



Precision at Colliders: Status and Perspectives

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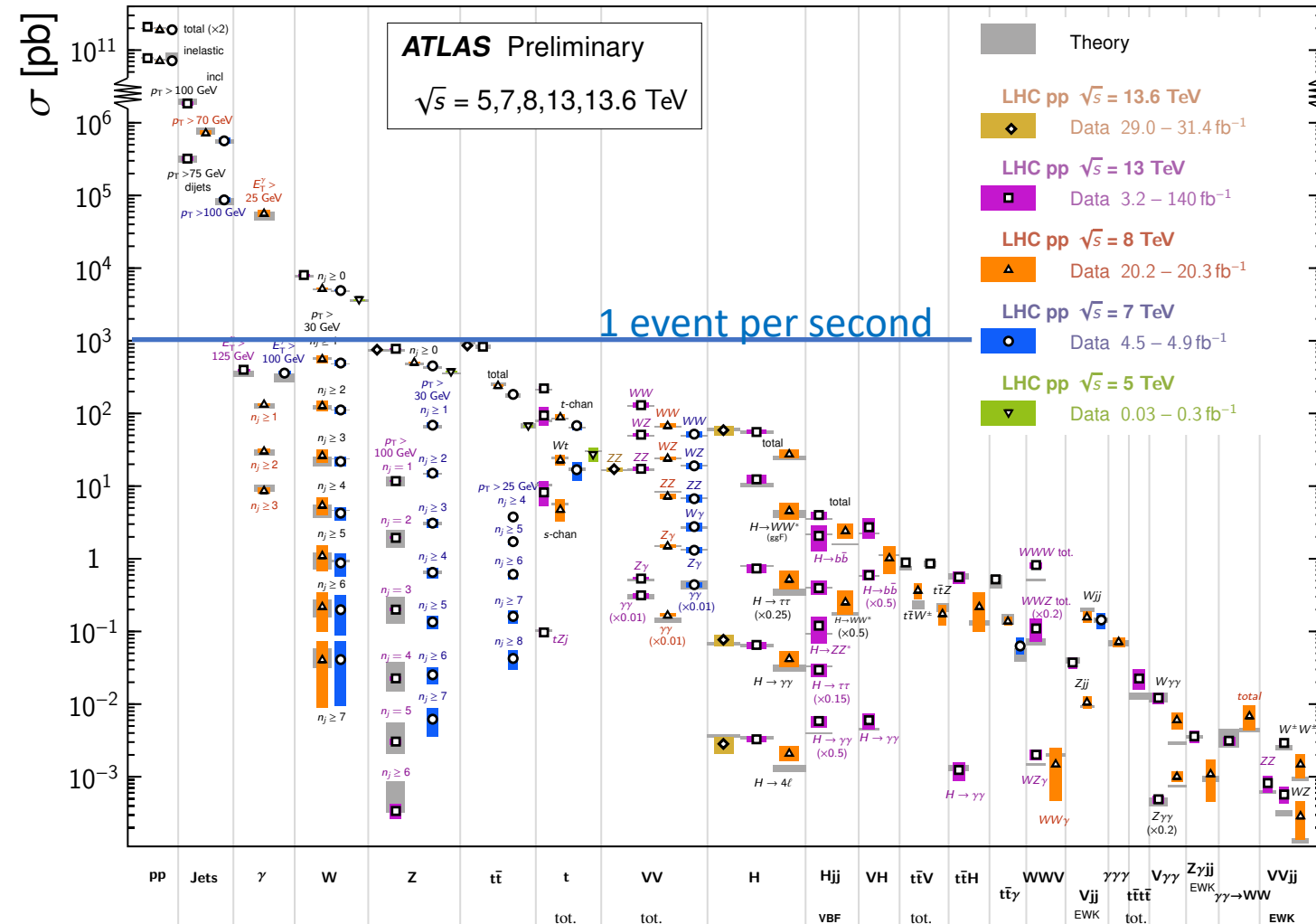


Precision physics at LHC

- Precision tests of the Standard Model
 - Measurements of masses and couplings
 - Searches for deviations
- Interplay of calculations and measurements
 - require precise theory
 - refine predictions through measurements (e.g. PDFs)

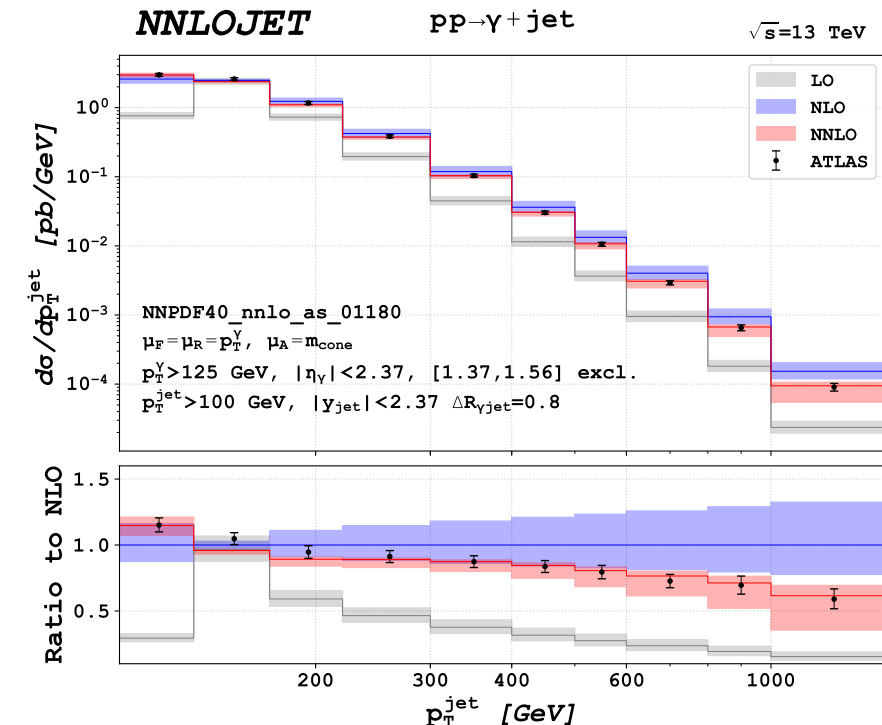
Standard Model Production Cross Section Measurements

Status: October 2023



Precision Predictions: State-of-the-art

- NNLO QCD predictions for $2 \rightarrow 2$ processes (NNLO revolution, 2015 $\rightarrow$)
 - accomplished during past 10 years on case-by-case basis
 - as parton-level event generators (full final state information)
 - computationally expensive
 - current frontier at NNLO: $2 \rightarrow 3$
- Typical size of corrections and uncertainty
 - NLO corrections: 10..100%, uncertainty: 10..30%
 - NNLO corrections: 2..15%, uncertainty: 3..8%
 - expect N3LO to yield uncertainty at level of 1%.



Ingredients to fixed order calculations

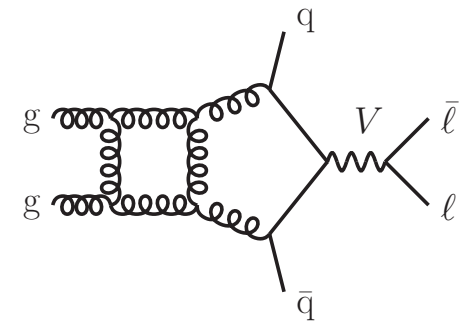
- Matrix elements with extra real (R) or virtual (V) partons

	Matrix elements	Parton evolution
LO	Born	1-loop
NLO	R, V	2-loop
NNLO	RR, RV, VV	3-loop
N3LO	RRR, RRV, RVV, VVV	4-loop

- Infrared singularities in all R-type and V-type subprocesses
 - sum of all subprocesses finite
 - require subtraction procedure to arrange IR cancellations between subprocesses
- Incoming hadrons: parton distributions
 - mass factorization of initial-state radiation and parton evolution

Two-loop matrix elements

- Analytical results for all massless $2 \rightarrow 2$ processes known
 - Di-jet production, V+jet, H+jet, vector boson pairs
- Recent progress on $2 \rightarrow 3$ amplitudes
 - all-massless partons (jets, photons) (S. Abreu, F. Febres Cordero, H. Ita, B. Page, V. Sotnikov; H. Chawdhry, M. Czakon, M. Lim, A. Mitov, R. Poncelet; B. Agarwal, F. Buccioni, F. Devoto, G. Gambuti, A. von Manteuffel, L. Tancredi)
 - V+2 jet and H+2 jet production (S. Abreu, F. Febres Cordero, H. Ita, M. Klinkert, B. Page, V. Sotnikov; S. Badger, H. Bayu Hartanto, J. Krys, S. Zoia)
- Top quark pair production
 - Numerical representation (P. Bärnreuther, L. Chen, M. Czakon, P. Fiedler)
 - Analytical results (M. Mandal, P. Mastrolia, J. Ronca, W. Bobadilla Torres)
- Numerical results for amplitudes with internal masses
 - Top quark loops in HH and H+jet production (S. Jones, M. Kerner et al.; J. Baglio et al., R. Bonciani et al.)
 - Mixed QCD-electroweak corrections (T. Armadillo et al., P. Bargiela, F. Caola, H. Chawdhry, X. Liu)



NNLO cross section

$$\begin{aligned} d\sigma_{\text{NNLO}} = & \int_{n+2} (d\sigma^{RR} - d\sigma^S) \\ & + \int_{n+1} (d\sigma^{RV} - d\sigma^T + d\sigma^{MF,1}) \\ & + \int_n \left(d\sigma^{VV} + d\sigma^{MF,2} + \int_2 d\sigma^S + \int_1 d\sigma^T \right) \end{aligned}$$

- Real and virtual contributions: $d\sigma^{RR}, d\sigma^{RV}, d\sigma^{VV}$
- Singular double real radiation: $d\sigma^S$
- Singular one-loop single real radiation: $d\sigma^T$
- Mass factorization terms: $d\sigma^{MF,1}, d\sigma^{MF,2}$
- Each line finite and free of poles → numerical implementation
- Implementation: parton-level event generator at NNLO

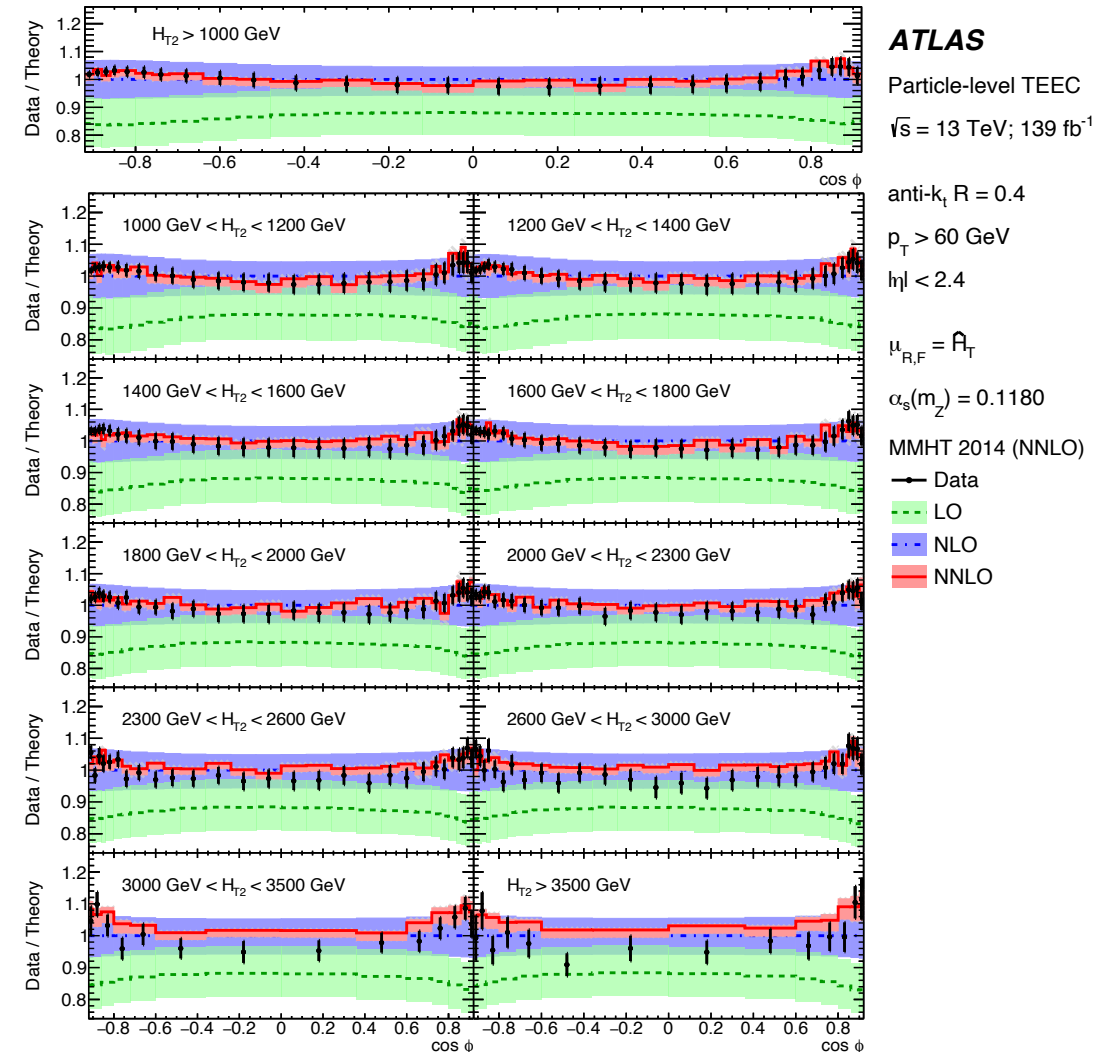
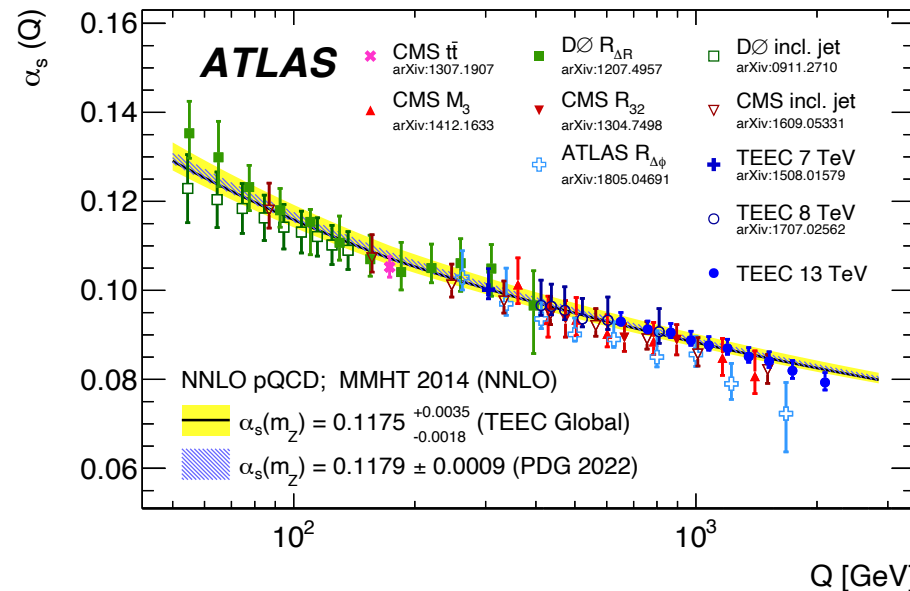
MATRIX: M.Grazzini, S.Kallweit, M.Wiesemann; MCFM: J.Campbell, T.Neumann, R.K.Ellis, S.Seth; NNLOCAL: V.Del Duca et al.; NNLOJET: A.Huss et al., MiNNLOPS: P.Monni, P.Nason, E.Re, M.Wiesemann, G.Zanderighi; GENEVA: S.Alioli et al.

Three-jet production

- 3-jet production and related event shapes at hadron colliders at NNLO

(M. Czakon, A. Mitov, R. Poncelet)

- using residue subtraction (M. Czakon)
- enabled ATLAS α_s determination based on energy-energy correlation functions



NNLOJET code

- **NNLO parton level event generator**
 - Based on antenna subtraction
- **Provides infrastructure**
 - Process management
 - Phase space, histogram routines
 - Validation and testing
 - Parallel computing (MPI) support for warm-up and production
 - ApplGrid/fastNLO interfaces
- **Processes implemented at NNLO**
 - $Z+(0,1)\text{jet}$, $\gamma+1\text{ jet}$, $H+(0,1)\text{jet}$, $W+(0,1)\text{jet}$
 - DIS-2j, LHC-2j
 - Typical runtimes: 60'000-250'000 core-hours

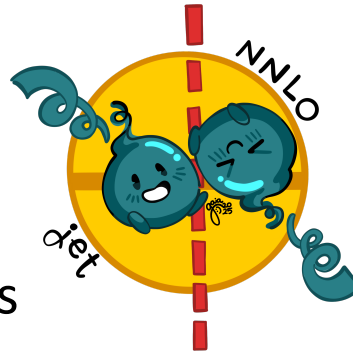
NNLOJET project:

A.Huss, L.Bonino, O.Braun-White, S.Caletti, X.Chen, J.Cruz-Martinez, J.Currie, W.Feng, G.Fontana, E.Fox, R.Gauld, A.Gehrmann-De Ridder, E.W.N.Glover, M.Höfer, P.Jakubcik, M.Jaquier, M.Löchner, F.Lorkowski, I.Majer, M.Marcoli, P.Meinzinger, J.Mo, T. Morgan, J.Niehues, J.Pires, C.Preuss, A.Rodriguez-Gracia, K.Schönwald, R.Schürmann, V.Sotnikov, G.Stagnitto, D.Walker, J.Whitehead, T.Z.Yang, H.Zhang, TG



NNLOJET code

- Open-source code release: NNLOJET v1.0.2
 - download from nnlojet.hepforge.org
- Runcard options
 - process/sub-process selection
 - generic histogramming
 - multi-run features: e.g. jet radius
 - example runcards for many processes
- Cluster workflow management: DOKAN
 - automated resource allocation
 - works with slurm and htcondor (lxplus)
 - combination of results, quality control

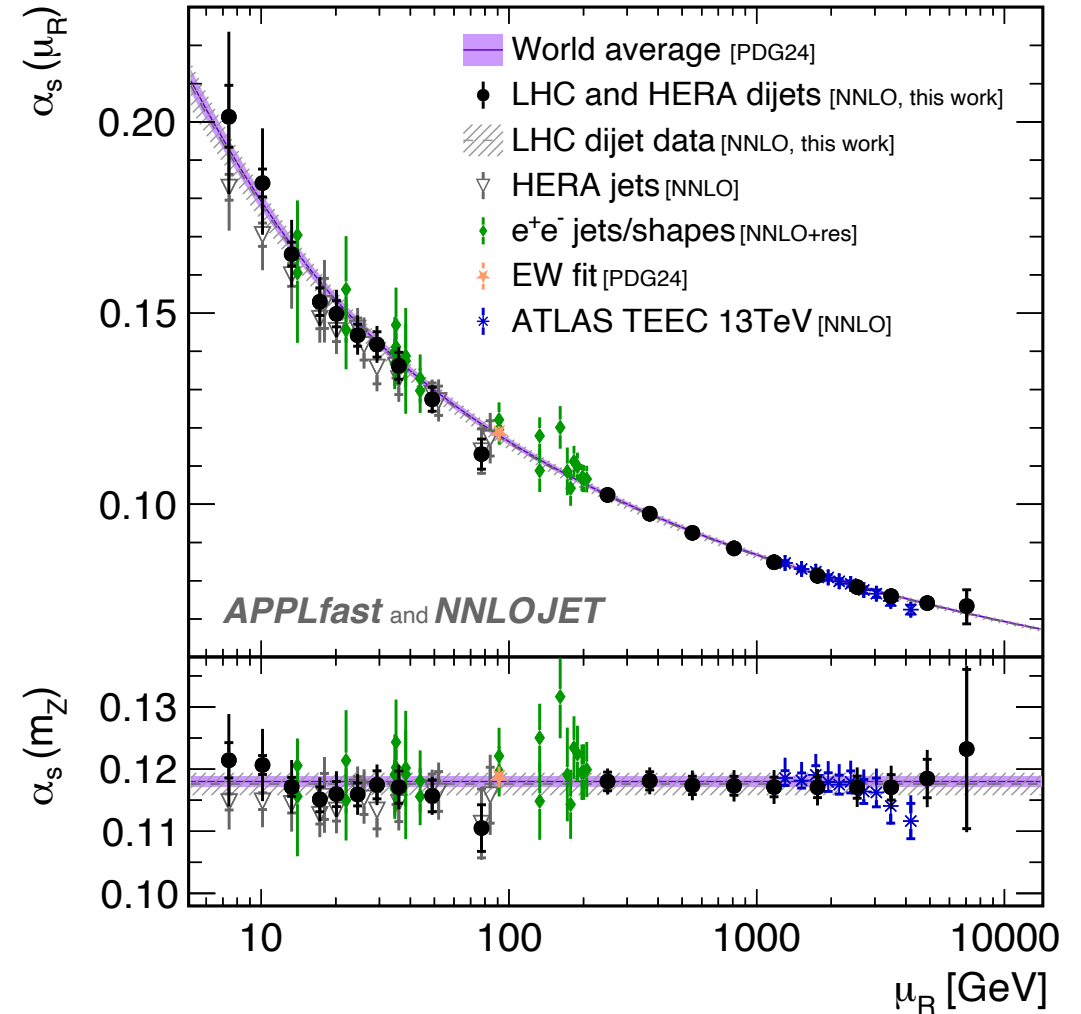


#	L0	R	V	RR	RV	VV
1	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	WRM A[0] D[4]	PRD A[1] D[0]	PRD A[0] D[1]
2	PRD A[0] D[1]	PRD A[1] D[0]	PRD A[0] D[1]	PRD A[0] D[1]	WRM A[1] D[3]	PRD A[0] D[1]
3	-	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[0]	WRM A[1] D[3]	PRD A[0] D[1]
4	-	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]
5	-	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
6	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]
7	-	-	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
8	-	-	-	PRD A[1] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
9	-	-	-	PRD A[0] D[0]	PRD A[1] D[0]	PRD A[0] D[1]
10	-	-	-	PRD A[1] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
11	-	-	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
12	-	-	-	PRD A[0] D[1]	-	-
13	-	-	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
14	-	-	-	PRD A[0] D[0]	-	-
15	-	-	-	PRD A[0] D[1]	WRM A[1] D[3]	PRD A[0] D[1]
16	-	-	-	PRD A[1] D[0]	WRM A[1] D[3]	PRD A[0] D[1]
17	-	-	-	PRD A[0] D[0]	WRM A[1] D[3]	PRD A[0] D[1]
18	-	-	-	PRD A[1] D[0]	-	-
19	-	-	-	PRD A[0] D[1]	-	-
20	-	-	-	PRD A[0] D[1]	-	-
21	-	-	-	WRM A[1] D[3]	-	PRD A[0] D[1]
22	-	-	-	PRD A[0] D[1]	-	PRD A[0] D[1]
23	-	-	-	PRD A[1] D[0]	-	PRD A[0] D[1]
24	-	-	-	WRM A[1] D[3]	-	PRD A[0] D[1]
25	-	-	-	WRM A[1] D[3]	-	-
26	-	-	-	PRD A[1] D[0]	-	-
27	-	-	-	PRD A[0] D[1]	-	-

Two-jet production

- 2-jet production at LHC
 - most basic hard QCD process
 - large cross section: high-statistics measurement
 - combine ATLAS/CMS data sets
 - group data according to $\mu=M_{2j}$
- correlation of α_s with PDFs
 - vary PDF through APPLgrid interface
 - combine with DIS 2-jet (HERA)
- reach: 7.4 GeV ... 7.0 TeV

$$\alpha_s(M_Z) = 0.1178 \pm 0.0014_{\text{exp, pdf}} \pm 0.0017_{\text{th}}$$

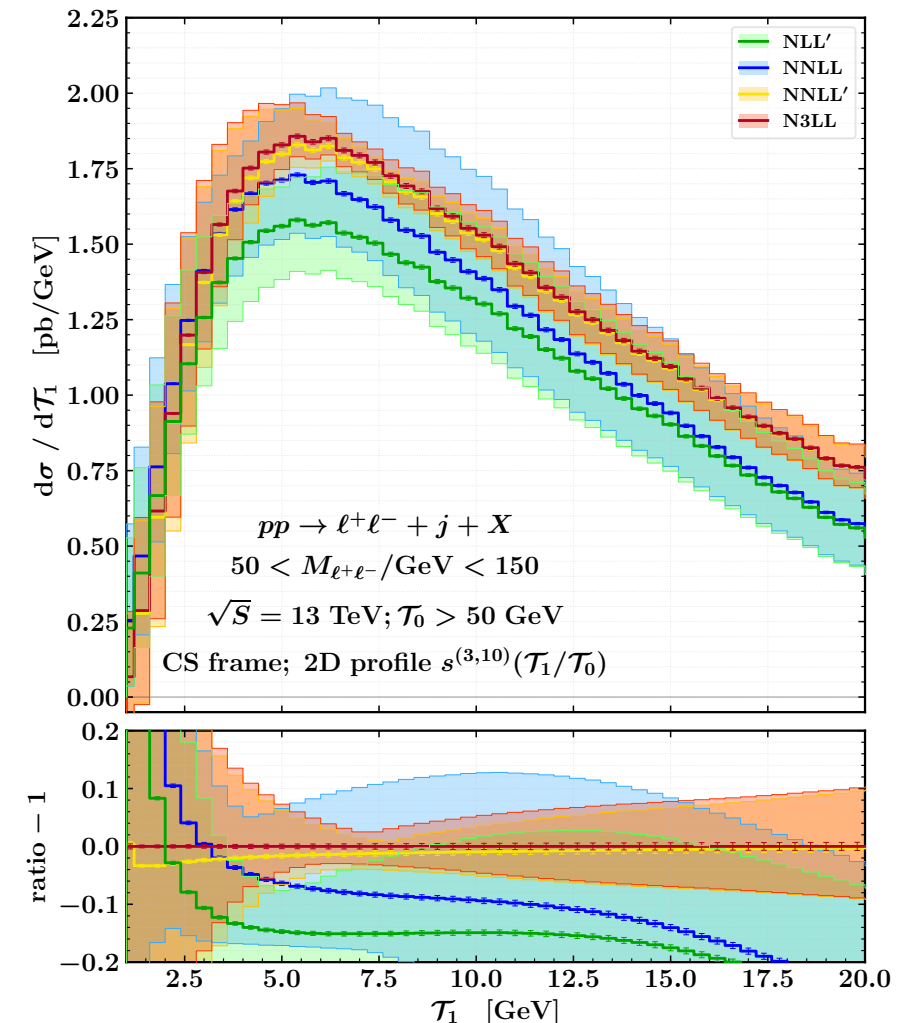


(APPLfast+NNLOJET: F. Ahamdova et al.)

Resummation

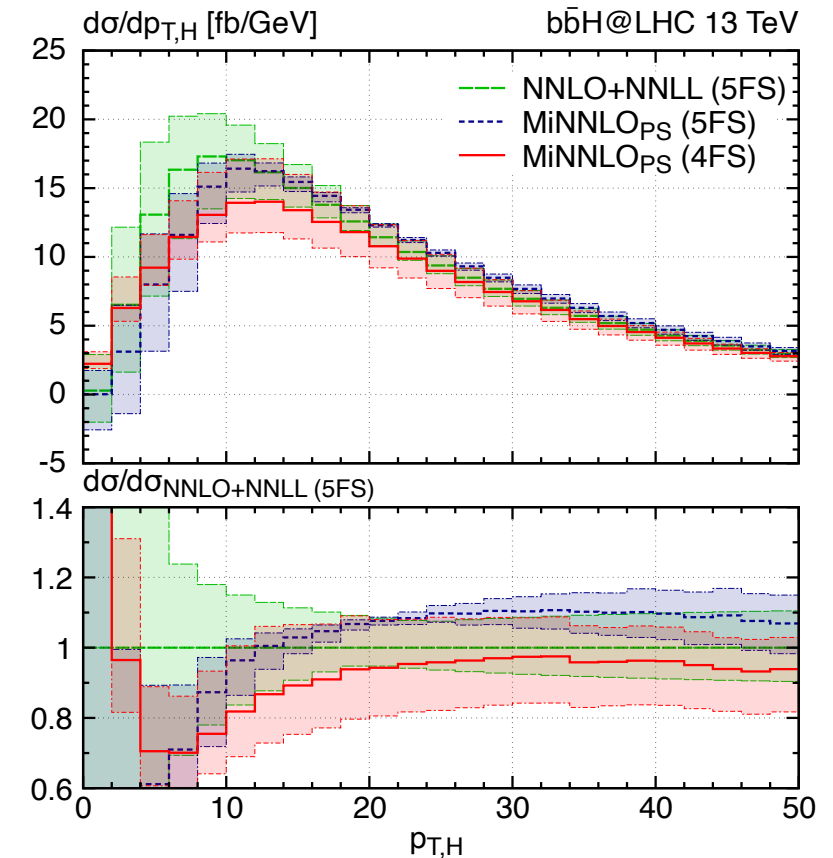
- generic resummation: parton shower
 - current parton shower accuracy: NLL
 - NNLL parton shower so far only for global observables in e^+e^- (PanScales: G.Salam et al.)
- analytic process-specific resummation
 - various approaches: direct space, SCET
 - transverse momentum (N3LL), threshold (N4LL), 0-Jettiness (N3LL), 1-jettiness (N3LL)

(1-jettiness: S.Alioli, G.Bell, G.Billis, A.Broggio, B.Dhenadi, M.Lim, G.Marinelli, R.Nagar, D.Napoletano, R.Rahn)



Matching fixed-order and resummation

- Obtain predictions over full kinematical range
- Parton shower matching
 - NLO+PS well-established
 - NNLO+PS for colour singlet processes (MiNNLO_{PS})
(Hbb: C.Biello, J.Mazzitelli, A.Sankar, M.Wiesemann, G.Zanderighi)
- Matching to analytic resummation
 - re-expand resummation formula
 - obtain full fixed-order logs for LO+NLL, NLO+NNLL, NNLO+N3LL,

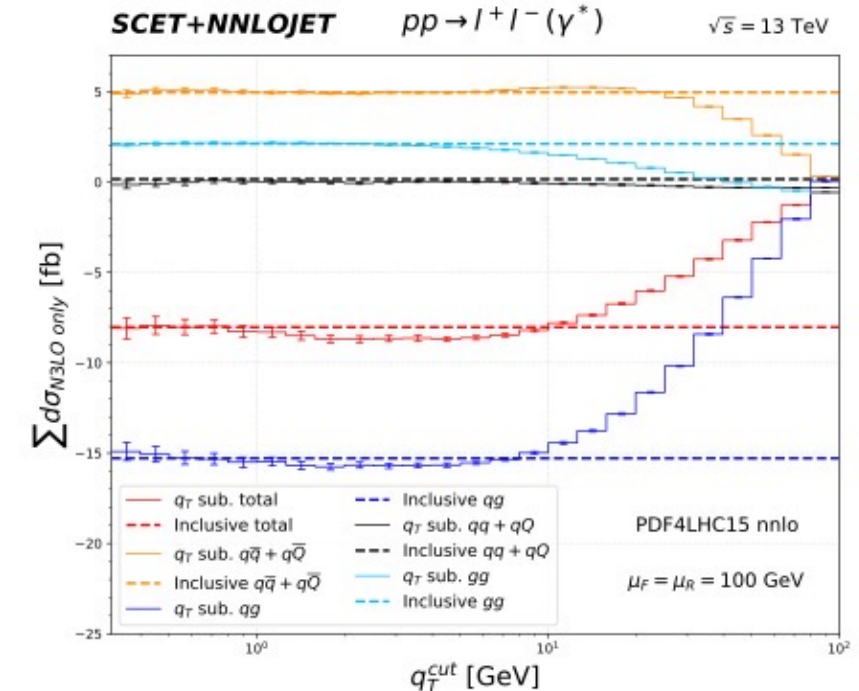


N3LO for Drell-Yan observables

Slicing parameter: transverse momentum (q_T slicing) [S.Catani, M.Grazzini]

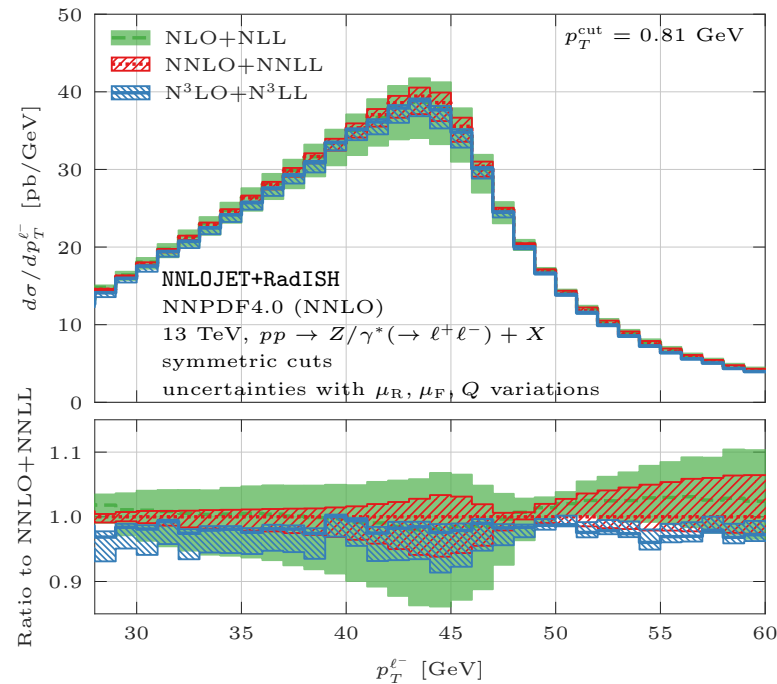
$$\frac{d\sigma_X^{N3LO}}{dO} = \mathcal{H}_{N3LO} \otimes \frac{d\sigma_X^{LO}}{dO} + \left[\int_{q_{T,X}} \frac{d\sigma_{X+j}^{NNLO}}{dO} - \frac{d\sigma_{X,CT}^{NNLO}}{dO}(q_T) \right]$$

- below-cut contribution from expansion of N3LL
 q_T resummation to $\mathcal{O}(\alpha_s^3)$ [W.Bizon, P.Monni, E.Re, P.Torrielli;
 S.Camrada, L.Cieri, G.Ferrera; T.Becher, T.Neumann; W.L.Ju, M.Schönherr]
- ingredients: three-loop soft and beam functions
 [Y.Li, H.X.Zhu; M.Ebert, B.Mistlberger, G.Vita; M.X.Luo, T.Z.Yang, Y.J.Zhu]
- check: independence on $q_{T,cut}$ slicing parameter
- check: reproduce inclusive coefficient functions
 (no ingredients or methodology in common!)
 [X.Chen, E.W.N.Glover, A.Huss, T.Z.Yang, H.X.Zhu, TG]



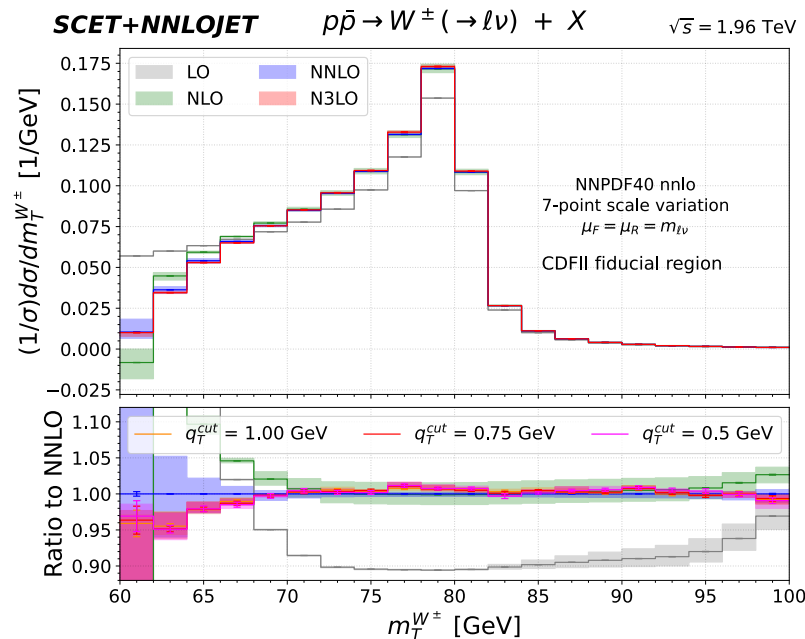
N3LO for Drell-Yan observables

Results: fiducial distributions



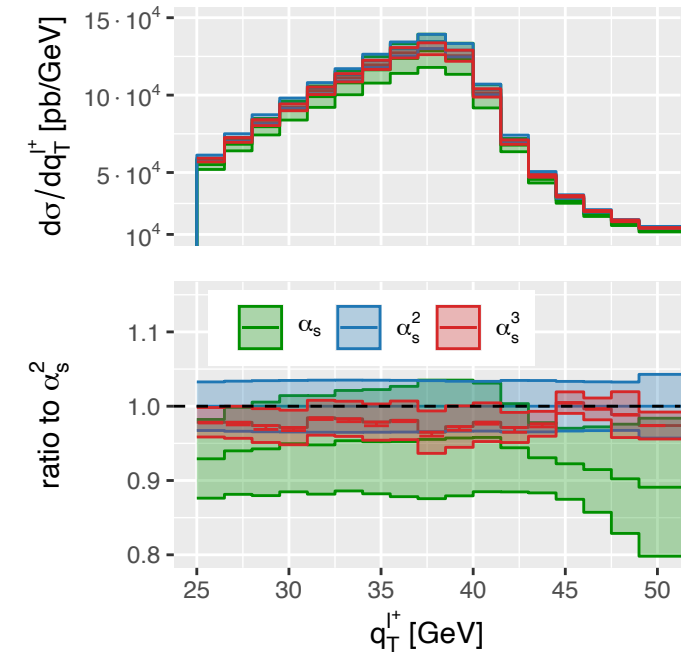
single lepton distribution in NC Drell-Yan,
matched to N3LL resummation (RadISH)
[X.Chen, E.W.N.Glover, A.Huss, P.F.Monni,
E.Re, L.Rottoli, P.Torrielli, TG]

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transverse mass distribution in
W boson production (CDF II cuts)
[X.Chen, E.W.N.Glover, A.Huss,
T.Z.Yang, H.X.Zhu, TG]

MITP P3H Program



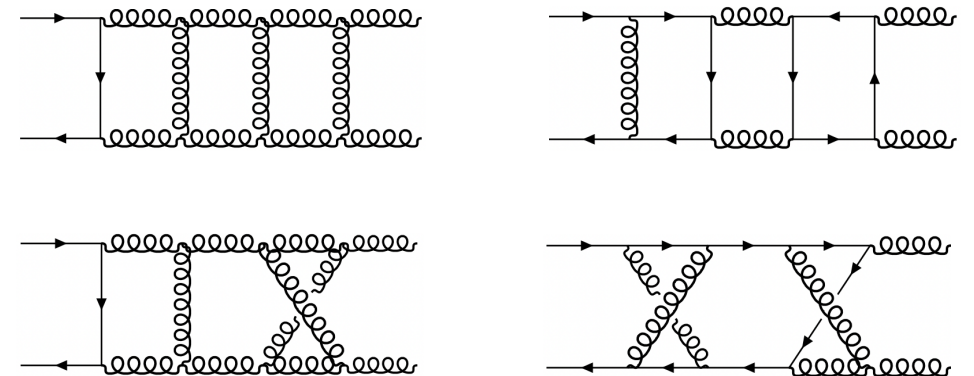
charged lepton distribution
in W boson production
(ATLAS 5.02 TeV)
[J.Campbell, T.Neumann]

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Towards N3LO predictions

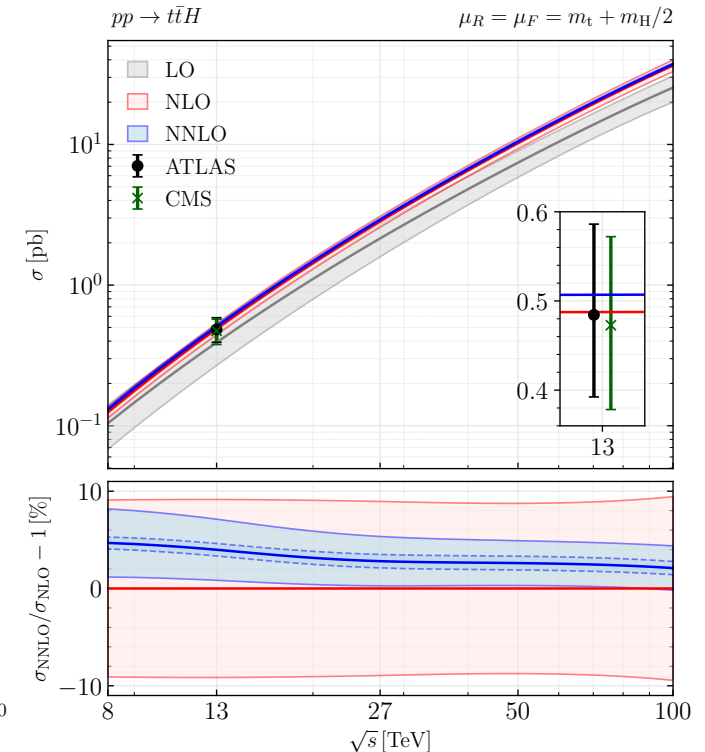
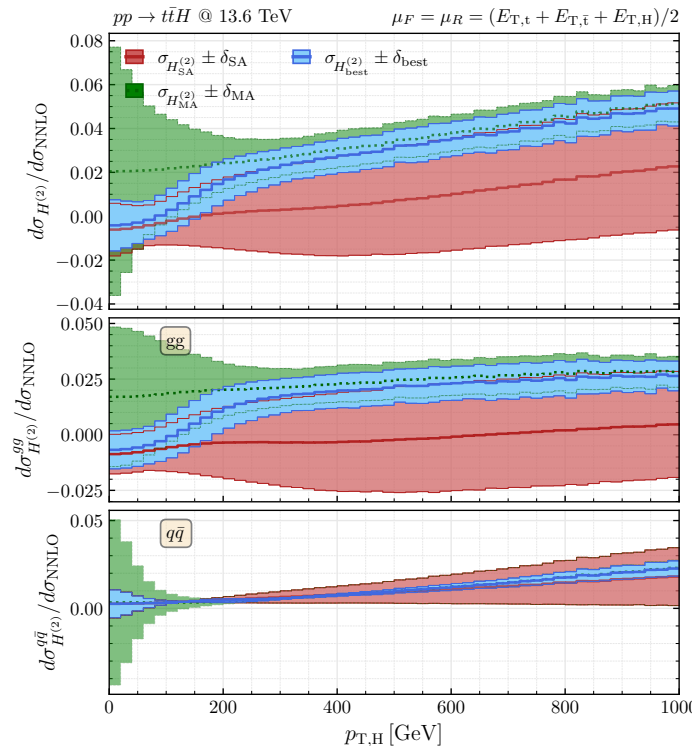
Three-loop amplitudes for $2 \rightarrow 2$ processes (VVV)

- algebraic complexity of integral reduction, computation of master integrals
- first results
 - four-parton amplitudes
(F.Caola, A.Chakraborty, G.Gambuti, A.von Manteuffel, L.Tancredi)
 - parton-photon amplitudes
(P.Bargiela, F.Caola, A.Chakraborty, G.Gambuti, A.von Manteuffel, L.Tancredi)
 - V+3-parton amplitudes (planar)
(P.Jakubcik, C.Mella, N.Syrrakos, L.Tancredi, TG)
 - H+3-parton amplitudes (leading colour)
(X.Chen, X.Guan, B.Mistlberger)



Computational complexity frontier

- multi-scale loop amplitudes are becoming complexity bottleneck
 - genuine (IR-subtracted) multi-loop contribution to cross section often small
 - approximate finite remainder
 - ... and quantify uncertainty
 - methods: leading colour, soft expansion, high-energy limit,...



($t\bar{t}H$: S.Devoto, M.Grazzini, S.Kallweit, J.Mazzitelli, C.Savoini)

Computational complexity frontier

- **Threshold expansion**

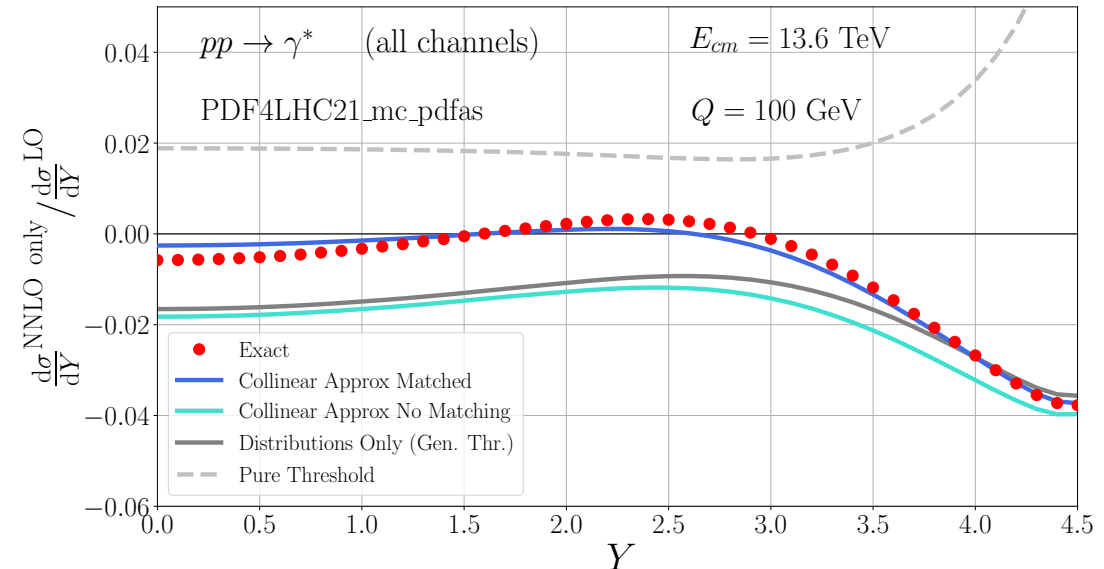
- expand (inclusive or differential) cross section around production threshold
- performed on individual subprocesses at fixed order
- yielded initial N3LO results for Higgs and vector boson production

(C.Anastasiou, C.Duhr, F.Dulat, F.Herzog, B.Mistlberger)

- slow convergence

- **Collinear expansion** (B.Mistlberger, G.Vita)

- expand differential cross section around collinear limit
- improve by inclusive matching



Beyond LHC: EIC

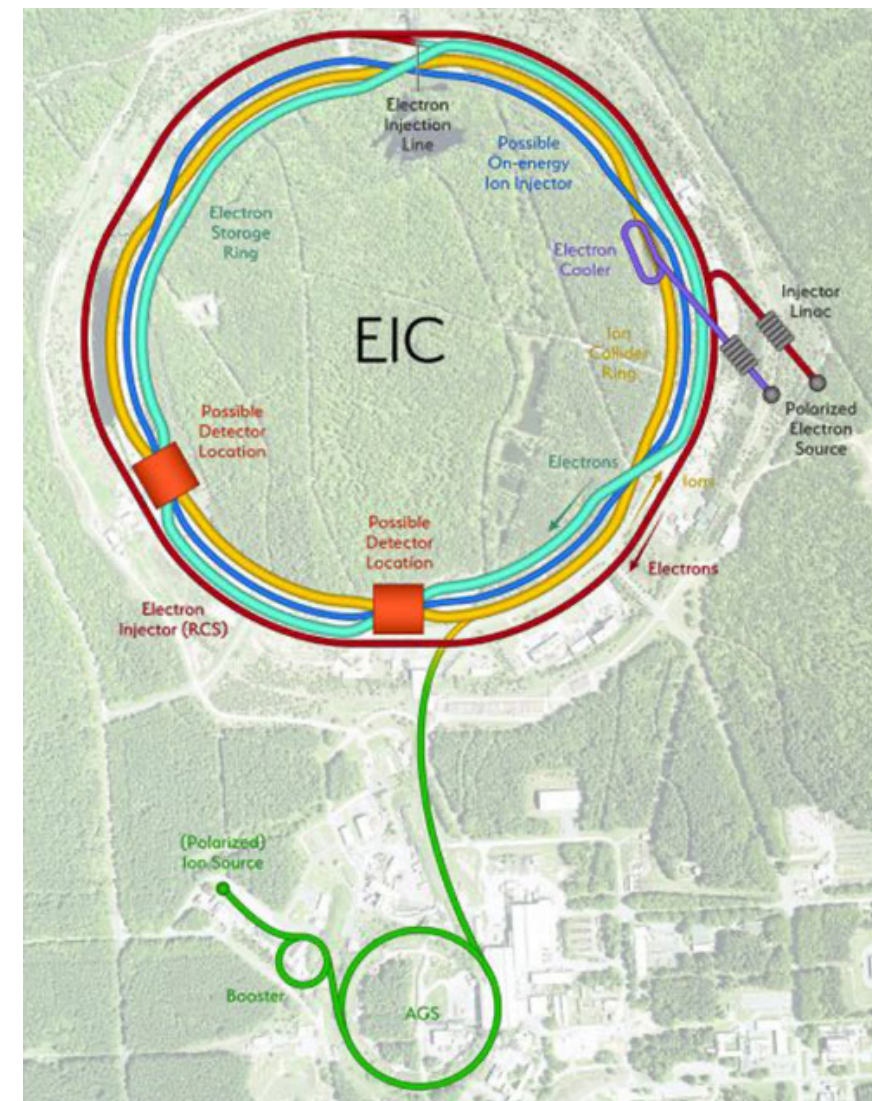
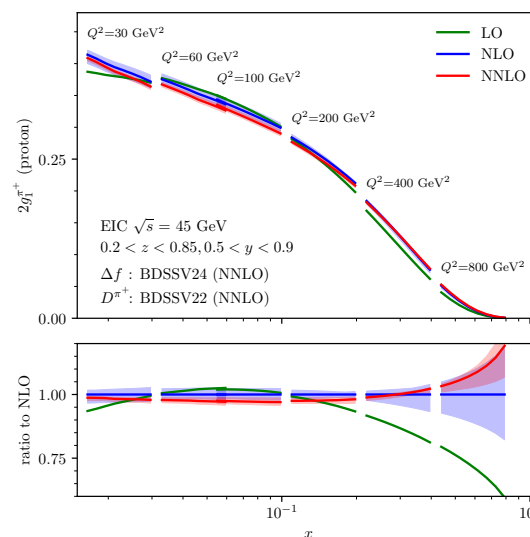
• Electron-Ion Collider at BNL

- High-luminosity: $10^{33} \dots 10^{34} \text{ cm}^{-2}\text{s}^{-1}$
- Centre-of-mass energy range: $40 \dots 140 \text{ GeV}$
- Full identification of hadronic final state
- precision QCD: hadron structure, fragmentation,
- Polarized collisions

• Theory challenges

- NNLO precision for benchmark processes
- identified hadrons, fully exclusive final states
- novel types of observables: TMD, GPD,

(SIDIS: L.Bonino, M.Löchner, K.Schönwald, G.Stagnitto, TG; S.Goyal, R.Lee, S.Moch, V.Pathak, N.Rana, V.Ravindran)



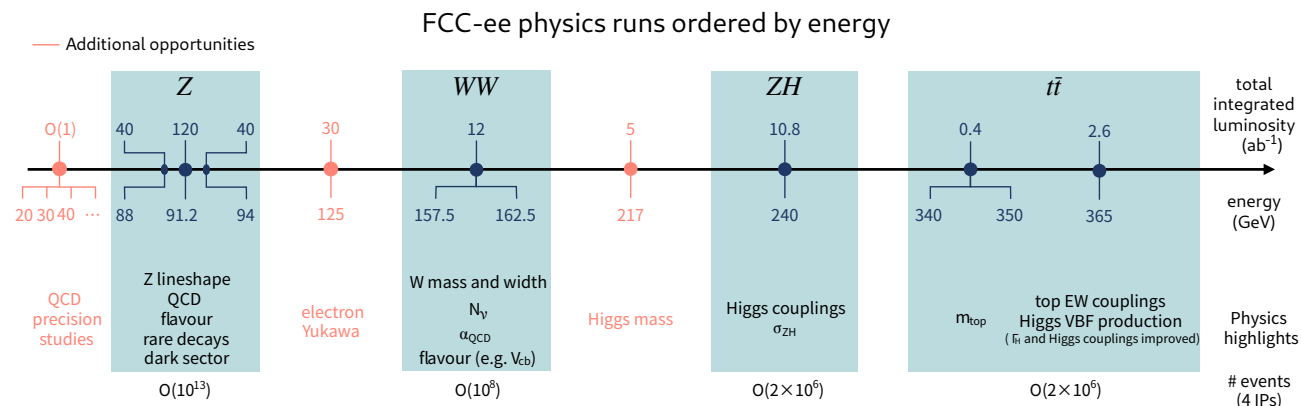
Beyond LHC: FCC-ee

- FCC-ee project at CERN

- ultrahigh-luminosity e^+e^- collider
- under consideration
- start of operation ~ 2045

- Will require new level of precision predictions

- electroweak NNLO corrections (multi-scale loop amplitudes)
- precision QCD: jets, hadrons, energy correlators
- novel types of observables
- transition perturbative $\leftrightarrow$ non-perturbative



Summary

- LHC is on a decade-long program of precision physics
- Ultimate precision challenge for QCD
 - predictions for complex final states at per-cent level accuracy
- Theory ready to face this challenge
 - NNLO predictions becoming the new standard
 - combine with parton showers and resummation
 - N3LO concepts, techniques and tools emerging
- Exciting opportunities for precision physics at FCC-ee and EIC
 - learn technical experience from LHC calculations
 - develop novel methods