Recent progress toward a new measurement of rubidium recoil using atom interferometry

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Physique quantique et applications

The fine-structure constant α

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \sim \frac{1}{137}$$

α : Dimensionless constant governing all electromagnetic interactions

Transition frequencies measurement

Muonium ground-state hyperfine splitting

$$\Delta\nu_{\mathrm{Mu}}(\mathrm{th}) = \Delta\nu_F \times \mathcal{F}(\alpha, m_e/m_\mu)$$

$$\Delta\nu_F = \frac{16}{3} c R_\infty Z^3 \alpha^2 \frac{m_e}{m_\mu}\left(1+\frac{m_e}{m_\mu}\right)^{-3}$$

Anomalous Magnetic Moment of the Electron

$$a_{\mathrm{e}}(\mathrm{theo}) \equiv a_{\mathrm{e}}(\exp)$$

Quantum Hall effect

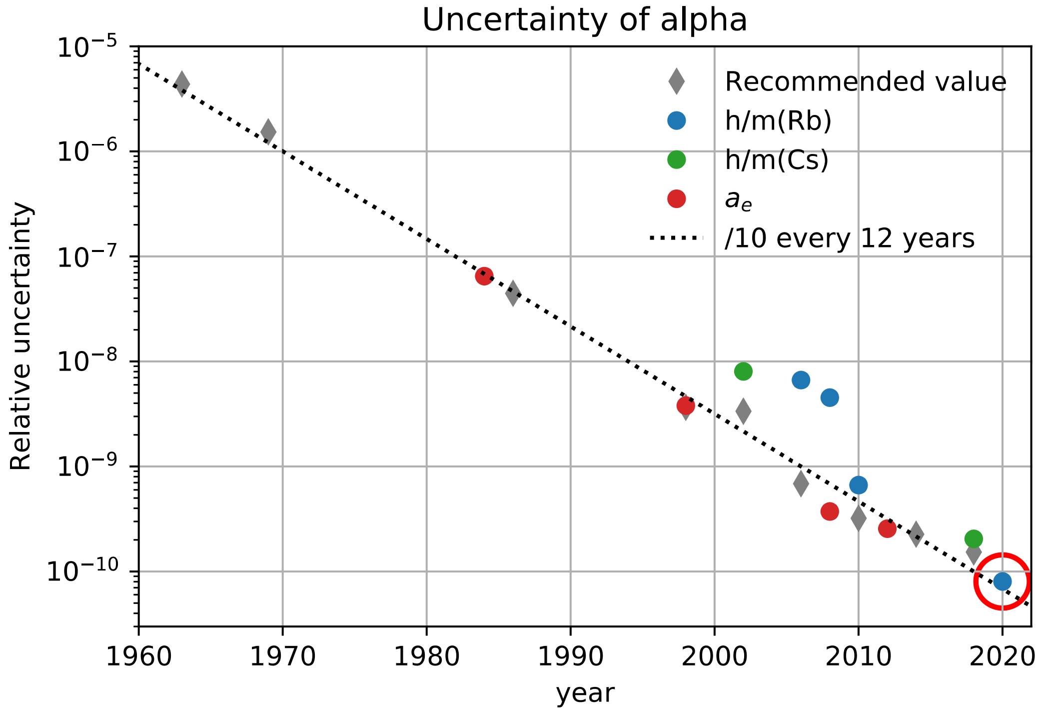
$$R_{\mathrm{K}} = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha}$$

Recoil measurement

$$\alpha^2 = \frac{2R_\infty}{c}\frac{m_{\mathrm{At}}}{m_{\mathrm{e}}}\frac{h}{m_{\mathrm{At}}}$$

Paris, Berkeley

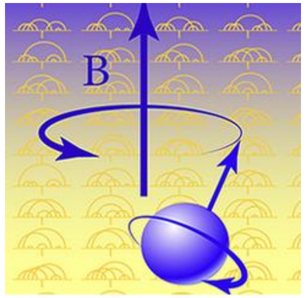
Test of the Standard Model, New physics ?



Test of the Standard at low energy scale

Electron magnetic moment ($\boldsymbol{g_e}$)

The most precisely measured quantity of a fundamental particle



$$\vec{\mu}_\mathrm{e} = -g_\mathrm{e}\mu_\mathrm{B}\frac{\vec{S}}{\hbar} \qquad \mu_\mathrm{B} = \frac{e\hbar}{2m_\mathrm{e}} : \text{Bohr magneton}$$

Measurement

$$\frac{g_e}{2} = 1.001\,159\,652\,180\,59\,(13) \quad [0.13\text{ ppt}]$$

X. Fan, et al., Phys. Rev. Lett. 130, 071801 (2023)

(See Gabrielse's talk)

=

or

≠

Standard Model prediction

$$\frac{g_e}{2} = 1 + \sum_{n=1}^{\infty} C_n \left(\frac{\alpha}{2}\right)^n + a_e(\text{Weak}) + a_e(\text{Hadron})$$

→ depends on α from atomic recoil measurement

Uncertainty in α is bottleneck for most precise tests of Standard Model

Electron magnetic's SM prediction

- Electron magnetic moment

$$\vec{\mu}_e = -g_e \mu_B \frac{\vec{S}}{\hbar} \qquad \mu_B = \frac{e\hbar}{2m_e} \text{ : Bohr magneton}$$

$$\frac{g_e}{2} = 1 + a_e$$

$$a_e(\mathrm{SM}) = a_e(\mathrm{QED}) + a_e(\mathrm{Hadron}) + a_e(\mathrm{Weak})$$

$$a_e(\mathrm{QED}) = \sum_{n=1}^{\infty} A_e^{(2n)} \left(\frac{\alpha}{\pi}\right)^n + \sum_{n=1}^{\infty} A_{\mu,\tau}^{(2n)} \left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right) \left(\frac{\alpha}{\pi}\right)^n$$

$$a_e[\alpha(\mathrm{Cs})] = 1\,159\,652\,181.59\,(23)(0)(3) \times 10^{-12}\ [0.20\ \mathrm{ppb}]$$

$$a_e[\alpha(\mathrm{Rb})] = 1\,159\,652\,180.238(82)(4)(30) \times 10^{-12}\ [0.075\ \mathrm{ppb}]$$

hadronic contribution
tenth-order QED
Fine-structure constant

- Recent update

- New evaluation of the tenth order
- New evaluation of hadronic contribution

Adopting the simple mean of KNT19 and KNT19/CMD-3

$$a^e_{\mathrm{HVP,\ LO}} = 1.89(3) \times 10^{-12}$$

$$a^e_{\mathrm{HVP,\ NLO}} = -0.2263(35) \times 10^{-12}$$

$$a^e_{\mathrm{HVP,\ NNLO}} = 0.02799(17) \times 10^{-12}$$

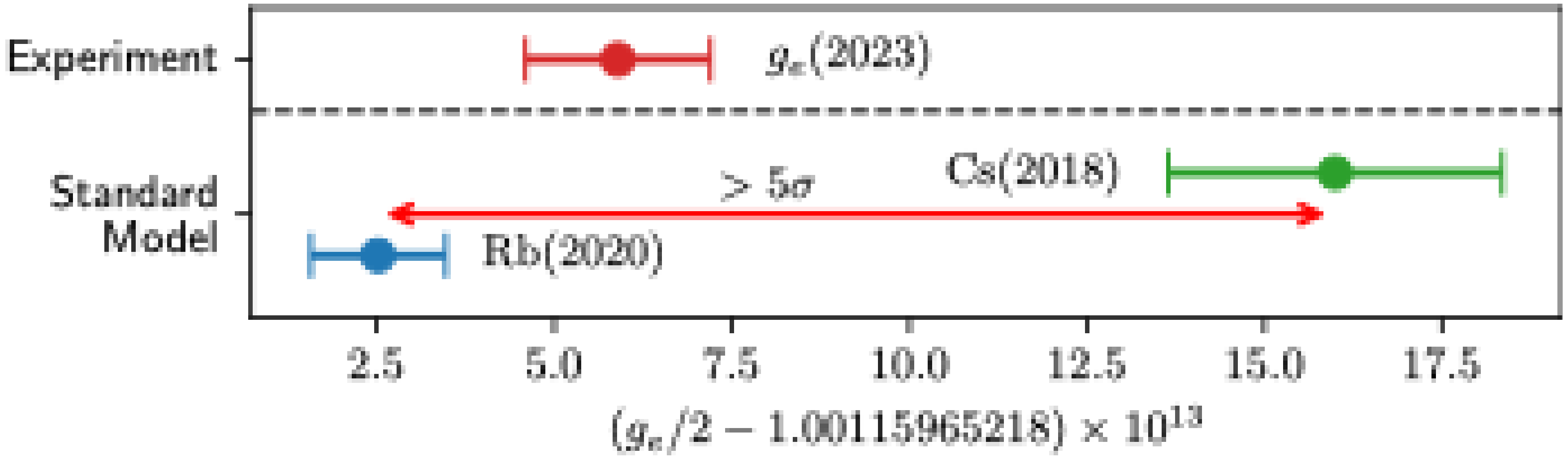
$$a^e_{\mathrm{HLbL}} = 0.0351(23) \times 10^{-12}$$

$$a^e_{\mathrm{EW}} = 0.03053(23) \times 10^{-12}$$

- R. Aliberti et al., https://arxiv.org/abs/2505.21476
- T. Aoyama, T. Kinoshita, M. Nio, Phys. Rev. D 2018, 97, 036001.
- S. Laporta, Phys. Lett. B 2017, 772, 232–238.
- T. Aoyama, T. Kinoshita and M. Nio, Atoms 2019, 7, 28.
- R.H. Parker et al, Science 2018, 360, 191–195.
- L. Morel et al., Nature 588, 61-68 (2020)

α from atomic recoil measurement: state-of-the art

So far only two group are measuring atomic recoil with atom interferometers - Berkeley and Paris - α at the level of 10^{-10}



Testing Standard Model at current precision of the electron measurement requires that this Cs/Rb discrepancy be resolved.

- R.H. Parker et al, Science 2018, 360, 191–195.
- L. Morel et al., Nature 588, 61-68 (2020)

Outline

- Measurement of the ratio *h*/*m* using atom interferometry based on Raman diffraction
- Recent work on the Paris experiment

Fine-structure constant from the photon recoil measurement

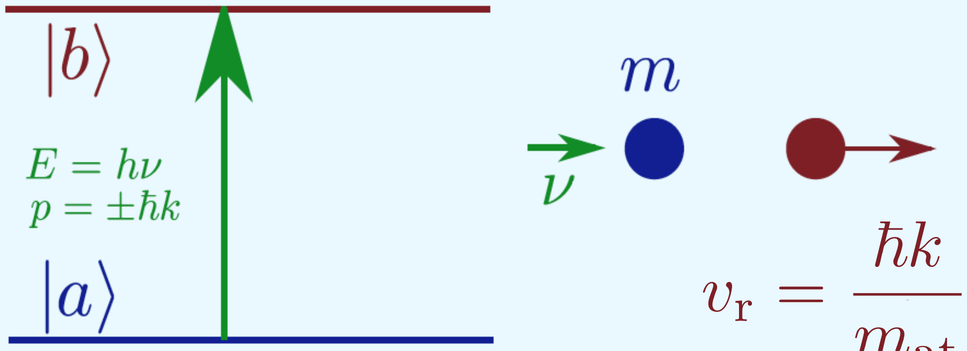
- Hydrogen atom: $hcR_\infty = \frac{1}{2} m_e \alpha^2 c^2$

$$\alpha^2 = \frac{2R_\infty}{c}\frac{h}{m_e} = \frac{2R_\infty}{c}\frac{A_r(\mathrm{at})}{A_r(\mathrm{e})}\frac{h}{m_{\mathrm{at}}}$$

Measured quantity	Relative uncertainty
Rydberg constant	1.9×10^{-12}
A_r (e)	1.8×10^{-11}
Ar(^{87}Rb)	7.0×10^{-11}

- Limitation: $\frac{h}{m_{\mathrm{at}}}$ (or absolute atomic mass in the new SI)

- G. Audi et al., 2014 Nuclear Data Sheets 120, 1-5 (2014)
- S. Sturm et al. Nature 506, 476-470 (2014),
- E. Tiesinga et al., Rev. Mod. Phys. 93, 025010

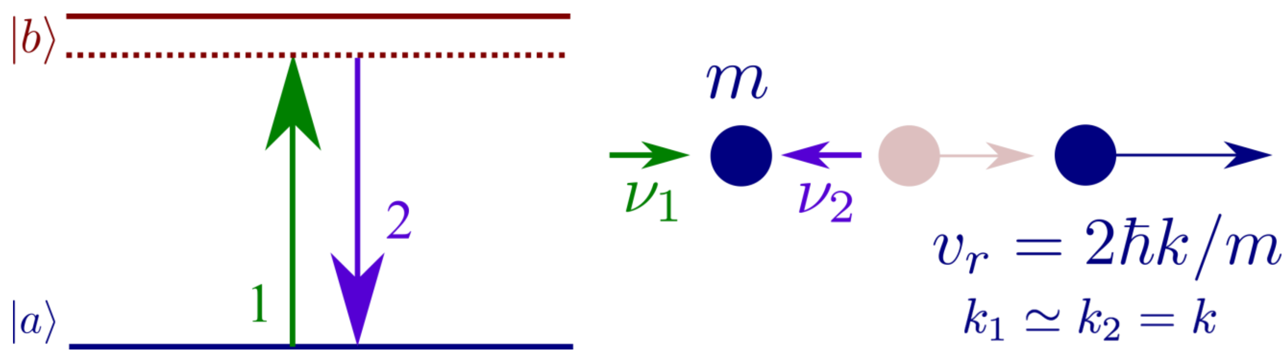


$$v_r = \frac{\hbar k}{m_{\mathrm{at}}}$$

$k = 2\pi/\lambda$: wave vector

$v_r = 5.9$ mm/s for ^{87}Rb and 3.5 mm/s for ^{133}Cs

Measurement of the recoil velocity



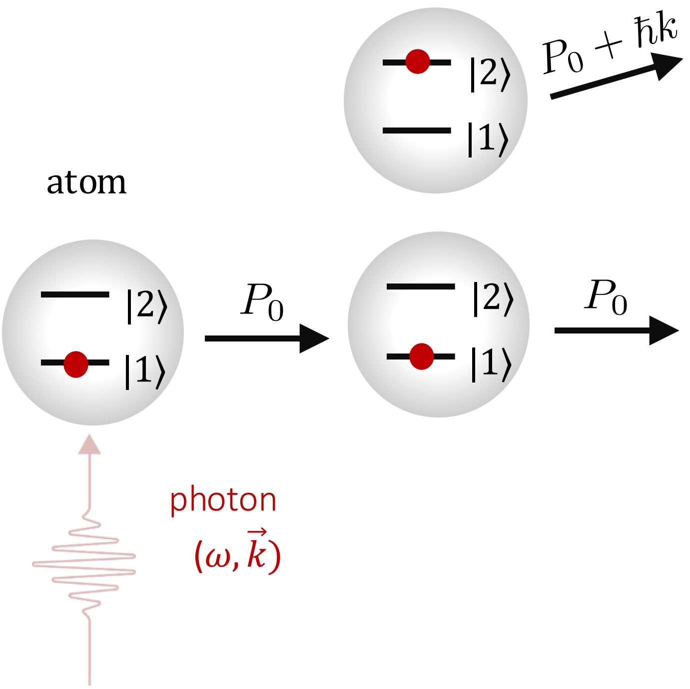
- Transfer to the atoms a large number *N* of photon momenta
  ⇒ Coherent acceleration in an optical lattice (Bloch oscillations)

- Quantum velocity sensor
  ⇒ Atom interferometer based on Raman transitions with a sensitivity: σ_v

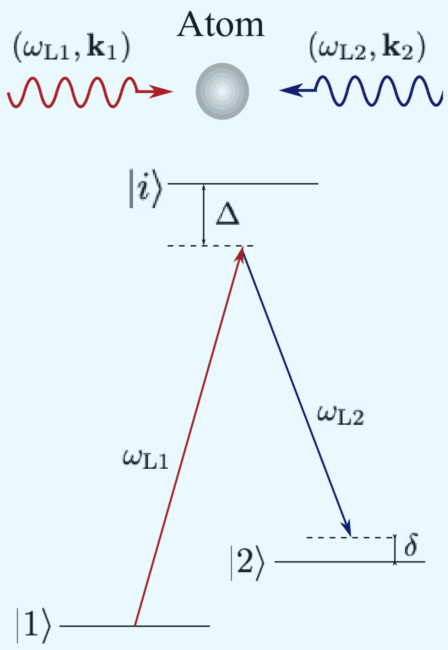
$$\sigma_{v_\mathrm{r}} = \frac{\sigma_v}{N}$$

Atom interferometry using Raman diffraction

- Atomic beam splitter



- Stimulated Raman transition



$\Delta \gg \delta$ and $\Delta \gg \Gamma$

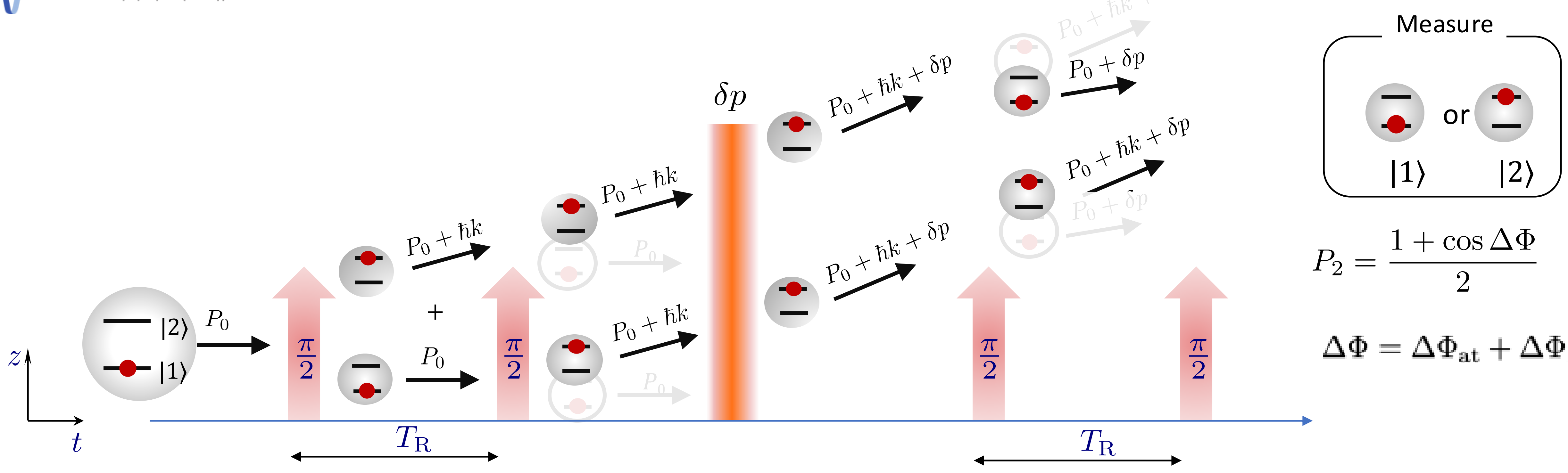
At resonance

$$\Psi(t) = \mathrm{e}^{-i\omega_1 t} \cos\left(\frac{\Omega t}{2}\right) |1, \mathbf{p}_0\rangle + \mathrm{e}^{i\Delta\phi_L} \mathrm{e}^{-i\omega_2 t} \mathrm{e}^{-i\pi/2} \sin\left(\frac{\Omega t}{2}\right) |2, \mathbf{p}_0 + \hbar\mathbf{k}\rangle$$

$$\Omega\tau = \frac{\pi}{2} \longrightarrow \Psi(\tau) = \frac{1}{\sqrt{2}} \left(\mathrm{e}^{-i\omega_1 \tau} |1, \mathbf{p}_0\rangle + \mathrm{e}^{i\Delta\phi_L} \mathrm{e}^{-i\omega_2 \tau} \mathrm{e}^{-i\pi/2} |2, \mathbf{p}_0 + \hbar\mathbf{k}\rangle\right)$$

- Contra-propagation laser beams: velocity sensitive Raman transitions
- The internal degrees of freedom are labelled by the external degrees

Velocity sensor based on atom interferometry using Raman diffraction



$$P_2 = \frac{1 + \cos \Delta\Phi}{2}$$

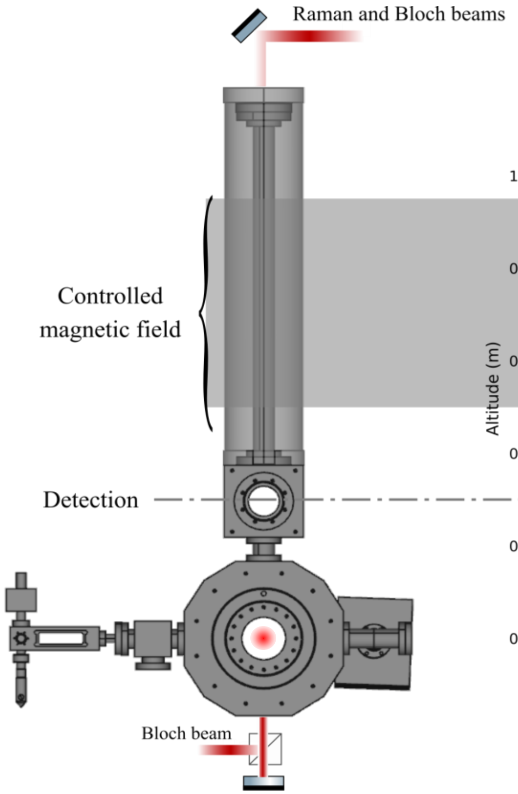
$$\Delta\Phi = \Delta\Phi_{at} + \Delta\Phi_L$$

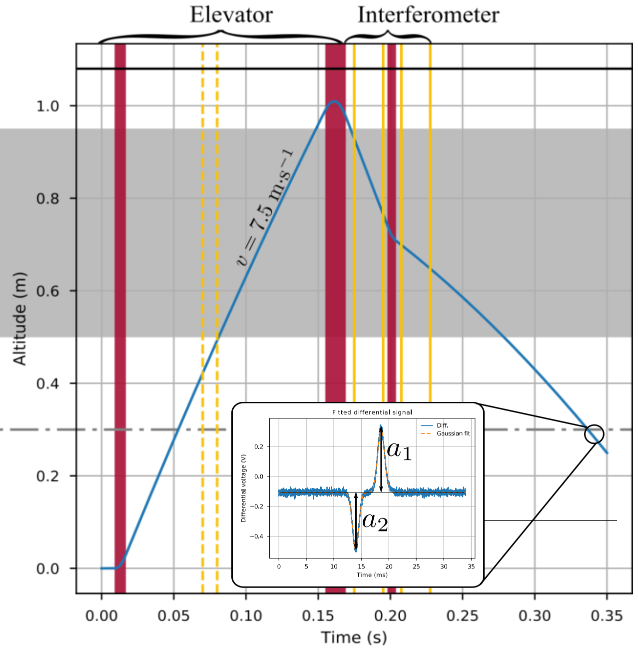
- Feynman Path Integral: $\Delta\Phi_{at} = \frac{1}{\hbar}\int \frac{m}{2}(v_A^2 - v_B^2)\,dt = \frac{m}{\hbar}\int \frac{v_A + v_B}{2}(v_A - v_B)\,dt$

- Atomic phase shift: $\Delta\Phi_{at} = T_R \times k \times \delta v + \Delta\Phi_{int}$ where $\delta v = N v_r$ and typically $N = 1000$
- We scan the phase of the laser to compensate (probe) the atomic phase: $\Delta\Phi_L = \Delta\nu T_R$

Experimental method

Optical molasses =10^8 atoms (^{87}Rb) @ T=4 μK, size = 3 mm

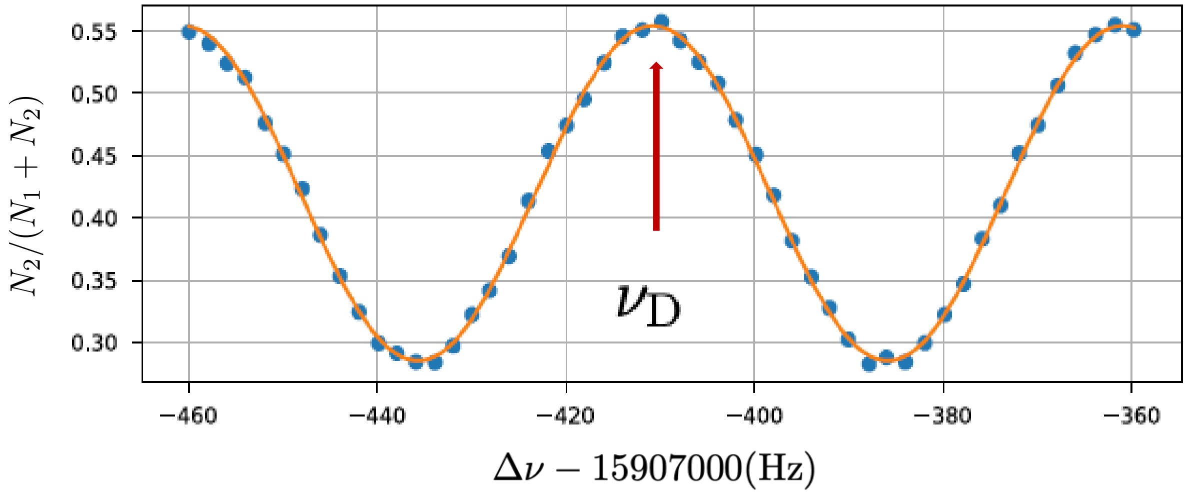




- Bloch pulses
  (Bloch oscillations in accelerated optical lattice)
- Raman laser pulses

- We scan the frequency of the laser to measure the Doppler shift due to $\delta v = N v_r$

$$\Delta\nu^0 = -15907410.770 \pm 0.047\mathrm{Hz}$$



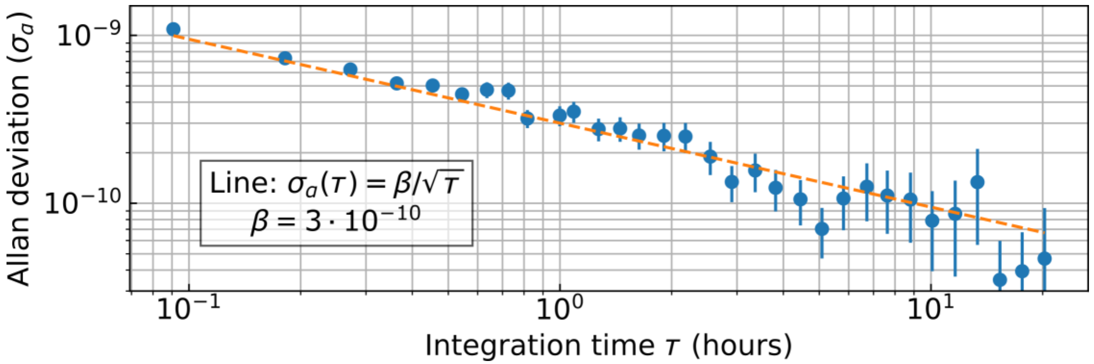
$$2\pi\nu_D = 2N k_R k_B \frac{\hbar}{m}$$

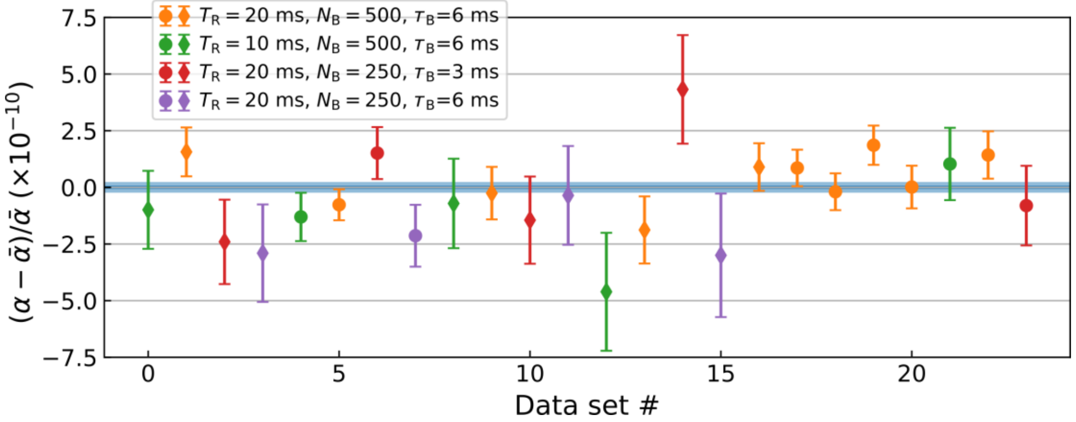
$$\sigma_v = 47\ \mathrm{mHz} \longrightarrow 20\ \mathrm{nm/s} \longrightarrow 3\times 10^{-9}\ \text{on}\ h/m \quad (1\mathrm{min})$$

2020 measurement

L. Morel et al., Nature 588, 61-68 (2020)

$$\alpha^{-1} = 137.035999206(11)$$





- Relative uncertainty of 8.1×10^{-11}
- Statistical uncertainty of 4.3×10^{-11} on 48h
- New systematic effects were considered

Source	Correction $[10^{-11}]$	Relative uncertainty $[10^{-11}]$
Gravity gradient	-0.6	0.1
Alignment of the beams	0.5	0.5
Coriolis acceleration		1.2
Frequencies of the lasers		0.3
Wave front curvature	0.6	0.3
Wave front distortion	3.9	1.9
Gouy phase	108.2	5.4
Residual Raman phase shift	2.3	2.3
Index of refraction	0	< 0.1
Internal interaction	0	< 0.1
Light shift (two-photon transition)	-11.0	2.3
Second order Zeeman effect		0.1
Phase shifts in Raman phase lock loop	-39.8	0.6
Global systematic effects	64.2	6.8
Statistical uncertainty		2.4
Relative mass of ^{87}Rb [16] : 86.909 180 531 0(60)		3.5
Relative mass of the electron [14] : $5.485\,799\,090\,65(16) \cdot 10^{-4}$		1.5
Rydberg constant [14] : $10\,973\,731.568\,160(21)\mathrm{m}^{-1}$		0.1
Total: $\alpha^{-1} = 137.035\,999\,206(11)$		8.1

Error budgets

Paris 2020

Source	Correction $[10^{-11}]$	Relative uncertainty $[10^{-11}]$
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Berkeley 2018

Effect	Section	δα/α (ppb)
This study		
Laser frequency	1	−0.24 ± 0.03
Acceleration gradient	4A	−1.79 ± 0.02
Gouy phase	3	−2.60 ± 0.03
Beam alignment	5	0.05 ± 0.03
Bloch oscillation light shift	6	0 ± 0.002
Density shift	7	0 ± 0.003
Index of refraction	8	0 ± 0.03
Speckle phase shift	4B	0 ± 0.04
Sagnac effect	9	0 ± 0.001
Modulation frequency wave number	10	0 ± 0.001
Thermal motion of atoms	11	0 ± 0.08
Non-Gaussian waveform	13	0 ± 0.03
Parasitic interferometers	14	0 ± 0.03
Total systematic error	All previous	−4.58 ± 0.12
Statistical error	N/A	±0.16
Other studies		
Electron mass (*16*)	N/A	±0.02
Cesium mass (*6, 15*)	N/A	±0.03
Rydberg constant (*6*)	N/A	±0.003
Combined result		
Total uncertainty in α	N/A	±0.20

Our strategy to solve 5 σ discrepancy

- New method to probe in-situ the spatial laser beam profile

  Better evaluation of the wave vectors ?

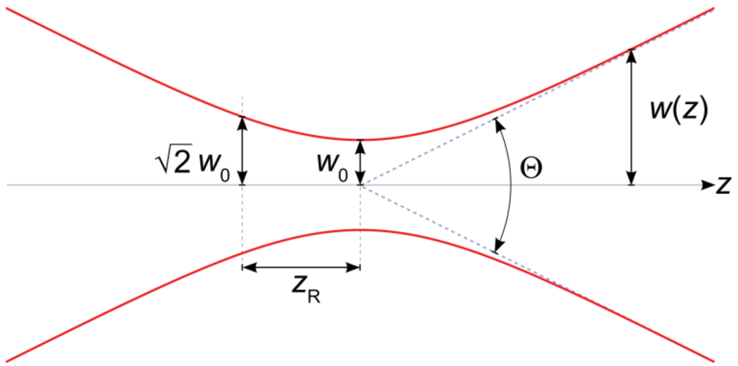
- New measurement using AI based Bragg diffraction (Berkeley scheme)

  Systematics due to atomic beam splitter methods

Photon momentum in a gaussian beam

$$2\pi\nu_{\mathrm{D}} = 2N k_{\mathrm{R}} k_{\mathrm{B}} \frac{\hbar}{m}$$

- Plane wave : $k = \frac{\omega}{c}$
  - Gaussian beam: Gouy phase and wavefront curvature $k_{\mathrm{eff},z} = k + \delta k$



$$\frac{\delta k}{k} = -\frac{2}{k^2 w^2(z)}\left(1 - \frac{\langle r^2\rangle}{w^2(z)}\right) - \frac{\langle r^2\rangle}{2R^2(z)}$$

Size of the atomic cloud

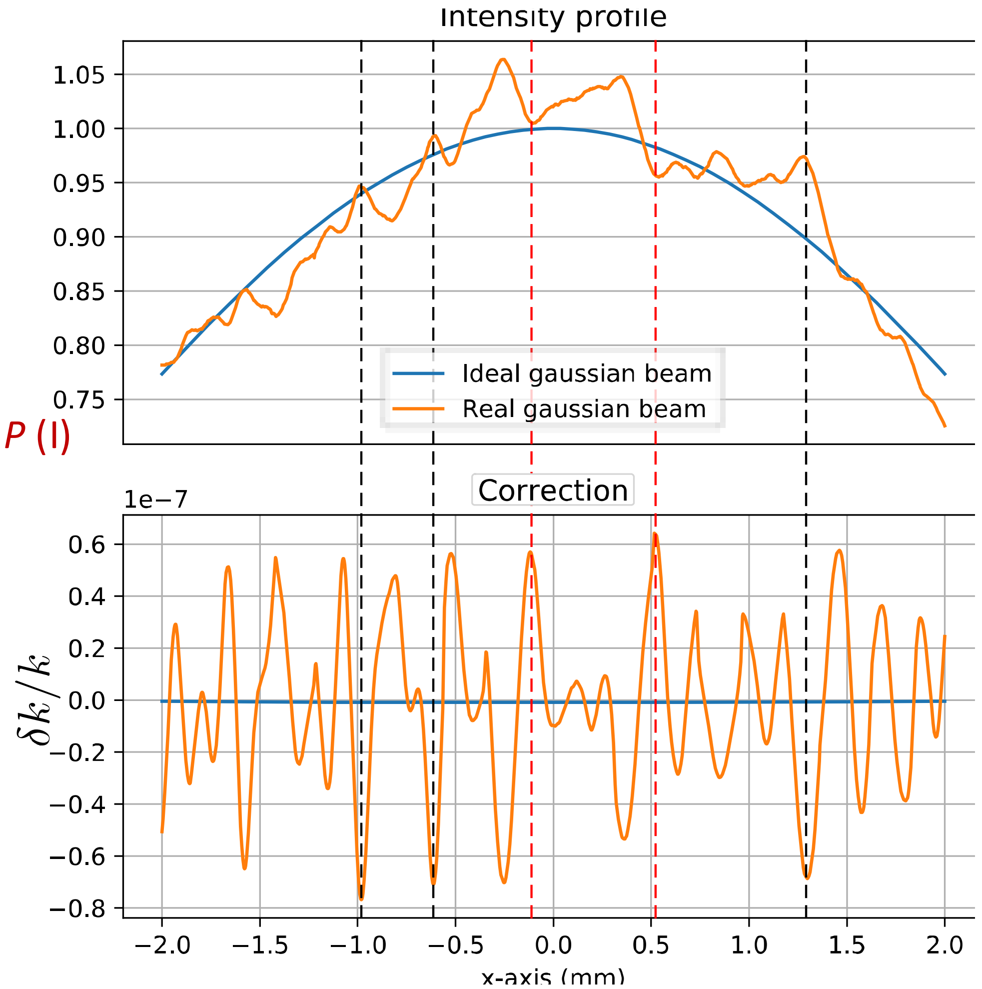
Curvature of the wavefront

- Related to the dispersion of wavevectors $\sim \frac{\Theta^2}{2}$ Effect on α: $(108.2 \pm 5.4) \times 10^{-11}$

Photon momentum in a distorted wavefront

$$E(\vec{r},t) = E_0(\vec{r},t)\ \mathrm{e}^{i(kz+\phi(\vec{r}))}; \quad k = \frac{2\pi\nu}{c}$$

$$\delta k = -\frac{1}{2}\left\|\vec{\nabla}_\perp \phi\right\|^2 + \frac{1}{4k}\frac{\Delta_\perp I}{I} \qquad (I \propto E_0^2)$$



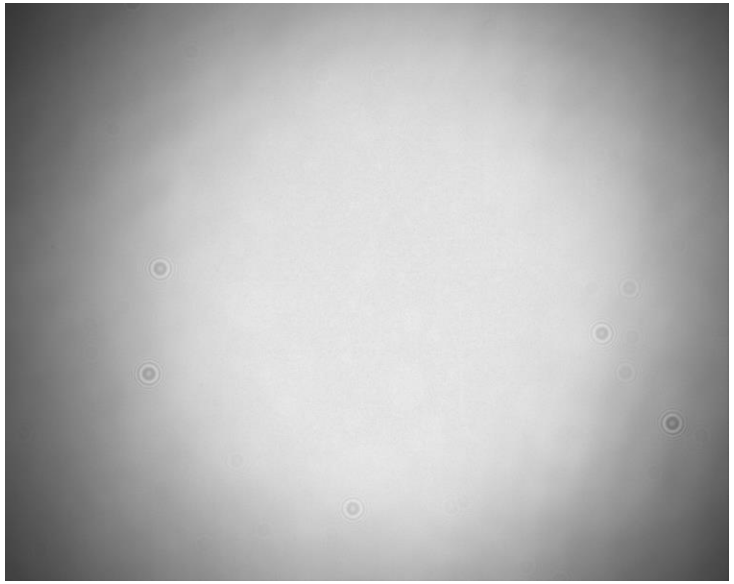
Correlation between the wavevector correction and the survival probability *P* (I) during Bloch oscillations (recoil transfer)

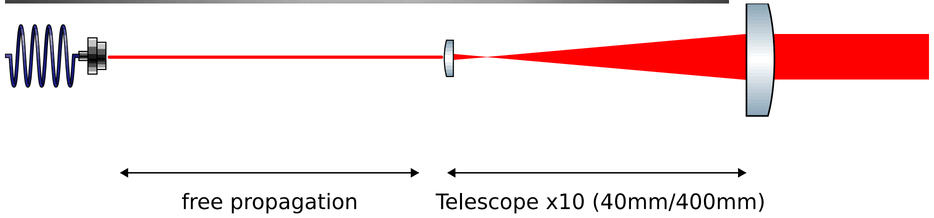
$$\langle \delta k \rangle = \frac{\langle \delta k\ P(I) \rangle}{\langle P(I) \rangle}$$

S. Bade et al., Phys. Rev. Lett. **121**, 073603 (2018)

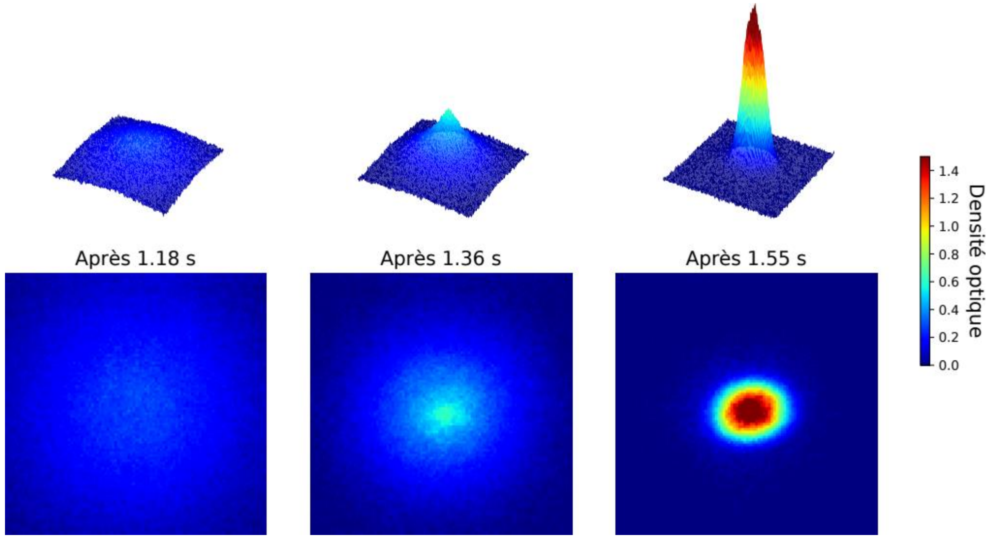
Clean Gaussian beam and Bose-Eintein condensate

- New optical setup





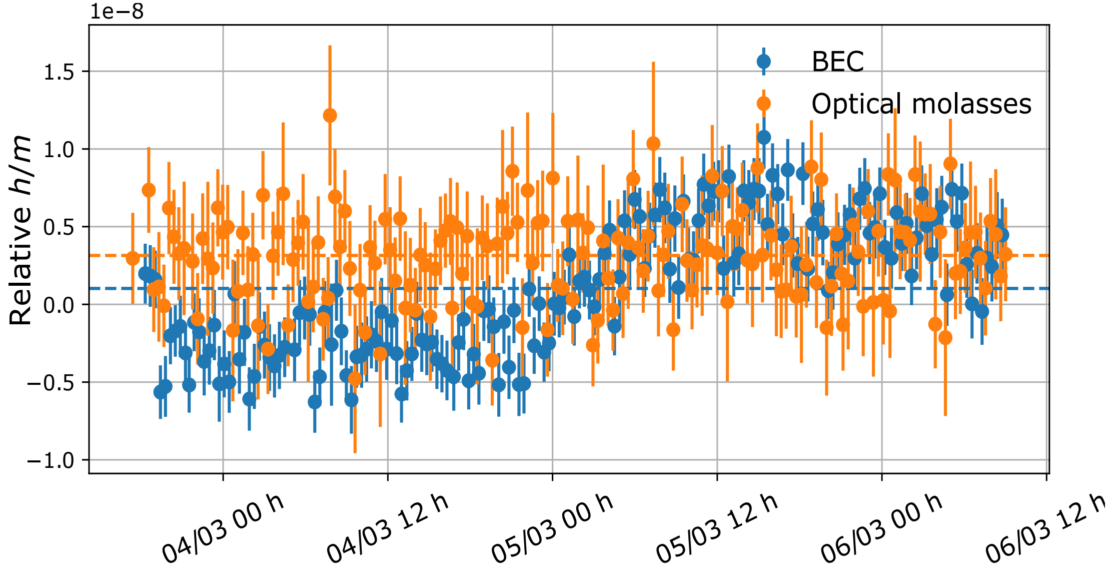
BEC = $2x10^5$ atoms@ 100 nK in F=1 m_F=0 in 3.6 s

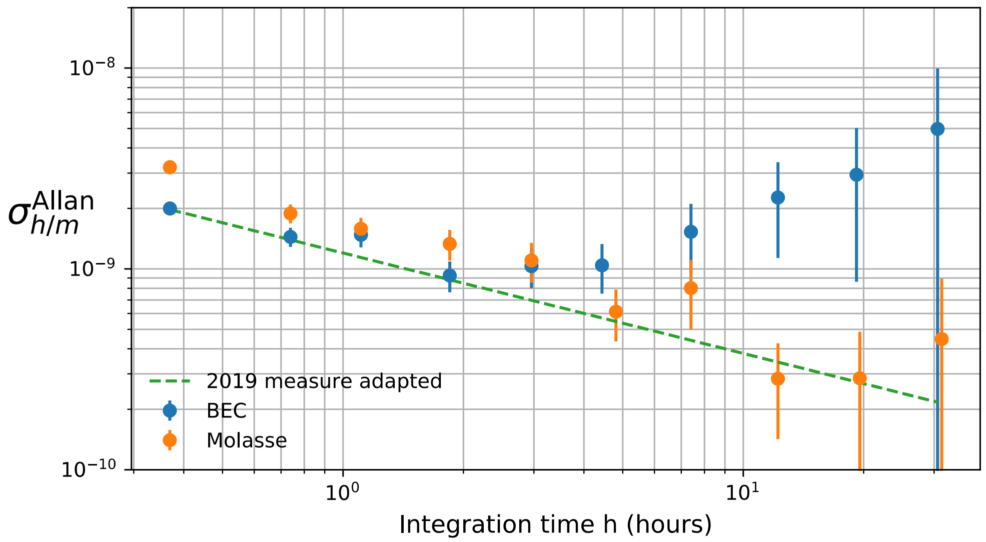


Size of the BEC= = 350 μm after 190 ms of free fall (start of the measurement sequence)

Preliminary measurement with Bose-Einstein condensate

- Measurements alternating between BEC and optical molasses



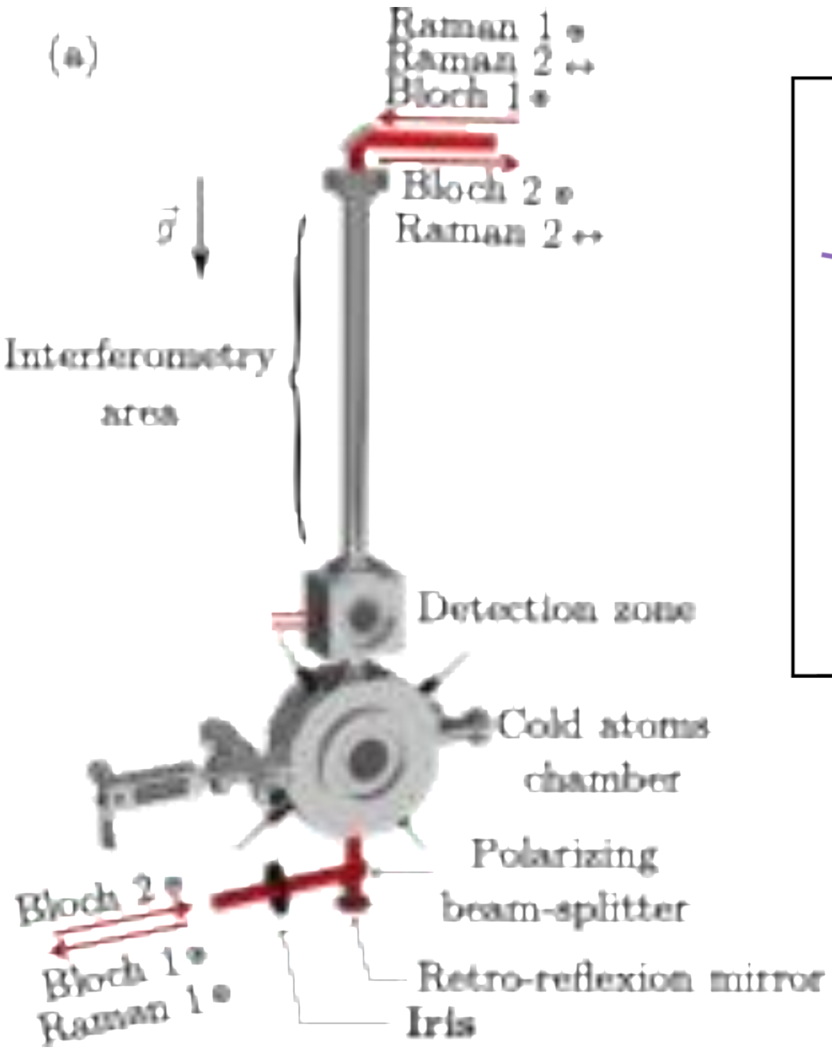


- Temporal fluctuation observed using the BEC (parasitic interference in the new optical setup)

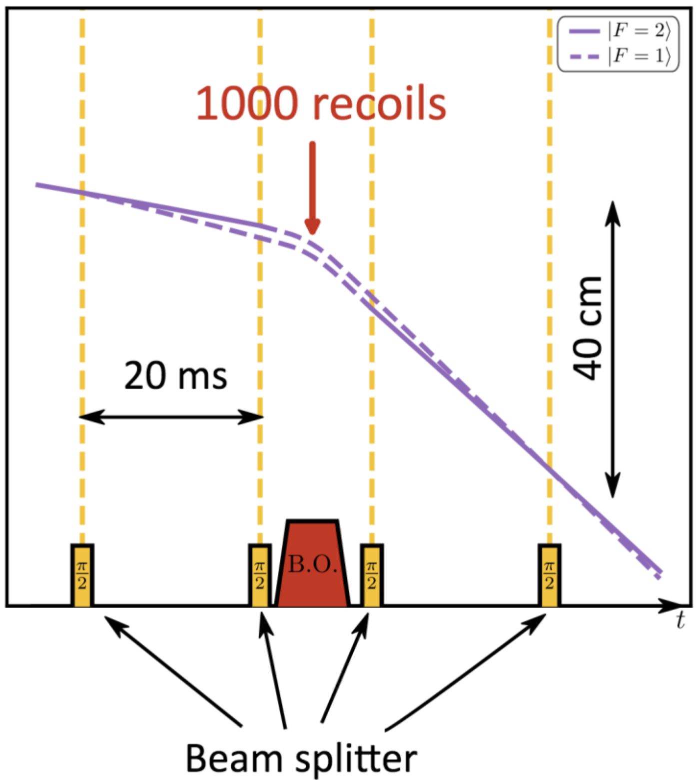
Size of the BEC= = 350 μm ; Size of the molasse = 3 mm

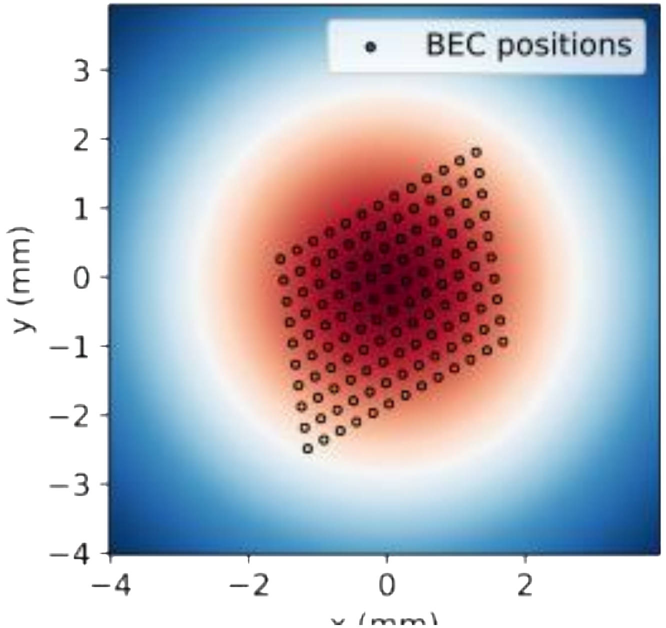
➢ Use BEC as probe to measure the spatial distribution of k-vectors and the intensity profile of laser beams in situ

Probing the spatial distribution of k-vectors *in situ* with the BEC



After 190 ms the size of the BEC = 350 μm





2D grid of 121 positions

How we construct the spatial distribution of k-vectors

$$2\pi\nu_D = \frac{\hbar}{m} N_\mathrm{B} \left(\vec{k}_\mathrm{B2} - \vec{k}_\mathrm{B1}\right) \cdot \left(\vec{k}_\mathrm{R2} - \vec{k}_\mathrm{R1}\right)$$

$$k_z = \hbar k_0 \left(1 + \vec{\kappa} \cdot \vec{u}_z\right)$$

k_0 is the wave vector of a plane wave and $\vec{u}_z$ is the propagation axis

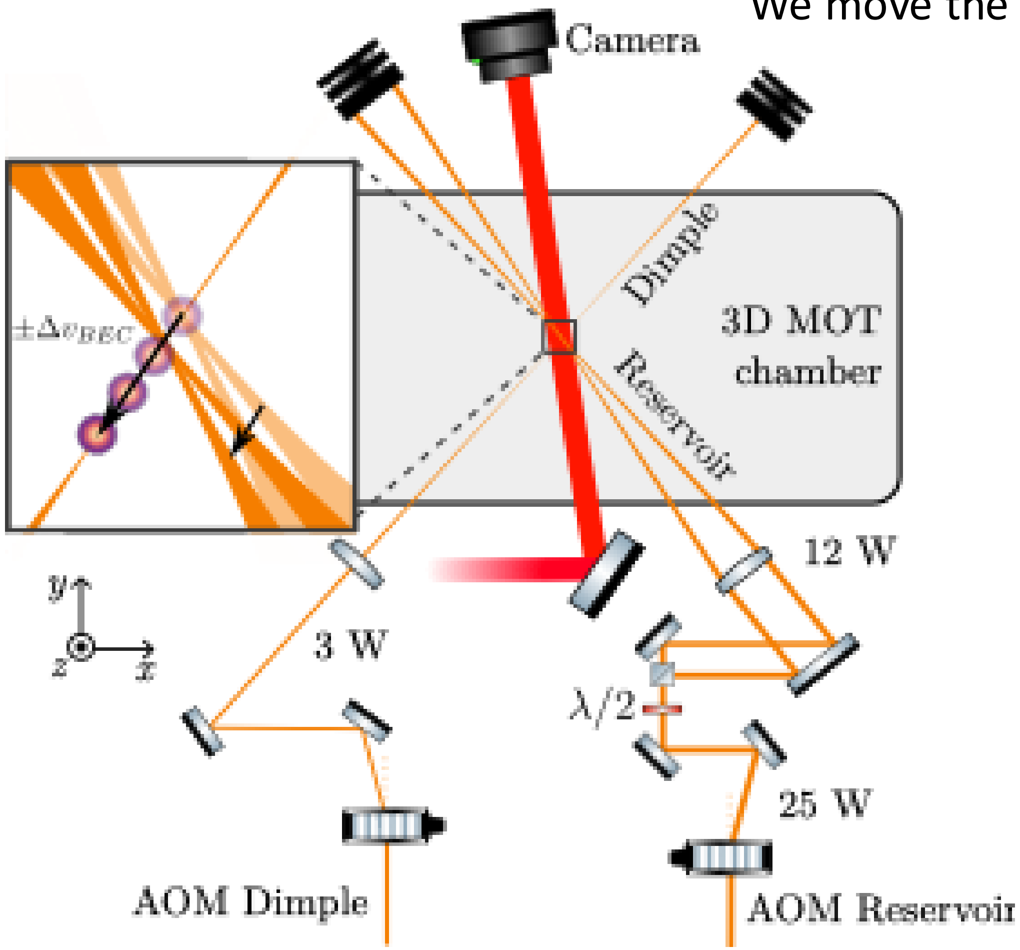
$$2\pi\nu_D \simeq 4N_\mathrm{B} \tfrac{\hbar}{m} k_{0\mathrm{B}} k_{0\mathrm{R}} \left(1 + \tfrac{1}{2}\vec{\kappa} \cdot \vec{u}_z\right)$$

$$\text{where } \vec{\kappa} = \left(\vec{\kappa}_\mathrm{B2} - \vec{\kappa}_\mathrm{B1} + \vec{\kappa}_\mathrm{R2} - \vec{\kappa}_\mathrm{R1}\right)$$

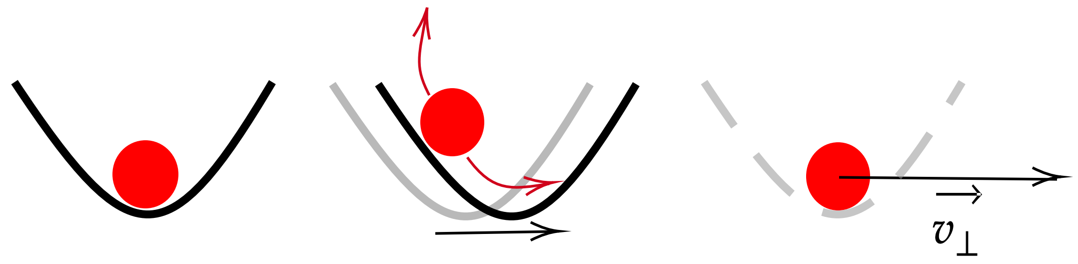
- Measurement of frequency ν_D directly provides the correction κ_z

How to move the BEC ?

We move the condensate by imparting a transverse velocity



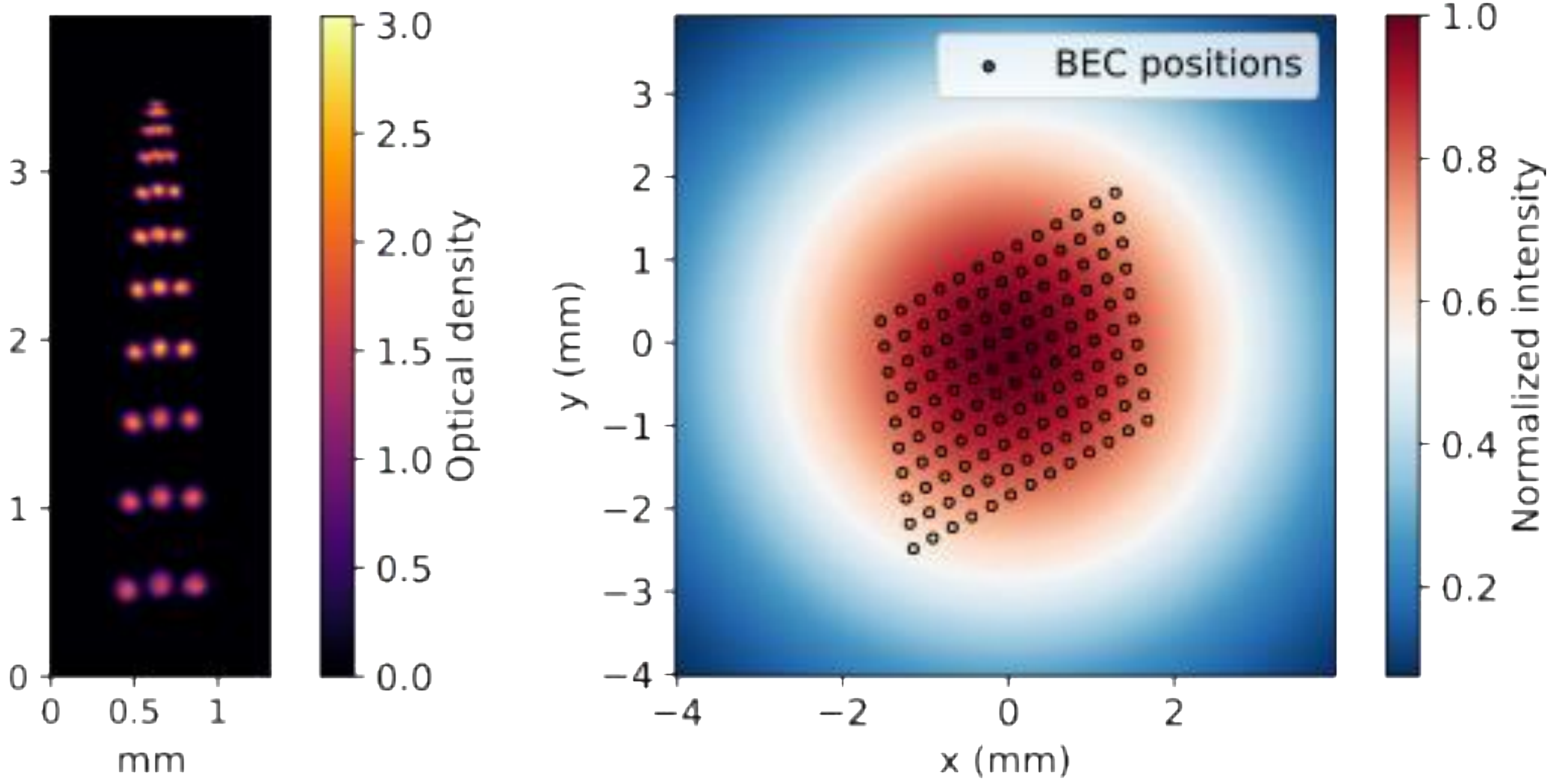
- The frequencies of the two AOMs (reservoir/dimple) are quickly shifted by few MHz, to displace the center of the trap.



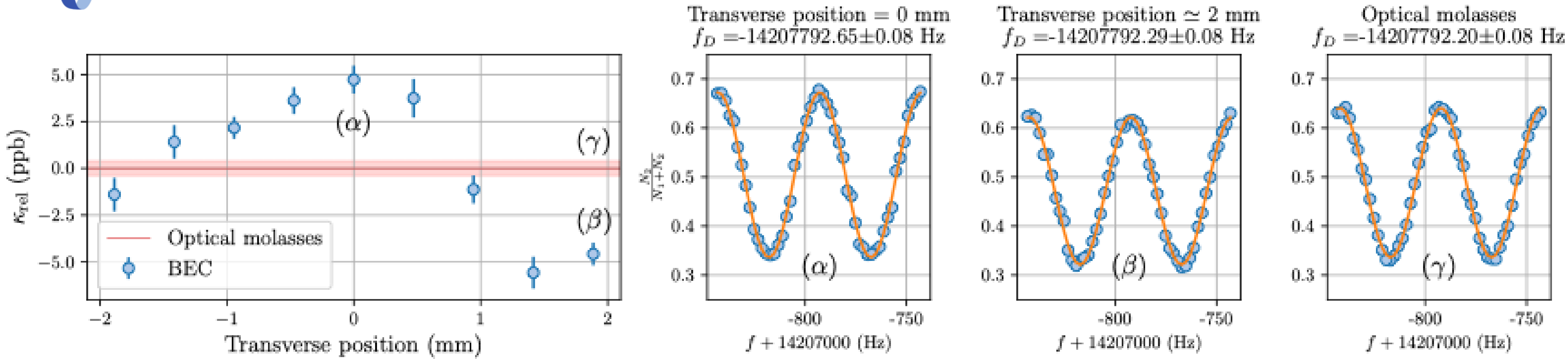
- After 10 ms, the laser beams are turned off, leaving the BEC with an initial transverse velocity

Control and calibration of transverse velocity

- We calibrate the BEC velocity by tracking the cloud trajectory using absorption imaging
- Maximum velocity of 10 mm/s along both x and y directions responds to a displacement of nearly 2 mm.
- The RMS cloud size of 350 µm.

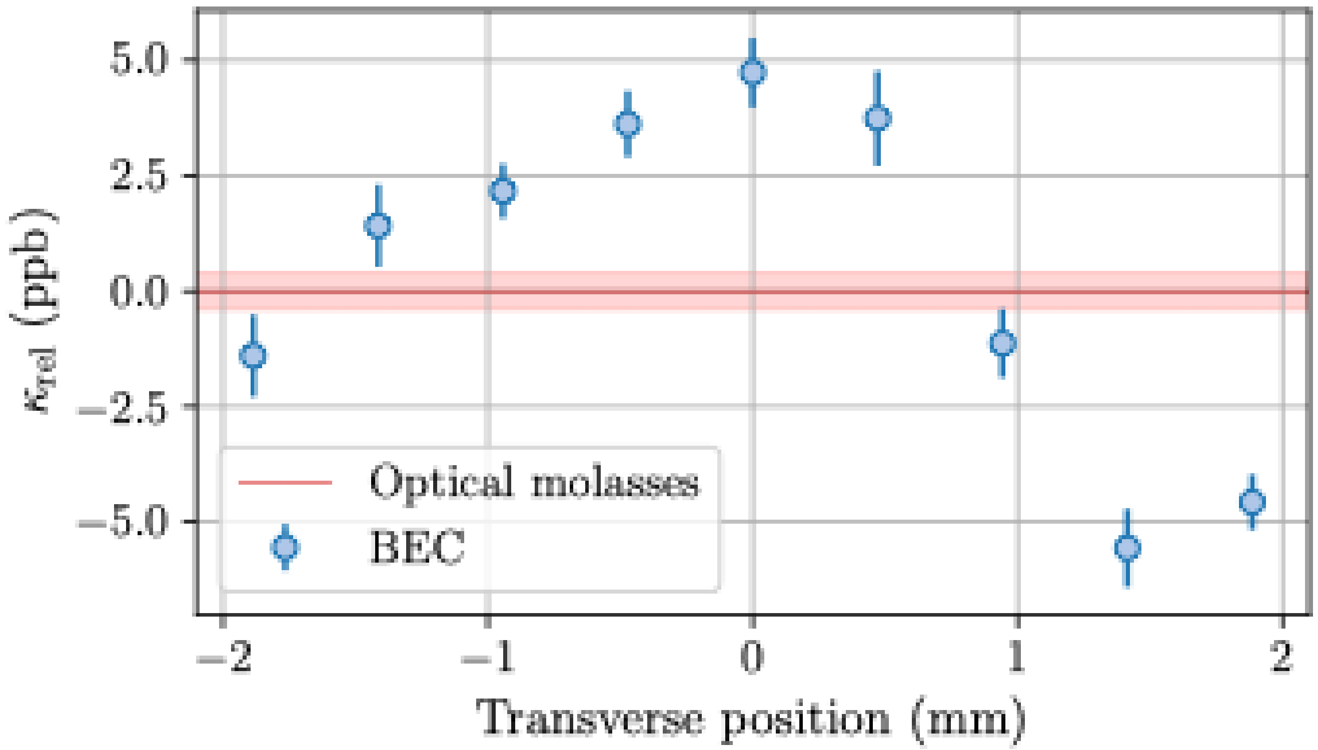


Measurement with « clean » Gaussian beam



- Typical uncertainty on ν_D is 80 mHz (2.8×10^{-9})
- Each data point represents an average 18 such measurements statistical uncertainty of 6×10^{-10} on κ
- Full data were acquired over 117 hours, using BEC and optical molasses alternately.
- We use as reference value the average of 53 values with optical molasses ($\sigma_r = 3.8 \times 10^{-10}$ with $\chi^2 = 1.2$)

Measurement with a « clean » Gaussian beam

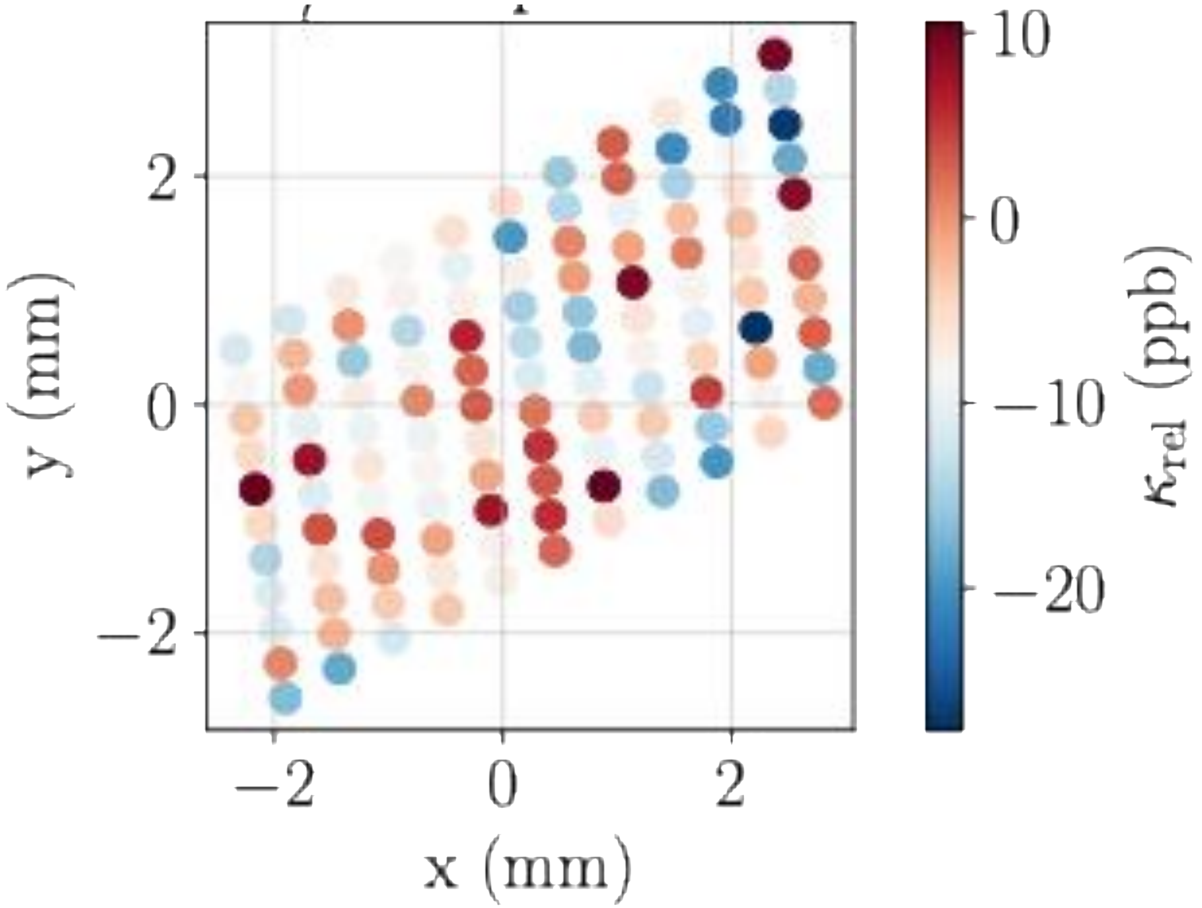


$$\kappa_z = -\frac{2}{k^2 w^2}\left(1 - \frac{r^2}{w^2}\right)$$

For displacements in the range *r* = 0 to 2 mm, and *waist*=5 mm, $\kappa_z \approx$ 1x10−9

➢ Local fluctuations in laser beam intensities induce a dispersion of *k*-vectors much larger than that expected from a simple Gaussian model.

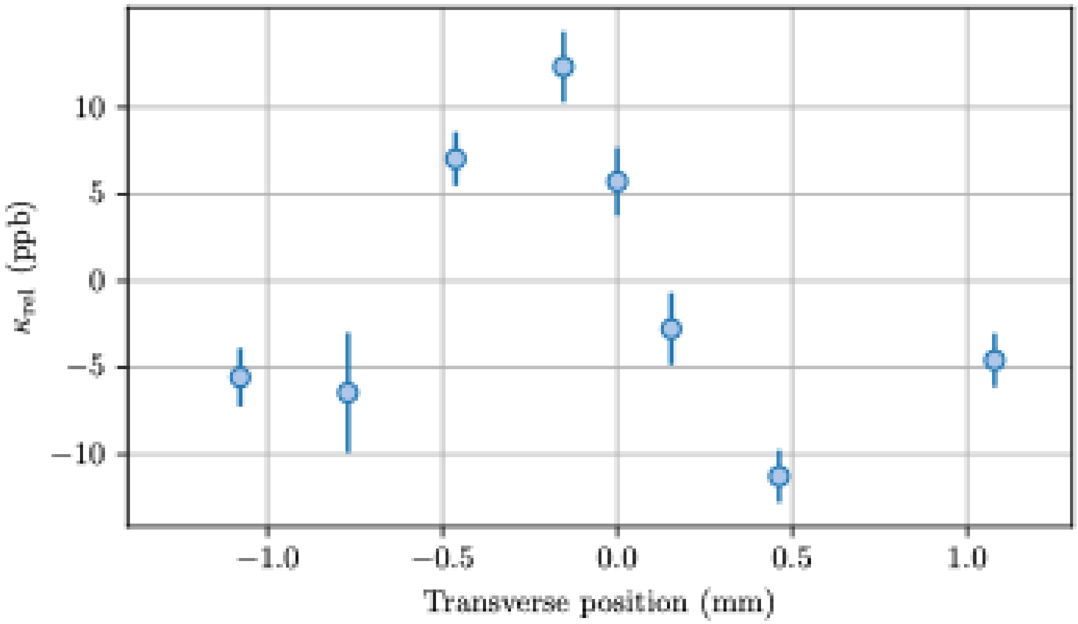
Measurement with a Gaussian beam on a 2 x 2 mm² grid

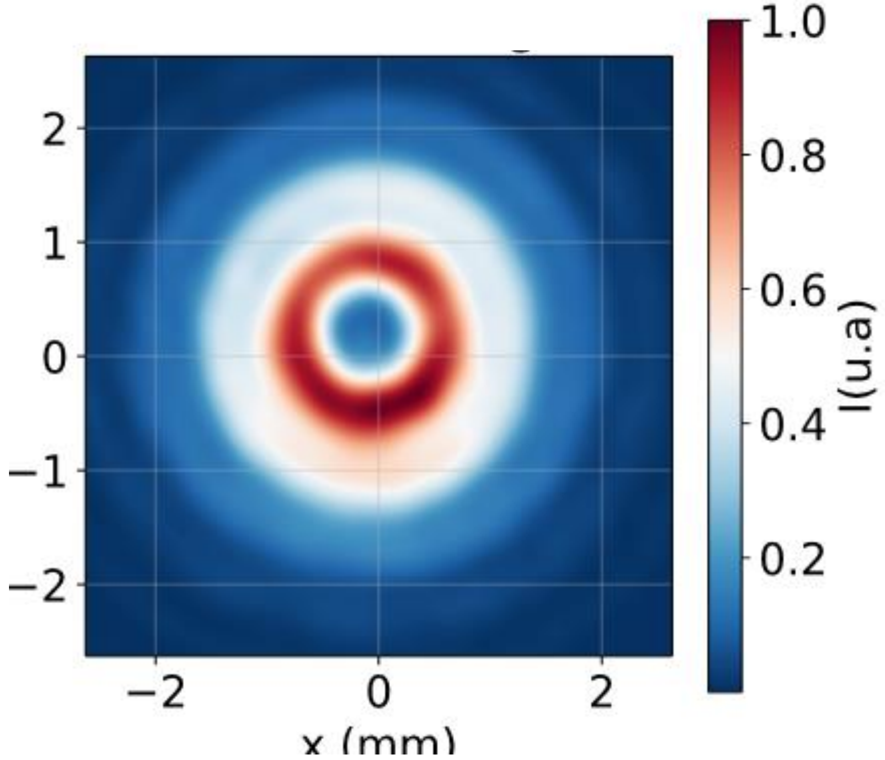


- Measurement using a 2D grid on 121 positions
- The average value matches the value obtained using an optical molasse.

Measurement by clipping the upward Bloch beam

- Spatial distribution of the *k*-vectors of the upward Bloch beam clipped by a 4 mm diameter iris





$$\kappa_z = -\frac{1}{2k_0^2}\left|\left|\vec{\nabla}_\perp \phi(\vec{r})\right|\right|^2 + \frac{1}{4k_0^2}\frac{\Delta_\perp I(\vec{r})}{I(\vec{r})},$$

➢ We observe locally an "extra recoil" where the photon recoil exceeds the nominal value $h\nu/c$

Model and Monte Carlo simulations

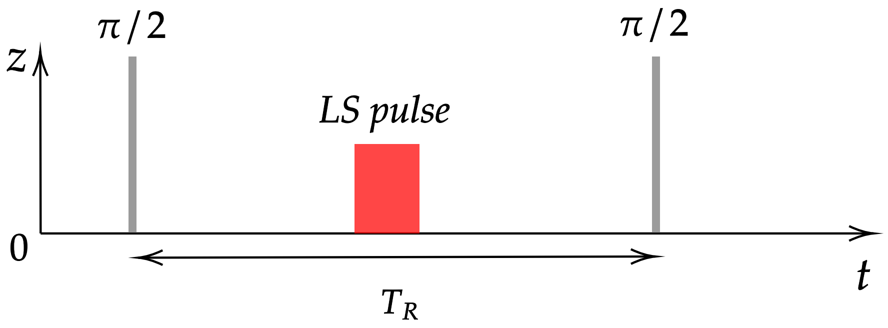
- Monte-Carlo approach: Simulate many classical atom trajectories with initial position/velocity dispersion, compute the interferometer phase shift and probability amplitude for each, then average over the atomic cloud.

- Evaluate the laplacian term at the position of the atoms, by numerically propagating a truncated Gaussian beam

➢ Need to know the intensity profile of laser beams

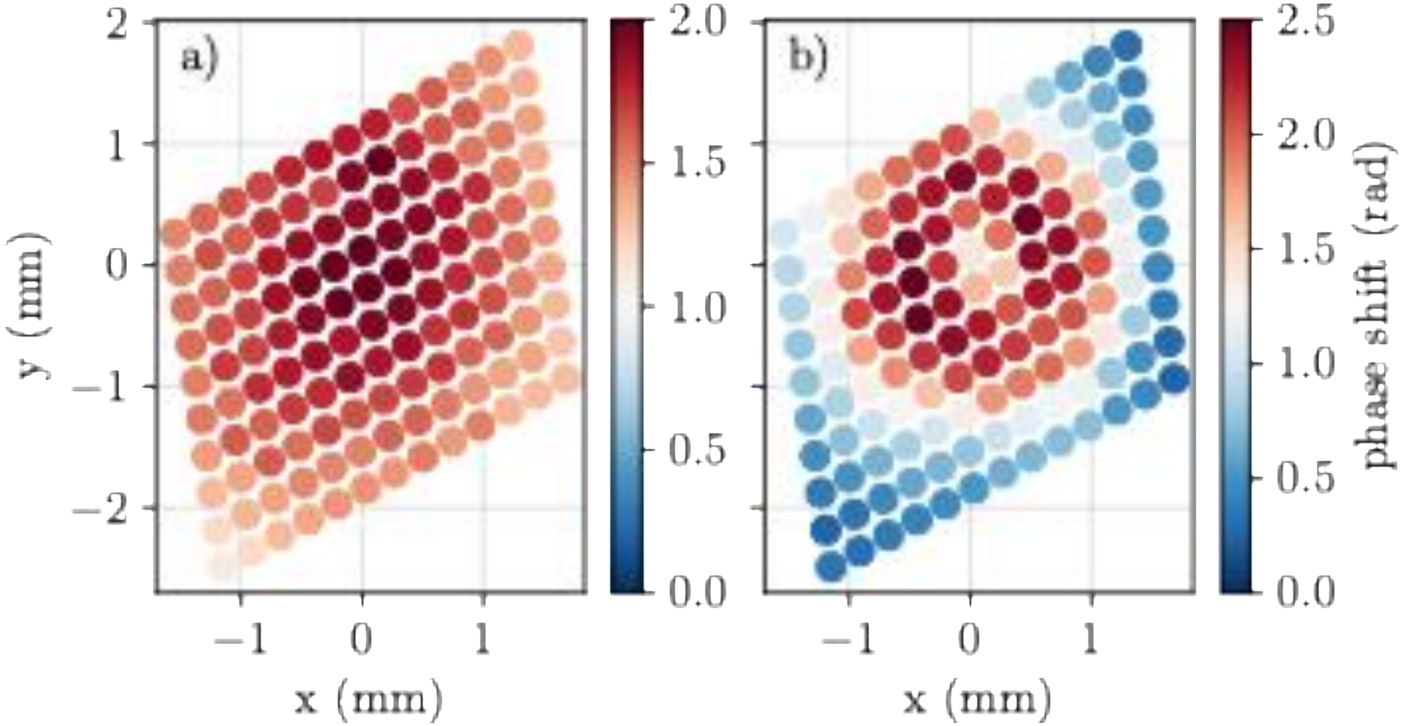
Measurement of intensity probe of the beam *in-situ*

- Ramsey sequence with a « the upward Bloch beam pulse » switched on in between the two π/2 pulses
- It induces a differential light shift proportional to the intensity



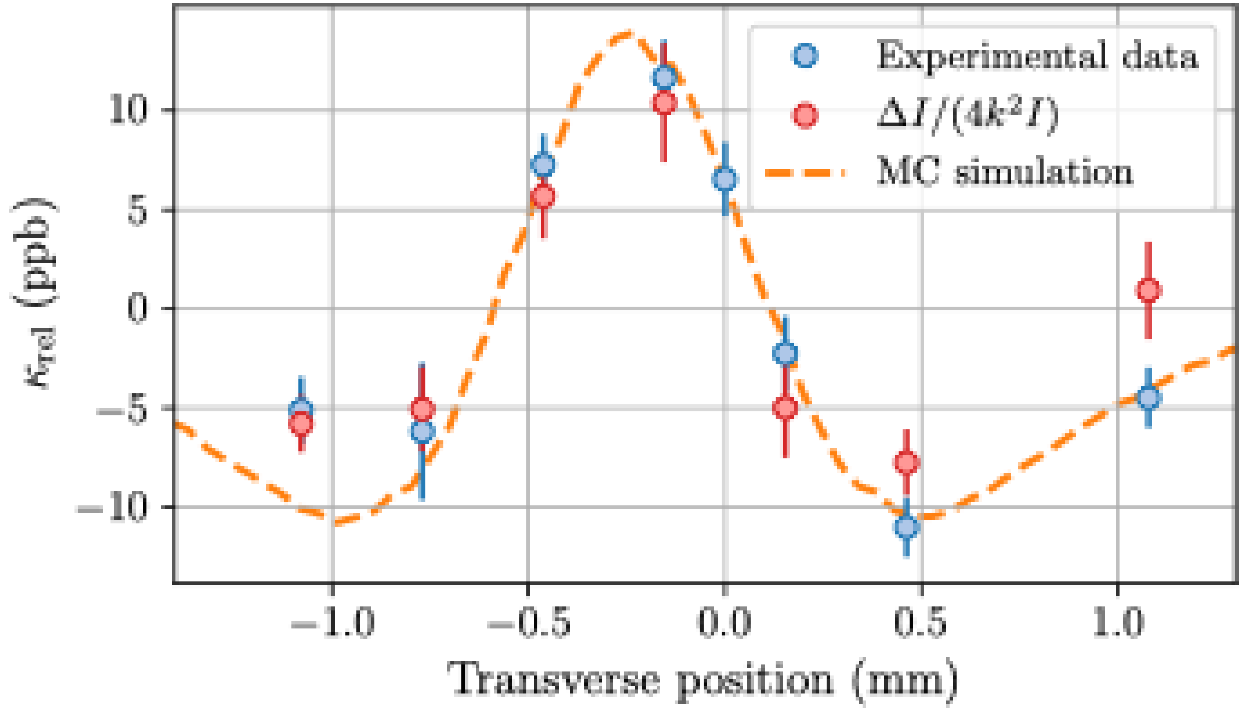
Intensity profile of the upward Bloch beam

- Gaussian beam
- Clipped beam



Measurement with clipped beam: Experiment/simulations

- Spatial distribution of the *k*-vectors of the upward Bloch beam clipped by a 4 mm diameter iris



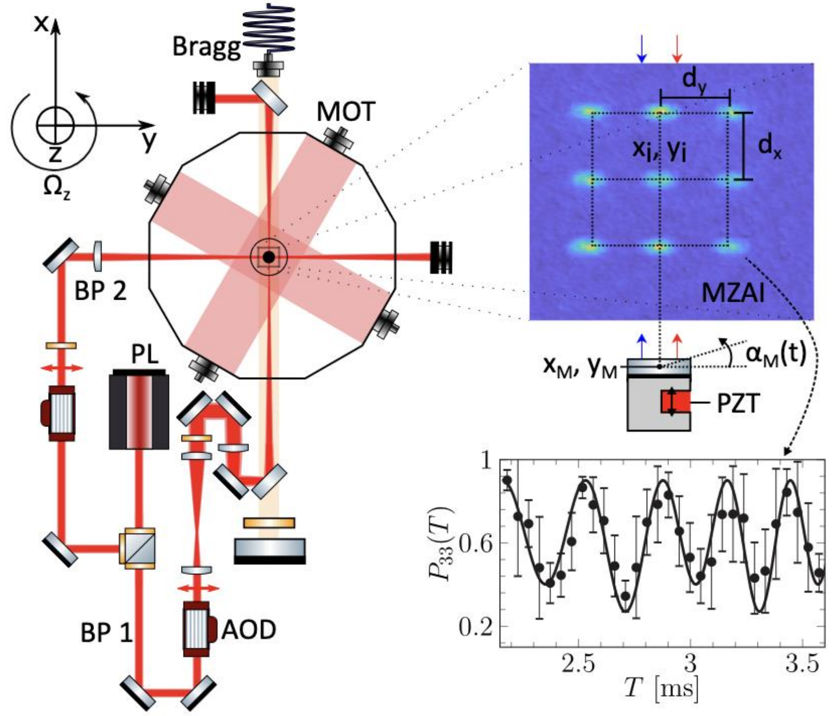
Around each point, we recorded the intensity on a 3x3 matrix of adjacent points and calculated the Laplacian of the intensity.

S. Gaudout, R. Si-Ahmed, C. Debavelaere, M. Door, P. Cladé and S. Guellati-Khelifa arXiv:2507.19157

Summary and Outlook

- A new method for measuring the spatial distribution of k-vectors and the laser intensity profile *in-situ*.
- Development of a full numerical simulation of the recoil velocity measurement protocol and a simple model to evaluate the correction due to laser intensity profiles. Both are in agreement with experimental data.
- Statistical uncertainty is limited by the time required to scan the full beam profile.

- **Outlook**:
  - The use of a 2D BEC array combined with imaging techniques would allow to probe simultaneously different regions of the laser beam's transverse profile.



K. Stolzenberg et al, Phys. Rev. Lett. **134**, 143601 (2025)

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