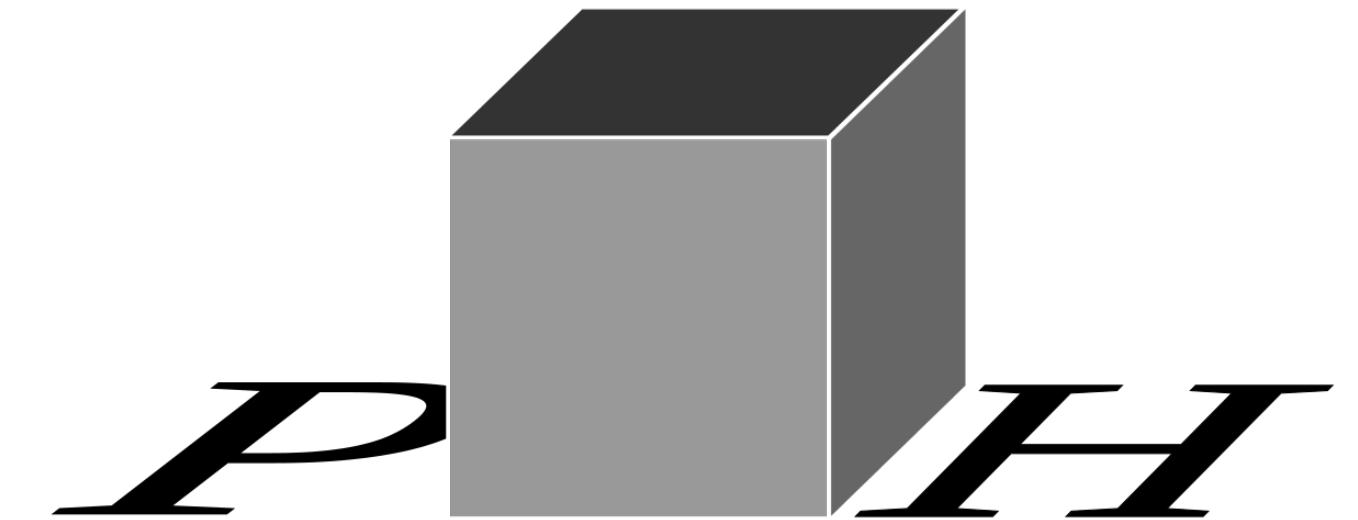




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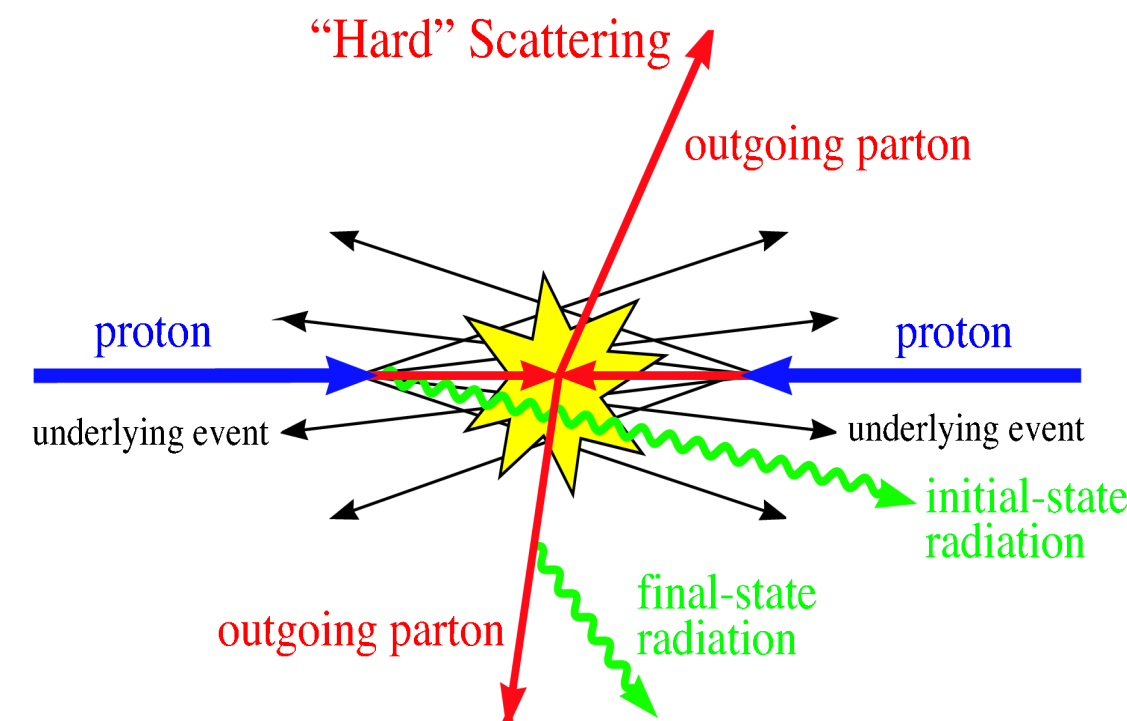
Particle Physics Phenomenology
after the Higgs Discovery

Power corrections to collider observables

Kirill Melnikov

Based on work done in collaboration with S. Makarov, P. Nason, M. Ozelik

In particle physics, we deduce information about the SM Lagrangian by comparing properties of hadrons, produced in collider processes, with theoretical predictions obtained using quark and gluon degrees of freedom. This mismatch leads to differences between partonic and hadronic cross sections, that we refer to as **power corrections**.



$$d\sigma^H = \sum_{ij} \int dx_1 dx_2 f_i(x_1) f_j(x_2) d\sigma_{ij}^{\text{part}}(x_1 P_1, x_2 P_2) \left[1 + \mathcal{O} \left(\frac{\Lambda_{\text{QCD}}^p}{Q^p} \right) \right].$$

There is no theory of power corrections and even the exponent $p > 0$ cannot be predicted from first principles for a given process or observable.

Numerically, such corrections cannot be large for “ordinary” observables, but **linear** $p = 1$ **power corrections** may still be relevant for “standard-candle” processes, and for studies of high-precision observables (the strong coupling constant, mass of the top quarks etc.).

$$\Lambda_{\text{QCD}} \sim 300 \text{ MeV}, \quad Q \sim 30 \text{ GeV} \Rightarrow \frac{\Lambda_{\text{QCD}}}{Q} \sim 10^{-2}$$

We want to study **linear power corrections** within the **renormalon model** where perturbatively-induced Landau singularity in the running QCD coupling constant is the **only** source of non-perturbative effects.

$$\int dk k^{p-1} \alpha_s(\mu) F(k) \Rightarrow \int dk k^{p-1} \alpha_s(k) F(k)$$

$$\alpha_s(k^2) \approx \frac{1}{2\beta_0} \frac{\Lambda_{\text{QCD}}^2}{k^2 - \Lambda_{\text{QCD}}^2}$$

$$N_f \rightarrow -\infty$$

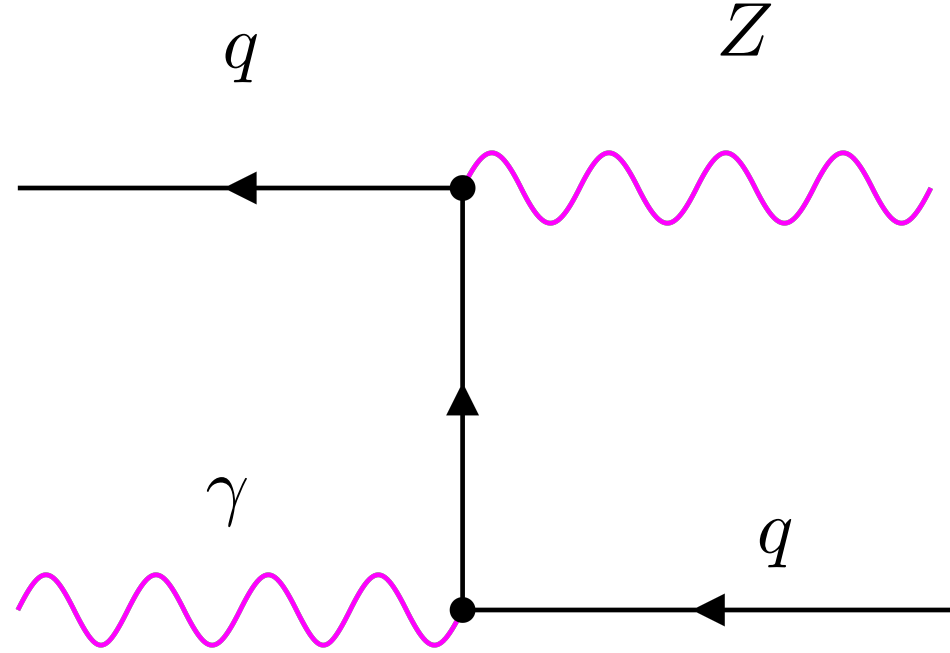
Without discussing the underpinnings of this model, I would like to make three remarks that are important for the rest of this talk:

- 1) within the renormalon model, **linear** power correction to a process X can be computed provided that a **linear term in the expansion of the NLO QCD corrections to X 's cross section in the small gluon mass is known**;
- 2) **linear** gluon-mass corrections are special because they **are non-analytic**. They only arise from the emission of real and virtual gluons **whose energies are comparable to their masses**. If the mass is small, the gluons are soft. Hence, to find a systematic way to compute the linear corrections, one needs to understand **soft-gluon emissions beyond the leading power**.
- 3) this set up can be used for processes that remain **abelian** through NLO; in other words, the non-abelian vertices are, strictly speaking, not allowed. I will discuss an example (top quark pair production in quark collisions) where this requirement can be lifted, although processes with on-shell gluons at tree level are still beyond reach.

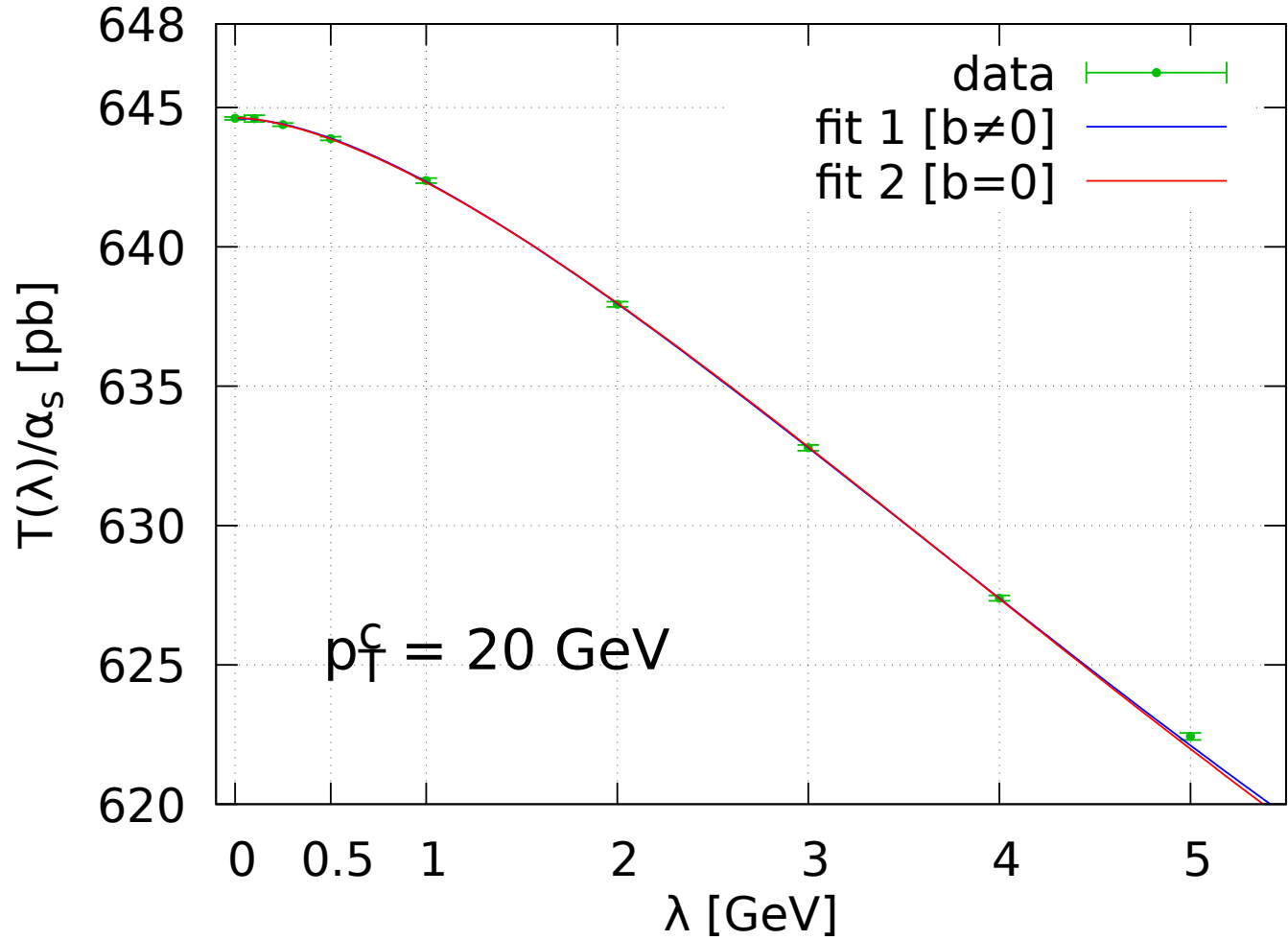
Earlier computations of power corrections to collider processes using this approach were performed in two distinct ways:

1) **for simple processes** (2 jet production at lepton colliders, DIS, Drell-Yan) NLO QCD corrections were first computed for an **arbitrary gluon mass and then** expanded assuming that the gluon mass is very small.

2) **for complex processes and observables**, studies of linear power corrections were performed **numerically**. A typical numerical result is shown below; it demonstrates (with a certain, not overwhelming confidence!) that no linear terms are present in the Z transverse-momentum distribution.



$p_T^c = 20 \text{ GeV}$	
fit 1	fit 2
$a = 644.60 \pm 0.02$	$a = 644.63 \pm 0.02$
$b = 0.009 \pm 0.004$	$b = 0$
$c = -0.063 \pm 0.008$	$c = -0.047 \pm 0.004$
$d = 0.341 \pm 0.005$	$d = 0.341 \pm 0.007$
$\chi^2/\text{ndf} = 0.12$	$\chi^2/\text{ndf} = 0.23$



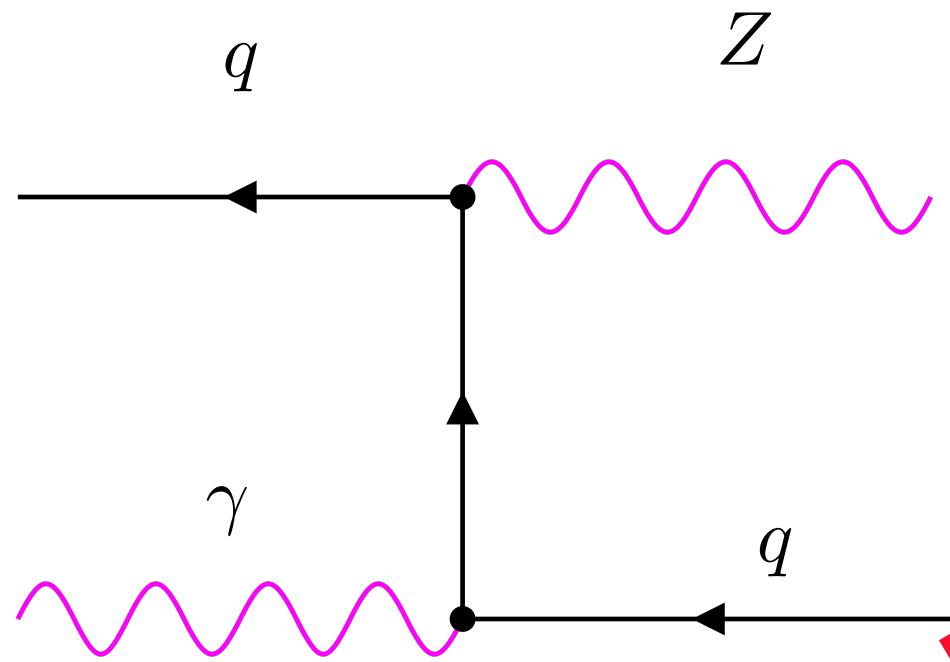
$$f(\lambda) = a \left[1 + b \left(\frac{\lambda}{p_T^c} \right) + c \left(\frac{\lambda}{p_T^c} \right)^2 \log^2 \left(\frac{\lambda}{p_T^c} \right) + d \left(\frac{\lambda}{p_T^c} \right)^2 \log \left(\frac{\lambda}{p_T^c} \right) \right]$$

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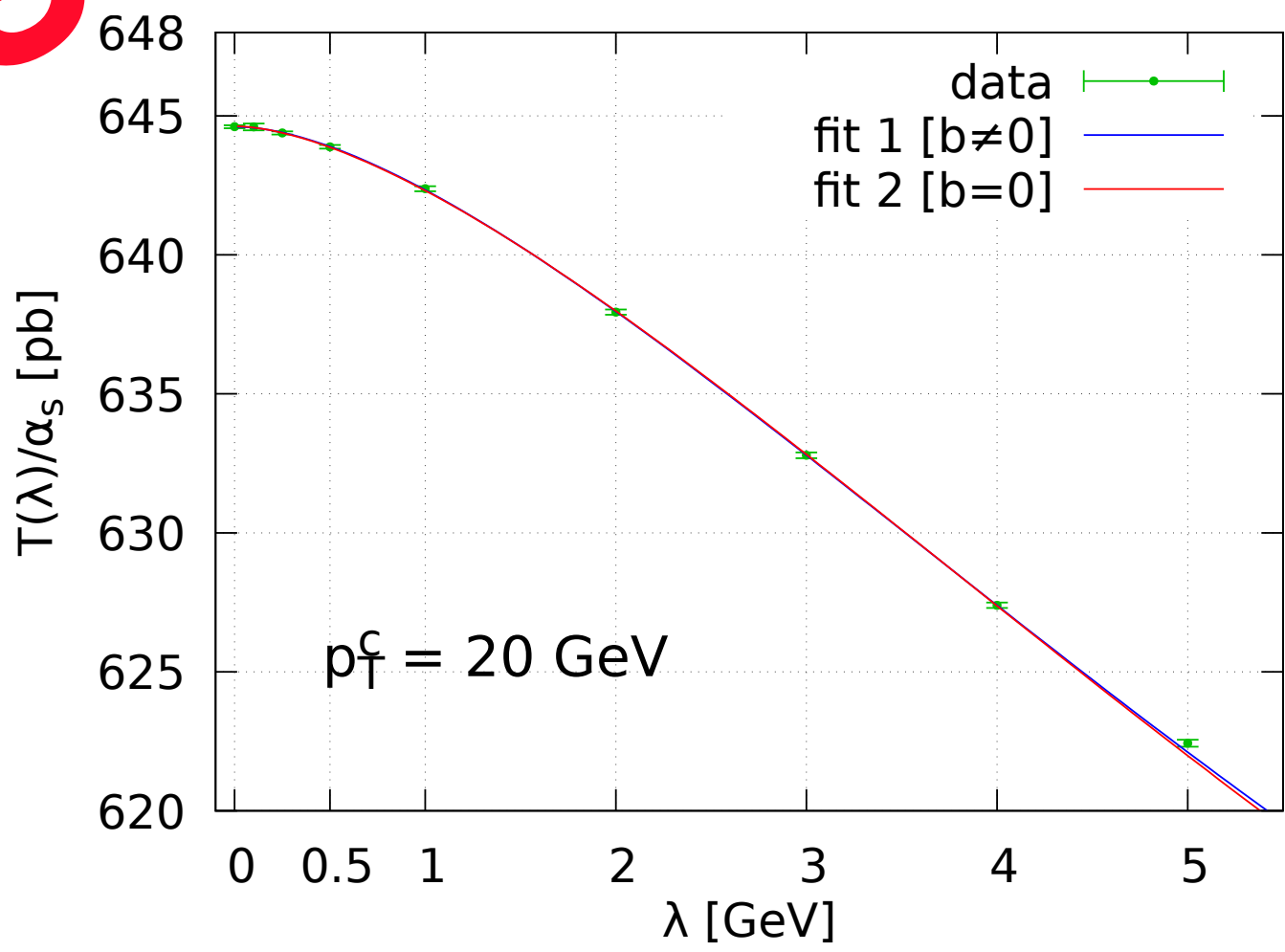
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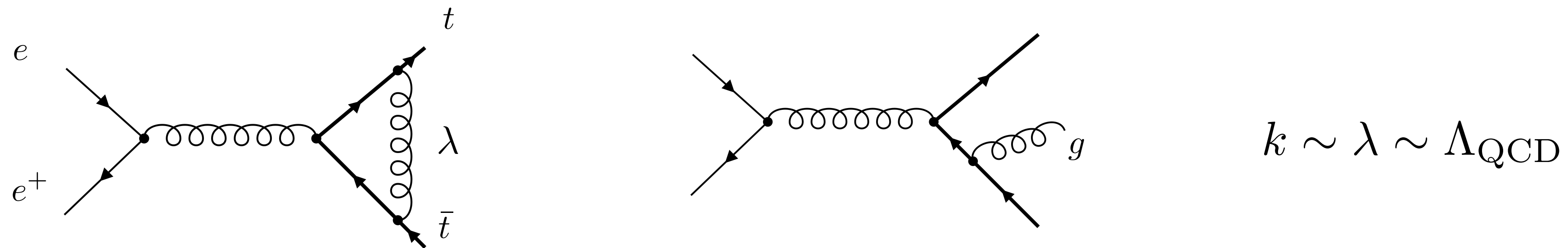
$$f(\lambda) = a \left[1 + b \left(\frac{\lambda}{p_T^c} \right) + c \left(\frac{\lambda}{p_T^c} \right)^2 \log^2 \left(\frac{\lambda}{p_T^c} \right) + d \left(\frac{\lambda}{p_T^c} \right)^2 \log \left(\frac{\lambda}{p_T^c} \right) \right]$$

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To do better, we need to understand **how dependences of partonic NLO QCD cross sections and observables on a tiny gluon mass arise, for arbitrary processes.**

For properly-defined observables, **the leading term** in the small-mass expansion is independent of the gluon mass. This is the consequence of the KLN theorem. Technically, this happens because of the cancellation of logarithmically-enhanced contributions in virtual and real-emission amplitudes, and because infra-red safe observable and the phase space for final-state particles are independent of the gluon mass.

These results follow from the soft (small gluon energy) expansion of the real and virtual amplitudes, the phase space and the observables.



$$d\sigma_{e^+e^- \rightarrow t\bar{t}} \sim d\sigma_0 \left(1 - c_1 \alpha_s \ln \frac{m_t}{\lambda} \dots \right)$$

$$d\sigma_{e^+e^- \rightarrow t\bar{t}+g} \sim d\sigma_0 c_1 \alpha_s \ln \frac{m_t}{\lambda} \dots$$

$$d\sigma_{e^+e^- \rightarrow t\bar{t}} + d\sigma_{e^+e^- \rightarrow t\bar{t}+g} \sim d\sigma_0 (1 + \alpha_s \mathcal{O}(\lambda^0))$$

Since the leading term is **independent** of the gluon mass, the linear term is the **first** correction. Hence, to compute it, we need to understand **next-to-leading terms in the soft expansion**.

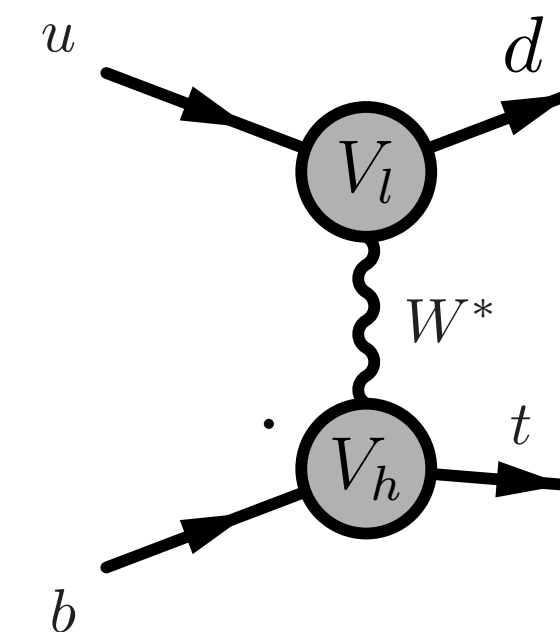
$$d\sigma_{\text{NLO}} = d\sigma_{\text{LO}} + \alpha_s \Delta\sigma(\lambda) \qquad \Delta\sigma(\lambda) \approx \Delta\sigma(0) + \underbrace{\lambda\Delta\sigma'} + \mathcal{O}(\lambda^2).$$

Calculation of NLO QCD cross sections requires well-known ingredients including

- real and virtual matrix elements;
- phase space;
- infra-red safe observable.

Each of these quantities depends on the gluon mass and we need to understand the (soft) expansion for each of them through next-to-leading power.

$$d\sigma = d\Phi_{\mathcal{O}(\lambda,k)} \times |\mathcal{M}|_{\mathcal{O}(\lambda,k)}^2 \times \mathcal{O}_{\mathcal{O}(k)}$$

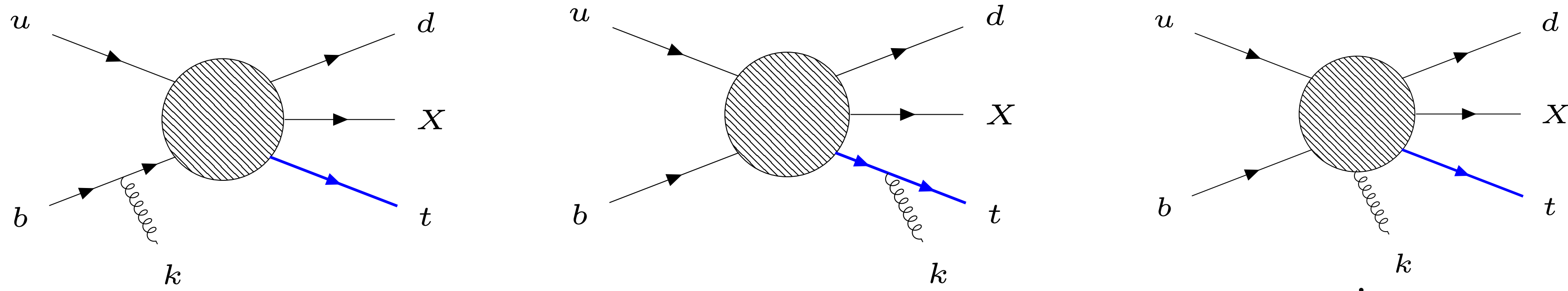


I will focus on processes that are similar to the t-channel single top production and then discuss other examples.

Matrix elements

$$d\sigma = d\Phi_{\mathcal{O}(\lambda,k)} \times |\mathcal{M}|_{\mathcal{O}(\lambda,k)}^2 \times \mathcal{O}_{\mathcal{O}(k)}$$

It follows from the [Low-Burnett-Kroll](#) theorem that next-to-leading soft corrections to the [real-emission amplitude squared](#) can be computed in a [process-independent manner](#). The LBK theorem exposes the dependence of the amplitude squared on the soft-gluon momentum (up to “hidden” dependencies in the leading term caused by the momentum non-conservation).



$$|\mathcal{M}(\dots, k)|^2 = -J^\mu J_\mu F_{\text{LO}}(q_t, p_b, q_d, p_u) - J_\mu L^\mu F_{\text{LO}}(q_t, p_b, q_d, p_u) + \mathcal{O}(k^0)$$

$$J^\mu = J_t^\mu + J_b^\mu, \quad L^\mu = L_t^\mu - L_b^\mu,$$

$$F_{\text{LO}} = \sum_{\text{spins}} |\mathcal{M}_{\text{LO}}|^2$$

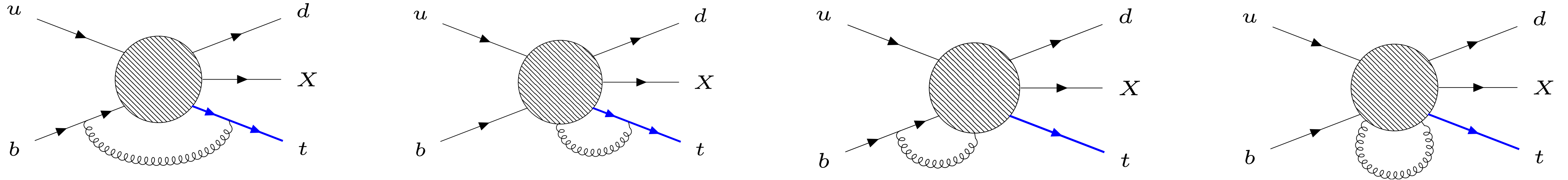
$$J_t^\mu = \frac{2q_t^\mu + k^\mu}{2q_t \cdot k + \lambda^2}$$

$$J_b^\mu = -\frac{2p_b^\mu - k^\mu}{2p_b \cdot k - \lambda^2}$$

$$L_t^\mu = J_t^\mu k^\nu \frac{\partial}{\partial q_t^\nu} - \frac{\partial}{\partial q_t^\mu}$$

$$L_b^\mu = J_b^\mu k^\nu \frac{\partial}{\partial p_b^\nu} + \frac{\partial}{\partial p_b^\mu}$$

A similar analysis can be performed for the [virtual corrections](#). One splits diagrams into groups according to how many times a virtual gluon couples to external lines. The expansion similar to the real-emission case is then constructed; in fact, very similar functions and their derivatives appear in the calculation.



$$\mathcal{M}_{\text{virt}} = \int \frac{d^4 k}{(2\pi)^4} \frac{-i}{k^2 - \lambda^2} \left[J_t^\alpha J_{b,\alpha} \bar{u}_t (\mathbf{N}(p_t, p_b, \dots) + k^\mu D_{p,\mu} \mathbf{N}(p_t, p_b, \dots)) u_b \right. \\ \left. - J_t^\alpha \bar{u}_t \mathbf{N}(p_t, p_b, \dots) \mathbf{S}_{b,\alpha} u_b + J_b^\alpha \bar{u}_t \mathbf{S}_{t,\alpha} \mathbf{N}(p_t, p_b, \dots) u_b - (J_t^\alpha + J_b^\alpha) \bar{u}_t D_{p,\alpha} \mathbf{N} u_b \right].$$

$$J_t^\alpha = \frac{2p_t^\alpha + k^\alpha}{d_t}, \quad \mathbf{S}_t^\alpha = \frac{\sigma^{\alpha\beta} k_\beta}{d_t}, \quad d_t = (p_t + k)^2 - m_t^2$$

$$J_b^\alpha = \frac{2p_b^\alpha + k^\alpha}{d_b}, \quad \mathbf{S}_b^\alpha = \frac{\sigma^{\alpha\beta} k_\beta}{d_b}, \quad d_b = (k + p_b)^2$$

$$D_p^\mu = \frac{\partial}{\partial p_{t,\mu}} + \frac{\partial}{\partial p_{b,\mu}}$$

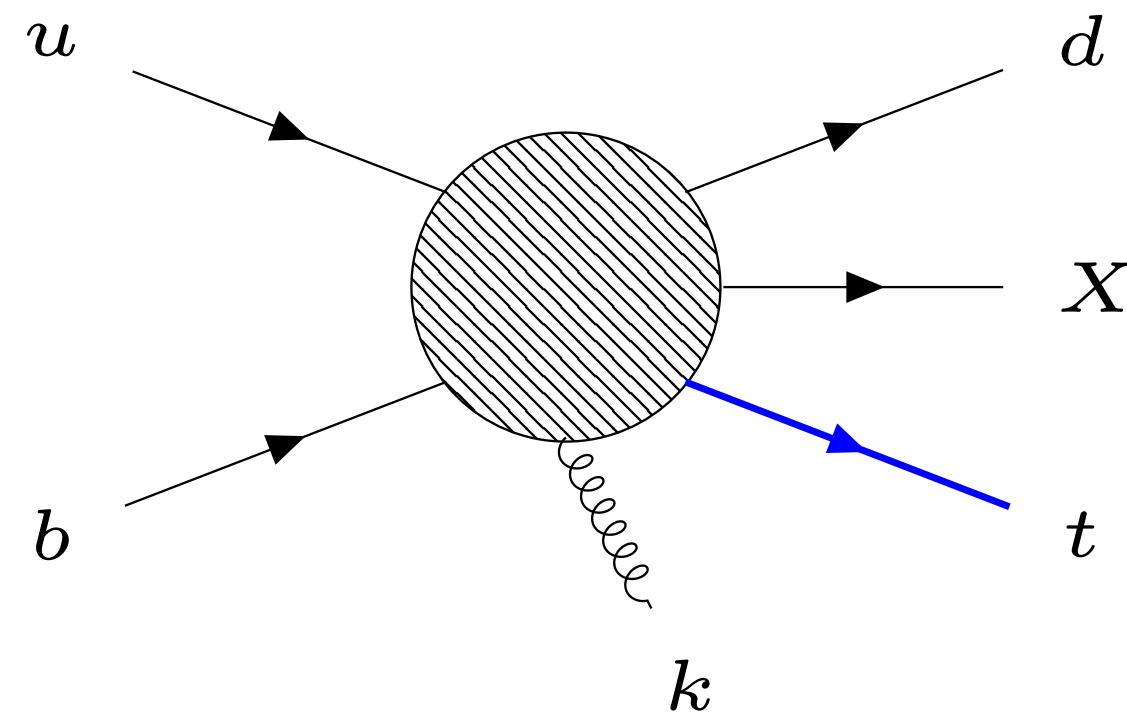
$$\delta[\mathcal{M}\mathcal{M}^+]_{\text{virt}} = \int \frac{d^4 k}{(2\pi)^4} \frac{-i}{k^2 - \lambda^2} \left[2J_t^\alpha J_{b,\alpha} F_{\text{LO}} + J_t^\alpha J_{b,\alpha} k^\mu D_{p,\mu} F_{\text{LO}} - (J_t^\alpha + J_b^\alpha) D_{p,\alpha} F_{\text{LO}} \right. \\ \left. + J_t^\alpha \text{Tr} \left[(D_{p,\alpha} \not{p}_t) \mathbf{N} \not{p}_b \bar{\mathbf{N}} \right] + J_b^\alpha \text{Tr} \left[(\not{p}_t + m_t) \mathbf{N} (D_{p,\alpha} \not{p}_b) \bar{\mathbf{N}} \right] \right] + \mathcal{O}(\lambda^2).$$

The dependence on the loop momentum is (again) clearly exposed in the eikonal currents, and the integration over the loop momentum can be performed in a process-independent way.

The phase space

$$d\sigma = d\Phi_{\mathcal{O}(\lambda,k)} \times |\mathcal{M}|_{\mathcal{O}(\lambda,k)}^2 \times O_{\mathcal{O}(k)}$$

Computing the real-emission contribution always involves integration over the phase space of final-state particles, which depends on the soft-gluon momentum. **One can factorize it from the rest of the phase space with (linear) power accuracy by redefining momenta of hard particles.** Once this is done, it becomes possible to integrate over the gluon momentum in a process-independent way with the next-to-leading power accuracy.



Original momenta

$$q_t = p_t - k + \frac{p_t k}{p_t p_d} p_d$$

$$q_t^2 = p_t^2 = m_t^2$$

Redefined momenta
(no dependence on k !)

$$q_d = p_d - \frac{p_t k}{p_t p_d} p_d$$

$$q_d^2 = p_d^2 = 0$$

New momentum, conservation condition

$$q_t + q_d + k + p_X = p_t + p_d + p_X$$

$$d\Phi(p_u, p_b; q_d, q_t, p_X, k) = d\Phi(p_u, p_b; p_d, p_t) \frac{d^4 k}{(2\pi)^3} \delta_+(k^2 - \lambda^2) \times \left(1 + \frac{k \cdot p_d}{p_t \cdot p_d} - \frac{k \cdot p_t}{p_t \cdot p_d} \right)$$

Radiative phase space with
the original momenta

Born phase space with
the redefined momenta

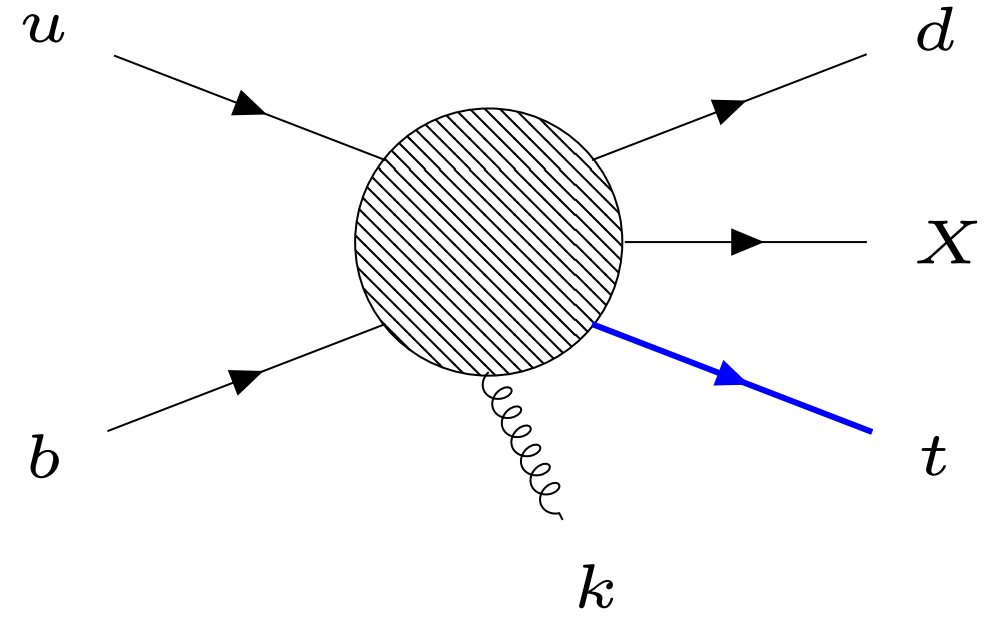
Factorized dependence on the gluon momentum

Integration over the gluon momentum does contain an upper boundary (not shown in the above formula). **However, the upper boundary only contributes at $\mathcal{O}(\lambda^2)$ and, therefore, plays no role for the linear power corrections.**

The phase space, the matrix element and the renormalization

$$d\sigma = d\Phi_{\mathcal{O}(\lambda,k)} \times |\mathcal{M}|_{\mathcal{O}(\lambda,k)}^2 \times O_{\mathcal{O}(k)}$$

To recap: using the LBK theorem, the momenta mapping and the phase-space factorization, we compute $\mathcal{O}(\lambda)$ contributions, that arise from the emissions of real and virtual gluons, to an **arbitrary process** of a single-top-production type. **If the put these contributions together, we find that the $\mathcal{O}(\lambda)$ corrections do not cancel.**



$$|\mathcal{M}|^2 = -J^\mu J_\mu F_{\text{LO}}(q_t, p_b, q_d, \dots) - J_\mu L^\mu F_{\text{LO}}(q_t, p_b, q_d, \dots).$$

$$q_t = p_t - k + \frac{p_t k}{p_t p_d} p_d \quad q_d = p_d - \frac{p_t k}{p_t p_d} p_d$$

$$d\Phi(p_u, p_b; q_d, q_t, p_X, k) = d\Phi(p_u, p_b; p_d, p_t) \frac{d^4 k}{(2\pi)^3} \delta_+(k^2 - \lambda^2) \times \left(1 + \frac{k \cdot p_d}{p_t \cdot p_d} - \frac{k \cdot p_t}{p_t \cdot p_d} \right)$$

$$\mathcal{T}_\lambda [\sigma_R] = \frac{\alpha_s C_F}{2\pi} \frac{\pi \lambda}{m_t} \int d\Phi_{\text{LO}} \left[\left(\frac{3}{2} - \frac{m_t^2}{p_d \cdot p_t} - \frac{m_t^2}{p_t \cdot p_b} \right) - \frac{m_t^2}{p_d \cdot p_t} p_d^\mu \left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu} \right) - \frac{m_t^2}{p_t \cdot p_b} p_b^\mu \left(\frac{\partial}{\partial p_b^\mu} + \frac{\partial}{\partial p_t^\mu} \right) \right] F_{\text{LO}}$$

$$\mathcal{T}_\lambda [\sigma_V] = \frac{\alpha_s C_F}{2\pi} \frac{\pi \lambda}{m_t} \int d\Phi_{\text{LO}} \left[\text{Tr} \left[\hat{p}_t \mathbf{N} \hat{p}_b \tilde{\mathbf{N}} \right] + \left(\frac{2p_t \cdot p_b - m_t^2}{p_t \cdot p_b} - \frac{m_t^2}{p_t \cdot p_b} p_b^\mu \left(\frac{\partial}{\partial p_{t,\mu}} + \frac{\partial}{\partial p_{b,\mu}} \right) \right) \right] F_{\text{LO}}$$

However, [real and virtual corrections are not the whole story](#). When massive particles are involved, linear power corrections [arise because of the renormalization](#) (the mass and the wave-function), [in the on-shell scheme](#). We have implicitly used the on-shell scheme because we did not consider self-energy corrections on external particles' lines.

$$Z_m = 1 + \frac{C_F g_s^2 m_t^{-2\epsilon} \Gamma(1 + \epsilon)}{(4\pi)^{d/2}} \left[-\frac{3}{\epsilon} - 4 + \frac{2\pi\lambda}{m_t} + \mathcal{O}\left(\frac{\lambda^2}{m_t^2}\right) \right], \quad Z_2 = 1 + \frac{C_F g_s^2 m_t^{-2\epsilon} \Gamma(1 + \epsilon)}{(4\pi)^{d/2}} \left[-\frac{1}{\epsilon} - 4 + 4 \ln \frac{m_t}{\lambda} + \frac{3\lambda\pi}{m_t} + \mathcal{O}\left(\frac{\lambda^2}{m_t^2}\right) \right].$$

It is also well-motivated (B-physics) to expect that the on-shell renormalization scheme introduces artificial power corrections, simply related to the fact that [the pole mass of a heavy quark has a linear renormalon](#). To get rid of this problem, we have to express our result through (any) short distance mass ($\overline{\text{MS}}$ etc.). The relation between any short-distance mass and the pole mass of a heavy quark contains universal linear gluon-mass term.

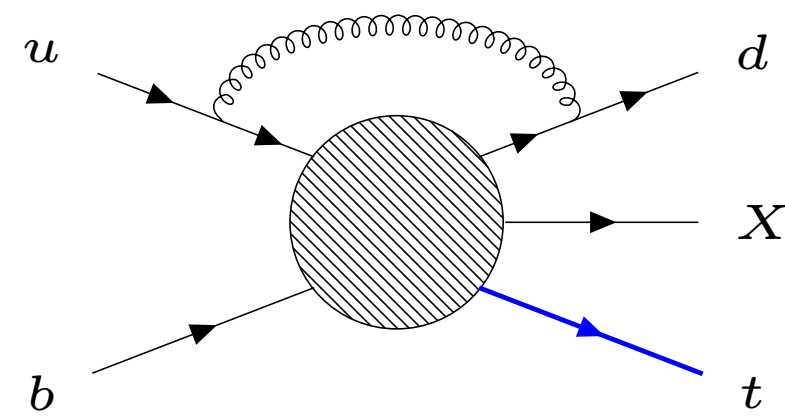
$$m_t = \tilde{m}_t \left(1 - \frac{C_F \alpha_s}{2\pi} \frac{\pi\lambda}{m_t} \right)$$

[An interesting technical question is how to “change the mass” when dealing with a general process](#), i.e. without making use of the explicit form of the matrix element and the phase space. As it turns out, this can also be done by [performing momenta redefinitions in the leading order cross section](#), similar to what we do with the real-emission contribution.

$$\sigma_{\text{LO}}(m_t) - \sigma_{\text{LO}}(\tilde{m}_t) = \frac{C_F \alpha_s}{2\pi} \frac{\pi\lambda}{m_t} \int d\Phi_{\text{LO}} \left[\frac{m_t^2}{p_d p_t} \left[1 + p_d^\mu \left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu} \right) \right] F_{\text{LO}} - m_t \text{Tr} \left[\mathbf{1} \mathbf{N} \not{p}_b \bar{\mathbf{N}} \right] \right. \\ \left. - m_t \text{Tr} \left[(\not{p}_t + m_t) \left(\frac{\partial \mathbf{N}^-}{\partial m_t} \not{p}_b \bar{\mathbf{N}} + \mathbf{N} \not{p}_b \frac{\partial \bar{\mathbf{N}}}{\partial m_t} \right) \right] \right]$$

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A remark about the massless quark line(s)



QCD emissions of real and virtual gluons off the upper ([massless](#)) line in the single-top production do not induce linear power corrections; [hence focusing on the massive line is sufficient](#). This is a general result; it applies to arbitrary processes with massless quark lines (abelian limit).

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Combining four different contributions — real, virtual, renormalization and the mass redefinition — we observe that cross sections for **arbitrary** single-top-like production processes **do not contain** linear power corrections.

$$\sigma = \sigma_{\text{LO}}(m_t) + \sigma_R + \sigma_V + \sigma_{\text{ren}} = \sigma_{\text{LO}}(\tilde{m}_t) + \delta\sigma_{\text{NLO}} \quad \delta\sigma_{\text{NLO}} = \sigma_R + \sigma_V + \sigma_{\text{ren}} + \delta\sigma_{\text{mass}}^{\text{expl}} + \delta\sigma_{\text{mass}}^{\text{impl}}$$

$$\mathcal{T}_\lambda \left[\delta\sigma_{\text{mass}}^{\text{expl}} + \delta\sigma_{\text{mass}}^{\text{impl}} \right] = \frac{C_F \alpha_s}{2\pi} \frac{\pi\lambda}{m_t} \int d\Phi_{\text{LO}} \left[\frac{m_t^2}{p_d p_t} \left[1 + p_d^\mu \left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu} \right) \right] F_{\text{LO}} - m_t \text{Tr} \left[\mathbf{1} \mathbf{N} \not{p}_b \bar{\mathbf{N}} \right] - m_t \text{Tr} \left[(\not{p}_t + m_t) \left(\frac{\partial \mathbf{N}}{\partial m_t} \not{p}_b \bar{\mathbf{N}} + \mathbf{N} \not{p}_b \frac{\partial \bar{\mathbf{N}}}{\partial m_t} \right) \right] \right]$$

$$\mathcal{T}_\lambda [\sigma_R] = \frac{\alpha_s C_F}{2\pi} \frac{\pi\lambda}{m_t} \int d\Phi_{\text{LO}} \left[\left(\frac{3}{2} - \frac{m_t^2}{p_d p_t} - \frac{m_t^2}{p_t p_b} \right) - \frac{m_t^2}{p_d p_t} p_d^\mu \left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu} \right) - \frac{m_t^2}{p_t p_b} p_b^\mu \left(\frac{\partial}{\partial p_b^\mu} + \frac{\partial}{\partial p_t^\mu} \right) \right] F_{\text{LO}}$$

$$\mathcal{T}_\lambda [\sigma_V] = - \frac{\alpha_s C_F}{2\pi} \frac{\pi\lambda}{m_t} \int d\Phi_{\text{LO}} \left[\text{Tr} \left[\not{p}_t \mathbf{N} \not{p}_b \bar{\mathbf{N}} \right] + \left(\frac{2p_t p_b - m_t^2}{p_t p_b} - \frac{m_t^2}{p_t p_b} p_b^\mu D_{p,\mu} \right) F_{\text{LO}} \right]$$

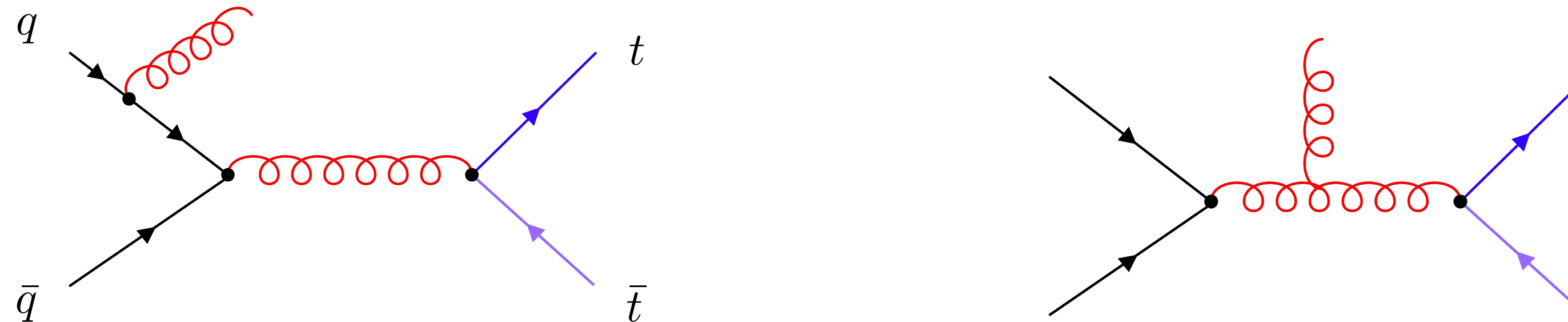
$$\mathcal{T}_\lambda [\sigma_{\text{ren}}] = \frac{\alpha_s C_F}{2\pi} \frac{\pi\lambda}{m_t} \int d\Phi_{\text{LO}} \left[\frac{3}{2} F_{\text{LO}} + m_t \text{Tr} \left[(\not{p}_t + m_t) \frac{\partial \mathbf{N}}{\partial m_t} \not{p}_b \bar{\mathbf{N}} \right] + m_t \text{Tr} \left[(\not{p}_t + m_t) \mathbf{N} \not{p}_b \frac{\partial \bar{\mathbf{N}}}{\partial m_t} \right] \right]$$

$$\mathcal{T}_\lambda [\delta\sigma_{\text{NLO}}] = \frac{\alpha_s C_F}{2\pi} \frac{\pi\lambda}{m_t} \int d\Phi_{\text{LO}} \left(F_{\text{LO}} - \text{Tr} \left[\not{p}_t \mathbf{N} \not{p}_b \bar{\mathbf{N}} \right] - m_t \text{Tr} \left[\mathbf{1} \mathbf{N} \not{p}_b \bar{\mathbf{N}} \right] \right) = 0$$

Generalization to top quark pair production

$$d\sigma = d\Phi_{\mathcal{O}(\lambda,k)} \times |\mathcal{M}|_{\mathcal{O}(\lambda,k)}^2 \times O_{\mathcal{O}(k)}$$

If **highly virtual** gluons are present in the Born matrix element, **the same approach** can be used; this is useful for the analysis of **top quark pair production in quark-antiquark annihilation**.



A required contribution of the three-gluon vertex will get reconstructed when the LBK theorem is applied. At the end of the day, a dipole-like structure of color-correlations emerges, similar to the eikonal functions but with next-to-soft accuracy.

$$|\mathcal{A}_{\text{real}}|^2 = -g_s^2 \sum_{i,j \in N} \eta_i \eta_j W_i^\mu W_{j,\mu} F_{\text{LO}}^{ij} + \mathcal{O}(k^0). \quad \eta_i = \pm 1.$$

$$F_{\text{LO}}^{ij} = \langle \mathcal{M}_0 | \vec{T}^i \cdot \vec{T}^j | \mathcal{M}_0 \rangle. \quad W_i^\mu = J_i^\mu + \frac{1}{2} L_i^\mu, \quad L_i^\mu = J_i^\mu k^\nu D_{i,\nu} - D_{i,\mu}$$

One can address the virtual contributions etc. in a similar way, and then repeat all the steps discussed in the single top production case. We find that **the linear power correction cancels out in the cross section** for top pair production in quark-antiquark annihilation, and that one can **arrange the cancellation to occur within each color dipole** (for example by performing **independent** momenta re-definitions).

Observables

$$d\sigma = d\Phi_{\mathcal{O}(\lambda,k)} \times |\mathcal{M}|_{\mathcal{O}(\lambda,k)}^2 \times O_{\mathcal{O}(k)}$$

We have seen that linear power corrections to many inclusive cross sections vanish. However, linear power corrections to **observables** are known to exist (e.g. shape variables in 2-jet production in e+e- annihilation etc.).

Our approach allows us to study complex processes and complex observables in a unified way, as I will now explain.

We consider top quark pair production processes and define an observable as an integral over a differential cross section. We analyze this integral using the same procedure as described earlier. The new element is that momenta mappings change observables.

$$O_X = \int d\sigma \, X(q_t) \qquad X(q_t) = X(p_t) + \frac{\partial X(p_t)}{\partial p_t^\mu} \left(\frac{p_t k}{p_t p_d} p_d^\mu - k^\mu \right)$$

For the purpose of computing linear power corrections, we then split $\mathcal{O}_X$ into two pieces

$$\mathcal{T}_\lambda[O_X] = \mathcal{T}_\lambda[O_X^{(1)}] + \mathcal{T}_\lambda[O_X^{(2)}]$$

The first one includes all corrections to the cross section that we discussed, and corrections to the observable, that are related to the change in the observable caused by the “mass redefinition”. The combination of these contributions vanishes, similar to the inclusive case.

$$\mathcal{T}_\lambda[O_X^{(1)}] = \mathcal{T}_\lambda \left[\int d\sigma \left(X(p_t) + \frac{\partial X(p_t)}{\partial p_t^\mu} \frac{p_t k}{p_t p_d} p_d^\mu \right) \right] \longrightarrow \mathcal{T}_\lambda[O_X^{(1)}] = 0$$

The second contribution survives. Since this contribution contains the momentum of the soft gluon explicitly, the corresponding cross section only needs to be known [in the leading soft approximation](#).

$$\mathcal{T}_\lambda[O_X^{(2)}] = -\mathcal{T}_\lambda \left[\int d\sigma \frac{\partial X(p_t)}{\partial p_t^\mu} k^\mu \right] = \mathcal{T}_\lambda \left[\int d\sigma_{\text{LO}} \frac{\partial X(p_t)}{\partial p_t^\mu} \int \frac{d^4 k}{(2\pi)^4} \delta_+(k^2 - \lambda^2) J_\nu J^\nu k^\mu \right]$$

Upon integration over gluon momentum, the correction can be re-absorbed into the observable, and described by a top-quark momentum shift. [In case of top quark pair-production, the shift is complex as it receives contributions from all color dipoles, analyzed using different momenta shifts.](#)

$$\mathcal{O}_X = \int d\sigma_{\text{LO}} X \left(p_t + \frac{\alpha_s}{2\pi} \sum_{\mathfrak{a}} C^{\mathfrak{a}} \delta p_{t,\mathfrak{a}} \right)$$

[Color structures for dipoles/monopoles:](#)

[Dipole momenta shifts:](#)

$$\delta p_{t,\mathfrak{a}} = \frac{\pi\lambda}{m_t} l_{\mathfrak{a}} \quad .$$

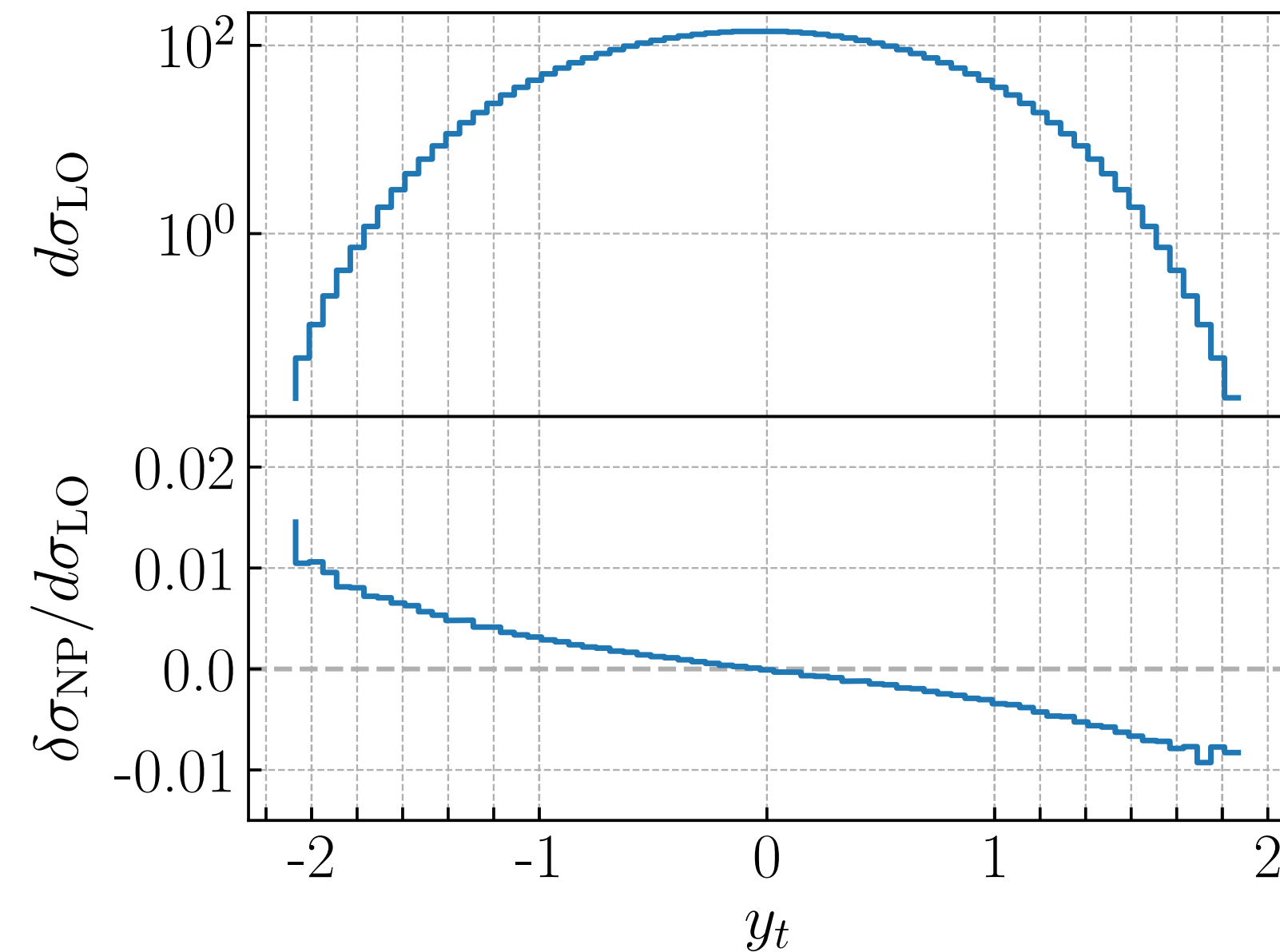
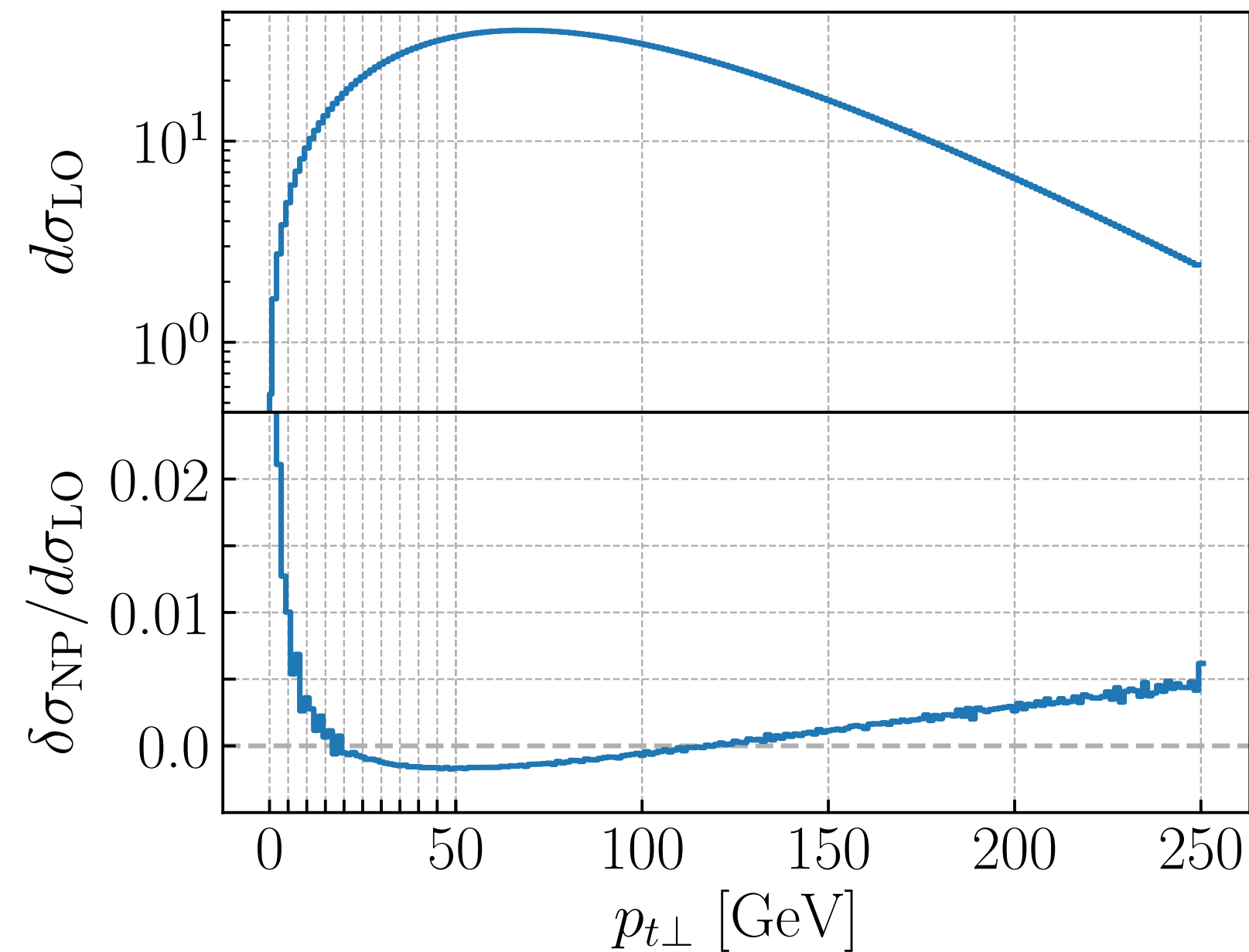
$$C^{tt} = C^{t\bar{t}} = C_F \qquad C^{\bar{t}q} = C^{t\bar{q}} = 2C_F - C_A$$

$$C^{tq} = C^{\bar{t}\bar{q}} = 2C_F - \frac{C_A}{2} \qquad C^{t\bar{t}} = C_F - \frac{C_A}{2}$$

$$l_{\mathfrak{a}}^\mu = \begin{cases} -p_t^\mu, & \text{for } (\mathfrak{a}) = (tt + \bar{t}\bar{t}), \\ 2(p_t p_{\bar{t}}) ((p_t p_{\bar{t}}) p_t^\mu - m_t^2 p_{\bar{t}}^\mu) / ((p_t p_{\bar{t}})^2 - m_t^4), & \text{for } (\mathfrak{a}) = (t\bar{t}), \\ 2p_t^\mu - 2m_t^2 p_q^\mu / (p_t p_q), & \text{for } (\mathfrak{a}) = (tq), \\ -2p_t^\mu + 2m_t^2 p_{\bar{q}}^\mu / (p_t p_{\bar{q}}), & \text{for } (\mathfrak{a}) = (t\bar{q}). \end{cases}$$

It is straightforward to apply the general formula to compute non-perturbative corrections to specific observables. Below I show shifts in the transverse momentum, and in the rapidity of the final-state top quark in the top quark pair production, in collisions of protons and anti-proton. In general, shifts are not large but they become enhanced close to edges of allowed kinematic regions.

$$\frac{\delta_{\text{NP}}[p_{t\perp}]}{p_{t\perp}} = \frac{\alpha_s}{2\pi} \frac{\pi\lambda}{m_t} \frac{(2C_F - C_A\tau)}{2(1-\tau)} \quad \delta_{\text{NP}}[y_t] = \frac{\alpha_s}{2\pi} \frac{\pi\lambda}{m_t} \left[(3C_A - 8C_F) \tau \cosh^2 y_t - (C_A - 2C_F) \frac{\tau(2-\tau)}{4(1-\tau)} \sinh(2y_t) \right]$$



$$\tau = 4m_t^2/s_{t\bar{t}}$$

Results are shown for the Tevatron where quark-antiquark annihilation channel dominates.

$$\alpha_s\lambda = \frac{0.4 \text{ GeV}}{C_F} = 0.3 \text{ GeV}$$

Top quark decays and linear power corrections

How does the top quark decay affect linear power corrections? There are two possible hierarchical relations between the width and the non-perturbative scale in QCD:

$$\Lambda_{\text{QCD}} \sim \lambda \ll \Gamma_t$$

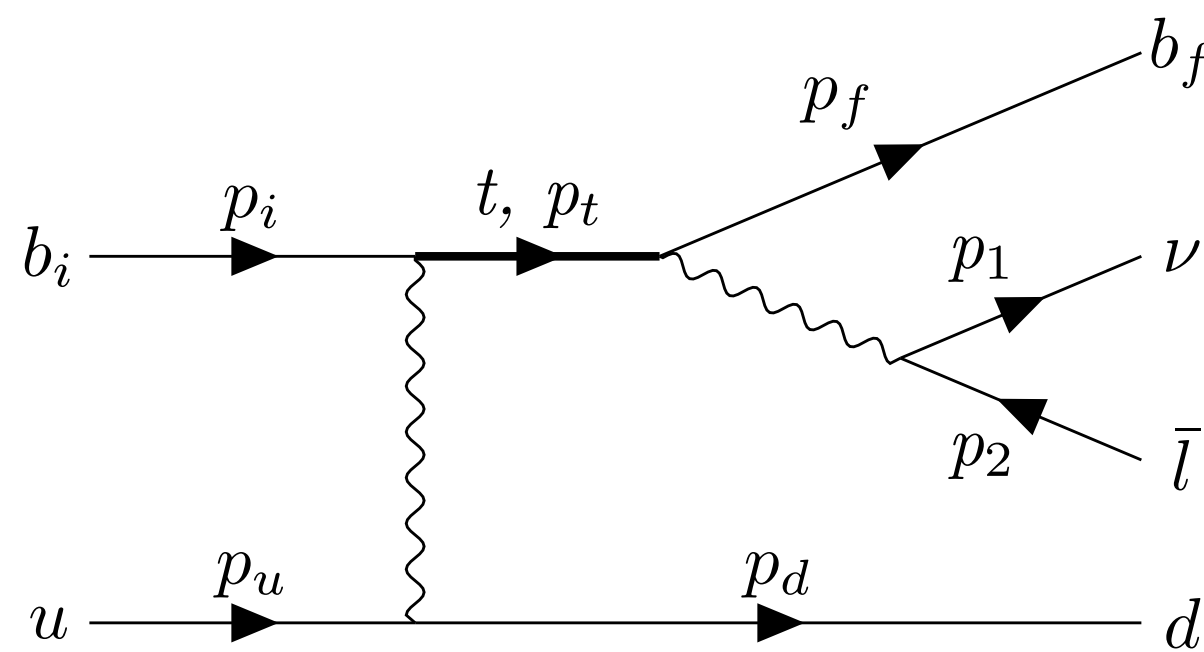
$$\Gamma_t \ll \Lambda_{\text{QCD}} \sim \lambda$$

The first is what happens in Nature; but we will study the second one because it is simpler (the narrow width approximation).

We will focus on the **polarization effects** in single top production and decay, which induce correlations between directions of the outgoing light jet (d-quark) from the production process, and a positron from the top decay process.

The cross section for the full process can be described either as a product of a cross section to produce polarized top quark followed by an unpolarized decay, or the other way around.

In the narrow width approximation, QCD corrections fully factorize between production and decay, so having the freedom to choose where to put top quark polarization is quite useful, for technical reasons.



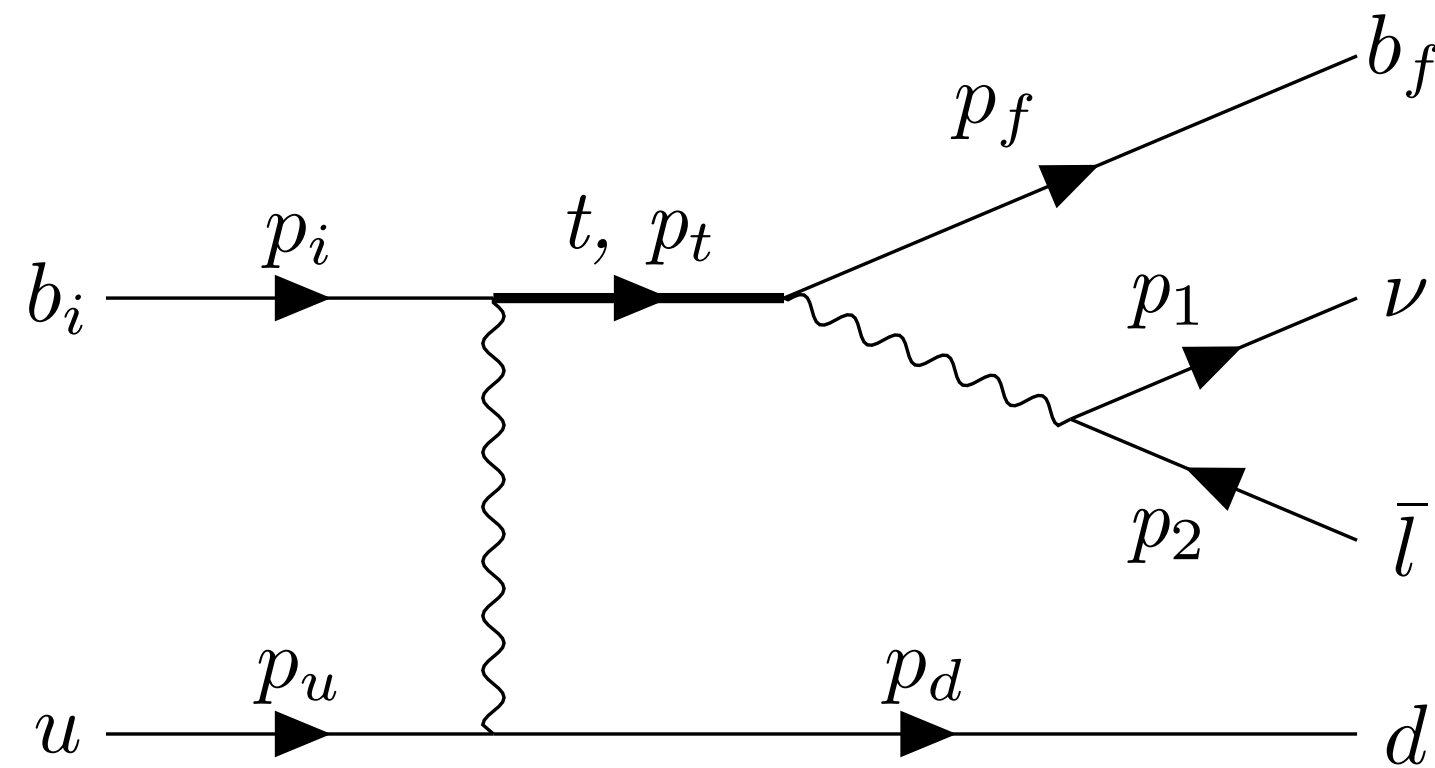
$$d\sigma_t d\Gamma_t(s_P) = d\sigma_t(s_D) d\Gamma_t = \frac{1}{2} d\sigma_t d\Gamma_t (1 - s_D \cdot s_P)$$

Following the same steps as described earlier, and making amendments to account for top quark polarization, and for the fact that the production-process kinematics and the decay kinematics are intertwined, we obtain the following result for the linear power corrections:

$$\mathcal{T}_\lambda[\mathcal{O}_X] = \frac{\alpha_s C_F}{2\pi} \frac{\pi\lambda}{m_t} \int \frac{d\Gamma_t d\sigma_t}{\Gamma_t} \left[- \frac{2m_t^2(p_i p_d)}{(p_t p_i)(p_t p_d)} s_{D,\mu} \omega_{id}^{\mu\nu} s_{P,\nu} X \right. \\ \left. + (1 - s_D \cdot s_P) \left[\left(p_t^\mu - \frac{2m_t^2}{(p_t p_i)} p_i^\mu \right) \frac{\partial X}{\partial p_t^\mu} - 2p_{2,\nu} \left(\omega_{td}^{\nu\mu} + \frac{m_t^2(p_d p_i)}{(p_t p_d)(p_t p_i)} \omega_{di}^{\nu\mu} \right) \frac{\partial X}{\partial p_2^\mu} \right] \right]$$

$$s_D^\mu = \frac{m_t}{p_2 \cdot p_t} p_2^\mu - \frac{1}{m_t} p_t^\mu \quad s_P^\mu = \frac{m_t}{p_d \cdot p_t} p_d^\mu - \frac{1}{m_t} p_t^\mu$$

$$\omega_{xy}^{\mu\nu} = \frac{p_x^\mu p_y^\nu - p_y^\mu p_x^\nu}{(p_x p_y)}$$



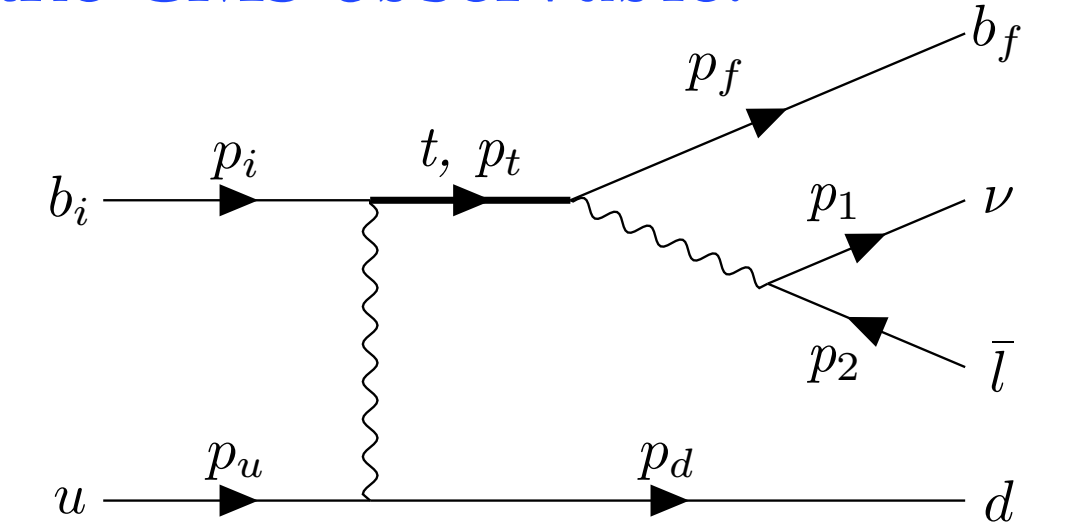
Note that there is a non-vanishing linear correction result to the polarized ``cross section'' itself, in addition to terms caused by the momenta-redefinitions that affect the observable X.

We can use this formula to analyze linear power corrections to a few observables used by CMS and ATLAS to study top quark polarization in single top quark production. We will show that some of these observables do receive linear power corrections and some do not.

The CMS collaboration studies asymmetry of light quark jets relative to the positron direction in the rest frame of the produced top quark. In the top quark rest frame, spin vectors are expressed through directions of the light jet and the positron. Using the derived formula, we find that **there are no linear power corrections to the CMS observable**.

$$X_{\text{CMS}} = \theta(-s_D s_P) - \theta(s_D s_P) \quad \rightarrow \quad \mathcal{T}_\lambda [X_{\text{CMS}}] = 0$$

$$s_D^\mu = (0, \vec{n}_2), \quad s_P^\mu = (0, \vec{n}_d).$$



The ATLAS collaboration uses more complicated observable which, as it turns out, **does receive linear power corrections**.

$$Q(\vec{n}_2, \{\vec{e}\}) = 4\theta(\vec{n}_2 \cdot \vec{e}_z) + 2\theta(\vec{n}_2 \cdot \vec{e}_x) + \theta(\vec{n}_2 \cdot \vec{e}_y)$$

$$\vec{e}_z = \vec{n}_d, \quad \vec{e}_y = \frac{\vec{n}_i \times \vec{n}_d}{|\vec{n}_i \times \vec{n}_d|}$$

$$\vec{e}_x = \vec{e}_y \times \vec{e}_z = \frac{\vec{n}_i \times \vec{n}_d}{|\vec{n}_i \times \vec{n}_d|} \times \vec{n}_d$$

$$\langle Q \rangle = \frac{1}{\sigma_t} \int d\sigma_t \left(\frac{9}{2} + \frac{\alpha_s C_F}{2\pi} \frac{\pi \lambda}{m_t} |\vec{n}_i \times \vec{n}_d| \right) \quad |\vec{n}_i \times \vec{n}_d| = \sqrt{\frac{4m_t^2 s t u}{(s - m_t^2)^2 (m_t^2 - t)^2}} = \frac{2m_t s p_{d\perp}}{(m_t^2 - t)(s - m_t^2)}$$

$$\frac{1}{\sigma_t} \int d\sigma_t |\vec{n}_i \times \vec{n}_d| = f_Q(s, m_t, m_W) = \frac{\pi m_t m_W \sqrt{s} \sqrt{s} (-m_t^4 + m_t^2 (m_W^2 + s) - 2m_t m_W \sqrt{s} \sqrt{s} + m_W^2 s)}{(m_t^2 - m_W^2)^2 (s - m_t^2)^2}$$

$$\lim_{s \rightarrow m_t^2} f(s, m_t, m_W) = \frac{\pi}{4} \approx 0.785$$

$$\lim_{s \rightarrow \infty} f(s, m_t, m_W) = \frac{\pi m_t m_W}{(m_t + m_W)^2} \approx 0.68$$

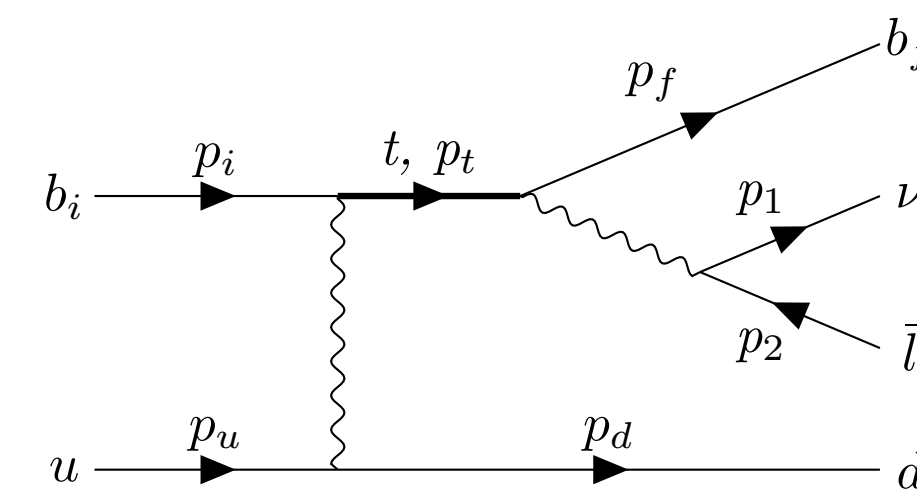
$$\langle Q \rangle \approx \frac{9}{2} + \frac{\alpha_s C_F}{2\pi} \frac{\pi \lambda}{m_t} \frac{\pi}{4}$$

The determination of the top quark mass with high (and defensible!) precision is an outstanding problem in hadron collider physics. Current measurements by the CMS and ATLAS collaborations claim precision that is comparable to Λ_{QCD} , but it is unclear if these results admit a clear interpretation. The central problem is to design observables which are not affected by linear power corrections.

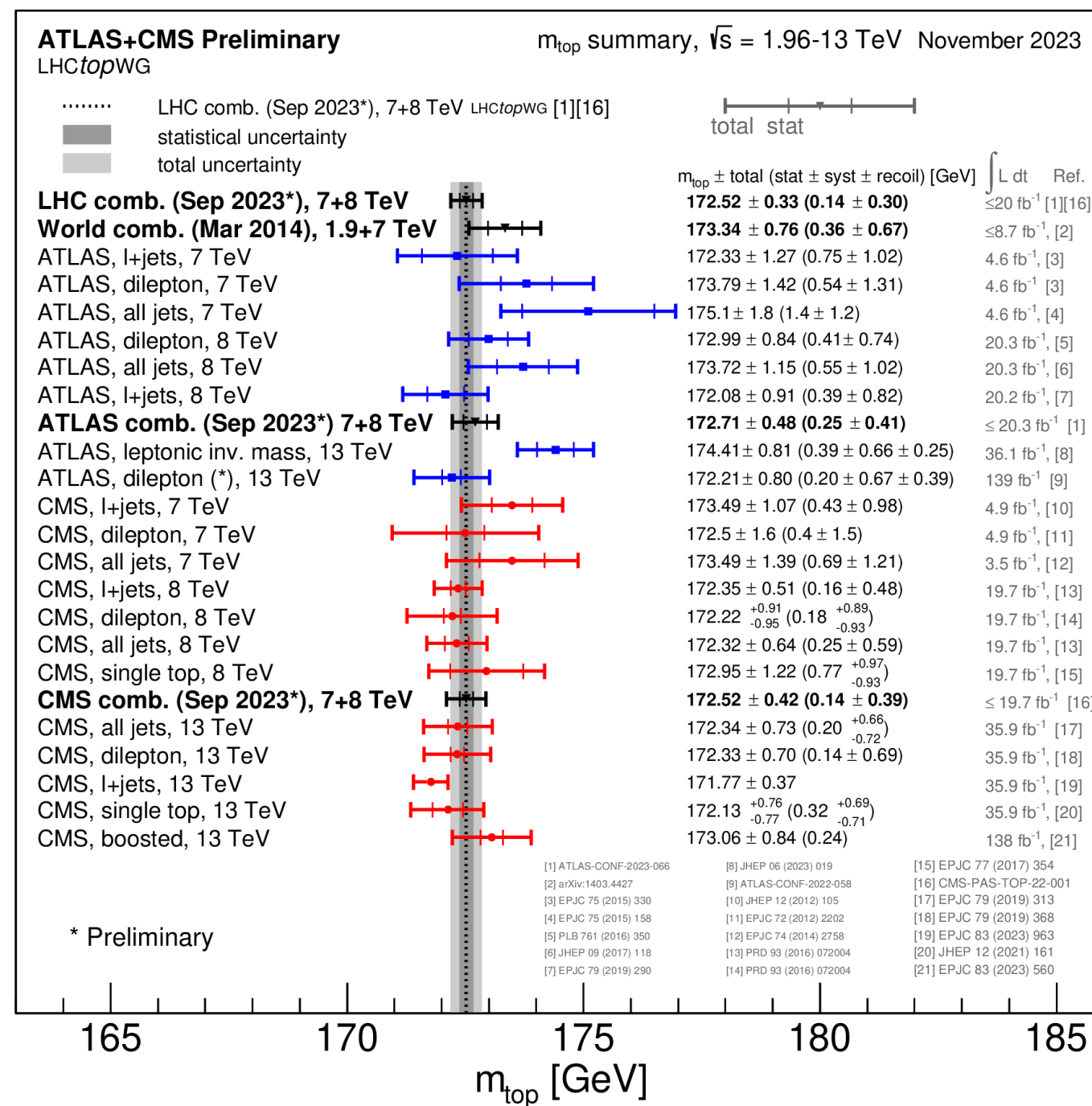
One possibility is to relate some kinematic features of a lepton from top quark decay to the top quark mass, however, it is unclear which “feature” is free from the linear power corrections...

We use out-of-the-collision plane component of the positron momentum $L_{\perp} = |\vec{p}_2 \cdot \vec{e}_y|$. Using the previously-derived formulas, we arrive at the following result which shows that the average value of this observable is correlated with the top quark mass:

$$\langle L_{\perp} \rangle = \frac{1}{\Gamma_t} \int d\Gamma_t L_{\perp} + \mathcal{O}(\lambda^2) = \frac{1}{2\Gamma_t} \int d\Gamma_t \frac{(p_2 p_t)}{m_t} = \frac{m_t^2 + m_W^2}{8m_t} + \mathcal{O}(\lambda^2)$$



$$\vec{e}_z = \vec{n}_d, \quad \vec{e}_y = \frac{\vec{n}_i \times \vec{n}_d}{|\vec{n}_i \times \vec{n}_d|}, \quad \vec{e}_x = \vec{e}_y \times \vec{e}_z = \frac{\vec{n}_i \times \vec{n}_d}{|\vec{n}_i \times \vec{n}_d|} \times \vec{n}_d$$

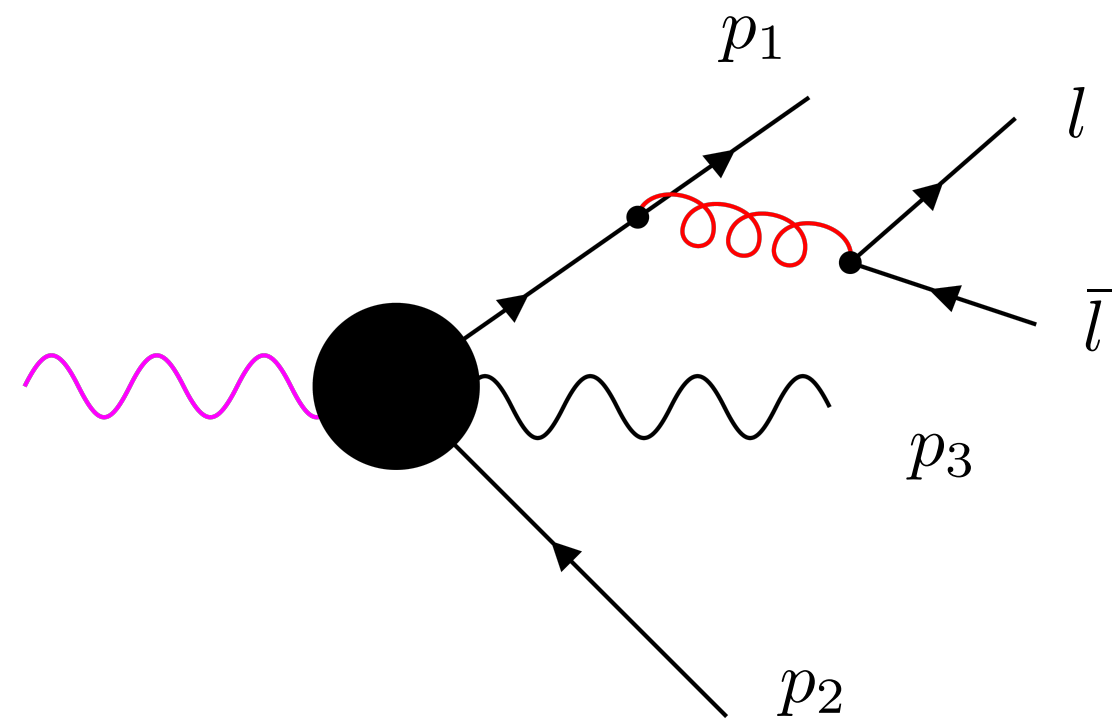


$$m_t = 172.52 \pm 0.33 \text{ GeV}$$

To summarise:

- 1) **linear** non-perturbative corrections $\mathcal{O}(\Lambda_{\text{QCD}}/Q)$ may become relevant for collider physics thanks to very high accuracy of both perturbative calculations and experimental measurements; there is no theory to calculate such corrections from first principles;
- 2) for processes **without on-shell gluons in Born diagrams** they can be studied within the renormalon model; within this model, linear power corrections can be computed once linear dependence of the NLO QCD corrections on the infinitesimal gluon mass is known;
- 3) such a dependence can be derived for **arbitrary processes** using the Low-Burnett-Kroll theorem for next-to-leading soft emissions, its generalization **to virtual corrections**, and **the momenta mappings** that factorise the dependence of the phase space on the soft gluon momentum with next-to-leading accuracy;
- 4) this approach can be used to prove cancellation of linear power corrections to cross sections of arbitrary (abelian) processes at colliders, without the need to compute one-loop corrections / real-emission contributions with the gluon mass exactly. **The use of the short-distance mass for a heavy quark is required for the cancellation;**
- 5) the linear corrections **to kinematic distributions** can be efficiently calculated using the same method;
- 6) **top quark decays can be included in the narrow width approximation.** This leads to the conclusion that polarization effects are affected by linear power corrections in general, and various observables designed to study polarization effects in top physics at the LHC may also be affected by such effects;
- 7) one can design an observable that depends on the momentum of the positron from top quark decay which has no linear power corrections and is strongly correlated with the top quark mass.

The second example that I want to discuss, is the calculation of power corrections to [shape variables](#), such as the C-parameter and the thrust, [in the 3-jet region](#). Such power corrections are important for the extraction of the strong coupling constant from shape variables.



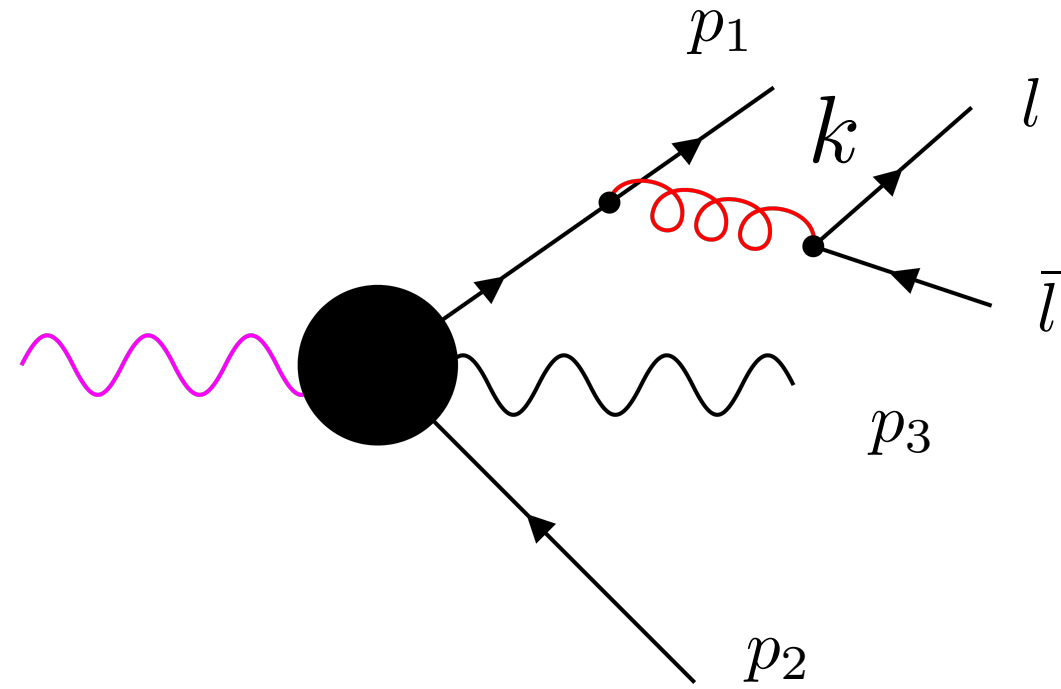
$$C = 3 - 3 \sum_{ij}^N \frac{(p_i p_j)^2}{(p_i q)(p_j q)}$$

$$T = \max_{\vec{n}} \frac{\sum_{i=1}^N |\vec{n} \cdot \vec{p}_i|}{\sum_{i=1}^N |\vec{p}_i|}$$

$$\Sigma(v) = \sum_F \int d\sigma_F \theta(V(\Phi_F) - v)$$

Since we cannot deal with on-shell gluons in Born diagrams, we consider production of two hard quarks and a hard photon as a gluon proxy.

Shape variables include sums over all final state partons. In the context of large- N_f calculation, we need to construct an observable using fermions from the soft gluon splitting and not the gluon itself.



$$\Sigma(v) = \sum_F \int d\sigma_F \theta(V(\Phi_F) - v)$$

$$C = 3 - 3 \sum_{ij}^N \frac{(p_i p_j)^2}{(p_i q)(p_j q)}$$

The non-vanishing result only appears because of the dependence of the observable on the soft quark momenta; this allows us to discard virtual contributions, phase-space modifications and next-to-soft corrections to the matrix elements. [This also allows us to generalize these results to true QCD jets by arguing that we only need to know soft emissions which are the same \(up to color charges\) for hard quarks and gluons.](#)

$$\mathcal{T}_\lambda[\Sigma(v; \lambda)] = \int d\sigma^b(\tilde{\Phi}_b) \delta(V(\{\tilde{p}\}) - v) \times \left[\mathcal{N} \mathcal{T}_\lambda [I_V(\{\tilde{p}\}, \lambda)] \right]$$

$$I_V(\{\tilde{p}\}, \lambda) = \int [dk] \frac{J^\mu J^\nu}{\lambda^2} \int [dl][d\bar{l}] (2\pi)^4 \delta^{(4)}(k - l - \bar{l}) \text{Tr} [\hat{l} \gamma^\mu \hat{\bar{l}} \gamma^\nu] [V(\{\tilde{p}\}, l, \bar{l}) - V(\{\tilde{p}\})]$$

Production

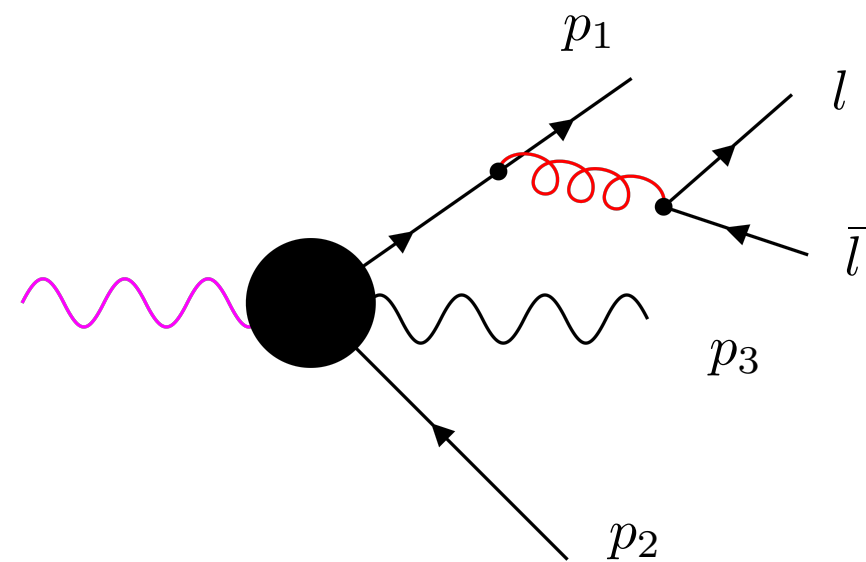
$$J^\mu = \frac{p_1^\mu}{p_1 k} - \frac{p_2^\mu}{p_2 k}$$

Decay phase space, matrix element squared and the observable

$$C(\{\tilde{p}\}, l, \bar{l}) - C(\{\tilde{p}\}) = \sum_{i=1}^3 \frac{(\tilde{p}_i l)^2}{(\tilde{p}_i q)(l q)} + (l \rightarrow \tilde{l}) .$$

It turns out that this integral can be written in a factorized form by choosing [a particular order of integration](#). The result for the C-parameter shown below is extremely compact. Similar results for the thrust value in 3-jet events can be obtained.

$$I_V(\{\tilde{p}\}, \lambda) = \int [dk] \frac{J^\mu J^\nu}{\lambda^2} \int [dl][d\bar{l}] (2\pi)^4 \delta^{(4)}(k - l - \bar{l}) \text{Tr} [\hat{l} \gamma^\mu \hat{\bar{l}} \gamma^\nu] [V(\{\tilde{p}\}, l, \bar{l}) - V(\{\tilde{p}\})]$$



$$C = 3 - 3 \sum_{ij}^N \frac{(p_i p_j)^2}{(p_i q)(p_j q)}$$

$$I_C = \frac{15}{128\pi^3} \frac{\lambda}{q} \frac{s_{12}^3}{1 - z_3} \left[\frac{1 + z_3}{2} K(c_{12}^2) - (1 - z_1 z_2) E(c_{12}^2) \right]$$

$$c_{12} = \cos \frac{\theta_{12}}{2}$$

$$qp_i = \frac{q^2}{2} (1 - z_i)$$

To obtain this result, one has to remove the delta-function by integrating over the transverse momentum of one of the quarks. Upon doing that, the result factorizes as follows

$$I_V(\{\tilde{p}\}, \lambda) = \int [dk] \frac{J^\mu J^\nu}{\lambda^2} \int [dl][d\bar{l}] (2\pi)^4 \delta^{(4)}(k - l - \bar{l}) \text{Tr} \left[\hat{l} \gamma^\mu \hat{\bar{l}} \gamma^\nu \right] [V(\{\tilde{p}\}, l, \bar{l}) - V(\{\tilde{p}\})]$$

$$I_C = \lambda F \times W_C$$

A universal, observable-independent constant that turns out to be the (famous) Milan factor

Observable-dependent function

$$F = 16\pi \int [dk] \frac{J_\mu J_\nu}{\lambda^3} \left\{ -2\tilde{l}^\mu \tilde{l}^\nu \frac{\lambda^8}{(2k\tilde{l})^5} - \frac{g^{\mu\nu} \lambda^6}{2(2k\tilde{l})^3} \right\} = -\frac{5}{64\pi}$$

$$W_C = -3 \int \frac{d\eta d\varphi}{2(2\pi)^3} \tilde{C}_{\alpha\beta} \frac{\tilde{l}^\alpha \tilde{l}^\beta}{(\tilde{l}q)}$$

$$\tilde{l}^\mu = \frac{p_1^\mu}{\sqrt{s}} e^\eta + \frac{p_2^\mu}{\sqrt{s}} e^{-\eta} + n^\mu.$$

$$\tilde{C}_{\alpha\beta} = \sum_{i=1}^3 \frac{p_i^\alpha p_i^\beta}{(p_i q)}$$

This factorization allows us to simplify the calculation of power corrections to shape variables significantly. Calculations for the C-parameter and the thrust turn out to be rather similar.

The universality of the “Milan” factor is explained; it generalises beyond relations between observables in the 2-jet limit; in fact, the same “factor” appears in the 3-jet case.

Generalisation of these calculations from the 3-jet to the N-jet kinematics is straightforward (abelian approximation, no gluons) and has been done.

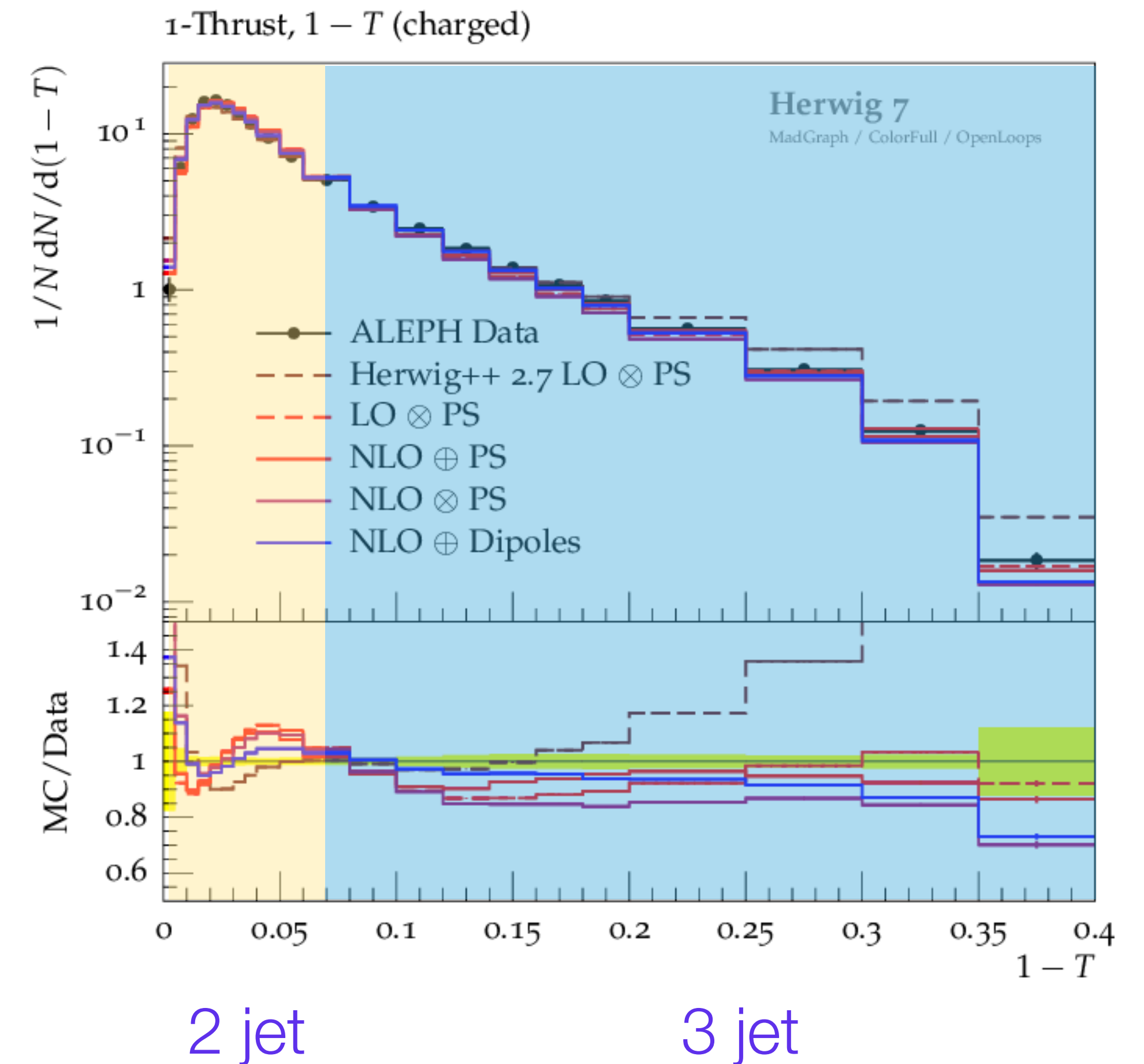
To see why this computation may be important, recall the existing problem with the determination of the strong coupling constant where values obtained from fits to event shapes turn out to be significantly smaller than values from more inclusive observables.

$$\alpha_s(M_Z) = \begin{cases} 0.1179 \pm 0.0010, & \text{PDG} \\ 0.1135 \pm 0.0010, & \text{thrust} \\ 0.1123 \pm 0.0015, & C - \text{parameter} \end{cases}$$

Non-perturbative corrections to event shapes were estimated in various ways for the 2-jet region where as measurements and fits include both 2-jet and 3-jet regions.

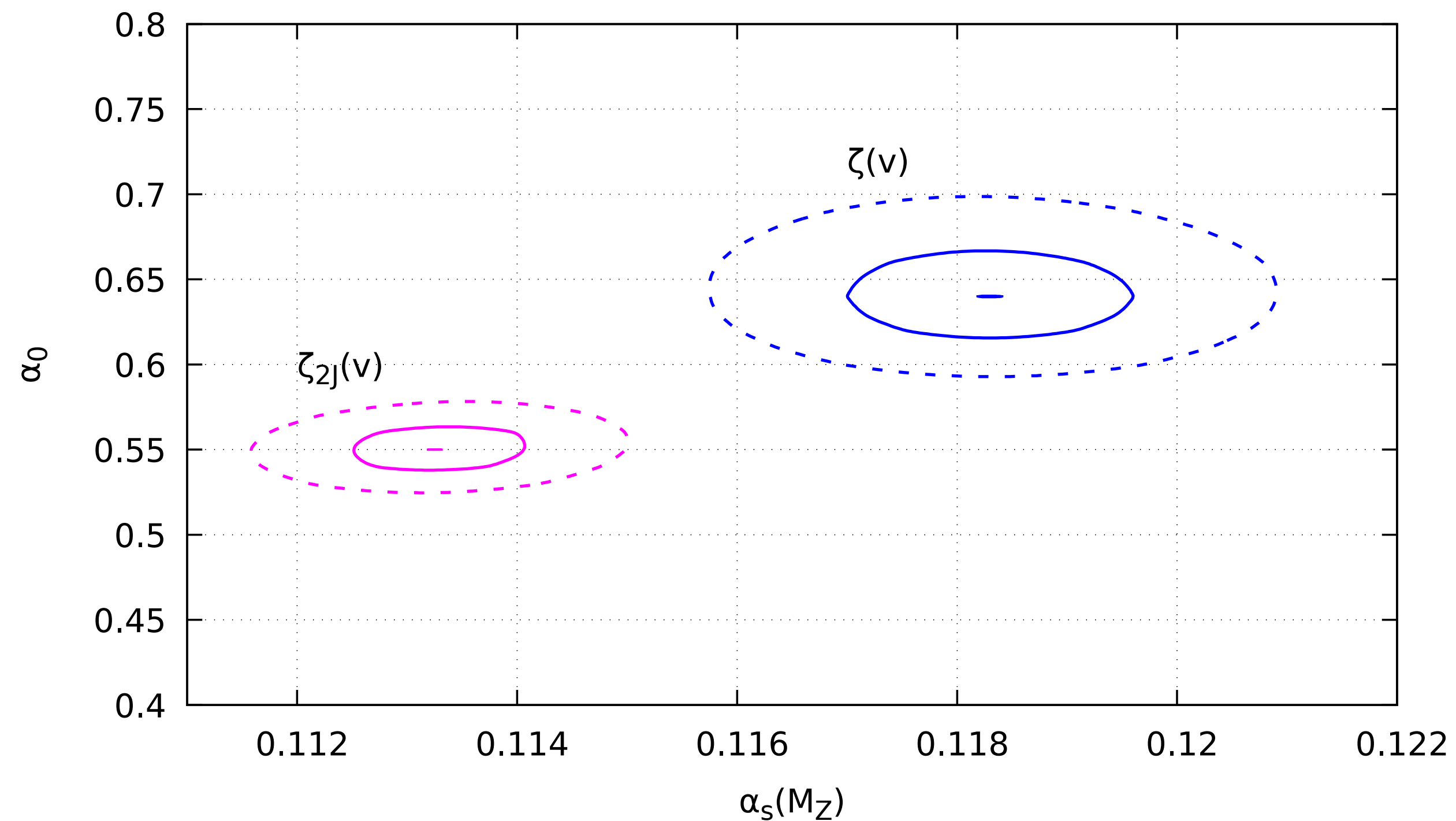
It is possible that knowledge of power corrections to shape variables in the 3-jet region may help to reconcile the different values of the strong coupling constant.

Luisoni, Monni, Salam



Belm et al. Herwig 7.0/7.3 release notes

The corresponding analysis was recently performed by Nason and Zanderighi and the conclusion seems to be that indeed the values the strong coupling constant moves in the right direction once power corrections are included.



Nason, Zanderighi