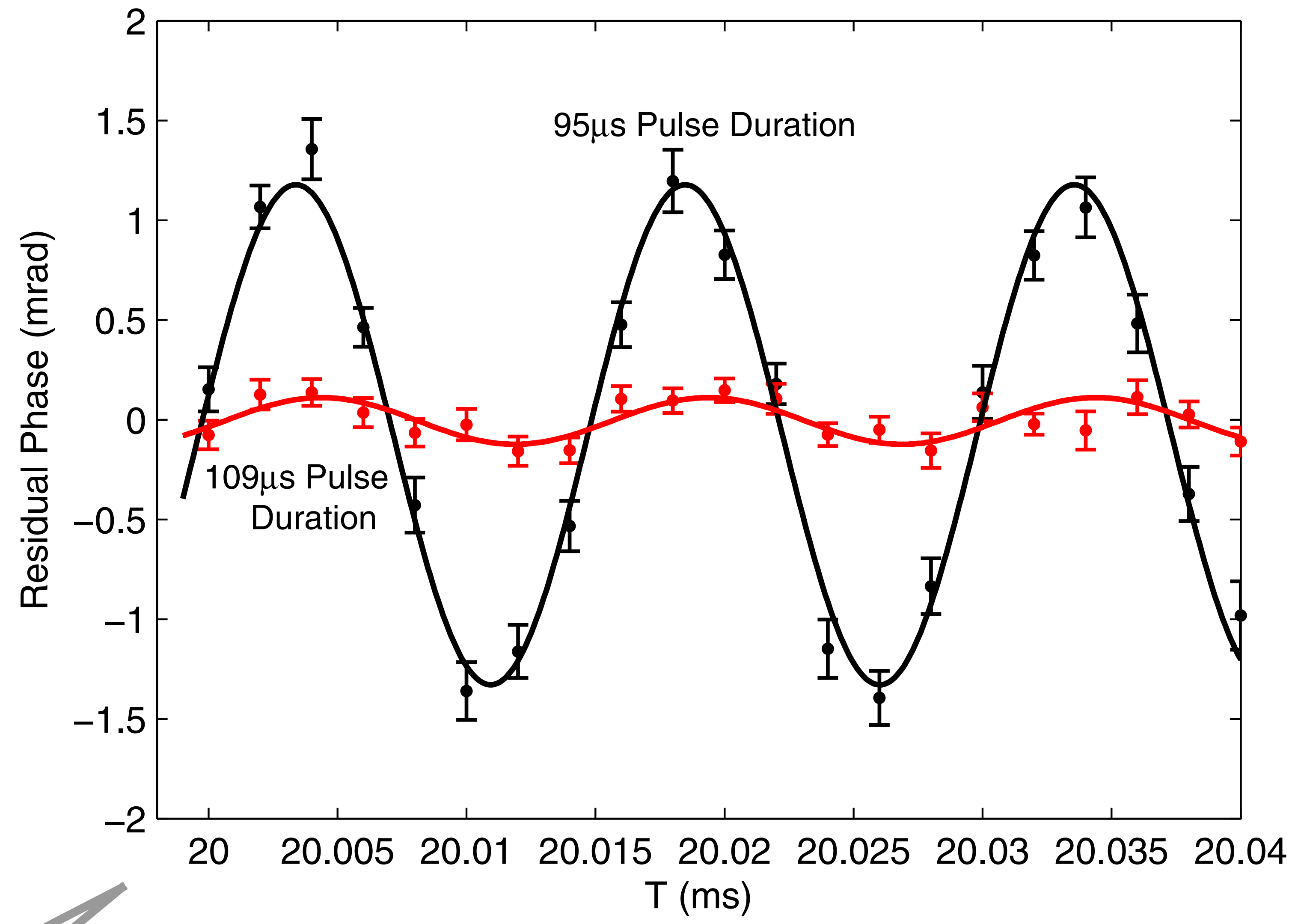
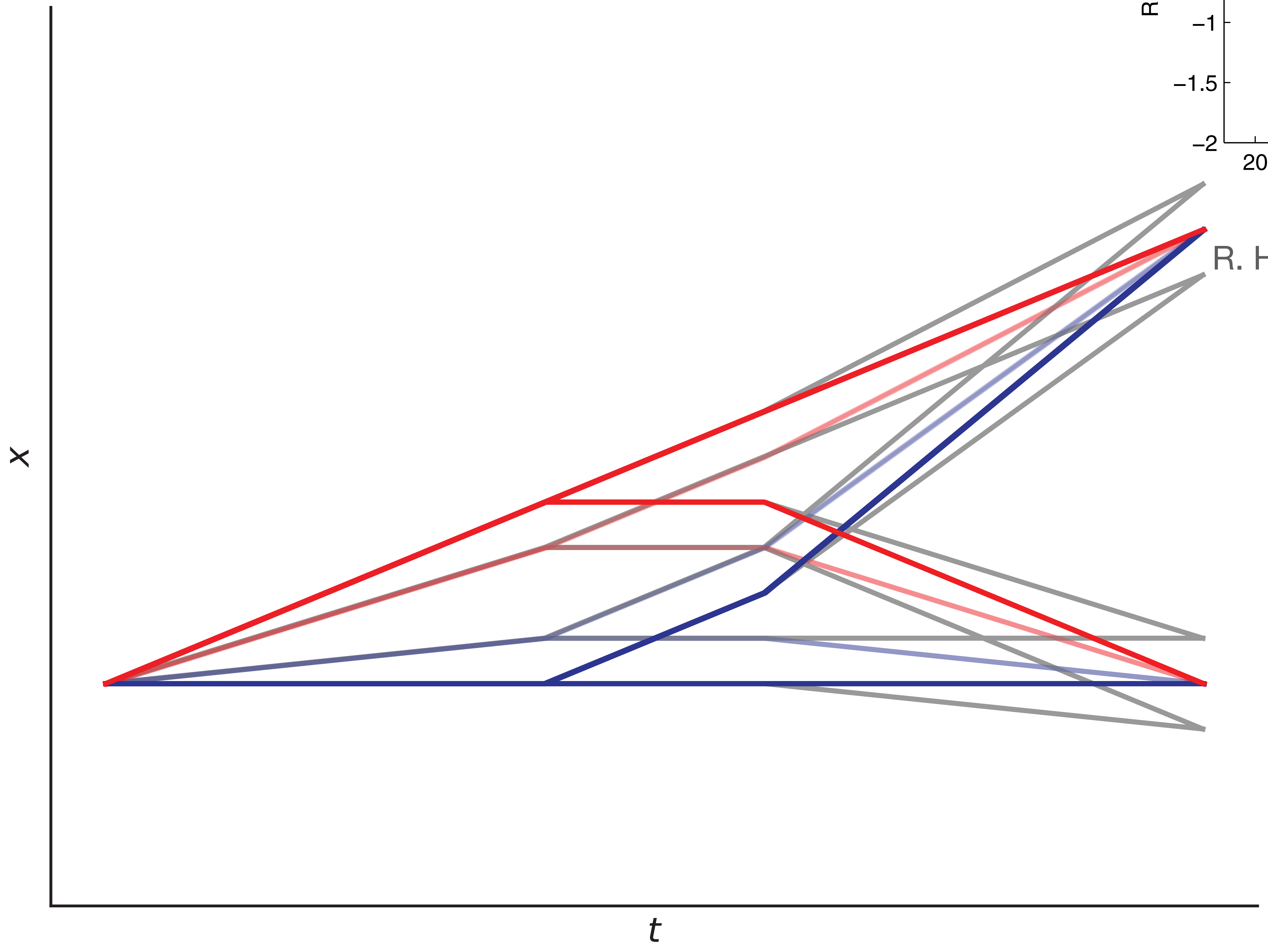


Parasitic ports

0 ± 0.03 ppb



R. H. Parker et al, Phys. Rev. A 94, 053618



Observation of Extra Photon Recoil in a Distorted Optical Field, S. Bade et al., Phys. Rev. Lett. (2018)

Gouy Phase

- $\boldsymbol{k}$ varies across Gaussian wavefront

$$\frac{\delta k_{\text{eff}}}{k_{\text{eff}}} = -\frac{\lambda^2}{2\pi^2 w_0^2}\left(1 - \frac{z_0^2}{z_{\mathrm{R}}^2} - \frac{r^2}{w_0^2}\right)$$

- Small scale intensity fluctuations lead to a shift in $\boldsymbol{k}$

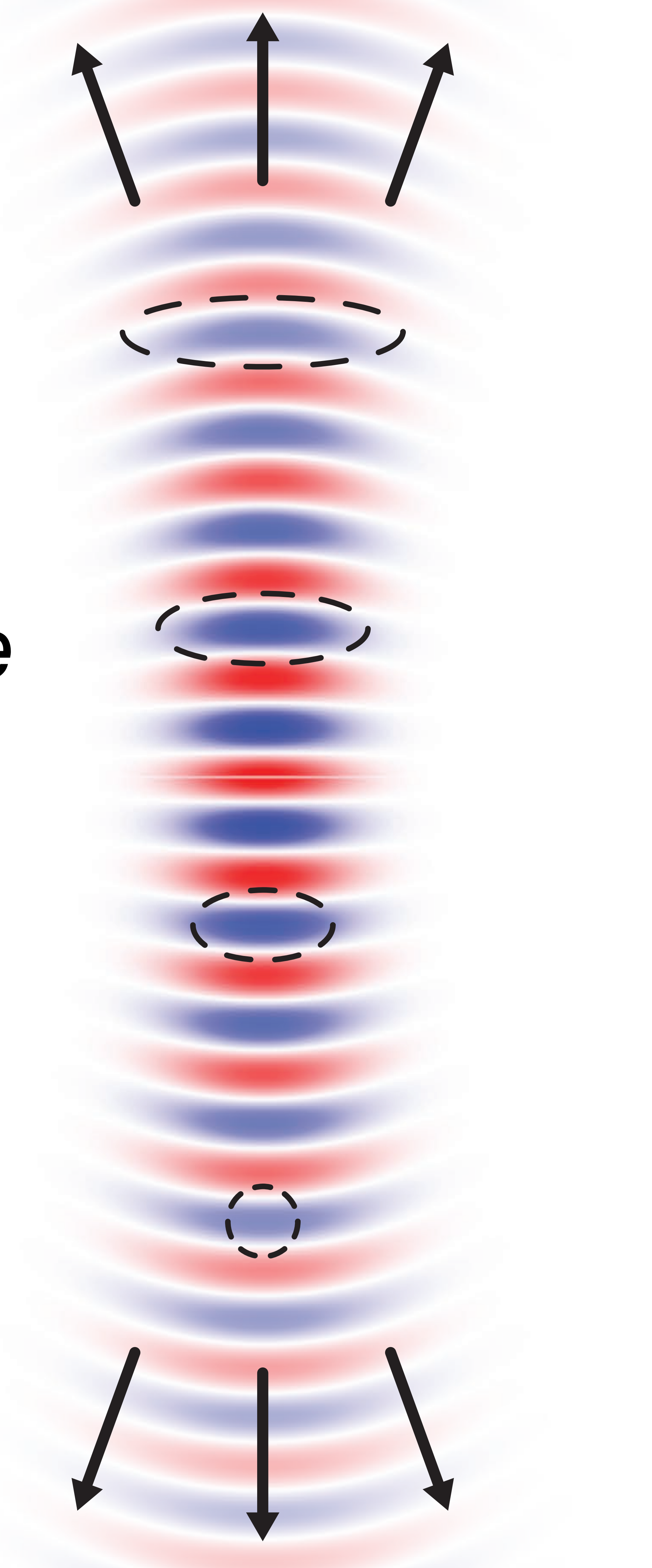
$$\frac{\delta k}{k} = -\frac{1}{2k^2}\frac{\boldsymbol{\nabla}_\perp^2 E}{E}$$

- Shift in k shifts $\omega_{\mathrm{r}} = \hbar k^2/2m$

- Also couples to thermal motion of atoms

phase front has curvature

$$k \neq \omega/c$$

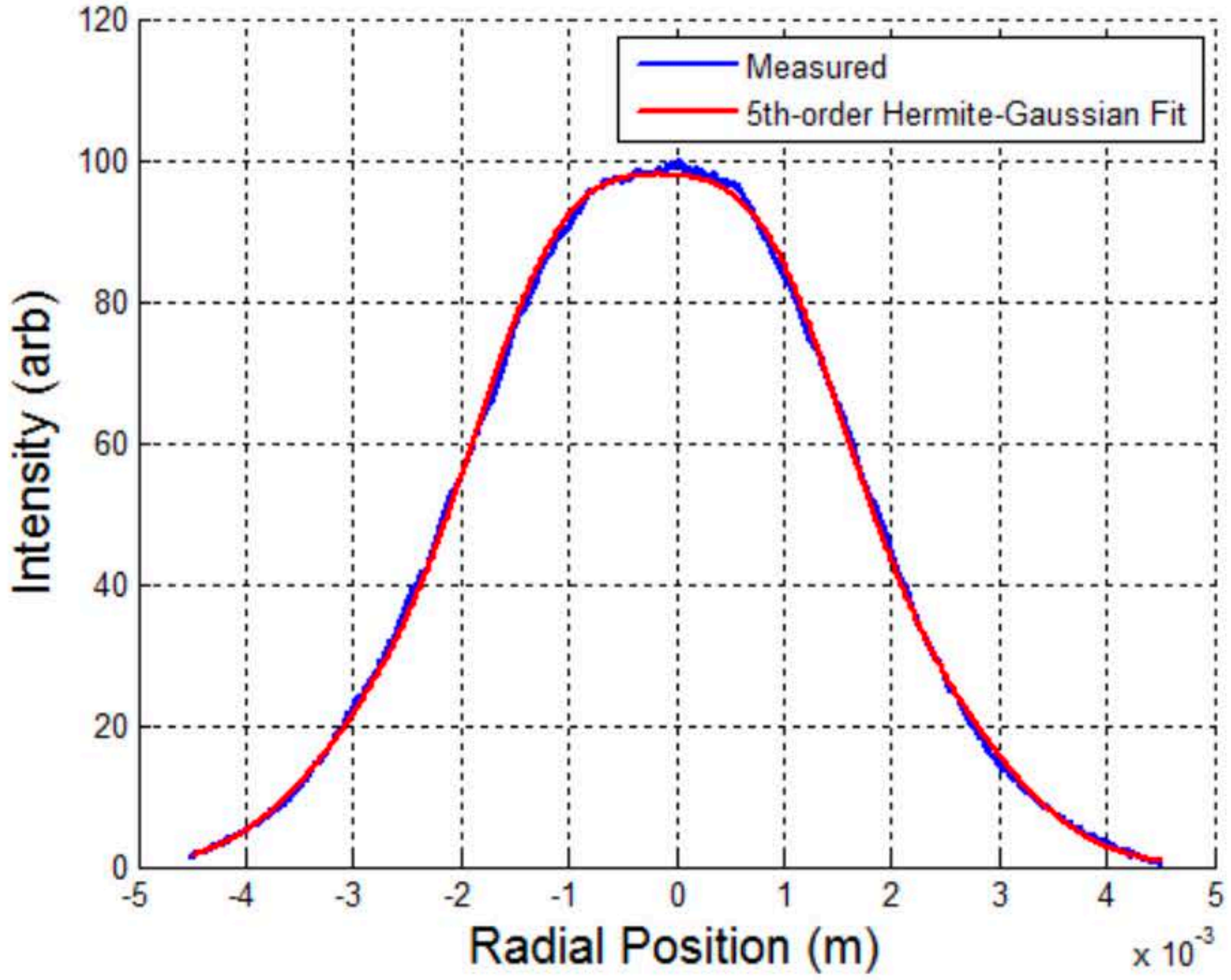


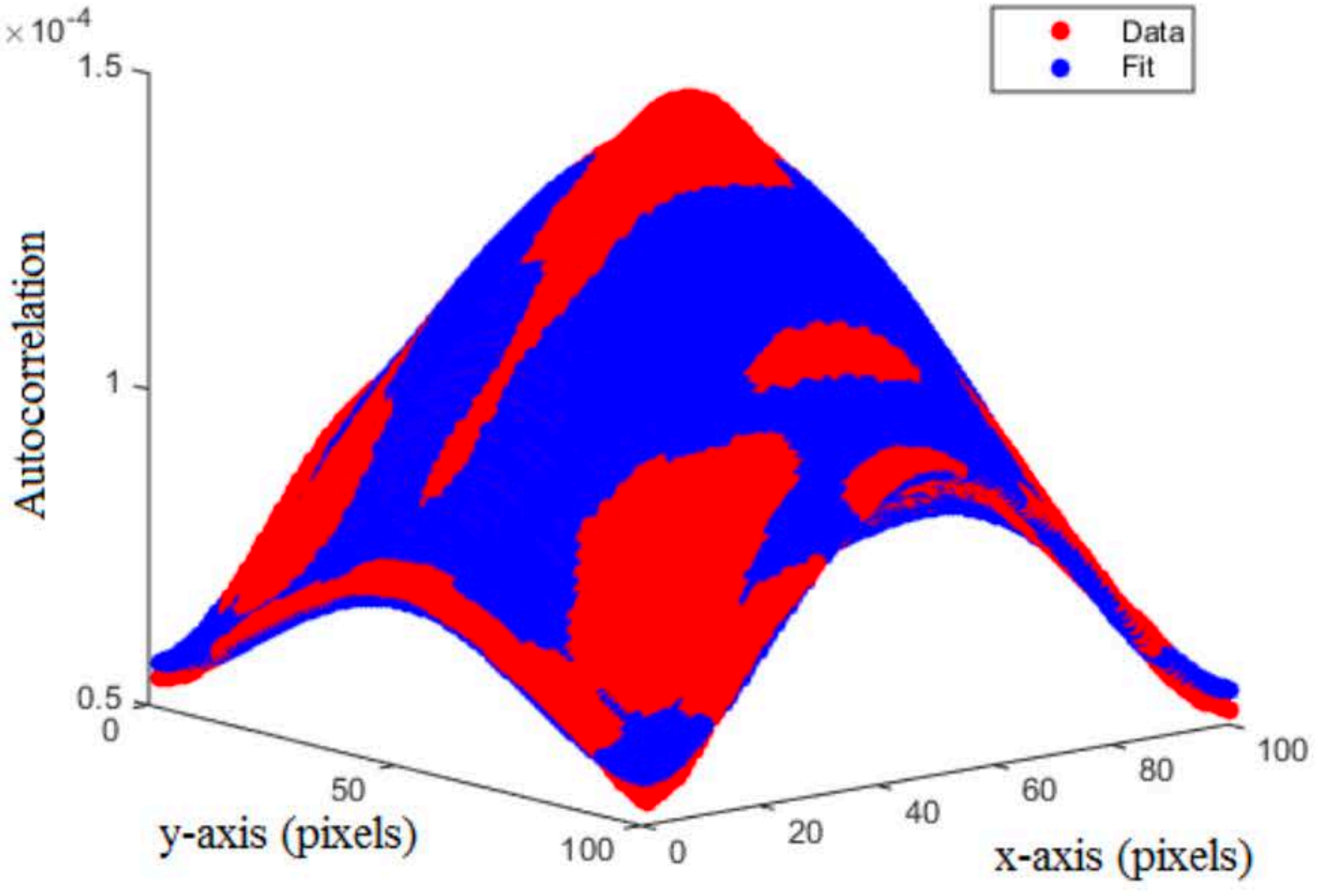
Observation of Extra Photon Recoil in a Distorted Optical Field, S. Bade et al., Phys. Rev. Lett. (2018)

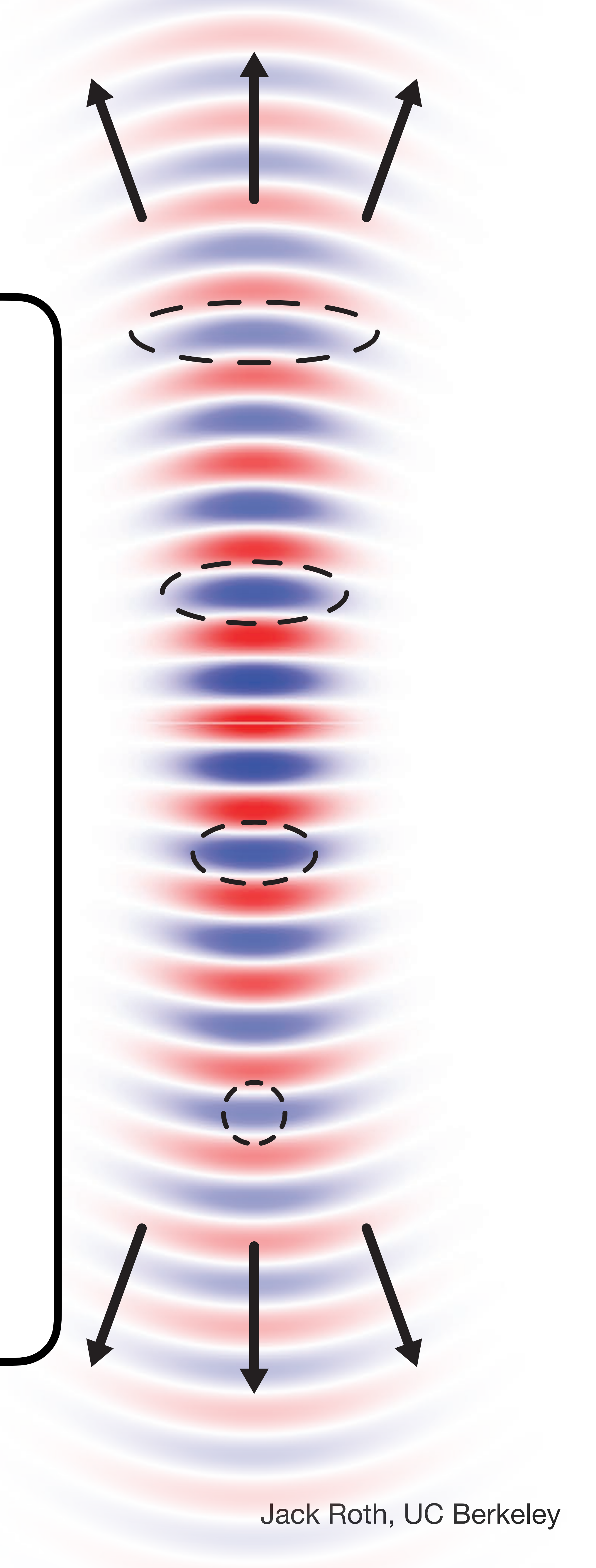
Gouy Phase

$\boxed{-2.60 \pm 0.03\ \text{ppb}}$

Beam profile measured, fed into Monte Carlo simulation





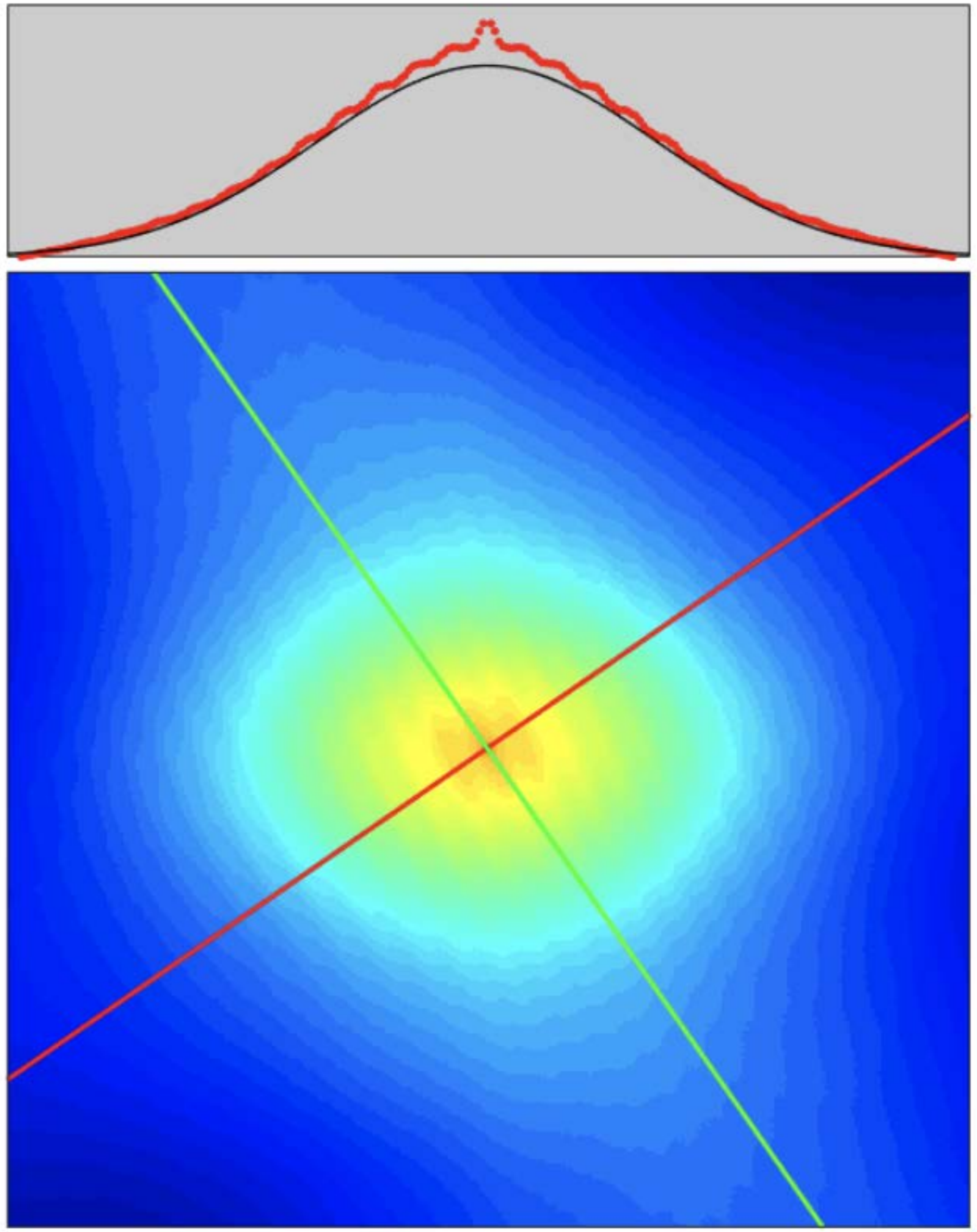


R. H. Parker et al., Science 360, 191-195 (2018)

Speckle

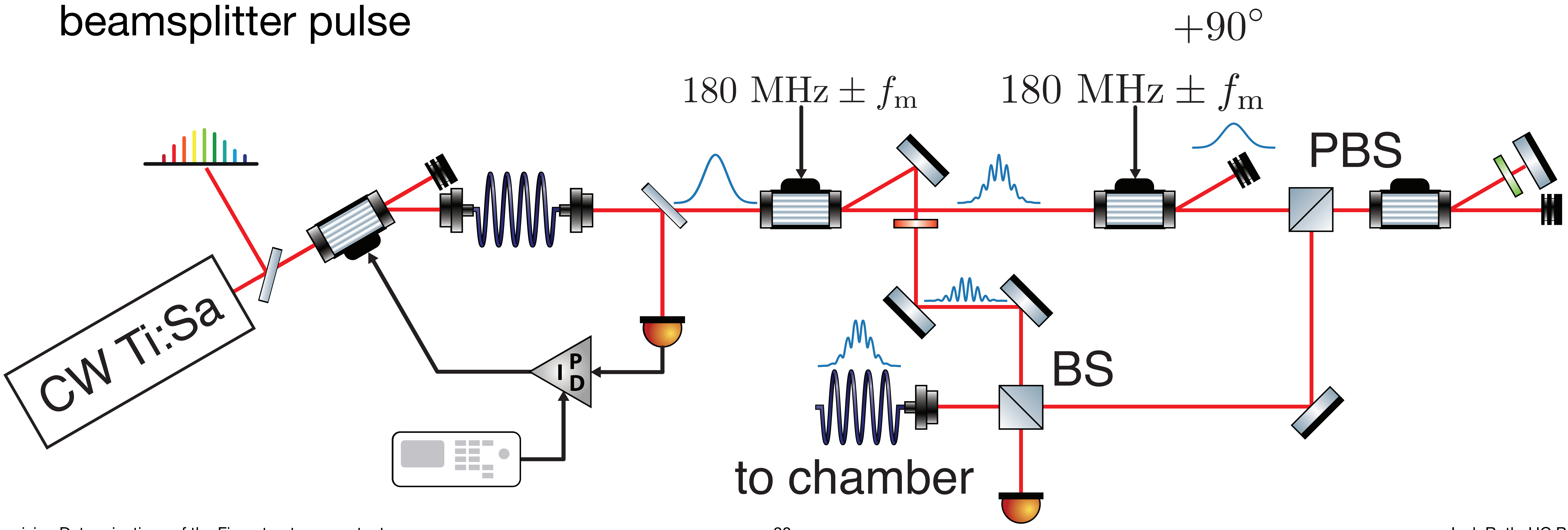
$\boxed{0 \pm 0.04\ \text{ppb}}$

- Causes small intensity ripples
- Causes shifts in $\mathbf{k}$
- Suppressed via apodizing filter



Non-Gaussian waveform

- Deviation of Gaussian intensity profile of beamsplitter pulse



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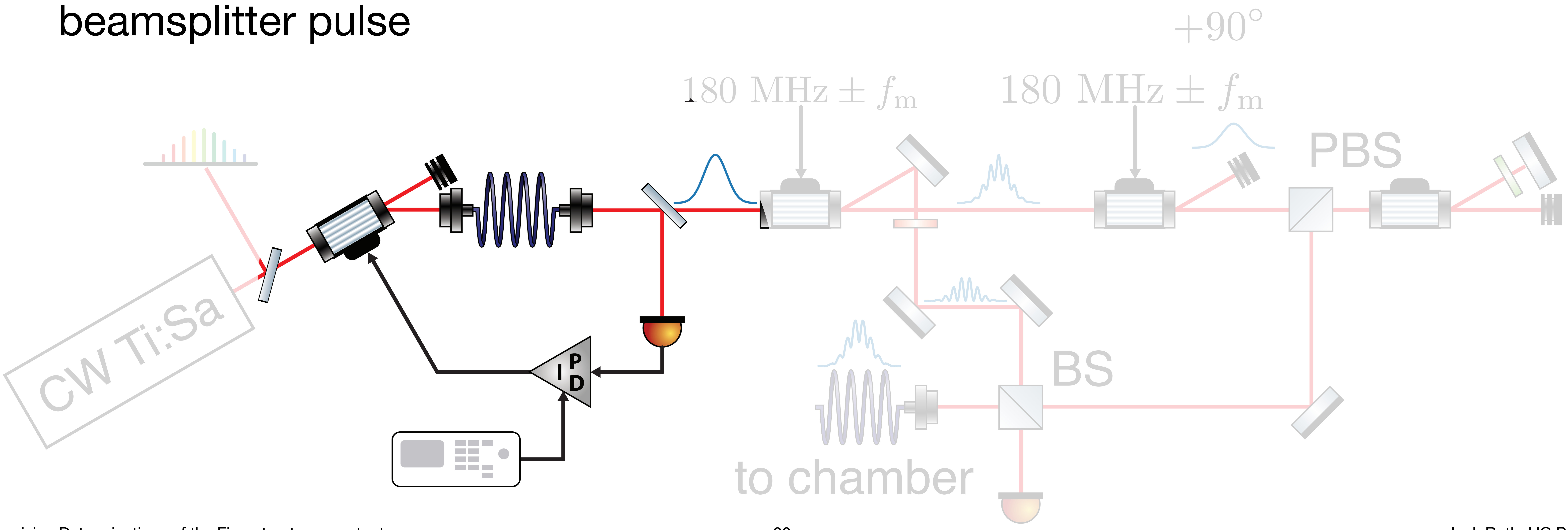

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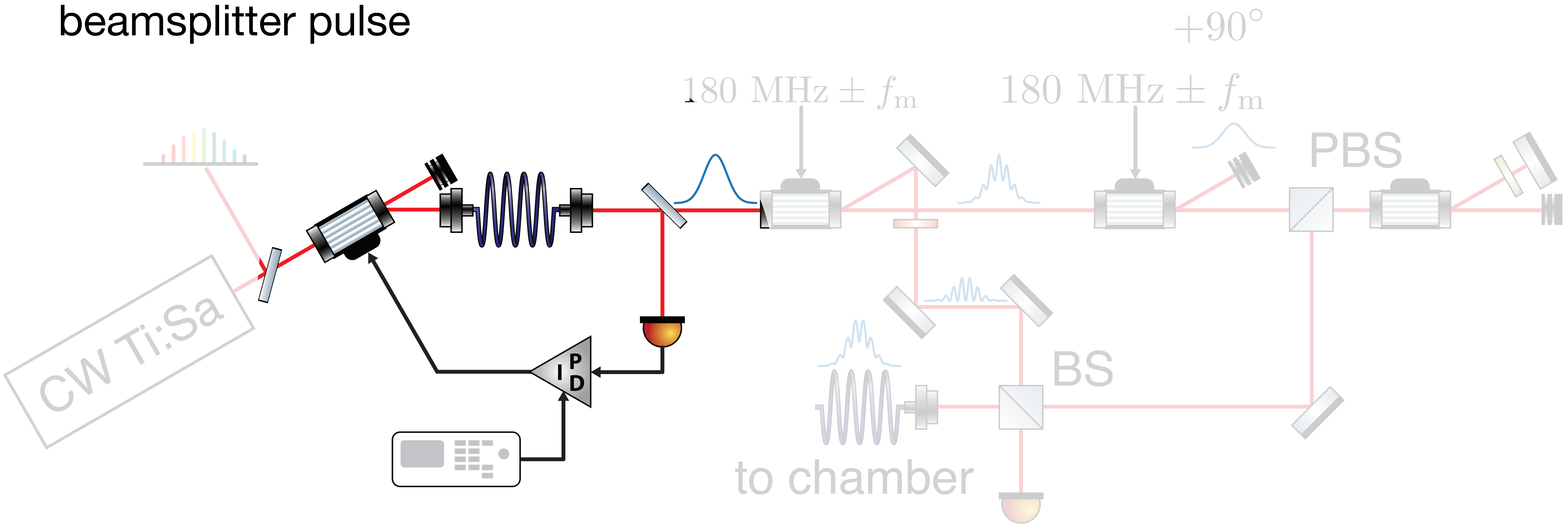

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- Approach to negating/understanding difficult systematics
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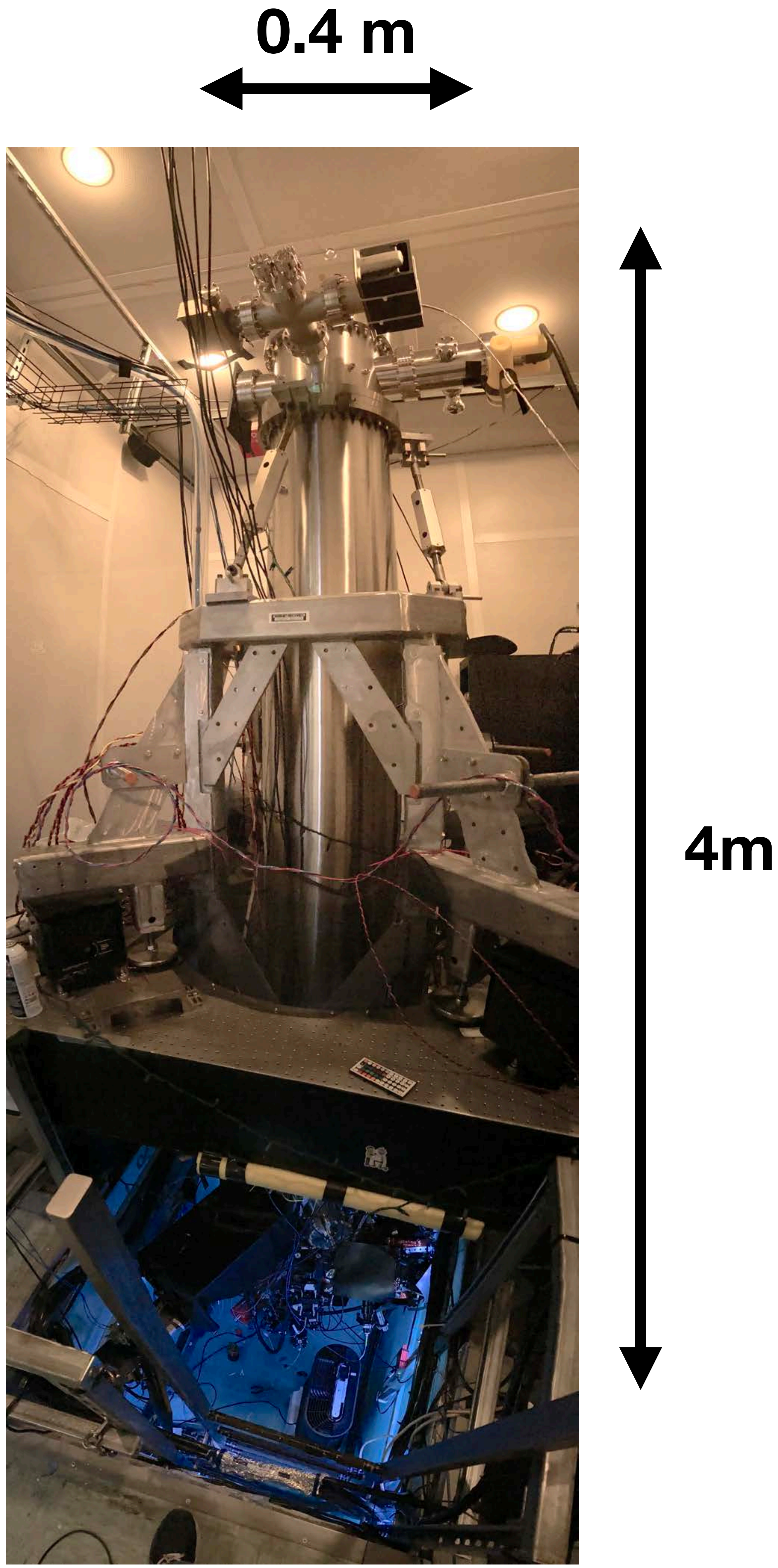
New apparatus

- New chamber
- New simulation
- New team



Chamber hardware upgrades

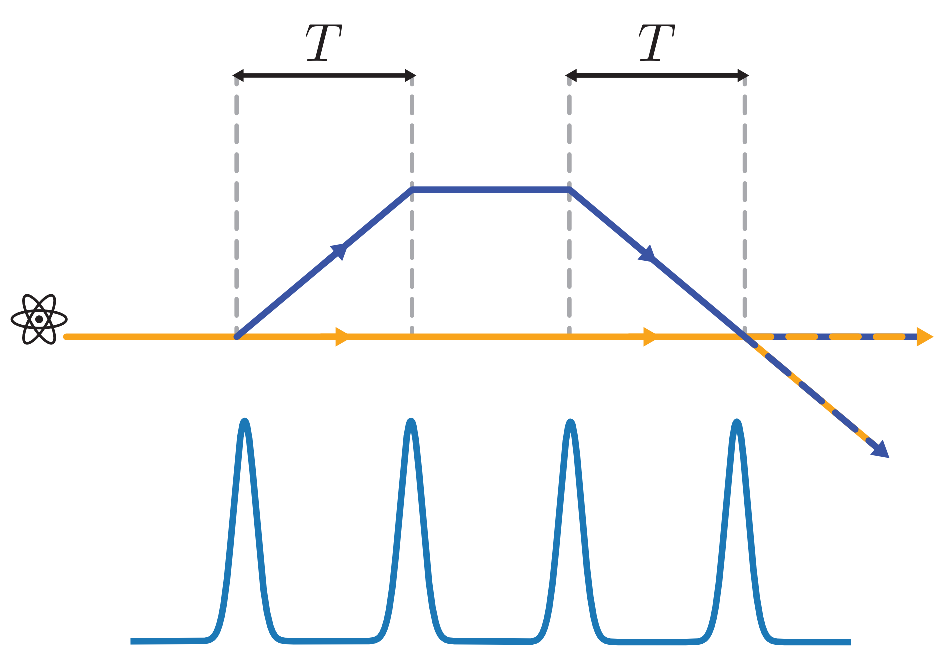
Larger beam → Reduced effect of thermal motion, Gouy phase, laser phase

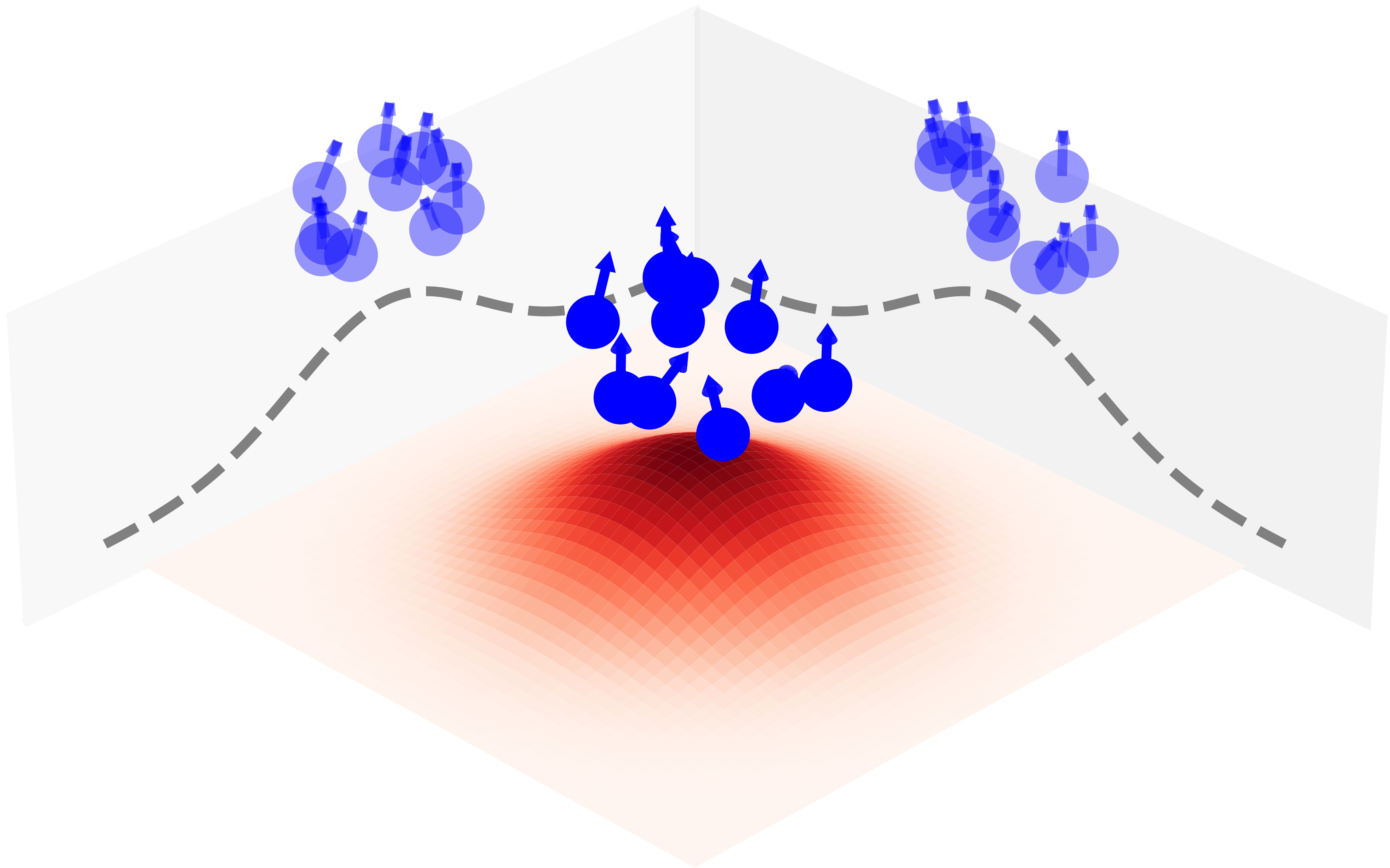
Larger chamber, baffles → Reduced speckle

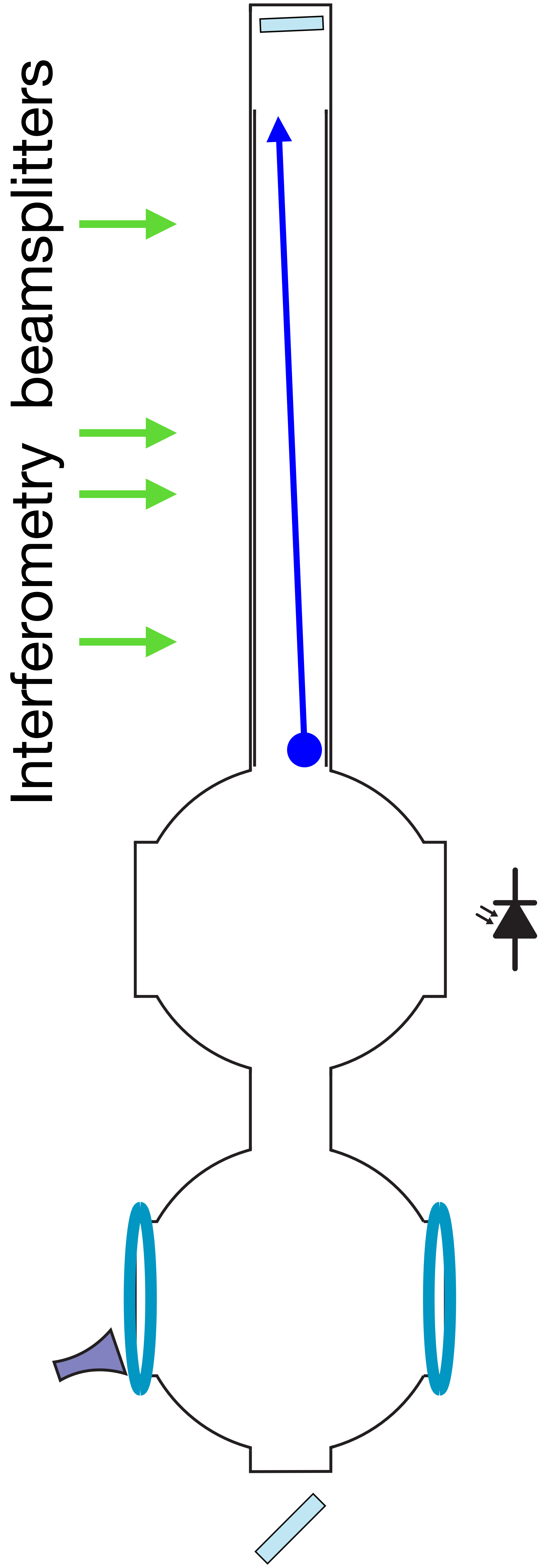
Atoms higher in chamber → Reduced beam imperfections

Simulation approach

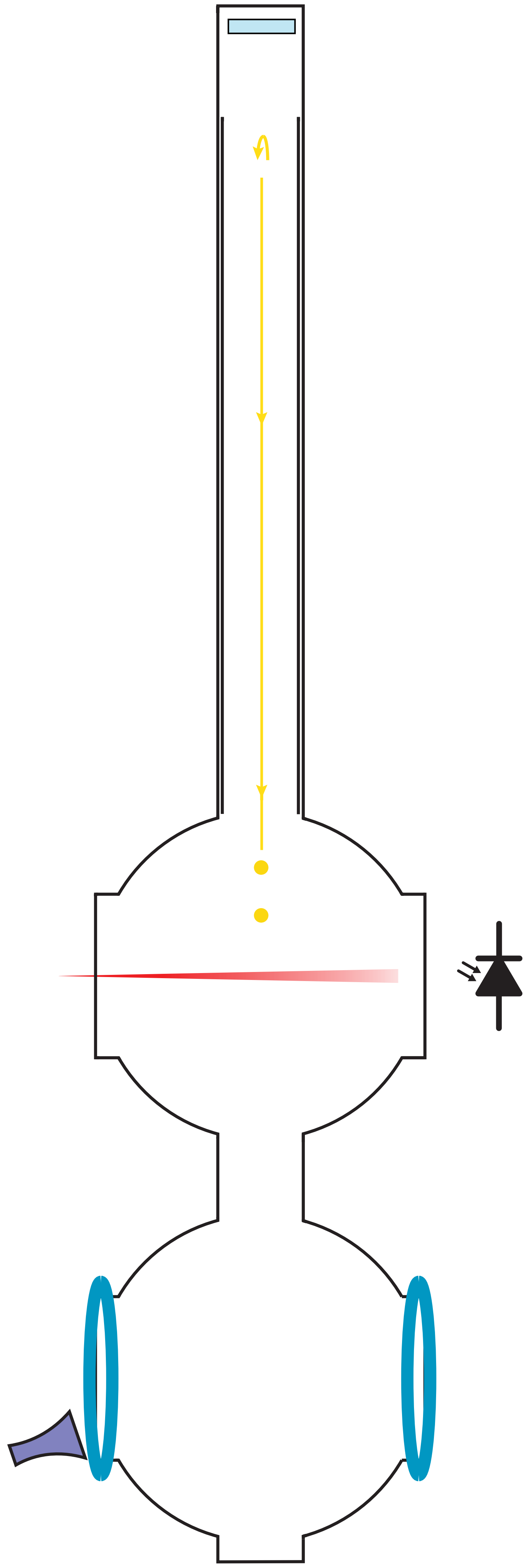
- One unified simulation for understanding all beam related systematics
- Attempting to verify each stage of the simulation before implementing next stage

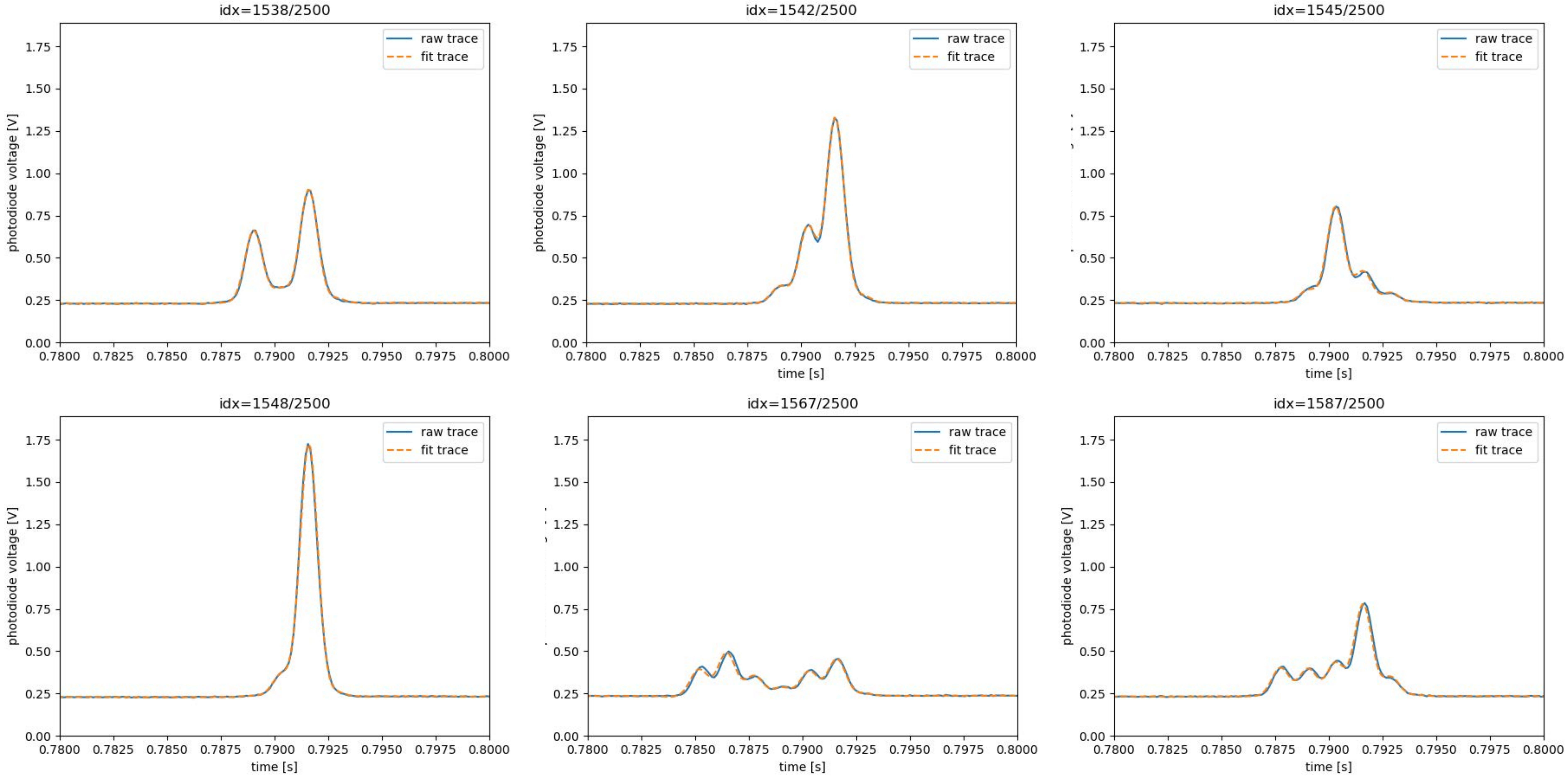






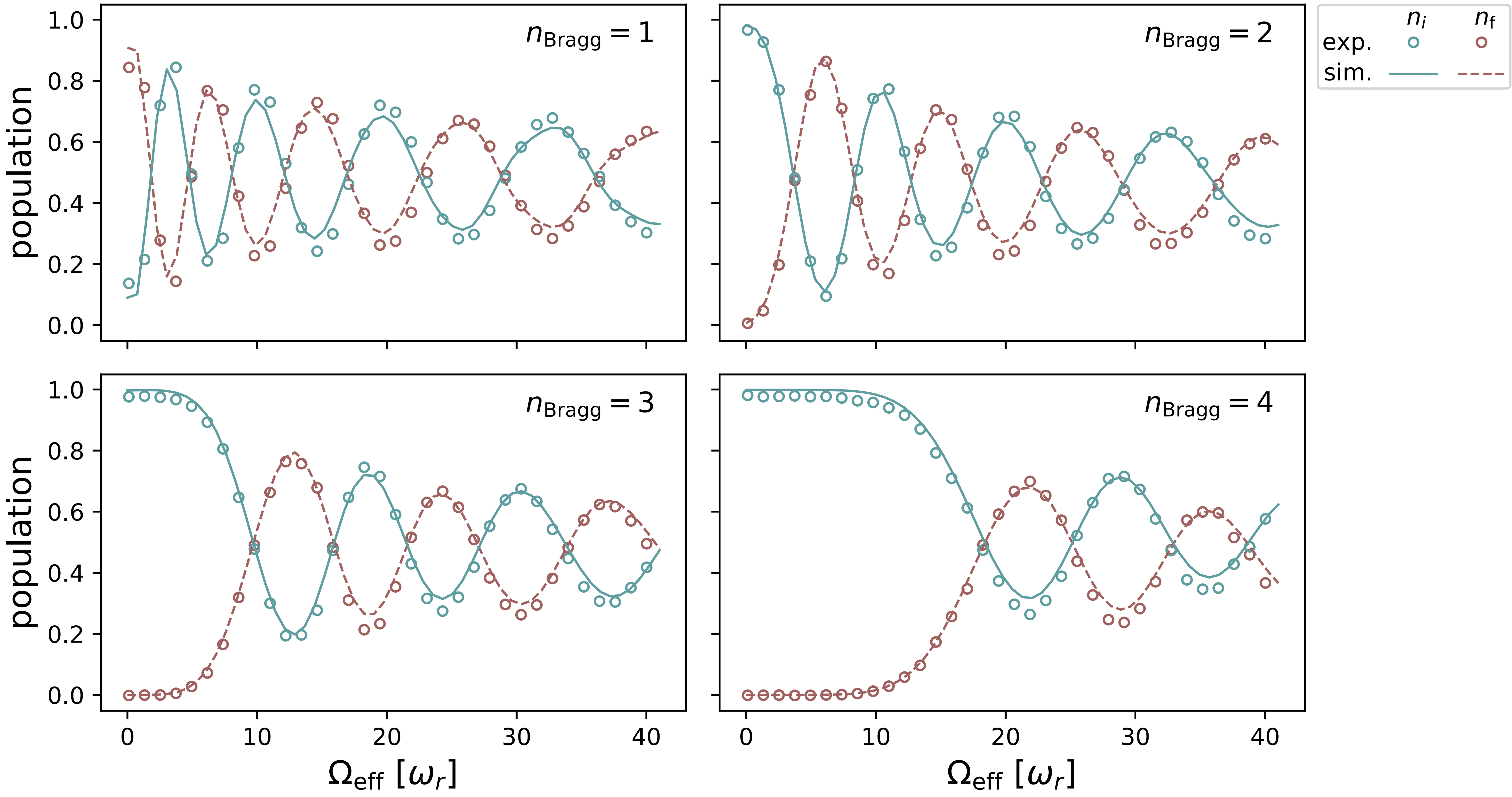
Matching simulation and experiment





Initial simulation results

- Completing simulation-experiment matching of single Bragg pulses
- Beginning to verify simulations of the full interferometer



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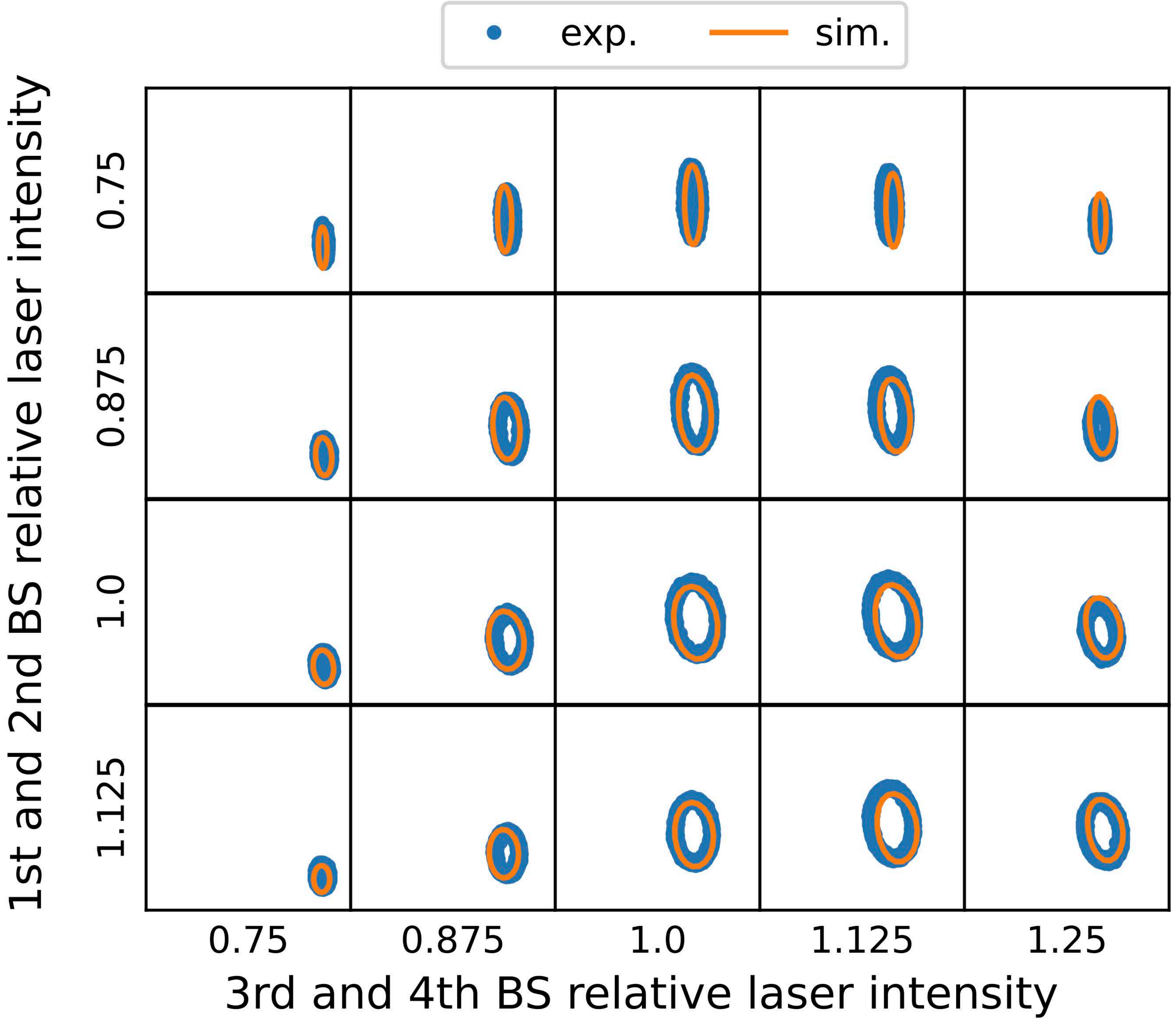
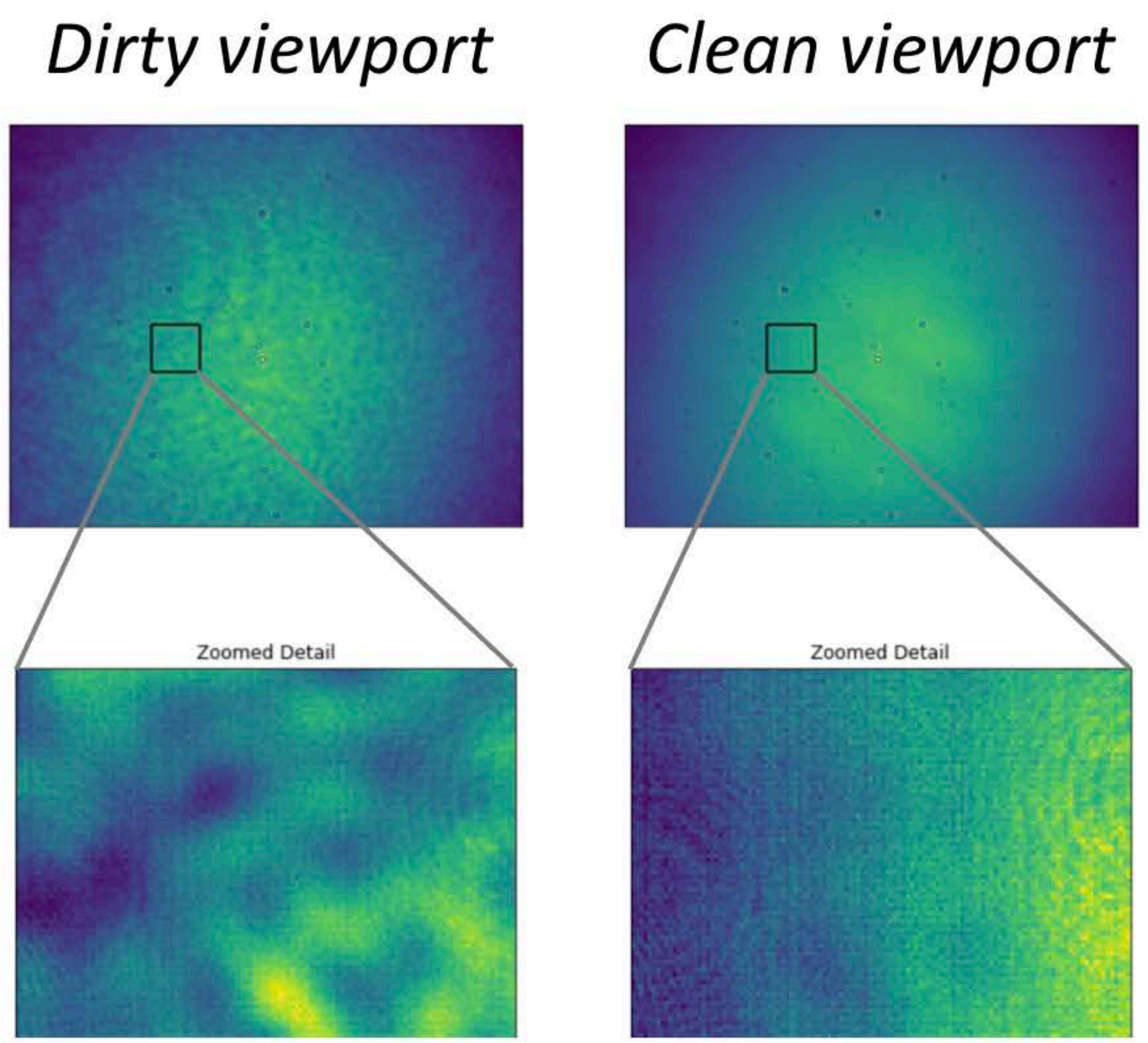


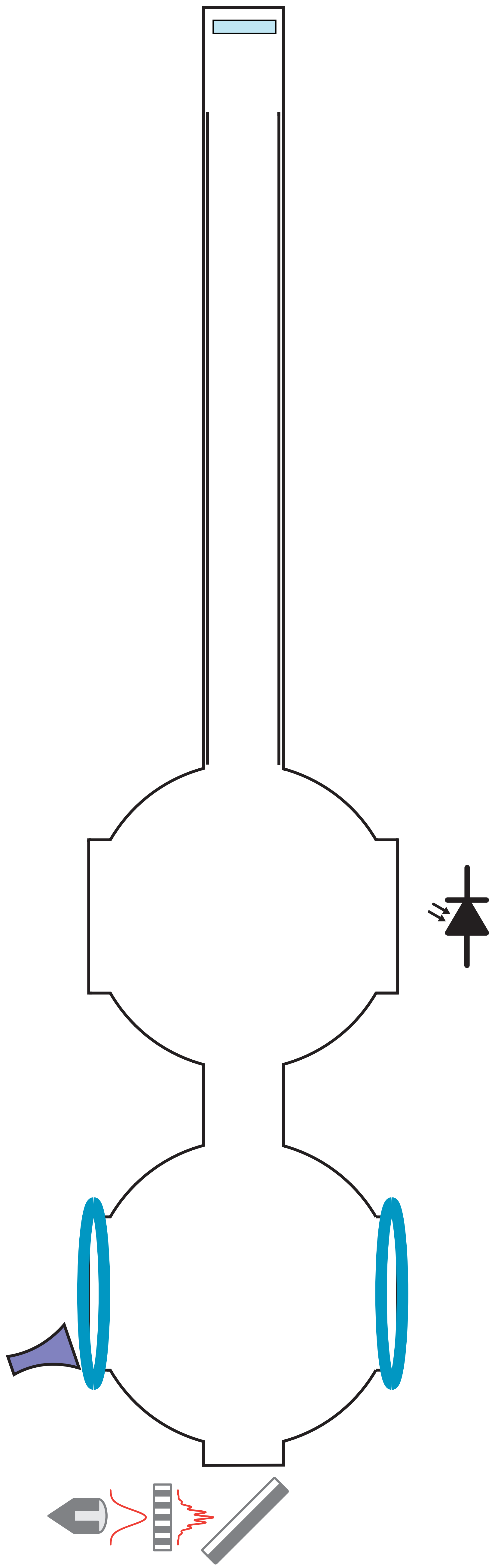
Outline

- ~~Review of the measurement technique~~
- ~~Review of systematic effects~~
- ~~Approach to negating/understanding difficult systematics~~
- Outlook

Outlook

- Experiment can now run 24/7, yields high quality datasets
- Next simulation-experiment matching
  - Gouy phase
  - Input cloud shaping
- New interferometer geometries?



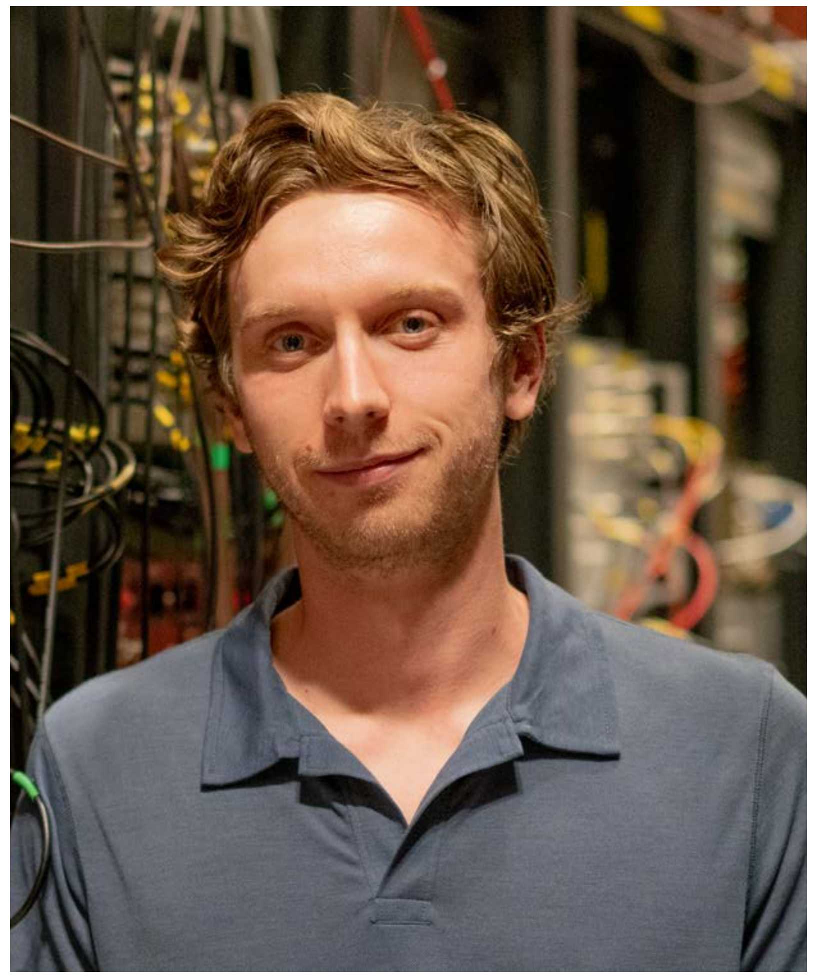


Team members

Holger Mueller

Madeline Bernstein

Andrew Christensen



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