Gravitational Waves From Primordial Fluctuations

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Leibniz Universität Hannover

Emmy Noether-Programm

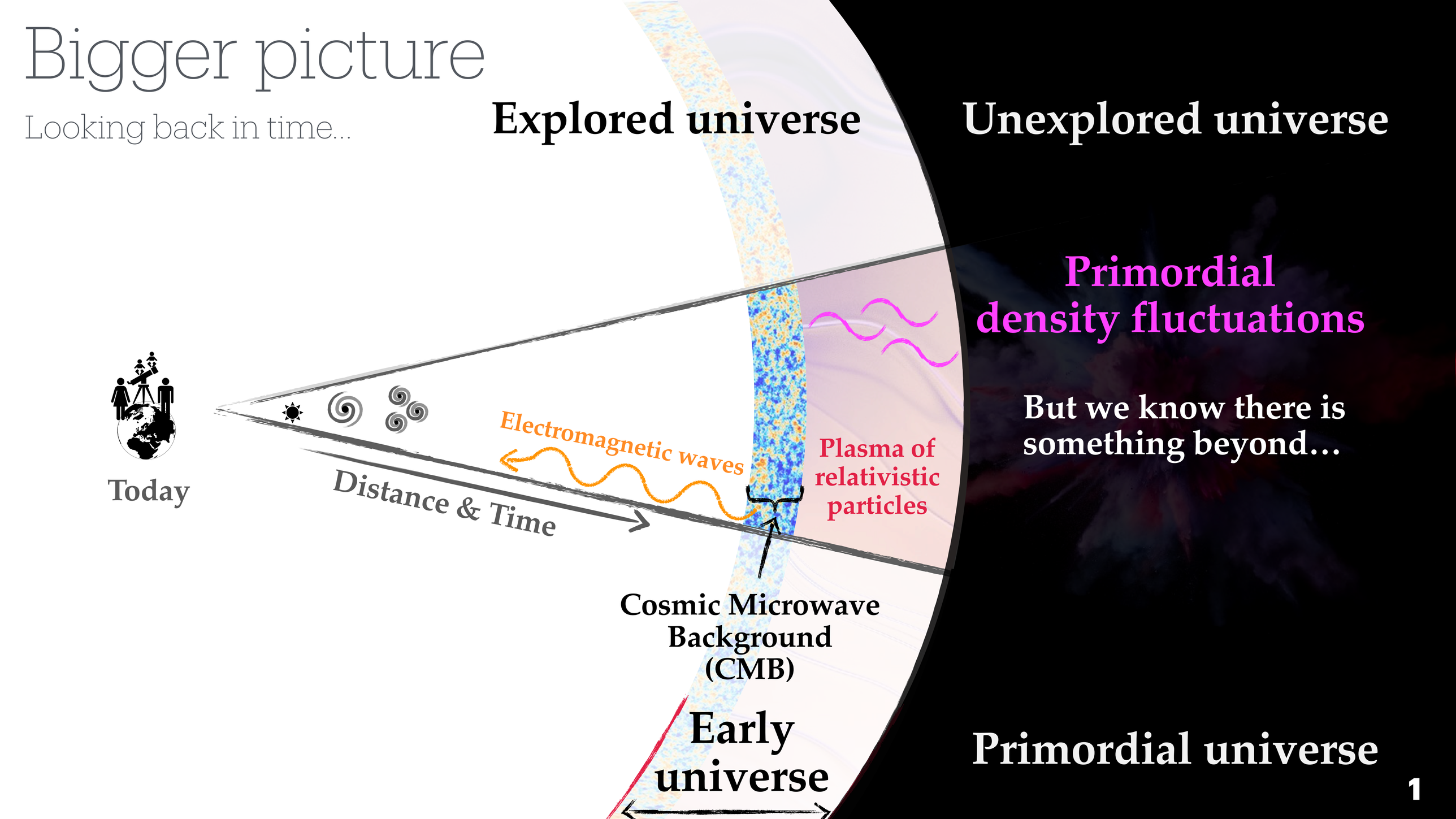
DFG Deutsche Forschungsgemeinschaft

Gravitational Waves From Primordial Fluctuations

Seeds of all structures we see in the Universe, like galaxies.

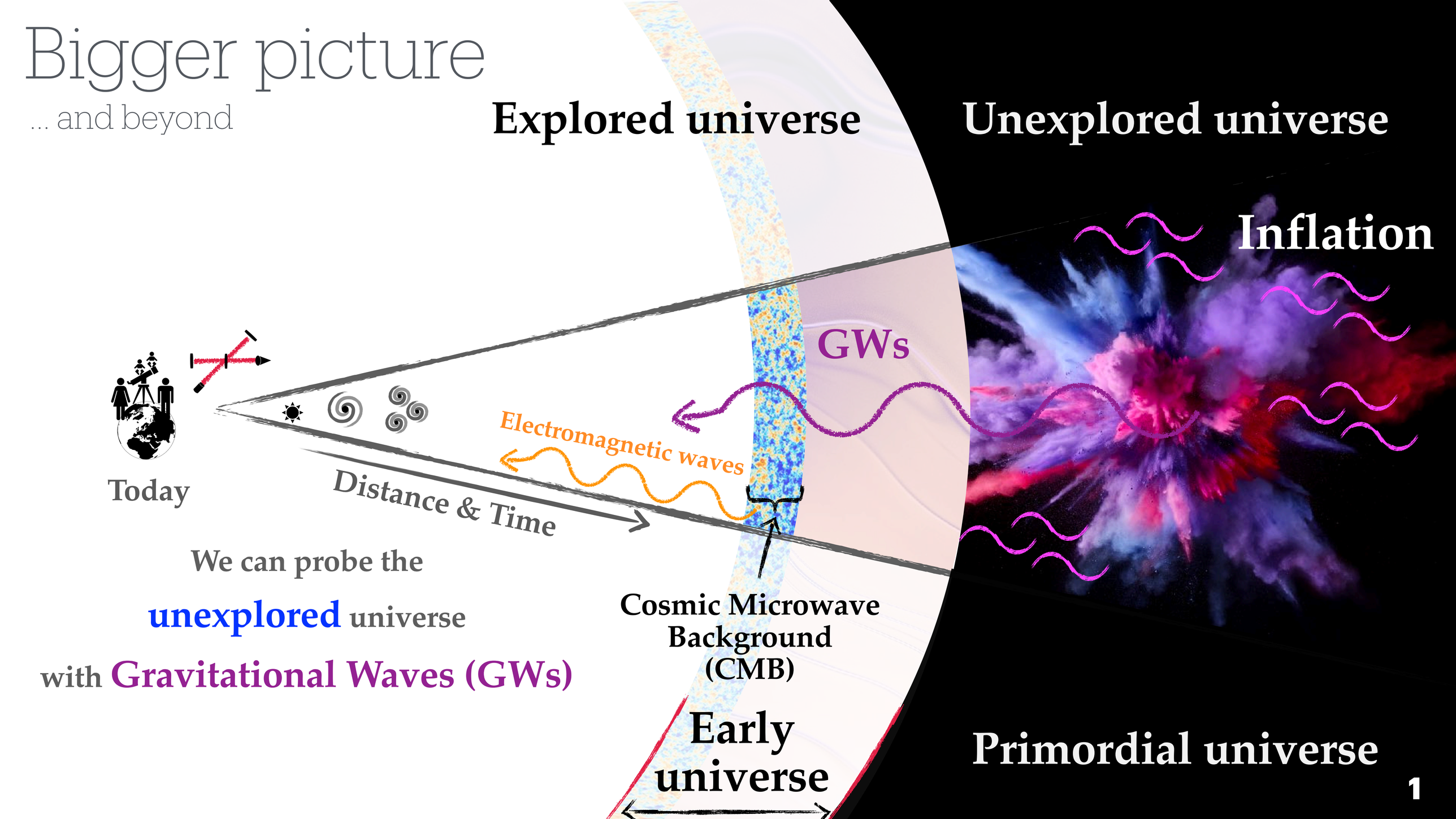
Bigger picture

Looking back in time...



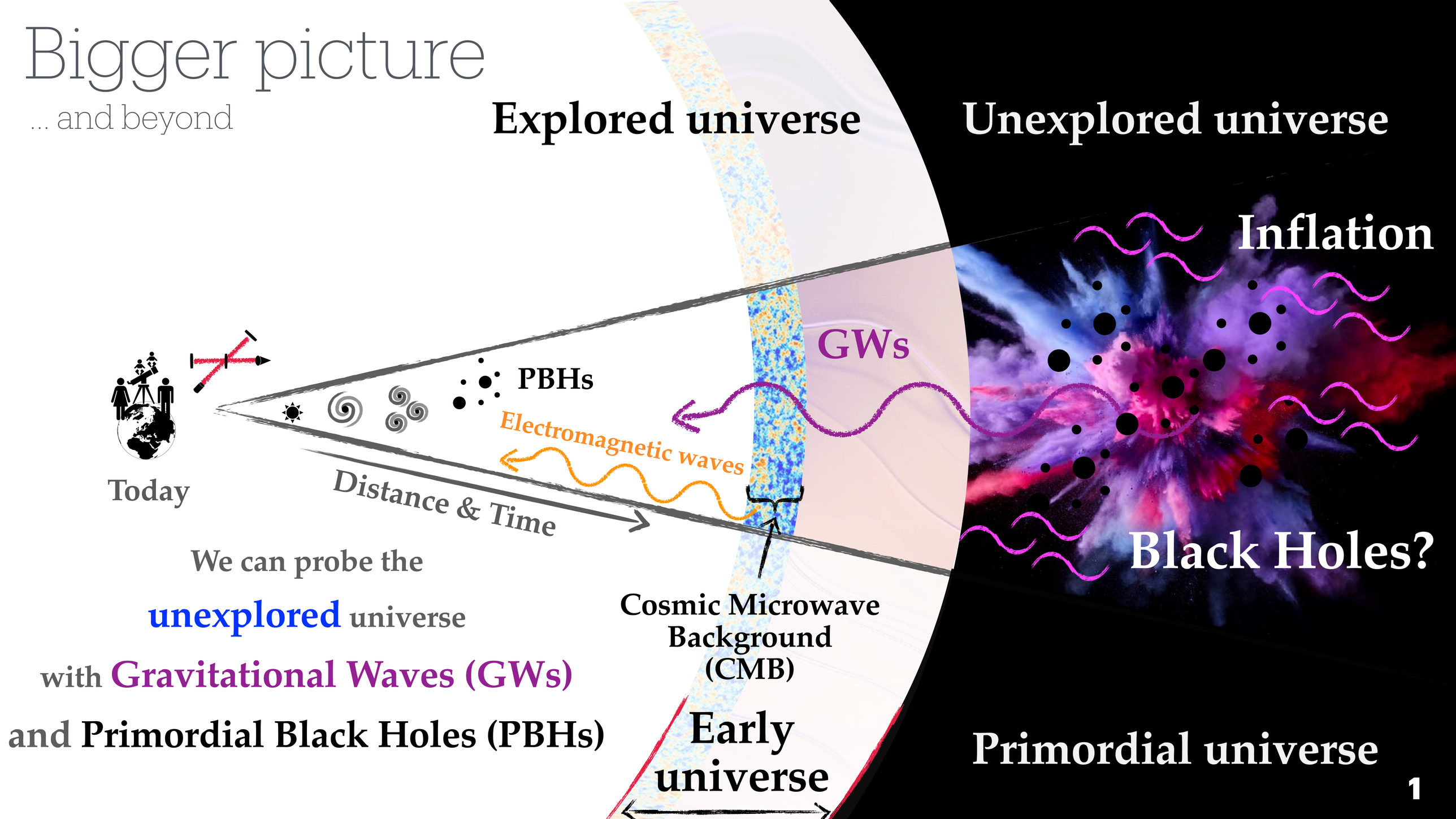
Bigger picture

... and beyond



Bigger picture

... and beyond



We can probe the

unexplored universe

with **Gravitational Waves (GWs)**

and **Primordial Black Holes (PBHs)**

Why are PBHs and GWs interesting?

We know BHs exist (LIGO/VIRGO/KAGRA + EHT)

- **Potential explanations of mysterious observations:** (wide PBH mass spectrum)
  - Dark matter? Seeds of Supermassive BHs?

Originated with Hawking in 1974

- **Relics of the primordial universe:** (Formation between $t \sim 10^{-36} - 1\,\mathrm{s}$ after Big Bang)
  - How was the universe in its initial stages?
    What kind of extreme processes could have formed PBHs and GWs?
- **We will get crucial tests in the next decades!**
  - Gravitational waves observations will favor/disfavor the presence of PBHs.

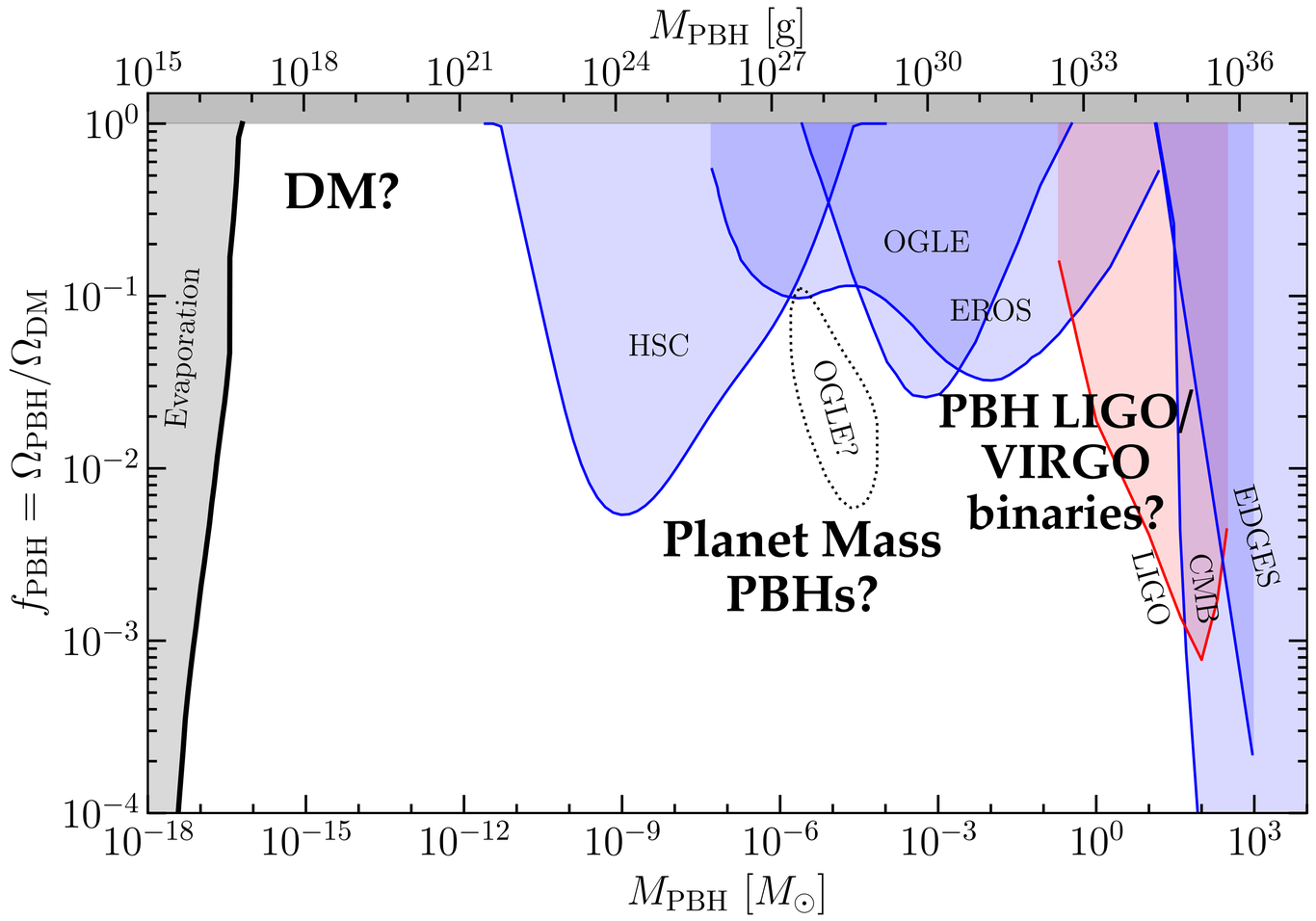
PBHs bounds or evidence?

Connection between GWB at PTAs and Planet-Mass PBHs: 2402.18965

Reheating by PBH evaporation?

$M < 10^9\,\mathrm{g}$

Modified Hawking evaporation? 2006.00011



SMBH seeds?

$M > 10^3\,M_\odot$

Source code https://github.com/bradkav/PBHbounds by Bradley J. Kavanagh

Why are PBHs and GWs interesting?

We know BHs exist (LIGO/VIRGO/KAGRA + EHT)

Our focus today

What would it entail to find evidence of PBHs and cosmic GWs?

- **Relics of the primordial universe:** (Formation between $t \sim 10^{-36} - 1\,\mathrm{s}$ after Big Bang)
  - How was the universe in its initial stages?
    What kind of extreme processes could have formed PBHs and GWs?
- **We will get crucial tests in the next decades!**
  - Gravitational waves observations will favor/disfavor the presence of PBHs.

Reviews and lectures

Scalar Induced Gravitational Waves Review

Guillem Domènech (INFN, Padua)

Sep 3, 2021

97 pages

Published in: *Universe* 7 (2021) 11, 398

Published: Oct 21, 2021

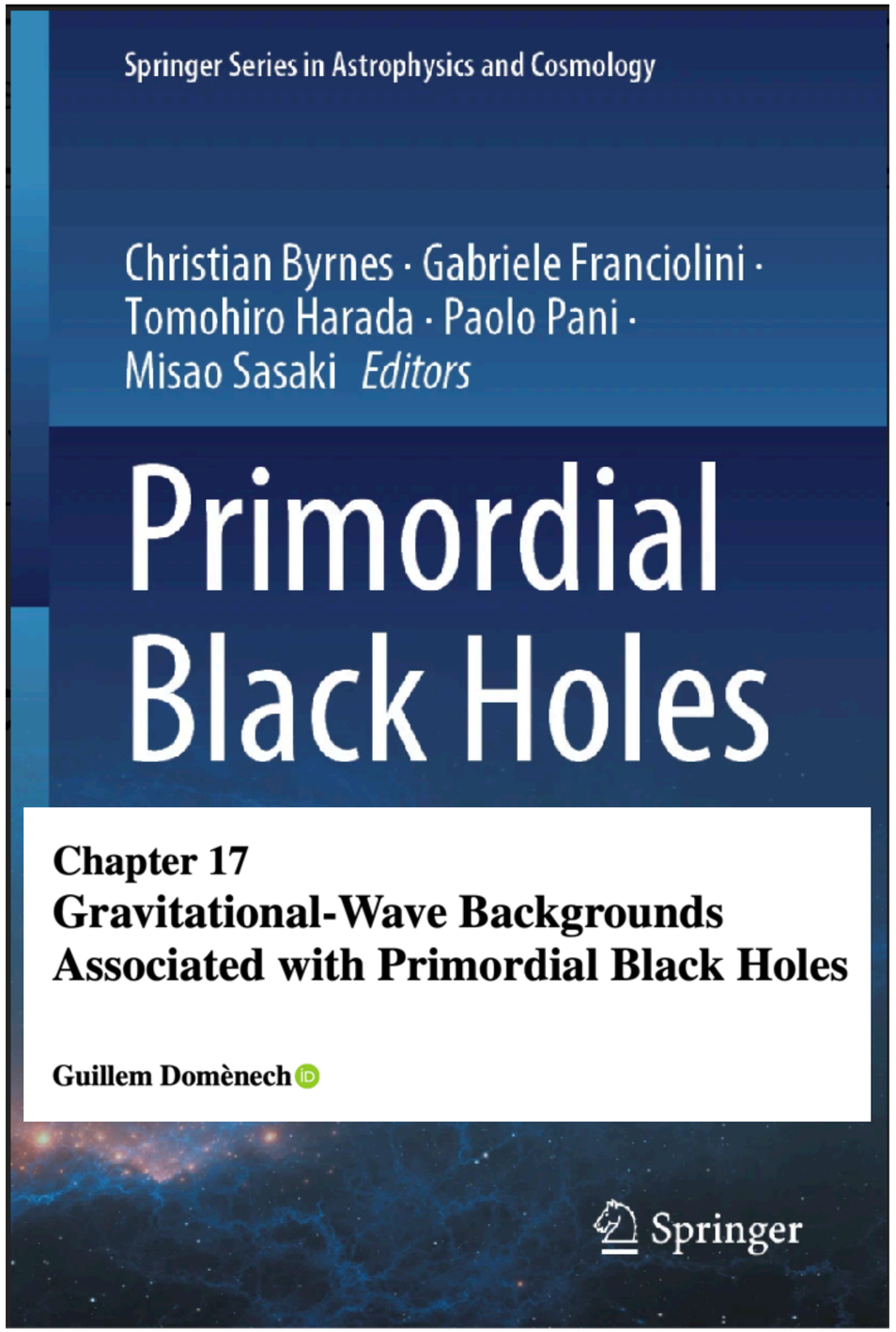
e-Print: 2109.01398 [gr-qc]

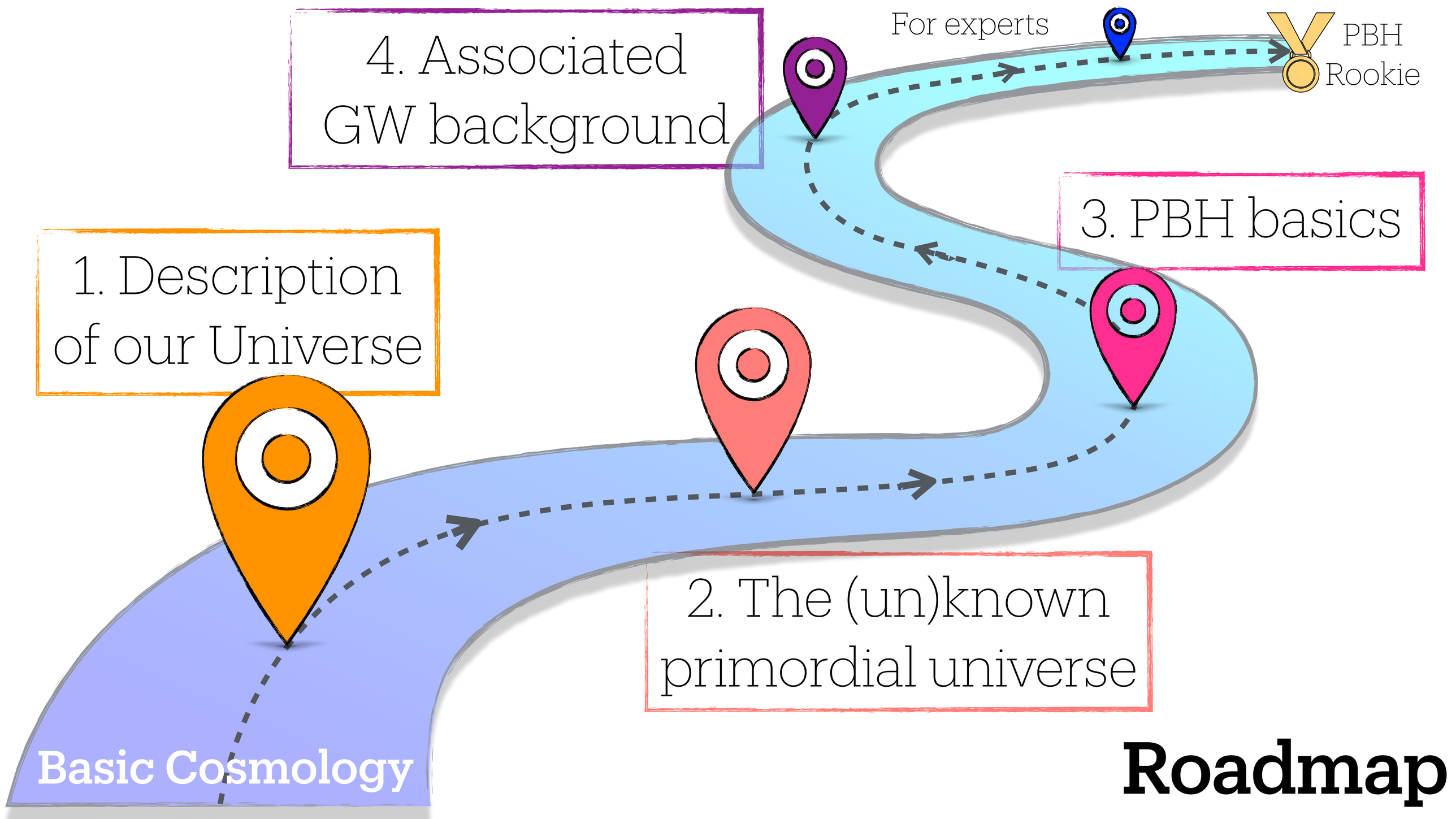
DOI: 10.3390/universe7110398

427 citations

Lectures on Gravitational Wave Signatures of Primordial Black Holes

Guillem Domènech[1‡]




4. Associated GW background
For experts
PBH Rookie
3. PBH basics
1. Description of our Universe
2. The (un)known primordial universe
Basic Cosmology

Roadmap

4. Associated GW background

For experts

PBH Rookie

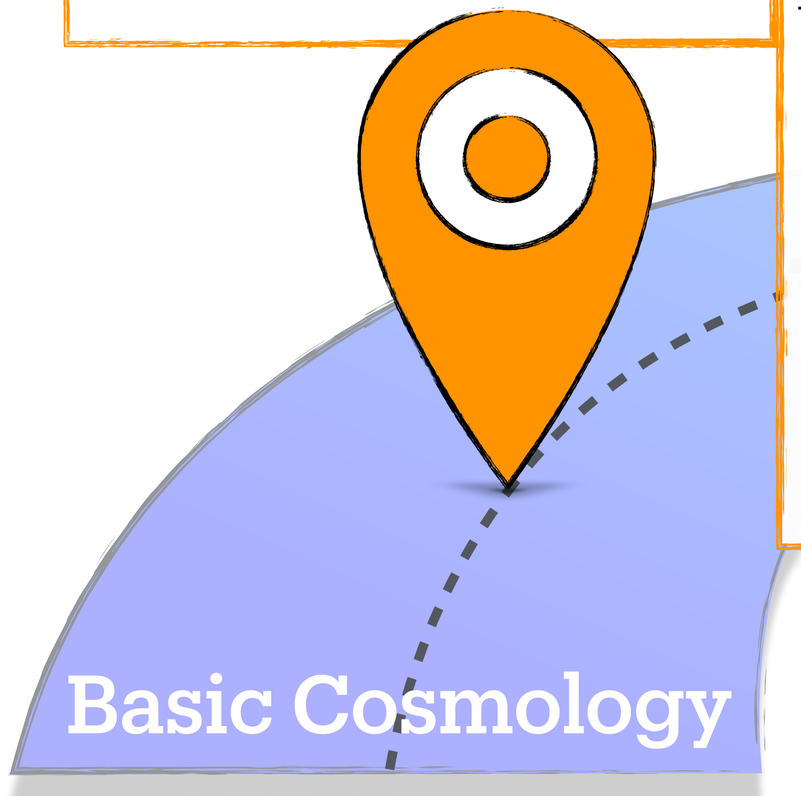
3. PBH basics

2. The (un)known primordial universe

Goal: grasp core assumptions & physics of cosmology

A. Composition (PBHs as DM?)

1. Description of our Universe

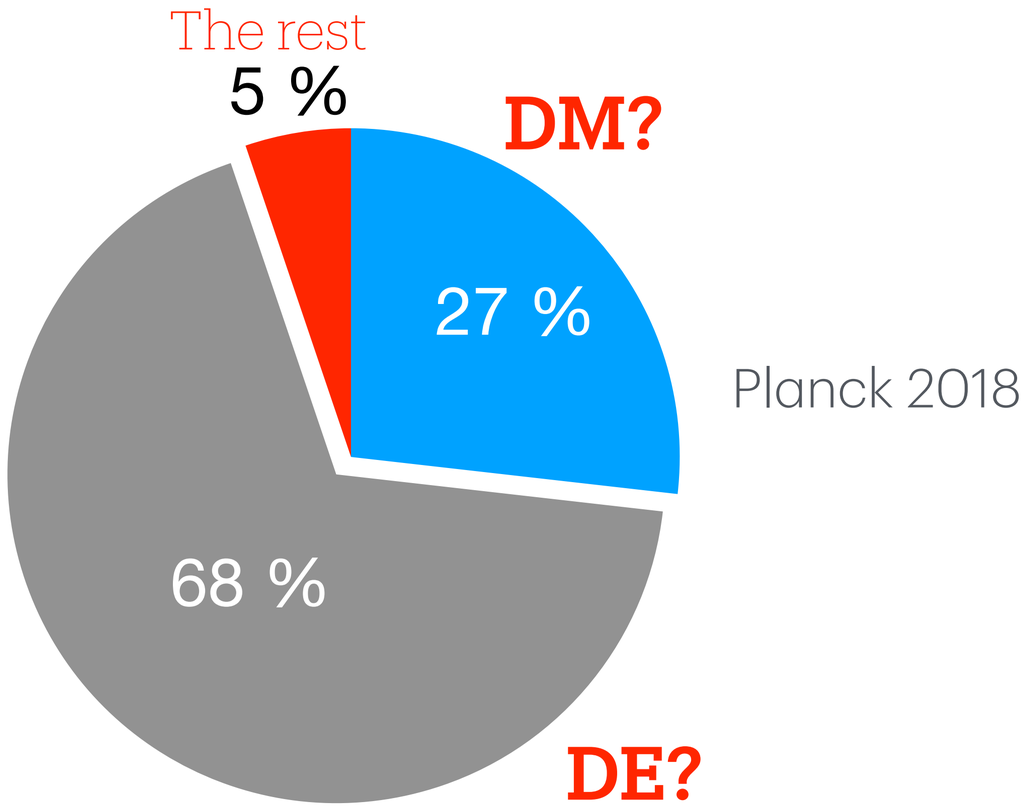


Basic Cosmology

Roadmap

Composition today

- We live in a **FLAT FLRW universe** ($R^{(3)} = 0$) with (at least)
  "Baryons" + Photons + Neutrinos + **DARK MATTER** + **DARK ENERGY**



Planck 2018

Question for you:

Suppose all Dark Matter is BHs.

How many now in this room?

Raise your hand if **NON-ZERO**.

- **HINT:** DM "Window" = Asteroid mass PBHs

$$M_{\mathrm{PBH}} \sim 10^{17-21}\,\mathrm{g}$$

Schwarzschild radius:

$$r_s \approx \mathcal{O}(\mathrm{pm} - \mathrm{nm})$$

Q1

PBHs as DM

- **BUT: How can DM be PBHs?**

DM "Window" = Asteroid mass PBHs

$$M_{\rm PBH} \sim 10^{17-21}\,{\rm g}$$

Schwarzschild radius:

$$r_s \approx \mathcal{O}({\rm pm} - {\rm nm})$$

Number of PBHs within Neptun orbit:

$$N_{\rm PBH,30AU} \approx \frac{\rho_{\rm DM,local}}{M_{\rm PBH}} \times V_{\rm 30AU} \sim 10^2 \times \left(\frac{M_{\rm PBH}}{10^{18}{\rm g}}\right)^{-1}$$

Earth cross-section:

$$\Gamma_{\rm PBH} \approx n_{\rm PBH,local} \times \pi \times (6400\,{\rm km})^2 \times v_{DM} \sim \frac{\rm impact}{\rm Gyr} \times \left(\frac{M_{\rm PBH}}{10^{18}{\rm g}}\right)^{-1}$$

Note: there are millions of asteroids in solar system

Sorry: no PBHs in this room today

SKY & TELESCOPE
THE ESSENTIAL GUIDE TO ASTRONOMY

Primordial Black Holes

ere you can find
ame in page 38)

ARE THEY OUT THERE?

Page 34

Death by a Primordial Black Hole

Avi Loeb · Follow
4 min read · Mar 7, 2025

ADVANCED SCIENCE NEWS

Hollow planets could help find primordial black holes

by Andrey Feldman | Dec 13, 2024

PBHs in the news

Are planet-killing black holes hiding inside your cat?

News By Robert Lea published December 3, 2024

SPACE.com

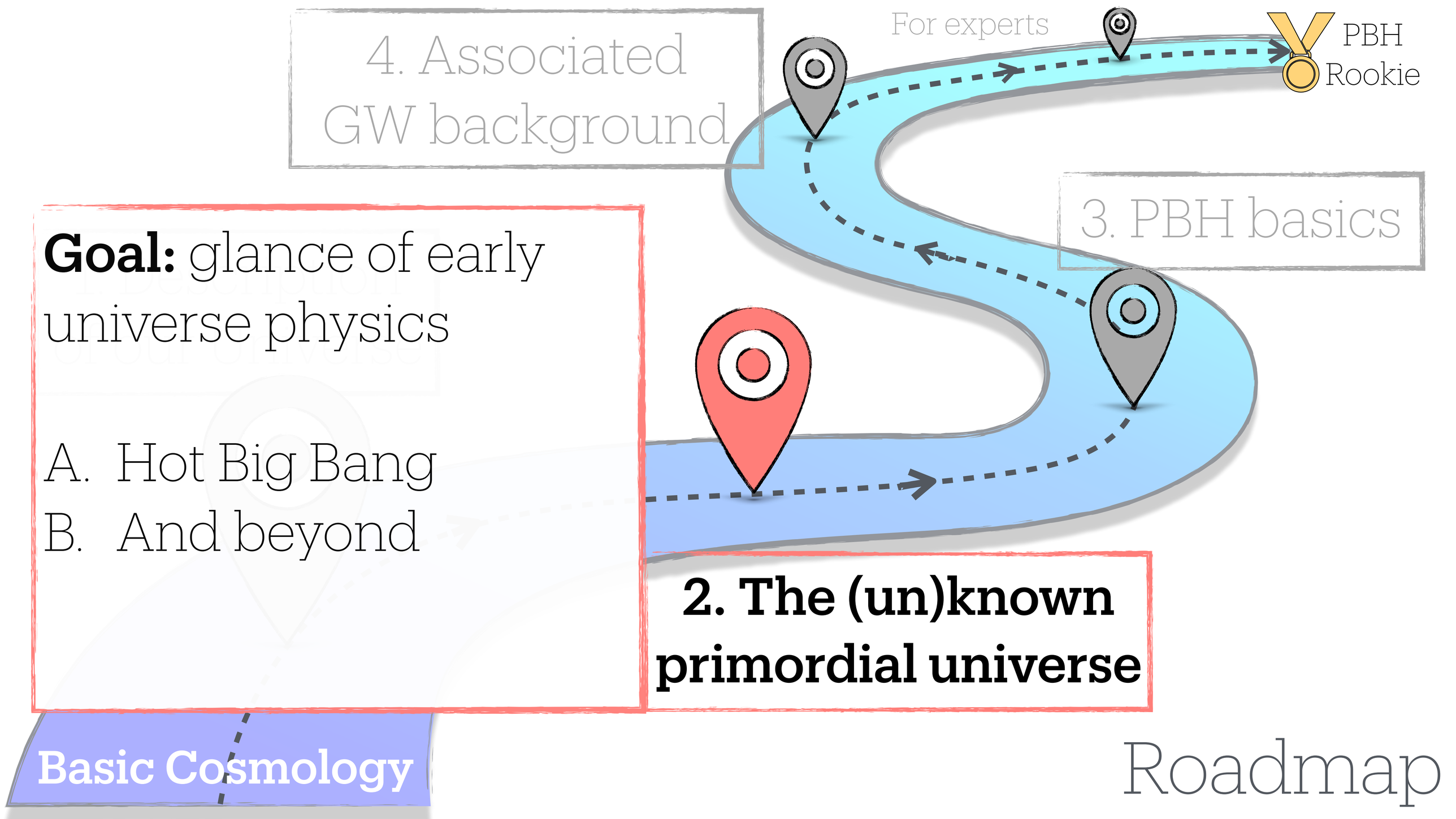
COSMOLOGY

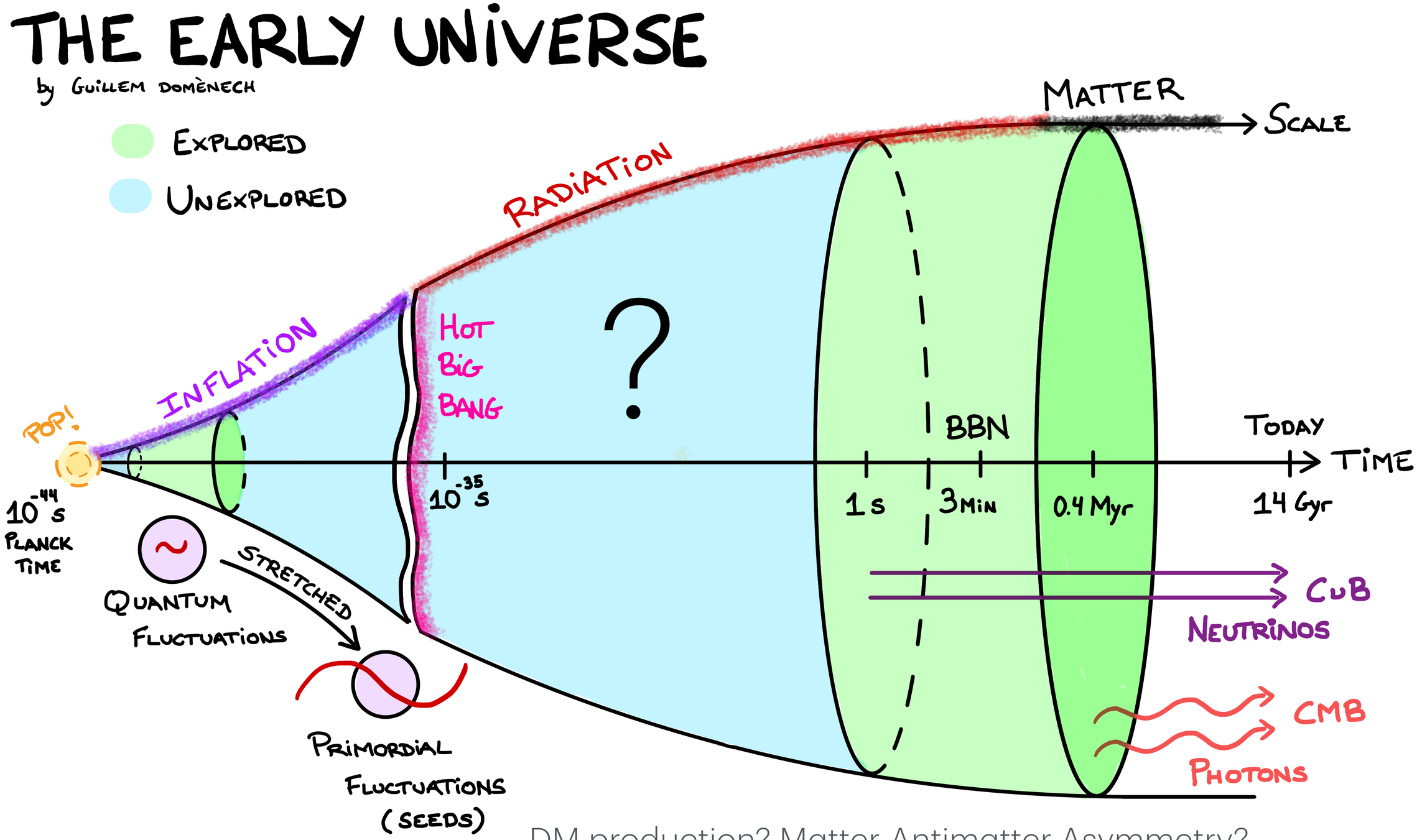
How we might finally find black holes from the cosmic dawn

ScienceNews

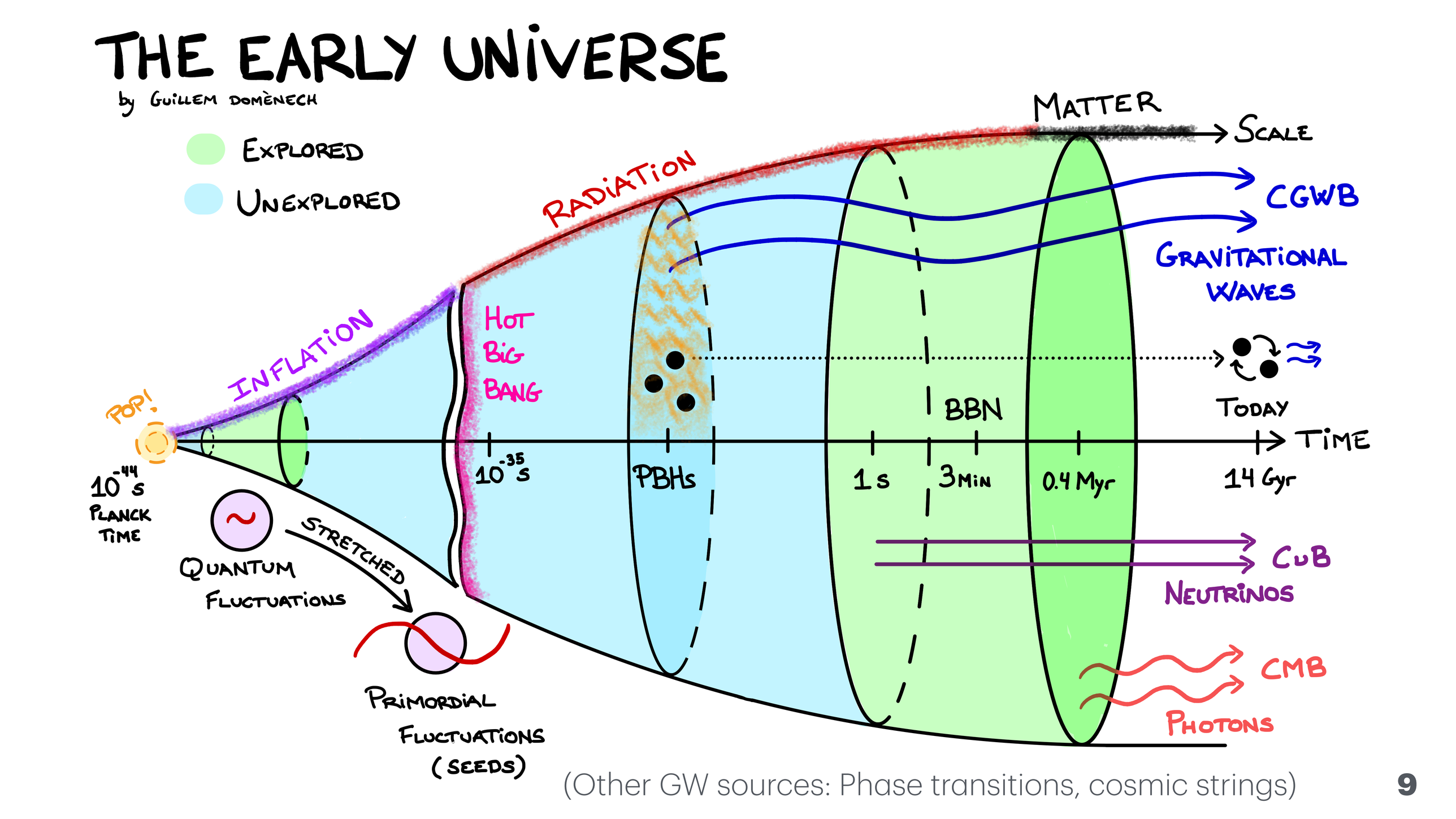
By Elizabeth Quill
JANUARY 13, 2025 AT 1:00 PM

Roadmap


For experts
PBH
Rookie
4. Associated GW background
3. PBH basics
Goal: glance of early universe physics
A. Hot Big Bang
B. And beyond
2. The (un)known primordial universe
Basic Cosmology

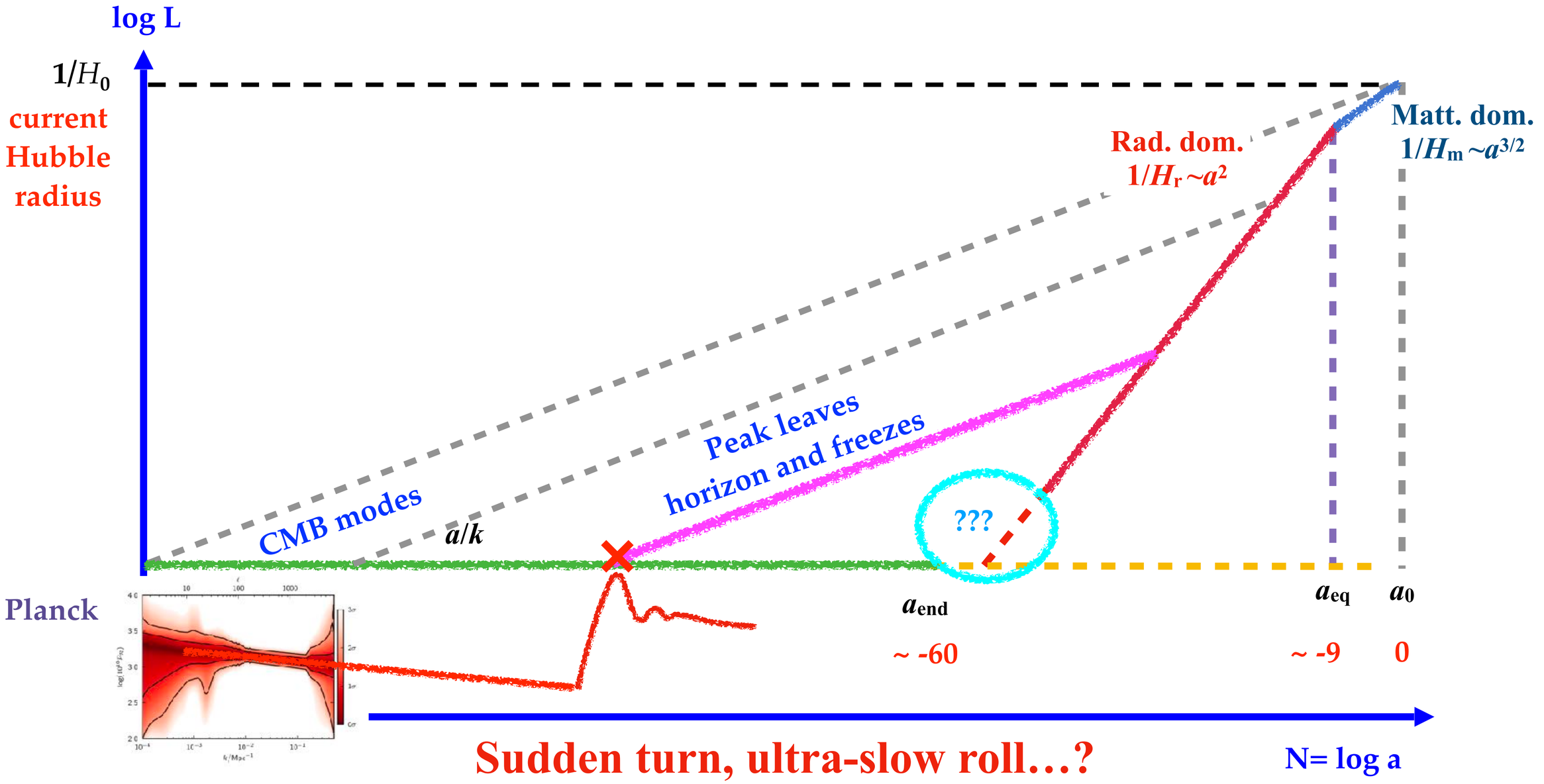


DM production? Matter-Antimatter Asymmetry?

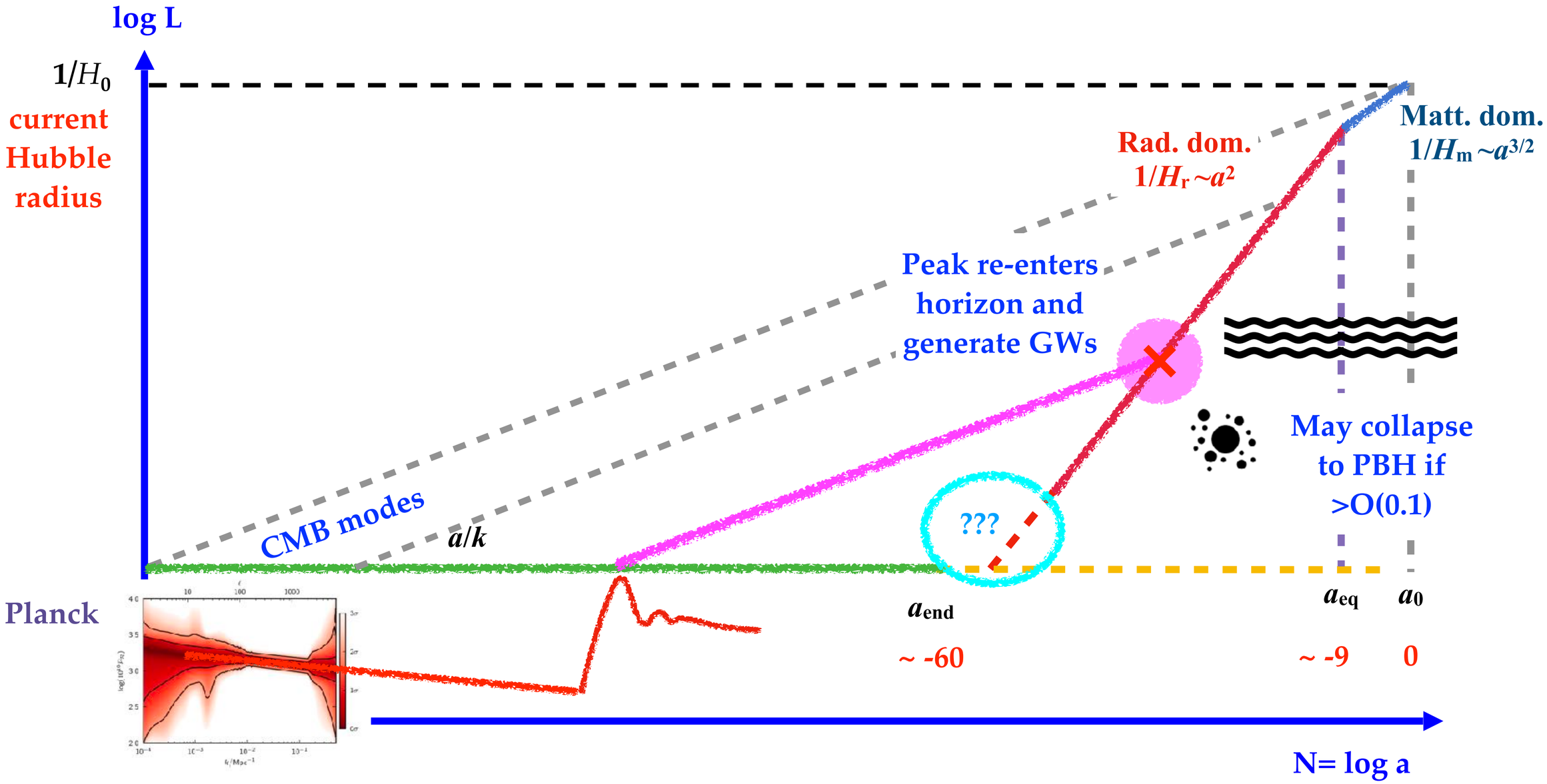


(Other GW sources: Phase transitions, cosmic strings)

Space-time diagram



Space-time diagram



Why is it important?

New way to observe the universe and explore yet unexplored periods

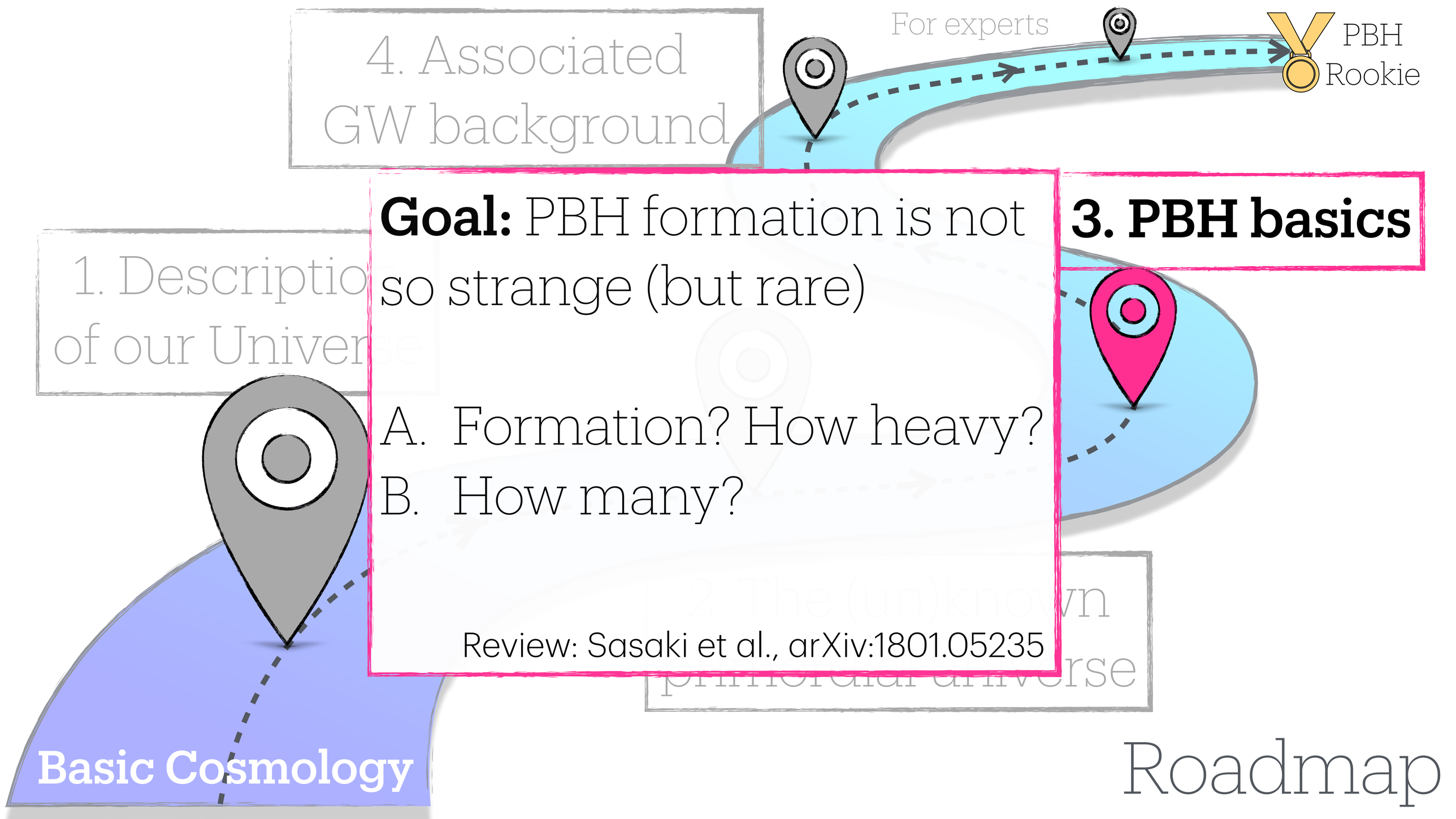
New tests of gravity and particle physics at extremely high energies

10^3 GeV — 10^{15} GeV

Exciting hints of Gravitational Wave Background from Pulsar Timing Arrays

GW frequency	Energy scale
MHz-GHz	10^{14} - 10^{17} GeV
Interesting future in High-Frequency GWs	
kHz	10^{11} GeV
LIGO/VIRGO/ET	~Now and 2035
Hz	10^8 GeV
DECIGO	
AION AEDGE	Uncertain
mHz	10^5 GeV
LISA	~2035
Astrometry	~ 2027
μHz	10^2 GeV
PTA	LIGO/VIRGO PBHs?
ongoing	QCD phase transition?
nHz	0.1 GeV

+ multi-messenger cosmology

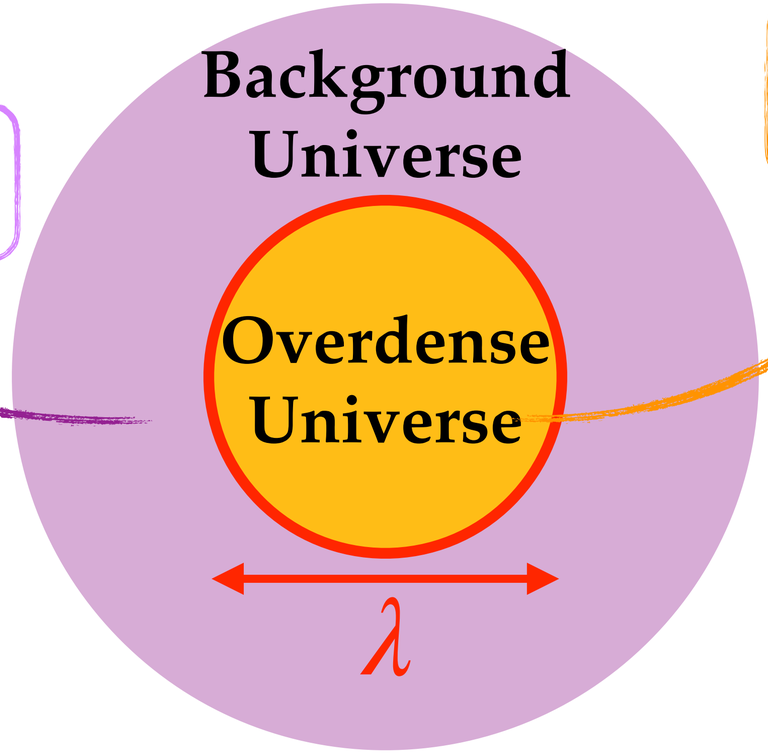

4. Associated GW background
For experts
PBH Rookie
1. Description of our Universe
Goal: PBH formation is not so strange (but rare)
A. Formation? How heavy?
B. How many?
Review: Sasaki et al., arXiv:1801.05235
3. PBH basics
2. The (un)known primordial universe
Basic Cosmology
Roadmap

How could PBH form?

Before collapse: $R_H \ll \lambda$

Friedmann Eq.

$$3H^2 = 8\pi G\rho$$



$$3H^2 + \frac{1}{2}R^{(3)} = 8\pi G(\rho + \delta\rho)$$

Constant H time-slice

$$\frac{1}{2}R^{(3)} = 8\pi G\delta\rho > 0$$

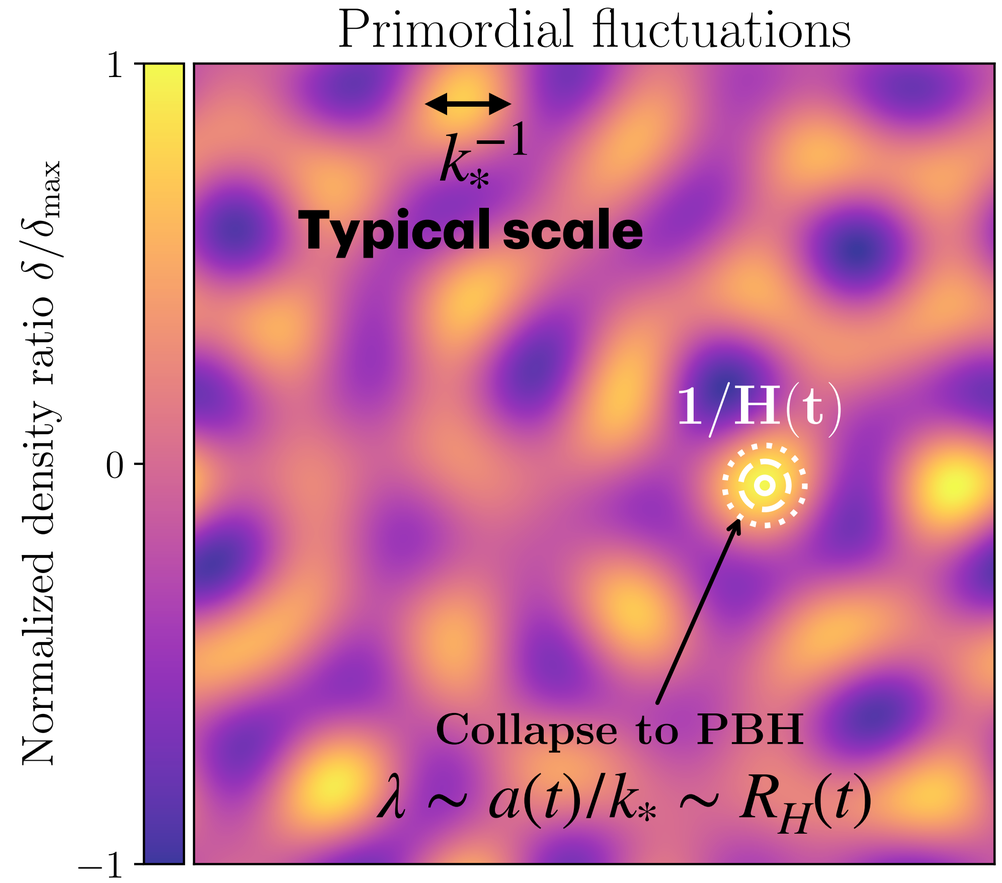
Possible re-collapse when $R_H \sim \lambda$

PBH mass ~ Mass inside Hubble Volume: $M_H(t) \sim \rho \times R_H^3 \sim 10^{38}\,\mathrm{g} \times \left(\frac{t}{1\mathrm{s}}\right)$

Curious coincidence: **Solar mass PBHs ~Time of QCD phase transition**

$M_{\mathrm{PBH}} \sim 2 \times 10^{30}\,\mathrm{kg}$ $\quad t \sim 10^{-5}\mathrm{s}$ $\quad T \sim 100\,\mathrm{MeV}$

How many?



$$\delta(x) \sim \int d^3k\, \delta_k\, e^{ikx} \;\; ; \;\; \langle \delta(x)\delta(x) \rangle \sim \int d\ln k\, \sigma^2$$

Initial conditions from inflation = Random Gaussian density fluctuations

$$P(\delta) \sim e^{-\frac{\delta^2}{2\sigma^2}}$$

σ = Root mean square,
typical size of fluctuations

$$\delta \equiv \frac{\delta\rho}{\rho}$$ **Density ratio**

How many?

Initial conditions from inflation = Random Gaussian density fluctuations

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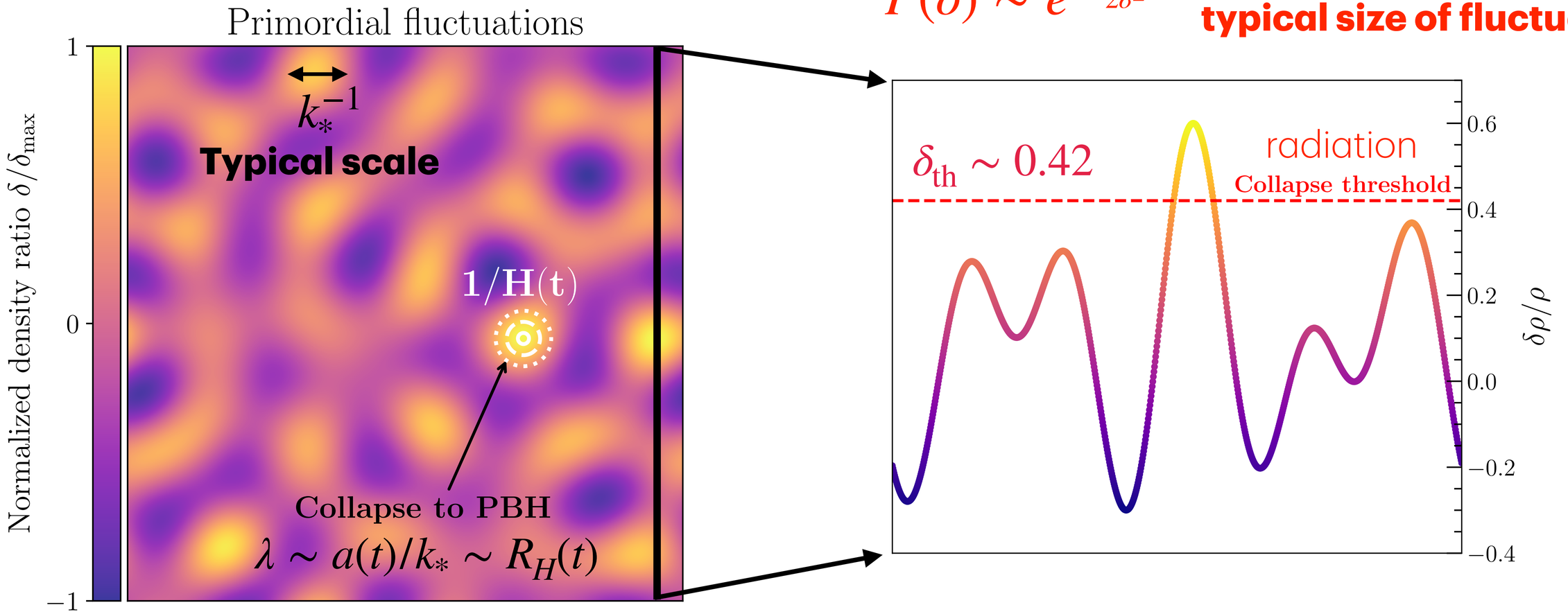
typical size of fluctuations



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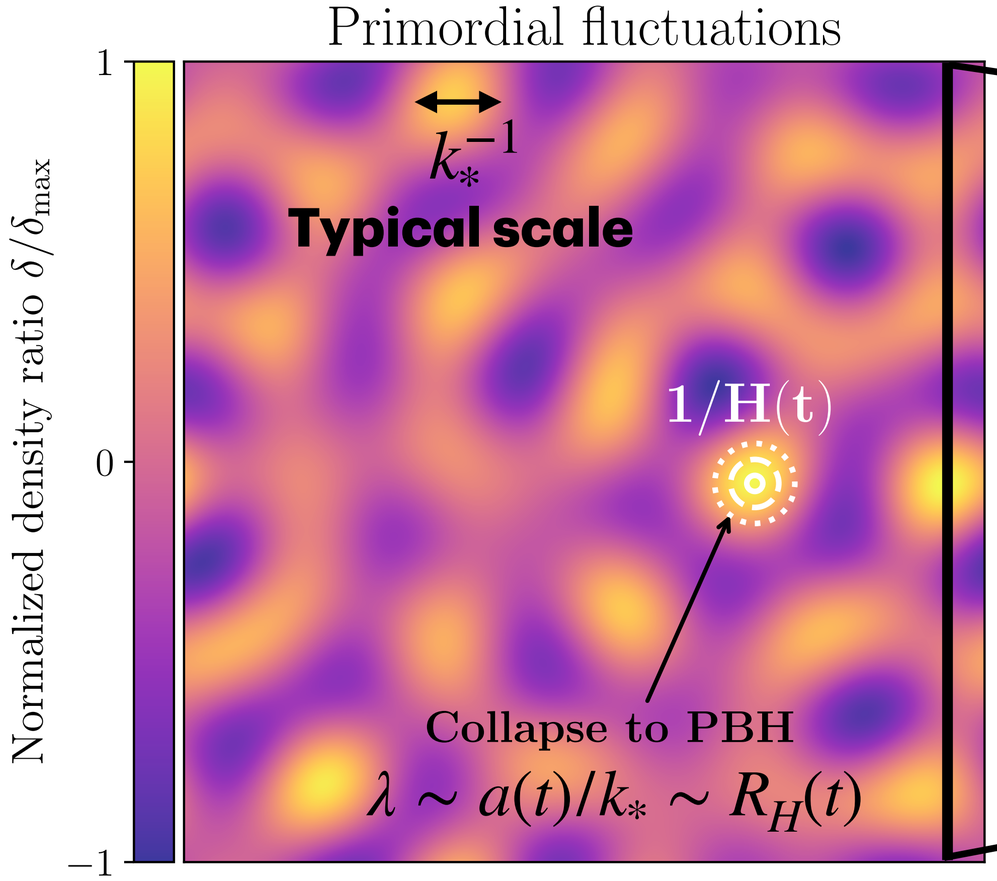
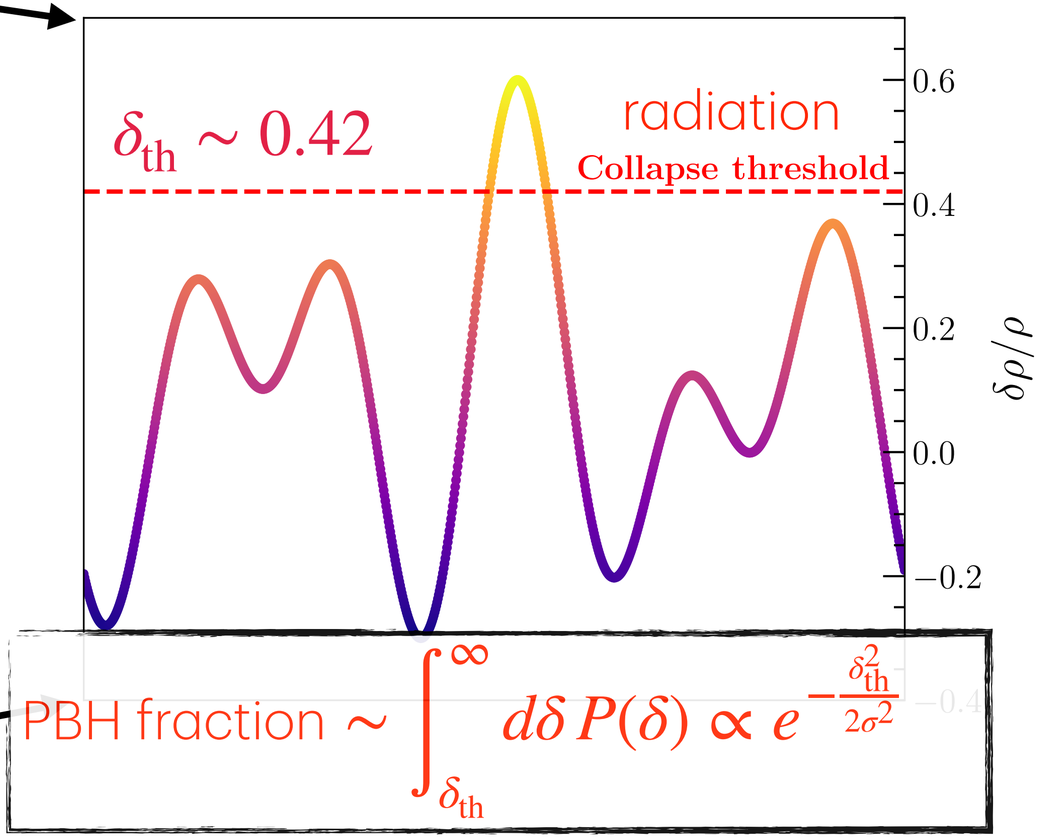
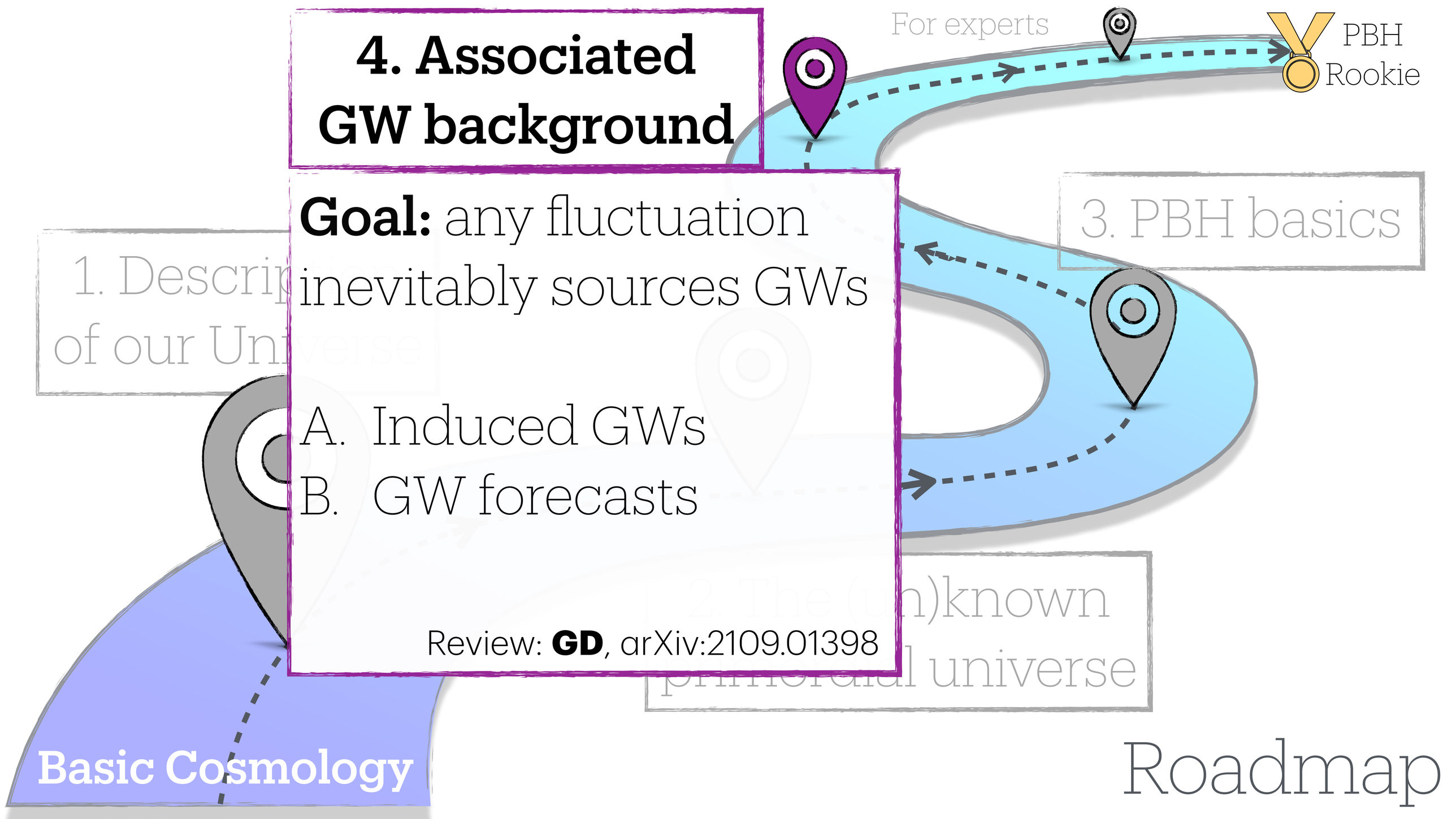




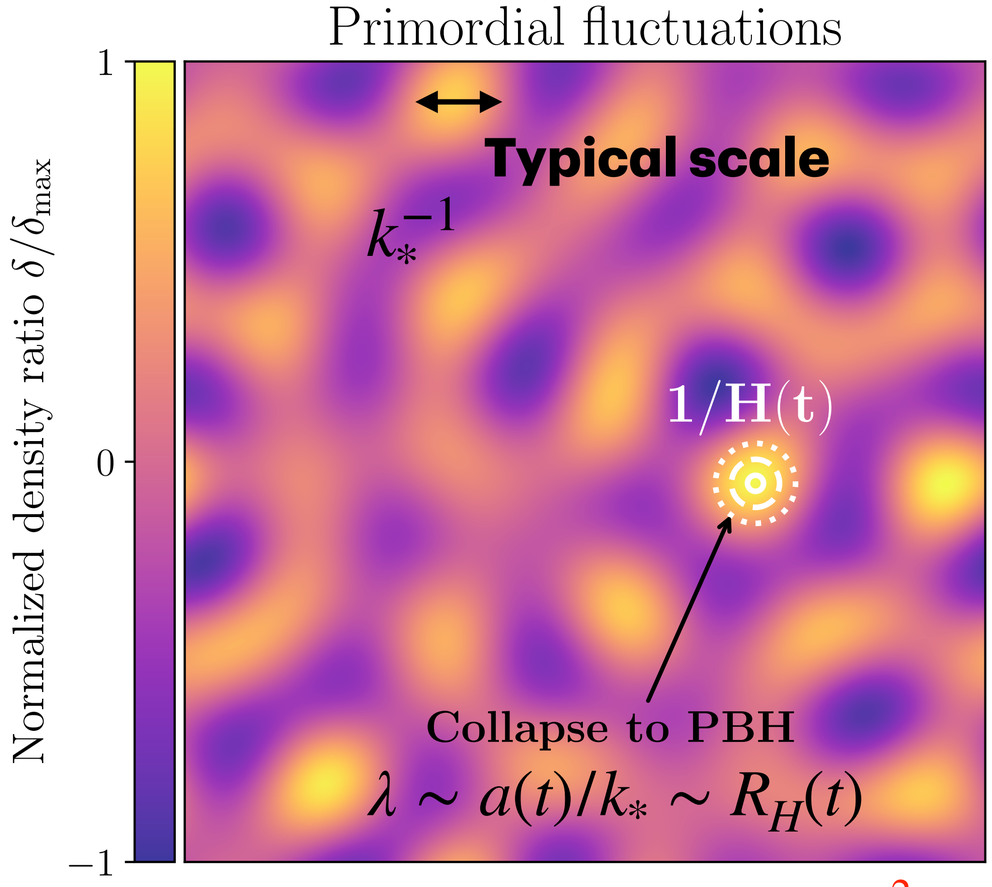
$$\text{PBH fraction} \sim \int_{\delta_{\rm th}}^{\infty} d\delta\, P(\delta) \propto e^{-\frac{\delta_{\rm th}^2}{2\sigma^2}}$$

$$\delta(x) \sim \int d^3k\, \delta_k\, e^{ikx} \;;\; \langle \delta(x)\delta(x)\rangle \sim \int d\ln k\, \sigma^2$$

Benchmark for significant # of PBHs $\sigma \sim 10^{-1}$


For experts
PBH Rookie
4. Associated GW background
Goal: any fluctuation inevitably sources GWs
A. Induced GWs
B. GW forecasts
Review: GD, arXiv:2109.01398
3. PBH basics
1. Descrip
of our Uni
2. The (un)known
primordial universe
Basic Cosmology
Roadmap

GWs induced by primordial fluctuations



PBH fraction $\sim e^{-\frac{\delta_{\rm th}^2}{2\sigma^2}}$

Rare fluctuations

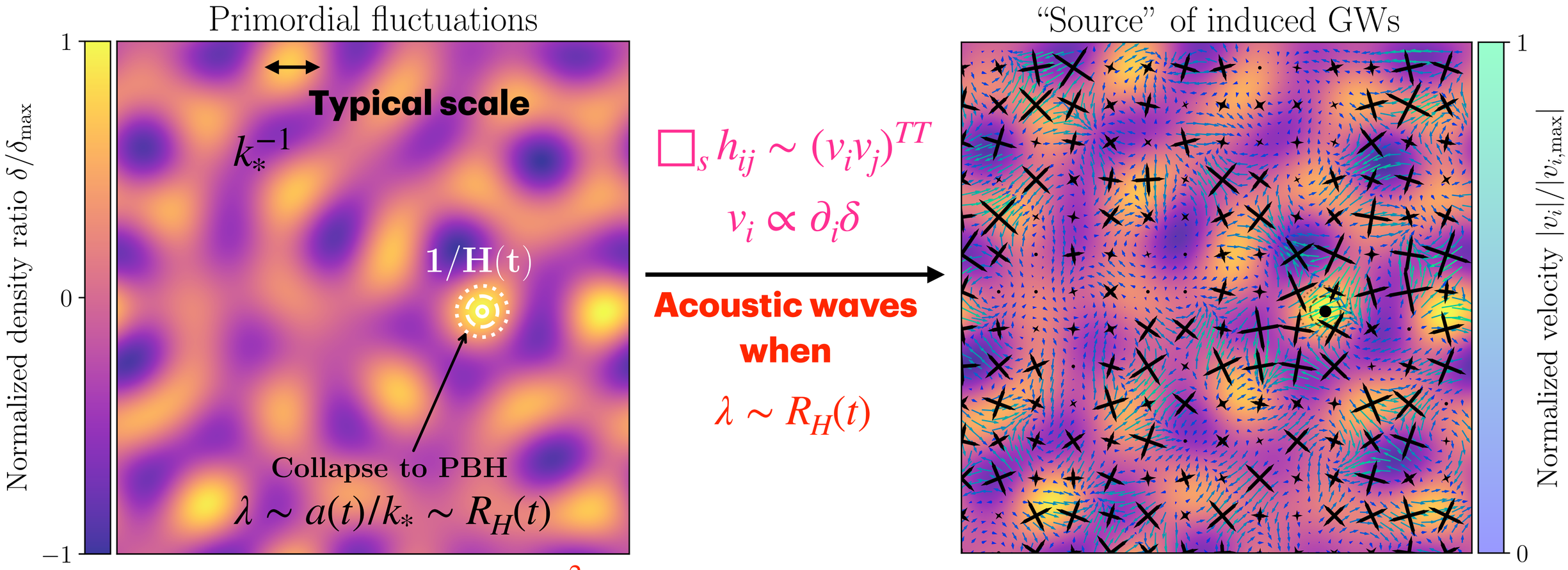
Anisotropies source GWs: $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$

$$\Box_s\, h_{ij} \approx T_{ij}^{TT} \sim 8\pi G\rho (v_i v_j)^{TT}$$

Velocity flows —> $v_i \propto \partial_i \delta$

$$\Box_s \equiv \frac{1}{a^3}\frac{d}{dt}\left(a^3 \frac{d}{dt}\right) - \frac{1}{a^2}\partial_k \partial^k$$

GWs induced by primordial fluctuations



PBH fraction $\sim e^{-\frac{\delta_{\rm th}^2}{2\sigma^2}}$

Rare fluctuations

VS

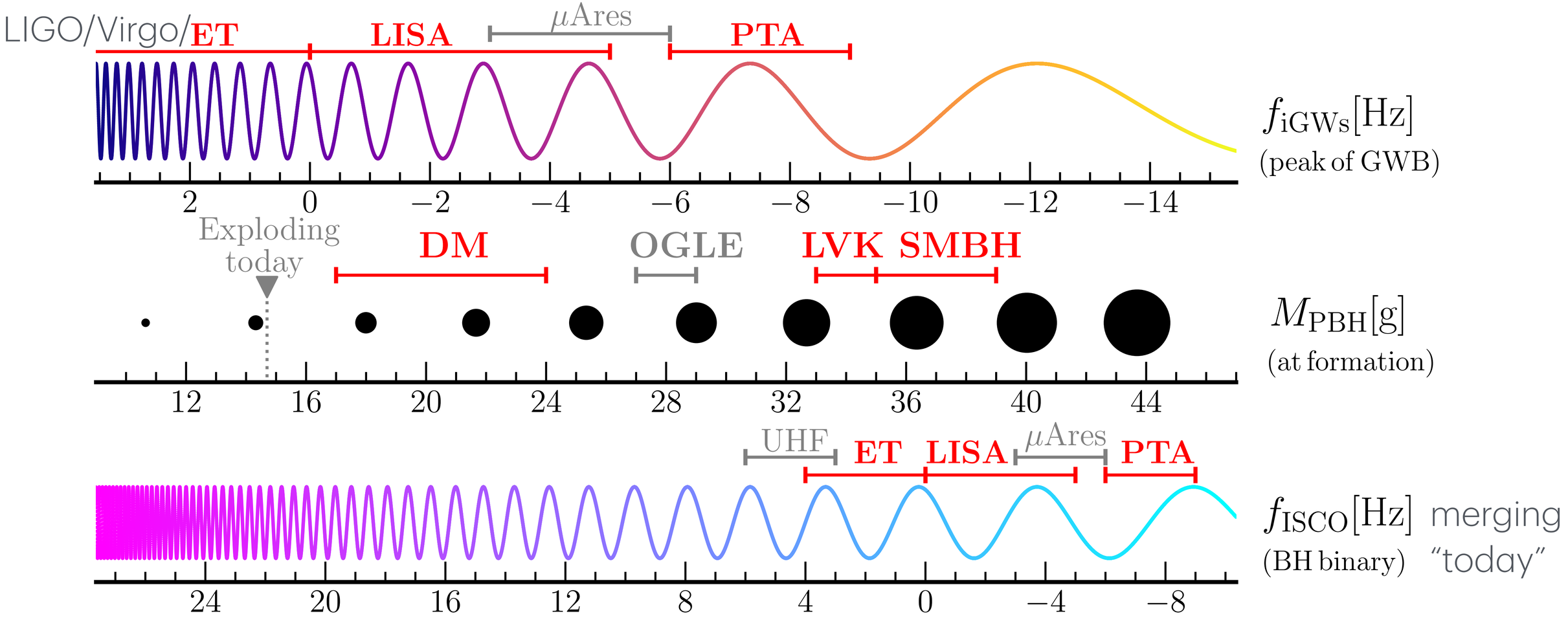
Strain variance: $\sigma_h \sim \sigma^2$

Typical fluctuations

PBH Mass <—> GW Frequency

$$\lambda \sim a(t)/k_* \sim a(t)/f_* \sim R_H(t) \sim 2GM_{\rm PBH}$$

GWs and PBHs from primordial curvature fluctuations – $\log_{10}$ scale



GW Forecasts

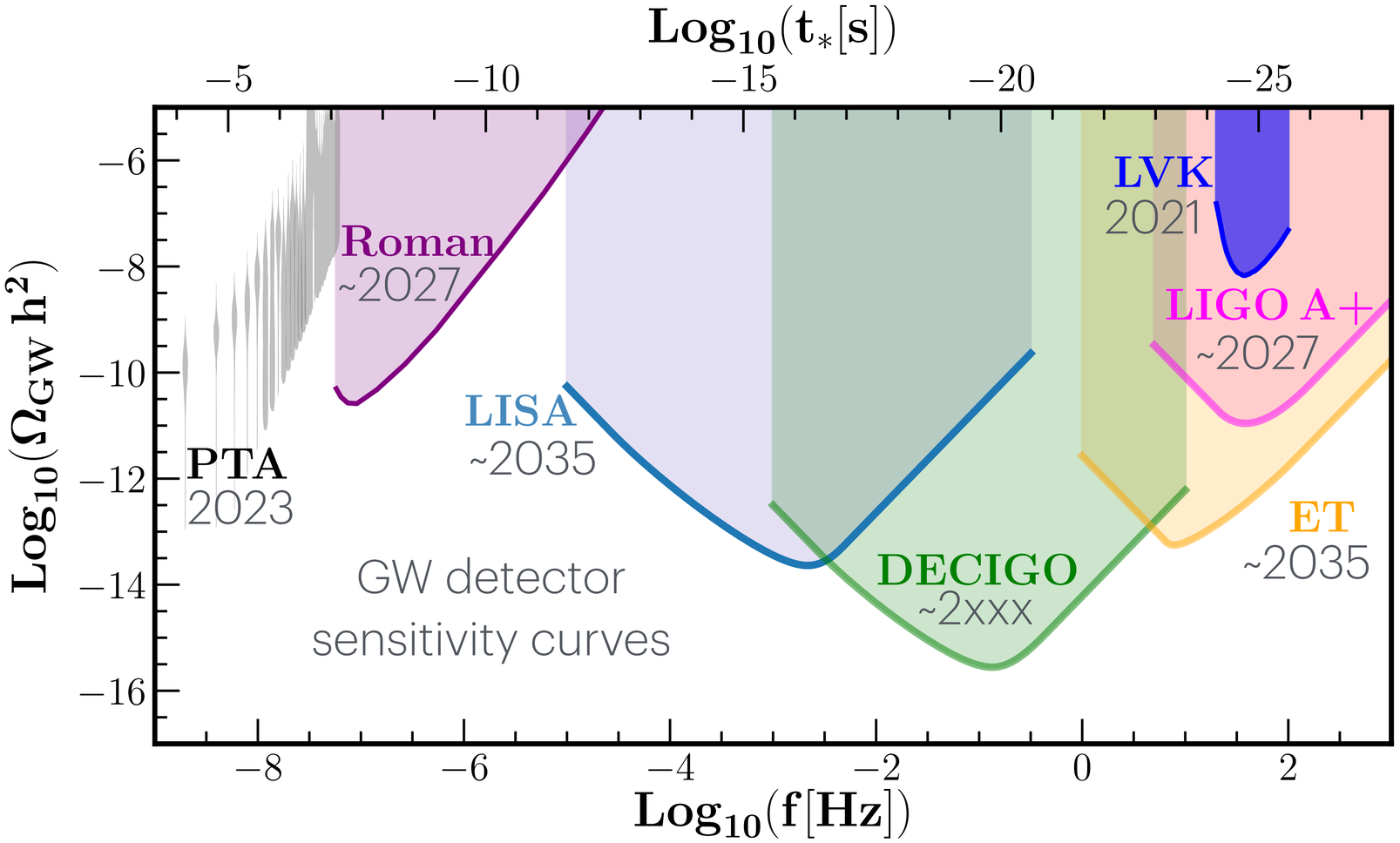
Peak GW amplitude:

$$\Omega_{\mathrm{GW},0}^{\mathrm{induced}} \sim \frac{1}{12}\Omega_{r,0}\sigma_h^2 \sim 10^{-5}\sigma^4$$

GW energy fraction per log f

Redshift

Primordial spectrum



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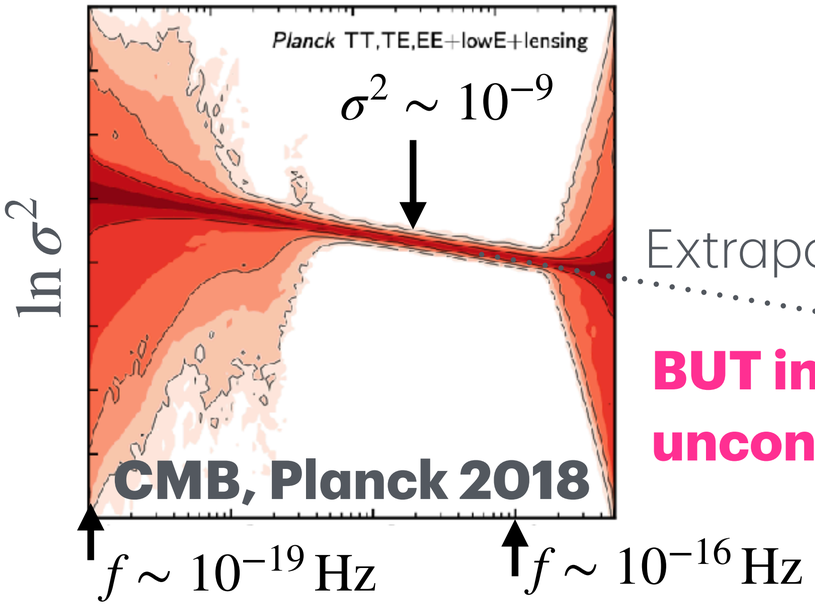
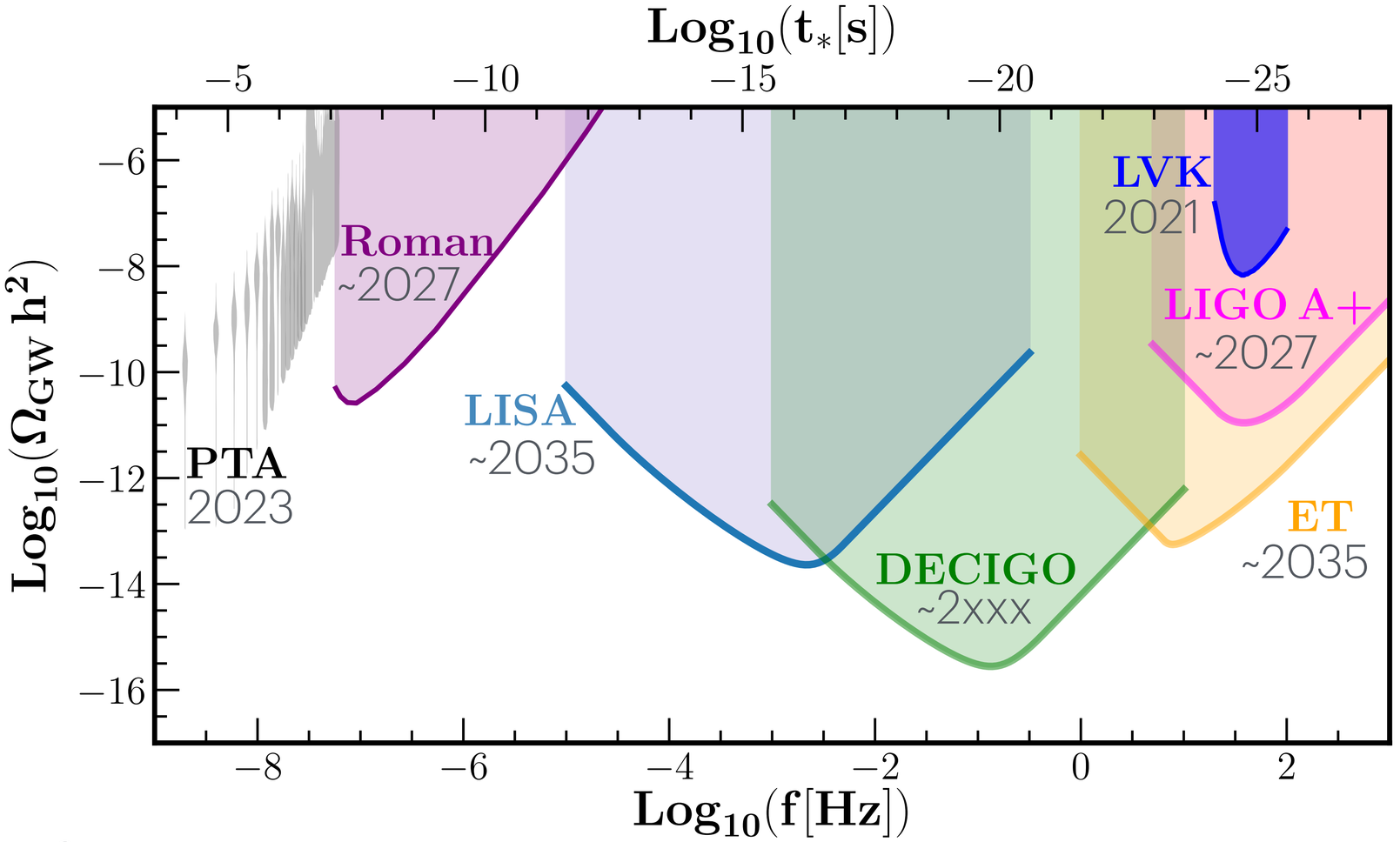


Extrapolation?

BUT in reality, unconstrained

$\Omega_{\mathrm{GW},0} \sim 10^{-23}$



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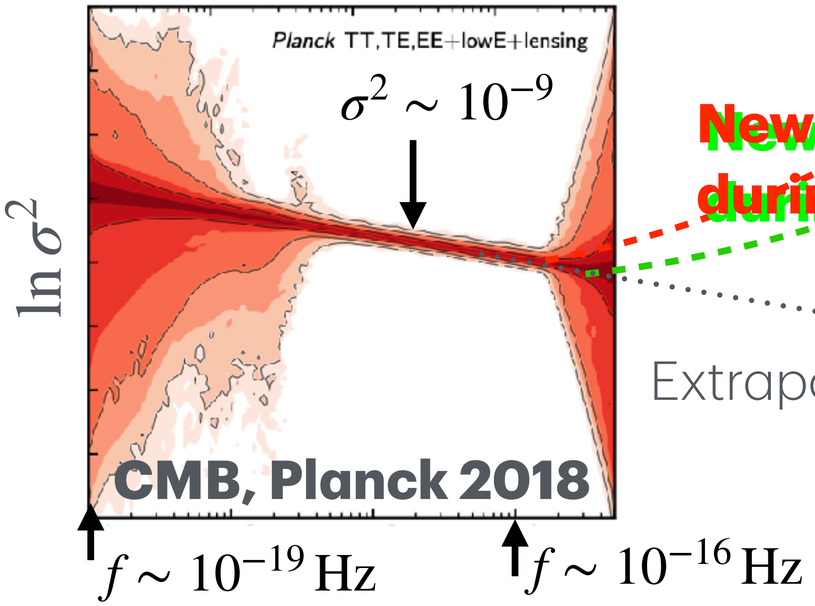
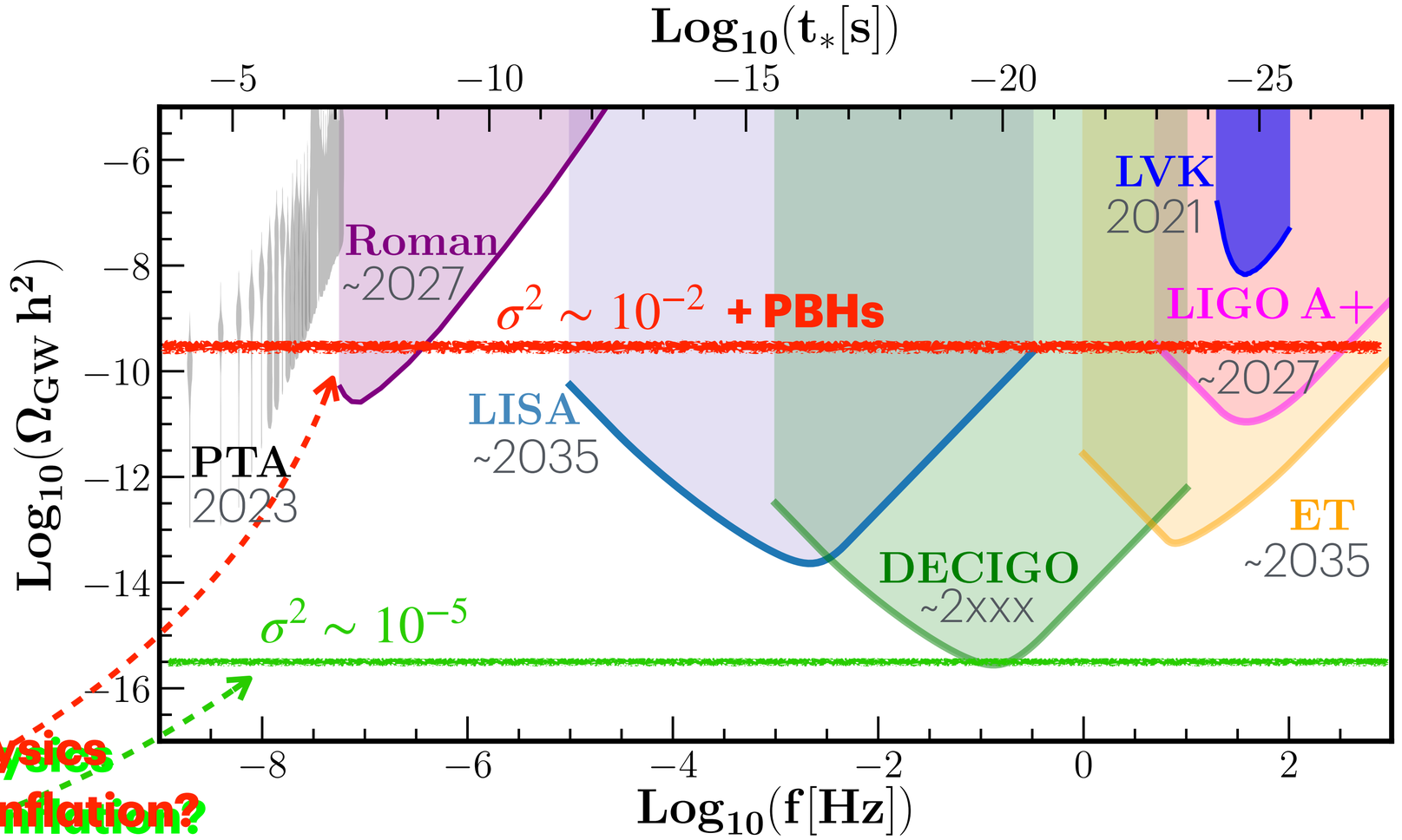


New physics during inflation?

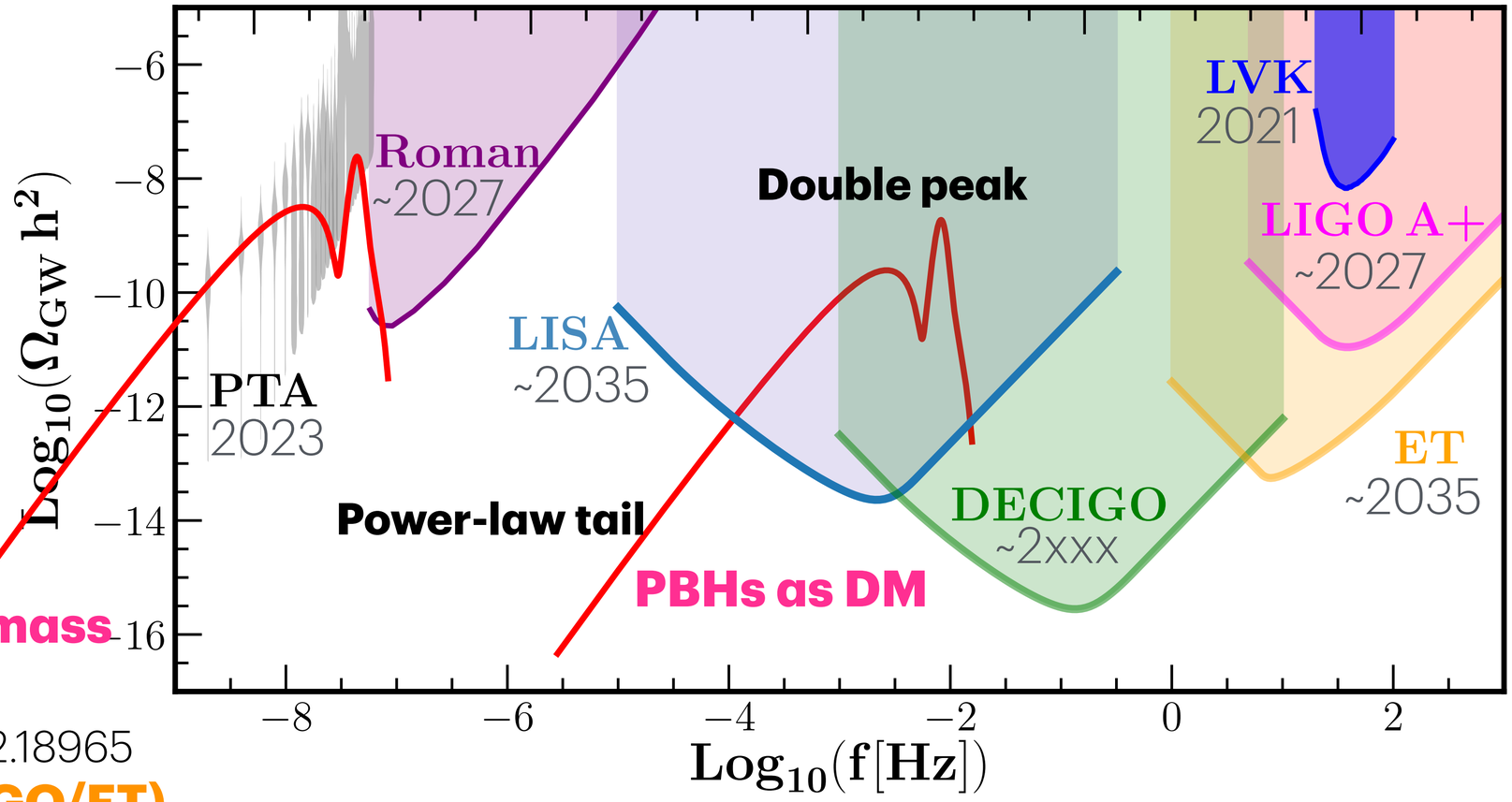
Extrapolation?

$\Omega_{\mathrm{GW},0} \sim 10^{-23}$



GW background associated with PBHs

GW spectrum can tell sources apart!

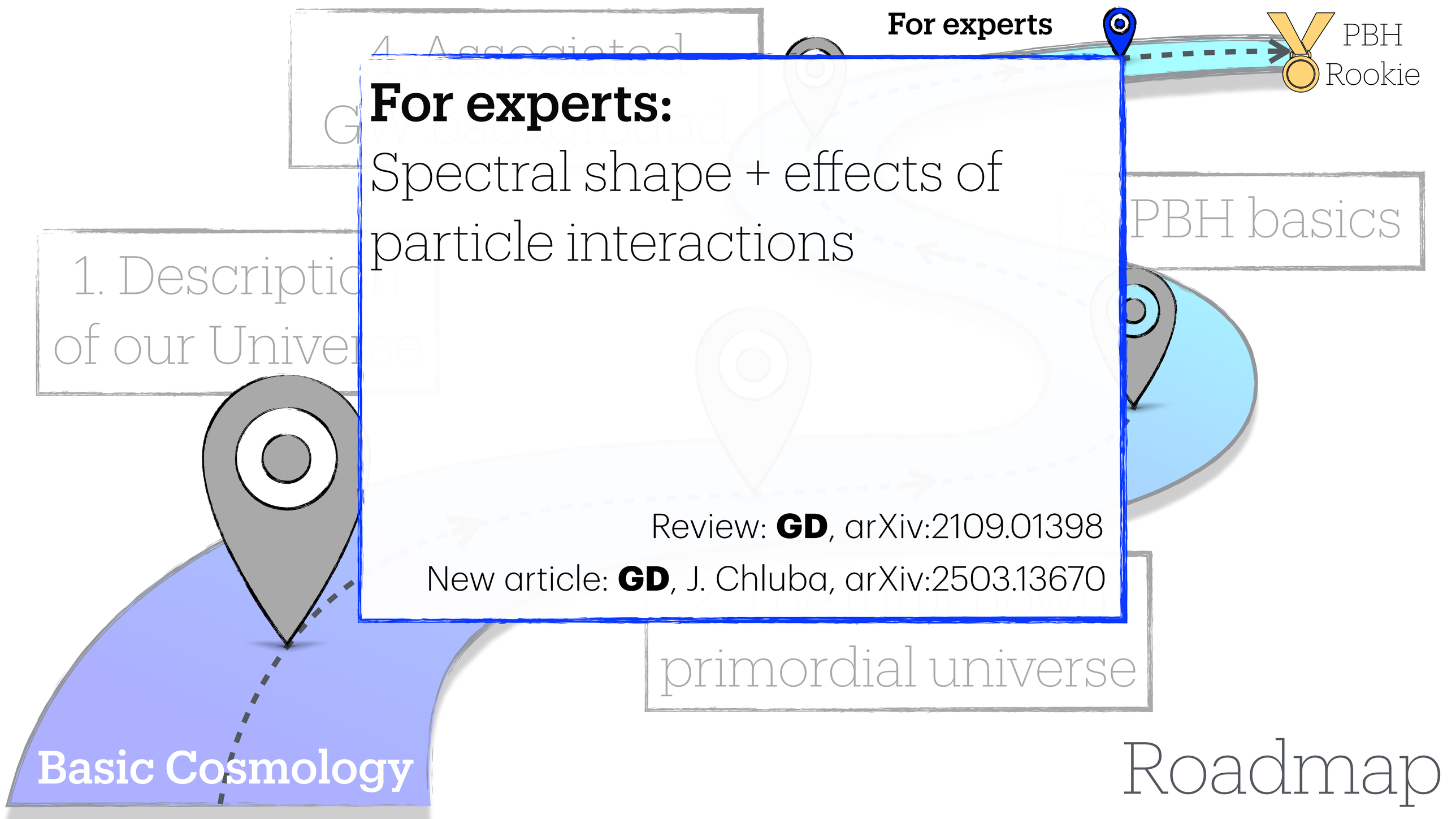


(Sub)-Solar mass PBHs?

GD+, arXiv:2402.18965

(Mergers at LIGO/ET)

If PBHs in universe, then there must be detectable GW background!



For experts:

Spectral shape + effects of particle interactions

Review: **GD**, arXiv:2109.01398

New article: **GD**, J. Chluba, arXiv:2503.13670

Spectral shape — Universal features

Sketch of spectrum calculations:

Kernel or Transfer function (cosmology dependent)

New solutions

GD:1912.05583

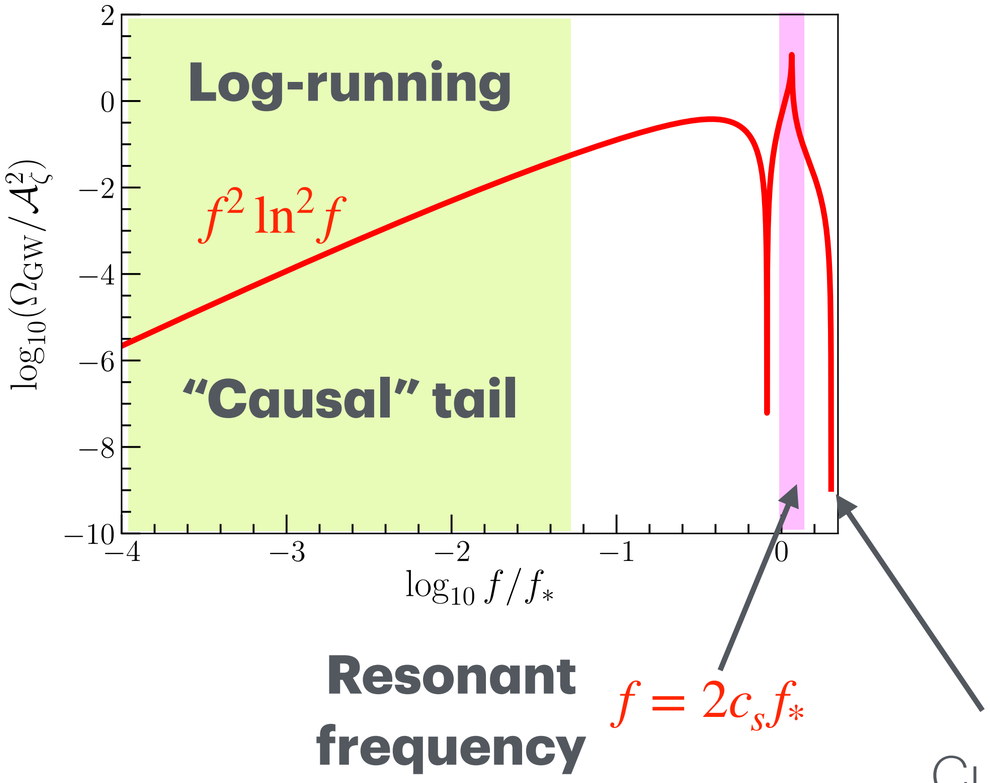
$$\sigma_h^2(k) = \int d^3q\, T(q, |k-q|) \times \sigma^2(q)\, \sigma^2(|k-q|)$$

primordial spectrum from inflation

Spectral shape — Universal features

Toy model: Inifinitely sharp (Dirac delta) primordial spectrum

$$\sigma^2(k) = \mathcal{A}_\zeta \delta(\ln(k/k_*))$$



Resonant frequency $f = 2c_s f_*$

Cut-off due to momentum conservation

Sketch of spectrum calculations:

Kernel or Transfer function (cosmology dependent)

New solutions

GD:1912.05583

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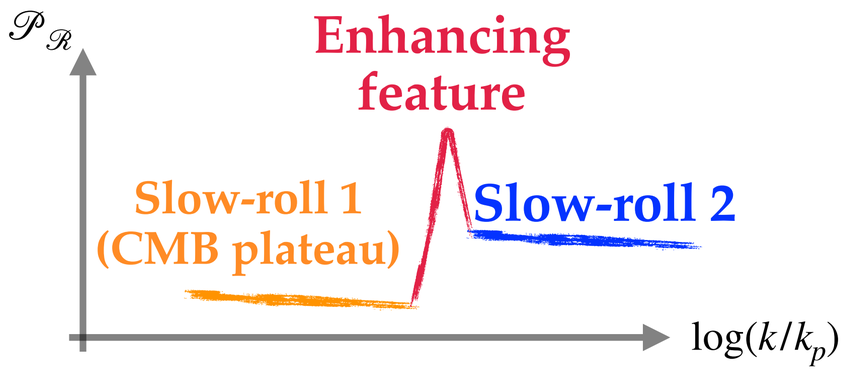
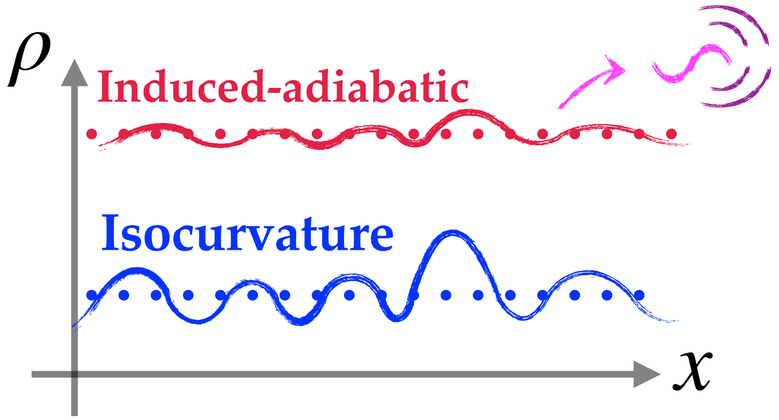
primordial spectrum from inflation

Cosmic inflation

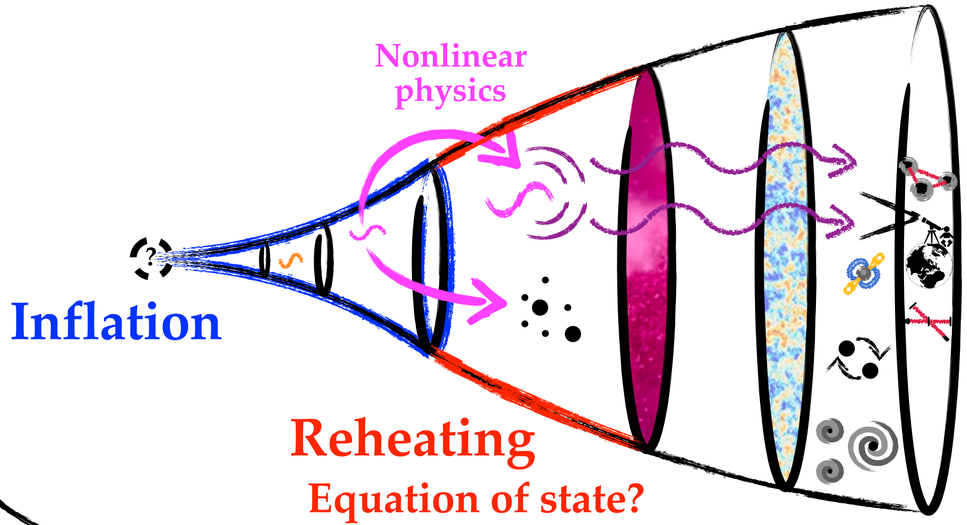


Initial conditions

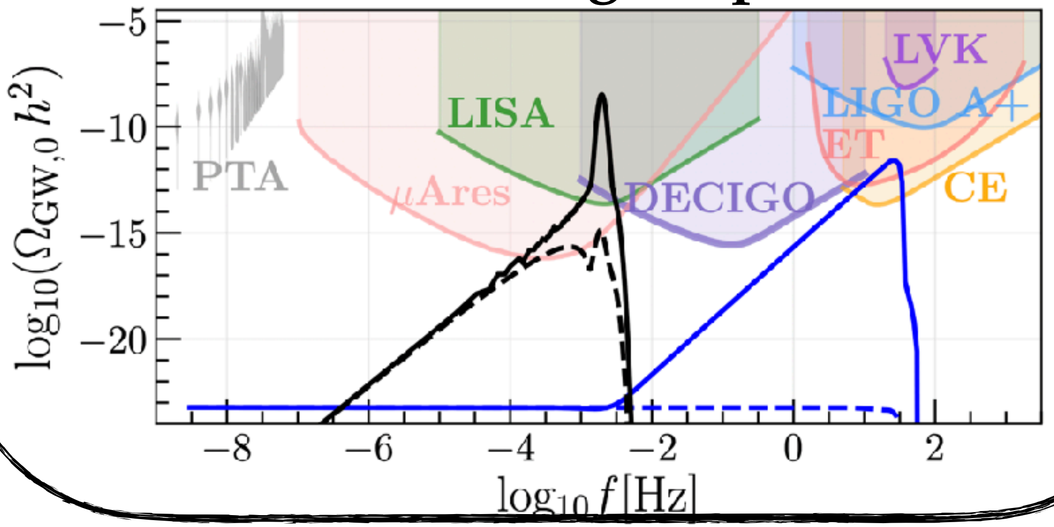


Induced GWs research

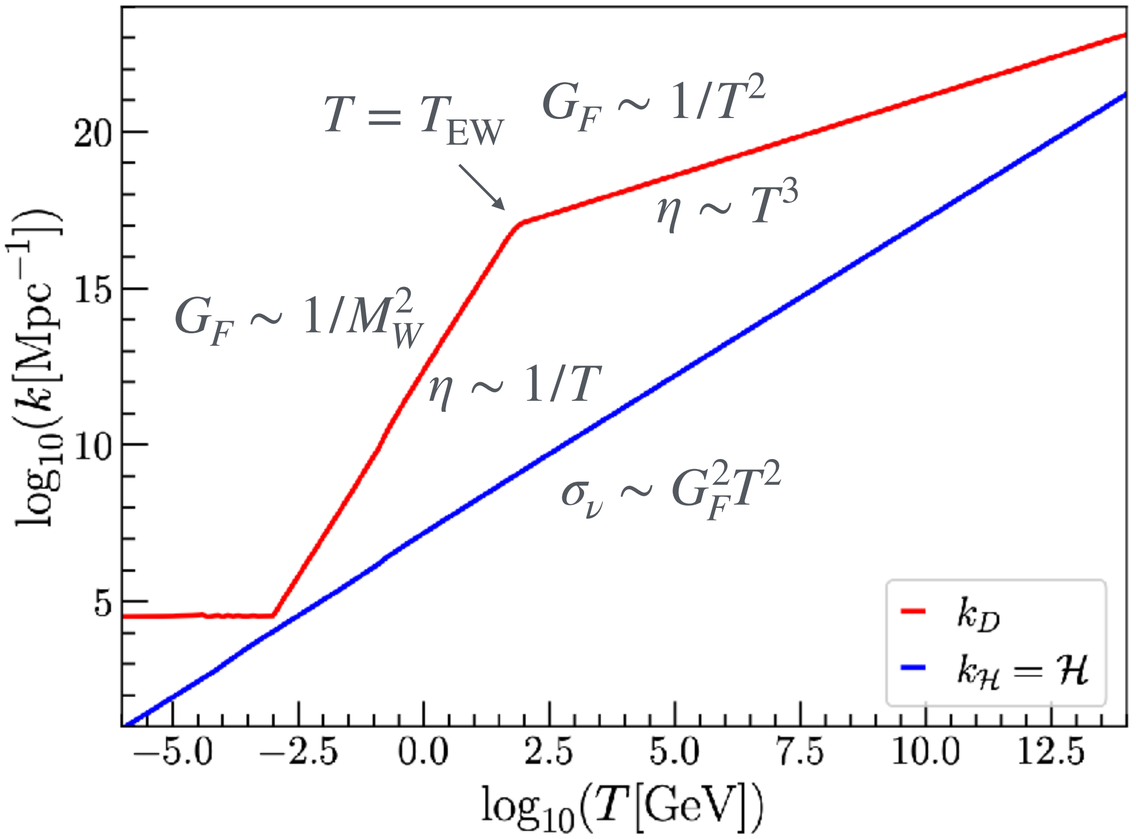
Content of the universe



Modified gravity/modified Hawking evap.



Dissipative effects - Damping of fluct.



$$\Phi(k,\tau) \approx \Phi_{\rm rad}(k,\tau) e^{-k^2/k_D^2(\tau)}$$

$$k_D^{-2}(\tau) = \int_0^\tau d\tilde{\tau}\, \Gamma_D(\tilde{\tau}) \approx \int_0^\tau d\tilde{\tau}\, \frac{2\eta(\tilde{\tau})}{3a(\rho+P)}$$

$$\eta(\tau) = \frac{16}{45}\rho_\gamma t_\gamma + \frac{4}{15}\sum_{j=X,\nu} \rho_j t_j \Theta(\tau_{j,\rm dec} - \tau)$$

$$t_\nu = (n_\nu \sigma_\nu)^{-1}$$

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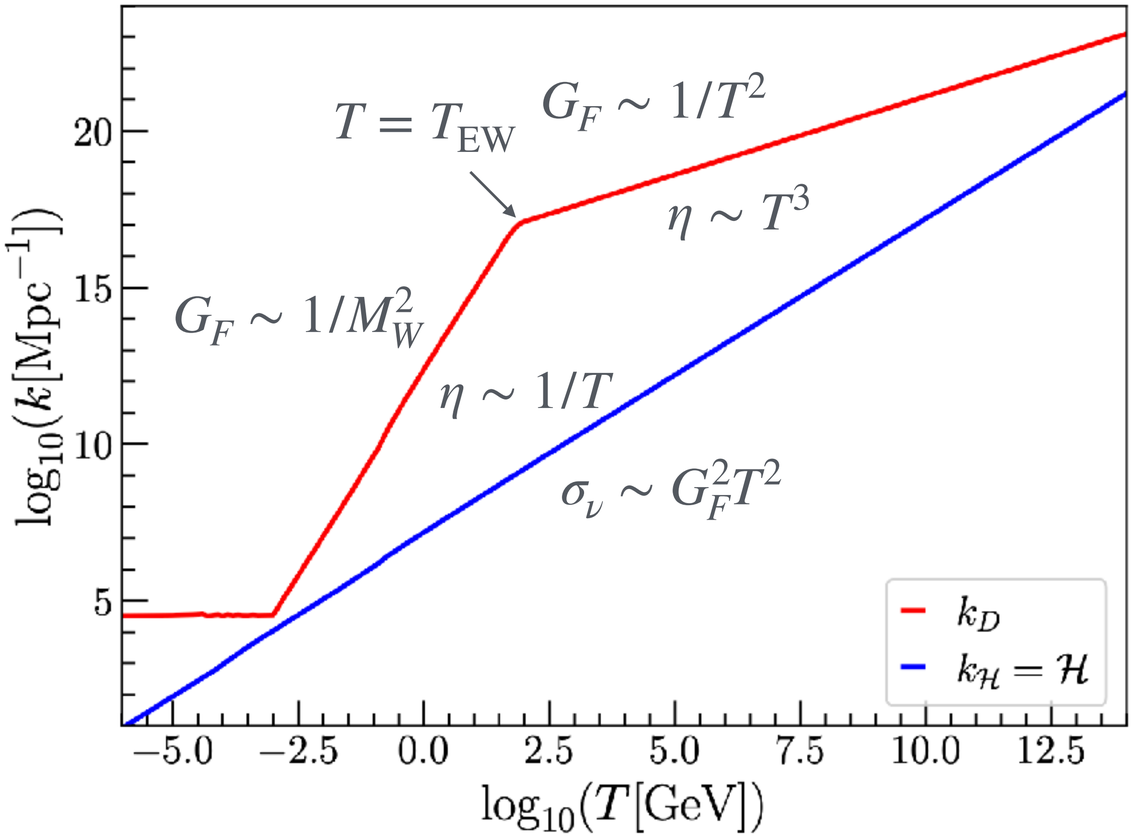

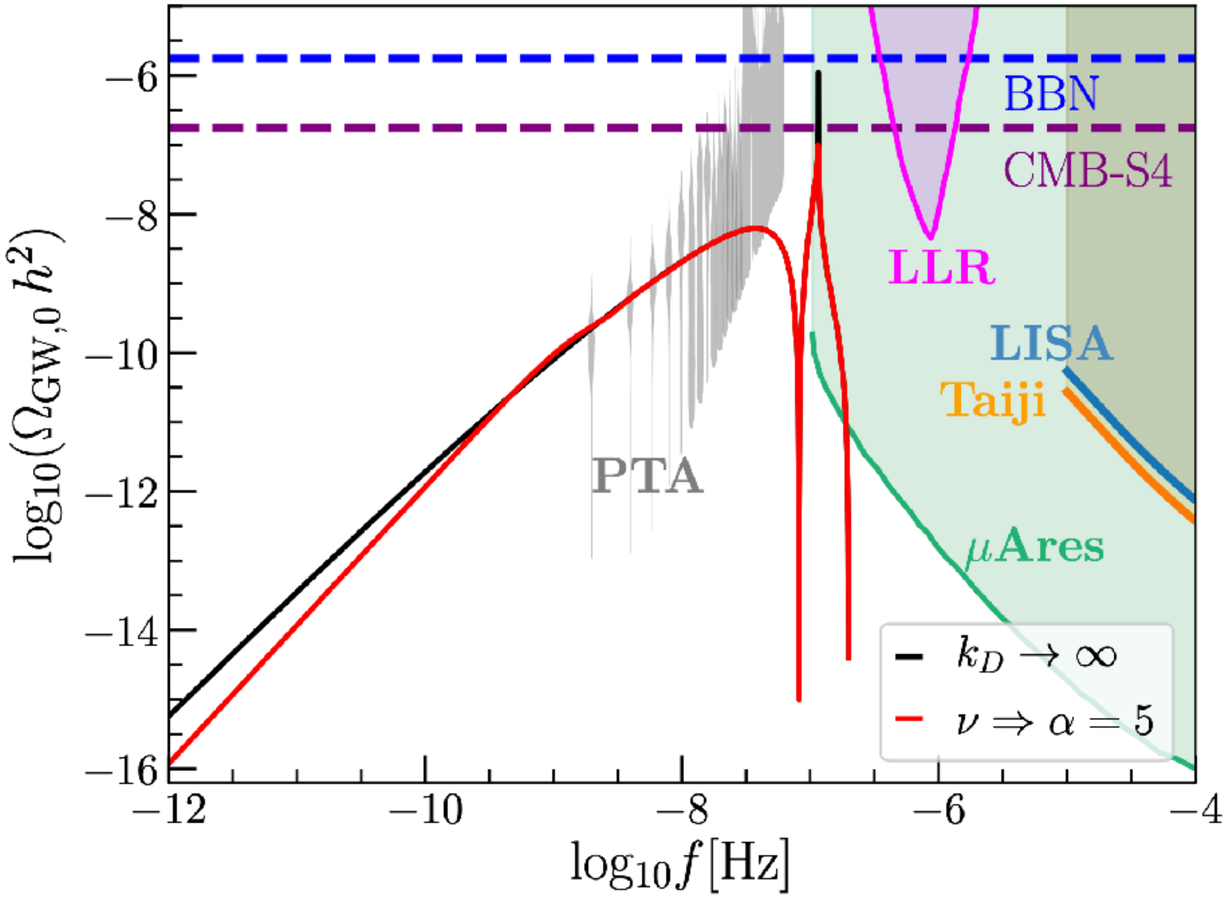

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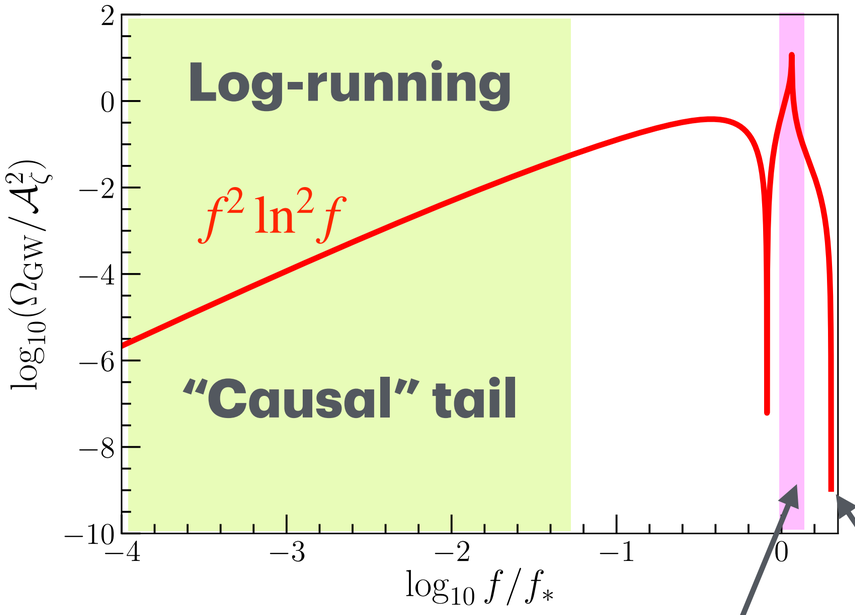
Spectral shape — Universal features

Toy model: Inifinitely sharp (Dirac delta) primordial spectrum

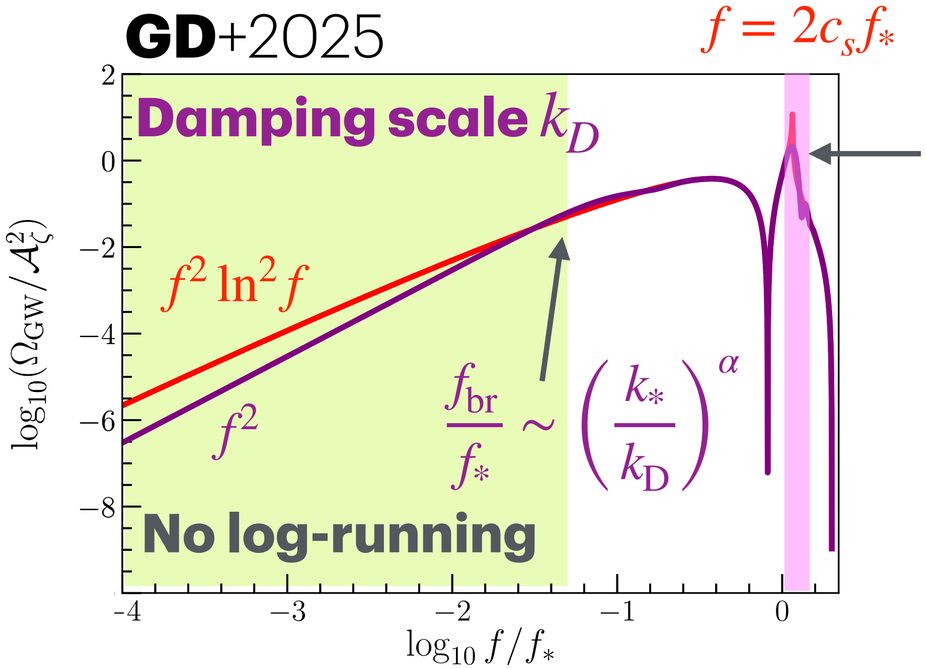
$$\sigma^2(k) = \mathscr{A}_\zeta \delta(\ln(k/k_*))$$



Resonant frequency $f = 2c_s f_*$

Cut-off due to momentum conservation

Particle Interactions (e.g. Neutrinos mean free path)



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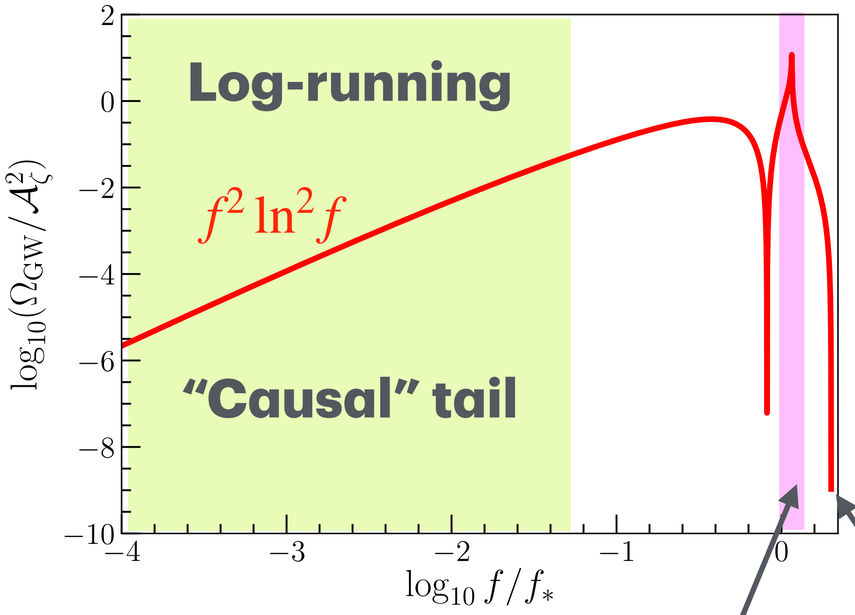

Resonant frequency $f = 2c_s f_*$

Particle Interactions (e.g. Neutrinos mean free path)

Finite width primordial spectrum (e.g. inflation model)

Cut-off due to momentum conservation

GD+2025

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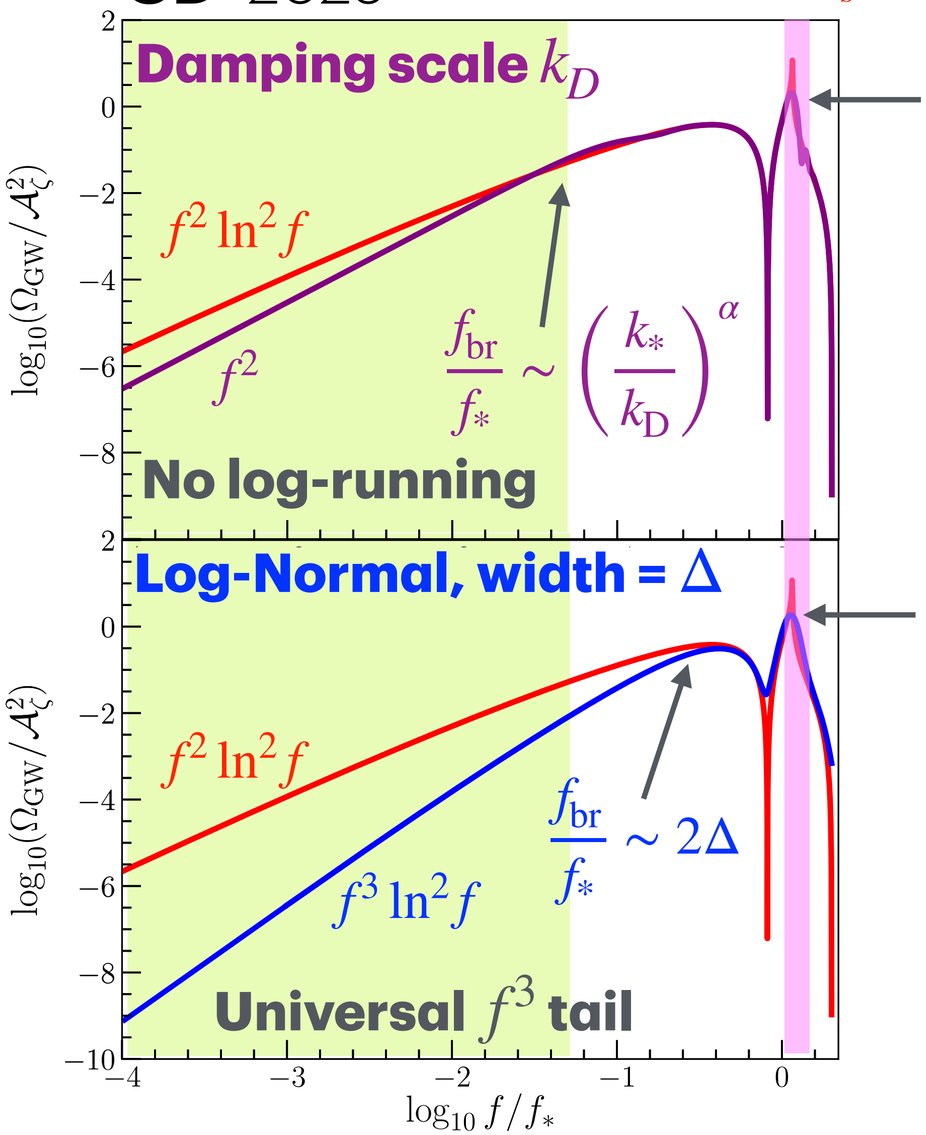

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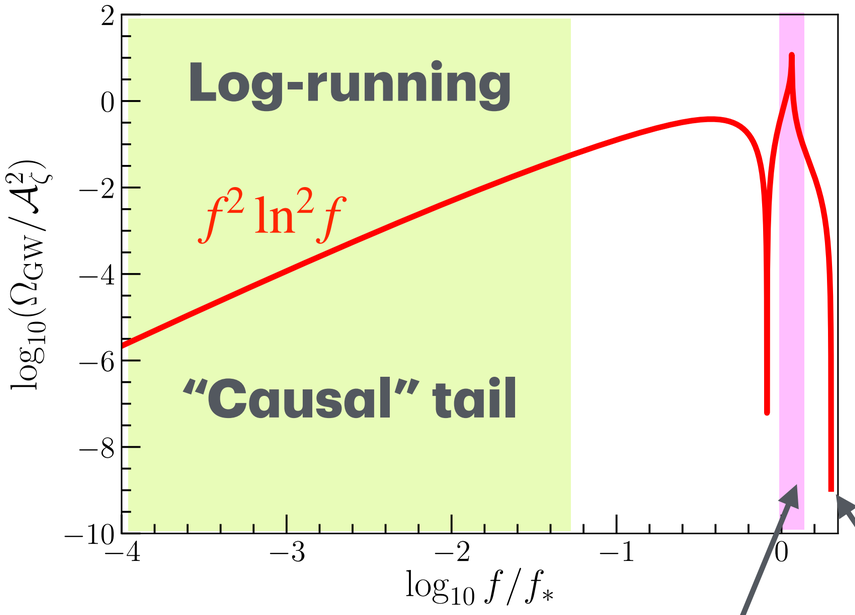


Resonant frequency $f = 2c_s f_*$

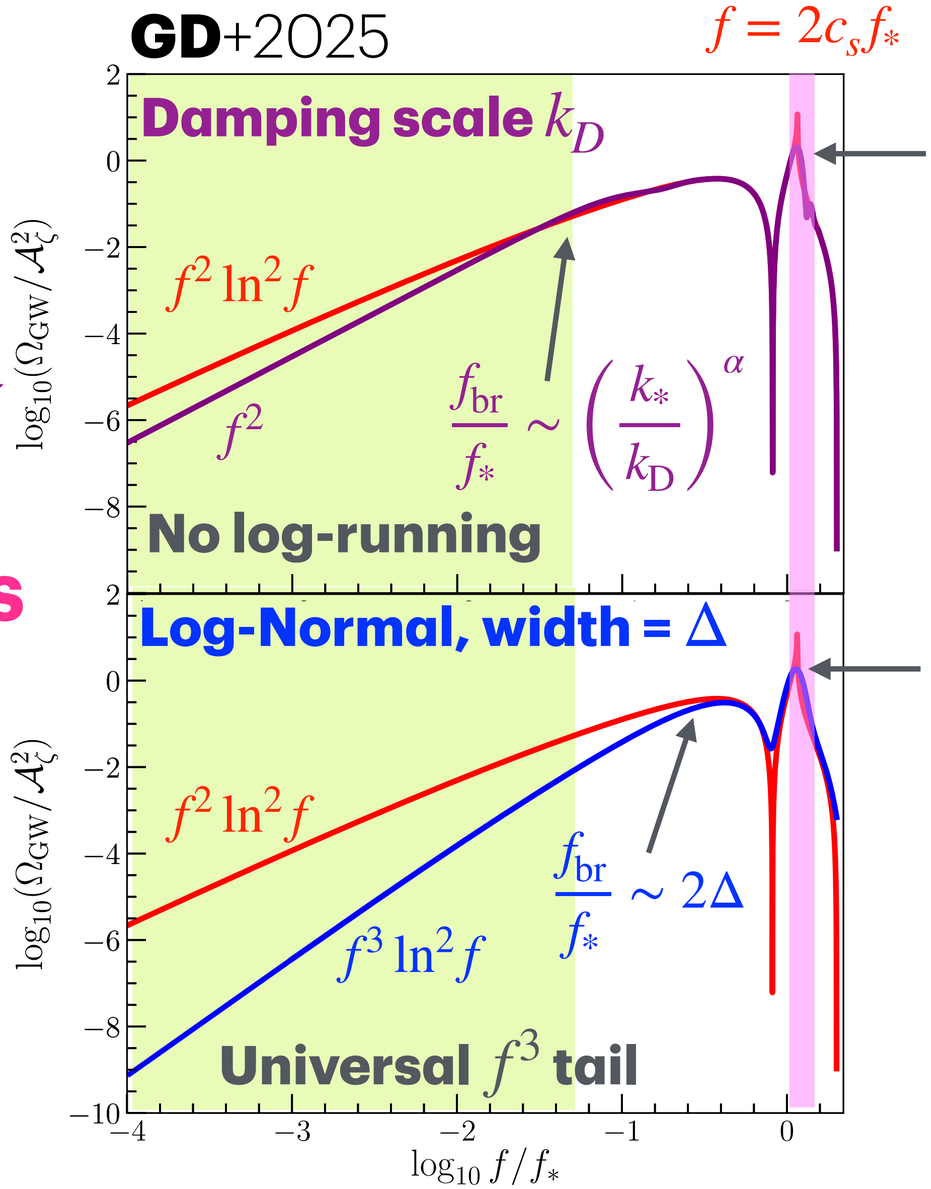
Cut-off due to momentum conservation

Particle Interactions (e.g. Neutrinos mean free path)

+ OTHER EFFECTS

Finite width primordial spectrum (e.g. inflation model)

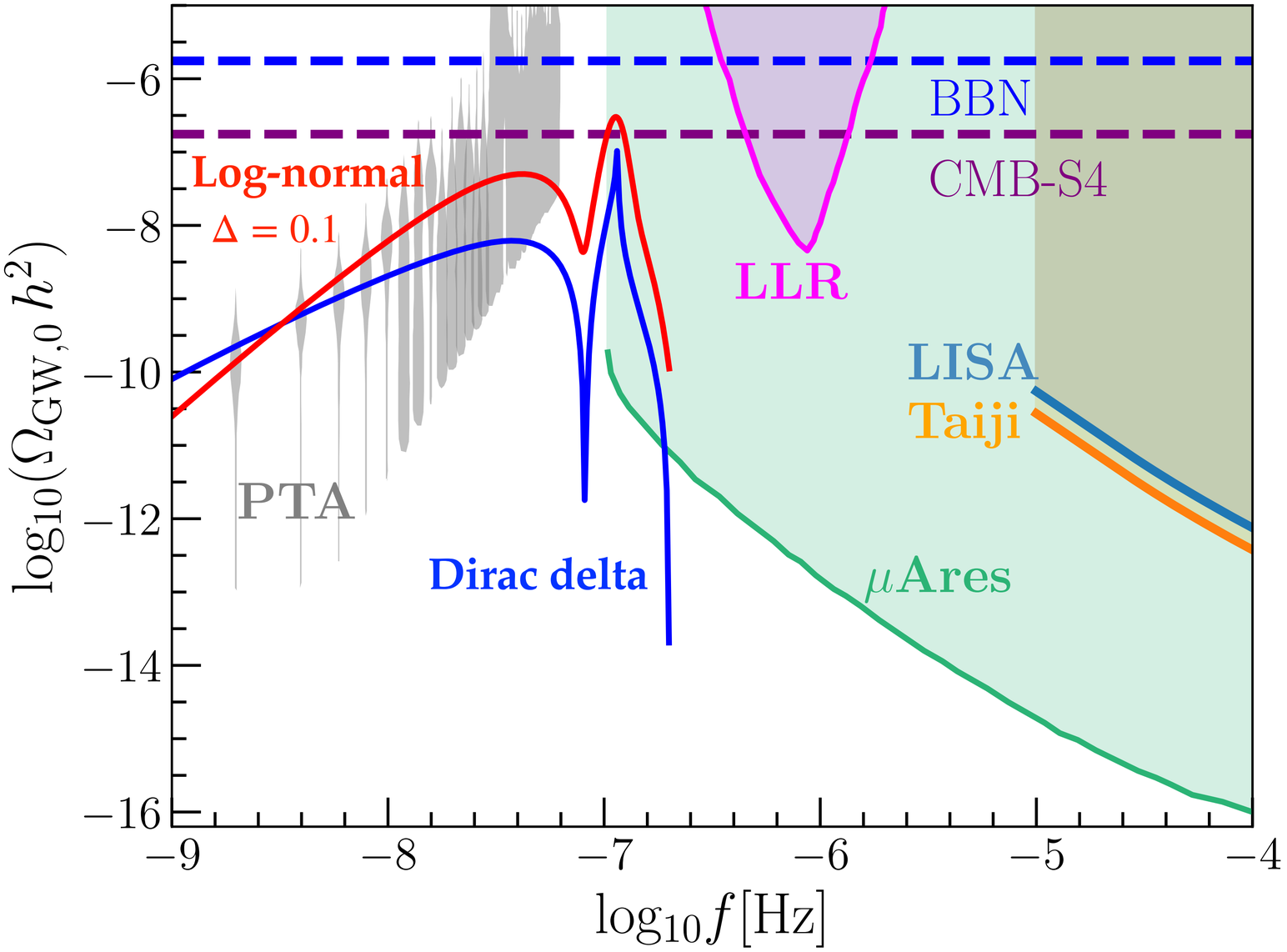
GD+2025



An interesting PBH mass window

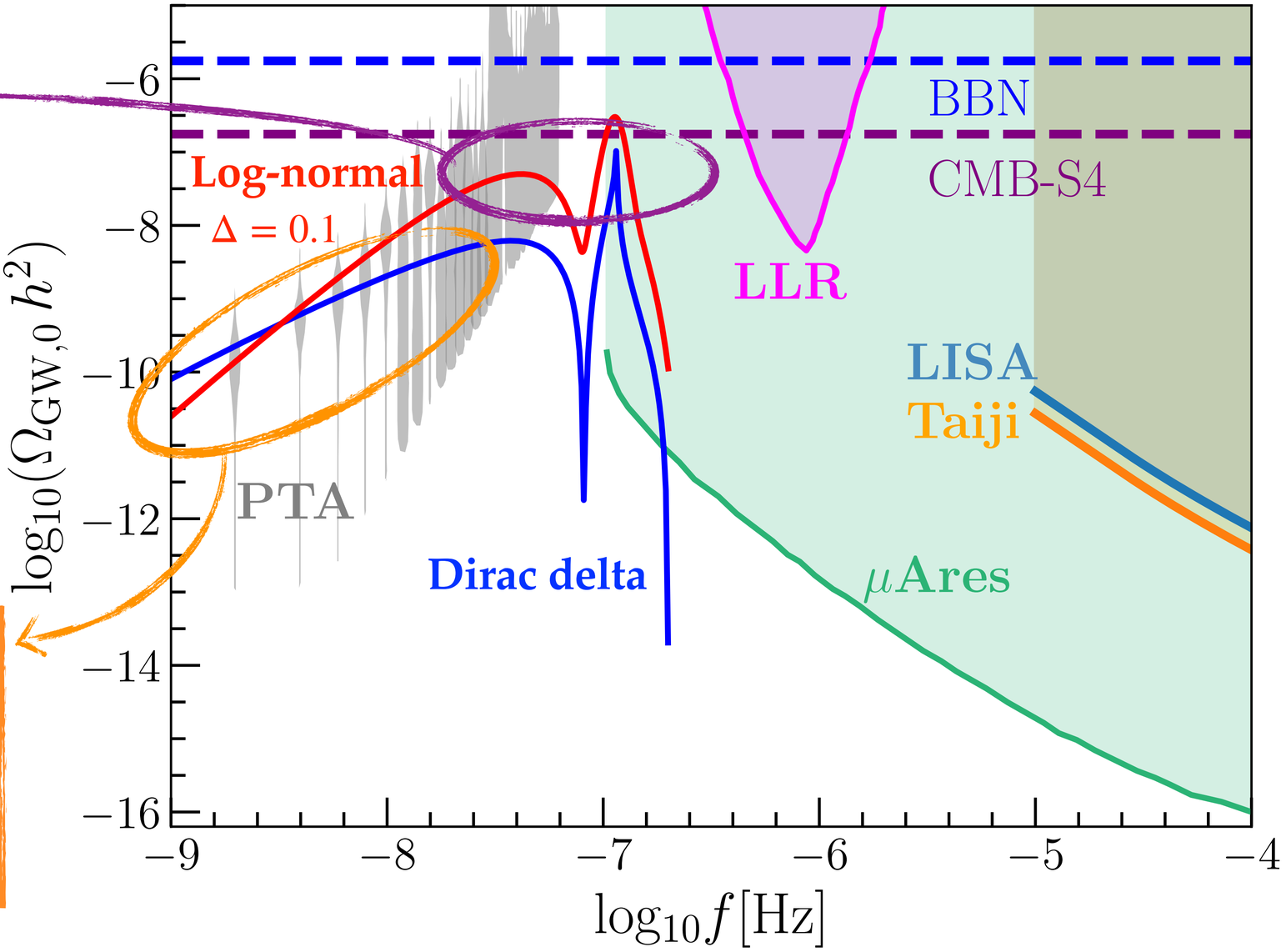
(Our best chance of finding PBH?)

Fit by eye


BBN
CMB-S4
Log-normal
$\Delta = 0.1$
LLR
LISA
Taiji
PTA
Dirac delta
μAres
$\log_{10}(\Omega_{\mathrm{GW},0}\, h^2)$
$\log_{10} f\,[\mathrm{Hz}]$
−6
−8
−10
−12
−14
−16
−9
−8
−7
−6
−5
−4

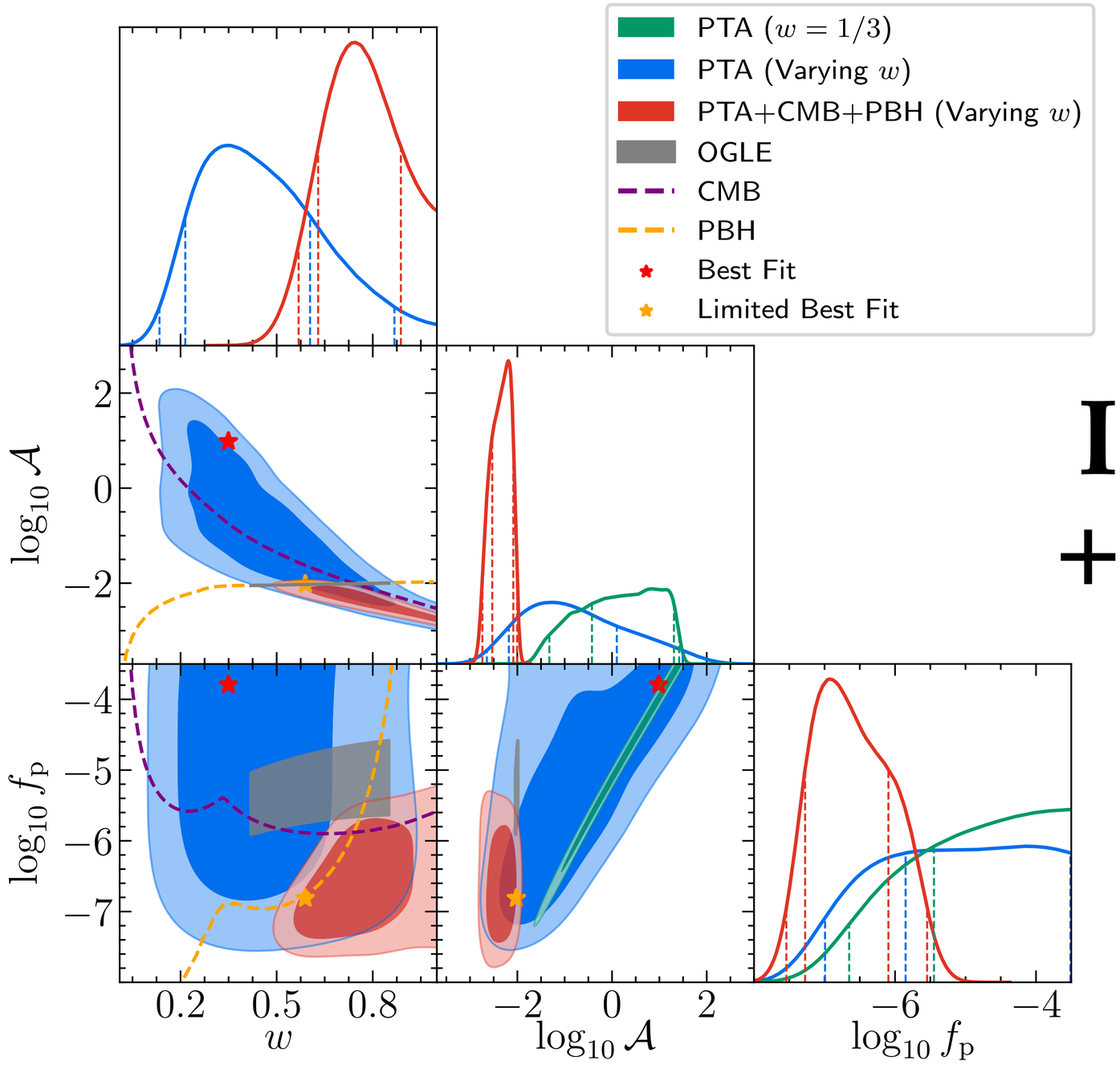
Fit by eye

$\mathcal{A}_{\mathcal{R}} \sim 0.1$



Low frequency tail

$$\Omega^{(\delta)}_{\mathrm{GW},w} \propto \mathcal{A}^2_{\mathcal{R}} \left(\frac{k_{\mathrm{p}}}{k_{\mathrm{rh}}}\right)^{-2b} \left(\frac{k}{k_{\mathrm{p}}}\right)^{2-2|b|}$$



$$\mathscr{P}_{\mathscr{R}} \sim \mathscr{A}_{\mathscr{R}}\delta(\ln(k/k_{\rm p}))$$

$$f_{\rm BBN} < f_{\rm rh} < f_{\rm PTA}$$

Including EoS + CMB + PBHs

$$\Omega^{(\delta)}_{{\rm GW},w}(k \ll k_{\rm p}) \approx \mathscr{A}^2_{\mathscr{R}} \left(\frac{k_{\rm p}}{k_{\rm rh}}\right)^{-2b} \left(\frac{k}{k_{\rm p}}\right)^{2-2|b|}$$

Domenech + 2402.18965

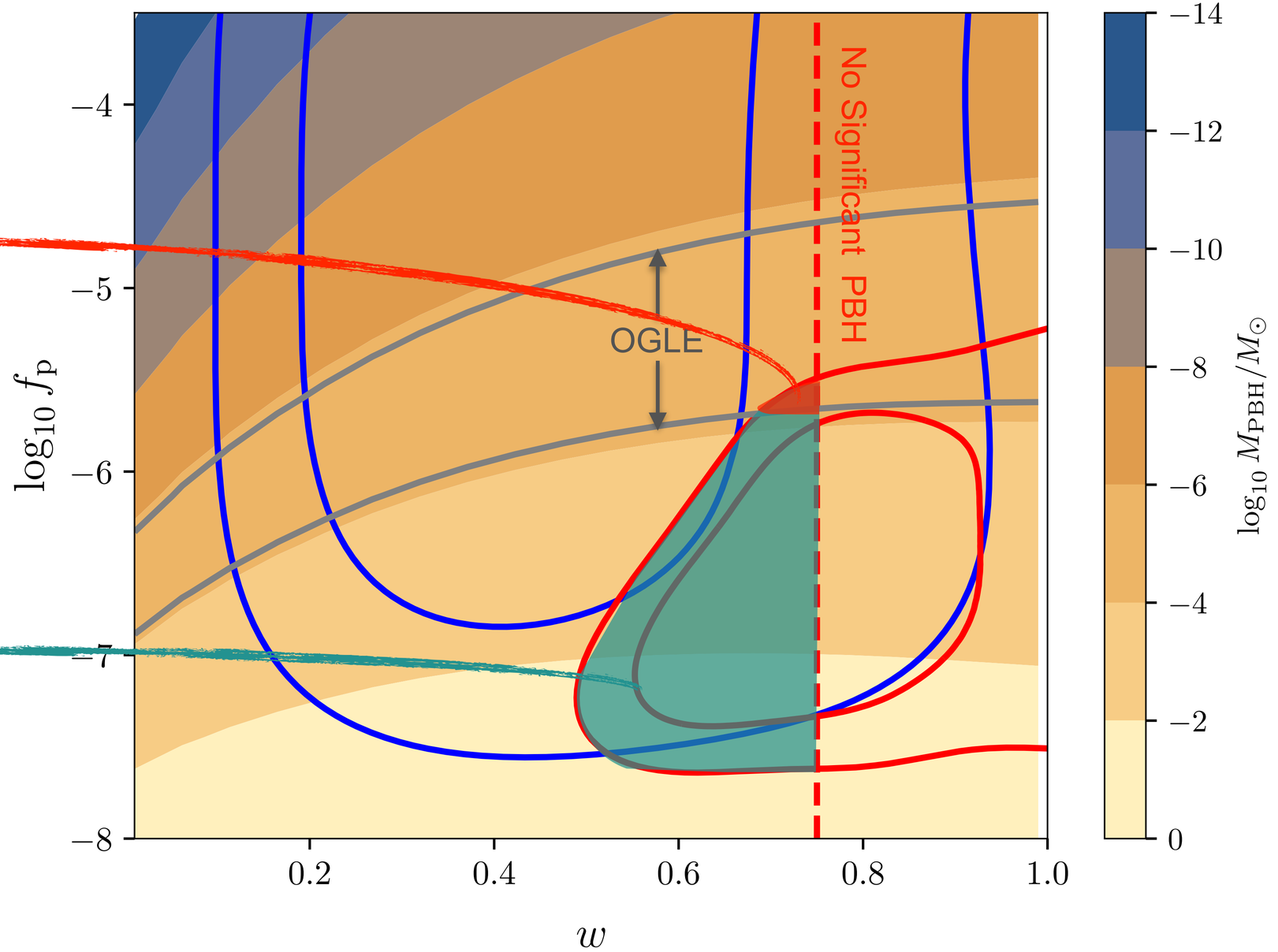
PTA results compatible with OGLE microlensing events?

Compatible with OGLE if

$$0.69 \lesssim w \lesssim 0.75$$

Interesting mass range

$$10^{-5} M_\odot \lesssim M_{\rm PBH} \lesssim 10^{-1} M_\odot$$



Implications for other GW detectors

Compatible PBH mass

$$10^{-5} M_\odot \lesssim M_{\rm PBH} \lesssim 10^{-1} M_\odot$$

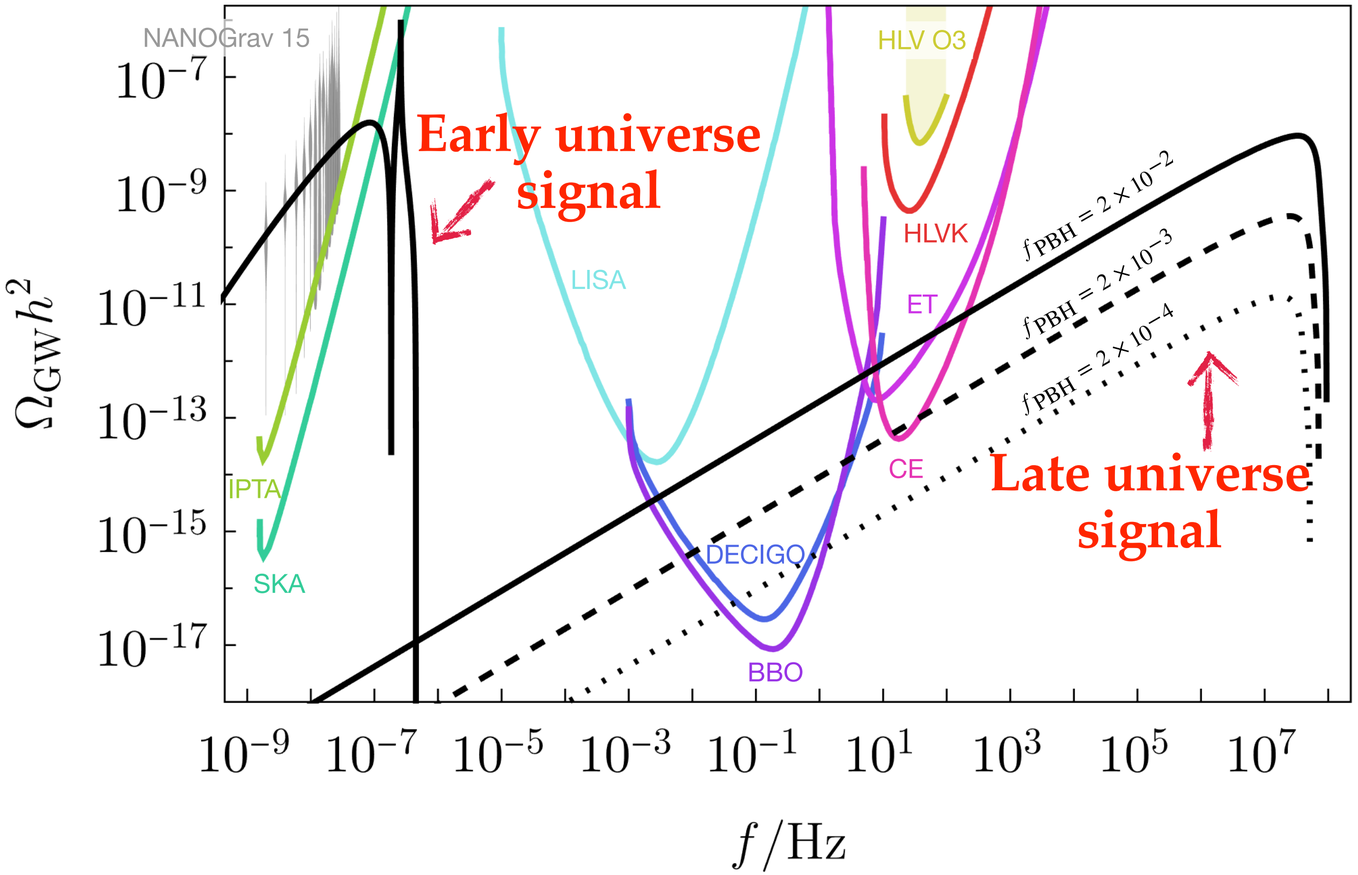


Figure from Inomata et al. 2306.17834

Compatible with OGLE if

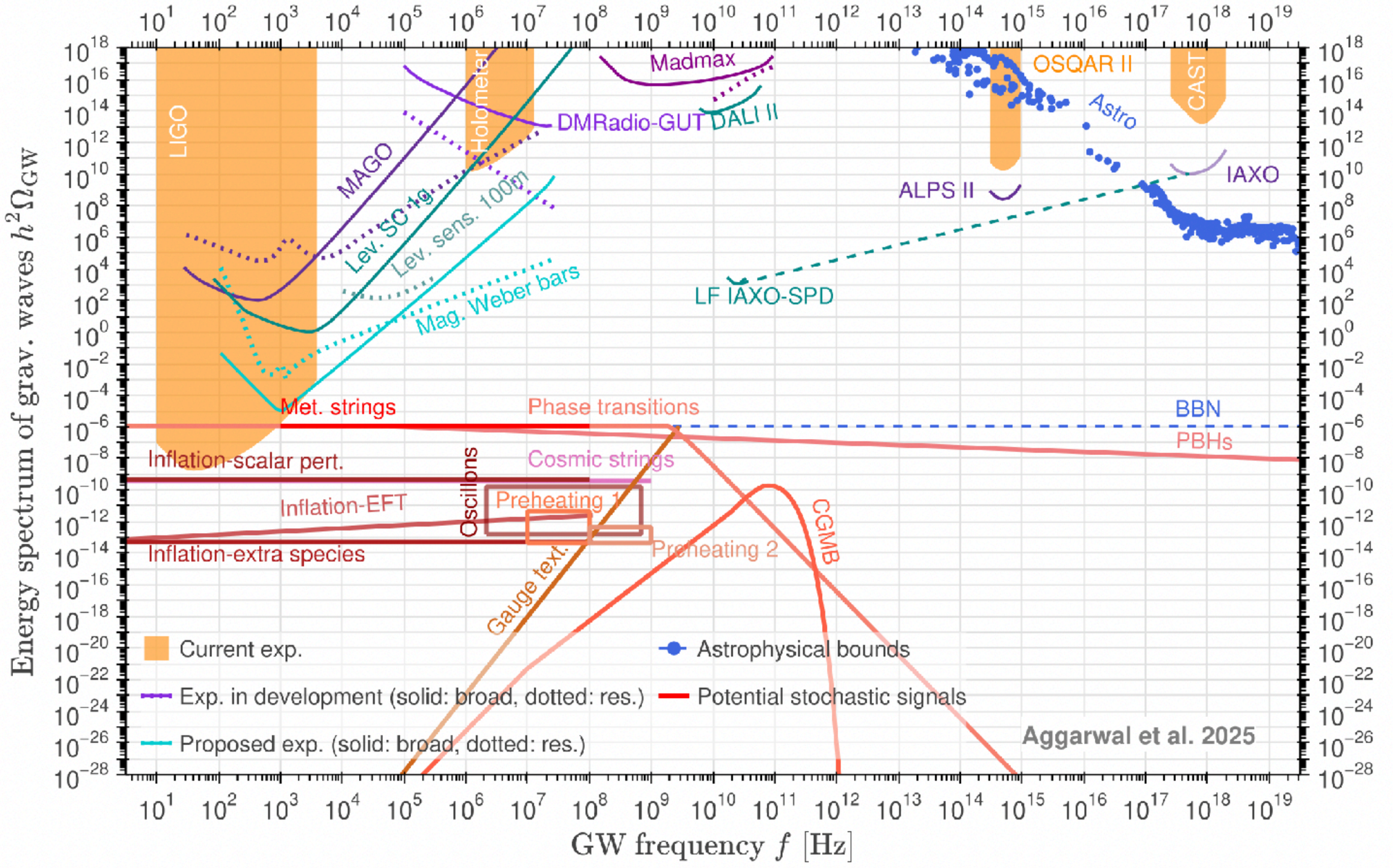
$$0.69 \lesssim w \lesssim 0.75$$

GW background from PBH binaries?

$$M_{\rm PBH} \sim 10^{-4} M_\odot$$

Ultra-high-frequency GWs?

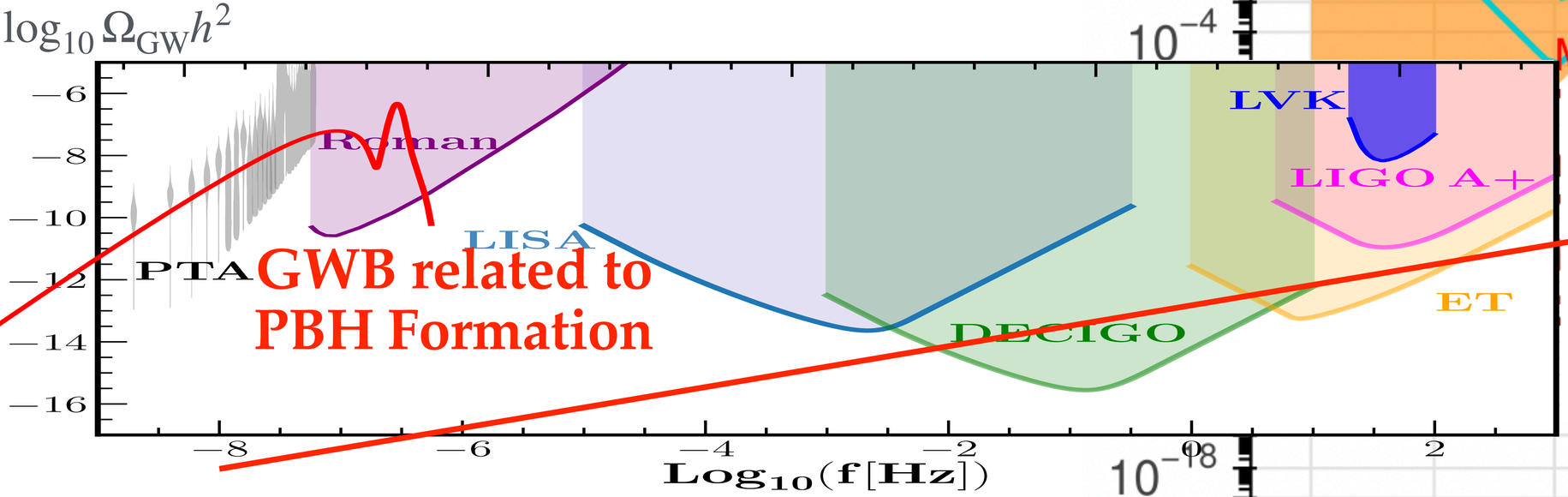
$$f_{\rm ISCO} \sim 10^7\,{\rm Hz}$$



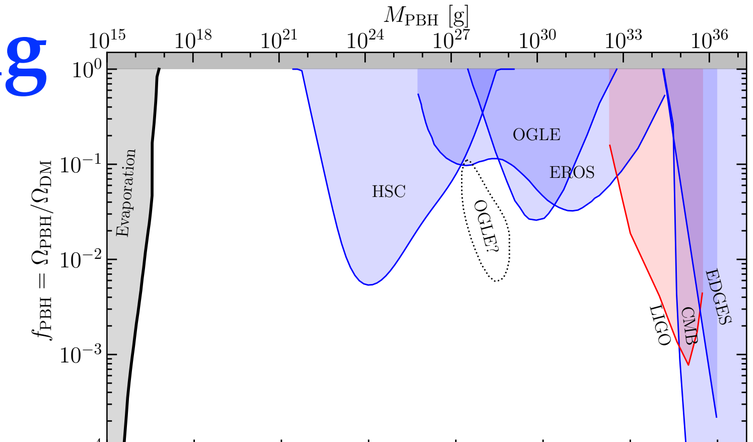
Aggarwal + 2501.11723

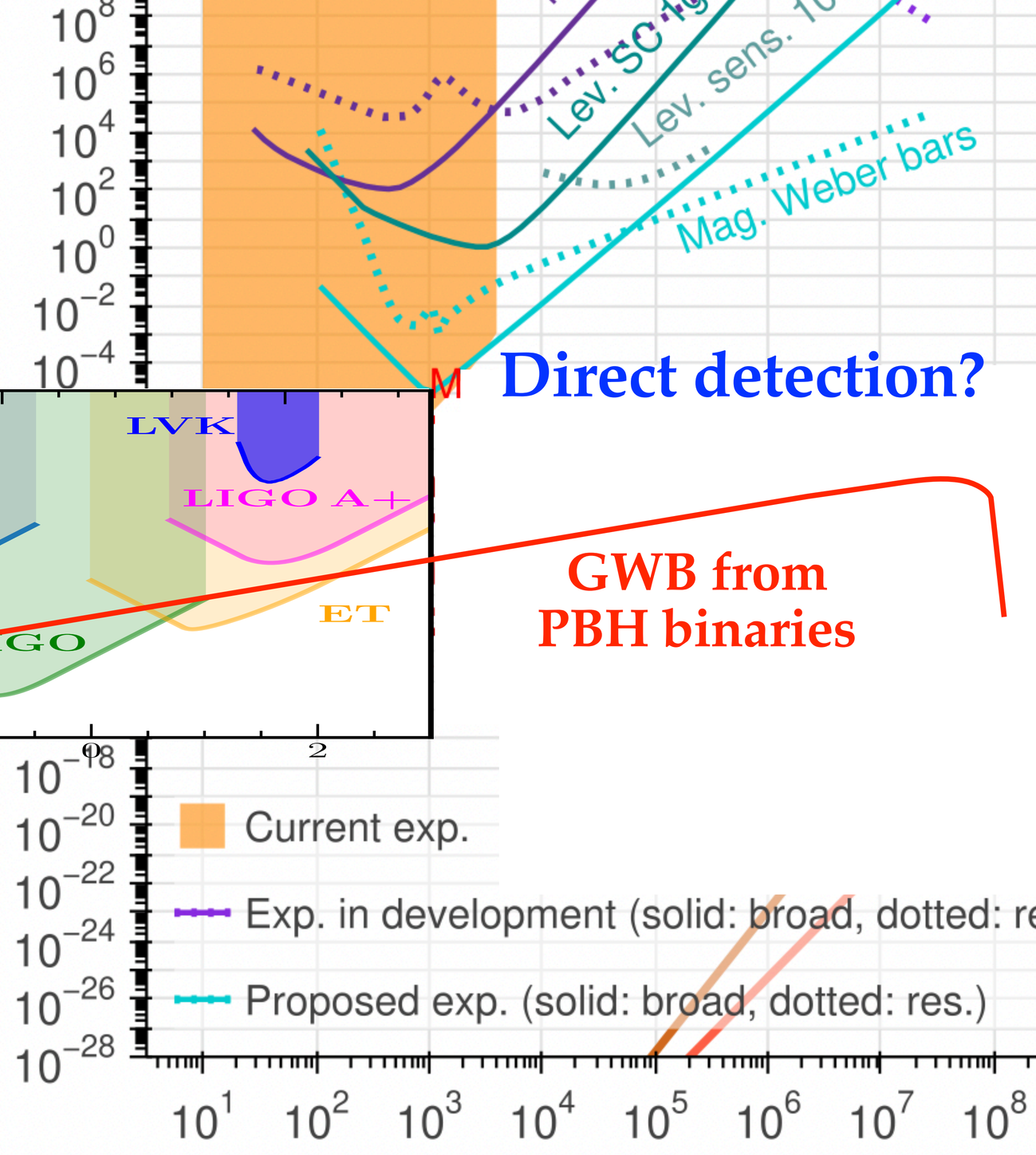
Best chance of finding PBH?

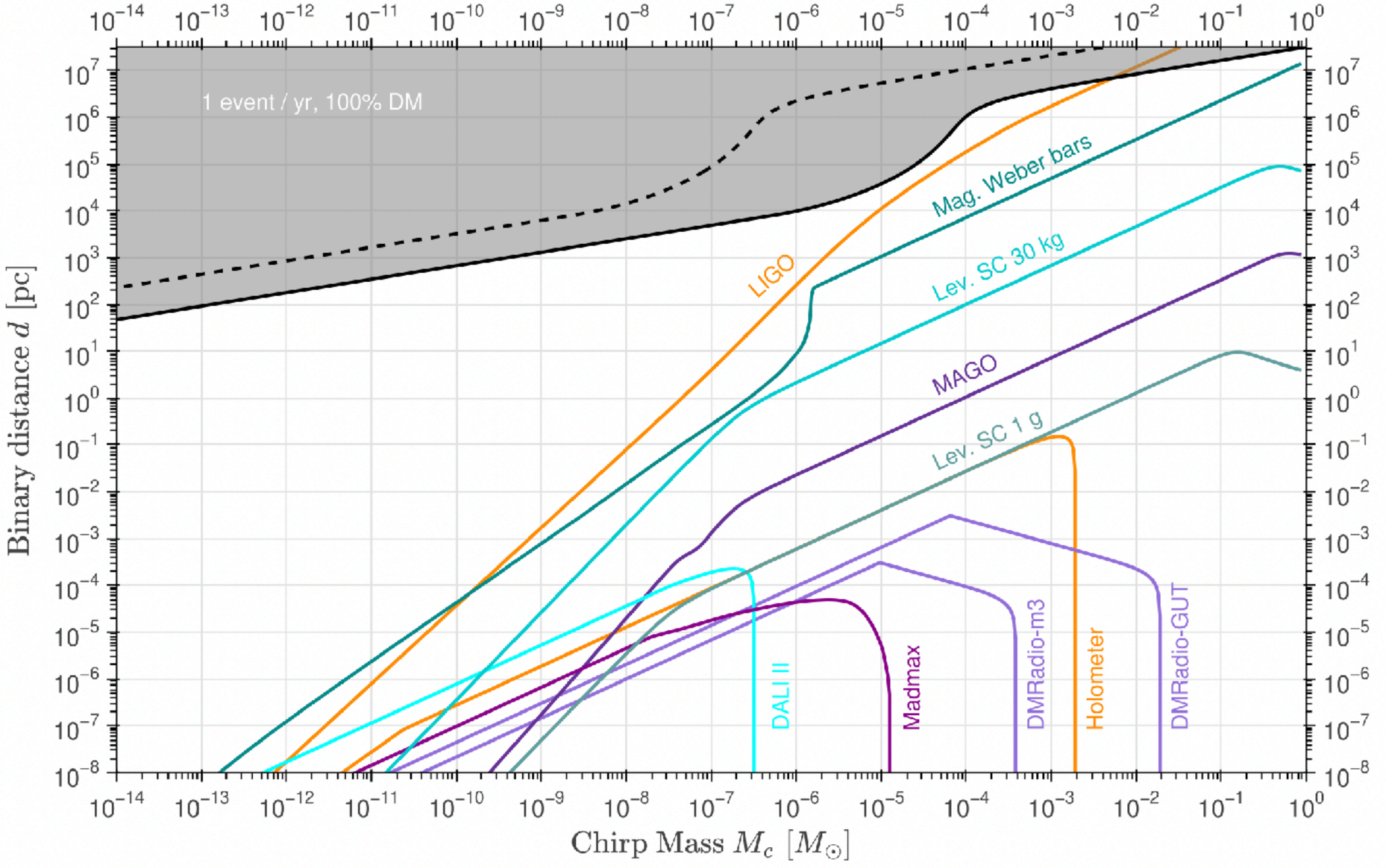
Astrometry GW signal?



+ Microlensing







Aggarwal + 2501.11723

Summary and conclusions

1. Potential new physics discoveries in cosmology (DM, beyond big bang)

2. We started to **explore the universe with GWs**

3. If **PBHs** in the universe => **induced GW** background!

4. The PBH scenario is **already being tested** with **PTAs** and **LVK**

   + other complementary observations (e.g. microlensing).

More GW detectors on the way:

We may find evidence of PBHs in the next two decades!