On gravitational wave signals from black hole binaries beyond the usual circular trajectory

Highly elliptic orbits, close hyperbolic encounters, boosted sources

Based on arXiv: 2303.06006, 2402.10706, 2404.08379, 2405.01297, 2412.01582

With Aurélien Barrau (LPSC), Juan Garcia Bellido (IFT), Camille Bonvin (Genève Univ.), Giulia Cusin (IAP), Thierry Grenet (Néel institute), Pierre Jamet (LPSC), Cyril Pitrou (IAP) and Martin Teuscher (LPSC).

For discussions on PBH merger rates:

See Antonio's talk next Tuesday!

Memory effect:

Contents

Review paper: arxiv:2501.11723

Gravitational Wave Memory of Primordial Black Hole Mergers

Silvia Gasparotto (Barcelona, IFAE), Gabriele Franciolini (CERN), Valerie Domcke (CERN)

May 2, 2025 arXiv:2505.01356

Part I

Some reminders on signals from usual circular Keplerian orbits

Usual binaries in circular Keplerian orbits

Operating at fixed frequency does not determine the masses!

- Fixed frequency ↔ fixed mass is true only at the ISCO: $(f_g)_{\rm ISCO} \simeq 4.4\text{kHz}\left(\frac{M_\odot}{m1+m2}\right)$

- **BUT Analyses cannot be restricted to the ISCO only**

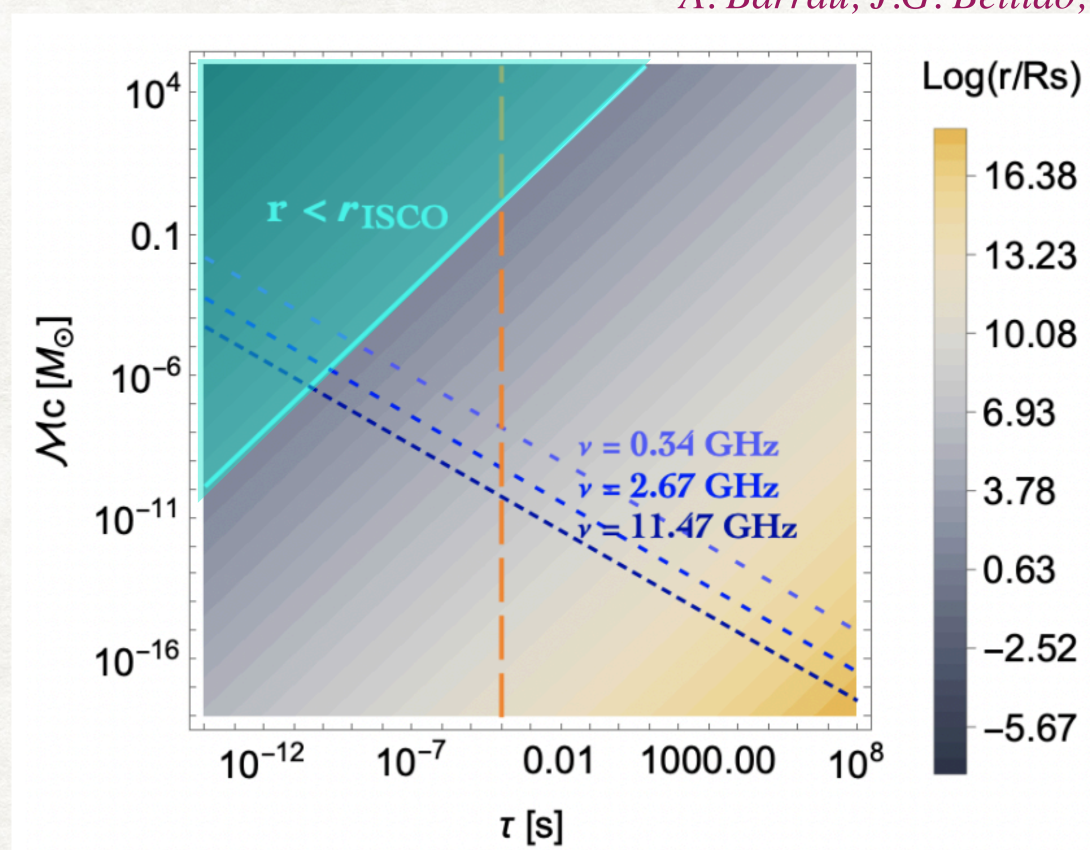
A. Barrau, J.G. Bellido, T. Grenet, K. M, arXiv:2303.06006

ν : Detector operating frequency

τ : Time to merger

$$\nu = \frac{1}{\pi}\left(\frac{5}{256}\frac{1}{\tau}\right)^{\frac{3}{8}}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{-\frac{5}{8}}$$

Chirp Mass: $M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$



Wide range of chirp masses accessible at a given frequency

Not even guaranteed that higher masses lead to better sensitivity.
The amplitude of the signal increases but its duration decreases

→ Depends on the experimental setup

Usual binaries in circular Keplerian orbits

- The frequency of GWs coming from binary systems drifts with time $\dot{f}(\nu) = \frac{96}{5}\pi^{\frac{8}{3}}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{\frac{5}{3}}\nu^{\frac{11}{3}}$

- Time during which the signal drifts in a detector frequency sensitivity bandwidth: $t_{\Delta\nu} \sim \frac{\Delta\nu}{\dot{f}(\nu)}$

Fast decrease of the signal duration with the mass: $t_{\Delta\nu} \propto \mathcal{M}_c^{-\frac{5}{3}}$

The heavier the BHs, the closer they are to their merging
(at fixed frequency)

Usual binaries in circular Keplerian orbits

Hunt for light primordial black hole dark matter with ultra-high-frequency gravitational waves

Nov. 2022

Gabriele Franciolini,[1,2] Anshuman Maharana,[3] Francesco Muia[4]

Contents

Ultra-high frequency gravitational waves: where to next ?

4–8 déc. 2023
CERN
Fuseau horaire Europe/Zurich

Entrer le t

Usual binaries in circular Keplerian orbits

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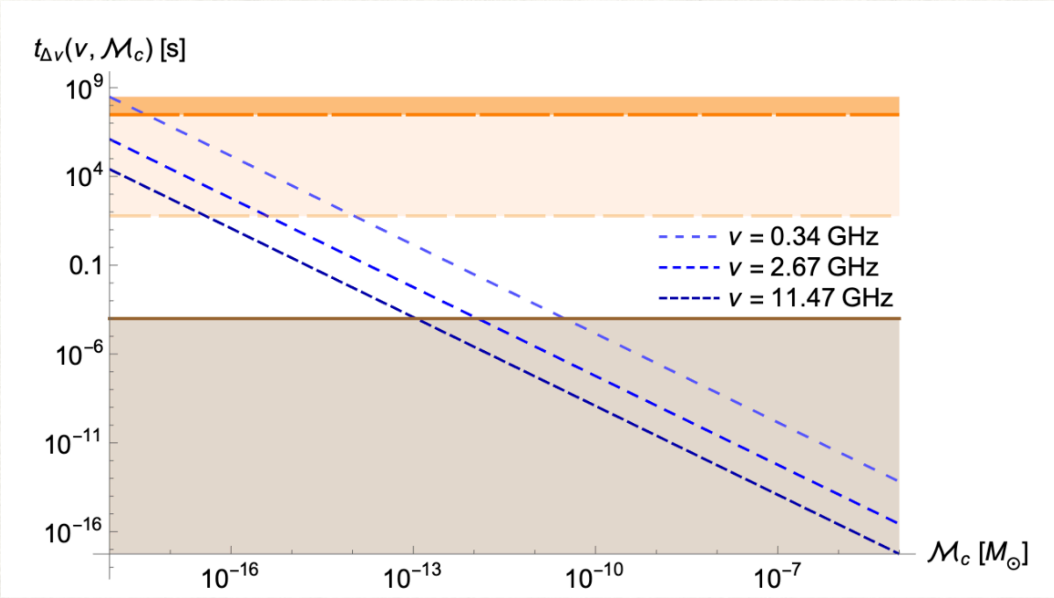
A. Barrau, J.G. Bellido, T. Grenet, K. M, arXiv:2303.06006

- Very small for high-frequency resonant detectors

Example case: GHz resonant cavities



Signal time duration in a resonant cavity

$$t_{\Delta\nu} \sim \frac{5}{96}\pi^{-\frac{8}{3}}\nu^{-\frac{8}{3}}Q^{-1}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{-\frac{5}{3}}$$

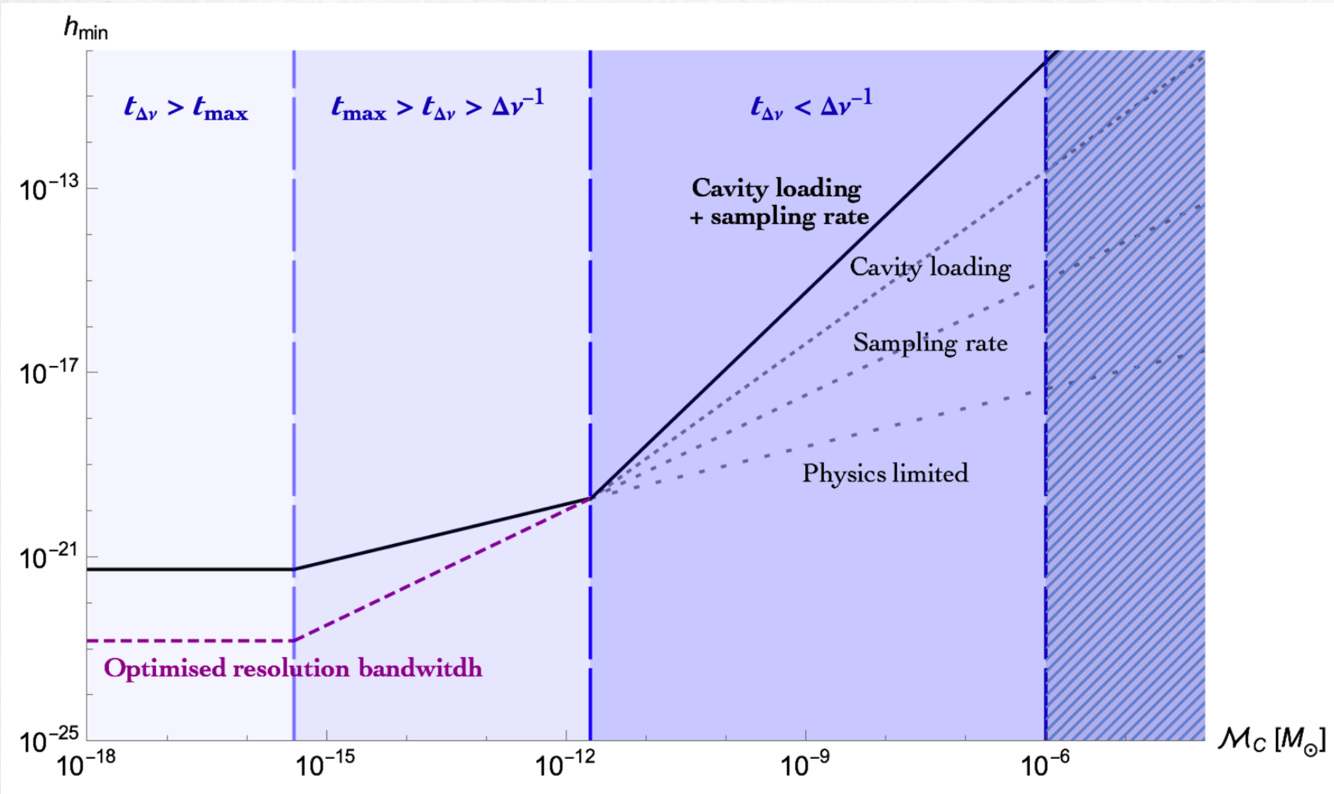
Sensitive only to a very small fraction of an orbit (for the highest masses accessible)

Usual binaries in circular Keplerian orbits

An illustrative example: GHz resonant cavities

A. Barrau, J.G. Bellido, T. Grenet, K. M, arXiv:2303.06006

Strain sensitivity



The sensitivity decreases with the chirp mass

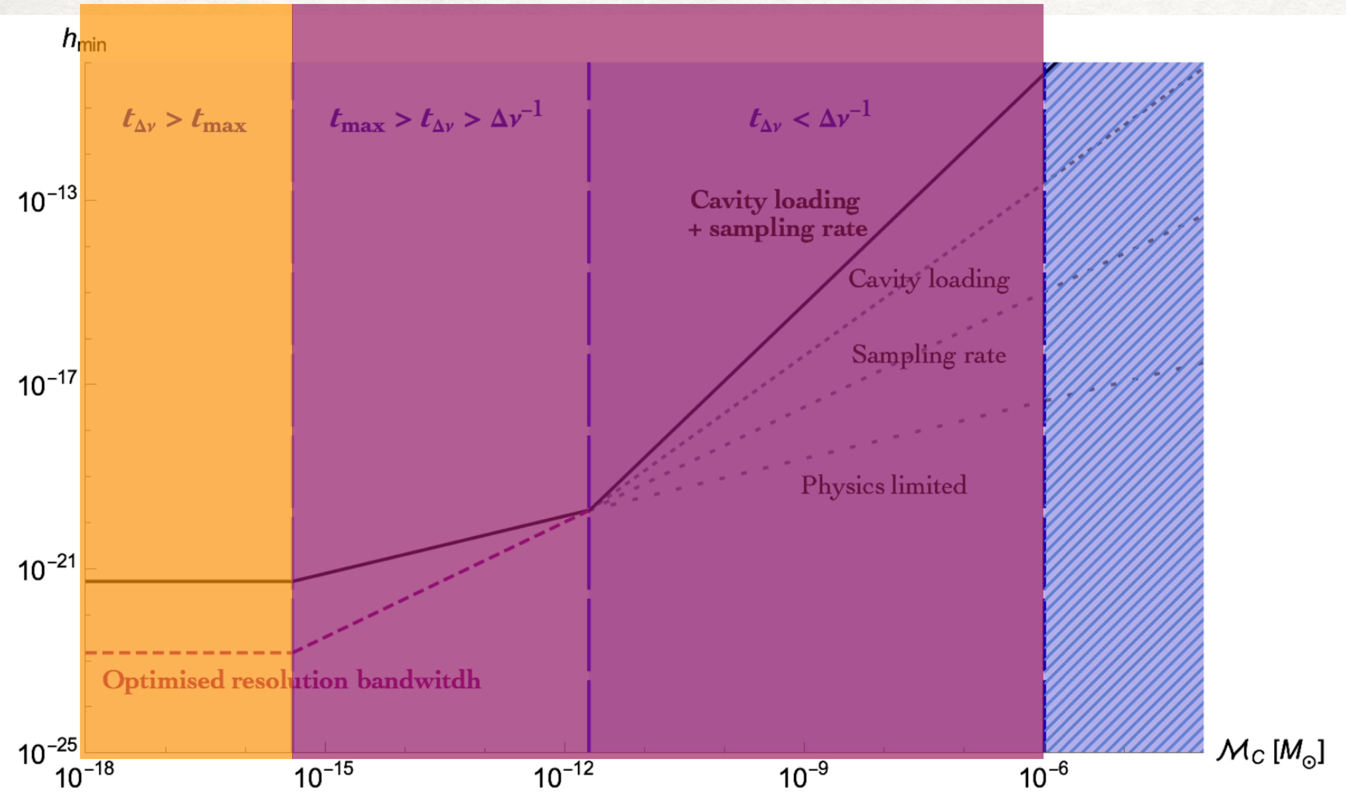
Usual binaries in circular Keplerian orbits

An illustrative example: GHz resonant cavities

A. Barrau, J.G. Bellido, T. Grenet, K. M, arXiv:2303.06006

Long lasting coherent signals

Transient signals



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Usual binaries in circular Keplerian orbits

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A. Barrau, J.G. Bellido, T. Grenet, K. M, arXiv:2303.06006

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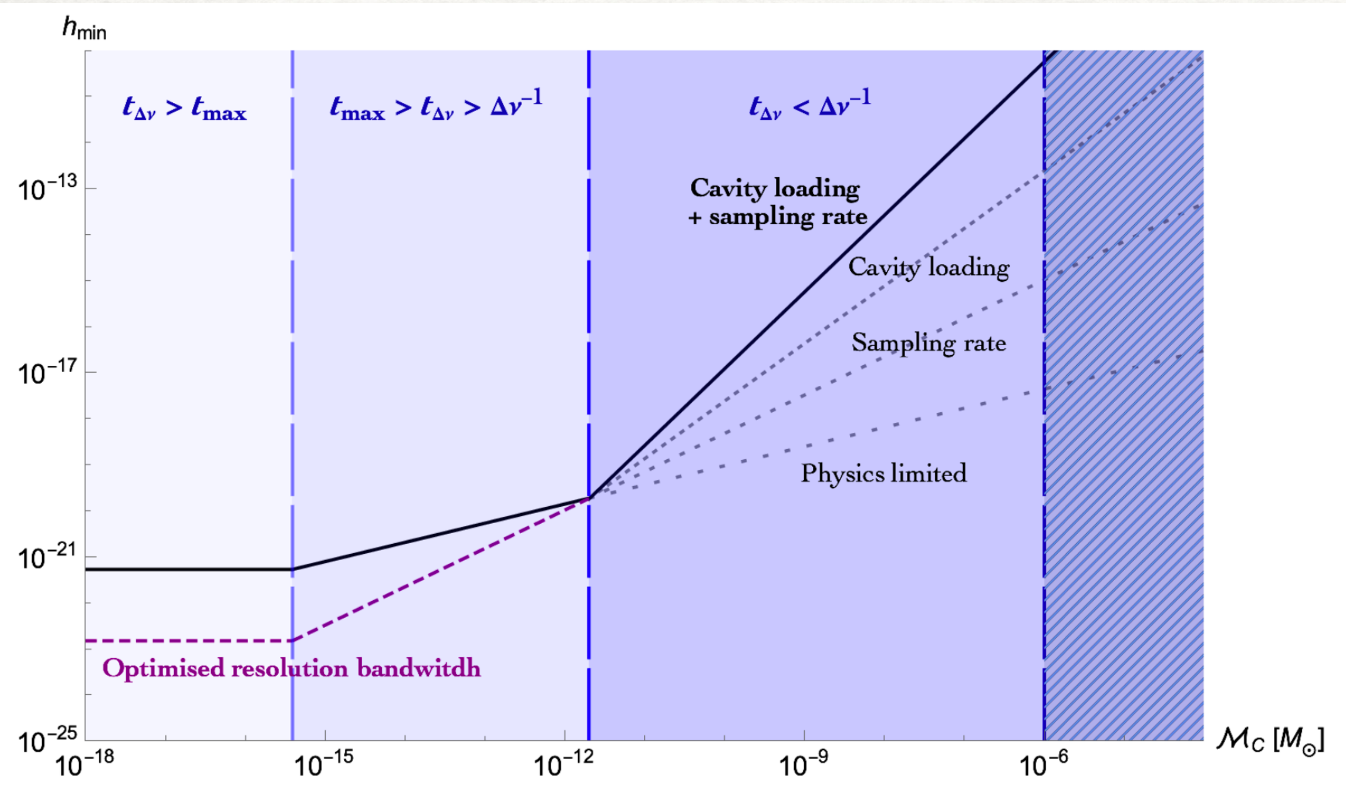

The sensitivity decreases with the chirp mass

Not the most relevant quantity ...

Usual binaries in circular Keplerian orbits

<u>An illustrative example: GHz resonant cavities</u>

A. Barrau, J.G. Bellido, T. Grenet, K. M, arXiv:2303.06006

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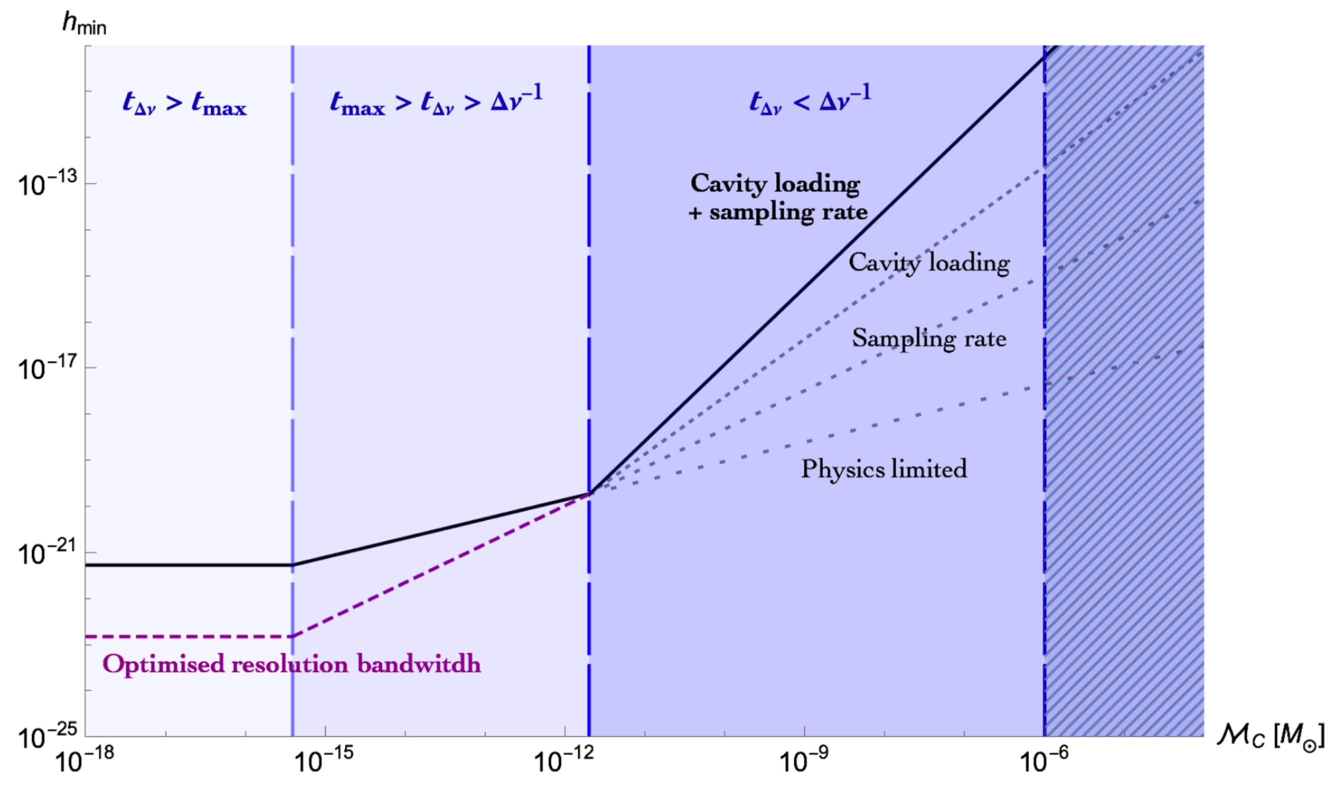
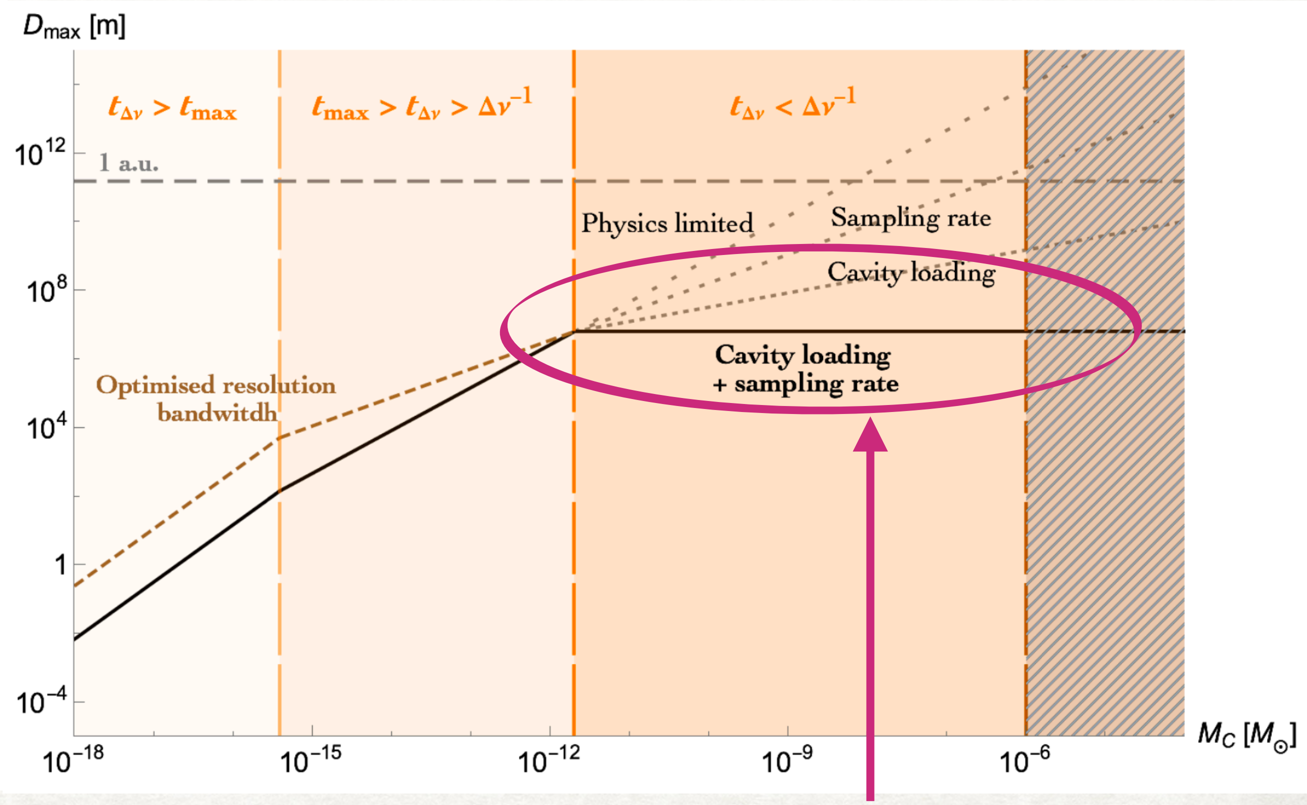


The sensitivity *decreases* *with the chirp mass*

Not the most relevant quantity …

Accessible distance

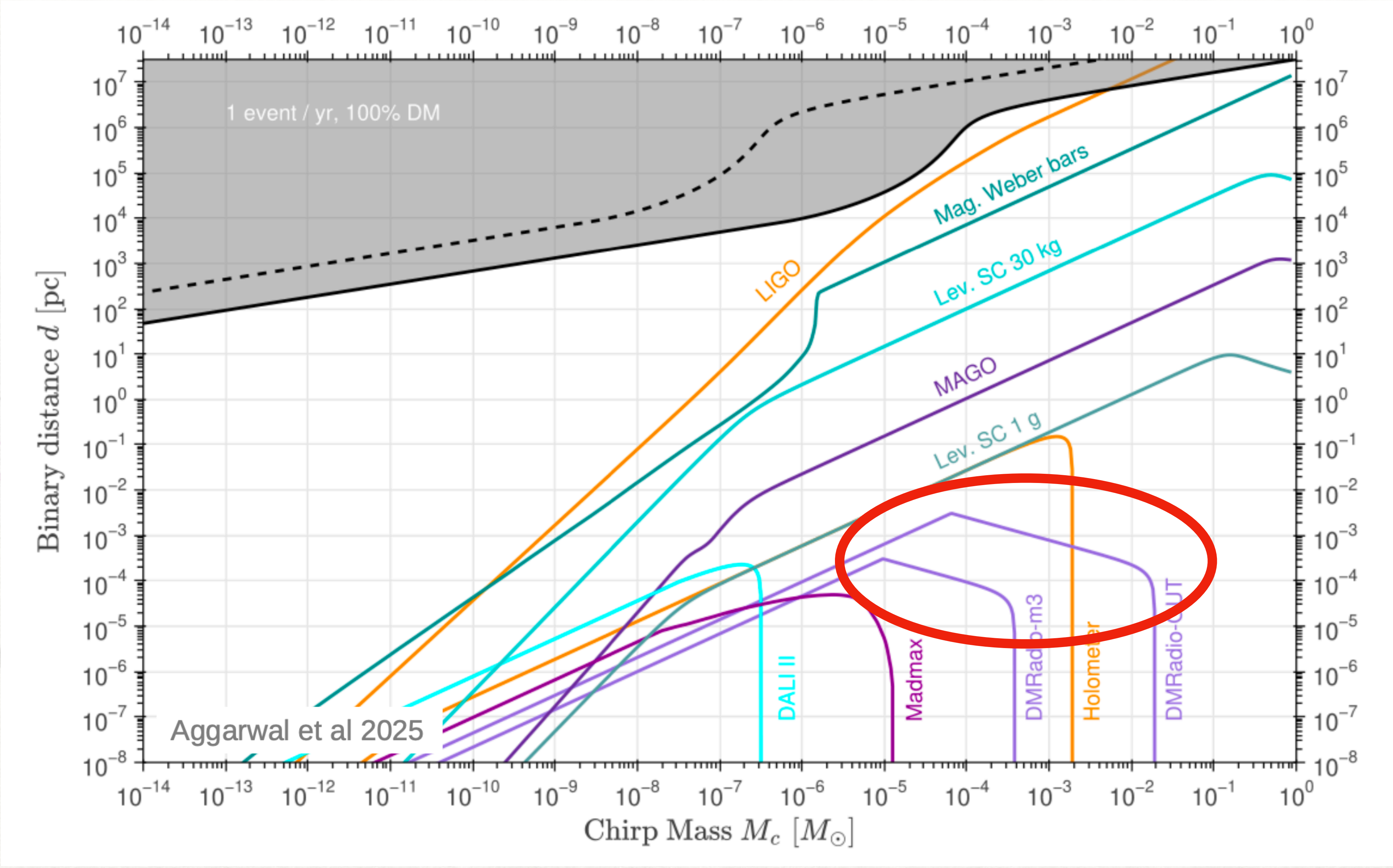
Physical quantity that matters in the end



The accessible distance does not depend on the chirp mass

Usual binaries in circular Keplerian orbits

Cf Valerie's talk on Monday



Usual binaries in circular Keplerian orbits

An illustrative example: GHz resonant cavities

A. Barrau, J.G. Bellido, T. Grenet, K. M, arXiv:2303.06006

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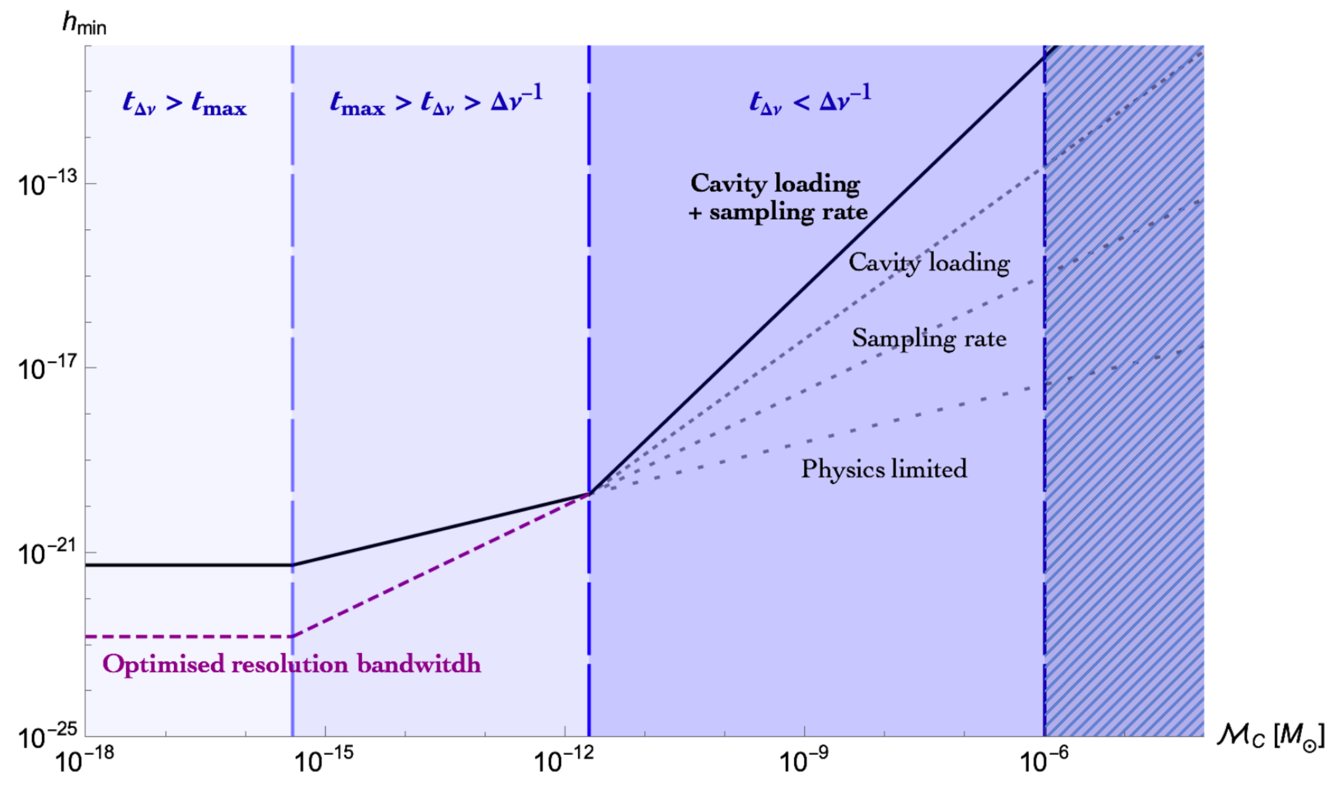

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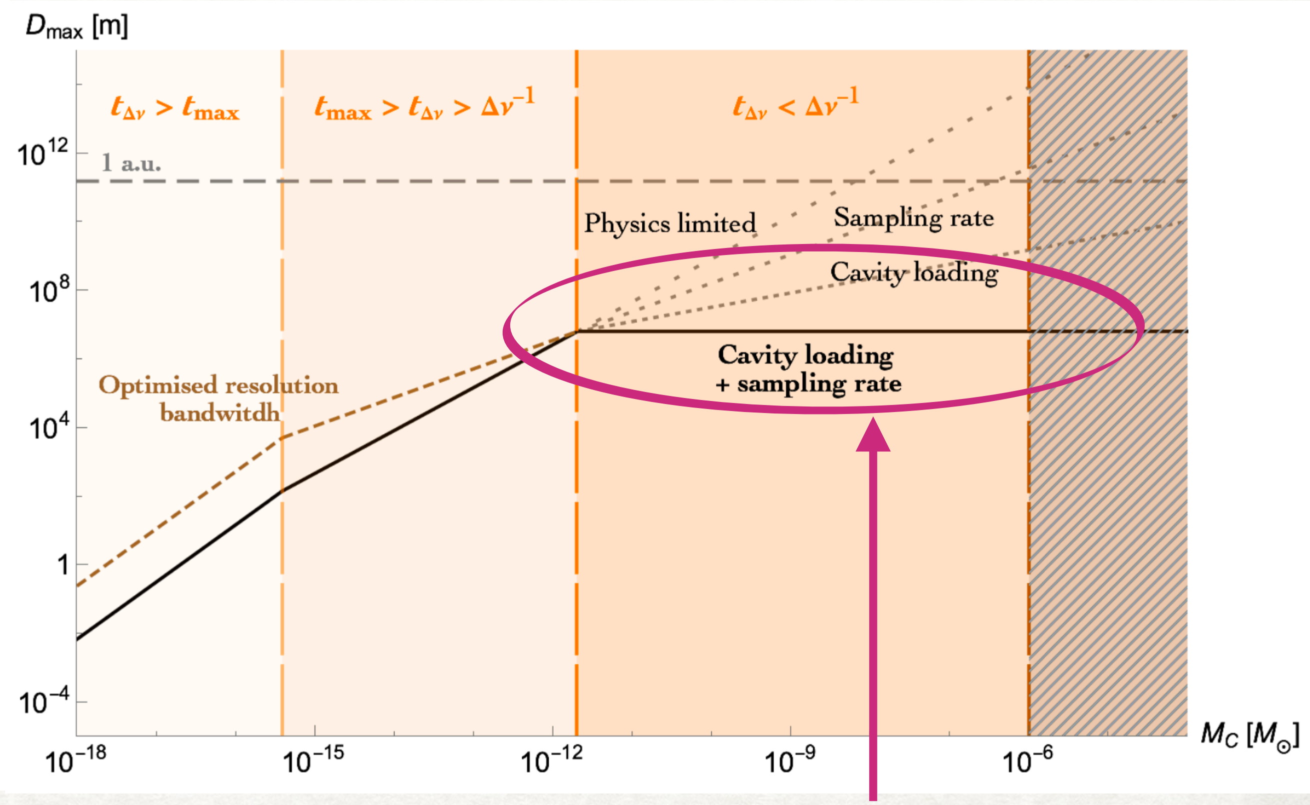


The accessible distance does not depend on the chirp mass

Unexpected behaviors can arise due to the competition between bigger but shorter signals

Not specific to resonant cavities

Usual binaries in circular Keplerian orbits

Take-away messages:

- **The frequency of the experiment does not fix the mass of the binary systems that could, in principle, be probed**

- **Time duration of the signal quickly decreases with the masses** $t_{\Delta\nu} \propto \mathcal{M}_c^{-\frac{5}{3}}$

  At fixed experimental frequency

  - Binaries generating the **higher strain** also generate the **shorter signals**
  - Competition between two effects ⟶ Which one wins in the end?

    ↪ The answer is **not trivial** and depends on the experimental setup

  - Lower chirp masses should *a priory* not be discarded

Part II

BH mergers beyond the usual circular trajectory:

Highly Eccentric Orbits

Naturalness of those events:

See Antonio's talk next Tuesday!

Highly eccentric black holes mergers

- **Average luminosity over one orbital period**

In the weak field, slow motion limit

$$L_{\rm GW}(M_1, M_2, f_p, e) = L_{\rm GW,circ}(M_1, M_2, f_p) \times \frac{1 + 73e^2/24 + 37e^4/96}{(1-e^2)^{7/2}}$$

P. C. Peters and J. Mathews, Phys. Rev. 131, 435 (1963)
M. Enoki, and M. Nagashima, arXiv:0609377

Amplification factor

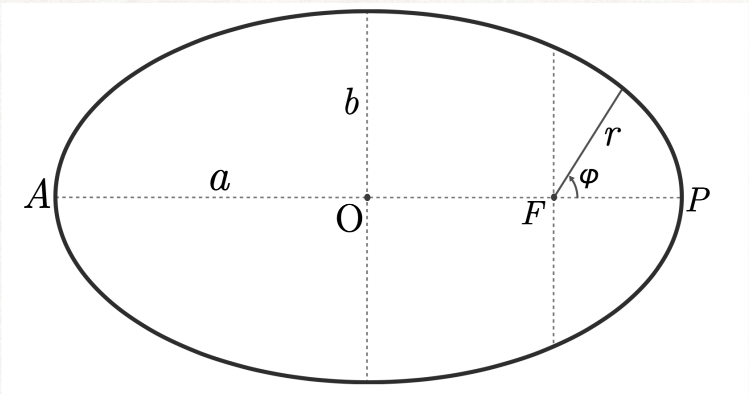
Does this extra factor increase the gravitational signal in a detector?

Naively: yes!

But ... not necessarily

Highly eccentric black holes mergers

P. Jamet, A. Barrau, K. M., arXiv:2412.01582



Parametrization used for an elliptic trajectory of semi-major axe a and semi-minor axe b.

Ellipse described by its **eccentricity**: $e = \sqrt{1 - \frac{b^2}{a^2}}$

Losses of energy and angular momentum:

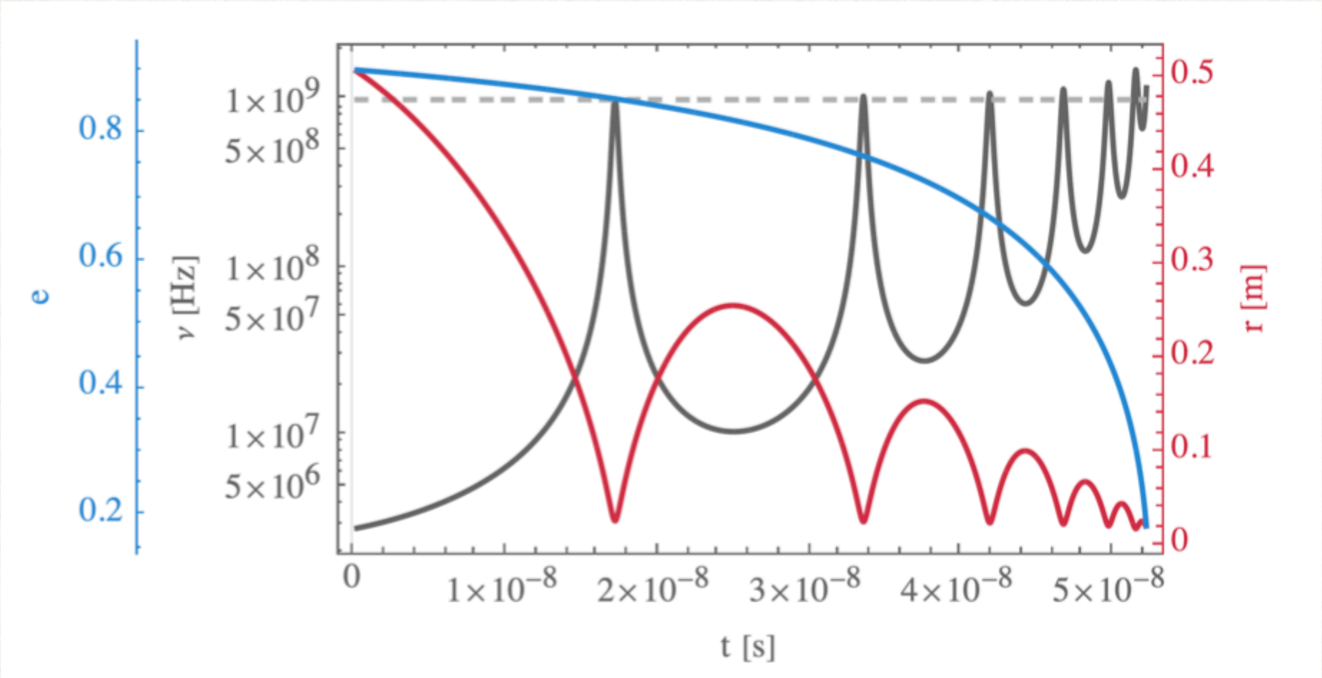
$$\dot{E}_{\text{avg}} = -\frac{\mu^2 G\kappa^3}{15c^5}\frac{1}{a^5}\frac{1}{(1-e^2)^{\frac{7}{2}}}\left(96 + 292e^2 + 37e^4\right)$$
$$\dot{L}_{\text{avg}} = -\frac{\mu^2 G\kappa^{\frac{5}{2}}}{15c^5}\frac{1}{a^{\frac{7}{2}}}\frac{1}{(1-e^2)^2}\left(96 + 84e^2\right)$$

Reduced mass:
$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$
$$\kappa \equiv G\mu$$

Orbital parameters evolutions:

$$\dot{a} = -\frac{\mu G\kappa^2}{15c^5}\frac{1}{a^3}\frac{1}{(1-e)^{\frac{7}{2}}}\left(192 + 548e^2 + 74e^4\right)$$
$$\dot{e} = -\frac{\mu G\kappa^2}{15c^5}\frac{1}{a^4}\frac{1}{(1-e)^{\frac{5}{2}}}\left(304 + 121e^2\right)$$

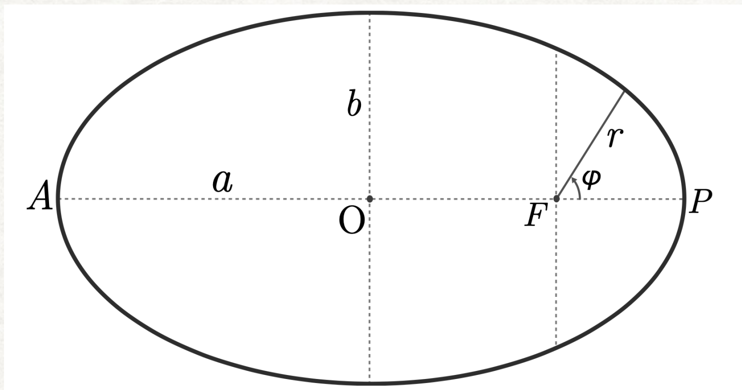
$$\dot{\varphi} = \sqrt{\frac{\kappa(1+e)}{a^3(1-e)^3}}\left(\frac{1 + e\cos(\varphi)}{1+e}\right)^2$$



The system first circularizes, then merges.

Highly eccentric black holes mergers

P. Jamet, A. Barrau, K. M., arXiv:2412.01582



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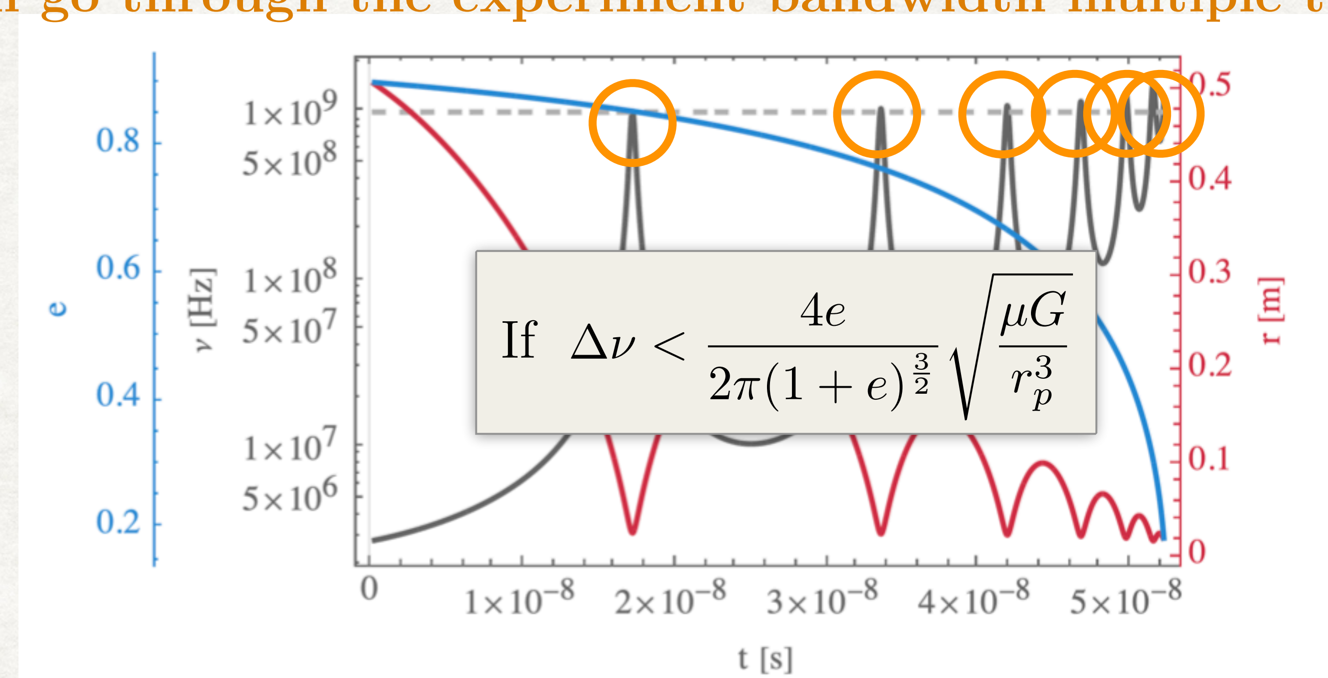
$$\dot{\varphi} = \sqrt{\frac{\kappa(1+e)}{a^3(1-e)^3}} \left(\frac{1 + e\cos(\varphi)}{1+e}\right)^2$$

Can go through the experiment bandwidth multiple times



The system first circularizes, then merges.

Highly eccentric black holes mergers

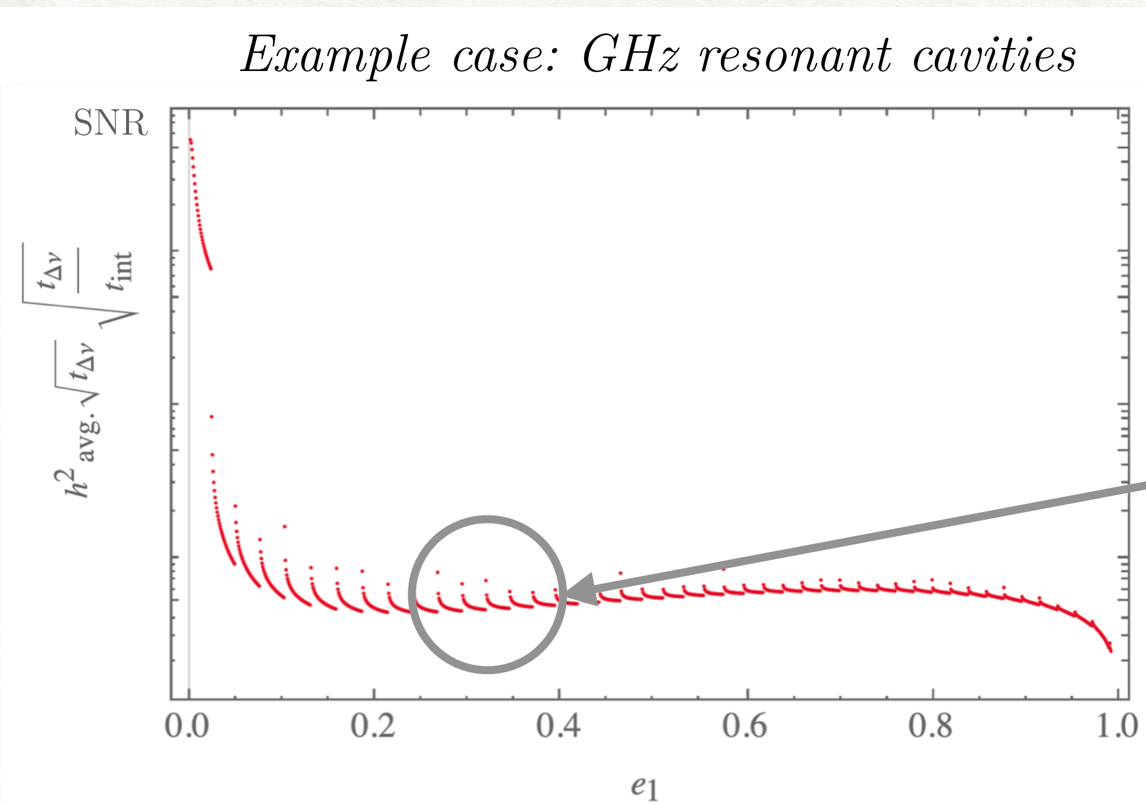
- **Does the eccentricity boost the signal?**

P. Jamet, A. Barrau, K. M., arXiv:2412.01582

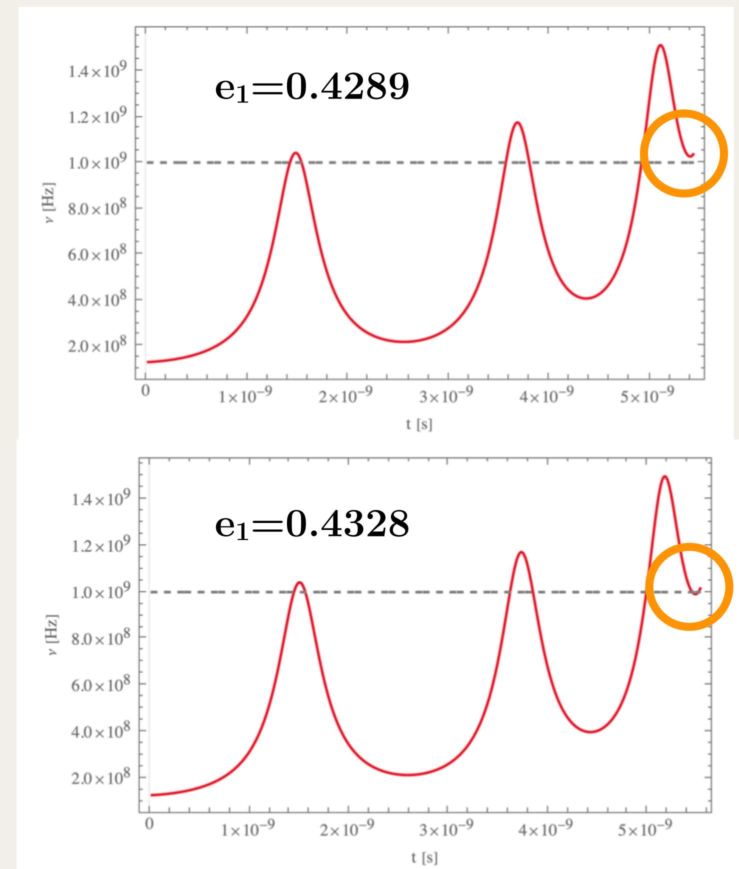
$$h_+ = -\frac{\mu G}{Dc^4}\left(\frac{\kappa\omega_p}{(1+e)^2}\right)^{\frac{2}{3}}\left[2e^2 + 5e\cos(\varphi) + 4\cos(2\varphi) + e\cos(3\varphi)\right]$$

$$h_\times = -\frac{\mu G}{Dc^4}\left(\frac{\kappa\omega_p}{(1+e)^2}\right)^{\frac{2}{3}}\sin(\varphi)\left[6e + 8\cos(\varphi) + 2e\cos(2\varphi)\right]$$

Example case: GHz resonant cavities



Why the jumps?



No crossing

Signal crosses the experimental bandwidth one more time

- The SNR is boosted for small eccentricities and damped for very high ones
- The circular orbit (e_1=0) leads to the best situation!
- When e increases, more energy is deposited in the detector, but more noise is also integrated!

Highly eccentric black holes mergers

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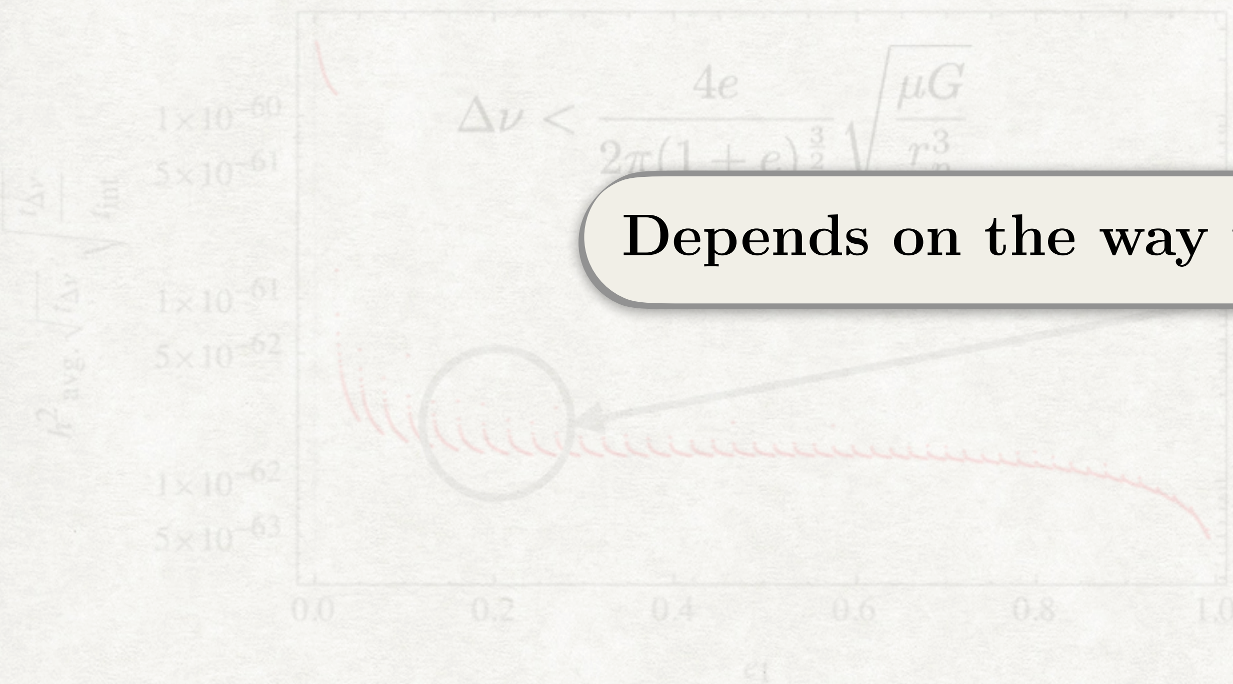
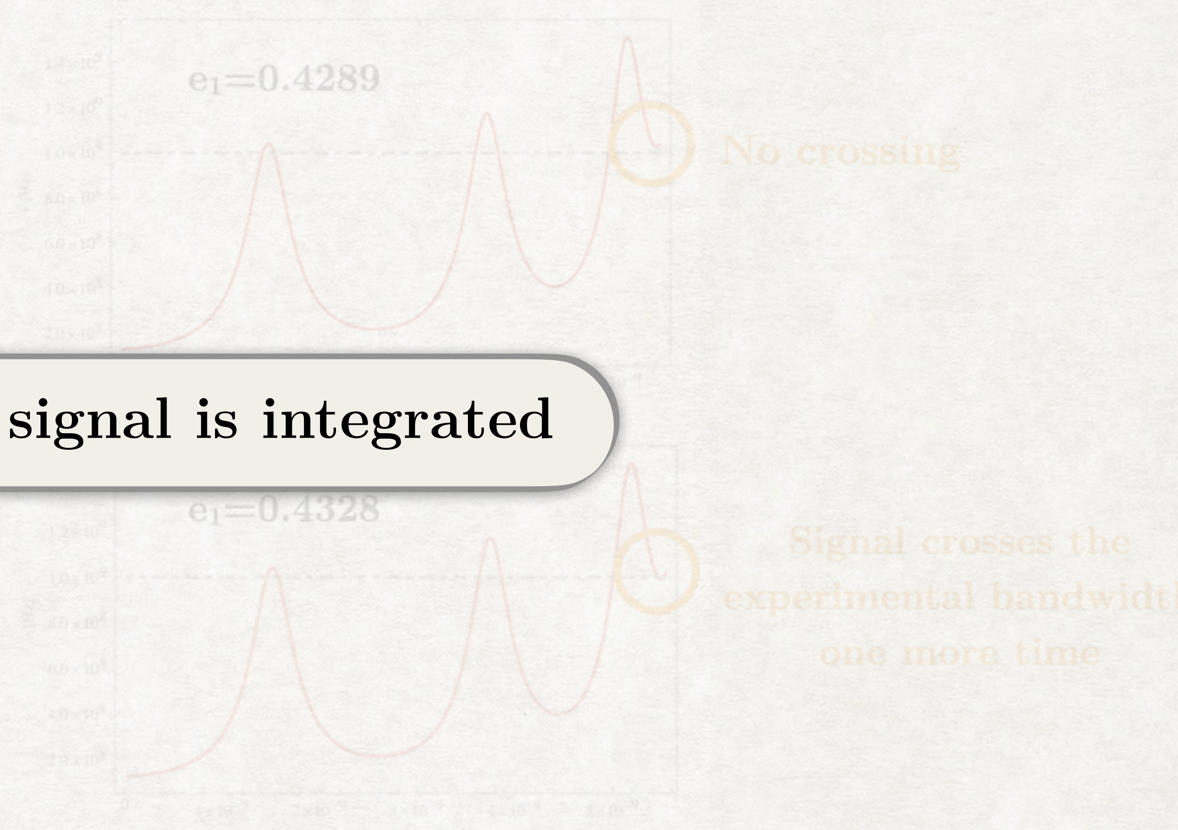
If the detector is not broadband enough:



Why the jumps?



Depends on the way the signal is integrated

- The SNR is boosted for small eccentricities and damped for very high ones
- The circular orbit (e_1=0) leads to the best situation!
- When e increases, more energy is deposited in the detector, but more noise is also integrated!

Part III

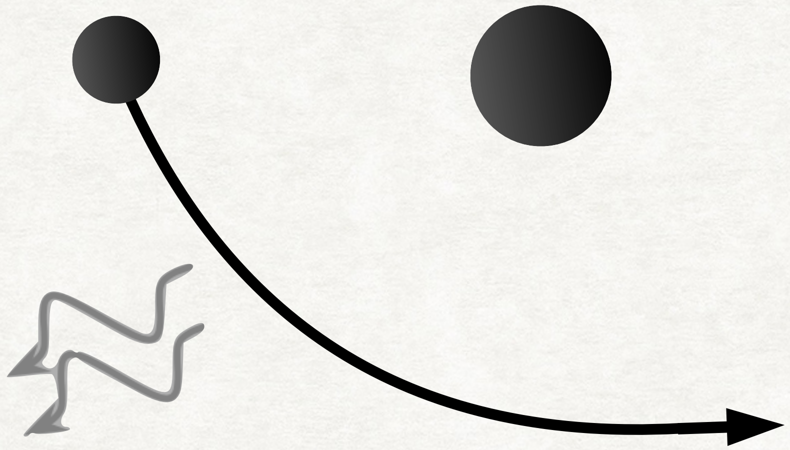
BH mergers beyond the usual circular trajectory:

Close Hyperbolic Encounters

Hyperbolic encounters

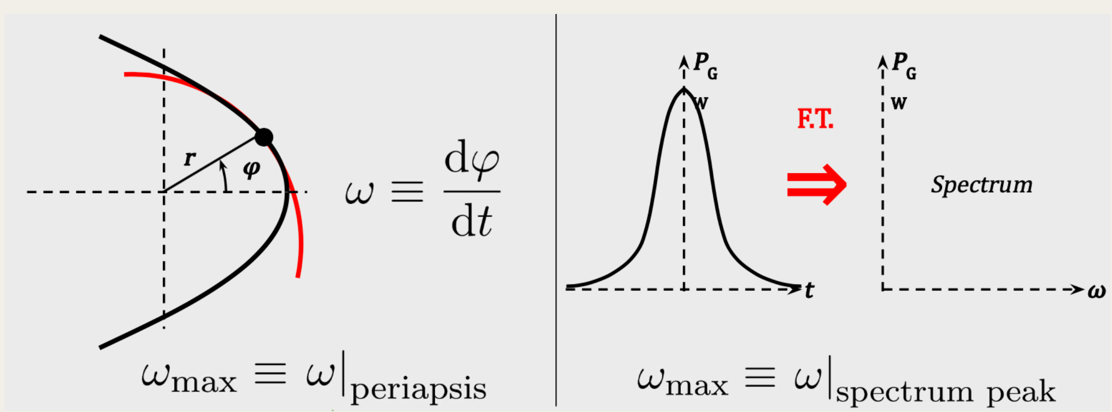
M. Teuscher, A. Barrau, J.G.Bellido, K. M., arXiv:2402.10706, 2404.08379

GW
bremsstrahlung



Studied e.g. in:
García-Bellido et al. arXiv:1711.09702, 2307.00915,
De Vittori et al. 1207.5359
M. Raidal, V. Vaskonen and H. Veermäe, arXiv:2404.08416

Aside: Definition of frequency for an aperiodic signal



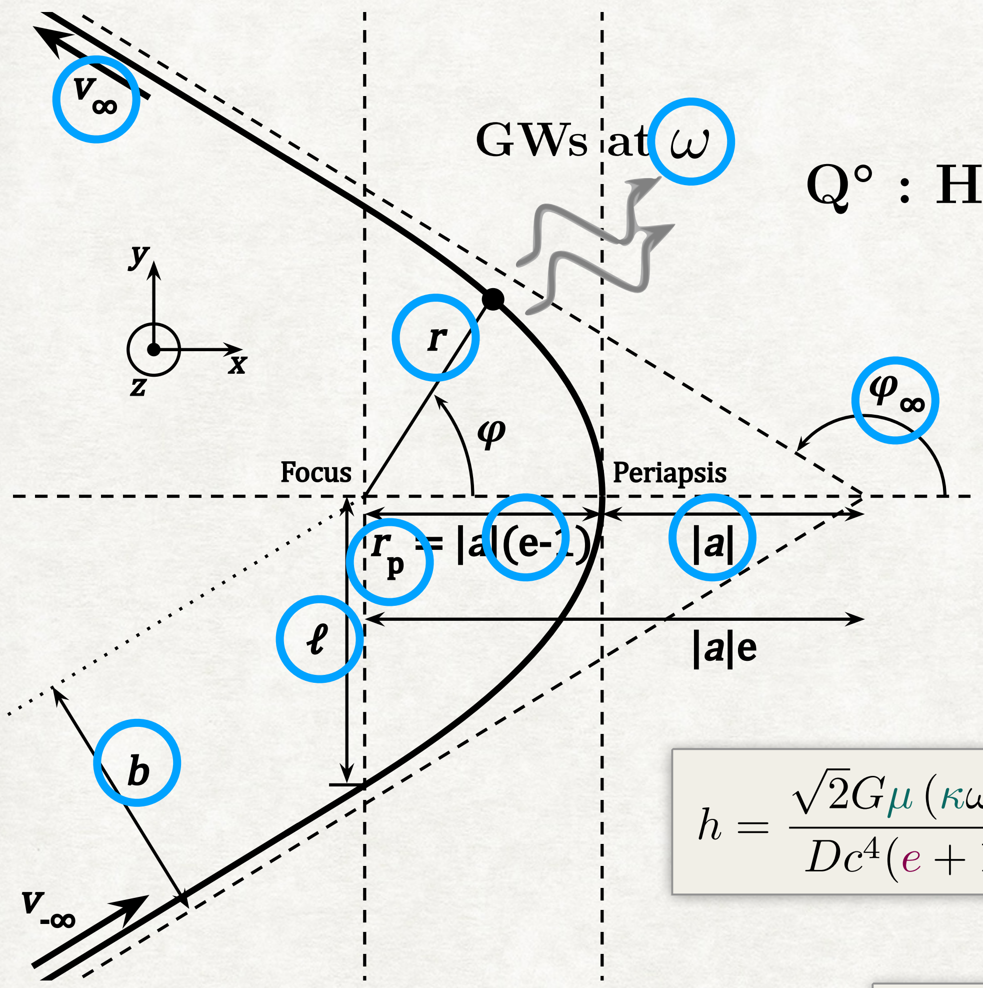
Both definitions are equivalent
García-Bellido et al. arXiv:1711.09702

And even: $r^3_{\min}\omega^2_{\max} = GM(e+1)$

Pseudo-Kepler law eccentricity

Hyperbolic encounters

M. Teuscher, A. Barrau, J.G.Bellido, K. M., arXiv:2402.10706, 2404.08379



Bunch of parameters that describe the system

↳ But only 3 d.o.f

Q° : How to cleverly pick them? *García-Bellido et al. arXiv:1711.09702*

↳ mass → constrained by physics

↳ eccentricity → easier computations

↳ frequency → fixed by the detector

↳ Additional constraint

↳ 2D parameter space: ($M=m_1+m_2$,e)

Strain

$$h = \frac{\sqrt{2}G\mu\left(\kappa\omega_p\right)^{2/3}}{Dc^4(e+1)^{4/3}}\left[8+13e^2+2e^4+2e\left(12+5e^2\right)\cos(\varphi)+13e^2\cos(2\varphi)+2e^3\cos(3\varphi)\right]^{1/2}$$

Power

$$P = \frac{4G\mu^2\left(\kappa\omega_p\right)^{4/3}}{15c^5(e+1)^{8/3}}\omega^2(\varphi)\left[24+13e^2+48e\cos(\varphi)+11e^2\cos(2\varphi)\right]$$

The angle phi parametrizes the position along the trajectory

Hyperbolic encounters

M. Teuscher, A. Barrau, J.G.Bellido, K. M., arXiv:2402.10706, 2404.08379

- **Searching for optimal emission scenarios**

The strain is maximal at periapsis

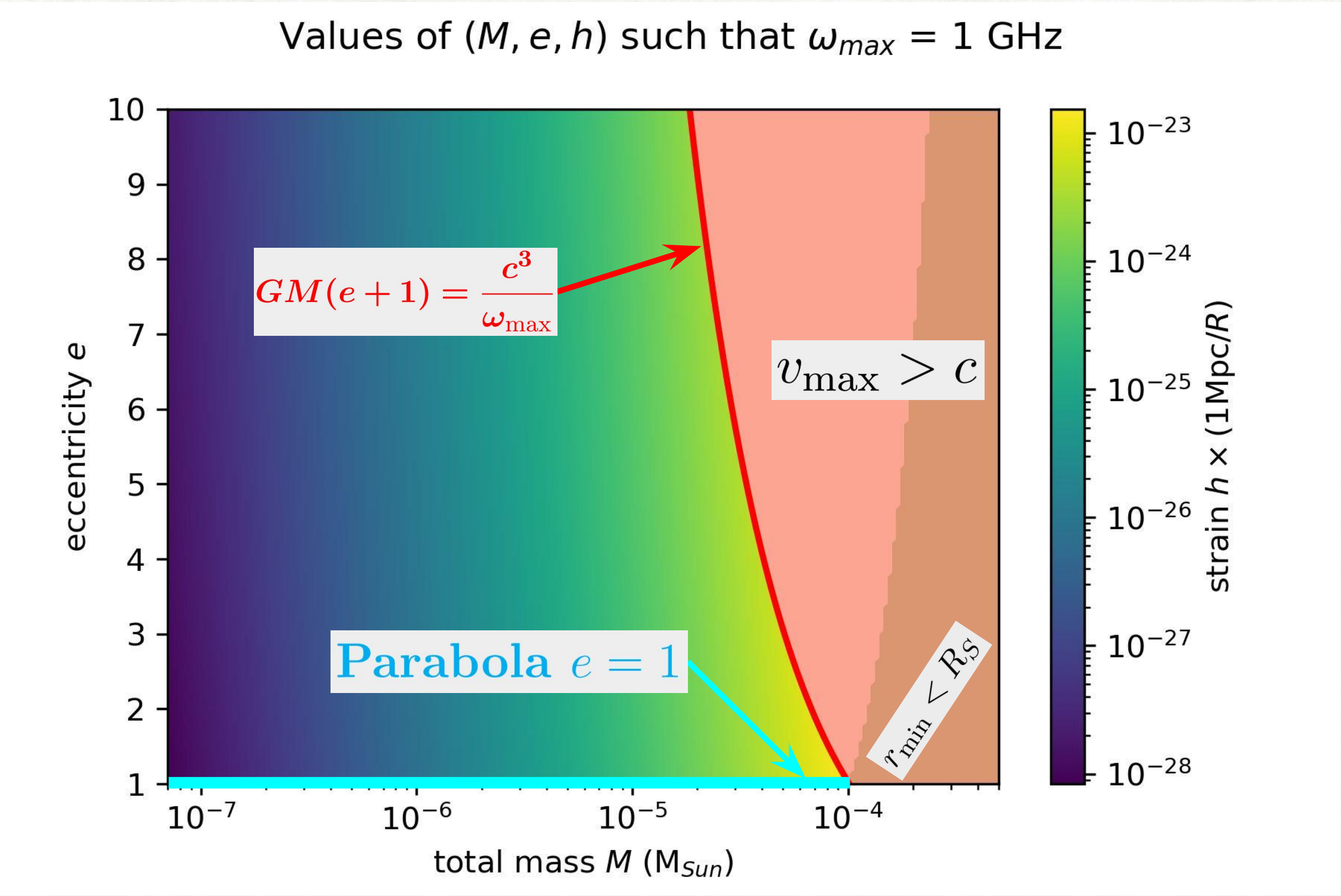
↳ So is the frequency

Strain and power at periapsis

$$h_p = 3.6 \times 10^{-25} \times \frac{4q}{(1+q)^2} \frac{G(e)}{G(1)} \left(\frac{M}{10^{-5} M_\odot}\right)^{\frac{5}{3}} \left(\frac{f_p}{1.6\ \text{GHz}}\right)^{\frac{2}{3}} \left(\frac{1\ \text{Mpc}}{D}\right)$$

$$P_p = 3.7 \times 10^{24} L_\odot \times \frac{1}{(1+e)^{\frac{2}{3}}} \left(\frac{4q}{(1+q)^2}\right)^2 \left(\frac{M}{10^{-5} M_\odot}\right)^{\frac{10}{3}} \left(\frac{f_p}{1.6\ \text{GHz}}\right)^{\frac{10}{3}}$$

with $q \equiv \frac{m_1}{m_2}$ $G(e) \equiv \frac{e+2}{(e+1)^{1/3}}$



At fixed M, the highest e maximizes the strain

BUT

The global maximal strain is obtained for the lowest value of e (and the highest value of M)

Hyperbolic encounters

M. Teuscher, A. Barrau, J.G.Bellido, K. M., arXiv:2402.10706, 2404.08379

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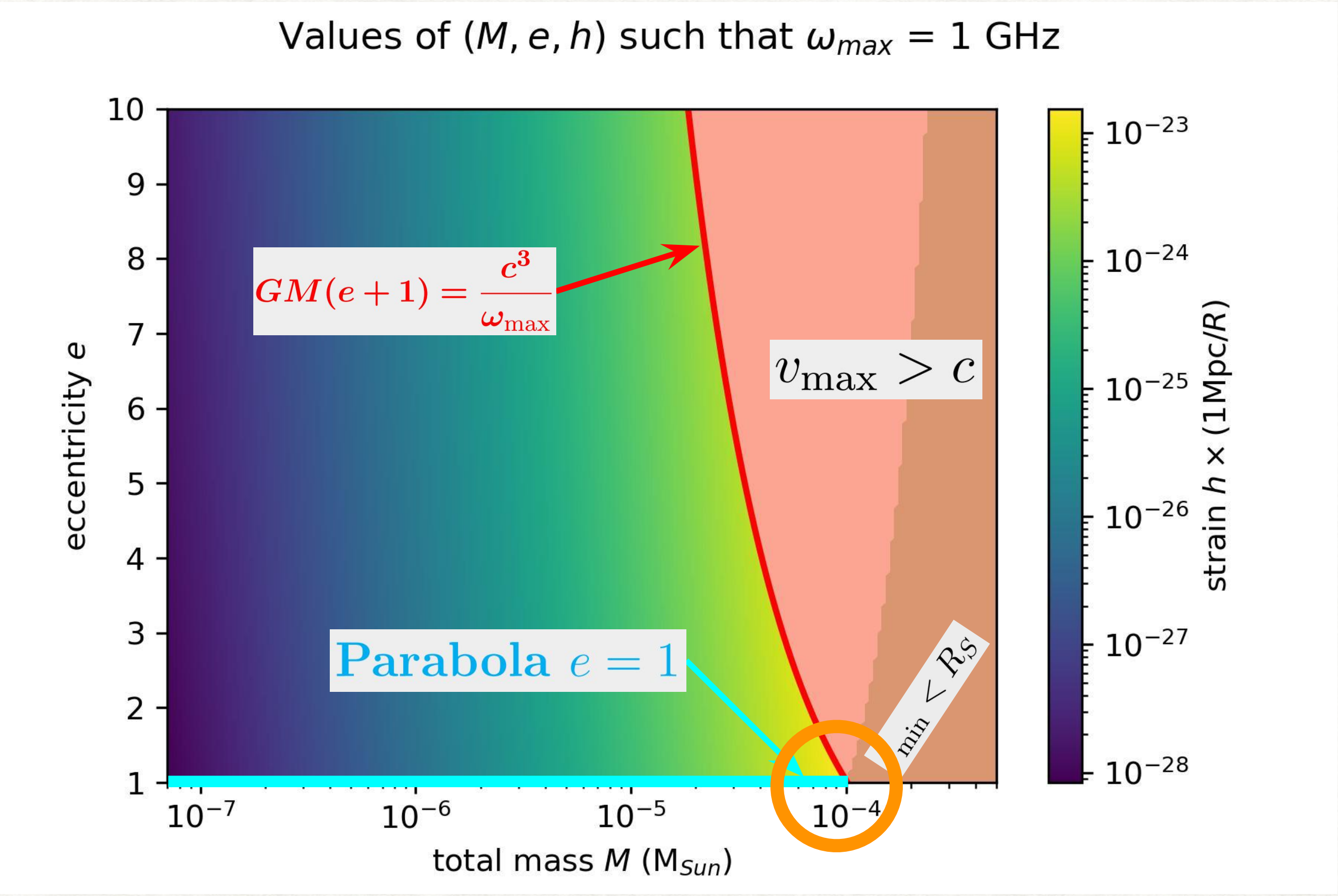

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Hyperbolic encounters

M. Teuscher, A. Barrau, J.G.Bellido, K. M., arXiv:2402.10706, 2404.08379

- **Burst duration**

Reminder: Event duration must be carefully assessed for proper sensitivity reckoning

Generic expression
Can be applied to any detector of bandwidth $\Delta\nu$ operating at ν .

$$t_{\Delta\nu}(\nu, \Delta\nu, e) = \frac{1}{\pi\nu}\sqrt{1+\frac{1}{e}\sqrt{\frac{\Delta\nu}{\nu}}}$$

Frequency is not monotonously increasing

The signal « drifts in, then back out»

- **GHz cavities signal-to-noise ratio**

$\nu \sim$ a few GHz, quality factor $Q = \frac{\nu}{\Delta\nu} \approx 10^5$

$\mathrm{SNR} = h^2 t_{\Delta\nu} \times \nu^3 Q \times [\text{other experimental parameters}]$

$$t_{\Delta\nu} \approx 5\times 10^{-13}\mathrm{s}\left(\frac{2.67\mathrm{GHz}}{\nu}\right)\left(\frac{10^5}{Q}\right)^{\frac{1}{2}}\left(\frac{1+e^{-1}}{2}\right)^{\frac{1}{2}}$$

Still horribly small

Distance at which an event can be observed:

$$D_{\max} = \frac{2.4\times 10^3\mathrm{km}}{\sqrt{\mathrm{SNR}}}\left(\frac{V_{\mathrm{cav}}}{1.83\times 10^{-3}\mathrm{m}^3}\right)^{\frac{5}{6}}\left(\frac{\eta}{0.1}\right)\left(\frac{B_0}{27\mathrm{T}}\right)\left(\frac{\nu}{2.67\mathrm{GHz}}\right)^{\frac{5}{3}}\left(\frac{Q}{10^5}\right)^{-\frac{1}{2}}\left(\frac{T}{0.4\mathrm{K}}\right)^{-\frac{1}{2}}\left(\frac{H(e)}{H(1)}\right)\left(\frac{M}{6.1\times 10^{-8}M_\odot}\right)^{\frac{5}{3}}\left(\frac{4\mu}{M}\right)$$

Most optimized trajectory:

$$D_{\max} = \frac{3.4\times 10^{-2}\mathrm{AU}}{\sqrt{\mathrm{SNR}}}\left(\frac{V_{\mathrm{cav}}}{1.83\times 10^{-3}\mathrm{m}^3}\right)^{\frac{5}{6}}\left(\frac{\eta}{0.1}\right)\left(\frac{B_0}{27\mathrm{T}}\right)\left(\frac{Q}{10^5}\right)^{-\frac{1}{2}}\left(\frac{T}{0.4\mathrm{K}}\right)^{-\frac{1}{2}}\left(\frac{4\mu}{M}\right)$$

Part IV

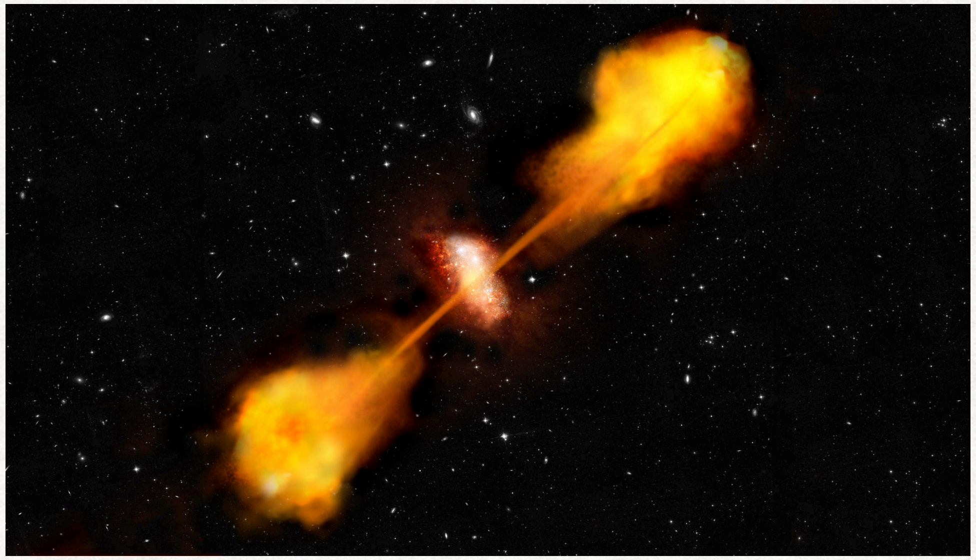
Boosted sources

Boosted sources

- **GW waveforms are usually derived assuming that the source and the observer are at rest with respect to the Hubble flow, i.e. that their relative peculiar velocity is zero.**

↪ GW sources move inside galaxies, and the galaxies themselves move with respect to the Hubble flow.

↪ This effect is usually small, but not always



HFGWs from jets in AGN?

↪ **A. Barrau, J. G. Bellido, J. Kopp, K.M.**

What are the effects of a boost on a GW signal?

G. Cusin, C. Pitrou, C. Bonvin, A. Barrau, K. M., arXiv:2405.01297

Boosted sources

My personal bet for the effect on the amplitude was:

- **Kinematic Doppler factor:** $\mathcal{D} \equiv \frac{1}{\gamma\left[1-\beta\cos(\theta)\right]}$

  $\gamma \equiv (1-\beta^2)^{-(1/2)}$

  θ : angle between the velocity vector and the line of sight

  Ghisellini et al. (1993) $\mathcal{D}_{\max} = \mathcal{D}(0^\circ) = (1+\beta)\gamma \sim 2\gamma$

- **Electromagnetic case:**

  Intensity enhancement: $I_\nu(\nu) = \mathcal{D}^3 I'_{\nu'}(\nu')$ — $'$ *: quantities in the rest frame of the source*

  Broad-band fluxes: $F = \mathcal{D}^4 F'$

- **Gravitational waves:**

$$\frac{dE}{dAdt} = \frac{c^3}{16\pi G}\langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle \sim \frac{c^3}{16\pi G}\omega^2 h^2$$

Enhanced as $\mathcal{D}^4 \Rightarrow$ $h = \mathcal{D}h' \sim 2\gamma h'$ *In the optimal case* $\theta = 0$

Boosted sources

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- **Eikonal approximation:** $h_{\mu\nu} = \Re\left(H_{\mu\nu} e^{i\omega\Phi}\right)$

  Phase Φ varies much faster than the wave amplitude $H_{\mu\nu}$
  ω : large dimensionless parameter

- **Einstein equations** $\Rightarrow -\omega^2 k_\beta k^\beta H_{\mu\nu} + i\omega\left[2k^\beta \nabla_\beta H_{\mu\nu} + \nabla_\beta k^\beta H_{\mu\nu}\right] + \mathcal{O}(\omega^0) = 0$

  In the geometric optics approximation

  Eikonal momentum: $k_\mu = \partial_\mu \Phi$

$$\Rightarrow \begin{cases} k_\beta k^\beta = 0 & k^\mu \text{ is a null vector} \\ 2k^\beta \nabla_\beta H_{\mu\nu} + \left(\nabla_\beta k^\beta\right) H_{\mu\nu} = 0 & (\star) \end{cases}$$

In the Eikonal approximation we can define trajectories for GWs which are null geodesics $x^\mu(\lambda)$

- **We split:**

$$H_{\mu\nu} = H \epsilon_{\mu\nu} \qquad H = \sqrt{H^\star_{\mu\nu} H^{\mu\nu}} \quad \epsilon_{\mu\nu}\epsilon^{\mu\nu} = 1 \quad k^\mu \epsilon_{\mu\nu} = 0$$

Amplitude (H) — Polarisation unit tensor ($\epsilon_{\mu\nu}$)

$\theta \equiv \nabla_\mu k^\mu$: Sachs scalar which characterizes the divergence of the geodesic beam

$$(\star) \Rightarrow \begin{cases} k^\mu \nabla_\mu H = -\frac{1}{2}\theta H \\ k^\alpha \nabla_\alpha \epsilon_{\mu\nu} = 0 \end{cases}$$

Conservation of the flux $\nabla_\alpha(H^2 k^\alpha) = 0$

Polarisation is parallel-propagated along the null vector k^μ

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- **The Sachs scalar can be written as:** $\theta = \nabla_\alpha k^\alpha = 2\frac{\mathrm{dlog}\chi}{d\lambda}$

« Propagation distance » $\chi = \sqrt{\frac{dA_0}{d\Omega_S}}$

$d\Omega_S$ **: solid angle of the emitted bundle of geodesics that is received by the observer**
dA_0 **: surface of the bundle at the observer location**

- **The strain scale as:** $H(\lambda) = \frac{H(\lambda_S)\chi(\lambda_S)}{\chi(\lambda)} \propto \frac{1}{\chi(\lambda)}$

- **Source ↔ Observer :** $D_A = \sqrt{\frac{dA_S}{d\Omega_O}}$ **Angular diameter distance**

Relates the solid angle at the observer location to the bundle surface at the source location

- **These two distances are related by:** $\chi = (1+z)D_A$ with $1+z = \frac{u_S^\mu k_\mu}{u_O^\mu k_\mu}$ the *source redshift*

- **Luminosity distance:** $D_L = (1+z)^2 D_A$ Governs the scaling of the GW energy density

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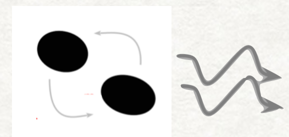
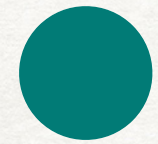
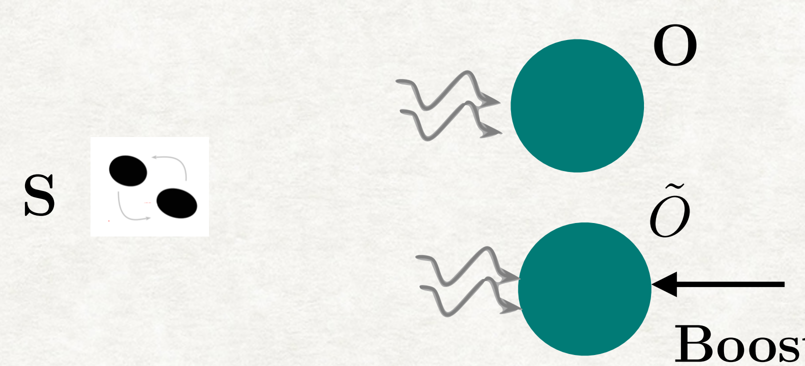
- **First, let us define a reference case (S,O):**

Reference source with frame S   Reference observer with frame O

- **Two separate situations must be considered!**

Situation A	**Situation B**
 **Boosted frame $\tilde{S}$ with 4-velocity $\tilde{u}^{\mu}_{S}$**	 **Boosted frame $\tilde{O}$ with 4-velocity $\tilde{u}^{\mu}_{O}$**
A source moving with respect to a reference source located at the same position when the signal is emitted	An observer (detector) moving with respect to a reference observer located at the same position when the signal is received

Those situations are not equivalent!

- **<u>Objective:</u>** Compare to the reference case (S,O) the following situations:

  Situation A: a signal is emitted in $\tilde{S}$ and received by O
  Situation B: a signal is emitted in S and received by $\tilde{O}$

Boosted sources

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Effects of a boost on the redshift (frequencies)

- **A signal emitted by the reference source and measured by the reference observer is seen with a reference redshift:**

$$1 + z_{\rm ref} = \frac{k_\mu u_S^\mu}{k_\mu u_O^\mu} = \frac{E_S}{E_O}$$

- **Source and observer Doppler shift factors:**

$$\mathcal{D}_O \equiv \frac{k_\mu \tilde{u}_O^\mu}{k_\mu u_O^\mu} = \frac{\tilde{E}_O}{E_O}$$

If the observer moves towards the source at relativistic speed:

$$\mathcal{D}_O \sim 2\gamma_O$$

$$\mathcal{D}_S \equiv \frac{k_\mu \tilde{u}_S^\mu}{k_\mu u_S^\mu} = \frac{\tilde{E}_S}{E_S}$$

If the source moves towards the observer at relativistic speed:

$$\mathcal{D}_S^{-1} \sim 2\gamma_S$$

- **When both the source and the observer move w.r.t the reference case:**

$$1 + \tilde{z} = \frac{\mathcal{D}_S}{\mathcal{D}_O}(1 + z_{\rm ref})$$

Boosted sources

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Effects of a boost on distances

- **Moving source (situation A):** $\mathcal{D}_O = 1,\ \mathcal{D}_S \neq 1$

$$\chi(\tilde{S} \to O) = \sqrt{\frac{dA_O}{d\tilde{\Omega}_S}} = \sqrt{\frac{dA_O}{d\Omega_S}}\sqrt{\frac{d\Omega_S}{d\tilde{\Omega}_S}} = \mathcal{D}_S \chi(S \to O)$$

The distance χ is affected by the source velocity

$$D_A(\tilde{S} \leftarrow O) = D_A(S \leftarrow O)$$

The distance D_A is not

- **Moving observer (situation B):** $\mathcal{D}_O \neq 1,\ \mathcal{D}_S = 1$

$$\chi(S \to \tilde{O}) = \chi(S \to O)$$

Does not depend on the observer velocity by definition

$$D_A(S \leftarrow \tilde{O}) = \mathcal{D}_O D_A(S \leftarrow O)$$

The angular distance is not the same

- **General case:**

$$\chi(\tilde{S} \to \tilde{O}) = \mathcal{D}_S \chi(S \to O)$$
$$D_A(\tilde{S} \leftarrow \tilde{O}) = \mathcal{D}_O D_A(S \leftarrow O)$$
$$D_L(\tilde{S} \to \tilde{O}) = \frac{\mathcal{D}_S^2}{\mathcal{D}_O} D_L(S \to O)$$

Does not break equivalence principle!

Situations A and B are not the same

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- **Effect of a boost on the amplitude of a gravitational wave:**

When both the source and the observer move w.r.t the reference situation:

$$H(\tilde{S} \to \tilde{O}) = \mathcal{D}_S^{-1} H(S \to O)$$

$$\sim 2\gamma_S H(S \to O)$$

The source velocity affects the strain amplitude but the observer velocity does not!

- **Effect of a boost on the energy density of a gravitational wave:**

$$\rho_{GW}(\tilde{S} \to \tilde{O}) = \mathcal{D}_O^2 \mathcal{D}_S^{-4} \rho_{GW}(S \to O)$$

$$\sim (2\gamma_O)^2 (2\gamma_S)^4 H(S \to O)$$

We recover that the energy density transforms as D_L^{-2}

In agreement with the definition of the luminosity distance

Boosted sources

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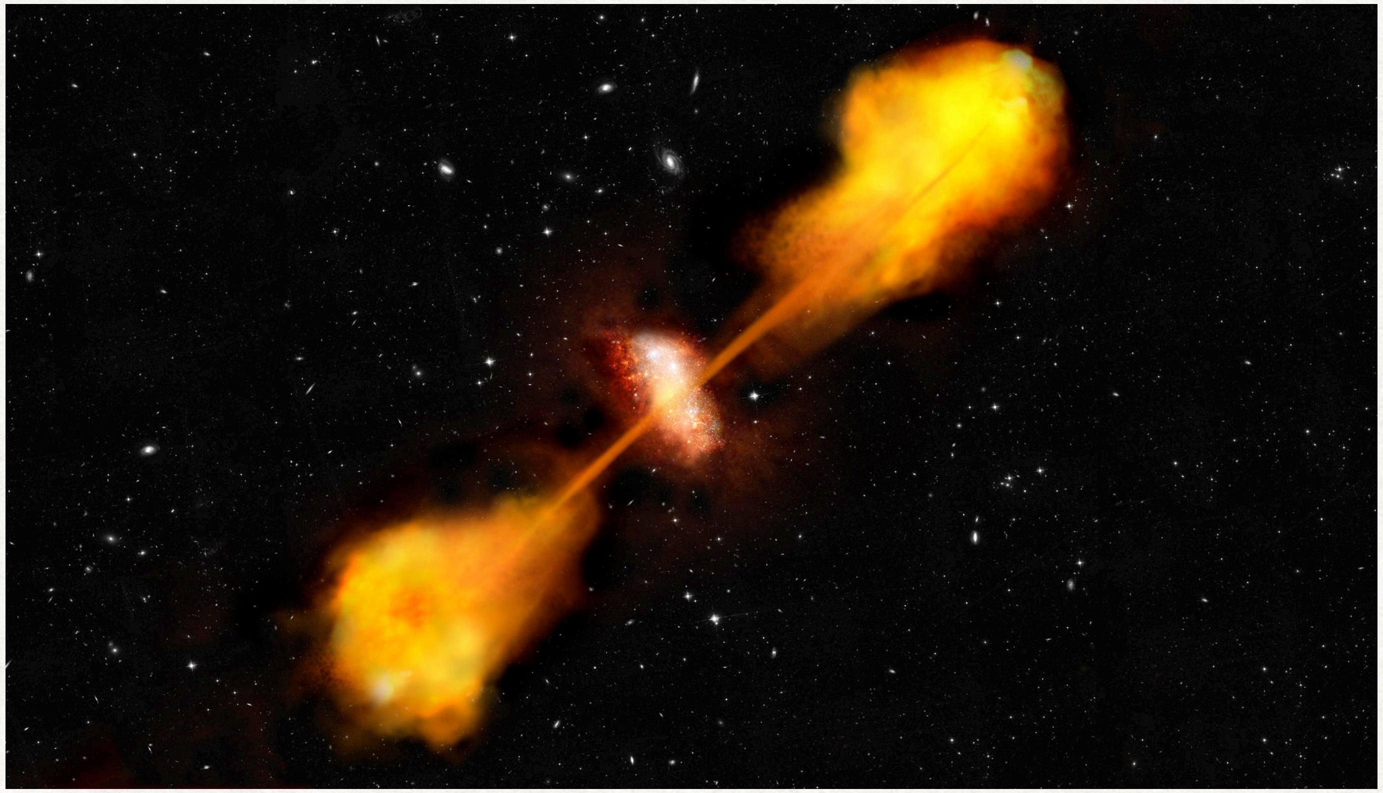
	boosted source	boosted observer	equations
$E_o = 2\pi f_o$	1	$\mathcal{D}_o$	(4.2)
$E_s = 2\pi f_s$	$\mathcal{D}_s$	1	(4.2)
$1 + z = \frac{E_s}{E_o} = \frac{\mathrm{d}\tau_o}{\mathrm{d}\tau_s}$	$\mathcal{D}_s$	$\mathcal{D}_o^{-1}$	(4.3), (4.31)
χ	$\mathcal{D}_s$	1	(4.8), (4.11), (4.14a)
D_A	1	$\mathcal{D}_o$	(4.9), (4.12), (4.14a)
D_L	$\mathcal{D}_s^2$	$\mathcal{D}_o^{-1}$	(4.10), (4.13), (4.14a)
H	$\mathcal{D}_s^{-1}$	1	(4.15), (4.16)
$\mathrm{d}\Omega_s$	$\mathcal{D}_s^{-2}$	1	(4.7)
$\mathrm{d}\Omega_o$	1	$\mathcal{D}_o^{-2}$	(4.7)
ρ_{GW}	$\mathcal{D}_s^{-4}$	$\mathcal{D}_o^2$	(4.19b)
$\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\Omega_o \mathrm{d}\log E_o}$	$\mathcal{D}_s^{-4}$	$\mathcal{D}_o^4$	(4.26)
$\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\Omega_o}$	$\mathcal{D}_s^{-4}$	$\mathcal{D}_o^4$	(4.28)
$\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\Omega_o \mathrm{d}E_o}$	$\mathcal{D}_s^{-3}$	$\mathcal{D}_o^3$	(4.25)

Boosted sources

Boosts can increase a GW signal



Are there relevant boosted sources?

Thank you!