Gravitational Waves in String Cosmology

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2310.11494 [hep-th], 2408.13803 [hep-th]; 2411.04186 [hep-ph], 2504.20994 [astro-ph.co]

With A. R. Frey, R. Mahanta, A. Maharana, F. Muia and F. Quevedo ; F: Revello (x2), A. Ghoshal

07/07/2025 - Quantum Sensing meets UHF-GWs - MITP

Target sensitivities: challenging but rewarding

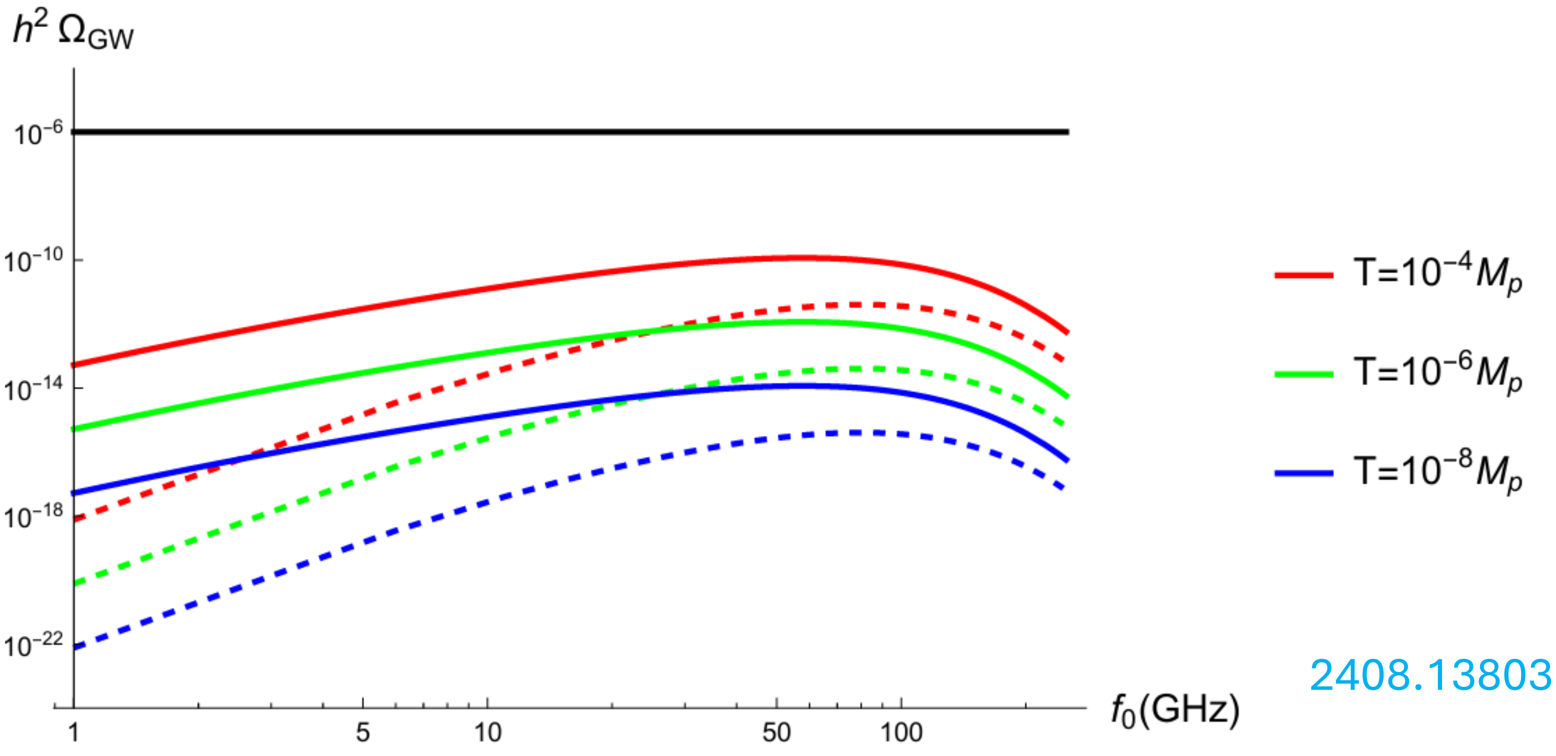
Tito D'Agnolo – Ellis '24

This talk:

- An UV completion for the CGMB
- Lessons on string theory
- More phenomenological opportunities

The results

A gas of highly excited fundamental strings (solid) predicts a larger amplitude than field theory (SM dashed)



Reheating temperature

= string scale

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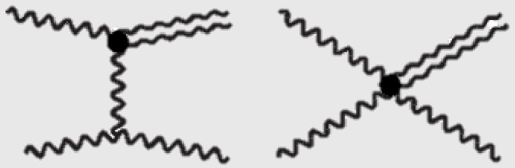
What are the UV completions of the Cosmic Gravitational Microwave Background?

(P)reheating in field theory	Xu et al'25 (& therein)
Stringy effects	Frey et al'24

CGMB: The SM as a GW factory

Thermal plasmas source GWs

Ghiglieri-Laine'15
Ghiglieri-Jackson-Laine-Zhu'20
Ringwald- Schütte-Engel -Tamarit'20



$$\frac{d}{d\log a}\left(\frac{d\rho_{\rm GW}(t)}{d\log f}a(t)^4\right) = \frac{T}{M_p}\rho_{\rm bath}(t)a(t)^4 F\left(\frac{f}{T}\right)$$

- Frequency today ⟺ CMB frequency
- UV sensitive!

The SM dominates…

In weakly coupled QFT,

$$F(f/T) \sim \alpha_s \sqrt{g_{*,\mathrm{UV}}} F_0(f/T)$$

$$h^2 \Omega_{\mathrm{GW}} \sim \alpha_s \frac{T}{M_p} g_{*,\mathrm{UV}}^{-5/6}$$

Extra dof are penalised!

Similar behaviour suggested at strong coupling

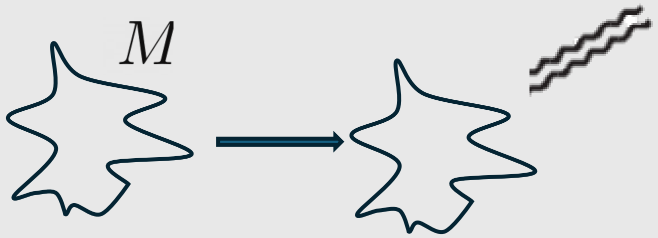
Castells-Tiestos – Casadelrrey-Solana’22

The SM dominates...

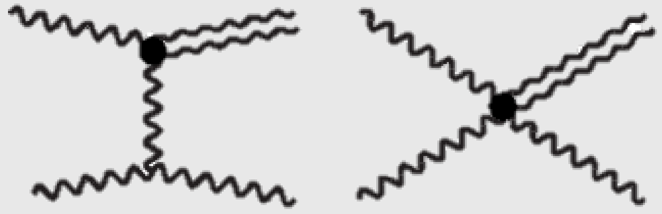
Except against strings!

A gas of highly excited strings predicts a larger amplitude.

Processes with 3 external legs are efficient:



$$\sim M/M_p$$



$$\sim gT/M_p$$

What we have learned

- String thermodynamics
- Semiclassical strings & interaction rates
- Strings out of equilibrium & cosmology
- GW emission & the spectrum
- Further opportunities

Bonus: cosmic (super)strings & varying tension

String thermodynamics

(What is the system?)

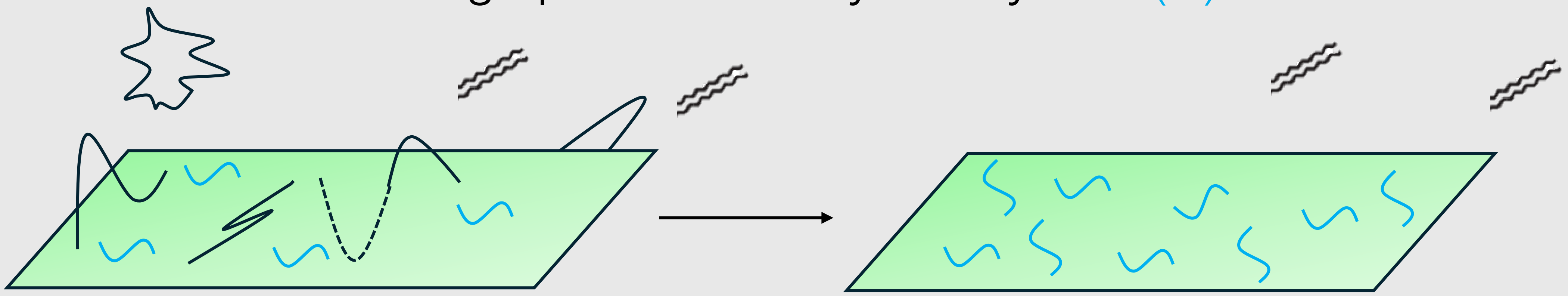
The scenario

We consider a gas of highly excited strings in branes

In string model building, massless open strings contain the SM

The gas produces out of equilibrium GWs

The strings predominantly decay into (B)SM.



Some string theory

The Hamiltonian of a string in Minkowski is, schematically:

$$H/M_s = \sum_{n=1}^{\infty} n\, a_n^{\dagger} \cdot a_n - 1$$

i.e: a system of harmonic oscillators

The massless states are spin 0, 1 and 2 particles.

The massive states are VERY degenerate (!)

String theory ingredients

At high energies, exponential density of states

$$d(E) \sim e^{\beta_H E} \,, \qquad \beta_H \sim \sqrt{\alpha'}$$

Strongly affects the thermodynamics! Equilibrium distributions

$$n(E) \sim e^{-(\beta - \beta_H)E} \equiv e^{-E/L}$$

Strings with masses much larger than T get excited

Thermodynamics and Cosmology

Important ratio:

$$\frac{\Gamma_{\mathrm{eq}}}{H} \gg 1$$

H determined by eq distribution

Γ_{eq} determined by near-equilibrium behaviour

Write **Boltzmann equations**

Semiclassical strings & interaction rates

(Goal: compute equilibration rates)

Towards Boltzmann equations

Exponentially large # states: hard

Strategy: coarse-grain over states with a given mass

Decay rates

We obtain tree-level decay rates after averaging:

Mañes'03

$$\frac{d\Gamma_o}{dl} \sim g_s$$

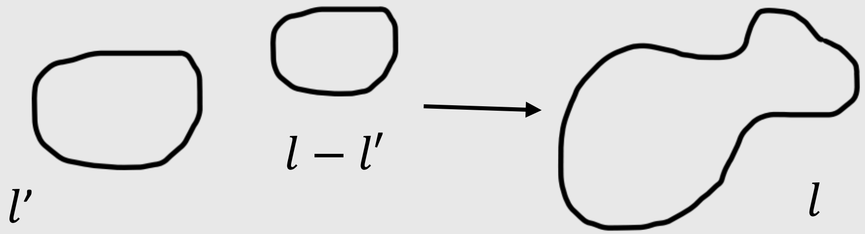
$$\frac{d\Gamma_{cl}}{dl} \sim g_s^2 L \left(\frac{L}{l(L-l)}\right)^{d/2}$$

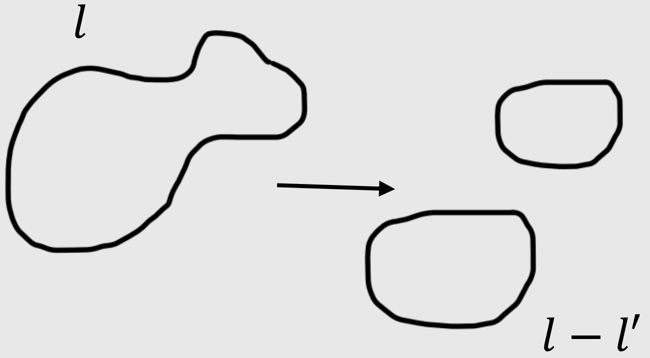
Applications to cosmic strings?

The Boltzmann equations

Similar computations / random walk arguments yield:

$$\frac{\partial n_c(l)}{\partial t} = \frac{n_c(l')l'\, n_c(l-l')(l-l')}{V} - l n_c(l) \left(\frac{l}{l'(l-l')} \right)^{d/2}$$





The Boltzmann equations

Similar computations / random walk arguments yield:

$$\frac{\partial n_c(l)}{\partial t} = \frac{\kappa}{2} \int_{l_c}^{l-l_c} dl' \left(\frac{n_c(l')l'\, n_c(l-l')(l-l')}{V} - l n_c(l) \left(\frac{l}{l'(l-l')} \right)^{d/2} \right)$$

$$+ \kappa \int_{l+l_c}^{\infty} dl' \left(l' n_c(l') \left(\frac{l'}{l(l'-l)} \right)^{d/2} - \frac{l n_c(l)(l'-l) n_c(l'-l)}{V} \right).$$

Equilibrium and detailed balance

Equilibrium distributions are obtained using *detailed balance*

$$\frac{n_c(l')l'\,n_c(l-l')(l-l')}{V} = l n_c(l)\left(\frac{l}{l'(l-l')}\right)^{d/2}$$

$$\bar{n}_c(l) = \frac{V}{l^{1+d/2}}e^{-l/L}$$

Perturbations & equilibration rates

$$\frac{\partial \delta n(l,t)}{\partial t} = -\kappa \left(\frac{l^2}{2} + lL\right) \delta n(l,t) + \kappa \int_0^l dl'\, l' \delta n(l',t) \left(e^{\frac{-(l-l')}{L}} - 1\right)$$

We found a family of zero energy solutions

$$\delta n(l,t) = \sqrt{\kappa}\sqrt{\frac{\pi(c+tL^2)}{2}}\,\frac{e^{-\frac{l}{L}+A(t)^2}}{L^2}\mathrm{Erf}\left(A(t), A(t) + \sqrt{\kappa}\sqrt{\frac{c+tL^2}{2}}\,\frac{l}{L}\right) - \frac{e^{-l/L}}{L^2}$$

Late times: $$\delta n(l,t) \sim \delta n(l,0) e^{-\kappa\left(\frac{l^2}{2}+lL\right)t}$$

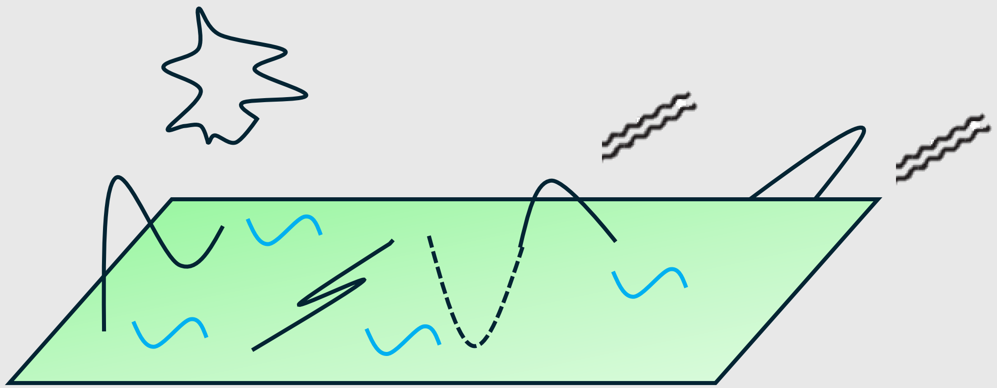
The Hagedorn phase in Cosmology

(i.e: does it actually work?)

In Cosmology

Open strings are essential:

- They dominate the ensemble
- They allow for equilibrium (closed string interactions are too weak)



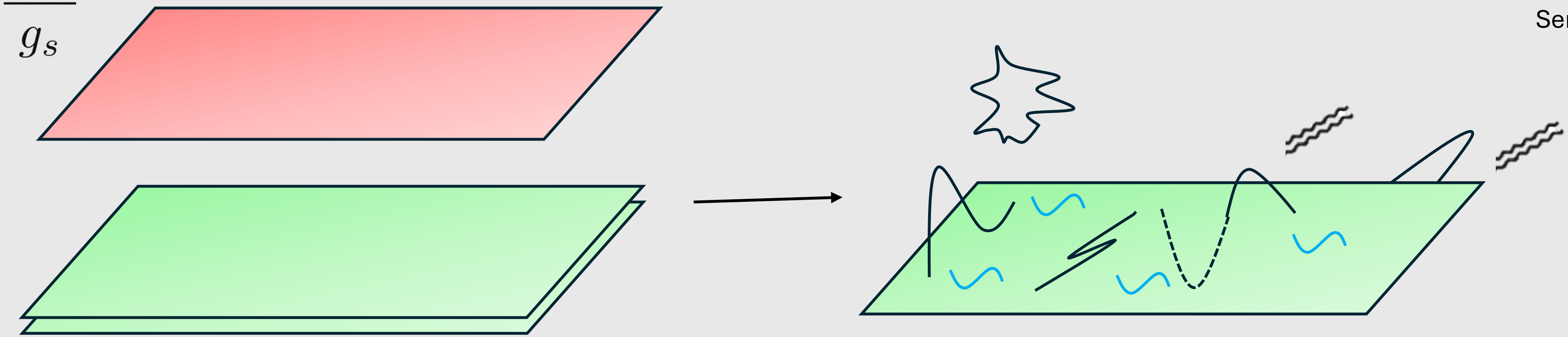
Embedding & consistency

Natural candidate: brane-antibrane inflation

Dvali-Tye'99
Burgess et al '01
Kachru et al '03

Sen'02-'04

$$V \sim \frac{M_s^4}{g_s}$$



Calculations under control if

$$\frac{H}{M_s} \ll 1 \to \frac{M_s}{M_p} \ll 1$$

GW emission & the spectrum

(At last!)

GW emission

The matrix element to compute is:

$$\mathcal{M}_{A\to B,g} = \frac{M_s^2}{2\pi M_p} e_{\mu\nu} \int d\sigma \left\langle B \left| e^{ik\cdot x(\sigma)} \sqrt{-\hat{\gamma}_{(0)}} x_a^\mu x_b^\nu \hat{\gamma}_0^{ab} \right| A \right\rangle .$$

C.f: photon-mediated transitions in Hydrogen:

$$\mathcal{M}_{A\to B,\gamma} \sim e\, e_\mu \left\langle B \left| e^{ik\cdot x(\sigma)} p^\mu \right| A \right\rangle .$$

GW emission

Coarse – graining over mass levels again:

Applications to cosmic strings?

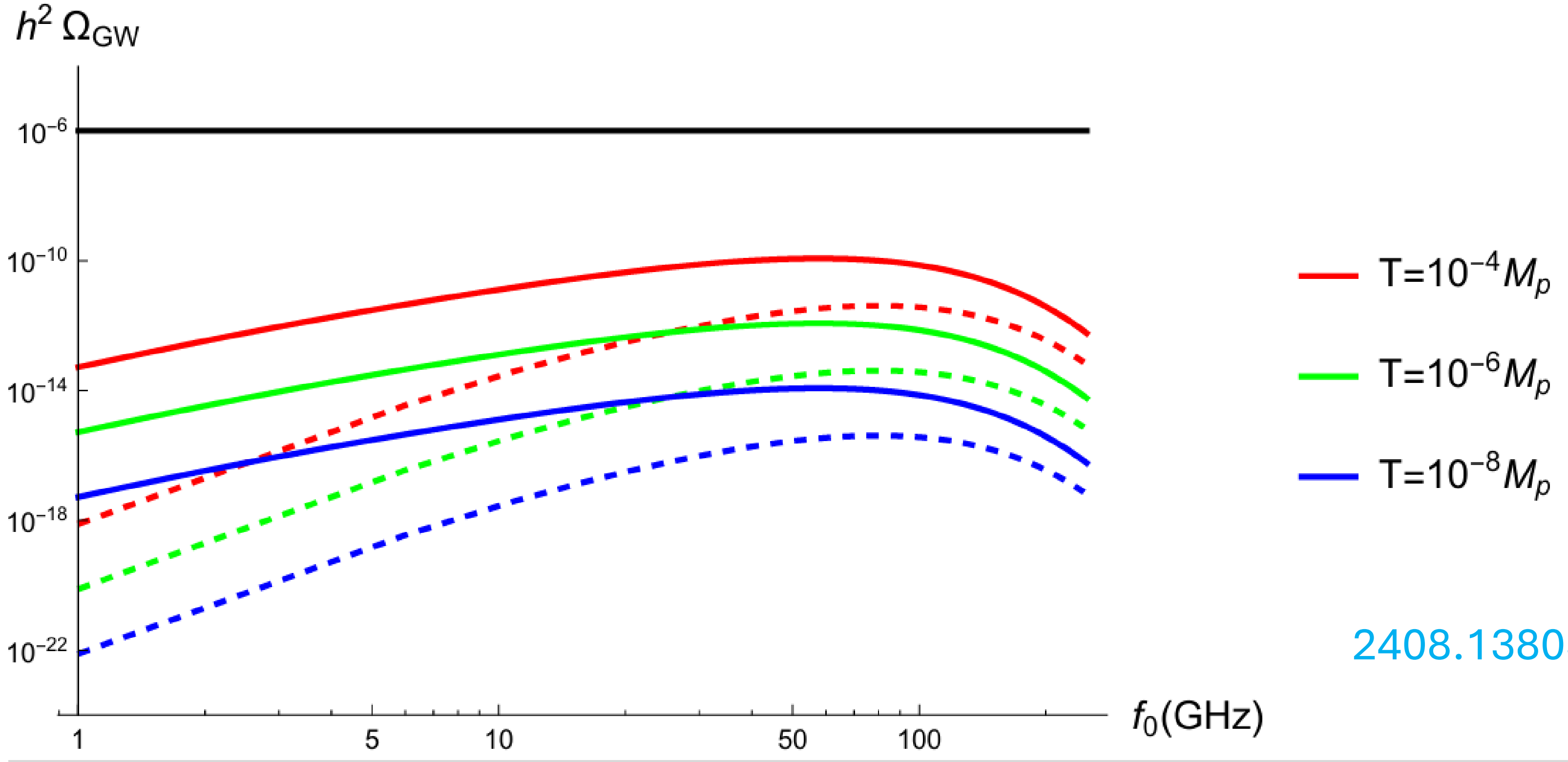
$$\frac{d\Gamma_{l\to g}}{d\omega\, dl} \simeq l \left(\frac{M_s}{M_p}\right)^2 \omega^2 \, \frac{e^{-\omega/T_H}}{\left(1 - e^{-\omega/2T_H}\right)^2}$$

Amati-Russo'99

$$\frac{d\rho_g}{dt} + 4H\rho_g = \int_{l_c}^{\infty} \omega \, \frac{d\Gamma_{l\to g}}{d\omega\, dl} \, \frac{n_o(l)}{V_{3D}} \, dl = \left(\frac{M_s}{M_p}\right)^2 I\left(\frac{\omega}{T_H}\right) \rho_o \, M_s$$

The spectrum today

Hagedorn phase (solid) vs SM (dashed) at TH=T:



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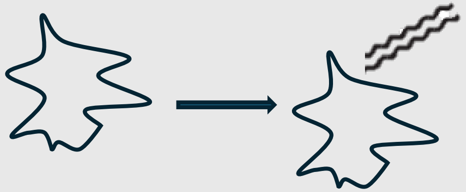
Comparison with the SM

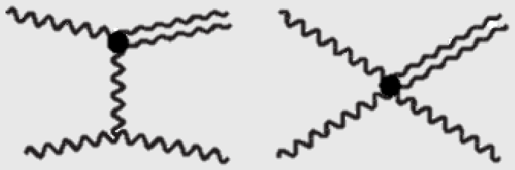
The generic behaviour is reproduced

$$\frac{d}{d\log a}\left(\frac{d\rho_{\rm GW}(t)}{d\log f}a(t)^4\right) = \frac{T}{M_p}\rho_{\rm bath}(t)a(t)^4F\left(\frac{f}{T}\right)$$

But F is less suppressed in string theory!



$\sim M/M_p$



$\sim gT/M_p$

Conclusions of the Hagedorn phase

- The string prediction dominates with similar peak frequency.
- Obvious further phenomenology: DM & axions.
- Possible challenge: early matter domination.
- We've learned about out of equilibrium string thermodynamics.
- Applications to cosmic strings & others?

Bonus:
Cosmic (super)strings
& varying tension

Cosmic strings & superstrings

- Field theory: topological defects from symmetry breaking in the early Universe.

Kibble'76

- String theory: highly excited strings/D1 branes/wrapped higher dimensional branes. Formed after brane-antibrane inflation?

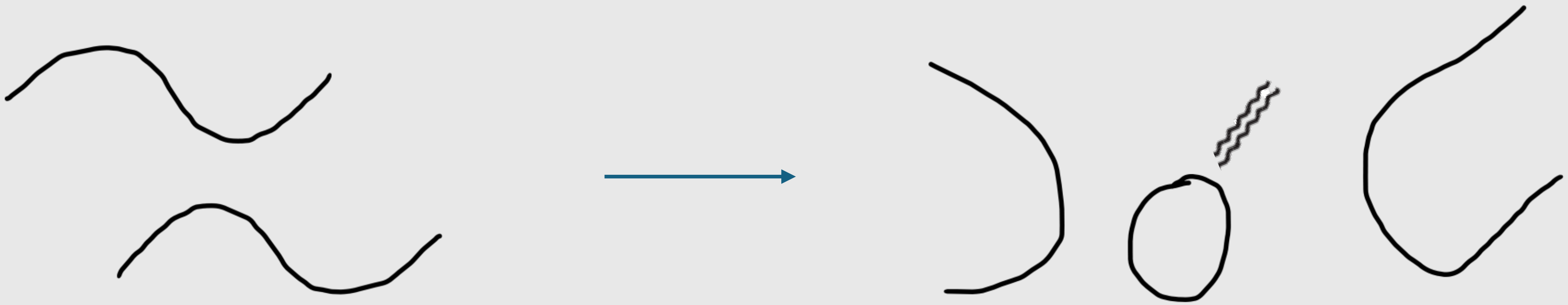
Sarangi-Tye'02

- Outstanding opportunity for probing pre-BBN physics!

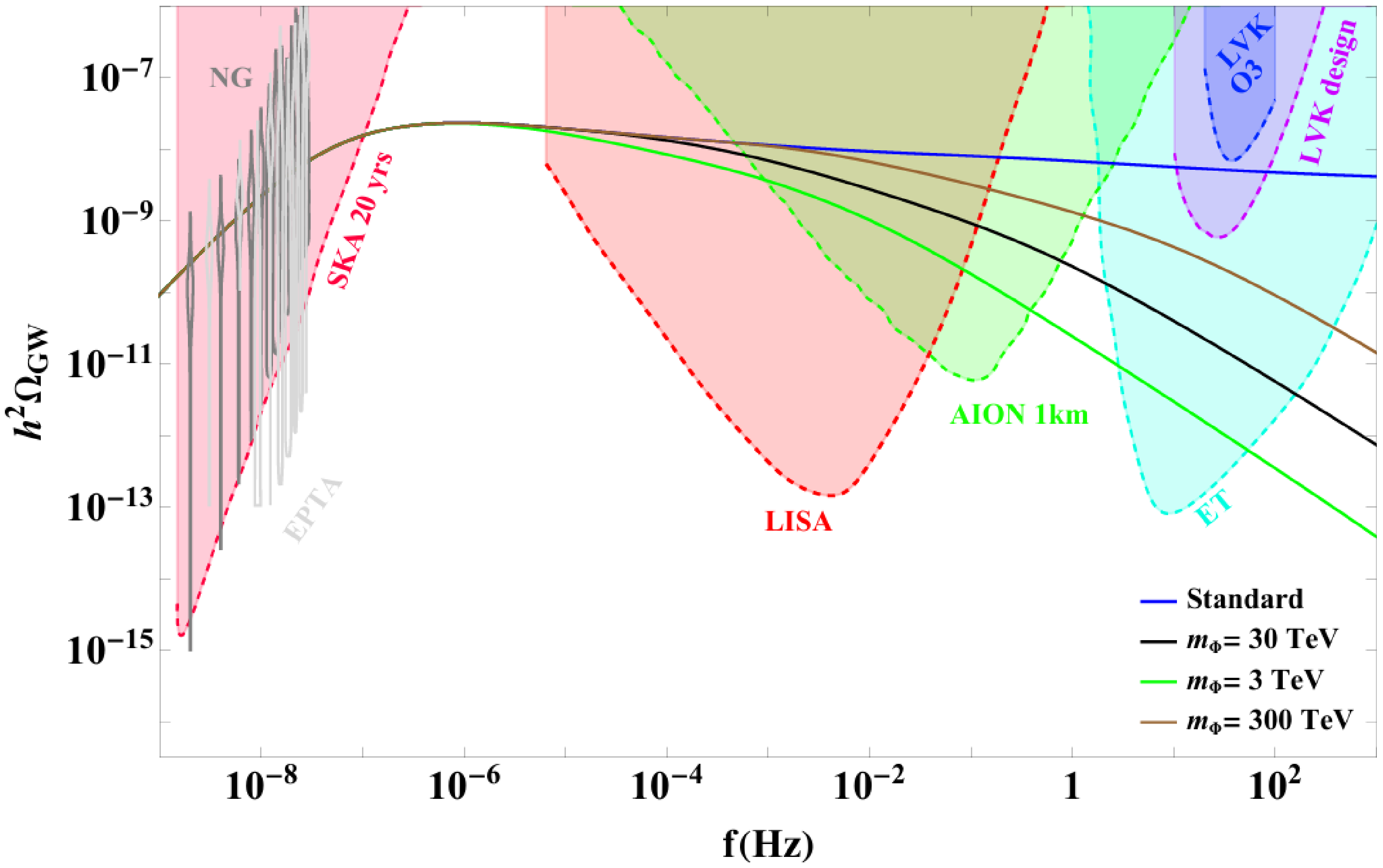
Cosmic strings & scaling

Cosmic strings track the energy density of the background:

- Slower dillution $\sim 1/a(t)^2$
- Energy loss via intercommutation



Pre-BBN physics with cosmic strings



The GW spectrum is sensitive to the equation of state of the background

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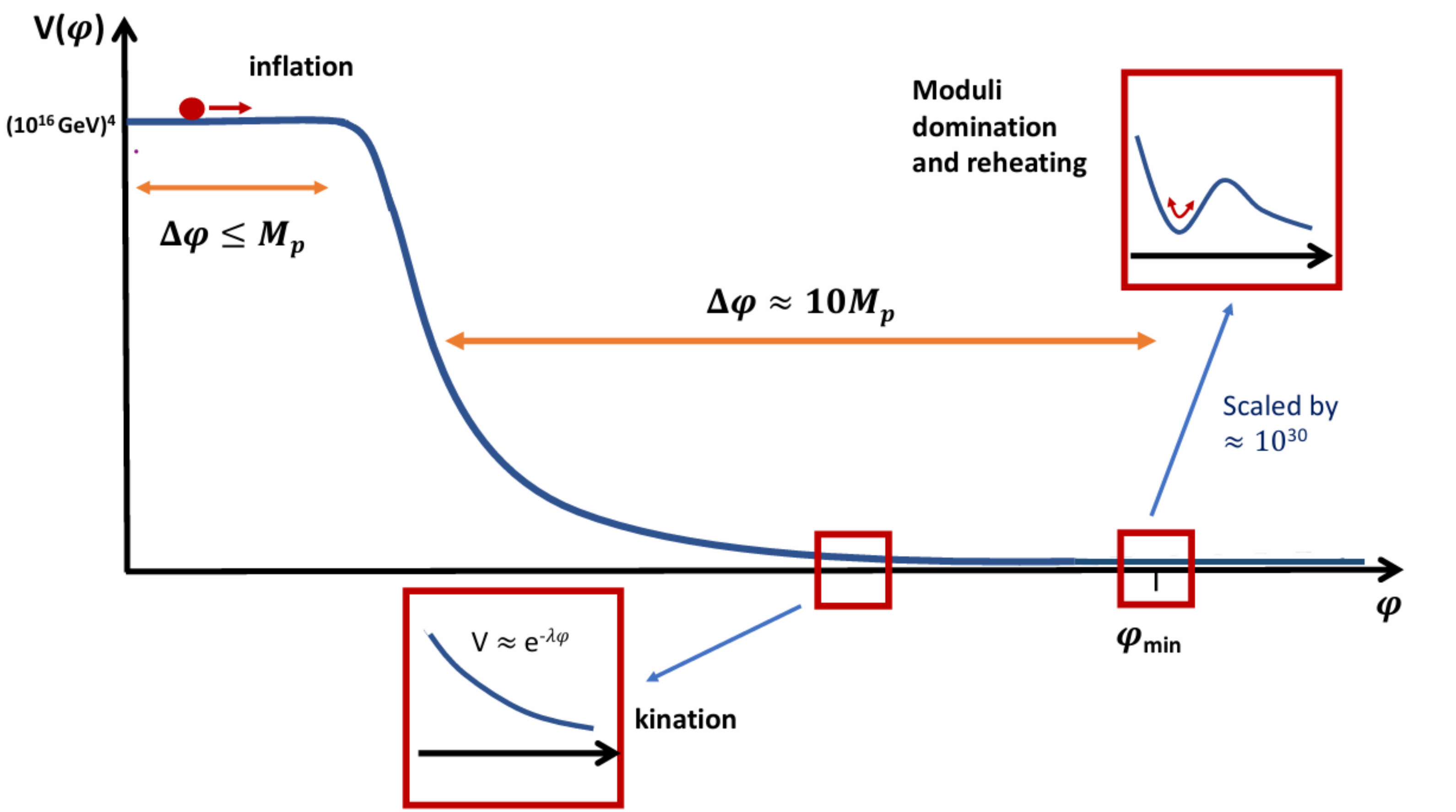
String theory & hierarchies



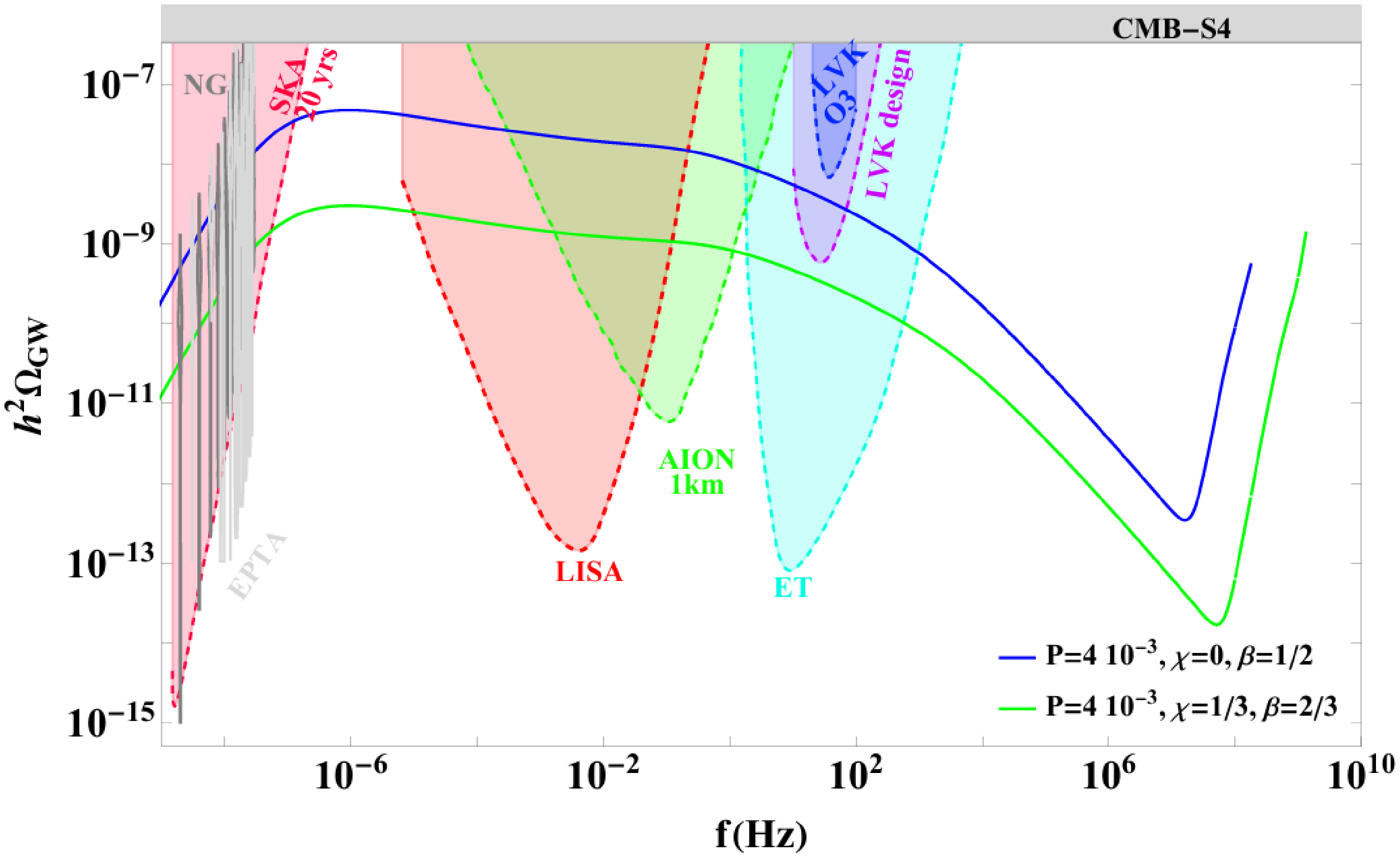
Hierarchies from VEVs of fields

Dynamical in the early Universe

$$G\mu = G\mu_0 e^{-\lambda(\phi-\phi_0)}$$

From 2303.04819

More tension: more GWs



Two effects:

- Larger relative fraction of CS
- Larger tension: more GWs/efold

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Conclusions

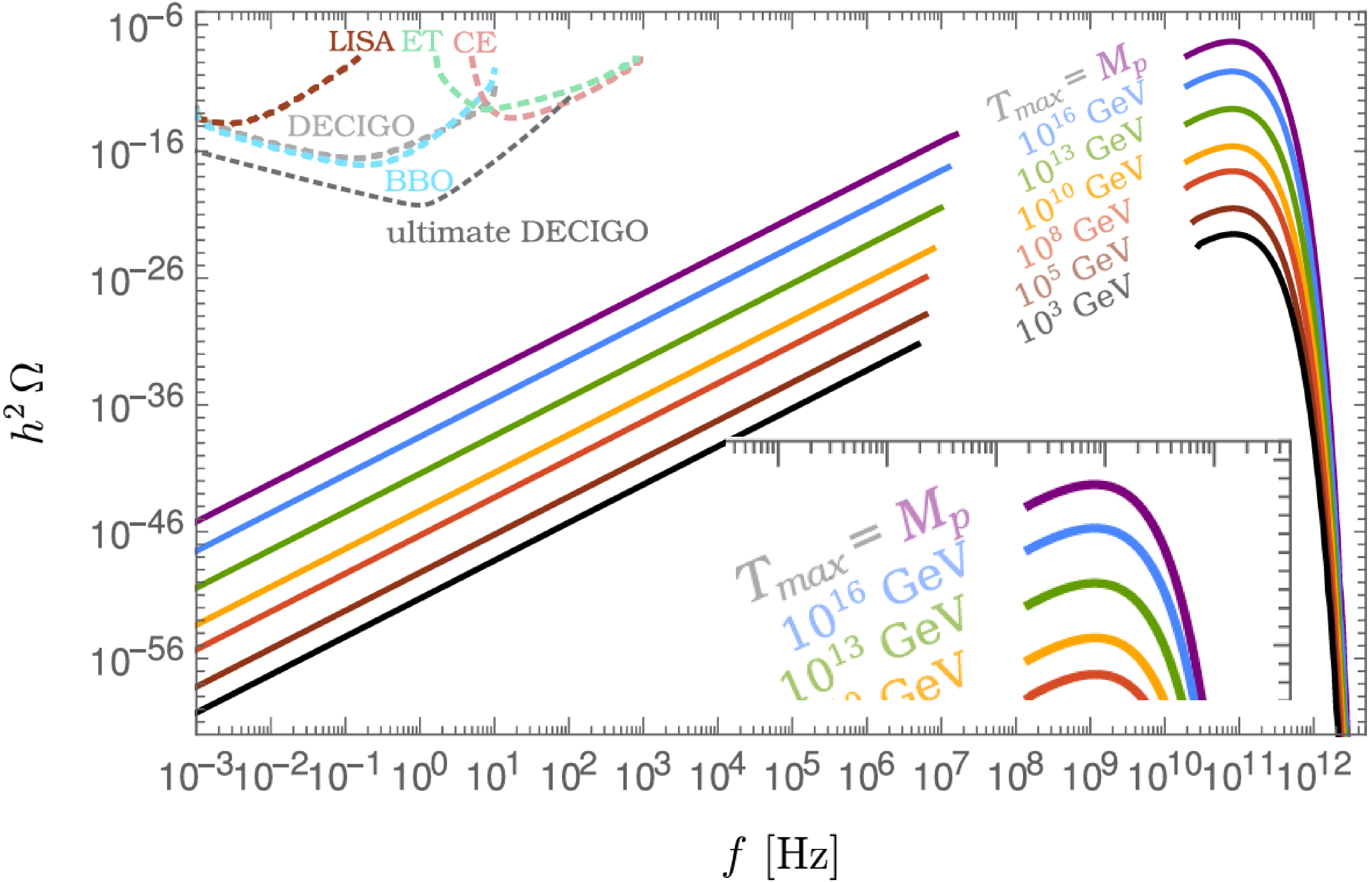
Reward is high

Meanwhile: learn lots of physics

Synergy with axions at source level?

More consequences of the scenarios e.g. DM

Thank you!


LISA
ET
CE
DECIGO
BBO
ultimate DECIGO
$T_{max} = M_p$
10^{16} GeV
10^{13} GeV
10^{10} GeV
10^{8} GeV
10^{5} GeV
10^{3} GeV
$h^2\,\Omega$
f [Hz]
10^{-6}
10^{-16}
10^{-26}
10^{-36}
10^{-46}
10^{-56}
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7} 10^{8} 10^{9} 10^{10} 10^{11} 10^{12}