Future Potential of High-Frequency Gravitational Wave Detection With Microwave Cavities

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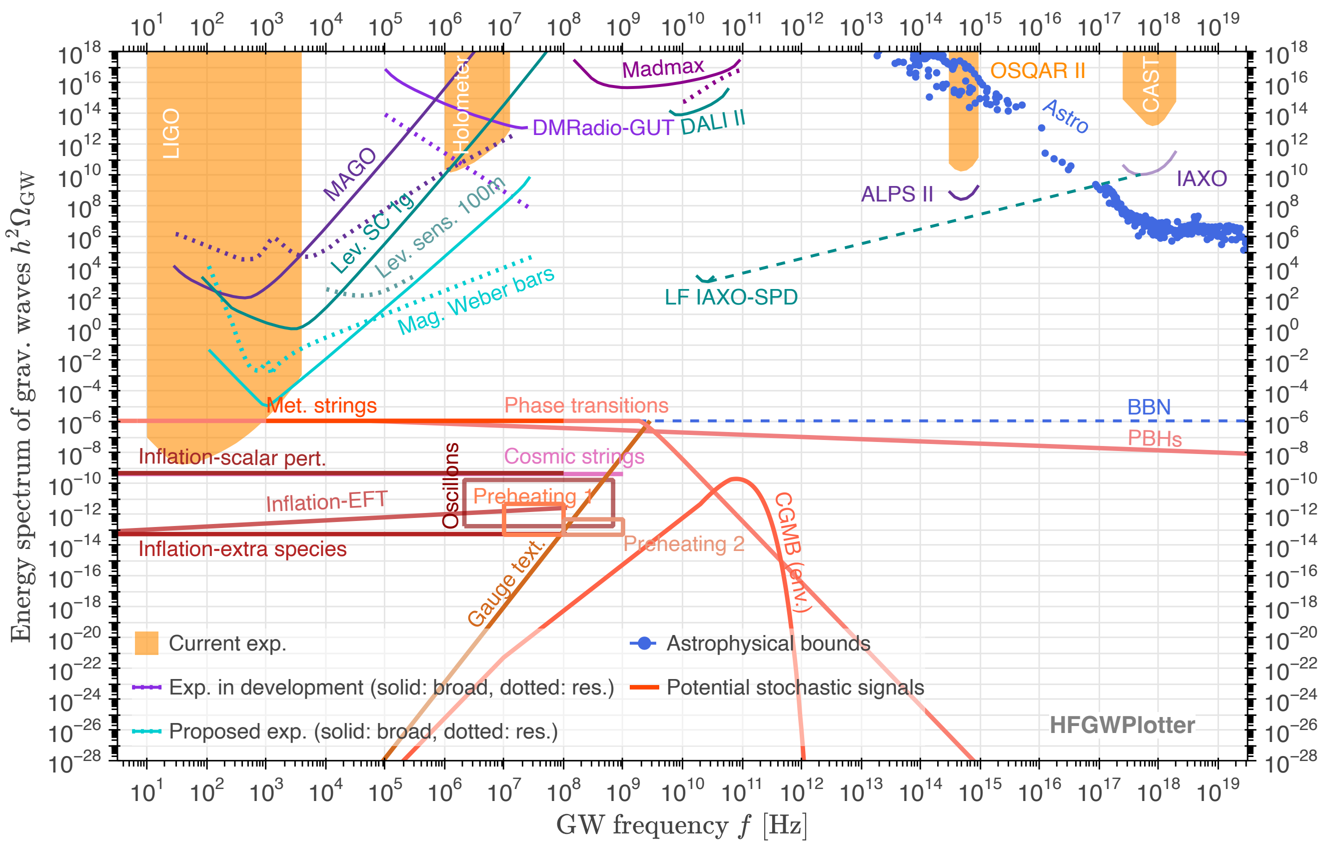
Quantum Sensing meets Ultra-high Frequency Gravitational Waves

Mainz Institute for Theoretical Physics

Why we need **a lot** better detectors



Problem: Ellis + D'Agnolo 2412.17897
GW detectors are constrained by
- (quantum) noise floor
- signal transfer function

Only options left:
- Drastically increase transfer function
- Maximally avoid quantum noise constraints

But scaling detectors up further is hard

$$\Omega_{\text{min}} \propto \frac{1}{\text{Mass}}, \frac{1}{\text{Volume}}, \frac{1}{(\text{EM Field})^2}$$

Can we even propose detectors which reach cosmological HFGWs?

How far can we push HFGW detectors?

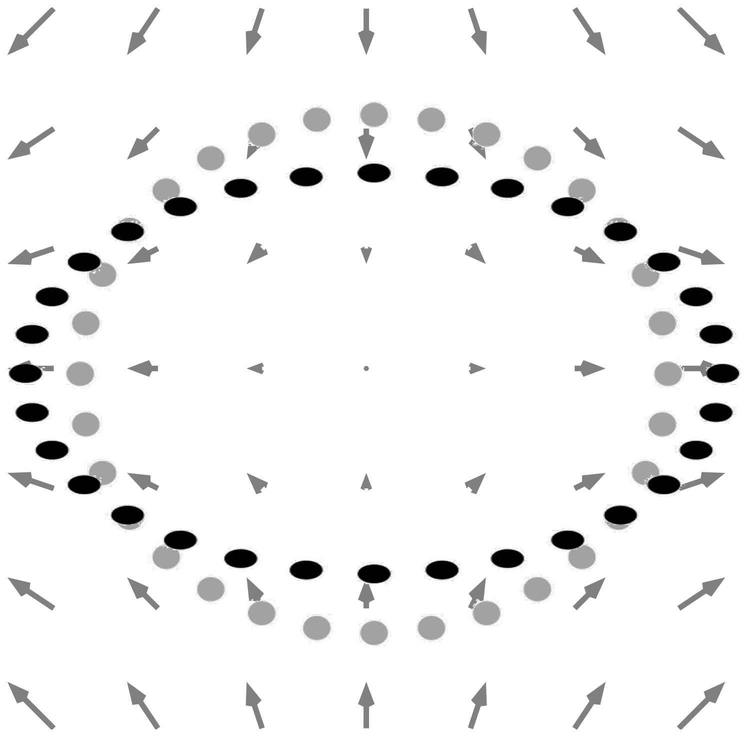
- **Position detection**
  - GW detection with cavities
  - MAGO optimization
- **Electromagnetic detection**
  - HFGWs on BREAD
  - Gertsenshtein MAGO
  - PD strain of general waveforms

Enhancing the signal transfer function

- **Quantum Enhancement**
  - Vacuum Squeezing
  - Back-action free amplification
  - Prospective sensitivity

Lowering the noise floor

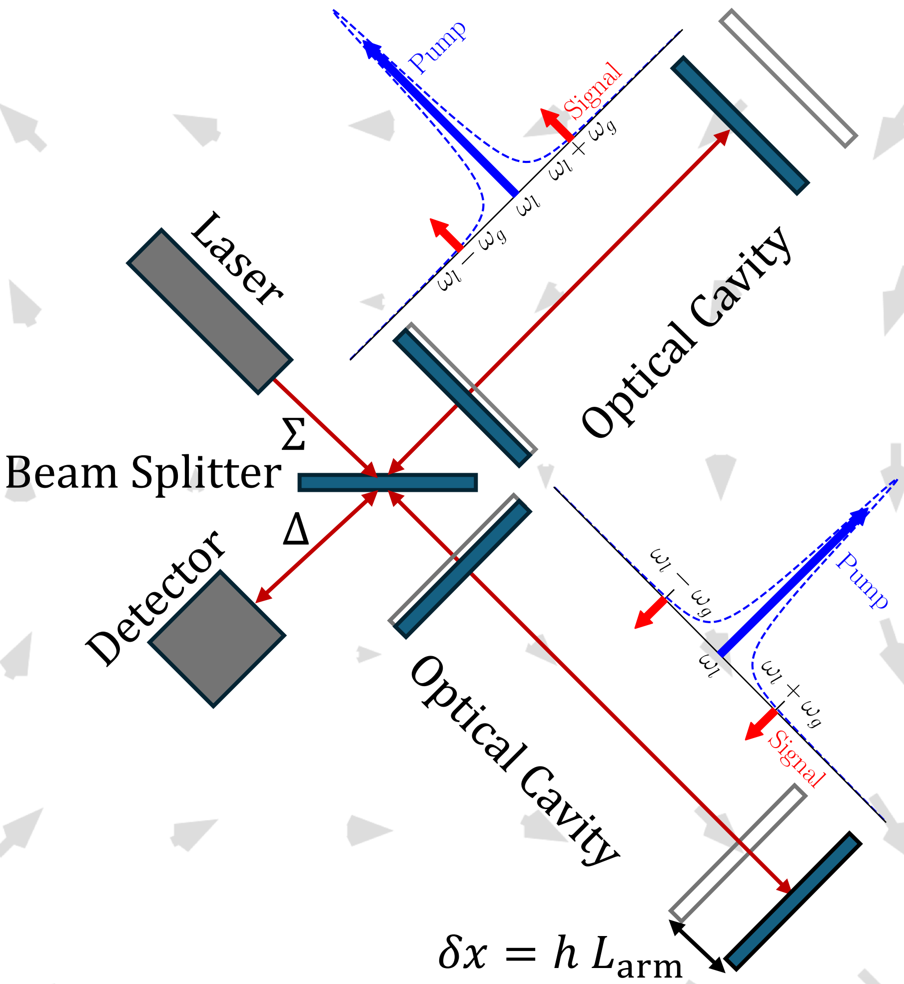
Position Detection



Optical Cavities in the Laboratory Frame

GW tidal force

$\boldsymbol{F}_{\mathrm{GW}} \propto (y, x, 0)e^{i\omega_g t}$



- Optical cavities loaded symmetrically with laser power
- GW perturbs cavity mirrors
- Mirror oscillation up-converts power anti-symmetrically in both cavities
- Beam splitter filters out differential signal

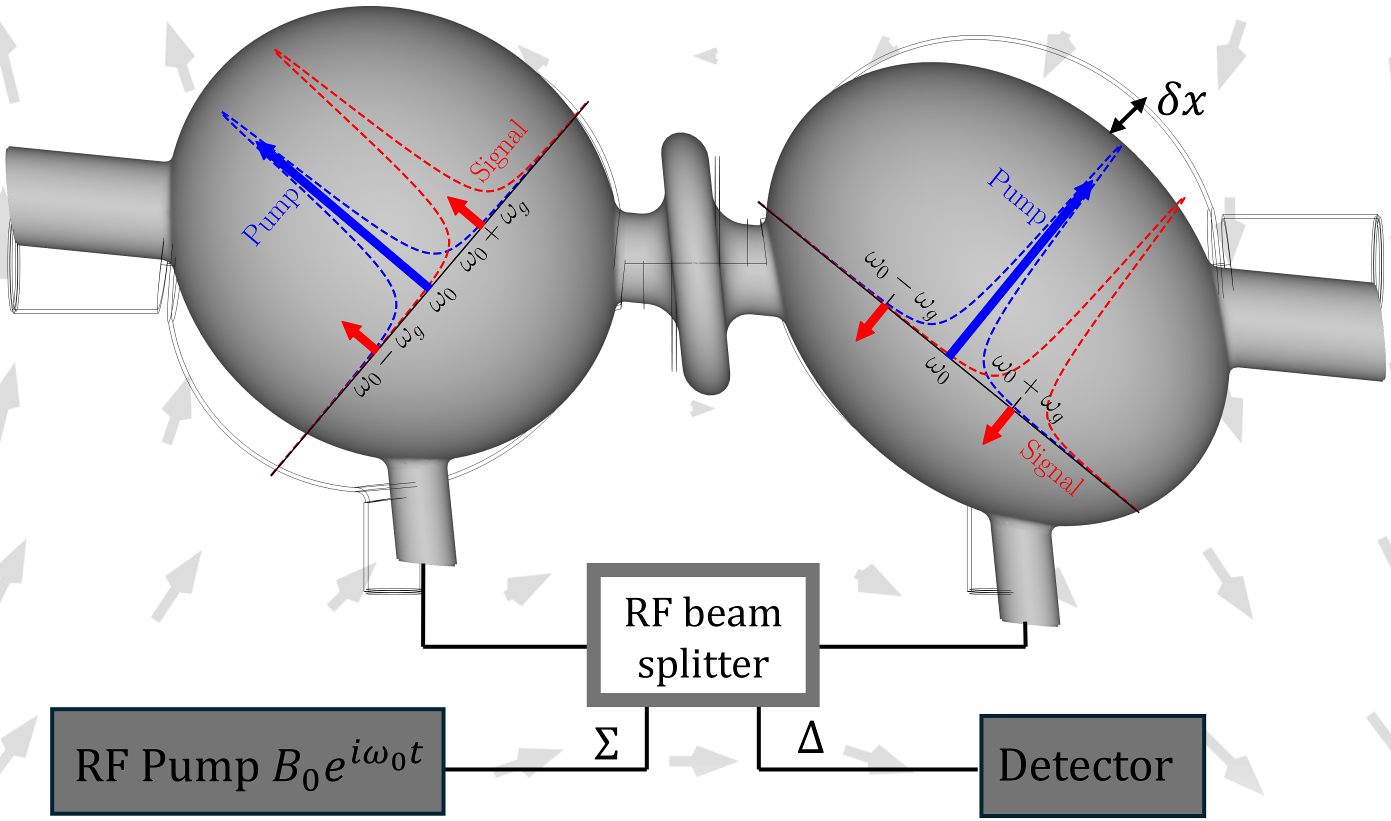
Position Detection

Microwave Cavities in the Laboratory Frame

GW tidal force

$\boldsymbol{F}_{\mathrm{GW}} \propto (y, x, 0)e^{i\omega_g t}$

Superconducting radio frequency (SRF) cavity



→Much smaller δx than laser interferometer

→ Signal **resonantly enhanced** $\propto Q \gtrsim 10^{10}$

→Additional enhancement from mech. resonance $\propto Q_m \approx 10^6$ possible

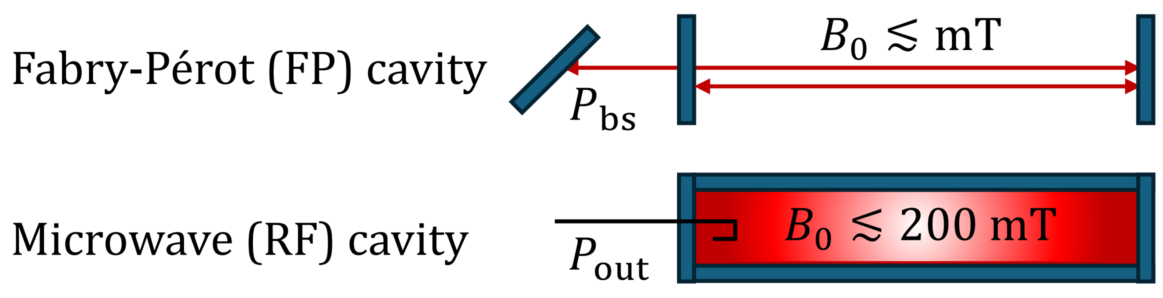
'MAGO' detector:
Ballantini et al. (2005)
Berlin et al. (2023)
Fischer, **TK** et al. (2025)

Optical Cavities vs Microwave Cavities

Rate of energy transfer — Transfer function (Resonance & Readout) — EM energy — Signal PSD — GW PSD

- Heuristically: $S_{\text{sig}}(\omega) \propto \omega_0\, U_0\, T(\omega)^2\, S_h(\omega)$ (Ellis + D'Agnolo 2412.17897)

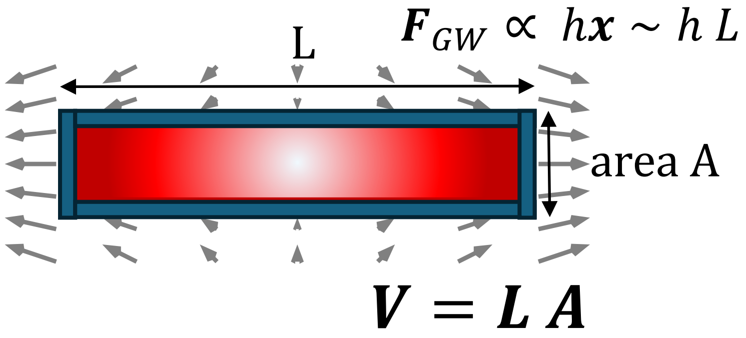
- Comparing the thermal & quantum limited sensitivities:
  - **Stored energy** in cavity → MAGO wins
  - EM **frequency** → LIGO wins



SRF cavities have the potential of more (GW sensitivity)/(Detector Size)

Position Detection

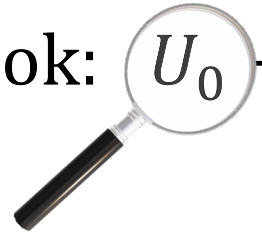
Length scaling



- Heuristically: $S_{\text{sig}}(\omega) \sim U_0 \sim L\,A$

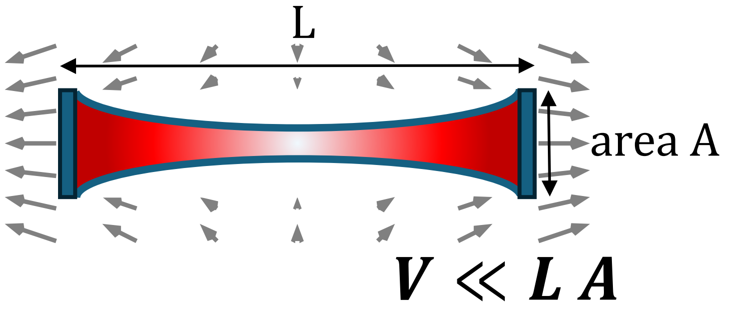
  $\Rightarrow h_{\min} \sim 1/\sqrt{L}$ (for RF **and** optical cavities)

- A closer look:



$$U_0 \to \frac{\left|\int dA\cdot\underbrace{\hat{h}\boldsymbol{x}}_{\sim\text{ GW tidal force}}\,(\boldsymbol{B}_0\cdot\boldsymbol{B}_1 - \boldsymbol{E}_0\cdot\boldsymbol{E}_1)\right|^2}{\underbrace{\max_A \boldsymbol{B}_0^2 \int dV\,\boldsymbol{B}_1^2}_{\text{Interaction volume } V_{\text{int}}}}\,\underbrace{B_{\text{crit}}^2}_{\approx\,0.2\text{ T for Nb RF cavity}}$$

$$\propto \frac{(LA)^2}{V} B_{\text{crit}}^2$$



$\Rightarrow h_{\min} \sim 1/L$ possible?

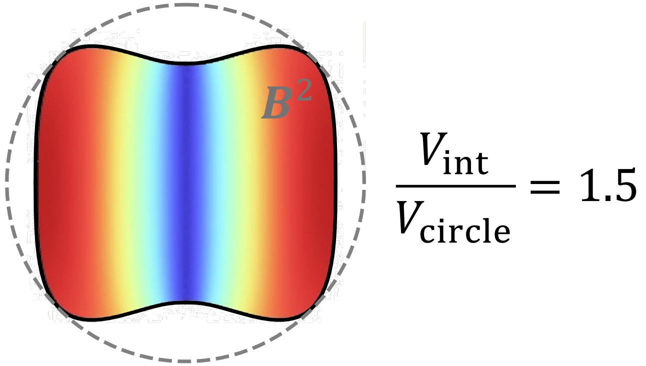
- Generic cavity geometries scale $h_{\min} \sim 1/\sqrt{L}$ with the cavity length
- Optimized geometries could unlock a $h_{\min} \sim 1/L$ scaling

Position Detection

Optimized Microwave Cavity

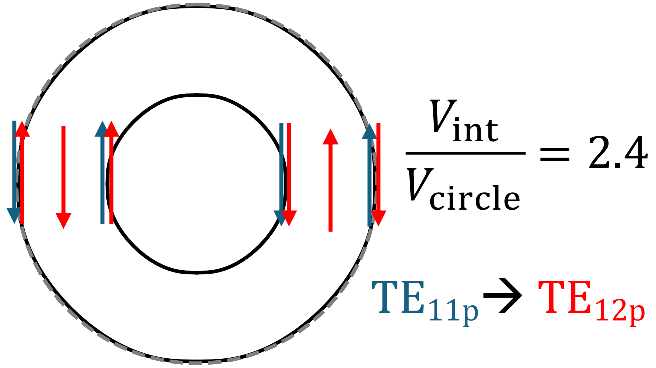
- $\boldsymbol{f_g}$ **in kHz – MHz:** Use symmetric and anti-symmetric oscillations of same mode in two-cell cavity ⇒ $V_{\text{int}} \propto \left|\int dA \cdot \hat{h}x\,(B^2 - E^2)\right|^2$

  →Example: Optimal 2D cavity that fits into circle
  (circular cavity has $V_{\text{int}} / V_{\text{circle}} = 1.1$)



- $\boldsymbol{f_g}$ **in MHz – GHz:** Different modes → more design flexibility

  → Example: Concentric cavity increases area with minimal cavity volume

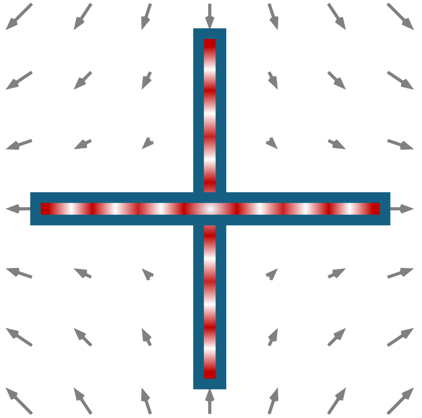


Optimized cavities can have higher sensitivity than the heuristic scaling

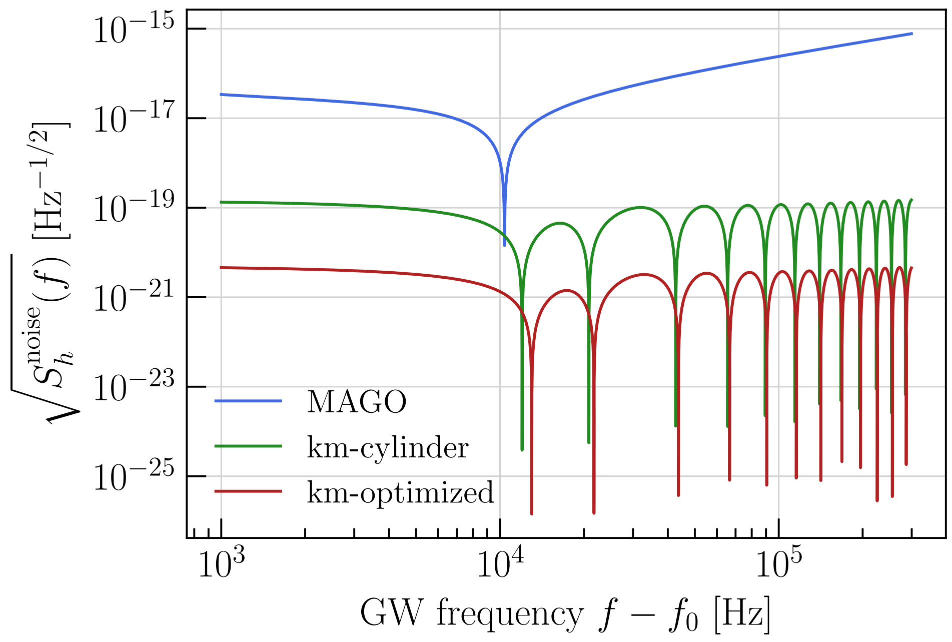
Position Detection

Maximal microwave cavity: $\mathcal{O}$(LIGO) MAGO

- Best way to increase sensitivity: long detectors
- Maximal length given through $f_g \leq 0.3\ \text{MHz} \frac{L_{\text{detector}}}{1\text{km}}$
- Density of resonant modes increases for long cylinders (optical cavities: Schnabel, Korobko arXiv:2409.03019)



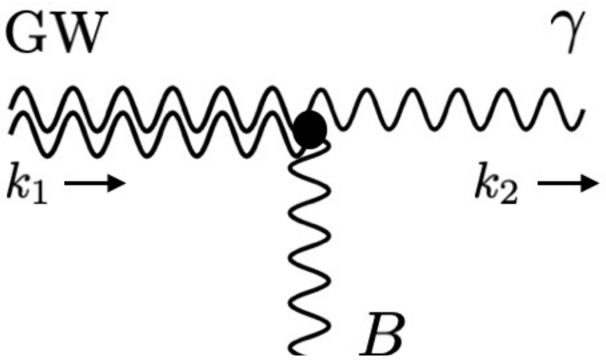
Completely unfeasible?
EU-XFEL uses 768 superconducting RF cavities over length of 1.7 km



$TE_{11p} \rightarrow TE_{11(2q+1)}$ transitions

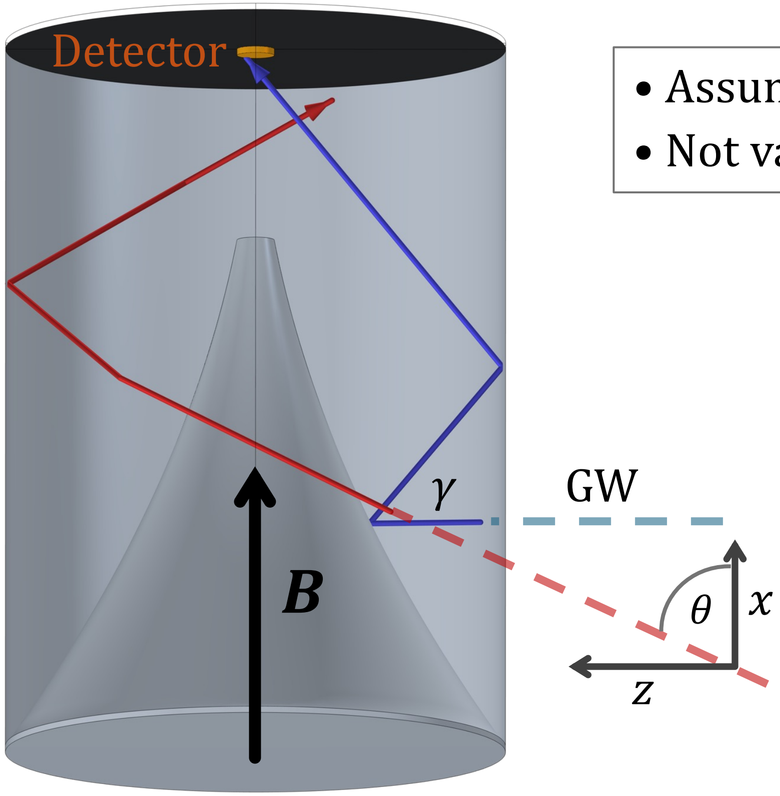
- Leaps in sensitivity are likely impossible with 'tabletop' experiments
- Optimal length scaling not found for arbitrarily long detectors (yet)

Electromagnetic detection



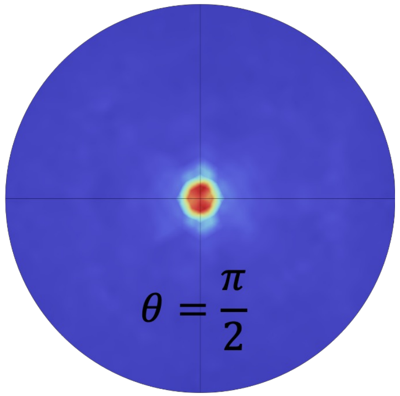
Electromagnetic detection

Dish Antennas: Broadband Focusing With BREAD

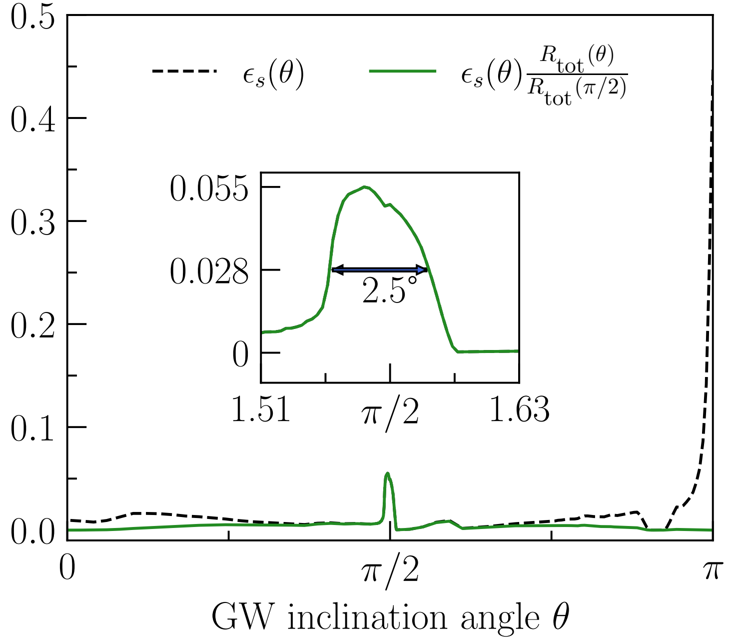


- Assuming ray optics $f_g L_{\text{detector}} \gg 1$
- Not valid for microwaves $f_g L_{\text{detector}} \simeq 1$

Ray heatmap on top of BREAD



Fraction of focused rays

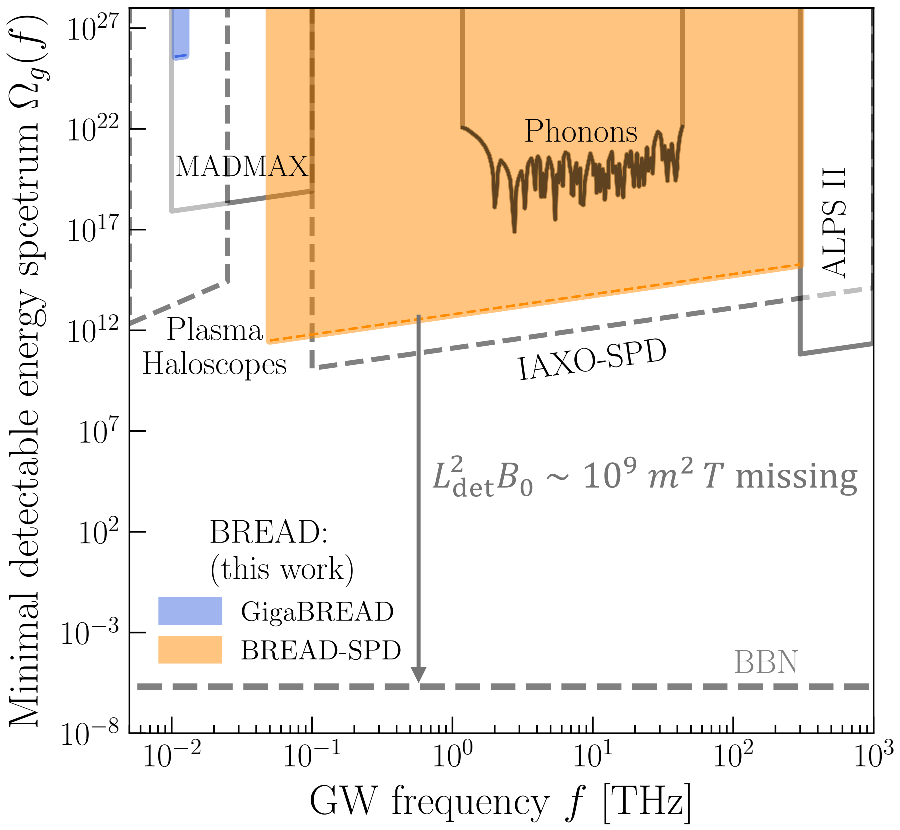


Coaxial dish antennas retain limited focusing abilities for GWs

Tom Krokotsch + **Rodolfo Capdevilla, Roni Harnik & Taegyun Kim** (arXiv:2505.21628)

Electromagnetic detection

Dish Antennas: GW Sensitivity of BREAD



- Small magnetic volume compensated by focusing
- Assume single photon detection (SPD) with dark count rate $R_D = 10^{-4}$ Hz
- Microwave detector (GigaBREAD) less sensitive due to
  - Worse focusing due to microwave resonances
  - Worse noise performance

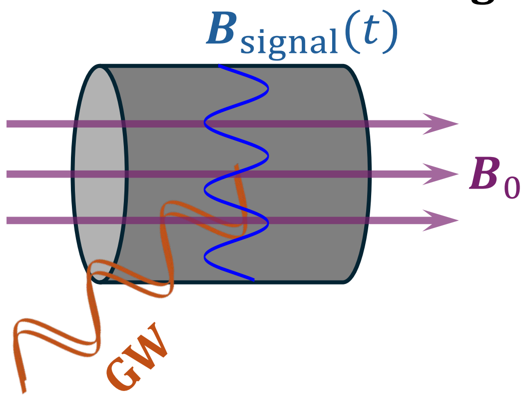
Remaining question: Are there antenna geometries which focus isotropic GW backgrounds better than BREAD?

Coaxial dish antennas benefit from larger field of view than ‘1D’ designs

Electromagnetic detection

Gertsenshtein Effect in Microwave Backgrounds

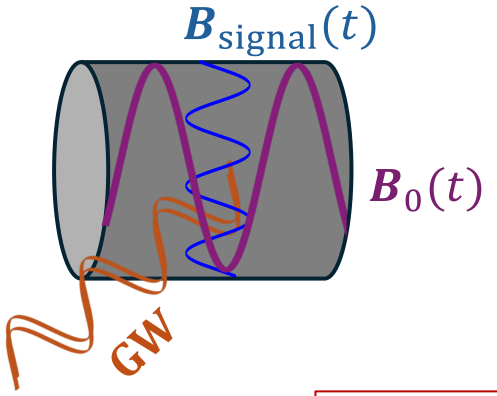
Conversion in **static magnetic field:**



Setup for axions searches like
ADMX (arXiv:2010.00169),
HAYSTAC (arXiv:1610.02580),
& more

→ Berlin, et al. (2021), arXiv:2112.11465

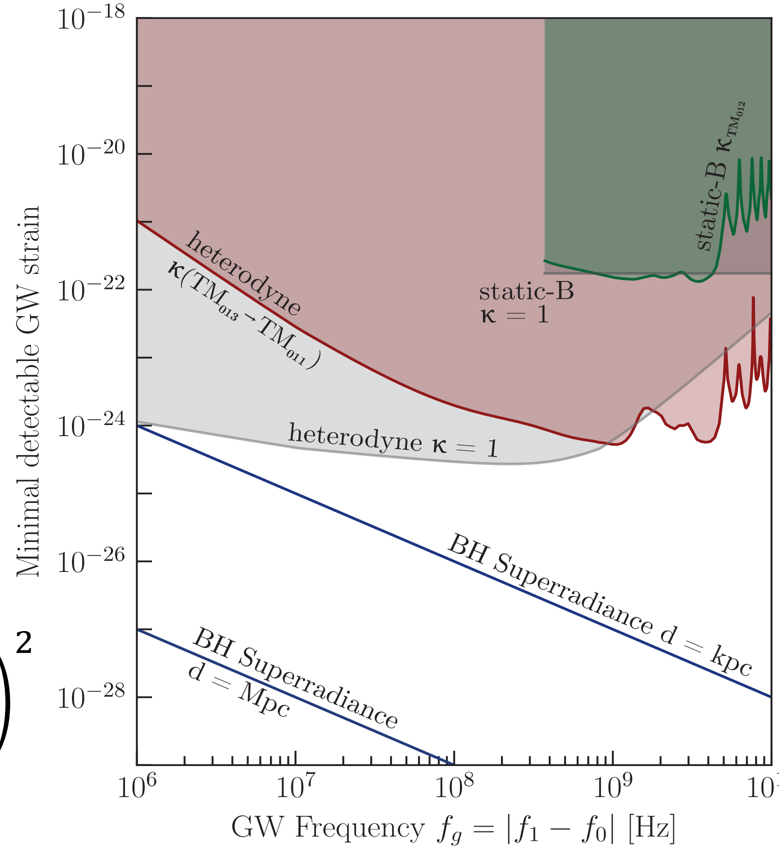
Conversion in **microwave background:**



Parametrically enhanced *on resonance*

$$\frac{\text{SNR}^{\text{SRF}}}{\text{SNR}^{\text{static B}}} \sim 600 \; \frac{1.8\text{ K}}{T_{\text{SRF}}} \frac{Q_{\text{SRF}}/10^{12}}{Q_{\text{NRF}}/10^{5}} \left(\frac{B_0(t)\;/0.2\text{ T}}{B_0/8\text{ T}} \; \frac{f_0+f_g}{2f_g} \right)^2$$

Scanning sensitivity



Up-conversion cavities can have higher GW sensitivity on resonance

Tom Krokotsch + **L. Fischer & W. Hillert & G. Moortgat-Pick** (to appear soon)

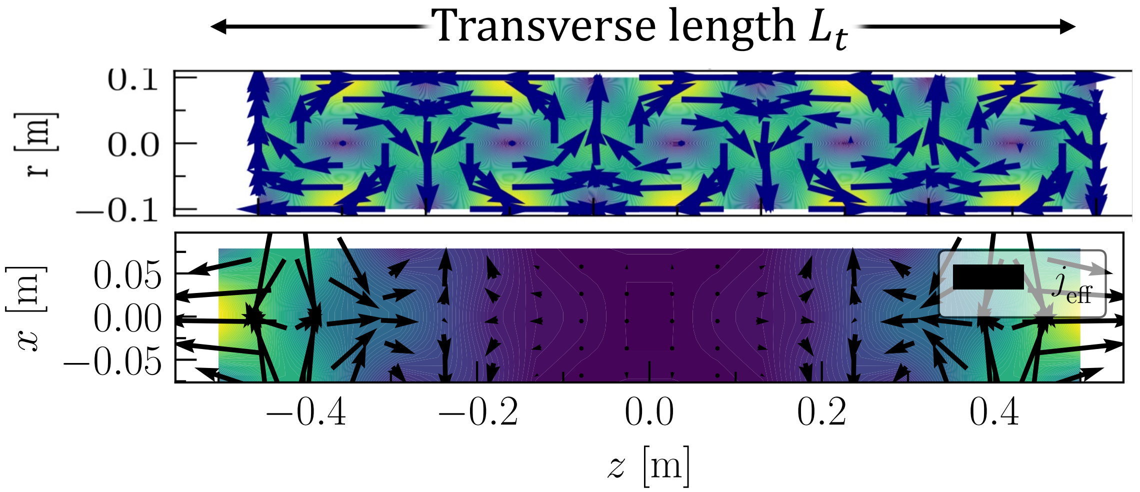
Electromagnetic detection

Length Scaling in SRF Cavities

$P_{\text{sig}} \approx \frac{1}{8}\frac{Q_1}{\omega_{\text{sig}}} \frac{\left|\int dV\, \boldsymbol{E}_{\text{sig}} \cdot \boldsymbol{J}_{\text{eff}}\right|^2}{\int dV\, \boldsymbol{E}_{\text{sig}}^2}$ with $J_{\text{eff}} \propto \omega_g^2\, L_t\, B_0\, h^{\text{TT}}$ $\Rightarrow$ Widening the detector leads to $h_{\text{min}} \sim \frac{1}{\sqrt{V}\, L_t}$

B field of pump mode TE_{115}

Effective current for plus-polarized GW in y-direction with f_g $\approx$ GHz



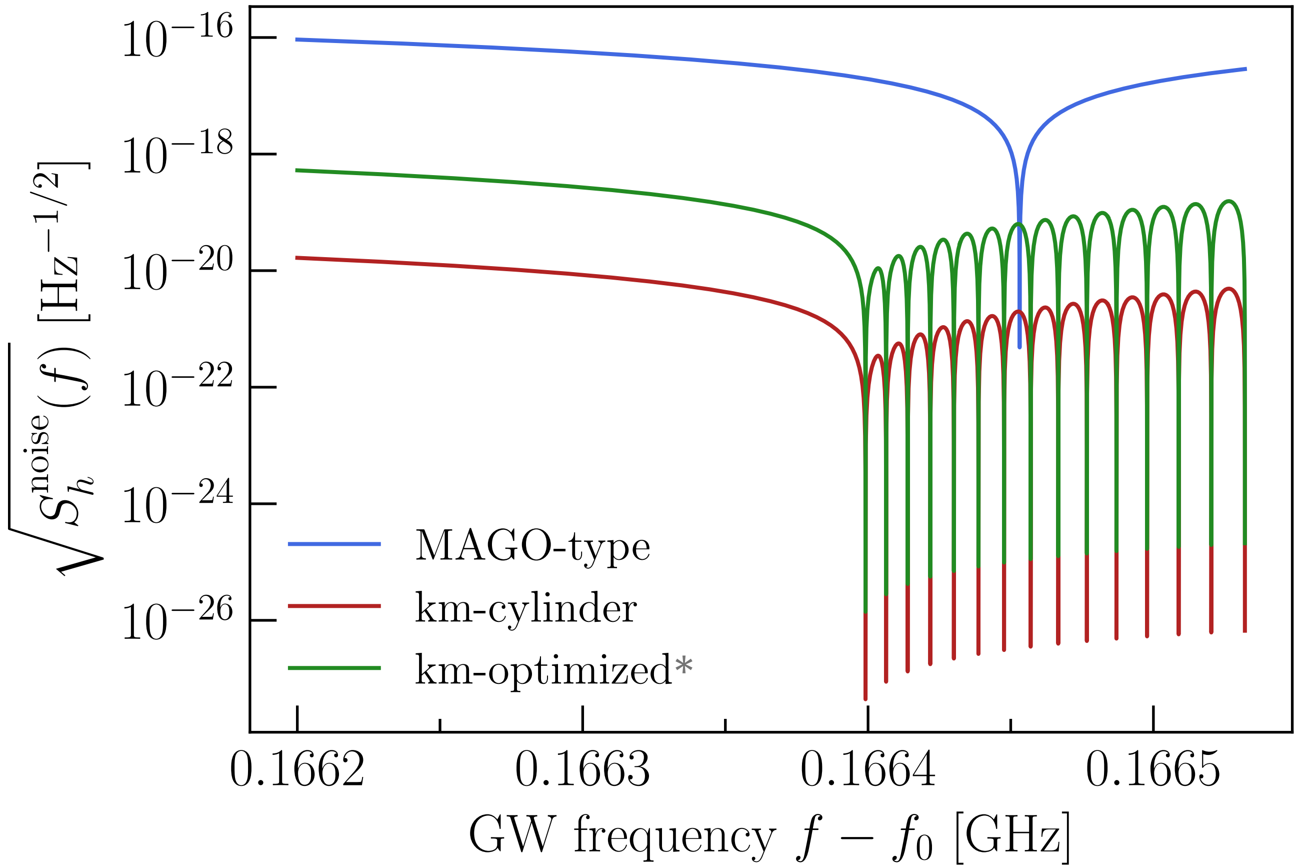
- Figure of merit conflicts with mechanical signal → Small volumes are not preferred
- No strong external magnetic fields needed

GW strain sensitivity of EM-SRF cavities increases rapidly with length

Tom Krokotsch + **L. Fischer & W. Hillert & G. Moortgat-Pick** (to appear soon)

Electromagnetic detection

Sensitivity of Microwave Cavities



*optimized only for mechanical signal

$TE_{11p} \rightarrow TE_{12(2q+1)}$ transitions

Electromagnetic detection

Proper Detector Frame: General GWs

Series expansion of PD metric

Fortini, Gualdi (1982)

Marzlin (1994)

$$h_{00}^{\mathrm{PD}} = -2\sum_{n=0}^{\infty}\frac{1}{(n+2)!}x^k x^l z^n\left[\partial_{z'}^n R_{0k0l}^{\mathrm{TT}}\right]_{z'=0}$$

$$h_{i0}^{\mathrm{PD}} = -2\sum_{n=0}^{\infty}\frac{n+2}{(n+3)!}x^k x^l z^n\left[\partial_{z'}^n R_{0kil}^{\mathrm{TT}}\right]_{z'=0}$$

$$h_{ij}^{\mathrm{PD}} = -2\sum_{n=0}^{\infty}\frac{n+1}{(n+3)!}x^k x^l z^n\left[\partial_{z'}^n R_{ikjl}^{\mathrm{TT}}\right]_{z'=0}$$

For monochromatic GW, expressed through exponentials in Berlin et al. (2021)

Assuming **general** plane GW $h_{ij}^{\mathrm{TT}}(t,\boldsymbol{x}) = h_{ij}^{\mathrm{TT}}(t-z)$ find equivalent expression:

$$h_{00}^{\mathrm{PD}} = \frac{x^k x^l}{z^2}\left[h_{kl}^{\mathrm{TT}}(t-z) - h_{kl}^{\mathrm{TT}}(t) + z\,\dot{h}_{kl}^{\mathrm{TT}}(t)\right],$$

$$h_{i0}^{\mathrm{PD}} = \frac{x^m z\,\delta_i^n - x^m x^n \delta_i^z}{z^2}\left[h_{mn}^{\mathrm{TT}}(t-z) + \frac{1}{z}\int_t^{t-z} h_{mn}^{\mathrm{TT}}(t')dt' + \frac{z}{2}\dot{h}_{mn}^{\mathrm{TT}}(t)\right],$$

$$h_{ij}^{\mathrm{PD}} = \frac{\delta_i^m \delta_j^n z^2 + x^m x^n \delta_i^z \delta_j^z - x^m z\left(\delta_i^n \delta_j^z + \delta_i^z \delta_j^n\right)}{z^2}\left[h_{mn}^{\mathrm{TT}}(t-z) + \frac{2}{z}\int_t^{t-z} h_{mn}^{\mathrm{TT}}(t')dt' + h_{mn}^{\mathrm{TT}}(t)\right]$$

→ Fully analytical if $\int dt\, h_{ij}^{\mathrm{TT}}(t)$ can be obtained.

→ Important application: GW chirps from compact binaries $h_{ij}^{\mathrm{TT}}(t,z) = A_{ij}(t-z)\, e^{-i\phi(t-z)}$

Electromagnetic detection

Proper Detector Frame: General GWs

What if the integral is not known analytically?

→ Evaluate infinite sum by expanding $h_{ij}^{TT}(t,z)$ for $z/t \ll 1$

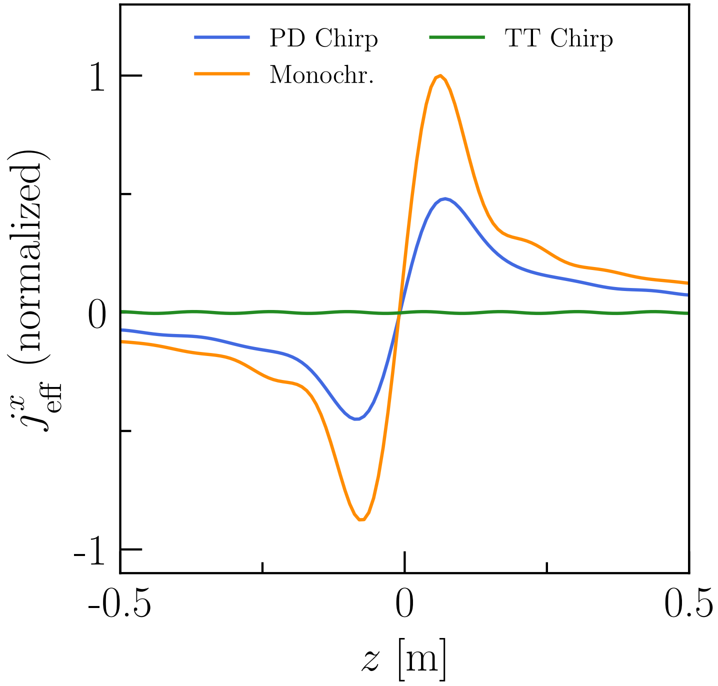
Most chirp signals fulfil $\phi(t) \gg 1$ long before merger.

→ Approximation is equivalent to monochromatic result in Berlin et al. (2021) under $\omega_g \to \omega_g(t)$ and $h_{ij}^{TT} \to A_{ij}(t-z)$

$$h_{00}^{PD} = -\omega_g^2 F(\omega_g z)\ x^k x^l h_{kl}^{TT} \,,$$

$$h_{i0}^{PD} = -\omega_g^2 \frac{F(\omega_g z) - iF'(\omega_g z)}{2}\left[h_{ki}^{TT} z x^k - h_{kl}^{TT} x^k x^l \delta_i^z\right] ,$$

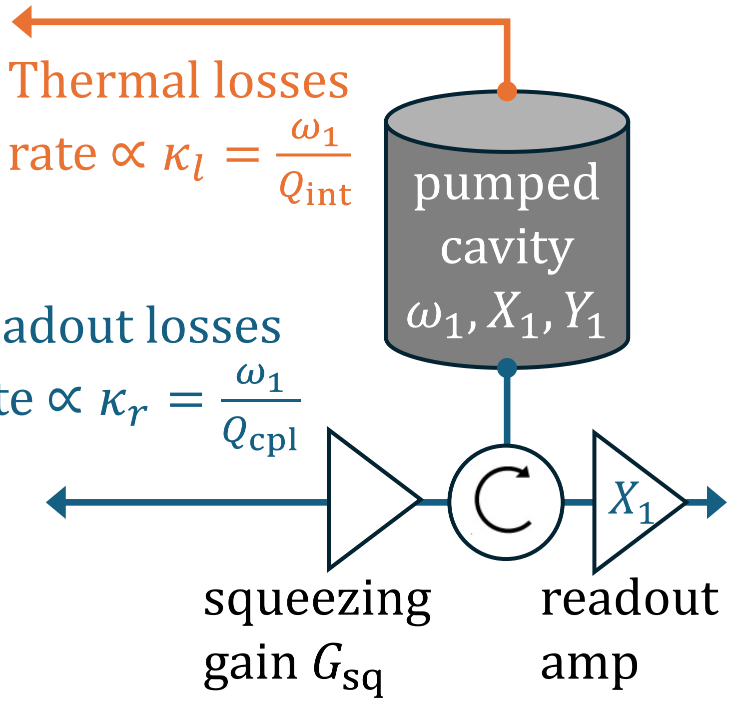
$$h_{ij}^{PD} = i\omega_g^2 F'(\omega_g z) \times \left[h_{ij}^{TT} z^2 + h_{kl}^{TT} x^k x^l \delta_i^z \delta_j^z - h_{il}^{TT} \delta_j^z z x^l - h_{kj}^{TT} \delta_i^z z x^k\right] ,$$



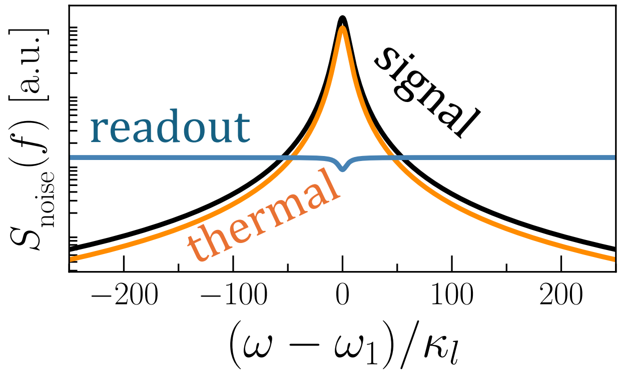
Quantum Enhancement

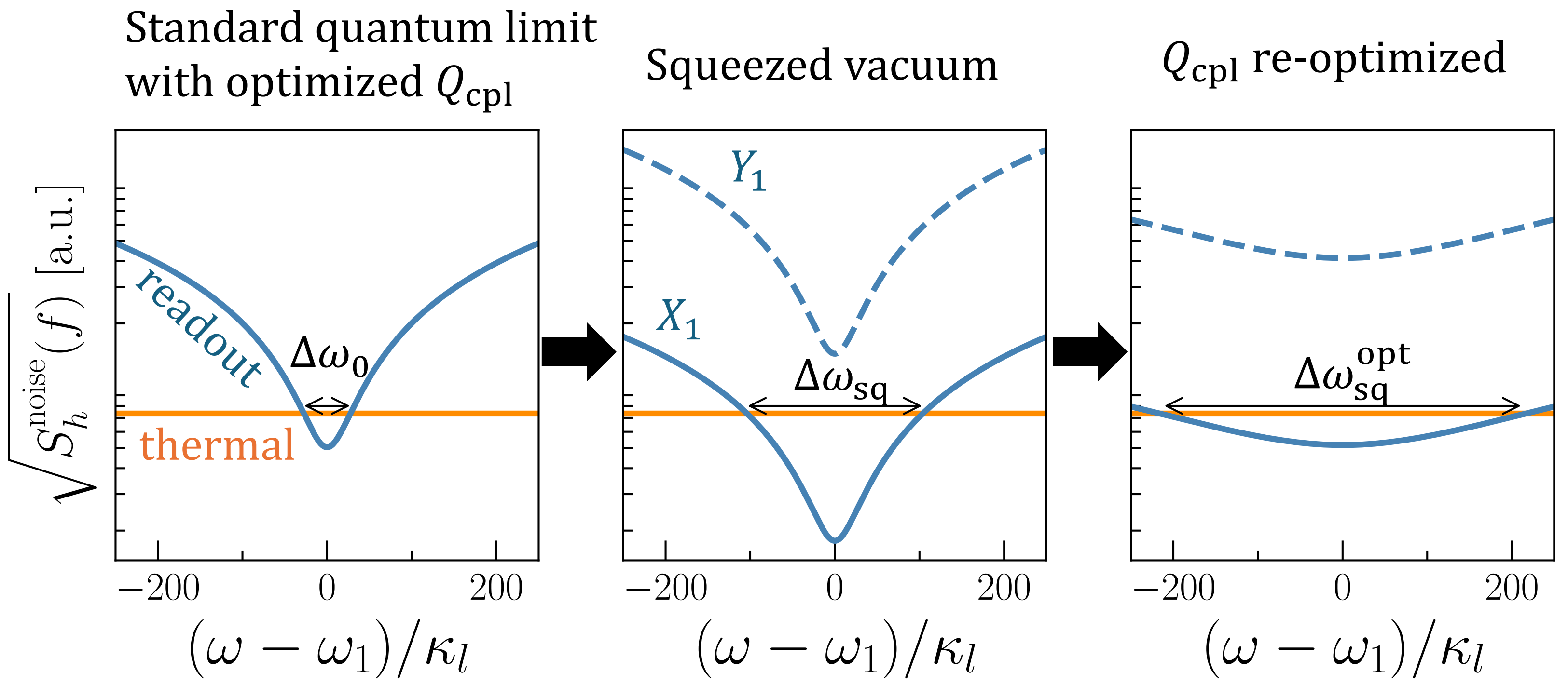
Quantum Enhancement

Vacuum Squeezing



Field quadratures:
$E_1 \propto X_1 \sin(\omega_1 t) + Y_1 \cos(\omega_1 t)$





Vacuum squeezing can increase the bandwidth but not the peak sensitivity

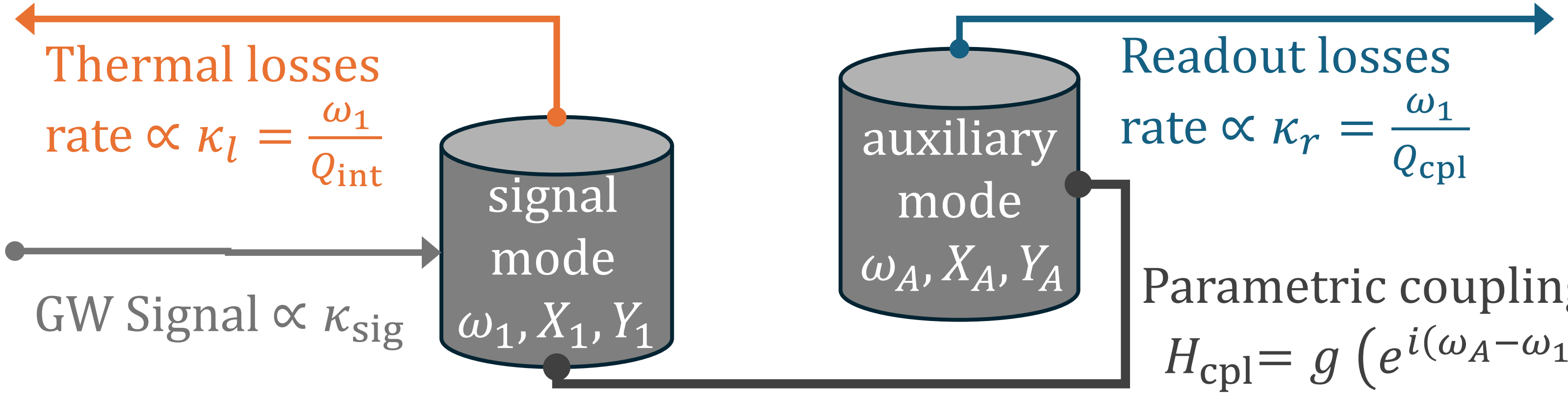
Quantum Enhancement

Back-action Evading (BAE) Amplification

Goal:

- Amplify fields in cavity (signal + noise) before readout noise is added
- Don't introduce additional (backaction) noise

See:
Clerk et al. 0810.4729
Wurtz et al. 2107.04147
Kuenstner et al. 2210.05576



Parametric coupling

$$H_{\text{cpl}} = g\left(e^{i(\omega_A-\omega_1)t}a_1^\dagger a_A + e^{i(\omega_A+\omega_1)t}a_1 a_A\right) = g\,X_1 X_A$$

state swapping — parametric amplification — rotating frame

Equations of motion:

$$\dot{X}_1 \propto \sqrt{\kappa_l}\xi_{\text{in}}^{\text{th}} + \sqrt{\kappa_{\text{sig}}}\xi_{\text{in}}^{\text{sig}}$$

$$\dot{Y}_1 \propto \sqrt{\kappa_l}\xi_{\text{in}}^{\text{th}} + \sqrt{\kappa_l}\xi_{\text{in}}^{\text{sig}} - 2\,i\,g\,X_A$$

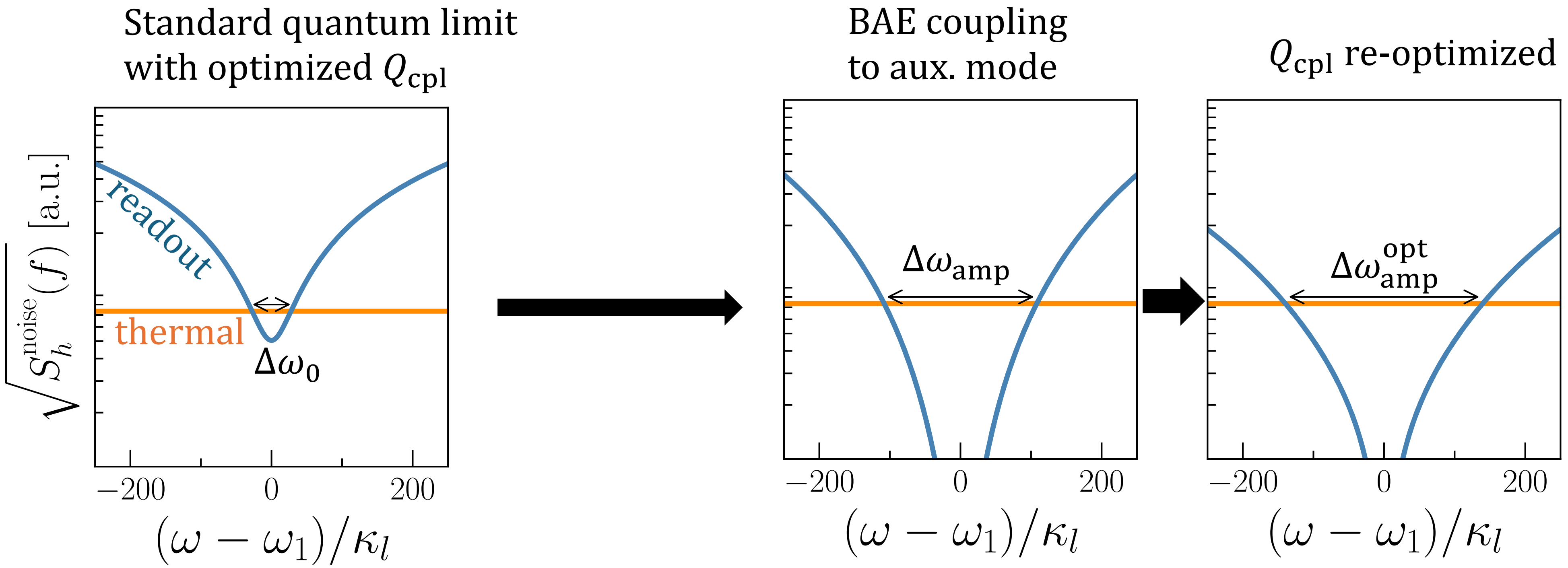
$$\dot{X}_A \propto \sqrt{\kappa_r}\xi_{\text{in}}^{\text{r}}$$

$$\dot{Y}_A \propto \sqrt{\kappa_r}\xi_{\text{in}}^{\text{r}} - 2\,i\,g\,X_1$$

signal → ; back-action ✗ ; read out

A parametric coupling to an auxiliary mode can noiselessly amplify the cavity fields

Back-action Evading (BAE) Amplification



$S_{\mathrm{th}}(\omega) \propto \frac{\kappa_l \kappa_r}{4\omega^2 + (\kappa_r + \kappa_l)^2}$

Sharper resonance, but amplified $S_{\mathrm{th}}(\omega) \propto \frac{4g^2 \kappa_l \kappa_r}{(4\omega^2 + \kappa_l^2)(4\omega^2 + \kappa_r^2)}$

BAE amplification of the cavity fields w.r.t. readout noise broadens the bandwidth

Quantum Enhancement

Comparing the Methods

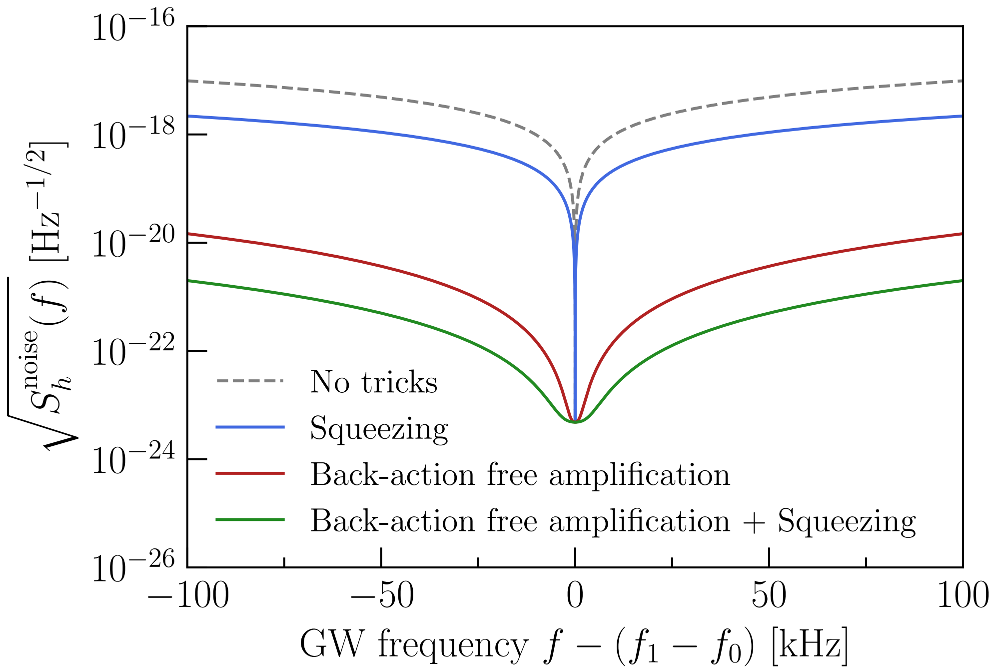
Squeezing: $\frac{\Delta\omega_{\mathrm{sq}}^{\mathrm{opt}}}{\Delta\omega_0} \simeq G_{\mathrm{sq}}$ for $T_{\mathrm{cavity}} \gg T_{\mathrm{readout}}$

Realistically: $G_{\mathrm{sq}} \approx 20$ (Malnou et al. 1809.06470)

BAE Amp.: $\frac{\Delta\omega_{\mathrm{amp}}^{\mathrm{opt}}}{\Delta\omega_0} \simeq \left(g\,\frac{Q_{\mathrm{int}}}{\omega_1}\right)^{2/3} \xrightarrow[\text{Chen et al. 2309.12387}]{\text{chain of } N \text{ circuits, large } N \text{ limit}} g\,\frac{Q_{\mathrm{int}}}{\omega_1}$

Realistically: $g \approx$ MHz (Lu et al. 2303.00959)

BAE Amp. + Squeezing: $\frac{\Delta\omega_{\mathrm{amp}}^{\mathrm{opt}}}{\Delta\omega_0} \simeq G_{\mathrm{sq}}^{1/3}\left(g\,\frac{Q_{\mathrm{int}}}{\omega_1}\right)^{2/3}$



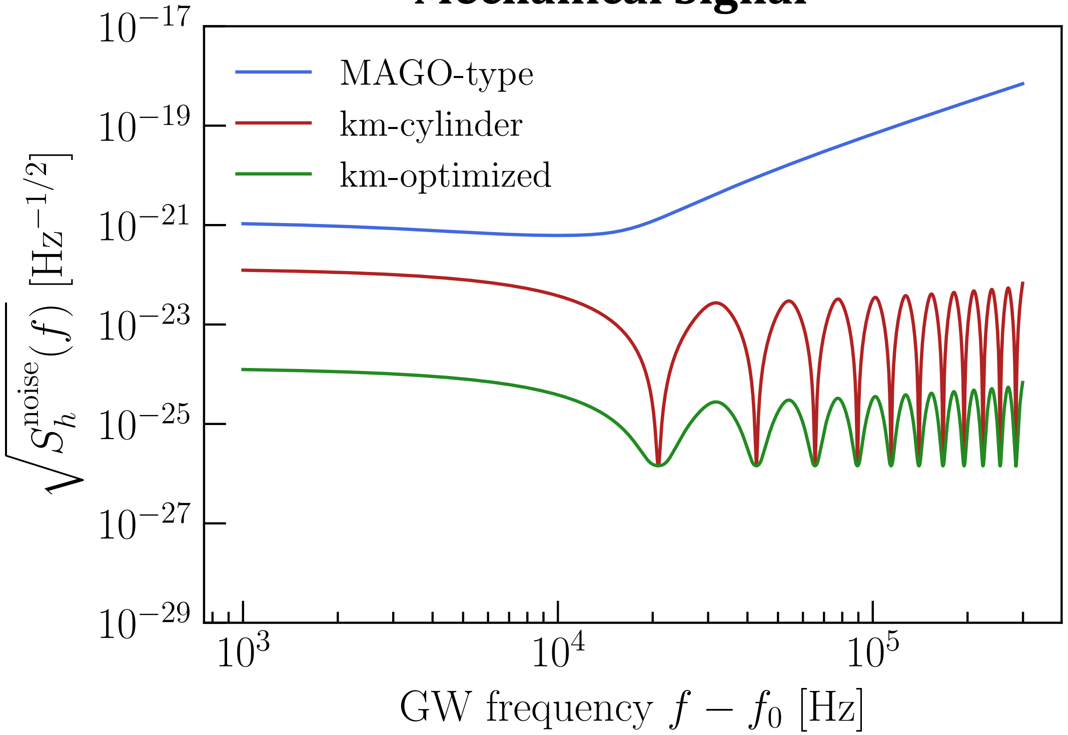
BAE amplification promises more bandwidth improvement than squeezing

Position Detection + Electromagnetic Detection + Quantum Enhancement

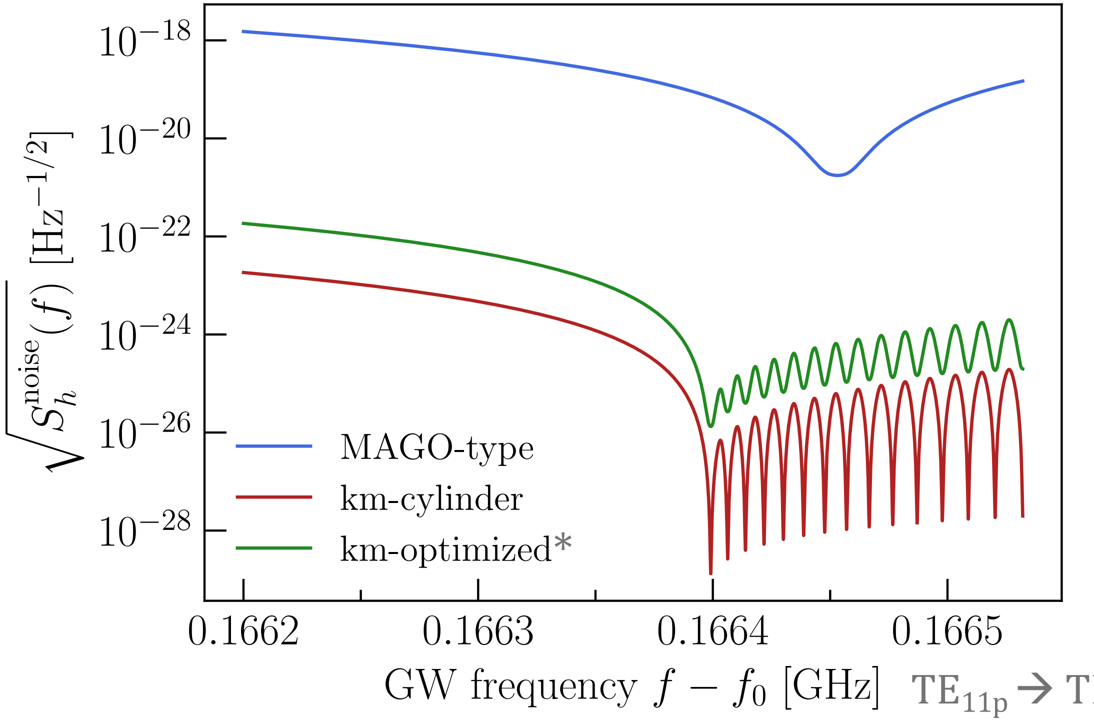
Maximal GW Strain Sensitivity of a SRF Cavity

Strain sensitivity using BAE amplification

Mechanical Signal



Electromagnetic Signal



$TE_{11p} \rightarrow TE_{12(2q+1)}$ transitions

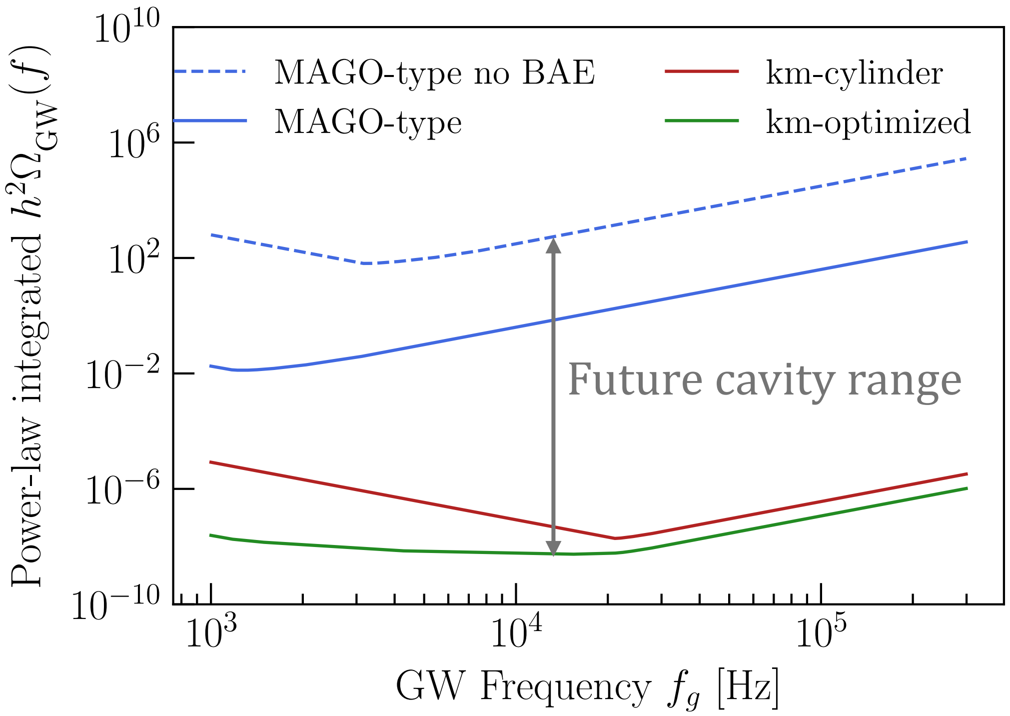
*optimized only for mechanical signal

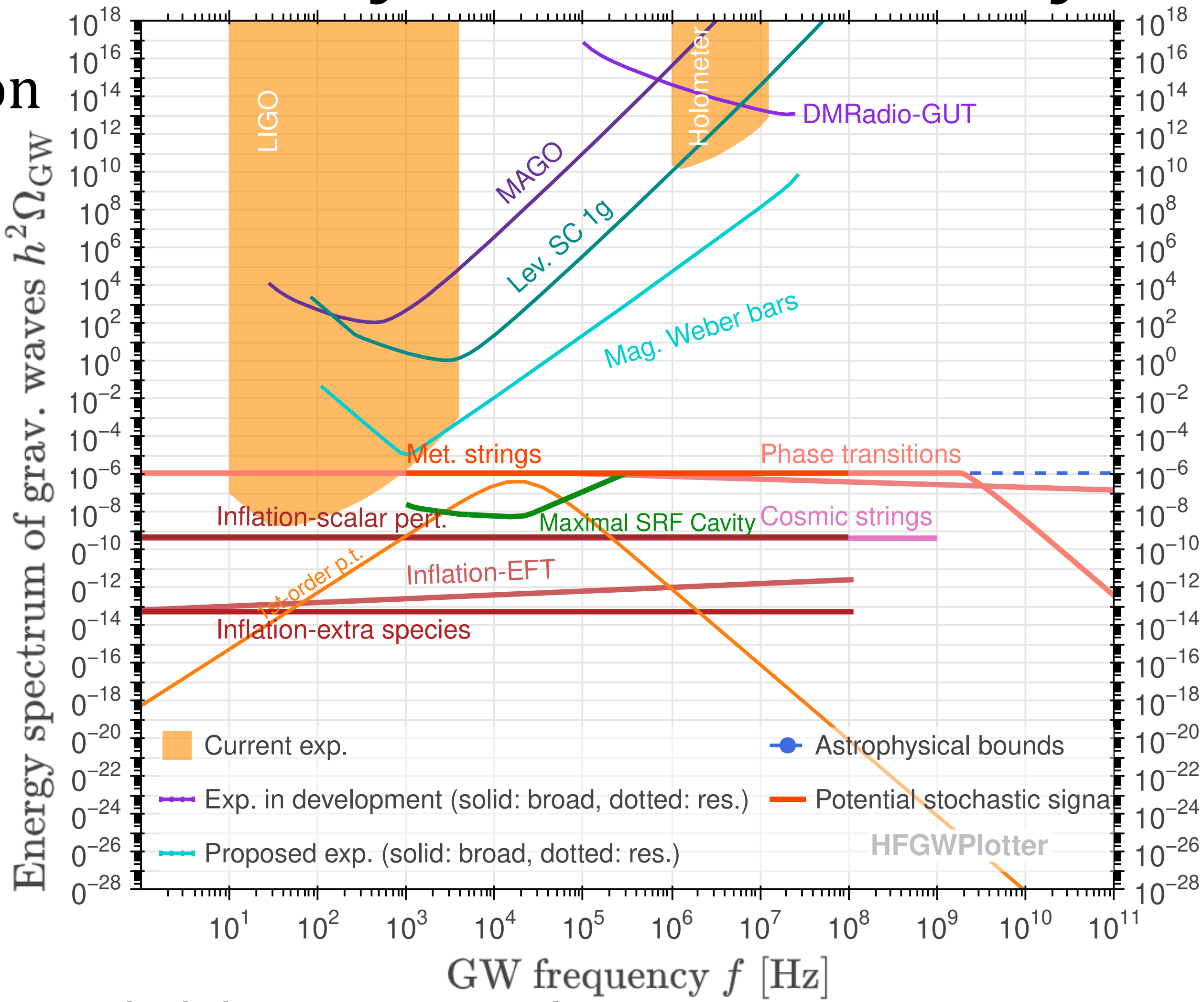
Position Detection + Electromagnetic Detection + Quantum Enhancement

Maximal GW Energy Sensitivity of a SRF Cavity

Energy sensitivity using BAE amplification

Mechanical Signal





Summary

- Cosmological HFGWs are out of reach for currently proposed detectors
- SRF cavities benefit from large stored energy
- Cavity length and geometry are key features to improve SRF detectors
- SRF cavities can simultaneously act as mechanical and electromagnetic GW detectors
- Quantum enhancement techniques are essential to widen bandwidth while maintaining peak sensitivity

Backup: Fabry Perot Cavities vs RF Cavities

Maggiore Eq. (9.220)

- $S_{\text{shot}}^{\text{LIGO}} \simeq \frac{\pi h f_L}{P_{bs}} \frac{f_p^2}{f_L^2}\left(1+\frac{f_g^2}{f_p^2}\right)$
- $S_{\text{SQL}}^{\text{MAGO}} \simeq \frac{\pi h f_1}{P_{\text{out}}} \frac{\Delta f_{\text{EM}}^2}{f_1^2}\left(1+\frac{\left(f_0+f_g-f_1\right)^2}{\Delta f_{\text{EM}}^2}\right)$
- Off resonant: Figure of merit
  - $f_1 P_{\text{out}} = 1.6\cdot 10^{17}\,\frac{\text{W}}{\text{s}}\frac{10^5}{Q_{\text{cpl}}}\frac{V}{\text{m}^3}\left(\frac{B_0}{0.2\text{ T}}\right)^2\left(\frac{f_1}{\text{GHz}}\right)^2$
  - $f_L P_{\text{bs}} = 2.8\cdot 10^{17}\,\frac{\text{W}}{\text{s}}\frac{f_L}{282\text{ THz}}\frac{P_{bs}}{1\text{ kW}}$
- On resonance: Figure of merit
  - $\frac{P_{\text{out}}Q_{\text{int}}Q_1}{f_1}\boxed{\frac{hf_1}{k_B T}} = 4\cdot 10^{12}\text{ J}\,\frac{10^5}{Q_1}\frac{10^{10}}{Q_{\text{int}}}\frac{V}{\text{m}^3}\left(\frac{B_0}{0.2\text{ T}}\right)^2\frac{f_1}{\text{GHz}}\frac{1.8\text{ K}}{T}$ (thermal noise dominates)
  - $\frac{P_{\text{bs}}Q_{FP}^2}{f_L} = 4.6\cdot 10^{13}\text{ J}\,\frac{282\text{ THz}}{f_L}\frac{P_{bs}}{1\text{ kW}}\left(\frac{Q_{FP}}{3.6\cdot 10^{12}}\right)^2$

Optical cavities:
- Laser frequency f_L
- Power on beam splitter P_{bs}
- FP cavity quality factor

$$\mathcal{F} = \frac{FSR}{f_l/Q_{FP}} \Rightarrow Q_{FP} = 2f_L L\,\mathcal{F}$$

- Pole frequency $f_p = \frac{1}{4\mathcal{F}L}$

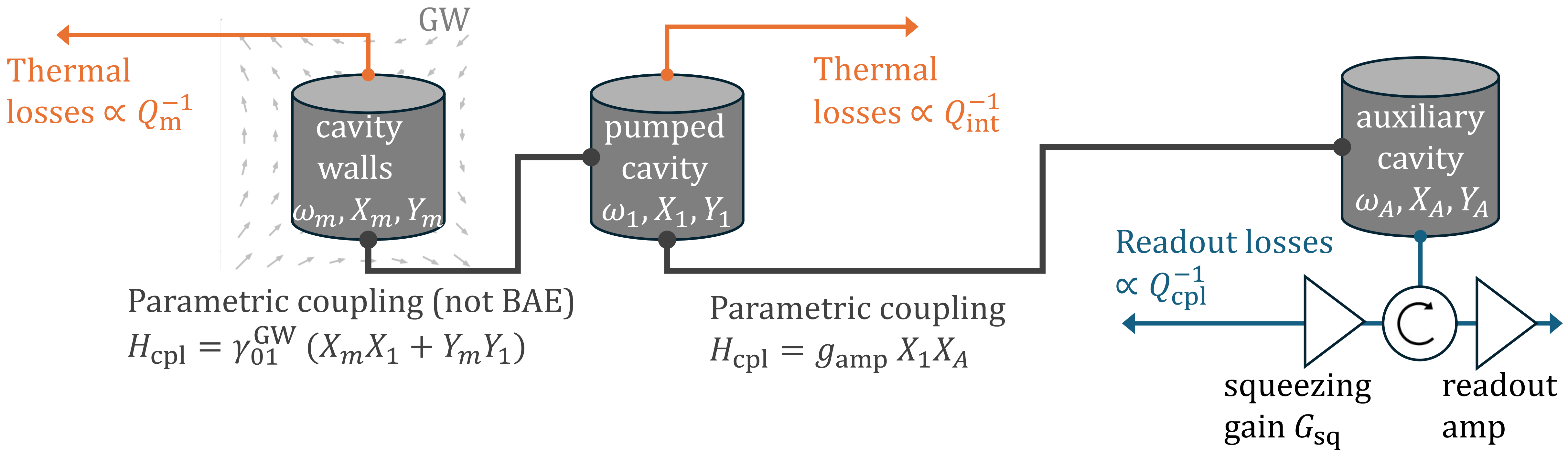
RF cavities:
- Signal frequency f_1
- Mode bandwidth $\Delta f_{\text{EM}} = \frac{f_1}{Q_1}$
- Out-coupled power $P_{\text{out}} = \frac{\omega_1}{Q_{\text{cpl}}}U_0$

	$\frac{\text{Optical Cavity Signal}}{\text{RF Cavity Signal}}$
Peak sensitivity	$\frac{P_{\text{bs}}Q_{\text{FP}}^2/f_L}{P_{\text{out}}Q_{\text{RF}}^2/f_{\text{RF}}}\frac{T_{\text{RF cavity}}}{f_{\text{RF}}} \approx 10$
High freq. limit	$\frac{f_L P_{\text{bs}}}{f_{\text{RF}}P_{\text{out}}} \approx 1$

For an *ideal* 1 m^3 SRF cavity without mech. resonances at the SQL vs. LIGO parameters.

Quantum Enhancement

Backup: Including Mechanical Resonances



$$S_{sig}^{BAE}(\omega) = \frac{4\,\kappa_r\,S_{f_{GW}}(\omega)\left(\gamma_{01}^{GW} g_{amp}\right)^2}{\left|\left[\left(\omega^2 + \frac{\kappa_m^2}{4}\right)\left(\omega^2 + \frac{\kappa_l^2}{4}\right) + \left(\gamma_{01}^{GW}\right)^2\right]\left(\omega^2 + \frac{\kappa_r^2}{4}\right)\right|^2}$$

$$S_{sig}^{0}(\omega) = \frac{4\,\kappa_r\,S_{f_{GW}}(\omega)\left(\gamma_{01}^{GW}\right)^2}{\left|\left(\omega^2 + \frac{\kappa_m^2}{4}\right)\left(\omega^2 + \frac{\kappa_l^2}{4}\right) + \left(\gamma_{01}^{GW}\right)^2\right|^2}$$

Quantum Enhancement

Comparing The Methods: Exact formulas

Squeezing: $$\frac{\Delta\omega_{\mathrm{sq}}^{\mathrm{opt}}}{\Delta\omega_0} \simeq G_{\mathrm{sq}} \text{ for } n_T^c \gg n_T^r$$

Realistically: $G_{\mathrm{sq}} \approx 20$ (HAYSTAC, Malnou et al. 1809.06470)

BAE Amp.: $$\frac{\Delta\omega_{\mathrm{BAE}}}{\Delta\omega_0} \simeq 2 \cdot 2^{1/3} \cdot \left(g \frac{Q_{\mathrm{int}}}{\omega_1} \frac{n_T^r + \frac{1}{2} + n_T^{\mathrm{amp}}}{n_T^c + \frac{1}{2}} \right)^{2/3} \xrightarrow{\text{chain of circuits}} g \frac{Q_{\mathrm{int}}}{\omega_1}$$

Realistically: $g \approx \mathrm{MHz}$ (Lu et al. 2303.00959)

BAE Amp. + Squeezing: $$\frac{\Delta\omega_{\mathrm{amp}}^{\mathrm{opt}}}{\Delta\omega_0} \simeq 2 \cdot 2^{1/3} \cdot G_{\mathrm{sq}}^{1/3} \left(g \frac{Q_{\mathrm{int}}}{\omega_1} \frac{n_T^r + \frac{1}{2} + n_T^{\mathrm{amp}}}{n_T^c + \frac{1}{2}} \right)^{2/3}$$

BAE amplification of the signal might increase the bandwidth more than squeezing